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Spatial Multiproduct Duopoly Pricing

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Spatial Multiproduct Duopoly Pricing

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Abstract

This paper studies the theoretical implications of location patterns in a spatial multiproduct duopoly model. It addresses the following question: To what extent do location patterns affect firms' pricing behaviour and market performance? I show that equilibrium prices are higher under a neighbouring location pattern. In other words, noncooperative Nash prices are higher when the organisational market structure is such that firms own strategically dependent outlets. In contrast, I demonstrate that an interlaced location pattern yields the most competitive Nash prices and the highest social welfare. Finally, I also show that from a social welfare point of view it is preferable to have three outlets rather than two. (JEL D43, L13)

Key Words: Spatial Model, Multiproduct Duopoly, Price Competition.

*I am indebted towards Louis Philips for detailed comments on the paper.

1 Introduction

The aim of this paper is to study the theoretical implications of location patterns in a spatial multiproduct duopoly model. Its goal is to address the following question: To what extent does a particular location pattern affect market power and social welfare in a spatial multiproduct duopoly? This question is addressed within the traditional framework developed by Hotelling's [1929] paper on spatial competition. The recent paper by Bensaid & de Palma [1994], first shows that different organisational structures may theoretically arise: A neighbouring location, an interlaced location and a mixed location pattern. In this work I show that a neighbouring location pattern yields less competitive prices both in the two-outlet and three-outlet cases. I demonstrate that for the two-outlet case, social welfare is not affected by location patterns while in the three-outlet case, social welfare is unambiguously higher under interlaced locations. I also show that from a social welfare point of view it is preferable to have three outlets (rather than two). In a related paper (Nero [1995a]) I show how these theoretical results can be tested using data on intra-European duopoly airline markets. The empirical results support the theoretical model since I find that fares are, *ceteris paribus*, higher on intra-European markets which exhibit a neighbouring location pattern in the time domain.

This paper is organised as follows. Section 2 presents the spatial multiproduct duopoly setting. It is assumed that locations are given and that consumers buy exactly one unit of the product (or service). The results of price competition are derived in Section 3 and Section 4, for the two-outlet and three-outlet cases, respectively. Finally, Section 5 concludes.

2 Price Competition and Location Choice in Multiproduct Duopoly

This section provides the general framework for the analysis of spatial multiproduct duopoly. The notation and terminology borrow heavily from Bensaïd & de Palma's [1994]. Firms and consumers are located on a circle with unit circumference ($L = 1$). In a nonspatial context, the length of the circle could be, e.g., 18 hours (from 6 am to 12 pm). Two firms, Firm *A* and Firm *B*, operate in the market, and entry by another potential firm is ruled out. Each multiproduct duopolist provides n ($n \geq 2$) outlets distributed on the circle with outlet locations denoted by x_k ($k = 1, \dots, 2n$) with $0 \leq x_1 \leq \dots \leq x_k \leq \dots \leq x_{2n} \leq 1$. Firm *A*'s (Firm *B*'s) marginal production cost are assumed to be constant and equal to c_A (c_B). The fixed cost per outlet and the sunk cost per firm are assumed to be zero. Duopolists set mill prices¹ p_k ($k = 1, \dots, 2n$) such that profits are maximised under noncooperative Nash behaviour. Products or services offered by the duopolists are homogenous except for the spatial dimension. In a nonspatial context, one can think of services differentiated in the time dimension. It is assumed that consumers are uniformly distributed around the circle and the population has mass D . They are identical apart from their location. Each consumer buys exactly one unit of the product (or service) so that total market demand is inelastic. It is assumed that consumers' reservation prices are sufficiently large so that consumers always buy the product. Each customer patronises the outlet for which the delivered price is the lowest. The delivered price is the sum of the mill price plus the utility loss incurred by not purchasing at the most preferred outlet. In a purely spatial context, the delivered price represents the sum of the f.o.b. mill price and the transportation costs incurred by the customer. It is assumed that the indirect utility function of a consumer located at x and purchasing good k is $v_k = y - p_k - \delta(x_k - x)^2$, where y represents the income of the consumer and $\delta > 0$ is the utility loss (transportation) rate. In other words, the farther a customer is located from its ideal outlet the larger

¹Accordingly, it is assumed that firms do not price discriminate across consumers.

the delivered price and utility loss. Clearly, each customer patronises the outlet where he obtains the maximal utility.

Let us assume that $x_{2n+1} = x_1$ and that $x_k < x_{k+1}$. The consumer who is indifferent between outlet k and $k + 1$ has a location z_k such that:

$$y - p_k - \delta(z_k - x_k)^2 = y - p_{k+1} - \delta(z_k - x_{k+1})^2. \quad (1)$$

Using (1), the location of the marginal consumer, $z_k(\cdot)$, is equal to

$$z_k = \frac{x_k + x_{k+1}}{2} + \frac{p_{k+1} - p_k}{2\delta(x_{k+1} - x_k)}, \quad k = 1, \dots, 2n. \quad (2)$$

By convention $z_0 = z_{2n} - 1$. Given (2) we can derive the market shares $S_k(\cdot)$ and therefore the demand for each outlet k . Clearly, the market shares will be a function of (a) the vector of locations $x = (x_1, \dots, x_{2n})$ and (b) the vector of prices $p = (p_1, \dots, p_{2n})$. The market shares $S_k(x, p)$ of outlet k is simply given by the following expression:

$$S_k(x, p) = z_k - z_{k-1}, \quad k = 1, \dots, 2n. \quad (3)$$

Assume that Firm A owns every outlet $k \in K_A$ with $K_A \cup K_B = \{1, \dots, 2n\}$. Given the assumptions on the fixed and sunk costs, Firm A 's profit is:

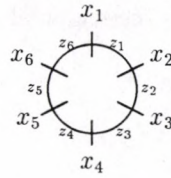
$$\Pi_A(x, p) = D\left(\sum_{k \in K_A} (p_k - c_A) S_k(x, p)\right). \quad (4)$$

Similarly, Firm B 's profit is:

$$\Pi_B(x, p) = D\left(\sum_{k \in K_B} (p_k - c_B) S_k(x, p)\right). \quad (5)$$

In order to solve the above multiproduct maximisation problem, two equilibrium concepts have been investigated in the literature: A **simultaneous** equilibrium where both firms choose product range-locations and prices simultaneously (Gabszewicz & Thisse [1986]) and a **two stage** equilibrium where both firms simultaneously choose product range-locations and then prices in a second stage (see e.g., Martinez-Giralt & Neven [1988]). The latter is called a **two stage subgame perfect Nash**

Figure 1: Symmetric Locations in the Three Outlet Case



equilibrium. Past research has shown that neither the simultaneous price-location equilibrium nor the two stage equilibrium exist within the above duopoly framework. In particular, Martinez-Giralt & Neven [1988] show that in the two stage duopoly model, firms always have an incentive to sell (or produce) a single (maximally differentiated) product at equilibrium. This is because, when a firm adds a second product, two contrasting effects are at work. On the one hand, the introduction of a second product allows the firm to increase its market share (market segmentation). On the other hand, it increases price competition. Martinez-Giralt & Neven [1988] show that the negative price effect outweighs the positive market share effect. Nero [1995b] recently shows that when duopolists face a ‘binding’ reservation price, i.e., when consumers have an elastic (downward sloping) demand and a finite and small reservation price, a multiproduct equilibrium may emerge as the result of a two stage game. This is because, contrary to the Martinez-Giralt & Neven’s [1988] result, the price effect turns out to be positive: The introduction of a second product allows duopolists to charge a higher mill price. Using a framework similar to Martinez-Giralt & Neven [1988], Bensaid & de Palma [1994] demonstrate that as soon as three or more firms compete in the market, multiproduct equilibria emerge. This occurs because a single product firm always has an incentive to introduce a second product when it faces two or more competing firms.

