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Abstract

This paper analyses a signalling model of managers' promotion from divisions to the CEO position. in both cases of a monopoly and a duopoly. Explicit and implicit incentives in the presence of asymmetric information are shown to induce managers to increase effort in order to signal high ability to owners. thereby raising their probability of promotion. Firms' performance in duopoly is shown to depend on their internal organisation. through owners' choice of incentives. in addition to demand and technology conditions.

JEL Codes: D82. J33. L13. L21

1 Introduction

This paper analyses managers incentives and derives consequences of owner-manager interactions for competition in oligopolistic markets. The major aim is to show that the effect of competition on market structure is ambiguous, depending critically on internal organisation and information flows within the firm. The focus is on promotion prospects in the firm's internal labour market.

Implicit incentives (reputation effects of promotion) have been shown to be important in the context of managers' compensation by Jensen and Murphy (1990). They show that managers' motivation does not stem primarily from direct compensation. This paper uses a signalling model[1] that captures an important feature of the reputation effects that is not analysed in previous work. An attempt is therefore made at formalising career concerns in internal labour markets, unlike existing literature which focuses on external labour markets. In this paper, signalling aspects of incentives are shown in the context of exogenous wages, and screening aspects are outlined by endogenising wages. This endogenisation of wages further enables me to provide a new explanation for the profit and sales maximisation objective of Vickers (1985).

Attempts to bring agency theory into oligopoly models are very recent and few. Thus Gal-Or (1993, p. 157) claims that "the relationship between competition in the product market and managerial compensation has been entirely overlooked". Vickers (1985) sets the agenda for the issue of strategic delegation in his 1985 paper "Delegation and the Theory of the Firm", where he shows that the delegation of decisions to a manager may have strategic advantages to the benefits of the owner. He illustrates his point by assuming an oligopoly in which all firms but one are profit maximisers. The remaining firm has management separate from ownership, where the owner has to set a compensation to managers for his interest (profit maximisation) to be served. This compensation is assumed to be a weighted sum of profits and sales, reflecting the claim in the literature[2] that managers may have a preference for sales. The firm that does not maximise profits directly is shown to be able to increase output at the expense of

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[1] Inspired by the model of limit pricing of Milgrom and Roberts (1982).

[2] See the managerial literature, and Baumol, Cyert and March in particular.

rivals, hence to earn larger profits than profit-maximisers (in Cournot competition, where strategic variables are substitutes hence reaction functions are downward-sloping).

Later, Fershtmann and Judd (1987) and Sklivas (1987), extend Vickers' point to study the effect on the weight of the objective function imposed on managers, of the type of competition or of uncertainty on demand and cost parameters in duopolies. Both find that it is more profitable to set greater weight to sales maximisation in Cournot competition with a homogenous good, whereas managers are overcompensated for profits in Bertrand competition with differentiated products. However, both papers avoid issues of asymmetry of information between owners and managers, and take compensation schemes as given rather than discussing the choice of optimal contract by the owner. This paper addresses this issue. The idea is to show that the weight on profit is just a result of managers' cost minimising activities. Once this point is made, I analyse how the owner can set incentives that induce managers to work more or less hard, which implies greater or smaller reductions in cost.

More recent papers introduce the issue of information asymmetry, in particular the uncertainty as to managers' ability yielding adverse selection problems. Thus Harrington and Senbongi (1995) consider a Cournot duopoly with homogenous good. Their novelty is to introduce adverse selection problems, as well as labour market constraints. Thus managers' income is the sum of their current income in the firm and their future income as valued on the managerial labour market. Therefore the paper includes the interesting feature of managers' reputation. The labour market assesses a manager's ability according to his realised performance. Hence managers have some incentive to signal high ability through high performance, in order to build a reputation for ability on the labour market. Compensation set by the owner is exogenous, assumed to be a function of gross profits. Senbongi and Harrington confirm that the consequence of the introduction of owner-manager agency problems in oligopolies is an intensification of competition.

Gal-Or (1993) formalises an interesting model of the interactions between internal organisation and market performance in oligopolies. She assumes both moral hazard and adverse selection, in that the owner observes neither managers' actions nor their ability. She studies a duopoly with differenti-

2

ated product. She assumes that the owner hires managers to supervise two functions, production and distribution. These activities can be done by one single manager, in which case the firm is specialised, or by two different managers, in which case the firm is departmentalised. A three-stage game is considered, where the owner decides on the internal organisation in the first stage (choosing either one or two managers), sets compensation in the second stage (as a function of observables only, namely sales volume and production cost). In the final stage, each sales manager observes demand and reports it to the owner. Her main results are that the firm's choice of internal organisation affects its strategic position and that compensation schemes depend on the extent of competition in the product market as well as on the internal organisation of the two firms through the degree of correlation between the uncertain demand schedules that they are facing.

However Gal-Or does not discuss alternative incentive contracts, but just assumes one form of contract. Harrington and Senbongi justify this drawback by saying that "at this early stage of trying to understand the interaction between agency theory and oligopoly theory", it is "best to have a minimum number of effects at play so as not to confound the analysis" (p98). It is true that the formalisation of agency models usually involves many parameters and complicated computations, which become easily untractable when added to oligopoly formalisation. However the important next step in the analysis of the above mentioned interactions is to examine more closely the determination of managers' rewards in light of both the agency problem and strategic competition in oligopolies. Martin (1993) makes this step forward by considering a Cournot principal agent model of the determinants of firm efficiency. Each firm in the oligopoly has a manager and an owner, the latter setting a cost target and fee schedule that induces the manager to choose the proper action and maximises the owner's expected profit. The main result is that the degree of firm efficiency is inversely related to the number of firms in the market, which contradicts previous findings of Leibenstein and his "x-inefficiency" theory. This shows that oligopoly theory is enriched by considering factors of firms' organisations rather than focusing on product market parameters.

2 Modelling implicit incentives

In this paper I model signalling features of implicit incentives which, given wages, induce managers to increase effort. Their ability is unknown to the owner who only observes output x, which is given by $x = a + e$. a is manager's ability, is given to them by nature and is either high (H) or low (L). e is effort which can take any positive value whatever the type H or L. The owner cannot observe e or a and therefore managers have to signal their ability to the owner in order to be considered for promotion.

Implicit incentives are incentives resulting from managers' career concerns rather than from wages. Career concerns occur in the presence of asymmetric information about managers ability, so that the labour market or employers have to use the observation of past and current output to update their beliefs about ability and then base future wages on these updated beliefs. Ability being unobservable, managers have to build a reputation for ability in order to either be promoted inside the firm (in the internal labour market), or to receive offers from other employers on the labour market (in the - usually competitive - external labour market), and thus obtain wage increases. This improvement of labour markets' or employers' perception of managers productivity is defined by Vickers and Meyer (1995) as a reputation effect and is an implicit incentive. Alternatively, the owner uses the compensation scheme to screen higher ability managers. Screening models are in fact signalling models, but are called screening because then the major role is that of the owner who decides on the contract in order that ability be revealed. This paper considers first an exogenous wage, so that the model is first purely signalling, and then endogenises wages, thereby taking a screening nature as termed in the literature.

Issues of promotion and career concern have been studied in the recent literature, with three main theories of promotion emerging. The first theory views promotions as a learning device. For instance, Holmstrom and Ricart i Costa (1986) develop a model based on learning about managerial talent in order to show how career concerns rather than effort aversion induces managers to follow the owners interests. The owners uses managers' performance as indicator of talent. Managers' decisions have two returns, one on the firm's value, and the other on the value of the managers' human capital, which increases with reputation. The incentive problem is then for

the owner to induce managers to care about both values, and centralised capital budgeting is shown to be a way to do this. However, as the relationship unfolds over time, ability becomes known to both the owner and the labour market. Hence the owner has to reward managers in order for them not to be bid away from the firm.

Another theory of promotion is that of rank-order tournaments. Thus Lazear and Rosen (1981) view salaries as prizes of promotion lotteries and grades as names for prizes. Besides, the model of Macleod and Malcomson (1988) is an interesting model of ability and adverse selection, where it is assumed that workers' ability is private information. Because performance measures are not verifiable, self-enforcing contracts are of the termination type: if the performance of a worker is not satisfactory, he is dismissed. Macleod and Malcomson show that the equilibrium rank hierarchy has a finite number of pay grades, despite a continuum of possible ability levels.

