Liquidity, Government Bonds and Sovereign Debt Crises

Francesco Molteni
Abstract
This paper analyzes the Eurozone financial crisis through the lens of sovereign bond liquidity. Using novel data, I show that following the emergence of sovereign risk, repo haircuts on peripheral government bonds sharply increased during the crisis, reducing their liquidity and amplifying the rise in their yields. I study the impact of this liquidity shock on asset prices and aggregate activity in a general equilibrium model with financial frictions. The model confirms the rise in the required returns of illiquid government bonds, predicts a substantial drop in economic activity and provides an additional mechanism for the transmission of sovereign risk.

Keywords
Repo, haircuts, liquidity shock, funding constraint

JEL codes
E44, E58, G12, G15, G23

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Why did countries at the periphery of the Eurozone (Greece, Ireland, Italy, Portugal and Spain) pay higher interest rates on public debt than countries in the core during the recent financial crisis? Because the creation of a monetary union has integrated the sovereign debt markets and eliminated the exchange rate risk, two main factors may explain this, credit risk and liquidity.

Credit risk derives from the government’s probability of default. The weak fiscal and macroeconomic fundamentals of a country induce investors to ask for higher compensation for holding government debt because of the possibility of suffering losses. In addition, fears of default and self-fulfilling dynamics in sovereign debt markets may also amplify the rise in government bond yields.

Liquidity is a broad concept, referred to in the traditional theories of Keynes (1936) and Hicks (1962) as the capacity of an asset to store wealth and protect its owner from a shortage of revenue, thus providing a means to smooth consumption. Modern corporate finance distinguishes between market liquidity and funding liquidity. Market liquidity is the facility to obtain cash by selling an asset; when frictions in the secondary market make it difficult to find a buyer the market liquidity is low and the price of the asset deviates from its fundamentals.

This paper instead focuses on the role of funding liquidity, which is the ease with which investors can obtain funding (Brunnermeier and Pedersen (2009)). As they typically borrow against an asset, funding liquidity is considered the ability of an asset to serve as collateral. I show that government bonds are the prime collateral securities in the European market of repurchase agreements (repos), which are becoming an essential source of funding for the banking system, especially since the onset of the crisis when the increase in counterparty credit risk led to a shift from the unsecured to secured funding. This forced borrowing banks to post securities in the interbank market, whose value exceeds the loan by a certain amount, the “haircut” (also called “initial margin” or “margin requirement”), which is the metric that I employ to measure funding liquidity. Given the value of a security, the lower the haircut the larger the amount of cash that the borrower can obtain by pledging this

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3Favero, Pagano and von Thadden (2010) and Manganelli and Wolewijk (2009) disentangle the impact of credit risk and market liquidity risk on the evolution of European government bond yields.
security.\footnote{Funding liquidity is determined by the level of the haircut, similarly to the loan-to-value which limits the amount of a mortgage loan. An increase in the level of the haircut reduces the funding of investors equivalently to a reduction in the loan-to-value in a mortgage contract.}

Prior to the crisis, the perceived safety of government bonds made them good collateral to back banks’ debt, their repo haircuts were low and their function as a medium of exchange compressed their yields. Nevertheless, I show that during the crisis the emergence of sovereign risk led to rises in repo haircuts on peripheral government bonds, reducing their liquidity and capacity to serve as collateral for secured borrowing. The funding of investors shrank along the lines of the mechanism emphasized by Gorton and Metrick (2012) for the U.S. liquidity crisis in 2007 - 2008, leading to a drop in investment. In order to reduce the contraction of their funding, leveraged investors shifted their portfolios towards the more liquid bonds of the core with lower haircuts, contributing to the widening of the yield spreads.\footnote{Banks could alternatively pledge government bonds for ECB refinancing operations, especially after the introduction of fixed-rate full allotment tender procedures, but paying a higher interest rate than the rate in the private repo market (see Mancini, Ranaldo and Wrampelmeyer (2016)).}

Using the narrative approach, I collected information on the variations in the haircuts applied by LCH Clearnet Ltd, the main European clearing house, which, during the crisis, adjusted the haircuts on Irish and Portuguese bonds as a protection against the increasing risk of default. I identify a liquidity shock as the innovation in the haircut in a high-frequency structural vector autoregressive model (SVAR), based on the one day delay between the announcement and the implementation of changes in haircuts.

Yields of Irish and Portuguese bonds increase significantly following a liquidity shock. This suggests that the returns on government bonds incorporate a “funding liquidity premium”.\footnote{These results are also consistent with empirical literature on finance. Bartolini et al. (2011), find that differences in the collateral values across asset classes contributes to explain yield spreads in the U.S.; Gårleanu and Pedersen (2011), show the emergence of a basis between a security and a derivative with the same cash flow but with different margin requirements.} Also, the yield spreads to government bonds with stable haircuts augment more than yields because of the flight-to-liquidity towards more liquid bonds. An important finding is that haircuts respond positively to an innovation in the yields, which indicates the presence of a dynamic interaction between sovereign risk, haircuts and yields. The enhanced levels of haircuts following the rise in sovereign risk induced investors to sell these securities, increasing their yields and CDS spreads, which in turn led to additional surges in the
haircuts, intensifying the feedback between the value of the collateral and the secured debt of investors (Kiyotaki and Moore (1997)).

Building on Del Negro et al. (2017), I study the effects of a liquidity shock in a general equilibrium model with financial frictions, using the series of the haircut on Irish bonds to calibrate the shock. Several papers incorporate the liquidity frictions proposed by Kiyotaki and Moore (2018) into general equilibrium models. They assume that privately issued assets are subject to a resaleability constraint, which limits their liquidity, whereas government bonds are perfectly liquid. I depart from this assumption and introduce two types of government bonds with different degrees of liquidity, in the spirit of Hicks (1939, pag. 146), i.e. long-term bonds that are subject to a liquidity constraint and short-term bonds that are not. A liquidity shock is a tightening of the constraint on long-term bonds, which increases the premium that investors are willing to pay for holding short-term bonds, equivalent to a rise in the repo haircut akin to Ashcraft, Gărleanu and Pedersen (2011) and Gărleanu and Pedersen (2011).8

Furthermore, in an extension of the model I consider the possibility that long-term government bonds are subject to a sovereign risk shock as in Bocola (2016) and I endogenize their liquidity as a negative function of the probability of default in order to study how liquidity can amplify the transmission of sovereign risk, investigating also the two-way interaction between liquidity and probability of default, as observed in peripheral countries of the Euro area.

The liquidity shock increases the return on long-term bonds and the yield to maturity because investors sell illiquid long-term bonds and buy liquid short-term bonds, consistent with the empirical findings. In addition, it has a quantitative large effect on macroeconomic variables in line with the evolution of the data for the Irish economy during the crisis. I also study to what extent non standard open market operations, which consist of swapping illiquid government bonds for highly liquid papers (short-term debt or money), can alleviate the contractionary effects of the liquidity shock. Finally, the two-way interaction between

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8 This shock differs from the liquidity shock of Jermann and Quadrini (2012) which affects the firms’ borrowing capacity.
9 The liquidity friction in long-term bonds breaks the irrelevance principle of Wallace (1981) and Eggertsson and Woodford (2003) for open market operations, since the government exchanges liquid assets for illiquid assets, thus modifying the composition of aggregate portfolio holdings. Chen Cúrdia and Ferrero
liquidity and probability of default generates a powerful amplification mechanism for the impact of the sovereign risk shock on asset prices and macroeconomic variables.

To summarize, this paper contributes to the study of the financial crises in peripheral countries of the Euro area showing the relevance of liquidity as amplification mechanism of the tensions in the sovereign debt markets.

The structure of the paper is as follows. Section 1 provides information about the European repo market and the haircuts on Irish and Portuguese bonds during the crisis. Section 2 shows the econometric strategy and the empirical results. Section 3 presents the theoretical model. Section 4 shows the calibration and the results of the numerical simulations. Section 5 concludes.

1 Funding liquidity of government bonds

This Section analyzes the European repo market during the crisis; it presents evidence of the importance of liquidity services provided by sovereign bonds and illustrates how the liquidity of peripheral government securities suddenly dried up. Figure 1 compares the dynamics of secured and unsecured borrowing in interbank transactions. There is a massive shift of banks’ funding from the unsecured to the secured segment, in particular after the onset of the global financial crisis following the rise in counterparty credit risk.\(^{10}\)

Furthermore, breaking down the repo market by types of arrangements we can observe that bilateral CCP-cleared repos steadily increased, while over-the-counter bilateral repos declined and tri-party repos account for a small share of the market, in contrast to the U.S. repo market where they are the largest component.\(^{11}\) The enhanced role of clearing houses increases the importance of the quality of collateral securities since they set repo haircuts as a function of their credit risk. Thus, a change in the credit risk is reflected in variations

(2012) introduce limits to arbitrage and market segmentation between short-term and long-term bonds in the preferred habitat framework to study the impact of the Asset Purchase Programmes. Reis (2017) evaluates the effect of Quantitative Easing, also assuming that short-term bonds are more liquid than long-term bonds, since they can be used as collateral in the interbank market together with reserves.

\(^{10}\) Looking at banks’ balance sheet, Table 3 in the Appendix A.2 show that repos are a considerable share of European banks’ funding.

\(^{11}\) Bilateral CCP-cleared repos are bilateral repos involving a clearing house or Central Clearing Counterparty (CCP), which bears the counterparty credit risk in the transactions between the cash borrower and the cash lender and sets the haircuts as a function of the credit risk of collateral securities. See Appendix A.1 for technical definitions used for repurchase agreements.
Figure 1: Interbank transactions in the European money market. On the left-hand panel, the solid line shows the evolution of the secured borrowing and the dashed line the evolution of unsecured borrowing in the European money market as total turnover. On the right-hand panel, the solid line represents the share of bilateral CCP-cleared repos, the dashed line the share of bilateral over-the-counter repos and the dotted line the share of tri-party repos in % of total repos. Source: European Money Market Survey.

... in haircuts affecting funding conditions in the European repo market.

Figure 2: Shares of collateral securities in the European repo market (in % of the total). EZ denotes the share of bonds issued by countries of the Eurozone. Source: European Repo Market Surveys.

