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Some Consequences of Temporal Aggregation of a VARIMA Process*

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Abstract

We study the effects of temporal aggregation on particular characteristics of the variables such as the presence of cointegration and serial correlation common features, or the possibility of adopting a conditional model for estimation, forecasting and policy analysis. An empirical example illustrates the main issues.

Key words: Temporal Aggregation, *VARIMA* Process, Invariance, Exogeneity, Cointegration, Cofeatures.
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1 Introduction

Econometric studies often look for particular characteristics of the variables under analysis, such as the presence of cointegration and serial correlation common features, the possibility of adopting a conditional model for estimation, forecasting and policy analysis, or the equality to a specified value of a certain parameter. The results are also usually interpreted as corroborating or falsifying or suggesting a set of economic propositions.

Yet, the temporal frequency of the economic propositions is quite often higher than that of the data which are employed in the econometric analysis. Hence, for these propositions to be relatable to the available data, their implications in terms of particular characteristics of the variables must be invariant to temporal aggregation.[1]

Many authors have analised this issue in the past. Among others, Campos et al. (1990) noticed that weak exogeneity can be lost through temporal aggregation, and a similar result was suggested for Granger non causality by Sims (1971) and Wei (1982). On the other hand, Granger (1990) argued that cointegration is invariant to temporal aggregation, while Granger and Siklos (1995) have shown that spurious cointegration can arise in particular cases. In this paper we deal with this topic in a more systematic way, summarising and generalising former results, and proposing some new ones. Most of analysis is conducted in the time domain, because this is the natural framework for economic models.

To start with, in Section 2 we briefly review and extend some results in Marcellino (1995a) concerning the relationship between the original and the temporally aggregated data generating process (DGP), under the assumption that the former belongs to the $VARMA$ class. It is highlighted that an MA component is usually introduced even if it is absent in the original DGP. It is also shown that the roots of the aggregated AR component are in general higher than the original ones, and its coefficients are subject to substantial modifications. This provides a general framework to

[1]We assume that the propositions of interest are referred to spatially aggregate variables, or that sufficiently spatially disaggregated data are available, in order to avoid issues of aggregation over agents.

deal with the effects of temporal aggregation on different notions of exogeneity, Granger noncausality and structural invariance. Not only it is possible to show that these properties are in general lost, but the exact effects of temporal aggregation can be spotted.

In Section 3, we deal with integrated variables, provide a formal proof of the invariance of integration and cointegration to temporal aggregation under certain conditions, and illustrate how spurious cointegration among the variables can be created when these conditions are not satisfied.[2] Invariance of the lag length and of weak exogeneity of some variables for the cointegration parameters are also discussed.

The effects of temporal aggregation on the Wold representation of the process and, in particular, on such characteristics as serial correlation common features and impulse response functions, are studied in Section 4.

Section 5 considers continuous time *DGP*s and contains some comments on the relationship between continuous and discrete time cointegration, and, more generally, continuous and discrete time temporal aggregation. The former issue has been analysed in Phillips (1991) and further developed by Comte (1994). We provide a simple refinement of a statement in Comte (1994) which ensures a similarity of results with respect to those in Phillips (1991) for the continuous time case, and to ours for the discrete time case.

In Section 6 we present an empirical example, in order to clarify and illustrate the theoretical findings. In particular, the relationship between a long term and a short term interest rate is analysed for different temporal aggregation schemes, and the validity of the theoretical predictions is successfully checked. It is also suggested that such a result can represent a general mispecification test for the models.

Finally, in Section 7 the main findings of the paper are restated and some directions of future research are proposed. The Appendix presents the proofs of the Propositions in the text.

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[2]We are mainly interested in the effects of temporal aggregation on integration and cointegration at frequency zero. Hence, when it is not otherwise specified, we use the terms integrated and cointegrated variables as shorthands for integrated and cointegrated at frequency zero variables.

2 Original and aggregated VARMA processes

Let us consider the n dimensional process $x = \{x_t\}_{t=0}^{\infty}$, whose elements satisfy the stochastic difference equations

$$G(L)x_t = S(L)\varepsilon_{xt}, \tag{1}$$

where L is the lag operator, $G(L) = I - G_1L - G_2L^2 - ... - G_gL^g$, $S(L) = I - S_1L - S_2L^2 - ... - S_sL^s$, the roots of $|G(L)| = 0$ and $|S(L)| = 0$ lie outside the unit circle and are not common, the Gs and Ss are $n \times n$ matrices of coefficients, $\varepsilon_{xt} \sim i.i.d.(0, \Upsilon_x)$, and, for simplicity, the initial conditions are set equal to zero. This process, which is said to follow a $VARMA(g,s)$ model or to be a $VARMA$ process, is often implied by economic theory, at least as a reduced form representation for the relationships among the variables.

It can happen that not all the realizations of the elements of x are observable, and we focus on the case where this is due to temporal aggregation, namely, to the frequency of data collection being higher than that of data generation. Such a situation seems to be rather frequent in economics, e.g., only quarterly or at most monthly data are available for many key macroeconomic variables, while their generating frequency is likely to be much higher.

Different temporal aggregation schemes can be applied to the original process in order to aggregate it. For example, if the elements of x are stock variables such as wealth or capital, then a proper aggregated process is $x_k = \{x_{tk}\}_{t=1}^{\infty}$, i.e., only the k^{th} elements of the original process are retained, where k is the frequency of aggregation. We refer to this situation as point-in-time sampling.

When the elements of x are flow variables such as consumption or investment, then the elements of the aggregated process are partial sums of the original ones, namely, $x_k = \{x_{tk} + x_{tk-1} + ... + x_{tk-k+1}\}_{t=1}^{\infty}$. It is also frequent to analyse such data as averages across a month or a quarter of interest or exchange rates, which are realizations of partial averages of the original elements, and in this case we would have $x_k = \{\frac{1}{k}x_{tk} + \frac{1}{k}x_{tk-1} + ... + \frac{1}{k}x_{tk-k+1}\}_{t=1}^{\infty}$. More generally, we can consider the aggregated process $x_k = \{\omega(L)x_{tk}\}_{t=1}^{\infty}$, where $\omega(L) = \omega_0 + \omega_1L + ... + \omega_{k-1}L^{k-1}$ is a scalar polynom which leads to the desired aggregation of the original elements, and we define this as average sampling. Notwithstanding the name, the weights are not required to be equal or to sum to one. In particular, $\omega(L) = 1$ corresponds to point-in-time sampling.

There are also situations where the data are likely obtained by applying different aggregation schemes to the variables under joint analysis, e.g., when the behaviour of investment, capital and an interest rate, or consumption, income and wealth is studied. This case is named mixed sampling in Marcellino (1995a), MM henceforth, and it encompasses the other two possibilities. Yet, it requires to introduce a rather cumbersome notation, and therefore, in the theoretical part of this paper, we will concentrate mainly on point-in-time and average sampling. The results are valid also for mixed sampling, *mutatis mutandis*.

The *DGP* of the aggregated process is characterised in the following Propositions:

Proposition 1. If it is possible to determine an $n \times n$ polynomial matrix of degree $gk - g$ in the lag operator, $B(L)$, such that the coefficients of the lags which are not multiple of k in the product $B(L)G(L)$ are equal to zero, then the *DGPs* of the point-in-time and average sampling temporally aggregated process x_k are, respectively, the *VARMA* models:

$$\begin{array}{c} C(Z)x_{kt} = P(Z)\varepsilon_{kxt} \\ C(Z)x_{kt} = Q(Z)\varepsilon_{kxt} \end{array}, \tag{2}$$

where the degree in Z of $C(Z)$, $P(Z)$, and $Q(Z)$ are reported in Table 1. □

Proposition 2. It is possible to determine a scalar polynom of degree $gkn - gn$ in the lag operator, $b(L)$, such that the *DGPs* of the point-in-time and average sampling temporally aggregated process x_k are, respectively, the *VARMA* models:

$$\begin{array}{c} c^*(Z)x_{kt} = P^*(Z)\varepsilon_{kxt} \\ c^*(Z)x_{kt} = Q^*(Z)\varepsilon_{kxt} \end{array}, \tag{3}$$

the degree in Z of $c^*(Z)$, which is a scalar polynom, $P^*(Z)$, and $Q^*(Z)$ can be obtained from Table 1 after substituting g with gn and s with $s + g(n - 1)$. □

[Table 1 about here]

These Propositions are proved in MM, where a sufficient condition for the existence of the $B(L)$ matrix in Proposition 1 is also stated, and general formulae for the determination of the coefficients of the aggregated AR and MA components are provided. When the condition in Proposition 1 is satisfied, it is possible to determine a much more parsimonious representation for the aggregated process. When it is not valid, it is always possible to resort to Proposition 2 to derive the aggregated DGP. In both cases, the orders in Table 1 should be more properly considered as upper bounds, because in particular situations, e.g., when $G(L) = G(L^k)$, lower values are obtained.[3]

Notice that the aggregated AR components are the same for point-in-time and average sampling, while, in general, the MA components differ. An MA component is usually introduced even if it is not present in the original DGP.

We can now analyse a first particular characteristic of interest, namely, the effects of temporal aggregation on the roots of the AR component.