These models either assume imperfect or private information on managers' ability. In the case of imperfect information, ability is initially unknown to the owner but then revealed over time as the game unfolds. This the case of the model of Gibbons and Murphy (1992), who extend the Fama (1980) and Holmstrom (1982) modelling of incentive contracts to show the implications of changes in implicit incentives (from career concerns) over the course of a manager's career for optimal explicit incentives (wages). In this paper ability is assumed to be private information. Managers know their ability, the owner cannot learn it perfectly and since promotion is profitable for managers they try to signal their ability in order to be included in the pool of candidates for promotion.

3 Signalling Model in a monopoly

3.1 Model

Consider a simple two-period model of a single corporation with N divisions, in the M-form of organisation. The role of the CEO is to advise, audit and allocate resources between divisions. The more efficient is a CEO, the more each division's unit cost is reduced. It is assumed that each CEO retires after one period. In the first period, the owner specifies a wage

schedule w_1 and w_2. In this section, wages are exogenous in the sense that the way the owner computes these wages is not formalised. Managers choose to work in the firm or not and then choose a level of effort hence of output and realise a profit. At the end of the first period, the owner observes profits and tries to infer managers ability in order to promote to the CEO position one of the high ability managers, at the beginning of the second period. Depending on the outcome of the signalling game, he may or may not be able to infer managers' abilities. Since high ability managers have the best skills for the CEO position, the owner prefers promoting one of them. However, in the case where he is unable to distinguish managers' abilities, the owner promotes one of the N managers at random to be CEO, since the CEO position must be filled in the second period[3]. Besides this, a new MBA is hired to run the vacant division. The game starts with a high ability manager at the CEO position.

Let N_H be the number of high ability managers ($N_H \leq N$, common knowledge). A manager learns his true type at the start of the game, although the owner remains uninformed. Ability is measured by a_H and a_L for high and low ability managers respectively, with $a_H > a_L$. The problem is that both types of managers can exert any effort levels $e \in [0, +\infty[$, so that the owner cannot infer ability by just observing output, which is given by $x = e + a$. The divisional profit accruing to the owner is given by $\Pi(x, a_T^*) - w$, where a_T^* is the effect of the CEO's ability on divisional profit (his contribution to the firm's performance), for $T = H, L$. Given their ability level, managers send the owner the signal x, and may choose any effort level so that $e_L + a_L$ may be greater than $e_H + a_H$ if $e_L > e_H$. Low ability managers may find it profitable to exert a high effort level in the second period if this conveys the information that they have high ability and can be promoted. The owner's prior probability that a manager has high ability is $\rho(a)$, a probability distribution over $A = \{a_L, a_H\}$, which is common knowledge. The owner receives the signal x and then takes action $s \in \{1:(\text{promote}), 0:(\text{no promotion})\}$.

Therefore I focus on the firm and its internal labour market. Contracts are signed at the beginning of the first period, by which the owner commits to

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[3]Implicit is the assumption that owners do not show any favouritism toward some managers, so that they just promote one of the N managers at random. Similarly, managers do not undertake any influence activity to increase their probability of promotion.

a wage w_1 for divisional managers and w_2 for the CEO. Both wages are set so that the participation constraints hold, that is higher than managers' outside opportunity wage (otherwise no managers would turn up to work in the firm). Thus in this model incentives are provided by the prospects of higher salary that follows from promotion. However, the model can be easily extended to formalise non monetary rewards, by assuming that managers derive satisfaction from being promoted, by an increase in their utility independent of the wage increase.

The managers' utility function is

$$U(w, e) = g(w) - C(e),$$

where $g(.)$ is continuous and strictly increasing in w, concave, reflecting managers risk aversion. $C(.)$ is the disutility of effort, a continuous and strictly convex function, with the property that $C(0) = 0$, $C'(\infty) = 0$. It is assumed that divisional profits are independent, and cannot side-contract. This assumption is crucial in this model and is realistic since multi-divisional firms like General Motors have very autonomous regional divisions. Most models in the multi-agent literature make this assumption. Observed profits can be either high or low, depending on the manager's effort. The owner's promotion decision depends on his beliefs about the manager's ability. Because he derives satisfaction from the wage increase, the manager wants to convey the information that he has high ability. The only indirect way for him to do so is to signal by exerting high effort hence generating high profit in the first period. The loss in first period utility may be offset by the second period gain in wage. High and low ability managers are assumed to have different utilities, $U^H = g(w) - C^H(e)$ and $U^L = g(w) - C^L(e)$. It is assumed that $C^H(e) \leq C^L(e)$ for all e, because a higher ability makes it easier to exert effort, but $C^H(0) = C^L(0) = 0$ (single-crossing property). It could be assumed that the levels of effort that minimise cost are different for the two types. This would amount to consider non-monetary rewards, such as social norms (Okuno-Fujiwara and Postlewaite, 1991) by which managers derive a social prestige from working for a large company or being promoted, inducing them to always play a minimum effort level through a loyalty they have for the company.

A rational owner, knowing that it is in the manager's interest to "lie", i.e. exert high effort in the first period to realise high output and convey the

information that he has high ability, will not necessarily infer high ability from an observation of x_H. In turn, the manager knows that the owner may reason like this, and so on. Hence the only way to solve this game is to look for a perfect Bayesian equilibrium. Strategies for both players are as follows. A pure strategy for the manager is a function x that gives his first period output as a function of his ability, his effort and his probability of promotion. A pure strategy for the owner is a function s that gives the owner's decision (promote:1 or not:0), as a function of the first period output of the manager. A perfect Bayesian equilibrium is then a strategy pair (x^*, s^*), and conjecture (x_c, s_c) such that:

- x^* maximises the manager's payoff given his beliefs s_c about the owner's strategy.

- s^* maximises the owner's payoff given his beliefs x_c about the manager's strategy.

- $(x^*, s^*) = (x_c, s_c)$, i.e. beliefs are correct.

Managers are assumed to always maximise their lifetime utility. As they do not need to keep a reputation in the last period of the game, they always exert their cost-minimising level of effort in the second period, namely zero effort[4]. In this context we have two kinds of equilibria, separating and pooling equilibria. In a separating equilibrium, the high ability (H) manager and the low ability (L) manager realise different output levels. The first period profit then fully reveals managers' abilities to the owner, and the owner promotes one of the high type managers. In a pooling equilibrium, a manager produces the same level of output whether he has high or low ability. The owner then learns nothing about the manager's ability from observing first period profit and his posterior beliefs are identical to his prior ones. He then either promotes one of the N managers at random.

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[4]An overlapping generation model with infinite number of periods, rather than finite (two) periods like in this paper, would not have the assumption of zero effort for divisional managers in the last period. I work in the simple case of two periods because this simplifies exposition and makes the point of this paper. In addition, finite models with zero effort in last periods are generally considered in the literature (e.g. Vickers and Meyer, 1995), for reasons of tractability.

3.2 Equilibria

3.2.1 Separating Equilibria

Two conditions are necessary for a separating equilibrium to exist. Namely, the high ability manager must not want to pick the low ability manager's output level, and vice versa. The owner then observes $x_H^* = e_H + a_H$ and $x_L^* = e_L + a_L$. Since the low type manager knows that he will not be promoted in a separating equilibrium, he has no incentive to expand effort, and therefore plays $e_L = 0$. The high type plays e_H, the value of which will be determined by the conditions for equilibrium to exist. The owner then learns the managers' ability and promotes one of the H type managers in the second period. This is the owner's best response to x^*, since he is sure to promote a H type manager. If the owner observes an output level different from x_H^* or x_L^*, he concludes that the manager concerned has low ability.

A separating equilibria is therefore defined by three conditions.

(i) Beliefs assign a unique vector of beliefs to each strategy profile;

(ii) managers play pure strategies that lead to a different action profile for each type;

(iii) beliefs are consistent with the managers strategies. An important property of the owners beliefs is that the owner assigns to each manager an identical probability of being high ability if managers play the same strategy.

Then one can derive the necessary conditions for existence of separating equilibria.

