Economics Department

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The Optimal Sequence of Privatization in Transitional Economies

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Abstract

The paper presents a simple model of a transitional economy with many state-owned enterprises (SOE) whose economic value is initially unknown. The lack of the information about value of SOE's impedes privatization by sales. The government decides on the optimal sequence of privatization of SOE's according to the following two criteria: (1) the initial privatization of some SOE's should provide the maximal informational gain; (2) an optimal privatization program should maximize the government's revenues from the privatization of all SOE's. The main results of the paper are two-fold: first, the optimal sequence of privatization is formally related to the structure of industrial relations within the economy, second, the paper characterizes computational methods that can solve the government's problem. It is shown that the capability of the government to define the optimal sequence of privatization is limited by the complexity of the problem and the efficiency of a solution method. The paper develops

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problem-solving techniques that help to overcome the complexity constraints. These techniques are characterized in terms of the efficient structure of an administrative organization responsible for privatization.

1. Introduction

The issue of privatization in transitional economies of Eastern Europe inevitably gives rise to three principal questions about the necessity, speed and sequencing of the privatization of state-owned enterprises. The main body of the literature on transition has concentrated so far on the first two questions. This might be the case because the last question, about the optimal sequencing of privatization, is meaningful only if privatization is proved to be necessary for successful transition and if its progress is gradual enough to make its sequencing non-trivial. However, recent advances in research about transition confirm the importance of the optimal sequencing of privatization.

It is no exaggeration to say that there exists a universal agreement about the necessity of privatization in transitional economies. The second question about the speed of privatization is more controversial. One of the most disputed points about transition is whether privatization should be a gradual process (exemplary for this point of view is Roland (1994)) or whether it should be accomplished swiftly (as argued by Frydman et al. (1994)). However, available evidence suggests that privatization in Eastern Europe proceeds slowly enough to make the question about the sequencing of privatization relevant. This paper thus takes as a given that private ownership is the long-run goal but that the divesture of the government's assets is not instantaneous, and will concentrate on the optimal sequencing of privatization. More precisely, I will look at the optimal sequencing of privatization in the context of the optimal plan of reforms designed by the government.

I am following here those researchers who view a coherent plan of transition as an important factor for the success of reforms. Transitional economies of Eastern Europe are quite complicated, heavily distorted systems that cannot be left to themselves to develop some sort of market capitalism from scratch. In the words of Frydman et al. (1993 p.76): "Paradoxically, the most important aspects of the transition to a spontaneously functioning market economy cannot be initiated by the market forces themselves. Indeed, the only force powerful enough to set the market forces in motion is the very state which is supposed to remove itself from the picture". How can the government proceed with the design of privatization?

There exists a growing body of recent economic literature that indicates a number of issues that the government should take into account in making a privatization decision. Glaeser et al. (1994) found that privatization of downstream firms should proceed before the privatization of upstream firms because downstream firms are subject to greater demand shocks. Their argument rests on the conjecture that private firms adapt to the shock better than state-owned firms; if the government can privatize only a limited number of firms at the beginning of transition process, it should privatize downstream firms first.

Roland (1994) looks at the political issues that determine the optimal sequence of privatization. He argues that any privatization plan designed by the government should be politically feasible, i.e. it should have enough support among voters. In order to build up a constituency of reforms privatization should start with the firms that are expected to be most profitable in private hands. First of all, privatization of such firms will not have any negative effect on employment, hence the government will not risk any political backlash at the beginning of the reforms process. Second, the government can raise revenues for the budget through the sale of potentially profitable firms. In general then, the initial success of reforms builds up support for further reforms.

However useful and interesting, this line of research largely ignores the fact that each individual firm operates in a complex framework of industrial relations within the economy; the question of the privatization of a given firm is considered within a partial equilibrium framework. As a consequence, the complex nature of the government's problem –

the designing of privatization reforms – is omitted from mathematical models. This paper is intended to bridge this gap.

I adopt a simple model of a transitional economy that allows for the existence of a large number of firms which are related through a complex network of technological links. The objective of the government – defining the optimal sequencing of privatization – is formalized as a constraint satisfaction optimization problem. Such a setup allows us to specify a mapping from the structure of industrial relations in the economy to the optimal sequence of privatization. However, finding a solution of the problem is a complex task. Because of the size of the problem none of the existing algorithms guarantees finding a solution in a reasonable amount of time. Overcoming complexity constraints motivates the development of problem-solving techniques mainly based on the decomposition of the problem. These problem-solving techniques can be related to the organization of the government's administration responsible for the development of a privatization program. It appears that the decentralized structure of the government's administration is optimal from the point of view of algorithmic efficiency.

The following section of the paper presents several stylized facts about transitional economies in Eastern Europe. The third section formalizes the argument and specifies the government's problem. Sections 4 and 5 explore solution methods and their complexity. Section 6 relates the solution methods to the structure of an administrative organization. Section 7 provides some discussion and possible extensions. Section 8 concludes.

2. Privatization in Transitional Economies: Some Stylized Facts.

There exists a substantial empirical literature about privatization in transitional economies. There are several case studies (for example, Boycko et al. (1994)) that describe recent privatization programs in several East European countries; other literature (Balcerowicz (1993), Begg at al.

(1992), Rostowski (1993)) provides useful information about the economic environments in these countries[1]. The model that I introduce later in the paper is based on the findings of this literature, so I will briefly summarize the main results that I use.

The first point is that designing a program for the privatization of public assets is a complex decision problem that involves many discretionary decisions about timing and method of privatization of state-owned enterprises. For example, in the Czech Republic there were two waves of voucher privatization of industrial firms, first in 1992-1993, then in 1994-1995. The first wave involved over 1000 companies, the second 861 companies. After the completion of the second wave of voucher privatization, an estimated 2,000 enterprises remain under state ownership while 1,426 are owned by the Czech privatization body – National Property Fund, and according to the Czech government, the final goal is to have only four enterprises under state ownership (EIU Country Report 1995).

In Russia the voucher program of mass privatization proceeded in one round from 1992 and ended formally before the end of 1994, but the privatization continued through conventional methods. A total number of 11,463 enterprises was involved in the mass privatization process at the end of December 1993. (Bornstein (1994)). In the middle of 1995 the Russian government started implementing the program of asset sales through public offering, direct sales and floating shares on the stock exchange.

In Poland, despite the significant progress in macroeconomic stabilization and liberalization, privatization is slower. The first attempts to privatize industrial enterprises through public offering took place in 1990, but since sales were considered largely to be a failure due to the lack of customers' interest, the privatization program was amended in favor of voucher privatization. A new mass privatization program started in 1995 with distribution of vouchers to the population. The program calls for initial privatization of 413 large industrial firms.

[1] I limit myself to the discussion on Poland, the Czech Republic, and Russia – countries that are probably best studied in the empirical economic literature on transition and are typical in terms of economic conditions in the region.

The second point is that the privatization program has to take into account obstacles and constraints that prevent the efficient transfer of the public property to the private sector. This issue is well developed in the literature about privatization (for example, Frydman et al. (1994) and Rostowski (1993)). I will concentrate on one of the most important problems that it identifies – the informational problem.

It is generally well-recognized that in a transitional economy one of the major obstacles to the sale of a public firm to private investor is insufficient information on the firm's economic value (Roland(1994) and Begg et al.(1992)). Economic valuation of the public firm is difficult because of the number of the microeconomic and macroeconomic factors, such as macroeconomic instability, the lack of the history of enterprise performance under market conditions, and enterprise debts. Referring to the later problem – indebtedness of state firms – Begg et al. (1992 p.26) point out that: "It is harder to privatize firms if their balance sheets are encumbered by inherited debt – indeed their accounting net worth may be negative, even if they can be expected to make operating profits".

Enterprise debts have two components: indebtedness to banks (state and private) and interfirm debts. The former problem and its possible solutions are described in Begg et al. (1992) and Rostowski (1993). I will concentrate on the later problem – the lack of information on the firm's profitability due to "interfirm credit", i.e. "a chain of mutual indebtedness among the companies along the production process" (Frydman et al. (1994)).

According to Frydman et al. (1994,p.84): "In this arrangement [interfirm credit] firms extend credit to their customers and become in turn indebted to their suppliers. Many of the firms show paper profits, but in fact a large proportion of their assets consists in indebtedness of other enterprises, which are in turn linked in this way to others etc. As a result, the solvency of any particular link in this chain is related to all the others, and no one really knows which are still viable." And further: "[the public firm] continued production and kept the unpaid debts on the asset side of the ledger, relying on its own ability to obtain supplies in the same fashion. Since these intercompany debts came to constitute a substantial portion of the assets of most state enterprises, they linked the solvency of the enterprise together, and made any evaluation of individual company very difficult (not only because its solvency may have been illusory, but also because the customers might not be able or willing to take its products if they were to be forced to pay for them in cash) (p. 131)".

Why is a state firm willing to extend interfirm credit? According to Rostowski (1993), state firms are ready to accumulate debts only to continue production because there does not exist an effective system of bankruptcy for insolvent public enterprises and because managers of the state firms "will attempt to avoid the conflicts with the workforce over nominal wages" (Rostowski (1993,p.10)). To finance the credit extensions the state firms do not usually pay their suppliers, which are in turn ready to extend credit to them: "...[in the post-communist economy] state enterprises become free to extend inter enterprise credit. If this is followed shortly after by a credit squeeze as a part of a stabilization program, state enterprises may initially be willing to muddle through and avoid adjustment by both taking on inter-enterprise debt and extending inter-enterprise credit without much concern about whether this is financially optimal or not, as long as there is a general belief in the impermanence of the credit squeeze. (Rostowski (1993,p.9))" The overall impression is then that state-owned firms behave as if they were the divisions of a single state-run giant enterprise where a single branch does not bear individual financial responsibility.

The debt problem comprises two aspects: first, firms have a stock of accumulated debt, and second, this stock grows due to the continuing extension of the interfirm credit. The later aspect is fundamental in the sense that even if the stock of the interfirm debt is written off in some way, the problem will reappear again when the new debt accumulates. Hence to find a permanent solution to the interfirm debt problem it is not enough to cancel in some way existing debt; it is instead necessary to find an arrangement that changes the credit extension practice of the firms. This is recognized by Begg et al. (1992) who consider a mix of government regulation, recapitalization of firms and privatization as a solution to the enterprise debt problem.

This brings us to the third point, that the privatization program may be designed to overcome the constraints that limit privatization in the first place. According to Frydman et al. (1994) a privatization plan should be designed so that the government will obtain additional information after the implementation of initial stages of the reform. "[Initial reforms] must reduce the unacceptably high level of uncertainty at the point of departure. This means that the design must contain mechanisms that reveal as much information as possible in the early stages of reform process (Frydman et al. (1994p.78))." Additional information may be generated if the debt problem in the transitional economy is alleviated. The remedy for the debt problem may be the initial property reforms. First, privatization of a state-owned firm signals the commitment of the government not to subsidize this firm and thus limits its incentives to extend credit to other firms. Second, privatization may change as well the behavior of firms that remain state-owned since privatization changes the economic environment where they operate.