Proposition 3. If $|G(L)| = 0$ has roots $L_j = \lambda_j$, $j = 1, ..., gn$, and $B(L)$ exists, then $|C(Z)| = 0$ has roots $Z_j = \lambda_j^k$, $j = 1, ..., gn$ and the $gn(k-1)$ roots of $|B(L)| = 0$ satisfy the equation $\prod_{j=1}^{gn}(\sum_{i=0}^{k-1} \lambda_j^{k-1-i} L^i) = 0$. Moreover, $c^*(Z) = 0$ has roots $Z_j = \lambda_j^k$, $j = 1, ..., gn$, and those of $b(L) = 0$ also satisfy the equation $\prod_{j=1}^{gn}(\sum_{i=0}^{k-1} \lambda_j^{k-1-i} L^i) = 0$. □

Thus, the roots of $|C(Z)| = 0$ or $c^*(Z) = 0$ are larger in absolute value than those of $|G(L)| = 0$, increase with the sampling frequency, and there can be changes in their signs for odd values of k. An interesting implication of this result is that the coefficients of the AR component in the DGP of x_k tend to zero when k diverges.

Another interesting topic regards the consequences of temporal aggregation on weak exogeneity, structural invariance and super exogeneity, as defined in Engle *et al.* (1983). Campos *et al.* (1990) argue that weak exogeneity can be not invariant through temporal aggregation. The following example illustrates this statement, shows that it is valid for the other

[3] An alternative approach to the derivation of the temporally aggregated DGP is suggested by Lütkepohl (1987). It is based on the application of a particular deterministic selection matrix to a reparameterized version of the original DGP. Even if it often leads to less parsimonious representations of the aggregated DGP, it can be shown to imply similar results to what we find in this paper by means of our method.

characteristics, but also points out the importance of a clear statement of the problem of interest.

Example 1. Let us assume that the variables y and z are generated by the structural model

$$\begin{matrix} y_t = \alpha z_t + \beta z_{t-1} + \eta_{yt} \\ z_t = \gamma y_{t-1} + \eta_{zt} \end{matrix}, \quad \begin{pmatrix} \eta_{yt} \\ \eta_{zt} \end{pmatrix} \sim i.i.d.N(0, \begin{pmatrix} \sigma_{yy} & \sigma_{yz} \\ \sigma_{yz} & \sigma_{zz} \end{pmatrix}), \tag{4}$$

where the first equation can represent the effects of a policy variable on a target variable, and the second equation is a control rule for the policy variable. The corresponding reduced form is

$$\begin{matrix} y_t = \alpha\gamma y_{t-1} + \beta z_{t-1} + \varepsilon_{yt} \\ z_t = \gamma y_{t-1} + \varepsilon_{zt} \end{matrix}, \quad \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{pmatrix} = \begin{pmatrix} \eta_{yt} + \alpha\eta_{zt} \\ \eta_{zt} \end{pmatrix}, \tag{5}$$

which is a $VAR(1)$ model.

If the parameter of interest is α, then z is weakly exogenous for α if it can be recovered from the parameters of the conditional model

$$y_t = az_t + by_{t-1} + cz_{t-1} + u_{yt}, \tag{6}$$

and a, b, c, and $var(u_{yt})$ are variation free from the parameters of the marginal model for z, γ and σ_{zz}. If there are modifications in the parameters of the marginal model, i.e., in the policy, rule, but those of the conditional model remain fixed, then the conditional model is structurally invariant. If the conditional model is structurally invariant and z is weakly exogenous, then it is said to be super exogenous for α. Weak exogeneity implies consistent and efficient estimation of α with the conditional model (6), and super exogeneity ensures that this is valid even if there are changes in the marginal model.

It turns out that $a = \alpha + \frac{\sigma_{yz}}{\sigma_{zz}}$, $b = -\gamma\frac{\sigma_{yz}}{\sigma_{zz}}$, $c = \beta$ and $var(u_{yt}) = \sigma_{yy} - \frac{\sigma_{yz}^2}{\sigma_{zz}}$. Hence, when $\sigma_{yz} = 0$, z is weakly exogenous for α. It is also super exogenous because the conditional model is structurally invariant. If the parameter of interest was a, then z would be weakly exogenous for a by construction. Even if $\sigma_{yz} \neq 0$, a and b can be invariant to changes in σ_{zz} if σ_{yz} adjusts so that $\frac{\sigma_{yz}}{\sigma_{zz}}$ is fixed. In this case z would be super exogenous for a with respect to modifications in σ_{zz}.

Let us now assume that point in time sampling with $k=2$ is applied to the original process. For the condition in Proposition 1 to be satisfied, it must be

$$B(L) = \begin{pmatrix} 1+\alpha\gamma L & \beta L \\ \gamma L & 1 \end{pmatrix}.$$

Then, we have

$$\begin{gathered} y_{kt} = (\alpha^2\gamma^2 + \beta\gamma) y_{kt-1} + \alpha\beta\gamma z_{kt-1} + \varepsilon_{kyt} \\ z_{kt} = \alpha\gamma^2 y_{kt-1} + \beta\gamma z_{kt-1} + \varepsilon_{kzt} \\ \begin{pmatrix} \varepsilon_{kyt} \\ \varepsilon_{kzt} \end{pmatrix} \sim i.i.d.N(0, \begin{pmatrix} \psi_{yy} & \psi_{yz} \\ \psi_{yz} & \psi_{zz} \end{pmatrix}), \end{gathered} \tag{7}$$

with $\psi_{yy} = (1+\alpha^2\gamma^2)\sigma_{yy} + (\alpha^2 + (\beta+\alpha^2\gamma)^2)\sigma_{zz} + 2(\alpha + \alpha\gamma(\beta+\alpha^2\gamma))\sigma_{yz}$, $\psi_{zz} = (1+\alpha^2\gamma^2)\sigma_{zz} + \gamma^2\sigma_{yy} + 2\alpha\gamma^2\sigma_{yz}$, $\psi_{yz} = \alpha\gamma^2\sigma_{yy} + (\alpha + \alpha\gamma(\beta+\alpha^2\gamma))\sigma_{zz} + (1+\alpha^2\gamma^2 + \gamma(\beta+\alpha^2\gamma))\sigma_{yz}$. It also follows that

$$y_{kt} = a_k z_{kt} + b_k y_{kt-1} + c_k z_{kt-1} + u_{kyt}, \tag{8}$$

where $a_k = \xi$, $b_k = \alpha^2\gamma^2 + \beta\gamma - \xi\alpha\gamma^2$, $c_k = \alpha\beta\gamma - \xi\beta\gamma$, $var(u_{kyt}) = \psi_{yy} - \xi\psi_{yz}$, $\xi = \frac{\psi_{yz}}{\psi_{zz}}$.

Hence, even if $\sigma_{yz} = 0$, z_k is no longer weakly exogenous either for α or for a. Moreover, even if the coefficients in (6) were invariant to changes in σ_{zz}, this is not true for (8). On the other hand, z_k is weakly exogenous for a_k, and the coefficients in (8) could be invariant to changes in ψ_{zz}. Finally, notice that z_k is instead both weakly and super exogenous for α if $\sigma_{yz} = 0$ and $\beta = 0$. □

It is simple to find examples which show that Granger noncausality is also not invariant through temporal aggregation, as Sims (1971) and Wei (1982) argue, and therefore strict and strong exogeneity, are also not invariant. A similar result also holds for the notions of encompassing exogeneity in Marcellino (1995b), which essentially require the conditional model to encompass the joint model with respect to the parameters or forecasts of interest, for the conditioning variables to be exogenous.

These results suggest that economic propositions which have implications in terms of exogeneity and Granger non causality of some variables, or assign specific values to particular parameters, should not in general be tested with aggregated data, because the validity or not of their implications could be due to the effects of temporal aggregation. Instead, if the generating frequency is stated in the economic propositions and the temporal aggregation scheme is known, Propositions 1 or 2 can be applied to derive the theoretical temporal aggregated *DGP*. Its capacity to provide a congruent statistical representation for the aggregated data would represent an indirect falsification or corroboration of the original economic statement.

3 Unit roots

In this Section we allow the autoregressive component in the *DGP* of the original variables to have unit roots, and we are interested in determining whether and how these roots are transferred through temporal aggregation. The results that we are going to present are valid also for a *VARIMAX* *DGP* whose conditioning variables follow themselves a *VARIMA* model. Such a *DGP* is equivalent to a *VARIMA* model with particular restrictions on his coefficients, and these restrictions are not relevant in this context, see, e.g., Lutkepohl (1991), MM.