When duopolists are assumed to provide more than one outlet, several location patterns may theoretically arise. Using Bensaid & de Palma’s [1994] terminology, I define the following three different types of location equilibria: An **interlaced** outlet equilibrium, a **neighbouring** outlet equilibrium and a **mixed** outlet equilibrium. For example, let us assume that every firm owns $n = 3$ outlets, as illustrated in Figure 1. An interla-

ced outlet equilibrium occurs when outlets' identity (ownership) alternates. This arises when Firm A owns every outlet $k \in K_A = \{1, 3, 5\}$ and Firm B owns every outlet $k \in K_B = \{2, 4, 6\}$. In this type of location it is as if each firm wishes to offer a 'product line' as broad as possible. A neighbouring outlet pattern characterises an equilibrium with all the outlets owned by a firm located next to each other. Then Firm A owns every outlet $k \in K_A = \{1, 2, 3\}$ and Firm B owns every outlet $k \in K_B = \{4, 5, 6\}$. Here each firm wishes to specialise on a segment of the 'product line'. Finally, a mixed outlet pattern combines the interlaced and neighbouring equilibria. A mixed outlet equilibrium may occur, e.g. when Firm A owns every outlet $k \in K_A = \{1, 2, 5\}$ and Firm B owns every outlet $k \in K_B = \{3, 4, 6\}$.

In conclusion, when one departs from the Hotelling's [1929] original assumptions, multiproduct equilibria are likely to emerge in models of spatial competition. Besides oligopolistic competition (see Bensaid & de Palma [1994]) and elasticity of the demand (see Nero [1995b]), additional features such as capacity constraints or economies of scope may provide more insight into spatial multiproduct competition. Unfortunately, as stressed by Greenhut et al. [1987], 'one of the major problems in the analysis of spatial competition is that a slight increase in model complexity can generate an intractable increase in mathematical complexity'. This is particularly true of any attempt to investigate the interactions between price competition **and** location choices among multiproduct firms. As a result, the analysis in this paper is confined to price competition under multiproduct duopoly. This implies that product selection in the first stage of the game is assumed to be given. Firms do not choose their locations but rather are automatically located equidistant from one another on the circle². One can argue that the assumption of exogenously given locations is rather restrictive on the grounds that firms generally control both price and product selection (location) variables. However, for some (differentiated) industries, like air transportation, where the selection of a particular location can be interpreted as the offered departure time, it stands to reason that firms do not always control the location or

²Tirole [1988] refers to the auctioneer picking the symmetric location pattern.

schedule variable: Once the slots are allocated, they cannot be shifted (at least not without cost). This is like having an infinite sunk cost to change location. For industries where the schedule -and the frequency of service- is the main element of differentiation, I believe that this simplification is not unrealistic, at least in the short run. In the case of intra-European airline markets, for example, the choice of the offered departure time greatly depends on local airport authorities which allocate available slots (see the related Nero's [1995a] paper for further details on the airline industry). From a modelling point of view, this simplification allows us to derive useful results which can subsequently be tested in an econometric model.

In summary, the focus of this paper is to study the theoretical implications of different location patterns on pricing, market performance and social welfare. In the next two sections I derive the theoretical results for different location patterns. For the sake of the analysis, I focus on two important cases: The two-outlet case (Section 3) and the three-outlet case (Section 4). The results are summarised in Proposition 1 and Proposition 2. Although the analysis of the two cases is quite similar, I derive the results separately in order to provide an interesting comparison. The results of this comparison are summarised in Proposition 3.

3 Spatial Multiproduct Duopoly Pricing: The Two-Outlet Case

Proposition 1 *Consider two duopolists, each owning two outlets. Locations are exogenously given. Two different symmetric location patterns are analysed: (1) An interlaced outlet location and (2) a neighbouring outlet location. Given the location pattern and the assumptions of Section 2, the noncooperative Nash prices are larger in a neighbouring outlet equilibrium. As a result, market performance (profit) is larger with neighbouring outlets. Social welfare, however, is identical under both location patterns.*

3.1 The Interlaced Outlet Equilibrium

For the ease of notation let outlet locations and mill prices be given by x_k^i and p_k^i , respectively (with $i = 1, 2$ and $k = A, B$). Figure 2 (see Appendix) depicts the candidate (symmetric) equilibria for the interlaced outlet location. Without loss of generality, x_A^1 can be set to zero by choice of normalisation. Using (2), the locations of the marginal consumer, $z_l(\cdot)$ ($l = 1, \dots, 4$), are

$$z_1(\cdot) = \frac{1}{8} + \frac{2}{\delta}(p_B^1 - p_A^1), \quad (6)$$

$$z_2(\cdot) = \frac{3}{8} + \frac{2}{\delta}(p_A^2 - p_B^1), \quad (7)$$

$$z_3(\cdot) = \frac{5}{8} + \frac{2}{\delta}(p_B^2 - p_A^2), \quad (8)$$

$$z_4(\cdot) = \frac{7}{8} + \frac{2}{\delta}(p_A^1 - p_B^2). \quad (9)$$

Given (6)-(9) we derive the market shares $S_k^i(\cdot)$ for each outlet. Notice that under the assumption of unitary (inelastic) demand there is a one to one mapping between the market shares and the aggregate demand. Market shares, $S_k^i(\cdot)$, are

$$S_A^1(\cdot) = z_1 + 1 - z_4 = \frac{1}{4} - \frac{4}{\delta}p_A^1 + \frac{2}{\delta}(p_B^1 + p_B^2), \quad (10)$$

$$S_B^1(\cdot) = z_2 - z_1 = \frac{1}{4} - \frac{4}{\delta}p_B^1 + \frac{2}{\delta}(p_A^1 + p_A^2), \quad (11)$$

$$S_A^2(\cdot) = z_3 - z_2 = \frac{1}{4} - \frac{4}{\delta}p_A^2 + \frac{2}{\delta}(p_B^1 + p_B^2), \quad (12)$$

$$S_B^2(\cdot) = z_4 - z_3 = \frac{1}{4} - \frac{4}{\delta}p_B^2 + \frac{2}{\delta}(p_A^1 + p_A^2). \quad (13)$$