Concerning the collateral composition, Figure 2 shows that government bonds are the predominant securities, accounting for around 80% of the total collateral pool. This share was stable during the crisis and represents a structural characteristic of the European repo market, different to the U.S. market where securities issued by the private sector account for a larger share (Krishnamurthy, Nagel and Orlov (2014)). Looking at the composition
of sovereign securities, German and French bonds are the largest share, while the fraction of Italian bonds reduced during the Eurozone crisis from 10.3% in December 2010 to 7% in December 2011. The collateral composition of the European repo market depends not only on the safety of collateral securities but also on their liquidity and the haircuts applied by European clearing houses to Italian bonds during that period rose.\footnote{Pelizzon et al (2016) and Armakolla et al. (2017).} Figure 3 shows that 10 year Irish and Portuguese government bonds experienced substantial increases in the haircuts applied by LCH Clearnet Ltd, which surged up to 80%, making these securities almost completely illiquid.\footnote{Delatte, Fouquau and Portes (2017) reports the sell-off in Irish bonds during the crisis driven by higher collateral requirements.}

To sum up, European government bonds have become an essential liquid instrument for banks, especially since the onset of the financial crisis, because they are needed to pledge collateral securities as guarantee of repayment in order to borrow in the interbank market. The escalation in sovereign risk increased the haircuts on peripheral bonds, leading investors to fire sell these securities increasing their required returns, which in turn entailed additional rises in haircuts. In the next Section, I investigate empirically this negative liquidity spiral in the Irish and Portuguese sovereign debt markets.
2 The impact of a liquidity shock

I study the dynamic relationship between haircuts and yields of Irish and Portuguese government bonds with a high frequency Bayesian vector autoregression (BVAR) model. It includes the series of haircuts applied by LCH Clearnet Ltd on 10 year government bonds ($h_t$), the logarithm of 5 year sovereign CDS spread ($CDS_t$) and the logarithm of the yield of 10 year government bonds ($yd_t$). For the variable $h_t$ the subscript $t$ refers to the day when the haircut becomes effective. Since sovereign CDS spreads and yield spreads to an AAA benchmark are part of the information set used by LCH Clearnet Ltd to settle the level of the haircuts, the VAR model allows us to deal with endogeneity problems and reverse causality issues.

The inclusion of the CDS spread as a measure of credit risk makes it possible to disentangle the liquidity channel from the sovereign risk channel and helps to identify a liquidity shock. The data source for CDS spreads and yields is Datastream.

The sample for Ireland covers the period from 01/11/2010 to 01/12/2011 at daily frequency. The sample size is the result of the data availability on changes in haircuts and is right censored to exclude the launch of the first large scale Long Term Refinancing Operation (LTRO) on December 2011. The sample for Portugal is shorter running from 01/04/2011 to 29/07/2011.

This exercise is close to the study of Pelizzon et al. (2017), who analyze the dynamic relation between credit risk and market liquidity on the Italian sovereign debt market. They find that this link is reinforced above 500 basis points in the sovereign CDS spread, because of changes in haircuts applied by LCH Clearnet. They also show that the threshold effect disappears after December 2011, after the onset of the large scale LTROs, providing abundant liquidity to the banking system and loosening the link between credit risk and market liquidity.

Let $y_t = [h_t, CDS_t, yd_t]'$ the vector of endogenous variables, I consider the following

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14See LCH.Clearnet Margining Approach (2011) [http://www.ecb.europa.eu/paym/groups/pdf/mmcg/Item_1_LCH_Margining.pdf?0fe79f1ef93461dc22568a4e185db44](http://www.ecb.europa.eu/paym/groups/pdf/mmcg/Item_1_LCH_Margining.pdf?0fe79f1ef93461dc22568a4e185db44)

15Other unconventional monetary policies, such as the Security Market Programme (SMP) and the refinancing operations with full allotment, were already active during this period and Ghysels et al. (2017) find that the SMP was effective in reducing the yields of peripheral government bonds. Because information regarding the implementation of the SMP is not public available, I cannot control for it but we can expect that this intervention has dampened the impact of variations in haircuts on government bond yields.
structural VAR (SVAR) model with orthogonalized residuals

\[ A_0 y_t = A_c + A_1 y_{t-1} + \ldots + A_p y_{t-p} + \epsilon_t \]  \hspace{1cm} (1)

In order to impose the necessary restrictions on the impact matrix \( A_0 \) to achieve the identification of structural shocks, I apply a Cholesky decomposition of the reduced form variance-covariance matrix, choosing \( \tilde{A}_0 \) as a lower triangular matrix with positive elements on the main diagonal.\(^\text{16}\) The recursive structure and zero restrictions on the contemporaneous coefficients find justification in the procedure through which LCH Clearnet Ltd decides and communicates to its members variations in haircuts, which is key for the identification of the liquidity shock.\(^\text{17}\) LCH Clearnet Ltd notifies all modifications in haircuts at the close of business through the Repo Clear Margin Rate Circulars; so their revision is applied one day after the publication of the circulars.

Thus, I place the haircut as the first variable in the VAR model. This Choleski ordering implies that the haircut does not respond within the period of impact to financial shocks relative to CDS spread and yield, but a shock to the haircut is allowed to affect CDS spread and yield instantaneously. This identification strategy is based on a similar assumption employed in the empirical literature on fiscal policy; that is, fiscal instruments do not react instantaneously to variations in macroeconomic variables, mainly economic activity, because of the outside lag, which is the delay between the decision and the implementation of a certain policy.\(^\text{18}\) Nevertheless, with low frequency data the implementations of fiscal policy can be anticipated by private agents, leading to a non-fundamental moving average representation.\(^\text{19}\)

In this application high frequency data rule out the possibility that market participants may react to announcements of changes in haircuts before their implementation. In this regard, it shares the High Frequency Identification (HFI) approach for monetary policy shocks.\(^\text{20}\)

\(^{16}\)The matrices of coefficients and covariances are estimated with Bayesian techniques using non-informative priors, the Appendix B.2 reports the details of the estimation procedure.

\(^{17}\)Figure 14 in the Appendix B.1 reports an example of the Circular; these documents provide information on the date of the announcement, date of implementation and variations in additional margins required.

\(^{18}\)Blanchard and Perotti (2002) assume that government spending is predetermined within the quarter imposing time restrictions in a SVAR. Romer and Romer (2010) and Ramey (2011) also place their narrative fiscal measures as first endogenous variable with a Choleski decomposition.

\(^{19}\)See Leeper, Walker and Yang (2013) and Ramey (2011).

\(^{20}\)The HFI approach addresses the problem of simultaneity that arises with low frequency data since within a period monetary policy both affects financial variables and responds to their movements.
Figure 4: Responses of government bond yield, CDS spread and haircut to a liquidity shock (Irish bonds). The solid line plots the posterior median and the dash lines are the 10th and 90th percentiles with 50,000 draws for which the first 20,000 are discarded as burn-in draws.

Figure 4 presents the impulse responses to a 1% increase in the haircut for Irish bonds. The solid line denotes the median estimate of the impulse responses and the dashed lines represent the range of the 90-% confidence band around the point estimates. The liquidity shock leads to a rise in government bond yield with the peak effect at 0.3% after 15 days from the impact period and an increase in CDS spread with the peak effect at 0.5% occurring after 16 days. The responses of all variables to a liquidity shock are very persistent, including the haircuts.  

Figure 5 shows the impulse responses to a 1% increase in the haircut for Portuguese bonds. Results are quantitatively similar to the impulse responses of Irish bonds, even though the confidence bands are larger. Yield and CDS spread increase with the peak effects to 0.4% after 10 days and 0.5% after 11 days from the impact period, and the effects of the shock in all the variables is long-lasting.

The previous findings can be interpreted as follows in the context of the crisis. If the

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\(^{21}\) I conducted several robustness checks considering different lag length, including the VIX index as exogenous variable to measure the global risk factor and estimating the model with Minnesota priors.
yields of Irish and Portuguese bonds are 10%, a jump of 10% in the haircuts applied by LCH Clearnet Ltd on these securities, which is the size of variations observed during the crisis, leads to an increase in the yields of Irish and Portuguese bonds by around 30 and 40 basis points.

In order to understand the consequences of rises in haircuts on the composition of the portfolios of investors, I also evaluate the impact of a liquidity shock on the yield spreads to government bonds whose haircuts were stable during the crisis. So I estimate alternative specifications where I replace the yields with the yield spreads to either French or German government bonds with the same maturity. Figures 6 and 7 show the impact of a 1% increase in the haircuts on Irish and Portuguese yield spreads. The yield spreads of Irish bonds to French and German bonds increase by around 0.4% and 0.6% respectively and the yield spreads of Portuguese bonds by around 0.5% relative to the other bonds. Because the responses of yield spreads to a liquidity shock are larger than the responses of yields, these results suggest that during the crisis European banks sold the bonds with enhanced haircuts.
and purchased bonds with stable haircuts.\footnote{This is also evident by looking at banks’ balance sheets. Acharya and Steffen (2015) in Table 2 report that between March 2010 and December 2010 non-Eurozone European banks increased their exposure to Italian, Portuguese and Spanish banks, but drastically reduced their exposure to Irish banks. During that period, the sovereign CDS spreads of Ireland and Portugal were almost identical, but the haircut on Irish bonds surged (see Figure 3). Instead the haircut on Portuguese bonds started to increase in 2011.}

Figure 6: Response of yield spreads to a liquidity shock (Irish bonds). The solid line plots the posterior median and the dash lines are the 10\textsuperscript{th} and 90\textsuperscript{th} percentiles with 50,000 draws for which the first 20,000 are discarded as burn-in draws.

Figure 7: Response of yield spreads to a liquidity shock (Portuguese bonds). The solid line plots the posterior median and the dash lines are the 10\textsuperscript{th} and 90\textsuperscript{th} percentiles with 50,000 draws for which the first 20,000 are discarded as burn-in draws.