3.1 Positive unit roots, integration and cointegration

A variable is said to be integrated of order d, $I(d)$, if it is non stationary because of the presence of positive unit roots in the AR component of its $ARMA$ DGP, but becomes stationary after differencing d times. When all the variables are $I(d)$, we can assume that their DGP is the $VARIMA(g,d,s)$ model

$$G(L)(1-L)^d x_t = S(L)\varepsilon_{xt} \tag{9}$$

or

$$G^{**}(L)x_t = S^{**}(L)\varepsilon_{xt} \tag{10}$$

where $G^{**}(L)$ is a diagonal matrix whose terms are $|G(L)|(1-L)^d$, $S^{**}(L) = G^a(L)S(L)$, and $G^a(L)$ is the adjoint matrix of $G(L)$. The aggregated process is obtained by premultiplying both sides of (9) or (10) by, respectively, $B(L)(1+L+...+L^{k-1})^d$ or $b(L)(1+L+...+L^{k-1})^d$. Hence, the aggregated AR component is $C(Z)(1-Z)^d$ or $c^*(Z)(1-Z)^d$, and in both cases x_k is still $I(d)$.

If there exist some linear combinations of $I(d)$ variables which are $I(b)$ with $b < d$, the variables are said to be cointegrated of order d, b, $CI(d, b)$, and the coefficients of the $I(b)$ linear combinations are grouped into the cointegration vectors, see, e.g., Engle and Granger (1987), Johansen (1988). It can also happen that linear combinations of different transformations of the original variables which involve the lag operator are $I(b)$, and in this case the variables are said to be polynomially cointegrated, see, e.g., Haldrup and Salmon (1994), Gregoir and Laroque (1994). Both for cointegrated and polynomially cointegrated variables it is evident that the $I(b)$ linear combinations remain such after temporal aggregation, as noted by Granger (1990). But one can wonder whether other linear combinations are created or some of the existing ones become collinear. We show in this subsection that this is not possible when the variables are $I(1)$, while in Marcellino (1995c) the same is proved for the $I(2)$ case and it is indicated how it can be demonstrated for the $I(d)$ case.

It is convenient to rewrite the $VARMA$ model in (1) in the Error Correction (EC) form:

$$\Delta x_t = -\Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{g-1} \Delta x_{t-g+1} + S(L)\varepsilon_t, \tag{11}$$

where $\Pi = G(1)$ and $\Gamma_i = -\sum_{j=i+1}^{g} G_j$, $i = 1, ..., g-1$. Then, the number of cointegration vectors is equal to the rank of Π, p, see, e.g., Johansen (1988), and they can be grouped in the columns of the matrix β, with

$$\Pi = \alpha\beta', \tag{12}$$

where α and β are $n \times p$ matrices with rank p, so that they are identified only up to non singular transformations.[4]

If we assume that the condition in Proposition 1 holds and premultiply both sides of (1) by $B(L)$, the EC representation of x_k is

$$\Delta x_{kt} = -\Pi_k x_{kt-1} + \Gamma_{k1} \Delta x_{kt-1} + ... + \Gamma_{kg-1} \Delta x_{kt-g+1} + H(Z)\varepsilon_{kt} \tag{13}$$

where $\Pi_k = C(1)$, $\Gamma_{ki} = -\sum_{j=i+1}^{g} C_{kj}$, $i = 1, ..., g-1$, and the number of cointegration vectors, p_k, is equal to the rank of Π_k. But

$$p_k = r(\Pi_k) = r(B(1)\Pi) \leq min[r(B(1)), r(\Pi)] = p, \tag{14}$$

[4]The following analysis remains valid even if some of the variables are stationary. In this case, cointegration vectors whose elements are all equal to zero except one must be allowed for, see Johansen (1991). When it is not assumed a priori that the variables are at most $I(1)$, a further condtion has to be verified, which ensures that the variables are not integrated of higher order than 1, see Johansen (1992b). The validity of this condition is also invariant to temporal aggregation, see Marcellino (1995c).

so that we should conclude that temporal aggregation can decrease the number of cointegration vectors. Yet, we have,

Proposition 4. $p_k = p$. □

Furthermore, all the cointegration vectors for x are such also for x_k, and viceversa. Actually, from (13) it is

$$\Pi_k = \alpha_k \beta' \tag{15}$$

with $\alpha_k = B(1)\alpha$ and $r(\alpha_k) = r(\alpha) = p$, because in the proof of Proposition 4 it is shown that $B(1)$ is invertible. If β' did not contain the cointegration vectors for x_k then $\beta' x_k$ would be $I(1)$ and therefore uncorrelated with Δx_k, but in this case α_k should be a matrix of zeros and its rank could not be p. On the other hand, if we started with the model for x_k we could obtain a different set of cointegration vectors, β_k'. But, because of (15), this could be rewritten as $Q\beta'$, where Q is a non singular $p \times p$ matrix, and therefore also $\beta_k' x = Q\beta' x$ would be stationary.

This discussion also highlights that when we attribute an economic interpretation to the magnitude of the reaction of Δx to the cointegration relationships, we should consider that such a reaction is aggregation dependent. Particularly important is the case where some of the elements of α are equal to zero but the corresponding elements of α_k are different from zero, and viceversa. This situation is also interesting from a statistical point of view, because it implies that the validity of a necessary condition for the weak exogeneity of a set of regressors for inference on the long run parameters, see, e.g., Johansen (1992a), can depend on the sampling frequency. The following example illustrates this issue.

Example 2. Consider the bivariate EC model with $p = 1$:

$$\begin{aligned} \Delta y_t &= -\alpha_1 (y_{t-1} - \beta_2 z_{t-1}) + \varepsilon_{yt} \\ \Delta z_t &= -\alpha_2 (y_{t-1} - \beta_2 z_{t-1}) + \gamma \Delta y_{t-1} + \varepsilon_{zt} \end{aligned} , \tag{16}$$

with $cov(\varepsilon_{yt}, \varepsilon_{zt}) = 0$, for simplicity. We have

$$G_1 = \begin{pmatrix} 1-\alpha_1 & \alpha_1\beta \\ \gamma - \alpha_2 & 1+\alpha_2\beta \end{pmatrix}; \quad G_2 = \begin{pmatrix} 0 & 0 \\ -\gamma & 0 \end{pmatrix};$$
$$\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}; \quad \beta = \begin{pmatrix} 1 \\ \beta_2 \end{pmatrix},$$

and for point-in-time sampling with $k = 2$ the condition in Proposition 1 is satisfied if:

$$B_1 = \begin{pmatrix} -(1-\alpha_1) & -\alpha_1\beta_2 \\ -(\gamma-\alpha_2) & -(1+\alpha_2\beta_2) \end{pmatrix};$$
$$B_2 = \begin{pmatrix} -(1+\alpha_2\beta_2)\alpha_1\beta_2\gamma & \alpha_1^2\beta_2^2\gamma \\ -(1+\alpha_2\beta_2)^2\gamma & \alpha_1\beta_2\gamma(1+\alpha_2\beta_2) \end{pmatrix} / dt,$$

where $dt = det(G_1) = 1 - \alpha_1 + \alpha_2\beta_2 - \alpha_1\beta_2\gamma$.

$\alpha_2 = 0$ is a necessary and in this case also sufficient condition for the weak exogeneity of z for β, and we wonder whether under this assumption α_{22}, the coefficient of the cointegration relationship in the equation for z_2, is also equal to zero or not, i.e., whether weak exogeneity is invariant to point-in-time sampling. Unfortunately, we find:

$$\alpha_{22} = (\gamma + \gamma/(1 - \alpha_1 - \alpha_1\beta_2\gamma))\alpha_1,$$

so that weak exogeneity is lost. □

Similarly, also in this case there can be changes in the patterns of Granger noncausality and in the super exogeneity status of the conditioning variables.[5] Moreover, a further implication of Proposition 3 is that the coefficients of the lagged differences in the ECM representation of x_k tend to zero when k increases.

The presence of a constant or of a deterministic trend in the DGP of x does not alter the results because, as in the stationary case, these deterministic regressors are simply transferred into the DGP for x_k, even if with different coefficients. Furthermore, there are cases where a constant or a trend can be restricted in such a way that they appear in the cointegration relationships but do not influence directly Δx_t. For example, if there is a constant, μ, in (1) but $\mu = \alpha\beta_0$,where β_0 is a $p \times 1$ vector, then a vector whose elements are all equal to one can be included in x, and Π can be rewritten as $\alpha(\beta, \beta_0')'$ (see, Johansen (1992c)). After temporal aggregation, $\mu_k = B(1)\mu$ but $\alpha_k = B(1)\alpha$, so that when the transformation in the preceding paragraph is possible for μ, it is also possible for μ_k. A similar reasoning holds for a trend.

The results that we have obtained so far are valid both for point-in-time and, in general, for average sampling, because they only rely on the AR

[5] Giannini and Mosconi (1992) deal in particular with Granger non causality in cointegrated systems.

component of the DGP of x_k and we have seen in Proposition 1 that this is equal for the two temporal aggregation schemes. Yet, for very particular weighting schemes, which involve differencing of the variables, there can be a decrease in the order of integration and a corresponding increase in the cointegration rank, and a similar reasoning holds for mixed sampling.

