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Equality of opportunity:  
Definitions and testable conditions, with an  
application to income in France (1979-2000)

**Arnaud Lefranc, Nicolas Pistoiesi and  
Alain Trannoy**



**EUROPEAN UNIVERSITY INSTITUTE**  
Robert Schuman Centre for Advanced Studies

**EUROPEAN UNIVERSITY INSTITUTE, FLORENCE**  
**ROBERT SCHUMAN CENTRE FOR ADVANCED STUDIES**

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## **Abstract**

We offer a model of equality of opportunity that encompasses different conceptions expressed in the public and philosophical debates. In addition to *circumstances* whose effect on outcome should be compensated and *effort* which represents a legitimate source of inequality, we introduce a third factor, *luck*, that captures the non-responsibility factors whose impact on outcome should be *even-handed* for equality of opportunity to be satisfied. Then, we analyse how the various definitions of equality of opportunity can be empirically identified, given data limitations and provide testable conditions. Definitions and conditions resort to standard stochastic dominance tools. Lastly, we develop an empirical analysis of equality of opportunity for income acquisition in France over the period 1979-2000 which reveals that the degree of inequality of opportunity tends to decrease and that the degree of risk of income distributions, conditional on social origin, appears very similar across all groups of social origins.

**Keywords:** Equality of opportunity, Income distribution, Luck, Stochastic dominance

**JEL:** D63, J62, C14

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# 1 Introduction

Most economic analysis of inequality, theoretical and empirical, rely on the assumption that equality of individual outcomes (*e.g.* welfare, income, health) is *per se* a desirable social objective. This is sometimes criticized for standing at odd with both public perceptions of inequalities and some developments in modern theories of justice. According to this criticism, a distinction must be drawn between morally or socially justified and unjustified inequalities. This has led egalitarian philosophers such as Rawls (1971), Dworkin (1981a; 1981b), Sen (1985), Cohen (1989) or Arneson (1989; 1990) to claim that distributive justice does not entail the equality of individual outcomes but only requires that individuals face equal *opportunities* for outcome. Despite the growing political audience of this view, few economic analysis have tried to assess the extent to which equality of opportunity is empirically satisfied.<sup>1</sup> Two major issues are likely to account for this state of affairs. First, how should equality of opportunity be characterized? In fact, no consensus has been reached, neither in the philosophical nor in the public debates, regarding how opportunities should be defined and in what sense they should be considered equal. In this paper we offer a model of equality of opportunity that encompasses several conceptions expressed in these debates. Second, how can equality of opportunity be empirically assessed? This requires that the determinants of individual outcomes be taken into account. However, these determinants are never fully observable. Hence, we analyze how the various conceptions can be empirically identified, given data limitations, and provide testable conditions for equality of opportunity. Lastly, we develop an empirical implementation of these conditions and examine the extent to which equality of opportunity is achieved in the distribution of income in France.

One important implication of the equal-opportunity view is that judgements about equality must take into account the determinants of individual outcomes. At least two sets of factors must be distinguished : on the one hand, factors that reflect individual responsible choices and are considered a legitimate source of inequality; on the other hand, factors beyond the realm of individual responsibility and that do not appear as socially or morally acceptable sources of inequality. Following the terminology introduced in Roemer (1998), we refer to the former determinants as *effort* and to the latter as *circumstances*. As most authors would agree, the principle of equality of opportunity essentially requires, that, given individual effort, circumstances do not affect individual prospects for outcome, or to paraphrase Rawls (1971, p.63), that individual with similar effort face “*the same prospects of success regardless of their*

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<sup>1</sup>Roemer et al. (2003), O’Neill et al. (2000), Checchi et al. (1999), Benabou and Ok (2001), Bourguignon et al. (2003), Goux and Maurin (2003), Alesina and La Ferrara (2005) and Checchi and Peragine (2005) who analyze equality of opportunity for income and Schuetz et al. (2005) who examine educational opportunities are some of the exceptions.



*initial place in the social system*". However, there remains considerable discussion regarding what factors should count as effort or circumstances.

A prominent view in these debates is the one expressed by John Roemer in a series of contributions.<sup>2</sup> <sup>3</sup> It claims that the definition of circumstances is a matter of political choice. Furthermore, once circumstances have been defined "*by society*"<sup>4</sup>, remaining differences in individual outcomes should be considered the result of effort. Hence, the distinction between circumstances and effort turns into a dichotomic partitioning of the determinants of outcome. As a consequence, requiring that, for a given level of effort, circumstances do not affect individual prospects for outcome, implies that individuals with similar effort should have *equal* outcomes.

This dichotomic approach lies at the heart of most economic analysis of equality of opportunity. However, it does not fully account for the diversity of the determinants of outcome and leads to a specific conception of equality of opportunity. Assuming that society has agreed on a given set of circumstances does not imply that the remaining determinants will reflect individual responsible choice and should be treated as effort. In this respect, international attitudes surveys, such as the one summarized in Figure 1, reveal two noteworthy differences across countries, if we are willing to identify "social injustice" with circumstances defined "by society". First, countries differ in their propensity to consider that bad economic outcomes reflect social injustice, which indicates that the definition of circumstances may vary across societies. Second, and more importantly, the figure also suggests that countries differ in their belief in the role of effort in shaping individual outcomes, over and beyond the influence of circumstances.<sup>5</sup> The assumption that the determinants of outcomes excluded from socially defined circumstances relate to individual effort provides a good approximation of US average beliefs. It does not however correspond to the social perception in many European countries, which emphasizes the role of luck in shaping individual success. Our purpose is to build a model of equality of opportunity flexible enough to encompass this diversity of perceptions. This requires to distinguish three generic determinants of individual outcomes : circumstances, effort and luck.

But how can luck be accounted for in the definition of equality of opportunity ? The extent to which the impact of non-responsibility factors should be compensated has been amply discussed in the philosophical literature. According to these debates, distributive justice requires

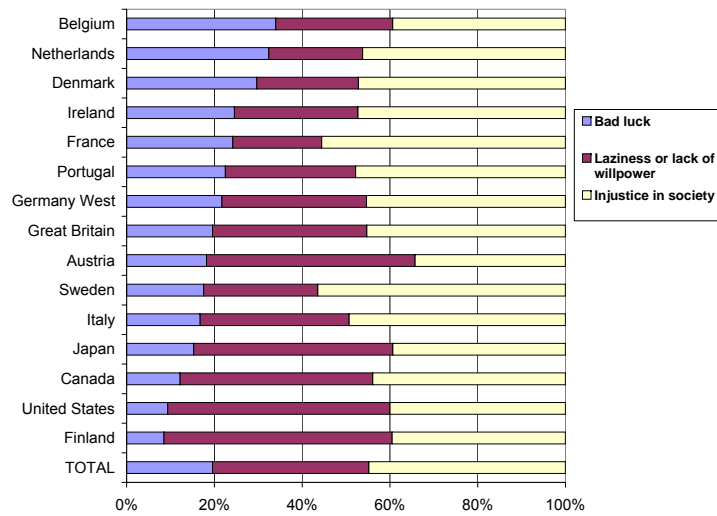
<sup>2</sup>For a theoretical discussion, see Roemer (1993; 1998) and for empirical applications Betts and Roemer (2006), Roemer et al. (2003) and Dardanoni et al. (2005).

<sup>3</sup>See Fleurbaey and Maniquet (2007) for a thorough discussion of alternative perspectives and related issues.

<sup>4</sup>Roemer (1993, p.149)

<sup>5</sup>For more detailed evidence, see among others Marshall et al. (1999), Corneo and Gruner (2002), Alesina and Glaeser (2004) and Alesina and Angeletos (2005).

Figure 1: Beliefs in the role of luck, effort and social injustice in bad economic outcomes



Source : World Values Survey (1990). Answers to the question : "Why are there people living in need ?". Authors' computations excluding the following answers : It is an inevitable part of modern progress; None of theses; Don't know.

that factors akin to circumstances, such as family and social background, do not lead, other things equal, to differences in outcome, and be compensated. However, owing to different moral demands, justice does not necessarily require that the impact of *all* non-responsibility factors be nullified. In some cases, luck may appear as a fair source of inequality provided that it is *even-handed*. Equality of opportunity does not entail that individual with similar effort reach equal outcomes. What equality of opportunity requires is that, given effort, no one face more favorable outcome prospects, as a result of luck, for reasons related to differential circumstances.

The first contribution of this paper is to offer a characterization of equality of opportunity consistent with this view. Given effort, the outcome prospects of an individual are summarized by the outcome distribution conditional on her circumstances. Our characterization rests on the idea that equality of opportunity prevails when individuals are indifferent between the distributions attached to all possible circumstances. To compare these distributions, we resort to the tools of stochastic dominance (first and second order) whose appeal is to encompass a wide range of preferences for uncertain outcomes. This leads us to distinguish two situations of interest, from the point of view of equality of opportunity. The first situation, which corresponds to a strict form of equality of opportunity, arises when the outcome distributions conditional on effort are equal. The second situation, which we refer to as weak equality of opportunity, arises when the outcome distributions conditional on effort cannot be ranked using first and second order stochastic dominance criteria : this corresponds to absence of unanimous preferences over

the range of possible circumstances.

The empirical implementation of these definitions of equality of opportunity would be straightforward if circumstances and effort were observable. However, in practice, this condition may not be easily met. The empirical assessment of equality of opportunity requires considerable information on the determinants of individual outcomes. And this information is not entirely available in existing data sets.<sup>6</sup> In most cases, not all the relevant aspects of individual effort can be measured and only a subset of the relevant circumstances can be observable. We discuss the consequences of these data limitations for the evaluation of equality of opportunity. The second contribution of the paper is to show that, conditional on further distributional assumptions, it is still possible in some cases to provide *testable* conditions for equality of opportunity when effort and circumstances are not fully observed.

We then develop an empirical analysis of equality of opportunity for income acquisition in France, using household surveys over the period 1979-2000. In this application, we assume that circumstances are defined by individual social background, measured by father's occupation and we compare income distributions conditional on social origin. Our analysis of these income distributions relies on non-parametric tests of stochastic dominance developed by Davidson and Duclos (2000). When comparing income distributions conditional on social origin, our analysis of equality of opportunity stands at the intersection of two strands of research on intergenerational mobility. First, a long tradition in sociology has analyzed the association between social origin and destination, using matrices of mobility among discrete social classifications.<sup>7</sup> Second a growing economic literature has recently focused on the correlation between parents' and children's income, concentrating on the mean impact of socio-economic origin on offspring's earnings.<sup>8</sup> Together with the sociological tradition, we capture social origin by using a discrete classification. However, we focus on offspring's income rather than social class of destination, a concern that is common to the economic analysis of intergenerational income mobility. Relative to this literature, one should emphasize that although we adopt a coarser description of socio-economic origin, our analysis of the full distribution of offspring's income allows for a rich description of the transmission of socio-economic status.

The rest of the paper is organized as follows. Section 2 discusses our characterization of equality of opportunity. We first review the various conceptions of equality of opportunity that have been discussed in recent philosophical debates. We then develop a comprehensive

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<sup>6</sup>In this respect, the imperfection of available data sets reflects a more fundamental informational constraint in liberal democracies.

<sup>7</sup>See for instance Boudon (1974), Erikson and Goldthorpe (1992) and Breen (2004).

<sup>8</sup>See for instance Bowles and Gintis (2002) and Solon (2002).

model that accommodates these various conceptions and discuss the identification of equality of opportunity when the relevant determinants of outcome are only partially observable. In section 3, we develop an empirical analysis of equality of opportunity for income in France.

## 2 Equality of opportunity : definitions and identification

### 2.1 Luck and equality of opportunity : a brief review

In the philosophical debates on equality of opportunity, the concept of luck refers to situations where individual control, choice or moral responsibility bears no relationship to the occurrence of outcomes.<sup>9</sup> <sup>10</sup>This broad concept includes the notions of circumstances and luck that we previously referred to. The general idea, shared by many authors, is that inequalities related to luck should be compensated, as they cannot be ascribed to personal responsibility. However, according to some authors, this egalitarian requirement may conflict with other values that should receive priority. This leads to distinguish several varieties of luck.

#### 2.1.1 Varieties of luck

Luck clearly appears as a multi-faceted notion that comprises a variety of empirical phenomena. Our goal is to draw attention to several ideal-type notions of luck that have been singled out in the debate about equality of opportunity, as potentially calling for different corrections. At least four different concepts of luck have been discussed, which can be illustrated by simple empirical examples. The four conceptions do not represent all possible concepts of luck nor are they independent from each other. Distinguishing these different types of luck seems useful for discussing whether and how luck should be neutralized.

First, consider two equally talented and motivated individuals whose outcome differ only because of differences in their family's social connections. In this situation individual actions and their results are pre-determined by antecedent factors (family and social origin). This illustrates the idea of *social lottery* developed by Rawls. Obviously, individuals have no control or choice over these factors. It is most probably the first candidate to be considered as a circumstance.<sup>11</sup> We propose to call it *social background luck*.

<sup>9</sup>See Lippert-Rasmussen (2005) for a discussion of the relationship between luck and distributive justice.

<sup>10</sup>In welfare economics, the analysis of social situations in the presence of risk has revealed a conflict between the *ex ante* and *ex post* approaches. See Diamond (1967), Hammond (1981), Broome (1991), Ben Porath et al. (1997), Gajdos and Maurin (2004) and Fleurbaey (2006). This perspective is however essentially absent from philosophical debates on equality of opportunity.