Indeed Stiglitz (1993 p.187) writes: "I want to argue that the government does have a marked disadvantage relative to private firms ... based on the inability of the government to make certain commitments, in particular, the commitment ... not to subsidy". If privatization means the commitment not to subsidize a privatized firm, then a privatized firm will not extend credit to its trade partners as it did before the privatization. Moreover, the initial privatization of some firms changes the economic environment in general – the economy cannot be regarded any more as a single giant enterprise owned and financed by the state.

The idea that the behavior of state firms depends not only on their internal structure including the ownership form but also on the economic environment where they operate is well recognized. According to Balcerowicz (1993): "... a change in the [internal] structure of one group of enterprises changes their behavior, and in this way the environment of other enterprises." In particular, privatization of some state enterprises may alter incentives of the firms remaining in the state property. The remaining state firms that have to deal with private firms do not regard themselves any more as a branch of a single state firm without individual financial responsibility. The perception of a private trade partner as an alien element in the established system of cross-subsidizations will make a state firm reluctant to extend credit to a private firm. Additionally, a private firm itself may be reluctant to accumulate unpaid debts because it would risk bankruptcy. The government which signalled its commitment not to subsidize a firm by privatizing it may be expected to be tougher in pursuing its liquidation than in the case of a state firm.

Initial privatization then should be aimed at isolating a state firm from trade with other state firms. In this way the privatization of some firms may help to overcome the problem of the interfirm cross-subsidization, and more generally the problem of deficient information in the transitional economy. The necessary initial mass of privatization that creates a "market environment" is achieved by voucher or "mass" privatization – a virtually free distribution of the public assets to the private sector that does not require information on the economic value of the distributed property. However, mass privatization has certain drawbacks, for example, the government essentially receives no revenues from privatization which we might well take to be an important consideration for the government. In fact, in most transitional economies the government usually prefers sales to give-away distribution at the later stages of a privatization program.

The above discussion suggests that the government can design a privatization scheme such that privatization in the first period reduces uncertainty about the firms' performance in the second period, thus making privatization in the second period profitable for the government. It is assumed that the government designs an optimal privatization plan according to this concept. It looks like very simple, even trivial exercise since the government's objective is assumed to be simple. However, the government needs to take thousands decisions about the privatization of individual firms in the first or in the second period and these decisions are interdependent – hence the government's problem is in reality a complex problem. It is difficult to say anything more about complexity of the optimal reform design without introducing formal argumentation. Hence I need to formulate a simple framework where the complex nature of the economic transformation design is explicitly recognized.

The mathematical model presented in the next section serves this purpose. It allows, first, the government's privatization strategy to be formalized and, second, to show that the government's ability to design an optimal reform plan is bounded by the complexity of the problem.

3. The model

Consider a closed transitional economy with n firms, each firm having a technology described by the production possibilities set (a set of all feasible production plans)

$$Y_i = \{(q_i^1, \ldots, q_i^k, \ldots, q_i^{m_i}, -q_1^i, \ldots, -q_j^i, \ldots, -q_{n_i}^i)\} \tag{1}$$

Let Γ be a set of all pairs $\{i,j\}$ such that $\{i,j\} \in \Gamma$ if and only if firm i buys output of j or j buys output of i, i.e. Γ is a set of trade relations between the firms in the economy. It is assumed that Γ is stationary. To make the definition of Γ more straightforward, suppose that under some additional set of assumptions we have a Leontieff economy with an input-output matrix A. Then $\{i,j\} \in \Gamma$ iff the i,j-component of the Leontieff input-output matrix $a_{ij} \neq 0$, or $a_{ji} \neq 0$.

Summing up (and inevitably simplifying) the discussion on inter-firm credit and privatization in the transitional economy in the previous section of the paper, I assume that there are two types of firms in the economy: state-owned and private. A state-owned firm is assumed to produce the output so far as it receives enough inputs, and supplies the output to other state firms irrespectively of whether the customer pays for the supplied commodities. However, it does not extend credit to a private firm. A private firm always requires its customers to pay for the supplied commodity and it always has to honor its financial obligations to the suppliers of inputs, i.e. it pays the bills. Formally,

(3.1) Assumption Firms i and j that trade with each other (that is, $i, j \in \Gamma$) create debt arrears against each other if and only if both firms are state- owned.

If a state firm i extends interfirm credit or does not pay for inputs, then the firm's i net flow of funds need not to be equal to the economic profits Π_i in the following accountancy identity:

$$\sum_{j=1}^{n_i} q_j^i p_j + \Pi_i = \sum_{k=1}^{m_i} q_i^k p_i \tag{2}$$

This is a situation when a state firm faces the "soft budget constraint" due to the extension of interfirm credit. Assumption 3.1 implies that the net flow of funds equals the economic profits Π_i if and only if one of the two conditions holds:

(a) firm i is a private firm;

(b) all firms that trade with firm i are private.

The difference between the net flows of funds and the profits of a state firm allows us to specify how the interfirm credit problem translates into the informational problem in the transitional economy. First, I make a distinction between public and private information in the model:

(3.2) Assumption A set Γ of trade relations in the economy and the net flows of funds for each firm in the economy are public information. A firm's technology and output quantities are the firm's private information.

Assumptions (3.1) and (3.2) imply that although the net flows of funds of a state firm are public information, they do not correspond to its economic profits because a state firm can extend interfirm credit.

Following the discussion in the previous part of the paper I assume that the government in the transitional economy wants to privatize state-owned firms and receive maximal revenues from privatization. The informational problem impedes, however, initial privatization through sales. When the economic value of a state firm is unknown, the government cannot privatize the firm by selling it to private investors. The only possibility for privatizing such a firm is by giving it away to private owners through the mass privatization scheme. Such a method of divesture entails the loss of the government revenues, hence the government wants to limit its use. On the other hand, initial privatization reveals information on the profitability of the remaining state firms. Hence, the initial give-away privatization clears the way to privatization through sales in the second period.

Without loss of generality, consider the simplest case described by the following assumption:

(3.3) Assumption (1) All firms in the economy are initially public.
(2) There are no restrictions on privatization of any firm in the economy.
(3) The government expects to incur a loss f^i of the budget revenues from the give-away privatization of the firm i.

Under these conditions the government will want to choose some firms for initial privatization, such that their privatization reveals all the information about the profitability of the remaining public firms and the loss of budget revenues is minimized.

I start formalizing the government's problem by defining a set of variables $X = \{x_i : i = 1 \dots n\}$ which describes an ownership structure in the economy. Variable x_i is equal to 1 if the government selects firm i for the give-away privatization program in the first period and $x_i = 0$ otherwise. A privatization plan l is defined then as an assignment of values – 1 for "private" and 0 for "state" – to each variable x_i in X.

Any privatization plan l should satisfy the following constraints: for any $\{i, j\} \in \Gamma$, firm i should be private, or firm j should be private, or both firms i and j should be private. The optimal privatization plan denoted by l^* should satisfy the constraints above and also minimize the total loss of the budget revenues by the government.

Consider the following example that introduces the definition of the government's objective:

(3.4) Example
The following economy has 3 firms and a set of trade relations $\Gamma = \{\{1,2\}, \{2,3\}, \{1,3\}\}$. It is further assumed that the give-away privatization of each firm in the first period leads to the equal loss of budget revenues normalized to 1, i.e. $f^i(1) = 1$ and $f^i(0) = 0$for $i = 1 \ldots 3$.

Define the set of variables $X = \{x_1, x_2, x_3\}$ so that $x_i = 1$ if firm i is to be privatized and $x_i = 0$ otherwise, $i = 1 \ldots 3$. A solution of the government's problem l^* is an assignment of values 0 or 1 to the variables $x_1 \ldots x_3$ such that this assignment minimizes $\sum_{i=1}^{3} x_i$ and satisfies the set of constraints of the following form: for any $\{i,j\} \in \Gamma$, $x_i = 1$ and $x_j = 0$, or $x_j = 1$ and $x_i = 0$ (one of firms i and j should be private), or $x_i = 1$ and $x_j = 1$ (both firms i and j should be private).

The example above essentially defines the government's objective – the optimal sequencing of privatization — in terms of a constraint satisfaction optimization problem (CSOP).

A CSOP is completely defined by specifying a set of variables together with the variable domains, a set of constraints and an objective function. Let X be a finite set of variables x_i ordered by an arbitrary total order. Let $d(x_i)$ (the domain of x_i) denote a set of values that the variable x_i can take.

The set of all possible assignments of values to the variables in the ordered set $X = \{x_i : i = 1 \ldots n\}$ is defined then as $\prod_{x_i \in X} d(x_i)$. Let a tuple u be one element of this set – an assignment of values to the variables in X.

A constraint c is defined then by its scope $s(c) \subseteq X$, i.e. the variables that it involves, and its extent $e(c) \subseteq \prod_{x_i \in s(c)} d(x_i)$. A tuple u is said to satisfy a constraint c if $RS(u, s(c)) \in e(c)$, where $RS(u, s(c))$ denotes the restriction of u to a subset $s(c)$ of X. For example, if $u = \{1,0,1\}$ is an assignment of values to the ordered set of variables $X = \{x_1, x_2, x_3\}$ and $s(c) = \{x_1, x_2\}$ is a subset of X, then $RS(u, s(c)) = \{1,0\}$. The assignment of values $\{1,0,1\}$ to the ordered set $\{x_1, x_2, x_3\}$

satisfies constraint c defined by its scope $s(c) = \{x_1, x_2\}$ and its extent $e(c) = \{(0,1),(1,0),(1,1)\}$, while the assignment $\{0,0,1\}$ does not satisfy this constraint.

Finally, an objective function maps every tuple u that satisfies all constraints in a given problem (a consistent assignment) to some measure $f(u)$ (f-value of u).

Returning to the privatization problem denote by L a set of all privatization plans l that satisfy the constraints of the following form: for any $\{i,j\} \in \Gamma$ (Γ is a set of trade relations in the economy) $x_i = 1$, $x_j = 0$, or $x_i = 0$, $x_j = 1$, or $x_i = 1$ and $x_j = 1$. The same set L can be alternatively defined as a set of tuples l, such that each l satisfies the set of binary constraints $C = \{c\}$ with scopes $s(c) = (x_i, x_j)$, where $\{i,j\} \in \Gamma$ and extents $e(c) = \{(0,1),(1,0),(1,1)\}$.

In the context of the government's objective, which is the minimization of the loss of budget revenues from privatization, a measure of optimality of a privatization plan $l \in L$ is clearly defined as the sum of f^i's for each firm earmarked for give-away privatization by this privatization plan; that is $f(l) = \sum_{x_i \in X} f^i(RS(l, x_i))$, where $f^i(1) = f^i$ and $f^i(0) = 0$. The optimal solution to the government's privatization problem is the privatization plan $l^* \in L$ with the minimal f-value.

Summing up, the government's objective is defined in terms of a CSOP as the following:

(3.5) Definition The government's privatization problem ($CSOP_P$) is defined as a constraint satisfaction optimization problem (X, C, f). X is a set of variables that indicate the form of ownership for each firm in the economy; each variable has a binary domain $d(x_i) = \{0,1\}$. C is a set of binary constraints $c_{i,j}$ with scopes $s(c_{i,j}) = (x_i, x_j)$, where $\{i,j\} \in \Gamma$ and extents $e(c_{i,j}) = \{(0,1),(1,0),(1,1)\}$. Then L is a set of assignments of values to the variables in X such that each $l \in L$ satisfies all the constraints in C. The objective function $f : L \rightarrow R^+$ is defined by $f(l) = \sum_{x_i \in X} f^i(RS(l, x))$, where $f^i(1) = f^i = const.$ and $f^i(0) = 0$. A $CSOP_P$ solution is defined by $l^* = \text{argmin}_{l \in L} f(l)$.