A possible issue for the identification of a liquidity shock is that market participants can anticipate the decision of the clearing house to change the haircuts after an increase in credit risk. In particular, LCH Clearnet Ltd published indicative thresholds at a 450 basis points spread at the 10 year maturity to the AAA benchmark, or at 500 basis points at a 5 year CDS spread as sovereign risks indicators.\footnote{See LCH.Clearnet Margining Approach (2011) \url{http://www.ecb.europa.eu/paym/groups/pdf/mmcg/Item_1_LCH_Margining.pdf} and LCH.Clearnet Management of Sovereign Credit Risk for RepoClear Services \url{http://secure-area.lchclearnet.com/member_notices/circulars/2010-10-05.asp}} However, the clearing house also states that these are key indicators to judge the credit risk of a security but do not trigger automatic...
actions for increases in haircuts and margin calls.\footnote{24}

In order to confirm that changes in haircut could not be anticipated, I perform two statistical tests, constructing a variable $h_t^*$ for the announced variations in haircuts on Irish and Portuguese bonds, where the subscript $t$ refers to the day of the announcement. First, I run the Hansen (2000) test to assess the presence of a threshold regressing $h_t^*$ on $CDS_t$.\footnote{25} Figure 15 in the Appendix B.1 displays the graph on the normalized likelihood ratio sequence as a function of the threshold in CDS spread for Irish bonds. The graph provides evidence of a significant threshold at 567 basis points, substantially higher than the 500 basis points threshold published by LCH Clearnet Ltd, as a key indicator of risk. For Portuguese bonds the test does not find a significant threshold. Second, I estimate a Granger causality test of the CDS spread on the announced variations in haircuts $h_t^*$. If it helps predict variations in $h_t^*$, market participants could anticipate their modifications by looking at this indicator of sovereign risk. Table 4 in the Appendix B.1 shows that the CDS spread fails to predict announced changes in haircuts applied by LCH Clearnet Ltd on Irish and Portuguese bonds, confirming that changes in haircuts did not automatically follow variations in these indicators and were in part discretionary. These tests suggest that variations in haircuts were to a large extent exogenous liquidity surprises.

Figure 8: Response of haircuts to a yield shock. The solid blue line plots the posterior median and the red dash lines are the $10^{th}$ and $90^{th}$ percentiles with 50,000 draws for which the first 20,000 are discarded as burn-in draws.

\footnote{Furthermore, the International Capital Market Association (2015) argues that “Although CCPs apply more rigorous risk management practices than many market users, their methodologies are often proprietary and therefore opaque, and it is not possible for members to scrutinize these methodologies, despite their critical dependence on them” (See “Frequently Asked Questions on Repos” pag. 25 https://www.icmagroup.org/Regulatory-Policy-and-Market-Practice/repo-and-collateral-markets/icma-ercc-publications/frequently-asked-questions-on-repo/).}

\footnote{LCH Clearnet Ltd does not specify which is the AAA benchmark for the yield spread, but the yield spreads of Irish and Portuguese bonds were above 450 basis points.”}
Furthermore, I evaluate the reaction of haircuts to a yield shock. Figure 8 shows that a 1% innovation in Irish and Portuguese bond yields leads to an increase in haircuts on these securities by around 0.5%, even though the confidence bands for the haircut on Portuguese bonds are large. Overall, these results confirm the presence of a dynamic interaction in the Irish and Portuguese sovereign debt markets between haircuts and yields, which generates a feedback loop between sovereign risk and liquidity. When the sovereign risk increases, the clearing house raises the haircuts on government bonds, which leads to a fire sell of these securities and a surge in their return and sovereign risk, which in turn leads to further raises in haircuts.

3 The model

The model is an infinite horizon economy populated by a continuum of households of measure one. The members of each household are either entrepreneurs or workers. The model incorporates nominal rigidities, since prices and wages are set in staggered contracts, and real rigidities with capital adjustment cost. Households allocate saving across financial assets characterized by different liquidity, which are equity, long-term and short-term sovereign bonds. Long-term bonds are subject to a liquidity shock that reduces the resources entrepreneurs can employ for their investment, pledging long-term bonds as collateral similarly to a rise in their haircut in the repo market. In response to this shock the government can implement an unconventional open market operation that consists of purchasing illiquid long-term bonds by issuing more liquid short-term bonds.

3.1 Households

3.1.1 Structure

Each household has an unit measure of members indexed $j \in [0, 1]$. At the beginning of each period all members are identical and hold an equal share of the household’s assets. They

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26 The response of haircuts to an innovation of yield spread to German bonds is quantitatively similar, but the confidence bands are narrower, especially for Portuguese bonds, probably because yield spreads are part of the information set used by LCH Clearnet Ltd to settle the haircuts on government bonds, also confirming that haircuts move in response of sovereign risk.

27 He and Milbradt (2004) study the feedback loop between liquidity and default for corporate bonds.
receive an idiosyncratic shock, i.i.d. across members and across time, which determines their profession. With probability $\gamma$ they are entrepreneurs and with probability $1 - \gamma$ they are workers.

By the law of large numbers $\gamma$ also represents the fraction of entrepreneurs in the economy. Each entrepreneur $j \in [0, \gamma)$ invests and each worker $j \in [\gamma, 1]$ supplies labor; both types return their earnings to the household and at the end of each period all members share consumption goods and assets, but resources cannot be reallocated among members within the period.

3.1.2 Preferences

The household’s objective is to maximize the utility function

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{c_s^{1-\sigma}}{1-\sigma} - \frac{\xi}{1+\eta} \int_{\gamma}^{1} h_s(j)^{1+\eta} dj \right]$$

where $E_t$ denotes the conditional expectation, $\beta$ is the subjective discount factor, $\sigma$ measures the degree of relative risk aversion, $\xi$ is a scaling parameter that can be chosen to match a target value for the steady state level of hours and $\eta$ is the inverse of the Frisch elasticity of the labor supply. Utility depends positively upon the sum of the consumption goods bought by household members ($c_t = \int_{0}^{1} c_t(j) dj$) and negatively upon the workers’ labor supply $h_t$.

3.1.3 Balance sheet

Households hold physical capital $k_t$ with price $q_t$ that depreciates for a fraction $\delta$ every period. They can sell claims on their capital to other households, $s_t^I$, which represents the only liabilities of households, and purchase claims on other households’ capital, $s_t^O$. Equity issued by the other households ($s_t^O$) and the unmortgaged capital stock ($k_t - s_t^I$) are assumed to yield the same returns, have the same value and liquidity and depreciate at the same rate, so they are perfect substitutes and can be summed together and defined as net equity. Table 1 summarizes the household’s balance sheet at the beginning of the period.

$$s_t = s_t^O + k_t - s_t^I$$
Table 1: Household’s balance sheet (financial assets)

<table>
<thead>
<tr>
<th>Asset</th>
<th>Liability</th>
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</thead>
<tbody>
<tr>
<td>Capital stock</td>
<td>Equity issued</td>
</tr>
<tr>
<td>q_tk_t</td>
<td>q_t s_t</td>
</tr>
<tr>
<td>Other’s equity</td>
<td></td>
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<tr>
<td>q_t n_t^P</td>
<td></td>
</tr>
<tr>
<td>Long-term bonds</td>
<td>Net worth</td>
</tr>
<tr>
<td>q^L_t b^L_t/p_t</td>
<td>q_t s_t + q^L_t b^L_t/p_t + q^S_t b^S_t/p_t</td>
</tr>
<tr>
<td>Short-term bonds</td>
<td></td>
</tr>
<tr>
<td>q^S_t b^S_t/p_t</td>
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</tr>
</tbody>
</table>

Households also own government debt with different maturities. Short-term bonds $b^S_t$ are one period securities purchased at time $t$ at price $q^S_t$ that pay an unit return at time $t+1$. The price of short-term bond is the inverse of the gross nominal interest rate

$$q^S_t = \frac{1}{r_t}$$ (4)

Long-term bonds $b^L_t$ are perpetuities with coupons which decay exponentially as in Woodford (2001) with price $q^L_t$. A bond issued at date $t$ pays $\lambda^{k-1}$ at date $t+k$, where $\lambda \in [0,1]$ is the coupon decay factor that parametrizes the maturity of long-term bonds, corresponding to $(1-\lambda \beta)^{-1}$. If $\lambda = 0$ these securities are one period zero coupon bonds and if $\lambda = 1$ they are consols. The gross yield to maturity at time $t$ on long-term bond is

$$y^d_t = \frac{1}{q^L_t} + \lambda$$ (5)

At the end of each period households receive the dividend per unit of capital ownership $r^K_t$, which is the sum of rental income of capital and profits of intermediate goods producers and capital goods producers.

### 3.1.4 Resources allocation

When the asset market and goods market open households allocate their resources and trade assets to finance new investments. The flow of fund constraint of household member $j$ is as

$^{28}$An alternative interpretation of the long-term debt is that it is a portfolio of zero coupon bonds with different maturities, whose weights decline geometrically with maturity (Hatchondo and Martinez (2009)).
follows

\[ c_t(j) + q_t^I i_t(j) + q_t^L \frac{b_{t+1}^L(j)}{p_t} + q_t^S \frac{b_{t+1}^S(j)}{p_t} + q_t[s_{t+1}(j) - i_t(j)] \]

\[ = \left[ 1 + \lambda q_t^L \right] \frac{b_t^L}{p_t} + \frac{b_t^S}{p_t} + [r_t^K + (1 - \delta)q_t]s_t + \frac{w_t(j)}{p_t}h_t(j) - t_t \]  

(6)

where \( p_t \) denotes the price level, \( q_t^I \) is the cost of one unit of new capital in terms of consumption goods, differing from 1 because of capital adjustment cost, \( w_t(j) \) is the nominal wage for workers j. According to the left side of the budget constraint, the household members allocate resources between purchase of non-storable consumption good, investment in new capital - if they are entrepreneurs - net purchase of equity, long-term bonds and short-term bonds. They finance their activities on the right side of the budget constraint with returns on financial assets (equity, long-term bonds and short-term bonds) and wages - if they are workers - net of taxes.

A key assumption of the model is the presence of the following financing constraints that limit the funding of new investment by entrepreneurs and determine the different liquidity of assets

\[ s_{t+1}(j) \geq (1 - \theta)i_t(j) + (1 - \delta)s_t \]  

(7)

\[ b_{t+1}^L(j) \geq (1 - \phi_t)b_t^L \]  

(8)

\[ b_{t+1}^S(j) \geq 0 \]  

(9)

Inequality (7) means that the entrepreneur can issue claims on the future output of investment but only for a fraction \( \theta \in [0, 1] \). This borrowing constraint implies that investment is partially funded internally and entrepreneurs have to retain \( 1 - \theta \) as their own equity. In addition, equity is assumed to be completely illiquid since entrepreneurs cannot sell it to obtain more resources to invest. Hence, the entrepreneurs’ equity holding at the start of the
period \( t+1 \) must be at least the sum of \((1 - \theta)i_t\) and depreciated equity \((1 - \delta)s_t\)\(^{29}\).