Example 3. Consider the simple $VARIX$ model:

$$\begin{aligned} y_t &= z_t + \varepsilon_{yt} \\ z_t &= z_{t-1} + \varepsilon_{zt} \end{aligned},$$

and assume that average sampling with $k = 2$ and $\omega(L) = 1 - L$ is applied, so that

$$\begin{aligned} y_{kt} &= z_{kt} + \varepsilon_{kyt} \\ z_{kt} &= \varepsilon_{kzt} \end{aligned}.$$

While the original variables are integrated and cointegrated with cointegration vector $(1,1)$, y_{kt} and z_{kt} are stationary and even white noise processes. □

Finally, we have

Proposition 5: Even if the condition in Proposition 1 is not satisfied, $p_k = p$. □

In the remaining of the paper, for simplicity, we will assume that the condition in Proposition 1 is satisfied. If it is not, similar results can be obtained by applying Proposition 2 to get the temporally aggregated DGP.

3.2 Negative and complex unit roots

It can now be worthwhile discussing the consequences of the presence of negative or complex unit roots as solutions of $|G(L)| = 0$, which can arise, for example, when there is a stochastic seasonal component in the DGP of the variables. A first point to be noticed is that in this case a full rank Π matrix is no longer related to stationarity of the variables. A second important consideration is that the existence of negative or complex unit roots is not invariant to temporal aggregation. Actually, they can be turned into positive unit roots, as can be simply derived from Proposition 3. Thus, it can happen that the frequency of integration and cointegration is modified as a consequence of temporal aggregation. The following example illustrates these statements.

Example 4. The DGP of y and z is hypothesised to be

$$\begin{aligned} y_t &= (1-\alpha_1)y_{t-1} + \alpha_1 z_{t-1} + \varepsilon_{yt} \\ z_t &= -z_{t-1} + \varepsilon_{zt} \end{aligned} \tag{17}$$

so that

$$G_1 = \begin{pmatrix} 1-\alpha_1 & \alpha_1 \\ 0 & -1 \end{pmatrix}, \quad \Pi = I - G_1 = \begin{pmatrix} \alpha_1 & -\alpha_1 \\ 0 & 2 \end{pmatrix},$$

and $r(\Pi) = 2$. But this should not be interpreted as evidence of stationarity of y and z because it is evident, e.g., that the variance of z grows linearly in time. Actually, the variables are integrated and cointegrated at frequency π.

If we now consider point-in-time-sampling for $k = 2$, we obtain

$$\begin{aligned} \Delta y_{2t} &= (2\alpha_1 - \alpha_1^2)y_{2t-1} - \alpha_1^2 z_{2t-1} + \varepsilon_{2yt} \\ \Delta z_{2t} &= \varepsilon_{2zt} \end{aligned} \tag{18}$$

and

$$\Pi_2 = I - G_1^2 = \begin{pmatrix} 2\alpha_1 - \alpha_1^2 & \alpha_1^2 \\ 0 & 0 \end{pmatrix},$$

so that $r(\Pi_2) = 1$. Now both y_2 and z_2 are integrated at frequency zero, and the rank of Π_2 does indicate the presence of a cointegration vector at this frequency. Hence, temporal aggregation has created spurious "long run" cointegration between the variables. □

A similar point was made by Granger and Siklos (1995).[6] However, they state (p. 361) that the non invariance of the frequency of integration and cointegration of the variables is related only to point-in-time and not to what we have called average sampling, temporal aggregation in their notation. But this seems not to be in general correct, as the following example shows.

Example 5. Let us assume that average sampling with $\omega(L) = (1 + L + L^2 + L^3)$ is applied to the univariate process

$$(1 + L^2)^2 z_t = \varepsilon_{zt},$$

[6] Granger and Siklos focus on a situation where the original variables have a unit root at different frequencies so that they cannot be cointegrated, when constant cointegration vectors are considered. As a result of temporal aggregation, the frequency of the unit root can become common to the variables, which can then also be cointegrated.

which is integrated of order 2 at frequency $\frac{\pi}{2}$. Then, the resulting temporally aggregated process is

$$z_{kt} = z_{kt-1} + \varepsilon_{kzt},$$

with $\varepsilon_{kzt} \sim i.i.d.(0, 4\gamma_z)$ because $\varepsilon_{kzt} = \varepsilon_{zt} + \varepsilon_{zt-1} - \varepsilon_{zt-2} - \varepsilon_{zt-3}$, and z_k is $I(1)$. If we also hypothesise, e.g., that

$$y_t = z_t + \varepsilon_{yt},$$

it turns out that y_k and z_k are cointegrated at frequency zero. □

The validity of their suggestion is related mainly to the coincidence of the stochastic seasonal component with the weighting scheme, e.g., when it is $g(L)(1 + L + L^2 + L^3)z_t = \varepsilon_{zt}$, $k = 4$ and $\omega(L) = (1 + L + L^2 + L^3)$. In this case, the stochastic seasonal component is simply lost with average sampling.

3.3 Some implications for testing

The findings that we have obtained so far have also interesting implications for testing for unit roots and cointegration. They imply that the results of the proper tests should in general be invariant to different average schemes and sampling frequency under the null hypothesis, a part from small sample bias. Particular hypotheses on the coefficients of the cointegration vectors can be also tested with temporally aggregated data.

Furthermore, in small samples temporal aggregation might also increase the power of the tests because of the increase in absolute value of the roots of $|G(L)| = 0$ which are not equal to one. Yet, this positive effect can be offset by the decrease in the number of available observations. Actually, for the univariate case, Pierse and Snell (1995) have shown that the asymptotic local power of a one-sided unit root test is independent of the frequency of aggregation, as long as the same data span is adopted. Instead, for fixed alternatives and small samples, an increase in the aggregate data span is required for the power to be the same. However, when a large enough set of observations is available, applying the tests to different temporally aggregated data can provide an additional useful tool for assessing whether economic variables are integrated and cointegrated.

Moreover, the possibility that MA errors are created as a result of temporal aggregation and the widespread adoption of temporally aggregated data in empirical analysis, suggest that tests for cointegration that allow the *DGP* of the variables to be infinite order *VAR* or *VARMA* models might outperform those tests which instead rely on a finite order *VAR* approximation of the *DGP*, see respectively, e.g., Saikkonen and Luukkonen (1995), Yap and Reinsel (1994) and Johansen (1991).

We could also think of trying to discriminate whether a unit root in the data under analysis is due to its presence in the *DGP* or to the effects of temporal aggregation on a negative or complex unit root. This is not in general possible, because it requires to know the *DGP* or at least to have the data at their true generating frequency. Yet, some partial insight can be gained by analysing the available higher frequency data.

4 Common trends and common cycles

We now analyse the effects of temporal aggregation on the presence of common trends and common cycles among the variables.

Following Engle and Granger (1987), Stock and Watson (1988) and Johansen (1991), if x is generated by (11) with $s = 0$ for simplicity, then

$$\Delta x_t = T(L)\varepsilon_t, \tag{19}$$

with

$$\frac{T(L)}{(1-L)I}G(L) = I, \tag{20}$$

or

$$x_t = T(1)\sum_{i=1}^{t}\varepsilon_i + \bar{T}(L)\varepsilon_t, \tag{21}$$

where $\bar{T}(L) = (T(L) - T(1))/(1-L)$, $T(1) = \beta_\perp(\alpha'_\perp\Psi(1)\beta_\perp)^{-1}\alpha'_\perp$, $\Psi(L) = (\Pi(L) - \Pi(1))/(1-L)$ (so that $-\Psi(1)$ is the derivative of $\Pi(z)$ with respect to z evaluated at $z = 1$), $\Pi(L) = (1-L)I - \sum_{j=1}^{g-1}\Gamma_j(1-L)L^j - \Pi L$, and $\alpha_\perp$ and $\beta_\perp$ are orthogonal to α and β. $T(1)$ can be decomposed into two $n \times n - p$ matrices of full column rank:

$$T(1) = OP', \quad O = \beta_\perp(\alpha'_\perp\Psi(1)\beta_\perp)^{-1}, \quad P = \alpha_\perp, \tag{22}$$

and therefore $n - p$ common trends, $f_t = \alpha'_\perp\sum_{i=1}^{t}\varepsilon_i$, drive the evolution of x.

After point-in-time sampling, the equivalent of (19) is:

$$\Delta x_{kt} = U(L^k)B(L)\varepsilon_t = U(Z)H(Z)\varepsilon_{kt}, \tag{23}$$

with

$$\frac{U(L^k)}{(1-L^k)I}B(L)G(L) = \frac{U(Z)}{(1-Z)I}C(Z) = I. \tag{24}$$

From (20), (23) and (24) it follows

$$r(T(1)) = r(U(1)B(1)) = r(U(1)H(1)),$$

so that the number of common trends among x_k is equal to that among x. This is true also for average sampling, and such a result could have been immediately obtained from the invariance of the number of cointegration vectors to temporal aggregation. Yet, this derivation is useful for the following discussion.