A solution of the government's problem defined by (3.5) specifies the optimal sequencing of privatization; it specifies which firms should be privatized immediately, in the first period, and which firms should be sold in the second period. The solution is defined by the set of trade relations in the economy and by the set of the expected losses of revenues because of give-away privatization in the first period.

However, finding a solution of the problem is a non-trivial task because of the computational time complexity of the algorithms that solve it. The following section describes the standard solution methods and identifies the complexity constraint faced by the government.

4. Solution Methods and Their Complexity

In example (3.4) it is easy to find a set of consistent assignments L by the exhaustive enumeration of all the possible assignments of values to the variables and checking which assignments satisfy the set of constraints C. After doing this, a CSOP solution – a consistent assignment with the minimal value of the objective function –can be found by direct comparison of f- values. Such a solution method is called the "generate-and-test" algorithm.

It is easy to see that the "generate-and-test" algorithm needs to process 2^n ($n = |X|$) candidate solutions before it finds a $CSOP_P$ solution. In the terminology of computer science such algorithm is said to have exponential time complexity[2]. Clearly, for small problems time complexity of the algorithm does not matter a lot. However, the government's optimal sequencing of privatization problem includes thousands

[2]More precisely, here and further down I use the term "time complexity" to refer to the worst case computational time of an algorithm. Computational time is measured in some simple computer operations, e.g. an operation that compares two sets of numbers and returns "true" if they are identical and "false" otherwise. Such a simple operation could be in turn easily decomposed into some number of elementary computer operations. Worst case time complexity is an upper bound of the computational time of an algorithm. Worst case time complexity is a sufficiently informative and easy to calculate measure; it is widely used in the computer science.

of variables as was suggested in the second section of the paper. Thus it is intuitively clear that the government faces a much more difficult decision-making problem than those considered in example (3.4).

The usual method for measuring the complexity of a problem is to look at the algorithm that solves this problem and see whether it finds a solution in a reasonable amount of time (Gibbons (1985)). If there exists an algorithm that solves a problem in the number of computational steps polynomial in the problem size (polynomial time algorithm or efficient algorithm) then this problem is tractable in general. A difference between polynomial and exponential time algorithm is illustrated by the following table (Gibbons (1985)), which lists the numbers of computational steps that algorithms with different time complexities need to perform to solve problems of the given size:

Time Complexity	Problem Size			
	2	8	128	1024
n	2	2^3	2^7	2^{10}
n^2	2^2	2^6	2^{14}	2^{20}
2^n	2^2	2^8	2^{128}	2^{1024}

$$\begin{aligned}\text{Note that } 2^{10} \text{ steps/second} &\approx 0.9 \times 2^{22} \text{ steps/hour}\\ &\approx 1.3 \times 2^{26} \text{ steps/day}\\ &\approx 0.9 \times 2^{35} \text{ steps/year}\\ &\approx 0.7 \times 2^{42} \text{ steps/century.}\end{aligned}$$

That is, the "generate-and-test" algorithm would solve, for example, a $CSOP_P$ with 1,000 variables for at least 2^{958} centuries if it can process 1,024 computational steps per second. Moreover, an exponential time algorithm may be so inefficient that, even with computational speed vastly increased, it would not be possible to solve significantly bigger problems. For example, if N_p is the size of the largest problem solvable by an algorithm with polynomial time complexity n^2, then using 2^{10} higher computational speed will allow to solve the problem of the maximal size $32N_p$. An analogous result for an algorithm with exponential time complexity that solves the problem of the maximal size N_e would be only $N_e + 10$.

However, the performance of the "generate-and-test" algorithm may be improved if the generation of an assignment of values is combined with the verification of constraints. This is the basic principle of a standard algorithm used for solving a CSOP – the pure backtracking (PB) algorithm.

The key concept needed to describe the PB is the notion of a search space. Informally, a search space is a concise way of characterizing all the assignments of values to the variables in a problem. A search space generated by an ordered set of variables $X = \{x_1, x_2, \ldots, x_n\}$ is a tree-structured graph. The children of the root of the graph are elements of $d(x_1)$ called first-level nodes. The children of any i-th level node $(1 \leq i \leq n-1)$ are elements of $d(x_{i+1})$. Level n nodes are leaves, i.e. they have no children. A path from the root to a leaf is a complete assignment of values to all the variables. A path from the root to an internal node at level i is a partial assignment up to the level i.

Note that all possible paths from the root to the leaves of a search space are generated by the "generate-and-test" procedure first and then tested for constraint violations. In contrast to "the generate-and-test" the PB does not generate all variable assignments first; instead, it combines finding of a complete assignment with the testing for a violation of constraints. Thus assignments that violate the constraints may be ruled out at the early stages of the search for a solution.

The PB algorithm starts from x_1 and traverses the search space depth first creating a consistent partial assignment of values to the variables. The PB checks relevant constraints each time when it moves from level i to level $i+1$ of the search space. The PB algorithm backtracks to the previous level when it cannot extend the assignment to include the next variable because of the violation of the constraint.

When the PB algorithm finds a complete consistent assignment of values it also backtracks and starts searching for another assignment. The PB algorithm keeps track of the visited nodes of the search space by creating a list of visited nodes for each level of the search space. When the PB backtracks from level i to level $i-1$ of the search space it clears all the lists from the level i to the last level. The PB algorithms stops when the list of visited nodes for the level 1 contains all the nodes at this level. The following example illustrates how the PB algorithm works[3].

(4.1) Example

Consider the problem defined in (3.4). The following table illustrates step-by-step how the PB algorithm finds a set of all consistent assignments[4].

step	assignment	l		$s(c)$
1	1	*	*	
2	1	1	*	(x_1, x_2)
3	1	1	1	(x_1, x_3), (x_2, x_3)
4	1	1	0	(x_1, x_3), (x_2, x_3)
5	1	0	*	(x_1, x_2)
6	1	0	1	(x_1, x_3),(x_2, x_3)
7	1	0	*	(x_1, x_3),(x_2, x_3)
8	0	*	*	
9	0	1	*	(x_1, x_2)
10	0	1	1	(x_1, x_3),(x_2, x_3)

The set of assignments that satisfy all the constraints is $\{(1,1,1), (1,1,0), (1,0,1), (0,1,1)\}$; the subset of solutions that satisfy the government's optimality criteria is $\{(1,1,0), (1,0,1), (1,0,1)\}$, which is the solution set of the $CSOP_P$.

In general, the PB algorithm may need to backtrack an exponential number of times before creating a complete consistent assignment[5]. However, in example (4.1) above the PB could always extend a consistent assignment of values to include the next variable. In other words the PB traversed the search space without backtracking. This is not a coincidence, backtrack-free performance of the PB hinges upon a special property of the $CSOP_P$. Since $e(c) = \{(0,1), (1,0), (1,1)\}, \forall c \in C$, the assignment of value 1 to any variable $x_i \in X$ is compatible with any

[3]For a more formal definition of PB see Tsang(1993).

[4]* is an empty assignment.

[5]For an extensive discussion on the sources of backtracking and possible ways to reduce backtracking see Tsang (1993) and Vassilakis (1996).

value assigned to another variable $x_j \in X$. Hence, any partial consistent assignment of values can be extended to include an additional variable labelled by 1 without violating any constraints.

It is easy to see then that the BP algorithm creates a consistent assignment of values to all variables x (assignment l) in computational time which is linear in the size of the problem $|X|$. In other words, the BP is an efficient method to find a single consistent assignment l. The source of inefficiency of the PB is in the second type of backtracking – backtracking after finding a complete consistent assignment of values. The number of such assignments and hence the number of times that the PB algorithm backtracks is bounded by an exponent.

(4.2) Proposition The worst case time complexity of finding a solution l^* of a $CSOP_P$ by the BP algorithm is $nr2^n$, where $n = |X|$ is the number of variables and $r = |C|$ is the number of constraints.
Proof: Finding an assignment l requires checking at most r constraints at each level of the search space (because the PB does not backtrack in this case), in total nr constraint checks. However, in the worst case the PB needs to generate 2^n assignments before it finds an optimal one – a $CSOP_P$ solution l^*. Q.E.D.

In fact, none of the existent computer algorithms can find a $CSOP_P$ solution in polynomial time. Existence of a polynomial time (efficient) algorithm should not be taken for granted for every problem. There exists a broad class of problems – the NP-complete problems – which are widely believed to be not tractable in general because no efficient algorithm is known for any member of this class. Moreover, each NP-complete problem can be transformed into any other problem of the same class, hence if an efficient algorithm was known for any of these problems, then such an algorithm would exist for any one of the others.

It could be proved that the government's problem $CSOP_P$ is an NP-complete problem[6]. It means that the government's ability to make

[6]In mathematical terms $CSOP_P$ is technically identical to the "minimal vertex cover" problem of the graph theory which is proved to be NP-complete (Gibbons

the optimal decision is restricted by the complexity of the problem, even though the government knows the solution method. For the large problem the complexity constraint is binding; the government is unable to set up an optimal privatization plan.

However, the government can set up a privatization plan l that does not necessarily satisfy the optimality criteria relatively easily; the complexity of finding a solution is not a problem for the government in this case[7]. However, such a privatization program would be biased towards give-away privatization and would lead to the excessive loss of government's revenues. Privatization in the first period would reveal all the information about the value of the firms but only few or none of the firms would remain for privatization through sales in the second period.

This result seems to be quite intuitive because the time and effort that the government can allocate to solve a given problem is limited in general and especially limited in the transitional economy, where the number of pressing economic problems is large. Hence it is reasonable to say that even if there exists a solution method to the government's problem it may not be useful in practical terms because the government cannot simultaneously consider all the different relevant aspects of the problem.

Summing up, existing solution methods do not guarantee that a large problem, such as the optimal sequencing of privatization problem, can be solved in practical terms because the complexity of the algorithm may be prohibitive. In terms of government decision-making, it means that in general the government cannot take for granted that it is able to find the optimal plan for privatization by establishing a set of criteria that such plan should satisfy and setting up an objective. The multiplicity of the relevant constraints implies that it might be prohibitively difficult for

(1985)). Moreover, a CSOP is an NP-complete problem in general (Tsang (1993)).

[7]On the other hand, this result is rather unstable with respect to the set of constraints that the government's privatization plan needs to satisfy. If, for example, $e(c)$ were modified for some $c \in C$ to become some subset of $\{(0,1),(1,0),(1,1)\}$, then finding an assignment that satisfies all the constraints become an NP-complete problem (Tsang (1993)). I will return to this issue later, in Section 7.

the government to find a privatization plan that satisfies all the criteria and optimizes the government's objective – the complexity constraint is binding. Overcoming the complexity constraint requires the organization of the solution procedure along the lines that I will specify next.

5. Overcoming Complexity Constraints

The complexity constraint is binding for a large problem. However, a relatively small problem can be solved even by an inefficient algorithm with exponential time complexity. This consideration points towards problem decomposition which may help to overcome the complexity constraint. The idea of dividing a large problem into several smaller problem is straightforward but its realization may be technically involved in a general case. Unless different parts of the original problem are completely independent from each other there will be links between the sub-problems.