The entrepreneur can acquire additional resources by disposing of a fraction \( \phi_t \in [0,1] \) of long-term bonds, so a resaleability constraint is imposed to keep the residual \((1 - \phi_t)\) of bonds (inequality 8). \((1 - \phi_t)\) is equivalent to the haircut in a repo transaction since it determines the amount of liquidity that the entrepreneur can obtain by pledging sovereign securities in the secured borrowing. In other words, the entrepreneur cannot borrow against the entire bond holding because of the presence of the haircut. An increase in the haircut is modeled as a negative shock to \( \phi_t \) which follows a stationary AR(1) process\(^{30}\).

Inequality 9 implies that short-term bonds are not subject to resaleability constraint and are fully liquid, but entrepreneurs cannot borrow from the government\(^{31}\). \( \phi_t \) is the key parameter of the model characterizing the liquidity of financial assets. We can think that it takes value 0 for equity, value 1 for short-term bonds and an intermediate value for long-term bonds.

The assumption on the diverse resaleability of equity and bonds reflects the different liquidity across assets observed in the European financial market and allows us to study the impact of a liquidity shock in a general equilibrium model with assets characterized by different liquidity\(^{32}\). In addition, the diverse maturity of government bonds introduces differences in liquidity of otherwise identical securities in a model which abstracts from sovereign risk, because short-term bonds are typically more liquid than long-term bonds.

At the end of the period, the assets of households are given by

\[
s_{t+1} = \int s_{t+1}(j) \, dj \tag{10}
\]

\(^{29}\)Nezefat and Slavik (2017) model a financial shock as a tightening in the credit conditions and a drop in \( \theta \) and assume that equity/capital is completely liquid. In this model the assumption that equity is illiquid means that entrepreneurs cannot issue equity on the unmortgaged capital stock and cannot sell any of the remaining equity to others.

\(^{30}\)The model is solved with the Newton-Raphson algorithm in order to take into account the ZLB and non-linear perfect foresight.

\(^{31}\)Similarly, inequalities 7 and 8 ensure that receipts from trading equity and long-term bonds are strictly positive, which prevents the entrepreneur from going short on these securities.

\(^{32}\)In Kiyotaki and Moore (2003) equity is illiquid because of the problem of adverse selection which can be solved by bundling equities together at a cost to avoid debtors selling a lemon. In this model, even though equity is riskless and I do not make explicit the reason why it is illiquid, it can be thought of as unbundled papers which do not circulate, long-term bonds as inside papers which circulate as a medium of exchange but with a transaction cost determined by \( \phi_t \) and short-term bonds as outside money which are liquid.
Next, the specific functions of entrepreneurs and workers are taken into account.

### 3.1.5 Entrepreneurs

The entrepreneur \( j \in [0, \gamma) \) does not supply labor, so \( H_t(j) = 0 \) in equation \( 6 \) to get her budget constraint. In order to acquire new capital she can either produce it at price \( q^l_t \) or buy it in the market at price \( q_t \). For the rest of the model I assume that \( q_t > q^l_t \) in order to focus on the economy where the funding constraints bind, thus limiting the ability of the entrepreneur to finance investments. In this case, entrepreneurs will use all the available liquidity for new investment to maximize the households’ utility. Accordingly, they minimize the equity holding by issuing the maximum amount of claims on the investment return. The entrepreneur also sells the maximum amount of bonds as allowed by constraints \( 8 \) and \( 9 \) because their expected returns are lower than those on new investment. As a result, in equilibrium the liquidity constraints all bind and the entrepreneur does not consume goods within the period \( c_t(j) = 0 \). This gives the solutions for entrepreneurs \( s_{t+1}(j), b^b_{L_{t+1}}(j), b^b_{S_{t+1}}(j), c_t(j) \) for \( j \in [0, \gamma) \) and plugging these values into equation \( 6 \) we can derive the function of investment for entrepreneurs

\[
i_t(j) = r^K_t s_t + [1 + \lambda \phi_t q^l_t] \frac{b^L_t}{p_t} + b^b_t \frac{b^S_t}{p_t} - t_t \]

Aggregating by entrepreneurs total investment is

\[
i_t = \int_0^\gamma i_t(j) \, dj = \gamma r^K_t s_t + [1 + \lambda \phi_t q^l_t] \frac{b^L_t}{p_t} + b^b_t \frac{b^S_t}{p_t} - t_t \]

18
Investment is a function of the maximum liquidity available for the entrepreneurs (the numerator) and the downpayment, which is the difference between the price of one unit of investment goods $q^I_t$ and the value of equity issued by the entrepreneur $\theta q_t$ (the denominator).

### 3.1.6 Workers

The worker $j \in [\gamma, 1]$ does not invest, so $i_t(j) = 0$. She supplies labor as demanded by firms at a fixed wage; the union representing each type of worker sets wages on a staggered basis. To determine the asset and consumption choices of workers, I first derive the household’s decision for $s_{t+1}, b^L_{t+1}, b^S_{t+1}$ and $c_t$, taking $w_t$ and $h_t$ as given. Knowing the solution for entrepreneurs, $s_{t+1}(j), b^L_{t+1}(j), b^S_{t+1}(j)$ and $c_t(j)$ can be determined for workers, given the aggregate consumption and asset holding.

### 3.1.7 The problem of households

To solve the model for the household, I aggregate the workers’ and entrepreneurs’ budget constraint

$$c_t + q^I_t i_t + q^I_t b^L_t \frac{b^L_t}{p_t} + q^S_t b^S_t \frac{b^S_t}{p_t} + q_t[s_{t+1} - i_t] = [1 + \lambda q^S_t] \frac{b^L_t}{p_t} + b^S_t + [r^K_t + (1 - \delta)q_t]s_t + \int_0^1 \frac{w_t(j)}{p_t} h_t(j) dj - t_t$$

Households maximize the utility function (3) by choosing $c_t, s_{t+1}, b^L_{t+1}$ and $b^S_{t+1}$ subject to the aggregate budget constraint and the investment constraint. The first order conditions for equity, long-term bonds and short-term bonds are respectively

$$U'_{c,t} = \beta E_t \left\{ U'_{c,t+1} \left[ \frac{r^K_{t+1} + (1 - \delta)q_{t+1}}{q_t} + \frac{\gamma(q_{t+1} - q^I_{t+1}) r^K_{t+1}}{q^I_{t+1} - \theta q_{t+1}} \right] \right\}$$

$$U'_{c,t} = \beta E_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[ \frac{1 + \lambda q^L_{t+1}}{q^L_t} + \frac{\gamma(q_{t+1} - q^L_{t+1}) 1 + \lambda q^L_{t+1} q^L_{t+1}}{q^L_{t+1} - \theta q_{t+1}} \right] \right\}$$

$$U'_{c,t} = \beta E_t \left\{ \frac{1}{\pi_{t+1}} U'_t \left[ r_t + \frac{\gamma(q_{t+1} - q^I_{t+1})}{q^I_{t+1} - \theta q_{t+1}} r_t \right] \right\}$$
where $\pi_t$ is the inflation rate defined as $\pi_t = \frac{p_{t+1}}{p_t}$. The choice of sacrificing one unit of consumption today to purchase a paper gives a payoff composed of two parts. The first is the returns on the assets, that is $\frac{r_{K,t+1}^L + \lambda q_{t+1}}{q_t}$ for equity, $\frac{1 + \lambda q_{t+1}^L}{q_t}$ for long-term bonds and $r_t$ for short-term bonds. The second part is a premium, deriving from the fact that papers in the entrepreneurs’ portfolio relax their investment constraint. This premium is a function of the leverage of entrepreneurs $\frac{1}{q_t - \theta q_t}$, the gap between $q_t$ and $q_t^L$, which represents the marginal value of relaxing the constraint, and the liquid returns of each asset. The bond holding eases the financing constraints more than the equity holding, which makes bonds more valuable for entrepreneurs. Further, long-term bonds pay a liquidity premium relative to short-term bonds, which depends on $\hat{\phi}_t$.

### 3.2 The Government

The government finances the debt repayment by the issue of new debt and net taxes $t_t$, so the intertemporal budget constraint is

$$q_t^L \frac{b_{t+1}^L}{p_t} + q_t^S \frac{b_{t+1}^S}{p_t} + t_t = (1 + \lambda q_t^L) \frac{b_t^L}{p_t} + \frac{b_t^S}{p_t}$$

The government can respond to the negative liquidity shock by purchasing long-term bonds and issuing short-term bonds, in this way it can modify the composition of the assets held by entrepreneurs.\(^{33}\) Since at the zero lower bound short-term debt is a close substitute for money, this policy can be thought of as an unconventional open market operation, where the supply of one-period bonds is proportional to the deviation of liquidity from its steady state

$$\frac{b_{t+1}^S}{p_t} = \psi_B (\phi_t - \phi)$$

where $\psi_B < 0$. A fiscal rule links taxes with the outstanding beginning-of-period long-term debt in deviations from the steady state to satisfy the government’s intertemporal budget.

---

\(^{33}\)This government policy differs from the liquidity facilities studied in Ashcraft, Gärleanu and Pedersen (2011), where the monetary authority, following a tightening in margin requirements of securities, lends against these securities at a lower haircut to alleviate the funding problems, here the government does not directly relax the financing constraint.
constraint.

\[ t_t - t = \psi_T \left( \frac{b_L}{p_t} - \frac{b_L}{p} \right) \]  \hspace{1cm} (22)

where \( \psi_T > 0 \) measures the elasticity of net taxes to variations in the size of the debt.\(^{34}\) The government sets the nominal interest rate, following a standard Taylor rule constrained by the zero lower bound condition

\[ r_t = \max \left\{ r_{\pi}^{\phi \psi_{\pi}} \left( \frac{y_t}{y} \right) \psi_{\psi}, 1 \right\} \]  \hspace{1cm} (23)

where \( \psi_{\pi} > 1 \) and \( \psi_y > 0. \)

3.3 Firms

The supply side of the economy is composed of intermediate and final goods firms, labor agencies and capital producers along the lines of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). Intermediate goods producers and labor unions in each period can change the price of intermediate goods and wages with probability \( 1 - \zeta_p \) and \( 1 - \zeta_w, \) and they have a price markup and a wage markup of \( \lambda_{\pi} \) and \( \lambda_w, \) respectively.\(^{35}\)

4 Numerical simulation

4.1 Parametrization

Table 2 reports the parameter values of the model calibrated at quarterly frequency. There are two sets of parameters. One is specific to the Irish economy for financial frictions, \( \phi \) and \( \theta, \) the maturity of public debt, \( \lambda, \) the coefficient for open market operations, \( \psi_B, \) and the share of long-term bonds over GDP at the steady state, \( \frac{b_L}{y}, \) the other is standard in the literature.

