Economics Department

Estimating Stochastic Volatility Models Through Indirect Inference

CHIARA MONFARDINI

ECO No. 96/22

EUI WORKING PAPERS

EUROPEAN UNIVERSITY INSTITUTE

EUROPEAN UNIVERSITY INSTITUTE, FLORENCE

ECONOMICS DEPARTMENT

EUI Working Paper **ECO** No. 96/22

Estimating Stochastic Volatility Models Through Indirect Inference

CHIARA MONFARDINI

BADIA FIESOLANA, SAN DOMENICO (FI)

Printed in Italy in July 1996
European University Institute
Badia Fiesolana
I – 50016 San Domenico (FI)
Italy

Estimating Stochastic Volatility Models through Indirect Inference

Chiara Monfardini*
European University Institute
Via dei Roccettini 9
I-50016 San Domenico di Fiesole (FI)- Italy
e-mail: monfardi@datacomm.iue.it
fax: +39 55 599887

June 1996

Abstract

We propose as a tool for the estimation of stochastic volatility models two Indirect Inference estimators based on the choice of an autoregressive auxiliary model and an ARMA auxiliary model respectively. These choices make the auxiliary parameter easy to estimate and at the same time allows the derivation of optimal procedures, leading to minimum variance Indirect Inference estimators. The results of some Monte Carlo experiments provide evidence that the Indirect Inference estimators perform well in finite sample, although less efficiently than Bayes and Simulated EM algorithms.

*I would like to thank the Department de la Recherche, CREST-INSEE, Paris, for giving me hospitality as a visiting student. I am indebted to Prof. Grayham Mizon for his encouragement and suggestions, and to Prof. Alain Monfort for his invaluable help. I also would like to thank Neil Shephard for kindly providing me with the set of his results. I am alone responsible for any remaining errors.

1 Introduction

In the last decade there has been a growing interest in time series models of changing variance, given the time varying volatility exhibited by most financial data. In the basic model, the autoregressive conditional heteroscedasticity (ARCH) model, introduced by Engle (1982), the conditional variance is assumed to be a function of the squares of past observations. This model has been extended in different directions, the most popular of which, the generalized ARCH (GARCH), lets the conditional variance depend on squared past observations and previous variances (see Bollerslev, 1986, and Taylor, 1986).

Another class of model is obtained by formulating a latent stochastic process for the variance. The resulting models are called stochastic volatility models (SV) and have been the focus of considerable attention in the recent years. SV models present two main advantages over GARCH models. The first one is their solid theoretical background, as they can be interpreted as discretized versions of stochastic volatility continuous-time models put forward from modern finance theory (see Hull and White (1987)). The second is their ability to generalize from univariate to multivariate series in a more natural way, as far as their estimation and interpretation are concerned. On the other hand, SV models are more difficult to estimate than the GARCH ones, due to the fact that it is not easy to derive their exact likelihood function.

For this reason, a number of econometric methods have been proposed in the literature to solve the problem of the estimation of SV models.some of which are aimed at achieving Maximum Likelihood estimation. An exhaustive presentation of the different estimation procedures and their properties can be found in Shephard (1996). Briefly, they include, among others, Generalized Methods of Moments (GMM)[1], Quasi-

[1]See Chesney and Scott (1989), Melino and Turnbull (1990), Duffie and Singleton (1993), Andersen (1993), Andersen and Sorensen (1994) and Jacquier, Polsen and Rossi for application of GMM method to SV models.

Maximum Likelihood (QML) method[2], Importance Sampling[3], Bayesian estimation[4], and a Simulated EM algorithm (SEM)[5]. The main conclusions which can be drawn from previous studies are the following. GMM is inefficient relative to QML, although the latter is in its turn a sub-optimal procedure. Importance Sampling techniques are very complicated even for the simplest model, therefore they do not seem suitable to be generalized. Bayes estimation seems to lead to large efficiency gain over QML[6], while SEM appears to be competitive with the Bayes estimator (cfr. Shephard, 1994).

A further approach which appears to be suitable for the estimation of both continuous-time and discrete-time stochastic volatility models is represented by the Indirect Inference procedure proposed by Gourieroux, Monfort and Renault (GMR) (1993). This approach requires the model on which inference is made to be easily simulated, which is the case of SV models, while the estimation is carried over an auxiliary model, carefully chosen for easy estimation. GMR indicate in their paper how the quasi-likelihood function formulated by Harvey, Ruiz and Shephard (1994) can be used as an auxiliary criterion to estimate a continuous-time SV model, while Engle and Lee (1994) apply the Indirect Inference methods to the same kind of model using as auxiliary criterion the likelihood function of a GARCH model. These proposals share the common idea of using discrete-time models in order to estimate continuous-time models provided by theoretical finance.

The purpose of this paper is to investigate further possibilities opened by the Indirect Inference approach into the matter of the estimation of SV models. Focusing on their discrete-time versions, and starting from the univariate case, two Indirect Inference methods relying on an autoregressive auxiliary model and an ARMA model are proposed. In the

---

[2]The QML procedure has been proposed by Harvey, Ruiz and Shephard (1994).

[3]Danielsson and Richard (1993) and Danielsson (1994).

[4]See Jacquier, Polson and Rossi (1994).

[5]This approach has been suggested by Kim and Shephard (1994), using Markov Chain Monte Carlo.

[6]This result is shown by Jacquier, Polson and Rossi (1994) and confirmed by some Monte Carlo evidence in Shephard (1994).

first method, the estimator is obtained by calibration of the estimate of the auxiliary parameter. The autoregressive representation allows easy computation of its pseudo- maximum likelihood (PML) estimates and, importantly, the derivation of an optimal procedure, leading to the minimum variance Indirect Inference estimator in the "class" of the Indirect Inference estimators relying on the same auxiliary model. Moreover, as the application of the Indirect Inference method based on the PML estimates calibration provides an indirect test of misspecification of the estimated model, a further objective of the paper is to study the finite sample properties of the test associated with the proposed indirect estimation procedure. Given the lack of tools for testing the adequacy of the SV specification, such a by product of the Indirect Inference methodology seems particularly attractive in this case. In the second approach, based on an ARMA model, the estimator is obtained through calibration of the score function, exploiting the fact that a closed form expression for the gradient can be derived. This way, the estimation of the auxiliary parameter, requiring in its turn numerical maximization, has not to be repeated during the numerical estimation process. Again, the simplicity of the auxiliary model allows the derivation of a minimum variance indirect estimator.

The performance of the proposed estimators are then evaluated through a series of Monte Carlo experiments in which the same process analysed by Shephard (1996) is used to generate the data. This makes it possible to perform an empirical comparison with some of the above mentioned alternative techniques, in particular QML, Bayes and SEM. As the latter method leads to an asymptotically efficient estimator, it will be also possible to evaluate the loss of efficiency implied by the Indirect Inference estimator. In absolute terms, the good performance of both Indirect Inference methods proposed evidenced by our results in terms of finite sample bias and variance of the estimates does suggest that both methods could be a useful tool for the estimation of Stochastic Volatility models. More particularly, the first approach based on calibration of the PML estimate of an AR auxiliary model seems preferable to the second one, based on calibration of the score function of an ARMA auxiliary model. As far as the comparison with other methods is concerned, from our evidence the two Indirect Inference methodologies seem to perform comparably with QML, while they present a loss of efficiency with respect to Bayes and SEM which is of acceptable size when one considers that the Indirect Inference methodology is very general and can be applied in cases in which alternative estimation methods are not feasible. Finally, we find that the finite sample properties of the Indirect Test are good for samples of realistic size for financial application, confirming the possibility of misspecification testing as an advantge of the Indirect Inference method over the alternative procedures.

The simplicity of the proposed approaches seem to be promising for generalization to more complicated models, including multivariate models and models in which the variance component exhibits more complicated structure than the one usually considered (e.g. autoregressive representation of order one). This feature constitutes an advantage over the Engle and Lee suggestion and could represent an advantage over the existing alternative estimation methods.