Lemma 1: The necessary condition for a separating equilibrium to exist is:

$$\frac{U^H(w_2,0) - U^H(w_1,0)}{U^H(w_1,0) - U^H(w_1,e_H)} \geq N_H \geq \frac{U^L(w_2,0) - U^L(w_1,0)}{U^L(w_1,0) - U^L(w_1,e_H)} - 1 \quad (1)$$

For simplicity assume payoffs are not discounted. The H type finds it optimal to exert e_H in the first period if his payoff from doing so exceeds the payoff from deviating. i.e. with

9

$$U^H(w_1, e_H) + \frac{1}{N_H}U^H(w_2, 0) + (1 - \frac{1}{N_H})U^H(w_1, 0) \geq 2U^H(w_1, 0)$$

That is,

$$\frac{U^H(w_2, 0) - U^H(w_1, 0)}{U^H(w_1, 0) - U^H(w_1, e_H)} \geq N_H$$

In the second period, the H type manager expends $e_H = 0$ because he no longer needs to build reputation. The condition for the L type manager is,

$$U^L(w_1, e_H + \Delta a) + \frac{1}{N_H + 1}U^L(w_2, 0) + (1 - \frac{1}{N_H + 1})U^L(w_1, 0) \leq 2U^L(w_1, 0),$$

where

$$\Delta a = a_H - a_L$$

because for x_H to be the same for both types of managers, the low type must realise

$$x_H = a_L + e_H + a_H - a_L = a_L + (e_H + \Delta a)$$

when the high type realises

$$x_H = a_H + e_H$$

Besides, since N_H is managers' private knowledge, the deviating low type manager is included in the pool of candidates for promotion; the owner believes that there are $(N_H + 1)$ high ability managers.

$$N_H \geq \frac{U^L(w_2, 0) - U^L(w_1, 0)}{U^L(w_1, 0) - U^L(w_1, e_H + \Delta a)} - 1$$

The first inequality implies that N_H must not be too large for the H type managers to be willing to separate. Otherwise their probability of promotion would be too low even after separation.

10

Similarly, the second inequality shows that as long as his disutility of effort is larger than his gain in utility due to a wage increase, the low ability manager will always be willing to separate. The high ability manager may exert strictly positive effort in order to separate, depending on the cost of pooling for the low ability manager. Notice that if the cost of effort to the low ability manager is very high, then the high types do not even need to expand effort and can play $e = 0$ at the separating equilibrium; that case is equivalent to the full information case.

Notice also that the low ability managers play zero effort at a separating equilibrium whatever the owners system of beliefs. To see this, assume the low type exerts $e' \neq 0$ at equilibrium. Then either he is still believed to be a low type and $e = 0$ is a dominating strategy, or he is believed to be low type with a probability $p < 1$, he is still not promoted but has a higher cost of effort $(C^L(0) = 0, C^L(e) > 0)$, hence lower utility. Therefore, in either cases he chooses $e = 0$ which is the dominating strategy.

It is assumed that managers' utility functions are separable and given by $U^T(w, e) = g(w) - C^T(e)$, $T = H, L$. Therefore, condition (1) above can be written as:

$$C^H(e_H) \leq \frac{(g(w_2) - g(w_1))}{N_H}$$

$$C^L(e_H + \Delta a) \geq \frac{(g(w_2) - g(w_1))}{(N_H + 1)}$$

Given that C^H and C^L are continuous and strictly increasing in e, with $C^H(e) \leq C^L(e)$ for all e, the single-crossing property holds and there exist separating equilibria e_H such that:

$$(C^L)^{-1} \left[\frac{g(w_2) - g(w_1)}{N_H + 1} \right] - \Delta a \leq e_H \leq (C^L)^{-1} \left[\frac{g(w_2) - g(w_1)}{N_H} \right] \quad (2)$$

On the other hand, if this condition is satisfied, low types managers will not play e_H because their resulting payoff will be less than the payoff from exerting zero effort in both period. High type managers find it profitable to exert limit effort, thereby building a reputation for high ability and

11

having some non zero probability of promotion. Hence condition (2) is also a sufficient condition; this leads to proposition 1.

Proposition 1: For utility functions additively separable, with continuous, strictly increasing effort cost functions, the single property holds and therefore there exist separating equilibria defined by inequality (2).

There is investment in reputation if the high ability managers exert at equilibrium an effort level higher than zero. The least-cost separating effort is higher than zero if

$$(C^L)^{-1} \left[\frac{g(w_2) - g(w_1)}{N_H + 1} \right] - \Delta a > 0$$

Therefore, for cost of effort functions such that

$$(C^L)^{-1} \left[\frac{g(w_2) - g(w_1)}{N_H + 1} \right] > \Delta a$$

, there is investment in reputation due to the signalling activity. I consider that case.

The belief structure under which this equilibrium holds can be explicitly defined by three conditions.

(i) If all managers play x_H such that (2) holds, owners believe all managers have high ability;

(ii) If one manager plays x_H satisfying (2) while all other managers exert zero effort, then that manager is believed to be a high type and all other managers to be low types, unless they also play x_H.

(iii) The owner believes any manager playing a strategy different from (i) and (ii) to be of low ability.

Given the structure of beliefs, if all high ability managers play the least-cost limit effort no one of them deviates to play a higher limit effort, because the owner will think that the deviating high ability manager is a low type.

A crucial assumption in this model is that a low type is much less efficient than a high type as a CEO. Therefore the owner earns much higher profits when the CEO is high ability than low ability, and it matters more to the owner which manager fills which job than how much effort is induced in the

12

first period by the investment in reputation. The effects of the incentive system on the owner's profit are first, an increase in the first period due to higher effort exerted to signal ability, and second, a higher profit due to the CEO being high ability. Since the second effect is assumed larger than the second one, the assumed structure of beliefs make sense. Otherwise, it could be assumed that the owner promotes for sure any upward deviating manager. Then both types would have higher incentives in the first period to increase effort, but with the drawback that for certain cost of effort functions, the low types may exert the highest first-period effort, and be promoted as CEO. This is appropriate if the first effect is higher than the second one.

To illustrate these general results, I now consider a specific example. Let $g(w) = w, C^H(e) = e$ and $C^L(e) = 2e$, so that low ability managers find it more costly to expand effort. Condition (2) can be rewritten

$$\frac{(w_2 - w_1)}{2(N_H + 1)} - \Delta a \le e_H \le \frac{(w_2 - w_1)}{N_H} \tag{3}$$

which implies that N_H must not be too small for the low type manager to find more profitable to separate on e_H. (3) defines an interval for separating equilibria $[e_0, e_1]^5$, such that

$$e_0 = \frac{(w_2 - w_1)}{2(N_H + 1)} - \Delta a \tag{4}$$

and

$$e_1 = \frac{(w_2 - w_1)}{N_H} \tag{5}$$

Since the high ability managers exerts the least-cost separating effort at equilibrium, $e_H = e_0$ at equilibrium. Besides this, high ability managers effort level is never zero at equilibrium unless the effort cost of low ability managers is so high that they cannot expand effort $e = a_H - a_L$, so that $x_L = x_H = a_H$. Consequently there exists a least-cost separating equilibrium $(x_H^* = a_H + e_0; x_L^* = a_L)$. such that e_0 is given by (4).

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[5]It can be shown that $e_o < e_1$ always holds.

For given wages, the owner has no incentive to deviate from his separating equilibrium strategy, because his resulting payoff Π_s is larger than his full information payoff, due to managers' overinvestment in reputation.

The owner's profit in the separating equilibrium is as follows.

First-period payoffs:

$$N_H.\Pi(e_H + a_H, a_H^*) + (N - N_H)\Pi(a_L, a_H^*) - Nw_1 - w_2$$

where the first term is the profit earned by the N_H divisions that are managed by high ability managers, the second term is the profit earned by the remaining divisions, which are managed by low ability managers, the third term is the wages paid to the divisional managers, and the final term is the wage of the CEO.

Second-period payoff:

$$N_H\Pi(a_H, a_H^*) + (N - N_H)\Pi(a_L, a_H^*) - Nw_1 - w_2$$

The first term is the profit earned by the N_H divisions that are managed by high ability managers. Since their ability has been revealed by their actions in the first period, and since this is a two-period model, they have no incentive to exert extra effort in the second period. This is an unrealistic implication of the model that would not arise in an overlapping generations model. Recognising this, I choose to work with a two-period model for simplicity. The remaining terms in the second-period payoff are interpreted in the same way as the corresponding terms in the first-period payoff.

Adding, the overall objective function of the owner is

$$\Pi_s = N_H\Pi(e_H + a_H, a_H^*) + 2(N - N_H)\Pi(a_L, a_H^*) + N_H\Pi(a_H, a_H^*) - 2Nw_1 - w_2 \tag{6}$$

The owners posterior probability is $\rho(a_H) = N_H/N$. The full information payoff is where all managers play their cost-minimising effort level (zero) and the owner promotes one high ability manager for sure. The owner then obtains,

$$\Pi_d = 2N_H.\Pi(a_H, a_H^*) + 2(N - N_H)\Pi(a_L, a_H^*) - 2Nw_1 - w_2 \tag{7}$$

14

Hence,

$$\Pi s - \Pi d = N_H.\Pi(e_H + a_H, a_H^*) - N_H.\Pi(a_H, a_H^*)$$

which is positive since effort is positively correlated with output, hence profit, and $e_H > 0$. This shows explicitly that the gain to the owner results exclusively from managers investment in reputation, and this establishes proposition 2, valid for a large class of parameters.