The key parameter characterizing the financial frictions is the liquidity constraint \( \phi_t. \)

---

\(^{34}\)As in Chen, Cúrdia and Ferrero (2012), short-term debt does not enter into the fiscal rule, so there is no fiscal reaction to the open market operations. However, quantitatively results do not change by including the short-term debt since the adjustment of taxes to debt is gradual, \( \psi_T \) is small. Cui (2016) analyzes the trade-off of issuing more liquid public debt financed via distortionary taxes.

\(^{35}\)The Appendix C reports additional details of the model and the optimality conditions.
Table 2: Parametrization

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<tr>
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<tr>
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<td>Relative risk aversion</td>
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<td>Inverse Frish elasticity</td>
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<td>Investment adjustment cost</td>
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<td>Probability of investment opportunity</td>
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<td>Depreciation rate</td>
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<td>Price and wage Calvo probability</td>
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<td>Price and wage steady-state markup</td>
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<td>Taylor rule output response</td>
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<td>Tax rule response</td>
<td>$\psi_T$</td>
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</tr>
<tr>
<td>Policy intervention</td>
<td>$\psi_B$</td>
<td>-0.247</td>
</tr>
</tbody>
</table>

Note: The table shows the parameter values of the model.

I calibrate its dynamics using the series of haircuts on Irish bonds presented in Section 2. The size of the liquidity shock is 0.40, equivalent to the rise in the haircut between the last quarter of 2010 and the first quarter of 2011, when the haircut jumped from 15% to 55%. The persistence of the shock $\rho^\phi$ is 0.985, which corresponds to the autoregressive coefficient of the liquidity shock $\phi_t$ estimated in an AR(1) process. The steady state value of liquidity is 0.85, corresponding to one minus the haircut in the post-crisis period.

The other parameter characterizing the financial frictions $\theta$ describes the fraction of investment financed externally. Since entrepreneurs represent broadly the banking system in channeling resources to the production sector of the economy, I consider $\theta$ as the ratio of Irish banks’ external finance over total assets, using aggregate banks’ balance sheet data. I compute the average of this ratio for the period between 1997 and 2010, which is around 0.3, which is the same parameter used by Kiyotaki and Moore (2018).

The parameter $\lambda$ pins down the duration of long-term bonds given by $(1 - \lambda \beta)^{-1}$. The price of equity at the steady state and $\lambda$ are selected to jointly determine the maturity of
government debt of about 7 years and the equity premium (6.9%) in line with the data.

For the steady state of the ratio of government bonds over GDP (ls) I consider the average of the general government gross financial liabilities as a percentage of GDP for the period between 1997 and 2010, which is 42%.\textsuperscript{36} The calibration of the parameter for open market operation ($\psi_B$) is based on the intervention of the European Central Bank on the Irish bond secondary market with the Expanded Asset Purchase Programme (APP).\textsuperscript{37}

There are some important differences with respect to Del Negro et al. (2017). $\theta$ is lower than in their model (0.79%), so the financing constraints of entrepreneurs are tighter because they cannot resell equity and they must finance a larger share of investment with their internal resources.\textsuperscript{38} In addition, in their calibration the size of the liquidity shock, based on the convenience yield of U.S. Treasuries, is smaller (-0.218) and the coefficient of their unconventional policy, which in their model exchanges government liquidity for illiquid equity is larger (-4.801).

Other parameters are standard in the literature, the discount factor $\beta = 0.99$, the inverse Frisch elasticity of labor supply $\eta = 1$, the relative risk aversion parameter $\sigma = 1$, the capital share $\alpha = 0.34$, the depreciation rate $\delta = 0.027$, the capital adjustment cost $Z''(1) = 0.75$ and the arrival rate of investment opportunity in each quarter $\gamma = 0.009$. For nominal rigidities, the degree of monopolistic competition in labor and product markets is calibrated symmetrically assuming a steady state markup of 10% ($\delta_p = \delta_w = 0.1$). The average duration of price and wage contracts is 4 quarters ($\zeta_p = \zeta_w = 0.75$). Concerning the monetary policy rule, the Taylor rule coefficient for inflation ($\psi_\pi$) and output gap ($\psi_y$) are 1.5 and 0.125, respectively. The Government finances most of the intervention through emission of new short-term debt and net taxes adjust slowly to net wealth position of the

\textsuperscript{36}This measure also includes government bonds with maturity of one year, but they account for only roughly 5% of the total gross government debt (Eurostat, Structure of Government Debt). For the series of the ratios of debt-to-GDP and banks’ external finance over total assets, the beginning of the sample is due to the availability of the data which are taken from the OECD Economic Outlook N. 102 and the Statistical Data Warehouse of the European Central Bank, respectively.

\textsuperscript{37}The monthly volume of Irish government bonds is around €0.7 bn, which corresponds to around 1% of Irish GDP. However, the calibrated value of the parameter of open market operation $\psi_B$ does not consider other interventions such as the Security Market Programme and LTROs which also swapped illiquid Irish bonds with liquid assets.

\textsuperscript{38}This economy where equity does not circulate and $\theta$ is relative low can be thought of in the terms of Kiyotaki and Moore (2003) as a region where there is a liquidity shortage and outside money (short-term bonds) and inside money (long-term bonds) are used as complementary means of savings and long-term bonds earn more than short-term bonds because of a liquidity premium.
Government ($\psi_T = 0.1$).

4.2 Results

4.2.1 The impact of the liquidity shock

I first analyze the economy in which the Government does not respond to the calibrated liquidity shock $\phi_t$, by setting $\psi_B = 0$. Figure 9 shows the response of the return on equity, return on long-term bonds and yield to maturity. The return on equity increases substantially by 15.8% and the return on long-term bonds by 0.8%, the yield to maturity increases by 0.02% (in annualized percentage points).

These results suggest that following a negative liquidity shock, entrepreneurs require a higher return because of the lower resaleability of long-term bonds, consistent with the empirical evidence. However, on impact the return on long-term bonds and yield to maturity fall because the portfolio of entrepreneurs is predetermined and the tightening of the resaleability constraint increases the demand for liquid assets, including the fraction of long-term government bonds which remains liquid, as pointed out by Shi (2015). 39

When the steady state parameter of long-term bonds is high, a bigger shock is necessary to generate a more pronounced rise in the return on long-term bonds by reducing the substitutability between long-term and short-term bonds. If I calibrate the magnitude of the liquidity shock to 0.65, which corresponds to the variations in the haircuts on Irish bonds from the last quarter of 2010 to the second quarter of 2011, the yield to maturity increases by around 0.3% and the return on long-term bonds by around 0.6%. 40

Figure 10 compares the response of output, inflation, consumption and investment to the liquidity shock with their evolution in the data for the Irish economy during the crisis. In the model, inflation is expressed in annualized percentage points, while the level of output, consumption and investment corresponds to percentage deviations from the steady state.

39 However, in contrast to Shi (2015) the return on equity increases following a tightening of the resaleability constraint, not on equity but on long-term bonds. So the model generates a positive comovement between asset prices and aggregate quantities. It is important to note the different response of the return on long-term bonds to a fall in $\phi_t$ and a fall in $\theta_t$ as reported in the Appendix. 39 When I simulate a negative shock in $\theta_t$, the return on long-term bonds falls because following a tightening in the borrowing constraint, the demand for liquid asset increases and entrepreneurs value more long-term bonds and thus their return decreases.

40 Since the economy is at the zero lower bound the response of spread between long-term and short-term bonds corresponds to the response of the return on long-term bonds.
The model suggests that the impact of the liquidity shock on aggregate variables is large and persistent. Output and inflation fall by -14.9% and -14.6% respectively.

![Graphs](image)

Figure 9: Response of returns on equity and long-term bonds, yield to maturity of long-term bonds and the nominal interest rate to the calibrated liquidity shock in annualized percentage points.

Breaking down the drop in output to consumption and investment, the last row of Figure 10 shows that they reduce respectively by 7.4% and 36.2%. The contraction of liquidity impacts directly on investment by tightening the entrepreneurs’ resaleability constraint and shrinking the liquidity they can obtain. Calibrating the liquidity shock with the same values as Del Negro et al. (2017), investment falls to -14% and output and consumption decrease to -5% and -1.5%, respectively. The effect of a liquidity crisis generated with a drop in $\phi_t$ is analogous to a tightening of margin requirements of leveraged investors in Ashcraft, Gărleanu and Pedersen (2011) and Gărleanu and Pedersen (2011), which increases the shadow cost of capital, raising the required returns of assets and lowering investment and output.

The model matches some properties of the data. Figure 10 also reports the percentage deviation of output, consumption and investment from a trend estimated from 1990Q1 to 2012Q3, using the Hodrick Prescott filter, normalized to zero in 2008Q3. Consumption and investment are expressed in real terms and output is the sum of these two components.
Figure 10: Response of output, inflation, consumption and investment to the calibrated liquidity shock (model versus data). Output, consumption and inflation are expressed in the log-deviation from steady state in percentage points. Inflation is expressed in annualized percentage points. The dashed lines represent the evolution of output, inflation, consumption and investment in the data. Output, consumption and investment are in percentage deviations from a trend estimated with the Hodrick Prescott filter from 1990Q1 to 2012Q3 and are normalized to zero in 2008Q3. Consumption is the private final consumption expenditure divided by the deflator of private final consumption expenditure. Investment is the gross capital formation divided by the deflator of gross capital formation. Output is the sum of real consumption and real investment. Inflation is the annualized quarterly inflation rate of the GDP deflator.