In general, we should be able to construct a solution of the whole problem from the sub-problem solutions. This requirement leads to the following two considerations. First, sub-problem solutions should be compatible with each other, second, each individual solution should be expandable to the solution of the whole problem. The compatibility of the solutions gives rise to constraints between the sub-problems of the following form: if two sub-problems share some common variables then these variables should be assigned identical values in each of the two sub-problems. These constraints insure that the sets of sub-problem solutions are compatible. For a sub-problem solution to be also expandable it is necessary that it does not violate any constraints in the rest of the problem. Hence every sub-problem should contain all the constraints whose scopes include the variables in the sub-problem.

The above principals that insure that the problem solution can be obtained from sub-problem solutions are realized in a hinge decomposition of the problem (Gyssens et al. (1994)). A hinge decomposition explores the structure of the problem; the problem constraints that define the structure of the problem are divided into sub-sets (hinges) to form a number of sub-problems. To insure compatibility of sub-problem solutions a hinge decomposition defines a set of constraints between the sub-problems – a set of "compatibility" constraints.

A hinge decomposition has an additional desirable property which is the special regular structure of the set of "compatibility" constraints. This property allows to find a problem solution relatively fast after all the sub-problems are solved. Thus the overall tractability of the problem will depend rather on the size of the largest sub-problem than on the size of the problem itself.

The hinge-decomposition techniques explore the problem structure. In the context of the government's problem, we do not usually expect that a firm has trade relations with every other firm in the economy. Because of physical distances or product specialization, firms form subgroups, such that a firm trades intensively with other firms within a subgroup, but relatively little outside. For example, the local food processing industry, retail and wholesale trade firms, services and utilities may form a closely related subgroup. Another intensively trading subgroup may, for example, be firms in the aerospace industry, which probably have little trade with food-processing firms. Such closely related groups of firms are best suited to define sub- problems with the minimal interactions with the others.

Every CSOP has an underlying structure called a CSOP graph defined by the scopes of the problem's constraints. A graph of a $CSOP = (X, C, f)$ is a graph $G = (X, S)$ with the set of nodes X and the set of edges $S = \{s(c) : c \in C\}$. Obviously the graph of a $CSOP_P$ has a set of edges which is identical to the set of trade relations in the economy.

A closely related group of variables and constraints defined for these variables corresponds to a connected sub-graph with the set of edges $S_i \subseteq S$: for any two edges s_0 and s_m in S_i there exists a sequence of edges s_0, s_1,..,s_{m-1}, s_m contained in S_i. Additionally, $S_i \subseteq S - S_j$ is called connected with respect to $S_j \subseteq S$ if for any two edges s_0 and s_m in S_i there exists a sequence of edges s_0, s_1,..,s_{m-1},s_m in S_i, such that $s_k \cap s_{k+1} \notin \bigcup S_j$ for $i = k, ..., m-1$. The maximal connected subsets of $S - S_j$ are called connected components with respect to S_j.

A hinge of a graph is a connected subset of edges, such that each connected (with respect to this subset) component of the graph intersects it within one of its edges. Formally:

(5.1) Definition [Gyssens et al. (1994)] Let $G = (X, S)$ be a connected graph and let h be either S or connected proper subset of S, containing at least two edges. Let $H_1, \ldots, H_m$ be connected components of $S - h$ with respect to h. Then h is called a hinge if for every $i = 1, \ldots, m$ there exists an edge $s_i \in h$ such that $(\bigcup H_i) \cap (\bigcup h) \subseteq s_i$. A hinge that does not contain other hinges is called a minimal hinge.

The crucial distinction between a hinge and any other subset of edges is that the variables shared by h and a connected component H_i belong to the separating edge s_i in h. Hence "interaction" links between hinges are limited in this sense.

Since a hinge is just a subset of constraints of the problem it is possible to define a set of consistent assignments of values to the variables contained in a hinge h_i. This set of assignments is denoted by L_i. Each set L_i can be found by the *PB* algorithm providing that the size of the hinge is small enough so that the problem is tractable even for an inefficient algorithm. The task of finding a set of consistent assignments for a hinge defines a sub-problem whose solution is the set L_i.

To insure compatibility of sub-problem solution sets define a set of constraints of the following form: variables x in a constraint shared by two different hinges should be assigned the same values in both hinges. A hinge decomposition is defined thus by the set of hinges and the set of such "compatibility" constraints. A hinge decomposition is characterized by its graph – a hinge tree. A set of nodes of a hinge tree corresponds to the set of hinges and a set of edges corresponds to the set of "compatibility" constraints. Gyssens et. al. (1994) present an algorithm that computes a tree-structured minimal hinge decomposition of a graph $G = (X, S)$ (hinge tree) in at most $|X||S|^2 = nr^2$ computational time.

A hinge decomposition is defined as the following:

(5.2) Definition [Gyssens et al. (1994)] Let $G = (X, S)$ be a graph. A hinge decomposition of G is a tree $HT = (X_h, A)$ with a set of nodes $X_h = \{h\}$ and a set of labeled arks $A = \{(h_i, h_j; label(h_i, h_j)\}$, such that:
(1) tree nodes $h \in X_h$ are minimal hinges of G,
(2) each edge in S is contained in at least one tree node,
(3) two adjacent tree nodes (h_i and h_j) share precisely one edge of S which is also the label of their connecting arc ($label(h_i, h_j)$), and their shared vertices are precisely the members of this edge,
(4) the vertices of X shared by two tree nodes are entirely contained within each node on their connecting path.

The following example provides an illustration of a hinge-tree decomposition of a CSOP.

(5.3) Example
Let $CSOP = (X, C, f)$ be given by $X = \{x_1, \dots, x_5\}$; $d(x_i) = \{0, 1\}$, for $i = 1, \dots, 5$; $C = \{c_1, \dots c_6\}$, where $s(c_1) \equiv s_1 = (x_1, x_2)$, $s_2 = (x_2, x_3)$, $s_3 = (x_1, x_3)$, $s_4 = (x_3, x_4)$, $s_5 = (x_3, x_5)$, $s_6 = (x_4, x_5)$, and $e(c_j) = \{(0,1), (1,0), (1,1)\}$ for $j = 1, \dots, 6$; $f(l) = \sum_{i=1}^{5} RS(l, x_i)$ for all $l \in L$ where L is a set of assignments that satisfy all the constraints in C.

A graph of the problem is defined as $G = (X, S)$, where $S = \{s_i : i = 1 \dots 5\}$. A hinge tree $HT = (X_h, A)$ decomposition of the graph $G = (X, S)$ is defined then as the following: $X_h = \{h_1, h_2, h_3\}$, where $h_1 = \{s_1, s_2, s_3\}$, $h_2 = \{s_3, s_5\}$, $h_3 = \{s_4, s_5, s_6\}$, and $A = \{(h_1, h_2; s_3)$, $(h_2, h_3; s_5)\}$.

Given its hinge decomposition any $CSOP$ can be transformed into a dual tree-structured problem – $CSOP'$. A dual problem is a constraint satisfaction optimization problem with the set of variables defined for minimal hinges and a set of constraints defined for all arcs that connect the hinges. Each hinge i defines a variable h_i with the set of values L_i, i.e. the domain of h_i $d(h_i) = L_i$. Each pair of adjacent hinges defines a "compatibility" constraint. A set of assignments of values to all hinges that satisfy a set of "compatibility" constraints is identical to the set of assignments L in the original problem - $CSOP$ because we do not add, delete, or modify any constraint or variable in the original problem transforming it into the dual problem. Hence the objective function $f : L \rightarrow R^+$ remains the same for the dual problem. Summing up:

(5.4) Definition Let $HT = (X_h, A)$ be a decomposition of $G = (X, S)$ of a $CSOP = (X, C, f)$. A dual tree-structured problem is defined as $CSOP' = (X_h, C', f)$, where X_h is a set of minimal hinges h_i of the hinge tree; the domain of a variable $h_i \in X_h$ is a set of solutions of the sub-problem defined for the minimal hinge h_i, i.e. $d(h_i) = L_i$. C' is a set of binary constraints c' defined by the scope $s(c') \subseteq X_h$, and extent $e(c') = \{(a, b) : a \in d'(h_i), b \in d(h_j), RS(a, x) = RS(b, x)$ for each $x \in label(h_i, h_j)\}$, i.e. variables x in the shared constraint should be assigned the same values in both adjacent hinges. An objective function $f : L \rightarrow R^+$ is defined as an objective function in the original problem.

Returning to example (5.3) define a dual optimization problem using hinge-tree decomposition.

(5.5) Example (5.3 continued)
For each of the minimal hinges in the hinge-tree decomposition find a set of assignments of values to the variables contained in the hinge that satisfy all the constraints that form the hinge. For example, hinge h_1 is formed by the set of constraints $C_{h1} = \{c_1, c_2, c_3\}$ and it contains the set of variables $X_{h1} = s(c_1) \bigcup s(c_2) \bigcup s(c_3) = \{x_1, x_2, x_3\}$. The set of assignments of values to the set of variables $\{x_1, x_2, x_3\}$ that satisfy the set of constraints $\{c_1, c_2, c_3\}$ is the following:

assignments:	α_1	α_2	α_3	α_4
x_1	1	0	1	1
x_2	1	1	0	1
x_3	1	1	1	0

The set of consistent assignments $\{\alpha_1, \ldots, \alpha_4\}$ can be found either by the PB algorithm or by the "generate-and-test" algorithm. We can define analogously the sets of consistent assignments $\{\beta_1, \ldots, \beta_5\}$ and $\{\gamma_1, \ldots, \gamma_4\}$ for hinges h_2 and h_3 respectively:

assignments:	β_1	β_2	β_3	β_4	β_5
x_1	1	0	0	1	1
x_3	1	1	1	1	0
x_5	1	1	0	0	1

assignments:	γ_1	γ_2	γ_3	γ_4
x_3	1	0	1	1
x_4	1	1	0	1
x_5	1	1	1	0

It is possible to define now a set of variables for the dual problem ($\{h_1, h_2, h_3\}$) with the following domains: $d(h_1) = \{\alpha_1, \ldots, \alpha_4\}$, $d(h_2) = \{\beta_1, \ldots, \beta_5\}$ and $d(h_3) = \{\gamma_1, \ldots, \gamma_4\}$. The set of "compatibility" constraints of the dual problem requires that shared variables x are assigned the same values in both adjacent hinges. For example, hinges h_1 and h_2 share two variables: x_1 and x_3; assignments α_1 and β_1 are compatible because both assign 1 to x_1 and 1 to x_3 while assignments α_1 and β_2 are not compatible because x_1 is assigned 1 by α_1 and 0 by β_2. In other words, there is a constraint c'_1 with the scope (h_1, h_2) such that the pair (α_1, β_1) belongs to the extent of this constraint. The whole set of constraints for the dual problem (C') is defined analogously: $C' = \{c'_1, c'_2\}$, $s(c'_1) = (h_1, h_2)$, $s(c'_2) = (h_2, h_3)$,

$$e(c'_1) = \{(\alpha_1, \beta_1), (\alpha_1, \beta_4), (\alpha_2, \beta_2), (\alpha_2, \beta_3), (\alpha_3, \beta_1), (\alpha_3, \beta_4), (\alpha_4, \beta_5)\}$$

$$e(c'_2) = \{(\beta_1, \gamma_1), (\beta_1, \gamma_3), (\beta_2, \gamma_1), (\beta_2, \gamma_3), (\beta_3, \gamma_4), (\beta_4, \gamma_4), (\beta_5, \gamma_2)\}$$

Finally, the objective function is defined as before: $f(l) = \sum_{i=1}^{5} RS(l, x_i)$.