In this set-up, Firm A's and Firm B's profits are

$$\Pi_A^I(\cdot) = D\left(\sum_{i=1}^{i=2}(p_A^i - c_A)S_A^i(\cdot)\right), \quad (14)$$

and

$$\Pi_B^I(\cdot) = D\left(\sum_{i=1}^{i=2}(p_B^i - c_B)S_B^i(\cdot)\right), \quad (15)$$

respectively, where the subscript 'I' stands for interlaced locations. The noncooperative Nash prices are solutions of the system of first order

conditions given by³:

$$\frac{\partial \Pi_k^I(\cdot)}{\partial p_k^i} = 0 \quad i = 1, 2 \quad k = A, B. \quad (16)$$

The solution of the above system yields the following equilibrium prices:

$$p_A^{1*} = p_A^{2*} = \frac{1}{16}\delta + \frac{1}{3}(c_B + 2c_A), \quad (17)$$

$$p_B^{1*} = p_B^{2*} = \frac{1}{16}\delta + \frac{1}{3}(c_A + 2c_B). \quad (18)$$

The equilibrium prices are an increasing function of the marginal costs. Notice that the higher the transportation rate δ , the higher the Nash prices. The latter results from the combination of f.o.b. mill pricing and totally inelastic demand, so that consumers accept any price. Furthermore, note that when marginal costs are identical, $c_A = c_B = c$, the price-cost margin is equal to $\delta/16$. Plugging the Nash prices (17)-(18) into the market boundaries (6)-(9), we can evaluate the market shares (10)-(13):

$$z_1^* = \frac{1}{8} + \frac{2(c_B - c_A)}{3\delta}, \quad z_2^* = \frac{3}{8} + \frac{2(c_A - c_B)}{3\delta},$$

$$z_3^* = \frac{5}{8} + \frac{2(c_B - c_A)}{3\delta}, \quad z_4^* = \frac{7}{8} + \frac{2(c_A - c_B)}{3\delta}.$$

and

$$S_A^{i*} = \frac{1}{4} + \frac{4(c_B - c_A)}{3\delta}, \quad i = 1, 2$$

$$S_B^{i*} = \frac{1}{4} + \frac{4(c_A - c_B)}{3\delta} \quad i = 1, 2.$$

Finally, after the appropriate substitutions, the profit functions (14)-(15) are equal to

$$\Pi_A^I = \frac{D}{288\delta} [16(c_A - c_B) - 3\delta]^2,$$

$$\Pi_B^I = \frac{D}{288\delta} [16(c_A - c_B) + 3\delta]^2.$$

When marginal costs are equal, we have that $\Pi_A^I = \Pi_B^I = \frac{1}{32}D\delta$.

³The existence of equilibrium depends on the quasiconcavity of the profit functions (14)-(15) and hence on the concavity of the demand functions (10)-(13). Since these latter are linear in prices, the existence is guaranteed.

3.2 The Neighbouring Outlet Equilibrium

Figure 3 (see Appendix) depicts the candidate (symmetric) equilibria for the neighbouring outlet location. The main difference with the previous case is that now each firm has outlets that are no longer isolated from each other. In an interlaced equilibrium, the price in each outlet is the same as if each were operated by a single firm. Here, the equilibrium is likely to be different. Using (2), the locations of the marginal consumer, $z_l(\cdot)$ ($l = 1, \dots, 4$), are

$$z_1(\cdot) = \frac{1}{8} + \frac{2}{\delta}(p_A^2 - p_A^1), \quad (19)$$

$$z_2(\cdot) = \frac{3}{8} + \frac{2}{\delta}(p_B^1 - p_A^2), \quad (20)$$

$$z_3(\cdot) = \frac{5}{8} + \frac{2}{\delta}(p_B^2 - p_B^1), \quad (21)$$

$$z_4(\cdot) = \frac{7}{8} + \frac{2}{\delta}(p_A^1 - p_B^2). \quad (22)$$

Given (19)-(22) the market shares $S_k^i(\cdot)$ for each outlet are

$$S_A^1(\cdot) = z_1 + 1 - z_4 = \frac{1}{4} - \frac{4}{\delta}p_A^1 + \frac{2}{\delta}(p_A^2 + p_B^2), \quad (23)$$

$$S_A^2(\cdot) = z_2 - z_1 = \frac{1}{4} - \frac{4}{\delta}p_A^2 + \frac{2}{\delta}(p_A^1 + p_B^1), \quad (24)$$

$$S_B^1(\cdot) = z_3 - z_2 = \frac{1}{4} - \frac{4}{\delta}p_B^1 + \frac{2}{\delta}(p_A^2 + p_B^2), \quad (25)$$

$$S_B^2(\cdot) = z_4 - z_3 = \frac{1}{4} - \frac{4}{\delta}p_B^2 + \frac{2}{\delta}(p_A^1 + p_B^1). \quad (26)$$

Firm A 's and Firm B 's profits with two neighbouring outlets are

$$\Pi_A^N(\cdot) = D \left(\sum_{i=1}^{i=2} (p_A^i - c_A) S_A^i(\cdot) \right), \quad (27)$$

and

$$\Pi_B^N(\cdot) = D \left(\sum_{i=1}^{i=2} (p_B^i - c_B) S_B^i(\cdot) \right), \quad (28)$$

respectively, where the subscript ' N ' stands for neighbouring locations. The noncooperative Nash prices are solutions of the system of first order conditions given by:

$$\frac{\partial \Pi_k^N(\cdot)}{\partial p_k^i} = 0 \quad i = 1, 2 \quad k = A, B. \quad (29)$$

The solution of the above system yields the following equilibrium prices:

$$p_A^{1*} = p_A^{2*} = \frac{1}{8}\delta + \frac{1}{3}(c_B + 2c_A), \quad (30)$$

$$p_B^{1*} = p_B^{2*} = \frac{1}{8}\delta + \frac{1}{3}(c_A + 2c_B). \quad (31)$$

Plugging the Nash prices (30)-(31) into the market boundaries (19)-(22), we can evaluate the market shares (23)-(26):

$$z_1^* = \frac{1}{8}, z_2^* = \frac{3}{8} + \frac{2(c_B - c_A)}{3\delta},$$

$$z_3^* = \frac{5}{8}, z_4^* = \frac{7}{8} + \frac{2(c_A - c_B)}{3\delta}.$$

and

$$S_A^{i*} = \frac{1}{4} + \frac{2(c_B - c_A)}{3\delta}, \quad i = 1, 2$$

$$S_B^{i*} = \frac{1}{4} + \frac{2(c_A - c_B)}{3\delta} \quad i = 1, 2.$$

Finally, after the appropriate substitutions, the profit functions (27)-(28) are equal to

$$\Pi_A^N = \frac{D}{144\delta} [8(c_B - c_A) + 3\delta]^2,$$

$$\Pi_B^N = \frac{D}{144\delta} [8(c_A - c_B) + 3\delta]^2.$$

When marginal costs are equal, then $\Pi_A^N = \Pi_B^N = \frac{1}{16}D\delta$.

