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DEPARTMENT OF ECONOMICS

Essays in Macro and Labor

João Brogueira de Sousa

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 09 October 2018

European University Institute
Department of Economics

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Abstract

This thesis contains four chapters that cover topics in macroeconomics and labor, and in theoretical asset pricing.Chapters one, two and three contain theoretical and quantitative work on macro and labor. Chapter one, written with Árpád Ábrahám, Ramon Marimon and Lukas Mayr, contains an evaluation of a policy proposal discussed by European Union policy makers and researchers over the last decades on the creation of a European Unemployment Insurance System (EUIS). Building on previous literature, we use a dynamic general equilibrium model to characterize the heterogeneous labor markets across EU countries. Our analysis shows that labor markets differ greatly across EU countries, implying different degrees of exposure of workers to unemployment risk, and different costs of providing unemployment insurance. With this model, we assess the potential benefits of a EUIS. We calculate the potential welfare gains from insuring against country-specific cyclical fluctuations in unemployment expenditures, and from jointly reforming currently national unemployment benefit systems. The results show that the potential gains from insuring short-term country-specific fluctuations at the European Union level are small, and that there could be significant positive improvements in all the countries studied in reforming the existing national unemployment insurance systems to a common EUIS, with country specific contribution rates to prevent long-term cross-country transfers.

The second chapter studies trends in schooling and lifetime labor supply in the United States. Life expectancy increased dramatically by almost 20 years for men born between 1850 and 1970 in the US. The allocation of time during the life also changed greatly. Hours worked per week for the population under 18 years old went from 20 hours to 5 hours on average, during the 1900s. Almost all the difference is explained by the increase in time spent in school. Later in life, despite the large increase in lifespan and years spent in school, total lifetime hours worked fell. The average length of retirement period increased fivefold to more than 10 years for the 1970 cohort. This chapter shows that the long-run trends can be rationalized with a life-cycle human capital acquisition model, where individuals optimally choose the years of schooling and of labor force participation. The observed increase in life expectancy alone would imply, according to the model, an increase in schooling years and in years of participation in the labor market, which is counterfactual. The analysis shows that in the model, an increase in the returns to schooling, in wages, and life expectancy, drives individuals' optimal time allocation decisions as in the data. In the chapter, I revisit empirical evidence on wages and on the decline in the cost of schooling during the first half of the 20th century, with the expansion of public provision of secondary education in the United States.

In the third chapter, in joint work with Julián Díaz-Saavedra and Ramon Marimon, we study how the introduction of an employment fund can enhance production efficiency and social welfare, and how it complements, and in part substitutes, the two classical systems of public insurance: pay-as-you-go pensions and unemployment insurance (UI). The employment fund is akin to the system introduced in Austria in 2003, that was planned to increase in job mobility and flexibility in the

labor market. We investigate the effects of this "Austrian Backpack" in a dynamic general equilibrium model with heterogeneous agents calibrated to the Spanish economy in 2014. A backpack (BP) employment fund is an individual (across jobs) transferable fund, which earns the economy interest rate as a return and is financed with a small payroll tax (a BP tax). The worker can use his BP savings if becomes unemployed or retires. The results in this chapter show that to complement the existing Spanish pension and UI systems with a 2% BP tax would be preferred to the status quo by more than 90% of the households of the calibrated economy; a percentage that can be higher with a more substantial BP (i.e. higher BP tax). The model presented in this chapter is a framework where other reforms (e.g. a partial, or complete, substitution of current unsustainable pension systems) can be quantitatively assessed.

Finally, the fourth chapter contains work developed with Fabian Schuetze about equilibrium conditions in the canonical Lucas asset pricing model. We provide conditions on investor's preferences, and the persistence and volatility of the dividend process under which equilibrium in the economy exists and is unique. The original paper of Lucas shows existence and uniqueness of equilibrium under the assumption of a bounded utility function. This chapter shows how to extend the argument in the case of constant relative risk aversion preferences and log-normally distributed dividends.

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Não teria escrito esta tese sem o apoio incondicional da minha família. Obrigado mãe, pai e Miana por estarem sempre comigo. A Susana pagou o maior preço durante estes cinco anos, e não o poderia ter feito com maior compreensão e carinho.

Contents

Chapter 1
On the Design of an European Unemployment Insurance System

This chapter was written with Árpád Ábrahám, Ramon Marimon and Lukas Mayr.

Abstract We assess the benefits of a potential European Unemployment Insurance System (EUIS) using a multi-country dynamic general equilibrium model with labour market frictions. Our calibration provides a novel diagnosis of the European labour markets, revealing the key parameters – in particular, job-separation and job-finding rates – that explain their different performance in terms of unemployment (or employment) and its persistence. We show that there are only small welfare gains from insuring against country-specific cyclical fluctuations in unemployment expenditures. However, we find that there are substantial gains from reforming currently suboptimal unemployment benefit systems. In spite of country differences, it is possible to unanimously agree on an EUIS with unlimited duration of eligibility, which eliminates the risk of not finding a job before the receipt of benefits ends, and a low replacement rate of 15%, which stabilizes incentives to work and save. We argue that such reforms are more effectively designed at the European level than at the national level because national governments do not take into account general equilibrium effects of their reforms on citizens in other countries. Concerns regarding the political feasibility of such a system are addressed through country-specific contribution payments that eliminate cross-country transfers. The resulting tax differences across countries may be the best statistic of their structural labour market differences, in terms of job creation and destruction, providing clear incentives for reform.

1.1 Introduction

The recent financial and sovereign debt crises have affected European labour markets asymmetrically both in terms of duration and severity of unemployment. In particular, stressed countries - such as Greece, Portugal and Spain - have experienced high levels of unemployment, making it very difficult, if not impossible, to provide adequate insurance for the unemployed and, at the same time, to satisfy the low-deficit (Fiscal Compact) commitments. This has raised interest in proposals for Europe-wide, or Euro-Area-wide, Unemployment Insurance schemes.[1]

Given the asymmetries and lack of perfect coordination of real business cycles across European countries,[2] a European Unemployment Insurance System (EUIS) can efficiently provide risk-sharing across national labour markets and, at the same time, reduce the countercyclical impact of unemployment expenditures on national budgets. Furthermore, it can provide three additional important benefits for the participant states. First, it can reduce the lasting recessionary effects which follow severe crisis, as it has happened in the euro crisis and recession; second, it can develop a much needed solidarity across national labour markets and, third, it can improve labour mobility and market integrations, since unemployment benefits, and the corresponding active policies of surveillance, do not need to be tied to a specific location.

However, the same asymmetries show that implementing a European Unemployment Insurance scheme may not be easy - or politically feasible - if it implies large and 'persistent transfers' across countries. In fact, these 'persistent transfers' are a good indicator of pending structural reforms; therefore, it is not just an issue of redistribution, it can also be a moral hazard problem: 'persistent transfers' may further delay costly, but needed, reforms.

Therefore, to assess the need, viability and possible design of an EUIS one needs to take into account its potential effects: on individual agents' employment and savings decisions; on the aggregate distribution of employment, unemployment and inactivity; on national budgets, in particular taxes to finance unemployment benefits; on insurance transfers across countries; on aggregate savings and investment and, ultimately, on social welfare. In other words, one needs to address these interrelated effects in order to answer a basic question: which unemployment risks need and should – and, if so, how they should – be shared across European countries?

This is a conceptual question that requires a quantitative answer. Unfortunately, with the exception of the works of Dolls et al. (2015) and Beblavy and Maselli (2014), there is very little quantitative evaluation of European Unemployment Insurance schemes. In particular, there is no modelling framework to analyse the key trade-offs of such schemes. In this paper we develop and calibrate – to European countries – a dynamic model to study these effects and provide a set of policy experiments and an implementable proposal.

Any model requires an adequate level of abstraction, in our case we need to effectively compare labour markets and unemployment policies of different countries. Regarding labour markets, Figure 1 ranks European countries using Eurostat data on average unemployment rates (and their variability) for different European countries (2001-2014). This is informative of the 'European labour market diversity' but it is too partial and crude an approximation to build a model just based on these statistics. Alternatively, a very detailed description of countries' labour markets and unemployment policies can be very informative but dilutes the main tradeoffs that should be at the core of a dynamic equilibrium model. Our approach is to study worker flows across the three states of employment, unemployment and inactivity. The corresponding transition matrices,

[1] In this paper we abstract from specific legal and institutional requirements; we will therefore refer to a European Unemployment Insurance System (EUIS) in reference to any possible transnational scheme that addresses the type of diversities which are present in the EU.

[2] For an overview on business cycles in the Euro Area see, for example, Böwer and Catherine (2006), Giannone et al. (2009) and Saiki and Kim (2014).

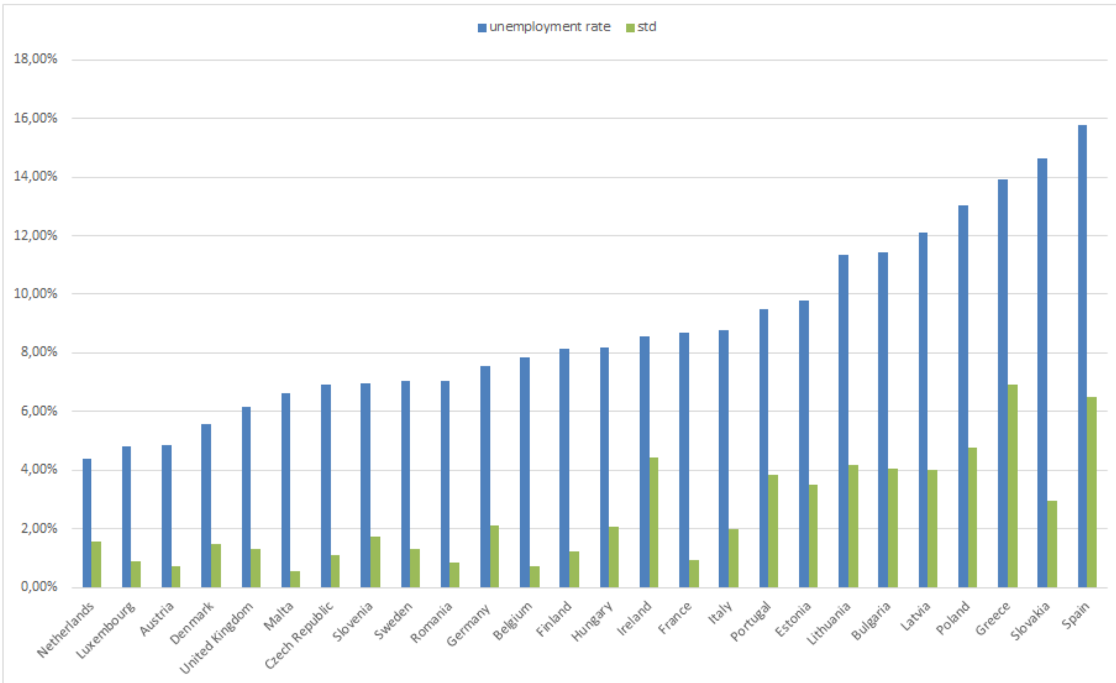

Fig. 1.1: Average European Unemployment Rates: 2001-2014

and associated steady-state distributions, are the pictures that describe our different economies. For example, using Eurostat quarterly data on worker flows (2010Q2-2015Q4), Figure 2 shows similarities and differences in terms of ‘persistency flows’: Employment to Employment (“E to E”, denoted E-E) versus Unemployment to Unemployment (“U to U”, denoted U-U). With the exception of three countries (Spain, Portugal and Slovenia), these ‘persistency flows’ show a strong correlation among European labour markets, with more important differences on U-U. The corresponding ranking, across this E-E vs. U-U axis (of all but three countries), is not the same as the ranking of unemployment rates of Figure 1. In steady-state, the transition matrix of flows for a given country defines its stationary distribution of employment, and the corresponding Figures 1 and 2 are just two snapshots of European labour markets. Behind the scattered plots lie possible differences in preferences, technologies and market institutions, and labour policies. We will assume that across EU countries citizens share (almost) the same preferences and that labour mobility is relatively low across countries (we assume it is nil) but that EU countries still differ in the other aspects – mainly, market institutions and labour policies.

We build on the work of Krusell et al. (2011) and Krusell et al. (2017), who calibrate the U.S. three-states flows with a dynamic general equilibrium model with labour market frictions, to analyse the diverse European labour markets. As in their calibration analysis, we generate worker-flows transition matrices and distributions across the three states as the outcome of a dynamic general

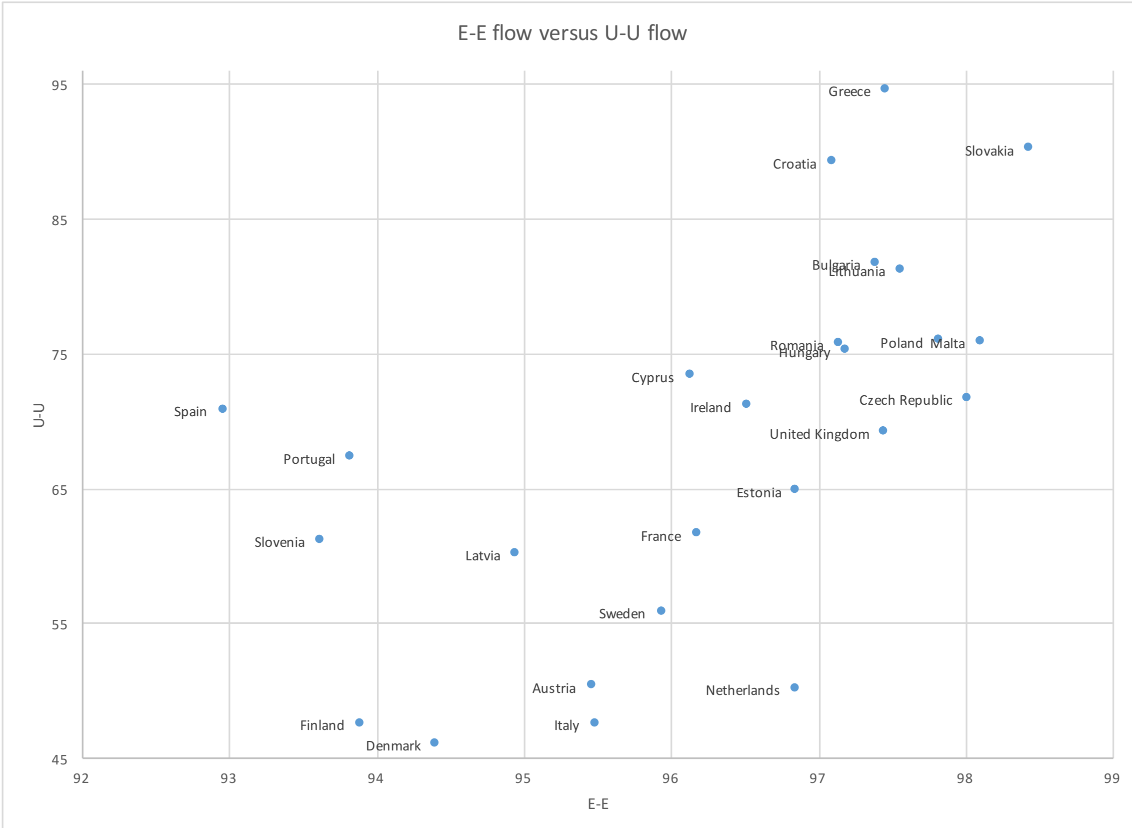

Fig. 1.2: Persistence of Employment and Unemployment

equilibrium. This requires us to set a few parameters on preferences and technology, and calibrate others to match flows and stocks, consistently with observed time series and the existing unemployment policies of a country. More specifically, our model economies are characterised by three sets of parameters: (*i*) generic parameters of preferences and technologies common to all economies – agents' discount factors, idiosyncratic productivity shock, etc.; (*ii*) country-specific structural parameters of their economies - for example, the job-separation and job-finding rates, which in turn are a summary of different factors determining job creation, destruction and matching, and (*iii*) the country-specific unemployment insurance policies, summarized in two – plus one – parameters; the two are the replacement ratio (unemployment benefits to wages) and the duration of unemployment benefits; the third is the unemployment payroll tax rate needed to balance the budget within a period. Section 2.3 describes our model.

Our calibration is a contribution in itself: it provides a novel diagnosis of the European labour markets, since it reveals the key parameters that explain their different performance – in terms of unemployment (or employment) and its persistence. Country-specific structural parameters – in particular, job-separation and job-finding rates – and not UI policy parameters, are the key parameters. Not surprisingly, the job-finding rates for unemployed and for inactive are aligned, but their ranking, while very significant to explain persistence, provides a partial picture of labour market performance: one needs to account for the job-separation rate – for example, the very high job-separation rate of Spain – to get a more accurate one. In contrast, the ‘technological’ dimension in which we allow countries to differ – the total factor productivity – is not a key parameter to account for labour market differences, it mostly accounts for average wage differences. The fact that differences in UI policy parameters do not correspond to differences in labour market performance does not mean they are not relevant: they are, for two related reasons. First, because they show interesting patterns: for example, countries with high unemployment rates –say, Spain, Portugal, Greece and Slovakia – have low replacement rates but, among them, only those with high job-separation rates have long average duration of unemployment benefits (Spain and Portugal), while long average duration of unemployment benefits and high job-separation rates are also characteristic of countries with low unemployment rates (Denmark and Finland). Second, they are relevant because different UI policies – and/or different distributions of employment – result in different payroll taxes, since in our calibration all national budgets balance. These tax differences also determine the desirability of UI policy changes, at the national or at the EU – or some other – level. It should be noted that our UI policy parameters are related, but not on a one-to-one basis, with reported replacement and duration rates. We account for the reported eligibility rates, but then we let the reported benefits and the existing unemployment rates and flows determine our calibrated UI parameters. Section 1.4 provides a more detailed description of our calibration procedures and results.

Our model and its calibration provide the framework for our policy experiments, the main goal and contribution of this paper. Perhaps the most frequently used argument in favor of an EUIS is that it may provide insurance against country specific large fluctuations in unemployment, which with limited fiscal capacity result in fluctuations in the tax burden associated with its financing. Our first experiment therefore targets a quantitative evaluation of the potential pure risk sharing benefits of an EUIS when one country suffers a severe negative shock. To this end, we compute the labour market and welfare consequences of a deep recession in two alternative scenarios: (i) the government is in financial autarky and needs to raise taxes on the employed in order to maintain a balanced UB budget; (ii) the country is insured against increased unemployment and can go through the recession without raising taxes. Otherwise, we assume that the unemployment insurance system remains the same in all remaining countries in both cases. We find that the risk sharing benefits resulting from the welfare differences of the second scenario with respect to the first one are small, and marginally higher for the employed, whose taxes are smoother, than for the unemployed, whose benefits have not changed. This experiment implies that although insurance benefits exist, their small size, questions the rationale for a EUIS as a “rainy day fund”, unless it rains very often.

In light of this result, one may doubt the desirability of a European unemployment insurance system. Even more so as the observed heterogeneity in labour market institutions suggests that the optimal benefit systems could differ substantially across European countries, making it difficult for governments to reach a common ground. To evaluate this claim, we compute the optimal unilateral reform of the unemployment benefit system (financed at the national level), separately for each country. We perform this exercise in partial equilibrium assuming that a single country does not affect equilibrium prices. We find that the optimal mix of replacement rate, and duration of unemployment benefits, is surprisingly similar across the countries studied. In all countries it is optimal to provide an unlimited duration of eligibility and the optimal replacement rates vary between 20% and 45%.

Despite similar optimal national unemployment insurance policies one may still argue that the small difference suffice to let countries reform their systems by themselves rather than to force them into a common European benefit scheme. We show that this argument is flawed because individual national governments do not internalize general equilibrium effects of their reforms on citizens in other European countries. In particular, we show that if all European countries would reform their system simultaneously and the capital market is required to clear at the union level, i.e. in general equilibrium, the very same UI benefit systems that seem optimal in partial equilibrium, are in fact welfare reducing in most of the countries. If national governments are benevolent but only towards the citizens of their own country, they would reform the benefit system towards a more generous one than what is optimal from a collective European perspective. Increasing the generosity of the UI benefit system in some European countries, reduces private savings and hence the aggregate, European, capital stock. As a consequence the marginal product of labour declines everywhere. This redistributes from poor agents, who derive most of their income from wages to rich agents with mainly capital income. Importantly, the common European capital market implies that this redistribution happens across all Europe.

The final contribution is to provide a better alternative: a common European Unemployment Insurance System (EUIS). We first show that a fully harmonized system which is jointly financed at the European level is unlikely to achieve unanimous support across member states as it would result in transfers from countries with structurally low unemployment to high unemployment countries. Interestingly, for some of the net payers, the welfare gains of such a reform are positive, suggesting that in these countries the current unemployment benefit systems are far from optimal. We then neutralize transfers through varying contribution payments across countries. We find that an EUIS with an unlimited duration and a replacement rate of 15% is welfare improving in *all* countries and almost unanimous. The unlimited duration insures agents against the risk of losing eligibility before the receipt of unemployment benefits ends. At the same time the low replacement rate stabilizes incentives to work and save, keeping the European capital stock and therefore wages high. A positive side effect of such a system with tax differences that eliminate cross-country transfers is that these differences may serve as an incentive device for individual countries to structurally reform weak labour market institutions.

Implementation

Although it is not the focus of this paper, it is worth to briefly consider how this EUIS proposal could be implemented. The basic idea is that it can be implemented through the existing national Unemployment Insurance Systems, it is for this reason we have only considered the common form of unemployment benefits defined by their ‘replacement and duration’. If the national funds had enough borrowing capacity, to provide the unemployment benefits without increasing the taxes in times of crisis, and enough commitment, to properly accumulate funds in normal and good times, the EUIS would only require policy commitment and coordination. However, not all (if any) existing national systems satisfy these requirements, in which case a mixed solution between the national UI funds and a central EU fund is in order.

The EUIS central fund can be hosted in the *European Stability Fund*[3] which would have contracts with participating countries stipulating (unemployment) countercyclical transfer between the national fund and the central fund as to guarantee the uniform unemployment benefits preserving smooth taxes within the limited borrowing capacity of the national fund. In other words, as with other *ESF* contracts, first there must be a country-risk assessment (an improved version of our calibration) to assess the country referential stable payroll tax rate and unemployment rate, as well as the thresholds unemployment rates determining country transfers to and from the central fund. The contract should be designed, as other ESF contracts, to guarantee that these transfers do not become permanent transfers. In fact, a stable system of payroll taxes and benefits results in fluctuating net revenues at the country level when, in additional to agents' idiosyncratic risk, there is also country risk (as in our first experiment in Section 1.5).

The mixed design of the EUIS means that the central fund absorbs these fluctuations beyond certain limits (given by unemployment rate thresholds), acting as a safe deposit when unemployment is relatively low and providing insurance when it is relatively high. Our reported structural differences across countries imply that constrained efficient contracts between participating countries and the fund should be country specific, but based on the same common principles. On a periodic basis – say, every seven years – the country risk assessment should be updated and the referential rates adapted accordingly, to make sure that transfers fulfil their stabilisation role without becoming persistent inflows or outflows, to or from the fund.

The remainder of this paper is organized as follows. The next section briefly discusses the current literature on the topic. In section 2.3 we present the model and in section 1.4 our calibration, which provides the basis for our policy experiments in section 1.5. Finally, section 1.6 concludes.

1.2 Literature Review

There are a few recent papers that also study different aspects of the design of a EUIS coming both from academic scholars and from policy institutions. In this section, we review briefly some of the most recent and relevant papers on this issue.

On the hand, Ignaszak et al. (2018) study the optimal provision of unemployment insurance in a federal state containing atomistic (and symmetric) regions. The focus of their paper is different from ours in three important dimensions. First, in their environment, the regions are ex ante identical, hence they cannot study the asymmetric effect of a EUIS on the different participating nations as we do. At the same time, their model allows for a rich interaction between federal and local policies as regional governments have a wide set of instruments, that they can use to respond to the introduction of new federal policies. Their main focus is indeed to study the crowding out

[3] See the ESF ADEMU proposal in Marimon and Cooley (2018, Chs. 2 and 12), based on Ábrahám *et al.* (2018) characterization of ESF constrained efficient contracts.

of regional incentives due to generous federal insurance schemes (moral hazard). The third difference is that their model does not allow for an intertemporal saving technology for any agents (households, regions or the union altogether). Our results show that general equilibrium effects of different unemployment insurance policies through the savings channel can be quantitatively very important.

On the other hand, Claveres and Clemens (2017) and Moyen et al. (2016)study unemployment insurance and international risk sharing in a two-region DSGE model with frictional labour markets and calibrate their model to the core and the periphery of the Euro-zone. In both papers, a supranational agency runs an unemployment insurance scheme that triggers transfers to recessionary countries but has zero transfers in expectation. Such a scheme allows recessionary countries to maintain unemployment benefits and simultaneously reduce taxes, thus dampening recessionary effects. Our model differs in many dimensions from these papers. First, our model features a higher degree of heterogeneity both across and within countries. In particular, our policy experiments are performed with ten countries of the Euro area instead of two regions. As we show, labour market institutions and consequently flows across employment, unemployment and inactivity are as heterogeneous across countries within the core (and the periphery) as across the core and the periphery. For example, we found that certain implementations of an EUIS have significantly different effects on Belgium and Germany, two core countries. In addition, the combination of endogenous savings decisions and idiosyncratic productivity shocks result in a non-degenerate distribution of wealth in our model. We show that this within country wealth heterogeneity is a key determinant for both the welfare effects of UI policies and for determining the general equilibrium channel of policies through precautionary savings. Finally, our paper provides an extensive welfare evaluation (across countries, employments states and wealth levels) of different EUIS implementations both with business cycle fluctuations and by studying the transition to a new steady states after a policy reform.

In contrast with the previous papers, Dolls et al. (2015) and Beblavy and Lenaerts (2017) take into account the rich heterogeneity within the Euro area. They provide quantitative exercises that measure the possibilities for intertemporal and interregional smoothing of unemployment benefits and social security contributions under different versions of a EUIS. Both papers present a set of counterfactual scenarios where household income and the evolution of labour markets are kept fixed during the period of study, and different specifications of a EUIS are considered. As in our paper, both studies find considerable interregional and intertemporal smoothing possibilities. In contrast with our paper, the lack of individual responses does not allow them to evaluate the effects of different insurance systems on labour markets, household consumption, individual savings and welfare. In addition, this implies that there are no equilibrium adjustments either and no effect on aggregate savings and capital accumulation.

Finally, Dullien et al. (2018) provide a concrete proposal to be discussed at the European Parliament following a similar approach as the two papers above. In contrast with our work, they only focus on the fund-contract aspect, applying the self-insurance and the reinsurance principles to the design of a EUIS which operates national funds and a joint ‘stormy day fund’ that is operational only when the country is hit by a severe crisis. Similarly to ours, their scheme is intended to be implemented on a voluntary basis and it has interesting countercyclical features, which can improve upon the current situation. However, the national contracts are not based on a country-specific risk-assessment, the final destination of the funds is not guaranteed and similarly the above papers the methodology does not allow to evaluate the impact on individual decisions and on equilibrium outcomes.

1.3 Model

Our model economy consists of a union of $I \in \mathbb{N}$ countries. We assume that the population in each country $i \in \{1,...,I\}$ is fixed and that there is no migration across countries. This implies that labor markets clear country by country. Capital, on the other hand, is perfectly mobile across countries. We assume that the union as a whole is a closed economy such that the (weighted) sum of the capital stocks in all countries equals the savings of all citizens in the union.

Each country is modeled along the lines of Krusell et al. (2011) and Krusell et al. (2017). Their model captures key economic decisions of agents regarding their labour market behaviour and is therefore suited to think about unemployment policy. In particular, in the model, given labour income taxes and unemployment benefits, agents with an opportunity to work are able to choose whether or not they work and agents currently not employed are able to choose whether or not to actively search for a job.

Timing and Preferences.

Time $t \in \{0,1,2,...\}$ is discrete. Each country is populated by a continuum of agents of measure n^i, where $\sum_{i=1}^{N} n^i = 1$. Preferences over consumption, labour supply and job search are given by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big[\log(c_t) - \alpha w_t - \gamma^i s_t \Big]. \tag{1.1}$$

Agents derive utility from consumption c_t and disutility from employment w_t and job search s_t. The parameter α captures the disutility of work and is assumed to be the same in each country. The parameter γ^i denotes the disutility of active job search and is varying across countries. In this way we capture that the governments' assistance in the search for a job differs across countries. The time discount factor $\beta \in (0,1)$ is the same for all citizens in the union. Workers can only choose to supply labor on the extensive margin, i.e. $w_t \in \{0,1\}$. Additionally, the search decision is also discrete: $s_t \in \{0,1\}$.

Markets and Technology.

The production sector is competitive. Firms, who produce according to a constant returns to scale technology, hire labour from the domestic labour market and pay a wage per efficiency unit of labour that equals the marginal product of labour. They rent capital from the international capital market at a price r_t and pay for the depreciation of capital; the total rental price equals the marginal product of capital, which is the same across countries. Workers supply labour in the domestic market. This market is characterized by frictions that affect workers' separations from jobs, and workers' access to a job opportunity. In what follows, these frictions are described in detail.

In the beginning of every period, agents who were employed in the previous period can loose their job with probability σ^i. The probability of finding a job while not employed depends on the search effort. An agent who is actively searching during period t finds an employment opportunity for period $t+1$ with probability λ_u^i; an agent who is not actively searching, with probability $\lambda_n^i < \lambda_u^i$. After loosing a job, agents who search may be eligible for unemployment benefits. The process that determines eligibility for unemployment benefits is described below. Note that the job arrival rates and the job separation rate are country specific. In this way we capture the heterogeneity in labour market institutions across Europe.

Agents are heterogeneous with respect to their labour productivity, denoted by $z_t \in Z = \{\bar{z}_1, \bar{z}_2, ..., \bar{z}_{n_z}\}$. Idiosyncratic productivity follows a first order Markov chain with transition probabilities $p(z'|z)$. This process is assumed to be the same in each country.

Agents cannot directly insure themselves against the idiosyncratic productivity risk, however they can save using a risk-free bond. The risk-free return is given by the international real interest rate r_t.

Production is given by the Cobb-Douglas technology:

$$F^i(K_t^i, L_t^i) = A_t^i (K_t^i)^\theta (L_t^i)^{1-\theta}, \tag{1.2}$$

where A_t^i denotes total factor productivity in country i, K_t^i the aggregate capital stock in country i and θ the capital share of output. L_t^i is aggregate labour in country i, measured in efficiency units. In what follows, we generally assume no aggregate (country-specific) shocks, i.e. $A_t^i = A^i$.[4]

Individual Labour Market States.

An agent can be employed, unemployed or inactive. The difference between unemployed and inactive agents is that the former exert search effort while the latter do not. Further, if an agent is unemployed he can either be eligible for unemployment benefits, in which case he receives a certain fraction of his potential income as a wage worker or he can be non-eligible, in which case he does not receive benefits and hence solely lives from his savings. This gives a total of four possible individual labor market states that an agent can attain, $x_t \in \{e, u^e, u^n, n\}$: employed, unemployed eligible, unemployed non-eligible, non-participating;

Unemployment Benefits.

Eligibility for unemployment benefits is partially determined by agent's endogenous decisions, partially by exogenous shocks. Only agents who are exogenously separated from their job are eligible for unemployment benefits, while agents who quit their job themselves are not eligible.

[4] We deviate from this assumption only in subsection 1.5.1.

Further, in order to maintain eligibility agents have to continuously exert search effort. Once an agent stops searching, she is non-eligible even if at some later time she starts searching again. Finally, in every period with some probability μ^i agents loose eligibility even if they search for a job. This is a parsimonious way to capture limited (and country-specific) duration of unemployment benefit receipt.[5] Non-eligibility is an absorbing state. The only way to regain eligibility is to find a job, be employed for some time and then be exogenously separated again.

An eligible unemployed agent in country i receives unemployment benefits $b_t^i(z_t)$ according to

$$b_t^i(z_t) = \bar{b}_t^i \omega_t^i z_t \tag{1.3}$$

where $\bar{b}_t^i$ is the replacement rate in country i, ω_t^i is the wage per efficiency unit of labour and z_t is the agent's current productivity level. The formula in (1.3) implies that an agent receives unemployment benefits according to his current labor market productivity. A more realistic assumption would be to have unemployment benefits depend on past labour earnings. We choose (1.3) to economize in the dimension of the state space of the model (avoiding the need to keep track of past productivity of currently unemployed agents), and because the process z_t is persistent, implying that current productivity is a good proxy for previous labor earnings.

Budget Sets.

In every period t, each agent in country i chooses a pair of consumption and savings from a budget set $B_t^i(a,z,x)$ that depends on his current assets, productivity and employment state as well as on current prices r_t and ω_t^i. The budget set of an agent who is employed in period t ($x_t = e$) is given by

$$B_t^i(a,z,e) = \Big\{(c,a') \in \mathbb{R}_+^2 : c + a' \leq (1+r_t)a + (1-\tau_t^i)\omega_t^i z\Big\}. \tag{1.4}$$

An employed agent finances consumption c and savings a' with current period's asset a inclusive of interest income $r_t a$ and income from work, net of the tax rate τ_t^i. An unemployed agent who is eligible for unemployment benefits faces the budget set

$$B_t^i(a,z,u^e) = \Big\{(c,a') \in \mathbb{R}_+^2 : c + a' \leq (1+r_t)a + b_t^i(z)\Big\}. \tag{1.5}$$

He does not have wage income but receives some fraction of his potential income as unemployment benefits.

Finally, both unemployed non-eligible and non-active agents finance consumption and next period's assets exclusively from savings:

$$B_t^i(a,z,u^n) = B_t^i(a,z,n) = \Big\{(c,a') \in \mathbb{R}_+^2 : c + a' \leq (1+r_t)a\Big\}. \tag{1.6}$$

[5] In reality this duration is not stochastic but fixed. However, implementing a fixed duration is computationally expensive as it requires to keep track of the periods that each unemployed agent already receives benefits. To economize on the state space we hence use this stochastic process as in Krusell et al. (2011) and Krusell et al. (2017).

Labor Market Decisions and Value Functions.

The individual optimization problem has a recursive representation. Denote the value of an individual in country i, period t, and state (a,z,x), by $V_t^i(a,z,x)$. The time index of the value function captures in a simple way that the current value depends on current and future prices and government policies, which both vary over time. Then the value of an agent in employment is given by

$$V_t^i(a,z,e) = \max_{(c,a')\in B_t^i(a,z,e)} \Bigg\{ \log(c) - \alpha + \beta \sum_{z'\in Z} p(z'|z) \Big[(1-\sigma^i) \max_{x'\in\{e,u^n,n\}} V_{t+1}^i(a',z',x') + \sigma^i \Big(\lambda_u^i \max_{x'\in\{e,u^e,n\}} V_{t+1}^i(a',z',x') + (1-\lambda_u^i) \max_{x'\in\{u^e,n\}} V_{t+1}^i(a',z',x') \Big) \Big] \Bigg\}. \quad (1.7)$$

The Bellman equation reflects the dynamics of the labour market. In the present period the worker derives utility from consumption but disutility of work. The continuation value takes into account that with probability $1-\sigma^i$ the agent will not be separated from the job. In this case he can choose between staying employed or to quit the job. In the latter case he can choose to stay inactive or to search for a new job. He will, however, not be eligible for benefits as he decided to leave the firm himself. Hence, if the worker does not get separated from his job he has three choices, $x' \in \{e,u^n,n\}$. With probability σ^i the worker is separated from his job. Then with probability λ_u^i he immediately gets matched with a new firm, in which case he again can choose between employment, unemployment and inactivity. If he chooses unemployment he is eligible for benefits since he was exogenously separated from the job. With probability $1-\lambda_u^i$ he does not immediately find a new job. In this case he can only choose between eligible unemployment and inactivity, i.e. $x' \in \{u^e,n\}$. Note that a worker who was separated from his job will get unemployment benefits for one period with certainty as long as he searches for a new job during this period.

Similarly, the value of an eligible unemployed agent in country i satisfies:

$$V_t^i(a,z,u^e) = \max_{(c,a')\in B_t^i(a,z,u^e)} \Bigg\{ \log(c) - \gamma^i + \beta \sum_{z'\in Z} p(z'|z) \Bigg[\lambda_u^i \Big((1-\mu^i) \max_{x'\in\{e,u^e,n\}} V_{t+1}^i(a',z',x') + \mu^i \max_{x'\in\{e,u^n,n\}} V_{t+1}^i(a',z',x') \Big) + (1-\lambda_u^i) \Big((1-\mu^i) \max_{x'\in\{u^e,n\}} V_{t+1}^i(a',z',x') + \mu^i \max_{x'\in\{u^n,n\}} V_{t+1}^i(a',z',x') \Big) \Bigg] \Bigg\}. \quad (1.8)$$

In the present period an unemployed agent incurs the utility cost of searching γ^i. While searching, a job offer for next period arrives with probability λ_u^i, in which case the agent can choose between employment, unemployment and inactivity. With the remaining probability $1-\lambda_u^i$ the agent does not receive a new offer and thus can only choose between unemployment and inactivity. Further the unemployed looses eligibility for benefits with probability μ^i and keeps eligibility with the remaining probability $1-\mu^i$.