<sup>11</sup>See for instance the discussion in Dardanoni, Fields, Roemer and Sanchez Puerta (2005).

Second, consider two fraternal twins whose outcome differ only because one of them genetically inherited a special talent. As in the previous example, the determinant of differential success, talent, lies beyond the realm of individual choice or control. One important difference with the previous form of luck is that a specific individual talent can be seen as constitutive of the individual, in the sense that it defines what person she is. This second example illustrates the notion of *constitutive luck*, or Rawls' idea of a *natural lottery*. It includes genetically inherited factors and we therefore propose to call it *genetic luck*.

Third consider two individuals with similar talent and social background. Their outcomes differ as a result of a lottery they could not escape. For instance, as a result of the Vietnam draft lottery, one of them is inducted into the Army and subsequently enjoys poor outcomes, but not the other. This is a special form of Dworkin's notion of *brute luck*, which represents a situation where the individual cannot reasonably impact the probability of an event taking place. This kind of luck can occur at any time over a life course. Vallentyne (2002) distinguishes two types of brute luck. *Initial brute luck* is defined as the set of factors that influence lifetime prospects up to the moment when individuals can be considered responsible for their choices and decisions. This roughly corresponds to Arneson (1990)'s idea of a "*canonical moment*" where individuals become responsible for their choices and preferences. By contrast, *later brute luck* denotes the luck factors that affect individual outcomes after the canonical moment. Our example illustrates *later brute luck*.

Fourth, consider two individuals who both have to choose among two lotteries. The outcome of the first lottery is certain. The outcome of the second is random. Assume that individuals make different choices and end up with different outcomes. The occurrence of outcomes partly escapes individual control, although by making different choices, one can influence the occurrence of outcomes. This corresponds to Dworkin's notion of *option luck*. This notion implies that risk is taken deliberately, is calculated, isolated, anticipated and avoidable.<sup>12</sup> We assume it is the case in our example and refer to it as *informed option luck*.

### 2.1.2 The requisites of equality of opportunity

Whether (and how), from an egalitarian perspective, these different varieties of luck ought to be compensated has been the subject of numerous papers. Their main (unconsensual) conclusion is that not all types of luck singled out in the previous paragraphs call for full compensation.

Almost all authors would agree that *social background luck* should be fully compensated,

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<sup>12</sup>Lippert-Rasmussen (2001) and Fleurbaey (2001) emphasize the strong informational requirements that underlie the notion of option luck : option luck presupposes that agents share similar subjective and objective probabilities of outcome occurrence.

resorting to the '*starting gate position*' argument : some deep inequalities of life prospects related to economic and social circumstances of birth cannot be justified by appeal to merit and desert (Rawls, 1971).<sup>13</sup> By full compensation, we mean that justice requires that outcomes be equal regardless of *social background luck*, other things being equal.

A similar argument applies to the effects of *genetic luck* on individual outcomes. However, given the constitutive nature of *genetic luck*, compensation of its impact may conflict with other ethical values. Hence, it has been claimed that *genetic luck* should not be compensated, owing to the libertarian principle of self-ownership which states that agents are entitled to the full benefit of their natural personal endowments (e.g. intelligence, beauty, strength) (Nozick, 1977, p.225). For some authors, this requirement should receive priority over other principles.<sup>14</sup> For instance, Vallentyne (1997) claims that "*there are several independent moral demands, that they include both a demand for self-ownership and a demand for equality, and that a very strong form of self-ownership [...] constrains the demands of equality*".

From a moral point of view, compensation for all forms of luck has also been contested on efficiency grounds. The cost of compensating for all forms of luck can obviously be quite high. Such compensation requires considerable (and costly) information on individual situations as well as strong redistribution which may lead to large distortions, in the case of option luck. If these costs are large enough, compensating for all forms of luck may diminish the overall well-being. This has led some authors to formulate a restricted requirement of justice, which only calls for the compensation of *initial brute luck* and avoids part of the cost of redistribution. According to Vallentyne (2002), justice only requires that the *initial* value of lifetime prospects be equal across individuals, where the initial value is computed at the onset of adulthood.<sup>15</sup> This requires compensating for *initial brute luck*. Of course, to the extent that *later brute luck* is related to initial brute luck, compensation for the latter implies (at least partial) compensation for the former. However, equalizing the value of *initial* lifetime prospects does not erase all the impact of *later brute luck* on individual outcomes and individual can still end up, *ex post*, with different outcomes as a result of brute luck. It simply makes sure that *later brute luck* is *ex ante* even-handed.

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<sup>13</sup>See Swift (2005) for a discussion of the legitimacy of parental influence on child's outcomes.

<sup>14</sup>The idea that several moral values could constrain the principle of equality is acknowledged by numerous authors. For instance, Cohen (1989) recognizes that the egalitarian principle may conflict with other values (for him individual responsibility) : "*I take for granted that there is something that justice requires people to have equal amounts of, not no matter what, but to whatever extent is allowed by values which compete with distributive equality*".

<sup>15</sup>According to Vallentyne, one advantage of this procedure is that the *ex ante* evaluation of life-time prospects takes into account the cost redistribution. This construct is in many ways similar to the one developed by Arneson (1989) who suggests that equality of opportunity should be defined by the equality of "*preference satisfaction expectations*".

Lastly, three distinct views are held regarding the compensation for *informed option luck*. To the extent that the risky outcomes of option luck are avoidable and result from individual choice, some authors have claimed that inequalities resulting from option luck should not be compensated, owing to the principle of natural reward which states that the consequences of individual choice should be maintained. Dworkin supports that idea. A second view, expressed for instance in Fleurbaey (1995), recommends full compensation of the outcomes of option luck. Two distinct arguments are given in favor of this proposal. First, the fact that pure option luck is an extremely restrictive notion of luck that is both very rarely met in practice and very difficult to assess empirically. Second, and more importantly, the fact that not compensating for the effect of option luck implies that small errors of choices involve disproportionate, and thus unfair, penalties for some individuals. Lastly, some authors take an intermediate stance and argue in favor of partial compensation of the effects of option luck. Vallentyne (2002), states that equity authorizes taxation of the results of good option luck to partly compensate individuals who suffered bad option luck. Fleurbaey (forthcoming) proposes to decompose the outcome of option luck into two components : on the one hand, the individual choice of a specific lottery, which belongs to responsibility factors; on the other hand, the intrinsic randomness of outcome attached to any lottery, which is akin to luck. According to Fleurbaey, the former component should not be compensated while the latter should to some extent be compensated. This view is close to the one of Le Grand (1991) who claims that individuals who make similar gambling decisions should enjoy equal well-being and therefore be fully insured and that individuals who make different decisions should only bear the consequences of their gambling over their expected well-being.

## 2.2 A model of equality of opportunity

The above section reveals the lack of agreement regarding how non-responsibility factors should be accounted for in the definition of equality of opportunity. However three main conclusions emerge from this analysis. First, the idea that *social background luck* should be included in the set of *circumstances* seems beyond dispute. Second, some non-responsibility factors could be excluded from the set of *circumstances*. Third, those non-responsibility factors excluded from the set of *circumstances* differ from individual effort to the extent that they do not necessarily relate to individual responsibility.

Our purpose is to build an economic model of equality of opportunity flexible enough to accommodate the diversity of positions held in ethical debates. It seems clear from the above

discussion that this model should incorporate three types of factors : *circumstances* denote the non-responsibility factors that are not considered a legitimate source of inequality; *effort* denotes the determinants of outcome that pertain to individual responsibility and that are consequently seen as a legitimate source of inequality; *luck* denotes the non-responsibility factors that are seen as a legitimate source of inequality as long as they affect individual outcomes in a neutral way, given circumstances and effort. Our aim is to develop a characterization of equality of opportunity<sup>16</sup> consistent with this view and that lends itself to empirically testable conditions.

In this paper, we take a neutral stance on the question of what factors should count as *circumstances*, *effort* or *luck*, which, in our view, pertains to moral or political debates. Ethical debates have emphasized the role of individual responsibility in defining *effort*. For this reason, and for ease of exposition, we largely retain this perspective in our discussion. One should however strongly emphasize that this perspective is in no way central to the analysis of this paper. As the previous section shows, other ethical principles may serve to define *effort* and to delineate the scope of legitimate inequalities.<sup>17</sup> The formal definitions of equality of opportunity provided below are also consistent with these alternative principles. We now start with a simplified model where effort is not considered, to emphasize our main concepts.

### 2.2.1 Circumstances and luck : a simplified model

**Definition of equality of opportunity** Consider the case where outcome is only determined by non-responsibility factors. Let  $y$  denote individual outcome and  $c$  denote the vector of *circumstances*. Let  $F()$  denote the cumulative distribution of outcome, which is assumed to be continuous. Let a *type* define the set of individuals with similar circumstances. We refer to the determinants of outcome not included in  $c$  as *luck*.<sup>18</sup> As will become clear, the factors included in *luck* only matter through their joint effect on outcome. More precisely, in this setting, the overall impact of luck can be measured by the level of outcome that an individual reaches, for by definition, lucky individuals are the one who enjoy higher outcomes. And the distribution of outcome conditional on circumstances,  $F(y|c)$ , measures how luck affects the outcomes of individuals of a given type. In fact, this distribution is precisely the distribution of opportunities for outcome offered to individuals of type  $c$ . It gives the odds of all possible outcomes that may *ex ante* occur for an individual of this type, as a result of the influence of

<sup>16</sup>Note that our aim is to provide an equity criterion consistent with equality of opportunity but not to develop a criterion that allows to rank social outcomes. See the discussion in Hild and Voorhoeve (2004).

<sup>17</sup>For instance, the principle of self-ownership implies that some non-responsibility factors could be included in effort.

<sup>18</sup>According to the analysis in section 2.1, the notion of luck considered in this simplified model cannot be *informed option luck*, for it requires that individual responsibility be explicitly modelled in the analysis of outcomes. This will be performed in the next section.



luck. Alternatively, without loss of generality, *luck* may be summarized by a scalar index  $l$ . In this case, let  $Y(c, l)$  denote the outcome function. Again, by the very definition of luck, this function must be strictly increasing in  $l$ . An example of such an index  $l$  may be defined by the “rank” where the individual sits in the distribution of outcome conditional on her circumstances :  $l = F(y|c)$ .<sup>19</sup> So arbitrarily defined,  $l$  measures the relative degree of luck, within a given type. And  $Y(c, l)$  expresses the outcome as a function of the individual circumstances and relative degree of luck. By construction,  $l$  is identically distributed across types, which does not imply that a given degree of luck is associated with similar outcomes regardless of circumstances.

Assume that the social planner’s objective is the following : circumstances *per se* should not lead to unequal outcomes; luck can lead to differences in individual outcomes as long as it remains neutral with respect to circumstances. Equality of opportunity so defined, is equivalent to require that individuals face similar prospects of outcome  $y$  regardless of their circumstances  $c$ . This leads to the following definition of equality of opportunity.

DEFINITION EOP 1

*Equality of opportunity is satisfied iff :*  $\forall(c, c'), F(|c) = F(|c')$ .

This condition makes sure that there is no inequality related to individual circumstances and that luck affects outcome in a similar ways regardless of circumstances. Another way to interpret this condition is to say that it requires individuals with the same degree of relative luck  $l$  to have equal outcomes, regardless of  $c$ , *i.e.*  $Y(c, l) = Y(l)$  for all  $c$ .

One should also note that definition EOP 1 does not place any restriction on the dispersion of outcome resulting, within type, from the influence of luck. One may further require that these distributions be equal to a *specific* income distribution  $F_\alpha$ , whose shape, indexed by some parameter  $\alpha$ , captures the preferences of the social planner regarding the equalization of the effect of luck. For instance, as discussed in section 2.1, some authors argue that the impact of luck should be fully compensated. In this case, they would require that  $F_\alpha$  be equal to a point mass distribution  $F_0$  which is equivalent to require that all the factors that account for luck be included among the set of *circumstances*. On the opposite, other authors call for non-compensation of the effect of some varieties of luck and would require that  $F_\alpha$  be the “natural” distribution of outcome resulting from luck, say  $F_1$ . An intermediate stance between each polar opinion could be taken and would demand partial compensation of the impact of luck. This would require that  $F_\alpha$  be some intermediate distribution between  $F_0$  and  $F_1$ . In that way, our definition of equality of opportunity is sufficiently flexible to encompass various view points

<sup>19</sup>A similar definition is adopted in Fleurbaey (forthcoming).

about the neutralization of luck.