3.3 Comparison of the Various Equilibrium Patterns in the Two-Outlet Case

3.3.1 Equilibrium Price Comparison

Table I (see page 11) summarises the results obtained in Section 3.1 and Section 3.2. For the sake of comparison, all values are expressed with a common denominator. It appears that the noncooperative Nash prices are larger in a neighbouring outlet equilibrium. In other words, the

interlaced outlet location yields more competitive Nash prices. This is a very intuitive result. Price competition is reduced as the number of Firm A's outlets which directly compete with Firm B's outlets decreases. Put differently, price competition is relaxed when firms own adjacent outlets.

Table I: **Equilibrium Price Comparison**

<i>Interlaced Equilibrium</i>	<i>Neighbouring Equilibrium</i>
$p_A^{1*} = p_A^{2*} = \frac{189}{3024} \delta + \frac{1}{3}(c_B + 2c_A)$	$p_A^{1*} = p_A^{2*} = \frac{378}{3024} \delta + \frac{1}{3}(c_B + 2c_A)$
$p_B^{1*} = p_B^{2*} = \frac{189}{3024} \delta + \frac{1}{3}(c_A + 2c_B)$	$p_B^{1*} = p_B^{2*} = \frac{378}{3024} \delta + \frac{1}{3}(c_A + 2c_B)$

3.3.2 Consumer Loss and Welfare

In this section I compare the two location patterns in terms of market performance ($\Pi_{A,B}^E$, $E = I, N$), consumer loss (CL^E) (f.o.b. mill price plus transport costs given the inelastic demand) and welfare loss (WL^E) (transport costs, i.e., disutility). For the sake of simplicity, let us assume that marginal costs are identical, i.e., $c_A = c_B = c$. Without loss of generality, let the marginal costs be zero.

Interlaced Outlet Location

Since all consumers pay the same f.o.b. mill price, $\frac{\delta}{16}$, total revenue is $\frac{D\delta}{16}$. Given the locations of the marginal consumers z_l^* ($l = 1, \dots, 4$) (see Section 3.1), transportation costs, T^I , are given by

$$T^I = D \left(8 \int_{s=0}^{s=1/8} [\delta s^2] ds \right). \tag{32}$$

Explicit evaluation of (32) yields $\frac{D\delta}{192}$, so that

$$CL^I = - \left(\frac{D\delta}{16} + \frac{D\delta}{192} \right) = - \left(\frac{13D\delta}{192} \right). \tag{33}$$

Neighbouring Outlet Location

Now all consumers pay the same f.o.b. mill price, $\frac{\delta}{8}$, and total revenue is $\frac{D\delta}{8}$. Given the locations of the marginal consumers z_l^* ($l = 1, \dots, 4$) (see Section 3.2), transportation costs, T^N , are

$$T^N = D \left(8 \int_{s=0}^{s=1/8} [\delta s^2] ds \right). \quad (34)$$

Notice that (34) is equal to (32), so that the transportation costs paid by consumers amount to $\frac{D\delta}{192}$. Finally,

$$CL^N = - \left(\frac{D\delta}{8} + \frac{D\delta}{192} \right) = - \left(\frac{25D\delta}{192} \right). \quad (35)$$

Table II (see page 12) summarises these results. For the sake of comparison, all values are expressed with a common denominator. Notice that when marginal costs are identical, each firm covers 50% of the market and industry profits are twice the profit of a single firm. The results of Table I and Table II provide a proof of Proposition 1.

Table II: Comparison of the Equilibrium Values

	<i>Interlaced Equilibrium</i>	<i>Neighbouring Equilibrium</i>
Industry Profit	$\frac{47628}{762048} D\delta$	$\frac{95256}{762048} D\delta$
Consumer Surplus (CL)	$-\frac{51597}{762048} D\delta$	$-\frac{99225}{762048} D\delta$
Welfare Loss (WL)	$-\frac{3969}{762048} D\delta$	$-\frac{3969}{762048} D\delta$

The aim of the next section is to provide a similar analysis for the three-outlet case.

4 Spatial Multiproduct Duopoly Pricing: The Three-Outlet Case

Proposition 2 *Consider two duopolists, each owning three outlets. Locations are exogenously given. Three different symmetric location patterns are analysed: (1) An interlaced outlet location, (2) a neighbouring outlet location, and (3) a mixed outlet location pattern. Given the location pattern and the assumptions of Section 2, the noncooperative Nash prices are larger in a neighbouring outlet equilibrium, ceteris paribus. The interlaced outlet equilibrium yields the more competitive Nash prices. Nash prices with the mixed outlet equilibrium are lower than with a neighbouring location but higher than with an interlaced location. Profits follow the same pattern, i.e., the highest profits are obtained under the neighbouring location whereas the lowest profits are obtained under the interlaced location. Finally, social welfare is higher under the interlaced location. Neighbouring location is the less socially desirable pattern.*

4.1 The Interlaced Outlet Equilibrium

Let outlet locations and mill prices be given by x_k^i and p_k^i , respectively with $i = 1, 2, 3$ and $k = A, B$. Figure 4 (see Appendix) depicts the candidate (symmetric) equilibria for the interlaced outlet location. Using (2), the locations of the marginal consumer, $z_l(\cdot)$ ($l = 1, \dots, 6$), are

$$z_1(\cdot) = \frac{1}{12} + \frac{3}{\delta}(p_B^1 - p_A^1), \quad (36)$$

$$z_2(\cdot) = \frac{3}{12} + \frac{3}{\delta}(p_A^2 - p_B^2), \quad (37)$$

$$z_3(\cdot) = \frac{5}{12} + \frac{3}{\delta}(p_B^2 - p_A^2), \quad (38)$$

$$z_4(\cdot) = \frac{7}{12} + \frac{3}{\delta}(p_A^3 - p_B^3), \quad (39)$$

$$z_5(\cdot) = \frac{9}{12} + \frac{3}{\delta}(p_B^3 - p_A^3), \quad (40)$$

$$z_6(\cdot) = \frac{11}{12} + \frac{3}{\delta}(p_A^1 - p_B^3). \quad (41)$$

Given (36)-(41) we derive the market shares $S_k^i(\cdot)$ for each outlet. Market shares, $S_k^i(\cdot)$, are

$$S_A^1(\cdot) = z_1 + 1 - z_6 = \frac{1}{6} - \frac{6}{\delta}p_A^1 + \frac{3}{\delta}(p_B^1 + p_B^3), \quad (42)$$

$$S_B^1(\cdot) = z_2 - z_1 = \frac{1}{6} - \frac{6}{\delta}p_B^1 + \frac{3}{\delta}(p_A^1 + p_A^2), \quad (43)$$

$$S_A^2(\cdot) = z_3 - z_2 = \frac{1}{6} - \frac{6}{\delta}p_A^2 + \frac{3}{\delta}(p_B^1 + p_B^2), \quad (44)$$

$$S_B^2(\cdot) = z_4 - z_3 = \frac{1}{6} - \frac{6}{\delta}p_B^2 + \frac{3}{\delta}(p_A^2 + p_A^3), \quad (45)$$

$$S_A^3(\cdot) = z_5 - z_4 = \frac{1}{6} - \frac{6}{\delta}p_A^3 + \frac{3}{\delta}(p_B^2 + p_B^3), \quad (46)$$

$$S_B^3(\cdot) = z_6 - z_5 = \frac{1}{6} - \frac{6}{\delta}p_B^3 + \frac{3}{\delta}(p_A^1 + p_A^3). \quad (47)$$