The value of the non-eligible unemployed is very similar. The only exception is that he will not be eligible for benefits next period with certainty,

$$V_t^i(a,z,u^n) = \max_{(c,a')\in B_t^i(a,z,u^n)} \left\{ \log(c) - \gamma^i + \beta \sum_{z'\in Z} p(z'|z) \left[\lambda_u^i \max_{x'\in\{e,u^n,n\}} V_{t+1}^i(a',z',x') + (1-\lambda_u^i) \max_{x'\in\{u^n,n\}} V_{t+1}^i(a',z',x') \right] \right\}. \tag{1.9}$$

Finally, the value for non-active (i.e. not actively searching) agents in country i is given by

$$V_t^i(a,z,n) = \max_{(c,a')\in B_t^i(a,z,n)} \left\{ \log(c) + \beta \sum_{z'\in Z} p(z'|z) \left[\lambda_n^i \max_{x'\in\{e,u^n,n\}} V_{t+1}^i(a',z',x') + (1-\lambda_n^i) \max_{x'\in\{u^n,n\}} V_{t+1}^i(a',z',x') \right] \right\}. \tag{1.10}$$

The value of the non-active is similar to the non-eligible unemployed. The difference is that a non-active does not suffer the disutility of search and has a lower probability of a receiving a job offer next period, i.e. $\lambda_n^i < \lambda_u^i$.

Definition of Partial and General Equilibrium.

We will now define two equilibria: (i) the partial equilibrium for a specific country i, which takes the union interest rate r_t as given; (ii) the general equilibrium for the union, for which the interest rate r_t is required to adjust such that aggregate savings equal aggregate capital in the union.

Individual state variables are assets $a \in \mathbb{R}_+$, idiosyncratic productivity $z \in Z$, and employment status $x \in \{e,u^e,u^n,n\}$. The aggregate state in country i is described by the joint measure ζ_t^i over assets, labor productivity status and employment status. Let $\mathscr{B}(\mathbb{R}_+)$ be the Borel σ-algebra of $\mathbb{R}_+$, $\mathscr{P}(Z)$ the power set over $Z=\{\bar{z}_1,\bar{z}_2,...,\bar{z}_{n_z}\}$ and $\mathscr{P}(X)$ the power set over $X=\{e,u^e,u^n,n\}$. Further, let $\mathscr{M}$ be the set of all finite measures over the measurable space $\{(\mathbb{R}_+\times Z\times X),\mathscr{B}(\mathbb{R}_+)\times\mathscr{P}(Z)\times\mathscr{P}(X)\}$.

Definition 1.1. Partial equilibrium in country i: Given sequences of interest rates $\{r_t\}_{t=0}^{\infty}$ and unemployment benefit policies $\{(\bar{b}_t^i,\mu_t^i)\}_{t=0}^{\infty}$ and given an initial distribution ζ_0^i, a partial equilibrium in country i is defined by a sequence of value functions $\{V_t^i\}_{t=0}^{\infty}$, consumption and savings decisions $\{c_t^i,a_{t+1}^i\}_{t=0}^{\infty}$, firm production plans $\{K_t^i,L_t^i\}_{t=0}^{\infty}$, payroll taxes $\{\tau_t^i\}_{t=0}^{\infty}$, wages $\{\omega_t^i\}_{t=0}^{\infty}$ and measures $\{\zeta_t^i\}_{t=1}^{\infty}$, with $\zeta_t^i\in\mathscr{M}$ $\forall t$, such that:

(i) Agents optimize: Given prices, unemployment benefit policies and tax rates, the value function V_t^i and the policy functions for consumption c_t^i and savings a_{t+1}^i satisfy the Bellman equations (2.20), (1.8), (1.9) and (1.10) with equality for each $t \geq 0$.

(ii) Firms optimize: Prices satisfy $r_t = F_K^i(K_t^i, L_t^i) - \delta$ and $\omega_t^i = F_L^i(K_t^i, L_t^i)$ for each $t \geq 0$.

(iii) The labour market clears:

$$L_t^i = \sum_{z \in Z} z \int_0^\infty \zeta_t^i(a,z,e)da \quad \forall t \geq 0 \tag{1.11}$$

(iv) The government budget clears:

$$\tau_t^i \omega_t^i L_t^i = \sum_{z \in Z} b_t^i(z) \int_0^\infty \zeta_t^i(a,z,u^e)da \quad \forall t \geq 0 \tag{1.12}$$

(v) The law of motion $\zeta_{t+1}^i = H_t^i(\zeta_t^i)$ holds for each $t \geq 0$: Thereby the function $H_t^i : \mathscr{M} \to \mathscr{M}$ can be explicitly written as follows:

$$\zeta_{t+1}^i(\mathscr{A} \times \mathscr{Z} \times \mathscr{X}) = \sum_{x \in X} \sum_{z \in Z} \int_0^\infty T_t^i((a,z,x); \mathscr{A} \times \mathscr{Z} \times \mathscr{X}) \zeta_t^i(a,z,x)da,$$

where $T_t^i((a,z,x); \mathscr{A} \times \mathscr{Z} \times \mathscr{X})$ describes the transition probability of moving from state (a,z,x) in period t to any state (a',z',x') such that $a' \in \mathscr{A} \subset \mathbb{R}_+$, $z' \in \mathscr{Z} \subset Z$, $x' \in \mathscr{X} \subset X$ in period $t+1$.[6]

Definition 1.2. General equilibrium in the union of countries: Given a collection of sequences of unemployment benefit policies $\{\{(\bar{b}_t^i, \mu_t^i)\}_{t=0}^\infty\}_{i=1}^I$ and given a collection of initial distributions $\{\zeta_0^i\}_{i=1}^I$, a general equilibrium in the union of countries is defined by sequences of value functions $\{\{V_t^i\}_{t=0}^\infty\}_{i=1}^I$, policy functions $\{\{c_t^i, a_{t+1}^i\}_{t=0}^\infty\}_{i=1}^I$, firm production plans $\{\{L_t^i, K_t^i\}_{t=0}^\infty\}_{i=1}^I$, payroll taxes $\{\{\tau_t^i\}_{t=0}^\infty\}_{i=1}^I$, wages $\{\{\omega_t^i\}_{t=0}^\infty\}_{i=1}^I$, measures $\{\{\zeta_t^i\}_{t=1}^\infty\}_{i=1}^I$, with $\zeta_t^i \in \mathscr{M}$, and by a sequence of interest rates $\{r_t\}_{t=0}^\infty$ such that all conditions of definition 1.1 are satisfied for each country $i \in \{1,2,...,I\}$ and in addition the capital market clears at the union level, i.e.

$$\sum_{i=1}^I n^i K_{t+1}^i = \sum_{i=1}^I n^i \sum_{x \in X} \sum_{z \in Z} \int_0^\infty a_{t+1}^i(a,z,x) \zeta_t^i(a,z,x)da \tag{1.13}$$

holds.

Definition 1.3. Stationary general equilibrium: A stationary general equilibrium is a general equilibrium in which all government policies, decision rules, value functions, aggregate variables and prices are constant in all countries of the union.

[6] The description of the transition function T_t^i is quite involved and therefore deferred to the appendix.

1.4 Calibration

We calibrate the model assuming that in $t = 0$ the union of I countries is in a stationary general equilibrium (see Definition 1.3 above). Hence, we assume that the Euro-Zone as a whole is a closed economy with no net capital in- or outflows. However, we want to note here that the structural calibrated parameters are not sensitive to this choice. In particular, if we do not require capital market clearing at the union level and consider any world interest rate within a reasonable range it does not affect the overall calibration much. Currently the countries we consider are the eleven countries that formed the original Euro-Zone in 1999 plus Estonia, Greece, Latvia, Slovenia and Slovakia.

The model presented in the previous section has three sets of parameters, which correspond to the three panels of Table 1.1. The upper panel describes technological and preference parameters that are common to all countries. In particular, we assume that in all countries the capital share of production θ, the depreciation of capital δ, the time discount factor β and the utility cost of work α is the same. Further, we assume that idiosyncratic productivity follows the same Markov process, for which we use a discretized version of an AR(1) process with persistence ρ_z and variance σ_z^2.

Parameter	Definition
θ	Capital share of output
δ	Capital depreciation rate
β	Discount factor
ρ_z	Persistence of productivity
σ_z^2	Variance of prod. shock
α	Utility cost of labor
A^i	Total factor productivity
γ^i	Utility cost of search
σ^i	Job separation rate
λ_u^i	Job finding rate for unemployed
λ_n^i	Job finding rate for inactive
μ^i	Prob. of loosing UB eligibility
$\bar{b}^i$	UB replacement rate

Table 1.1: Model parameters.

The middle and lower panels display parameters that are specific to each country. The middle panel includes parameters that capture - in a reduced form - different labour market institutions: total factor productivity A^i (which affects wage differences across countries), the cost of job search γ^i, the exogenous job separation rate σ^i, as well as the job arrival rates λ_u^i and λ_n^i. The lower panel contains parameters that define country specific unemployment benefit policies $(\mu^i, \bar{b}^i)$.

In total our model has $6 + I \times 7$ parameters.The three sets of parameters constitute a hierarchical structure in the degree to which policy can influence them. The unemployment benefit policy parameters $(\mu^i, \bar{b}^i)$ can be changed relatively easy by governments, while it takes more complex labour market reforms to change the institutional parameters $(A^i, \gamma^i, \sigma^i, \lambda_u^i, \lambda_n^i)$ and it is very hard, if not impossible, to change the parameters of the first panel. Given the scope of this paper, in the policy experiments below we only vary unemployment benefit policies (and how these are financed), though we want to explicitly mention here that the institutional parameters are not set in stone and can be changed through structural labour market reforms.

A central aspect of our analysis are the transitions between employment, unemployment and inactivity. Flow statistics are a useful measure since they quantify the aggregate transitions between labour market states in the data. In order to calibrate the model, we therefore use estimated quarterly transition probabilities, and the corresponding three average labour market stocks, generously provided by Etienne Lalé. Lalé and Tarasonis (2017) estimate these transition probabilities using quarterly data on prime-age workers (25-54) in the EU countries, from 2004 until 2013. [7] Data on unemployment benefits in EU Member States is taken from Esser et al. (2013), and data on population and average labour earnings from Eurostat.

1.4.1 Calibration strategy

We now describe in detail how the model is calibrated. First, we set the technological parameters θ, δ, ρ_z and σ_z to the quarterly counterparts of Krusell et al. (2017), who use monthly data for the US economy to estimate them. We discretize the AR(1) process for individual productivity process by 5 different productivity states using the Tauchen method. We set the discount factor β to 0.99, implying a subjective discount rate close to one percent per quarter.

The policy related parameters are chosen as follows. The parameter μ^i, which is the conditional probability of remaining eligible for UB next period, is also the inverse of the average duration of unemployment benefits in the model. We therefore set $1/\mu^i$ to the maximum duration of eligibility acoording to the law in country i. As described above, we model the eligibility process in this way because it allows for a simpler representation and a reduction in the dimensionality of the state space. For the unemployment benefit replacement rates, we set $\bar{b}^i$ to the data equivalents in Esser et al. (2013).

The remaining five country specific parameters A^i, γ^i, σ^i, λ_u^i and λ_n^i are calibrated in order to match the following five data moments: the differentials of average wages across countries,[8] the share of unemployed individuals in the population, the employment-to-employment, the unemployment-to-employment, and the non-active to employment flows. Finally, we set the common utility cost of work parameter α such that the population-weighted average of the fraction of employed agents in the Union matches the data.

Table 1.2 lists the common parameters, and table 1.3 contains the country specific parameters for the calibrated European countries. We also report the tax rates τ^i that clear the government budget in each country.

[7] The underlying data is from the EU-SILC dataset, except Germany which comes from the GSOEP.

[8] We picked Germany, the largest country in the European Union, as our reference country. So TFP in Germany is equal to one and for the other countries it is calibrated in order to match wages relative to German wages.

Parameter	Definition	Value
θ	Cobb-Douglas capital weight	0.3
δ	Capital depreciation rate	0.01
ρ_z	Persistence of individual productivity	0.89
σ_z^2	Variance of individual productivity	0.08
α	Utility cost of work	0.89
β	Discount factor	0.99

Table 1.2: Common Parameters. Time period is 1 quarter; r clears the EU capital market.

	A^i	γ^i	σ^i	λ_u^i	λ_n^i	$\bar{b}^i$	$1/\mu^i$	$\tau^i(\%)$
Austria	0.92	0.63	0.04	0.25	0.08	0.28	2.27	0.92
Belgium	1.01	0.60	0.02	0.10	0.06	0.37	19.70	2.13
Germany	1.00	0.01	0.01	0.10	0.10	0.23	3.94	0.45
Estonia	0.57	0.35	0.03	0.17	0.10	0.46	3.86	2.94
Spain	0.81	0.68	0.06	0.17	0.04	0.33	7.80	4.43
Finland	0.97	0.40	0.05	0.21	0.18	0.36	7.58	3.75
France	0.93	0.30	0.02	0.16	0.06	0.35	7.88	1.90
Greece	0.82	0.90	0.04	0.17	0.03	0.65	3.94	5.60
Ireland	1.05	0.55	0.03	0.13	0.05	0.36	3.94	1.97
Italy	0.92	0.48	0.03	0.13	0.04	0.09	2.58	0.25
Luxembourg	1.15	0.95	0.02	0.17	0.04	0.27	3.94	0.53
Latvia	0.45	0.30	0.04	0.16	0.07	0.57	2.95	1.60
Netherlands	0.87	0.03	0.01	0.14	0.13	0.35	3.50	1.00
Portugal	0.69	0.49	0.06	0.15	0.09	0.36	5.91	4.98
Slovenia	0.77	0.14	0.01	0.14	0.05	0.65	1.97	0.98
Slovakia	0.54	0.25	0.03	0.13	0.07	0.08	1.97	0.16

Table 1.3: Country specific parameters.

1.4.2 Quality of the Fit

In this section we investigate how well the model fits the European labour markets. In the calibration described above, several labour market moments were targeted. These are shown in Figures 1.3 to 1.6. In Figure 1.3 we observe that the average unemployment rate in Spain, Greece, Latvia and Portugal is much higher than the European average, while in Austria, Germany, Luxembourg and the Netherlands it is lower. The persistence of employment (Figure 1.4) is high in almost all countries. The exceptions are Spain, Finland and Portugal who have substantial flows out of employment in each quarter. The flows from unemployment to employment (Figure 1.5) are quite heterogeneous across European countries. Interestingly, it is lowest in Germany, a country with rather low structural unemployment. By contrast, Austria, which has similar average unemployment rates as Germany, has the highest flow from unemployment to employment. We observe substantial heterogeneity also in the flows from inactivity to employment (Figure 1.6). For example, in Finland this flow is much higher than in the other countries.

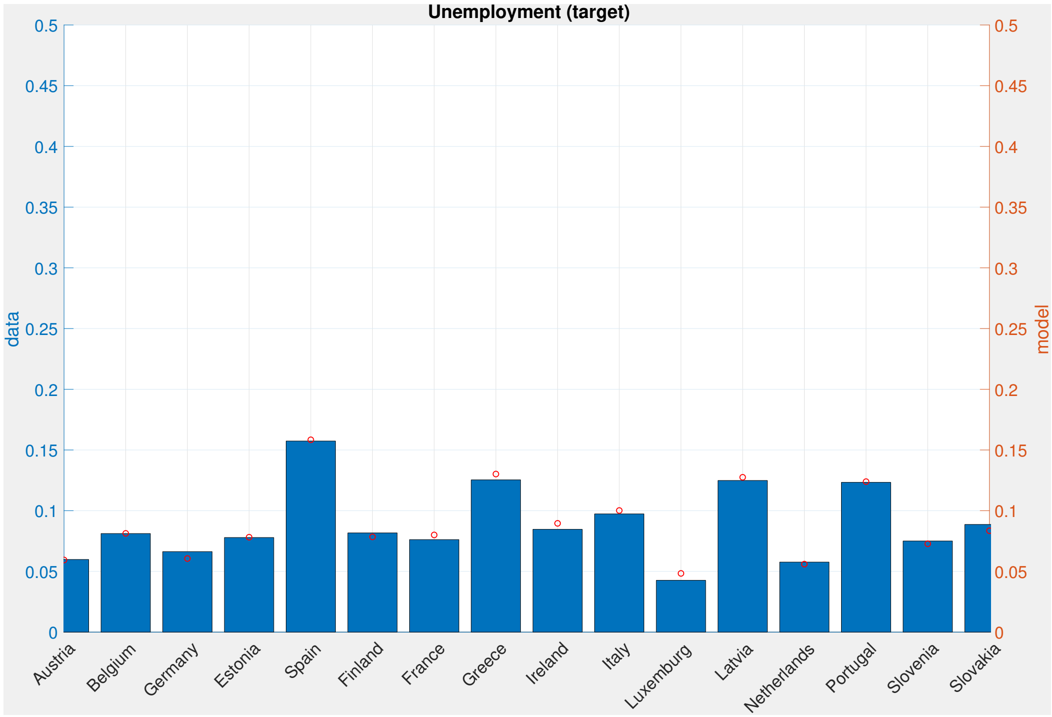

Fig. 1.3: Unemployment.

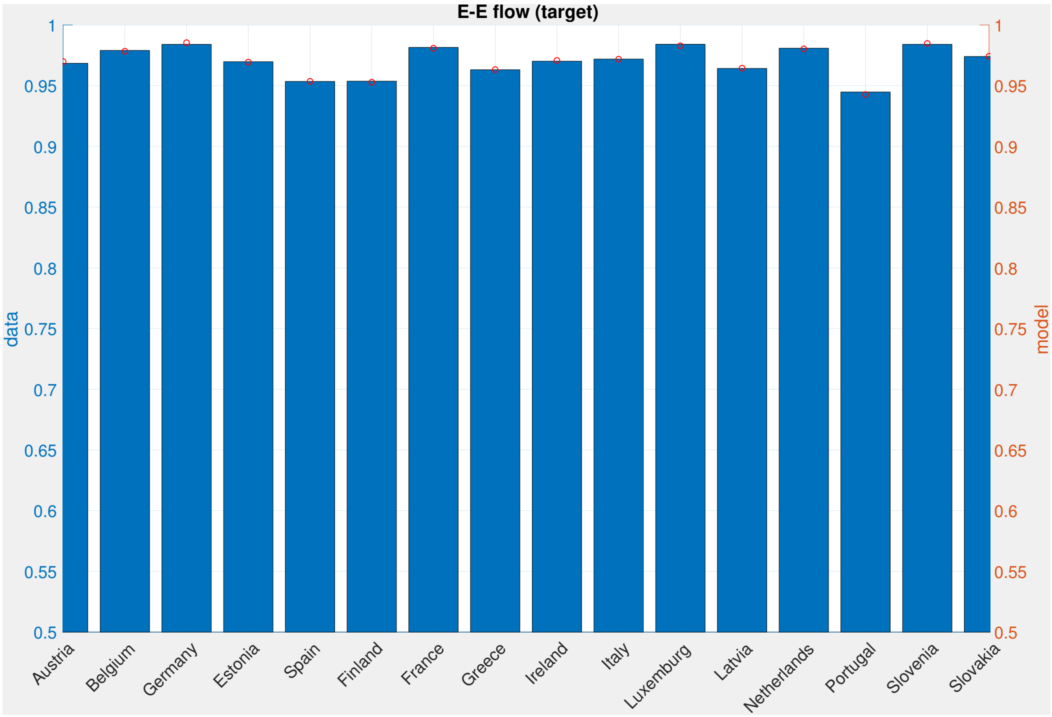

Fig. 1.4: Employment-Employment Flows.

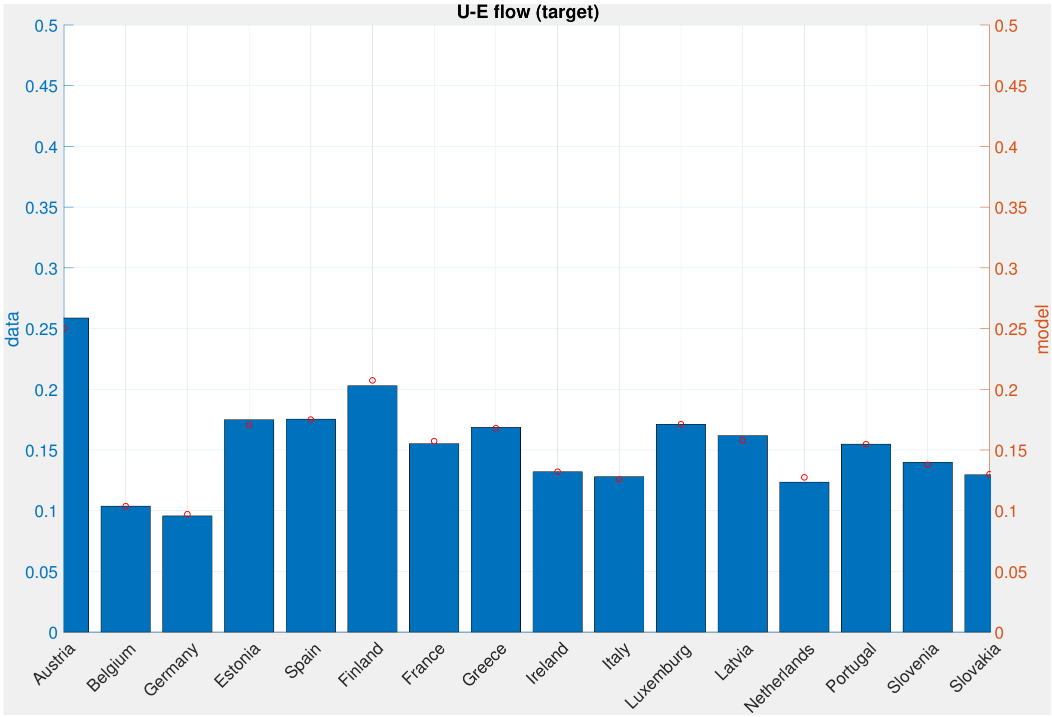

Fig. 1.5: Unemployment-Employment Flows.

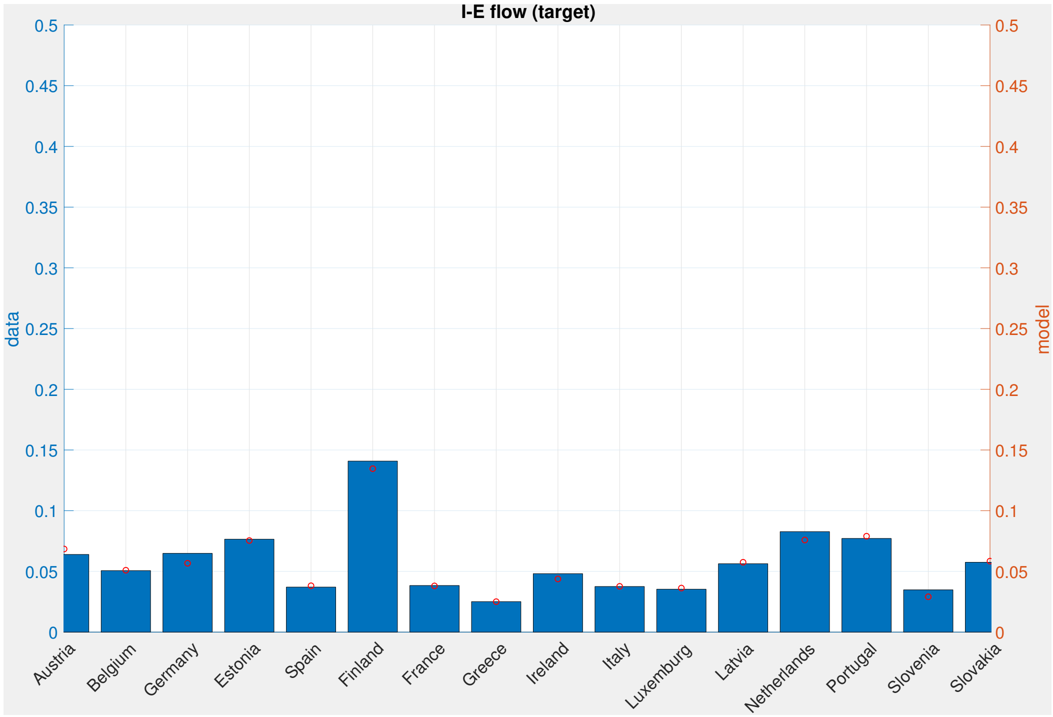

Fig. 1.6: Inactivity-Employment Flows.

The employment ratios were not targeted country by country, but the union average was. At the country level, the comparison with the data is shown in Figure 1.7. The model does very well in replicating the heterogeneity in stocks of employment that we observe in the data.

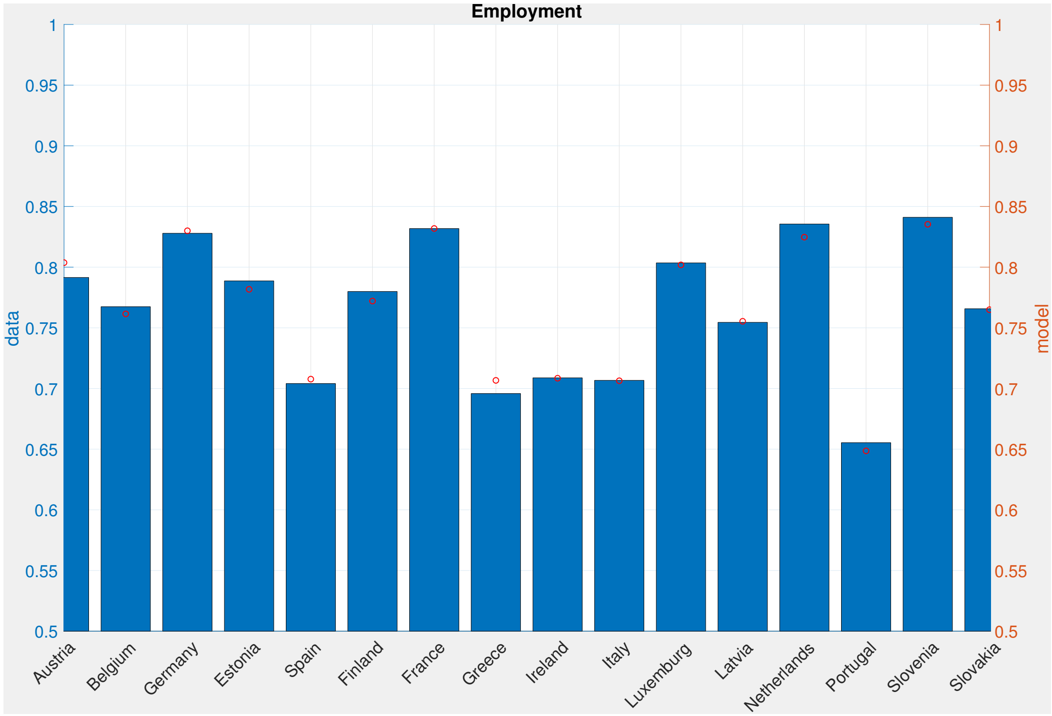

Fig. 1.7: Employment.

Given that the model shares of employed and unemployed agents is in line with the data counterparts, the model unemployment rate is also as in the data (Figure 1.8). Another important moment in the model is the average persistence of an unemployment spell in each country, which is not targeted directly by the calibration. The model predictions is shown in Figure 1.9. By and large the model performs well also along this dimension, though there are some cases where it underpredicts (Estonia, Greece, Portugal), and one where it overpredicts (Slovenia) the average duration of an unemployment spell by more than one quarter.

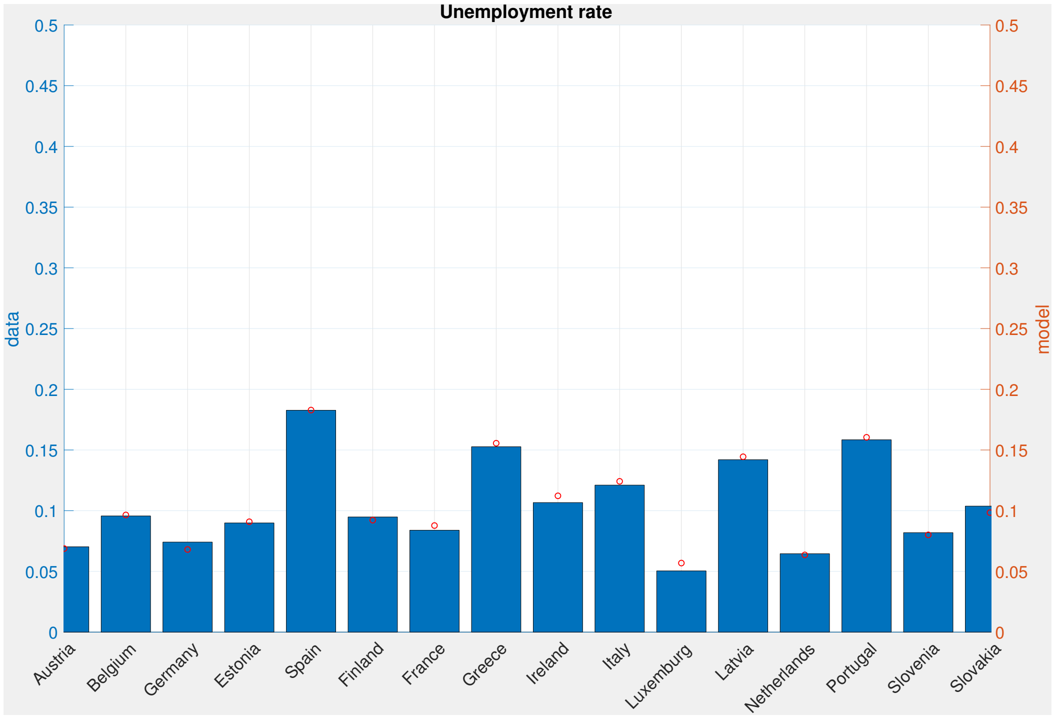

Fig. 1.8: Unemployment Rate.

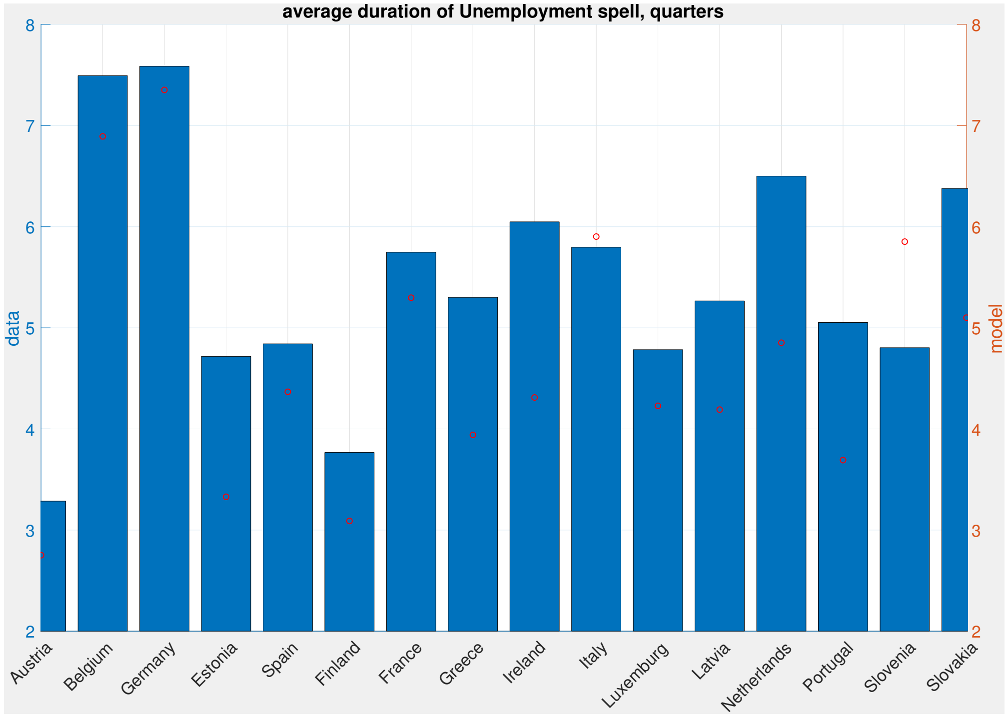

Fig. 1.9: Average Unemployment Duration.

For completeness, the persistence of inactivity is shown in Figure 1.10 below. Again, the model does a good job in replicating the data with only minor deviations.

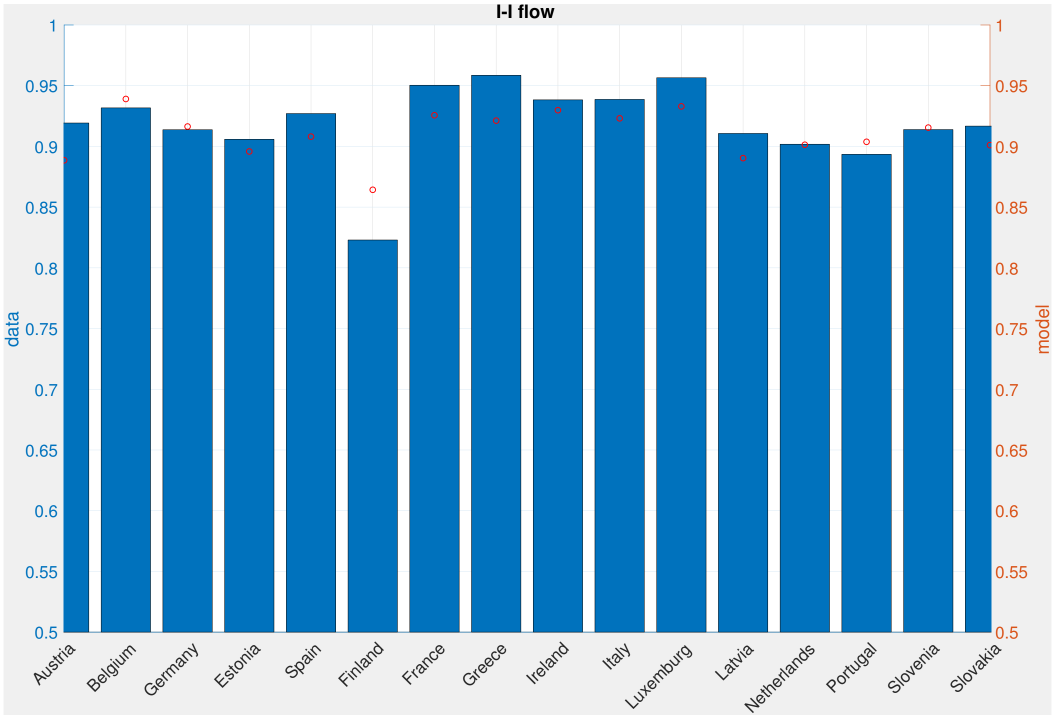

Fig. 1.10: Inactivity to Inactivity Flows.

1.4.3 Diversity of Labour Market Institutions

Our calibration makes apparent that labour market institutions vary substantially across countries. We visualize this in the Figures 1.11 to 1.13. Figure 1.11 shows the job arrival rate for non-searchers (λ_n^i, horizontal axis) and searchers (λ_u^i, vertical axis) for each of the calibrated economies. We observe that these two rates are correlated but their values differ substantially across countries. For example, in Finland, the Netherlands and Estonia the job finding rate for not actively searching individuals is higher than 10%, while in Greece, Italy, Spain and Luxembourg it is lower than 5%.

Figure 1.12 plots average the job finding rate for non-employed on the x-axis, but this time against the job separation rate σ^i on the y-axis. It hence gives an idea of the rigidity of the respective labour markets. Countries in the southeast have the high job finding and low separation rates. Countries in the northwest corner have high job destruction risk and low chances of finding a job while not working. Note for instance that while Germany and Spain have similar job finding rates for the non-employed, job destruciton in Spain in roughly 5 times higher, contributing to higher unemployment in Spain.

Finally, Figure 1.13 shows that the countries also differ substantially with respect to their unemployment benefit system. It plots the replacement rate vs. the average duration for which unemployed are eligible to receive benefits.