The paper is structured as follows: Section 2 recalls the main features of Indirect Inference and describe some details related to the particular case under scrutiny, Section 3 contains the results of the Monte Carlo experiments, Section 4 is devoted to the analysis of the performance of the misspecification test, Section5 concludes.

2 Stochastic Volatility and Indirect Inference

2.1 The model

Let us introduce with general notation the model object of inference as:

$$M = \left\{ g(w_t \mid w^{t-1}; \theta), \theta \in \Theta \subset R^p \right\} \qquad (1)$$

where $w^{t-1} = \{w_{t-1}, w_{t-2}, ...\}$. Notice that we limit our consideration to the pure autoregressive case, i.e. the model does not contain exogenous variables. In order to particularize model 10 for univariate discrete-time stochastic volatility models, let w_t be a bivariate vector, say (y_t, h_t), and let its probability distribution conditional on the past, say M^{sv}, be uniquely determined by the two expressions:

$$\begin{aligned} y_t &= \exp\left\{\tfrac{1}{2}h_t\right\} u_t, & u_t &\sim I.I.N.(0,1) \\ h_t &= \mu + \rho h_{t-1} + v_t, & v_t &\sim I.I.N.(0,\sigma^2) \end{aligned} \tag{2}$$

$t = 1...T$, where the two error terms, u_t and v_t are assumed to be independent of one other and ρ is in modulus less than one to ensure stationarity. This model specifies the variance of the observable variable y_t to be a function of the unobservable h_t, which follows a first order autoregressive process. Let us call the parameters of interest θ, with $\theta' = (\mu, \rho, \sigma^2)$. Computation of the likelihood function associated with model 2 , in order to achieve estimation of θ, is difficult due to the presence of the latent variables h_t, as it requires computation (impossible analytically) of a T-dimensional integral of the joint density of $\underline{w}_T = \{w_t, t = 1...T\}$ with respect to $h_1...h_T$. On the other hand, it is easy to simulate values of $\underline{y}_T = \{y_t, t = 1...T,\}$ from M^{sv}, for a given value of the parameter vector θ and a given initial condition $w_0 = (y_0, h_0)$.

2.2 The method

The first step in the Indirect Inference approach (see Gourieroux, Monfort and Renault, 1993) is to choose an auxiliary criterion, $Q_T\left(\underline{y}_T, \beta\right)$, with $\beta \in B \subset R^q$, whose maximization leads to an estimate of β:

$$\widehat{\beta}_T = \arg\max_{\beta \in B} Q_T\left(\underline{y}_T, \beta\right). \tag{3}$$

It is assumed that the criterion converges to a deterministic limit, which is a function of the distribution defined in M^{sv}, therefore of θ,as well as of β. This limit is indicated by $Q_\infty\ (\theta, \beta)$ and the value of β which maximizes it, which in its turn depends on θ, is called the binding function:

$$b(\theta) = \arg\max_{\beta \in B} Q_\infty(\theta, \beta)$$

It is assumed that $b(\theta)$ is injective and that the above maximum, $b(\theta)$, when evaluated at the true value of θ, θ_0, is unique. $b(\theta_0) = \beta_0$ is the pseudo-true value of β, and it is the limit toward which the estimate $\widehat{\beta}_T$ converges.

The second step of the estimation procedure amounts to deriving an estimate of the binding function through simulation of the observations $\underline{y}_T$ by drawing from the distribution defined by 2. Let us denote by $\underline{y}_{TH}(\theta) = \left\{y^h(\theta),\ h = 1...TH\right\}$[7] a simulated vector for the y's, which can be obtained for a particular value of the parameter θ and a given initial condition h_0. After replacing the original observation with the simulated ones in 3, the (functional) estimator of the binding function is given by:

$$\widetilde{\beta}_{TH}(\theta) = \arg\max_{\beta \in B} Q_T\left[\underline{y}_{TH}(\theta), \beta\right]. \tag{4}$$

With the above notation, the indirect inference estimator of θ is defined as:

$$\widetilde{\theta}_T^H = \arg\min_{\theta \in \Theta} \left[\widehat{\beta}_T - \widetilde{\beta}_{TH}(\theta)\right]' \widehat{\Omega}_T \left[\widehat{\beta}_T - \widetilde{\beta}_{TH}(\theta)\right] \tag{5}$$

i.e. it is chosen so as to make the pseudo maximum likelihood estimators $\widehat{\beta}_T$ and $\widetilde{\beta}_{TH}(\theta)$ as close as possible. Notice that the estimator will be a function of the weighting matrix $\widehat{\Omega}_T$, a positive definite matrix converging to a deterministic positive definite matrix Ω.

Gourieroux, Monfort and Renault show that under the assumptions above mentioned $\widetilde{\theta}_T^H$ is a consistent estimator of θ_0 and that, under some

[7] Given the absence of exogenous variables for our model M^{sv}, we introduce here the second version of the indirect estimator of Gourieroux, Monfort, Renault (1993) based on a single simulated path of length TH, while the first version uses H simulated paths of length T. The authors show in their appendix the asymptotic eqivalence of the two approaches.

further conditions, it is asymptotically normally distributed, when H is fixed and T goes to infinity. Moreover, they provide the expression of the optimal choice of the matrix Ω, i.e. the choice which minimizes the asymptotic variance-covariance matrix of the indirect estimator $\tilde{\theta}_T^H$. Let us define the following matrices:

$$I_0 = \lim_{T\to\infty} \underset{0}{V} \left\{ \sqrt{T} \tfrac{\partial Q_T}{\partial \beta} \left[y^h(\theta_0), \beta_0 \right] \right\}$$
$$J_0 = \operatorname*{plim}_{T\to\infty} - \tfrac{\partial^2 Q_T}{\partial \beta \partial \beta'} \left[y^h(\theta_0), \beta_0 \right]$$

where $\underset{0}{V}$ indicates variance with respect to the true distribution of the y's process. Notice that the term containing the limits of the covariances between the scores vectors, usually indicated by K_0, is equal to zero since the model does not contain exogenous variables[8]. With the above notation, the asymptotic variance-covariance matrix of $\tilde{\theta}_T^H$ is given by:

$$W(H,\Omega) = \left(1 + \frac{1}{H}\right) d(\theta_0,\Omega) \frac{\partial b'}{\partial \theta}(\theta_0) \Omega J_0^{-1} I_0 J_0^{-1} \Omega \frac{\partial b}{\partial \theta'}(\theta_0)\ d(\theta_0,\Omega)$$

where:

$$d(\theta_0,\Omega) = \left[\frac{\partial b'}{\partial \theta}(\theta_0) \Omega \frac{\partial b}{\partial \theta'}(\theta_0) \right]^{-1}$$

so that the optimal choice of Ω is:

$$\Omega^* = J_0 I_0^{-1} J_0 \qquad (6)$$

The optimal indirect estimator, say $\hat{\theta}_T^H$, can be computed by substituting for $\hat{\Omega}_T$ in 5 a consistent estimator of Ω^*, say $\hat{\Omega}_T^*$. $\hat{\theta}_T^H$ is asymptotically normal with variance-covariance matrix: given by:

$$W_H^* = W(H,\Omega^*) = \left(1 + \frac{1}{H}\right) \left[\frac{\partial b'}{\partial \theta}(\theta_0) J_0 I_0^{-1} J_0 \frac{\partial b}{\partial \theta'}(\theta_0) \right]^{-1}$$

Alternatively, according to the proposal of Gallant and Tauchen (1992), it is possible to implement the indirect inference procedure by

[8] Cfr. Gourieroux, Monfort and Renault (1993).

calbrating the parameter of interest θ through the score function, i.e. choosing the value of θ which makes the score function of the auxiliary model as close as possible to 0:

$$\tilde{\tilde{\theta}}_T^H = \arg\min_{\theta \in \Theta} \frac{\partial Q_T}{\partial \beta'} \left[\underline{y}_{TH}(\theta), \hat{\beta}_T\right] \; \hat{\Sigma}_T \; \frac{\partial Q_T}{\partial \beta} \left[\underline{y}_{TH}(\theta), \hat{\beta}_T\right] \tag{7}$$

where $\hat{\Sigma}_T$ converges to a positive definite matrix Σ. Gourieroux, Monfort and Renault show that $\tilde{\tilde{\theta}}_T^H(\Sigma)$ is asymptotically equivalent to $\tilde{\theta}_T^H(J_0 \Sigma J_0)$, so that the minimum variance estimator is obtained when $\Sigma^* = J_0^{-1} \Omega^* J_0^{-1} = I_0^{-1}$.