Even without placing any further restriction on the distribution of income, the situation characterized by EOP 1 appears as a situation of *strong* equality of opportunity. This condition is very stringent and may not easily be satisfied in practice. Consequently one may wonder whether all situations where EOP 1 is violated should be considered equivalent from the point of view of equality of opportunity.<sup>20</sup>

Assume that EOP 1 is not satisfied for two types with circumstances  $c$  and  $c'$ . Two situations can arise. First, for all relative degrees of luck, one type, say  $c$ , always gets higher outcome than the other ( $\forall l, Y(c, l) \geq Y(c', l)$  and the inequality is strict for some levels of  $l$ ). Second, one type gets higher outcomes for some degrees of luck while the other type gets higher outcomes for other degrees of luck (for instance, unlucky type- $c$  do better than unlucky type- $c'$  but lucky type- $c$  do worse than lucky type- $c'$ ). Now consider the hypothetical situation of someone who would be given the option to choose between circumstances  $c$  and  $c'$ , without knowing her degree of luck. This is a typical case of choice under risk. In the first case, the outcome distribution associated with  $c$  stochastically dominates the one associated with  $c'$  (see below for a definition). There is a large agreement among specialists of decision theory (Starmer, 2000) to say that in this case, consistent preferences under risk, should lead to choose  $c$  over  $c'$ .<sup>21</sup> In the second case, there is no such unanimous preference for  $c$  over  $c'$ . The first situation represents a clear case of inequality of opportunity while the second corresponds to a weak form of equality of opportunity, where no set of circumstances yield an unambiguous advantage over the other.<sup>22</sup>

This idea can be formalized using the well-known definition of first-order stochastic dominance (FSD):

DEFINITION (FIRST ORDER STOCHASTIC DOMINANCE)

$F(\cdot|c)$  strictly stochastically dominates  $F(\cdot|c')$  at the first order ( $F(\cdot|c) \succ_{FSD} F(\cdot|c')$ ) iff:

$$\forall y, F(y|c) \leq F(y|c') \quad \text{and} \quad \exists y \mid F(y|c) < F(y|c').$$

Weak equality of opportunity can be defined as the situation where no type strictly dominates any other according to FSD.

<sup>20</sup>Empirically, this question seems highly relevant. For instance Dardadoni et al. (2005) and O'Neil et al.(2000) both test a condition close to EOP 1 and conclude that it is violated. However, they do not offer a formal ranking criterion for situations in which this condition is violated.

<sup>21</sup>This consensus reaches well beyond the Expected Utility Theory. There is also empirical support for that view. In some experiments (Birnbaum and Navarette, 1998) individual choices may not accord with the first-order stochastic dominance criteria. However, as argued by Starmer (2000) this may occur in situations where stochastic dominance is opaque to the agent.

<sup>22</sup>Van de Gaer (1993) and Van de Gaer et al. (2001) already proposed the use of stochastic dominance tools to define equality of opportunity in a different framework.

DEFINITION EOP 2 (WEAK EQUALITY OF OPPORTUNITY)

*Equality of opportunity is satisfied iff :*  $\forall c \neq c', F(\cdot|c) \not\prec_{FSD} F(\cdot|c')$ .

Avoidance of first-order stochastic dominance is however a very weak requirement to define equality of opportunity and one may object that the condition stated in EOP 2 is not restrictive enough. For instance, it would consider that equality of opportunity prevails between  $c$  and  $c'$  in the case where all agents of type  $c$  do worse than those of type  $c'$  except for the one with the highest relative degree of luck. One may provide a more restrictive definition of weak equality of opportunity by resorting to the criterion of second order stochastic dominance. We discuss this criterion in the next section, in a welfarist framework.

**A welfarist foundation** The analysis in Arneson (1989) and Vallentyne (2002) suggests an alternative way to define equality of opportunity, in a welfarist framework. They propose to use the *expected value* of future prospects as the relevant metric for evaluating opportunities. This is coherent with the idea that, from an *ex ante* perspective, given individual circumstances, luck, and consequently outcomes, may be seen as random processes. In this context, equality of opportunity can be defined by the equality of the expected value of future prospects across individuals. To perform this, one can use a specific Von Neumann-Morgenstern utility function  $u$  and compute the expected utility of the opportunities for outcome offered to a given type. In this case, equality of opportunity is defined by the following proposition :

DEFINITION EOP 3

*Equality of opportunity is satisfied iff :*  $\forall(c, c'), \int_y u(y)dF(y|c) = \int_y u(y)dF(y|c')$ .

However, the question of what utility function to choose remains open. One may of course resort to a specific utility function, but in this case, the definition of equality of opportunity will lack generality. Ideally, one would like the characterization of equality of opportunity to hold for a sufficiently broad class of utility functions. In the case where there is a natural ordering for the outcome under consideration, as is the case for income, it is reasonable to focus on monotone increasing utility functions. In this context, it is obvious that the expected value of future prospects attached to different circumstances  $c$  will be equal, for all possible increasing utility functions if and only if the income distributions for these circumstances are equal. Hence, we get a welfarist foundation to EOP 1.

It is commonly assumed that the Von Neumann-Morgenstern utility function exhibit risk-aversion, which corresponds to the case where  $u()$  is concave. Under this assumption, in cases where EOP 1 is not satisfied, it is possible to provide a least partial ranking of types than

the one implied by definition EOP 2, by resorting to the criterion of second-order stochastic dominance (SSD). It is well known<sup>23</sup> that the expected value derived from a distribution  $F(Y|c)$  will be greater than the one derived from  $F(y|c')$  for all increasing concave utility functions if and only if  $F(Y|c)$  stochastically dominates  $F(y|c')$  at the second order, where second-order stochastic dominance is defined by :

DEFINITION (SECOND-ORDER STOCHASTIC DOMINANCE)

$F(.|c)$  strictly stochastically dominates  $F(y|c')$  at the second order ( $F(.|c) \succ_{SSD} F(.|c')$ )

iff:  $\forall x \in \mathbb{R}_+, \int_0^x F(y|c)dy \leq \int_0^x F(y|c')dy$  and  $\exists x \mid \int_0^x F(y|c)dy < \int_0^x F(y|c')dy$ .

It is also well-known that SSD is equivalent to generalized Lorenz dominance, more precisely:

$$F(.|c) \succeq_{SSD} F(.|c') \iff \forall p \in [0, 1] \quad GL_{F(.|c)}(p) \geq GL_{F(.|c')}(p)$$

where  $GL_{F(.|c)}(p)$  denotes the value at  $p$  of the generalized Lorenz curve for the distribution  $F(.|c)$ .

Using these notations, we get the following definition of weak equality of opportunity under risk aversion:

DEFINITION EOP 4 (WEAK EQUALITY OF OPPORTUNITY UNDER RISK AVERSION)

Equality of opportunity is satisfied iff :  $\forall c \neq c', \quad F(.|c) \not\prec_{SSD} F(.|c')$ .

Note that using second-order stochastic dominance leads to a more restrictive definition of equality of opportunity than the one provided by definition EOP 2.

### 2.2.2 General model : circumstances, luck and effort

We now develop a general model that takes into account a third determinant of individual outcomes : *effort*. *Circumstances*, denoted by a vector  $c$ , consist of the determinants of outcome that are not seen as a legitimate source of inequality and whose effect on outcome should be compensated; *effort*, denoted by a scalar  $e$ , includes the determinants that are considered, without any restriction, a legitimate source of inequality; *luck*, denoted by a scalar  $l$ , comprises those determinants that are seen as a legitimate source of inequality if they affect outcome in a neutral way.<sup>24</sup>

In this context, what does equality of opportunity require ? Since inequalities related to effort are morally acceptable, the requirement of equality of opportunity should only apply

<sup>23</sup>The requirement that choices under risk be consistent with the principle of second-order stochastic dominance (SSD) stated below does not require that the Von Neumann - Morgenstern axioms be satisfied. Machina (1982) proved that this property is valid under more general conditions within the context of non-expected utility theories.

<sup>24</sup>Extending the framework of the previous section,  $l$  may be defined by the rank where the individual sits in the distribution of outcome conditional on her circumstances and her effort:  $l = F(y|c, e)$ .

among individuals with similar effort. Hence equality of opportunity requires that individuals with similar effort face similar prospects for outcome, regardless of their circumstances. This is equivalent to say that given effort the distribution of outcome should not depend on circumstances. This criterion extends that of the previous section<sup>25</sup> and can be formalized in the following definition :

DEFINITION EOP 5

*Equality of opportunity is satisfied iff:*  $\forall(c, c') \forall e, \quad F(\cdot|c, e) = F(\cdot|c', e).$

The above definition simply requires that, given effort, the effect of luck be independent of circumstances.

As already mentioned, the criterion of individual responsibility, put forward by Cohen (1989), Arneson (1989) and Roemer (1993), offers a moral principle that can serve to define effort. This perspective is, however, in no way essential to our analysis. As suggested by the discussion in section 2.1, alternative principles, such as the principle of self-ownership, may serve to define our generic notion of effort. What matters to our analysis is that inequalities originating in differential effort are seen as legitimate and do not call for compensation. To give another illustration, if we consider that effort is defined by talent and ability, definition EOP 5 leads to the Rawlsian conception of “*fair equality of opportunity*”, defined as a situation where “*those who are at the same level of talent and ability, and have the same willingness to use them, should have the same prospects of success regardless of their initial place in the social system*” (Rawls, 1971, p.63).

The present model also allows to analyze equality of opportunity in the presence of option luck. In this case, the effort variable should comprise the individual preference towards risk that condition her choice of a particular lottery. Under this interpretation, the condition in EOP 5 implies that individuals with similar preferences should face similar lotteries independently of their circumstances. This is consistent with the view held by Dworkin. The compensation for option luck suggested by Le Grand (1991) would amount to place further restrictions on the effect of luck, conditional on effort, *i.e.* it would require that  $F(\cdot|c, e)$  be a mass point distribution, at the expected value of well-being for a given choice of lottery.

It is also possible to formalize a weak notion of equality of opportunity under risk aversion that extends EOP 4:

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<sup>25</sup>This definition also formalizes the discussion in Arneson (1989), where the author suggests that, for equality of opportunity to hold, expected welfare should be equal across individuals, only to the extent that they exercise the same degree of responsibility.

DEFINITION EOP 6

*Weak Equality of opportunity under risk aversion is satisfied iff:*

$$\forall c \neq c' \forall e, \quad F(\cdot|c, e) \not\prec_{SSD} F(\cdot|c', e).$$

## 2.3 Empirical identification

Once circumstances and effort have been defined, it is straightforward to examine whether the requirements of EOP 5 or EOP 6 are empirically satisfied, provided that outcome, circumstances and effort are observable. However, in many cases, some of that information may be missing. In this section we discuss to what extent equality of opportunity can be assessed when effort or circumstances are not fully observable to the empirical analyst.

### 2.3.1 Unobservability of effort

We first consider the case where only outcome and circumstances are observable. Hence we only observe the distribution of outcome conditional on circumstances  $F(y|c)$  and can only assess whether EOP 1 or EOP 4 are satisfied. However, in the general case, the conditions for equality of opportunity are stated in terms of the distribution of outcome conditional on circumstances and effort,  $F(y|c, e)$ , and not in terms of  $F(y|c)$ . Letting  $G(\cdot)$  denote the cumulative distribution of  $e$ , the relationship between the two is given by :

$$F(y|c) = \int_e F(y|c, e) dG(e|c) \tag{1}$$

Without additional conditions, EOP 5 will not imply any restriction on the observable conditional distributions  $F(y|c)$ . One interesting case arise when the distribution of effort is independent of  $c$ . In this case, it is straightforward to show that EOP 5 implies that the conditional distributions  $F(y|c)$  should be independent of  $c$ . Hence EOP 1 is a necessary condition for EOP 5. This is summarized by the following proposition :

PROPOSITION 1

*If  $\forall c, G(e|c) = G(e)$  then : EOP 5  $\implies$  EOP 1.*

**Proof :** If for all  $c$ ,  $G(e|c) = G(e)$  and if EOP 5 is satisfied, then, from equation 1 we get :  
 $F(y|c) = \int_e F(y|e) dG(e)$ . Hence  $F(\cdot|c)$  is independent of  $c$  and EOP 1 is satisfied. ■

Whether the condition that effort be distributed independently of circumstances is satisfied may at first appear as an empirical matter. For instance, one may claim that effort should be

defined by some ideal objective measure of how hard someone works. In this case, depending on the set of circumstances, it may or may not be the case that diligence be distributed independently of  $c$ . And equality of opportunity consistent with this definition of effort may or may not be assessed without observation of effort.

One may however object that the case where effort would be correlated to circumstances reflects an inconsistent definition of effort, when effort is restricted to the determinants of outcome that the individual is considered responsible for. As Roemer argues convincingly, if we are to take seriously the idea that individuals are not responsible for their circumstances, the definition of effort needs to be purged of any residual influence of circumstances. This leads to a relativist conception according to which effort is, *by construction*, distributed independently of circumstances. This is the conception put forward by Roemer : "*The choice of a degree of effort (as measured by the percentile of effort levels within a type) as the relevant metric for how hard a person tried, is justified by a view that, if we could somehow disembody individuals from their circumstances, then the distribution of the propensity to exert effort would be the same in every type*".<sup>26</sup> This makes it clear that the question of whether  $e$  is distributed independently of  $c$  is not solely, nor primarily, an empirical matter related to the identification of equality of opportunity under partial observation of the determinants of outcome. It pertains to more fundamental ethical debates.<sup>27</sup> Of course, not everyone would subscribe to the relativist view of effort. It is far from obvious that Roemer's argument carries over to the case where effort also includes non-responsibility factors such as genetic or option luck. If one resort to an "absolutist" view of effort, assessing equality of opportunity requires, in general, that effort be observable.<sup>28</sup>

Roemer's conception of effort can be formalized by a function of some objectively measurable effort variable  $e$  and circumstances  $c$ ,  $e^\perp(e, c)$ , such that the distribution of  $e^\perp(e, c)$  is independent of  $c$ . A natural candidate is  $e^\perp(e, c) = G(e|c)$ , the rank in the distribution of effort conditional on circumstances  $c$ .<sup>29</sup> Whether effort should also be purged of the influence of luck is an opened question that has not been addressed in the normative literature. Two opposite arguments seem relevant from an ethical perspective. On the one hand, luck is beyond individual responsibility, which suggest that its influence on effort should be nullified. In this case, effort would be defined by  $e^{\perp\perp} = G(e | c, l)$ . On the other hand, luck differs from circumstances in the

<sup>26</sup>Roemer (1998, p.15)

<sup>27</sup>Along similar lines, see the example developed in Cohen (1989, pp917-921).