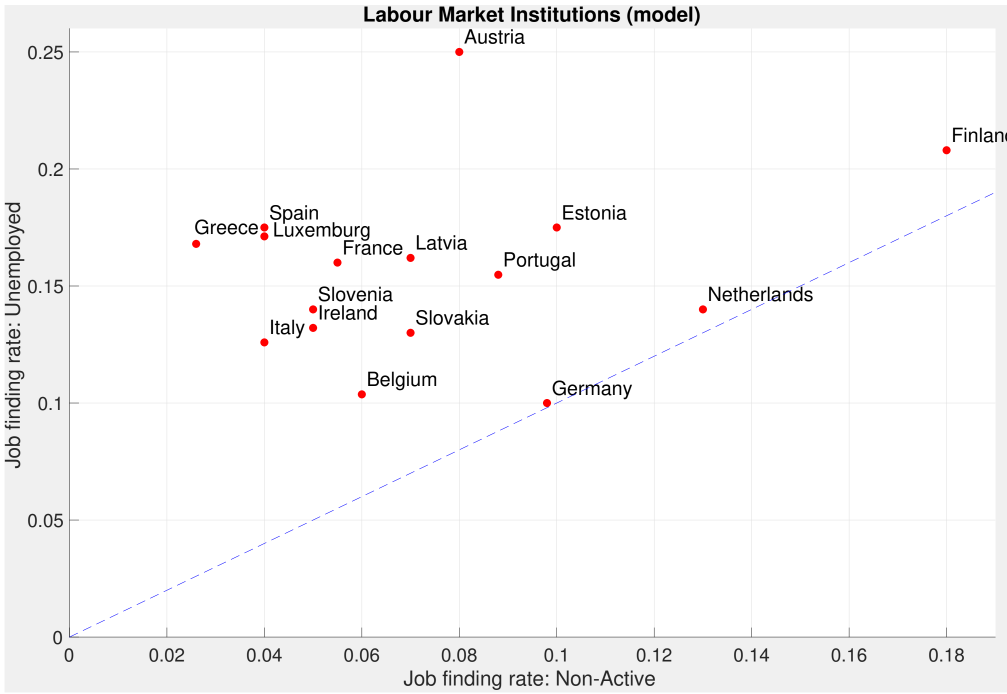

Fig. 1.11: Job Arrival Rates

1.5 Policy Experiments

Based on our calibration, which initializes the economy in $t = 0$, we are now able to perform several experiments and analyze the evolution of countries' labour markets and other macroeconomic variables under different configurations of unemployment policy for $t \geq 1$. In this part of the analysis we restrict the set of countries to a number of ten: Austria, Belgium, Germany, Spain, Finland, France, Ireland, Italy Luxembourg and Netherlands. These countries account for more than 90% of the Euro-Zone's total GDP.

As we pointed out earlier, a key motivation for an EUIS would be that it may provide insurance against country specific fluctuations in unemployment and, in particular, against fluctuations in the tax burden associated with its financing. In Subsection 1.5.1, we directly study the pure insurance effects of such policy from the viewpoint of individual European countries. In this experiment,

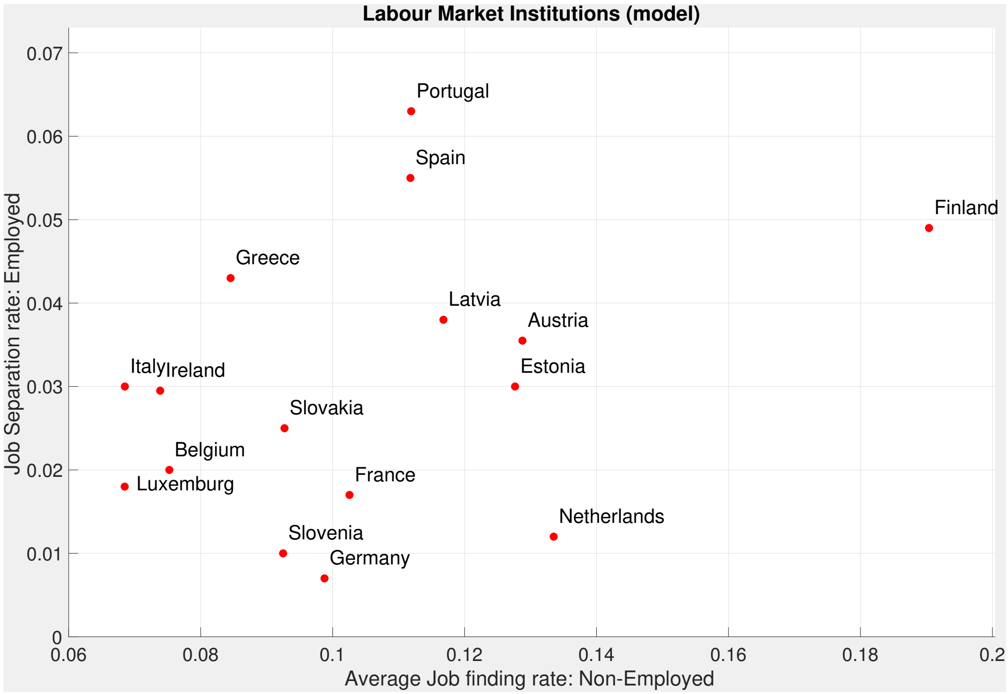

Fig. 1.12: Labour Market Rigidity

the EUIM only insures countries against country-specific fluctuations in unemployment, while keeping the benefit systems at the status quo. We will see that the welfare benefits of such an insurance system are very small.

As we have seen above, the ten countries are quite heterogeneous in their labour market institutions, suggesting that the individually optimal benefit systems could vary substantially across countries. If this was the case and in light of the result that the benefits of insuring against country specific shocks are small, one may doubt the desirability of a common European unemployment insurance system. In Subsection 1.5.2, we ask what the countries' individually optimal benefit systems are and whether an agreement to a common harmonized European benefit system could actually be achieved. For this means we compute the optimal unilateral reform separately for each country (in partial equilibrium). We find that the optimal unemployment benefit systems are surprisingly similar. In all countries it is optimal to provide an unlimited duration of eligibility and the optimal replacement rates vary between 20% and 45%.

Abstracting from other political constraints, one may argue that these slight differences are sufficient to implement reforms at the country level rather than at the union level. We show that such an argumentation is only true if one assumes that the Euro-Zone as a whole is a small open economy. If, on the other hand, prices are affected by unemployment policy, it neglects spill over effects across countries. In particular, we show that if all European countries would reform their system simultaneously and the capital market is required to clear at the union level, i.e. in general

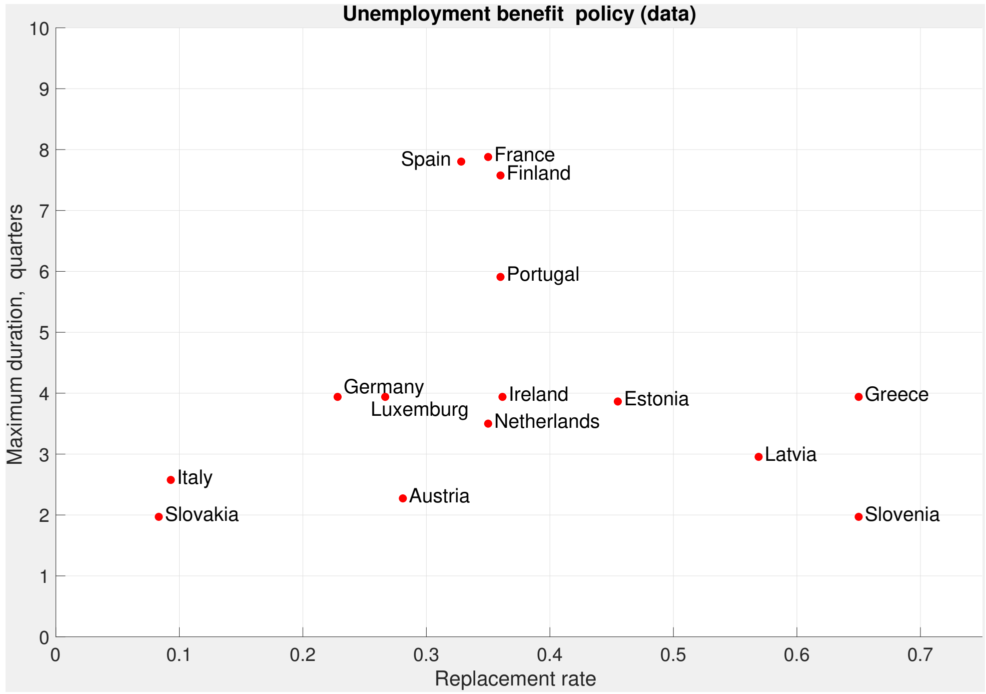

Fig. 1.13: National Unemplyoment Benefit Systems.

equilibrium, the very same UI benefit systems that are optimal in partial equilibrium turn out to be actually welfare worsening in most of the countries. The reason for this result is that increasing the generosity of the UI benefit system in almost all countries, reduces private savings and hence the aggregate, European, capital stock. As a consequence the marginal product of labour declines everywhere. This is bad for poor agents, who, irrespective whether they are currently employed or not, derive most of their lifetime income from wages.

In reality, the responsiveness of prices to policy changes is likely in between the two extreme cases we consider - the Euro-Zone as small open vs. a closed economy. The point we want to make is that if prices are at least to some degree affected by unemployment policy, lack of coordination across European member states may result in detrimental reforms, thus providing a rationale for centralizing unemployment policy at the European level. In the last subsection 1.5.3, we therefore search for a common harmonized European benefit system that is welfare improving in all countries of the union in general equilibrium. We consider two versions of this experiment that differ in the way the governments' budgets clear. First, we consider a fully harmonized system, where the tax rate in each country is the same and the ten individual government budget constraints (1.12) are replaced by a single European one. Such a system, by construction, results in transfers from countries with structurally low unemployment rates to countries with high unemployment and thus does not achieve unanimous agreement among member states. Interestingly, for some of the net payers, the welfare gains of such a reform are positive, suggesting that in these countries the current unemployment benefit system is far from optimal. In the second version of this experiment the tax rates which finance the harmonized system vary across countries such that each government's budget constraint (1.12) is satisfied. Such a system eliminates cross-country transfers and we show that with a replacement rate of 15% all countries are better off than in the status quo.

1.5.1 Insuring Country Level Fluctuations Only

As we mentioned above, the main argument for an EUIM can be that it may provide insurance against country level fluctuations in unemployment. This insurance might be very valuable as European countries (especially recently after the crisis) have a hard time to finance the increasing fiscal burden of unemployment using debt because of tighter deficit requirements. In the following experiment we provide the "best chance" for these insurance benefits to realise as we assume that individual countries have no access to any debt or savings to smooth out unemployment fluctuations.

The experiment is constructed as follows. At time $t = 0$ the country is in its steady state. At the end of this period, when all decisions are already made, it becomes aware that at $t = 1$ it is hit by a completely unanticipated negative shock. After the shock hits the country returns back to its steady state in a deterministic and gradual way.

Similarly to Krusell et al. (2017), we model shocks as hitting simultaneously TFP (A) and exogenous labor market flows (σ, λ_u and λ_n).[9] In particular, a deep recession will be modelled as a drop in TFP and job arrival rates and a rise in the separation rate. We model economic fluctuations in this way, because it is well-known that fluctuations of TFP alone are not able to generate large enough fluctuations of unemployment if output fluctuations are reasonable. This issue is amplified in our framework by the fact that job creation and job destruction are not modelled endogenously.

Given all these assumptions, note that after the shock is realised the economy is following a deterministic pattern and eventually converges back to its steady state. Hence, after the realisation of the shock agents have perfect foresight when solving their dynamic optimisation problems. We consider two cases: financial autarky and insurance through the EUIS. In financial autarky, along the transition the tax rate needs to adjust to balance the government budget constraint every period. In the case of the EUIS, we assume that countries can get full insurance against the rise in unemployment expenditure and thus can leave the tax rate at its steady state value. We assume that the shock is a zero probability event and therefore comes at a complete surprise to agents. This assumption also implies that leaving the tax rate with the EUIS at the steady state value is actuarially (from an ex-ante perspective) fair.

We want to note here that the zero probability assumption serves one purpose: To calculate an *upper bound* for the actual welfare gains that a EUIS would achieve when its sole purpose was to insurance against country level fluctuations in unemployment expenditures. If we relax this assumption and assume that the shock happens with some positive probability, an actuarially fair

[9] Note that in order to economize on notation we suppressed the time subscript in these parameters in the description of our model. In most of our analysis these parameters are indeed treated as constant. Only in the present subsection we deviate from this assumption.

EUIS would imply a higher tax rate than the steady state tax rate, i.e. countries would have to pay an insurance premium. This reduces consumption and thus welfare. It also would imply that agents would prepare themselves for the possibility of such a shock through higher savings, in which case the smoothing of taxes is less valuable than in case of the fully unanticipated shock.

We calculate the welfare effect of the introduction of this EUIS by comparing the welfare of autarky and the EUIS of each individual at the end of period $t = 0$, i.e. after learning that shocks will occur next period. That we calculate the welfare gains conditional on the negative shock happening again provides an upper bound for the actual insurance gains.

The argument that we want to make with this exercise is that even in a highly stylized scenario which in several dimensions is constructed in a way to increase welfare gains of an EUIS relative to what we would expect in reality, the gains of insuring country level fluctuations are small.

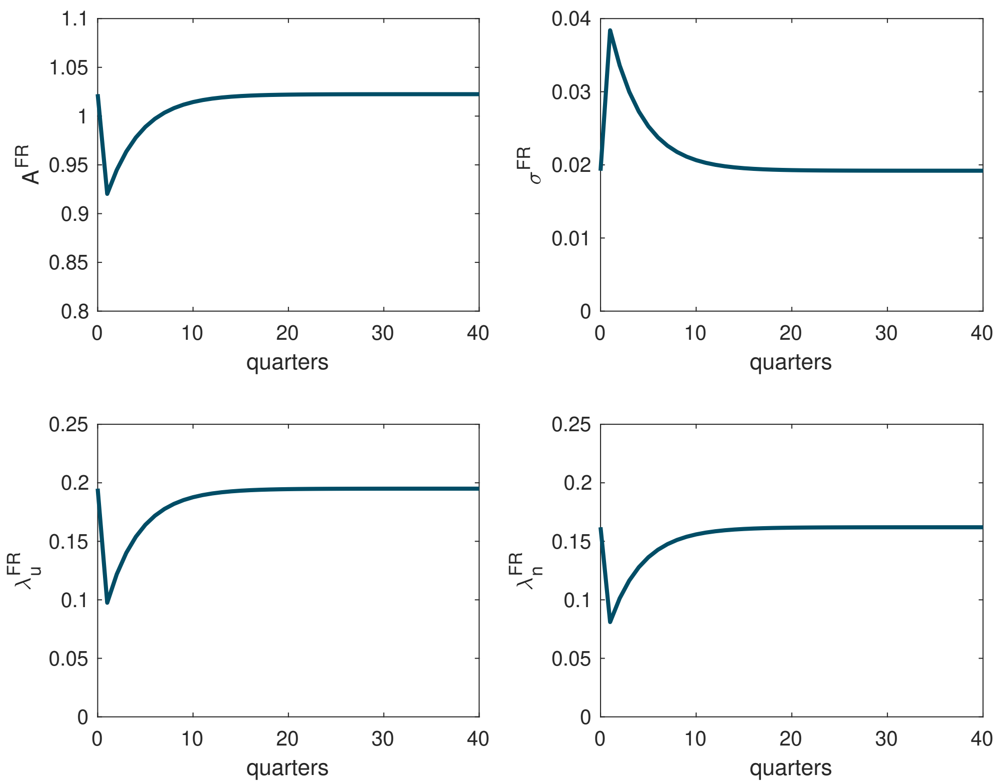

Fig. 1.14: Shock process in France

The Shocks. The combination of shocks has the following structure. Consider first total factor productivity in country i. At $t = 0$ the country is in steady state, i.e. $A_0^i = A^i$. At $t = 1$ a negative shock of size ε_A hits,

$$A_1^i = (1-\varepsilon_A)A^i.$$

The shock has persistence ρ_A and moves back to the steady state in a gradual and deterministic way,

$$\log(A_t^i) = \rho_A \log(A_{t-1}^i) + (1-\rho_A)\log(A^i) \quad \text{for } t \geq 1.$$

Similarly, the job separation rate and the job arrival rates are hit in $t = 1$,

$$\sigma_1^i = (1+\varepsilon_\sigma)\sigma^i$$
$$\lambda_{u,1}^i = (1-\varepsilon_{\lambda_u})\lambda_u^i$$
$$\lambda_{n,1}^i = (1-\varepsilon_{\lambda_n})\lambda_n^i.$$

After that they gradually return back to their steady state values, i.e. for $t \geq 1$

$$\sigma_t^i = \rho_\sigma \sigma_{t-1}^i + (1-\rho_\sigma)\sigma^i$$
$$\lambda_{u,t}^i = \rho_{\lambda_u}\lambda_{u,t-1}^i + (1-\rho_{\lambda_u})\lambda_u^i$$
$$\lambda_{n,t}^i = \rho_{\lambda_n}\lambda_{n,t-1}^i) + (1-\rho_{\lambda_n})\lambda_n^i$$

holds.

We consider a deep recession with TFP dropping by 10% ($\varepsilon_A = 0.1$), the job separation rate doubling ($\varepsilon_\sigma = 1$), and the job finding rates being reduced by half ($\varepsilon_{\lambda_u} = \varepsilon_{\lambda_n} = 0.5$). We further assume that $\rho_A = \rho_\sigma = \rho_{\lambda_u} = \rho_{\lambda_n} = 0.75$. Figure 1.14 depicts the evolution of the shock for the case of France.

The shock induces changes in labour markets, which are depicted in Figure 1.15. To some extent these responses are driven directly by the exogenous shock. For example a higher separation rate reduces employment by construction. But to a substantial degree they result from endogenous decisions of agents. For example, we observe that unemployment decreases at impact and only later rises above its steady state value (middle panel) and that at the same time inactivity increases at impact and gradually decreases later (lower panel). The reason is that because of lower wages and a lower likelihood to find a job even when searching, many agents are not willing to incur the utility loss of searching and instead decide not to participate. Only later, when economic conditions improved, they start searching for a job again. Further, many not separated agents decide to quit working because of the reduction in wages. In the upper panel we see that already during the first period (before the shock hits but after agents learned about the coming recession) employment declines.

If the country is in financial autarky, this mechanism is amplified through a rise in taxes, distorting incentives to (try to) work further. Figure 1.16 shows how taxes in France would evolve under autarky (solid line) as opposed to the case in which the country is fully insured against fluctuations in benefit expenditures (dashed line). In France such a shock would result in a gradual increase in the payroll tax that is from 1.9% to about 3.3% at the peak of the recession.

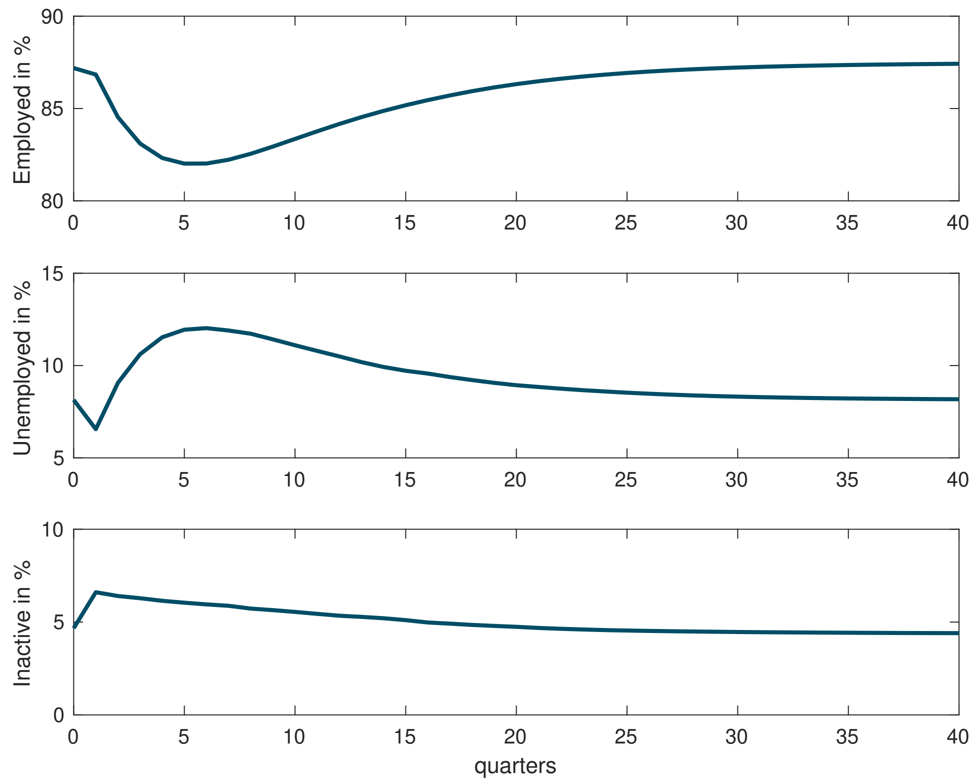

Fig. 1.15: Labor Market in France

We performed this very same exercise for all our ten countries. The tax rates in autarky for all countries are shown in Figure 1.17. We saw before that the steady state taxes which finance the country specific unemployment benefit systems differ substantially across country. This is a consequence of both different unemployment benefit policies and different labour market institutions that determine job creation and destruction. As we can see in Figure 1.17 these difference not only affect the steady state level of taxes but also their responses to shocks.

Table 1.4 shows the welfare gains of insuring against country level fluctuations in taxes. Remember that they are computed conditional on the shock happening. One obvious feature of these results is the very small magnitude of the average welfare gains. This is due to the fact that most welfare gains come from the small improvement of consumption smoothing for the employed. In fact, the only reason why also unemployed and non-active have positive welfare gains is because they may be employed, and thus paying taxes, in the future.

This exercise shows that risk-sharing by itself does not provide a strong rationale for the introduction of an EUIS. In light of this result one may doubt the desirability of a common European

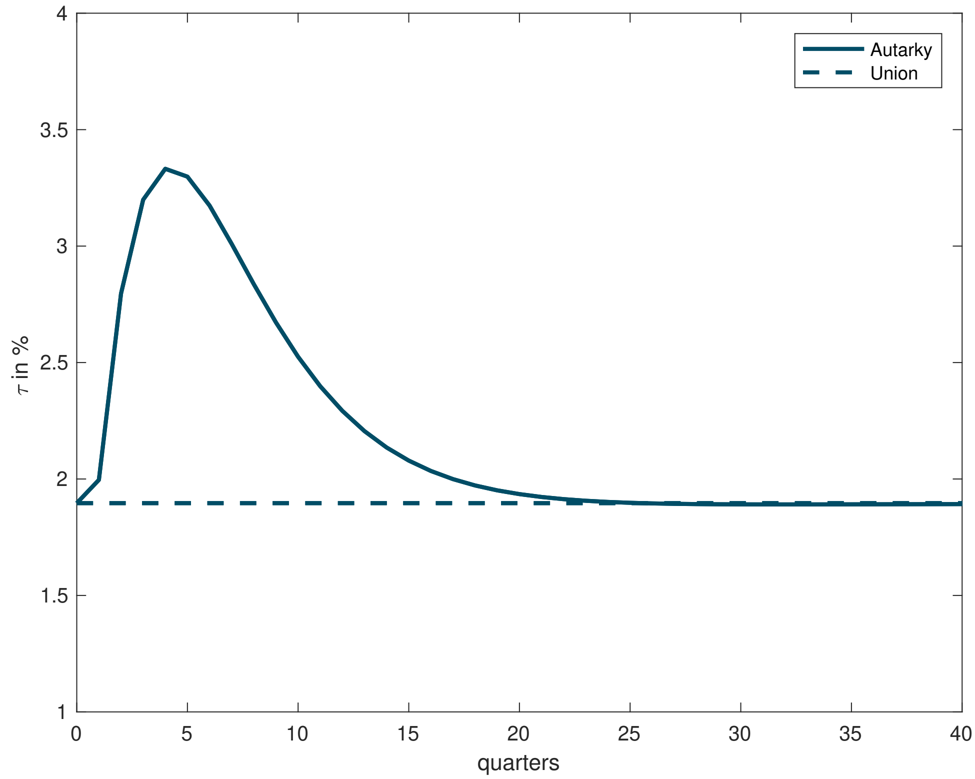

Fig. 1.16: Taxes in France

unemployment benefit scheme. Especially, since the observed heterogeneity in labour market institutions (see section 1.4) suggests that the optimal benefit system could differ substantially across countries, making it difficult to reach a consensus across Europe. In the next section we want to evaluate this claim and analyze how different optimal benefit systems are across countries.

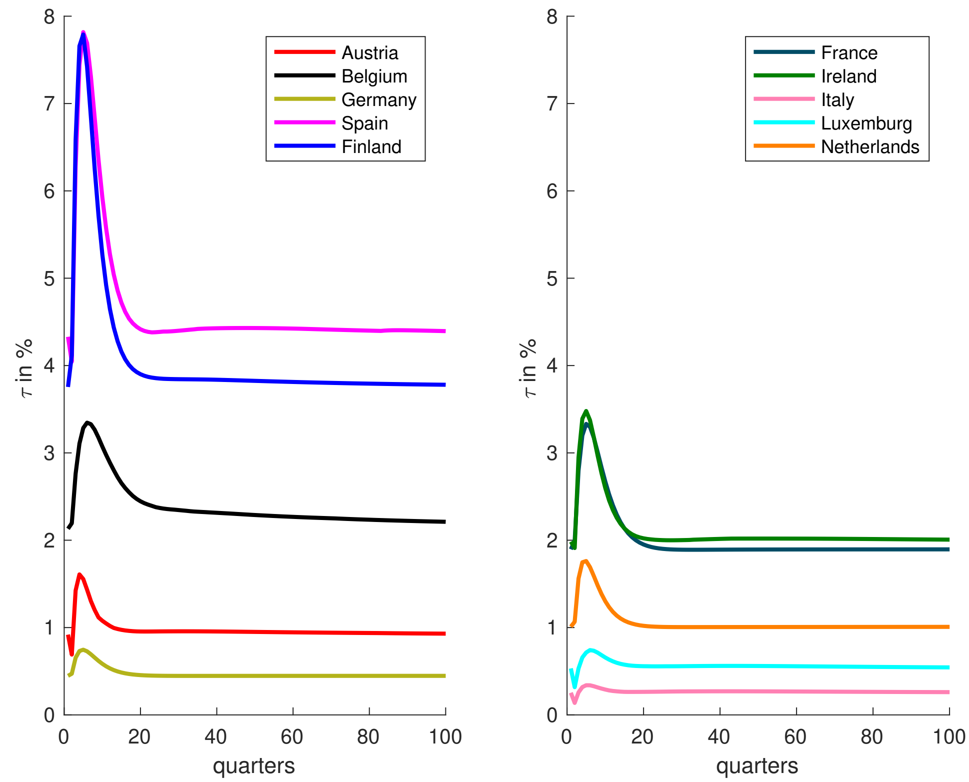

Fig. 1.17: Taxes in Autarky

	Employed	Un. Eligible.	Un. Non-Elig.	Non-Active	Total
Austria	0.05	0.04	0.04	0.03	0.04
Belgium	0.18	0.14	0.17	0.10	0.15
Germany	0.02	0.01	0.01	0.00	0.02
Spain	0.26	0.18	0.20	0.09	0.21
Finland	0.27	0.20	0.27	0.16	0.24
France	0.10	0.06	0.06	0.02	0.09
Ireland	0.11	0.06	0.08	0.04	0.09
Italy	0.01	0.01	0.01	0.01	0.01
Luxembourg	0.02	0.02	0.02	0.01	0.02
Netherlands	0.04	0.02	0.02	0.01	0.04

Table 1.4: Welfare gains (in % CEV) of insuring country level fluctuations

1.5.2 National Reforms of the Unemployment Benefit System

Before trying to harmonize European unemployment insurance systems, it is worthwhile to compute the optimal unemployment insurance system individually for each country. This gives us an idea whether the substantial heterogeneity in labour market institutions across Europe actually allow for such a harmonization or whether they make it impossible to reach a consensus across Europe. In the following subsection 1.5.2.1 we therefore compute the optimal national unemployment benefit reforms separately for each country, in partial equilibrium. It turns out that optimal national benefit systems are similar but not identical. To be specific, all countries find an unlimited duration of benefit receipt optimal while optimal replacement rates vary between 20% and 45%.

One might think that even if optimal policies are similar, countries are still better off by reforming the system to their individual optimum rather than to join a common European scheme. In subsection 1.5.2.2 we show that in such a scenario it is important to take into account that the European Union has a common capital market. In particular, we show that sticking to the partial equilibrium analysis would result in large imbalances between capital and savings across Europe, unlikely to be sustained in reality. The reason is that the optimal partial equilibrium reforms are rather generous, resulting in a large decline in precautionary savings. In section 1.5.2.3 we show that in general equilibrium this would result in a substantial decline in the capital stock. The induced changes in equilibrium prices, in particular a substantial decrease in Europe-wide wages would render seven out of the ten reforms that seem optimal in partial equilibrium, actually welfare worsening. This suggests that optimal unemployment insurance should be designed at the broader European level.

1.5.2.1 Optimal Unilateral Reform

In this section we compute the optimal reform for each country separately. For each country i we ask the question: What is the optimal unilateral once-and-for-all change in $(\bar{b}^i, \mu^i)$ if only country i was to change its benefit system and the other countries would stick to the status quo? This analysis is done in partial equilibrium, i.e. we assume that a single country does not affect the equilibrium interest rate when changing its unemployment benefit policy even thought the savings decisions of its citizens change. This implies that the marginal product of capital and hence the capital-labour ratio is pinned down by the interest rate and as a consequence also wages are unaffected by the change in policy.

We assume that the government maximizes the utilitarian welfare of its citizens. Formally, the government in country i chooses a pair of policy parameters $(\bar{b}^i_1, \mu^i_1)$ with $\bar{b}^i_t = \bar{b}^i_1$ and $\mu^i_t = \mu^i_1$ for all $t \geq 1$ such that social welfare is maximized,[10]

$$\max_{(\bar{b}^i_1, \mu^i_1)} SW(\bar{b}^i_1, \mu^i_1) = \max_{(\bar{b}^i_1, \mu^i_1)} \sum_{x \in X} \sum_{z \in Z} \int_0^\infty V^i_0(a, z, x; \bar{b}^i_1, \mu^i_1) \zeta^i_0(a, z, x) da.$$

[10] Here we add the policy parameters as arguments in the value function to make it explicit that the values depend on policy parameters.

Thereby, individually optimal policies, firm production plans and taxes adjust such that all equilibrium conditions in Definition 1.1 are satisfied. Note that for each individual we compute the value in the initial period and therefore take into account the whole transitional dynamics to the new steady state.

In order to be able to interpret the welfare gains associated with the policy reform, we translate them into consumption equivalent variation. In particular, $\Delta^i(a,z,x)$ defines the per period percentage increase in consumption that you would need to give an individual with initial state (a,z,x) when the benefit system is kept at the status quo such that he is indifferent between this status quo and the optimal reform. The aggregate welfare gain is then defined as

$$\Delta^i = \sum_{x \in X} \sum_{z \in Z} \int_0^\infty \Delta^i(a,z,x)\zeta^i(a,z,x)da.$$

Similarly, we define the aggregate welfare gain of the employed, unemployed eligible, unemployed non-eligible and inactive as

$$\Delta^i_x = \frac{\sum_{z \in Z} \int_0^\infty \Delta^i(a,z,x)\zeta^i(a,z,x)da}{\sum_{z \in Z} \int_0^\infty \zeta^i(a,z,x)da} \quad \text{for } x \in \{e, u^e, u^n, n\}.$$

Table 1.5 shows the current benefit policy and the optimal reform in each country along with the taxes that finance this policy. For the optimal reform we report the new steady state taxes τ^i_∞. Note, however, that along the transition taxes vary in order to clear the government budget period by period.

We see that despite the substantial heterogeneity in labour market institutions, optimal unemployment benefit policies are surprisingly similar. In particular, in all countries an unlimited duration of eligibility is optimal. This policy eliminates the risk of not finding a job before losing eligibility. There is some variation in optimal replacement rates but all are in the range of 20 to 45 percent.[11] The main difference are the tax rates that finance the rather similar benefit policies.

Country	Status Quo			Optimal Reform			Δ
	$1/\mu^i_0$	b^i_0	$\tau^i_0(\%)$	$1/\mu^i_1$	b^i_1	$\tau^i_\infty(\%)$	
Austria	2	0.28	0.92	∞	0.45	4.61	1.51
Belgium	20	0.37	2.13	∞	0.25	1.36	0.16
Germany	4	0.23	0.45	∞	0.25	3.15	1.40
Spain	8	0.33	4.43	∞	0.45	11.26	2.27
Finland	8	0.36	3.75	∞	0.20	0.90	0.33
France	8	0.35	1.90	∞	0.35	3.75	1.18
Ireland	4	0.36	1.97	∞	0.45	8.37	1.72
Italy	3	0.09	0.25	∞	0.45	9.22	3.23
Luxembourg	4	0.27	0.53	∞	0.45	3.52	1.07
Netherlands	4	0.35	1.00	∞	0.25	4.53	0.89

Table 1.5: Optimal National Reforms of the Benefit System

[11] Our optimization routine optimized over increments of 0.05 in the replacement rate dimension.

For example, there are five countries for which a replacement rate of 45% is optimal. In Ireland, Italy and Spain the more generous benefit scheme comes along with large increases in the tax rate, which eventually reach 8.4%, 9.2%, and 11.3%, respectively. In Austria and in Luxembourg, on the other hand, the same benefit system is optimal but the tax rates only reach values of 4.6% and 3.5%, respectively. The reason for these are the structurally different labour market institutions. In particular, in section 1.4 we saw that in Spain the job separation rate is much higher, while in Ireland and Italy the job finding rates are much lower then in the other countries.

The last column shows the aggregate welfare gains. In Belgium this gain is less than 0.2% of consumption equivalent variation as the current benefit system is not too far from the optimal one. In particular the duration of eligibility is much higher than in all the other countries. Still there are some gains from extending the duration to infinity and slightly reducing the replacement rate. On the other extreme the welfare gains for Italy are large, about 3.2% CEV. According to our model the current benefit system in Italy with an average duration of eligibility of 3 quarters and a replacement rate of less than 10% is way too restrictive.

Of course, not every individual in a country benefits from the reform in the same way. Table 1.6 shows the welfare gains at a more dis-aggregated level. We see that in countries where the unemployment system becomes more generous (Austria, Spain, France, Ireland, Italy, Luxembourg) the main beneficiaries are the eligible unemployed as they are the ones who are most directly affected. However, even if the gains are smaller, also the other agents benefit from the reform as they might be eligible unemployed in the future.

	Employed	Un. Eligible.	Un. Non-Elig.	Non-Active	Total
Austria	1.40	3.06	2.62	1.57	1.51
Belgium	0.13	0.28	0.70	0.18	0.16
Germany	1.35	5.20	1.42	1.11	1.40
Spain	2.07	4.16	3.87	1.74	2.27
Finland	0.34	0.14	0.51	0.37	0.33
France	1.11	2.63	1.44	0.84	1.18
Ireland	1.54	4.52	4.37	1.40	1.72
Italy	3.06	9.42	6.15	2.50	3.23
Luxembourg	0.94	4.05	2.56	1.12	1.07
Netherlands	0.85	2.98	0.79	0.71	0.89

Table 1.6: Welfare gains (in % CEV) of individually optimal reforms

Finland has the most restrictive reform. Even if the duration of eligibility is extended, the replacement rate is substantially reduced to 20%. As a consequence the eligible unemployed benefit the least from the reform. The other agents benefit more through the reduction in taxes. This is directly true for the employed. But also the non-eligible unemployed and the inactive like the reform more than the eligible unemployed. The reason is that these agents will sooner earn wage income and pay taxes than they will receive unemployment benefits. Remember that the only way to gain eligibility for benefits for these agents is by going through employment and being exogenously separated from the job.

1.5.2.2 Simultaneous Reforms - The Euro-Zone as a Small Open Economy

Remember that we computed the optimal reform above sequentially, each time from the perspective of an individual country, which takes interest rates and wages as given. We treated this single country as a small open economy and assumed that eventual capital in- or outflows resulting from the reform are absorbed from outside the country.

We now ask the question: What happens if all countries would simultaneously reform their system to the individually optimal reforms computed above?

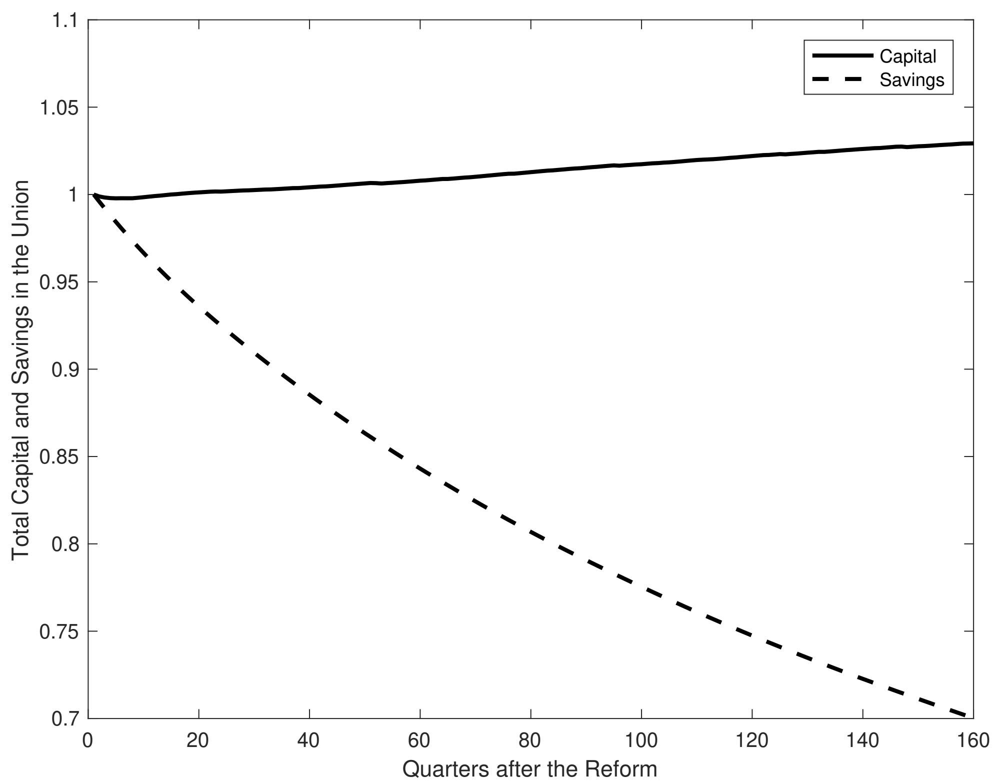

Fig. 1.18: Capital and Savings in the Union under Open Economy Assumption

To answer this question an assumption on how the capital market clears is necessary. An extreme assumption would be that the Euro-Zone as a whole is a small open economy and therefore prices would not change even if all countries would change their unemployment benefit systems at the same time.[12] In such a case the answer is simple: The same set of reforms as the one computed above (Table 1.5) is optimal and it will result in the same welfare gains (Table 1.6). However, the reforms would imply large imbalances between total European savings and total European capital. Figure 1.18 shows that in this scenario private savings decline substantially while the capital stock actually rises as a result of the reforms. Forty years after the reform about one third of the total European capital stock is financed from outside the union. The unlimited duration of eligibility reduces the risk of individuals substantially and therefore leads to a reduction in precautionary savings. On the other hand, Figure 1.19 shows that the new policies, on aggregate, induce many inactive agents to start searching. As some of them eventually find jobs also employment and thus aggregate effective labour increases. As a consequence the marginal product of capital increases and firms demand more capital.