Let us now deal with the effects of temporal aggregation on the presence of common cycles. As linear combinations of $I(1)$ variables can be stationary, other linear combinations can be such that the first differences of the variables are innovations, see Vahid and Engle (1993a). If we assume that there are s such so called cofeature combinations and group their coefficients into the $n \times s$ matrix of cofeature vectors γ, then

$$\gamma' \Delta x_t = \gamma' \varepsilon_t \tag{25}$$

or

$$\gamma' x_t = \gamma' T(1) \sum_{i=1}^{t} \varepsilon_i. \tag{26}$$

Vahid and Engle (1993a) show that the following conditions are equivalent and sufficient for the existence of cofeature vectors:

$$\begin{gathered} \gamma' \bar{T}_i = 0, i \geq 0; \\ \gamma' T_i = 0, i > 0; \\ \gamma' \Gamma_i = 0, i = 1, ..., g-1, \quad \text{and} \quad \gamma' \Pi = 0; \\ \gamma'(I - G_1) = 0, \quad \text{and} \quad \gamma' G_i = 0, i \geq 2. \end{gathered} \tag{27}$$

Moreover, they prove that $\bar{T}_i$ can be decomposed for all i into two $n \times n-s$ matrices F and $\bar{T}_i^*$ such that

$$\bar{T}(L)\varepsilon_t = F \bar{T}^{*'}(L)\varepsilon_t,$$

and they call common cycles (CC) the $n-s$ elements of $g_t = \bar{T}^{*'}(L)\varepsilon_t$. Hence, x admit the representation

$$x_t = Of_t + Fg_t.$$

Finally, the number of cointegration and cofeature vectors are linked by the relationship

$$s \leq n - p$$

and the two types of linear combinations of x are linearly independent.

Let us assume now that x_k is obtained by point-in-time sampling from x, that the s_k cofeature vectors in x_k are grouped in γ_k, and that γ^* is a $n \times (s_k - s)$ matrix with full column rank. We have

Proposition 6. $s_k \geq s$ and $\gamma_k = (\gamma \quad \gamma^*)$. □

Hence, point-in-time sampling can increase but not decrease the number of cofeature vectors, and the cofeature vectors for x are such also for x_k.

Instead, in the case of average sampling the transmission of common cycles is no longer possible. Actually, we have seen that the generating process of x_k has also an $MA(1)$ component when that of x is a pure random walk. Thus, $\gamma'\Delta x_t = \varepsilon_t$ implies $\gamma'\Delta x_{kt} = u_{kt}$ where u_{kt} is $MA(1)$.

However, Engle and Vahid (1993b) indicate that the presence of cofeature vectors such that the resulting linear combinations of Δx_t have an MA representation of lower order than that of Δx_t, implies the existence of non synchronous common cycles ($NSCC$) among the variables. Hence, average sampling determines a shift from common cycles to non synchronous common cycles of lag 1, $NSCC(1)$. More generally, if there are $NSCC(h)$, i.e., $\gamma'\Delta x_t = \varepsilon_t$ where ε_t is $MA(h)$, we can use the results in Table 1 with $g = 1, s = h$ in order to obtain the order of the $NSCC$ in x_k, h_k, both for point-in-time and for average sampling. Notice that, except in the case of average sampling with CC in x, $h_k \leq h$. Moreover, the presence of additional cofeature vectors in x_k can determine an increase in the number of $NSCC$ and/or a decrease in the otherwise expected h_k.

Example 6. Consider Example 1 in Vahid and Engle (1993a):

$$y_t + \delta z_t = \varepsilon_{yt}, \quad \varepsilon_{yt} = \varepsilon_{yt-1} + \eta_t,$$
$$y_t + \beta z_t = \varepsilon_{zt}, \quad \varepsilon_{zt} = \rho\varepsilon_{zt-1} + \xi_t, \quad |\rho| < 1,$$

which is a bivariate system with one cointegration vector, $(1, \beta)$, and one cofeature vector, $(1, \delta)$. Assume that x_2 is obtained by point-in-time sampling from x. It follows that

$$y_t + \delta z_t = \varepsilon_{2yt}, \quad \varepsilon_{2yt} = \varepsilon_{2yt-1} + \eta_{2t},$$
$$y_t + \beta z_t = \varepsilon_{2zt}, \quad \varepsilon_{2zt} = \rho^2 \varepsilon_{2zt-1} + \xi_{2t},$$

where η_{2t} and ξ_{2t} are white noise, and therefore there are still one cointegration and one cofeature vectors which are also equal to those for x. If instead average sampling is applied, η_{2t} and ξ_{2t} become $MA(1)$ so that there is again one cointegration vector but the same cofeature vector is associated to a $NSCC(1)$. □

When there are some negative or complex unit roots in the autoregressive component of the DGP there cannot be common cycles among the variables, at least according to their current definition which is stated only for stationary or $I(1)$ variables. It follows that if the variables become $I(1)$ as a consequence of temporal aggregation, the number of common cycles can increase, e.g., it is equal to one in Example 5. Thus, in summary, the total number of cofeature vectors should not decrease with temporal aggregation and, in general, $h_k \leq h$.

To conclude, we notice that the impulse response functions (IRF) of the temporally aggregated process are in general different from those of the original process, even if they have to satisfy the singularity restrictions which are imposed by the transmission of common trends and common cycles. Thus, in the interpretation of the IRF, and in particular when a structural interpretation is required, see, e.g., Giannini (1992), their non complete invariance to temporal aggregation should be kept into account.

5 Continuous time DGP

In this Section we deal with the effects of temporal aggregation on unit roots when x_k is obtained by either point-in-time or average sampling from either the exact discretization or continuous averaging, x, of a continuous time $VAR(g)$ process, X. The first out of the four cases, namely, point in time sampling from the exact discretization of X, is dealt with in an interesting paper by Comte (1994), which we rely upon and refer to for further details.

Following Comte, we write the DGP of X as

$$dD^{g-1}X_t = (\Phi_0 X_t + \Phi_1 DX_t + ... + \Phi_{g-1} D^{g-1} X_t)dt + \omega dW \tag{28}$$

where Φ_i and ω are $n \times n$ matrices, DX is the derivative of X in the mean square sense and W is an n-dimensional Brownian Motion. The DGP of x_k is then $VARMA(g, g-1)$:

$$x_{t+gk} = F_{k1}x_{t+(g-1)k} + F_{k2}x_{t+(g-2)k} + ... + F_{kg}x_t + \varepsilon_{kt}. \tag{29}$$

The $ng \times ng$ companion form matrices which are associated to (28) and (29) are

$$\bar{\Phi} = \begin{pmatrix} 0 & I & 0 & ... & 0 \\ 0 & 0 & I & ... & 0 \\ ... & & & & \\ 0 & 0 & 0 & ... & I \\ \Phi_0 & \Phi_1 & \Phi_2 & ... & \Phi_{g-1} \end{pmatrix}; \quad \bar{F}_k = \begin{pmatrix} F_{k1} & F_{k2} & ... & F_{kg-1} & F_{kg} \\ I & 0 & ... & 0 & 0 \\ 0 & I & ... & 0 & 0 \\ ... & & & & \\ 0 & 0 & ... & I & 0 \end{pmatrix}$$

and Comte proves that they are linked by the relationship

$$\bar{F}_k = \bar{T}_k\, e^{\bar{\Phi}k}\, \bar{T}_k^{-1}, \quad \bar{T}_k = ([e^{(g-i)\bar{\Phi}k}]_{1j})_{1 \leq i,j \leq g}. \tag{30}$$

He then shows that X are integrated and cointegrated when Φ_0 has reduced rank. Moreover, when $\bar{\Phi}$ is diagonalizable and has only negative real or zero eigenvalues (where the latter correspond to those in Φ_0),

$$p_c \leq p = p_k \tag{31}$$

where p_c is the number of continuous time cointegration relationships, and p and p_k are the numbers of discrete time cointegration relations in x and x_k.

The first part of (31) seems to be in contrast both with Phillips (1991), who would suggest $p_c = p$, and with our findings for discrete time, in the sense that temporal aggregation might create additional cointegration relationships among the variables. However,

Proposition 7. Under the assumptions in Comte (1994), $p_c = p$. □

Moreover, we have demonstrated that $p = p_k$ even if x is not the exact discretization of a continuous process. Hence, under the proper assumptions on the continuous time DGP of X, both x and x_k have the same properties of integration and cointegration of the original variables. This result is valid for average sampling from x as well, because we have seen that average sampling from a discrete time DGP determines the same AR component as point-in-time sampling.

If we now assume that

$$x_t = \int_0^1 X(t+1-\tau)d\tau,$$

the DGP of x becomes $VARMA(g, g)$ but its AR component is the same as in (29) with $k = 1$ (see, e.g., Christiano and Eichenbaum (1987)). Therefore, the AR component in the DGP of x_k is equal to that in (29), and the results on the transmission of unit roots that we have reported for point-in-time and average sampling from the exact discretized process are valid also for this case.

Finally, under the assumptions that we have made so far,

Proposition 8. Spurious cointegration at frequency zero cannot arise from temporal aggregation of a continuous time $VARIMA$ process. □

6 An empirical example

We now discuss a simple empirical example in order to illustrate some of the theoretical results that we have derived so far. The original data are monthly observations on a long term and a short term interest rate for Canada, *canli* and *cansi*, respectively, and the sample period is 1961:1-1993:11 so that there are 395 available observations.[7] They are graphed in Figure 1. We first analyse these data under the assumption that their generating process can be approximated by means of a $VAR(g)$, where g is chosen according to the result of a recursive $F-$test for the significance in both equations of the g^{th} lag of at least one of the two variables, starting with $g = 24$. Then, we study what are the effects of different temporal aggregation schemes on some of the main characteristics of interest, such as the presence of common trends and common cycles. The main references for the tests which are employed in this Section are Johansen (1988,1991,1992a) for cointegration, and Vahid and Engle (1993a, 1993b), and Engle and Kozicky (1993) for

[7] *Canli* and *cansi* are, respectively, the 10 years Government bond yield and the 90 day deposit rate. The series have been taken from the $OECD$ database through TSM (codes: *7mh2camn* and *7mc2camn*). Notice that these data are likely to be themselves temporally aggregated.

cofeatures. All the calculations were performed by means of PCGIVE 8.0 and PCFIML 8.0 (see Doornik and Hendry (1994a, 1994b)).