<sup>28</sup>For instance, equality of opportunity is sometimes defined as the absence of discrimination on irrelevant characteristics. In the case of labor market discrimination, the relevant effort variable would be some measure of individual productivity, and there are no reasons to expect it to be distributed independently of circumstances. For a full discussion of the two notions of effort and their consequences for the assessment of equality of opportunity, see Lefranc and Trannoy (2006).

<sup>29</sup>A technical condition is required for  $e^\perp(e, c)$  to be properly defined. The distribution of  $e$  conditional on  $c$  should not exhibit any mass point.

sense that we want to compensate circumstances but not luck, as long as it remains neutral. If luckier individuals exercise more effort, their higher outcome may be seen as legitimate. In this case, effort should only be purged of the effect of circumstances. These two points of view lead to different conceptions of equality of opportunity. However it is important to emphasize that they yield similar testable restrictions, in the case where effort is not observable. Consequently, without loss of generality, we will adopt the second view and define relative effort as effort net of the influence of circumstances, *i.e.*  $e^\perp(e, c) = G(e|c)$ .

The definitions of equality of opportunity given in the previous section are general and do not rest on a specific view of effort. A definition of equality of opportunity consistent with the relative view of effort can be obtained by substituting  $e^\perp$  for  $e$  in the definition of EOP 5 or EOP 6. By construction, if EOP 5 is satisfied for  $e^\perp$ , then, by integration, the outcome distributions, conditional on circumstances alone, should be equal for all values of  $c$ , which is summarized by the following proposition :

COROLLARY

For  $e = e^\perp$  : EOP 5  $\implies$  EOP 1.

Hence EOP 1 is a necessary condition for EOP 5 under the relative view of effort. Note however that, as in proposition 1, it is not a sufficient condition. EOP 5 requires that individuals with similar effort have similar opportunities. Now consider two values of circumstances  $c$  and  $c'$ . Assume that the opportunities offered to low effort type- $c$  individuals are offered to high effort type- $c'$  individuals and *vice versa*. In this case, EOP 1 will be satisfied but not EOP 5.

We now turn to the assessment of weak equality of opportunity, as defined by EOP 6. Consistent with the relativist view of effort, we assume that effort is independent of circumstances. The consequences of the unobservability of effort are more serious for asserting EOP 6 than for EOP 5. In a nutshell, weak equality is defined by some inequalities, conditional on effort and circumstances, that do not survive well to integration over effort levels. Hence, in general, it is not possible to assess EOP 6 without observing effort. However, one special case can be singled out that can be assessed without observing effort. This situation, which we refer to as Strong inequality of opportunity, is defined by :



DEFINITION SIOP (STRONG INEQUALITY OF OPPORTUNITY)

*Strong inequality of opportunity is satisfied iff:*

$\exists(c, c')$  such that :

$$\forall e, \quad F(\cdot|c, e) \succeq_{SSD} F(\cdot|c', e)$$

and

$$\exists e \text{ such that } F(\cdot|c, e) \succ_{SSD} F(\cdot|c', e).$$

In terms of equality of opportunity, SIOP appears as the worst situation, since the opportunities offered to type  $c$  dominate those offered to type  $c'$ , whatever the value of effort. This represents as a particularly strong deviation from EOP 6. EOP 6 states that, whatever the value of effort, it is never possible to rank the opportunities offered by different circumstances using stochastic dominance. On the contrary, SIOP states not only that EOP 6 does not hold but also requires that the ranking of circumstances be the same for all levels of effort, at least for a pair of types.<sup>30</sup> When effort is independent of circumstances, we have the following proposition :

PROPOSITION 2

*If  $\forall c, G(e|c) = G(e)$  then : EOP 4 is a sufficient condition for the non-occurrence of SIOP.*

**Proof :** Assume that for all  $c$ ,  $G(e|c) = G(e)$  and that SIOP is satisfied. Then, from equation 1 and the definition of second-order stochastic dominance, we get for a pair  $(c, c')$ :  $F(\cdot|c) \succ_{SSD} F(\cdot|c')$ . Hence EOP 4 is violated. ■

### 2.3.2 Partial observability of circumstances

We now consider the case where the vector of circumstances is only partially observable. As before, we also assume that effort is not observable. The vector of observable circumstances is denoted by  $c_1$  and the vector of unobservable circumstances by  $c_2$ . In this case, it is still possible to provide a necessary condition for EOP 5 when effort is independent of circumstances. This is summarized by the following proposition :

PROPOSITION 3

*If  $\forall c, G(e|c) = G(e)$  then : EOP 5  $\implies \forall(c_1, c'_1), \quad F(\cdot|c_1) = F(\cdot|c'_1)$ .*

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<sup>30</sup>Note that the negation of EOP 6 would simply yield :  $\exists(c, c') \exists e$  such that  $F(\cdot|c, e) \succ_{SSD} F(\cdot|c', e)$

**Proof :** If EOP 5 is satisfied, we have :  $\forall(c_1, c'_1), \forall c_2, \forall e, \forall y, \quad F(y|c_1, c_2, e) = F(y|c'_1, c_2, e)$ .

Furthermore, if  $e$  is distributed independently of circumstances, integrating over values of  $e$  implies :  $\forall(c_1, c'_1), \forall c_2, \quad F(\cdot|c_1, c_2) = F(\cdot|c'_1, c_2)$

Integrating over values of  $c_2$  implies :  $\forall(c_1, c'_1), \quad F(\cdot|c_1) = F(\cdot|c'_1)$ . ■

The reciprocal is however not true. Loosely speaking, proposition 3 simply states that the values of  $F(\cdot|c_1, c_2)$  and  $F(\cdot|c'_1, c_2)$  are equal on average, where the average is computed over values of  $c_2$ . Of course, this does not imply that the CDF are equal for all values of  $c_2$ . Hence, under partial observability of the circumstances, the condition in proposition 3 is only a necessary condition for equality of opportunity as defined in EOP 5.

Again, assessing EOP 6 is not achievable under partial observability of circumstances. Consider two values  $c_1$  and  $c'_1$  of the vector of observable circumstances. Even the case where there is SIOP between  $(c_1, c_2)$  and  $(c'_1, c_2)$  for all values of  $c_2$  does not imply that  $F(\cdot|c_1) \succ_{SSD} F(\cdot|c'_1)$ , since  $c_2$  need not be distributed independently of observable circumstances. However one special case of strong inequality of opportunity can be assessed, which corresponds to the case where, for all possible effort, the set of options offered to individuals with circumstances  $c_1$  dominates the one offered to individuals with circumstances  $c'_1$  whatever the value of their unobservable circumstances  $c_2$ . That is :

$$\forall(c_2, c'_2), \forall e \quad F(\cdot|c_1, c_2, e) \succ_{SSD} F(\cdot|c'_1, c'_2, e) \quad (\text{SSIOP})$$

Again, it is possible to provide a sufficient condition for the non-occurrence of SSIOP, which is given in the following proposition :

PROPOSITION 4

*If  $\forall c, G(e|c) = G(e)$  then : EOP 4 is a sufficient condition for the non-occurrence of SSIOP.*

**Proof :** Integrating the condition in SSIOP over effort and unobservable circumstances implies that  $F(\cdot|c_1) \succ_{SSD} F(\cdot|c'_1)$ . Hence if effort is independent of circumstances and SSIOP holds, EOP 4 is violated. ■

### 2.3.3 Roemer's model : circumstances and effort

Roemer's model appears as a special case of the above setting in which circumstances are observable, effort is not observable and luck plays no role.<sup>31</sup> Hence outcome can be expressed

<sup>31</sup>For a complete discussion of the conditions of identification of equality of opportunity in Roemer's model, see O'Neill, Sweetman and Van De Gaer (2000).

as a function  $Y(c, e^\perp)$  that only depends on effort and circumstances. Since luck plays no role in this model, EOP 5 implies that individuals with similar effort should receive equal outcomes regardless of circumstances. This is summarized by the following proposition :

DEFINITION EOP 7 (ROEMER'S DEFINITION)

*Equality of opportunity is satisfied in Roemer's model iff :*

$$\forall(c, c'), \forall e^\perp, \quad Y(c, e^\perp) = Y(c', e^\perp).$$

In this case, EOP 1 is of course still a necessary condition for Roemer's definition EOP 7. Roemer further assume that the outcome function is strictly increasing in effort. In this case, although effort is not directly observable, relative effort  $e^\perp$  can be inferred from the observation of outcome and circumstances. For an individual with circumstances  $c$ ,  $e^\perp$  is in fact equal to the rank  $p$  where she sits in the conditional outcome distribution,  $F(y|c)$ . Since effort can be inferred, we have the following proposition :

PROPOSITION 5

*Under Roemer's assumptions : EOP 7  $\iff$  EOP 1.*

**Proof :** Let  $Q(p|c)$  denote the quantile function associated to the distribution  $F(y|c)$  and defined by  $p = F(Q(p|c)|c)$ . If outcome is a strictly increasing function of effort, the definition of  $e^\perp$  implies that  $Y(c, e^\perp) = Q(e^\perp|c)$ . Hence, we have : EOP 7  $\iff$   $(\forall(c, c'), \forall p \in [0, 1], Q(p|c) = Q(p|c')) \iff$  EOP 1. ■

Consequently, under the assumption that outcome is strictly increasing in effort, EOP 1 is a necessary *and sufficient* condition for EOP 7. For similar reasons, EOP 4 is a necessary and sufficient condition for avoiding SIOP. These two results require, however, that circumstances be fully observable. If they are only partially observable, the results of the previous subsection apply.

Of course, if, conditional on circumstances, outcome is determined by effort and luck, the rank in the distribution of outcome conditional on  $c$  can no longer serve to identify effort, even if we assume that outcome is strictly increasing in effort, as the rank reflects the joint impact of luck and effort. This explains why, in the general model, we only have necessary (but not sufficient) conditions for EOP 5.

### 2.3.4 Summary

To summarize the main conclusions of this section, equality of opportunity can be empirically assessed using the conditions EOP 1 and EOP 4, even in the case where effort is not observable. To appraise EOP 1, we need to compare the cumulative distributions of income, conditional on observed circumstances. Since EOP 1 is only a necessary condition for equality of opportunity in the general model, we can only draw firm conclusions in the case where the cumulative distributions are not found equal. This case indicates that equality of opportunity, as defined by EOP 1, EOP 5 or EOP 7, is violated. The situation where the cumulative distributions are found equal is only indicative of equality of opportunity : we can only conclude to equality of opportunity if we are willing to consider that the determinants of outcome excluded from the circumstances only reflect luck (which corresponds to the model of section 2.2.1) or effort (which corresponds to the model of section 2.3.3).

EOP 4 requires to compare the generalized Lorenz curves associated with observed circumstances. When comparing two generalized Lorenz curves, three situations can occur: (a) the two curves are identical, (b) the two curves intersect, (c) one curve lies above the other. Case (a) is equivalent to the equality of the cumulative distributions, which has already been discussed. Case (b) implies that strong inequality of opportunity, as defined by SIOP, is not satisfied. It also implies that weak equality of opportunity is satisfied if we are willing to assume that the determinants of outcome excluded from the circumstances resort to luck or to effort alone. Lastly, case (c) is suggestive of a deviation from equality of opportunity for two reasons : first, EOP 1 is not satisfied; second we cannot even rule out the situation of SIOP. Case (c) corresponds to the situation of second-order stochastic dominance of one distribution over the other. A special case of this situation is the case of first-order stochastic dominance. This case is worth investigating in its own right, since first-order stochastic dominance signals a strong deviation from equality of opportunity, since the opportunity sets offered to individuals with different circumstances can be ranked whatever the attitude of the decision maker towards risk.

## 3 Empirical application : income in France, 1979-2000

In this section, we analyze equality of opportunities for income in France. To this end, we examine whether income distributions conditional on social origin are equal or exhibit stochastic dominance patterns. We first present the data and the statistical procedure used in the analysis. We then discuss the results.

### 3.1 Data

The data come from the French household survey "Budget des Familles" (BdF) conducted by the French national statistical agency (INSEE). Five waves of the survey have been collected (1979, 1984, 1989, 1994 and 2000), each on a sample of about 12,000 households. We use all available waves.<sup>32</sup> For each household, the data provide detailed information on all sources of income and expenditures and enable to identify the household's social background. Sample summary statistics are presented in appendix table A-1.

#### 3.1.1 Main variables

All waves of the BdF data contain information on the social origin of heads of household and their spouse. Both are asked to report the one-digit occupational group of their two parents.<sup>33</sup> From these four variables, it would be possible to build a detailed classification of social origin. Given the size of our samples, the use of a detailed classification would lead to small sub-samples, and to inaccurate estimations of the income distributions conditional on social origin. For this reason, we only use information on the occupational group of the household head's to define individual circumstances. This leads to distinguish the following six social origins: farmers, artisans, higher-grade professionals, lower-grade professionals, non-manual workers and manual workers.<sup>34</sup>

The outcome variable we focus on is household standard of living. The BdF data provide a detailed record of all income sources including wage and labor income, asset income, transfers (pensions, unemployment benefits, child support, welfare benefits) as well as income and property taxes. We consider two measures of family income. The first one is primary income, which includes labor and asset income, and unemployment and pension benefits. The second one corresponds to disposable income and is equal to primary income plus redistributive transfers minus taxes. In both cases, we normalize family income by family size using the OECD equivalence scale. To make income measures comparable over time, income is expressed in constant terms (2002 Euros) using the consumer price index.