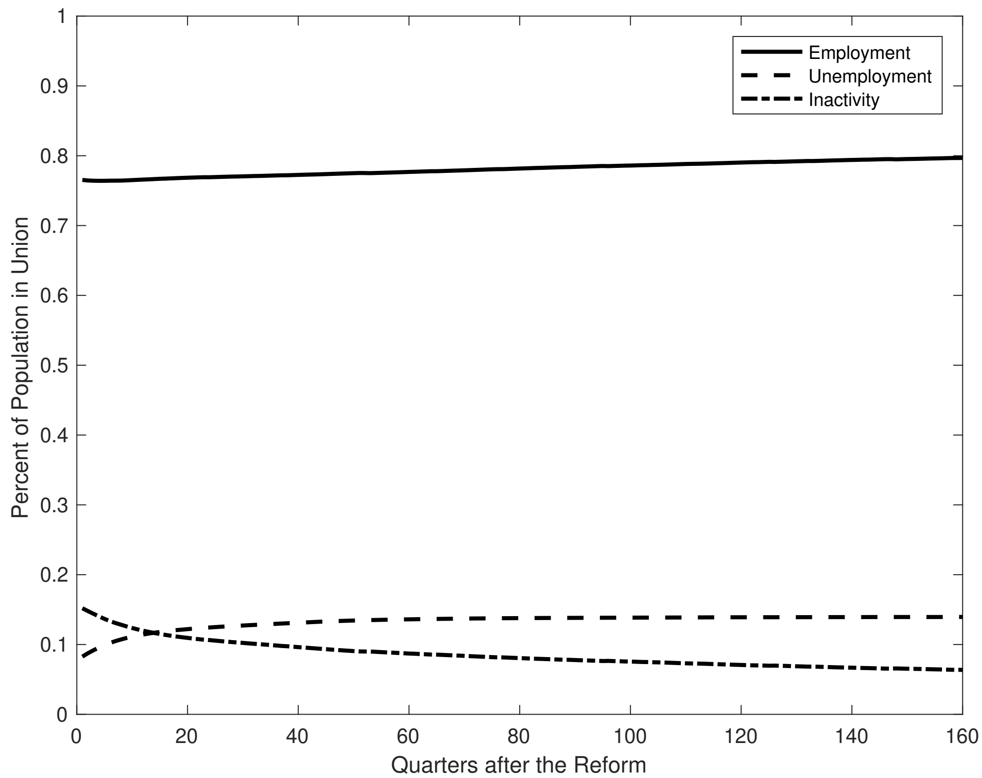

Fig. 1.19: Aggregate Labour Market States under Open Economy Assumption

[12] Note that in such a case, also at $t = 0$ the capital market need not clear and one could instead pick another world interest rate. We experimented with interest rates in a reasonable range and this did not affect optimal policies much.

1.5.2.3 Simultaneous Reforms - The Euro-Zone as a Closed Economy

While one can challenge the view that the Euro-Zone as a whole is a closed economy, one can certainly reject the hypothesis that it is too small to have any impact on the interest rate. In particular, the imbalances in savings and capital which we see in Figure 1.18 are way beyond what we observe in the data.

Therefore, we now assume that there cannot be imbalances in capital and savings for the union as a whole. In particular, we solve for the path of interest rates $\{r_t\}_{t=1}^{\infty}$ that clears the capital market at the union level (condition (1.13)). This change in the interest rates will change optimal firm production plans and optimal individual behaviour and it will change the equilibrium path of wages in all countries.

The resulting welfare effects are strikingly different. In Table 1.7 we observe that general equilibrium effects not only have negative effects in eight countries, in seven out of the ten countries they even reverse the sign of the welfare gains. Only in Belgium and in Finland agents benefit from general equilibrium effects. In Luxembourg the gains are still positive but substantially lower than under the open economy assumption.

	Employed	Un. Eligible.	Un. Non-Elig.	Non-Active	Total
Austria	-0.39	1.12	0.43	-0.21	-0.30
Belgium	1.16	0.96	1.19	1.03	1.12
Germany	-0.82	2.72	-1.20	-0.82	-0.77
Spain	-0.39	1.43	0.72	-0.36	-0.17
Finland	1.83	1.53	1.96	1.69	1.78
France	-0.44	0.98	-0.33	-0.51	-0.37
Ireland	-0.31	2.32	1.55	-0.21	-0.12
Italy	-0.32	5.05	1.56	-0.32	-0.10
Luxembourg	-0.01	2.76	1.08	0.24	0.10
Netherlands	-3.09	-1.48	-3.95	-3.15	-3.09

Table 1.7: Welfare gains (in % CEV) of Reforms under Closed Economy Assumption

How can these results be explained? First, note that the aggregate capital stock is now substantially lower than in the open economy case. Capital is now solely financed by the saving of Euro-Zone citizens and as mentioned above the reforms on average reduce savings. Figure 1.20 depicts the evolution of capital (savings) when the capital market clears at the union level.

This reduction in the capital stock causes an increase in the marginal product of capital but reduces the marginal product of labour. As a consequence, interest rates rise but wages decline after the reform (see Figure 1.21). Note that wages vary across countries but their relative proportions are unaffected. To be specific, for any pair $i, j \in \{1, 2, ..., I\}$ and any $t \geq 0$ it holds that $w_t^i / w_t^j = A^i / A^j$. This implies that wages decline in all countries of the union.

The movements in prices are good for agents who derive most of their net present live time income from capital but bad for those whose main income source are wages. Welfare gains are computed in percent of initial consumption implying that if an initially poor and an initially rich agent experience the same consumption increase in absolute value, the welfare gain for the poor

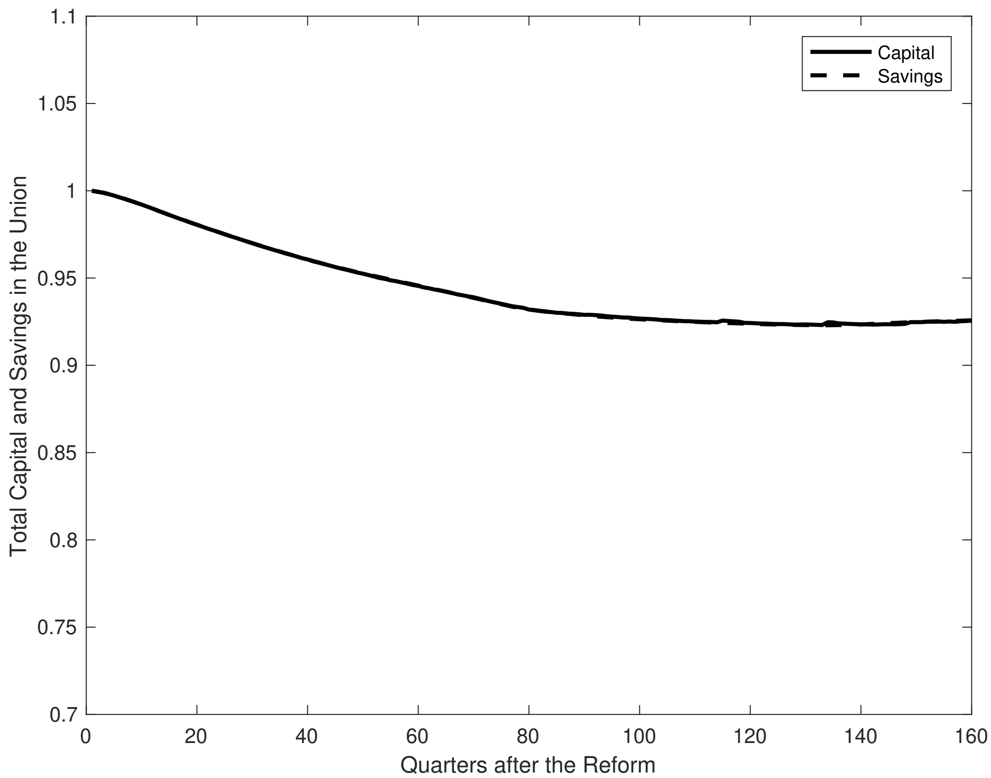

Fig. 1.20: Capital and Savings in the Union under Closed Economy Assumption

agent will be higher. Hence, what matters mostly for the aggregate welfare effects are the changes in the income composition of poor agents. Agents with little assets do not gain much from the higher interest rate but they lose substantially from lower wages. In the right panel of Figure 1.21 we see that in the long run wages decrease by about 1.5%, the same order of magnitude as the average decline in welfare through GE effects. It is important to note that by and large all groups in a country lose or benefit from general equilibrium effects in a similar way (compare Tables 1.6 and 1.7). At first glance this seems surprising since non-eligible unemployed and non-active agents currently do not have wage income while the other two groups do, the employed directly and the eligible unemployed indirectly as their unemployment benefits are proportional to wages. The reason for this result is that mobility across labour market states is higher than across wealth. For example, a currently poor non-eligible unemployed will sooner be employed and get wage income than he will be rich enough to derive most of his lifetime income from capital returns.

Now why are there two countries, Belgium and Finland, who benefit from the general equilibrium effects? Because these two countries are relatively rich on average and have a relatively equal

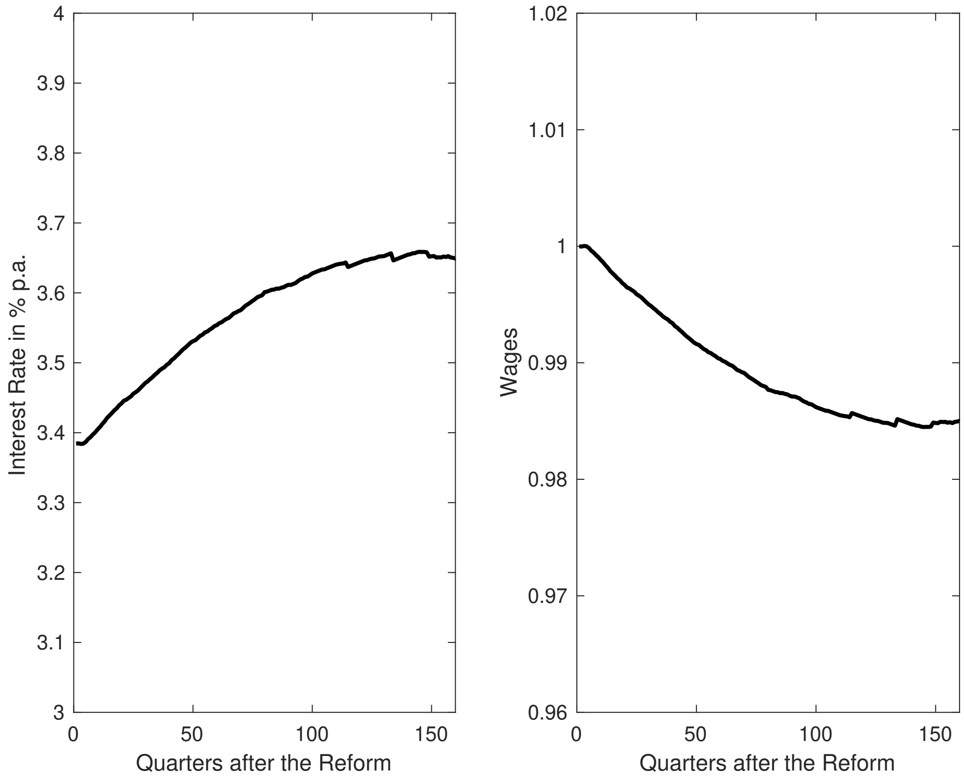

Fig. 1.21: Equilibrium Prices under Closed Economy Assumption

distribution of wealth. This means that the wealth owned by consumption poor agents is higher than in the other countries and therefore their consumption losses are not big enough to offset the overall consumption gains.

The general equilibrium feedback effects from prices in turn change the behaviour of agents. For example, in Figure 1.22 we observe that the decline in inactivity is muted compared to the open economy case and that employment declines, while before it was increasing. Higher interest income allows inactive agents to run down their assets at a lower pace and at the same time lower wages make employment less attractive.

In sum, the results of this section are twofold: (i) According to our model current national unemployment benefit systems are not optimal and for some countries there is a large potential for welfare improvement through policy reform; (ii) An integrated European capital market causes unemployment policies in some European countries to affect the citizens of other European countries even under the assumption that labor is not mobile. The latter implies that lack of coordination

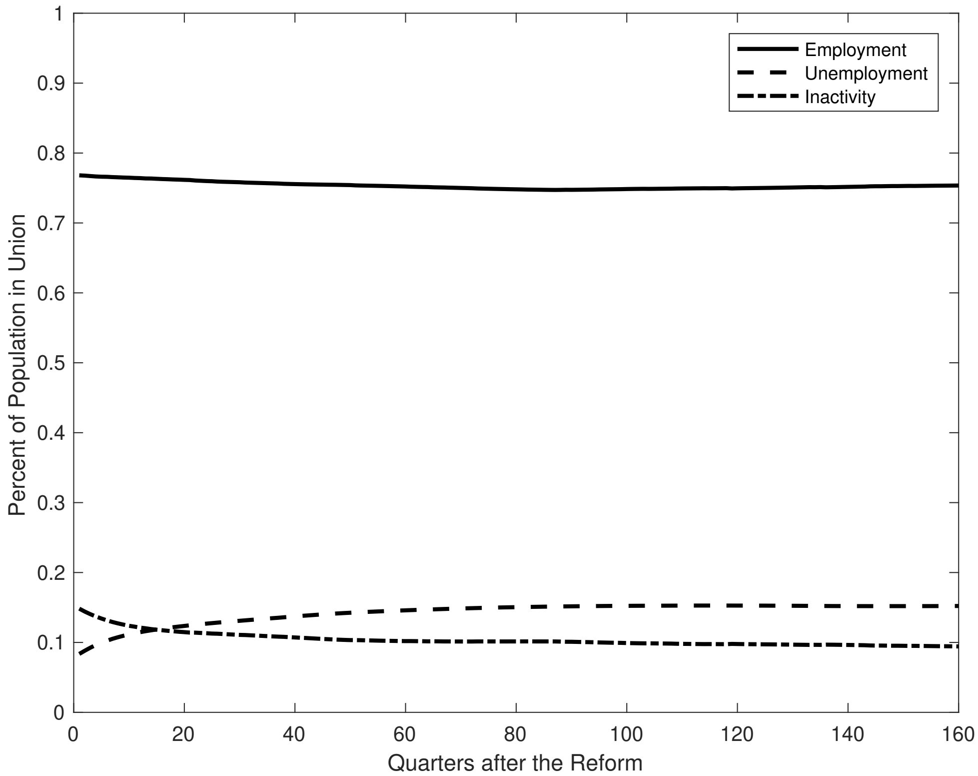

Fig. 1.22: Aggregate Labour Market States under Closed Economy Assumption

across member states may result in detrimental reforms and thus provide a rationale to centralize unemployment policy at the European level. This is the direction we explore next.

1.5.3 Harmonized European Unemployment Insurance Scheme

Can we find a harmonized European unemployment benefit system that is welfare improving in all countries of the union? We find that the answer to this question depends on how such a system is financed. If contribution payments vary such that cross-country transfers are eliminated, we find that a harmonized benefit system with an unlimited duration and a replacement rate of 15% is welfare improving in all countries. On the other hand, we could not find such a system, when it is financed jointly at the union level.

1.5.3.1 Joint Financing on the Union Level

Let us first consider jointly financed benefit systems. In this experiment, we replace individual countries' budget constraints with a common European one. Instead of I government budget constraints (equation (1.12)) which solve for I different tax rates, there is only one tax rate that clears the union budget constraint

$$\tau_t \sum_{i=1}^{I} \omega_t^i L_t^i = \bar{b} \sum_{i=1}^{I} \omega_t^i \sum_{z \in Z} z_t \int_0^\infty \zeta_t^i(a, z, u^e) da \quad \forall t \geq 0. \tag{1.14}$$

Note that both the tax rate and the replacement rate are independent of i. As we mentioned above not every country would agree to this form of financing no matter what the benefit system is.

In Table 1.8 we show the results for the benefit system with an unlimited duration of eligibility and a common replacement rate of 15%. We see that while most countries gain from the reform, there are three exceptions, all of whom are net payers: Austria, France and the Netherlands. Interestingly, not all countries which are net payers lose from the reform. In particular the welfare gains in Belgium, Germany, Finland and Ireland are positive even though these countries pay substantial transfers, between 0.14% and 0.87% of their respective GDP. This result is another indicator that current unemployment benefit policies are far from optimal in some of the countries.

Country	Status Quo			Optimal Reform			Δ	Transfer/GDP
	$1/\mu_0^i$	$\bar{b}_0^i$	$\tau_0^i(\%)$	$1/\mu_1^i$	$\bar{b}_1^i$	$\tau_\infty^i(\%)$		
Austria	2	0.28	0.92	∞	0.15	1.47	-0.29	-0.51
Belgium	20	0.37	2.13	∞	0.15	1.47	0.25	-0.71
Germany	4	0.23	0.45	∞	0.15	1.47	0.20	-0.14
Spain	8	0.33	4.43	∞	0.15	1.47	1.45	0.74
Finland	8	0.36	3.75	∞	0.15	1.47	1.04	-0.87
France	8	0.35	1.90	∞	0.15	1.47	-0.06	-0.16
Ireland	4	0.36	1.97	∞	0.15	1.47	0.68	-0.09
Italy	3	0.09	0.25	∞	0.15	1.47	0.87	0.29
Luxembourg	4	0.27	0.53	∞	0.15	1.47	-0.40	-0.64
Netherlands	4	0.35	1.00	∞	0.15	1.47	0.17	0.07

Table 1.8: Harmonized Benefit System Financed Jointly

1.5.3.2 Country-Specific Contribution Payments

Let us next consider the case of varying contribution payments across countries, which clear each country's government budget constraint separately. To be specific, in this experiment we require that condition (1.12) holds for each $i \in \{1, 2, ..., I\}$. Table 1.9 shows that in this case the very same benefit system (unlimited duration, replacement rate of 15%) that we considered above is welfare improving in all countries of the union.

To understand why this mix of taxes and benefits is welfare improving in all countries it is worthwhile to remember three findings of our analysis so far: (i) European countries have structurally different labour market institutions which as a consequence lead to very different long term averages in employment, unemployment and inactivity (section 1.4); (ii) Despite these differences, the optimal unilateral reforms are surprisingly similar with an unlimited duration of unemployment benefits and replacement rates between 20% and 45% (section 1.5.2.1). (iii) If all countries were to reform their benefit systems by themselves to their respective individual optima this would result in a substantial decline in the capital stock. As a consequence wages across Europe would be lower and especially poor agents, who derive most of their lifetime income from labour, would experience large welfare losses (section 1.5.2.3).

Country	Status Quo			Optimal Reform			Δ
	$1/\mu_0^i$	$\bar{b}_0^i$	$\tau_0^i(\%)$	$1/\mu_1^i$	$\bar{b}_1^i$	$\tau_\infty^i(\%)$	
Austria	2	0.28	0.92	∞	0.15	0.73	0.23
Belgium	20	0.37	2.13	∞	0.15	0.45	0.92
Germany	4	0.23	0.45	∞	0.15	1.27	0.38
Spain	8	0.33	4.43	∞	0.15	2.53	0.62
Finland	8	0.36	3.75	∞	0.15	0.22	2.03
France	8	0.35	1.90	∞	0.15	1.23	0.11
Ireland	4	0.36	1.97	∞	0.15	1.34	0.79
Italy	3	0.09	0.25	∞	0.15	1.90	0.60
Luxembourg	4	0.27	0.53	∞	0.15	0.55	0.32
Netherlands	4	0.35	1.00	∞	0.15	1.57	0.08

Table 1.9: Optimal Harmonized Benefit System Financed at the Country Level

These three findings help in the design of a collectively optimal unemployment benefit system. The first finding says that for a system to be politically sustainable we need varying contribution payments across countries. As we have seen, a common, jointly financed, system would result in transfers from countries with structurally low unemployment to countries with structurally high unemployment. The former, at least some of them, would not participate in such a scheme. We therefore demand from the system that it eliminates cross-country transfers. This is achieved through varying tax rates that clear government budgets at the national level, i.e. that satisfy condition (1.12). We again observe that in countries with structurally weak labour market institutions such as Spain or Italy these tax rates are substantially higher than in other countries with more efficient institutions such as Belgium or Finland.

The second finding suggests that it makes sense to harmonize the benefit system and we indeed find that an unlimited duration of eligibility and a replacement rate of 15% is welfare improving in all countries. As before the unlimited duration eliminates the risk of losing eligibility before finding a job. However, the replacement rate is lower than in any of the unilaterally optimal reforms computed above, where the minimal replacement rate was the Finnish one with 20%. The third finding explains why this is the case. We have seen that a too generous benefit system discourages savings, which in general equilibrium reduces wages, the main income source of poor agents. In Figure 1.23 we see that in the long-run the capital stock is reduced by only about 2% and 3%, while before it was between 7% and 8%. As a consequence the effects on prices are muted as can be seen in Figure 1.24. In particular, the wage decline is only about half to what we observed in Figure 1.21.

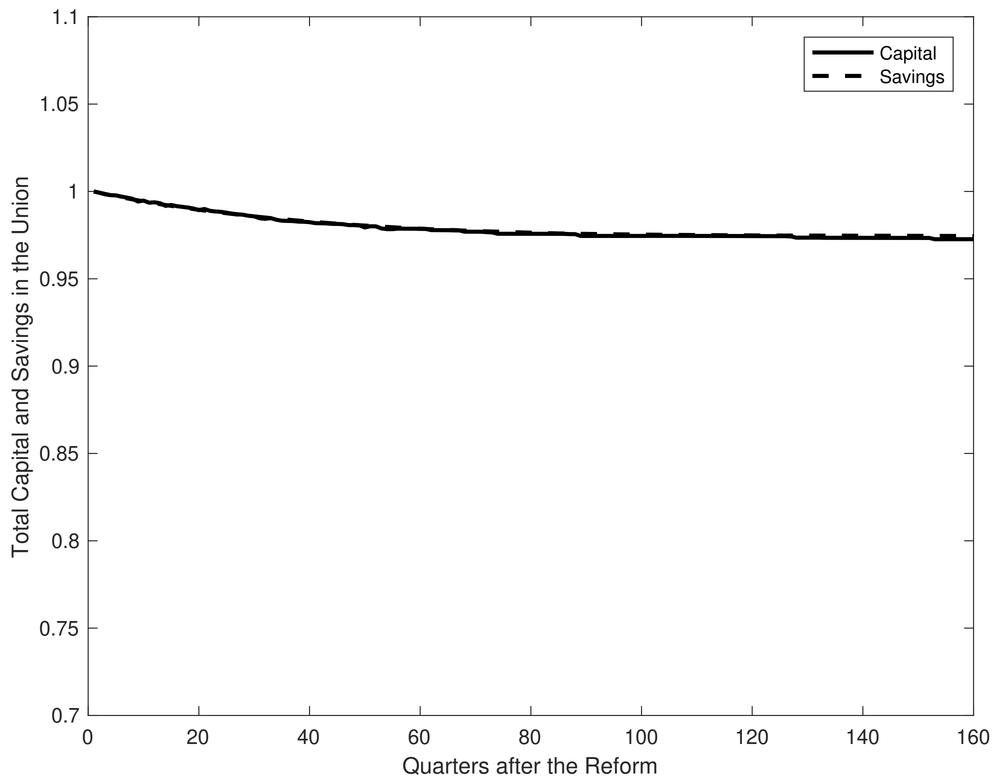

Fig. 1.23: Capital and Savings with Harmonized UI Benefit System

Both, less generous unemployment benefits and higher wages, make employment more and inactivity less attractive and as a consequence aggregate employment is stabilized (Figure 1.25).

As before, the welfare effects are not only heterogeneous across countries but also across different groups within each country. This is shown in Table 1.10. We see that while the size varies, the sign is positive almost everywhere. In fact, only one single group does not benefit from the reform, the eligible unemployed in Belgium. The reason is that the current benefit system in Belgium has already a rather high duration of eligibility (20 quarters) and its current replacement rate is much higher than after the reform (37% vs. 15%). What stands out are the high welfare gains in Finland. In Table 1.10 we see that the tax rate after the reform is much lower than before. Together with strong labour market institutions that make the duration of non-employment very short, at least

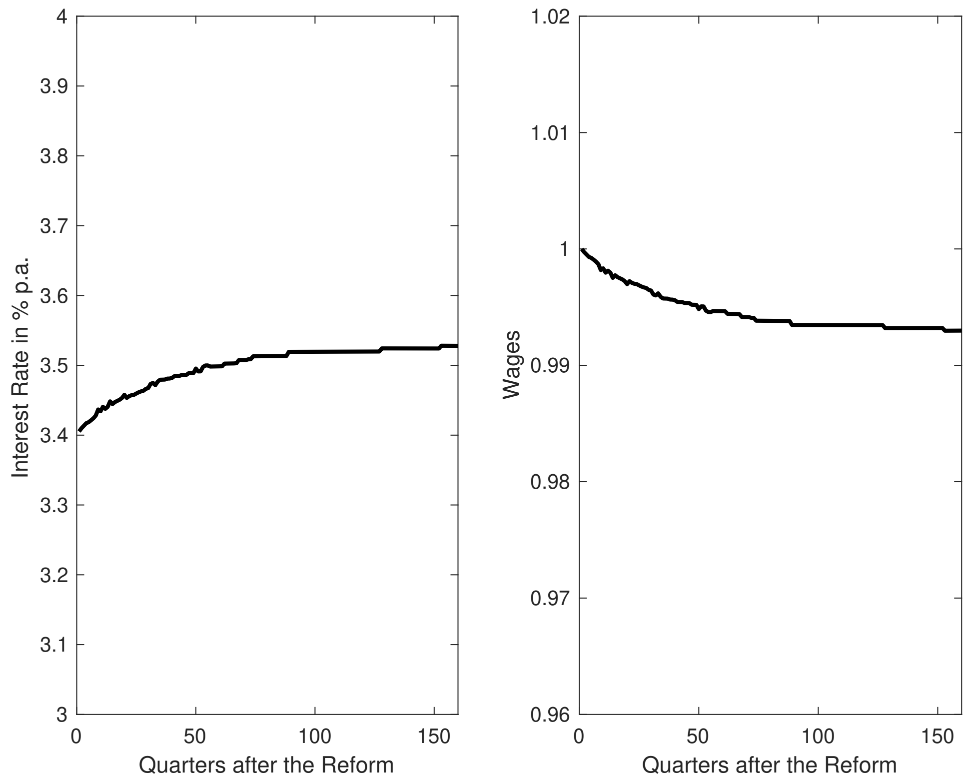

Fig. 1.24: Equilibrium Prices with Harmonized UI Benefit System

for people who are willing to work, these tax reductions lead to substantial welfare gains for all groups.

	Employed	Un. Eligible.	Un. Non-Elig.	Non-Active	Total
Austria	0.21	0.56	0.47	0.24	0.23
Belgium	1.05	-0.35	0.15	0.81	0.92
Germany	0.34	2.34	0.44	0.28	0.38
Spain	0.65	0.44	0.65	0.58	0.62
Finland	2.09	1.64	1.83	1.90	2.03
France	0.11	0.15	0.18	0.09	0.11
Ireland	0.77	1.07	1.31	0.73	0.79
Italy	0.48	2.73	1.58	0.51	0.60
Luxembourg	0.30	0.85	0.51	0.33	0.32
Netherlands	0.06	0.91	0.13	0.03	0.08

Table 1.10: Welfare gains in (in % CEV) with Harmonized UI Benefit System

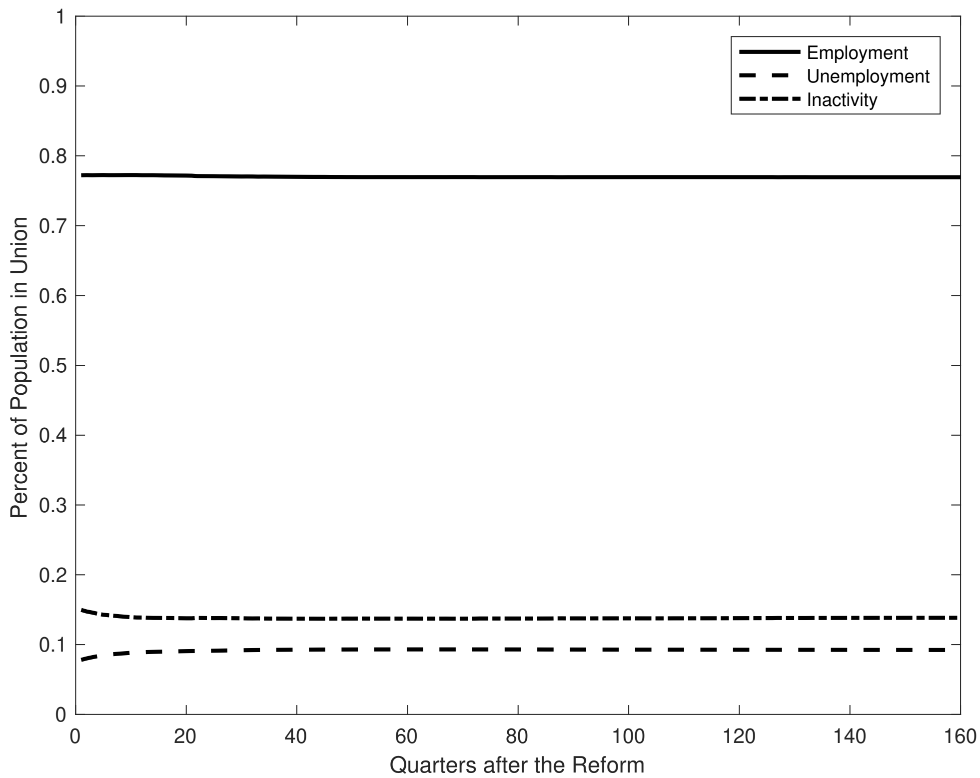

Fig. 1.25: Aggregate Labour Market States with Harmonized UI Benefit System

1.6 Conclusion

This paper is aimed at assessing the value of a European Unemployment Insurance System (EUIS) and, in particular, how it should be designed as a constrained efficient mechanism. We take as a constraint the current labour market institutions which determine differences in job destruction and the likelihood to receive offers by the unemployed (searching for a job) and the inactive (not actively searching), we also limit the scope of unemployment insurance contracts to contracts defined by their coverage duration and their replacement rate. Our work provides a quantitative proof of the potential gains that market reforms – not just labour market reforms – can achieve in many European countries. In fact, the first contribution of this paper is to provide a novel diagnosis of European labour markets. The second, which is almost a corollary of the first, is to show quantitatively that country-specific structural parameters play a determinant role in explaining the different performance of labour markets across the EU.

Based on this calibration we perform a set of policy experiments. We show that the gains from pure risk-sharing (i.e. absent UI reforms) are very limited but that substantial welfare gains can be achieved by reforming the existing UI systems within European countries. Even if, as we document, labour markets are very different, almost surprisingly the (parameterised) UI systems that maximise welfare are very similar: unemployment benefits duration should be unlimited and replacement rates more similar across countries than what they are now. We show that unemployment benefit reforms in some European countries affect the citizens of other countries through general equilibrium effects which result from a common capital market. In particular, the decline in precautionary savings associated with more generous unemployment benefit systems causes a reduction in the aggregate, European, capital stock. This in turn causes a decline in wages all over Europe. This means that reforms which may seem optimal at the national level can be detrimental at the European level once general equilibrium effects are taken into account.

Finally, we show that a common, less generous, European benefit system can tackle this problem. We find that a harmonized benefit system with an unlimited duration and a replacement rate of 15% is welfare improving in all countries, when it is financed by country specific contribution payments. The welfare gains are relatively large for all countries, and almost unanimous within countries, even without accounting for the risk-sharing gains that countries would have if, in addition to agents' idiosyncratic risk we also had aggregate country risks. That is, we required that each country runs a balanced budget, thereby eliminating permanent cross-country transfers. With country risks and no aggregate European risk, at the steady-state constant – but differential – taxes would also provide risk-sharing, with short-run cross-country transfers across the EUIS, possibly with the support of a centralized fund as we discussed in the Introduction. Even with European aggregate risk the EUIS would play a major stabilising role: taxes would not be constant, unless the fund has borrowing capacity (which it should), but still provide risk-sharing across countries and agents. In any case, the resulting tax differences across countries reflect their structural labour market differences, in terms of job creation and destruction. These tax differences also provide clear incentives for labour market reforms.

In sum, by increasing welfare across European citizens the proposed EUIS can also be an important cohesive EU institution. There is no need to wait for European labour markets to converge to implement the EUIS. In fact, it can promote national labour market reforms and European labour market integration.

Acknowledgements

Appendix

Transition Function

The transition function $T_t^i((a,z,x);\mathscr{A}\times\mathscr{Z}\times\mathscr{X})$ describes the probability that an agent, who is in state (a,z,x) in period t, is in any state $\{(a',z',x') : a'\in\mathscr{A}, z'\in\mathscr{Z}, x'\in\mathscr{X}\}$ in period $t+1$. This function is quite involved as it captures exogenous shocks and endogenous decisions of the agent.

Next period's assets $a'(a,z,x)$ are purely endogenous as they are chosen from the agent in period t and not subject to any shock. Next period's productivity level z' is purely exogenous and depends on the Markov transition probabilities. Next period's employment state $x' \in \{e, u^e, u^n, n\}$ depends on a combination of exogenous shocks (job separation, job finding) and endogenous decisions (work, search), which in turn depend on assets and individual productivity.

We can write the transition function as

$$T_t((a,z,x);\mathscr{A}\times\mathscr{Z}\times\mathscr{X}) = \mathbb{1}_{a^i_{t+1}(a,z,x)\in\mathscr{A}} \cdot \sum_{z'\in\mathscr{Z}} p(z'|z)\Big\{\mathbb{1}_{e\in\mathscr{X}} \cdot xe\big(a^i_{t+1}(a,z,x),z'\big) + \mathbb{1}_{u^e\in\mathscr{X}} \cdot xu^e\big(a^i_{t+1}(a,z,x),z'\big) + \mathbb{1}_{u^n\in\mathscr{X}} \cdot xu^n\big(a^i_{t+1}(a,z,x),z'\big) + \mathbb{1}_{n\in\mathscr{X}} \cdot xn\big(a^i_{t+1}(a,z,x),z'\big)\Big\},$$

where $xe\big(a^i_{t+1}(a,z,x),z'\big)$ describes the probability of moving from labor market state $x \in \{e,u^e,u^n,n\}$ into employment, conditional on saving $a^i_{t+1}(a,z,x)$ and on drawing productivity shock z'. Similarly, $xu^e(.)$ is the conditional probability of moving into unemployment and being eligible for benefits, and so on.

It is useful to define the decision to search for a job next period, conditional on being not eligible for unemployment benefits by

$$s_{t+1}(a',z',0) = \begin{Bmatrix} 1 & \text{if } \arg\max\limits_{x'\in\{u^n,n\}} V^i_{t+1}(a',z',x') = u^n \\ 0 & \text{else} \end{Bmatrix}$$

and the decision to search for a job conditional on being eligible for unemployment benefits by

$$s(_{t+1}a',z',1) = \begin{Bmatrix} 1 & \text{if } \arg\max\limits_{x'\in\{u^e,n\}} V^i_{t+1}(a',z',x') = u^e \\ 0 & \text{else} \end{Bmatrix}$$

Similarly, define the decision to work next period, conditional on being not eligible for unemployment benefits by

$$w_{t+1}(a',z',0) = \begin{Bmatrix} 1 & \text{if } \arg\max\limits_{x'\in\{,e,u^n,n\}} V^i_{t+1}(a',z',x') = e \\ 0 & \text{else} \end{Bmatrix}$$

and the decision to work conditional on being eligible for unemployment benefits by

$$w_{t+1}(a',z',1) = \begin{Bmatrix} 1 & \text{if } \arg\max\limits_{x'\in\{,e,u^e,n\}} V^i_{t+1}(a',z',x') = e \\ 0 & \text{else} \end{Bmatrix}$$

The conditional transition probability from employment into employment is then given by

$$ee\big(a^i_{t+1}(a,z,x),z'\big) = (1-\sigma^i)w^i_{t+1}(a^i_{t+1}(a,z,e),z',0) + \sigma^i\lambda^i_u w^i_{t+1}(a^i_{t+1}(a,z,e),z',1).$$

There are two possibilities how an agent, who is employed in period t, is also employed in $t+1$: (i) the agent does not get separated, which happens with probability $1-\sigma^i$ and does not quit his job, which is the case if the work decision $w^i_{t+1}(a^i_{t+1}(a,z,e),z',0)=1$. Since job quitters are not eligible for benefits the last entry of the work decision is zero; (ii) the agent gets separated from his job (with probability σ^i) but immediately finds a new job (with probability λ^i_u) and decides to work. In case of exogenous separation the agent would be eligible for unemployment benefits, therefore the last entry in the work decision is equal to one. One can observe that this conditional probability is a mixture of exogenous probabilities and endogenous decisions.

