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EUI Working Paper ECO No. 94/43

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Printed in Italy in November 1994
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November 1994

Abstract

An alternative way of modelling Stackelberg warfare – i.e. a strategic fight for market leadership – is suggested which avoids the inconsistencies of the prevalent disequilibrium interpretation. In a reputation game Stackelberg warfare occurs as an equilibrium choice for two scenarios. In the first scenario a firm challenges its rival by playing Stackelberg leader. The other firm plays warfare to discipline its opponent and to make it return to the Cournot equilibrium. In the second scenario both firms simultaneously play warfare in the initial phase of the game until one of them concedes and accepts to be the follower.

*Paper presented at the Ninth Annual Meeting of the European Economic Association, September 1994 in Maastricht (Netherlands).

†I am grateful to Andrzej Baniak, Michael Begg, James Mirrlees, Louis Phlips, Günther Rehme and in particular to Stephen Martin for helpful suggestions and comments.

1 Introduction

In *Marktform und Gleichgewicht* Heinrich von Stackelberg (1934) introduced the leader and follower concept to oligopoly theory. Since then, the asymmetric Stackelberg equilibrium constituted by a leader and a follower has found wide acceptance in economics, including areas other than oligopoly.

Stackelberg himself was rather skeptical whether this equilibrium would be reached and maintained in real markets. His conjecture was that in duopoly both firms would try to capture the more profitable position of the leader. Although in this situation (subsequently called Stackelberg[1] warfare) profits are drastically deteriorated he thought that both firms fighting for leadership would be the "normal case" in duopoly. According to him, the leader and follower equilibrium would occur only as an "exceptional case".

On the contrary, the ensuing literature, more and more influenced by rising game theory, found the Stackelberg equilibrium quite appealing and was reluctant to use the warfare concept. The latter remained restricted to a few lines in the textbooks as the case in which both firms in duopoly take the action of a leader. No single publication in economic literature explicitly deals with Stackelberg warfare or deepens Stackelberg's analysis with respect to this part. Recently Stackelberg warfare even vanished from the textbooks: the latest editions of microeconomic or industrial economic textbooks have dropped this topic. Why is Stackelberg's idea of warfare in oligopoly theoretically unattractive though fighting for market leadership is clearly an empirically important phenomenon[2]?

Following Fellner (1949) Stackelberg warfare is interpreted as a disequilibrium in a strict game theoretic sense. Only by mistake can both firms in a market choose to become the Stackelberg leader. But this concept is theoretically inconsistent: in a warfare disequilibrium firms

[1]Stackelberg referred to Bowley's (1924, p.38) conjectural variations approach as similar to his concept of warfare. The criticism in this paper made on Stackelberg warfare applies unrestrictedly to Bowley "solution".

[2]See the last section for recent empirical examples of Stackelberg warfare.

hold wrong conjectures about their rival which are not corrected or revised. However, equilibrium requires both strategies and conjectures to be consistent with each other (Friedman (1983)). It is due to this flaw that Stackelberg warfare has not received any attention in the literature.

In the present paper an alternative approach towards Stackelberg warfare is suggested. It is based on the theory of reputation as introduced by Kreps and Wilson (1982a) and Milgrom and Roberts (1982). Dynamic games of asymmetric information with reputational effects have been successfully used to explain the strategic quest for dominance by firms, most prominently predatory pricing, price wars and limit pricing[3]. While in games of perfect information these phenomena are difficult to demonstrate, the information based reputation games offer a new potential of explanation.

In the manner of these models Stackelberg warfare as an equilibrium choice is proved in this paper – first for a scenario with one-sided uncertainty, the "discipline" scenario. It is shown that a firm with private information about its type can credibly and profitably challenge its opponent by deviating from the symmetric Cournot equilibrium and by playing Stackelberg leader. The provoked firm, faced with the Stackelberg leader action of its opponent, does not accept to be follower in the long run. It plays Stackelberg warfare in order to make the other firm withdraw from its Stackelberg leader position, to discipline it. An equilibrium is derived in which both firms engage in warfare knowingly that the opponent does the same. They sacrifice short run losses in order to gain in the long run. Both firms play Stackelberg warfare: the first firm to augment and maintain its reputation, the opponent to deprive its rival of it. Strikingly, after warfare has occurred firms are likely to end up in a symmetric Cournot equilibrium. However, warfare occurs as an equilibrium choice rather than a result of mistakes.

A similar result holds if both firms have private information about their types. Given two-sided uncertainty both firms play Stackelberg warfare in a war of attrition game. This is called the "concession" sce-

[3]See Martin (1993, Ch.4) or Roberts (1987) for surveys on such "Battles for market share".

nario. Both firms play Stackelberg warfare in the initial phase of the game until one of them concedes and accepts to be the follower. The game ends in a Stackelberg leader and follower equilibrium and again warfare occurs as an equilibrium choice.

The rest of the paper is organized in the following way. In Section 2 discusses Stackelberg's approach and the game theoretic critique made upon the disequilibrium interpretation of Stackelberg warfare. In Section 3 the reputation game with one-sided uncertainty is presented. Section 4 discusses the case of two-sided uncertainty. Section 5 concludes.

2 Stackelberg Warfare as a Disequilibrium?

The original idea of Stackelberg duopoly is based on the assumption that the Stackelberg leader (correctly) conjectures that its rival is acting according to the Cournot behavioural assumption. The Stackelberg follower chooses its action taking the leader's decision as given to, i.e., to be follower only implies adaptive behaviour and not any concrete action. Stackelberg analyses a scenario (1934, pp. 16–24) in which two firms have the choice to take one of the roles. In Figure 1 his approach is represented as a simple 2x2 matrix game.

		Firm 2	
		St.-Leader	*St.-Follower*
Firm 1	*St.-Leader*	π_W, π_W	π_L, π_F
	St.-Follower	π_F, π_L	π_C, π_C

Figure 1: Stackelberg's duopoly game.

There are four possible outcomes. If one firm chooses "Leader" (indicated by the subscript L) and the other "Follower" (F) then a Stackelberg equilibrium results. There are two Stackelberg equilibria with either firm 1 or firm 2 as the leader. If both firms choose "Follower" the Cournot (C) equilibrium results. The situation of Stackelberg warfare (W) arises if both firms choose to be the leader.

For equilibrium analysis the nature of oligopoly competition has to be specified. It is assumed that firms are symmetric and have downward sloping reaction functions. These result generally, but not always in models of quantity competition. A differentiated price setting model may yield downward sloping reaction functions, too. However, henceforth it will be referred to output as firms' actions.

With downward sloping reaction functions the Stackelberg leader gets a higher profit than the follower. Also the Cournot outcome is preferred to being follower. The worst case for both firms is the warfare situation (Gal-Or (1985), Dowrick (1986)). Hence, given downward sloping reaction functions two Nash equilibria result: the Stackelberg solutions. Neither the Cournot outcome, nor Stackelberg warfare can describe an equilibrium.[4] Both can occur only by mistake, as a disequilibrium[5].