Inflation is the annualized percentage change in the GDP deflator in deviation from the inflation target of 2%. Output contracts by -11.6% and investment reduces much more than consumption, respectively -30.5% and -5.3%, in line with the theoretical findings. The model overpredicts the reductions in investment and consumption, which are deeper and more persistent, but these differences are not far from what happened in Ireland during the crisis. Finally, in the data the drop in inflation is also very large (-16.8%) and persistent and it is closer to the prediction of the model.
4.2.2 The effects of open market operations

I consider now the case in which the Government reacts to the drop in $\phi_t$ by issuing more short-term bonds. Since at the zero lower bound short-term debt shares the characteristics of money with a fixed interest rate (Tobin (1969)), we can consider this intervention as an unconventional open market operation which consists of buying long-term debt by selling short-term bonds or issuing reserves. Figure 11 shows the difference in aggregate variables and returns on assets in the model without intervention and in the model with intervention following a negative liquidity shock. The fall in output and inflation is reduced by around 0.04% and 0.03%, respectively. Decomposing the gain of output into investment and consumption, Figure 11 shows that almost all the benefit of the intervention derives from a lower reduction in investment, while consumption is barely affected. Furthermore, the impact on asset prices is limited, with a reduction in the spike of equity by 0.03% and almost no effect on the returns on long-term bonds.

Figure 11: The effect of the open market operation. The figure shows the difference between the responses of output, inflation, investment, consumption and returns on equity and long-term bonds to the calibrated liquidity shock with and without the open market operation.

Because these results differ markedly from the findings of Del Negro et al. (2017), I also
calibrated the intervention with their coefficients. Although quantitatively the gain on output and inflation are more substantial (0.8% and 0.6% respectively), they represent roughly half of the gain obtained by Del Negro et al. (2017) in their simulation of unconventional open market operation which consists of purchasing equity following a tightening in its resaleability constraint. The reason is the lower substitutability of assets because of the higher value of $\phi$.

4.3 Sovereign risk and liquidity

This Section studies the interaction between sovereign risk and liquidity, departing from the assumption of riskless government bonds. I model sovereign risk on long-term bonds as Bocola (2016), assuming that short-term bonds are in zero net supply, therefore I abstract from the implications of sovereign risk for the maturity structure of the government debt.\footnote{See Arellano and Ramanarayanan (2012), Broner, Lorenzoni and Schmukler (2013).}

Sovereign risk follows an exogenous process which increases the probability of default but I do not consider the possible materialization of the default. The focus is on the consequences of a rise in sovereign risk rather than the causes and on its transmission via a tightening of the resaleability constraint of long-term bonds. So I compare the impact of a sovereign risk shock with exogenous and endogenous liquidity. I endogenize the parameter $\phi_t$ as a function of the probability default, thus a positive sovereign risk shock, which increases the probability of default, reduces the liquidity of government bonds. Furthermore, I also consider the possibility of a two-way interaction between sovereign risk and liquidity, in which the reduction of liquidity leads to an additional increase in the probability of default.

The probability of default is defined as a logistic function

$$p^d_t = \frac{\exp(S + \rho_S S_t)}{1 + \exp(S + \rho_S S_t)}$$

(24)

where $S_t$ is an AR(1) process, $S$ is the steady state value of $S_t$ and $\rho_S$ is the autoregressive coefficient of $S_t$

$$S_t = (1 - \rho_S) \log(S) + \rho_S S_{t-1} + \sigma_S \varepsilon_t$$

(25)
The liquidity $\phi_t$ depends negatively on the probability of default

$$\phi_t = (1 - \mu \, p_d^t)$$  \hspace{1cm} (26)
where $\mu_\phi > 1$ is the parameter which determines how the probability of default affects the liquidity of long-term bonds. The presence of sovereign risk modifies the first order condition for long-term bonds (equation 18) and the government budget constraint (equation 20) since the return on long-term bonds also incorporates a risk premium

$$U'_{c,t} = \beta \mathbb{E}_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[ (1 - \mu_p p_t^d) \frac{1 + \lambda q_{L,t+1}^L}{q_t^L} + \frac{\gamma (q_{t+1} - q_{L,t+1}^L)}{q_t^{L+1} - \theta q_{t+1}^L} \right] \right\}$$ (27)

$$q_t^L b_{L,t+1}^L + t_t = (1 - \mu_p p_t^d) (1 + \lambda q_t^L) \frac{b_t^L}{p_t}$$ (28)

where $\mu_p > 0$ is the parameter which determines how the probability of default affects the risk premium on long-term bonds. Finally, I also consider the two-way interaction between the probability of default and liquidity by modifying the logistic function of the probability of default

$$p_t^d = \frac{\exp \{ S + \rho_S [S_t + (1 - \phi_t)] \}}{1 + \exp \{ S + \rho_S [S_t + (1 - \phi_t)] \}}$$ (29)

Figure 12 shows the responses of the returns on equity and long-term bonds to a positive sovereign risk shock in the model with exogenous liquidity, endogenous liquidity and with the interaction between liquidity and probability of default. I set $z = -1.8, \sigma = 0.85, \rho_z = 0.95, \mu_p = 0.01, \mu_\phi = 5.15$.

When the liquidity parameter is exogenous and does not respond to the sovereign risk shock an increase in the probability of default raises the returns on equity and long-term bonds. Figure 13 shows that the decline in asset prices has a negative wealth effect on consumption, while investment reacts with a small decline. This experiment also allows us to assess how variations in the return on equity affect investment, since $\phi_t$ is constant. Kiyotaki and Moore (2003) note that investment can react to a reduction in the price of equity in two opposite ways. On the one hand, a lower price of equity reduces the external finance, on the other hand a higher return on equity increases the available internal finance and the results seem to suggest that in this model the first effect dominates.
When liquidity is endogenous and falls in response to the higher probability of default, the sovereign risk shock has a stronger impact on macroeconomic variables. This effect is not due to a sharper increase in the return on long-term bonds, but to the tightening of the resaleability constraint, which reduces the ability of entrepreneurs to finance investment and also leads to a larger fall in the price of equity. However, the interaction between liquidity and probability of default leads to higher returns on equity and long-term bonds and a stronger contraction in consumption and investment.

5 Conclusions

This paper has studied the liquidity channel of the Eurozone sovereign debt crises. It has shown that government securities play a key role as collateral in the secured interbank market, which is a primary source of funding for banks. Nevertheless, I show that, during the crisis, repo haircuts on Irish and Portuguese government bonds substantially augmented following the rise in sovereign risk.

I analyze the impact of increases in haircuts on sovereign debt markets by identifying a liquidity shock with a high frequency SVAR model. The liquidity shock increases the government bond yields, CDS spreads and yield spreads significantly, suggesting that investors adjust their portfolio towards more liquid bonds in response to the rise in haircuts. Also, the haircuts respond positively to an innovation in the yields indicating the presence of a dynamic interaction between yields and haircuts.

I also investigate the impact of a liquidity shock on asset prices and macroeconomic variables with a general equilibrium model with financial frictions calibrated for Ireland. The model predicts a rise in the return on long-term bonds and yield to maturity because investors sell fewer liquid long-term bonds and purchase more liquid short-term bonds, consistent with the empirical findings. Furthermore, the model exhibits a substantial fall in output, investments, consumptions and deflation, similar to the data for the Irish economy, and allows us to study the effects of non standard open market operations in response to the liquidity shock. It also shows how the reduction in the liquidity of government bonds amplifies the transmission of sovereign risk to aggregate variables, in particular with the
presence of a two-way interaction between liquidity and sovereign risk, as observed in the Irish and Portuguese bonds markets during the crisis.

A Data Appendix

A.1 Definitions for repurchase agreements (repos)

A repo transaction is an agreement between two parties on the sale and subsequent repurchase of securities at an agreed price. It is equivalent to a secured loan, with the main difference that the legal title of securities passes from the cash borrower to the cash lender, who may re-use them as collateral in other repo transactions. In order to protect the lender from the risk of a reduction in the value of collateral, repos involve overcollateralization and the difference between the value of the loan and the value of collateral is the haircut or initial margin. For instance, at time t, the cash borrower (securities dealer, commercial bank, hedge fund) posts €100 security as collateral and receives a €95 loan from the cash lender (commercial bank, investment fund, money market fund) with a haircut of 5%. At time t+k, the borrower returns the cash with an interest of 1% (the repo rate) and receives back the collateral. If repo is used to finance the purchase of a security, the haircut is equivalent to the inverse of the leverage. In order to hold €100 security the investor can borrow up to €95 from the repo lender and must finance the remaining €5 with its own capital, so the maximum leverage is 20.

The determinants of haircuts vary according to the repo structure. In repos that are not cleared by a Central Clearing Counterparty (CCP), the haircut reflects mainly the credit-worthiness of the borrower. Instead in repos involving a CCP which bears the counterparty credit risk, haircuts are settled on the basis of the CCP’s internal rules and depend on the market risk of collateral.

A.2 Data on repos

Because of the lack of comprehensive information on the European repo market, I use different sources. First, Bankscope, which provides banks’ balance sheet data at annual frequency showing the amount of repos and reverse repos held by credit intermediaries. It allows to
compare different funding instruments, but lacks important breakdowns (such as counterparty, maturity and currency) preventing a more granular analysis and does not distinguish between repos in the interbank market from ECB monetary policy operations. Table 3 shows the funding structure of the largest banks for which Bankscope report information on repos. I consider 2010 in order to avoid the large-scale LTROs implemented in 2011 and 2012.

Table 3: Funding structure of European commercial banks in percentage of total liabilities (2010)

<table>
<thead>
<tr>
<th>Bank</th>
<th>Deposits</th>
<th>Interbank</th>
<th>LT debt</th>
<th>Repos</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP Paribas</td>
<td>26.62</td>
<td>7.07</td>
<td>6.19</td>
<td>10.48</td>
</tr>
<tr>
<td>Barclays Bank Plc</td>
<td>23.41</td>
<td>5.89</td>
<td>9.89</td>
<td>13.26</td>
</tr>
<tr>
<td>Banco Santander</td>
<td>45.04</td>
<td>4.69</td>
<td>16.92</td>
<td>9.60</td>
</tr>
<tr>
<td>Société Générale</td>
<td>24.37</td>
<td>7.62</td>
<td>8.74</td>
<td>9.58</td>
</tr>
<tr>
<td>UBS AG</td>
<td>24.13</td>
<td>2.13</td>
<td>10.04</td>
<td>12.52</td>
</tr>
</tbody>
</table>

Note: The Table displays the structure of European commercial banks in percentage of total liabilities for 2010. Legend: Deposits = customer deposits; Interbank = interbank deposits; LT debt = long-term debt; Repos = repurchase agreements. Source: Bankscope.

Second, the European Repo Market Survey published semi-annually by the International Capital Market Association (ICMA) since 2001, which asks a sample of 67 banks in Europe for the value of their repo contracts that were still outstanding at close of a certain business date, excluding repos transacted with central banks as part of official monetary policy operations.