Similarly, we can define the other conditional probabilities: The probability of moving from employment to unemployment and being eligible for benefits is

$$eu^e\left(a^i_{t+1}(a,z,x),z'\right)=\sigma^i s^i_{t+1}(a^i_{t+1}(a,z,e),z',1)\left[(1-\lambda^i_u)+\lambda^i_u\left(1-w^i_{t+1}(a^i_{t+1}(a,z,e),z',1)\right)\right].$$

Eligibility next period requires that the worker is exogenously separated, which happens with probability σ^i and that the agent is actively searching for a job, i.e. $s^i_{t+1}(.)=1$. There are again two possibilities to be unemployed next period: (i) With probability $1-\lambda^i_u$ the agent does not immediately find a new job (ii) with probability λ^i_u the agent immediately finds a new job but he decides not to accept the offer ($w^i_t(.)=0$).

The conditional probability of moving from employment into unemployment and being eligible for benefits is equal to the probability of not being separated (once you are separated you are automatically eligible for benefits), given that the agent decides to quit $w(.)=0$ and to search for a new job $s(.)=1$:

$$eu^n\left(a^i_{t+1}(a,z,x),z'\right)=(1-\sigma^i)\left(1-w^i_{t+1}(a^i_{t+1}(a,z,e),z',0)\right)s^i_{t+1}(a'(a,z,e),z',0).$$

Finally, the conditional probability of moving from employment into inactivity is given by

$$\begin{aligned}eu^n\left(a^i_{t+1}(a,z,x),z'\right)=&(1-\sigma^i)\left(1-w^i_{t+1}(a^i_{t+1}(a,z,e),z',0)\right)\left(1-s^i_{t+1}(a^i_{t+1}(a,z,e),z',0)\right)+\\&\sigma^i\left(1-\lambda^i_u+\lambda^i_u\left(1-w^i_{t+1}(a^i_{t+1}(a,z,e),z',1)\right)\right)\left(1-s^i_{t+1}(a^i_{t+1}(a,z,e),z',1)\right).\end{aligned}$$

The agent can become inactive either if he does not get exogenously separated but decides to quit working and searching (fist line) or if he gets separated and does not search for a new job (second line).

We now described all possibly cases for an agent who is employed in period t, i.e. $x_t=e$. In an analogous way this can be done for all other initial labor market states, i.e. for $x_t\in\{u^e,u^n,n\}$.

References

Beblavy, M. and K. Lenaerts (2017): Feasibility and Added Value of a European Unemployment Benefits Scheme, CEPS Special Report.

Beblavy, M. and I. Maselli (2014): An Unemployment Insurance Scheme or the Euro Area: A Simulation Exercise of Two Options, CEPS Special Report.

Böwer, U. and G. Catherine (2006): Determinants of Business Cycle Synchronization across Euro Area Countries, ECB Working Paper No. 587.

Claveres, G. and M. Clemens (2017): Unemployment Insurance Union, German Institute for Economic Research (DIW).

Dolls, M., C. Fuest, D. Neumann and A. Peichl (2015): An Unemployment Insurance Scheme for the Euro Area? A Comparison of Different Alternatives Using Micro Data, CESifo Working Paper No. 5581.

Dullien, S., J. Fernandez, M. Lopez, G. Maass, D. del Prado and J. von Weizsacker (2018): Fit for Purpose: German-Spanish proposal for a robust European Unemployment Insurance, Friedrich Ebert Stiftung, Madrid.

Esser, I., T. Ferrarini, K. Nelson, J. Palme and O. Sjöberg (2013): Unemployment Benefits in EU Member States, Technical report, Directorate-General for Employment, Social Affairs and Inclusion, EC.

Giannone, D., M. Lenza and L. Reichlin (2009): Business Cycles in the Euro Area, ECB Working Paper No. 1010.

Ignaszak, M., P. Jung and K. Kuester (2018): Federal Unemployment Insurance: Theory, and an Application to Europe, mimeo, Technical University of Dortmund.

Krusell, P., T. Mukoyama, R. Rogerson and A. Şahin (2011): A Three State Model of Worker Flows in General Equilibrium, *Journal of Economic Theory* 146, 1107–1133.

Krusell, P., T. Mukoyama, R. Rogerson and A. Şahin (2017, November): Gross Worker Flows over the Business Cycle, *American Economic Review 107 (11)*, 3447–76.

Lalé, E. and L. Tarasonis (2017): The Life-cycle Profile of Worker Flows in Europe, Working Paper.

Moyen, S., N. Stahler and F. Winkler (2016): Optimal Unemployment Insurance and International Risk Sharing, Deutsche Bundesbank Discussion Paper No 33/2016.

Saiki, A. and S. H. Kim (2014): Business Cycle Synchronization and Vertical Trade Integration: A Case Study of the Eurozone and East Asia, DNB Working Paper No. 407.

Chapter 2
Schooling and Lifetime Labor Supply

Abstract This paper presents an empirical and theoretical investigation of the trends in schooling and lifetime labor supply during the 20th century in the United States. It shows that a life-cycle human capital model, where individuals optimally choose both the years of schooling and of labor force participation, is consistent with the long-run trends. The observed increase in life expectancy alone would imply, according to the model, an increase in schooling years and in years of participation in the labor market, a counterfactual result. The analysis shows that an increase in schooling time and a decline in lifetime labor supply is explained by an increase in life expectancy, returns to schooling and wages. I revisit evidence on decline in the cost of schooling during the first half of the 1900s, with the expansion of public provision of secondary education in the United States.

2.1 Introduction

The 20th century saw major demographic and economic developments in the most advanced economies. In the United States, life expectancy at age 5 climbed from 52.5 years for men born in 1850 to 70.7 years for men born in 1970. The allocation of time during the life-cycle displayed major changes too. From 1900 to 2000, market work in the population group under 18 years old steadily declined from 20 average working hours per week to below 5 hours, in per capita terms. Almost all the difference is compensated by the increase in time spent in school, that increased sharply during the first half of the 20th century. In 1900, the population group over 65 years old worked on average 20 hours per week. One hundred years later, the same group devotes less than 5 hours per week to market work, and dedicates instead most of the difference to leisure activities (Ramey and Francis (2009)). Interestingly, in the prime-age period (25-54 years old group), weekly hours of work per capita were remarkably stable from 1900 to the 2000s.[1] Figures 2.1 to 2.3 show data on market work, schooling and leisure of different age groups in the US since 1900 until 2005, measured in hours per capita.[2] Figure 2.2 illustrates the dramatic increase in time spent

[1] The period of the Great Depression and the second World War show exceptional swings.

[2] Data for the period 1900 to 1958 is covered in Kendrick (1961) and Kendrick (1973), and from 1958 to 2005 in Ramey and Francis (2009). The dataset for the entire period is available at the Ramey and Francis (2009) AEA website.

in school at ages younger than 18, that offsets hours of work in this age group. Goldin and Katz (1999) calculates that around 70% of the increase in years of the education in the US population between 1900 and 1970 was due to high school graduation. I provide further data on the 1900s US 'high school movement' below. These large shifts motivated a long research literature, and recent papers study the consequences of the demographic evolution for economic growth and public policy programs (e.g. Cooley and Henriksen (2018) and Scheiner (2018)). In figure 2.3, data on leisure consumption explains how most of the decline in hours of work in the 65+ age group was compensated. Costa (1998) provides an overview of the trends in retirement during this period.

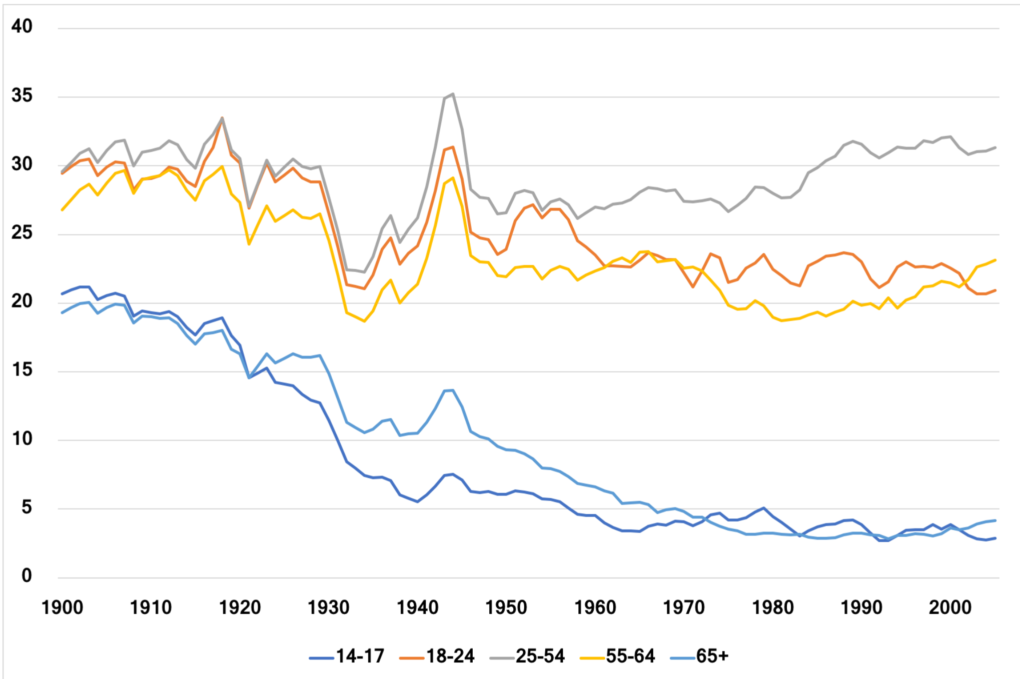

Fig. 2.1: Average weekly hours of market work per capita for different age groups.

In this paper, I show that a model of human capital investment, as in Ben-Porath (1967), enriched such that the decision problem about the allocation of time considers schooling, labor supply and a retirement period, provides a rational for the data trends described above. In particular, the model shows that when individuals optimally decide jointly on how long to go to school *and* how long to participate in the labor market, an increase in life expectancy, in the return to schooling and in wages implies longer time in school, a decline in lifetime labor supply, and a longer retirement period, as in the data. In contrast, a longer life horizon alone increases both schooling years and market participation - a counterfactual result. Previous literature identifies the increase in life expectancy as the driving force behind the observed changes in schooling years and lifetime labor supply (see Cervellati and Sunde (2013)). Finally, I review the evidence on the sharp decline in

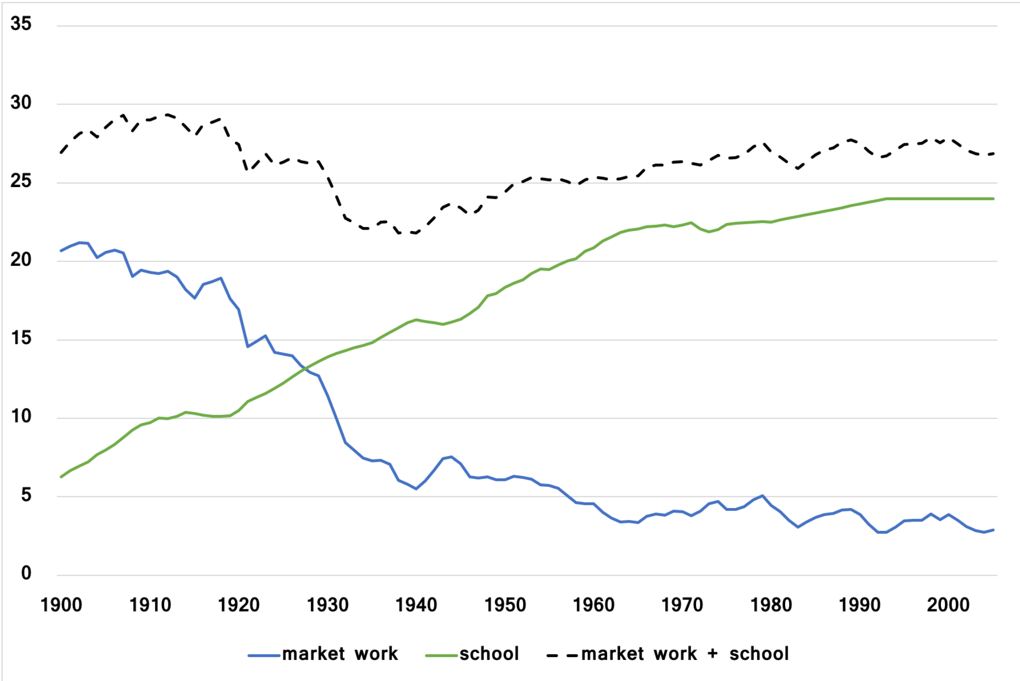

Fig. 2.2: Average weekly hours of market work and school per capita for young 14-17 years old.

the cost of schooling in the beginning of the 1900s, after the expansion of public expenditures on primary and secondary education in the United States, which together with increase in real wages during the same period, is consistent with the proposed mechanism.

2.2 Empirical Evidence and Related Literature

This section presents the empirical evidence that is addressed in the theoretical and quantitative section below, and discusses related literature. Figure 2.4 shows total lifetime hours of work and years of schooling for cohorts of men born in the decades from 1850 to 1970 in the US.

Figure 2.4 replicates one of the contributions of Moshe (2009) and is at the center of the discussion of subsequent literature. It shows a continuous decline in total hours of labor supply and a clear increase, since the 1880 cohort, in schooling years of the subsequent cohorts. The evolution of total lifetime hours of work, seen in figure 2.4, is affected by changes in average hours conditional on working (intensive margin), and changes in the number of years of market participation (extensive margin). The trends in figure 2.1 show stable hours per capita in the prime-age group and declining hours in the 65+ group, suggesting an important role of changes in the participation rates at older ages. Using cohort estimates, Moshe (2009) shows that participation declines

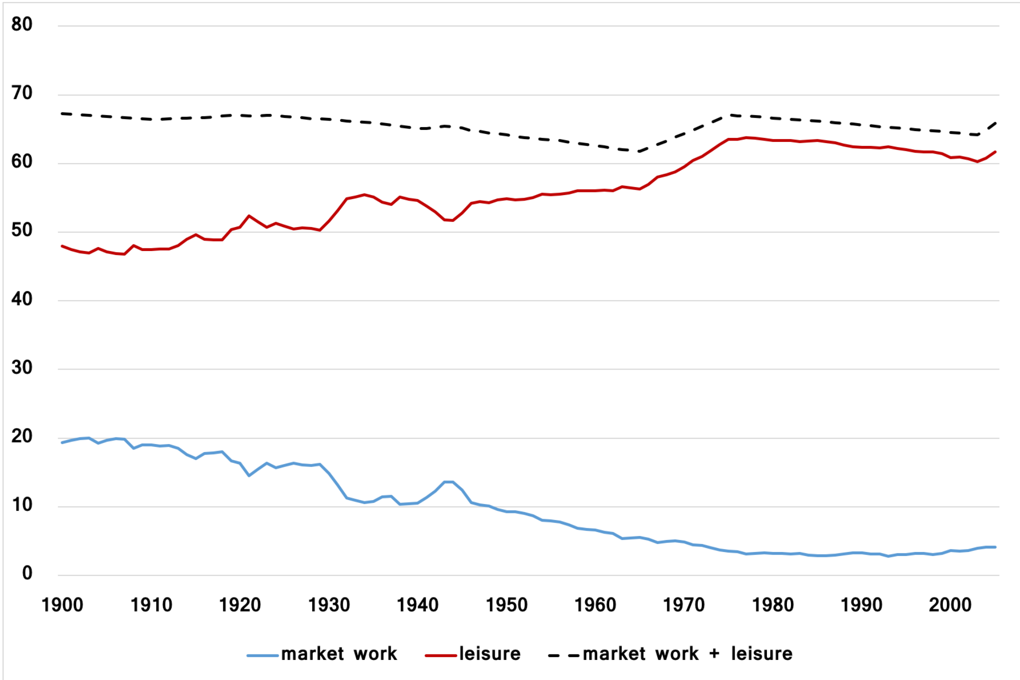

Fig. 2.3: Average weekly hours of market work and leisure time per capita for 65+ years old.

after age 65, smoothly in the 1840 cohort estimate (around 50% participation at age 75) but more sharply the older the cohort (below 20% at age 75, for the 1930 cohort). Looking specifically at the retirement period, Lee (2001) provides estimates of average duration of the retirement period enjoyed by the subsequent cohorts. These estimates, plotted in figure 2.5, help dissect the long-run (cross-sectional) changes in labor supply per capita shown in figure 2.1.[3] By subtracting to the evolution of life expectancy (black dots in figure 2.6), years of schooling and length of the retirement period, I obtain a measure of the evolution of average years of labor force participation (grey dots), shown in figure 2.6.

To summarize, these figures provide the cohort-based estimates that correspond to the long-run trends displayed in figures 2.1, 2.2 and 2.3. They show that over time, total lifetime labor supply declined due to increases in time dedicated to formal education at younger ages, and to increases in the length of the retirement period dedicated to leisure consumption. During this period, life expectancy increased sharply by more than 10 years.

Previous literature has studied the evolution of life expectancy, schooling and lifetime labor supply discussed above. Moshe (2009) documents the relationship between schooling years and total hours of work, shown in figure 2.4 above, and argues that the predictions of the canonical Ben-Porath (1967) model of lifetime human capital investment and labor supply are at odds with

[3] Lee (2001) provides cohort estimates and period estimates of the expected length of the retirement period at age 20. Figure 2.5 shows the average of the two estimates, which is used throughout the paper as the average length of the retirement period.

Fig. 2.4: Expected total working hours over lifetime at age 5, for different cohorts of men (left axis); and schooling years (right axis). This figure replicates Fig. 10 in Moshe (2009).

this evidence. According to the 'Ben-Porath mechanism', an increase in life expectancy implies an increase in school time if and only if total lifetime hours of labor supply increase. Since the latter is not observed in the data, life expectancy alone cannot drive the changes in schooling and labor supply choices. Cervellati and Sunde (2013) further investigates the mechanism by exploring the role of changes in the age profile of survival rates during the period. The paper identifies a set of theoretical assumptions that are necessary for the Moshe (2009) 'if and only if' condition to hold, among these the assumption of a rectangular survival function.[4] This assumption, albeit counterfactual, is important in the analysis. Cervellati and Sunde (2013) shows that in the Ben-Porath model, the benefits of schooling may increase following an increase in the survival probability at working ages, even if life expectancy does not change. Interestingly, the survival rate for prime age individuals increased significantly in the older cohorts. In Cervellati and Sunde (2013) it is shown that the standard model, calibrated with realistic age profiles of survival probabilities, is consistent with an increase in schooling and a decline in total lifetime hours of labor supply, i.e. with figure 2.4.

[4] A survival function gives the survival probability at each age. A rectangular shaped survival function implies a probability one of surviving up to a certain age (usually set to the average life expectancy), and zero after that. The other assumptions behind the Moshe (2009) condition are:

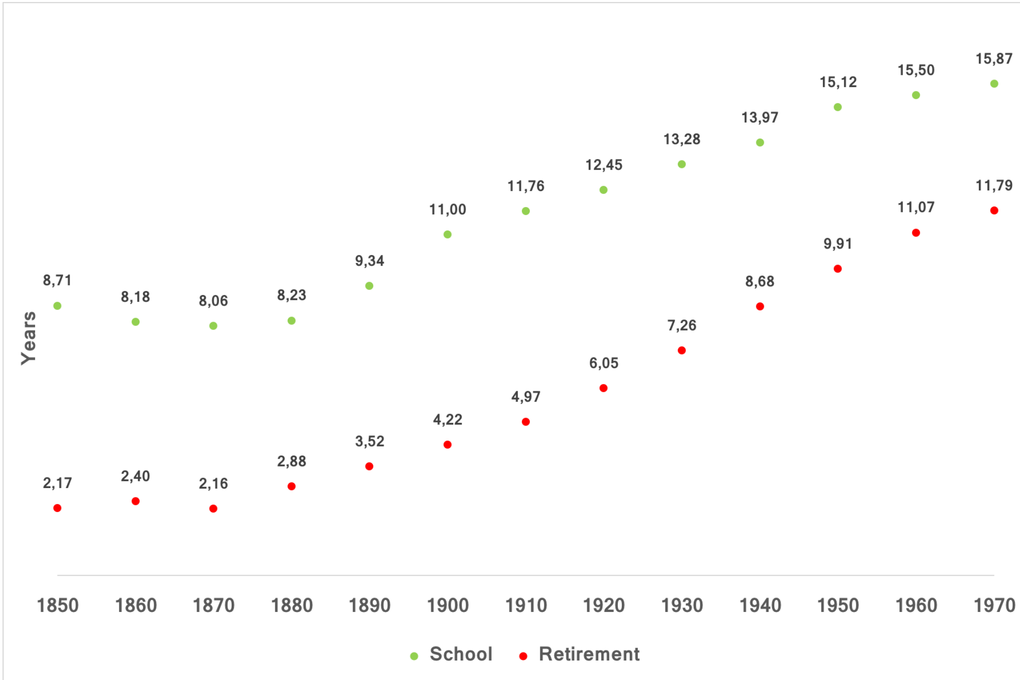

Fig. 2.5: Estimates of school years is from Moshe (2009). Retirement years are the mid-range values of the upper and lower bound estimates in Lee (2001).

As documented above, the decline in lifetime labor supply since 1900 is not given by a uniform decline of hours worked over the life-cycle. Hours worked per capita are stable in the prime-age group, and decline in younger and older age groups. With a longer life span, schooling years almost double and the average duration of the retirement period increased more than fivefold between 1850 and 1970, as shown in figure 2.5. The ‘Ben-Porath mechanism’ discussed in Moshe (2009) and Cervellati and Sunde (2013) is not tested against these observations. Moshe (2009) identifies the puzzle in figure 2.4, which is about total hours of lifetime labor supply. Cervellati and Sunde (2013) shows that the Ben-Porath mechanism is consistent with the decline in total hours of labor supply. Given that the Ben-Porath (1967) model is a life-cycle model about the timing of human capital investment and labor supply, the question about whether the theory is consistent with the trends in schooling, labor supply and retirement - figures 2.4, 2.5 and 2.6 - remains without an answer.

In this paper, I tackle this question by considering a life-cycle Ben-Porath model that includes a joint decision on schooling, labor supply and retirement period. In contrast to Moshe (2009) and Cervellati and Sunde (2013), where the decision about retirement is absent, in the model presented below agents optimally decide jointly on how long to stay in school and how long to participate in the labor market. The model can then speak to figures 2.5 and 2.6. When doing so, I revisit the role of changes in life expectancy, returns to school and wages on the observed long-run trends.

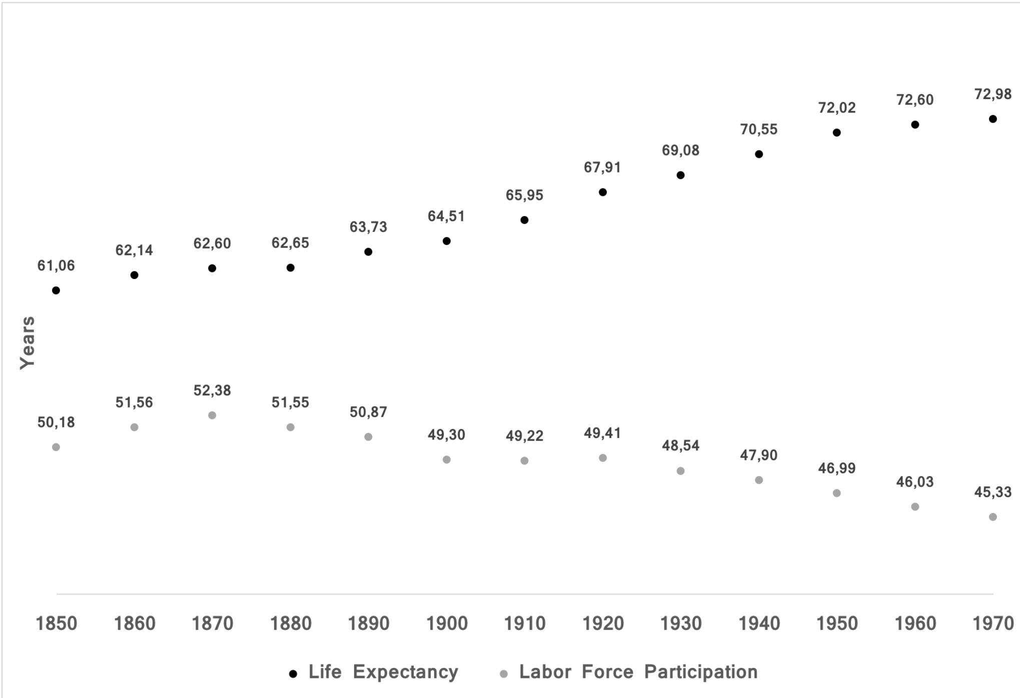

Fig. 2.6: Using the estimates of life expectancy at age 20 (black dots) and length of retirement in Lee (2001), and school years from Moshe (2009), I calculate years of labor force participation (grey dots).

The following section introduces the model. In addition to the human capital investment decision of the standard Ben-Porath (1967) theory, individuals decide on the optimal duration of their career in the labor market.[5] Next, I solve the model and evaluate it quantitatively by comparing it to the data discussed above.. The paper concludes with a review of empirical evidence on the sharp decline in the cost of schooling and rise in labor productivity during the 1900s in the United States.

2.3 Model

The model described below is designed to capture the individual problem of optimizing the allocation of time during the life-cycle. During his lifetime, an individual derives utility from the consumption of goods and leisure. Apart from leisure consumption, the agent can use his time endowment to invest in human capital and to provide labor services. Higher human capital means

[5] Shin et al. (2012) uses a model that includes an individual life-cycle problem with a schooling and retirement decision, as the model used here, to study the role of taxes and retirement policies in explaining US-Europe differences in lifetime labor supply.

higher productivity in the labor market and higher wage income, but requires diverting time from the labor market to school. Higher lifetime income allows for higher consumption of goods and leisure. Because human capital depreciates with time, human capital investment is optimally carried out more intensively at the beginning of life, while leisure consumption is instead backloaded. I compare the predictions of the model after an increase in life duration, the wage rate and the return to human capital investment, represented by an increase in the productivity of human capital production, and compare them with the data shown in the previous section.

2.3.1 Preferences and Technology

Age is continuous and represented by $a \in [0,T]$. An individual maximizes discounted lifetime utility from consumption of goods $c(a)$ and leisure $l(a)$:

$$\int_0^T e^{-\rho a} U(c(a), l(a)) da; \tag{2.1}$$

ρ is a discount factor. At each age, he is endowed with one unit of time that can be allocated to human capital investment $x(a)$, to labor supply $n(a)$, or to leisure consumption $l(a)$:

$$x(a) + n(a) + l(a) = 1 \tag{2.2}$$

I assume that human capital investment and labor supply are decided along the intensive margin, $n(a), x(a) \in [0,1]$, while leisure is consumed at the extensive margin, $l(a) \in \{0,1\}$. An interpretation of this assumption is provided at the end of this section.

Human capital depreciates at a rate δ and, given an initial level $h(0)$, evolves over time according to:

$$\dot{h}(a) = f(z, h(a), x(a)) - \delta h(a). \tag{2.3}$$

z is a productivity parameter. Below, I assume that f is a strictly increasing and concave function of the time spent in school. A linear production function is one of the assumptions used in Moshe (2009) that Cervellati and Sunde (2013) shows need to be imposed to derive the necessary conditions in Moshe (2009). The agent can borrow and lend at the interest rate r, and can not die with debt: $b(T) \geq 0$. His flow budget constraint is:

$$\dot{b}(a) + c(a) = y(a) + rb(a). \tag{2.4}$$

$y(a)$ is labor market income. It is determined by the wage rate w and labor supply in efficiency units, $h(a)n(a)$: $y(a) = wh(a)n(a)$. The agent starts life with an initial asset holding $b(0)$.

In the following, I describe the solution to the problem of maximizing 2.1 subject to 2.2, 2.3 and 2.4, given an initial endowment $h(0)$ and $b(0)$. Since human capital takes time to accumulate, the solution displays higher human capital investment at earlier ages and leisure consumption at older ages. We can interpret $x(a) = 1$ as the schooling period and the leisure choice as a retirement decision. The analysis below focuses on how changes in the life horizon, T, wages w and the return to human capital investment, parametrized by z, affect the allocation of time over the life-cycle.

2.3.2 Solution

To simplify the analysis, and because my focus is on the time allocation decision, I assume $\rho = r = 0$. Furthermore, I assume that preferences are described by a separable utility function, so that the marginal utility of consumption is constant at all ages:

$$U(c,l) = u(c) + v(l), \tag{2.5}$$

u is a strictly increasing and concave function, with $\lim_{c\to 0} u_1(c) = \infty$, and v (which is evaluated at $l = 0$ and $l = 1$ only) taken on non-negative values. The separability assumption, together with free borrowing and lending and no discounting, imply that the consumption of goods is constant over the life-cycle.[6] The discrete leisure decision amounts to the choice of the share of the total lifetime endowment of time dedicated to leisure consumption.[7] This formulation allows to clearly study how changes in life expectancy, technology and prices impact the optimal decision of schooling time and labor force participation, and compare the model predictions with the data presented above.

The problem can be studied in two steps. First, for a given retirement age R, the human capital investment and labor supply decisions solve a lifetime income maximization problem. Second, given the optimal labor and human capital investment policies conditional on R, I solve for the optimal goods and leisure consumption decisions that pin down R.

2.3.2.1 Income maximization problem

Given a retirement age $R < T$, $l(a) = 0$ from $a = 0$ to R, and $l(a) = 1$ for $a \in [R,T]$. The objective is to maximize:

$$\int_0^R y(a)da = \int_0^R wh(a)(1-x(a))da, \tag{2.6}$$

subject to

$$\dot{h}(a) = f(z,h(a),x(a)) - \delta h(a), \tag{2.7}$$

given $h(0)$. Here I am using the condition $x(a) + n(a) = 1$ to eliminate $n(a)$. I assume that gross human capital production is given by:

$$f(z,h(a),x(a)) = z(h(a)x(a))^{\gamma},\ \gamma \in (0,1), z > 0. \tag{2.8}$$

[6] This simplification is in line with Moshe (2009) and Cervellati and Sunde (2013).

[7] A general proof of the property of the solution according to which leisure is backloaded and consumed continuously from a certain age R until T can be found in Shin et al. (2012). In the present environment, with no time discounting and zero interest rate, a sufficient condition is that human capital depreciates, $\delta > 0$.

Empirical estimates indicate decreasing to constant returns to schooling.

At an interior choice of $n(a)$, and denoting by $q(a)$ the co-state variable, the solution is described by:

$$wh(a) = q(a)\gamma z h(a)(h(a)x(a))^{\gamma-1}, \tag{2.9}$$

$$\dot{q}(a) = -w(1-x(a)) - q(a)[\gamma z x(a)(h(a)x(a))^{\gamma-1} - \delta], \tag{2.10}$$

with $q(R) = 0$. Rearranging and substituting out $wn(a)$, one obtains the law of motion of $q(a)$:

$$\dot{q}(a) = -w + \delta q(a). \tag{2.11}$$

The solution to the differential equation, with terminal condition $q(R) = 0$, is

$$q(a) = \frac{w}{\delta}\left(1 - e^{-\delta(R-a)}\right) := \frac{w}{\delta}m(a). \tag{2.12}$$

The shadow price of human capital increases with the wage rate w and decreases with depreciation δ, and decays with age a until R. At age R, human capital has no value since the agent will not supply labor services from then until T. Having solved for $q(a)$, one can solve explicitly for the human capital investment policy using 2.9:

$$h(a)x(a) = \left(\gamma\frac{z}{\delta}m(a)\right)^{\frac{1}{1-\gamma}} \tag{2.13}$$

Note that, for given R, the wage rate w does not influence human capital investment. The reason is that, at the margin, an increase in w increases the return on human capital by as much as the increase the opportunity cost of not working. In contrast, changes in z influence investment in human capital and, as shown below, labor supply and hence leisure consumption. Given 2.8 and the optimal investment decisions, human capital evolves according to:

$$\dot{h}(a) = z\left(\gamma\frac{z}{\delta}m(a)\right)^{\frac{\gamma}{1-\gamma}} - \delta h(a). \tag{2.14}$$

With an initial stock of $h(0)$, at age a the human capital stock is

$$h(a) = e^{-\delta a}h(0) + z^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{\delta}\right)^{\frac{\gamma}{1-\gamma}}\int_0^a e^{-\delta(a-t)}m(t)^{\frac{\gamma}{1-\gamma}}dt. \tag{2.15}$$

As seen before, conditional on R, changes in the wage rate per efficiency unit have no effect on the stock of human capital, whereas increases in the productivity level z increase human capital at all ages. For an arbitrary choice R, conditions 2.13 and 2.15 further allow to evaluate the impact of changes in z on the human capital investment decision. Using 2.15 to eliminate $h(a)$ from 2.13 and dividing by the term $z^{1/(1-\gamma)}$:

$$\left[\frac{e^{-\delta a}h(0)}{z^{1/(1-\gamma)}} + \left(\frac{\gamma}{\delta}\right)^{\gamma/(1-\gamma)} \int_0^a e^{-\delta(a-t)} m(t)^{\gamma/(1-\gamma)} dt\right] x(a) = \left(\frac{\gamma}{\delta}\right)^{1/(1-\gamma)} m(a)^{1/(1-\gamma)} \tag{2.16}$$

The expression above shows that, for a fixed R (so that the effective discount factor on human capital $m(a)$ does not change) an increase in z leads to an increase in the allocation of time to human capital formation, $x(a)$, at all ages $a \in [0,R)$. The conditions describing the solution of the human capital investment problem when the constraint $x(a) \leq 1$ is binding are slightly different and are derived in the Appendix. The analysis above continues to hold. Below I interpret the period $a \in [0,s]$ when $x(a) = 1$ as the schooling period, s.

I now turn to the optimal timing of leisure consumption, $R-T$, and how it reacts to changes in T, wages and productivity. To study these interactions I consider the second step of the solution procedure.

2.3.2.2 Goods and leisure consumption

The second step amounts to finding the optimal level of consumption, $\bar{c}$, and fraction of time allocated to leisure consumption, $T-R$. For that purpose, it is useful to rewrite the problem by expressing lifetime utility and lifetime resources as functions of $\bar{c}$ and R only.

$$\begin{aligned}\int_0^T U(c(a),l(a))da &= \int_0^T u(\bar{c}) + v(l)da \\ &= \int_0^T u(\bar{c})da + \int_R^T v(1)da \\ &= Tu(\bar{c}) + (T-R)\chi \end{aligned} \tag{2.17}$$

To ease notation, I normalized $v(0) = 0$ and $v(1) = \chi$. Lifetime resources are given by lifetime labor market income plus initial assets:

$$\int_0^T y(a)da + b(0) = \int_0^R wh(a)(1-x(a))da + b(0) \tag{2.18}$$

For each age a, conditions 2.13 and 2.15 are functions of R only, through the discounting function

$$m(a) = 1 - e^{-\delta(R-a)}, \tag{2.19}$$

which allows to replace $h(a)$ and $h(a)n(a)$ in 2.18 and write an expression for lifetime resources as a function of R alone plus initial assets $b(0)$. I denote lifetime labor market income by $W(R)$, and abstract from initial assets by setting $b(0) = 0$.[8]

$$W(R) = \int_0^R wh(a)(1-x(a))da. \tag{2.20}$$

[8] $W(R)$ depends on $h(0)$.