Firm A 's and Firm B 's profits can be expressed as

$$\Pi_A^I(\cdot) = D \left(\sum_{i=1}^{i=3} (p_A^i - c_A) S_A^i(\cdot) \right), \quad (48)$$

and

$$\Pi_B^I(\cdot) = D \left(\sum_{i=1}^{i=3} (p_B^i - c_B) S_B^i(\cdot) \right), \quad (49)$$

respectively, where the subscript 'I' stands for interlaced locations. The noncooperative Nash prices are solutions of the system of first order conditions

$$\frac{\partial \Pi_k^I(\cdot)}{\partial p_k^i} = 0 \quad i = 1, 2, 3 \quad k = A, B. \quad (50)$$

and equal to

$$p_A^{1*} = p_A^{2*} = p_A^{3*} = \frac{1}{36}\delta + \frac{1}{3}(c_B + 2c_A), \quad (51)$$

$$p_B^{1*} = p_B^{2*} = p_B^{3*} = \frac{1}{36}\delta + \frac{1}{3}(c_A + 2c_B). \quad (52)$$

Equilibrium prices are again an increasing function of both the marginal costs and the transportation rate δ . Notice that when marginal costs are identical, $c_A = c_B = c$, the price-cost margin is $\delta/36$. This is the equilibrium price that would result if each outlet were operated by a single

firm. Plugging the Nash prices (51)-(52) into the market boundaries (36)-(41), we can evaluate the market shares (42)-(47):

$$z_1^* = \frac{1}{12} + \frac{c_B - c_A}{\delta}, z_2^* = \frac{3}{12} + \frac{c_A - c_B}{\delta}, z_3^* = \frac{5}{12} + \frac{c_B - c_A}{\delta},$$

$$z_4^* = \frac{7}{12} + \frac{c_A - c_B}{\delta}, z_5^* = \frac{9}{12} + \frac{c_B - c_A}{\delta}, z_6^* = \frac{11}{12} + \frac{c_A - c_B}{\delta},$$

and

$$S_A^{i*} = \frac{1}{6} + \frac{2(c_B - c_A)}{\delta}, \quad i = 1, 2, 3$$

$$S_B^{i*} = \frac{1}{6} + \frac{2(c_A - c_B)}{\delta} \quad i = 1, 2, 3.$$

Finally, the profit functions (48)-(49) are equal to

$$\Pi_A^I = \frac{D}{72\delta} [12(c_B - c_A) + \delta]^2,$$

$$\Pi_B^I = \frac{D}{72\delta} [12(c_B - c_A) - \delta]^2.$$

When marginal costs are equal, we have $\Pi_A^I = \Pi_B^I = \frac{1}{72} D\delta$.

4.2 The Neighbouring Outlet Equilibrium

Figure 5 (see Appendix) depicts the candidate (symmetric) equilibria for the neighbouring outlet location. From Figure 5 it appears that the outlet located at $x_A^2(x_B^2)$ is fully isolated from Firm B's (Firm A's) outlets. Using (2), the locations of the marginal consumer, $z_l(\cdot)$ ($l = 1, \dots, 6$), are

$$z_1(\cdot) = \frac{1}{12} + \frac{3}{\delta}(p_A^2 - p_A^1), \quad (53)$$

$$z_2(\cdot) = \frac{3}{12} + \frac{3}{\delta}(p_A^3 - p_A^2), \quad (54)$$

$$z_3(\cdot) = \frac{5}{12} + \frac{3}{\delta}(p_B^1 - p_B^3), \quad (55)$$

$$z_4(\cdot) = \frac{7}{12} + \frac{3}{\delta}(p_B^2 - p_B^1), \quad (56)$$

$$z_5(\cdot) = \frac{9}{12} + \frac{3}{\delta}(p_B^3 - p_B^2), \quad (57)$$

$$z_6(\cdot) = \frac{11}{12} + \frac{3}{\delta}(p_A^1 - p_B^3). \quad (58)$$

Given (53)-(58), the market shares $S_k^i(\cdot)$ for each outlet are

$$S_A^1(\cdot) = z_1 + 1 - z_6 = \frac{1}{6} - \frac{6}{\delta}p_A^1 + \frac{3}{\delta}(p_A^2 + p_B^3), \quad (59)$$

$$S_A^2(\cdot) = z_2 - z_1 = \frac{1}{6} - \frac{6}{\delta}p_A^2 + \frac{3}{\delta}(p_A^1 + p_A^3), \quad (60)$$

$$S_A^3(\cdot) = z_3 - z_2 = \frac{1}{6} - \frac{6}{\delta}p_A^3 + \frac{3}{\delta}(p_B^1 + p_A^2), \quad (61)$$

$$S_B^1(\cdot) = z_4 - z_3 = \frac{1}{6} - \frac{6}{\delta}p_B^1 + \frac{3}{\delta}(p_B^2 + p_A^3), \quad (62)$$

$$S_B^2(\cdot) = z_5 - z_4 = \frac{1}{6} - \frac{6}{\delta}p_B^2 + \frac{3}{\delta}(p_B^1 + p_B^3), \quad (63)$$

$$S_B^3(\cdot) = z_6 - z_5 = \frac{1}{6} - \frac{6}{\delta}p_B^3 + \frac{3}{\delta}(p_A^1 + p_B^2). \quad (64)$$

As before, Firm *A* and Firm *B*'s problem reduces to maximising

$$\Pi_A^N(\cdot) = D\left(\sum_{i=1}^{i=3}(p_A^i - c_A)S_A^i(\cdot)\right), \quad (65)$$

and

$$\Pi_B^N(\cdot) = D\left(\sum_{i=1}^{i=3}(p_B^i - c_B)S_B^i(\cdot)\right), \quad (66)$$

respectively, where the subscript 'N' stands for neighbouring locations. The noncooperative Nash prices are solutions of the system

$$\frac{\partial \Pi_k^N(\cdot)}{\partial p_k^i} = 0 \quad i = 1, 2, 3 \quad k = A, B. \quad (67)$$

and equal to

$$p_A^{1*} = p_A^{3*} = \frac{6}{72}\delta + \frac{1}{3}(c_B + 2c_A), \quad p_A^{2*} = \frac{7}{72}\delta + \frac{1}{3}(c_B + 2c_A), \quad (68)$$

and

$$p_B^{1*} = p_B^{3*} = \frac{6}{72}\delta + \frac{1}{3}(c_A + 2c_B), \quad p_B^{2*} = \frac{7}{72}\delta + \frac{1}{3}(c_A + 2c_B). \quad (69)$$