Proposition 2: For exogenous wages, the owner is better-off in the separating equilibrium relative to the full information equilibrium, thanks to the high ability managers' investment in reputation.

3.2.2 Pooling Equilibria

In a pooling equilibrium both types play the same level of output. One such equilibrium is one in which $x_H^* = x_L^* = x_p$. High ability managers in this case do not find it profitable to expand effort in order to be separated, and they let low ability managers imitate them. Conditions for this to occur are given below. The owner's best response is:

$s^*(x \neq x_p) =$ "no promotion"

$s^*(x = x_p) =$ "promote one manager at random"

Suppose the owner follows s^*. s^* is a best response to x^* since the owner would not promote a manager realising an output different from x_p who may be a low ability manager. The structure of beliefs supporting this equilibrium is the following.

(i) If all managers play the same output, the owner promotes one of them at random.

(ii) If one (or more) manager plays a different output from all other managers, the owner believes he has low ability, unless he (they) play the separating output level. Is x^* a utility-maximising response for managers ? Formally, the condition for managers to find it optimal to follow this strategy is as follows.

Lemma 2: The necessary condition for pooling equilibria to exist is

$$C^L(e_{pL}) \leq \frac{(g(w_2) - g(w_1))}{N} \qquad (8)$$

15

Managers implicit incentive to increase effort is e_P.

In the pooling equilibrium, high and low ability managers realise the same output level but they exert different effort levels:

$$x_p = e_{PH} + a_H = e_{PL} + a_L$$

So that

$$e_{PL} = e_{PH} + (a_H - a_L)$$

The low type manager finds it optimal to mimic his high type counterpart in the first period if his payoff from doing so is higher than his payoff from deviating, given that all other managers pool on x_P. The condition is,

$$U^L(w_1, e_{PL}) + \frac{1}{N} U^L(w_2, 0) + (1 - \frac{1}{N}) U^L(w_1, 0) \geq 2 U^L(w_1, 0)$$

That is, given the additive separability assumptions for the utility function,

$$N \leq \frac{U^L(w_2, 0) - U^L(w_1, 0)}{U^L(w_1, 0) - U^L(w_1, e_{PL})}$$

$$C^L(e_{pL}) \leq \frac{(g(w_2) - g(w_1))}{N} \tag{9}$$

Deviation means exerting zero effort level in both period. The reason is that the assumed out-of-equilibrium beliefs are that the owner thinks that any manager playing an effort level different from the equilibrium level are likely to be low types. Again the above inequality shows that managers exert an effort higher than zero, thereby pooling, only if the gain from doing so is higher than its cost.

Similarly, the H manager follows the equilibrium strategy if and only if

$$U^H(w_1, e_{PH}) + \frac{1}{N} U^H(w_2, 0) + (1 - \frac{1}{N}) U^H(w_1, 0) \geq 2 U^H(w_1, 0)$$

That is,

$$N \leq \frac{U^H(w_2, 0) - U^H(w_1, 0)}{U^H(w_1, 0) - U^H(w_1, e_{PH})}$$

16

or

$$C^{H}(e_{pH}) \leq \frac{(g(w_2) - g(w_1))}{N} \tag{10}$$

Since $e_{PH} < e_{PL}$ and $C^{L}(e) \geq C^{H}(e)$ for all e, (7) implies (8). Hence lemma 2.

Since the single-crossing condition holds, pooling equilibria do exist. The interval of pooling effort levels is then defined by e such that (9) holds. This is $e \in [a_H - a_L, e_2]$, with

$$e_2 = (C^{L})^{-1} \left[\frac{g(w_2) - g(w_1)}{N} \right] \tag{11}$$

The least-cost pooling effort for high ability manager is zero, hence $x_p = a_H$, implying an effort level of $(a_H - a_L)$ for low ability managers. In addition if all managers pool on x_P such that condition (9) holds, then no manager deviates because he would get a lower utility. Hence proposition 3.

Proposition 3: The interval for pooling equilibria is given by e_{PH} and e_{PL} such that

$$e_P = e_{PH} + (a_H - a_L)$$

$$e_P \in [a_H - a_L, e_2],$$

with

$$e_2 = (C^{L})^{-1} \left[\frac{g(w_2) - g(w_1)}{N} \right]$$

The low ability manager invests in reputation in that equilibrium.

Pooling and separating equilibria should now be compared, in order to check the coherence of the results. e_P is given by (9), and e_H by (2). Since the effort cost functions are continuous and strictly increasing, so are their inverse functions. In addition, for $N \geq N_H + 1$,

$$\frac{g(w_2) - g(w_1)}{N_H + 1} \geq \frac{g(w_2) - g(w_1)}{N}$$

Consequently, the minimum possible separating effort is higher than the maximum pooling effort, and the intervals for pooling and for separating

17

equilibria do not intersect.[6] For given effort costs, the higher the proportion of high ability managers in the firm, the more pooling is likely since the probability of promotion of high ability managers is low anyhow. The lower the number of divisions, and the lower the cost of effort for the low ability manager, the more likely is a pooling equilibrium to arise.

The owner's profit in the least-cost pooling equilibrium is as follows.

The first-period payoff is

$$N\Pi(a_H, a_H^*) - w_2 - Nw_1$$

where all divisional managers realise output $x_p = a_H$.

The second-period payoff is

$$\frac{N_H}{N}\left[N_H.\Pi(a_H, a_H^*) + (N - N_H)\Pi(a_L, a_H^*)\right] +$$

$$\frac{N - N_H}{N}\left[N_H.\Pi(a_H, a_L^*) + (N - N_H)\Pi(a_L, a_L^*)\right] - w_2 - Nw_1$$

where the first term is the payoff got from promoting a high ability manager, time the probability of choosing a high ability manager, the second term is the profit obtained when promoting a low ability manager, times the probability of choosing such a CEO. The last terms are the wages given to managers in the second-period.

Adding, the pooling equilibrium payoff to the owner is

$$
\begin{aligned}
\Pi p = \ & N.\Pi(a_H, a_H^*) - 2w_2 - 2Nw_1 + \\
& \frac{N_H}{N}\left[N_H.\Pi(a_H, a_H^*) + (N - N_H)\Pi(a_L, a_H^*)\right] + \\
& \frac{N - N_H}{N}\left[N_H.\Pi(a_H, a_L^*) + (N - N_H)\Pi(a_L, a_L^*)\right]
\end{aligned}
\tag{12}
$$

The payoff to the owner is increased in the first period, relative to the case of known ability, because the L manager exerts higher effort; however in the second period the expected payoff to the owner is reduced because

---

[6]I do not consider other effort levels because I focus on pure strategy equilibria.

he may promote a low ability manager. who realises a low output in the second period. His posterior x is $1/N$, equal to his prior probability.

In order to compare this with the separating equilibrium, one has to compute

$$
\begin{aligned}
\Pi s - \Pi p = {}& N_H.\Pi(e_H + a_H, a_H^*) + 2(N - N_H)\Pi(a_L, a_H^*) + N_H.\Pi(a_H, a_H^*) - \\
& \frac{N_H}{N}[N_H.\Pi(a_H, a_H^*) + (N - N_H)\Pi(a_L, a_H^*)] - \\
& \frac{N - N_H}{N}[N_H.\Pi(a_H, a_L^*) + (N - N_H)\Pi(a_L, a_L^*)]
\end{aligned}
$$

One can rewrite this

$$
\begin{aligned}
& N_H.\Pi(e_H + a_H, a_H^*) - N.\Pi(a_H, a_H^*) + \\
& \frac{N - N_H}{N}[(2N - N_H).\Pi(a_L, a_H^*) - (N - N_H)\Pi(a_L, a_L^*)] + \\
& N_H \frac{N - N_H}{N}[\Pi(a_H, a_H^*) - \Pi(a_H, a_L^*)]
\end{aligned}
$$

Hence the payoff in the separating equilibrium is higher than in that in the pooling equilibrium if

$$
\begin{aligned}
& N_H.\Pi(e_H + a_H, a_H^*) + \frac{N - N_H}{N}[(2N - N_H).\Pi(a_L, a_H^*) - (N - N_H)\Pi(a_L, a_L^*)] \\
& N_H \frac{N - N_H}{N}[\Pi(a_H, a_H^*) - \Pi(a_H, a_L^*)] > N.\Pi(a_H, a_H^*)
\end{aligned}
$$