Optimal consumption of goods and leisure is the solution to

$$\max_{\{\bar{c},R\}} \quad \left\{Tu(\bar{c}) + (T-R)\chi\right\} \tag{2.21}$$

subject to the lifetime budget constraint:

$$T\bar{c} \leq W(R). \tag{2.22}$$

With Φ denoting the multiplier on constraint 2.22, the first order conditions are:

$$u_1(c) - \Phi = 0, \tag{2.23}$$

$$-\chi + \Phi W_R(R) = 0, \tag{2.24}$$

where u_1 is the first derivative of u and W_R represents the first derivative of W with respect to R. Assuming 2.22 holds at equality, jointly these imply:

$$\chi = u_1(W(R)/T)W_R(R). \tag{2.25}$$

The condition above has a clear interpretation. At the optimal leisure choice, the marginal utility gain of an increase in leisure consumption $(T-R)$ is balanced with the marginal increase in lifetime income W_R, priced by average lifetime consumption $W(R)/T$. The derivative of lifetime income with respect to the retirement choice is:

$$\begin{aligned} W_R(R) = \frac{d}{dR}W(R) = \frac{d}{dR}\left[\int_0^R wh(a)(1-x(a))da\right] = \\ = w\left[h(R) + \int_0^R \frac{d}{dR}h(a)da\right] - \\ + w\left[h(R)x(R) + \int_0^R \frac{d}{dR}h(a)x(a)da\right] = \\ = wh(R) + w\int_0^R h(a)da - w\int_0^R \frac{d}{dR}h(a)x(a)da. \end{aligned} \tag{2.26}$$

The first term is the marginal increase in income at the retirement age, R. The second term captures the increase of the human capital life-cycle profile due to a marginal increase in R. We saw above that, according to expression 2.15, increasing R raises human capital at all ages. The last term, which contributes negatively, summarizes the additional cost of human capital formation through the increase in human capital investment.

In the following two sections, I calibrate the model presented above and replicate the quantitative exercise in section 4 of Cervellati and Sunde (2013). In the quantitative exercise, I ask the model what are the effects of an increase in life expectancy T, wages w and returns of schooling z on schooling time and lifetime labor supply. I compare these effects with the estimated changes in time allocation choices during the 1900s.

2.3.3 Numerical solution

The model has 7 parameters: T, z, γ, δ, w, θ, χ. To specify their values, I take two cohorts of men born in 1880 and in 1930 in the United states, and use the corresponding estimated average life expectancy, schooling years and length of retirement period. In figure 2.6 above, life expectancy for cohort 1880 is 65.6 and for cohort 1930 is 73 years.[9] From figure 2.5, I take the average years of schooling for men born in 1880 and for men born in 1930, and the Lee (2001) estimates of the expected length of the retirement period. I use these estimates to first calibrate the model to the 1880 cohort and then evaluate the model predictions about the observed relative changes schooling and labor supply of the 1930 cohort. Table 2.1 summarizes.

	1880	1930
life expectancy	65.6	73.0
years of schooling	8.2	13.3
length of retirement period	2.9	7.2
years on the market	54.5	52.5

Table 2.1: Data: the last row is given by the first row minus the second and the third rows.

Accordingly, I set $T = 65.6$ for the 1880 cohort, normalize $w = 1$, and choose z and χ to match the average years of schooling (8.2) and on the labor market (54.5) given in table 2.1. For the remaining parameters that govern the human capital production technology, γ and δ, I take the two values used in Shin et al. (2012).[10] Finally, I use the utility function 2.27 with $\theta = 2.0$, which is standard in the literature and the same specification as in Cervellati and Sunde (2013).

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta} \tag{2.27}$$

T	δ	γ	z	θ	χ
65.6	0.024	0.68	.052	2.0	1.6

Table 2.2: Calibration to 1880 cohort.

Table 2.2 shows the implied model for the 1880 cohort, which I label M_{1880}:

[9] I choose the 1880 and 1930 cohorts to directly compare the results to the ones in Cervellati and Sunde (2013), which uses the same decades. High child mortality rates imply life expectancy at age 5 quite low, so I use age 20 instead.

[10] Shin et al. (2012) targets more recent data on the age profile of wages to discipline the human capital production technology. So far I couldn't find estimates of the age profile of labor earnings for the early 1900s period, so I stick to the values γ and δ from that paper.

	Data	M_{1880}
life expectancy	65.6	65.6
years of schooling	8.2	8.4
years on the market	54.5	54.6

Table 2.3: Model of the 1880 cohort.

2.3.4 Quantitative exercise

The following table presents the main results of the paper. I perform the following experiment: Relative to the 1880 cohort, I compute the impact of increasing life expectancy (T) and returns to schooling (z, w) on schooling years and lifetime labor supply. Recall that in the quantitive exercise in Cervellati and Sunde (2013), the evolution of life expectancy explains the observed changes in schooling and total hours worked. The question is whether in a human capital investment model in which schooling time and retirement are optimally chosen, the same holds relative to the data in figures 2.5 and 2.6.

For better readability, I introduce the following notation. $M_{1930}(T,z,w)$ denotes model predictions for the 1930 cohort, for given life expectancy T, productivity of human capital technology z and wage rate w confronting individuals born in 1930. $\bar{z} = .052$ and $\bar{w} = 1$ are the parameter values corresponding to the 1880 cohort, as in table 2.2. The third column in table 2.4 shows the model predictions when z and w are fixed at the 1880 levels, and life expectancy increases to $T = 73$ years. The fourth column, in addition to an increase in life expectancy to 73 years, shows the model predictions after a 10% increase in $z = 1.1 * \bar{z}$ and $w = 1.1 * \bar{w}$, relative to the 1880 cohort.[11]

	1880	1930	$M_{1930}(73.0,\bar{z},\bar{w})$	$M_{1930}(73.0,1.1*\bar{z},1.1*\bar{w})$
life expectancy	65.6	73.0	73.0	73.0
years of schooling	8.2	13.3	12.7	14.7
years on the market	54.5	52.5	58.1	51.7

Table 2.4: The exercise shows the effect of increasing life expectancy between 1880 and 1930 on schooling and labor supply.

The exercise shows that the 7 year increase in life expectancy has a large impact both on schooling and on labor force participation. Years of schooling increase 5.1 years between the 1880 cohort and the 1930 cohort; and 4.5 according to model in the third column of the 1930 cohort. Years on the market decline in the data, from 54.5 to 52.5. In contrast, the model predicts an expansion of labor force participation to 58.1 years after the increase in life expectancy. This is consistent with the theoretical tradeoff described in the previous section: an increase in life expectancy increases

[11] A 10% increase is likely to be a conservative assumption on the increase in wages and school returns over the 1880-1930 period. See the discussion at the end of this section.

the value of labor force participation the retirement age, delaying retirement (recall Figure 2.7). Longer time working raises the incentive to acquire human capital in school, which increases the schooling period. Accordingly, the combined change in the schooling period and leisure consumption at older ages is such that lifetime labor supply increases almost 4 years. Relative to the data in figure 2.6, the model predicts - with increasing life expectancy alone - that the theoretical counterparts of the grey dots in the figure would be trending upward, instead of declining as in the data. As a consequence, the implied length of the retirement period in the third column (73.0-12.7-58.1=2.2 years in the model vs 7.3 in the data) declines relative to the 1880 cohort (2.9 years), which is counterfactual given the positive trend in figure 2.5.

The second step of the exercise, under the fourth column in table 2.4, shows the effect of increasing T, w and z on schooling and labor force participation. As previewed before, under the baseline calibration of the human capital production technology and with $\theta = 2.0$, increasing z leads to an increase in the schooling period and an increase in leisure consumption, implying a decrease in lifetime labor force participation to 51.7 (which compares with 52.5 in the data). School increases by more 1.4 years according to the model, relative to the data on the 1930 cohort. For a given retirement choice R, an increase in the wage rate w has no impact on the schooling period and hence on human capital $h(R)$. Both $wh(R)$ and $W(R)$ increase in proportion to the increase in w. With $\theta > 1$, the curve in Figure 2.7 shifts left and the retirement age decreases. The two effects jointly imply more time in school and less time on the market. Despite over predicting the joint effect of increases in life expectancy and in the returns to school on schooling time by 1.4 years, the predicts a decline in lifetime labor supply, in line with the data. To conclude, the joint effect of life expectancy and increasing returns to school return is consistent with the data trends in figures 2.5 and 2.6.

I now provide some motivation for the increase in the returns to schooling, represented by increase in z above. Evidence on the sharp decline in the cost of schooling during the first half of the 1900s abounds. Goldin and Katz (1999) documents the US ‘high school movement’ between 1910 and 1940. About 70 percent of the observed increase in years of education of adults in the US between 1900 and 1970 was due solely to high school attendance. During this period, state and local (public) expenditures in elementary and secondary tripled from about 1% to 3% of regional income (Fig. 1, Goldin and Katz (1999)). In 1970, the national level was 4.5 percent, with more than half of the total increase between 1910 and 1925. Rangazas (2002) estimates that real spending per pupil in primary and secondary public education increases by a factor of 4 between 1880 and 1940, implying an increase from 1.0% to 2.4% of GDP over the same period. The long-run annual average of aggregate productivity growth was 1.6% from 1870 to 1972. As a result of the increase in years of schooling, Rangazas (2002) estimates that worker productivity rose 1.7 to 2 times from 1870 to 1970.

2.4 Concluding remarks

In this paper, I revisit the long-run trends about the evolution of the years of schooling, lifetime labor supply and retirement in the 20th century in the US. I document that hours worked per capita

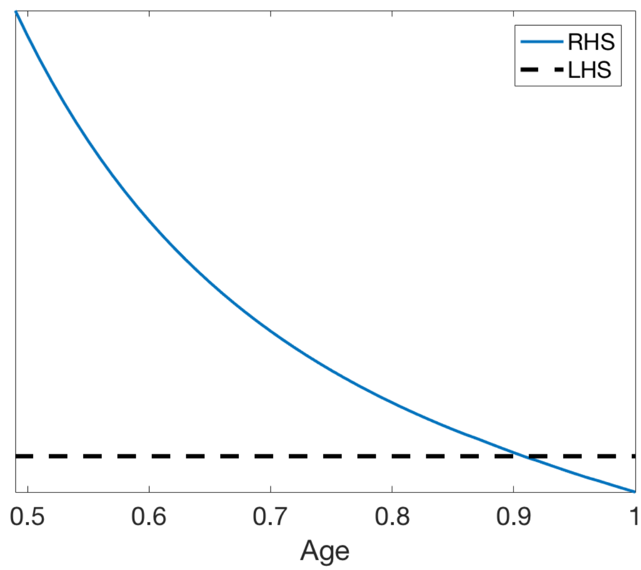

Fig. 2.7: This figure ilustrates equation 2.25. Horizontal axis: age, with $T = 1$. Vertical axis: left and right hand sides of equation 2.25. The intersection between the two lines determines the solution R.

declines sharply at younger and older ages, an remain stable during the prime-age period. Younger people spend much more time obtaining formal education, and older people enjoy leisure time in much longer periods of retirement after leaving the labor force. In the second part of the paper, I show that the standard Ben-Porath (1967) life-cycle model of human capital and labor supply, extended as to include both an optimal choice of the school period and length of labor force participation, is consistent with the stylized facts. I investigate the role of increasing life expectancy, the decline of the cost of schooling through the expansion of public provision of schools in the US during the first half of the 1900s, and improvements in productivity. The results are consistent with the empirical evidence presented in Moshe (2009) that stresses the importance of an increase in the returns to schooling in explaining the observed patterns of time allocation. To fully address the evidence on the evolution of the age-profile of survival rates and its consequences for optimal time allocation decisions in Cervellati and Sunde (2013), further research should include survival risk in the stylized model presented above. As noted before, the fact that survival probabilities increased the most for the age group of 65+ years, raises a quantitative question regarding the overall effect of increasing survival rates on labor supply among the 65+ age group as well as on total lifetime labor supply. I am working on the answer to that question, that will be included in a future version of this paper.

Acknowledgements I thank Lukas Mayr, Fabian Schuetze and especially Ramon Marimon for helpful comments.

Appendix

This section describes the evolution of human capital $h(a)$ when early life includes a schooling period, i.e. $x(a) = 1$ for some ages a.

$$\max \int_0^R wh(a)(1-x(a))da \tag{2.28}$$

subject to

$$\dot{h}(a) = z\,(h(a)x(a))^{\gamma} - \delta h(a) \quad [q(a)] \tag{2.29}$$

$$h(a)(1-x(a)) \geq 0 \quad [\lambda(a)] \tag{2.30}$$

First order conditions:

$$-wh(a) + q(a)\gamma z h(a)(h(a)x(a))^{\gamma-1} - \lambda(a)h(a) = 0 \tag{2.31}$$

$$\dot{q}(a) = -w(1-x(a)) - q(a)\left[\gamma z(h(a)x(a))^{\gamma-1} - \delta\right] - \lambda(a)(1-x(a)) \tag{2.32}$$

$$\lambda(a)h(a)(1-x(a)) = 0 \tag{2.33}$$

In the schooling period $n(a) = 1$ and $\lambda(a) \geq 0$. The conditions simplify to:

$$-w + q(a)\gamma z(h(a))^{\gamma-1} - \lambda(a) = 0 \tag{2.34}$$

$$\dot{q}(a) = -q(a)\left[\gamma z(h(a)x(a))^{\gamma-1} - \delta\right] \tag{2.35}$$

$$\lambda(a) \geq 0 \tag{2.36}$$

Together these imply

$$\dot{q}(a) = \delta q(a) - w - \lambda(a) \tag{2.37}$$

From the law of motion of capital, when $x(a) = 1$:

$$\dot{h}(a) = zh(a)^{\gamma} - \delta h(a) \tag{2.38}$$

Using 2.34 and the equation above, we obtain

$$w + \lambda(a) = \gamma q(a)\left[\frac{\dot{h}(a)}{h(a)} + \delta\right] \tag{2.39}$$

Hence, we have

$$\frac{\dot{q}(a)}{q(a)} + \gamma\frac{\dot{h}(a)}{h(a)} = (1-\gamma)\delta \tag{2.40}$$

References

Ben-Porath, Y. (1967): The Production of Human Capital and the Life Cycle of Earnings, *Journal of Political Economy 75 (4)*, 352–365.

Cervellati, M. and U. Sunde (2013): Life Expectancy, Schooling, and Lifetime Labor Supply: Theory and Evidence Revisited, *Econometrica 81 (5)*, 2055–2086.

Cooley, T. and E. Henriksen (2018): The demographic deficit, *Journal of Monetary Economics 93 (C)*, 45–62.

Costa, D. L. (1998): *The Evolution of Retirement: An American Economic History, 1880-1990*, University of Chicago Press.

Goldin, C. and L. F. Katz (1999): Human Capital and Social Capital: The Rise of Secondary Schooling in America, 1910-1940, *The Journal of Interdisciplinary History 29 (4)*, 683–723.

Kendrick, J. W. (1961): *Productivity Trends in the United States*, National Bureau of Economic Research, Inc.

Kendrick, J. W. (1973): *Postwar Productivity Trends in the United States, 1948–1969*, NBER.

Lee, C. (2001, Dec): The expected length of male retirement in the United States, 1850–1990, *Journal of Population Economics 14 (4)*, 641–650.

Moshe, H. (2009): Longevity and Lifetime Labor Supply: Evidence and Implications, *Econometrica 77 (6)*, 1829–1863.

Ramey, V. A. and N. Francis (2009, July): A Century of Work and Leisure, *American Economic Journal: Macroeconomics 1 (2)*, 189–224.

Rangazas, P. (2002): The Quantity and Quality of Schooling and U.S. Labor Productivity Growth (1870–2000), *Review of Economic Dynamics 5 (4)*, 932 – 964.

Scheiner, L. (2018): The Long-term Impact of Aging on the Federal Budget, Working Paper 40, Hutchins Center on Fiscal and Monetary Policy, Brookings Institution.

Shin, Y., A. Seshadri and R. Manuelli (2012): Lifetime Labor Supply and Human Capital Investment, 2012 Meeting Papers 946, Society for Economic Dynamics.

Chapter 3
Introducing an Austrian Backpack in Spain

This chapter was written with Julián Díaz-Saavedra and Ramon Marimon.

Abstract In an overlapping generations economy with incomplete insurance markets, the introduction of an employment fund – akin to the one introduced in Austria in 2003, also known as 'Austrian backpack'– can enhance production efficiency and social welfare, if it complements, and in part substitutes, the two classical systems of public insurance: pay-as-you-go pensions and unemployment insurance (UI). We show this in a calibrated dynamic general equilibrium model with heterogeneous agents of the Spanish economy (2014). A 'backpack' (BP) employment fund is an individual (across jobs) transferable fund, which earns the economy interest rate as a return and is financed with a small payroll tax (a BP tax). The worker can use the fund if he or she becomes unemployed or retires. To complement the existing Spanish pension and UI systems with a 2% BP tax would be preferred to the status quo by more than 90% of the households of the calibrated economy, a percentage that can be higher with a more substantial BP (i.e. higher BP tax). Our model also provides a framework where other reforms – e.g. a partial, or complete, substitution of current *unsustainable* pension systems – can be quantitatively assessed.

3.1 Introduction

The reform of existing public insurance systems is an ongoing issue which the last euro crisis and recession has only exacerbated, and which has been the focus of extensive research in economics and other social sciences. Most of this research has focused on the possible reform of separate specific systems: either the pay-as-you-go pension system or the unemployment insurance system (UI), but not on their interaction. However, there is an interesting reform that, in our opinion, deserves further analysis: this is the introduction of an *employment fund*, akin to the one introduced in Austria in 2003 – also known as the 'Austrian backpack'. This is the enquiry that we start in this paper: the effect of introducing such a fund to complement and partially, or even completely, substitute existing public insurance systems.

The basic features of a 'backpack' (BP) employment fund are: it is a fund own by the employee which accumulates, with a basic payroll (BP) tax, while working; it is transferable across jobs and can be finally used as a pension fund; usually it earns a market interest rate (i.e. it can be privately managed), but there may be restrictions in its use (e.g. additional individual contributions may be restricted and the worker may only be able to use it if he or she is unemployed, inactive or retired). While different forms of private employment funds are not a novelty in some countries – Austria being the leading example and it is has been proposed in Spain – such funds are not common as part of the public insurance policy. One example of a private funding scheme is the TIAA-CREF (Teachers Insurance and Annuity Association-College Retirement Equities Fund), which is a non-profit employment fund founded by Andrew Carnegie in 1918 and nowadays serving over 5 million active and retired employees; it has played, and plays, an important role in enhancing mobility among university professors across US universities. However, it is a retirement fund not designed to provide unemployment insurance, while the 'backpack' provides both forms of insurance. Obviously, privately saved assets can also play this double role. However, there are two features that distinguish the 'backpack' : first, and foremost, its character of 'forced savings', and, second, possibly a favourable tax treatment (both are common features of fully-funded pension systems). Our benchmark 'backpack' does not allow additional private contributions, the worker can only draw from it if not employed, and its returns are taxed (as any capital gains) but backpack assets are not taxed as part of the income.

The 'Austrian backpack' was introduced in 2003 as part of a broader labour reform. In particular, it was the socially agreed exchange for the elimination of the existing system of severance payments and, in fact, the 'backpack tax' of 1.5% was set according to this tradeoff. Kettemann *et al.* (2017) have shown (with a structurally estimated search-and-matching model) that this reform spiked job-creation and lowered unemployment. Nevertheless, the main effect of the reform came from the elimination of severance payments. We are interested in analysing the effect of just introducing the 'backpack' , therefore we do not pair its introduction to another labour reform. To our best knowledge, this analysis is missing and it is the focus of this paper.

As mentioned, there is a wide related literature on the reform of public insurance systems; nevertheless our work builds directly on, and integrates, two models: the model of Díaz-Giménez and Díaz-Saavedra (2009) and Díaz-Giménez and Díaz-Saavedra (2017), developed to study pension system reforms in Spain using overlapping generations general equilibrium models, and the model with job creation and destruction with search frictions and three employment states (employed, unemployed and inactive) of Krusell et al. (2011), further developed in Ábrahám et al. (2018) to study unemployment insurance reforms in Europe. The latter shows that there is ample room to improve existing European UI systems even within the limits of their current design in which unemployment benefits (UB) are determined by their duration and the replacement rate. In particular, unlimited UB duration emerges as a welfare-improving feature when agents have limited insurance possibilities. In sum, in our model economy agents – which we refer to as households – can differ by their age, education, and productivity, and they decide how much to save and consume, as well as their employment status, which also depends on the rates of job creation and destruction – i.e. agents can also differ by their assets and employment status and are subject to idiosyncratic risks.

In our benchmark economy, households can insure against their idiosyncratic risks privately, through their savings, but there is also public unemployment insurance and a pay-as-you go pension system, both financed with payroll taxes. An aggregate production function and a government that must balance the budget close the model. The model is calibrated to the Spanish economy (in 2014) with the distinct – welfare improving – feature of having unlimited unemployment benefits[1].

We introduce a 'backpack' , similar to the one introduced in Austria, with a 2% 'backpack tax' , and compute the new steady state. Even if it is a relatively small backpack, the effects within the economy are relatively large in terms of macroeconomic aggregates (higher capital and output) and prices (lower interest rates and higher wages), as well as in terms of households' decisions underlying the macroeconomic effects (substitution of private for backpack savings) and as a result of these effects (high productivity agents work more). Overall the economy is more productive and agents benefit from higher consumption. The final result is that there are substantial welfare gains, in terms of consumption-equivalent values and, importantly, of support for the change when agents are asked whether they would prefer to be in an economy with the 'backpack' . The fact that, at this stage, we only 'add the backpack' makes the comparison of steady states more meaningful since we do not violate any existing claims that retirees or unemployed may have on the current public insurance system. We also show that a 'bigger backpack' , can provide even higher welfare gains.

The next section presents our model economy, Section 3 its calibration and Section 4 the results.

3.2 The Model Economy

In this section we introduce the baseline model economy. We study an overlapping generations model economy with heterogeneous households, a representative firm, and a government. The model economy is an enhancement of framework in Díaz-Giménez and Díaz-Saavedra (2009), with job creation and destruction and dynamic work and search decisions as in Ábrahám et al. (2018).

3.2.1 The Households

Households in our baseline economy are heterogeneous and differ in their age, $j \in J$; in their education, $h \in H$; in their temporary productivity level $z \in \mathscr{Z}$; in their employment status, $l \in \mathscr{L}$, and in their assets, $a \in A$; Sets J, H, $\mathscr{Z}$, $\mathscr{L}$, and A, are all finite sets and we use $\mu_{j,h,z,l,a}$ to denote the measure of households of type (j,h,z,l,a). They also differ in their claims to different social insurance systems: unemployment benefits $ub \in UB$ and retirement pensions $p \in P$. In Section 3.4, we introduce an employment fund (the Austrian backpack) as yet another social insurance policy. In this section we describe the model that will be matched with the Spanish economy in 2014 and we do not make reference to the backpack system. We think of a household in our model as a

[1] This distinct feature also helps to reduce the number of states since our detailed heterogenous agents structure already results in 6,804,000 agent types!

single individual, even though we use the two terms interchangeably. To calibrate the model, we use individual data of persons older than 20 in the Spanish economy.

Age. Individuals enter the economy at age 20, the duration of their lifetimes is random, and they exit the economy at age 100 at the latest. Therefore $J = \{20, 21, ..., 100\}$. The parameter ψ_j denotes the conditional probability of survival from age j to age $j+1$. The notation makes explicit that the probabilities depend on age j, but not on education or other factors.

Fertility and immigration. In our model there is no demographic change and, therefore, economy fertility rates and immigration flows are not accounted for.

Education. Households can either be high school dropouts with $h = 1$, high school graduates who have not completed college $h = 2$, or college graduates denoted $h = 3$. Therefore $H = \{1, 2, 3\}$. A household's education level is exogenous and determined forever at the age of 20.

Labor market productivity. Individuals receive an endowment of efficiency labor units every period. This endowment has two components: a deterministic component, denoted $\varepsilon_{h,j}$ and a stochastic component, denoted by z. The deterministic component depends on the household's age and education, and we use it to characterize the life-cycle profiles of earnings. The stochastic component is independently and identically distributed across the households, and we use it to generate earnings and wealth inequality within the age cohorts. This component does not depend on the age or the education of the households, and we assume that it follows a first order, finite state, Markov chain with conditional transition probabilities given by Γ:

$$\Gamma \left[z'|z\right] = \Pr\left\{z_{t+1} = z'|z_t = z\right\}, \text{ with } z, z' \in Z. \tag{3.1}$$

Every period, agents who have not retired yet receive a new realization of z. Accordingly, the labor productivity is given by $\varepsilon_{h,j}z$. An individual may or may not have a job opportunity in any given period. If he does and decides to work, he works for a given number of hours denoted e_j, that depend on age.[2] His gross labor earnings during that period, y, are given by $\varepsilon_{h,j}z$ times the economy-wide wage rate w:

$$y = \omega \varepsilon_{h,j} z e_j. \tag{3.2}$$

Labor market status. In the model economy, an agent is either employed, unemployed, non-active or retired. Every individual enters the economy with a job opportunity. An individual with a job at hand in the beginning of the period, who decides to work, is employed in that period and his labor market status is denoted by $l = e$. An agent without a job or who decides not to work, can search for a new job for the next period - becoming unemployed -, or choose not to search and remain non-active in that period. An unemployed household who is eligible for unemployment benefits is labeled u^e, and if he is not eligible labeled $l = u^n$. An agent not working and also not actively searching for a job, who has not retired yet, is non-active and denoted with $l = n$. The eligibility criteria is defined below. Every worker faces a positive probability of becoming jobless next period, and every jobless agent (who has not retired) receives a job offer with some probability. The probability of receiving a job offer is higher for job-searchers than for non-active agents. Once a household has reached the early retirement age, it decides whether to retire. The retirement decision is irreversible and there is no mandatory retirement age.

[2] All the details on the calibration of the process $\varepsilon_{h,j}$ and e_j are provided in Section 3.3

Workers. A worker provides labor services and receives a compensation that depends on his efficiency labor units. Workers face a probability of loosing their job at the end of the period, denoted σ_j. This probability is age dependent, and we use it to generate the observed labor market flows between employment and non-employment states within age cohorts. With probability $1-\sigma_j$, the worker keeps his job, draws a productivity according to Γ, and decides whether to continue working or to quit.
Unemployed. Unemployed agents who lost a job at the end of the previous period and exercise a search effort are entitled to collect unemployment benefits determined by the function b, specified below. At the end of each unemployment period, an unemployed receives a job offer with probability λ_j^u. This probability is again age dependent, and we use it to generate the observed labor market flows between unemployment and employment.
Non-Active. Agents that are without a job and do not actively search for a new one are labeled non-active. Those agents are not eligible for unemployment benefits, and receive a job offer for next period with a lower probability than an unemployed agent, $\lambda_j^n < \lambda_j^u$. This probability is also age dependent, and we use it to generate the observed labor market flows between non-activity and employment.
Retirees. Households who are R_0 years old or older and have a job decide whether to to retire and collect the retirement pension. They take this decision after observing their current labor productivity. If they decide to retire, they loose the endowment of labor efficiency units for ever and exit the labor market.
Insurance Markets. An important feature of the model is that there are no insurance markets for the stochastic component of the endowment shock nor for unemployment risk. We model different public insurance systems that help agents in the economy to smooth consumption in face of these shocks.
Assets. Households in our model economy differ in their asset holdings, which are constrained to being non-negative. The absence of insurance markets give the households a precautionary motive to save. They do so by accumulating real assets which take the form of productive capital, denoted $a \in A$. For computational reasons below we restrict A to be a discrete set $A = \{a_1, a_2, ..., a_n\}$.
Unemployment Benefits. Eligibility for unemployment benefits is conditional on having lost a job at some point in the past and not having started a new job yet, and on continuously searching for a job. Eligibility does not expire as long as this condition is met, and non-eligibility is an absorbing state. Eligible agents receive unemployment benefits given $ub = b_0\bar{y}$, where b_0 is a replacement rate and $\bar{y}$ is the average labor earnings in the economy. Unemployment benefits are financed with payroll taxes, described below.
Pensions. Pension payments differ for each educational group,

$$p_h = p_b\bar{y}_h, \tag{3.3}$$

where $\bar{y}_h$ is the average earnings of households in educational group h during the last N_b years before the first retirement age, R_0, and p_b is a replacement rate. Specifically, $\bar{y}_h$ is computed as:

$$\bar{y}_h = \frac{1}{N_b}\sum_{j=R_0-N_b}^{R_0-1}\bar{y}_{jh} \tag{3.4}$$

where $\bar{y}_{jh}$ is the average gross labor earnings of workers aged j and with education h. Finally, in this model we assume that there are no early retirement penalties, nor minimum or maximum pensions. Pension payments are financed with payroll taxes.

Other transfers. In addition to the social insurance systems described before, households receive a small transfer from the government, denoted t_r.

Preferences. Households derive utility from consumption and leisure, and disutility from search effort. Given our assumption of a discrete choice of work and search, the period utility is described by a utility flow from consumption and the utility cost of time allocated to market work and to job search. Non-active and retired agents dedicate all the time endowment to leisure consumption. Accordingly, lifetime utility is given by

$$\mathbb{E}\sum_{j=20}^{100}\beta^{j-20}\psi_{j-1}\left[\frac{c^{1-\sigma}}{1-\sigma}-\alpha w-\gamma s\right], \tag{3.5}$$

where β is a time discount factor, c is consumption and σ the curvature parameter, α and γ are respectively the work and job search utility costs. ψ_j is the survival probability, and $\psi_{19}=1$. w equals 1 in periods of work and is zero otherwise, and similarly for the search decision s.

3.2.2 The Firm

In our model economy there is a representative firm. Aggregate output depends on aggregate capital, K, and on the aggregate labor input, L, through a constant returns to scale, Cobb-Douglas, aggregate production function of the form

$$Y = K^{\theta}(AL)^{1-\theta} \tag{3.6}$$

where A denotes an exogenous labor-augmenting productivity factor. Factor and product markets are perfectly competitive and the capital stock depreciates geometrically at a constant rate, δ.

3.2.3 The Government

The government taxes capital income, household income and consumption, and it confiscates unintentional bequests. It uses its revenues to finance an exogenous flow of public consumption, and to make transfers to households other than pensions. In addition, the government runs a pay-as-you-go pension system and provides unemployment benefits. The consolidated government and pension system budget constraint is then:

$$G+P+U+T_r = T_k+T_p+T_y+T_c+E \tag{3.7}$$

where G denotes government consumption, P denotes total pension payments, U denotes unemployment benefits, T_r denotes government transfers other than pensions, T_k, T_p, T_y, and T_c, denote the revenues collected by the capital income tax, the payroll tax, the household income tax, and the consumption tax, and E denotes unintentional bequests.

Capital income taxes. Capital income taxes are given by $\tau_k y_k$, where τ_k is the tax rate on gross capital income y_k.

Payroll taxes. Payroll taxes are proportional to before-tax labor earnings: $\tau_p y$.

Consumption taxes. Similarly, consumption taxes are simply $\tau_c c$, where τ_c is the consumption tax rate and c is consumption.

Household income taxes. In this model, we assume a simplified income tax formula according to which the income tax is proportional to the income level: $\tau_y y_b$, where τ_y is a tax rate parameter and y_b is the tax base. The income tax base depends on the employment status. If a household is employed,

$$y_b = (1-\tau_p)y + r(1-\tau_k)a + t_r. \tag{3.8}$$

For the unemployed and non-active agents,

$$y_b = r(1-\tau_k)a + t_r, \tag{3.9}$$

and for a retired household:

$$y_b = r(1-\tau_k)a + t_r + p. \tag{3.10}$$

In these expressions, p is the retirement pension, that depends on h as explained before.

3.2.4 The Backpack System

Our baseline model, calibrated to the Spanish economy in 2014, does not feature the employment fund that we denote by Backpack system. Hence, even though to simplify the model description we omit the backpack system in the next section, we explain it here to fix ideas. It is at the center of our analysis in the policy experiments discussed in Section 3.4. The backpack system is a fully-funded social security program, similar to the system introduced in Austria in 2003. Every individual starts without backpack claims. For every period of employment, a worker sees a small portion τ_b (around 1.5% in Austria) of his labor income deducted and invested into a personal employment-linked savings account, which is remunerated at the market rate of return, r. If b_t is the level of backpack assets at the beginning of an employment period, then next period's backpack evolves according to:

$$b_{t+1} = \tau_b y_t + (1 + r(1-\tau_k))b_t, \tag{3.11}$$

When a worker looses his job, his backpack assets are added to his private savings that he can use to finance consumption (present or future, as he can choose to save the backpack assets). In contrast, if a worker chooses to quit his job while still in the labor force, he keeps the backpack but cannot withdraw. In that period, the backpack evolves according to

$$b_{t+1} = (1 + r(1 - \tau_k))b_t. \tag{3.12}$$

Upon retirement, backpack assets are added to private savings and households can use them as they please. The aggregate amount of backpack assets is invested in the capital market and adds to the supply of productive capital in the economy.

3.2.5 The Households' Decision Problem

The households' problem is described recursively. To simplify the exposition, we describe first the consumption and savings decision facing each household after his labor market decision has been taken. Given the value of having a job opportunity or being jobless, the optimal consumption and savings decisions determine the value of working, searching, inactivity, and retirement. With those values, we in turn describe the optimal labor market decisions.

We start with individuals that are younger than the minimum retirement age, specifically $j < R_0$. In this way we can abstract, for now, from the retirement decision. An individual of age j and education level h, private savings a, stochastic productivity z, and is currently employed (w), faces the following optimization problem:

$$W_{h,j}(a,z) = \max_{(c,a') \in B_h^e(a,z)} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \alpha + \beta \psi_j \sum_{z' \in Z} \Gamma(z'|z) \Big[(1-\sigma_j) J_{h,j+1}(a',z') + \sigma_j L_{h,j+1}(a',z',1) \Big] \right\}, \tag{3.13}$$

subject to the budget constraint B_h^e given by

$$B_h^e(a,z) = \Big\{ (c,a') : c, a' \geq 0, \text{ and } (1+\tau_c)c + a' + \tau_y y_b + \tau_p y \leq (1 + r(1-\tau_k))a + y + t_r \Big\}. \tag{3.14}$$

with $y = \omega \varepsilon_{h,j} z e_j$ and $y_b = (1-\tau_p)y + r(1-\tau_k)a + t_r$.

Equation (3.13) above reads in the following way: the first two terms inside the curly brackets reflect the flow utility from consumption and the utility cost of work, α. The continuation value, discounted by β times the probability of survival until age $j+1$, given by ψ_j, reflects all the possible continuation histories of the realization of the stochastic component $z' \in \mathscr{Z}$, and two distinct labor market outcomes. With probability $1-\sigma_j$, the worker keeps the job in the next period, which has a value $J_{h,j+1}$ that also depends on next period's assets a' and the new realization of idiosyncratic productivity z'. With probability σ_j, the worker looses his job and starts next period jobless, with value $L_{h,j+1}$. This value depends on whether the household is eligible for unemployment compensation. After a period of work and a separation shock, the agent is eligible and hence the last argument of L equals 1, denoting eligibility for unemployment benefits (0 denotes not eligible, below). Savings a' are optimally chosen today and z' follows the Markov chain described in (3.1).