Summing up, the dual tree-structured problem $CSOP'$ is defined by the triple (X_h, C', f) specified above.

Let $CSOP'_P$ be a dual problem defined for $CSOP_P$. The dual optimization problem can be solved, in principal, by the same method as the original problem; first we apply the PB algorithm to find a set of assignments L and, second, we find an optimal assignment $l^* \in L$ by the direct comparison of all the assignments in L. However, this would not be efficient because we did not use the acyclic structure of the dual problem. Remember that the source of inefficiency of the BP algorithm was identified as being in the exhaustive enumeration of all sub-optimal assignments. The acyclic structure of the dual problem can be used to avoid such an enumeration and thus improve the performance of the PB algorithm in solving the dual problem. The acyclic structure of the set of constrains allows us to apply the principle of dynamic programming to rule out suboptimal assignments before actually finding them.

However, to apply the principle of dynamic programming we should be able to find f-values of partial assignments. The necessary step then is to make sure that the f-value of a problem solution $l \in L$ is equal to the sum of f-values of the sub-problems' solutions, i.e. we need to find a function ψ^i defined on each hinge (variable) of the dual problem such that

$$f(l) = \sum_{h_i \in X_h} \psi^i(h_i) = \sum_{x \in X} f^i(RS(l,x)) \tag{3}$$

We get the necessary result if ψ^i is defined on some subset of variables x contained in a hinge h_i, such that these subsets form a partition of X[8]. More precisely, define

$$\psi^i(h_i) = \sum_{x \in dom_i} f^i(RS(l,x)) \tag{4}$$

where

$$dom_i = \{x : x \in (\bigcup h_i - \bigcup h_j) \text{ and } h_j \text{ is a parent of } h_i\}$$

A set $\{dom_i\}$ can be created by a simple algorithm that compares each variable x in h_i with each variable in the parent node h_j. If the

[8]Note that otherwise the objective function would be defined as

$$f(l) = \sum_{h_i \in X_h} \psi^i(h_i) = \sum_{x \in X} a^i f^i(RS(l,x)),$$

where $a^i \geq 0$

number of variables x contained in the largest minimal hinge of the HT is bounded by m and the number of minimal hinges in the HT is bounded by $|X_h|$, then the time complexity of the algorithm is bounded by $m^2|X_h|$ computational steps.

The next proposition shows that the set of dom_i forms a partition of X.

(5.6) Proposition $\{dom_i : h_i \in X_h\}$ forms a partition of X.
Proof: To complete the proof it is necessary to show that the following two conditions are satisfied:

$$\bigcup_{h_i \in X_h} dom_i = X \text{ and} \tag{5}$$

$$dom_i \cap dom_j = \emptyset, \forall h_i, h_j \in X_h, h_i \neq h_j \tag{6}$$

Suppose the first condition (5) does not hold. If there exists x such that $x \notin dom_i, \forall h_i \in X_h$ then it is either the case that $x \notin \bigcup h_i, \forall h_i \in X_h$, which is impossible by the second property of the HT, or $x \in \bigcup h_j$ where h_j is the parent node of h_i $\forall h_i \in X_h$ which is impossible because the root of the HT has no parents.

To establish the second condition it is sufficient to show that if $x \in dom_i$ then $x \notin dom_j$. Suppose $x \in dom_j$, then by the fourth property of the HT x should belong to all nodes of the HT on the path that connects h_i and h_j. Any such path includes the parent node of h_i in the HT, which is a contradiction. Q.E.D.

Returning to the problem specified by (5.3) and (5.5) define $f : L \to R^+$ as an additively-separable function.

(5.7) Example (5.3 continued)
Define $dom_3 = \{x_4\}$, $dom_2 = \{x_5\}$, $dom_1 = \{x_1, x_2, x_3\}$. Now we can define functions ψ^i on dom_i for each hinge: $\psi^i(h_i) = \sum_{x \in dom_i} f^i(RS(l, x))$ for $i = 1, 2, 3$. The $\{dom_i\}$ forms a partition of X hence

$$f(l) = \sum_{i=1}^{3} \psi^i(h_i) = \sum_{x \in X} f^i(RS(l, x)).$$

A tree-structured dual problem with additively-separable objective function can be solved by an efficient algorithm which is explained in appendix A. The main idea of the algorithm is the following: first, define starting from the leaves of the hinge tree an optimal value function (algorithm (A.2)), second, use the modified PB algorithm (algorithm(A.5)) to create an optimal assignment of values (a CSOP solution). The detailed description of the algorithms is presented in the appendix and the following example illustrates the main steps of the solution procedure.

(5.8) Example (5.5 continued)
Consider the dual problem $CSOP'$ defined in (5.5) with the objective function f defined in (5.7). The problem has a simple tree-structured graph with three nodes and two arks. In order to identify which node is the root of the tree it is necessary to define a total order on the set of nodes such that each node has at most one parent node (a width-one order which is formally defined in appendix A). Let h_3 be a node of the highest order and h_1 be a node of the lowest order. Node h_1 is then a root of the tree and h_3 is a leaf.

For leaf h_3 define the values of optimal value function v for each element in the domain of h_3 by setting them equal to their ψ-values (step 1 of algorithm A.2 in appendix A):

h_3	γ_i	$\psi^i(\gamma_i)$	$v(\gamma_i)$
	γ_1	1	1
	γ_2	1	1
	γ_3	0	0
	γ_4	1	1

For node h_2 the values of optimal value function v are defined as the sum of the ψ-value of an assignment to h_2 and the v-value of the best assignment to h_3 which is compatible with the given assignment to h_2 (step k of algorithm (A.2)). For example, $\beta_1 \in d(h_2)$ is compatible both with $\gamma_1 \in d(h_3)$ and $\gamma_3 \in d(h_3)$ (see example (5.5)). The optimal value function takes the value 1 for γ_1 and 0 for γ_3: $v(\gamma_1) = 1$ and $v(\gamma_3) = 0$. The ψ-value of β_1 is 1. Hence $v(\beta_1) = \psi(\beta_1) + v(\gamma_3) = 1$. The rest of the v-values for the elements of $d(h_2)$ (assignments to h_2) is defined in the same way:

h_2	β_i	$\psi^i(\beta_i)$	$v(\beta_i)$
	β_1	1	1
	β_2	1	1
	β_3	0	1
	β_4	0	1
	β_5	1	2

The values of the optimal value function for h_1 are defined analogously using v-values for h_2 and ψ-values for h_1 (step k of algorithm (A.2)):

h_1	α_i	$\psi^i(\alpha_i)$	$v(\alpha_i)$
	α_1	3	4
	α_2	2	3
	α_3	2	3
	α_4	2	4

After defining the optimal value function we can use it to find an optimal solution without enumerating all possible sub-optimal assignments (algorithm (A.5)). We start from the root of the tree (h_1) and select an assignment to the root ($\alpha_i \in d(h_1)$) which has an optimal v-value (step 1 of algorithm (A.5)). We proceed to the children of the root and find a set of assignments which are consistent with the optimal assignment to the root; among them we select one with the optimal v-value (step k of algorithm (A.5)). We repeat this step until each node (variable) h_i is assigned an optimal value. Such an assignment is a solution of the optimization problem (both $CSOP'$ and $CSOP$).

For example, take $\alpha_2 \in d(h_1)$ which has the minimal v-value among $\{\alpha_i : i = 1 \dots 4\}$; α_2 is compatible with β_2 and β_3 in $d(h_2)$. Take β_2 which has the minimal v-value among $\{\beta_2, \beta_3\}$[9]; β_2 is compatible with γ_1 and

[9] Since $v(\beta_2) = v(\beta_3)$ in this particular case we can take either β_2 or β_3.

γ_3 in $d(h_3)$. In the set $\{\gamma_1, \gamma_3\}$ γ_3 has the minimal v-value hence h_3 is assigned γ_3. The complete assignment $(\alpha_2, \beta_2, \gamma_3)$ is a solution of the dual problem; the corresponding solution of the $CSOP$ in example (5.3) is the assignment of values $(0,1,1,0,1)$ to the tuple of variables $(x_1 \dots x_5)$[10].

Summing up, it is shown that a $CSOP$ with an arbitrary graph structure can be transformed into an equivalent tree-structured $CSOP'$. The transformation involves finding a hinge-tree decomposition of a $CSOP$ graph, finding all consistent assignments of values in the sub-problems associated with each minimum hinge in the hinge tree and formulating the dual tree-structured problem $CSOP'$, using the sub-problem solutions. After doing this $CSOP'$ is solved by algorithms (A.2) and (A.5). Is this method significantly better for solving the government's problem than a simple (unaided) BP algorithm?

The next proposition estimates the efficiency of the method.

(5.9) Proposition The worst case time complexity of finding a solution of $CSOP_P$ using the problem-decomposition techniques is bounded by $nr^2 + mr^2 2^m + m^2 r + r(2^m + 2)$, where $n = |X|$ is the number of variables, $r = |C|$ is the number of constraints in the original problem and m is the number of variables in the largest minimal hinge of a HT.

Proof: The total computational time consists of the cost of formulating the dual problem $(CSOP'_P)$ and of the cost of solving it. The cost of formulating a dual problem is composed from three components: finding a HT decomposition of the graph G, finding sets L_i defined for each minimal hinge h_i, and finally finding a partition of the variables $\{dom_i\}$.

The cost of finding a HT decomposition of the graph $G = (X, S)$ is bounded by $|X||S|^2 = nr^2$, where $n = |X|$ and $r = |S| = |C|$ (Gyssens et al. (1994)).

The cost of finding one set L_i is bounded by $|\bigcup h_i|\ |h_i| 2^{|\bigcup h_i|}$ (Proposition (4.2)), where $|\bigcup h_i|$ and $|h_i|$ are the number of variables and the

[10]The complete set of $CSOP'$ solutions is: $\{(\alpha_2, \beta_2, \gamma_3),\ (\alpha_2, \beta_3, \gamma_4),\ (\alpha_3, \beta_1, \gamma_3),\ (\alpha_3, \beta_4, \gamma_4)\}$; these establish solutions of the original problem $\{(0,1,1,0,1), (0,1,1,1,0), (1,0,1,0,1), (1,0,1,1,0)\}$.

number of constraints in the hinge h_i respectively. Let m be the number of variables in the largest minimal hinge. Since the number of minimal hinges in the HT and the number of constraints in the largest hinge are both bounded by the number of constraints in the original problem $r = |C|$, the cost of finding all sets L_i is bounded by r^2m2^m.

Finally, the cost of defining a partition $\{dom_i\}$ is bounded by m^2r, m and r defined as before.