Consequently, our data set provide a comprehensive measure of one fundamental individual outcome : living standard. It also offer a characterization of an important determinant of this outcome, that most authors would agree to include among the set of relevant individual

<sup>32</sup>We use sample weight to ensure sample-representativeness.

<sup>33</sup>For every survey, except 1979, it is the occupational group when the respondent was 16. In 1979, it is the last occupational group of the parents.

<sup>34</sup>These six groups correspond to the French INSEE job classification. Children of artisans also include the children of small proprietors. The occupational groups of the 1979 survey have been recoded to account for the change in the occupational classification that occurred in 1982.

circumstances. On the contrary, it offers no measure of individual effort or luck and may be criticized for providing only an incomplete description of circumstances. Therefore, the empirical assessment of equality of opportunity that our data allow corresponds to the situation analyzed in section 2.3.2.

### 3.1.2 Sample selection rules

Within a given survey wave, changes in the social structure over time imply that the age composition will differ across groups of different social origin. For instance, the rise in the share of higher-grade professionals and the fall in the share of farmers implies that children of higher-grade professionals (respectively farmers) will be younger (resp. older) than the average. To avoid this composition effect, our sample is limited to households whose head was between 30 and 50 years old at the time of the survey. We also exclude households whose head was retired or student at the time of the survey. Another advantage of this sample selection rule is that household income will be more representative of their lifetime income (Grawe, 2006). This leads to samples of about 4 000 households in each wave.

The early waves of the survey exhibit a high rate of non-response to the questions pertaining to income earned and taxes paid. In some case, information is missing for one or several income items. In others, respondents only report some income items in bracketed form. In these data, non-response cannot be considered random. It is correlated with the occupation of the head of household as well as with other socio-demographic characteristics, and is stronger for self-employed workers (farmers and artisans) than for wage earners. Hence, ignoring missing data would lead to a biased view of the income prospects conditional on social origin. For this reason, in case of non-response, household income has been imputed, using the simulated residuals method. For observations with missing or bracketed data, we predict income using an estimated income equation. This equation is estimated on those households who report an income (in level or in brackets), and income is regressed on observable characteristics (age, sex, occupational group, last diploma, consumption, nationality, family composition, geographic area of living ...)<sup>35</sup>. In case of missing data, we also draw a residual term that is added to the predicted income. This procedure is implemented to impute primary and disposable income. The income distribution estimated using this imputation procedure appear consistent with the distributions obtained from administrative data. This is true, in particular, for farmers' income that was especially badly reported in the first waves of the survey.<sup>36</sup>

<sup>35</sup>Detailed equations are available upon request.

<sup>36</sup>As we will discuss, part of the results reported below are driven by changes in the income distribution

### 3.2 Statistical Inference : General principles

Here, we explain the general principles of our statistical methodology. The details of the stochastic dominance tests we implement are presented in the Appendix. Our samples allow to build income distribution conditional on social origin and test whether conditions EOP 1 or EOP 4 are satisfied. Assessing equality or stochastic dominance relationships is a demanding exercise. It requires that the *entire* outcome distributions (or some integral of them) be compared for all possible circumstances. One should also bear in mind that these distributions need to be estimated and compared, in our case using samples of relatively small size. Hence, special attention must be paid to the statistical robustness of the conclusions drawn from sample data. In this context, performing parametric tests of stochastic dominance is likely to yield fragile conclusions. On the contrary, our empirical analysis rests on non-parametric stochastic dominance tests developed in Davidson and Duclos (2000), which lead to robust conclusions.

The empirical procedure we implement is the following. For all possible pairs of circumstances  $c$  and  $c'$ , we perform three tests independently : (1) we test the null hypothesis of equality of the distributions of types  $c$  and  $c'$ ; (2) we test the null of first-order stochastic dominance of the distribution of type  $c$  over type  $c'$  and *vice-versa*; (3) we test the null of second-order stochastic dominance of the distribution of type  $c$  over type  $c'$  and *vice-versa*.

Test (1) corresponds to condition EOP 1 and tests (2) and (3) to EOP 4. For any pair of types, we interpret the joint results of these tests in the way summarized below. Of course, this interpretation is only tentative and one should keep in mind the caveats discussed in section 2.3.4.

- If we fail to reject the null of test (1), we say that equality of opportunity is supported, since EOP 1 is satisfied.
- Else, if test (2) or (3) accepts dominance of one distribution over the other but not the other way round (*e.g.*  $F(|c) \succeq_{SSD} F(|c')$  and  $F(|c') \not\prec_{SSD} F(|c)$ ) we say that equality of opportunity is violated, since neither EOP 1 nor EOP 4 are satisfied.
- Else, if test (3) rejects dominance of each distribution over the other (*i.e.*  $F(|c) \not\prec_{SSD} F(|c')$  and  $F(|c') \not\prec_{SSD} F(|c)$ ) we say that weak equality of opportunity is supported, since EOP 4 is satisfied but not EOP 1.

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of farmers, estimated from our data. Consequently, we were concerned that part of these evolutions may reflect spurious changes in income distribution related to changes in reporting behavior. Hence we compared the estimated income distribution of farmers to the ones obtained from agricultural national accounts. As documented in Lefranc *et al.* (2004), it turns out that our estimates are strongly consistent with those obtained from administrative sources.

- Else, if test (2) or (3) conclude that the two distributions dominate each other ((*i.e.*  $F(|c) \succeq_{SSD} F(|c')$  and  $F(|c') \succeq_{SSD} F(|c)$ )), we give priority to the result of test (1) since it is a more powerful test of equality of distributions for any significance level. Hence, we say that only weak equality of opportunity is supported, since EOP 4 is satisfied but not EOP 1.

Lastly, one should note that, given our interpretation, conclusions of test (2) and (3) cannot contradict since the null of (2) is included in the null of (3). Thus the conjunction of the results of the three tests interpreted in this way cannot be inconsistent.

### 3.3 Results

In this section, we first report the results of the tests of equality and stochastic dominance, for the income distributions conditional on our partition of social origin. While we use all available waves, we only report in the main tables and figures the results for 1979 and 2000, since our discussion mostly focus on the initial and terminal waves. Results for other years are reported in the appendix.

Three main conclusions emerge from this analysis. First, equality of opportunity is not satisfied : for most pairwise comparisons of types, we find evidence of stochastic dominance relationships; overall, a clear hierarchy of the different groups of social origin emerges. Second, the *pattern* of inequality of opportunity is stable over time : the *relative* ranking of types remains almost constant across the period 1979-2000. Third, the *degree* of inequality of opportunity decreases over time : while the ranking of types is unchanged, the income distributions of the different types come closer together over the period.

We then examine what factors account for inequality of opportunity. As discussed in decision theory, two factors may contribute to stochastic dominance between two income lotteries : differences in the expected return of the two lotteries and differences in the degree of risk of the two lotteries. In our case, the expected return corresponds to the mean income for each type and risk corresponds to within type inequality. Our results indicate that the degree of risk is very similar for all types. This is true over the entire period. On the contrary, the returns differ markedly across types. Since the evolution of these returns is a key determinant of the narrowing of the income prospect gap, we finally analyze the determinants of the changes in these returns over time.



### 3.3.1 A reduction in the degree of inequality of opportunity

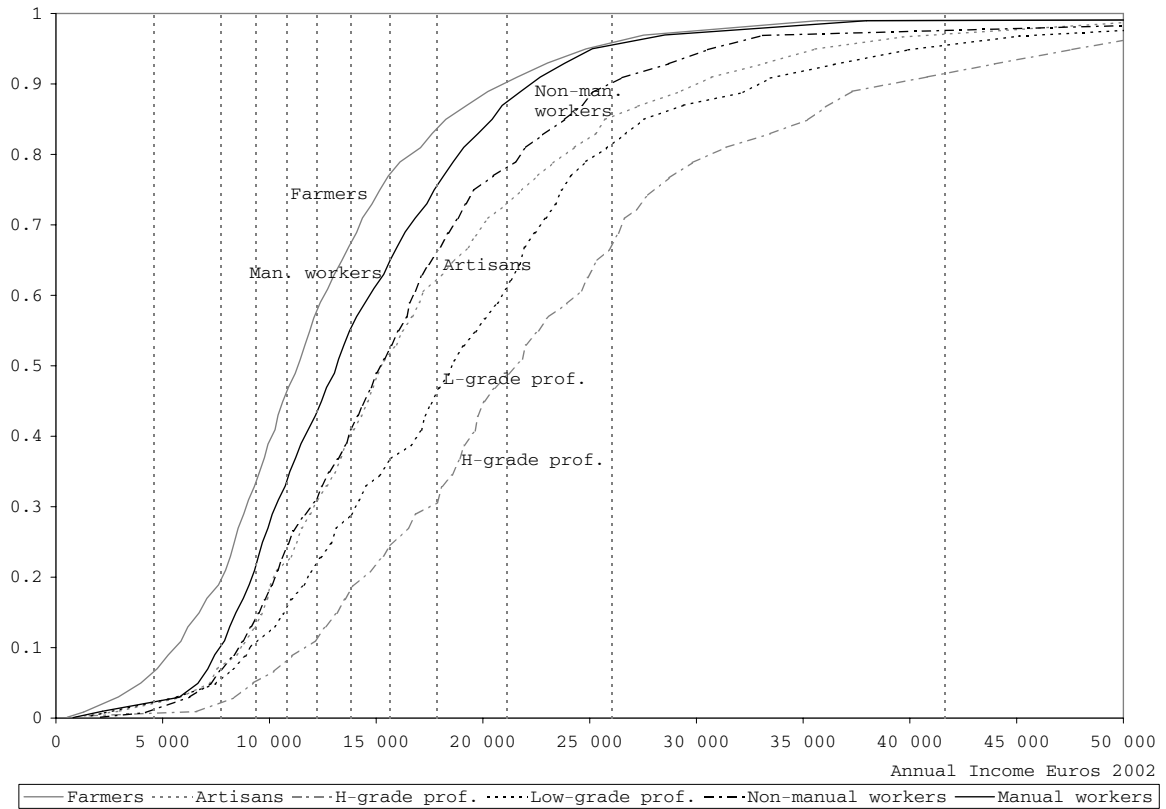
The cumulative distribution functions and the generalized Lorenz curves conditional on social origin are given in Figures 2 and 3 for 1979 and 2000. For 1979, a particularly clear ranking of social types emerges. Children of higher-grade professionals stand out as the most advantaged type : their conditional distribution dominates by far those of other social groups. Children of lower-grade professionals come next, followed by children of artisans and children of non-manual workers. In fact, the income distributions of the latter two groups seem very close, especially in the first half of the distribution. Lastly, at the bottom of the social hierarchy, come the children of manual workers and the children of farmers. The income distribution of the children of farmers, in 1979, is, by far, dominated by all other social backgrounds.

This “visual” ranking is strongly supported by the results of the tests of equality and stochastic dominance. These results are presented in Table 1, for primary and disposable income. In 1979, in all but one pair-wise comparisons, the equality of the conditional income distributions is rejected. Without ambiguity, this indicates that EOP 5 is not satisfied. The only two types who apparently face equal opportunities are the children of non-manual employees and of artisans, although one should keep in mind, here and in the rest of the paper, that EOP 1 is only a necessary condition for EOP 5. Furthermore in all other pair-wise comparisons, the tests indicate that one distribution dominates the other, which suggests that strong inequality of opportunity, as defined by SIOP, may prevail. One should also note that in all these cases, stochastic dominance is satisfied at the first order, which implies that a ranking of social types can be achieved without assuming risk aversion. Lastly, in 1979, the impact of taxes and transfer on inequality of opportunity, is very limited. The gap between the generalized Lorenz curves is slightly lower for disposable income than for primary income (see figure 3), but stochastic dominance relationships are not affected by redistribution.

