The Propagation of Uncertainty Shocks: Rotemberg vs. Calvo

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Joonseok Jason Oh
European University Institute
joonseok.oh@eui.eu
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Abstract

This paper studies the effects of uncertainty shocks on economic activity, focusing on inflation. Using a VAR, I show that increased uncertainty has negative demand effects, reducing GDP and prices. I then consider standard New Keynesian models with Rotemberg-type and Calvo-type price rigidities. Despite the belief that the two schemes are equivalent, I show that they generate different dynamics in response to uncertainty shocks. In the Rotemberg model, uncertainty shocks decrease output and inflation, in line with the empirical results. By contrast, in the Calvo model, uncertainty shocks decrease output but raise inflation because of firms’ precautionary pricing motive.

Keywords: uncertainty shocks, inflation, Rotemberg pricing, Calvo pricing

JEL Classification: C68, E31, E32

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1 Introduction

Recently, uncertainty has received substantial attention in the wake of the Great Recession and the subsequent slow recovery. Many researchers have argued that uncertainty is an important factor in determining business cycle fluctuations. In a New Keynesian framework, increased uncertainty leads to a decrease in aggregate demand because of precautionary saving motives and time-varying markups. While the impact of uncertainty on aggregate demand is well understood, the effects on inflation have not been yet explored in the literature.

In this paper, I study how increased uncertainty affects economic activity, concentrating in particular on inflation. Firstly, I conduct a structural vector autoregression (VAR) analysis on quarterly U.S. macroeconomic data. I consider eight widely cited U.S. uncertainty measures from the literature. These eight measures can be categorized into four groups: (i) macroeconomic uncertainty, (ii) financial uncertainty, (iii) survey-based uncertainty, and (iv) policy uncertainty. The VAR analysis shows that an exogenous increase in any of these uncertainty indices results in significant falls in output and prices. In other words, uncertainty shocks act in the same way as aggregate demand shocks.

To explain these empirical findings, I compare two standard New Keynesian models with the most common sticky price assumptions: the Rotemberg (1982)-type quadratic price adjustment cost and the Calvo (1983)-type constant price adjustment probability. In the Rotemberg model, a firm can adjust its price whenever it wants after paying a quadratic adjustment cost. On the other hand, in the Calvo model, each firm may reset its price only with a constant probability each period, independent of the time elapsed since the last adjustment. Although the two assumptions have different economic intuitions, the predictions of the New Keynesian model are robust against the pricing assumption up to a first-order approximation around a zero-inflation steady state. For this reason, there is a widespread agreement in the literature that the pricing assumption is innocuous for the dynamics of the standard New Keynesian model. However, by employing a third-order perturbation, I show that the Rotemberg and Calvo models generate very different results in response to uncertainty shocks. In particular, I separately consider five different sources of uncertainty shocks in the models: (i) preference uncertainty, (ii) productivity uncertainty, (iii) markup uncertainty, (iv) government spending uncertainty, and (v) interest rate uncertainty. In all cases, increased uncertainty leads to a decrease in inflation in the Rotemberg model, and to an increase in inflation in the Calvo model, while still resulting in a decrease in output in both models. This result is important because inflation stabilization is one of the main goals of monetary policy. For this reason, it is important to understand which propagation mechanism holds in the data.

Uncertainty shocks have two effects on firms: an aggregate demand effect and a precautionary pric-
ing effect, as pointed out by Fernández-Villaverde et al. (2015). Increased uncertainty induces risk-averse households to consume less. The fall in aggregate demand lowers the demand for labor and capital, which decreases firms’ marginal costs. In the Rotemberg model, only the aggregate demand effect is at work for firms. To be specific, since their pricing decision is symmetric, all firms behave as a single representative firm. Thus, the firms are risk-neutral concerning their pricing decision: the firms’ marginal profit curve, a function of the reset price, is constant. Therefore, the decrease in marginal costs induces firms to lower their prices. Consequently, inflation decreases in the Rotemberg model. On the other hand, in the Calvo model, both the precautionary pricing effect as well as the aggregate demand effect are operative when an uncertainty shock hits. The Calvo pricing assumption generates heterogeneity in firms’ prices. This implies that firms are risk-averse regarding their pricing decision: the firms’ marginal profit curve is strictly convex. Thus, higher uncertainty induces firms which are resetting their prices to increase them so as to self-insure against being stuck with low prices in the future. If firms lower their prices, they may sell more but at negative markups, thereby incurring losses. As a result, inflation increases in the Calvo model. Using a prior predictive analysis, I show that the predictions of the two models are robust against the exact model parametrization and the different sources of uncertainty. Therefore, the Rotemberg model is more consistent with the empirical evidence than the Calvo model.

**Related Literature** This paper is related to three main strands of literature. First of all, this paper contributes to the literature that studies the propagation of uncertainty shocks in New Keynesian models. This is the first paper which highlights the different responses to uncertainty shocks in the Rotemberg and Calvo models. The following papers which assume the Rotemberg pricing argue that uncertainty shocks reduce output and inflation in the same way as negative demand shocks: Bonciani and van Roye (2016), Leduc and Liu (2016), Basu and Bundick (2017), Cesa-Bianchi and Fernandez-Corugedo (2018), and Katayama and Kim (2018). On the contrary, Born and Pfeifer (2014) and Mumtaz and Theodoridis (2015), which adopt Calvo pricing, argue that uncertainty shocks result in a decrease in output but an increase in inflation, i.e., negative supply shocks. Exceptionally, Fernández-Villaverde et al. (2015) study an inflationary effect of uncertainty shocks in a Rotemberg-type New Keynesian model. However, this result is obtained because, in contrast to the abovementioned literature, their price adjustment cost directly affects firms’ marginal costs. Basu and Bundick (2017) attribute this discrepancy to different sources of shocks and calibrations. However, I show that the primary reason for the different results found in the literature is the adopted assumption of price stickiness.

Second, this paper organizes the literature that looks at the empirical impact of uncertainty shocks on inflation. Caggiano et al. (2014), Fernández-Villaverde et al. (2015), Leduc and Liu (2016), and Basu
and Bundick (2017) argue that uncertainty shocks empirically induce a decrease in inflation. On the other hand, Mumtaz and Theodoridis (2015) find an inflationary effect of uncertainty shocks, and Carriero et al. (2018) and Katayama and Kim (2018) find an insignificant response of inflation to uncertainty shocks. However, they all use different uncertainty measures and time spans. Hence, I study eight widely cited U.S. uncertainty measures and, to avoid parameter instability, I start my sample only after the beginning of Paul Volcker’s mandate as the Federal Reserve Chairman. I find that any kind of uncertainty has a negative effect on inflation.

Lastly, this paper adds to the literature that studies the difference between the Rotemberg and Calvo models. This is the first paper which compares the two models in terms of uncertainty shocks. Nisticó (2007) and Lombardo and Vestin (2008) compare the welfare implications of the two models. Ascari et al. (2011) and Ascari and Rossi (2012) investigate the differences between the two models under a positive trend inflation rate. Ascari and Rossi (2011) study the effect of a permanent disinflation in the Rotemberg and Calvo models. More recently, Boneva et al. (2016), Richtera and Throckmorton (2016), Eggertsson and Singh (2018), and Miao and Ngo (2018) investigate the differences in the predictions of the Rotemberg and Calvo models with the zero lower bound for the nominal interest rate. Sims and Wolff (2017) study the state-dependent fiscal multipliers in the two models under a Taylor rule in addition to periods where monetary policy is passive. Moreover, Born and Pfeifer (2018) discuss the mapping between Rotemberg and Calvo wage rigidities.