By proposition (A.8) the cost of finding a solution of a tree-structured $CSOP$ is bounded by $n'(k'+2)$, where k' is the size of the largest domain and n' is the number of variables in the dual problem. For the dual problem $CSOP'_P$ the size of the largest domain k' will be equal to the size of the largest set L_i and it is bounded by 2^m, where m is defined as before and 2 is the size of domains $d(x)$. The number of variables in the dual problem equals to the number of minimal hinges in the HT and it is bounded by r. Hence the cost of finding a solution to the dual problem is bounded by $r(2^m+2)$.

Overall, the total cost of solving $CSOP_P$ using the decomposition method is bounded by the sum of all the components: $nr^2 + mr^22^m + m^2r + r(2^m+2)$. Q.E.D.

The time complexity of finding a HT of the graph and solving the dual problem $CSOP'$ is bounded by a polynomial. However, finding a set of assignments defined for the minimal hinge is bounded by exponential time, hence it is the most "expensive" part of the algorithm. In other words, the leading term in the cost expression derived in proposition (5.9) above is 2^m. The leading term in the time cost expression is most informative about the time complexity of an algorithm for large inputs. "It is the rate of growth of the running time that really interests us. We therefore consider only the leading term of a formula since the lower order terms are relatively insignificant for large [input size]. We also ignore the leading term's constant coefficient, since constant factors are less significant than the rate of growth in determining computational efficiency for large inputs" (Cormen et al. (1990 p.10)). Hence, the overall complexity of solving the problem using the problem-decomposition techniques can be roughly measured by the size of the largest minimal hinge in the HT which is uniquely defined for each graph (Gyssens et al.(1994)).

The decomposition of the privatization problem along the lines specified above helps to overcome the complexity constraints unless the size of the largest minimal hinge is equal to the size of the whole problem. Remember that by proposition (4.2) the cost of finding a $CSOP_P$ solution by the PB algorithm is roughly measured by the leading term of the time expression 2^n. When the size of the largest sub-problem (m) is less than the size of the whole problem (n) problem-decomposition techniques provide a clear efficiency gain.

6. Problem Decomposition and an Efficient Administrative Structure

Countries in transition that implemented mass privatization programs had to set up administrative organizations responsible for making privatization decisions. Although the details of the administrative organizations differ from country to country, there is a common feature in their structure – the decentralization of decision-making. Given that the problem decomposition helps to overcome the complexity of the privatization problem the decentralization of the administrative structure may be rationalized as a method that improves administrative efficiency.

In the same way as an algorithm uses limited computational resources to solve $CSOP_P$, an administrative organization utilizes limited administrative resources to produce a required output – administrative decisions. The efficiency of an algorithm and administrative efficiency both matter because resources that they utilize are limited. While an inefficient algorithm makes the computational cost of solving a problem prohibitive, inefficient bureaucracy is unable to make an administrative decision.

In the context of the privatization problem an efficient administrative organization should be able to produce a decision about the optimal sequencing of privatization (a $CSOP_P$ solution). It was shown that finding the optimal sequencing of privatization is a difficult decision-making task; the methods which can solve it have exponential time complexity. However, the complexity may be reduced if we divide the whole task into several sub-problems.

The result on the efficiency of the decomposition of the problem is close to the following argument made by Herbert Simon with respect to decision-making in organizations: "The division of labor is quite as important in organizing decision-making as in organizing production, but what is being divided is different in the two cases. From the information processing point of view, division of labor means factoring the total system of decisions that need to be made into relatively independent subsystems, each one of which can be designed with only minimal concern for its interactions with the others. The division is necessary because the processors that are available to organizations whether humans or computers are very limited in their processing capacity in comparison with the magnitude of decision problems that organizations face. The number of alternatives that can be considered, the intricacy of the chains of consequences that can be traced – all these are severely restricted by the limited capacities of the available processors" Simon (1976,p.293). Following Simon's advice we have found a solution method that divides the whole problem into relatively independent sub-problems – the hinge-decomposition method.

How can we characterize the solution method in organizational terms? In general, an administrative structure specifies which administrative units are responsible for making given decisions and how the information (the initial input information and the information about decisions already taken) passes between different units (Simon 1976). We can also characterize the structure of a solution method in the same terms. Hence we can specify a mapping from a solution method (algorithm) to an administrative structure of an organization responsible for a decision-making task.

The decision about the optimal sequencing of privatization in the model consists of a large number of decisions about the privatization of individual firms. The initial input information for the privatization problem corresponds to the set Γ of trade relations in the economy and the set $\{f^i\}$ of the expected losses of revenues due to give-away privatization. Hence the structure of an administrative organization responsible for privatization is characterized by three factors: the authority which makes a decision about the privatization of individual firm, the information which is used to make this decision and further application of the decisions.

The centralized administrative structure thus is characterized as one administrative unit that processes all the individual decisions itself using the whole set of input information. We have seen that this corresponds (with obvious simplifications) to the manner in which the simple PB and the "generate-and-test" algorithms operate. On the other hand, considering the hinge-decomposition method we can identify separate administrative units with individual hinges. Each unit makes decisions about the problem variables contained in the hinge using only the information that concerns these variables (the set of constraints that forms the hinge). However, the division of the decision-making tasks and the coordination of individual units requires a central administrative authority.

The existence of a central administrative unit implies some degree of centralization of decision-making but it is not equivalent to the centralized administrative structure. The important thing is that the detailed information, the information about domains of the variables and extents of the constraints, is stored and processed locally. Only the information about the structure of the problem and sub-problem solutions has to be communicated to the center.

The efficiency gain from applying the hinge-decomposition solution method corresponds then to the efficiency gain from the decentralization of the administrative organization. Thus we can rationalize a decentralized administrative structure from the point of view of algorithmic efficiency. The efficient administrative structure is determined by a hinge-tree decomposition of the problem which depends, in turn, on the structure of trade relations in the economy. Since a hinge-tree decomposition of the problem is not uniquely defined there is no one-to-one correspondence between the administrative structure and the structure of trade relations. However, the size of the largest sub-problem (the size of the largest hinge in a hinge-tree decomposition) is uniquely defined for a given problem. Hence we can predict the size of the largest administrative unit in the efficient administrative organization given the structure of the problem.

Summing up, there exists a direct link between the efficiency of solution methods and the efficient organization of the government's decision-making process. The structure of the decision-making process determines then the efficient administrative structure. We can predict in the framework of the model that the efficient structure of the administrative organization responsible for privatization is a decentralized one.

7. Extensions

The results of the paper so far have been developed in the extremely simplified framework defined by assumptions (3.1) and (3.2), which restrict the model to the binary constraints with the special extent type $e(c) = \{(0,1),(1,0),(1,1)\}$. This is clearly a simplification and real-life governments may need to take into account many more considerations when designing the optimal reform plan. For example, the privatization of some firms may not be politically feasible unless a large part of the shares goes to the enterprise insiders (political constraints, specified and studied by Boycko et al.(1994)), or the government may worry about monopolistic behavior and possible collusion if privatization creates excessively concentrated ownership.

We can show that additional considerations may be captured in the CSOP framework by a more detailed specification of the variable domains and a more general specification of the constraints. Hence a more general problem can be specified in the framework of the same mathematical model. What implications will it have for the efficiency of solution methods? This section briefly presents several possible extensions and shows that the efficiency gain from the decomposition of the problem is sufficiently robust with respect to modifications of the basic model.

The literature on privatization in Eastern Europe often stresses the "political constraints" that any successful privatization plan should meet. First, some enterprises may be considered "too big to fail", hence the government may want to postpone their privatization. This type of constraint is easily modeled in the CSOP framework by assigning a very large loss value f^i to the relevant variable x_i, or, alternatively, by introducing a constraint c with the scope $s(c) = x_i$ and the extent $e(c) = 0$.

Second, enterprise insiders in some cases may have enough political influence to block privatization of their firm, thus the government needs to buy their support giving them assets on preferential terms. To capture this consideration the domains $(d(x_i))$ need to have more than two values to describe different types of privatization. For example, the Russian mass privatization law provided three basic privatization options (modes) which differ mainly by the degree of preferential treatment for insiders (Boycko et al. (1994), Bornstein (1994)); in this case the domain $d(x_i)$ may be extended to include four values: one for state ownership and three values for the three different privatization modes.

Other concerns of the government, for example, a possible collusive agreement between the privatized firms in oligopolystic market, can be modeled by introducing additional multivariate constraints and by re-specifying the variable domains to distinguish, for example, privatization with the option to regulate the privatized firm.

The examples above demonstrate that additional considerations can be accommodated in the CSOP framework, first, by modifying variable domains and extents of the constraints and, second, by introducing more (possibly multivariate) constraints. It is easy to see that the first option does not affect relative efficiency gains from the decomposition of the problem.

A hinge-tree decomposition of the problem can be found without knowing the extents of constraints and the variable domains. A hinge-tree decomposition depends only on the problem structure (scopes of the constraints). Hence the size of the largest minimal hinge in a HT decomposition (which characterizes the complexity of solving the problem) does not depend on a more detailed specification of the domains and the extents.

On the other hand, introducing additional constraints adversely affects the efficiency gain achieved by the decomposition because it may affect the size of the largest minimal hinge in the HT decomposition. This is a rather intuitive result. Indeed, the more considerations the government wants to incorporate into reform design, the more difficult is to find a solution; it becomes more difficult to divide the problem into relatively independent sub- problems.

Multivariate constraints can be easily accommodated into a CSOP as well. Multivariate constraints transform the problem structure into a hypergraph, i.e. a pair (X, S), where $S = \{s : s \subseteq X\}$. However, this does not change the solution methods. We can still find a set of consistent assignments by the PB algorithm; the algorithm for finding a hinge tree decomposition remains also the same (Gyssens (1994)). Introducing additional multivariate constraints in the basic model has the same effect as introducing additional binary constraints: additional constraints may affect the size of the largest hinge in the HT.

In general then, we can analyze more complicated problems using the CSOP framework of the basic model. The problem decomposition techniques developed in the paper will still provide an efficiency gain. Hence we can rationalize a decentralized administrative structure in a more general, richer environment. However, with more additional considerations (constraints) that the government wants to incorporate into the optimal reform plan the government's problem becomes more complex (in the sense of computational complexity) and difficult to solve.

8. Conclusions and Further Research

A simple model developed in the paper is based on the idea that an optimal plan for privatization should be designed by the government so that mass privatization in the early stage of reforms helps to overcome initial informational problem in the transitional economy. The model explicitly recognizes that a decision about the privatization of one firm will affect the incentives and behavior of other firms – in other words, it is intended to emphasize a systematic approach to privatization and its implications.

I take the point of view that mass privatization hardens the budget constraints of privatized firms; it also indirectly hardens the budget constraints of the firms that are linked by trade relations to the privatized firms. The hardening of the budget constraints improves the information about the profitability of all firms and thus makes easier the sales of the remaining state-owned firms to private investors. However, mass privatization leads to the loss of revenues from the divesture of the state-owned assets and the government is assumed to take this into consideration. Thus mass privatization should be limited and selective.

The optimal privatization plan includes the privatization of some set of enterprises by the give-away methods of the mass privatization in the first period and privatization of the rest of the firms by conventional methods, which create revenues for the government in the second period. Finding a set of firms earmarked for privatization in the first period defines the problem of the optimal sequencing of privatization. This problem is defined mathematically in the paper as a constraint satisfaction optimization problem (CSOP). This specification maps the structure of trade relations in the economy into the optimal privatization plan.