Notice that $p_A^{2*} > p_A^{1*} = p_A^{3*}$, and $p_B^{2*} > p_B^{1*} = p_B^{3*}$. Firm *A*'s (Firm *B*'s) market power is higher at location x_A^2 (x_B^2) since, at that location, there is a reduction in the drive to compete for the marginal consumers located

at z_1 and z_2 (z_4 and z_5). Plugging the Nash prices (68)-(69) into the market boundaries (53)-(58), we obtain

$$\begin{aligned} z_1^* &= \frac{3}{24}, z_2^* = \frac{5}{24}, z_3^* = \frac{10\delta + 24(c_B - c_A)}{24\delta}, \\ z_4^* &= \frac{15}{24}, z_5^* = \frac{17}{24}, z_6^* = \frac{22\delta - 24(c_A - c_B)}{24\delta}, \end{aligned}$$

and

$$\begin{aligned} S_A^{1*} &= S_A^{3*} = \frac{5\delta + 24(c_B - c_A)}{24\delta}, S_A^{2*} = \frac{1}{12}, \\ S_B^{1*} &= S_B^{3*} = \frac{5\delta + 24(c_A - c_B)}{24\delta}, S_B^{2*} = \frac{1}{12}. \end{aligned}$$

The profit functions are

$$\begin{aligned} \Pi_A^N &= \frac{D}{864\delta} [288(c_B - c_A)(2c_B - 2c_A + \delta) + 37\delta^2], \\ \Pi_B^N &= \frac{D}{864\delta} [288(c_B - c_A)(2c_B - 2c_A - \delta) + 37\delta^2]. \end{aligned}$$

With equal marginal costs, $\Pi_A^N = \Pi_B^N = \frac{37}{864} D\delta$.

4.3 The Mixed Outlet Equilibrium

In this section, I investigate how the equilibrium prices are affected under the mixed outlet equilibrium. Let us focus on the mixed outlet equilibrium represented in Figure 6 (see Appendix). Each firm now owns two adjacent outlets and one interlaced outlet. The mixed outlet equilibrium is likely to be a combination of the previous two cases. The locations of the marginal consumer, $z_l(\cdot)$ ($l = 1, \dots, 6$), are

$$z_1(\cdot) = \frac{1}{12} + \frac{3}{\delta}(p_A^2 - p_A^1), \quad (70)$$

$$z_2(\cdot) = \frac{3}{12} + \frac{3}{\delta}(p_B^1 - p_B^2), \quad (71)$$

$$z_3(\cdot) = \frac{5}{12} + \frac{3}{\delta}(p_B^2 - p_B^1), \quad (72)$$

$$z_4(\cdot) = \frac{7}{12} + \frac{3}{\delta}(p_A^3 - p_B^2), \quad (73)$$

$$z_5(\cdot) = \frac{9}{12} + \frac{3}{\delta}(p_B^3 - p_A^3), \quad (74)$$

$$z_6(\cdot) = \frac{11}{12} + \frac{3}{\delta}(p_A^1 - p_B^3). \quad (75)$$

while the market shares, $S_k^i(\cdot)$, are

$$S_A^1(\cdot) = z_1 + 1 - z_6 = \frac{1}{6} - \frac{6}{\delta}p_A^1 + \frac{3}{\delta}(p_A^2 + p_B^3), \quad (76)$$

$$S_A^2(\cdot) = z_2 - z_1 = \frac{1}{6} - \frac{6}{\delta}p_A^2 + \frac{3}{\delta}(p_A^1 + p_B^1), \quad (77)$$

$$S_A^3(\cdot) = z_5 - z_4 = \frac{1}{6} - \frac{6}{\delta}p_A^3 + \frac{3}{\delta}(p_B^2 + p_B^3), \quad (78)$$

$$S_B^1(\cdot) = z_3 - z_2 = \frac{1}{6} - \frac{6}{\delta}p_B^1 + \frac{3}{\delta}(p_A^2 + p_B^2), \quad (79)$$

$$S_B^2(\cdot) = z_4 - z_3 = \frac{1}{6} - \frac{6}{\delta}p_B^2 + \frac{3}{\delta}(p_B^1 + p_A^3), \quad (80)$$

$$S_B^3(\cdot) = z_6 - z_5 = \frac{1}{6} - \frac{6}{\delta}p_B^3 + \frac{3}{\delta}(p_A^1 + p_A^3). \quad (81)$$

Firm *A* and Firm *B*'s problem is to maximise

$$\Pi_A^M(\cdot) = D\left(\sum_{i=1}^{i=3}(p_A^i - c_A)S_A^i(\cdot)\right), \quad (82)$$

and

$$\Pi_B^M(\cdot) = D\left(\sum_{i=1}^{i=3}(p_B^i - c_B)S_B^i(\cdot)\right), \quad (83)$$

respectively, where the subscript 'M' stands for mixed locations. The noncooperative Nash equilibrium prices are solutions of the system of 6 first order conditions

$$\frac{\partial \Pi_k^M(\cdot)}{\partial p_k^i} = 0 \quad i = 1, 2, 3 \quad k = A, B. \quad (84)$$

and equal to

$$p_A^{1*} = \frac{18}{378}\delta + \frac{1}{3}(c_B + 2c_A), \quad (85)$$

$$p_A^{2*} = \frac{19}{378}\delta + \frac{1}{3}(c_B + 2c_A), \quad (86)$$

$$p_A^{3*} = \frac{13}{378}\delta + \frac{1}{3}(c_B + 2c_A), \quad (87)$$

and

$$p_B^{1*} = \frac{19}{378}\delta + \frac{1}{3}(c_A + 2c_B), \quad (88)$$

$$p_B^{2*} = \frac{18}{378}\delta + \frac{1}{3}(c_A + 2c_B), \quad (89)$$

$$p_B^{3*} = \frac{13}{378}\delta + \frac{1}{3}(c_A + 2c_B). \quad (90)$$

In contrast with the previous cases, the f.o.b. mill price is different at each location $x_{A,B}^i$ ($i = 1, 2, 3$). Indeed, $p_A^{2*} > p_A^{1*} > p_A^{3*}$, and $p_B^{1*} > p_B^{2*} > p_B^{3*}$. It is interesting to observe that locations x_A^3 and x_B^3 face a lower mill price. At these locations Firm B's (Firm A) outlets surround the outlet located at $x_A^3(x_B^3)$. As a result, the rivalry to compete for the marginal consumers located at z_4, z_5 and z_6 is more important, ceteris paribus. Plugging the Nash prices (85)-(90) into the market boundaries (70)-(75), we obtain