2.3 The proposed auxiliary models

As the estimation procedure involves the numerical minimization of a quadratic form, it is desirable that the estimation of the auxiliary parameter β is quite simple to perform, possibly without resorting to further numerical methods. On the other hand, the auxiliary model should be chosen so that it reflects at least some features of the original model. With these considerations in mind, notice that squaring y_t in model 2 and taking the logarithmic transformation gives:

$$\ln y_t^2 = h_t + \ln u_t^2$$

where the first term of the right hand side follows a first order autoregressive process, while the second is a non gaussian white noise (involving a transformation of a gaussian white noise which does not preserve normality), or a zero order non gaussian autoregressive process. The sum of the two terms is a non gaussian ARMA(1,1) process in the covariance sense, i.e. its autocovariance function has the pattern of an ARMA(1,1). A first idea is to use a gaussian autoregressive representation of a given order, i.e. an AR(m), for $\ln y_t^2$, in order to approximate its ARMA nature. The second possibility is to use directly an ARMA model as auxiliary.

2.3.1 The AR(m) auxiliary model

A first possibility is to consider the following auxiliary model:

$$\ln y_t^2 = \beta_0^* + \beta_1^* \ln y_{t-1}^2 + \beta_2^* \ln y_{t-2}^2 + ... + \beta_m^* \ln y_{t-m}^2 + \varepsilon_t, \qquad \varepsilon_t \sim I.I.N(0, \tau^2) \tag{8}$$

whose parameters $\beta^* = (\beta_0^*, \beta_1^*, \beta_2^*, ...\beta_m^*)'$ and τ^2 can be easily estimated through the Maximum Likelihood method, based on the sequential factorization of the density of $\ln y_t^2$ given its past and conditioning on the first m observations.

Let $x_t = \ln y_t^2$, $\underline{x} = (x_{m+1}, x_{m+2}, ...x_T)'$, a vector $(T-m, 1)$, $X_{-m} = (\underline{1}, \underline{x}_{-1}, \underline{x}_{-2}, ...\underline{x}_{-m})$, a matrix $(T-m, m)$, whose columns are $\underline{1}$, a vector of ones, and the lagged vectors $\underline{x}_{-l} = (x_{m+1-l}, x_{m+2-l}, ...x_{T-l})', l = 1.....m$. Denoting the whole auxiliary parameter, of dimension $(m+2, 1)$ as $\beta = (\beta^{*\prime}, \tau^2)'$, the criterion function corresponding to 3 becomes the average conditional log likelihood function:

$$Q_T(\underline{x}, \beta) = \left[-\frac{1}{2}\ln(2\pi\tau^2) - \frac{1}{2\tau^2(T-m)} (\underline{x} - X_{-m}\beta^*)' (\underline{x} - X_{-m}\beta^*) \right] \tag{9}$$

leading to the estimators $\widehat{\beta}^* = (X'_{-m}X_{-m})^{-1}X'_{-m}\underline{x}$, and $\widehat{\tau}^2 = \frac{\widehat{\varepsilon}'\widehat{\varepsilon}}{T-m}$, with $\widehat{\varepsilon} = \underline{x}$-$X_{-m}\,\widehat{\beta}^*$. These Pseudo Maximum Likelihood estimators are directly computable for both the original and the simulated observations. Given the simplicity of the evaluation of the PMLE in this autoregressive auxiliary model, it is convenient to combine it with the first indirect inference method, which involves the calibration of the PMLE themselves (see 5).

Although the estimation process could, for simplicity, be implemented by using an arbitrary positive definite matrix as weight in the quadratic form[9], the auxiliary criterion in 9 allows an easy computation an estimator of the optimal matrix Ω^*, and makes it possible to obtain

[9]Often the identity matrix is chosen, as the derivation of the optimal matrix Ω^*can be quite complicated.

directly an optimal procedure. Accordingly, let us write the criterion associated with the whole sample, Q_T, as a sum of components associated with the single observations, i.e. $Q_T(.) = \frac{1}{T-m} \sum_{t=m+1}^{T} q_t$, with $q_t = -\frac{1}{2}\left[\ln(2\pi\tau^2) + \frac{1}{\tau^2}(x_t - \beta_0 - \beta_1 x_{t-1} - ..... - \beta_m x_{t-m})^2\right]$, and write, consequently, $I_0 = \lim_{T\to\infty 0} V \left\{\frac{1}{\sqrt{T-m}} \sum \frac{\partial q_t}{\partial \beta}\right\}$. If the scores associated with the single observations are uncorrelated over time[10], a consistent estimator of the above quantity is given by:$\widehat{V}_T^0 = \frac{1}{T-m} \sum_{t=1}^{T-m} \frac{\partial q_t}{\partial \beta}\frac{\partial q_t}{\partial \beta'}\rfloor_{\beta=\widehat{\beta}}$. In our case, as the auxiliary model is misspecified, the scores associated with the single observations, $\frac{\partial q_t}{\partial \beta}$, are likely not to be martingale differences[11] and a consistent estimate of the variance matrix of the scores of the data, I_0, can be obtained using the Newey and West (1987) formula, which takes into account the correlations of the scores over time:

$$\widehat{I}_T =: \widehat{V}_T^0 + \sum_{k=1}^{K} (\widehat{V}_T^k + \widehat{V}_T^{-k})(1 - \frac{k}{K+1}) \tag{10}$$

with:

$$\widehat{V}_T^k = \frac{1}{T-m} \sum_{t=k+1}^{T-m} \frac{\partial q_t}{\partial \beta}\frac{\partial q_{t+k}}{\partial \beta'}$$

where K is a bandwidth which is a function of T and grows slowly enough with the sample size in order to ensure consistency of the above estimator.

As far as J_0 is concerned, it can be estimated, as usual, with the empirical second derivative matrix $\frac{\partial^2 Q_T}{\partial\beta\partial\beta'}$, i.e. a consistent estimator of it is:

$$\widehat{J}_T = \begin{bmatrix} -\frac{1}{(T-m)\widehat{\tau^2}} X'_{-m} X_{-m} & \underline{0} \\ \underline{0}' & -\frac{1}{2\widehat{\tau^4}} \end{bmatrix}$$

where $\underline{0}$ is a $(m+1, 1)$ vector of zeros. The above quantities lead to the consistent estimator of the optimal matrix :

$$\widehat{\Omega}_T^* = \widehat{J}_T \widehat{I}_T^{-1} \widehat{J}_T.$$

[10] This is usually found in practice, but needs to be checked.

[11] $\frac{\partial qt}{\partial \beta}$ is not, in general, a martingale difference with respect to the $\sigma - field$ generated by the past y_{t-1}, ...,cfr Gourieroux and Monfort (1995).