This simple model allows us to identify the difficulties that the government faces if it wants to specify the optimal sequencing of privatization. It is shown that the problem may be impossible to solve because the government has to take many related decisions and these decisions should be compatible with each other (the government's problem is shown to be NP-complete). This result allows us to formally specify the notion of complexity of the optimal reform design and recognize that complexity constraints are binding for the optimal sequencing of privatization problem.

The paper also describes a method that allows us to alleviate the complexity problem. The main feature of this method is the decomposition of the whole problem into a set of sub-problems. In the context of the privatization design, decomposition is understood as dividing the whole task into a number of sub- problems along the lines that make them relatively independent. However, solutions to individual sub-problems should be compatible with each other in order to generate an optimal global solution. Hence the role for coordination. In fact, it is shown that decomposition and coordination substantially reduce the complexity of reform design, and thus may be key factors of the successful transition.

Decomposition has two further implications for the structure of the government's decision-making. First, there is a place for a decentralized informational structure. Indeed, decomposition of the global problem requires only limited knowledge – information about the structure of industrial relations in the economy; then solutions of the local sub-problems are computed using only local information. Second, decentralization implies the possible gradualism of reforms in the sense that sequential implementation of optimal sub-problem solutions will create an optimal global solution as long as the individual local solutions are coordinated. However, in each economic region or in each closely related by technological links industry there exists a minimum number of firms that should be privatized to achieve the government's objective. This provides a justification for the "big bang" privatization programs, that call for the immediate privatization of a large number of firms.

Given the importance of the organization of the government's decision-making, a tentative normative implication of the paper would be that the government should invest into an efficient (along the lines specified in the paper) organization of its different administrative branches responsible for different parts of the privatization reform. More precisely, a single local agency should be responsible for the privatization of a group of state firms that are held together by their location or technological specialization. Local administrative units should be subordinated, however, to the central government body to insure the coordination of their decisions.

The paper emphasizes the efficient method of the reform design, but says little so far about the instantiation of the optimal privatization plan, i.e. it does not specify a concrete privatization plan which says what firms are optimal to privatize first. Such results are pending upon the data on the structure of industrial relations in the economy (input-output matrix of the economy) and on some plausible estimation of the losses that the government can expect from the privatization of each firm in the first period. This is an important task for future research.

Appendix A. An Efficient Algorithm for a Tree-Structured Problem

The pure backtracking algorithm solves a CSOP in exponential computational time and a CSOP is an NP-complete problem in general. The source of inefficiency of the BP algorithm is identified in section 4 as being in the exhaustive enumeration of all consistent assignments. We can avoid it for a CSOP with a tree-structured set of constraints by using the solution method presented in this section.

The efficient performance of the PB algorithm in finding a single assignment l implies that the source of inefficiency of the PB is not an extensive backtracking due to the search of the branches of a search space that do not contain consistent assignments, but rather the fact that the PB finds all consistent assignments including a lot of assignments with suboptimal f-values. If we are interested only in the optimal solution (a CSOP solution), we do not want the algorithm to generate assignments that lead to suboptimal solutions. Hence for better performance in solving the CSOP an algorithm has to rule out branches of the search space that do not contain optimal solutions.

It appears that this can be achieved by combining a consistency check with optimization at each computational step that extends a partial consistent assignment to include a new variable, i.e. by combining a step of the BP algorithm with the optimization step. Moreover, if a problem has a special, acyclical structure of constraints, there exists an efficient algorithm that finds a CSOP solution in polynomial computational time.

Consider a class of problems (X, C, f) with an additively-separable objective function $f(l) = \sum_{x_i \in X} f^i(RS(l, x_i))$ and a tree-structured graph $G = (X, S)$ (where $S = \{s(c) : c \in C\}$). Clearly the dual problem defined for the government's privatization problem ($CSOP'_P$) belongs to this class.

In a tree-structured problem each variable interacts with the rest of the problem through a limited number of constraints. This property allows us to extend a consistent labeling by doing a limited number of consistency checks at each step. Moreover, a computational step that extends a consistent labeling could be combined with optimization due to the additive separability property of the objective function. In fact, an optimal assignment l^* may be composed step by step, using the methods of dynamic programming.

I begin formalizing the argument by defining a tree-structured graph (X, S) and its components. First, specify an order of nodes in the graph.

(A.1) Definition A width-one order $\prec$ of the set of nodes X of a graph $G = (X, S)$ is a total order on set X such that for each node x_i there is at most one node $x_j \prec x_i$ connected to x_i by an edge of the graph $((x_i, x_j) \in S)$.

In other words, under a width-one order each node of a tree-structured graph is connected by an edge with at most one node of lower order. A width-one order can be generated by depth-first or breadth-first traversal of a tree-structured graph in time proportional to the number of nodes in the graph (Leeuwen (1990)).

Let X be ordered by an arbitrary width-one order. The node of the smallest order x_0 is called a root of the tree. For each $x \in X$ define a set of nodes that are children of x: $CH(x) = \{y \in X : x \prec y, (x, y) \in S\}$. In turn, for each node y in $CH(x)$, x is called a $parent(y)$ node. Each node $x \in X$ also defines a sub-tree in G with the root node x. Let $ST(x)$ denote a set of nodes of the sub-tree: $ST(x) = \bigcup_{x \preceq z} CH(z)$ [11].

A width-one order also defines tree levels. The root node x_0 is called a 0-level node, children of the root are called first-level nodes, etc. Let X^t denote a set of t-level nodes with a typical element $x_{t,i}$.

Now I introduce some more CSOP-related concepts that have standard counterparts in dynamic programming. For any value $a_{t,i} \in d(x_{t,i})$ define a set of $x_{t+1,j}$ values consistent with $a_{t,i}$: $CV(x_{t+1,j}, a_{t,i}) = \{a_{t+1,j} \in d(x_{t+1,j}) :$ if $\exists\ c \in C$ such that $s(c) = (x_{t,i},\ x_{t+1,j})$ then $(a_{t,i}, a_{t+1,j}) \in e(c)\}$. For any value $a_{t,i}$ of the variable $x_{t,i}$ define a set of partial consi-

[11] A set of edges of the sub-tree is $\{(x_i, x_j) : x_i, x_j \in ST(x), (x_i, x_j) \in S\}$.

stent assignments of values to the nodes in the sub-tree $ST(x_{t,i})$ (or paths in the search space): $Path(a_{t,i}) = \{(a_{t,i}, \ldots, a_{m,k}, \ldots, a_{m+1,j}, \ldots, a_{T,|X^T|}) : a_{m,k} \in d(x_{m,k}),\ a_{m+1,j} \in CV(x_{m+1,j}, a_{m,k}),\ x_{m,k} \in ST(x_{t,i})\}$. Let a partial consistent labeling $u(a_{t,i})$ be a typical element of the $Path(a_{t,i})$.

With each value $a_{t,i} \in d(x_{t,i})$ associate a value $v^*(a_{t,i})$ which is equal to the value of the objective function evaluated on the best partial consistent labeling of the nodes in $ST(x_{t,i})$:

$$v^*(a_{t,i}) = \min_{u(a_{t,i}) \in Path(a_{t,i})} \sum_{x \in ST(x_{t,i})} f^i(RS(u(a_{t,i}), x)) \tag{7}$$

Then the optimal partial consistent labeling of a sub-tree with the root node $x_{t,i}$ and value $a_{t,i} \in d(x_{t,i})$ fixed is defined by

$$u^*(a_{t,i}) = \arg \min_{u(a_{t,i}) \in Path(a_{t,i})} \sum_{x \in ST(x_{t,i})} f^i(RS(u(a_{t,i}), x)) \tag{8}$$

By construction $l^* = \arg\min_{a_0 \in d(x_0)} f(u^*(a_0))$.

Function v^* can be defined by a recursive method using the principle of optimality. More precisely, the following algorithm defines a function v which is proved to be identical to v^* by propositions (A.3) and (A.4).

(A.2) Algorithm (defining the optimal value function v)

Input: $CSOP = (X, C, f)$

Output: function $v : \bigcup_{x \in X} d(x) \times x \rightarrow R^+$

Method:

Step 1.

For each leaf node x_{leaf} set $v(a_{leaf}) = f^i(a_{leaf})\,, \forall a_{leaf} \in d(x_{leaf})$.

Step k.

At this point v - values are defined for all values of all variables in X from X^T to X^{t+1}. For $a_{t,i} \in d(x_{t,i})$ find $CV(x_{t+1,j}, a_{t,i}), \forall x_{t+1,j} \in CH(x_{t,i})$ and set

$$v(a_{t,i}) = f^i(a_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} \min_{a_{t+1,j} \in CV(x_{t+1.j}, a_{t,i})} v(a_{t+1,j}) \tag{9}$$

If $CV(x_{t+1.j}, a_{t,i}) = \emptyset$ remove $a_{t,i}$ from $d(x_{t,i})$ [12]. Repeat the step k for all $a_{t,i} \in d(x_{t,i})$ and all $x_{t,i} \in X^t$.

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[12] For $CSOP_P$ it is easy to see that $CV(x_{t+1,j}, a_{t,i}) \neq \emptyset$ for all $x_{t+1,j} \in CH(x_{t,i}), \forall a_{t,i} \in d(x_{t,i}),\ \forall x_{t,i} \in X$, hence no $a_{t,i}$ will be removed.

A.3 Proposition
Function v^* satisfies recursive equation (9).

Proof: By definition v^* is a unique function satisfying the following two conditions:

$$v^*(a_{t,i}) \leq \sum_{x \in ST(x_{t,i})} f^i(RS(u(a_{t,i}), x)) \tag{10}$$

for all $u(a_{t,i}) \in Path(a_{t,i})$ and

$$v^*(a_{t,i}) = \sum_{x \in ST(x_{t,i})} f^i(RS(u(a_{t,i}), x)) \tag{11}$$

for some $u(a_{t,i}) \in Path(a_{t,i})$.

Since the recursive equation (9) is equivalent to

$$v(a_{t,i}) = \min_{a_{t+1,j} \in CV(x_{t+1,j}, a_{t,i})} \left(f^i(a_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} v(a_{t+1,j}) \right) \tag{12}$$

we can establish the result by demonstrating that v^* satisfies (12).

Function v^* satisfies (12) if the following two conditions hold:

$$v^*(a_{t,i}) \leq f^i(a_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} v^*(a_{t+1,j}) \tag{13}$$

for all $a_{t+1,j} \in CV(x_{t+1,j}, a_{t,i})$, and

$$v^*(a_{t,i}) = f^i(a_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} v^*(a_{t+1,j}) \tag{14}$$

for some $a_{t+1,j} \in CV(x_{t+1,j}, a_{t,i})$.

First show that (10) and (11) imply (13). Take some value $a_{t,i}$ and some values $a_{t+1,j} \in CV(x_{t+1,j}, a_{t,i})$, for all $x_{t+1,j} \in CH(x_{t,i})$. By definition of $Path(a_{t,i})$, $\exists u(a_{t,i}) \in Path(a_{t,i})$ such that $a_{t+1,j} = RS(u(a_{t,i}), x_{t+1,j})$ and $Path(a_{t+1,j}) \subseteq RS(Path(a_{t,i}), (x_{t+1,j}, \ldots, x_{T,|X^T|})$.