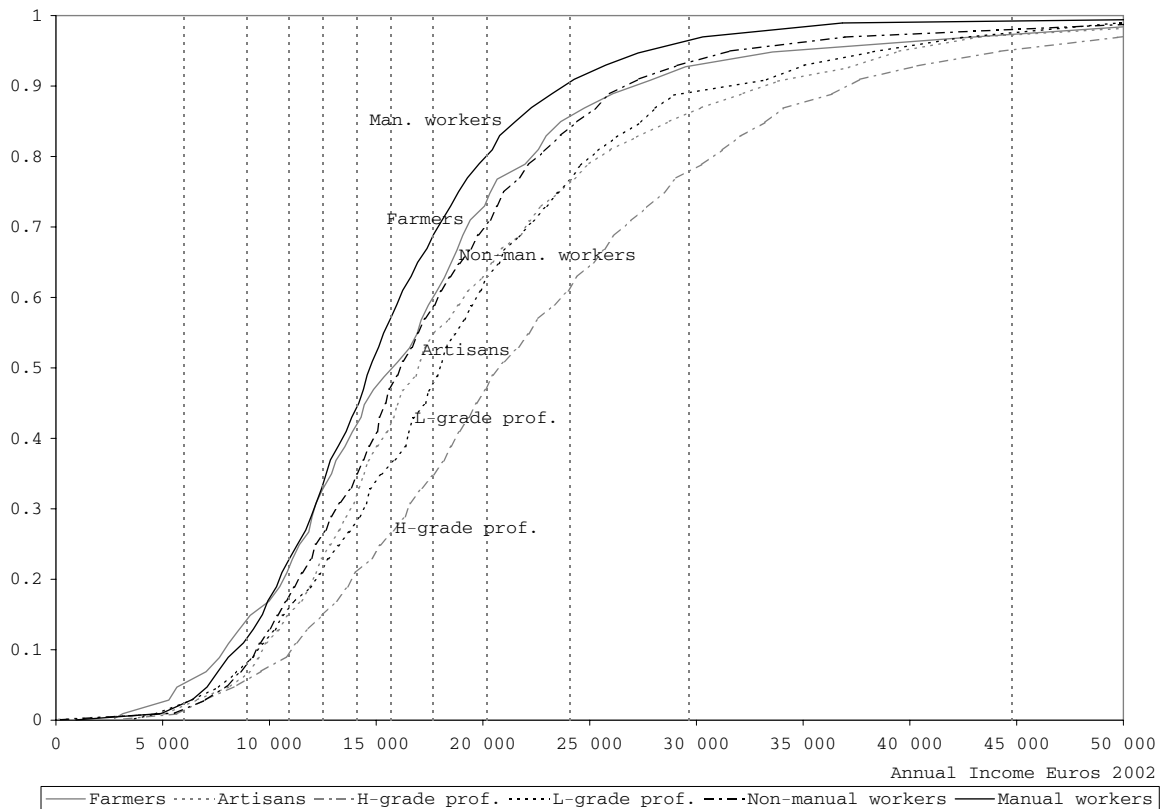
The pattern of stochastic dominance relationships exhibits small changes between 1979 and 2000. The results of the tests for primary income are given in Table 1. Four important features can be underlined. First, the dominant position of the children of higher-grade professionals remains unchallenged during the entire period: in every wave their income distribution dominates those of all other groups. Second, the hierarchy of intermediate groups tends to weaken. This is in great part due to an improvement of the relative ranking of the children of artisans : in 1979, this type was dominated by children of lower-grade professionals and their opportunities were equal to those offered to children of non-manual workers; this is no longer the case after 1994. In 2000, the opportunities offered to children of artisans are equal to those offered to chil-

Equality of opportunity: Definitions and testable conditions, with an application to income in France (1979-2000)  
 Figure 2: Income distributions by social background - disposable income

A- 1979



B- 2000



Notes : The occupational group refers to social origin. H-grade prof. : higher-grade professionals; L-grade prof. : lower-grade professionals; Non-man. workers : non-manual workers

dren of lower-grade professionals and dominate those of children of non-manual workers. In this intermediate group, one can also notice a change in the relative ranking of children of lower-grade professionals and non-manual workers. In 1989 and 1994, the income distribution of these two groups are equal, although it is no longer true in 2000. The third important phenomenon that takes place over this period occurs at the bottom of the hierarchy. In 2000 the conditional distribution of the children of farmers dominates the distribution of children of manual workers. This group is now dominated by all the others social backgrounds. Overall, a three-levels hierarchy persists over the entire period. It is dominated by the children of higher grade professionals. In the middle comes an intermediate group that includes the children of lower-grade professionals, artisans and non-manual workers. At the bottom, come the children of farmers and manual workers.

While the ranking of social backgrounds has changed, one may nevertheless be tempted to conclude that equality of opportunity has not made any good progress over the period 1979-2000. In all waves, equality of opportunity is rejected in at least 80% of all pair-wise comparisons. In 2000, EOP 1 is only satisfied in one case and EOP 4 prevails in another one. Hence, a strict ranking of all types is almost always possible. Despite this fact, the comparison of the generalized Lorenz curves for the first and the last wave (see figure 3) clearly indicates that the income distributions of the different types have come closer together between 1979 and 2000. This suggests that the change at work is more cardinal than ordinal. While inequality of opportunity continues to prevail, the degree of inequality of opportunity seems to weaken. We now turn to the analysis of this cardinal change. To do so, we analyze the degree of risk and the return attached to the different conditional income distributions.

### 3.3.2 The risk of social lotteries

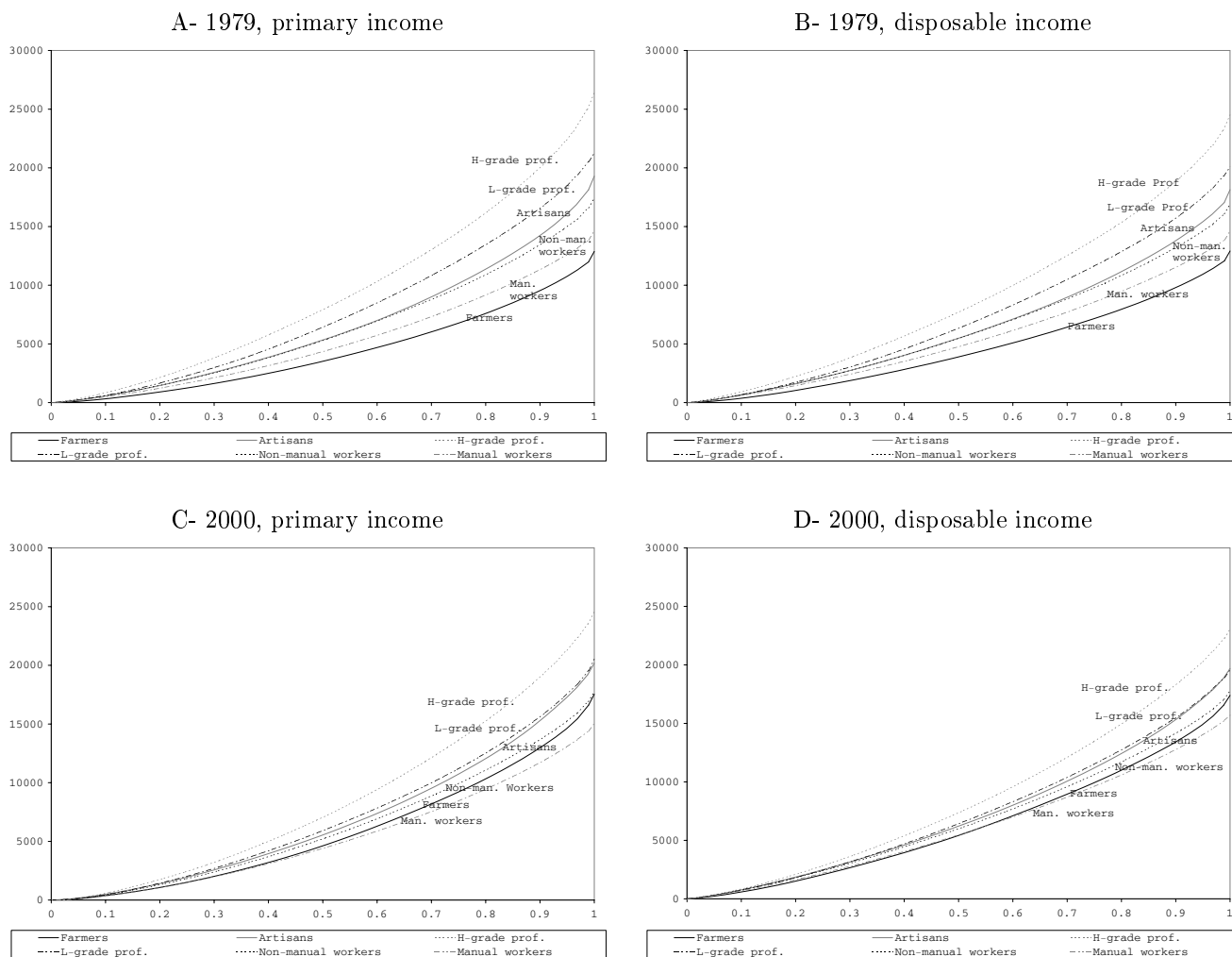
As already discussed, stochastic dominance relationships among income lotteries can arise because of differences in their expected return or in their degree of risk. In the present context, the expected return is defined as the mean income conditional on social origin and the risk corresponds to within-type inequalities. Note that in the general model of section 2 the income lotteries offered to individuals correspond to the income distribution conditional on their circumstances and effort. Return and risk should be computed from these distributions. In our case, we do not observe individual effort. Hence, within-type inequalities will reflect the joint influence of luck and effort.<sup>37</sup> This remark should be kept in mind throughout this section.

The degree of risk of the different social lotteries can be analyzed using the Lorenz curves of

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<sup>37</sup>Effort can nevertheless be seen as a source of moral hazard and is therefore akin to a form of risk.

Figure 3: Generalized Lorenz curves in 1979 and in 2000



Notes : Income in Euros 2002 is represented on the  $y$ -axis. The occupational group refers to social origin. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

Table 1: Stochastic dominance tests

A- Primary Income						
<b>1979</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	< <sub>1</sub>	=	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>2000</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	?	> <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	=	> <sub>1</sub>	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
B- Disposable Income						
<b>1979</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	< <sub>1</sub>	=	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>2000</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	> <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	=	> <sub>1</sub>	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>

Notes : The occupational group refers to social origin. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

Each element in the table indicates the result of the comparison of the income distribution of the groups in row and column using the tests presented in section 3.2 :

><sub>i</sub>: the row dominates the column for order *i* stochastic dominance ;

<<sub>i</sub>: the column dominates the row for order *i* stochastic dominance;

=: the distributions are equal;

?: the distributions cannot be ranked using first and second order stochastic dominance.

the distributions of income conditional on social origin. Consider two lotteries  $A$  and  $B$ . The lottery  $A$  is less risky than  $B$  if its Lorenz curve is always above the curve of  $B$ . In this case, lottery  $A$  is said to Lorenz-dominate lottery  $B$ . If the two Lorenz curves are identical then the two social lotteries are equally risky. Consequently, to compare the degree of risk of the different social lotteries, we can resort to the same testing procedure as for stochastic dominance.

Table 2 displays the results of the tests of Lorenz dominance. The Lorenz curves of the different social lotteries are very similar. For primary income, tests conclude to the equality of the Lorenz curves, in 9 comparisons (out of 15) in 1979, 5 in 1984, 12 in 1989, 11 in 1994 and 10 in 2000. The tests conclude to dominance in only one case in 1979, 3 in 1984, 2 in 1994 and 0 in 1989 and 2000. Hence, the different social lotteries exhibit very similar degrees of risk.

A slightly different picture emerges from the analysis of the Lorenz curves of disposable income, for the end of the period. First, in 2000 (as well as in 1994), the Lorenz curve of the children of manual workers dominates that of all most other groups, with the exception of the children of non-manual workers and lower-grade professionals. In other words, the income distribution for children of manual workers exhibits the smallest degree of risk. Second, at the end of the period, the income distribution of children of farmers and, to a lesser extent, artisans, tend to exhibit more risk than those of children of wage earners.

Overall, these results suggest that social origin mostly influences the distribution of outcomes as a scale factor. On the other hand, the combined influence of effort and luck is such that the relative prospects are roughly similar across types. This implies that income conditional on social origin can be represented by the following multiplicative model:

$$y_{ic} = E(y | c)\epsilon_i \quad (2)$$

Where  $y_{ic}$  represents income of individual  $i$  with social origin  $s$ ,  $E(y | c)$  is the income mean conditional on  $c$  and  $\epsilon_i$  a random term whose distribution is independent of social origin.

The strong similarity in the degree of risk attached to the different social backgrounds explains why in most cases, first-order stochastic dominance is a sufficient criterion for ranking conditional income distributions.<sup>38</sup> In fact, in this context, equality of opportunity can be assessed by relying solely on comparisons of mean conditional incomes. This situation is a priori quite rare.

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<sup>38</sup>The rare exceptions arise when comparing children of non-wage earners to children of wage earners. Due to the higher risk in the distribution for the former group at the end of the period, expected return and degree of risk sometimes point to opposite directions. For instance in 1994, as will be discussed in the next section, the mean disposable income is 5% larger for children of artisans than for children of lower-grade professionals. However, the stochastic dominance test turns out to be inconclusive due to the higher degree of risk of the lottery of children of artisans.

Table 2: Lorenz dominance tests

A- Primary Income						
<b>1979</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	?	=	=	=	<
Artisans	-	-	=	?	?	?
H-grade prof.	-	-	-	=	=	?
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=
<b>2000</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	?	?	?	<
Artisans	-	-	=	=	=	?
H-grade prof.	-	-	-	=	=	=
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=
B- Disposable Income						
<b>1979</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	?	=	=	<	<
Artisans	-	-	=	=	=	=
H-grade prof.	-	-	-	=	=	?
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=
<b>2000</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	=	?	<	<
Artisans	-	-	=	=	=	<
H-grade prof.	-	-	-	=	=	<
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=

Notes : The occupational group refers to social origin. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

Each element in the table indicates the result of the comparison of the Lorenz curves of the groups in row and column using the criterion of Lorenz dominance :

><sub>i</sub>: the row dominates the column for Lorenz dominance ;

<<sub>i</sub>: the column dominates the row for Lorenz dominance;

=: the Lorenz curves are equal;

?: the Lorenz curves cannot be ranked using Lorenz dominance.

### 3.3.3 The expected return of social lotteries

We now compare the expected returns of the different social lotteries. These expected returns can be summarized by the mean income conditional on social origin.