2.3.2 The ARMA(1,1) auxiliary model

A second possibility is to choose directly an ARMA(1,1) model as auxiliary one, avoiding the approximation level introduced by the purely autoregressive representation. This amounts to postulate:

$$\ln y_t^2 = \alpha_0^* + \alpha_1^* \ln y_{t-1}^2 + \omega_t - \alpha_2^* \omega_{t-1}, \qquad \omega_t \sim I.I.N(0, \nu^2). \tag{11}$$

Letting $x_t = \ln y_t^2$ and $\alpha = (\alpha_0^*,\ \alpha_1^*,\ \alpha_2^*,\ \nu^2) = (\alpha^{*\prime},\ \nu^2)$, we get the following average loglikelihood function conditional to the starting values $(x_0,\ \omega_0)$:

$$Q_T(\underline{x}, \alpha) = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln \nu^2 - \frac{1}{T 2\nu^2}\sum_{t=1}^{T} \omega_t(\alpha^*)^2 \tag{12}$$

The sequence $\{\omega_1, \omega_2, ..., \omega_T\}$ can be derived by the recursive expression:

$$\omega_t = x_t - \alpha_0^* - \alpha_1^* x_{t-1} + \alpha_2^* \omega_{t-1} \tag{13}$$

setting the initial values equla to their expected value, i.e.: $x_0 = \frac{\alpha_0^*}{1-\alpha_1^*}$, $\omega_0 = 0$. In order to get the PMLE $\widehat{\alpha} = (\widehat{\alpha}^{*\prime},\ \widehat{\nu}^2)$ it is necessary to resort to numerical optimization of the above conditional likelihood. This makes the calibration of the PMLE computationally cumbersome. On the other hand, it can be noticed that the gradient $\frac{\partial Q_T}{\partial \alpha}$ can be analitically derived by iterating on expression 13, getting:

$$\frac{\partial Q_T}{\partial \alpha} = \begin{bmatrix} \frac{\partial Q_T}{\partial \alpha^*} \\ \frac{\partial Q_T}{\partial \nu^2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T\nu^2}\sum_{t=1}^{T} \omega_t \frac{\partial \omega_t}{\partial \alpha^*} \\ -\frac{1}{2\nu^2} + \frac{1}{T 2\nu^4}\sum_{t=1}^{T} \omega_t^2 \end{bmatrix}$$

where:

$$\frac{\partial \omega_t}{\partial \alpha^*} = \begin{bmatrix} \frac{\partial \omega_t}{\partial \alpha_0^*} \\ \frac{\partial \omega_t}{\partial \alpha_1^*} \\ \frac{\partial \omega_t}{\partial \alpha_2^*} \end{bmatrix} = \begin{bmatrix} -1 \\ -x_{t-1} \\ \omega_{t-1} \end{bmatrix} + \alpha_2^* \frac{\partial \omega_{t-1}}{\partial \alpha^*}$$

Therefore, the indirect inference estimator can be obtained through the second method outlined in the previous section, i.e. by minimization of the quadratic form in the score function 7. Moreover, the analytical expression of the gradient above allows for the estimation of the optimal weighting matrix, $\widehat{\Sigma}^{*} = \widehat{I}_T^{-1}$, for which the Newey-West formula in 10 can be used.

3 Monte Carlo results

3.1 Some evidence on the performance of the Indirect Inference estimators

In our Monte Carlo experiment we take the same data generating process as Shephard (1996), in order to provide both some evidence on the performance of the Indirect Inference method, and the basis for the comparison with Quasi Maximum Likelihood, SEM and Bayes estimation.

The observations $\{y_t, t = 1.....T\}$ are generated by the SV model 2 with $\theta_0 = (\mu_0, \rho_0, \sigma_0)' = (0, 0.9, 0.316)'$, for $T = 1000, T = 2000$. As far as the sample sizes considered are concerned, it is important to emphasize that inference in stochastic volatility models is quite demanding in terms of sample information required, due to the presence of a latent structure governing the variance of the model. This is the reason why all applications are concerned with quite long financial series (T is hardly found to be inferior to 1000)[12]. On the other hand, it is well known that many financial time series are available with a large number of observations. The series are simulated setting the initial value for the h_t process equal to its mean, i.e. we put $h_0 = 0$.

The number of drawings H determining the length of the simulated series $\underline{y}_{TH}(\theta)$ is set respectively equal to 16 for $T = 1000$ and to 8 for $T = 2000$, so that in both cases the resulting simulated series $\underline{y}_{TH}(\theta)$ is of the same size. The minimization problem to be solved to get the

[12]For this reason, differently from Shephard, the sample size $T = 500$ has not been considered in the experiment.

indirect inference optimal estimator, as described in the previous section, has been implemented numerically using the procedure "Optmum" of Gauss 3.1. In particular, the BFGS method has been used, which is a quasi-Newton method as it exploits both first order and second order derivative information, but relies on approximation of the Hessian matrix. Numerical computation of the derivatives of the objective function offered by the same library has been used. As starting values for the algorithm, the true parametric vector θ_0 has been chosen throughout the experiments[13]. These extremely good starting values allow a considerable time reduction in the length of the experiment, as they ensure that the algorithm will start from a point close enough to the minimum of the function to be minimized[14]. The order m of the autoregressive process used as auxiliary model in the first case turned out to be a relevant choice for the performance of the estimation procedure. Given the absence of any theoretical criterion to help such a choice, we proceeded on empirical grounds. After some experimenting, some evidence was found in favour of discarding values of m inferior to 10, while $m = 10$ appeared to be a satisfactory choice[15]

Table 3.1 and 3.2 display mean, bias and standard deviation of the estimated values of the parameters over 200 replications of the Monte Carlo experiment, for $T = 1000$ and $H = 16$, for the two Indirect Inference estimators considered. Tables 3.3 and 3.4 contain the same information relating to the case $T = 2000$ and $H = 8$.

Despite the limited number of replications performed, the results displayed do indicate the good performance of the methods proposed in

[13]We have performed some sensitivity analysis and verified that perturbing the starting values to $\theta^{(0)} = (0.5, 0.5, 0.5)'$ did not change the outcome of the minimization problem.

[14]Note that θ_0 corresponds to the minimum of the limit of the criterion function as T goes to infinity, while the indirect inference estimator corresponds to the minimum of a finite sample objective function.

[15]The order of the autoregressive process is likely to be quite high in order to lead to a sufficiently good approximation to a model containing a moving average component. With values of m lower than 10 we observed a quite high frequency of false maxima of the criterion function (ρ very close to 1 and σ very close to 0).

Ind. Inf.1 (AR)			
	$\widehat{\mu}$	$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
	0.00144	0.86856	0.33323
Bias			
	0.00144	-0.03144	- 0.01700
St. Dev.			
	0.01961	0.09867	0.15480

Table 1: *True values* : $\mu_0 = 0, \rho_0 = 0.9, \sigma_0 = 0.31623$. $T = 1000, H = 16$.

Ind. Inf.2 (ARMA)			
	$\widehat{\mu}$	$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
	-0.00547	0.86372	0.36578
Bias			
	-0.00547	-0.03628	0.04955
St. Dev.			
	0.02325	0.09454	0.14789

Table 2: *True values* : $\mu_0 = 0, \rho_0 = 0.9, \sigma_0 = 0.31623$. $T = 1000, H = 16$.

Ind. Inf.1 (AR)			
	$\widehat{\mu}$	$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
	0.00059	0.88764	0.31917
Bias			
	0.00059	-0.01236	-0.00294
St. Dev.			
	0.01073	0.05852	0.10891

Table 3: *True values* : $\mu_0 = 0, \rho_0 = 0.9, \sigma_0 = 0.31623$. $T = 2000, H = 8$.

Ind. Inf.2 (ARMA)			
	$\widehat{\mu}$	$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
	0.00207	0.88670	0.33565
Bias			
	0.00207	-0.01330	0.01942
St. Dev.			
	0.01099	0.06864	0.10869

Table 4: *True values* : $\mu_0 = 0, \rho_0 = 0.9, \sigma_0 = 0.31623$. $T = 2000, H = 8$.

terms of finite sample bias and variance of the estimators. Moreover, the results evidence in favour of the first approach, especially as far as the bias is concerned. This means that the autoregressive representation, which is the more easiliy generalizable to the multivariate case, is a sufficiently good auxiliary model.[16]. Notice that the length of the simulated series H, has been kept quite low, in order not to make the simulation experiment too burdensome. A gain in efficiency has to be expected for higher values of it that can without problems be considered in applications,while further experimenting on this possibility would be of interest, but very demanding in terms of computational time. The mean and the bias of the estimated parameters over the replications show their proximity to the theoretical values for finite sample sizes which are reasonable ones for the model under analysis (T greater than 1000). A remarkable improvement of precision is observed in both cases as T is increased from $T = 1000$ to $T = 2000$.

3.2 Comparison with alternative estimation methods

In Tables 3.5 to 3.8 the results obtained by Shephard for the same number of replications (200) are reported for comparison purposes. Quasi Maximum Likelihood, Bayes and SEM[17] results are available for $T = 1000$, while only SEM is for $T = 2000$. Moreover, available results do not include the estimation of the intercept μ.