The remainder of the paper is structured as follows. Section 2 provides the VAR-based empirical evidence. Section 3 presents the two New Keynesian models. Section 4 explains the parametrization and the solution method. Section 5 compares the quantitative results. Section 6 investigates the robustness of the results. Finally, Section 7 concludes.

2 Empirical Evidence

In this section, I empirically investigate the impacts of uncertainty shocks on economic activity.

2.1 Measuring Uncertainty

Measuring uncertainty is inherently difficult. Ideally, one would like to know the subjective probability distributions over future events for economic agents. As this is almost impossible to quantify directly, there exists no agreed measure of uncertainty in the literature. For my analysis, I take eight widely cited U.S. uncertainty measures from the literature similarly to Born et al. (2018). Considering this wide range of uncertainty proxies has the advantage that I am able to capture different kinds of uncertainty, such as
specifically, the eight uncertainty measures are (i) the macro uncertainty proxy measured by jurado et al. (2015) and ludvigson et al. (2019), (ii) the time-varying volatility of aggregate TFP innovations estimated by a stochastic volatility model (born and PFEifer, 2014; Fernald, 2014; bloom et al., 2018), (iii) the financial uncertainty proxy estimated by ludvigson et al. (2019), (iv) stock market volatility (VXO) studied by bloom (2009) and basu and Bundick (2017), (v) the consumers’ perceived uncertainty proxy (concerning vehicle purchases) proposed by leduc and Liu (2016), (vi) the firm-specific uncertainty proxy using the dispersion of firms’ forecasts about the general business outlook constructed by bachmann et al. (2013), (vii) the economic policy uncertainty index constructed by baker et al. (2016), and (viii) the monetary policy uncertainty index constructed by baker et al. (2016).
Table 1: Correlations

<table>
<thead>
<tr>
<th>Uncertainty Indices</th>
<th>MU</th>
<th>TU</th>
<th>FU</th>
<th>VXO</th>
<th>CSU</th>
<th>FSU</th>
<th>EPU</th>
<th>MPU</th>
</tr>
</thead>
<tbody>
<tr>
<td>MU</td>
<td>1</td>
<td>0.26***</td>
<td>0.66***</td>
<td>0.59***</td>
<td>0.27***</td>
<td>0.23***</td>
<td>0.34***</td>
<td>0.21**</td>
</tr>
<tr>
<td>TU</td>
<td>-</td>
<td>1</td>
<td>0.19**</td>
<td>0.09</td>
<td>0.46***</td>
<td>-0.04</td>
<td>0.28***</td>
<td>-0.02</td>
</tr>
<tr>
<td>FU</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>0.86***</td>
<td>0.20**</td>
<td>0.23***</td>
<td>0.37***</td>
<td>0.36***</td>
</tr>
<tr>
<td>VXO</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>0.20**</td>
<td>0.29***</td>
<td>0.38***</td>
<td>0.50***</td>
</tr>
<tr>
<td>CSU</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.27***</td>
<td>0.70***</td>
<td>0.27***</td>
</tr>
<tr>
<td>FSU</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.02</td>
<td>0.20**</td>
</tr>
<tr>
<td>EPU</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>0.48***</td>
</tr>
<tr>
<td>MPU</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

Macro Variable

| ΔGDP     | -0.56*** | -0.21** | -0.31*** | -0.34*** | -0.39*** | -0.26*** | -0.37*** | -0.20** |

Note: Numbers are pairwise unconditional time-series correlation coefficients. I test the hypothesis of no correlation against the alternative hypothesis of a nonzero correlation where *** denotes 1%, ** 5%, and * 10% significance levels, respectively. Abbreviations: macro uncertainty (MU), TFP uncertainty (TU), financial uncertainty (FU), stock market volatility (VXO), consumers’ survey-based uncertainty (CSU), firms’ survey-based uncertainty (FSU), economic policy uncertainty (EPU), and monetary policy uncertainty (MPU). ΔGDP is the quarterly growth rate of GDP. The sample period is 1985Q1 to 2017Q3.

I present the evolution of the eight measures from 1985Q1 to 2017Q3 in Figure 1. These eight measures can be categorized by four groups: (i) macroeconomic uncertainty, (ii) financial uncertainty, (iii) survey-based uncertainty, and (iv) policy uncertainty. Each category incorporates two indices respectively. For comparison, each series has been demeaned and standardized. The uncertainty indices are strongly counter-cyclical. Most of them increase noticeably before and during recessions while they are rather low during periods of stable economic expansion. Moreover, as shown in Table 1, there is generally a sizable degree of comovement between the uncertainty indices, consistent with Born et al. (2018).

2.2 VAR Analysis

Following the existing literature of Bloom (2009), Fernández-Villaverde et al. (2015), Leduc and Liu (2016), and Basu and Bundick (2017), I estimate a structural four-lag VAR model with a constant on quarterly U.S. macroeconomic data from 1985Q1 to 2017Q3:

\[ AY_t = c + \sum_{j=1}^{L} B_j Y_{t-j} + \epsilon_t, \]

where \( \epsilon_t \) is a vector of unobservable zero mean white noise processes. The vector \( Y_t \) comprises 7 variables: (i) the uncertainty measure, (ii) real GDP per capita, (iii) real consumption per capita, (iv) real investment per capita, (v) hours worked per capita, (vi) the GDP deflator, and (vii) the quarterly average of the effective

\(^1\)The time span is determined by the availability of the monetary policy uncertainty index.
Figure 2: Empirical Responses to Uncertainty Shocks: Macroeconomic and Financial Uncertainty Measures

Note: The solid lines represent median responses of the variables to a one-standard-deviation innovation to each uncertainty index. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. Abbreviations: macro uncertainty (MU), TFP uncertainty (TU), financial uncertainty (FU), and stock market volatility (VXO). The sample period is 1985Q1 to 2017Q3.

Since the sample includes a period during which the federal funds rate hits the zero lower bound, I use the shadow federal funds rate constructed by Wu and Xia (2016) from 2009Q1 to 2015Q4, which is not bounded below by zero and is supposed to summarize the stance of monetary policy. With the exception of the federal funds rate and the shadow rate, all other variables enter the VAR in log levels. To identify uncertainty shocks, I use a Cholesky decomposition with the uncertainty measure ordered first. This ordering is based on the assumption that uncertainty is not affected on impact by the other endogenous

federal funds rate.\(^2\) Since the sample includes a period during which the federal funds rate hits the zero lower bound, I use the shadow federal funds rate constructed by Wu and Xia (2016) from 2009Q1 to 2015Q4, which is not bounded below by zero and is supposed to summarize the stance of monetary policy. With the exception of the federal funds rate and the shadow rate, all other variables enter the VAR in log levels. To identify uncertainty shocks, I use a Cholesky decomposition with the uncertainty measure ordered first. This ordering is based on the assumption that uncertainty is not affected on impact by the other endogenous

\(^2\)I use data on GDP, consumption, investment, hours worked, price, and the interest rate. My data set comes from the FRED database of St. Louis Fed. GDP is real GDP (GDPC1). Consumption is the sum of real consumptions on nondurable goods and services (PCNDGC96 and PCESVC96). Investment is the sum of real consumption on durable goods and real private fixed investment (PCDGCC96 and FPIC1). Hours worked are measured by hours of all persons in the business sector (HOABS). Price is based on the GDP deflator (GDPDEF). To convert them to per-capita terms, I use the quarterly average of the civilian non-institutional population (CNP16OV). The short-term interest rate corresponds to the quarterly average of the effective federal funds rate (FEDFUNDS) and the Wu and Xia (2016)’s shadow rate.
variables in the VAR.\footnote{I also check a Cholesky decomposition with the uncertainty measure ordered last. The associated impulse response functions are consistent regardless of the ordering of the uncertainty measure. I display them in Appendix A.2.} This assumption is supported by Angelini et al. (2019). They argue that uncertainty is an exogenous source of decline of economic activity.