[Figure 1 about here]

The model appears to have no negative or complex unit roots, so that the theoretical references will be for this case. From Table 2a, it is evident that there is one cointegration vector which indicates a one to one correspondence between *canli* and *cansi*, even if the former tends to be higher than the latter because of the existence of a risk premium. Moreover, *canli* is weakly exogenous for the parameters of the cointegration vector. Instead, the presence of common cycles among the two interest rates, i.e., of a linear combination of their first differences which is $MA(0)$, is rejected. The possibility of non-synchronous common cycles is also rejected (only the result for $NSCC(1)$ is reported in Table 2b).

[Tables 2a, 2b about here]

Quarterly data can be obtained by point-in-time, average, or mixed sampling from the original observations, and the suffices *qp*, *qa* and *qm* are added to *canli* and *cansi* to identify the temporally aggregated variables. In the case of average sampling, we consider a three period average of the observations, while to construct the mixed sampling quarterly sample *canliqp* and *cansiqa* are employed. The resulting variables, for which 131 observations are available (1961:1-1993:3), are graphed in Figure 2.

[Figure 2 about here]

In all cases there is still one cointegration vector, and the hypothesis that its coefficients are equal to those for the monthly series is always accepted, see Tables 3a, 4a, and 5a. Also weak exogeneity of *canli* seems still valid. Instead, some non-synchronous common cycles are now detected, as Tables 3b,4b, and 5b highlight.

[Tables 3a, 3b, 4a, 4b, 5a, 5b about here]

The results that we have obtained so far are in a substantial agreement with the theoretical findings, because the cointegration rank and vectors do not change and the number of cofeature vectors does not decrease. One can wonder whether these conclusions remain unchanged in the empirical example by further aggregating the data. In order to still have a large enough number of observations, we move to halfyearly data which, in accordance to the former notation, are labeled by adding the suffices *hp*, *ha*, and *hm* to *canli* and *cansi*. Therefore, this time, in the case of point-in-time sampling only one every six observations is retained, for average sampling a six period average is considered, and the mixed sampling halfyearly sample is formed by *canlihp* and *cansiha*. There are 65 observations for each variable (1961:1-1993:1), and these are graphed in Figure 3.

[Figure 3 about here]

Tables 6a, 7a, and 8a show that there is still one cointegration vector and the hypothesis of equality to that for monthly variables cannot be rejected. However, *canli* is no longer weakly exogenous for the coefficients of this cointegration vector, i.e., we are in a situation similar to that in Example 2. Also the number of cofeature vectors does not decrease, even if there is a change in the order of the obtainable MA linear combination, which is in agreement with the theoretical relationship $h_k \leq h$, see Tables 6b, 7b, and 8b.[8]

[Tables 6a, 6b, 7a, 7b, 8a, 8b about here]

In conclusion, the empirical example is in agreement with and illustrates many of the theoretical suggestions. However, it can be worthwhile stressing again that with a small sample size or when there are heavy misspecifications in the model, the empirical results could differ from the theoretical ones. For example, if only 8 lags were considered in the analysis of the original monthly data, no cointegration would be found and hence it would seem that temporal aggregation has increased the cointegration rank. As a consequence, when the original and aggregated sample sizes are large enough and the models for the aggregated variables appear to be correctly

[8]Notice that only one lag is chosen to model *canliha* and *cansiha* and the presence of one cointegration vector implies the presence of one common cycle, and the coefficients of the cofeature vector are orthogonal to the loadings, α.

specified, the validity of the theoretical predictions can be also seen as an additional diagnostic check on the model for the original variables.[9]

7 Conclusions

The adoption of temporally aggregated data in empirical analysis renders it important to study the invariance to aggregation of particular characteristics of the variables. Given that it has become quite common to look for long run relationships among economic variables by means of cointegration analysis, and the possibility of studying short run relationships by means of cofeature analysis is also receiving increasing attention, we have focused on the "transmission" of common trends and common cycles through temporal aggregation. The main results are that, if only positive unit roots are allowed for, the cointegration rank is invariant to temporal aggregation. Instead, when the possibility of negative or complex unit roots in the AR component of the DGP of the variables is considered, temporal aggregation can, spuriously, increase the number of long run cointegration relationships. It also determines, in general, changes in the frequency of common cycles, and the number of cofeature vectors is expected to increase but not decrease.

Invariance of the cointegration rank is also associated with invariance of the cointegration vectors, while there are changes in the loadings of the cointegration relationships and, more generally, in the dynamic structure of the system and in the MA component. This can lead to modifications in

[9] Actually, there are some signals of misspecification also in the models that we have adopted. In particular, the residuals seem to be non normal and, maybe, heteroskedastic. Transforming the data with the logarithmic operator and including step dummies for the periods 80:6-82:6, 88:7-90:12 and impulse dummies for 71:1, 81:5, 83:4, improves the outcome of the diagnostic tests, which, though, still reject the null hypothesis of normality and heteroskedasticity. Moreover, in this case the hypothesis of joint stationarity of the original monthly variables is accepted, and other specifications which differ mainly for the number and timing of the dummies lead in general to the same conclusion. But the results of the tests for cointegration and for the significance of the dummies should be compared with proper critical values, which are not available, see, e.g., Perron (1989) and Banerjee et al. (1992) for the univariate case. However, even if the specification of a fully congruent model for the series under examination is difficult, what is more interesting for our discussion is that the conclusions of the tests for cointegration are still in general invariant to temporal aggregation. Thus, the validity of the theoretical results on the transmission of common trends and cycles can be a powerful necessary condition for correct specification even if, as the other criteria, it is not suffiicient.

the patterns of Granger non causality and in the weak, strong and super exogeneity status of some variables. The impulse response functions of the temporally aggregated variables are also expected to be rather different from the original ones. Thus, these particular characteristics of the original DGP should not be directly tested with temporally aggregated data, while long run restrictions in general can.

An alternative possibility is the derivation of the theoretical aggregated generating model, given the original DGP and a particular aggregation scheme. If it is compatible with the available aggregated data, then the original model is corroborated.

As an example, the relationship between a long term and a short term interest rate is analysed for different temporal aggregation schemes, and the empirical results are in agreement with the theoretical findings. This could not be true when the model which is adopted to represent the variables is heavily misspecified, or when there are problems of inaccuracy in the testing procedures because of the small number of available observations.

Another possible explanation for a mismatch between empirical and theoretical results could be that the initial assumptions are not correct. For example, the original variables could be generated by a non linear model or the aggregated variables could have been obtained by means of a different temporal aggregation scheme, e.g., average sampling with time varying weights. An extension of the contents of this paper in these directions seems to be an interesting subject of future research.

Appendix

Proof of Proposition 3.

We have $|C(Z)| = |C(L^k)| = |B(L)G(L)| = |B(L)||G(L)|$. Thus, if we consider $C(L^k)$ as a function of L, $|C(L^k)| = 0$ has gkn roots which are equal to the gn roots of $|G(L)| = 0$ plus the $gn(k-1)$ roots of $|B(L)| = 0$. But if we consider $C(L^k)$ has a function of $L^k = Z$, then $|C(Z)| = 0$ has only gn roots and $|C(\lambda_j^k)| = 0$ for $j = 1, ..., gn$ implies that these roots are just λ_j^k, $j = 1, ..., gn$.

If we consider again $C(L^k)$ as a function of L, it also follows that $\prod_{j=1}^{gn}(\lambda_j^k - L^k) = 0$. This admits the decomposition

$$\prod_{j=1}^{gn}(\lambda_j^k - L^k) = \prod_{j=1}^{gn}(\sum_{i=0}^{k-1}\lambda_j^{k-1-i}L^i)\prod_{j=1}^{gn}(\lambda_j - L) = 0,$$

so that the $gn(k-1)$ roots of $|B(L)| = 0$ have to satisfy the equation $\prod_{j=1}^{gn}(\sum_{i=0}^{k-1}\lambda_j^{k-1-i}L^i) = 0$.

$c^*(Z)$ is equal to $b(L)|G(L)|$, where $b(L)$ is such that the coefficients of the lags which are not multiple of k in $b(L)|G(L)|$ are zero. Hence, reasoning as before, the roots of $c^*(Z)$ roots are $Z_j = \lambda_j^k$, $j = 1, ..., gn$, and those of $b(L)$ satisfy the equation $\prod_{j=1}^{gn}(\sum_{i=0}^{k-1}\lambda_j^{k-1-i}L^i) = 0$. □

Proof of Proposition 4.