Tables 3.5-3.7 refer to $T = 1000$, and show, as evidenced by Shephard (1996), that Bayes and SEM are competitive and both outperform QML as far as the efficiency of the estimator is concerned. However, SEM has the advantage of not being conditional on the selection of a

[16]However, the second method behaves better in computational terms. This can be inferred from the fact that with the first method about the 6% of the replications with $T = 1000$ and the 2% with $T = 2000$ were discarded as convergence was not reached within 40 iterations of the minimization algorithm, while with the second method we observed one such case out of 200 with $T = 1000$ and no one with $T = 2000$.

[17]We report only one of the cases for the SEM method analysed by Shephard, i.e. the one which best approximates the exact problem, corresponding, in his notation, to $M = 10$.

QML			
		$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
		0.86732	0.34809
Bias			
		-0.03268	0.03186
St. Dev.			
		0.09950	0.15773

Table 5: *True values* : $\mu_0 = 0, \rho_0 = 0.9, \sigma_0 = .31623. \quad T = 1000.$

BAYES			
		$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
		0.87866	0.33563
Bias			
		-0.02134	0.0194
St. Dev.			
		0.04963	0.09212

Table 6: *True values* : $\mu_0 = 0, \rho_0 = 0.9, \sigma_0 = .31623. \quad T = 1000.$

SEM			
		$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
		0.89004	0.30328
Bias			
		-0.00996	-0.01295
St. Dev.			
		0.03877	0.05551

Table 7: *True values* : $\mu_0 = 0, \rho_0 = 0.9, \sigma_0 = .31623. \quad T = 1000.$

SEM			
		$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
		0.89905	0.29940
Bias			
		-0.00095	-0.01683
St. Dev.			
		0.02405	0.03977

Table 8: *True values* : $\mu_0 = 0, \rho_0 = 0.9, \sigma_0 = .31623. \quad T = 2000.$

prior distribution. Comparing with Table 3.1, the performance of the indirect inference method proposed appears to be satisfactory in terms of bias of the estimates, for which it is sligthly better than QML and comparable to Bayes and SEM, while these two latter methodols are more efficient than the indirect inference one. Comparison of Tables 3.3-3.4 and Table 3.6, for which $T = 2000$, confirms the substantial gain in efficiency of the SEM method relative to the Indirect Inference ones. This is not surprising from a theoretical point of view, as SEM provides a close approximation to the Maximum Likelihood estimator, and is therefore asymptotically efficient. However, the loss of efficiency of the Indirect Inference estimators has the counterpart of a greater generality and applicability in cases in which the alternative estimators are difficult or impossible to compute.

3.3 Further Monte Carlo evidence on the performance of the estimator

In order to get more extended results, some Monte Carlo experiments have been performed using the AR auxiliary model and generating the observations according to stochastic volatility models estimated in the literature, i.e. taking as true parameter vector the estimated parameter value for some economic time series. This way it is hoped that the characteristics of the method enlightened by the Monte Carlo analysis, although model-specific, refer to "plausible" cases encountered in practice.

We refer therefore to an univariate model estimated by Shephard (1995) for the explanation of the following Japanese-Yen/Deutsche Mark exchange rate (Font: DATASTREAM, 1/1/86 to12/04/94, 2160 daily observations). Consequently, the observations are generated from model 2 with $\theta_0 = (-1.14, 0.967, 0.43)'$[18] . Notice that the proximity of the generated series to the non-stationary case, due to the high value of ρ_0, makes it possible that during the numerical algorithm some inadmissible

[18] μ_0 and ρ_0 are equal to the values of the estimates obtained through the SIEM algorithm, while our σ_0 is the square root of the corresponding estimate in Shephard's application.

Ind. Inf.1 (AR)			
	$\widehat{\mu}$	$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
	-1.32409	0.96167	0.43574
Bias			
	-0.18409	-0.00533	0.00574
St. Dev.			
	0.47676	0.01379	0.07471

Table 9: *True values* : $\mu_0 = -1.14, \rho_0 = 0.967, \sigma_0 = 0.43 \quad T = 1000, H = 16$.

region of the parametric space is entered ($\rho \geq 1, \sigma^2 = 0$), and causing the alghorithm to break down. Therefore, it turned out to be fundamental to perform a constrained minimization (imposing $\rho < 1$). The order of the autoregressive auxiliary model, m, was set to 10, and the true parameter vector was fixed again as starting value for the numerical minimization.

Table 3.9 contains the results obtainted in correspondence of $T = 1000, H = 16$, while Table 3.10 refers to $T = 2000, H = 8$. This set of experiments does confirm the good performance of the indirect inference estimator found in the previous case, in terms of both finite sample variance and standard deviation of the estimates.

4 Misspecification testing through Indirect Inference

It is well known that diagnostic checking in estimated SV models is very poor and limited to the Box-Ljung statistic to check absence of residual autocorrelation[19]. Therefore the possibility of exploiting any additional

[19]This requires the application of some filter in order to get the series of the residuals.

Ind. Inf.1 (AR)			
	$\widehat{\mu}$	$\widehat{\rho}$	$\widehat{\sigma}$
Mean			
	-1.21663	0.96493	0.43288
Bias			
	-0.07663	-0.00207	0.00288
St. Dev.			
	0.33675	0.00974	0.05491

Table 10: *True values* : $\mu_0 = -1.14, \rho_0 = 0.967, \sigma_0 = 0.43$ $T = 2000, H = 8$.

tool to test the appropriateness of the stochastic volatility formulation is of particular importance. The Indirect Inference methodology opens some interesting possibility in this direction.

A first possibility is to test the estimated SV model against a GARCH model for the same series, through a Simulated Encompassing Test for non-nested models. Dhaene, Gourieroux and Scaillet (1995) provide the theory for the case in which one of the model to be tested against the other is estimated through indirect inference. Their method is feasible to test both the null hypothesis that a GARCH model encompasses a SV one and vice-versa, as in both cases computation of the indirect inference estimate of the SV model parameter is required once. This comparison is interesting as SV models have been introduced in the literature as alternative to GARCH models and existing comparison between the two are simply based on the estimated maximum of likelihood function, without any testing (cfr Shepard, 1994).

A second possibility, to which we draw attention, is directly provided as a by-product of the estimation process, as an indirect specification test can be based on the optimal value of the quadratic form. More precisely, proposition 6 of the Indirect Inference paper of Gourieroux, Monfort and Renault (1993) contains the following result:

$$\xi_T = \frac{TH}{1+H} \underset{\theta \in \Theta}{Min} \left[\hat{\beta}_T - \tilde{\beta}_{TH}(\theta)\right]' \hat{\Omega}_T^* \left[\hat{\beta}_T - \tilde{\beta}_{TH}(\theta)\right]$$

where $\hat{\Omega}_T^*$ is a consistent estimator of Ω^*, is asymptotically distributed as a $\chi^2_{(q-p)}$, where $q = dim(\beta)$ and $p = dim(\theta)$, under the hypothesis of correct specification of the original model.

Therefore, a test statistic for the null hypothesis of correct specification of the stochastic volatility model M^{sv} can be evaluated simply by appropriate multiplication of the optimal value of the objective function of the indirect inference procedure. Rejection of the null hypothesis based on the critical region: C=$\left\{\xi_T > \chi^2_{(1-\alpha),(q-p)}\right\}$, leads to a test of asymptotic level α.

4.1 The performance of the indirect specification test

As usual, when a test is based on an asymptotic distribution, the issue of evaluating its finite sample behaviour is one of the fundamental steps towards its "safe" application. In order to achieve this, the stored values of the objective function of the 200 replications of the two different Monte Carlo experiments in the previous sections have been used. The critical values of reference in our case are $\chi^2_{0.90,9} = 14.684$, $\chi^2_{0.95,9} = 16.919$, $\chi^2_{0.99,9} = 21.666$ for a test of asymptotic level equal to 0.10, 0.05, 0.01 respectively. Indicating by $\xi_T^{(r)}$, $r = 1...200$ the (scaled) minimum value of the quadatic form in the r-th replication of the experiment, estimation of the empirical rejection frequency $\hat{P}_T^{(\alpha)}$ is based on the percentage of values $\xi_T^{(r)} > \chi^2_{(1-\alpha),9}$.