**Trends in conditional mean income** Mean incomes conditional on social origin are given in Table 3. This table confirms the evolution apparent in Figure 3 : mean conditional incomes tend to converge between 1979 and 2000. The ratio between the mean income of the most advantaged group (children of higher-grade professionals) and the least advantaged one (children of farmers in 1979 and of manual workers in 2000) falls from 1.89 to 1.46 for disposable income and from 2.04 to 1.63 for primary income. The advantage of children of higher-grade professionals falls by one half relative to the least advantaged group and diminishes relative to every other social background. More generally, the increase in the mean income of the different social groups is inversely related to their initial rank.

Table 3: Mean income conditional on social origin or destination

A- Mean income conditional on social origin				
	Primary Income		Disposable Income	
	1979	2000	1979	2000
Farmers	12 874	17 541	12 914	17 395
Artisans	19 295	20 174	18 137	19 691
H-grade prof.	26 375	24 543	24 490	23 033
L-grade prof.	21 225	20 511	20 055	19 534
Non-man. workers	17 379	17 720	16 884	17 747
Manual workers	14 612	15 008	14 592	15 709
Mean Income	16 503	18 313	16 070	18 180

B- Mean income conditional on social destination				
	Primary Income		Disposable Income	
	1979	2000	1979	2000
Farmers	9 367	17 858	9 614	17 449
Artisans	17 090	19 833	15 797	18 972
H-grade prof.	28 513	30 642	26 104	27 604
L-grade prof.	19 048	20 185	18 304	19 672
Non-man. workers	14 009	13 327	13 999	14 199
Manual workers	12 264	13 113	12 738	14 357
Mean Income	16 503	18 313	16 070	18 180

Notes : incomes in 2002 Euros. In panel A, the occupational group refers to social origin; in panel B, to social destination. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

The increase in mean income is particularly strong for children of non-wage earners: children



of farmers increase their mean income by 34.6%; children of artisans by 8.5%. As a result children of non-wage earners improve their relative position relative to all other social backgrounds. This contributes to the fall in the degree of inequality of opportunity since the initial ranking of both groups was relatively low : in 1979, children of artisans ranked third and children of farmers ranked last. The results for non-wage earners should be interpreted with caution due to the high rate of non-response discussed in section 3.1.2. However, the growth in non-wage earners mean income reported here is very close to what is observed using non-declarative sources such as national accounts. Moreover, most of this increase occurs during the 1990's, a decade for which we performed very few imputations. Hence, it is most unlikely that the erosion of higher-grade professionals' position and the improvement of non-wage earners' lot documented here arise from a statistical artifact. They reflect important changes in the extent of inequality of opportunity that need to be analyzed.

**Decomposition** Two factors can explain this fall in the dispersion of mean incomes conditional on social origin. First it may originate from an increase in social mobility : if mobility increases, conditional mean incomes converge, as the distribution of social class destinations conditional on social origin come closer together. Second, it may come from a reduction in the dispersion of mean incomes conditional on social destination. We refer to the first effect as the mobility effect, and to the second as the return effect.

As documented in Table 3, mean incomes conditional on social destination tend to partly converge between 1979 and 2000. Higher-grade professionals experience slower income growth than other groups and non-wage earners experience faster growth. This indicates that the return effect contributes to the narrowing of the dispersion of mean incomes conditional on social origin. However, it only accounts for part of the evolution. The observed reduction in mean income gaps is much stronger when conditioning on social origin than when conditioning on social destination. For instance the ratio between the mean disposable income of higher-grade professionals and manual workers only falls from 2.04 to 1.92. When conditioning on social origin instead of destination, it falls from 1.67 to 1.46.

The evolution of social mobility is summarized by the mobility matrices given in Table 4. The matrices indicate a rise in social mobility for several social groups. For example, children of higher-grade professionals see their probability of becoming higher-grade professionals fall from 51% in 1979 to 44% in 2000, while in the meantime the proportion of higher-grade professionals in the total population has increased. Children of farmers experience a large increase in upward mobility : while their probability of becoming farmer remains unchanged, the probability that

Table 4: Occupational group transition matrices

A- 1979							
	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers	Total
Farmers	.225	.074	.047	.116	.14	.396	.216
Artisans	.009	.234	.192	.219	.147	.195	.139
H-grade prof.	.009	.033	.51	.261	.133	.05	.069
L-grade prof.	.000	.045	.287	.401	.138	.127	.073
Non-man. workers	.006	.057	.144	.295	.197	.298	.111
Manual workers	.005	.083	.064	.209	.164	.474	.389
Total	.053	.093	.134	.218	.156	.343	1.00

B- 2000							
	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers	Total
Farmers	.218	.049	.100	.159	.146	.325	.105
Artisans	.004	.118	.205	.251	.191	.229	.122
H-grade prof.	.005	.044	.415	.318	.119	.097	.150
L-grade prof.	.001	.055	.222	.349	.175	.186	.085
Non-man. workers	.007	.056	.137	.275	.232	.290	.184
Manual workers	.005	.047	.066	.192	.198	.490	.351
Total	.028	.058	.166	.243	.184	.318	1.00

Notes : the table gives the distribution of social destination (in column) conditional on social origin. The rows and columns labelled 'Total' give the column (resp. row) marginal distribution. Example: in 2000 sample, 10.5% of the population are children of farmers, 2.8% are farmers, and 4.9% of the children of farmers are artisans. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

they become higher-grade or lower-grade professionals increases between 1979 and 2000 and the probability that they become manual or non-manual workers decreases. Again, the mobility effect accounts for part of the change in mean incomes conditional on social origin.

The contribution of the mobility and the return effect to the observed evolution can be identified using the Oaxaca-Blinder decomposition<sup>39</sup>. The mean income in year  $t$ , conditional on social origin  $j$  can be expressed as:

$$\bar{y}_j^t = \sum_k \alpha_{jk}^t \bar{y}_{jk}^t$$

where  $k$  denotes social destination,  $\alpha_{jk}^t$  is the probability of destination  $k$  conditional on origin  $j$  and  $\bar{y}_{jk}^t$  denotes the mean conditional on social origin and destination. The change in mean income conditional on social origin between  $t$  and  $t'$  can be written as :

<sup>39</sup>Oaxaca (1973); Blinder (1973)

Table 5: Decomposition of the evolution of the mean income

	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
	4481	1 553	-1 457	- 521	863	1 118
	Change in mean income (1979-2000)					
	4481	1 553	-1 457	- 521	863	1 118
	First decomposition (in %)					
Return effect	83	122	38	-75	121	117
Mobility effect	17	-22	62	175	-21	-17
	Second decomposition (in %)					
Return effect	85	73	38	-100	118	115
Mobility effect	15	17	62	200	-18	- 15

Notes : Change in mean income in Euros 2002. The occupational group refers to social origin. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

$$\Delta \bar{y}_j = \Delta \alpha_j \bar{y}_j^t + \Delta \bar{y}_j \alpha_j^t \quad (3)$$

$$\Delta \bar{y}_j = \Delta \alpha_j \bar{y}_j^{t'} + \Delta \bar{y}_j \alpha_j^t \quad (4)$$

where  $\alpha_j$  and  $\bar{y}_j$  denote the vectors  $(\alpha_{j1} \cdots \alpha_{jK})$  and  $(\bar{y}_{j1} \cdots \bar{y}_{jK})$ . The first term on the right hand-side is the mobility effect; the second is the return effect. As is well-known, the decomposition is not unique and depends on the choice of the reference period.

The results of the two decompositions for the mean primary incomes are given in Table 5. For children of farmers, artisans, manual and non-manual workers, the evolution of the mean conditional income is mostly driven by a positive return effect. For example, the mean income of farmers has increased by 81.4% during the period 1979-2000, which has been beneficial to the 22% of children of farmers who become farmers themselves. The mobility effect is more modest and does not affect these groups in similar ways. Children of farmers have benefited from changes in social mobility. On the contrary, children of non-manual workers experienced a fall in the probability to become higher- or lower-grade professionals and tend to lose from the change in mobility. The same holds true, to a lesser extent, for the children of manual workers. For children of artisans, the mobility effect is ambiguous and depends on the reference period. This reflects the fact that during the period 1979-2000, this category experienced both a rise in the probability to become wage-earners and a rise in the wage of artisans relative to wage earners.

The mobility effect has a stronger impact on the evolution of the mean income of children of higher-grade and lower-grade professionals. The latter group mostly loses as a result of the evolution of social mobility : their probability of becoming higher or lower grade professionals decreases over time. This is partly compensated by the return effect. On the contrary, both effects contribute to the fall of the advantage of children of higher-grade professionals, although the contribution of the mobility effect is larger.

In summary, the mobility effect mostly contributed to the fall of the advantage of the higher ranking types, while the improvement of the lot of lower ranking types mostly stems from the return effect.

## 4 Conclusion

While different ethical positions can be defended, regarding how to substantively define equality of opportunity, our analysis indicates that any definition relies on a partition of the determinants of individual outcomes into three distinct groups : effort, which includes the determinants that are seen as a legitimate source of outcome differences; circumstances, which consist of the determinants that should not lead, other things equal, to differences in outcome; luck, which comprises the determinants that are seen as a fair source of inequality provided that they are *even-handed*, with respect to circumstances. In this perspective, the generic model of equality of opportunity developed in this paper appears as a general model, that can encompass a variety of specific conceptions of equality of opportunity, depending on the precise empirical characterization of the above three sets of factors. This model also makes clear that, once these different sets have been delineated, there may still be several ways to define equality of opportunity, which correspond to the strong and weak criteria introduced here.

Whatever the precise conception adopted, empirically assessing equality of opportunity turns out to be a data-demanding exercise. To many, equality of opportunity appears as a more desirable social objective than equality of outcome because it takes into account the determinants of observed outcomes. The obvious drawback of this alternative conception is that making equality of opportunity judgments ideally requires that all the relevant determinants of outcome be observable. Of course, this condition will rarely be met and assessing equality of opportunity will most likely take place under conditions of imperfect information on the relevant determinants. In this paper, we exhibit two testable conditions of equality of opportunity that can be used in this context. One is a necessary condition for the strong form of equality of opportunity. The other is a sufficient condition for the weak form. Only in very restrictive cases is it possible to

exhibit a necessary and sufficient condition. Hence, in almost all cases, imperfect information implies that equality of opportunity cannot be fully assessed. This is probably the price to pay if we are willing to develop a rich enough view. It is also important to emphasize that these two conditions still allow, in our empirical application, to provide a valuable assessment of equality of opportunity in France. Even with limited information on the determinants of outcomes, these conditions indicate that equality of opportunity is clearly violated. This conclusion would therefore remained unchanged if we had access to a richer data set.

On the empirical side, this paper reveals that social inheritance is a deeply rooted source of inequality in France over the period 1979-2000. Differences in social origin translate into significant gaps of living conditions. Equality of opportunity in income acquisition does not prevail, neither for primary income nor for disposable income. However, the degree of inequality of opportunity tends to decrease. During this period, the average gap between the most advantaged social group, the children of higher-grade professionals, and the least advantaged one fall by one half.<sup>40</sup> The explanation of this evolution is addressed in a companion paper (Lefranc et al. (2006)). We show that this reduction in inequality of opportunity does not arise from a decrease in the degree of transmission of economic advantage from one generation to the next. Over time, children tend to face more equal opportunities because of a fall in inequality of outcomes among their parents.

Lastly, this paper has underlined an important phenomenon: the risk of social lotteries appears very similar across the different groups of social origin. As a first approximation, the influence of social origin on opportunities for income, in France, can be summarized by a scale factor : individual income is determined by the product of a random variable - distributed independently of social origin- and the mean income conditional on social origin. Whether a similar determination of income opportunities is also at work in other countries is a question that would be worth investigating. Future research should also analyze the theoretical explanation and consequences of this important stylized fact.

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<sup>40</sup>This conclusion confirms those of Vallet (2004) who notices a slight increase in social mobility in France.

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## Appendix

### A- Statistical tests

Stochastic dominance relationships can be easily expressed and statistically tested by making use of poverty indices. Formally the poverty index at the order  $g$ , for a poverty line  $z$ , is defined by :

$$D^g(z) = \int_0^z (z-x)^{g-1} dF(x)$$

For example  $D^1(z)$  is the proportion of people whose income is below  $z$ ;  $D^2(z)$  measures for the mean poverty gap, *i.e.* the mean amount that should be given to people below the poverty line  $z$ , to reach this threshold.

The link between poverty indices and stochastic dominance has been established by Foster and Shorrocks (1988). They have shown that stochastic dominance at the order  $g$  of distribution  $F_A$  over  $F_B$  is equivalent to the situation where the poverty index of order  $g$  for  $F_A$  is smaller than the poverty index for  $F_B$ , for all poverty lines. Letting  $\succeq_{SD_g}$  denote stochastic dominance at the order  $g$ , we have, for  $g \in \mathbb{N}_+$  :

$$F_A(x) \succeq_{SD_g} F_B(x) \Leftrightarrow \forall z \in \mathbb{R}_+ D_A^g(z) \leq D_B^g(z)$$

Consequently, tests of stochastic dominance are equivalent to test of inequality for poverty indices. The procedure adopted here follows Davidson and Duclos (2000) and consists in testing such inequalities for a fixed number,  $k$ , of poverty lines. The poverty lines used here are the deciles and 95<sup>th</sup> percentile of the overall household income distribution.<sup>41</sup>

For a given poverty line  $z$ , an unbiased and asymptotically normal estimator for the poverty index  $\hat{D}^g(z)$  is given by<sup>42</sup> :

$$\hat{D}^g(z) = \frac{1}{N(g-1)!} \sum_{i=1}^N (z-x_i)^{g-1} I(x_i \leq z)$$

where  $i$  denotes the  $i^{\text{th}}$  observation,  $N$  denotes the sample size and  $I(\cdot)$  is an indicator function equal to 1 when its argument is true, 0 otherwise. For some fixed set of poverty lines  $\{z_1, \dots, z_k\}$ , let  $\hat{\mathbf{D}}^g$  denote the vector of poverty indices  $(\hat{D}^g(z_1) \dots \hat{D}^g(z_k))$  and  $\Sigma$  its asymptotic variance-covariance matrix<sup>43</sup>.