A household currently unemployed and eligible for unemployment benefits solves the following problem:

$$U_{h,j}(a,z,1) = \max_{(c,a')\in B_h^{u^e}(a,z)} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \gamma + \beta\psi_j \sum_{z'\in Z} \Gamma(z'|z) \Big[\lambda_j^u J_{h,j+1}(a',z') + (1-\lambda_j^u) L_{h,j+1}(a',z',1) \Big] \right\}, \tag{3.15}$$

subject to

$$B_h^u(a,z,1) = \Big\{ (c,a') : c,a' \geq 0, \text{ and } (1+\tau_c)c + a' + \tau_y y_b + \leq (1+r(1-\tau_k))a + ub + t_r \Big\}, \tag{3.16}$$

where $y_b = r(1-\tau_k)a + t_r$ (also in (3.18), (3.20) and (3.25) below). Similarly, an unemployed household who is not eligible for benefits solves

$$U_{h,j}(a,z,0) = \max_{(c,a')\in B_h^{u^n}(a,z)} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \gamma + \beta\psi_j \sum_{z'\in Z} \Gamma(z'|z) \Big[\lambda_j^u J_{h,j+1}(a',z') + (1-\lambda_j^u) L_{h,j+1}(a',z',0) \Big] \right\}, \tag{3.17}$$

with the important difference that in this case there are no unemployment benefits:

$$B_h^u(a,z,0) = \Big\{ (c,a') : c,a' \geq 0, \text{ and } (1+\tau_c)c + a' + \tau_y y_b \leq (1+r(1-\tau_k))a + t_r \Big\}. \tag{3.18}$$

Note that eligibility for unemployment benefits is represented by the last argument in the value functions U and L, and the expression for the budget constraint of the unemployed: 1 means the agent is eligible for benefits, a 0 means he is not. An agent who is currently unemployed, eligible for benefits, and does not find a job for next period will continue to be eligible (and will collect unemployment benefits if he decides to continue searching for a job). In contrast, an agent who is currently non-eligible (because he quit the job or because we was not eligible last period) will continue to be non-eligible next period. Finally, households may decide to stay out of the labor market, by not working and not actively searching for a job. The problem is:

$$N_{h,j}(a,z) = \max_{(c,a')\in B_h^n(a)} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta\psi_j \sum_{z'\in S} \Gamma(z'|z) \Big[\lambda_j^n J_{h,j+1}(a',z') + (1-\lambda_j^n) L_{h,j+1}(a',z',0) \Big] \right\}, \tag{3.19}$$

For non-active households, the flow utility comes only from consumption, and the job arrival rates are lower than for the unemployed and given by λ_j^n. As in the previous case, consumption and investment are supported solely by past savings and government transfers:

$$B_h^n(a) = \Big\{(c,a') : c,a' \geq 0, \text{ and } (1+\tau_c)c + a' + \tau_y y_b \leq (1+r(1-\tau_k))a + t_r\Big\}. \tag{3.20}$$

To close the description of the household's problem when $j < R_0$, the values of having a job or being jobless at the beginning of a period pin down the work and search decicions:

$$J_{h,j}(a,z) = \max_{w\in\{0,1\}} \Big\{wW_{h,j}(a,z) + (1-w)L_{h,j}(a,z)\Big\}, \tag{3.21}$$

$$L_{h,j}(a,z,1) = \max_{s\in\{0,1\}} \Big\{sU_{h,j}(a,z,1) + (1-s)N_{h,j}(a,z)\Big\}, \tag{3.22}$$

$$L_{h,j}(a,z,0) = \max_{s\in\{0,1\}} \Big\{sU_{h,j}(a,z,0) + (1-s)N_{h,j}(a,z)\Big\}. \tag{3.23}$$

We consider now the the problem of the retiree and the retirement decision. Retired individuals are not in the labor market and have no endowment of efficiency units of labor. They finance consumption with past private savings, pension payments (and with backpack savings if there is a backpack system). The problem is a standard consumption-savings decision, with survival risk and a certain maximum attainable age, assumed to be $j = 100$. At age $j = 99$, the continuation value is zero because the agent exist the economy next period with probability one. Before that, the retired household solves:

$$Vr_{h,j}(a) = \max_{(c,a')\in B_h^r(a)} \left\{\frac{c^{1-\sigma}}{1-\sigma} + \beta\psi_j Vr_{h,j+1}(a')\right\}, \tag{3.24}$$

subject to

$$B_h^r(a) = \Big\{(c,a') : c,a' \geq 0, \text{ and } (1+\tau_c)c + a' + \tau_y y_b \leq (1+r(1-\tau_k))a + p_h + t_r\Big\}. \tag{3.25}$$

The problem depends on state a and on h, the education level, because the pension payments depend on h: p_h. From the first retirement age R_0 on, an employed household compares the value of retirement $Vr_{h,j}$ with the value of working one additional period, depending on his current labor market productivity. In addition to his current endowment of efficiency units, the decision depends on his savings and on his education level that, as we have assumed, determines is pension payments. The retirement decision is given by:

$$r(j,h,a,z) = \operatorname*{argmax}_{\rho\in\{0,1\}} \Big\{(1-\rho)J_{h,j}(a,z) + \rho Vr_{h,j}(a)\Big\}. \tag{3.26}$$

3.2.6 Steady-state dynamics

The steady-state dynamics of the economies under study have the following characterisation. Given a distribution of households entering the economy ($j = 20$ and $a = 0$; say, at T) they all receive a job opportunity and make their consumption, asset and employment decisions. These households' decisions together with their survival probabilities define the distribution of this cohort the following year ($T+1$) at $j = 21$, but it also the distribution of households of $j = 21$ at T. Similarly, for $j = 22, ..., 100$; that is, the different cohorts coexisting at T mirror the evolution of the distribution of households entering the economy at T up to the end of their potential survival $j = 100$. In other words, the decisions that agents of generation T make through their live are already made in the year they enter the labour market by older agents if they have the same state. By construction, this is a steady-state distribution, which is our benchmark distribution. Different economies simply expose the T cohort distribution to different public insurance systems and, therefore, all the cohorts coexisting at T behave as if the given system was in place when they entered the economy.

3.2.7 Definition of Stationary Equilibrium

Let $j \in J$, $h \in H$, $z \in \mathscr{Z}$, $l \in \mathscr{L}$, and $a \in A$, and let $\mu_{j,h,z,l,a}$ be a probability measure defined on $\Re = J \times H \times \mathscr{Z} \times \mathscr{L} \times A$.[3] Then, a stationary competitive equilibrium for this economy is a government policy, $\{G, P, T_r, U, T_k, T_s, T_y, T_c, E\}$, a household policy, $\{c(j,h,z,l,a)$, $w(j,h,z,l,a)$, $s(j,h,z,l,a)$, $r(j,h,a,z)$, $a^{'}(j,h,z,l,a)\}$, a measure, μ, factor prices, $\{r, \omega\}$, macroeconomic aggregates, $\{C, Y, K, L\}$, and a function, Q, such that:

(i) The government policy satisfy the consolidated government described in Expression (3.7).
(ii) Firms behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities $r = f_1(K, AL) - \delta$ and $\omega = f_2(K, AL)$.
(iii) Given the government policy, and factor prices, the household policy solves the households' decision problem defined in Expressions (3.13), through (3.26).
(iv) The stock of capital, consumption, the aggregate labor input, pension payments, unemployment benefit payments, lump-sum transfers, tax revenues, and accidental bequests are obtained aggregating over the model economy households as follows:

[3] For convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.

$$K = \int a \, d\mu$$
$$C = \int c \, d\mu$$
$$L = \int \varepsilon_{jh} z e_j \, d\mu$$
$$P = \int p_h \, d\mu$$
$$U = \int ub \, d\mu$$
$$T_r = \int t_r \, d\mu$$
$$T_c = \int \tau_c c \, d\mu$$
$$T_k = \int \tau_k y_k \, d\mu$$
$$T_p = \int \tau_p y \, d\mu$$
$$T_y = \int \tau_y y_b \, d\mu$$
$$E = \int (1 - \psi_j) a' \, d\mu$$

where all the integrals are defined over the state space $\Re$.

(vi) The goods market clears:

$$C + \int (a' - (1-\delta)a) d\mu + G = F(K, AL). \quad (3.27)$$

(vii) The law of motion for μ_j is:

$$\mu_{j+1} = \int_{\Re} Q d\mu_j. \quad (3.28)$$

Describing function Q formally is complicated because it specifies the transitions of the measure of households along its five dimensions: age, education level, productivity, employment status, and assets holdings. An informal description of this function is the following: We assume that new-entrants, who are 20 years old, enter the economy with a job opportunity, that they draw the stochastic component of their endowment of efficiency labor units from its invariant distribution, and that they own zero assets. Their educational shares are exogenous. The evolution of μ_{jh} is exogenous, it replicates the the distribution by age and education of the Spanish population in our calibration target year, 2014. The evolution of μ_z is governed by the conditional transition probability matrix of its stochastic component. The evolution of μ_l, is governed by the exogenous probabilities of find/loss a job, by the endogenous employment and search decisions, and by the optimal decision to retire. The evolution of μ_a is determined by the optimal savings decision and by the changes in the population.

3.3 Calibration

To calibrate our model economy we do the following: First, we choose a calibration target country —Spain in this article— and a calibration target year —2014 in this article. Then we choose the initial conditions and the parameter values that allow our model economy to replicate as closely as possible selected macroeconomic aggregates and ratios, distributional statistics, and the institutional details of our chosen country in our target year. We describe these steps in the subsections below.

3.3.1 Initialising the steady-state

In order to determine the steady-state, first we choose as an initial distribution of households $\mu_0 = \mu_{2014}$; that is, we take μ_{jh} at year 2014 directly from the Spanish Institute of Statistics (INE). The initial distribution of households implies an initial value for the capital stock. This value is $K_{2014} = 8.8025$. The initial distribution of households and the initial survival probabilities determine the initial value of unintentional bequests, E_{2014}. Finally, we must also specify the initial values for the productivity process, A_{2014}. Since A_{2014} determines the units which we use to measure output and does nothing else, we choose $A_{2014} = 1.0$.

3.3.2 Parameters

Once the initial conditions are specified, to characterize our model economy fully, we must choose the values of a total of 40 parameters. Of these 40 parameters, 4 describe the household preferences, 3 describe the age profile of hours worked, 21 the process on the endowment of efficiency labor units, 2 the production technology, 4 the pension system rules, and 6 the remaining components of the government policy. To choose the values of these 40 parameters we need 40 equations or calibration targets which we describe below.

3.3.3 Equations

To determine the values of the 40 parameters that identify our model economy, we do the following. First, we determine the values of a group of 21 parameters directly using equations that involve either one parameter only, or one parameter and our guesses for (K, L). To determine the values of the remaining 19 parameters we construct a system of 19 non-linear equations. Most of these equations require that various statistics in our model economy replicate the values of the corresponding Spanish statistics in 2014. We describe the determination of both sets of parameters in the subsections below.

3.3.3.1 Parameters determined solving single equations

The life-cycle profile of earnings. We measure the deterministic component of the process on the endowment of efficiency labor units independently of the rest of the model. We estimate the values of the parameters of the three quadratic functions that we describe in Expression (3.29), using the age and educational distributions of hourly wages reported by the *Instituto Nacional de Estadística* (INE) in the *Encuesta de Estructura Salarial* (2010) for Spain.[4] This procedure allows us to identify the values of 9 parameters directly.

$$\varepsilon_{jh} = \xi_{1h} + \xi_{2h} j - \xi_{3h} j^2 \tag{3.29}$$

Age profile of hours worked. To characterize the age profile of full-time hours, we use data from the Encuesta de Empleo del Tiempo 2010 provided by the Spanish National Institute of Statistics (INE). We impute hours supplied to the market as a fraction of the total time endowment and we assume that the time endowment is 14 hours a day, 7 days a week. We then parameterize the average allocated time to market activities per age with the help of quadratic curves. That is, we estimate e_j as:

$$e_j = a_1 + a_2 j - a_3 j^2 \tag{3.30}$$

Full hours worked rise initially and then they level off. Starting in the 50s, hours decline slightly. This procedure allows us to identify the values of 3 parameters directly.

The pension system. In 2014 in Spain, the payroll tax rate paid by households was 28.3 percent and it was levied only on the first 50,358 euros of annual gross labor income. Since we omit the tax cap, we impose that all gross earnings pay pension contributions. We also impose that payroll tax collections are used to finance both pension payments and unemployment benefits. This implies that the payroll tax rate in our model economy is 0.2954.

Our choice for the number of years used to compute the retirement pensions in our benchmark model economy is $N_b = 17$. This is because in 2014 the Spanish *Régimen General de la Seguridad Social* took into account the last 17 years of contributions prior to retirement to compute the pension. Finally, our choice for the first retirement age is $R_0 = 61$.

Government policy. To specify the government policy, we must choose the values of government consumption, G_t, of the tax rate on capital income, τ_k, of the tax rate on income, τ_y, and of the tax rate on consumption, τ_c.

Preferences. Of the three parameters in the utility function, we choose the value of only σ directly. Specifically, we choose $\sigma = 2.0$.

Technology. According to the Spanish National Institute of Statistics data (INE), the capital income share in Spanish GDP was 0.4702 in 2014. Consequently, we choose $\theta = 0.4702$.

[4] Since we only have data until age 64, we estimate the quadratic functions for workers in the 20–64 age cohort and we project the resulting functions from age 65 onwards.

Adding up. So far we have determined the values of 21 parameters either directly or as functions of our guesses for (K, L) only. We report their values in Table 3.1.

3.3.3.2 Parameters determined solving a system of equations

We still have to determine the values of 19 parameters. To find the values of those 19 parameters we need 19 equations. Of those equations, 14 require that model economy statistics replicate the value of the corresponding statistics for the Spanish economy in 2014, 4 are normalization conditions, and the last one is the government budget constraint that allows us to determine the value of T_r/Y^* residually.

Aggregate Targets. According to the BBVA database, in 2014 the value of the Spanish capital stock was 3,194,730 million euros.[5] According to the *Instituto Nacional de Estadística* (INE) in 2014 the Spanish Gross Domestic Product at market prices was 1,037,250 million euros. Dividing these two numbers, we obtain $K/Y = 3.08$, which is our target value for the model economy capital to output ratio.

According to the INE, Private Consumption plus indirect taxes was 586,080 million euros in 2014 (see Díaz-Giménez and Díaz-Saavedra (2017) for an explanation). That same year, pension payments were 114,410 million euros and unemployment benefits ammounted 24,570 million euros. Consequently, the ratios of these variables to GDP at market prices are 56.4, 11.1, and 2.4 percent.

Finally, and according to the Encuesta de Población Activa (INE), in Spain in 2014 there were 36,804,100 people aged 20+ years old. That same survey reports that 17,264,850 were workers and 5,436,500 were unemployed. Consequently, these numbers imply that the share of workers was 46.9 percent and the share of unemployed were 14.8 percent.

Distributional Targets. We target the 3 Gini indexes and 5 points of the Lorenz curves of the Spanish distributions of earnings, income and wealth. We have taken these statistics from the Spanish National Institute of Statistics (INE), the OECD, and Budría and Díaz-Giménez (2006), and we report them in bold face in Table 3.3. Castañeda et al. (2003) argue in favor of this calibration procedure to replicate the inequality reported in the data. These targets give us a total of 8 additional equations.

Normalization conditions. In our model economy there are 4 normalization conditions. The transition probability matrix on the stochastic component of the endowment of efficiency labor units process is a Markov matrix and therefore its rows must add up to one. This gives us three normalization conditions. We also normalize the first realization of this process to be $z(1) = 1$.

The Government Budget. Finally, the government budget is an additional equation that allows us to obtain residually the government transfers to output ratio, T_r/Y^*.

The Parameters. The 14 parameters determined by the system are the following:

[5] This number can be found at $http://www.fbbva.es/TLFU/microsites/stock09/fbbva\_stock08\_index.html$.

- Preferences: β, α, and γ.
- Technology: δ.
- Stochastic process for labor productivity: $z(2), z(3), z_{11}, z_{12}, z_{21}, z_{22}, z_{32}$, and z_{33}.
- Pension system: ϕ.
- Fiscal policy: b_0.

3.3.3.3 Methodology

To solve this system of equations we use a standard non-linear equation solver. Specifically, we use a modification of Powell's hybrid method, implemented in subroutine DNSQ from the SLATEC package.

The DNSQ routine works as follows

1. Choose the weights that define the loss function that has to be minimized
2. Choose a vector of initial values for the 14 unknown parameters
3. Solve the model economy
4. Update the vector of parameters
5. Iterate until no further improvements of the loss function can be found.

Table 3.4 provides the parameter values of our calibration and of their accuracy.

3.4 Comparing economies with different public insurance systems

In this section, we compare economies that differ in terms of the public insurance systems that are available to households. Our baseline economy, described above, features an unemployment insurance (UI) system and a pension system for retirees, as in Spain. We compare this economy to an economy where there is only the pay-as-you-go pension system but no unemployment insurance; one in which in addition to the pension system, there is a backpack system (but no UI); and finally one in which all the three systems are available (denoted BP Economy). The backpack system introduced is parametrized with a contribution rate of $\tau_b = 2.0\%$ of labor earnings, as described in Section 3.2.4. The description of household's problem changes slightly, with the backpack b_t becoming a relevant state variable. With the law of motion of the backpack described in 3.2.4, it is straightforward to extended the optimization problem presented in 3.2.5 to account for the new state variable.

The introduction of the backpack system affects individual decisions and aggregate variables. There are several direct effects of the backpack system at the individual level. The backpack is a substitute for private insurance, and discourages savings. It helps to smooth consumption after a job loss, when jobless individuals can withdraw from their backpack; and it affects the allocation of consumption over the life-cycle, since workers can use the accumulated backpack savings to finance consumption after retirement.

Since we assume that there are no annuity markets in the economy, households leave accidental bequests when they die. Such bequests are wasted at the individual level, therefore an increase in private savings or in backpack savings generates additional bequests.[6] In addition, since the backpack system is financed with taxation of labor income, it affects the labor supply decision. It allows to reduce individual tax payments, since both backpack assets and private assets are subject to capital taxation, backpack income is not subject to household income taxation whereas capital income is. Shifting savings from private capital holdings into the backpack account allows for a reduction in tax payments. Finally, at the retirement age, the value of the accumulated backpack assets affects the retirement decision. The worker faces the tradeoff between leaving the labor force and collecting the retirement pension and cashing out backpack savings, or prolonging the career and continuing accumulating backpack savings while working.

The resolution of these individual tradeoffs affect aggregate outcomes. The introduction of the backpack influences aggregate investment and the capital stock in the economy, depending on the offsetting effects on private savings and accumulation of backpack assets. It affects aggregate labor supply, depending on the impact of taxes and labor supply incentives through the employment-linked savings technology. By distorting the allocation of capital and labor, it affects aggregate output and consumption, that may increase or decrease if there is under- or over-accumulation of capital in the economy. As a consequence, the equilibrium market prices of capital and labor may change, again influencing savings and labor supply decision of households. The introduction of different policies and the corresponding household and market forces also impact the government budget constraint. Longer retirement periods and higher average wages increases pension payments and hence social security taxes on workers. Higher unemployment and wages also increase taxes, necessary to finance unemployment benefit expenditures. More production, consumption, and private investment enlarge the tax base and, for a given level of government expenditures, imply lower tax rates. These changes in government policy further influence households decisions and welfare, and are accounted for in the results presented below.

When comparing the different scenarios, there are a few policy parameters that we change and others we intentionally keep fixed. Specifically, we fix the level of all tax rates except for the payroll tax, that adjusts across economies to clear the social security budget (payroll tax collections finance unemployment benefits and pensions). We also fix the level of the individual lump sum transfer. Therefore, given that all other tax rates are fixed, changes in total tax revenue are thrown into or fished from the sea (in the form of wasteful government consumption). In the next subsection, we evaluate social welfare across the different economies by calculating the implied *consumption equivalent variation* (CEV) in the expected lifetime utility of individuals entering the economy at age 20, relative to the benchmark model that is calibrated to the Spanish economy in 2014. We calculate the welfare changes for specific groups in the population, depending on education, labor market productivity, and labor market status. After we introduce the backpack system in the model, we further dissect the welfare consequences by isolating the more important channels discussed above.

[6] This is in contrast with the tax-transfer pension system that directly transfers resources from young workers to old retirees hence helps avoid accidental bequests.

3.4.1 Unemployment Insurance and Backpack System

Compared with the baseline economy, replacing the existing unemployment benefit system with a backpack system (BP) with a contribution rate of 2% has a large impact in the economy. Households enter the economy without backpack claims. Absent an unemployment insurance policy, young households need to privately insure the risk of loosing a job early in life. Aggregate savings increase substantially, more than 20% relative to the baseline economy. The return on capital goes down from 5.3% in the baseline to 3.9% in the economy with no unemployment insurance. Relative to total output, this increase in precautionary savings raises investment 3.7% and the share of output allocated to private consumption goes down by more than 2%.

Table 3.5: Aggregates and output ratios in the baseline economy and in the economy without UI but with BP system (Only BP).

	Values			Y Ratios*		
	Baseline	Only BP	%Δ	Baseline	Only BP	$\Delta p.p.^a$
Y	2.3565	2.5763	9.32	100.00	100.00	0.00
K	8.7809	10.8108	23.11	3.72	4.19	0.47
L	0.7311	0.7200	–1.52	31.02	27.94	–3.08
H	0.2115	0.2086	–1.38	8.97	8.09	–0.88
C	1.0690	1.1124	4.05	45.36	43.17	–2.19
G	0.5182	0.5295	2.18	21.99	20.55	–1.45
I	0.7693	0.9344	21.46	32.54	36.26	3.72

Only BP is the alternative economy without UI but with BP system. Y: output, K: capital stock, L: labor in efficiency units, H hours of labor supply, C: aggregate private consumption plus consumption tax collections, G: government consumption, I: investment. The ratio K/Y measured in model units and not in percentage terms.
[a] : Difference in percentage points.
$*$: As a share of output at factor cost.

The over-accumulation of capital in the economy is unproductive and costly in terms of social welfare, as Table 3.6b below shows. Low productivity workers are hurt the most. Nevertheless, this capital over-accumulation effect is even stronger if the unemployment system is eliminated and there is no ‘backpack’ to replace it, as Table 3.6a below shows.

Table 3.6: Consumption Equivalent Variation (%) in average lifetime utility relative to the baseline economy, in an alternative scenario:

(a) Without Unemployment Insurance.

Education	Labor Productivity		
	Low	Medium	High
Dropouts	-18.73	-11.54	-6.76
High School	-20.23	-12.41	-6.58
College	-24.91	-17.04	-10.12

(b) With only a Backpack System.

Education	Labor Productivity		
	Low	Medium	High
Dropouts	-18.42	-11.20	-6.30
High School	-19.96	-12.05	-6.10
College	-24.62	-16.71	-9.54

3.4.2 Pension System and Backpack System

In contrast to the previous results, adding the backpack system in the baseline economy (that features unemployment insurance and the pay-as-you-go pension system) implies significant positive welfare gains for all demographic groups. As described above, the introduction of the backpack account affects individual decisions, aggregate quantities and prices. Households partially substitute private savings by saving through the backpack system. While private asset holdings go down, the backpack savings that are added to the supply of capital in the economy imply that, at the new equilibrium, the aggregate capital stock is 3.56% larger. Hours worked slightly decrease, but high productivity workers work more, such that the aggregate effective labor increases. This implies that the level of output increases by 2%. In the baseline economy, capital is productive at the margin and the interest rate is relatively high. Hence the backpack system, by slightly increasing the capital stock, improves the allocation of resources in the economy. A larger capital stock decreases the interest rate to 5.1% and makes labor more productive at the margin, therefore the wage rate increases slightly more than 1%. Aggregate consumption increases by 1.21%. Relative to output, aggregate consumption declines slightly (less than half a percentage point).

Table 3.7: Aggregates and output ratios in the baseline economy and in the economy with the BP system (BP Economy).

	Values			Y Ratios*		
	Baseline	BP Economy	%Δ	Baseline	BP Economy	$\Delta p.p.^a$
Y	2.3565	2.4035	1.99	100.00	100.00	0.00
K	8.7809	9.0942	3.56	3.72	3.78	0.06
L	0.7311	0.7355	0.60	31.02	30.60	–0.42
H	0.2115	0.2113	–0.10	8.97	8.79	–0.17
C	1.0690	1.0820	1.21	45.36	45.02	–0.34
G	0.5182	0.5179	–0.06	21.99	21.54	–0.55
I	0.7693	0.8036	4.45	32.54	33.43	0.89

The BP Economy is the alternative economy with BP system. Y: output, K: capital stock, L: labor in efficiency units, H hours of labor supply, C: aggregate private consumption plus consumption tax collections, G: government consumption, I: investment. The ratio K/Y measured in model units and not in percentage terms.
[a] : Difference in percentage points.
$*$: As a share of output at factor cost.

At older ages unemployment declines, bringing additional utility from leisure, and the share of retired households declines. Workers move to the pension system later by delaying retirement (albeit some of them are inactive) and consuming out of the accumulated backpack savings.

The breakdown of the welfare gains across education and productivity types is presented in Table 3.8.

Table 3.8: Consumption Equivalent Variation (%) in average lifetime utility in the economy with a backpack system (BP), relative to the baseline economy.

Education	Labor Productivity		
	Low	Medium	High
Dropouts	1.59	1.36	1.29
High School	1.52	1.40	1.56
College	1.44	1.48	1.41

Table 3.9: Labor Market Shares (% of population).

	W	U	I	R
Spain[a]	46.9	14.8	14.2	24.1
Baseline	54.1	14.9	5.7	25.2
Only BP	53.6	14.3	7.0	25.1
BP Economy	54.3	14.7	6.3	24.7

[a]Spanish data is from 2014, retired households include those collecting other pensions than retirement pensions (disability pensions, widowhood pensions, etc.) W: workers, U: unemployed, I: inactive, R: retirees. All shares correspond to people aged 20+.

The fact that there are relatively less unemployed and retired households in the backpack economy implies a smaller burden on the social security budget. Accordingly, payroll taxes decline which, together with the income tax exemption on backpack asset income, eases the distortionary effects of taxation relative to the baseline economy. Table 3.10 shows the decline in the payroll tax τ_p, and the decline in capital income tax collection. Nevertheless, total tax revenues are slightly higher in the BP Economy (0.89 vs. 0.87). Note that the government budget constraint is satisfied in both economies, with the increase in tax collection being compensated by higher government expenditures.

Table 3.10: Policy Parameters and tax revenue ratios in the baseline economy and in the economy with the BP system (BP Economy).

	Tax Rates			Tax Revenues			Revenue Y Ratios (%)*		
	Baseline	BP Economy	%Δ	Baseline	BP Economy	%Δ	Baseline	BP Economy	$\Delta p.p^a$
τ_c	0.2320	0.2320	0.00	0.2480	0.2511	1.25	10.52	10.44	–0.08
τ_y	0.1213	0.1213	0.00	0.2018	0.1987	–1.54	8.56	8.26	–0.30
τ_k	0.1111	0.1111	0.00	0.0521	0.0519	–0.39	2.21	2.15	–0.06
τ_p	0.2965	0.2942	–0.78	0.3684	0.3658	–0.71	15.63	15.21	–0.42
τ_b	–	0.0200	–	–	0.0253	–	–	1.06	–

τ_c: consumption tax rate, τ_y: household income tax rate, τ_k: capital income tax rate, τ_p: payroll tax, τ_b: backpack contribution tax.
[a] : Difference in percentage points.
* : As a share of output at factor cost.

Regarding inequality, moving from 1.1 to 1.3 increases wealth inequality since low earners (those who receive the stochastic productivity shock 1) reduce their holdings of liquid assets proportionately more than those medium and high earners. Specifically, the Gini of Wealth increases from 0.6663 to 0.6795 (see Table 3.3). Nevertheless, that there is no significant variation in earnings inequality, or in that of pensions. Consequently, income inequality slightly increases from

0.3582 to 0.3588. In sum, there is an overall welfare improvement with a moderate increase in inequality.

Table 3.11: The Distributions of Earnings, Income, and Wealth*

		Bottom	Quintiles					Top
	Gini	10	1st	2nd	3rd	4th	5th	10
The Earnings Distributions (%)								
Spain	0.35	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Baseline	0.39	3.2	7.7	10.8	13.0	20.5	48.1	31.9
BP Economy	0.39	3.2	7.7	10.8	12.9	20.4	48.1	32.1
The Income Distributions (%)								
Spain	0.35	2.1	6.3	12.1	17.2	23.7	40.7	25.0
Baseline	0.36	2.9	6.9	12.0	15.4	21.6	43.9	29.0
BP Economy	0.36	2.9	6.9	11.9	15.5	21.8	43.9	29.2
The Wealth Distributions (%)								
Spain	0.57	0.0	0.9	6.6	12.5	20.6	59.5	42.5
Baseline	0.66	0.0	0.3	3.0	7.5	20.1	69.0	46.1
BP Economy	0.68	0.0	0.3	2.7	7.0	20.0	69.9	46.8

*The source for the Spanish data of earnings and income are the Spanish National Institute of Statistics (INE) and the OECD. The source for the Spanish data of wealth is the 2004 *Encuesta Financiera de las Familias Españolas* as reported in Budría and Díaz-Giménez (2006). The model economy statistics correspond to 2014.

3.4.2.1 Welfare Decomposition

As explained above, the introduction of the backpack system affects households directly by influencing their consumption-savings and labor market decisions, and also indirectly through changes in prices and taxes. We decompose the total effect presented in Table 3.8 by conducting the following exercise.

First, starting from the baseline economy, we solve for the (partial equilibrium) steady state with a backpack policy in place with a 2% contribution rate, and with the backpack income taxed the same way as private savings. In addition, we fix the factor prices at the baseline level, and the payroll tax is also fixed at the baseline. We compare lifetime utility for different groups in this economy with the corresponding values of the baseline economy. The difference, expressed in percentage change of consumption equivalent units, is shown in 3.12a. We then solve for the steady state in which the payroll tax adjusts, such that the decrease in pension payments and unemployment benefit expenditures is compensated with a decrease in τ_p. The additional welfare gains of this decrease in taxes are in 3.12b. In the BP economy, backpack income is not subject to household income tax. 3.12c shows the impact of this tax exemption, positive for all demographic groups. Finally, the increase in capital and labor supply affects the equilibrium capital-labor ratio and hence prices. As explained above, in the baseline economy the return on capital is relatively high. When these prices adjust, high productivity households suffer with a lower return on their savings, and low productivity workers enjoy an upgrade in their labor income with increasing wages. The effect of prices in shown in 3.12d. Adding up all the contributions, we obtain the total welfare effect of the introduction of the backpack system, presented in Table 3.8.

Table 3.12: Decomposition of welfare effects of the backpack system.

(a) With the BP, fixed prices, taxing backpack income, fixing all taxes at the Baseline.

Education	Labor Productivity		
	Low	Medium	High
Dropouts	0.81	0.80	1.11
High School	0.78	0.83	1.35
College	0.77	0.92	1.12

(b) Lower payroll tax.

Education	Labor Productivity		
	Low	Medium	High
Dropouts	+0.25	+0.25	+0.27
High School	+0.25	+0.25	+0.27
College	+0.26	+0.26	+0.26

(c) Backpack income tax exemption.

Education	Labor Productivity		
	Low	Medium	High
Dropouts	+0.17	+0.19	+0.24
High School	+0.16	+0.17	+0.21
College	+0.12	+0.12	+0.11

(d) Price effects.

Education	Labor Productivity		
	Low	Medium	High
Dropouts	+0.36	+0.12	–0.33
High School	+0.33	+0.15	–0.27
College	+0.29	+0.18	–0.08

We also measure the consumption equivalent variation in welfare for different labor market groups, when comparing the baseline economy with the backpack economy. We calculate the gain in lifetime utility for employed, unemployed and inactive agents from the perspective of age 20. For retirees, we compute the CEV of the change in the remaining lifetime value at the first retirement age. The breakdown is presented in Table 3.13 for workers, discriminating across the different productivity levels, and for the other groups in Table 3.14.

Table 3.13: Consumption Equivalent Variation (%) in the economy with a backpack system (BP), relative to the baseline economy: workers.

Education	Labor Productivity		
	Low	Medium	High
Dropouts	1.59	1.36	1.29
High School	1.52	1.40	1.56
College	1.44	1.48	1.41

Table 3.14: Consumption Equivalent Variation (%) in the economy with a backpack system (BP), relative to the baseline economy: unemployed, inactive and retirees.

Labor Status	Education		
	Dropouts	High School	College
Unemployed	1.34	1.31	1.25
Inactive	1.46	1.45	1.44
Retirees	0.92	1.13	1.82

3.4.3 Expanding the Backpack System

After studying the implications of a backpack system with a small (2%) contribution tax, we began to solve for the allocative and welfare effects of larger systems. As part of our next steps, we will investigate what is the best system for the calibrated Spanish economy, taking into account how the backpack system interacts with the other two insurance policies in the baseline economy: unemployment insurance and pay-as-you-go pension system. As a first step, we set as the backpack contribution rate, τ_b, the share of the payroll tax that implicitly finances unemployment insurance expenditures in the baseline economy. This implies $\tau_b = 4.57\%$. The resulting economy (denoted BP 2) is described in the tables below.

Table 3.15: Aggregates and output ratios in the baseline economy and in the economy with the BP system with $\tau_b = 4.57\%$ (BP 2).

	Values			Y Ratios*		
	Baseline	BP 2	%Δ	Baseline	BP 2	$\Delta p.p.^a$
Y	2.3565	2.4376	3.44	100.00	100.00	0.00
K	8.7809	9.2553	5.40	3.72	3.79	0.07
L	0.7311	0.7433	1.66	31.02	30.49	–0.52
H	0.2115	0.2122	0.33	8.97	8.70	–0.29
C	1.0690	1.0955	2.47	45.36	44.94	–0.42
G	0.5182	0.5156	–0.51	21.99	21.15	–0.84
I	0.7693	0.8268	7.47	32.54	33.91	1.37

The ratio K/Y measured in model units and not in percentage terms.
[a] : Difference in percentage points.
$*$: As a share of output at factor cost.

Table 3.16: Policy Parameters and Ratios (Baseline and BP 2)

	Tax Rates			Tax Revenues			Revenue Ratios (%)*		
	Baseline	BP 2	%Δ	Baseline	BP 2	%Δ	Baseline	BP 2	$\Delta p.p^a$
τ_c	0.2320	0.2320	0.00	0.2480	0.2542	2.50	10.52	10.42	–0.10
τ_y	0.1213	0.1213	0.00	0.2018	0.1939	–3.92	8.56	7.95	–0.61
τ_k	0.1111	0.1111	0.00	0.0521	0.0524	0.57	2.21	2.15	–0.06
τ_p	0.2965	0.3049	2.83	0.3684	0.3741	1.54	15.63	15.34	–0.29
τ_b	–	0.0457	–	–	0.0589	–	–	2.41	–

[a] : Difference in percentage points.
* : As a share of output at factor cost.

Note that the larger backpack system increases the capital stock by more than 5%. As a percentage of output, investment increases 1.37% and consumption actually falls. The increase in wages raise the cost of pensions and payroll taxes have to increase. In addition to the introduction of the backpack tax, this has a negative effect on labor supply. These results suggest that "forced savings" dimension of the backpack system may become too large close to $\tau_b = 4.57\%$, which is something we will investigate in future work.

Nevertheless, the aggregate level of consumption in the expanded backpack economy increases, bringing additional welfare improvements for all demographic groups and for more than 95% of the households, when compared to the status quo.

Table 3.17: Consumption Equivalent Variation (Baseline and BP 2, Newborns, %)

Education	Labor Productivity		
	Low	Medium	High
Dropouts	1.78	1.57	1.37
High School	1.74	1.65	1.78
College	1.47	1.66	1.60

3.4.3.1 Voting

In all the results discussed above, we compared different steady states. For each steady state, we described changes in utility from the perspective of a newborn, who is about to enter the economy at age 20. However, the policy changes may have different impacts for households of different ages. We therefore count all the agents who are better off in the reformed economy, relative to the baseline, according to the distribution $\mu_{j,h,z,l,a}$. As we saw above, replacing the current unemployment insurance policy with a small backpack system as large negative consequences: over-accumulation of capital and reduction in households' consumption out of total output, and corresponding reduction in welfare among the different demographic groups. If we ask each household in the economy if they prefer that scenario to the baseline economy, only 25% households do so. In contrast, as we saw, the backpack economy improves the allocation of capital and labor, and increases consumption for households. More than 92% of households backpack economy with $\tau_b = 2\%$. If we increase the contribution rate to $\tau_b = 4.57\%$, the fraction of households who prefer the backpack economy to the baseline increases to 96%.

3.5 Conclusions

We have shown that there can be important allocative and welfare gains in introducing a 'backpack' , system in an economy with a pay-as-you-go pension system and unemployment insurance with unlimited benefits. The main mechanism behind these gains is the partial substitution of private savings by backpack savings, while total savings and capital increase; as a result, the economy interest rate decreases while wages increase. Associated with this change there is a better allocation of employment, with a lower share of unemployed and retirees and a higher percentage of high productive agents within the employed. Effectively, there is a more efficient allocation of savings in the economy, with a shift from pure transfers – to the unemployed and retirees – to savings and, therefore, investment in productive capital. Nevertheless, unemployed are better off by the better prospects that the economy offers and retirees are better, since in our economy pension benefits are linked to productivity, which is higher in the 'backpack economy'. Similarly, while the price changes – higher wages and lower interest rates – unambiguously benefit low and medium productivity workers, high productivity workers are hurt by the lower return on their savings; nevertheless, the positive effect of lower payroll taxes and a better employment allocation (e.g. later retirement age) results in an overall benefit from the reform.

It is quite a remarkable result that, in our economy with a very detailed heterogeneous agent structure, if all the agents from age 20 to 100 are asked if they prefer the current public savings system or one with a relatively small 'backpack,' more than 90% choose the latter. Further robustness checks are needed, but, in any case, all these effects and results would not had been possible to capture without a calibrated exercise such as the one that we have carried out.

Our results also suggest new ones. Within the current single-country analysis: from further improving our calibration and performing further robustness checks, to sharpen the potential gains of an introduction of a 'backpack' since, as we have seen, a 'bigger backpack' , can be better, but we have not solved which one would be the best for our Spanish benchmark economy, keeping the existing pay-as-you-go and (improved) unemployment benefit systems, nor have we explored yet the partial substitution of the existing pension system by an even bigger 'backpack'. But, as it has been done in Ábrahám et al. (2018), we can extend our analysis to other European economies, In fact, as we have mentioned in the introduction, employment (retirement) funds, such as TIAA-CREF, have played, and play, a major role in improving labor mobility (e.g. across US universities). An *European backpack* can be a great complement to existing public savings systems, not only by the effects shown here at the national level but, possibly even more importantly, by improving labor mobility across the European Union.

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References

Ábrahám, A., J. a. Brogueira de Sousa, R. Marimon and L. Mayr (2018): On the Design of an European Unemployment Insurance System, ADEMU Working Paper 2018/106.

Díaz-Giménez, J. and J. Díaz-Saavedra (2009): Delaying retirement in Spain, *Review of Economic Dynamics 12 (1)*, 147 – 167.

Díaz-Giménez, J. and J. Díaz-Saavedra (2017): The future of Spanish pensions, *Journal of Pension Economics and Finance 16 (2)*, 233–265.

Krusell, P., T. Mukoyama, R. Rogerson and A. Şahin (2011): A Three State Model of Worker Flows in General Equilibrium, *Journal of Economic Theory* 146, 1107–1133.