If $B(1)$ is invertible, $\Pi_k = B(1)\Pi$ implies

$$p = r(\Pi) \leq min[r(B^{-1}(1), r(\Pi_k)] = r(\Pi_k) = p_k.$$

But it is also $p \geq p_k$, so that it must be $p_k = p$. Hence, we aim at showing that $B(1)$ is invertible, i.e., that $|B(1)| \neq 0$. From Proposition 3, which is still valid because the hypothesis of stationarity is not required in its proof, we know that the roots of $|B(L)| = 0$ satisfy the equation $\prod_{j=1}^{gn}(\sum_{i=0}^{k-1}\lambda_j^{k-1-i}L^i) = 0$, so that for $B(1)$ not to be invertible we should have $\sum_{i=0}^{k-1}\lambda_j^{k-1-i} = 0$ for some j, which is not possible when $|\lambda_j| > 1$ or $\lambda_j = 1$ for all j. □

Proof of Proposition 5.

To start with, it is useful to consider the equivalent restricted stationary *VAR* representation of the *ECM* in (11) which has been proposed by Mellander *et al.* (1992),

$$A(L)y_t = \eta_t, \tag{32}$$

with $A(L) = M(\Gamma(L)M^{-1}D(L) + \alpha^* L)$, $y_t = F(L)Mx_t$, $M = (\beta_\perp \quad \beta)'$, $\alpha^* = (0 \quad \alpha)$, $D(L) = \begin{pmatrix} I_{n-p} & 0 \\ 0 & (1-L)I_p \end{pmatrix}$, $F(L) = \begin{pmatrix} (1-L)I_{n-p} & 0 \\ 0 & I_p \end{pmatrix}$. Moreover, $G(L) = M^{-1}A(L)F(L)M$, $\eta_t = MS(L)\varepsilon_t$, and $p = rank\Pi = rankG(1) = rankF(1)$.

We define $z_t = Mx_t$, where the number of cointegration vectors for z_t is equal to that for x_t being M non singular, and we have:

$$A(L)y_t = A(L)F(L)Mx_t = A(L)F(L)z_t = \eta_t,$$

or

$$|A(L)|F(L)z_t = a(L)F(L)z_t = A^a(L)\eta_t. \tag{33}$$

Now, from Proposition 2, the DGP of z_{kt} can be obtained by pre-multiplication of both sides of (33) by $b(L)F_k(L)$, where $b(L)$ is a scalar polynom such that all the terms which are not the coefficients of a multiple of L^k in $b(L)a(L) = d(L)$ are equal to zero, and

$$F_k(L) = \begin{pmatrix} (1 + L + ... + L^{k-1})I_{n-p} & 0 \\ 0 & I_p \end{pmatrix}.$$

It follows that the aggregate AR component is $d(Z)F(Z)$.

Thus, being $d(1) \neq 0$ from Proposition 3, the number of cointegration vectors in z_{kt} and $x_{kt} = M^{-1}z_{kt}$ is equal to the rank of $F(1)$, which is equal to the number of cointegration vectors in z_t and in x_t. □

Proof of Proposition 6.

From (25), if we define $z_t = \gamma' x_t$, we have $z_t = z_{t-1} + \varepsilon_t$. Thus,

$$z_{kt} = z_{kt-1} + \varepsilon_{kt}$$

and s_k is at least equal to s because $\gamma' \Delta x_{kt}$ is a random walk.

Moreover, from (23), $\gamma^* U(1)H(1) = \gamma^*$ does not necessarily imply $\gamma^* U(1)B(1) = \gamma^* T(1) = \gamma^*$, and therefore $s_k \geq s$. □

Proof of Proposition 7.

We simply continue the proof of Proposition 7 in Comte (1994), which is rather long and is not reported to save space. He arrives at showing, in our notation, that $p = n - r(S(1))$, where

$$S(1) = RDR^{-1}$$

and D is an $ng \times ng$ diagonal matrix whose diagonal has the first $n - p_c$ elements equal to one and the last $ng - (n - p_c)$ elements equal to zero. Thus, he notices that

$$r(S(1)) \leq min[r(R), r(D)] = r(D) = n - p_c$$

and therefore

$$p \geq p_c.$$

However, by properly partitioning R and D, we have

$$RD = \begin{pmatrix} R_1 & R_2 \\ R_3 & R_4 \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} R_1 & 0 \\ R_3 & 0 \end{pmatrix}$$

and $r(RD) = n - p_c$ because all the columns of R are linearly independent. Then,

$$r(S(1)R) \leq min[r(S(1)), r(R)] = r(S(1)).$$

Finally,

$$r(S(1)) \geq r(S(1)R) = r(RD) = n - p_c,$$

but $r(S(1) \leq n - p_c$ and therefore $r(S(1)) = n - p_c$ and $p_c = p$. □

Proof of Proposition 8.

Phillips (1991) has provided a proof of this result in the frequency domain. We propose an explanation in the time domain, which also highlights the relationship between the roots of the continuous and discrete time AR components. To this aim, notice that the roots of $|F_k(Z)| = |I - F_{k1}Z - F_{k2}Z^2 - ... - F_{kg}Z^g| = 0$ are equal to the reciprocal of the eigenvalues of $\bar{F}_k$. Equation (30) implies that $\bar{F}_k$ is similar to $e^{\bar{\Phi}k}$ and $e^{\bar{\Phi}k}$ is diagonalizable, in particular,

$$e^{\bar{\Phi}k} = (\sum_{i=0}^{\infty} \frac{\bar{\Phi}^i}{i!})^k = (\sum_{i=0}^{\infty} \frac{P\Lambda^i P^{-1}}{i!})^k = (P(\sum_{i=0}^{\infty} \frac{\Lambda^i}{i!})P^{-1})^k = Pe^{\Lambda k}P^{-1},$$

where Λ is a diagonal $ng \times ng$ matrix whose elements are the eigenvalues of $\bar{\Phi}$, which are all real negative or equal to zero by assumption. Thus, unit roots as solution of $|F_k(Z)| = 0$ are determined only from zero eigenvalues in $\bar{\Phi}$, and other eigenvalues are transformed into stationary roots. □

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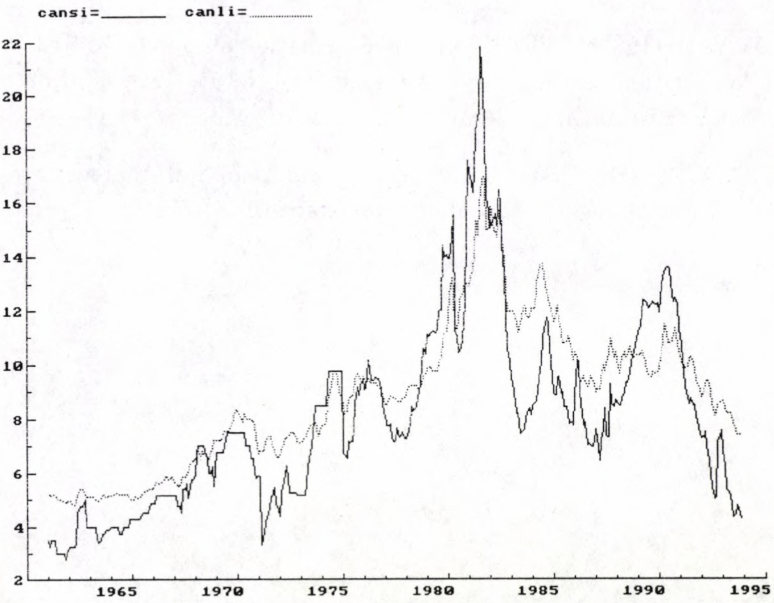

Figure 1: Graph of the original monthly data.

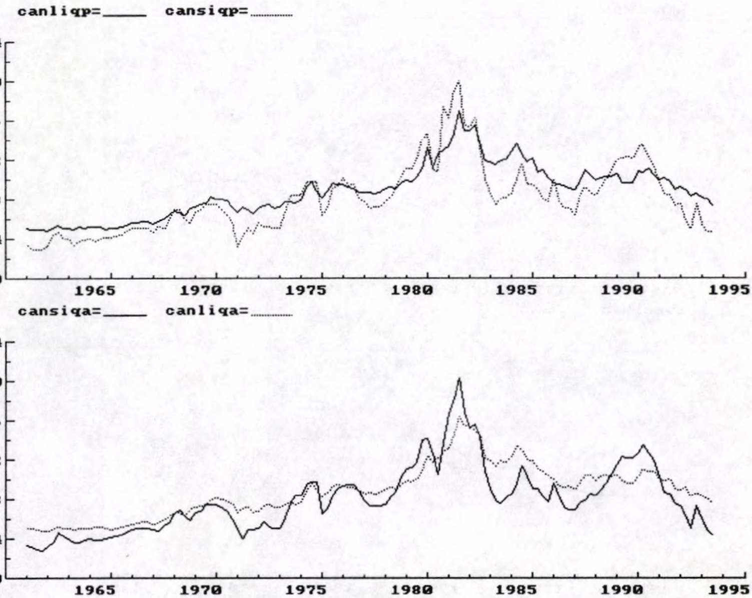

Figure 2: Graph of aggregated quarterly data.