However, the disequilibrium interpretation is criticized in the literature. Firstly, this explanation lacks a realistic interpretation. It is hard to imagine that firms – say producers of a new generation of RAM microchips – would install too large capacities as a result of being erroneous about their role in the market. Generally it can be assumed that firms know their position in the market very well. If they engage in a fight to become the Stackelberg leader, then they do so consciously.

Secondly, this explanation of warfare contains a theoretical inconsistency. Friedman (1983, p.116) criticizes this approach and concludes:

[4]The fact that the Cournot solution is not an equilibrium of this game must appear as an awkward result. One will not doubt the reasonableness and usefulness of the Cournot equilibrium as a consequence of this game.

[5]With upward sloping reaction functions (which result in general under price competition) being follower is preferred to being leader. Further Stackelberg warfare is preferred to being leader. The worst outcome is the Cournot case (Gal-Or (1985), Dowrick (1986)) which is the equilibrium of the game in Figure 1. Thus, also with upward sloping reaction functions warfare can occur as a result of mistakes.

> *"... each thinks that the other is a follower and accordingly behaves as a leader. This has been said to cause a Stackelberg disequilibrium, but in fact it is just another untenable situation that cannot arise."*

The crucial point is that firms have wrong conjectures about their rivals. The choice of the role of a leader is inseparably connected to a particular action (the leadership output), no matter what the opponent does. In the case of two Stackelberg leaders firms start with the wrong conjecture that their opponent plays follower and they do not correct it. Yet, equilibrium requires both actions and conjectures to be consistent with each other. This is obviously true for the two Stackelberg equilibria and the Cournot equilibrium (by default), but not for the warfare situation.

Recent contributions to the literature[6] no longer base the leader-follower model on behavioral assumptions (duopoly-*roles*), but on a sequence of moves. The Stackelberg leader produces its quantity first and the follower, moving second, has to take this quantity as given. Leadership has its origins in a historical differentiation among the firms rather than in a decision to "become" Stackelberg leader. Such timing games (Hamilton and Slutsky (1990)) avoid the inconsistencies described, however, they cannot provide a rationale for Stackelberg warfare either.

3 The "Discipline" Scenario

3.1 The Model

The following model is based on the theory of reputation as introduced by Kreps and Wilson (1982a) and Milgrom and Roberts (1982). Reputation effects rely on asymmetric information about the firms' types. This captures the realistic fact that firms are unlikely to be absolutely certain of their opponents' options, motivations or behaviour. The introduction of uncertain types might also refer to the possibility of irrational behaviour

[6] See Phlips (1994) for a survey.

of firms. Earlier contributions to the literature show that given only a very small amount of uncertainty about players' types can lead to drastic changes in equilibrium behaviour.

Following Kreps and Wilson (1982a) two types of firms, a "weak" one and a "strong" one, are assumed. Here the two types differ with respect to their preferences about Stackelberg warfare, expressed in the following ranking about the profits in the standard duopoly constellations:

$$\text{“}weak\text{” } type: \ \pi_L > \pi_C > \pi_F > 0, \ \ \pi_F > |\pi_W|, \tag{1}$$

$$\text{“}strong\text{” } type: \ \bar{\pi}_L > \bar{\pi}_W > \bar{\pi}_C > \bar{\pi}_F > 0, \tag{2}$$

where the subscripts L, C, F, W indicate the profit of the Stackelberg leader, the Cournot equilibrium, the follower and Stackelberg warfare respectively.

The ranking of the "weak" type results if both firms have downward sloping reaction functions (Gal-Or (1985), Dowrick (1986)). The assumption $|\pi_W| < \pi_F$ for negative values of π_W ensures positive values in the equilibrium derived below. This is, however, not restrictive for most oligopoly models.

The "strong" firm "enjoys" warfare. It might, for example, have employed a manager who has a different objective from the owner of the firm (Vickers (1985)). The manager might follow a policy that is not profit maximizing from the point of view of the owner, but rather maximizes sales – or a combination of profits and sales – according to an incentive contract. (It is thus somewhat sloppy to speak of the $\bar{\pi}$ as "profits"; strictly speaking the utility of firm owner (or manager) is meant.) The objective of the firm is assumed to be such that it prefers the warfare quantity/profit outcome to the Cournot one and thus it is a dominant strategy for the "strong" firm to produce the Stackelberg leader output.

Call the two duopolists firm 1 and firm 2 and assume that firm 2 has prior beliefs that firm 1 is "strong":

$$Prob.(firm\ 1 = \text{“}strong\text{”}) = p. \tag{3}$$

These beliefs are common knowledge. It is also common knowledge that firm 2 is "weak" with probability 1 (Section 4 deals with the case of two-sided uncertainty).

The game the firms are playing is based on a pre-market announcement subgame. This might appear somewhat stylized, however, firms announcing business targets like market shares or sales for a new product or a future period of time is a common phenomenon in the real world. This subgame is based on the following steps.

- The starting point of the game is simply that symmetric firms play an equally symmetric Cournot equilibrium.

- In this model deviations from the symmetric Cournot equilibrium can be expected from the "strong" firm 1 as well as from the "weak" firm 1 trying to mimic the "strong" type's behaviour. In both cases firm 1 will announce production of the output of a Stackelberg leader. Thus, the first move is by firm 1 deciding to announce either the Cournot or the Stackelberg leader quantity.

- Since firm 2 cannot do better than in a Cournot equilibrium it comes only into play if firm 1 announces the Stackelberg leader quantity. Firm 2 then has to decide whether to accept being follower (with the appropriate quantity) or to try to discipline firm 1 and announce also the Stackelberg leader quantity. This would mean warfare with the goal of making firm 1 withdraw from the Stackelberg leader position.

- Given that firm 2 announced playing Stackelberg warfare, firm 1 might in fact want to withdraw its announcement to play Stackelberg leader: both firms return to the symmetric Cournot equilibrium (firm 2 agrees to this, because it cannot do better than in a Cournot equilibrium). However, firm 1 might also not want to withdraw from its announced Stackelberg leader quantity. Then warfare is actually played.

- Finally quantities are set according to the announcements (Cournot, Stackelberg leader/follower, or warfare).

This announcement subgame needs justification with respect to two points. To begin with, the quantities announced (and so the quantities indicated in the extensive form in Figure 2) are the only subgame perfect quantities. Firstly, according to (2) the "strong" firm produces the Stackelberg leader output under all circumstances. Thus, the "weak" firm 1 has to do the same when trying to mimic this behaviour. Secondly, if firm 2 accepts being follower, then the follower quantity is the optimal response to firm 1's Stackelberg leader output. The only undetermined action is the warfare output of firm 2. Though it will occur as an equilibrium action in this game it is not determined by subgame perfection. Firm 2 is *assumed* to produce the Stackelberg leader output to keep the game symmetric. However, any quantity which disciplines the "weak" firm 1 in the long run (i.e. which pushes its profits below π_C) works out in the equilibrium derived below. If the warfare actions were asymmetric in this sense it would only be necessary to introduce different π_W^1 and π_W^2 for the "weak" firm 1 and firm 2.