I display the impulse responses of GDP and prices to each uncertainty shock in Figure 2 and 3.\footnote{I display the full sets of empirical impulse response functions in Appendix A.1. All kinds of uncertainty shocks have similar adverse demand effects on economic activity: GDP, consumption, investment, hours worked, prices, policy rate all decrease in response to uncertainty shocks.} For each variable, the solid line denotes the median estimate of the impulse response and the shaded area represents the range of the one-standard-error bootstrapped confidence bands around the point estimates. Each uncertainty shock causes significant declines in GDP and prices. These results imply that uncertainty shocks act like aggregate demand shocks, consistently with Caggiano et al. (2014), Fernández-Villaverde et
al. (2015), Leduc and Liu (2016), and Basu and Bundick (2017).

3 Models

In this section, I outline two standard New Keynesian models with different price setting assumptions. Both economies are populated by identical infinitely-lived households. There are also a continuum of identical competitive final goods firms and a continuum of monopolistically competitive intermediate goods firms. Lastly, there are fiscal and monetary authorities.

3.1 Households

The representative household maximizes the following lifetime utility:

\[
E_0 \sum_{t=0}^{\infty} \beta^t A_t U(C_t, N_t),
\]

(2)

\[
U(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{\chi N_t^{1+\eta}}{1+\eta},
\]

(3)

where \(E_0\) is the conditional expectation operator, \(\beta\) is the subjective discount factor, \(C_t\) denotes consumption, and \(\gamma\) measures the degree of relative risk aversion. \(N_t\) denotes labor supply, \(\eta\) denotes the inverse elasticity of labor supply, and \(\chi\) indicates disutility from working. \(A_t\) is an exogenous preference shock which follows a stationary AR(1) process:

\[
\log A_t = \rho_A \log A_{t-1} + \sigma_A^A \varepsilon_t^A,
\]

(4)

where \(0 \leq \rho_A < 1\) and \(\varepsilon_t^A \sim N(0,1)\).

Every period, the household faces the following budget constraint:

\[
P_t C_t + P_t I_t + \frac{B_{t+1}}{R_t} = B_t + W_t N_t + R_t^k K_t - P_t T_t + P_t \Pi_t,
\]

(5)

where \(P_t\) is the price level, \(I_t\) is investment, \(B_t\) is one-period nominal bond holdings, \(R_t\) is the gross nominal interest rate, \(W_t\) is the nominal wage rate, \(R_t^k\) is the nominal rental rate of capital, \(K_t\) is capital stock, \(T_t\) is a lump-sum tax, and \(\Pi_t\) is profit income.

In addition, the capital stock evolves according to:

\[
K_{t+1} = (1-\delta) K_t + \left(1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right)^2 I_t,
\]

(6)

\(^{5}\)In Appendix B, I report the equilibrium conditions in the two models.
where $\delta$ is the depreciation rate and $\kappa$ controls the size of adjustment costs when the level of investment changes over time, as proposed by Christiano et al. (2005).

### 3.2 Final Goods Firms

The final good $Y_t$ is aggregated by the constant elasticity of substitution technology:

$$Y_t \equiv \left( \int_0^1 Y_t(i)^{(1-\varepsilon)/\varepsilon} di \right)^{\varepsilon}, \tag{7}$$

where $Y_t(i)$ is the quantity of intermediate good $i$ used as an input and $\varepsilon$ is the elasticity of substitution for intermediate goods. The cost minimization problem for the final goods firm implies that the demand for intermediate good $i$ is given by:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \tag{8}$$

where $P_t(i)$ is the price of intermediate good $i$. Finally, the zero-profit condition implies that the price index is expressed as:

$$P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} di \right)^{1/1-\varepsilon}. \tag{9}$$

### 3.3 Intermediate Goods Firms

There is a continuum of monopolistically competitive firms, indexed by $i \in [0, 1]$, which produce differentiated intermediate goods. Each intermediate goods firm produces its differentiated good $i$ using the following Cobb-Douglas production function:

$$Y_t(i) = Z_t K_t(i)^{\alpha} N_t(i)^{1-\alpha} - \Phi, \tag{10}$$

where $\alpha$ denotes capital income share and $\Phi$ denotes the fixed cost of production. $Z_t$ is an exogenous productivity shock which follows a stationary AR(1) process:

$$\log Z_t = \rho_Z \log Z_{t-1} + \sigma_Z^2 \varepsilon_t^Z, \tag{11}$$

where $0 \leq \rho_Z < 1$ and $\varepsilon_t^Z \sim N(0, 1)$.

Cost minimization implies that all intermediate goods firms have the same capital-to-labor ratio and the same marginal cost:

$$\frac{K_t(i)}{N_t(i)} = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t}, \tag{12}$$
MC = \frac{1}{Z_t} \left( \frac{W_t}{1-\alpha} \right)^{1-\alpha} \left( \frac{R_t^k}{\alpha} \right)^{\alpha}. \tag{13}

3.4 Two Price Setting Mechanisms

To model price stickiness, I introduce Rotemberg (1982)’s and Calvo (1983)’s price setting mechanisms. Intermediate goods firms have market power and set prices to maximize their discounted profits. They face frictions in adjusting prices and, thus, prices are sticky.

3.4.1 Rotemberg Model

Rotemberg (1982) assumes that each intermediate goods firm $i$ faces costs of adjusting price, which are assumed to be quadratic and zero at the steady state. Therefore, firm $i$ sets its price $P_t(i)$ to maximize profits given by:

$$
\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left( \frac{P_{t+j}(i)}{P_{t+j}} - \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \right) Y_{t+j}(i) - \phi \left( \frac{P_{t+j}(i)}{P_{t+j-1}(i)} - 1 \right)^2 Y_{t+j},
$$

subject to its demand in Equation (8), where $\Lambda_{t,t+j} \equiv \beta^j \frac{A_{t+j}}{A_t} \left( \frac{C_{t+j}}{C_t} \right)^{-\gamma}$ is the stochastic discount factor for real payoffs of the households, and $\phi$ is the adjustment cost parameter which determines the degree of nominal price rigidity. $M_t$ is an exogenous markup shock which follows a stationary AR(1) process:

$$
\log M_t = \rho_M \log M_{t-1} + \sigma^M_M \varepsilon^M_t,
$$

where $0 \leq \rho_M < 1$ and $\varepsilon^M_t \sim N(0,1)$.