Lastly, the Euro Money Market Survey, a yearly survey published by the ECB covering 101 banks, which breaks down the repo market into CCP-based, over-the-counter bilateral, and triparty repos.

A.3 Narrative approach

I collected data on haircuts applied to 10 year government bonds by LCH Clearnet Ltd, which is the largest European clearing house, reading the Repo Clear Margin Rate Circulars published in the website of LCH Clearnet Ltd. Figure 14 shows an example of the Circulars indicating the date of the announcement and the date of the implementation of changes in haircuts.
Figure 14: Example of Repo Clear Margin Rate Circular

Management of Sovereign Credit Risk for RepoClear Service

In accordance with the Sovereign Credit Risk Framework and in response to the yield differential of 10-year Irish government debt against a AAA benchmark, LCH.Clearnet Ltd has revised the risk parameters for Irish government bonds cleared through the RepoClear service. The additional margin required for positions in Irish government bonds will consequently be increased to 5% for long positions, the amount will be adjusted for the current bond price. Short positions will pay a proportionately lower margin.

1. This decision is based solely on publicly available yield spread data and in no way represents a forward-looking market view. LCH.Clearnet will continue to monitor yield spreads closely and keep the parameters under close review in accordance with the Sovereign Credit Risk Framework.
2. The additional margin will be reflected in a margin call payment on Friday 25 March 2011.
3. Report 74 (available on the LCH.Clearnet Member Reporting website) will detail any further changes in the margin levels charged under this Framework.
4. This circular supersedes LCH.Clearnet Ltd Circular No 2748 dated 06 December 2010.
5. For further information please contact either Tom Chapman (tom.chapman@lch.Clearnet.com) +44(0)2075430330 or Lauren Arnold (lauren.arnold@lch.Clearnet.com) +44(0)2072873730.

Christopher Jones
Executive Director, Head of Risk Management

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B Econometric model

B.1 Statistical tests for the anticipation of the liquidity shock

The threshold test statistics is plotted for the regression \( h_t^* = \alpha_0 + \alpha_1 CDS_t \) The figure displays a graph of the normalized likelihood sequences as a function of the threshold \((CDS_t)\). The dotted line plots the 95% critical value.

Table 4: Granger causality tests of CDS spreads on announced variations in haircuts

<table>
<thead>
<tr>
<th>Country</th>
<th>F statistics</th>
<th>Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ireland</td>
<td>2.43</td>
<td>3.88</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.44</td>
<td>3.96</td>
</tr>
</tbody>
</table>

Note: The number of lags is selected using the Bayesian Information Criterion (BIC) considering a maximum of 6 lags. Tests are performed at the significance level of 0.05. If the F statistics is lower than the critical value, we accept the null hypothesis that \( CDS_t \) does not Granger-cause \( h_t^* \).
B.2 Bayesian estimation

Given N different variables in a vector $y_t = (y_{1t}, ..., y_{Nt})'$, consider the following VAR:

$$y_t = B_1 y_{t-1} + ... + B_p y_{t-p} + u_t$$

where $t = 1, ..., T$ and $u_t \sim (0, \Sigma_u)$. Each equation has $M = Np + 1$ regressors. By collecting the coefficient matrices in the $N \times M$ matrix $B = \begin{bmatrix} B_c, B_1, ..., B_p \end{bmatrix}$ and defining $x_t = (1, y'_{t-1}, ..., y'_{t-p})'$ as a vector containing an intercept and $p$ lags of $y_t$, the VAR can be written as

$$Y = X B + E$$

where $Y = [y_1, ..., y_T]'$, $X = [x_1, ..., x_T]'$, and $B = [B_1, ..., B_T]'$ are matrices of dimension $T \times N$, $T \times M$ and $T \times N$ respectively. Defining the following OLS estimations $\hat{B} = (X'X)^{-1}X'Y$, $\hat{b} = \text{vec}(\hat{B})$, $\hat{S} = (Y - X\hat{B})'(Y - X\hat{B})$, we consider the non-informative version of the natural conjugate prior

$$b|\Sigma, y \sim N(b^*, V^*)$$

$$\Sigma^{-1}|y \sim W(S^{-1}, T - K)$$

where $V^* = \Sigma_u \otimes (X'X)^{-1}$ and $b^* = \hat{b} + Q$. $Q$ is the Choleski factor of $V^*$, i.e. $V^* = QQ'$. The structural VAR is the following

$$A_0 y_t = A_c + A_1 y_{t-1} + ... + A_p y_{t-p} + \epsilon_t \quad (B.1)$$

where $A_0$ is the impact matrix, $A_j = A_0 B_j$, and $\epsilon_t$ are the structural shocks with diagonal covariance matrix $\Sigma_c$. Reduced form residuals can be expressed as a linear combination of structural shocks $u_t = A_0 \epsilon_t$ and $\Sigma_u = A_0 \Sigma_c A_0$. Normalizing the variances of the structural shocks to one (i.e. $E(\epsilon_t \epsilon'_t) = I$) gives $\Sigma_u = A_0 A'_0$.
C Additional Model Details and Optimality Conditions

C.1 Final and Intermediate Good Firms

Competitive final goods producers combine differentiated intermediate goods $y_t(i)$, for $i \in [0, 1]$ into a single homogeneous final good $y_t(i)$, using a constant return to scale technology.

\[ y_t = \left[ \int_0^1 y_t(i)^{1+\lambda_x} \, di \right]^{1+\lambda_x} \]  
\hspace{1cm} (C.1)

where $\lambda_x > 0$. They take input prices $p_t(i)$ and output prices $p_t$ as given and choose $y_t(i)$ to maximize profits

\[ p_t y_t - \int_0^1 p_t(i) \, y_t(i) \, di, \]

The solution to the profit maximization gives their demand for the generic $i^{th}$ intermediate good

\[ y_t(i) = \left[ \frac{p_t(i)}{p_t} \right]^{-\frac{1+\lambda_x}{\lambda_x}} y_t \]  
\hspace{1cm} (C.2)

The zero profit condition for competitive final goods producers implies that the aggregate price level is

\[ p_t = \left[ \int_0^1 p_t(i)^{-\frac{1}{\lambda_x}} \, di \right]^{-\lambda_x} \]  
\hspace{1cm} (C.3)

Monopolistically competitive intermediate goods producers hire labor services from households and rent capital from entrepreneurs to produce the intermediate goods choosing (i) the optimal amount of inputs, (ii) optimal price setting in case its price can be adjusted.

(i) The intermediate goods producer i chooses the optimal amount of inputs to minimize the costs

\[ r^K_t k_t(i) + w_t h_t(i) \]
subject to the production technology

\[ y_t(i) = z_t \ k_t(i)^\alpha \ h_t(i)^{1-\alpha} - \Omega \]  

(C.4)

where \( \alpha \in (0,1) \) is the share of capital, \( z_t \) is an aggregate technology and \( \Omega \) is fixed cost of production, \( k_t(i) \) denotes the capital services and \( h_t(i) \) the quantity of labor hired by the \( i^{th} \) intermediate goods producer. Defining \( W_t = \frac{w_t}{p_t} \) the real wage, the first order condition implies that capital labor ratio is independent of firm-specific variables

\[ \frac{k_t(i)}{h_t(i)} = \frac{k_t}{h_t} = \frac{\alpha W_t}{1 - \alpha r_t^K} \]  

(C.5)

The Lagrange multiplier on the constraint is the real marginal cost which is also independent of firm specific variables

\[ mc_t(i) = mc_t = \frac{1}{z_t} \left( \frac{r_t^K}{\alpha} \right)^\alpha \left( \frac{W_t}{1 - \alpha} \right)^{1-\alpha} \]  

(C.6)

(ii) Intermediate goods producers set prices \( p_t(i) \) subject to Calvo (1983) scheme frictions. With probability \( 1 - \zeta_p \), the firm can reset its price and with probability \( \zeta_p \) cannot. By the law of large numbers, the probability of changing the price corresponds to the fraction of firms that rest the price, so in each period a randomly selected fraction of firms \( 1 - \zeta_p \) chooses the price \( \tilde{p}_t(i) \) to maximize the present discount value of profit

\[ \mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} \ v_s \left[ \frac{\tilde{p}_t(i)}{p_s} - mc_s \right] y_s(i) = 0 \]

subject to (C.2). The first order condition is

\[ \mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} \ v_s \left[ \frac{\tilde{p}_t(i)}{p_s} - (1 + \lambda_x)mc_s \right] y_s(i) = 0 \]

We focus on a symmetric equilibrium in which all firms choose the same price \( \tilde{p}_t(i) = \tilde{p}_t \).

Let \( \tilde{p}_t = \tilde{p}_t/p_t \) the optimal relative price. The first order condition for optimal price settings
becomes
\[
\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} c_s \left( \frac{\bar{p}_t(i)}{\pi_s} - (1 + \lambda \pi) mc_s \right) \left( \frac{\bar{p}_t}{\pi_{t,s}} \right)^{-\frac{1 + \lambda \pi}{\lambda \pi}} y_s = 0 \quad (C.7)
\]

By the law of large numbers, the probability of changing the price coincides with the fraction of firms who change the price in equilibrium. From the zero profit condition (C.3) inflation depends on the optimal reset price according to
\[
1 = (1 - \zeta_p) \bar{p}_t^{\frac{1}{\lambda \pi}} + \zeta_p \left( \frac{1}{\pi_t} \right) \quad (C.8)
\]

Plugging (C.8) into (C.7) the price setting rule becomes
\[
\left( \frac{1 - \zeta_p \pi_t^{\frac{1}{\lambda \pi}}}{1 - \zeta_p} \right)^{-\lambda \pi} \frac{x^p_{1t}}{x^p_{2t}} = \left( \frac{1 - \zeta_p \pi_t^{\frac{1}{\lambda \pi}}}{1 - \zeta_p} \right)^{-\lambda \pi} \frac{x^p_{1t}}{x^p_{2t}} \quad (C.9)
\]

where \(x^p_{1t}\) and \(x^p_{2t}\) are expected present value of real marginal cost and real marginal revenue as

\[
x^p_{1t} = c_t^{-\sigma} y_t mc_t + \beta \zeta_p \mathbb{E}_t \left( \frac{1 + \zeta_p}{\pi_{t+1}} x^p_{1t} \right) \quad (C.10)
\]

\[
x^p_{2t} = \frac{1}{1 + \zeta_p} c_t^{-\sigma} y_t + \beta \zeta_p \mathbb{E}_t \left( \frac{1}{\pi_{t+1}} x^p_{2t} \right) \quad (C.11)
\]