The second point of justification concerns the credibility of the announcements. Clearly, the announcements themselves do not involve any commitment. For example, if warfare results from the announcements firm 2 could strictly increase its profit by deviating from its announcement and by producing the Stackelberg follower quantity. However, such behaviour would not be rational. As will become clear in the equilibrium derived below firm 2 does not do so because it would not learn anything about firm 1: only actually committing to the warfare output can make firm 1 withdraw from its leadership output. Similarly, firm 1 does not deviate from its announcements because it would lose its reputation. Thus, in equilibrium firms cannot gain by not carrying out the announced quantities.

The structure of the game is summarized as in Figure 2. It is – apart from Nature's move – repeated a finite number of times. Periods are numbered backwards in time: $n = N, N-1, \ldots, 1$. The prior beliefs p are updated in every period such that p_n denotes the posterior in period n. The discount factor is denoted by δ, with $0 < \delta \leq 1$. To make it profitable for the "weak" firm 1 to engage in warfare even over only two

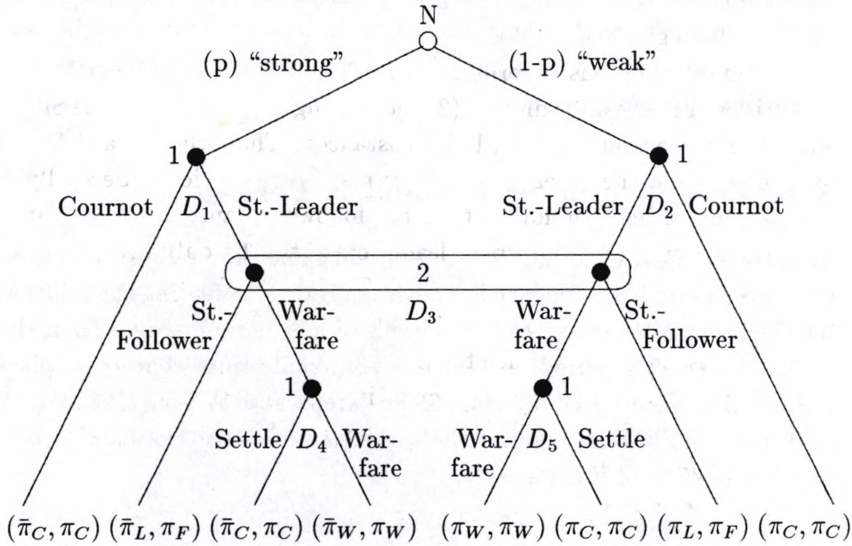

Figure 2: The extensive form of the warfare game.

periods it is assumed without loss of generality (see the Appendix for a generalization of this assumption) that

$$\pi_W + \pi_L \delta \geq \pi_C(1 + \delta). \tag{4}$$

Stackelberg warfare might occur due to the following effect. Firm 1 could make use of the asymmetric information. Though being in fact "weak" it produces the Stackelberg leader output pretending to be "strong". Even if firm 2 also plays Stackelberg warfare this might be a profitable strategy for firm 1: engaging in warfare might convince firm 2 that firm 1 is actually "strong", while settling the fight is definitely proof of being "weak".

Firm 2 also has – after a provocation by firm 1 – an incentive to engage in warfare. If firm 2 continues playing follower it cannot discover whether firm 1 is "strong" or not. Only if firm 2 plays warfare it can force the troublemaker to settle the fight and so to reveal itself as being "weak" type. Following this strategy causes losses in the period(s) of warfare, but makes it possible to reach the higher profit of the Cournot equilibrium for the remainder of the game. Hence, both firms intuitively might take warfare as an equilibrium action. This presumptive equilibrium is derived in detail in the next section.

3.2 Equilibrium for 1, 2 and N Periods

To solve for the equilibrium of the N-period game the equilibria of games of 1 and 2 periods are analysed. This is equivalent to finding the solution for the last and the next to last periods of a N-period game. Then the equilibrium for N periods is stated. The equilibrium concept applied is that of sequential equilibrium as in Kreps and Wilson (1982b). A sequential equilibrium requires the specification of strategies and beliefs over the periods of the game.

To determine the equilibrium of a single play of this game (or the final stage of an N-period game) is straightforward. To begin with, the "strong" firm 1 plays Stackelberg leader at decision node D_1 in Table 3 because this is a dominant strategy. Also, if it is "weak", firm 1 plays Stackelberg leader at D_2 if its type is not known with probability 1.

If firm 2 plays Stackelberg leader at D_3 it gets π_W with probability p_1 and π_C with probability $(1 - p_1)$. From playing follower a profit π_F results. Maximizing its expected profit firm 2 prefers to play Stackelberg leader if $p_1\pi_W + (1 - p_1)\pi_C > \pi_F$. Solving for p_1 (and calling this value α_1) one obtains

$$p_1 < \frac{\pi_C - \pi_F}{\pi_C - \pi_W} = \alpha_1. \tag{5}$$

Firm 2 plays follower if the inequality is reversed.

Given that $p_1 < \alpha_1$ and firm 2 plays Stackelberg leader the "strong" firm 1 chooses warfare because it strictly prefers the warfare to the Cournot outcome. For the "weak" firm 1 at decision node D_5 it cannot pay to engage in warfare because there are no further periods in which it could use its reputation.

Now imagine a game of two periods. Given that firm 2 plays Stackelberg warfare in period 2 the "weak" firm 1 might gain by adhering to warfare: according to condition (4) the cost of fighting, the warfare outcome in period two, is lower or equal than the benefits, the leadership profit in period 1. This is a necessary condition. A sufficient condition has to include firm 2's behaviour which depends on the beliefs p_2.

Assume firstly that $p_2 > \alpha_1$. Then playing warfare in period 2 is sufficient to make firm 2 follow in period 1 according to equation (5). Hence, the "weak" firm 1 plays Stackelberg warfare in period 2, and in period 1 a leader and follower equilibrium results.

Secondly, if $p_2 \leq \alpha_1$ it cannot be an equilibrium for the "weak" firm 1 to play warfare with probability 1 or to settle with probability 1. In the former case the posterior probability would not be sufficient to make firm 2 follow in period 1 and in the latter case firm 1's strategy would induce firm 2 to play Stackelberg warfare at D_3 with probability 1. Hence, firm 1 of the "weak" type plays a mixed strategy. The mutual dependence of strategies requires that in a mixed strategy equilibrium firm 1 randomizes such that firm 2 is indifferent between playing Stackelberg leader and follower in period 1. By the same token firm 2 plays mixed in period 1 such that firm 1 is indifferent about its actions in period 2.