I assume that $\Pi(., a_H^*) - \Pi(., a_L^*) \geq \Pi(a_H, .) - \Pi(a_L, .)$, so that the difference in profit due to different CEO ability is higher than or equal to the difference in profit due to the different ability of the divisional manager. This implies for the owner the importance in promoting a high ability manager as CEO. Then the above inequality is equivalent to

$$
N_H.\Pi(e_H + a_H, a_H^*) + (N - N_H)[\Pi(a_H, a_H^*) - \Pi(a_H, a_L^*)] > N[\Pi(a_H, a_H^*) - \Pi(a_L, a_L^*)]
$$

Given the above assumption on profit differences,

19

$$(N - N_H)\left[\Pi(a_H, a_H^*) - \Pi(a_H, a_L^*)\right] > (N - N_H)\left[\Pi(a_H, a_H^*) - \Pi(a_L, a_H^*)\right]$$

Besides this,

$$N_H.\Pi(e_H + a_H, a_H^*) > N_H\left[\Pi(a_H, a_H^*) - \Pi(a_L, a_H^*)\right]$$

Adding the latter two inequalities, one obtains

$$\Pi s > \Pi p$$

Hence proposition 4.

Proposition 4: The owner is better-off in a separating equilibrium than in a pooling equilibrium.

4 Screening Model in the Monopoly

4.1 Wage endogenisation in the monopoly

Consider a fixed number of divisions. The owner can then set first and second period wages to induce a separating equilibrium. The owners maximisation problem is then, for $w_i \in [w_0, w_m], i = 1, 2$ (w_m is the maximum wage that the owner could possibly pay)[7]:

$$Max_{w_1, w_2}\Pi s = N_H.\Pi(e_H + a_H, a_H^*) + 2(N - N_H)\Pi(a_L, a_H^*) + N_H.\Pi(a_H, a_H^*) - 2Nw_2 \quad (13)$$

subject to

$$(a)\ C^L(e_H)\ =\ \frac{g(w_2) - g(w_1)}{N_H + 1} \quad (14)$$
$$(b)\ w_1\ \geq\ w_0$$

---

[7]The owner cannot set a wage lower than w_0, because managers would never choose to work for the firm, having the possibility to work for higher wage elsewhere.

Since managers minimise effort, they will separate on the least-cost separating effort, hence (a). Since C^L is convex and continuous, $(C^L)^{-1}$ exists and has continuous first and second derivatives . Also Πs has continuous first and second derivatives. 14(a) implies

$$\frac{\partial e_H}{\partial w_1} = -\frac{g'(w_1)}{N_H+1}(C^L)^{-1'}\left(\frac{g(w_2)-g(w_1)}{N_H+1}\right)$$

$$\frac{\partial e_H}{\partial w_2} = \frac{g'(w_2)}{N_H+1}(C^L)^{-1'}\left(\frac{g(w_2)-g(w_1)}{N_H+1}\right)$$

The Lagrangian of the above maximisation problem is (12)

$$L = N_H\Pi(e_H+a_H,a_H^*)+2(N-N_H)\Pi(a_L,a_H^*)+N_H\Pi(a_H,a_H^*)-2Nw_1-2w_2+\lambda(w_1$$

The Kuhn-Tucker conditions are

$$(a)\frac{\partial L}{\partial w_1} = N_H\frac{\partial\Pi(e_H+a_H,a_H^*)}{\partial e_H}\frac{\partial e_H}{\partial w_1}-2N+\lambda=0 \qquad (15)$$

$$(b)\frac{\partial L}{\partial w_2} = N_H\frac{\partial\Pi(e_H+a_H,a_H^*)}{\partial e_H}\frac{\partial e_H}{\partial w_2}-2=0$$

$$(c)\frac{\partial L}{\partial \lambda} = w_1-w_0\geq 0; \lambda(w_1-w_0); \lambda\geq 0$$

Equation 15(a) implies

$$\lambda = -N_H\frac{\partial\Pi(e_H+a_H,a_H^*)}{\partial e_H}\frac{\partial e_H}{\partial w_1}-2N+\lambda=0+2N$$

Since $\frac{\partial e_H}{\partial w_1}<0$, $\lambda>0$, the constraint is binding and $w_1=w_0$. The first period wage is determined on the external managerial labour market, where divisional managers are hired in the first period.

λ can be interpreted as the marginal loss in profit due to the lower incentive to expand effort. Since this is positive, the owner sets the first period wage at the lowest possible level, namely w_0.

w_2 is set where (b) holds, that is where

21

$$N_H \frac{\partial \Pi(e_H + a_H, a_H^*)}{\partial e_H} \frac{\partial e_H}{\partial w_2} = 2 \qquad (16)$$

Since Πs is a composition of functions $g(.)$, $(C^L)^{-1}$, $\Pi(.)$,, which are continuous and differentiable in R^+, Πs is also continuous and differentiable on $[w_0, w_m]$, a compact set. By the Weierstrass theorem, (b) has solutions. For this solution to be a maximum, the second order condition must be negative.

I assume the second order condition is met[8]. This leads to the following result.

Proposition 5: The wage contract (w_1, w_2), where

$$w_1 = w_0$$

and w_2 is solution to

$$\frac{\partial \Pi(e_H + a_H, a_H^*)}{\partial e_H} \frac{\partial e_H}{\partial w_2} = \frac{2}{N_H} \qquad (17)$$

induces a separating equilibrium.

To illustrate this result, consider the example where $g(w) = w$ and $C^L(e) = 2e$. Profit is specified as follows. Managers activity is aimed at minimising divisional marginal costs, so that the latter are given by $c = \beta - a - e$. Each division faces identical demand, which is given by $p = A - q$, where q is the quantity produced in each division. Then managers set simultaneously effort and quantity produced to maximise each divisions gross profit $\Pi(x)$, with $\Pi(e) = (A - \beta + a + e)^2/4$, where a is the ability of the CEO to coordinate the firms activities. $\Pi(e_H + a_H, a_H^*)$ is then given by

$$\Pi(e_H + a_H, a_H^*) = \frac{(A - \beta + a_H + a_H^* + e_H)^2}{4}$$

Therefore (17) can be solved for w_2

---

[8] Alternatively, Hestenes (quoted by Takayama, 1990), shows that:
- If there exists solutions to the optimisation problem,
- The rank of $[\frac{\partial(w_2 - w_0)}{\partial w_2}, \frac{\partial(C^L)^{-1}(.) - e_H}{\partial w_2}]$ is equal to 2,

Then the second order conditions hold.

22

$$w_2 = w_0 + 2(N_H + 1)\left[\frac{8(N_H + 1)}{N_H} - (A - \beta + a_H + a_H^*)\right]$$

A numerical example is $A = 10$, $\beta = 5$, $a_H = 2$, $a_H^* = 1$, and $N_H = 5$ out of 10 divisions. Then

$$w_2 = w_0 + 19.2$$

which is quite a substantial wage increase necessary to make high ability managers investment in reputation worthwhile. The separating equilibrium limit effort is then $e_H = 1.6$.

Corollary 1: For $g(w) = w$ and $C^L(e) = 2e$, the wage contract (w_1, w_2) where

$$w_1 = w_0$$

$$w_2 = w_0 + 2(N_H + 1)\left[\frac{8(N_H + 1)}{N_H} - (A - \beta + a_H + a_H^*)\right]$$

induces high ability managers to separate on the limit effort

$$e_H = \frac{8(N_H + 1)}{N_H} - (A - \beta + a_H + a_H^*)$$

In order to see how will w_2 respond to an exogenous change in some of the parameters, I carry out some comparative statics. These will ascertain how w_2 would have differed from what it is in this equilibrium had the parameters been slightly different.