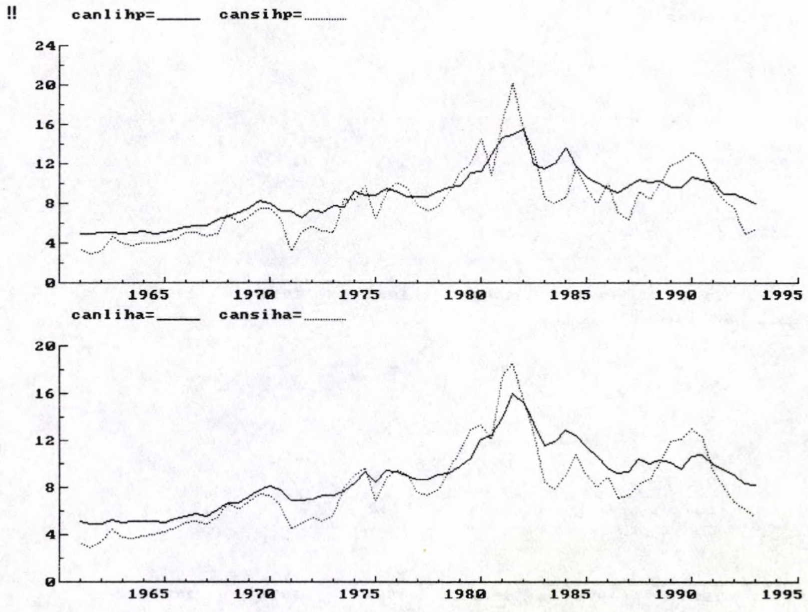

Figure 3: Graph of halfyearly aggregated data.

Table 1: DGP of x_k when x is VARMA(g,s) and B(L) exists.

Point-in-time sampling	Average sampling
$VARMA(g, g-1-q)$ for $qk < g-s \leq (q+1)k$ $q = 0, 1, ..., g-1$	$VARMA(g, g-q)$ for $qk < g-s+1 \leq (q+1)k$ $q = 0, 1, ..., g$
$VARMA(g, g)$ for $g = s$	$VARMA(g, g)$ for $g = s$
$VARMA(g, g+q)$ for $qk \leq s-g < (q+1)k$ $q = 0, 1, ...$	$VARMA(g, g+1+q)$ for and $qk \leq s-1-g < (q+1)k$ $q = 0, 1, ...$

Table 2a: Cointegration analysis

H_0	*test* :	$\lambda - \max$	95%	*Trace*	95%
$p = 0$		17.47	15.7	21.91	20.0
$p \leq 1$		4.44	9.2	4.44	9.2
cointegration relationship		$cansi - \beta_1 canli + \beta_2$	$\beta_1 = 1.05$ $\beta_2 = 1.322$		
LR test for $\beta_1 = 1, \beta_2 = 1.322$	$\chi^2(2)$	2.01[0.36]			
LR test for weak exogeneity	$\chi^2(1)$	0.97[0.32]			
number of lags	12				

Table 2b: Cofeature analysis

Test for common cycle	$\chi^2(22)$	95.105[0.00]
Test for $NSCC(1)$	$\chi^2(20)$	55.356[0.00]

Table 3a: Cointegration analysis

H_0	$test:$	$\lambda - \max$	95%	$Trace$	95%
$p = 0$		18.59	15.7	21.92	20.0
$p \leq 1$		3.321	9.2	3.321	9.2
cointegration relationship		$cansiqp - \beta_1 canliqp + \beta_2$	$\beta_1 = 1.063$ $\beta_2 = 1.427$		
LR test for $\beta_1 = 1, \beta_2 = 1.322$	$\chi^2(2)$	0.01[0.99]			
LR test for weak exogeneity	$\chi^2(1)$	0.89[0.34]			
number of lags	4				

Table 3b: Cofeature analysis

Test for common cycle	$\chi^2(6)$	15.81[0.015]
Test for $NSCC(1)$	$\chi^2(4)$	7.79[0.0996]
cofeature relationship	$canliqp - \gamma_1 cansiqp + \gamma_2$	$\gamma_1 = 0.25 \quad (s.e. = 0.10)$ $\gamma_2 = 0.02 \quad (s.e. = 0.04)$

Table 4a: Cointegration analysis

H_0	*test* :	$\lambda - \max$	95%	*Trace*	95%
$p = 0$		22.89	15.7	26.18	20.0
$p \leq 1$		3.289	9.2	3.289	9.2
cointegration relationship		$cansiqa - \beta_1 canliqa + \beta_2$	$\beta_1 = 1.082$ $\beta_2 = 1.619$		
LR test for $\beta_1 = 1, \beta_2 = 1.322$	$\chi^2(2)$	0.12[0.94]			
LR test for weak exogeneity	$\chi^2(1)$	2.91[0.08]			
number of lags	6				

Table 4b: Cofeature analysis

Test for common cycle	$\chi^2(10)$	23.74[0.008]
Test for $NSCC(1)$	$\chi^2(8)$	16.8[0.032]
Test for $NSCC(2)$	$\chi^2(6)$	8.21[0.22]
cofeature relationship	$canliqa - \gamma_1 cansiqa + \gamma_2$	$\gamma_1 = 0.14 \quad (s.e. = 0.13)$ $\gamma_2 = 0.02 \quad (s.e. = 0.04)$

Table 5a: Cointegration analysis

H_0	$test:$	$\lambda - \max$	95%	$Trace$	95%
$p = 0$		23.3	15.7	26.43	20.0
$p \leq 1$		3.13	9.2	3.13	9.2
cointegration relationship		$cansiqm - \beta_1 canliqm + \beta_2$	$\beta_1 = 1.086$ $\beta_2 = 1.632$		
LR test for $\beta_1 = 1, \beta_2 = 1.322$	$\chi^2(2)$	0.15[0.92]			
LR test for weak exogeneity	$\chi^2(1)$	2.65[0.10]			
number of lags	6				

Table 5b: Cofeature analysis

Test for common cycle	$\chi^2(10)$	20.03[0.029]
Test for $NSCC(1)$	$\chi^2(8)$	9.82[0.28]
cofeature relationship	$canliqm - \gamma_1 cansiqm + \gamma_2$	$\gamma_1 = 0.19 \quad (s.e. = 0.14)$ $\gamma_2 = 0.02 \quad (s.e. = 0.05)$

Table 6a: Cointegration analysis

H_0	$test:$	$\lambda-\max$	95%	$Trace$	95%
$p=0$		28.81	15.7	33.02	20.0
$p\leq 1$		4.21	9.2	4.21	9.2
cointegration relationship		$cansihp-\beta_1 canlihp+\beta_2$	$\beta_1=1.176$ $\beta_2=2.402$		
LR test for $\beta_1=1,\beta_2=1.322$	$\chi^2(2)$	2.21[0.33]			
LR test for weak exogeneity	$\chi^2(1)$	3.97[0.046]			
number of lags	4				

Table 6b: Cofeature analysis

Test for common cycle	$\chi^2(6)$	8.35[0.21]
cofeature relationship	$canlihp-\gamma_1 cansihp+\gamma_2$	$\gamma_1=-0.037 \quad (s.e.=0.075)$ $\gamma_2=0.05 \quad (s.e.=0.12)$

Table 7a: Cointegration analysis

H_0	*test* :	$\lambda - \max$	95%	*Trace*	95%
$p = 0$		17.97	15.7	21.84	20.0
$p \leq 1$		3.87	9.2	3.87	9.2
cointegration relationship		$cansiha - \beta_1 canliha + \beta_2$	$\beta_1 = 1.162$ $\beta_2 = 2.395$		
LR test for $\beta_1 = 1, \beta_2 = 1.322$	$\chi^2(2)$	0.52[0.77]			
LR test for weak exogeneity	$\chi^2(1)$	4.3107[0.0379]			
number of lags	1				

Table 7b: Cofeature analysis

cofeature relationship	$canliha - \gamma_1 cansiha + \gamma_2$	$\gamma_1 = -0.89$ $(s.e. = 1.33)$ $\gamma_2 = 0.07$ $(s.e. = 0.27)$

Table 8a: Cointegration analysis

H_0	$test:$	$\lambda - \max$	95%	$Trace$	95%
$p = 0$		30.02	15.7	34.19	20.0
$p \leq 1$		4.168	9.2	4.168	9.2
cointegration relationship		$cansihm - \beta_1 canlihm + \beta_2$	$\beta_1 = 1.147$ $\beta_2 = 2.148$		
LR test for $\beta_1 = 1, \beta_2 = 1.322$	$\chi^2(2)$	1.59[0.45]			
LR test for weak exogeneity	$\chi^2(1)$	5.029[0.025]			
number of lags	4				

Table 8b: Cofeature analysis

Test for common cycle	$\chi^2(6)$	9.32[0.15]
cofeature relationship	$canlihm - \gamma_1 cansihm + \gamma_2$	$\gamma_1 = -0.00$ $(s.e. = 0.11)$ $\gamma_2 = 0.05$ $(s.e. = 0.12)$

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