Let β_2 be the conditional probability that the "weak" firm 1 plays warfare in period 2. Bayes' rule for updating beliefs after the occurrence of warfare (see equations (A.4) in the Appendix) in period 2 gives

$$p_1 = p_2/(p_2 + \beta_2(1 - p_2)). \tag{6}$$

In equilibrium β_2 is such that inequality (5) holds with equality (i.e., firm 2 is indifferent in period 1). Solving (6) for β_2 gives

$$\beta_2 = \left(\frac{1 - p_1}{p_1}\right)\left(\frac{p_2}{1 - p_2}\right). \tag{7}$$

Since warfare occurs in period 2 $p_1 = \alpha_1$ and so

$$\beta_2 = \left(\frac{1 - \alpha_1}{\alpha_1}\right)\left(\frac{p_2}{1 - p_2}\right) = \left(\frac{\pi_F - \pi_W}{\pi_C - \pi_F}\right)\left(\frac{p_2}{1 - p_2}\right). \tag{8}$$

So far firm 1's behaviour was studied under the assumption that firm 2 plays Stackelberg warfare in period 2. Under which circumstances will firm 2 do so? From 2's point of view, the total probability of firm 1 choosing warfare in period 2 is

$$p_2 + (1 - p_2)\beta_2 \;=\; p_2 \left(\frac{\pi_C - \pi_W}{\pi_C - \pi_F} \right) \;=\; \frac{p_2}{\alpha_1}. \tag{9}$$

If firm 2 chooses Stackelberg warfare at D_3 it gets π_W with probability p_2/α_1. After this event and in equilibrium firm 2's posterior beliefs p_1 are such that it has the expected profit of a follower π_F in period 1. With probability $1 - (p_2/\alpha_1)$ firm 2 receives π_C in period 2. Then firm 1 is known to be "weak" and firm 2 gets the Cournot payoff also in period 1. For this expected profit to be larger than the opportunity (the follower payoff in both periods) $(p_2/\alpha_1)(\pi_W + \pi_F\delta) + (1 - p_2/\alpha_1)\pi_C(1+\delta) > \pi_F(1+\delta)$ has to hold. Solving for p_2 (which becomes α_2) gives

$$p_2 \;<\; \frac{(\pi_C - \pi_F)^2(1+\delta)}{(\pi_C - \pi_W)(\pi_C(1+\delta) - \pi_F\delta - \pi_W)} = \alpha_2. \tag{10}$$

Finally, firm 2's mixed strategy (denoted by γ) in period 1 has to be derived. If $p_2 < \alpha_2 < \alpha_1$ firm 2 plays Stackelberg leader and the "weak" firm 1 randomizes with β_2 in period 2. With probability β_2 warfare arises and firm 2's posterior beliefs in period 1 are α_1. In equilibrium, firm 2 plays a mixed strategy in period 1 such that firm 1 is indifferent (in terms of expected profits) in period 2: $\beta_2(\pi_W + \gamma\pi_C\delta + (1-\gamma)\pi_L\delta) + (1-\beta_2)\pi_C(1+\delta) = \pi_C(1+\delta)$. Rearranging terms gives

$$\gamma = \frac{\pi_W + \pi_L\delta - \pi_C(1+\delta)}{(\pi_L - \pi_C)\delta}. \tag{11}$$

Condition (4) ensures that $0 < \gamma \leq 1$.

Now strategies and beliefs for an N-period version of the game are formulated in a proposition.

Proposition 1. *The following strategies and beliefs constitute a sequential equilibrium.*

Beliefs of firm 2.

- If firm 1 plays Stackelberg leader and firm 2 plays follower in period n then $p_{n-1} = p_n$.

- If firm 1 plays Cournot or settles a warfare situation in period n then $p_{n-1} = 0$.

- If warfare occurs in period n then $p_{n-1} = max\ (\alpha_{n-1}, p_n)$.

Behaviour of firm 1.

- A "strong" firm 1 plays Stackelberg leader at decision node D_1 and warfare at D_4.

- A "weak" firm 1 at D_2 plays Stackelberg leader if it has not played Cournot or settled before. Otherwise it plays Cournot.

- Given $n > 1$ and $p_n > \alpha_{n-1}$ it pays a "weak" firm 1 to play warfare at D_5 with probability 1. If $p_n \leq \alpha_{n-1}$ firm 1 mixes: with probability β_n warfare occurs, with $(1 - \beta_n)$ it settles the conflict. If $n = 1$ firm 1 of the "weak" type does not play warfare.

Behaviour of firm 2.

- If $p_n < \alpha_n$ Firm 2 plays Stackelberg leader.

- If $p_n = \alpha_n$ firm 2 plays Stackelberg leader with probability γ. Given that *prior* beliefs $p = \alpha_n$ firm 2 plays Stackelberg leader with probability 1.

- If $p > \alpha_n$ firm 2 plays follower.

The probabilities α_n, β_n and γ are given by

$$\alpha_n = \prod_{i=1}^{n} \left(\frac{(\pi_C - \pi_F) \sum_{j=0}^{i-1} \delta^j}{(\pi_C - \pi_F) \sum_{j=0}^{i-1} \delta^j + \pi_F - \pi_W} \right), \tag{12}$$

$$\beta_n = \left(\frac{1 - \alpha_{n-1}}{\alpha_{n-1}} \right) \left(\frac{p_n}{1 - p_n} \right), \tag{13}$$

$$\gamma = \frac{\pi_W + \pi_L \delta - \pi_C (1 + \delta)}{(\pi_L - \pi_C)\delta}. \tag{14}$$

Proof. *See the Appendix.*

This equilibrium can be proven to be unique *on* the equilibrium path (with one exception) by using the concept of admissible equilibria as part of Kohlberg and Mertens' (1986) strategic stability (see Govindan (1992)). The exception is the definition of the equilibrium path if prior beliefs $p = \alpha_n$. Any mixed strategy of firm 2 will work for this case.

3.3 Properties of Equilibrium Behaviour

What sequence of decisions by the firms over the time horizon does the equilibrium behaviour as specified imply? In this section the answer to this problem is given in a second proposition. Afterwards the result is illustrated with a numerical example.

Proposition 2. *Let $j(p)$ denote the first period in which firm 2 plays Stackelberg leader (and hence in which warfare might occur), that is $j(p) = max\ (j | p \leq \alpha_j)$. Then*

- *for any prior beliefs p the game begins with a sequence of periods from N to $j(p)$ in which firm 1 can exercise Stackelberg leadership provided the game is sufficiently long.*
- *Stackelberg warfare occurs in period $j(p)$ with probability $\beta_{j(p)}$. In the limit*

$$\lim_{p \to 0} Prob.(Warfare\ in\ j(p)) = \lim_{p \to 0} \beta_{j(p)} = 1.$$

- *If firm 1 is "weak" the game ends in the remaining periods $j(p) - 1$ to 1 in a Cournot equilibrium with a probability of* $1 - (1 - \gamma)\prod_{i=1}^{j(p)-1} \beta_i$. *In the limit*

$$\lim_{p \to 0} Prob.(Cournot) = \lim_{p \to 0} 1 - (1-\gamma) \prod_{i=1}^{j(p)-1} \beta_i = 1.$$

- *If firm 1 is "strong" the game has firm 1 as a Stackelberg leader over all periods.*

Proof. *See the Appendix.*

Part two and three of the proposition are stated for the limit $p \to 0$. Note that – in contrast to signalling games – there is no discontinuity of equilibrium behaviour if $p = 0$. Assume that $p = 0$ and firm 1 plays out of equilibrium Stackelberg leader. Then warfare occurs with probability 1 in the initial period N and the game evolves according to the mixed strategy behaviour specified in Proposition 1. Similarly, if $p > 0$ but $j < N$ warfare occurs with probability β_j and then the game evolves as in Proposition 1.