The first-order condition associated with the optimal price is given by:

$$
\left(1 - \varepsilon\right) \left( \frac{P_t(i)}{P_t} \right)^{1-\varepsilon} + \varepsilon \frac{MC_t}{P_t} M_t \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} - \phi \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{P_t(i)}{P_{t-1}(i)} Y_t
+ \phi E_t \Lambda_{t,t+1} \left( \frac{P_{t+1}(i)}{P_t(i)} - 1 \right) \frac{P_{t+1}(i)}{P_t(i)} Y_{t+1} = 0. \tag{16}
$$

Since all intermediate goods firms face an identical profit maximization problem, they choose the same price $P_t(i) = P_t$ and produce the same quantity $Y_t(i) = Y_t$. In a symmetric equilibrium, the optimal pricing rule implies:

$$
\phi \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} = \phi E_t \Lambda_{t,t+1} \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} Y_{t+1} + 1 - \varepsilon + \varepsilon \frac{MC_t}{P_t} M_t. \tag{17}
$$
3.4.2 Calvo Model

According to the stochastic time dependent rule proposed by Calvo (1983) and Yun (1996), in each period an intermediate goods firm $i$ keeps its previous price with probability $\theta$ and resets its price with probability $1 - \theta$. The firm that gets the chance to set its price, chooses its price $P^*_t(i)$ to maximize:

$$\max_{P^*_t(i)} E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} \left( \frac{P^*_t(i)}{P^t_{t+j}} \frac{MC_{t+j}^i}{M_{t+j}^i} \right) Y_{t+j}(i),$$

subject to its demand in Equation (8).

The first-order condition with respect to the optimal price is given by:

$$E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} \left( 1 - \varepsilon \right) \left( \frac{P^*_t(i)}{P^t_{t+j}} \right)^{1-\varepsilon} + \varepsilon \frac{MC_{t+j}^i}{P^t_{t+j}} M_{t+j}^i \left( \frac{P^*_t(i)}{P^t_{t+j}} \right)^{-\varepsilon} Y_{t+j} = 0. \quad (19)$$

The optimal reset price, $P^*_t = P^*_t(i)$, is the same for all firms resetting their prices in period $t$ because they face the identical problem above. This implies that the optimal reset price is:

$$P^*_t = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} P^t_{t+j} \varepsilon MC_{t+j}^i P^t_{t+j} M_{t+j}^i Y_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} P^t_{t+j}^{\varepsilon - 1} Y_{t+j}}. \quad (20)$$

Finally, I rewrite Equation (9) describing the dynamics for the aggregate price level:

$$P_t = \left( (1 - \theta) P_t^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \quad (21)$$

3.5 Fiscal and Monetary Authorities

The fiscal authority runs a balanced budget and raises lump-sum taxes to finance government spending $G_t$, which is given by:

$$G_t = T_t. \quad (22)$$

The government spending $G_t$ follows a stationary AR(1) process:

$$\log G_t = (1 - \rho_G) \log G + \rho_G \log G_{t-1} + \sigma_G^G \varepsilon_t^G, \quad (23)$$

where $0 \leq \rho_G < 1$ and $\varepsilon_t^G \sim N(0,1)$. $G$ is the deterministic steady-state government spending.

The monetary authority conducts monetary policy using the short-term nominal interest rate as the
policy instrument. The gross nominal interest rate \( R_t \) follows a conventional Taylor rule:

\[
\log R_t = (1 - \rho_R) \log R + \rho_R \log R_{t-1} + (1 - \rho_R) (\phi_\pi (\log \pi_t - \log \pi) + \phi_Y (\log Y_t - \log Y)) + \sigma_t^R \varepsilon_t^R,
\]

where \( 0 \leq \rho_R < 1, \phi_\pi > 1, \phi_Y \geq 0, \) and \( \varepsilon_t^R \sim N(0,1). \) \( \pi_t \equiv \frac{P_t}{P_{t-1}} \) is the gross inflation rate. \( R, \pi, \) and \( Y \) are the deterministic steady-state values of the corresponding variables.

### 3.6 Market Clearing

In the Rotemberg model with the symmetric equilibrium, aggregate output satisfies:

\[
Y_t = Z_t K_t^\alpha N_t^{1-\alpha} - \Phi,
\]

and the equilibrium in the goods market requires:

\[
Y_t = C_t + I_t + G_t + \frac{\phi}{2} \left( \frac{P_t}{P_{t-1}} - 1 \right)^2 Y_t.
\]

On the other hand, in the Calvo model where the equilibrium is not symmetric, aggregate output satisfies:

\[
\Delta_t Y_t = Z_t K_t^\alpha N_t^{1-\alpha} - \Phi,
\]

where \( K_t = \int K_t(i) \, di \) and \( N_t = \int N_t(i) \, di. \) \( \Delta_t = \int \left( \frac{P^*_t(i)}{P_t} \right)^{-\varepsilon} \, di \) is relative price dispersion and can be rewritten as the following recursive form:

\[
\Delta_t = (1 - \theta) \left( \frac{P^*_t}{P_t} \right)^{-\varepsilon} + \theta \left( \frac{P_t}{P_{t-1}} \right)^{\varepsilon} \Delta_{t-1}.
\]

The equilibrium in the goods market for the Calvo model is given by:

\[
Y_t = C_t + I_t + G_t.
\]

### 3.7 Uncertainty Shock Processes

I consider the following uncertainty shock processes:

\[
\log \sigma_t^X = (1 - \rho_{\sigma^X}) \log \sigma^X + \rho_{\sigma^X} \log \sigma_{t-1}^X + \sigma_{t-1}^X \varepsilon_t^\sigma,
\]
where $X \in \{A, Z, M, G, R\}$, $0 \leq \rho_{X} < 1$, and $\varepsilon_{\tau}^{X} \sim N(0, 1)$ is a second-moment uncertainty shock. An increase in the volatility of the shock process increases the uncertainty about the future time path of the stochastic process. All stochastic shocks are independent.

4 Parametrization and Solution Method

The two models are parameterized to a quarterly frequency. Table 2 provides a summary of the key parameters. To make sure that the differences in the Rotemberg and Calvo models hold independent of the parametrization, I conduct a prior predictive analysis as in Pappa (2009). This exercise formalizes, via Monte Carlo methods, standard sensitivity analysis. Firstly, I fix a zero inflation steady state ($\pi = 1$) and a zero profit steady state ($\Pi = 0$). I draw the values of the following 32 parameters uniformly: the discount factor ($\beta$), the risk aversion ($\gamma$), the inverse labor supply elasticity ($\eta$), the steady-state hours worked ($N$), the capital depreciation rate ($\delta$), the investment adjustment cost parameter ($\kappa$), the elasticity of substitution between intermediate goods ($\varepsilon$), the capital income share ($\alpha$), the Calvo price duration ($\theta$), the steady-state government spending share ($\frac{G}{Y}$), the coefficients of the Taylor rule ($\phi_{\pi}$ and $\phi_{Y}$), and the coefficients of the shock processes ($\rho_{X}, \sigma^{X}, \rho_{\sigma X},$ and $\sigma^{\sigma X}$). The parameters are allowed to vary over the ranges reported in Table 2. The ranges are based on theoretical and practical considerations. I impose the following 3 parameters to be fixed according steady state considerations and the first-order equivalence of the two models: the labor disutility parameter ($\chi$), the production fixed cost ($\Phi$), and the Rotemberg price adjustment cost parameter ($\phi$).