The hypothesis of dominance at the order  $g$  between two distributions  $F_A$  and  $F_B$  can be expressed in vectorial notations. Let  $\delta = (\mathbf{D}_B^g - \mathbf{D}_A^g)$  the difference of the vectors of poverty indices. Stochastic dominance at the order  $g$  can be test as :  $\mathbf{H}_0 : \delta \in \mathbb{R}_+^k$  versus  $\mathbf{H}_1 : \delta \notin \mathbb{R}_+^k$ . The test statistic is constructed from the estimated vector  $\hat{\delta} = (\hat{\mathbf{D}}_B^g - \hat{\mathbf{D}}_A^g)$ , whose asymptotic variance-covariance matrix is given by  $\frac{\Sigma_A}{N_A} + \frac{\Sigma_B}{N_B}$  under the hypothesis of independence between distributions  $A$  and  $B$ .

The null hypothesis is defined by a set of  $k$  constraints. Two approaches can be followed to conduct this test. The first consist in testing each of the  $k$  constraints separately. The intersection of  $k$  sub-hypothesis is tested in each point where the distributions are compared. This kind of test, called "intersection-union", is used for example in Bishop et al. (1992). However, Dardanoni et Forcina (1999) and Davidson and Duclos (2000) have demonstrated that this test procedure has relatively low power, since it ignores the covariance structure of the vector  $\hat{\delta}$ . The second approach, that will be followed here, amounts to simultaneously test the  $k$  constraints, using a Wald test and explicitly taking into account the covariance structure of the estimated poverty indices differences vector. The general principle of the test amounts to compare the distances between the estimated vector  $\hat{\delta}$  to the sets defining respectively the

<sup>41</sup>In order to use non-stochastic poverty lines, these percentiles are computed from administrative tax records (the Revenus Fiscaux data) that are independent of the BDF survey.

<sup>42</sup>In our application this formula has been adapted to account for sample weights.

<sup>43</sup>See Davidson and Duclos (2000), theorem 1. p.1441 for the expression of this matrix.

null hypothesis and the alternative hypothesis. Of course, Wald tests can also be used to test the equality of two distributions.

**Equality tests** The Wald test for the equality of two distributions is relatively easy to implement and resort to a  $\chi^2$  test. The null hypothesis is given by  $\mathbf{H}_0 : \boldsymbol{\delta} = 0$ . One can show (Beach and Davidson, 1983; Davidson and Duclos, 2000) that under the null hypothesis, the vector  $\hat{\boldsymbol{\delta}}$  is asymptotically normal and we have :

$$\hat{\boldsymbol{\delta}} \sim \mathcal{N}\left(0, \frac{\boldsymbol{\Sigma}_A}{N_A} + \frac{\boldsymbol{\Sigma}_B}{N_B}\right)$$

Hence the test statistic  $T_1$  has the following asymptotic distribution under the null hypothesis:

$$T_1 = \hat{\boldsymbol{\delta}}' \left( \frac{\boldsymbol{\Sigma}_A}{N_A} + \frac{\boldsymbol{\Sigma}_B}{N_B} \right)^{-1} \hat{\boldsymbol{\delta}} \sim \chi_k^2$$

**Stochastic dominance tests** Stochastic dominance tests are more complex to implement since in this case, the set corresponding to the null hypothesis is defined by an inequality constraint. The hypothesis are given by:  $\mathbf{H}_0 : \boldsymbol{\delta} \in \mathbb{R}_+^k$  against  $\mathbf{H}_1 : \boldsymbol{\delta} \notin \mathbb{R}_+^k$ . The Wald test statistic with such constraints has been developed by Kodde and Palm (1986) and Wolak (1989). For this set of hypotheses, the test statistic  $T_2$  is defined by:

$$T_2 = \min_{\boldsymbol{\delta} \in \mathbb{R}_+^k} \|\hat{\boldsymbol{\delta}} - \boldsymbol{\delta}\|$$

with  $\|x\| = x' \boldsymbol{\Sigma}^{-1} x$ . Kodde and Palm (1986) have shown that the statistic  $T_2$  is distributed as a mixture of  $\chi^2$  distributions:

$$T_2 \sim \bar{\chi}^2 = \sum_{j=0}^k w(k, k-j, \boldsymbol{\Sigma}) Pr(\chi_j^2 \geq c)$$

with  $w(k, k-j, \boldsymbol{\Sigma})$  the probability that  $k-j$  elements of  $\boldsymbol{\delta}$  are strictly positive. The distribution of the  $\bar{\chi}^2$  distribution have not been tabulated, but lower and upper bounds of critical values are available. When these bounds do not allow to reach a conclusion we estimate the critical values of the statistic  $T_2$  using Monte-Carlo simulation<sup>44</sup>.

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<sup>44</sup>We draw 10.000 multivariate normal vectors with mean 0 and covariance matrix  $\boldsymbol{\Sigma}$ , and compute the proportion of vectors with  $j$  positive elements (for  $j \in (0, k)$ ). This proportion is an estimate of the weight  $w(k, j, \boldsymbol{\Sigma})$ .

**B- Summary statistics and results for intermediate waves**

Table A-1: Sample summary statistics

	1979	1984	1989	1994	2000
Occupation of the father (%)					
Farmers	21.65	18.23	15.69	12.95	10.55
Artisans	13.93	11.29	12.13	13.61	12.26
H-grade prof.	6.95	6.97	7.92	15.25	15.05
L-grade prof.	7.37	9.63	11.45	8.13	8.52
Non-man. workers	11.18	14.03	12.2	16.28	18.49
Manual workers	38.92	39.85	40.6	33.79	35.13
Obs Imputed	377	233	158	149	0
Mean Income					
before imputation	16182	16428	17071	18276	18180
after imputation	16070	16590	17161	18178	18180
Obs	4231	4428	3529	4644	3984

Notes : mean income in Euros 2002. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

Table A-2: Stochastic dominance tests - Primary Income

<b>1979</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	< <sub>1</sub>	=	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>1984</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	< <sub>1</sub>	=	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>1989</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	=	=	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	=	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>1994</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	?
Artisans	-	-	< <sub>1</sub>	=	> <sub>1</sub>	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	=	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>2000</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	?	> <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	=	> <sub>1</sub>	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>

Notes : The occupational group refers to social origin. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

Each element in the table indicates the result of the comparison of the income distribution of the groups in row and column using the tests presented in section 3.2 :

><sub>i</sub>: the row dominates the column for order *i* stochastic dominance ;

<<sub>i</sub>: the column dominates the row for order *i* stochastic dominance;

=: the distributions are equal;

?: the distributions cannot be ranked using first and second order stochastic dominance.

Table A-3: Stochastic dominance tests - Disposable Income

<b>1979</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	< <sub>1</sub>	=	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>1984</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>1989</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	< <sub>1</sub>	?	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>1994</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	> <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	?	> <sub>1</sub>	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	=	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>
<b>2000</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	< <sub>1</sub>	> <sub>1</sub>
Artisans	-	-	< <sub>1</sub>	=	> <sub>1</sub>	> <sub>1</sub>
H-grade prof.	-	-	-	> <sub>1</sub>	> <sub>1</sub>	> <sub>1</sub>
L-grade prof.	-	-	-	-	> <sub>1</sub>	> <sub>1</sub>
Non-man. workers	-	-	-	-	-	> <sub>1</sub>

Notes : The occupational group refers to social origin. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

Each element in the table indicates the result of the comparison of the income distribution of the groups in row and column using the tests presented in section 3.2 :

><sub>i</sub>: the row dominates the column for order *i* stochastic dominance ;

<<sub>i</sub>: the column dominates the row for order *i* stochastic dominance;

=: the distributions are equal;

?: the distributions cannot be ranked using first and second order stochastic dominance.

Table A-4: Mean income conditional on social origin and destination

A- Mean income conditional on social origin										
	Primary Income					Disposable Income				
	1979	1984	1989	1994	2000	1979	1984	1989	1994	2000
Farmers	12 874	14 079	14 058	16 839	17 541	12 914	14 018	14 219	16 728	17 395
Artisans	19 295	17 846	19 199	21 328	20 174	18 137	16 814	18 464	20 214	19 691
H-grade prof.	26 375	26 126	26 435	25 038	24 543	24 490	23 694	23 975	22 946	23 033
L-grade prof.	21 225	21 752	20 688	20 122	20 511	20 055	20 292	19 748	19 156	19 534
Non-man. workers	17 379	18 195	19 753	18 633	17 720	16 884	17 486	18 970	18 053	17 747
Manual workers	14 612	15 320	15 440	15 406	15 008	14 592	15 252	15 305	15 588	15 709
Mean Income	16 503	17 154	17 676	18 774	18 313	16 070	16 590	17 161	18 178	18 180

B- Mean income conditional on social destination										
	Primary Income					Disposable Income				
	1979	1984	1989	1994	2000	1979	1984	1989	1994	2000
Farmers	9 367	8 939	9 765	13 377	17 858	9 614	9 221	10 221	13 571	17 449
Artisans	17 090	14 545	19 853	21 432	19 833	15 797	13 584	18 229	19 801	18 972
H-grade prof.	28 513	29 492	29 868	31 423	30 642	26 104	26 410	26 952	28 198	27 604
L-grade prof.	19 048	19 961	20 247	19 890	20 185	18 304	18 959	19 285	19 077	19 672
Non-man. workers	14 009	14 680	14 054	14 008	13 327	13 999	14 620	14 198	14 354	14 199
Manual workers	12 264	12 548	12 673	12 975	13 113	12 738	13 141	13 364	13 752	14 357
Mean Income	16 503	17 154	17 676	18 774	18 313	16 070	16 590	17 161	18 178	18 180

Notes : income in 2002 Euros. In panel A, the occupational group refers to social origin; in panel B, to social destination. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

Table A-5: Lorenz dominance tests - Primary Income

<b>1979</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	?	=	=	=	<
Artisans	-	-	=	?	?	?
H-grade prof.	-	-	-	=	=	?
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=
<b>1984</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	?	<	<	<
Artisans	-	-	=	?	=	=
H-grade prof.	-	-	-	?	?	?
L-grade prof.	-	-	-	-	?	?
Non-man. workers	-	-	-	-	-	=
<b>1989</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	?	=	=	?
Artisans	-	-	=	=	=	=
H-grade prof.	-	-	-	=	=	?
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=
<b>1994</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	=	=	=	=
Artisans	-	-	=	<	=	<
H-grade prof.	-	-	-	=	?	?
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=
<b>2000</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	?	?	?	<
Artisans	-	-	=	=	=	?
H-grade prof.	-	-	-	=	=	=
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=

Notes : The occupational group refers to social origin. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

Each element in the table indicates the result of the comparison of the Lorenz curves of the groups in row and column using the criterion of Lorenz dominance :

><sub>i</sub>: the row dominates the column for Lorenz dominance ;

<<sub>i</sub>: the column dominates the row for Lorenz dominance;

=: the Lorenz curves are equal;

?: the Lorenz curves cannot be ranked using Lorenz dominance.

Table A-6: Lorenz dominance tests - Disposable Income

<b>1979</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	?	=	=	<	<
Artisans	-	-	=	=	=	=
H-grade prof.	-	-	-	=	=	?
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=
<b>1984</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	=	<	<	<
Artisans	-	-	=	?	=	=
H-grade prof.	-	-	-	=	=	=
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=
<b>1989</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	=	=	?	<
Artisans	-	-	=	=	?	?
H-grade prof.	-	-	-	=	?	?
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	?
<b>1994</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	=	<	<	<
Artisans	-	-	=	<	<	<
H-grade prof.	-	-	-	?	<	<
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=
<b>2000</b>	Farmers	Artisans	H-grade prof.	L-grade prof.	Non-man. workers	Manual workers
Farmers	-	=	=	?	<	<
Artisans	-	-	=	=	=	<
H-grade prof.	-	-	-	=	=	<
L-grade prof.	-	-	-	-	=	=
Non-man. workers	-	-	-	-	-	=

Notes : The occupational group refers to social origin. H-grade prof. : higher-grade professionals. L-grade prof. : lower-grade professionals. Non-man. workers : non-manual workers.

Each element in the table indicates the result of the comparison of the Lorenz curves of the groups in row and column using the criterion of Lorenz dominance :

$>_i$ : the row dominates the column for Lorenz dominance ;

$<_i$ : the column dominates the row for Lorenz dominance;

=: the Lorenz curves are equal;

?: the Lorenz curves cannot be ranked using Lorenz dominance.



**Author Contact Information:**

Arnaud Lefranc  
Robert Schuman Centre for Advanced Studies, European University Institute and THEMA, Université  
de Cergy-Pontoise  
Email: arnaud.lefranc@iue.it

Nicolas Pistolesi  
THEMY, Université de Cergy-Pontoise  
Email: Nicolas.pistolesi@eco.u-cergy.fr

Alain Trannoy  
EHESS, GREQAM-IDAP  
Email: alain.trannoy@ehess.cnrs-mrs.fr