Firms' expected profits per period in periods $n > j(p)$ are $E(\pi_1) = \pi_L$ for the "weak" firm 1 and $E(\pi_2) = \pi_F$ for firm 2. *Average* expected payoffs per period for the last $j(p)$ periods are $E(\pi_1) = \pi_C,\ \ E(\pi_2) = \pi_F$. This follows from Propositions 1 and 2. Note that firm 2 has a positive expected profit at all stages of the game. For this reason it does not choose to exit the market even if it makes zero or negative profits in the period(s) of warfare. The expected profits indicate that the "weak" firm 1 gains from its type being private information while firm 2 looses profits.

In what follows the result is illustrated with a numerical example for a "weak" firm 1. In this example (see Table 1) the profits are $\pi_L = 5, \pi_C = 2, \pi_F = 1, \pi_W = 0$ and $\delta = 0.95$. Prior beliefs are assumed to be $p = 0.031$.

For these values $j = 20$, i.e., in period 20 firm 2 is willing to choose the Stackelberg leader action for the first time. (In period 21 firm 1 would not play Stackelberg warfare with probability 1. But since firm 2 does not yet fight in 21 the leader/follower pattern results in this period with probability 1.) In period 20 firm 1 plays the aggressive strategy with a probability of 0.939, so warfare is likely to occur. If the Cournot equilibrium results in this period (which is the case with a likelihood of 0.061) posterior beliefs $p_{19} = 0$ and firms play Cournot for the rest of the game.

Period	p_n	Firm 2 play St.-leader if:	Firm 1 play warfare if:	Market Structure
N	p_N	$p_N < \alpha_N$	$p_N > \alpha_{N-1}$	L/F
N-1	0.031	$0.031 < \alpha_{N-1}$	$0.031 > \alpha_{N-2}$	L/F
⋮				
22	0.031	$0.031 < 0.028$	$0.031 > 0.030$	L/F
21	0.031	$0.031 < 0.030$	$0.031 > 0.032$	L/F
			else with $\beta_{21} = 0.968$	
20	0.031	$0.031 < 0.032$	$0.031 > 0.34$	W (0.939)
			else with $\beta_{20} = 0.939$	C (0.061)
– if warfare occurred in 20 –				
19	0.034	$0.034 < 0.034$	$0.034 > 0.037$	W (0.273)
		else with	else with $\beta_2 = 0.916$	L/F (0.702)
		$\gamma = 0.298$		C (0.025)
⋮				
– if L/F occurred in 4 –				
3	0.192	$0.192 < 0.245$	$0.192 > 0.33$	W (0.482)
			else with $\beta_3 = 0.482$	C (0.518)
– if Cournot occurred in 3 –				
2	0	$0 < 0.33$	$0 > 0.5$	
			else with $\beta_2 = 0$	C
1	0	$0 < 0.5$	Do not play warfare	C

Table 1: A numerical example of the warfare game. Numerical values are $\pi_L = 5, \pi_C = 2, \pi_F = 1, \pi_W = 0, \delta = 0.95$ and $p = 0.031$.

Given that warfare occurred in 20 posterior beliefs $p_{19} = 0.034$. Now also firm 2 plays a mixed strategy: with a probability of $\gamma = 0.298$ it plays Stackelberg leader and with $1 - \gamma = 0.702$ follower. Firm 1 mixes with $\beta_{19} = 0.916$ – given that firm 2 played Stackelberg leader. Hence, in period 19 warfare results with a probability of 0.273, a leader/follower market structure with a probability of 0.702 and a Cournot equilibrium with a probability of 0.025.

In the same manner the game proceeds over the remaining periods: after an instance of warfare beliefs held by firm 2 are updated and firm 2 mixes with γ, after an instance of leadership beliefs are carried forward and firm 2 plays Stackelberg leader with a probability of 1. Firm 1 mixes in each period with β_n – if no Cournot equilibrium has resulted from such mixed behaviour.

For a Cournot equilibrium to occur there is a positive probability in each period. Moreover, this probability is increasing as the game moves on towards its final periods. As proved in Proposition 2 the likelihood that the game ends in a Cournot equilibrium approaches 1 in the limit as $p \to 0$. In the numerical example Cournot occurs arbitrarily in period 3 for the first time and then for the rest of the game. The total probability that the example game does end in a Cournot equilibrium is 0.977.

4 The "Concession" Scenario

In this section the analysis is extended to the case that *both* firms have private information about their type, i.e., both firm are possibly "strong". Assuming the same preferences as stated in (1) and (2) a "strong" type will take the action of a Stackelberg leader under any circumstances. Therefore any failure to set the Stackelberg leader output reveals a firm as "weak". Hence, a challenge by just one firm (as above for the one-sided case) is no longer appropriate. Both "weak" firms will play Stackelberg leader in the beginning of the game in order to augment their reputation for being "strong".

Differently to the game in Section 3 firms of the "weak" type now have two actions to choose between: continue to play Stackelberg leader or to concede. If both firms play Stackelberg leader then warfare arises. The firm which concedes first accepts the position of a follower and the other firm can exercise Stackelberg leadership for the remainder of the game.

This is clearly a game of "chicken" (or a "war of attrition"). This particular type of chicken game lacks a fixed premium at the end (unlike many chicken games in the literature), but one which is decreasing in time towards the end. The preferences for π_W are private information and beliefs about these are updated every period. Hence, this game is a non stationary war of attrition with asymmetric information. The chicken game is extensively treated in the literature (see e.g. Hendricks *et al.* (1988)). A two type game with asymmetric information is analysed by Kreps and Wilson (1982, Section 4) and Fudenberg and Kreps (1987). Their solution is applied here.

Modelling this setup in discrete time (like the model above) leads to recursions. Therefore the game is assumed to be in continuous time. Time goes backwards over an interval which is normalized to the length of one: $t \in [1, 0]$.

Suppose, for example, that firm 1 concedes first, at time t. That is, firms play warfare for $(1-t)$ and firm 2 is Stackelberg leader for a time of length t. Then profits are

$$\pi_1 = (1-t)\pi_W + t\pi_F \quad (15)$$

$$\pi_2 = (1-t)\pi_W + t\pi_L. \quad (16)$$

From this it follows that for "weak" firms it is best to have the opponent concede (since $\pi_L > \pi_F$) and second best to concede oneself. To stick to warfare is costly for a "weak" firm because the opportunity (the follower profit) is available at any time and $\pi_F > \pi_W$.