From (10) follows that

$$v^*(a_{t,i}) \leq f^i(a_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} \sum_{x \in ST(x_{t+1,j})} f^i(RS(u(a_{t,i}), x)) \tag{15}$$

for all $u(a_{t+1,j}) \in Path(a_{t+1,j})$.

From (11) follows that

$$v^*(a_{t+1,j}) = \sum_{x \in ST(x_{t+1,j})} f^i(RS(u(a_{t+1,j}), x)) \tag{16}$$

for some $u(a_{t+1,j}) \in Path(a_{t+1,j})$. Combining (15) and (16) get (13).

It remains to show that (10) and (11) establish (14). Take some value $a_{t,i}$ and select some $u(a_{t,i}) \in Path(a_{t,i})$ such that equality (11) holds for this $u(a_{t,i})$ or equivalently

$$v^*(a_{t,i}) = f^i(a_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} \sum_{x \in ST(x_{t+1,j})} f^i(RS(u(a_{t,i}), x)) \tag{17}$$

Take $a_{t+1,j} = RS(u(a_{t,i}), x_{t+1,j})$ and $u(a_{t+1,j}) = RS(u(a_{t,i}), ST(x_{t+1,j}))$ for all $x_{t+1,j} \in CH(x_{t,i})$. Since the inequality (10) holds for all $u(a_{t+1,j}) \in Path(a_{t+1,j})$ it holds for the selected $a_{t+1,j}$ and $u(a_{t+1,j})$ as well, i.e.

$$v^*(a_{t+1,j}) \leq \sum_{x \in ST(x_{t+1,j})} f^i(RS(u(a_{t+1,j}), x)) \tag{18}$$

From (17) and (18) follows that

$$v^*(a_{t,i}) \geq f^i(a_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} v^*(a_{t+1,j}) \tag{19}$$

for the selected values $a_{t+1,j} \in CV(a_{t,i})$. Finally, since it is proved that (13) is true for all values $a_{t+1,j} \in CV(a_{t,i})$, (13) and (19) establish (14). Q.E.D.

A.4 Proposition

If v is defined by the recursive equation (9) then $v = v^*$.

Proof: Show that (13) and (14) establish (10) and (11).

Take some $a_{t,i}$ and show that (13) implies (10). By (13)

$$v(a_{t,i}) \leq f^i(a_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} v(a_{t+1,j}) \tag{20}$$

for all $a_{t+1,j} \in CV(x_{t+1,j}, a_{t,i})$ and

$$v(a_{t+1,j}) \leq f^i(a_{t+1,j}) + \sum_{x_{t+2,k} \in CH(x_{t+1,j})} v(a_{t+2,k}) \tag{21}$$

for all $a_{t+2,k} \in CV(x_{t+2,k}, a_{t+1,j})$ or

$$v(a_{t,i}) \leq f^i(a_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} \left(f^i(a_{t+1,j}) + \sum_{x_{t+2,k} \in CH(x_{t+1,j})} v(a_{t+2,k}) \right) \tag{22}$$

for all $a_{t+2,k} \in CV(x_{t+2,k}, a_{t+1,j})$, $\forall x_{t+1,j} \in CH(x_{t,i})$. This and further analogous steps create $Path(a_{t,i})$ such that

$$v(a_{t,i}) \leq \sum_{x \in ST(x_{t,i})} f^i(RS(u(a_{t,i}), x)) \tag{23}$$

for all $u(a_{t,i}) \in Path(a_{t,i})$, that is establish (10). Analogously get (11) by iterating (14). Q.E.D.

After finding v-values that uniquely define the optimal value function v^* it is relatively easy to define recursively, starting from the root of the tree, an optimal path that solves the CSOP.

(A.5) Algorithm (defining the optimal assignment)
Input: $CSOP = (X, C, f)$ and function v defined by the (A.2).
Output: An optimal path $(a_0^*, \ldots, a_{T,|X^T|}^*)$.
Method:
Step 1.
Set the value of x_0 equal to

$$a_0^* = \arg \min_{a_0 \in d(x_0)} v(a_0)$$

Step k .
At this point a partial consistent optimal labeling is created for all the variables starting from 0-level of the tree to level t. Find the set $CV(x_{t+1,j}, a_{t,i}^*)$ for $a_{t,i}^* \in d(x_{t,i})$, and define

$$a_{t+1,j}^* = \arg \min_{a_{t+1,j} \in CV(x_{t+1,j}, a_{t,i}^*)} v(a_{t+1,j}), \text{ for all } x_{t+1,j} \in CH(x_{t,i}) \tag{24}$$

Repeat step k for all $x_{t,i} \in X^t$.

Now it remains to show that the optimal path defined by algorithm (A.5) is a CSOP solution. From (9) and (24) it follows that

$$v(a^*_{t,i}) = f^i(a^*_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} v(a^*_{t+1,j}) \tag{25}$$

And since v is identical to v^*

$$v^*(a^*_{t,i}) = f^i(a^*_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} v^*(a^*_{t+1,j}) \tag{26}$$

The following propositions (A.6) and (A.7) establish that a path $u^*(a_{t,i})$ solves problem (7) iff its elements satisfy recursive equation (26).

A.6 Proposition
If $u^*(a^*_{t,i})$ solves the problem

$$\min_{u(a^*_{t,i}) \in Path(a^*_{t,i})} \sum_{x \in ST(x_{t,i})} f^i(RS(u(a^*_{t,i}), x)) \tag{27}$$

then

$$v^*(a^*_{t+k,i}) = f^i(a^*_{t+k,i}) + \sum_{x_{t+k+1,j} \in CH(x_{t+k,i})} v^*(a^*_{t+k+1,j}) \tag{28}$$

for all $k = 0, 1, \ldots, T-(t+1)$, where $a^*_{t+k,j} = RS(u^*(a^*_{t,i}), x) \; \forall \; x_{t+k,j}$.
Proof: Since $u^*(a^*_{t,i})$ attains the minimum of the problem (27) the following equality holds true

$$v^*(a^*_{t,i}) = f^i(a^*_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} \sum_{x \in ST(x_{t+1,j})} f^i(RS(u^*(a^*_{t,i}), x)) \tag{29}$$

Also the by definition of v^*

$$v^*(a^*_{t+1,j}) \leq \sum_{x \in ST(x_{t+1,j})} f^i(RS(u(a^*_{t+1,j}), x)) \tag{30}$$

for all $u(a^*_{t+1,j}) \in Path(a^*_{t+1,j})$. By construction

$$u(a^*_{t+1,j}) \in RS\left(Path(a^*_{t,i}), ST(x_{t+1,j})\right).$$

Then (29) and (30) imply that

$$v^*(a^*_{t,i}) \geq f^i(a^*_{t,i}) + \sum_{x_{t+1,j} \in CH(x_{t,i})} v^*(a^*_{t+1,j}). \tag{31}$$

Finally, since (13) holds for all values $a_{t+1,j} \in CV(a_{t,i})$ and by propositions A.3 and A.4 $v \equiv v^*$, (13) and (31) establish (28) for $k = 0$. By induction get the same result for $k = 1, \ldots, T-(t+1)$. Q.E.D.

A.7 Proposition

If elements of $u^*(a^*_{t,i}) \in Path(a^*_{t,i})$ satisfy

$$v^*(a^*_{t+k,i}) = f^i(a^*_{t+k,i}) + \sum_{x_{t+k+1,j} \in CH(x_{t+k,i})} v^*(a^*_{t+k+1,j}) \tag{32}$$

for all $k = 0, 1, \ldots, T-(t+1)$ where $a^*_{t+k,j} = RS(u^*(a^*_{t,i}), x)$, $\forall x_{t+k,j}$, then $u^*(a^*_{t,i})$ solves the problem

$$\min_{u(a^*_{t,i}) \in Path(a^*_{t,i})} \sum_{x \in ST(x_{t,i})} f^i(RS(u(a^*_{t,i}), x)) \tag{33}$$

Proof: Iterating (32) gives

$$v^*(a^*_{t,i}) = \sum_{x \in ST(x_{t,i})} f^i(RS(u^*(a^*_{t,i}), x)) \tag{34}$$

and by definition of v^*

$$v^*(a^*_{t,i}) \leq \sum_{x \in ST(x_{t,i})} f^i(RS(u(a^*_{t,i}), x)) \tag{35}$$

for all $u(a^*_{t,i}) \in Path(a^*_{t,i})$. Then (34) and (35) establish (33). Q.E.D.

Taking into account the first step of the algorithm (A.5) we get that an optimal path defined by (A.5) solves the problem

$$\min_{a_0} \left\{ \min_{u(a_0) \in Path(a_0)} \sum_{x \in X} f^i(RS(u(a_0), x)) \right\} \tag{36}$$

which is identical to $\min_{l \in L} f(l)$. That is, (A.5) finds a CSOP solution. Hence a tree-structured $CSOP_P$ can be solved by the iterative optimization method (Algorithms (A.2) and (A.5)).

The efficiency of algorithms (A.2) and (A.5) is accessed by the following proposition.

(A.8) Proposition The overall computational time of algorithms (A.2) and (A.5) is bounded by $n(k+2)$, where $n = |X|$ is the number of variables and k is the size of the largest domain in a $CSOP$.

Proof: First, we need to perform $n = |X|$ computational steps to obtain a width-one order of nodes for the graph $G = (X, S)$ (Leeuwen (1990)). Next apply algorithms (A.2) and (A.5).

Time complexity of algorithm (A.2):

Let an elementary operation be defined as finding an element of $CV(x_{t+1,j}, a_{t,i})$ with the minimum v-value. This is the most costly operation of the algorithm which involves checking one constraint $c \in C$ (because of the tree structure of the problem) with the scope $s(c) = (x_{t+1,j}, x_{t,i})$ and finding the minimum element of the set. At each step k the algorithm performs $|CH(x_{t,i})|$ elementary operations and it needs to perform $\sum_{t=0}^{T} \sum_{i=0}^{|X^t|} |d(x_{t,i})|$ such steps before it stops. Hence the time complexity of the algorithm is $\sum_{t=0}^{T} \sum_{i=0}^{|X^t|} |d(x_{t,i})||CH(x_{t,i})|$ elementary operations. If the maximum number of variables in the domain is k, then the worst case time complexity of the algorithm is bounded by $\sum_{t=0}^{T} \sum_{i=0}^{|X^t|} k|CH(x_{t,i})|$. Since $\sum_{t=0}^{T} \sum_{i=0}^{|X^t|} |CH(x_{t,i})| = r$, where r is the number of constraints in the problem, the worst case time complexity of the algorithm is bounded by kr or by kn [13] where $|X| = n$, i.e. it is polynomial in the size of the problem.

Time complexity of algorithm (A.5):

The algorithm essentially performs the same operations as algorithm (A.2) but uses only one value $a^*_{t,i}$ from each $d(x_{t,i})$. Hence it requires at most $\sum_{t=0}^{T} \sum_{i=0}^{|X^t|} |CH(x_{t,i})|$ elementary steps defined as before and its worst case time complexity is bounded by n.

[13] In the acyclic graph the number of edges is bounded by the number of nodes.

Hence the overall computational time of algorithms (A.2) and (A.5) is bounded by $n(k+2)$. Q.E.D.

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