I solve the two models using a third-order approximation to the equilibrium conditions around their respective deterministic steady states. To solve the models, I use the Dynare software package developed by Adjemian et al. (2011) and the pruning algorithm designed by Andreasen et al. (2018). Then, I repeat this procedure 10,000 times. I construct the impulse response functions of the endogenous variables to uncertainty shocks for each draw and rearrange them in ascending order. Lastly, I generate pointwise 68% probability bands between the 84 and 16 percentiles in both models.

---

6Fernández-Villaverde et al. (2011) explain that in the third-order approximation, in contrast to first and second-order approximations, the innovations to the stochastic volatility shocks enter independently the approximated policy functions.

7As discussed by Fernández-Villaverde et al. (2011), a third-order approximation moves the ergodic means of the endogenous variables of the model away from their deterministic steady-state values. Hence, I compute the impulse responses in percent deviation from the stochastic steady state of each endogenous variable while keeping the level of corresponding standard shocks constant.
Table 2: Quarterly Parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Range</th>
<th>Value/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>[0.985, 0.995]</td>
<td>0.99</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Risk aversion</td>
<td>[1, 4]</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse labor supply elasticity</td>
<td>[0.25, 2]</td>
<td>1</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Labor disutility parameter</td>
<td>$N = [0.2, 0.4]$</td>
<td>$N = \frac{1}{3}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>[0.01, 0.04]</td>
<td>0.025</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Investment adjustment cost parameter</td>
<td>[0, 6]</td>
<td>3</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Elasticity of substitution between goods</td>
<td>[6, 31]</td>
<td>11</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital income share</td>
<td>[0.2, 0.4]</td>
<td>0.33</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Production fixed costs</td>
<td>$\Pi = 0$</td>
<td>$\Pi = 0$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Steady-state inflation</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Calvo probability of keeping price unchanged</td>
<td>[0.5, 0.9]</td>
<td>0.75</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Rotemberg price adjustment cost parameter</td>
<td>$\frac{\theta(\varepsilon - 1)}{(1 - \theta)(1 - \beta\theta)}$</td>
<td>$\frac{\theta(\varepsilon - 1)}{(1 - \theta)(1 - \beta\theta)}$</td>
</tr>
<tr>
<td>$G$</td>
<td>Steady-state government spending</td>
<td>$G = [0.1, 0.3]$</td>
<td>$G = 0.2$</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>Coefficient of inflation target in the Taylor rule</td>
<td>[1.1, 3]</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_{Y}$</td>
<td>Coefficient of output target in the Taylor rule</td>
<td>[0, 0.5]</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_X$</td>
<td>Persistence of level shocks</td>
<td>[0.5, 0.99]</td>
<td>0.9 ($\rho_R = 0.7$)</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>Volatility of level shocks</td>
<td>[0.005, 0.015]</td>
<td>0.01</td>
</tr>
<tr>
<td>$\rho_{\epsilon X}$</td>
<td>Persistence of uncertainty shocks</td>
<td>[0.5, 0.99]</td>
<td>0.7</td>
</tr>
<tr>
<td>$\sigma_{\epsilon X}$</td>
<td>Volatility of uncertainty shocks</td>
<td>[0.2, 0.8]</td>
<td>0.5</td>
</tr>
</tbody>
</table>

5 Quantitative Results

In this section, I quantitatively investigate the effects of uncertainty shocks on macroeconomic variables in the Rotemberg and Calvo models. I plot the pointwise 68% probability bands for the impulse response functions of output and inflation to each uncertainty shock in the Rotemberg (blue solid bands) and Calvo (red dashed bands) models in Figure 4. The figure shows that increased uncertainty has negative effects on output in both models. It increases inflation in the Calvo model. On the other hand, even though the bands of inflation slightly contain the zero line in the Rotemberg model, higher inflation generally decreases in response to uncertainty shocks as compared to the Calvo model.\(^8\) Hence, this exercise shows that the pricing assumptions are the main reason behind the different inflation responses and that the result is robust against different parameterization and sources of uncertainty. In the following subsections, I am going to explain why the effects of uncertainty shocks on inflation are different in the two models.

5.1 Households’ Precautionary Decision: Rotemberg and Calvo

I display the pointwise 68% probability bands for the impulse response functions of the endogenous variables to a productivity uncertainty shock only in the Rotemberg (blue solid bands) and Calvo (red dashed bands)

---

\(^8\)Fasani and Rossi (2018) show that in the Rotemberg model, uncertainty shocks can have inflationary or deflationary effects depending on the monetary policy rule.
models in Figure 5. The effects of the other uncertainty shocks are qualitatively similar and are displayed in Appendix C.1.

Increased uncertainty induces a precautionary saving effect on risk-averse households. This implies that when uncertainty increases, households want to consume less and save more. To save more, households would like to invest and work more. Since the fall in consumption implies a decline in aggregate demand, this decreases output. Lower output decreases the marginal products of capital and labor, thus leading to a fall in the demand for capital and labor. Consequently, this reduces the rental rates and wages, and thus decreases firms’ marginal costs. To investigate the firms’ pricing decision, I rewrite Equation (16) from
Figure 5: Pointwise 68% Probability Bands to Productivity Uncertainty Shock in Rotemberg and Calvo Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
recursive form to infinite sum form:

\[
\left\{ E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left( 1 - \varepsilon \right) \left( \frac{P_{t+j}(i)}{P_{t+j}} \right)^{1-\varepsilon} + \varepsilon \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \left( \frac{P_{t+j}(i)}{P_{t+j}} \right)^{-\varepsilon} \right\} Y_{t+j}
\]

\[- \phi \left( \frac{P_{t}(i)}{P_{t-1}(i)} - 1 \right) \frac{P_{t}(i)}{P_{t-1}} Y_t = 0. \quad (31)\]

Since all intermediate goods firms solve an identical profit maximization problem, they choose the same price \( P_t(i) = P_t \). In a symmetric equilibrium, the optimal pricing rule implies:

\[
\left\{ E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left( 1 - \varepsilon + \varepsilon \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \right) Y_{t+j} \right\} - \phi \left( \frac{P_{t}}{P_{t-1}} - 1 \right) \frac{P_{t}}{P_{t-1}} Y_t = 0. \quad (32)\]

Following Equation (32), when the marginal costs of the intermediate goods firms decrease, they lower their prices to stimulate the demand for output. This corresponds to a decrease in inflation. However, the prices do not decrease as much as the marginal costs due to the price adjustment costs. This implies an increase in price markups over marginal costs. Aggregate demand falls after all. Consequently, since the equilibrium is demand-determined, output, consumption, investment, and hours worked decrease. Under the Taylor rule, the monetary authority lowers the nominal interest rate to alleviate the adverse effects of uncertainty.