The first-order condition on w_2 is

$$\frac{\partial\Pi(e_H + a_H, a_H^*)}{\partial e_H}\frac{\partial e_H}{\partial w_2} = \frac{2}{N_H}$$

or,

$$\Pi_1 \frac{\partial e_H}{\partial w_2} = \frac{2}{N_H} \tag{18}$$

23

Now, take the derivative of (16) with respect to a_H.

$$\Pi_{11} \frac{\partial e_H}{\partial w_2} \left[\frac{\partial e_H}{\partial w_2} \frac{\partial w_2}{\partial a_H} + 1 \right] + \Pi_1 \frac{\partial^2 e_H}{\partial w_2^2} \frac{\partial w_2}{\partial a_H} = 0$$

$$\frac{\partial w_2}{\partial a_H} \left[\Pi_{11} (\frac{\partial e_H}{\partial w_2})^2 + \Pi_1 \frac{\partial^2 e_H}{\partial w_2^2} \right] = -\Pi_{11} \frac{\partial e_H}{\partial w_2} \tag{19}$$

The term in bracket is negative by the sign of the second order condition for profit maximisation,

and the RHS is negative, which results in

$$\frac{\partial w_2}{\partial a_H} > 0 \tag{20}$$

This means that the second period wage increases with the ability of the high types. The owner finds it more profitable to induce a separating equilibrium for greater ability level of the high types.

Now, take the derivative of (16) with respect to N_H. This is

$$\frac{\partial w_2}{\partial N_H} \left[\Pi_{11} (\frac{\partial e_H}{\partial w_2})^2 + \Pi_1 \frac{\partial^2 e_H}{\partial w_2^2} \right] = -\frac{2}{(N_H)^2} \tag{21}$$

Since the term in square brackets is negative,

$$\frac{\partial w_2}{\partial N_H} > 0 \tag{22}$$

The interpretation is that the more numerous the high ability managers, the lower the probability of promotion for each of them. Hence at the separating equilibrium the second period wage has to be higher to induce the same limit effort.

4.2 Alternative incentive contract

In the case where owners were not concerned about choosing a CEO among divisional managers, this section analyses whether the owner would be better-off offering the wage contract (w_1, w_2) in the first period and give

24

the wage increase to all managers revealing high type. Then in a separating equilibrium, managers revealing high type will be promoted for sure. This raises the likeliness of a separating equilibrium, as shown in the following lemma.

The condition for the high ability managers to separate on a limit effort e_H in this case is

$$U^H(w_1, e_H^2) + U^H(w_2, 0) \geq 2U^H(w_1, 0)$$

That is, given the assumption of additively separable utility,

$$C^H(e_H^2) \leq g(w_2) - g(w_1) \tag{23}$$

For the low ability manager,

$$U^L(w_1, e_H^2 + \Delta a) + U^L(w_2, 0) \leq 2U^L(w_1, 0)$$

That is,

$$C^L(e_H^2 + \Delta a) \geq g(w_2) - g(w_1) \tag{24}$$

Hence lemma 3.

Lemma 3: In the case where all managers revealing high ability are given a wage increase, separating equilibria are defined by conditions (23) and (24).

In addition, since the assumption about utility functions are the same as in the previous sections, the conditions for existence of separating equilibria are verified, leading to the following proposition.

Proposition 6: There exists a least-cost separating equilibrium ($x_H^* = e_0^2; x_L^* = 0$), such that

$$e_0^2 = (C^L)^{-1} [g(w_2) - g(w_1)] - \Delta a$$

where e_0^2 measures high ability managers' implicit incentives.

Now I need to compute the wage contract(s) enforcing a separating equilibrium, and then compare the owners equilibrium profit in both cases of promotion of one or all high ability managers.

The owner's payoff when he gives the wage increase to all high ability managers is Πs^2 such that

$$\Pi s^2 = N_H.\Pi(e_H^2 + a_H, a_H^*) + 2(N - N_H)\Pi(a_L, a_H^*) + N_H.\Pi(a_H, a_H^*) \tag{25}$$
$$-(2N + 1 - N_H)w_1 - (N_H + 1)w_2$$

The owner sets w_1 and w_2 to maximise Πs^2 subject to the constraints

$$(a)\ C^L(e_H^2) = g(w_2) - g(w_1)$$
$$(b)\ w_1 \geq w_0$$

The Lagrangian for this problem is

$$L = N_H\Pi(e_H^2 + a_H, a_H^*) + 2(N - N_H)\Pi(a_L, a_H^*) + N_H\Pi(a_H, a_H^*)$$
$$-(2N + 1 - N_H)w_1 - (N_H + 1)w_2 + \mu(w_1 - w_0)$$

Kuhn Tucker conditions are

$$(a)\frac{\partial L}{\partial w_1} = N_H\frac{\partial\Pi(e_0^2 + a_H, a_H^*)}{\partial e_0^2}\frac{\partial e_0^2}{\partial w_1} - (2N + 1 - N_H) + \mu = 0 \tag{26}$$

$$(b)\frac{\partial L}{\partial w_2} = N_H\frac{\partial\Pi(e_0^2 + a_H, a_H^*)}{\partial e_0^2}\frac{\partial e_0^2}{\partial w_2} - (N_H + 1) = 0$$

$$(c)\frac{\partial L}{\partial \mu} = w_1 - w_0 \geq 0; \mu(w_1 - w_0); \mu \geq 0$$

This problem is solved in the same way as in the previous section, leading to the solution

$$w_1 = w_0$$

and w_2 such that

26

$$\frac{\partial \Pi(e_0^2 + a_H, a_H^*)}{\partial e_0^2} \frac{\partial e_0^2}{\partial w_2} = \frac{N_H + 1}{N_H} \qquad (27)$$

Let owners' profit be Πs^1 and second period wage w_2^1 in the case in which they promote one manager, Πs^2 and w_2^2 when they reward all managers revealing high type. These can be computed as

$$\begin{aligned}
\Pi s^1 &= N_H \Pi(e_H + a_H, a_H^*) + 2(N - N_H)\Pi(a_L, a_H^*) + N_H \Pi(a_H, a_H^*) \\
&\quad -2N w_1 - 2 w_2
\end{aligned}$$

$$\begin{aligned}
\Pi s^2 &= N_H \Pi(e_0^2 + a_H, a_H^*) + 2(N - N_H)\Pi(a_L, a_H^*) + N_H \Pi(a_H, a_H^*) \\
&\quad -(2N + 1 - N_H)w_1 - (N_H + 1)w_2
\end{aligned}$$

This yields

$$\begin{aligned}
\Pi s^2 - \Pi s^1 &= N_H \left[\Pi(e_0^2 + a_H, a_H^*) - \Pi(e_H + a_H, a_H^*) \right] - (N_H + 1)w_2^2 - 2w_2^1 \\
&\quad +(N_H - 1)w_0
\end{aligned}$$

Therefore if w_2^1 and w_2^2 are such that

$$N_H \left[\Pi(e_0^2 + a_H, a_H^*) - \Pi(e_H + a_H, a_H^*) \right] > (N_H + 1)(w_2^2 - w_0) - 2(w_2^1 - w_0)$$

It is more profitable for the owner to reward one high ability manager.

As an example, if $g(w) = w$, $C^L(e) = 2e$, $C^H(e) = e$, the equilibrium wage contract and implied limit effort in the case of one promotion are given by Corollary 1. The wage contract inducing a separating equilibrium in the case in which the owner rewards all managers revealing high ability is,

$$w_2^2 = w_0 + 2\frac{N_H + 1}{N_H} - 2(A - \beta + a_H + a_H^*)$$

and the induced limit effort is

27

$$e_0^2 = \frac{N_H + 1}{N_H} - (A - \beta + a_H + a_H^\star)$$

Hence

$$\Pi s^2 - \Pi s^1 = -4(N_H + 1)\left[2\frac{(N_H + 1)}{N_H} - (A - \beta + a_H + a_H^\star)\right] < 0$$

Therefore the owner finds it more profitable to promote one high ability manager only. The reason for this is that the gain to the owner from managers investment in reputation is offset by the increase in the wage cost resulting from the scheme which promotes all managers signalling high ability.

Proposition 7: For $g(w) = w$, $C^L(e) = 2e$, $C^H(e) = e$, it is more profitable for the owner to promote only one manager rather than offer a wage increase to all managers that reveal high ability.

5 Model in a Duopoly

The duopoly has two firms, each with N divisions, hence N managers. Each manager competes in a Cournot duopoly with the manager heading a similar division in the rival firm. Strategic considerations affect the contracts offered to managers. The two firms are denoted by $i = 1, 2$. Their cost functions are negatively correlated with effort, and demand is given exogenously, as in the previous section. The signalling game is the same as in section II. Since both firms are symmetric and face the same demand and technology conditions, Bayesian updating should lead to the same conclusion. Hence I assume that both owners form the same beliefs as to managers' types after observing managers' actions[9].

Once again two types of equilibria, pooling and separating, arise. It is assumed that divisional managers cannot change firm during the two periods considered (the contract they sign at the beginning of the first period stipulates that they work for at least two periods in the same firm).