While p still stands for the prior on firm 1's type, let q denote the prior probability that firm 2 is "strong"

$$Prob.(firm\ 2 = \text{“}strong\text{”}) = q. \quad (17)$$

These prior beliefs are common knowledge. Both firms update their posterior assessments concerning the toughness of the opponent at time t: the history of the game is summarized in the posteriors p_t and q_t.

In equilibrium firms play with "stopping rules", a date at which to concede with a certain probability if the opponent has not given in yet. Call these probabilities for an engagement in warfare $\beta_t(t, p_t, q_t)$ and $\gamma_t(t, p_t, q_t)$ for firm 1 and firm 2 respectively. Each player's stopping rule has to be optimal given the probability distribution over the other player's type.

Now consider firms' decision to stick to warfare over the period $(t, t-h)$, h being small. In equilibrium a firm must be indifferent (up to terms of $o(h)$) between conceding immediately at t and waiting until $(t-h)$ and then conceding:

$$h\pi_W + (1-q_t)\beta_t(t, p_t, q_t)h\pi_L t = h\pi_F, \tag{18}$$

$$h\pi_W + (1-p_t)\gamma_t(t, p_t, q_t)h\pi_L t = h\pi_F. \tag{19}$$

Based on this calculation the following equilibrium can be derived.

Proposition 3. *In a game of two "weak" firms with prior beliefs of p and q respectively for being "strong"*

- *at $t = 1$ the firm with the higher prior plays warfare with probability 1 and the other firm according to $\beta_1(1, p, q)$ or $\gamma_1(1, p, q)$ respectively. If $p = q$ both firms play warfare with their mixed strategy.*

- *Given that neither firm has conceded at time $t < 1$, firms continue playing warfare in t with a probability of $\beta_t(t, p_t, q_t)$ and $\gamma_t(t, p_t, q_t)$ where*

$$\beta_t(t, p_t, q_t) = \frac{\pi_F - \pi_W}{(1-q_t)\pi_L\, t},$$

$$\gamma_t(t, p_t, q_t) = \frac{\pi_F - \pi_W}{(1-p_t)\pi_L\, t}.$$

- *Posterior beliefs if neither firm has conceded are given by*

$$p_t = q_t = q \left(\frac{1}{t}\right)^{\frac{\pi_F - \pi_W}{\pi_L}} \quad ; \quad q < p,$$

$$p_t = q_t = p \left(\frac{1}{t}\right)^{\frac{\pi_F - \pi_W}{\pi_L}} \quad ; \quad q \geq p.$$

Proof. *See the Appendix.*

At time 1 the game starts with priors p and q. If $p > q$ firm 1 plays Stackelberg leader with probability 1. Firm 2 plays Stackelberg leader with probability $\gamma_1(p,q)$ and concedes with $1 - \gamma_1(p,q)$. If the mixed strategy calls for firm 2 to concede a Stackelberg equilibrium with firm 1 as a leader remains until time 0. If firm 2 fights equilibrium requires that posterior probabilities move according to $p_t = q_t$. Similarly, if the game starts with priors $q > p$ firm 2 plays warfare with probability 1 and firm 1 mixes with $\beta_1(p,q)$.

In either case, given that warfare results in $t = 1$ both firms play mixed strategies and $p_t = q_t$ follows for the posterior beliefs. Again at any time t if one firm concedes, the Stackelberg equilibrium results, if warfare results posteriors are updated increasingly. Clearly, only if both firms are "strong" they reach $p_t = q_t = 1$ at some $t > 0$. "Weak" firms on the other hand will concede before $t = 0$ (see Kreps and Wilson (1982a)).

In contrast to the model in Section 3 firms do not end up in a Cournot but in Stackelberg constellation. Simultaneous concession (and only that would imply a Cournot equilibrium) is a probability zero event. There is neither a profitable possibility (in terms of expected profits) for the firm which conceded first to play warfare again in order to make the Stackelberg leader withdraw from its position as in the "discipline" scenario.

5 Concluding Remarks

Stackelberg's (1934) original interpretation of warfare in duopoly is that of a strategic fight for leadership in which firms engage consciously. On the contrast, the interpretation of Stackelberg warfare as a disequilibrium, a result of mistakes, ignores the element of strategic fighting completely. Both approaches, Stackelberg's original one as well as the disequilibrium interpretation, fail to provide a rationale for warfare in duopoly because of the static nature of the frameworks. In this paper a rationale for an engagement in Stackelberg warfare was proved. Warfare arises if either one or both firms in duopoly, based on reputation effects, strategically strive for leadership.

A striking result of the paper is that for the "discipline" scenario firms are likely to end up in a Cournot equilibrium, i.e., in a symmetric situation in which reputation effects are no longer at work. This paradoxical phenomenon of a fight for dominance with an eventually symmetric outcome in the end is known from the literature. Mailath (1989) shows in a game with simultaneous signalling that price wars may occur as an equilibrium. Similarly to the result in this paper, these price wars do not affect ultimate market shares. In contrast to this, in the case of two sided uncertainty one firm can manage to maintain the position of a Stackelberg leader based on reputation effects.

Stackelberg warfare is obviously similar to predation. The structure of conflict is similar in both cases. However, in the case of warfare both firms deliberately accept losses for strategic reasons while in the case of predation the preying firm's action makes the market unprofitable for the other firm. Consequently, predation is prohibited by competition law while warfare is not. Here the analysis of Stackelberg warfare has an implication for competition policy. Two recent empirical papers deal with Stackelberg warfare and predatory pricing. Phlips and Moras (1993) discuss the AKZO decision of the Commission of the European Community. While the Commission imposed a fine on AKZO for predatory "abuse of a dominant position" Phlips and Moras find evidence of active competition in form of Stackelberg warfare. Dodgson, Katsoulacos and Newton (1993) investigate alleged cases of predation in the UK bus industry. Similar to Phlips and Moras their findings are in some cases that firms deliberately chose actions which denied profitable outcomes. They conclude that Stackelberg warfare and not predatory behaviour happened. Generally, when trying to identify cases of predation it has to be checked carefully whether firms did not consciously take decisions which deny profitable outcomes in the market (i.e. Stackelberg warfare), though – ex post – one firm might look like the grim predator.

Appendix

Generalization of Profit Relations

Condition (4) imposed on the relation of the profits seems to restrict the generality of the model. In particular, the condition is not fulfilled for the widely used linear demand curve. Generalizing (4), let k denote the number of periods that is required for a profitable engagement in warfare by a "weak" firm 1:

$$k = \min(\pi_W + \pi_L \sum_{i=1}^{k} \delta^i \geq \pi_C \sum_{i=0}^{k} \delta^i). \tag{A.1}$$

In (4) it is assumed that $k = 1$ to keep to model clearly arranged. It is, nevertheless, easy to describe what happens if this unhappy restriction is relaxed [7].