5.2 Firms’ Precautionary Decision: Calvo

Apart from the aggregate demand effect of uncertainty shocks discussed above, uncertainty shocks have an additional effect on firms’ pricing decision in the Calvo model. Equation (19) can be rewritten as follows:

\[
\left\{ E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} \left( 1 - \varepsilon \right) \left( \frac{P_{t+j}(i)}{P_{t+j}} \right)^{1-\varepsilon} + \varepsilon \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \left( \frac{P_{t+j}(i)}{P_{t+j}} \right)^{-\varepsilon} \right\} Y_{t+j} = 0. \quad (33)\]

The optimal reset price, \( P^*_t = P^*_t(i) \), is the same for all firms resetting their prices in period \( t \). This implies the following optimal pricing condition:

\[
\left\{ E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} \left( 1 - \varepsilon \right) \left( \frac{P^*_t(i)}{P_{t+j}} \right)^{1-\varepsilon} + \varepsilon \frac{MC_{t+j}}{P_{t+j}} M_{t+j} \left( \frac{P^*_t(i)}{P_{t+j}} \right)^{-\varepsilon} \right\} Y_{t+j} = 0. \quad (34)\]

Therefore, the associated equilibrium is not symmetric.

Similarly to Fernández-Villaverde et al. (2015) and Born and Pfeifer (2019), without loss of generality, I explain firms’ pricing decision in this model by using the steady-state period marginal profit function under
Figure 6: Steady-State Expected Period Marginal Profits in Rotemberg and Calvo Models

Note: The period marginal profit is a function of the reset price.

the specific values of parametrization in Table 2. Under certainty, this function is as follows:

\[ MP^C = \left(1 - \varepsilon\right) \left(\frac{P^*_\text{certainty}}{P}\right)^{1-\varepsilon} + \varepsilon \frac{MC}{P} \left(\frac{P^*_\text{certainty}}{P}\right)^{-\varepsilon} \right) Y. \] (35)

I assume that the aggregate price \( P \) is equal to 1. Figure 6(b) displays that the \( MP^C \) is strictly convex in the reset price. This feature comes from the existence of the relative price dispersion. Economically, this implies that firms set their prices risk-aversely like households discussed above. Under uncertainty, the steady-state expected period marginal profit function is as follows:

\[ EMP^C = q \left(1 - \varepsilon\right) \left(\frac{P^*_\text{uncertainty}}{P^l}\right)^{1-\varepsilon} + \varepsilon \frac{MC}{P} \left(\frac{P^*_\text{uncertainty}}{P^l}\right)^{-\varepsilon} \right) Y \]

\[ + (1 - q) \left(1 - \varepsilon\right) \left(\frac{P^*_\text{uncertainty}}{P^h}\right)^{1-\varepsilon} + \varepsilon \frac{MC}{P} \left(\frac{P^*_\text{uncertainty}}{P^h}\right)^{-\varepsilon} \right) Y. \] (36)

In this case, I assume that the aggregate price is either \( P^l = 0.95 \) or \( P^h = 1.05 \) with probability \( q = \frac{1}{2} \). Figure 6(b) shows that to maximize their profits, the optimal price under uncertainty (\( P^*_\text{uncertainty} = 1.02 \)
is higher than that under certainty ($P^\text{certainty} = 1$), applying Jensen’s inequality. The firms which increase their prices will sell fewer goods but at higher price markups. In contrast, the firms which lower their prices may sell more but at negative markups, thereby incurring losses. Thus, when uncertainty increases, firms increase their prices to self-insure against being stuck with low prices in the future. Therefore, price markups increase by more. This precautionary pricing decision increases inflation and decreases output. Under the Taylor rule, the monetary authority increases the nominal interest rate to stabilize the increase in inflation.

On the other hand, those profit curves have zero curvature in the Rotemberg model as shown in Figure 6(a): 

\[ MP^R = EMP^R = \left(1 - \varepsilon + \varepsilon \frac{MC}{P}\right) Y = 0. \] \hspace{1cm} (37)

Equation (37) implies that whatever the shocks realization is, all firms change their prices equally in the Rotemberg model.\(^9\) This means that they do not face the trade-off present in the Calvo model where being an expensive firm is preferred to being a cheap one.

In sum, due to the precautionary pricing effect, inflation increases in the Calvo model, while it decreases in the Rotemberg model. Moreover, output, consumption, investment, and hours worked in the Calvo model decrease by more than those in the Rotemberg model. Thus, the Rotemberg model is qualitatively consistent with the empirical findings with respect to the transmission of uncertainty shocks. The opposite response of inflation to uncertainty shocks would prompt different monetary policy reactions. For this reason, understanding which propagation mechanism holds in the data becomes important.

6 Robustness Checks

To examine the robustness of my results, I conduct several robustness checks in this section.

6.1 Elasticity of Substitution between Intermediate Goods

I show how important the elasticity of substitution between intermediate goods, $\varepsilon$, is for the responses of inflation to increased uncertainty. I display the steady-state expected period marginal profit functions for four levels of the elasticity of substitution ($\varepsilon = 6, 11, 21, \text{ and } 31$) in the Rotemberg and Calvo models in Figure 7. These values imply a 20%, 10%, 5%, and 3.3% markup, respectively. As shown in Figure 7(a), the changes in $\varepsilon$ do not have any effects on the marginal profits in the Rotemberg model. This confirms that unlike the Calvo model, uncertainty shocks do not have the precautionary pricing effects in the Rotemberg

\(^9\)One may argue that when capital is accumulated by the Rotemberg-type firms, this forward-looking behavior can induce a precautionary pricing behavior. However, due to a symmetric equilibrium, the capital accumulation by firms does not have any effects on their pricing behavior. In other words, those behaviors are independent from each other in the Rotemberg model. See Basu and Bundick (2017).
model. By contrast, as the elasticity becomes higher, the marginal profit curve becomes more convex in the Calvo model as shown in Figure 7(b). This means that firms become more risk-averse regarding their pricing decision. The more convex curve amplifies the precautionary pricing effect. Hence, the optimizing price increases for higher levels of $\varepsilon$ in the Calvo model.

Furthermore, I conduct an impulse response function analysis. In this exercise, I set the specific values of parametrization in Table 2. Moreover, I fix $\phi = 116.5$ in the Rotemberg model to evaluate the effects of the changes in $\varepsilon$ only. Figure 8 displays the impulse responses of inflation to five different uncertainty shocks for four levels of the elasticity of substitution ($\varepsilon = 6, 11, 21$, and $31$) in the Rotemberg (blue line) and Calvo (red line) models. In the Rotemberg model, higher level of $\varepsilon$ means less differentiation between the goods. As in differentiated Bertrand competition (Hotelling, 1929), less differentiation implies that firms lower their prices because they compete more vigorously. Therefore, inflation decreases by more in response to uncertainty shocks given the higher elasticity. However, in the Calvo model, the higher elasticity amplifies the precautionary pricing effect discussed above. Thus, the responses of inflation to uncertainty shocks.
shocks are amplified for higher levels of $\varepsilon$. Exceptionally, inflation decreases in response to a government spending uncertainty shock in the Calvo model under the specific parametrization. This is because the drop in inflation triggered by the decrease in aggregate demand is not outweighted by the increase in inflation due to the precautionary pricing behavior of firms. Nevertheless, the feature of that higher elasticity amplifies the precautionary pricing behavior is preserved.