We found the following results concering the size of the test:

Experiment 1: $\theta_0 = (0, 0.9, 0.316)'$

- $\hat{P}^{(0.10)}_{1000} = \underset{(0.026)}{0.165}$ $\quad$ $\hat{P}^{(0.05)}_{1000} = \underset{(0.021)}{0.100}$ $\quad$ $\hat{P}^{(0.01)}_{1000} = \underset{(0.013)}{0.035}$

- $\widehat{P}^{(0.10)}_{2000} = \underset{(0.025)}{0.150}$ $\widehat{P}^{(0.05)}_{2000} = \underset{(0.017)}{0.065}$ $\widehat{P}^{(0.01)}_{2000} = \underset{(0.009)}{0.015}$

Experiment 2: $\theta_0 = (-1.14, 0.967, 0.43)'$

- $\widehat{P}^{(0.10)}_{1000} = \underset{(0.026)}{0.165}$ $\widehat{P}^{(0.05)}_{1000} = \underset{(0.020)}{0.090}$ $\widehat{P}^{(0.01)}_{1000} = \underset{(0.011)}{0.025}$

- $\widehat{P}^{(0.10)}_{2000} = \underset{(0.020)}{0.090}$ $\widehat{P}^{(0.05)}_{2000} = \underset{(0.016)}{0.060}$ $\widehat{P}^{(0.01)}_{2000} = \underset{(0.011)}{0.025}$

The standard errors in brakets are evaleuated as: $se = \sqrt{\frac{\widehat{P}(1-\widehat{P})}{200}}$. It can be noticed that while for $T = 1000$ the test tends in both the experiments considered to over-reject the true null hypothesis of correct specification, for a sample size $T = 2000$, the performance of the indirect test is already sufficiently good for it to be a valid base for an evaluation of the stochastic volatility specification. Moreover, the decrease of the empirical rejection frequencies towards the theoretical sizes 0.05 and 0.01, when T is increasesd from 1000 to 2000, suggests that for bigger values of the sample size (still realistic in financial applications) the test is likely to reach its asymptotic level.

5 Conclusions

The estimation of Stochastic Volatility (SV) models is an challenging field of research given the difficulty which one encounters when deriving their exact likelihood function. While these models are difficult to estimate, they can be very easily simulated, and this characteristic makes the Indirect Inference methodology a good candidate for their estimation. The crucial step of the Indirect Inference procedure is the choice of an auxiliary model, which must be easy to estimate and in the same time should reflect some features of the original one. The observation that the logarithmic transformation of the square of a SV process has an ARMA (1,1) autocovariance function pattern is the basis for the choice of either a finite autoregressive representation or an ARMA representation as auxiliary models. Beside the approximating nature for the original model, the proposed auxiliary models exhibit two further nice characteristics. The first one can be very easily estimated by Pseudo Maximum Likelihood, and can be used to calibrate the PML estimate, the second, whose estimation requires in its turn numerical maximization, can be the basis for a score calibration based procedure, which exploits its recursive structure. In both cases, simplicity of the auxiliary model proposed allows the derivation of an optimal Indirect Inference Procedure, leading to a minimum variance Indirect Inference estimator.

The performance of the two Indirect Inference estimators in finite samples of realistic dimension for financial series, on which SV models are usually estimated, is evaluated through some Monte Carlo experiments. The proposed estimators are found to have good properties in terms of closeness of the estimated parameters to the theoretical values and standard deviations of the estimates, and the first approach seems to outperforms the second one. A byproduct of our experiments is the comparison with QML, Bayes and SEM estimators evaluated by Shephard (1996), leading to the conclusion that the Indirect Inference estimator based on PMLE calibration and AR auxiliary model performs slightly better than the QML one, while it is out performed by Bayes and SEM as far as efficiency is concerned. A further possibility open by the Indirect Inference procedure is the derivation of a misspecification test for the estimated model based on the optimal value of the objective function. The finite sample behaviour of such a test is found to be good for samples of 2000 observations, a size encountered in practice in financial applications.

Our results suggest that the choice of an autoregressive auxiliary model could be an useful one in the application of the Indirect Inference procedure to the estimation of SV models on real financial series. In particular, the advantage of the procedure is likely to be assessed for the estimation of more sophisticated SV models, including the assumption of a more complicated structure of the process describing the variance component and/or the multivariate case, which would imply the choice of a VAR auxiliary model..

References

[1] **Andersen,T. (1993)**, Return volatility and Trading Volume: an Information flow interpretation of stochastic volatility. Unpublished paper, Finance Department, Northwestern University.

[2] **Andersen,T. and Sorensen,B. (1994)**, GMM Estimation of a Stochastic Volatility Model : a Monte Carlo Study. Unpublished paper, Finance Department, Northwestern University.

[3] **Bollerslev,T. (1986)**, Generalized Autoregressive Condidional Heteroskedasticity. *Journal of Econometrics*, 51, 307-327.

[4] **Chesney,M. and Scott,L.O. (1989)**, Pricing European Options: a Comparison of the Modified Black-Scholes Model and a Random Variance Model. *Journal of Financial and Qualitative Analysis*, 267-284.

[5] **Danielsson,J. (1994)**, Stochastic Volatility in Asset Prices: Estimation with Simulated Maximum Likelihood. *Journal of Econometrics*, 61, 375-400.

[6] **Danielsson,J. and Richard,J.F. (1993)**, Accelerated Gaussian Importance Sampler with Application to Dynamic Latent Variable Models. *Journal of Applied Econometrics*, 8,

[7] **Duffie,D. and Singleton,K.J. (1993)**, Simulated Moments Estimation for Markov Models of Asset Process. *Econometrica*, 61, 929-952.

[8] **Engle,R.F. (1982)**, Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of the United Kingdom Inflation. *Econometrica*, 50, 987-1007.

[9] **Engle,R.F and Lee,G.G.J. (1994)**, Estimating Diffusion Models of Stochastic Volatility. Unpublished paper, Department of Economics, University of California at San Diego.

[10] **Gallant,R. and Tauchen (1994)**, Which Moments to Match? Working paper, Duke University and the University of North Carolina at Chapel Hill.

[11] **Gourieroux,C., Monfort,A. and Renault,E. (1993)**, Indirect Inference, *Journal of Applied Econometrics*, 8, S85-S118.

[12] **Gourieroux,C., and Monfort,A. (1995)**, Testing, Encompassing, and Simulating Dynamic Econometric Models, *Econometric Theory*, 11,195-228.

[13] **Harvey,A.C., Ruiz,E. and Shepard,N. (1994)**, Multivariate Stochastic Variance Models. *Review of Economic Studies*, 61, 247-264.

[14] **Hull,J. and White,A. (1987)**, The Pricing of Options on Assets with Stochastic Volatilities. *Journal of Finance,* 42, 281-300.

[15] **Jacquier,E., Polson,N.G. and Rossi,P.E. (1994)**, Bayesian Analysis of Stochastic Volatility Models (with discussion). *Journal of Economics and Business Statistics*, 12, 371-417.

[16] **Kim,S. and Shephard,N. (1994)**, Stochastic Volatility: Optimal Likelihood Inference and Comparison with ARCH models. Unpublished paper, Nuffield College, Oxford.

[17] **Melino,A. and Turnbull,S.M. (1990)**, Pricing Foreign Currency Options with Stochastic Volatility, *Journal of Economietrics*, 45, 239-265.

[18] **Shephard,N. (1996)**, Statistical Aspects of ARCH and Stochastic Volatility, in "Likelihood, Time Series with Econometric and Other Applications", D.R. Cox, D.V. Hinkley and O.E. Barndorff-Nielsen editors, Chapman & Hall.

[19] **Taylor,S.J. (1986)**, *Modelling Financial Time Series*. Chichester, John Wiley.