$$z_1^* = \frac{23}{252}, z_2^* = \frac{63\delta + 252(c_B - c_A)}{252\delta}, z_3^* = \frac{103}{252}, z_4^* = \frac{137\delta + 252(c_A - c_B)}{252\delta},$$

$$z_5^* = \frac{189\delta + 252(c_B - c_A)}{252\delta}, z_6^* = \frac{241\delta - 252(c_A - c_B)}{252\delta},$$

and

$$S_A^{1*} = \frac{34\delta + 252(c_B - c_A)}{252\delta}, S_A^{2*} = \frac{40\delta + 252(c_B - c_A)}{252\delta},$$

$$S_A^{3*} = \frac{52\delta + 252(2c_B - 2c_A)}{252\delta}, S_B^{1*} = \frac{40\delta + 252(c_A - c_B)}{252\delta},$$

$$S_B^{2*} = \frac{34\delta + 252(c_A - c_B)}{252\delta}, S_B^{3*} = \frac{52\delta + 252(2c_A - 2c_B)}{252\delta}.$$

Finally,

$$\Pi_A^M = \frac{D}{11907\delta} [3969(c_A - c_B)(4c_A - 4c_B - \delta) + 256\delta^2],$$

$$\Pi_B^M = \frac{D}{11907\delta} [3969(c_A - c_B)(4c_A - 4c_B + \delta) + 256\delta^2].$$

It is straightforward to note that when marginal costs are equal, $\Pi_A^M = \Pi_B^M = \frac{256}{11907} D\delta$.

4.4 Comparison of the Various Equilibrium Patterns in the Three-Outlet Case

4.4.1 Equilibrium Price Comparison

Table III summarises the results obtained in Section 4.1, Section 4.2 and Section 4.3.

Table III: Equilibrium Price Comparison

<i>Interlaced Location</i>	<i>Mixed Location</i>	<i>Neighbouring Location</i>
$p_A^{1*} = \frac{84}{3024} \delta + \frac{(c_B+2c_A)}{3}$	$p_A^{1*} = \frac{144}{3024} \delta + \frac{(c_B+2c_A)}{3}$	$p_A^{1*} = \frac{252}{3024} \delta + \frac{(c_B+2c_A)}{3}$
$p_A^{2*} = \frac{84}{3024} \delta + \frac{(c_B+2c_A)}{3}$	$p_A^{2*} = \frac{152}{3024} \delta + \frac{(c_B+2c_A)}{3}$	$p_A^{2*} = \frac{294}{3024} \delta + \frac{(c_B+2c_A)}{3}$
$p_A^{3*} = \frac{84}{3024} \delta + \frac{(c_B+2c_A)}{3}$	$p_A^{3*} = \frac{104}{3024} \delta + \frac{(c_B+2c_A)}{3}$	$p_A^{3*} = \frac{252}{3024} \delta + \frac{(c_B+2c_A)}{3}$
$p_B^{1*} = \frac{84}{3024} \delta + \frac{(c_A+2c_B)}{3}$	$p_B^{1*} = \frac{152}{3024} \delta + \frac{(c_A+2c_B)}{3}$	$p_B^{1*} = \frac{252}{3024} \delta + \frac{(c_A+2c_B)}{3}$
$p_B^{2*} = \frac{84}{3024} \delta + \frac{(c_A+2c_B)}{3}$	$p_B^{2*} = \frac{144}{3024} \delta + \frac{(c_A+2c_B)}{3}$	$p_B^{2*} = \frac{294}{3024} \delta + \frac{(c_A+2c_B)}{3}$
$p_B^{3*} = \frac{84}{3024} \delta + \frac{(c_A+2c_B)}{3}$	$p_B^{3*} = \frac{104}{3024} \delta + \frac{(c_A+2c_B)}{3}$	$p_B^{3*} = \frac{252}{3024} \delta + \frac{(c_A+2c_B)}{3}$

The noncooperative Nash prices are larger in a neighbouring outlet equilibrium. The interlaced outlet equilibrium yields the more competitive Nash prices. Nash prices with the mixed outlet equilibrium are lower than with a neighbouring location but higher than with an interlaced location. Consequently, prices are increasing as the number of Firm A's (Firm B's) neighbouring outlets increases. This is an appealing result. Price competition is reduced as the number of Firm A's outlets which directly compete with Firm B's outlets decreases. Put differently, as the number of neighbouring outlets increases, each firm coordinates its pricing policy in order to avoid 'cannibalization' between its own outlets. As a result, market power increases as the number of neighbouring outlets increases.

4.4.2 Consumer Loss and Welfare

In this section, I compare the three location patterns in terms of market performance ($\Pi_{A,B}^E$, $E = I, N, M$), consumer loss (CL^E) and welfare loss (WL^E). Again, assume that $c_A = c_B = c = 0$.

Interlaced Outlet Location

Since all consumers pay the same f.o.b. mill price, $\frac{\delta}{36}$, total revenue (price paid by all consumers) is $\frac{D\delta}{36}$. Given the locations of the marginal consumers z_l^* ($l = 1, \dots, 6$) (see Section 4.1), transportation costs, T^I , are given by

$$T^I = D \left(12 \int_{s=0}^{s=1/12} [\delta s^2] ds \right). \quad (91)$$

Evaluating expression (91) yields $\frac{D\delta}{432}$. Consequently,

$$CL^I = -\left(\frac{D\delta}{36} + \frac{D\delta}{432} \right) = -\left(\frac{13D\delta}{432} \right). \quad (92)$$

Neighbouring Outlet Location

In this case things are a bit more complicated since **not** all consumers pay the same f.o.b. mill price (see Section 4.2). Indeed, in equilibrium, $\frac{20}{24} \cong 83.3\%$ of consumers pay a f.o.b. mill price equal to $6\delta/72$, while $\frac{4}{24} \cong 16.6\%$ of consumers pay a price equal to $7\delta/72$. The latter price is paid by the consumers located at proximity of x_A^2 and x_B^2 . It is straightforward to show that total revenue amounts to $\frac{37D\delta}{432}$. Given the locations of the marginal consumers z_l^* ($l = 1, \dots, 6$), transportation costs, T^N , are given by

$$T^N = D \left(4 \int_{s=0}^{s=1/24} [\delta s^2] ds + 4 \int_{s=0}^{s=2/24} [\delta s^2] ds + 4 \int_{s=0}^{s=3/24} [\delta s^2] ds \right). \quad (93)$$

Explicit evaluation of (93) yields $\frac{D\delta}{288}$, so that

$$CL^N = -\left(\frac{37D\delta}{432} + \frac{D\delta}{288} \right) = -\left(\frac{77D\delta}{864} \right). \quad (94)$$