### 6.2 Rotemberg Price Adjustment Costs

I show the importance of non-linearity when choosing two different types of price adjustment costs. One ($AC_1^1$) is scaled by aggregate output $Y$ as in Bonciani and van Roye (2016), Leduc and Liu (2016), Basu and Bundick (2017), and Katayama and Kim (2018). The other ($AC_1^2$) is scaled by individual output $Y_i(i)$.
as in Fernández-Villaverde et al. (2015):

$$AC^1_t(i) = \frac{\phi}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t,$$

$$AC^2_t(i) = \frac{\phi}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t(i).$$

(38)

(39)

In a symmetric equilibrium ($P_t(i) = P_t$ and $Y_t(i) = Y_t$), the respective optimal pricing rules imply:

$$\phi \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} = \phi E_t \Lambda_{t,t+1} \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}Y_{t+1}}{P_tY_t} + 1 - \varepsilon + \varepsilon \frac{MC_t}{P_t}M_t,$$

$$\phi \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} = \phi E_t \Lambda_{t,t+1} \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}Y_{t+1}}{P_tY_t} + 1 - \varepsilon + \varepsilon \frac{MC_t}{P_t}M_t + \frac{\varepsilon \phi}{2} \left( \frac{P_t}{P_{t-1}} - 1 \right)^2.$$

(40)

(41)

The two optimal pricing conditions above are equivalent up to a first-order approximation.

By using these two Rotemberg models, I conduct a prior predictive analysis under the parametrization in Table 2. Then, I plot the pointwise $68\%$ probability bands for the impulse response functions of output and inflation to each uncertainty shock in the Rotemberg 1 (blue solid line) and Rotemberg 2 (green dashed line) models in Figure 9. The figure shows significant differences between the two models. Unlike in the Rotemberg 1 model, output decreases by more and inflation increases in the Rotemberg 2 model. Interestingly, the responses in the Rotemberg 2 model are similar to those in the Calvo model. However, the propagation mechanisms of uncertainty shocks in the two models are totally different.

The different responses in the two Rotemberg models depend on how price adjustment costs are scaled. The Rotemberg 2 optimal pricing rule (41) has an additional quadratic price adjustment cost term $\frac{\varepsilon \phi}{2} \left( \frac{P_t}{P_{t-1}} - 1 \right)^2$ relative to the Rotemberg 1 optimal pricing rule (40). Therefore, the Rotemberg 2 model adds one further effect to the aggregate demand channel of uncertainty shocks already present in the Rotemberg 1 model as discussed in Section 5.1. To be specific, when marginal costs decrease due to the aggregate demand effect of uncertainty shocks, intermediate goods firms would like to lower their prices to stimulate the demand for output. However, the change in prices raises the quadratic price adjustment cost term. If the increase in the cost term dominates the decrease in marginal costs, firms would increase their prices. Consequently, inflation can increase in response to uncertainty shocks in the Rotemberg 2 model as in Fernández-Villaverde et al. (2015). This is a different channel of uncertainty propagation from the precautionary pricing channel of the Calvo model.

\[I display the full sets of model impulse response functions in Appendix C.2.\]
6.3 Density of Inflation

An additional and simple way of confirming the firms’ precautionary behavior mechanism in the Calvo model is to show the density of inflation. Using the specific values of parametrization in Table 2 and the policy functions, the Rotemberg and Calvo models are simulated separately for 20,000 periods in response to each uncertainty shock considered above. I then identify periods of increased uncertainty and finally plot the histograms of inflation to each uncertainty shock for the Rotemberg 1 (blue bar), Rotemberg 2 (green bar), and Calvo (red bar) models in Figure 10. I confirm important differences in the shape of the distributions.\footnote{In Appendix D, I show that there are little differences in the distributions of inflation to level shocks in the three models.} Consistently, in the Rotemberg 1 model, the densities of inflation are left-skewed. On the other hand, in the Rotemberg 2 and Calvo models, the densities of inflation are right-skewed. Moreover, the densities of inflation are more right-skewed in the Calvo model than in the Rotemberg 2 model. These results are
consistent with the impulse response function analysis discussed above.

7 Conclusion

This paper contributes to our understanding of the role of different sticky price assumptions in the propagation of uncertainty shocks. An important contribution of this paper is to show that in contrast to the Calvo model, the Rotemberg model does not generate a precautionary pricing effect of uncertainty shocks. For this reason, the response of inflation to uncertainty shocks is opposite in the Rotemberg and Calvo models. This result has important implications for monetary policy. Depending on the model adopted, the implied policy responses to higher uncertainty are qualitatively different. The implications of the Rotemberg model are qualitatively more consistent with the empirical findings than those of the Calvo model. However, from
a quantitative perspective, in both models uncertainty shocks have much smaller effects on macro aggregates than those shown by the empirical evidence. To bring the theoretical models closer to the data, future research should focus on understanding the amplification channels of uncertainty shocks.
References


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Appendices

A Full Sets of Empirical Impulse Response Functions

A.1 First Cholesky Ordering

In Figure A.1 to A.8, I display the empirical impulse responses to each uncertainty shock under a Cholesky decomposition with the uncertainty measure ordered first.

A.2 Last Cholesky Ordering

In Figure A.9 to A.16, I display the empirical impulse responses to each uncertainty shock under a Cholesky decomposition with the uncertainty measure ordered last.
Figure A.1: Empirical Responses to Uncertainty Shocks: Macro Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation innovation to macro uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.2: Empirical Responses to Uncertainty Shocks: TFP Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.3: Empirical Responses to Uncertainty Shocks: Financial Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.4: Empirical Responses to Uncertainty Shocks: Stock Market Volatility

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.5: Empirical Responses to Uncertainty Shocks: Consumers’ Survey-Based Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.6: Empirical Responses to Uncertainty Shocks: Firms’ Survey-Based Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.7: Empirical Responses to Uncertainty Shocks: Economic Policy Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.8: Empirical Responses to Uncertainty Shocks: Monetary Policy Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.9: Last Cholesky Ordering: Macro Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.10: Last Cholesky Ordering: TFP Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.11: Last Cholesky Ordering: Financial Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.12: Last Cholesky Ordering: Stock Market Volatility

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.13: Last Cholesky Ordering: Consumers’ Survey-Based Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.14: Last Cholesky Ordering: Firms’ Survey-Based Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.15: Last Cholesky Ordering: Economic Policy Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
Figure A.16: Last Cholesky Ordering: Monetary Policy Uncertainty

Note: The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The shaded area around each solid line represents the one-standard-error bands for the estimated median impulse responses. The sample period is 1985Q1 to 2017Q3.
B Equilibrium Conditions in Two New Keynesian Models

In this section, I write the equilibrium conditions in the Rotemberg-type and Calvo-type New Keynesian models, respectively.