To begin with, what remains unchanged for $k > 1$ is the beginning of the game. For small prior beliefs p there is a time horizon N such that firm 1 can be Stackelberg leader over a number of times until - according to the notation of Proposition 2 - period j.

Different are the final periods of the game. The "weak" firm 1 will not play warfare in the last $k - 1$ periods and this is known by firm 2. Hence the total probability that firm 1 plays warfare equals the probability p_n that firm 1 is "strong" for $n < k$.

Now think of a period n (where $(j > n > k-1)$ in which firm 2 plays Stackelberg leader. The "weak" firm 1 needs $k-1$ subsequent periods of Stackelberg leadership to make warfare in j a profitable strategy. Thus, it is not sufficient that the mixed strategy β_n makes firm 2 indifferent in

[7] Take the example of a linear demand duopoly with constant marginal cost. The inverse demand curve is given by $p = a - b(x_1 + x_2)$. In equilibrium and for $\delta = 1$ profits of $\pi_L = (a-c)^2/8b, \pi_C = (a-c)^2/9b, \pi_F = (a-c)^2/16b$ and $\pi_W = 0$ result. Plugging these profits in (A.1) $k = 9$ follows. This means that a firm, after an instance of warfare, needs 8 subsequent periods of Stackelberg leadership to break even with the profit of 9 periods of Cournot equilibrium in a model with linear demand.

period $n-1$ only. Equilibrium requires this to happen over $k-1$ periods until $n-(k-1)$. In turn firm 2's mixed strategy γ in equilibrium has to make firm 1 indifferent over this part of the game.

That is, equilibrium strategies in $(n-2,\ldots,n-(k-1))$ also depend on what happened in n. More abstractly speaking in games of $k > 1$ the beliefs p_n are no longer a sufficient statistic for the equilibrium strategies. The latter will depend on p_n and the history of the game over the last k periods. Rewriting the model such that the history from $n,\ldots,n-(k-1)$ is included would not change Propositions 1 and 2 structurally. The results derived hold for any constellations of profit functions with downward sloping reaction functions.

Proof of Proposition 1

Bayesian consistency of beliefs. If firm 1 plays Stackelberg leader and firm 2 plays follower nothing is learned about firm 1. So the beliefs of period n are carried forward to stage $n-1$,

$$p_{n-1} = p_n. \tag{A.2}$$

If firm 1 plays Cournot at decision node D_1 or settles a warfare situation it is known to be "weak". Since a "strong" firm would not follow such behaviour firm 2 updates

$$p_{n-1} = 0. \tag{A.3}$$

If firm 1 plays warfare Bayes' rule gives

$$\begin{aligned} p_{n-1} &= Prob.(\text{“}strong\text{”}|warfare) \\ &= \frac{Prob.(warfare|\text{“}strong\text{”})\,Prob.(\text{“}strong\text{”})}{P.(war.|\text{“}strong\text{”})\,P.(\text{“}strong\text{”}) + P.(war.|\text{“}weak\text{”})\,P.(\text{“}weak\text{”})} \\ &= \frac{1\cdot p_n}{1\cdot p_n + \beta_n(1-p_n)}. \end{aligned} \tag{A.4}$$

Substituting β_n as in equation (13) gives

$$p_n = \alpha_n. \tag{A.5}$$

Hence, firm 2's beliefs are consistent with firm 1's strategies and Bayes' rule.

Behaviour of firm 2. In period n the total probability – from firm 2's point of view – that firm 1 engages in warfare is

$$p_n + \beta_n(1 - p_n) = p_n/\alpha_{n-1}. \tag{A.6}$$

Thus, if firm 2 plays Stackelberg leader in n warfare occurs with p_n/α_{n-1} and a Cournot equilibrium results with $1 - p_n/\alpha_{n-1}$. In the former case firm 2's posterior beliefs are α_{n-1} and the profit of a follower is expected for the remainder of the game. Hence, firm 2's expected profit from playing Stackelberg leader in n is

$$E(\pi_2^n) = (p_n/\alpha_{n-1})(\pi_W + \pi_F \sum_{i=1}^{n-1} \delta^i) + (1 - p_n/\alpha_{n-1})(\pi_C \sum_{i=0}^{n-1} \delta^i) \tag{A.7}$$

$E(\pi_2^n)$ has to be larger than playing the alternative (the follower option) for the rest of the game:

$$E(\pi_2^n) > \pi_F \sum_{i=0}^{n-1} \delta^i. \tag{A.8}$$

Rearranging terms and plugging in β_n yields

$$p_n < \prod_{i=1}^{n} \left(\frac{(\pi_C - \pi_F)\sum_{j=0}^{i-1} \delta^j}{(\pi_C - \pi_F)\sum_{j=0}^{i-1} \delta^j + \pi_F - \pi_W} \right) = \alpha_n. \tag{A.9}$$

If the inequality is reversed firm 2 is better off playing follower. Equilibrium behaviour requires firm 2 to play a mixed strategy γ if $p_n = \alpha_n$. In this case equation (A.8) becomes an equality and firm 2 is indifferent between playing Stackelberg leader and follower. Hence, any mixed strategy is optimal and thus γ is. In the same way playing Stackelberg leader with probability 1 is optimal given the case that *prior* beliefs (i.e., without an instance of warfare) $p = \alpha_n$.

Behaviour of firm 1. A "strong" firm 1 is better off playing warfare than Cournot. Hence it plays Stackelberg leader at decision node D_1 and warfare at D_3.

For a "weak" firm 1 to play warfare in period n obviously requires (4) and so $n > 1$ as necessary conditions. Given that

$$p_n > \alpha_{n-1} \tag{A.10}$$

firm 2 does neither play Stackelberg leader in period n nor $n-1$. Hence the Stackelberg leader payoff is feasible for firm 1 in n which is strictly preferred to the Cournot outcome. Thus, firm 1 plays warfare. In equilibrium the "weak" firm 1 randomizes if

$$p_n \leq \alpha_{n-1}. \tag{A.11}$$

Firm 1 mixes with β_n such that firm 2 is indifferent between playing leader or follower in $n-1$ and firm 2 randomizes in a mixed strategy equilibrium with γ in $n-1$ such that firm 1 is indifferent in period n. It was already shown above that playing the mixed strategy β_n makes firm 2 indifferent in $n-1$. It remains to determine γ. The "weak" firm 1's expected profit from randomizing with β_n is

$$E(\pi_1^n) = \beta_n[\pi_W + \gamma E(\pi_1^{n-1}) + (1-\gamma)(\pi_L\delta + E(\pi_1^{n-2})] + (1-\beta_n)\pi_C \sum_{i=0}^{n-1} \delta^i. \tag{A.12}$$

In equilibrium this expected profit has to equal the one from playing the opportunity (that is Cournot)

$$E(\pi_1^n) = \pi_C \sum_{i=0}^{n-1} \delta^i. \tag{A.13}$$

Plugging this into (A.12) and solving for γ gives

$$\gamma = \frac{\pi_W + \pi_L\delta - \pi_C(1+\delta)}{(\pi_L - \pi_C)\delta}. \tag{A.14}$$