Table 3.1: Parameters determined solving single equations

	Parameter	Value
Parameters determined directly		
Earnings Life-Cycle		
	$\xi_{1,1}$	0.9189
	$\xi_{1,2}$	0.8826
	$\xi_{1,3}$	0.5064
	$\xi_{2,1}$	0.0419
	$\xi_{2,2}$	0.0674
	$\xi_{2,3}$	0.1648
	$\xi_{3,1}$	0.0006
	$\xi_{3,2}$	0.0008
	$\xi_{3,3}$	0.0021
Age Profile of Hours Worked		
	a_1	0.3649
	a_2	0.0025
	a_3	0.00005
Preferences		
Curvature	σ	2.0000
Technology		
Capital share	θ	0.4702
Public Pension System		
Number of years of contributions	N_b	17
First retirement age	R_0	61
Parameters determined by guesses for (K,L)		
Public Pension System		
Payroll Tax Rate	τ_p	0.2954
Government Policy		
Government consumption	G	0.5181
Capital income tax rate	τ_k	0.1116
Consumption tax rate	τ_c	0.2330
Income tax Rate	τ_y	0.1214

Table 3.2: Macroeconomic Aggregates and Ratios in 2014 (%)

	C/Y^{*}[a]	P/Y^{*}	U/Y^{*}[b]	K/Y^{*}[c]	W[d]	I[e]
Spain	56.4	11.1	2.4	3.08	46.9	14.6

[a] Variable Y^* denotes GDP at market prices.
[b] The ratio U/Y^* is the Unemployment benefits as a share of Output at market prices.
[c] The target for K/Y^* is in model units and not in percentage terms.
[d] Variable W is the share of workers in the Spanish population with 20+ years old.
[e] Variable I is the share of inactive in the Spanish population with 20+ years old.

Table 3.3: The Distributions of Earnings, Income, and Wealth*

		Bottom	Quintiles					Top
	Gini	10	1st	2nd	3rd	4th	5th	10
The Earnings Distributions (%)								
Spain	**0.35**	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
The Income Distributions (%)								
Spain	**0.35**	2.1	**6.3**	**12.1**	17.2	23.7	**40.7**	25.0
The Wealth Distributions (%)								
Spain	**0.57**	0.0	**0.9**	6.6	12.5	20.6	**59.5**	42.5

*The source for the Spanish data of earnings and income are the Spanish National Institute of Statistics (INE) and the OECD. The source for the Spanish data of wealth is the 2004 *Encuesta Financiera de las Familias Españolas* as reported in Budría and Díaz-Giménez (2006).

Table 3.4: Initial Values, Final Values, Weights, and Errors.

Parameter	Initial Value	Final Value	Statistic	Weight (%)	Target	Result	Error (%)
β	0.9750	1.0010	K/Y^*	300	3.08	3.38	9.74
α	0.7412	0.7799	W (%)	500	46.90	54.11	15.37
γ	0.4412	0.6999	I (%)	300	14.59	5.72	–60.80
δ	0.0760	0.0730	$(C+T_c)/Y^*$ (%)	300	56.41	50.42	–10.62
ϕ	0.7570	0.8800	P/Y^* (%)	200	11.12	11.89	6.92
b_0	0.1529	0.2300	U/Y^* (%)	200	2.41	2.18	–9.55
$z(2)$	1.5082	2.3491	GY	700	0.35	0.39	11.42
$z(3)$	5.6303	5.6877	GE	800	0.35	0.36	2.85
z_{11}	0.9990	0.9918	2QY (%)	500	12.10	12.0	–0.83
z_{12}	0.0005	0.0081	4QW (%)	1	20.60	20.11	-2.38
z_{21}	0.0007	0.0275	1QW (%)	1	0.90	0.32	–64.45
z_{22}	0.9992	0.9724	5QY (%)	5	40.70	43.90	7.86
z_{32}	0.0001	0.0000	1QY (%)	5	6.30	6.90	9.52
z_{33}	0.9991	0.9999	GW	700	0.57	0.66	15.78

Chapter 4
Existence and Uniqueness of Equilibrium in Lucas' Asset Pricing Model when Utility is Unbounded

This chapter was written with Fabian Schuetze.

Abstract This note presents a proof of the existence of a unique equilibrium in a Lucas (1978) economy when the utility function displays constant relative risk aversion and log dividends follow a normally distributed autoregressive process of order one with positive autocorrelation. We provide restrictions on the coefficient of relative risk aversion, the discount factor and the conditional variance of the consumption process that ensure the existence of a unique equilibrium.

4.1 Introduction

We prove the existence of a unique equilibrium in a Lucas (1978) economy when the utility function displays constant relative risk aversion (CRRA) and log dividends follow a normally distributed AR(1) process with positive auto-correlation. The equilibrium in the economy is characterized by a pricing function for the Lucas tree and a value function for the representative consumer. Under the assumption of a bounded utility function, Lucas proves the existence of a unique equilibrium by showing that the pricing and value functions are fixed points of functional equations. Lucas resorts to the sufficient conditions of Blackwell (1965) to document that Banach's fixed point theorem (e.g. p. 176 of Ok (2007)) guarantees the existence of a unique solution to each of the functional equations. Alas, Blackwell's conditions do not hold when the utility function displays the CRRA property. The conditions require utility being bounded in the sup-norm, which does not hold when the consumption space is equal to the positive real numbers and the investor displays CRRA preferences. Fortunately, Blackwell's conditions are only sufficient. We exploit the extension of Blackwell's conditions by Boyd (1990) and document under which circumstances Banach's theorem can be applied. In particular, we provide a joint restriction on the coefficient of relative risk aversion, the discount factor and the conditional variance of the consumption process under which an equilibrium in the economy exists and is unique.

Our solution method serves the same purpose as the local contraction methods (see Martins-da Rocha and Vailakis (2010), Matkowski and Nowak (2011) and the references therein), which provide conditions under which a functional equation has a unique solution in an unbounded setting. In short, local contraction arguments rewrite the domain of the elements of the functional space under consideration as a countable union of always increasing compact subsets. A function is then said to be bounded if it is bounded in every such subset of its domain. Local boundedness is implied by restrictions on the co-domain of the transition function for the state variable. In non-stochastic problems, the strictest version of such restrictions is simply to require tomorrow's state to be an element of the same subset of the state space as today's state variable. For stochastic problems, Matkowski and Nowak (2011) write the state space as a sequence of increasing (in the sense of inclusion) subspaces. The probability distribution that characterizes the transition of the state is such that with probability one tomorrow's state lies in the strictly larger but smallest subspace relative to the smallest subspace that includes today's state. These assumptions about how the state traverses imply that if today's value function is locally bounded, tomorrow's value function is locally bounded too. In our application, the state traverses according to a log-normal distribution. This probability distribution has an unbounded support, precluding the application of the argument outlined above.

This note is a complement to Kamihigashi (1998), who provides sufficient conditions for uniqueness of equilibrium prices in a Lucas (1978) economy, presuming existence of the equilibrium. He shows that utility functions which are discontinuous and unbounded below can lead to non-uniqueness of asset prices, and that a transversality condition is not sufficient to guarantee uniqueness. The sufficient conditions for uniqueness of equilibrium in Kamihigashi (1998) are in the form of a bound on the growth rate of marginal utility when consumption goes to zero and to infinity, independently of the process governing dividends. CRRA utility functions satisfy such a bound on the growth rate of marginal utility and hence the equilibrium is unique. In contrast, we formulate the optimization problem recursively as in Lucas (1978). We show that, for a particular endowment process, an equilibrium exists and is unique: the value of the problem and the asset pricing function are uniquely defined, and a transversality condition on the value function holds. Alvarez and Stokey (1998) analyses dynamic programming problems with homogeneous return functions and transition functions that are homogeneous of degree one. Using similar arguments as theirs, we consider an optimization problem with a transition function that is not homogeneous of degree one.

The structure of this note is as follows: In the next Section, we provide a brief description of the economy and a definition of equilibrium in the asset pricing problem. Section 3 presents the main result of the note and discusses an extension to Blackwell's sufficient conditions for a given operator on a metric space to be a contraction that is useful in our argument. We prove the existence and uniqueness of the equilibrium and study properties of the pricing function in Section 4. Section 5 concludes.

4.2 The economy and definition of equilibrium

We describe the equilibrium as in Lucas (1978). Let us denote next period's share holdings by $x' \in \mathbf{X}$, with $\mathbf{X} = [\underline{x}, \bar{x}]$, $0 < \underline{x} < 1 < \bar{x}$, and current consumption by $c \in \mathbb{R}_+$. Let $y \in \mathbf{Y} = \mathbb{R}_{++}$ be the current dividend of a Lucas-tree. The transition equation for next period's dividend is $G(y,z') = y^{\alpha} z'$ with $\alpha \in (0,1)$ and $\log z' \sim N(0,\sigma^2)$, $\sigma > 0$. Let Q be the probability density over z'. Instantaneous utility is given by $u(c) = c^{1-\gamma}/(1-\gamma)$, with $\gamma > 0$; $\beta \in (0,1)$ is a discount factor. We use the following equilibrium definition:

Definition 1 *An equilibrium is a continuous function* $p(y) : \mathbf{Y} \to \mathbb{R}_+$ *and a continuous function* $v(y,x) : \mathbf{Y} \times \mathbf{X} \to \mathbb{R}_+$ *such that:*

$$v(y,x) = \max_{c,x' \in \Gamma(y,x)} \left\{ u(c) + \beta \int_Z v(G(y,z'),x') Q(dz') \right\} \tag{4.1}$$

with

$$\Gamma(y,x) = \left\{ (c,x') \in \mathbf{Y} \times \mathbf{X} : c + p(y)x' \leq yx + p(y)x \right\},$$

and

$$\textit{for each } y,\ v(y,1) \textit{ is attained by } c = y \textit{ and } x' = 1, \textit{and satisfies}$$

$$\lim_{t\to\infty} \mathbb{E}_0 \left[\beta^t v(x_t, y_t) \right] = 0$$

where $\mathbb{E}_0$ *is the expectation operator conditional on the initial period information, which is the initial endowment* $y_0 \in \mathbf{Y}$ *and asset holdings* $x_0 \in \mathbf{X}$.

4.3 Sufficient conditions for the existence of a unique equilibrium

The following proposition is the crux of our note:

Proposition 1 *Take* $\beta \in (0,1)$, $\sigma \in (0,\infty)$. *Suppose that for all* $\gamma \in (0,1)$:

$$\beta \left[0.5 + \int_1^{\infty} \left(z'\right)^{1-\gamma} Q(dz') \right] < 1. \tag{4.2}$$

Alternatively, for all $\gamma > 1$, *suppose that:*

$$\beta \left[\int_0^1 \left(z'\right)^{1-\gamma} Q(dz') + 0.5 \right] < 1. \tag{4.3}$$

Finally, suppose that $\log(\beta) + (1-\gamma)^2\sigma^2/2 < 0$. *Then there exists a unique equilibrium. That is:*

(1) There exists a unique non-negative continuous pricing function $p(y)$,
(2) There exists a unique function $v(y,x)$,

in accordance with Definition 1.

The inequalities (4.2) and (4.3) will be described at the end of this section. Given a value function v, we first study the existence of a unique pricing function. We begin by deriving a variant of an Euler equation following Lucas (1978), given by (4.4) below. Let us assume that for each y, $v(y,x)$ is an increasing, concave and differentiable function with respect to x. Defining $f(y) = p(y)\partial u(y)/\partial y$ and using the equilibrium conditions $x = x' = 1$ and $c = y$ allows formulating the stochastic Euler equation as:

$$f(y) = h(y) + \beta \int_Z f(G(y,z'))Q(dz'), \tag{4.4}$$

$$\text{with } h(y) = \beta \int_Z \left[\frac{\partial u(G(y,z'))}{\partial y'} G(y,z') \right] Q(dz') = \beta y^{\alpha(1-\gamma)} \exp\left((1-\gamma)^2\sigma^2/2\right).$$

Lucas uses Blackwell's sufficient conditions to show that the operator T, defined such that (4.4) is equivalent to $Tf = f$, is a contraction and then applies Banach's fixed point theorem. To employ Blackwell's conditions, Lucas assumes that the utility function u and thereby the function h is bounded with the sup-norm. With CRRA utility and a dividend process in $\mathbb{R}_{++}$, the function h is unbounded with the sup-norm. Importantly, boundedness is a characteristic that is closely linked to the employed metric. In the following subsection we study a norm with respect to which the function h in (4.4) is bounded.

4.3.1 A weighted norm approach

Boyd (1990) extends Blackwell's sufficient conditions by generalizing the metric from a sup-norm to a weighted sup-norm. We denote the set of continuous functions $f : \mathbf{Y} \to \mathbb{R}_+$ by $\mathbf{S}$, and take $\varphi \in \mathbf{S}$ with $\varphi > 0$. Then f is φ-bounded with respect to the weighted sup-norm $||f||_\varphi = \sup_{y\in\mathbf{Y}} \{|f(y)|/\varphi(y)\}$ if $\exists\ B \in \mathbb{R}_+$ such that $||f||_\varphi < B$. Let $\mathbf{S}_\varphi \subset \mathbf{S}$ be the set of continuous and φ-bounded functions. Let us define the metric $d_\varphi(f,g) := ||f-g||_\varphi$ on $\mathbf{S}_\varphi$. Note that $(\mathbf{S}_\varphi, d_\varphi)$ is a complete metric space (see e.g. Theorem 12.2.8 in Stachurski (2009)). We have the following lemma, which is a corollary to the Weighted Contraction Mapping Theorem in Boyd (1990):

Lemma 1 (Boyd's sufficient conditions) *Let $T : \mathbf{S}_\varphi \to \mathbf{S}$ and suppose:*

1. *(monotonicity) T is monotone, that is $\forall f,g \in \mathbf{S}_\varphi$, $f \geq g$ implies $Tf \geq Tg$;*
2. *(discounting) For any $A \in \mathbb{R}_{++}$, there exists $\theta \in (0,1)$ such that: $T(f + A\varphi) \leq Tf + \theta A\varphi$;*
3. *(self-map) $T(0) \in \mathbf{S}_\varphi$.*

Then T is a contraction in $(\mathbf{S}_\varphi, d_\varphi)$ with modulus θ.

The discounting and self-map property of operator T involve the weighting function explicitly. The self-map property requires the function h to be bounded with a weighted sup-norm: $||h||_\varphi < B$ for some $B \in \mathbb{R}_+$. To develop some intuition about our proposal for such a φ, we consider the functional form of $h(y) = \kappa y^{\alpha(1-\gamma)}$, where κ is a positive constant given by β, γ and σ. For a given $\alpha \in (0,1)$, if $\gamma < 1$, h is a strictly increasing and concave function; alternatively, if $\gamma > 1$, h is strictly decreasing and convex. Any positive continuous function φ which is weakly above h in $\mathbf{Y}$ is a weighting function that makes h φ-bounded. One such function is given by:

$$\varphi(y) = \kappa \cdot \max\left\{1, y^{1-\gamma}\right\}. \tag{4.5}$$

When $\gamma \in (0,1)$, the function above is equal to κ while $0 < y \leq 1$ and then grows strictly above h for all $y > 1$. When $\gamma > 1$, the weighting function in (4.5) is above h while $0 < y < 1$ and stays at κ when $y \geq 1$.

In our framework, the restriction implied by Boyd's discounting property is equivalent to:

$$\beta \frac{\int_Z \varphi(G(y,z'))Q(dz')}{\varphi(y)} < 1, \forall y \in \mathbf{Y}. \tag{4.6}$$

Similarly to the discussion of dynamic programming techniques with homogeneous return functions in Alvarez and Stokey (1998), inequality (4.6) places a bound on the expected growth rate of the weighting function. If φ was a constant function, (4.6) would be trivially satisfied (since $\beta < 1$). However, as discussed above, in order to bound h the weight φ has to be above h, that decreases or increases with y depending on γ. This observation motivates using another weighting function arbitrarily close to and greater than h: relative to the weight in (4.5), such a weighting function increases the range of parameters β, γ, σ that satisfy condition (4.6). Instead, for the sake of simplicity, we proceed with our analysis using (4.5) and illustrate the restriction on the parameter space imposed by (4.6). To that end, take $\gamma \in (0,1)$. To evaluate (4.6), for a given $y \in \mathbf{Y}$, consider realizations of z' in the interval $(0, y^{-\alpha})$. Since in this interval $G(y,z') = y^{\alpha} z' < 1$, $\varphi(G(y,z')) = 1$. Conversely, for $z' \in [y^{-\alpha}, \infty)$, $\varphi(G(y,z')) = G(y,z')^{1-\gamma}$. The following lines place an upper bound on the left hand side of (4.6):

$$\begin{aligned}
\beta \frac{\int_Z \varphi(G(y,z'))Q(dz')}{\varphi(y)} &= \beta \frac{\int_0^{y^{-\alpha}} Q(dz') + y^{\alpha(1-\gamma)} \int_{y^{-\alpha}}^{\infty} (z')^{1-\gamma} Q(dz')}{\varphi(y)} \\
&\leq \begin{cases} \dfrac{\beta \left[\int_0^{y^{-\alpha}} Q(dz') + \int_{y^{-\alpha}}^{\infty} z'^{1-\gamma} Q(dz')\right]}{1} & \text{if } 0 < y < 1, \\ \dfrac{y^{\alpha(1-\gamma)} \beta \left[\int_0^{y^{-\alpha}} Q(dz') + \int_{y^{-\alpha}}^{\infty} z'^{1-\gamma} Q(dz')\right]}{y^{(1-\gamma)}} & \text{if } y \geq 1, \end{cases} \\
&\leq \beta \left[\int_0^{y^{-\alpha}} Q(dz') + \int_{y^{-\alpha}}^{\infty} z'^{1-\gamma} Q(dz')\right] \quad \forall y \in \mathbf{Y} \\
&\leq \beta \left[\int_0^{1} Q(dz') + \int_{1}^{\infty} z'^{1-\gamma} Q(dz')\right] \quad \forall y \in \mathbf{Y} \\
&= \beta \left[0.5 + \int_1^{\infty} z'^{1-\gamma} Q(dz')\right].
\end{aligned}$$

In the numerator, after the first equality sign, we split the support of z' in two intervals to consider the two branches in (4.5) separately. The inequalities in the second line hold as $1 > y^{\alpha(1-\gamma)}$ for $0 < y < 1$ and $y^{\alpha(1-\gamma)} \geq 1$ for $y \geq 1$. The inequality in the third line follows since $y^{\alpha(1-\gamma)} < y^{1-\gamma}$ for $y > 1$. Finally, the last inequality holds because the term in square brackets is the greatest at $y = 1$. Accordingly, inequality (4.2) of Proposition 1 guarantees that, if $\gamma < 1$, Boyd's discounting property is satisfied. A similar arguments holds for values $\gamma > 1$, when inequality (4.3) holds. As an illustration, the shaded region in Figure 4.1 documents which parameter pairs (σ, γ) satisfy conditions (4.2) and (4.3), when β is 0.99. When $\gamma = 1$, $u(c) = \log(c)$ and a solution to (4.4) can be calculated analytically. A lower value of β enlarges the admissible region. In the following, the

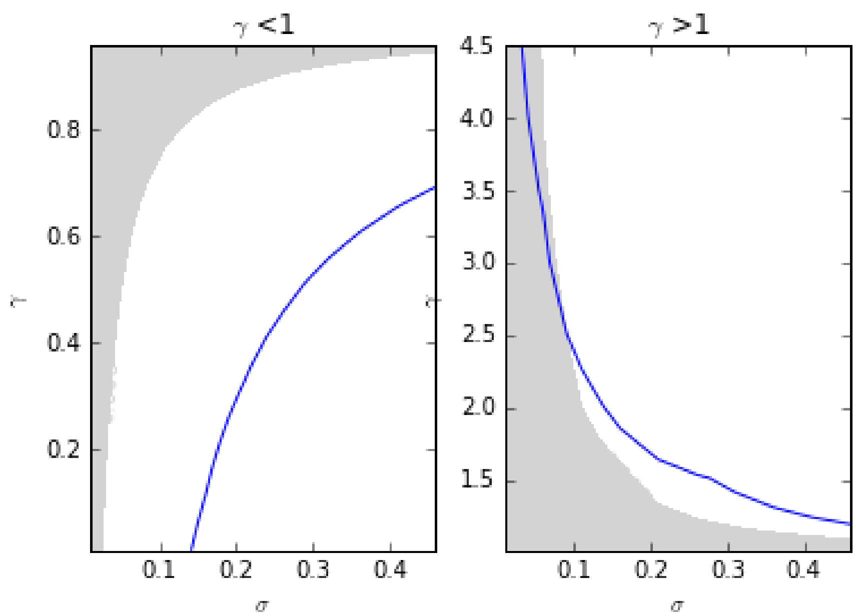

Fig. 4.1: Illustration of the set of (γ, σ) values satisfying the conditions in Proposition 1. The grey regions represent the combinations of (γ, σ) satisfying inequalities 4.2 and 4.3 in Proposition 1. The region above (respectively below) the blue line corresponds to parameter combinations that satisfy the additional restriction in Proposition 1, when $\gamma < 1$ (resp. $\gamma > 1$). β is 0.99.

proof of Part (1) of Proposition 1 will be completed.

4.4 Proof of the Proposition

In this section we formally prove the existence and uniqueness of equilibrium in the economy described in Section 2. Following Proposition 1,we proceed in two steps. We first solve for a unique pricing function, p, taking as given a value function v (part (1) in Proposition 1). Then, taking p as given, we complete the proof by showing that there is a unique value function v (part (2)), in accordance with Definition 1.

Proof. (Part (1) of Proposition 1.) The proof begins by showing that T is a map from $\mathbf{S}_\varphi$ to $\mathbf{S}$. Take some $f \in \mathbf{S}_\varphi$. As both h and the integral over f are continuous, Tf is continuous.[1] Hence $T : \mathbf{S}_\varphi \to \mathbf{S}$. The operator T is monotone since for any $f, g \in \mathbf{S}_\varphi$ with $f \geq g$, $\int f(G(y,z'))Q(dz') \geq \int g(G(y,z'))Q(dz')$, so $Tf \geq Tg$ and 1 of Lemma 1 holds. Under assumption (4.2), condition 2 of Lemma 1 is satisfied for $0 < \gamma < 1$, as the argument in section 4.3.1 shows. It remains to be shown that this condition holds for $\gamma > 1$. For these values of γ, we observe the following:

$$
\begin{aligned}
\beta \frac{\int_Z \varphi(G(y,z'))Q(dz')}{\varphi(y)} &= \beta \frac{y^{\alpha(1-\gamma)} \int_0^{y^{-\alpha}} (z')^{1-\gamma} Q(dz') + \int_{y^{-\alpha}}^{\infty} Q(dz')}{\varphi(y)} \\
&\leq \begin{cases} \beta \dfrac{y^{\alpha(1-\gamma)} \left[\int_0^{y^{-\alpha}} z'^{1-\gamma} Q(dz') + \int_{y^{-\alpha}}^{\infty} Q(dz')\right]}{y^{1-\gamma}} & \text{if } 0 < y < 1, \\ \beta \dfrac{\left[\int_0^{y^{-\alpha}} z'^{1-\gamma} Q(dz') + \int_{y^{-\alpha}}^{\infty} Q(dz')\right]}{1} & \text{if } y \geq 1, \end{cases} \\
&\leq \beta \left[\int_0^{y^{-\alpha}} z'^{1-\gamma} Q(dz') + \int_{y^{-\alpha}}^{\infty} Q(dz')\right] \forall y \in \mathbf{Y} \\
&\leq \beta \left[\int_0^{1} z'^{1-\gamma} Q(dz') + \int_{1}^{\infty} Q(dz')\right] \quad \forall y \in \mathbf{Y} \\
&= \beta \left[\int_0^{1} z'^{1-\gamma} Q(dz') + 0.5\right].
\end{aligned}
$$

The reasoning for each condition is analogous to the one made for the case $0 < \gamma < 1$. Under (4.3), condition 2 in Lemma 1 holds for $\gamma > 1$. The third condition of Lemma 1 requires h to be bounded with the weighted sup-norm. Hence, since:

$$
||h||_\varphi = \beta \exp\left((1-\gamma)^2\sigma^2/2\right) \sup_{y\in\mathbf{Y}} \left\{ \frac{y^{\alpha(1-\gamma)}}{\varphi(y)} \right\} = \beta \exp\left((1-\gamma)^2\sigma^2/2\right),
$$

$T(0) \in \mathbf{S}_\varphi$ and point 3 of Lemma 1 holds. Note that, under Lemma 1, the operator T is a self-map (maps the space $\mathbf{S}_\varphi$ into itself).[2] Concluding, since T is a contraction over $(\mathbf{S}_\varphi, d_\varphi)$, Banach's fixed point theorem guarantees that a unique function $f \in \mathbf{S}_\varphi$ satisfying (4.4) exists. The solution is non-negative and continuous. Therefore the pricing function $p(y) = f(y)/u'(y)$ is non-negative and continuous as well.

---

[1] Here is a sketch of the proof for continuity: Take any sequence $y_n \to y$ and define the difference $|T(y_n) - T(y)| = |h(y_n) - h(y) + \beta \int (f(G(y_n,z')) - f(G(y,z')))\, Q(dz')|$. Successive application of the triangle inequality shows that the difference converges to zero as n increases.

[2] See proof in p.6, Section 3, in Boyd (1990).

After having shown that there exists a unique pricing function p given v, the converse remains to be shown. The following argument completes the proof of Proposition 1.

Proof. (Part (2) of Proposition 1). We define the operator H such that:

$$Hv(y,x) = \max_{c,x' \in \Gamma(y,x)} \left\{ u(c) + \beta \int_Z v(G(y,z'),x')Q(dz') \right\}. \tag{4.7}$$

The weighting function, now denoted by ϕ, can be defined similarly as:

$$\phi(y,x) = \kappa \cdot \max\{1, y^{1-\gamma}\}. \tag{4.8}$$

Let us assume that the function p is as in part (1) of Proposition 1, and $p \in \mathbf{S}_\phi$.[3] To prove that a unique fixed point H exists, one can resort to Lemma 1 to show that H is a contraction and then use Banach's theorem to establish existence and uniqueness of the fix point. We begin by showing $H : \mathbf{S}_\phi \to \mathbf{S}$. For any $v \in \mathbf{S}_\phi$ and u continuous, $u(c) + \beta \int_Z v(G(y,z'),x)Q(dz')$ is continuous. Since for each (y,x) the budget correspondence is compact valued and continuous, Berge's theorem (Theorem 3.6 in Stokey and Lucas (1989)) guarantees that Hv is continuous. Hence $H : \mathbf{S}_\phi \to \mathbf{S}$. Monotonicity of H holds. Discounting of H can then be established as in the proof of part (1) of Proposition 1, above. Finally one needs to show that H has the self-map property. In mathematical terms, $H(0) \in \mathbf{S}_\phi$ if there is some $B \in \mathbb{R}_+$ such that:

$$\sup_{(y,x) \in \mathbf{Y} \times \mathbf{X}} \left\{ \frac{|\max_{c,x' \in \Gamma(y,x)} u(c)|}{\phi(y,x)} \right\} < B. \tag{4.9}$$

To show that (4.9) holds we consider two cases: $\gamma \in (0,1)$ and $\gamma > 1$. For any $\gamma \in (0,1)$ condition (4.9) is equivalent to:

$$\sup_{(y,x) \in \mathbf{Y} \times \mathbf{X}} \left\{ \frac{(p(y)x + xy)^{1-\gamma}}{\phi(y,x)} \right\} < B \Leftrightarrow \sup_{y \in [1,\infty)} \left\{ \left(\frac{p(y)}{y} + 1 \right)^{1-\gamma} \right\} < B. \tag{4.10}$$

Now we note that, by definition, $p(y) = f(y)y^\gamma$ and as shown in Part (1) of the proof, for all $y \in [1,\infty)$ (and all $x \in \mathbf{X}$): $f(y)y^\gamma \leq \phi(y,x)y^\gamma = \kappa y$. Therefore it follows that

$$\sup_{y \in [1,\infty)} \left\{ \left(\frac{p(y)}{y} + 1 \right)^{1-\gamma} \right\} \leq (\kappa + 1)^{1-\gamma} < B \tag{4.11}$$

holds for some $B \in \mathbb{R}_+$, which proves that (4.9) holds for all $\gamma \in (0,1)$. The proof for the complementary case $\gamma > 1$ follows directly from (4.9) and for brevity we omit the argument.[4] Concluding, we showed that H is a contraction and Banach's fixed point theorem establishes that it has a unique solution.

[3] $\mathbf{S}_\phi$ is defined as $\mathbf{S}_\varphi$, with φ replaced by ϕ.

[4] In the case $\gamma > 1$ the assumption $x \geq \underline{x}$ with $\underline{x} > 0$ is needed to show that H has the self-map property.

At this stage, a characterization of function f that solves (4.4) is in order. The following lemma documents some of its properties:

Lemma 2 *Take $f \in \mathbf{S}_\varphi$ such that $Tf = f$. Then:*

1. *For any $f_0 \in \mathbf{S}_\varphi$, $||T^n f_0 - f||_\varphi \to 0$ as $n \to \infty$;*
2. *Suppose $0 < \gamma < 1$. Then, both h and f are strictly increasing and concave. Suppose otherwise $1 < \gamma$. Then both h and f are strictly decreasing and convex.*

Proof. Point 1 of Lemma 2 follows directly from the fact that T is a contraction on a complete metric space and hence, for brevity, will not be proved here. Point 2 is proved in the Appendix of this note.

Properties of v assumed in Section 4.3 can be shown by arguments similar to Lucas (1978) (see propositions 1 and 2). The proof that the fixed point v of the operator H satisfies the limit condition in Proposition (1) is in Appendix B. This concludes the proof of Proposition 1.

4.5 Conclusion

We present a proof of the existence and uniqueness of equilibrium in a pure exchange economy of Lucas (1978), when the utility function takes the CRRA form and the dividend stream follows an autoregressive process of order one with positive autocorrelation. An interesting extension of the argument presented in this note, that we leave for future research, is to consider the case in which innovations affect the growth rate of dividends instead of the level.

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Appendix

4.6 Proof of Part 2 of Lemma 2

Proof. (Part 2 of Lemma 2) As in the main text, denote by S_φ the set of continuous and φ-bounded functions. The set S'_φ is the set of continuous, φ-bounded, non-decreasing and concave functions, and $S''_\varphi \subset S'_\varphi$ imposes additionally strict monotonicity and concavity. We want to show that the contraction operator T maps any function $\tilde{f} \in S'_\varphi$ into the subset S''_φ. As the solution to the functional equation is characterized by $Tf = f$ and S'_φ is a closed set, if the operator T transforms any non-decreasing and concave function into a strictly increasing and concave function, then f is strictly increasing and concave (Corollary 1 of the Contraction Mapping Theorem in Stokey and Lucas (1989), p.52). To show the desired result, we suppose first that h is strictly increasing and concave and pick any $\tilde{f} \in S'_{\varphi}$. To begin, let us study whether $T\tilde{f}$ is strictly increasing. For any pair $\hat{y}, y \in Y$ with $\hat{y} > y$, the function $T\tilde{f}$ satisfies:

$$\begin{aligned} T\tilde{f}(\hat{y}) &= h(\hat{y}) + \beta \int_Z \tilde{f}(G(\hat{y}, z'))Q(dz') \\ &> h(y) + \beta \int_Z \tilde{f}(G(y, z'))Q(dz') \\ &= T\tilde{f}(y). \end{aligned}$$

The inequality holds because G and h are strictly increasing and $\tilde{f}$ is non-decreasing. Hence, $T\tilde{f}$ is strictly increasing. To analyse concavity, define $y_\omega = \omega y + (1-\omega)y'$, for any $y, y' \in Y$, $y \neq y'$, and $0 < \omega < 1$. The strict concavity form of h and G, together with $\tilde{f}$ being concave, ensure that:

$$\begin{aligned} T\tilde{f}(y_\omega) &= h(y_\omega) + \beta \int_Z \tilde{f}(G(y_\omega, z'))Q(dz') \\ &> \omega \left[h(y) + \beta \int_Z \tilde{f}(G(y, z'))Q(dz') \right] + (1-\omega) \left[h(y') + \beta \int_Z \tilde{f}(G(y', z'))Q(dz') \right] \\ &= \omega T\tilde{f}(y) + (1-\omega)T\tilde{f}(y'). \end{aligned}$$

The function $T\tilde{f}$ is strictly concave. Taken together, we know that for any $\tilde{f} \in S'_{\varphi}$, $T\tilde{f} \in S''_{\varphi}$. Hence, f (such that $Tf = f$) must be an element of the set S''_{φ}, guaranteeing that f has the same functional form as h. Now, suppose h is convex and falling. We could again define the operator T as $Tf(y) = h(y) + \beta \int_Z f(G(y, z'))Q(dz')$ and study into which subset a candidate solution is mapped into. To facilitate analysis though, take a different route. Look at the modified operator $Tf_- = h_- + \beta \int_Z f_-(G(y, z'))Q(z')$, with $h_- = -h$ and $f_- = -f$. Under the same assumptions guaranteeing a unique solution to the original contraction mapping, there exists a unique solution to the modified contraction mapping. As h_- is strictly increasing and concave, the proof above applies to the modified contraction mapping. As f_- is strictly increasing and concave, f is strictly decreasing and convex and inherits the properties of h.

4.7 Limit condition on v

Let us take $v \in S_\phi$ sucht that $Hv = v$, with the operator H as defined in Section 4.4 of the main text. Our initial aim is to characterise lower and upper bounds on v in the functional space S_ϕ. We will now argue that when $v \geq 0$ (respectively ≤ 0), v can be bounded below (resp. above) using the zero function and above (resp. below) using function ϕ (respec. $-\phi$). To this end, we consider any $\gamma \in (0,1)$. Define the set $S'_\phi = \left\{ f \in S_\phi : f \geq 0 \right\}$. This is a closed subset of S_ϕ (its complement in S_ϕ is open). We pick any $f \in S'_\phi$. Since the utility function u takes on positive values, $Hf \geq 0$ Thus, since f was arbitrary, $H : S'_\phi \to S'_\phi$. Then by Corollary 1 of the CMT (p.52) in Stokey and Lucas (1989) $v \in S'_\phi$, i.e. $v \geq 0$. A similar argument shows that for any $\gamma > 1$, $v \leq 0$. The remainder of this section shows that the discounted expected vale of the upper bound converges to zero, implying that $\lim_{t\to\infty} \mathbb{E}_0 [\beta^t v(x_t, y_t)] = 0$. We consider any $\gamma \in (0,1)$ (For $\gamma > 1$, the condition is equivalent, as the constant in the bound changes the sign of the bound.) Then for any t, $x_t \in X$ and $y_t \in Y$:

$$\begin{aligned}
\mathbb{E}_0 \left[\beta^t v(x_t, y_t)\right] &\leq \mathbb{E}_0 \left[\beta^t \phi\right] \\
&= \kappa \beta^t \, \mathbb{E}_0 \left[\mathbb{E}_{t-1} \left[max \left\{1, y_t^{1-\gamma}\right\}\right]\right] \\
&\leq \kappa \beta^t \, \mathbb{E}_0 \left[0.5 + y_{t-1}^{\alpha(1-\gamma)} \int_0^\infty Q(dz')(z')^{1-\gamma}\right] \\
&\leq \kappa \beta^t \left[0.5 + y_0^{1-\gamma} \exp\left(t(1-\gamma)^2 \sigma^2/2\right)\right] \\
&= \kappa \left[\beta^t 0.5 + y_0^{1-\gamma} \exp\left(t\left[\log(\beta) + (1-\gamma)^2 \sigma^2/2\right]\right)\right]
\end{aligned}$$

The first line follows from the fact that $v \in S_\phi$. The second line uses the definition of ϕ and the law of iterated expectations. The third line bounds the the term in brackets. The strategy is identical to the one used in the proof of Proposition 1. The fourth lines iterates until time zero and uses the fact that $\alpha \in (0,1)$. The fifth line factors β^t in. The entire sums converges to zero with $t \to \infty$ if $\log(\beta) + (1-\gamma)^2\sigma^2/2 < 0$. The proof for $\gamma > 1$ is analogous.

References

Alvarez, F. and N. L. Stokey (1998): Dynamic Programming with Homogeneous Functions, *Journal of Economic Theory 82 (1)*, 167–189.

Blackwell, D. (1965): Discounted Dynamic Programming, *The Annals of Mathematical Statistics 36 (1)*, 226–235.

Boyd, J. H. (1990): Recursive utility and the Ramsey problem, *Journal of Economic Theory 50 (2)*, 326–345.

Kamihigashi, T. (1998): Uniqueness of asset prices in an exchange economy with unbounded utility, *Economic Theory 12 (1)*, 103–122.

Lucas, Jr., R. E. (1978): Asset Prices in an Exchange Economy, *Econometrica 46 (6)*, 1429–1445.

Martins-da Rocha, V. F. and Y. Vailakis (2010, May): Existence and Uniqueness of a Fixed Point for Local Contractions, *Econometrica 78 (3)*, 1127–1141.

Matkowski, J. and A. S. Nowak (2011): On discounted dynamic programming with unbounded returns, *Economic Theory 46 (3)*, 455–474.

Ok, E. A. (2007, January): *Real Analysis with Economic Applications*, Princeton University Press, Princeton.

Stachurski, J. (2009): *Economic Dynamics*, MIT Press.

Stokey, N. L. and R. E. Lucas (1989): *Recursive methods in economic dynamics*, Harvard University Press, Cambridge, Mass.