EUI WORKING PAPERS

EUI Working Papers are published and distributed by the European University Institute, Florence

Copies can be obtained free of charge – depending on the availability of stocks – from:

The Publications Officer
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI)
Italy

Please use order form overleaf

Publications of the European University Institute

To The Publications Officer
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI) – Italy
Telefax No: +39/55/4685 636
E-mail: publish@datacomm.iue.it

From Name . . . .
Address. . . . .
. . . .
. . . .
. . . .
. . . .

❒ Please send me a complete list of EUI Working Papers
❒ Please send me a complete list of EUI book publications
❒ Please send me the EUI brochure Academic Year 1996/97

Please send me the following EUI Working Paper(s):

No, Author . . . .
Title: . . . .
No, Author . . . .
Title: . . . .
No, Author . . . .
Title: . . . .
No, Author . . . .
Title: . . . .

Date . . . .

Signature . . . .

Working Papers of the Department of Economics Published since 1994

ECO No. 94/1
Robert WALDMANN
Cooperatives With Privately Optimal Price Indexed Debt Increase Membership When Demand Increases

ECO No. 94/2
Tilman EHRBECK/Robert WALDMANN
Can Forecasters' Motives Explain Rejection of the Rational Expectations Hypothesis? *

ECO No. 94/3
Alessandra PELLONI
Public Policy in a Two Sector Model of Endogenous Growth *

ECO No. 94/4
David F. HENDRY
On the Interactions of Unit Roots and Exogeneity

ECO No. 94/5
Bernadette GOVAERTS/David F. HENDRY/Jean-François RICHARD
Encompassing in Stationary Linear Dynamic Models

ECO No. 94/6
Luigi ERMINI/Dongkoo CHANG
Testing the Joint Hypothesis of Rationality and Neutrality under Seasonal Cointegration: The Case of Korea *

ECO No. 94/7
Gabriele FIORENTINI/Agustín MARAVALL
Unobserved Components in ARCH Models: An Application to Seasonal Adjustment *

ECO No. 94/8
Niels HALDRUP/Mark SALMON
Polynomially Cointegrated Systems and their Representations: A Synthesis *

ECO No. 94/9
Mariusz TAMBORSKI
Currency Option Pricing with Stochastic Interest Rates and Transaction Costs: A Theoretical Model

ECO No. 94/10
Mariusz TAMBORSKI
Are Standard Deviations Implied in Currency Option Prices Good Predictors of Future Exchange Rate Volatility? *

ECO No. 94/11
John MICKLEWRIGHT/Gyula NAGY
How Does the Hungarian Unemployment Insurance System Really Work? *

ECO No. 94/12
Frank CRITCHLEY/Paul MARRIOTT/Mark SALMON
An Elementary Account of Amari's Expected Geometry

ECO No. 94/13
Domenico Junior MARCHETTI
Procyclical Productivity, Externalities and Labor Hoarding: A Reexamination of Evidence from U.S. Manufacturing *

ECO No. 94/14
Giovanni NERO
A Structural Model of Intra-European Airline Competition *

ECO No. 94/15
Stephen MARTIN
Oligopoly Limit Pricing: Strategic Substitutes, Strategic Complements

ECO No. 94/16
Ed HOPKINS
Learning and Evolution in a Heterogeneous Population *

ECO No. 94/17
Berthold HERRENDORF
Seigniorage, Optimal Taxation, and Time Consistency: A Review *

ECO No. 94/18
Frederic PALOMINO
Noise Trading in Small Markets *

ECO No. 94/19
Alexander SCHRADER
Vertical Foreclosure, Tax Spinning and Oil Taxation in Oligopoly

*out of print

ECO No. 94/20
Andrzej BANIAK/Louis PHLIPS
La Pléiade and Exchange Rate Pass-Through

ECO No. 94/21
Mark SALMON
Bounded Rationality and Learning; Procedural Learning

ECO No. 94/22
Isabelle MARET
Heterogeneity and Dynamics of Temporary Equilibria: Short-Run Versus Long-Run Stability

ECO No. 94/23
Nikolaos GEORGANTZIS
Short-Run and Long-Run Cournot Equilibria in Multiproduct Industries

ECO No. 94/24
Alexander SCHRADER
Vertical Mergers and Market Foreclosure: Comment

ECO No. 94/25
Jeroen HINLOOPEN
Subsidising Cooperative and Non-Cooperative R&D in Duopoly with Spillovers

ECO No. 94/26
Debora DI GIOACCHINO
The Evolution of Cooperation: Robustness to Mistakes and Mutation

ECO No. 94/27
Kristina KOSTIAL
The Role of the Signal-Noise Ratio in Cointegrated Systems

ECO No. 94/28
Agustín MARAVALL/Víctor GÓMEZ
Program SEATS "Signal Extraction in ARIMA Time Series" - Instructions for the User

ECO No. 94/29
Luigi ERMINI
A Discrete-Time Consumption-CAP Model under Durability of Goods, Habit Formation and Temporal Aggregation

ECO No. 94/30
Debora DI GIOACCHINO
Learning to Drink Beer by Mistake

ECO No. 94/31
Víctor GÓMEZ/Agustín MARAVALL
Program TRAMO "Time Series Regression with ARIMA Noise, Missing Observations, and Outliers" - Instructions for the User

ECO No. 94/32
Ákos VALENTINYI
How Financial Development and Inflation may Affect Growth

ECO No. 94/33
Stephen MARTIN
European Community Food Processing Industries

ECO No. 94/34
Agustín MARAVALL/Christophe PLANAS
Estimation Error and the Specification of Unobserved Component Models

ECO No. 94/35
Robbin HERRING
The "Divergent Beliefs" Hypothesis and the "Contract Zone" in Final Offer Arbitration

ECO No. 94/36
Robbin HERRING
Hiring Quality Labour

ECO No. 94/37
Angel J. UBIDE
Is there Consumption Risk Sharing in the EEC?

ECO No. 94/38
Berthold HERRENDORF
Credible Purchases of Credibility Through Exchange Rate Pegging: An Optimal Taxation Framework

ECO No. 94/39
Enrique ALBEROLA ILA
How Long Can a Honeymoon Last? Institutional and Fundamental Beliefs in the Collapse of a Target Zone

ECO No. 94/40
Robert WALDMANN
Inequality, Economic Growth and the Debt Crisis

*out of print

ECO No. 94/41
John MICKLEWRIGHT/
Gyula NAGY
Flows to and from Insured
Unemployment in Hungary

ECO No. 94/42
Barbara BOEHNLEIN
The Soda-ash Market in Europe:
Collusive and Competitive Equilibria
With and Without Foreign Entry

ECO No. 94/43
Hans-Theo NORMANN
Stackelberg Warfare as an Equilibrium
Choice in a Game with Reputation Effects

ECO No. 94/44
Giorgio CALZOLARI/Gabriele
FIORENTINI
Conditional Heteroskedasticity in
Nonlinear Simultaneous Equations

ECO No. 94/45
Frank CRITCHLEY/Paul MARRIOTT/
Mark SALMON
On the Differential Geometry of the Wald
Test with Nonlinear Restrictions

ECO No. 94/46
Renzo G. AVESANI/Giampiero M.
GALLO/Mark SALMON
On the Evolution of Credibility and
Flexible Exchange Rate Target Zones *

***

ECO No. 95/1
Paul PEZANIS-CHRISTOU
Experimental Results in Asymmetric
Auctions - The 'Low-Ball' Effect

ECO No. 95/2
Jeroen HINLOOPEN/Rien
WAGENVOORT
Robust Estimation: An Example *

ECO No. 95/3
Giampiero M. GALLO/Barbara PACINI
Risk-related Asymmetries in Foreign
Exchange Markets

ECO No. 95/4
Santanu ROY/Rien WAGENVOORT
Risk Preference and Indirect Utility in
Portfolio Choice Problems

ECO No. 95/5
Giovanni NERO
Third Package and Noncooperative
Collusion in the European Airline
Industry *

ECO No. 95/6
Renzo G. AVESANI/Giampiero M.
GALLO/Mark SALMON
On the Nature of Commitment in Flexible
Target Zones and the Measurement of
Credibility: The 1993 ERM Crisis *

ECO No. 95/7
John MICKLEWRIGHT/Gyula NAGY
Unemployment Insurance and Incentives
in Hungary *

ECO No. 95/8
Kristina KOSTIAL
The Fully Modified OLS Estimator as a
System Estimator: A Monte-Carlo
Analysis

ECO No. 95/9
Günther REHME
Redistribution, Wealth Tax Competition
and Capital Flight in Growing
Economies

ECO No. 95/10
Grayham E. MIZON
Progressive Modelling of
Macroeconomic Time Series: The LSE
Methodology *

ECO No. 95/11
Pierre CAHUC/Hubert KEMPF
Alternative Time Patterns of Decisions
and Dynamic Strategic Interactions

ECO No. 95/12
Tito BOERI
Is Job Turnover Countercyclical?