Hence, the proposition follows.□

Proof of Proposition 2

For $\delta < 1$ rewrite α_n such that

$$\frac{1}{\alpha_n} = \prod_{i=1}^{n}\left(1 + \left(\frac{\pi_F - \pi_W}{\pi_C - \pi_F}\right)\left(\frac{1-\delta}{1-\delta^i}\right)\right) > \left(1 + \left(\frac{\pi_F - \pi_W}{\pi_C - \pi_W}\right)(1-\delta)\right)^n. \tag{A.15}$$

Then, for a positive δ and for $\lim_{n\to\infty} 1/\alpha_n = \infty$ and so one gets $\lim_{n\to\infty} \alpha_n = 0$. For $\delta = 1$ divide denominator as well as the numerator of α_n by $(\pi_C - \pi_W)$. Then $1/\alpha_n$ becomes

$$\frac{1}{\alpha_n} = \left(\frac{i+c}{i}\right)^n \tag{A.16}$$

where $c = (\pi_F - \pi_W)/(\pi_C - \pi_F)$. Taking logarithms gives

$$\ln(1/\alpha_n) = \sum_{i=1}^{n} (\ln(i+c) - \ln(i)). \tag{A.17}$$

By the mean value theorem

$$\frac{\ln(i+c) - \ln(i)}{c} = \frac{1}{i'}; \quad i \leq i' \leq i+c \tag{A.18}$$

and since $i \leq i+c \leq 2i$ at some finite i it follows that

$$\ln(1/\alpha_n) = \sum_{i=1}^{n} \frac{c}{i'} \geq c \sum_{i=1}^{n} \frac{1}{i+c} \geq c \sum_{i=1}^{n} \frac{1}{2i}. \tag{A.19}$$

Now $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{i} = \infty$ implies $\lim_{n\to\infty} ln(1/\alpha_n) = \infty$ and so $\lim_{n\to\infty} \alpha_n = 0$ also for $\delta = 1$. Hence, for any positive p one can find a time horizon N such that $\alpha_N < p$.

In period $j(p)$ firm 2 plays Stackelberg leader with probability 1. (The total probability that Stackelberg warfare occurs *at least once* is equal to the probability that it occurs precisely in j since further occurrences of warfare in the game tree are only possible if warfare happens in j.) Firm 1 of the "weak" type mixes with

$$\beta_{j(p)} = \left(\frac{1 - \alpha_{j(p)-1}}{\alpha_{j(p)-1}}\right)\left(\frac{p}{1-p}\right) \tag{A.20}$$

by definition of $j(p)$. Also

$$\alpha_{j(p)+1} < p \leq \alpha_{j(p)} \tag{A.21}$$

follows from the definition of $j(p)$ (otherwise firm 2 would play Stackelberg leader one period earlier or later). Modifying (A.20) gives

$$\lim_{p\to 0} \beta_j = \lim_{p\to 0} \left(\frac{1 - \alpha_{j-1}}{1-p}\right) \lim_{p\to 0} \left(\frac{p}{\alpha_{j(p)-1}}\right). \tag{A.22}$$

Clearly, $p \to 0$ implies $j(p) \to \infty$, then $\alpha_{j(p)} \to 0$ and so $\alpha_{j(p)-1}, \alpha_{j(p)+1} \to 0$. The first term on the right hand side in (A.22) is clearly positive and $\to 1$. The second term on the right hand side is also positive. Since numerator and denominator $\to 0$ with the same speed of convergence,

$$\lim_{p \to 0} Prob.(Warfare\ in\ j(p)) = 1 \tag{A.23}$$

follows.

In the remaining periods $(j-1, \ \ldots, \ 1)$ the probability that the Cournot equilibrium does *not* occur in a particular period n is β_n. The probability that a Cournot equilibrium does not result in the last period (given that Cournot has not been played up to then) is $1-\gamma$. Thus, the probability that the game moves into a Cournot equilibrium in some period between $j-1$ and 1 is

$$Prob.(Cournot) = 1 - (1-\gamma)\prod_{i=1}^{j-1} \beta_i. \tag{A.24}$$

Since

$$1 - (1-\gamma)\prod_{i=1}^{j-1} \beta_i > 1 - (1-\gamma)(\beta_{j-1})^{j-1} \tag{A.25}$$

and for $\beta_{j-1} < 1$

$$\lim_{j \to \infty} 1 - (1-\gamma)(\beta_{j-1})^{j-1} = 1 \tag{A.26}$$

the third part of Proposition 2

$$\lim_{p \to 0} Prob.(Cournot) = 1 \tag{A.27}$$

follows. This completes the proof.□

Proof of Proposition 3

The proof is adopted (with modifications necessary for this model) from Kreps and Wilson (1982a, Section 4).

Dividing (18) and (19) by h it is straightforward to solve for $\beta_t(t, p_t, q_t)$ and $\gamma_t(t, p_t, q_t)$.

Over the interval $(t, t-h)$ the conditional probability a firm is "strong", given that it has not conceded yet is according to Bayes' rule

$$p_{t-h} = p_t/(p_t + (1-p_t)\gamma_t h) = p_t/1 - (1-p_t)\gamma_t h), \tag{A.28}$$

$$q_{t-h} = q_t/(q_t + (1-q_t)\beta_t h) = q_t/1 - (1-q_t)\gamma_t h). \tag{A.29}$$

Rearranging terms gives

$$\frac{p_t - p_{t-h}}{h} = p_t(1-p_t)\gamma_t = p_t \frac{\pi_F - \pi_W}{\pi_L t} = \dot{p}_t = \frac{dp}{d(-t)}, \tag{A.30}$$

$$\frac{q_t - q_{t-h}}{h} = q_t(1-q_t)\beta_t = q_t \frac{\pi_F - \pi_W}{\pi_L t} = \dot{q}_t = \frac{dq}{d(-t)} \tag{A.31}$$

It follows that $dp_t/dq_t = p_t/q_t$. Integrating $dp_t/p_t = dq_t/q_t$ yields $p = q$ in equilibrium. From this the initial equilibrium behaviour at $t = 1$ is obtained.

Now integrating $\dot{p}_t/p_t = (\pi_F - \pi_W)/\pi_L t$ (and similarly for $\dot{q}_t/q_t$) one obtains

$$p_t = t^{-\frac{\pi_F - \pi_W}{\pi_L}} k \tag{A.32}$$

$$q_t = t^{-\frac{\pi_F - \pi_W}{\pi_L}} k', \tag{A.33}$$

The constants of integration k and k' have to be chosen such that $p_1 = q_1 = p; p > q$ and $p_1 = q_1 = q; q > p$. This is obtained by $k = k' = p; p > q$ and $k = k' = q; q > p$. This gives the terms for q_t and p_t and hence the Proposition follows.□

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