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[9]This assumption might be simplistic; at this stage of my research it is enough to make my point.

Since duopolists are assumed symmetric. equilibria are the same as in the monopoly case. Hence only separating equilibria are envisaged below. What differ in the duopoly are the equilibrium wage contracts.

5.1 Wage determination

Consider the condition for separating equilibria. The high ability managers in each firm find it optimal to exert limit effort e_H in the first period if their payoff from doing so exceeds the payoff from deviating,

$$U^H(w_1, e_H) + yU^H(w_2, 0) + (1 - y)U^H(w_1, 0) \geq 2U^H(w_1, 0)$$

That is,

$$\frac{U^H(w_2, 0) - U^H(w_1, 0)}{U^H(w_1, 0) - U^H(w_1, e_H)} \geq N_H$$

In the second period, the H type manager expends $e_H = 0$ because he no longer needs to build reputation. The condition for the L type manager is,

$$U^L(w_1, e_H + \Delta a) + yU^L(w_2, 0) + (1 - y)U^L(w_1, 0) \leq 2U^L(w_1, 0)$$

or

$$N_H \geq \frac{U^L(w_2, 0) - U^L(w_1, 0)}{U^L(w_1, 0) - U^L(w_1, e_H + \Delta a)} - 1$$

Therefore as in section 3, The least-cost separating equilibrium is

$$C^L(e_H) = \frac{g(w_2) - g(w_1)}{N_H + 1} - \Delta a$$

Owners can then set w_2 to force a separating equilibrium. In the duopoly, the resulting higher effort will shift the firm's reaction function and increase output and profit at equilibrium at the expense of the rival firm. Assume again that managers activities aim at reducing the firms marginal costs[10],

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[10]This is what is usually assumed in the literature (see Levinthal, 1988).

so that as x increases, the marginal cost of the firm reduces. Also assume both firms in each duopoly face the same demand and technology. Demand is linear and given by $p = A - q_1 - q_2$, while costs are given by $c(q_i) = (\beta - x_i)q_i - W$, where W is a vector of wages paid to managers, and $i = 1, 2$.

Then the duopoly equilibrium is at the crossing point of reaction functions, which are computed as

$$q_i = \frac{A - \beta + x_i - q_j}{2}; i, j = 1, 2; i \neq j \tag{28}$$

where $x_i, i = 1, 2$, are the divisional output in each firm, such that

$$x_i = a_H + a_H^* + e_i$$

a_H^* is the CEO's contribution to the divisional profits, assumed the same in the two firms (for simplicity). Then the profit in each division is the usual Cournot duopoly profit,

$$\Pi_i = \frac{(A - \beta + 2x_i - xj)^2}{9} \tag{29}$$

for $i, j = 1, 2, i \neq j$. This shows that divisional profits depend on the limit effort exerted in the competing division of the rival firm.

In firm 1, the owner's payoff in a separating equilibrium is

$$\begin{aligned}
\Pi s_1 =\ & N_H \Pi(e_H(w_2^1) + a_H, a_H^*, e_H(w_2^2)) \tag{30} \\
& + 2(N - N_H)\Pi(a_L, a_H^*, e_H(w_2^2)) \tag{31} \\
& N_H \Pi(a_H, a_H^*, e_H(w_2^2)) - 2w_2^2 - 2Nw_1^1
\end{aligned}$$

where

- w_1^1 is the first period wage in firm 1;

- w_2^1 the second period wage in firm 1;

- w_2^2 the second period wage in firm 2;

30

- $e_H(w_2^i), i = 1, 2$, the limit effort of high ability managers as a function of the second period wage in each firm 1 and 2.

The owner of firm 1 therefore set wages to maximise Πs_1 subject to the participation constraint. The Lagrangian can be written as

$$
\begin{aligned}
L &= N_H \Pi(e_H(w_2^1) + a_H, a_H^*, e_H(w_2^2)) \\
&\quad + 2(N - N_H)\Pi(a_L, a_H^*, e_H(w_2^2)) \\
&\quad N_H \Pi(a_H, a_H^*, e_H(w_2^2)) - 2w_2^2 - 2N w_1^1 \\
&\quad + \gamma(w_1^1 - w_0)
\end{aligned}
$$

The Kuhn-Tucker conditions are

$$
\frac{\partial L}{\partial w_1^1} = N_H \frac{\partial \Pi(e_H(w_2^1) + a_H, a_H^*, e_H(w_2^2))}{\partial w_1^1} - 2N + \gamma = 0 \qquad (32)
$$

$$
\begin{aligned}
\frac{\partial L}{\partial w_2^1} &= N_H \frac{\partial \Pi(e_H(w_2^1) + a_H, a_H^*, e_H(w_2^2))}{\partial e_H(w_2^1)} \frac{e_H(w_2^1)}{\partial w_2^1} \\
&\quad + N_H \frac{\partial \Pi(e_H(w_2^1) + a_H, a_H^*, e_H(w_2^2))}{\partial w_2^2} \frac{\partial w_2^2}{\partial w_2^2} \\
&\quad + 2(N - N_H) \frac{\partial \Pi(a_L, a_H^*, e_H(w_2^2))}{\partial w_2^2} \frac{\partial w_2^2}{\partial w_2^1} \\
&\quad + N_H \frac{\partial \Pi(a_H, a_H^*, e_H(w_2^2))}{\partial w_2^2} \frac{\partial w_2^2}{\partial w_2^1} - 2 = 0
\end{aligned} \qquad (33)
$$

$$
\frac{\partial L}{\partial \gamma} = w_1 - w_0 \geq 0; \gamma(w_1 - w_0); \gamma \geq 0 \qquad (34)
$$

Equation (32) implies that

$$
\gamma = -N_H \cdot \frac{\partial \Pi(e_H(w_2^1) + a_H, a_H^*, e_H(w_2^2))}{\partial w_1^1} + 2N
$$

Since

$$
\frac{\partial \Pi(e_H(w_2^1) + a_H, a_H^*, e_H(w_2^2))}{\partial w_1^1} < 0
$$

31

we have

$$\gamma > 0$$

Therefore the constraint is binding and

$$w_1^1 = w_0$$

By symmetry,

$$w_1^2 = w_0$$

Equation (33) then gives a reaction function in wages, $w_2^1(w_2^2)$, that will then be solved for equilibrium second period wages. From (33)

$$\frac{\partial w_2^2}{\partial w_2^1} \left(\begin{array}{c} N_H \frac{\partial \Pi(e_H(w_2^1)+a_H, a_H^*, e_H(w_2^2))}{\partial w_2^2} \\ +2(N-N_H)\frac{\partial \Pi(a_L, a_H^*, e_H(w_2^2))}{\partial w_2^2} + N_H \frac{\partial \Pi(a_H, a_H^*, e_H(w_2^2))}{\partial w_2^2} \end{array} \right)$$

$$= 2 - N_H \frac{\partial \Pi(e_H(w_2^1) + a_H, a_H^*, e_H(w_2^2))}{\partial e_H(w_2^1)} \frac{e_H(w_2^1)}{\partial w_2^1}$$

Given (28),

$$\frac{\partial \Pi(., e_H(w_2^2))}{\partial w_2^2} < 0$$

Therefore the term in the large brackets is negative. It follows that

$$\frac{\partial w_2^1}{\partial w_2^2} \le 0$$

if and only if,

$$N_H \frac{\partial \Pi(e_H(w_2^1) + a_H, a_H^*, e_H(w_2^2))}{\partial e_H(w_2^1)} \frac{e_H(w_2^1)}{\partial w_2^1} \le 2 \qquad (35)$$

$$\frac{N_H}{N_H + 1} \Pi_1 g'(w_2)(C'^{L})^{-1'} \left(\frac{g(w_2) - g(w_1)}{N_H + 1} \right) \le 2$$

32

Since

$$\frac{N_H}{N_H + 1} < 1$$

$$\Pi_1 g'(w_2)(C^L)^{-1'} \left(\frac{g(w_2) - g(w_1)}{N_H + 1} \right) < 1$$

Therefore (35) holds and the slope of the reaction function in the wage space is negative.

Proposition 8: There exists a wage contract (w_1 , w_2) such that

$w_1 = w_0$

w_2 *is given by (33).*

that induces a separating equilibrium, in which high ability managers exert limit effort. In the duopoly, strategic interactions imply that wages set in one firm depend (negatively) on the wages set in the rival firm.