The evolution of the real wage is given by
\[
\frac{w_t}{w_{t-1}} = \frac{\pi_{wt}}{\pi_t} \quad (C.12)
\]

where \(\pi_{wt} = \frac{w_t}{w_{t+1}}\) is defined as the wage inflation. Since the ratio of capital-output is independent of firm-specific factors, the aggregate production function is

\[
z_t k_t^{\alpha} h_t^{1-\alpha} = \int_0^1 y_{it} \, di = \sum_{s=0}^{\infty} \zeta_p (1 - \zeta_p)^{t-s} \left( \frac{\bar{p}_{t-s}}{\pi_{t-s,t}} \right)^{-\frac{1 + \lambda \pi}{\lambda \pi}} y_t \quad (C.13)
\]
where \( k_t = \int_0^1 k_{it} \, di \) and \( h_t = \int_0^1 h_{it} \, di \). The effect of price dispersion is

\[
\Delta_t = \sum_{s=0}^{\infty} \zeta_p (1 - \zeta_p)^{t-s} \left( \frac{P_{t-s}}{\pi_{t-s,t}} \right)^{-\frac{1+\lambda_p}{\lambda_\pi}}
\]

(C.14)

Using (C.8), \( \Delta_t \) can be expressed recursively

\[
\Delta_t = \zeta_p \Delta_{t-1}^{\frac{1+\lambda_p}{\lambda_\pi}} + (1 - \zeta_p) \left( \frac{1 - \zeta_\pi \pi_t^{\lambda_\pi}}{1 - \zeta_\pi} \right)^{1+\lambda_p}
\]

(C.15)

The aggregate production function becomes

\[
z_t \alpha \cdot h_t^{1-\alpha} - \Omega = \Delta_t y_t
\]

(C.16)

### C.2 Labor market

The labor market is symmetric to the goods market. Competitive labor agencies aggregate differentiated \( j \) labor inputs into a homogeneous single labor service \( h_t \) according to the technology

\[
h_t = \left[ \left( \frac{1}{1 - \gamma} \right)^{\lambda_w} \int_{\gamma}^{1} h_t(j)^{\frac{1}{1+\lambda_w}} \right]^{1+\lambda_w}
\]

(C.17)

where \( \lambda_w > 0 \). They choose \( h_t(j) \) to maximize their profits

\[
w_t h_t - \int_{\gamma}^{1} w_t(j) h_t(j) dj
\]

subject to (C.17) and taking wages specific \( w_t(j) \) as given. The first order condition yields the demand for the \( j^{th} \) labor inputs

\[
h_t(j) = \frac{1}{1 - \gamma} \left[ \frac{w_t(j)}{w_t} \right]^{\frac{1+\lambda_w}{\lambda_w}} h_t
\]

(C.18)

The aggregate wage index \( w_t \) comes from the zero profit condition for labor agencies

\[
w_t = \left[ \frac{1}{1 - \gamma} \int_{\gamma}^{1} w_t(j)^{-\frac{1}{\lambda_w}} dj \right]^{-\lambda_w}
\]

(C.19)
Labor unions represent all types of workers and set the wage rate \( w_t(j) \) for the specific labor input \( j \) subject to the Calvo scheme frictions on a staggered basis, taking as given the demand for their specific labor input. Each period, labor agencies are able to reset the wage with probability \( 1 - \zeta_w \), with probability \( \zeta_w \) they cannot and the wage remains fixed. By the law of large numbers, the probability of changing the wage corresponds to the fraction of workers whose wages change. Households supply whatever labor is demanded at that wage. If labor agencies can modify the wage, they will choose the wage \( \tilde{w}_t \) to maximize

\[
\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} \left[ \left( \frac{c_s^{1-\sigma}}{1-\sigma} - \frac{\xi}{1+\eta} \int_1^1 h_s(j)^{1+\nu} \; dj \right) \right]
\]

subject to (16) and (C.18). Focusing on a symmetric equilibrium in which all agencies choose the same wage, the first order condition for this problem is

\[
\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} c_s^{-\sigma} \left\{ \tilde{W}_t \frac{\omega}{\pi_{t,s}} \left[ \left( \frac{\tilde{W}_t}{\pi_{t,s} W_s} \right)^{-1+\lambda_w} H_s \right]^{\eta} \right\} \left( \frac{\tilde{W}_t}{\pi_{t,s} W_s} \right) h_s(j) = 0
\]

from (C.19) the law of motion of real wage is

\[
W_t^{\gamma_w} = (1 - \zeta_w)\tilde{w}_t - \frac{1}{\gamma_w} + \zeta_w \left( \frac{W_{t-1}}{\pi_t} \right)^{-\frac{1}{\gamma_w}}
\]

Using (C.22), (C.21) can be written

\[
\frac{1 - \zeta_w}{1 - \zeta} \left( \frac{1}{x_{wl}^{\gamma_w}} \right)^{-\lambda_w(1+\pi_w)\eta} = \frac{x_{wl}^{w_t}}{x_{wl}^{2t}}
\]

where \( x_{wl}^{w_t} \) and \( x_{wl}^{2t} \) are the expected present value of marginal disutility of work and real marginal wage revenue as

\[
x_{wl}^{w_t} = \frac{\omega}{(1-\gamma)\eta} \tilde{h}_t^{1+\eta} + \beta \zeta_w \mathbb{E}_t \left( \frac{(1+\lambda_w)(1+\eta)}{\lambda_w^w} \tilde{W}_t^{w_t} x_{wl}^{2t+1} \right)
\]

\[
x_{wl}^{2t} = \frac{1}{1+\lambda_w} c_t^{-\sigma} \tilde{W}_th_t + \beta \zeta_w \mathbb{E}_t \left( \frac{1}{\pi_{wl+1}^{\gamma_w}} x_{wl+1}^{2t+1} \right)
\]
C.3 Capital good producers

The creation of new capital is delegated to competitive capital goods producers who transform consumption goods into investment goods. They choose the amount of investment goods to maximize the profits taking the price of investment goods $q^t_I$ as given

$$d^t_I = \left\{ q^t_I - \left[ 1 + Z \left( \frac{i_t}{t} \right) \right] i_t \right\} i_t$$  (C.26)

The price of investment goods differs from the price of consumption goods because of the adjustment costs, which depends on the deviations of actual investment from its steady state value. $Z(.)i_t$ reflects the adjustment cost. We assume that $Z(1) = Z'(1) = 0$ and $Z'' \left( \frac{i_t}{t} \right) > 0$. The first order condition for this problem is

$$q^t_I = 1 + Z \left( \frac{i_t}{t} \right) + Z' \left( \frac{i_t}{t} \right) \frac{i_t}{t}$$  (C.27)

C.4 Market clearing and equilibrium

The market clearing conditions for composite labor and capital use are

$$h_t = \int_0^1 h_t(i)di$$

and

$$k_t = \int_0^1 k_t(i)di$$

The law of motion of capital is

$$k_{t+1} = (1 - \delta)k_t + i_t$$  (C.28)

We consider the following identity equation between capital and net equity

$$k_{t+1} = s_{t+1}$$  (C.29)
The resource constraint can be expressed as

\[ y_t = c_t + \left[ 1 + Z \left( \frac{i_t}{t} \right) \right] i_t \] (C.30)

Finally, considering the aggregate expression for \( d_t \) and \( d_t^i \), the investment function can be written as

\[ i_t = \gamma \frac{r_t^K s_t + [1 + \lambda \phi_t q_t^{L_t} b_{L_t}^{L_t}]^b_t}{q_t^l - \theta q_t} \] (C.31)

To solve the model, I define \( B_{t+1}^{L_t} = b_{t+1}^{L_t} \) and \( B_{t+1}^{S_t} = b_{t+1}^{S_t} \) as real long-term and short-term bonds. There are 5 endogenous state variables: the aggregate capital stock, the real long-term bond, the real short-term bond, and the real wage rate and the effect of price dispersion from the previous period \((k_t, B_t^{L_t}, B_t^{S_t}, W_t, \Delta_t)\). The recursive competitive equilibrium is defined as 9 endogenous quantities \((i_t, c_t, y_t, k_{t+1}, s_{t+1}, B_{t+1}^{S_t}, B_{t+1}^{L_t}, h_t, t_t)\) and 16 prices \((q_t, q_t^L, q_t^S, q_t^i, r_t, W_t, \pi_t, \pi_{wt}, m_c, x_{1t}, x_{2t}, x_{1t}, x_{w}, x_{w}, \Delta_t, y_t)\) as a function of state variables \((k_t, B_t^{S_t}, B_t^{L_t}, W_{t-1}, \Delta_{t-1}, z_t, \phi_t)\), which satisfies the 25 equilibrium conditions \((4, 5, 15, 17, 18, 19, 20, 22, 23, 21, C.5, C.6, C.9, C.10, C.11, C.12, C.15, C.16, C.23, C.24, C.25, C.27, C.28, C.29, C.30)\). Once all market clearing conditions and the government budget constraint are satisfied, the household budget constraint \((16)\) is satisfied by Walras’ Law.

**D The impact of a borrowing constraint shock**

I compare the asset price implications of the liquidity shock with another financial shock, a tightening in the borrowing constraint of entrepreneurs that corresponds to an exogenous drop in \( \theta \). For the purpose of comparison, the borrowing constraint shock follows an AR(1) process with the same dynamic properties of the liquidity shock for the purpose of comparison and a steady state value of \( \theta = 0.55 \).

The return on equity falls in response to the shock to the borrowing constraint. Even though a fall in \( \theta_t \) reduces the supply of equity (Kiyotaki and Moore (2003)), it also generates a larger contraction in the demand, leading to a fall in the price and an increase in the return.
on equity. In contrast to a liquidity shock, the return on long-term bonds decreases after a shock to the borrowing constraint because they become more valuable for entrepreneurs to finance investment. In other words, a drop in $\theta_t$ increases the demand for liquid assets to finance the larger downpayment, raising the value of long-term bonds.

![Graph showing impulse responses to a liquidity shock and a borrowing constraint shock.](image)

Figure 16: Impulse responses to a liquidity shock and a borrowing constraint shock.

References