ECO No. 95/13
Luisa ZANFORLIN
Growth Effects from Trade and
Technology *

ECO No. 95/14
Miguel JIMÉNEZ/Domenico
MARCHETTI, jr.
Thick-Market Externalities in U.S.
Manufacturing: A Dynamic Study with
Panel Data

*out of print

ECO No. 95/15
Berthold HERRENDORF
Exchange Rate Pegging, Transparency, and Imports of Credibility

ECO No. 95/16
Günther REHME
Redistribution, Income cum Investment Subsidy Tax Competition and Capital Flight in Growing Economies *

ECO No. 95/17
Tito BOERI/Stefano SCARPETTA
Regional Dimensions of Unemployment in Central and Eastern Europe and Social Barriers to Restructuring

ECO No. 95/18
Bernhard WINKLER
Reputation for EMU - An Economic Defence of the Maastricht Criteria

ECO No. 95/19
Ed HOPKINS
Learning, Matching and Aggregation

ECO No. 95/20
Dorte VERNER
Can the Variables in an Extended Solow Model be Treated as Exogenous? Learning from International Comparisons Across Decades

ECO No. 95/21
Enrique ALBEROLA-ILA
Optimal Exchange Rate Targets and Macroeconomic Stabilization

ECO No. 95/22
Robert WALDMANN
Predicting the Signs of Forecast Errors *

ECO No. 95/23
Robert WALDMANN
The Infant Mortality Rate is Higher where the Rich are Richer

ECO No. 95/24
Michael J. ARTIS/Zenon G. KONTOLEMIS/Denise R. OSBORN
Classical Business Cycles for G7 and European Countries

ECO No. 95/25
Jeroen HINLOOPEN/Charles VAN MARREWIJK
On the Limits and Possibilities of the Principle of Minimum Differentiation *

ECO No. 95/26
Jeroen HINLOOPEN
Cooperative R&D Versus R&D-Subsidies: Cournot and Bertrand Duopolies

ECO No. 95/27
Giampiero M. GALLO/Hubert KEMPF
Cointegration, Codependence and Economic Fluctuations

ECO No. 95/28
Anna PETTINI/Stefano NARDELLI
Progressive Taxation, Quality, and Redistribution in Kind

ECO No. 95/29
Ákos VALENTINYI
Rules of Thumb and Local Interaction *

ECO No. 95/30
Robert WALDMANN
Democracy, Demography and Growth

ECO No. 95/31
Alessandra PELLONI
Nominal Rigidities and Increasing Returns

ECO No. 95/32
Alessandra PELLONI/Robert WALDMANN
Indeterminacy and Welfare Increasing Taxes in a Growth Model with Elastic Labour Supply

ECO No. 95/33
Jeroen HINLOOPEN/Stephen MARTIN
Comment on Estimation and Interpretation of Empirical Studies in Industrial Economics

ECO No. 95/34
M.J. ARTIS/W. ZHANG
International Business Cycles and the ERM: Is there a European Business Cycle?

ECO No. 95/35
Louis PHLIPS
On the Detection of Collusion and Predation

ECO No. 95/36
Paolo GUARDA/Mark SALMON
On the Detection of Nonlinearity in Foreign Exchange Data

*out of print

ECO No. 95/37
Chiara MONFARDINI
Simulation-Based Encompassing for Non-Nested Models: A Monte Carlo Study of Alternative Simulated Cox Test Statistics

ECO No. 95/38
Tito BOERI
On the Job Search and Unemployment Duration

ECO No. 95/39
Massimiliano MARCELLINO
Temporal Aggregation of a VARIMAX Process

ECO No. 95/40
Massimiliano MARCELLINO
Some Consequences of Temporal Aggregation of a VARIMA Process

ECO No. 95/41
Giovanni NERO
Spatial Multiproduct Duopoly Pricing

ECO No. 95/42
Giovanni NERO
Spatial Multiproduct Pricing: Empirical Evidence on Intra-European Duopoly Airline Markets

ECO No. 95/43
Robert WALDMANN
Rational Stubbornness?

ECO No. 95/44
Tilman EHRBECK/Robert WALDMANN
Is Honesty Always the Best Policy?

ECO No. 95/45
Giampiero M. GALLO/Barbara PACINI
Time-varying/Sign-switching Risk Perception on Foreign Exchange Markets

ECO No. 95/46
Víctor GÓMEZ/Agustín MARAVALL
Programs TRAMO and SEATS
Update: December 1995

***

ECO No. 96/1
Ana Rute CARDOSO
Earnings Inequality in Portugal: High and Rising?

ECO No. 96/2
Ana Rute CARDOSO
Workers or Employers: Who is Shaping Wage Inequality?

ECO No. 96/3
David F. HENDRY/Grayham E. MIZON
The Influence of A.W.H. Phillips on Econometrics

ECO No. 96/4
Andrzej BANIAK
The Multimarket Labour-Managed Firm and the Effects of Devaluation

ECO No. 96/5
Luca ANDERLINI/Hamid SABOURIAN
The Evolution of Algorithmic Learning: A Global Stability Result

ECO No. 96/6
James DOW
Arbitrage, Hedging, and Financial Innovation

ECO No. 96/7
Marion KOHLER
Coalitions in International Monetary Policy Games

ECO No. 96/8
John MICKLEWRIGHT/ Gyula NAGY
A Follow-Up Survey of Unemployment Insurance Exhausters in Hungary

ECO No. 96/9
Alastair McAULEY/John MICKLEWRIGHT/Aline COUDOUEL
Transfers and Exchange Between Households in Central Asia

ECO No. 96/10
Christian BELZIL/Xuelin ZHANG
Young Children and the Search Costs of Unemployed Females

ECO No. 96/11
Christian BELZIL
Contiguous Duration Dependence and Nonstationarity in Job Search: Some Reduced-Form Estimates

*out of print

ECO No. 96/12
Ramon MARIMON
Learning from Learning in Economics

ECO No. 96/13
Luisa ZANFORLIN
Technological Diffusion, Learning and Economic Performance: An Empirical Investigation on an Extended Set of Countries

ECO No. 96/14
Humberto LÓPEZ/Eva ORTEGA/Angel UBIDE
Explaining the Dynamics of Spanish Unemployment

ECO No. 96/15
Spyros VASSILAKIS
Accelerating New Product Development by Overcoming Complexity Constraints

ECO No. 96/16
Andrew LEWIS
On Technological Differences in Oligopolistic Industries

ECO No. 96/17
Christian BELZIL
Employment Reallocation, Wages and the Allocation of Workers Between Expanding and Declining Firms

ECO No. 96/18
Christian BELZIL/Xuelin ZHANG
Unemployment, Search and the Gender Wage Gap: A Structural Model

ECO No. 96/19
Christian BELZIL
The Dynamics of Female Time Allocation upon a First Birth

ECO No. 96/20
Hans-Theo NORMANN
Endogenous Timing in a Duopoly Model with Incomplete Information

ECO No. 96/21
Ramon MARIMON/Fabrizio ZILIBOTTI
'Actual' Versus 'Virtual' Employment in Europe: Is Spain Different?

ECO No. 96/22
Chiara MONFARDINI
Estimating Stochastic Volatility Models Through Indirect Inference

*out of print