If the owner in one firm increases the second period wage, limit effort by its high ability managers increases, implying a rise in their divisional output hence a shift of their reaction functions to the right, and an increase in profit at the expense of the rival in the divisional duopoly. This result is as expected, and leads to an endogenisation of the objective and incentive function in the model of Vickers (1985), also considered in Feshtmann and Judd (1987), Sklivas (1987), and others.

5.2 Endogenisation of the profit and sales maximisation objective

Vickers analyses a duopoly with one manager-owned firm and one firm where management is separated from control. In the latter firm, the owner sets an incentive scheme which is a linear combination of profits and sales. Hence managers are asked to maximise $\phi\Pi + (1 - \phi)R$, where $R = pq$ is the revenue from sales. Vickers show that setting $\phi < 1$ actually leads the firm to earn larger payoff than its direct profit-maximising rival. The reason

is that this incentive scheme amounts to inducing managers to reduce marginal cost by ϕ[11]:

$$\phi\Pi + (1 - \phi)R = (p - \phi c)q$$

where c is the marginal cost. Thus a firm which does not maximise directly profit ends up earning larger profit than a firm which directly aims at maximum profit. However, Vickers avoids issues of wage determination and does not analyse precisely the asymmetry of information between the manager and the owner.

Now let the owner of one firm in the duopoly increase the promised second period wage. This induces high ability managers at a separating equilibrium to raise the limit effort they exert, with, writing Δ for variations, $\Delta e = \Delta g(w_2)/(N_H + 1)$, producing a shift of firm 1's reaction function to the right. Reaction functions are indeed given by (28), and the increase in the wage induces q_1^* to increase by $2\Delta e/3$, while q_2^* reduces by $\Delta e/3$. The incentive ϕ that would produce the same shift of firm 1s reaction function is just $\phi c_1 = c_1'$, where c_1 is firm 1's cost before the wage increase, and c_1' the cost after the wage increase. Therefore

$$\phi = \frac{c_1'}{c_1} = \frac{\beta - a_H - a_H^* - (C^L)^{-1}\left[\frac{g(w_2') - g(w_1)}{N_H + 1}\right]}{\beta - a_H - a_H^* - (C^L)^{-1}\left[\frac{g(w_2) - g(w_1)}{N_H + 1}\right]} \qquad (36)$$

$\phi < 1$ because $w_2' > w_2$, and the objective function assumed by Vickers is endogenised. The owner setting an incentive scheme based on wages and promotion induces his managers not to maximise profit, but this turns out to be profitable. ϕ is higher, the higher the effect of the wage increase on limit effort, the latter effect increasing with managers willingness to build reputation, i.e. with the extent of managers implicit incentives. Hence the empirical evidence on regular wage increases given to divisional managers and CEOs can be interpreted as a mean for the owner to induce cost-minimising activities, which increases the firms profit at the expense

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[11] In the model of Vickers (1985), managers' objective function is $O = [p - c + \phi]q$, so that the incentive enters the function additively rather than multiplicatively, as in the function I consider. The effect of the incentive on managers' cost reducing activities in both cases are similar, except than one is additive rather than multiplicative.

of rivals.

6 Conclusions

This model shows that signalling aspects of implicit incentives induce managers to exert effort above their cost of effort-minimising level in order to build a reputation and therefore have a higher probability of promotion. This confirms the observation in reality that top executives do sometimes work very hard in order to climb the hierarchical ladder. This is especially the case for Japanese managers. Also, this result accords with the findings in signalling games[12] that overinvestment tends to occur in all equilibria. Here managers exert higher effort levels than in the full information case in both separating and pooling equilibria, high managers in the former and low ability ones in the latter. The model of this paper can therefore be seen as a reinterpretation of the familiar Spence signalling model to the context of internal labour markets and promotion.

It has been shown that in both cases of a single firm and a duopoly, delegation and asymmetric information is actually profitable to the owner. This is due to the promotion system implying the need for managers to signal their ability, in order to have some chance of promotion. Since higher effort is assumed to produce higher output, the divisional profit of the owner increases as a result of this signalling activity. Limit effort in the separating equilibrium even unambiguously makes the owner better-off, earning a higher profit.

Moreover, this model provides insights as to the need to consider features of firms' internal organisation to account for the relative performance of firms in oligopoly models, rather than focusing exclusively on parameters representing demand and rivals' behaviour. Thus it is shown that a firm (or its owner) can induce an increase in its divisional output and profit at the expense of the rival in the divisional duopoly, by increasing the return to managers' investment in reputation (by raising the second-period wage). The strategic advantage of delegation in oligopolies outlined by Vickers (1985) is therefore given a new and more precise explanation.

---

[12]Spence (1976).

Further research is as follows. First, the assumption that divisional profits are independent may be questionable. The case of correlation of some divisions' outcomes should be further investigated. Second, uncertainty as to demand or cost parameters may be worth studying. A high effort may lead to low profits because of events beyond managers' control, such as an unexpected slump or strike. Lastly, beliefs structures different from that assumed in the model might be investigated, such as the case of uncorrelated beliefs.

References

[1] Akerlof G., 1976, "The Economics of Caste and of the Rat Race and Other Woeful Tales", *Quarterly Journal of Economics*, Symposium 1976, 599-617.

[2] Fama E.F., 1980, "Agency Problems and the Theory of the Firm", *Journal of Political Economy*, 88, 288-307.

[3] Fershtmann C. and Judd K.L., 1987, "Equilibrium Incentives in Oligopoly", *American Economic Review*, 77, 927-940.

[4] Fudenberg, D. and Tirole J., 1989, "Non Co-operative Game Theory for Industrial Organisation: an Introduction and Overview", in *Handbook of Industrial Organisation*, Chapter 5, edited by R. Schmalensee and R. D. Willig, 1989.

[5] Gal-Or E., 1993, "Internal Organisation and Managerial Compensation in Oligopolies", *International Journal of Industrial Organisation*, 11, 157-183.

[6] Gibbons R. and Murphy K., 1992, "Optimal Incentive Contracts in the Presence of Career Concerns: Theory and Evidence", *Journal of Political Economy*, 100, 468-505.

[7] Harrington J.E. and Senbongi S., 1995, "Managerial Reputation and the Competitiveness of an industry", *International Journal of Industrial Organisation*, 13, 95-110.

[8] Holmstrom B., "Managerial Incentive Schemes - A Dynamic Perspective", in *Essays in Economics and Management in Honour of Lars Wahlbeck*. Helsinki: Swenska Handelshgkolan, 1982.

[9] Holmstrom B. and Ricart i Costa J., 1986, "Managerial Incentives and Capital Management", *Quarterly Journal of Economics*, 101, 836-60.

[10] Jensen M.L. and Murphy K., 1990, "Performance pay and top management incentives", *Journal of Political Economy*, 98, 225-264.

[11] Kreps D. and Wilson R., 1982, "Reputation and imperfect information", *Journal of Economic Theory*, 27, 253-79.

[12] Lazear E. and Rosen S., 1981, "Rank-Order Tournaments as Optimal Labour Contracts", *Journal of Political Economy*, 89, 841-64.

[13] Levinthal D., 1988, "A survey of agency models of organisations", *Journal of Economic Behaviour and Organisation*, 9,153-85.

[14] Macleod W. and Malcomson J., 1988, "Reputation and Hierarchy in Dynamic Models of Employment", *Journal of Political Economy*, 96, 832-54.

[15] Martin S., 1993, "Endogenous Firm Efficiency in a Cournot Principal-Agent Model", *Journal of Economic Theory*, 59, 445-450.

[16] Milgrom P. and Roberts J., 1982, "Limit Pricing and Entry under Incomplete Information: An Equilibrium Analysis", *Econometrica*, 50, no. 2, 443-59.

[17] Okuno-Fujiwara M. and Postlewaite A., 1991, "Social Norms and Random Matching Games", *Games and Economic Behaviour*, 9, 79-109.

[18] Sklivas S.D., 1987, "The Strategic Choice of Management Incentives", *Rand Journal of Economics*, 18, 452-458.

[19] Spence M., *Market signalling: Information Transfer in Hiring and Related Screening Processes*, Harvard University Press, Cambridge, 2nd Ed. 1981. (first published in 1974).

[20] Spence M., 1976, "Competition in salaries, credentials and signalling prerequisites for jobs", *Quarterly Journal of Economics*, 91, 51-74.

[21] Vickers J., 1985, "Delegation and the theory of the firm", *Economic Journal*, 95, 138-47.

[22] Vickers J. and Meyer M., "Performance Comparisons and Dynamic Incentives", CEPR Discussion Paper No. 1107, January 1995.

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