B.1 Rotemberg Model

\[ \phi (\pi_t - 1) \pi_t = \phi E_t \Lambda_{t,t+1} (\pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} + 1 - \varepsilon + \varepsilon mc_t M_t \]  

(B.1)

\[ Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} - \Phi \]  

(B.2)

\[ Y_t = C_t + I_t + G_t + \frac{\phi}{2} (\pi_t - 1)^2 Y_t \]  

(B.3)

\[ \Lambda_{t,t+1} = \beta E_t \frac{A_{t+1}}{A_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \]  

(B.4)

\[ w_t = mc_t (1 - \alpha) \frac{Y_t + \Phi}{N_t} \]  

(B.5)

\[ r_t^k = mc_t \alpha Y_t + \Phi \]  

(B.6)

\[ \frac{\chi N_t^\eta}{C_t^{-\gamma}} = w_t \]  

(B.7)

\[ 1 = q_t \left( 1 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) + E_t I_{t,t+1} q_{t+1} \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \]  

(B.8)

\[ q_t = E_t I_{t,t+1} \left( r_t^k + q_{t+1} (1 - \delta) \right) \]  

(B.9)

\[ K_{t+1} = (1 - \delta) K_t + \left( 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \]  

(B.10)

\[ 1 = E_t \Lambda_{t,t+1} \frac{R_t}{\pi_{t+1}} \]  

(B.11)

\[ \log R_t = (1 - \rho_R) \log R + \rho_R \log R_{t-1} + (1 - \rho_R) \left( \phi_X (\log \pi_t - \log \pi) + \phi_Y (\log Y_t - \log Y) \right) + \sigma_t^R \varepsilon_t^R \]  

(B.12)

\[ \log A_t = \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A \]  

(B.13)

\[ \log Z_t = \rho_Z \log Z_{t-1} + \sigma_t^Z \varepsilon_t^Z \]  

(B.14)

\[ \log M_t = \rho_M \log M_{t-1} + \sigma_t^M \varepsilon_t^M \]  

(B.15)

\[ \log G_t = (1 - \rho_G) \log G + \rho_G \log G_{t-1} + \sigma_t^G \varepsilon_t^G \]  

(B.16)

\[ \log \sigma_t^X = (1 - \rho_{\sigma X}) \log \sigma_X + \rho_{\sigma X} \log \sigma_{t-1}^X + \sigma_t^X \varepsilon_t^X, \quad X = A, Z, M, G, R \]  

(B.17)
B.2 Calvo Model

\[
\left( \frac{1 - \theta \pi_t^{\varepsilon_t}}{1 - \theta} \right)^{\frac{1}{\varepsilon_t}} = \frac{\varepsilon}{\varepsilon - 1} CC_t
\] (B.18)

\[
BB_t = Y_t + \theta E_{t} \Lambda_{t,t+1} \pi_{t+1}^{\varepsilon_t - 1} BB_{t+1}
\] (B.19)

\[
CC_t = m_c M_t Y_t + \theta E_{t} \Lambda_{t,t+1} \pi_{t+1}^{\varepsilon_t} CC_{t+1}
\] (B.20)

\[
\Delta_t Y_t = Z_t K_t N_t^{1-\alpha} - \Phi
\] (B.21)

\[
\Delta_t = (1 - \theta) \left( \frac{1 - \theta \pi_t^{\varepsilon_t}}{1 - \theta} \right)^{\frac{1}{\varepsilon_t}} + \theta \pi_t^{\varepsilon_t} \Delta_{t-1}
\] (B.22)

\[
Y_t = C_t + I_t + G_t
\] (B.23)

\[
\Lambda_{t,t+1} = \beta E_{t} \frac{A_{t+1}}{A_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}
\] (B.24)

\[
w_t = m_c (1 - \alpha) \frac{\Delta Y_t + \Phi}{N_t}
\] (B.25)

\[
r_t^k = m_c \alpha \frac{\Delta Y_t + \Phi}{K_t}
\] (B.26)

\[
\frac{\chi N_t^\alpha}{C_t^{\varepsilon_t}} = w_t
\] (B.27)

\[
1 = q_t \left( 1 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) + E_{t} \Lambda_{t,t+1} q_{t+1} \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2
\] (B.28)

\[
q_t = E_{t} \Lambda_{t,t+1} \left( r_t^k + q_{t+1} (1 - \delta) \right)
\] (B.29)

\[
K_{t+1} = (1 - \delta) K_t + \left( 1 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \right) I_t
\] (B.30)

\[
1 = E_{t} \Lambda_{t,t+1} \frac{R_t}{\pi_{t+1}}
\] (B.31)

\[
\log R_t = (1 - \rho_R) \log R + \rho_R \log R_{t-1} + (1 - \rho_R) \left( \phi_\pi \left( \log \pi_t - \log \pi \right) + \phi_Y \left( \log Y_t - \log Y \right) \right) + \sigma_R^R \varepsilon_t^R
\] (B.32)

\[
\log A_t = \rho_A \log A_{t-1} + \sigma_A^A \varepsilon_t^A
\] (B.33)

\[
\log Z_t = \rho_Z \log Z_{t-1} + \sigma_Z^Z \varepsilon_t^Z
\] (B.34)

\[
\log M_t = \rho_M \log M_{t-1} + \sigma_M^M \varepsilon_t^M
\] (B.35)

\[
\log G_t = (1 - \rho_G) \log G + \rho_G \log G_{t-1} + \sigma_G^G \varepsilon_t^G
\] (B.36)

\[
\log \sigma_t^X = (1 - \rho_{\sigma_X}) \log \sigma_X + \rho_{\sigma_X} \log \sigma_{t-1}^X + \sigma_{X}^X \varepsilon_t^X,
\quad X = A, Z, M, G, R
\] (B.37)
C  Full Sets of Model Impulse Response Functions

C.1  Rotemberg 1 vs. Calvo

I display the pointwise 68% probability bands for the impulse response functions of the endogenous variables to each uncertainty shock in the Rotemberg 1 (blue solid bands) and Calvo (red dashed bands) models in Figure C.1 to C.4.

C.2  Rotemberg 1 vs. Rotemberg 2

I display the pointwise 68% probability bands for the impulse response functions of the endogenous variables to each uncertainty shock in the Rotemberg 1 (blue solid bands) and Rotemberg 2 (green dashed bands) models in Figure C.5 to C.9.
Figure C.1: Pointwise 68% Probability Bands to Preference Uncertainty Shocks in Rotemberg and Calvo Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and real wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
Figure C.2: Pointwise 68% Probability Bands to Markup Uncertainty Shocks in Rotemberg and Calvo Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and real wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
Figure C.3: Pointwise 68% Probability Bands to Government Spending Uncertainty Shocks in Rotemberg and Calvo Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and real wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
Figure C.4: Pointwise 68% Probability Bands to Interest Rate Uncertainty Shocks in Rotemberg and Calvo Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and real wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
Figure C.5: Pointwise 68% Probability Bands to Preference Uncertainty Shocks in Rotemberg 1 and Rotemberg 2 Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and real wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
Figure C.6: Pointwise 68% Probability Bands to Productivity Uncertainty Shock in Rotemberg 1 and Rotemberg 2 Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
Figure C.7: Pointwise 68% Probability Bands to Markup Uncertainty Shocks in Rotemberg 1 and Rotemberg 2 Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and real wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
Figure C.8: Pointwise 68% Probability Bands to Government Spending Uncertainty Shocks in Rotemberg 1 and Rotemberg 2 Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and real wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
Figure C.9: Pointwise 68% Probability Bands to Interest Rate Uncertainty Shocks in Rotemberg 1 and Rotemberg 2 Models

Note: The bands of output, consumption, investment, hours worked, real marginal cost, and real wage are plotted in percent deviations from their stochastic steady states. The bands of inflation and nominal interest rate are plotted in annualized percentage point deviations from their stochastic steady states.
D Density of Inflation to Level Shocks

I plot the histograms of inflation to each level shock for the Rotemberg 1 (blue bar), Rotemberg 2 (green bar), and Calvo (red bar) models in Figure D.1. In contrast to the case of uncertainty shocks, there are little differences in the distributions of inflation to level shocks in the three models.

Figure D.1: Histograms of Inflation to Level Shocks in Rotemberg and Calvo Models

Note: Sturges’ rule is used to determine the number and width of the bins. The response of inflation is plotted in annualized percentage point deviation from its stochastic steady state.