Mixed Outlet Location

In equilibrium, consumers pay three different f.o.b. mill prices (see Section 4.3). It can be shown that, $\frac{68}{252} \cong 27.0\%$ of consumers pay a price equal to $18\delta/378$, $\frac{80}{252} \cong 31.7\%$ pay a price equal to $19\delta/378$, and finally $\frac{104}{252} \cong 41.3\%$ of consumers pay a price equal to $13\delta/378$. The latter, more competitive price is paid by the consumers located at proximity of x_A^3 and x_B^3 . Hence, total revenue amounts to $\frac{512D\delta}{11907}$. Given the locations of the marginal consumers z_l^* ($l = 1, \dots, 6$), transportation costs, T^M , are

$$T^M = D \left(2 \int_{s=0}^{s=\frac{11}{252}} [\delta s^2] ds + 2 \int_{s=0}^{s=\frac{19}{252}} [\delta s^2] ds + 4 \int_{s=0}^{s=\frac{21}{252}} [\delta s^2] ds + 2 \int_{s=0}^{s=\frac{31}{252}} [\delta s^2] ds \right) \quad (95)$$

Explicit evaluation of (95) yields $\frac{545D\delta}{190512}$. It follows that

$$CL^M = - \left(\frac{512D\delta}{11907} + \frac{545D\delta}{190512} \right) = - \left(\frac{8737D\delta}{190512} \right). \quad (96)$$

Table IV summarises these results. Notice that when marginal costs are identical, each firm covers 50% of the market and industry profits are twice the profit of a single firm. Neighbouring locations give rise to significantly higher profits. Social welfare (or average transport costs) is higher under the interlaced location. The results of Table IV suggest that neighbouring location is the less socially desirable pattern. Notice that all results were obtained with outlets assumed to be equispaced. Therefore, in equilibrium, different transportation costs (or welfare) arise because different mill prices are charged by duopolists. The results of Table III and Table IV (see page 23) provide a proof of Proposition 2.

In summary, the results of Proposition 1 and Proposition 2 show that both prices and market performance are higher under a neighbouring location pattern. It is important to note that the interlaced location and neighbouring location constitute the two benchmark cases. When the number of outlets is larger than two, the mixed location equilibrium yields intermediate results⁴. I would like to stress that the results of Proposition 1 and of Proposition 2 are likely to hold under alternative

⁴As the number of outlet increases, a multiplicity of different mixed equilibria emerges.

Table IV: Comparison of the Equilibrium Values

	<i>Interlaced Location</i>	<i>Mixed Location</i>	<i>Neighbouring Location</i>
Industry Profit	$\frac{21168}{762048} D\delta$	$\frac{32768}{762048} D\delta$	$\frac{65268}{762048} D\delta$
Consumer Loss (CL)	$-\frac{22932}{762048} D\delta$	$-\frac{34948}{762048} D\delta$	$-\frac{67914}{762048} D\delta$
Welfare Loss (WL)	$-\frac{1764}{762048} D\delta$	$-\frac{2180}{762048} D\delta$	$-\frac{2646}{762048} D\delta$

assumptions, in particular, with an elastic (downward sloping) demand and/or a two stage location-then-price equilibrium with more than two firms. However, it still remains to study the sensitivity of this result to a non uniform demand distribution.

Finally, a last result is derived from the comparison between the two-outlet case (Section 3) and the three-outlet case (Section 4).

Proposition 3 *Given the results of Proposition 1 (Section 3) and Proposition 2 (Section 4), we have that both noncooperative Nash prices and profits are higher in the two-outlet case, ceteris paribus. Social welfare, however, is higher in the three-outlet case.*

The proof of Proposition 3 directly follows from the comparison of Tables (I)-(II) and Tables (III)-(IV). Given the inelastic demand assumption, it is not surprising that prices and profits are higher in the two-outlet case. This arises because two contrasting effects are at work when a third outlet is introduced. The introduction of the third outlet has a positive market share effect (through a better market segmentation) but a negative price effect due to the increase in price competition. Here, we show that the negative price effect dominates the positive market share effect. This latter result is similar to Martinez-Giralt & Neven [1988]. Interestingly, notice that duopolists would prefer to provide three outlets under a neighbouring pattern rather than two outlets under an

interlaced pattern. Finally, observe that with quadratic utility loss, social welfare is higher in the three-outlet case. This occurs because the introduction of a third outlet allows consumers to incur lower average transportation costs.

5 Conclusion

This paper studies the theoretical implications of location patterns in a spatial multiproduct duopoly model. In particular, it addresses the following question: To what extent do location patterns affect firms' pricing behaviour and market performance? I show that equilibrium prices are higher under a neighbouring location pattern. In other words, noncooperative Nash prices are higher when the organisational market structure is such that firms own strategically dependent outlets. In contrast, I demonstrate that an interlaced location pattern yields the most competitive (Nash) prices. Finally, I also show that social welfare is higher under an interlaced location pattern. These results appear to be robust within the standard assumptions of the spatial model. For example, Bensaid & de Palma [1994] obtain similar results using a two stage location-then-price game. However, it still remains to check the sensitivity of these results to a non uniform demand distribution.

The analysis of a spatial multiproduct (duopoly) model has many potential applications in Industrial Organisation. In a product differentiation interpretation, a neighbouring location pattern arises when each firm specialises on a segment of the 'product line'. In a nonspatial context, a neighbouring location pattern arises when, for example, in a given duopoly market an airline provides all the morning flights and its rival provides all the afternoon flights. In a related paper (Nero [1995a]), an empirical model is derived from the present theoretical model. In particular, the empirical model explicitly controls for the location pattern effect using data on intra-European duopoly airline markets. The principal empirical result suggests that the neighbouring location pattern hypothesis cannot be rejected with data on intra-European airline markets.

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Appendix

Figure 2: Interlaced Locations with two Outlets

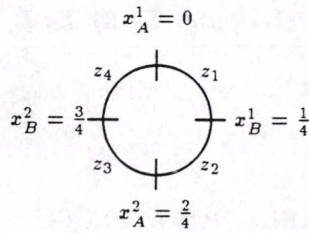


Figure 3: Neighbouring Locations with two Outlets

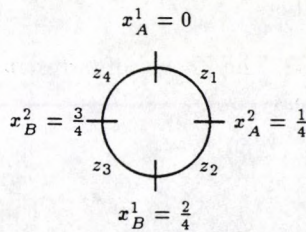


Figure 4: Interlaced Locations in the Three Outlet Case

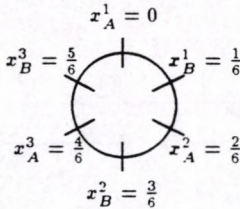


Figure 5: Neighbouring Locations with three Outlets

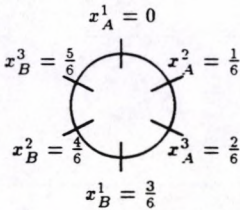
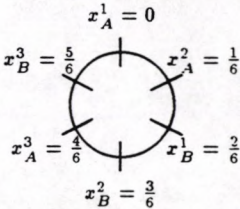


Figure 6: Mixed Locations with three Outlets





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