Asset Bubbles and Product Market Competition

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Abstract
This paper studies the interactions between rational bubbles and product market competition. It offers two main insights. The first is that, by providing an entry or production subsidy, asset bubbles may reduce entry barriers and force incumbents to reduce markups. The second is that imperfect competition relaxes the conditions for the existence of rational bubbles. When they have market power, firms restrict output and investment to enjoy monopoly rents. This depresses the interest rate, making rational bubbles possible even when capital accumulation is dynamically efficient. I use anecdotal evidence from the British railway mania of the 1840s and the dotcom bubble of the late 1990s to support the model's hypotheses and predictions.

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1   Introduction

"With valuations based on multiples of revenue, there's ample incentive to race for growth, even at the

cost of low or even negative gross margins."

"Dotcom history is not yet repeating itself, but it is starting to rhyme", Financial Times, March 12, 2015

Stock markets often experience fluctuations that seem too large to be entirely driven by fundamentals. Major historical events include the Mississippi and the South Sea bubbles of 1720 or the British *railway mania* of the 1840s. A more recent example is that of the US stock market during the so called *dotcom* bubble: between October 1995 and March 2000, the NASDAQ Composite index increased by almost sixfold to then collapse by 77% in the following two years.[1] One common aspect among most of these episodes is that they seem to be concentrated on a particular market or industry and to be associated with increased competition in the sector where they appear.[2] The *dotcom* bubble of the late 1990s constitutes a good example in this regard. In an environment characterized by soaring prices of technology stocks, many internet firms appeared and went public.[3] Furthermore, since the valuation of young firms is typically based on metrics of size (revenues or market shares) and not on profits, the new *dotcoms* often sought rapid growth and engaged in aggressive commercial practices, such as advertisement overspending or extremely low penetration prices. For instance, some online delivery companies appearing around this period (such as *Kozmo.com* or *UrbanFetch*) provided their services completely for free. Some firms would even make money payments to attract consumers: the advertising company *AllAdvantage.com* paid internet users to display advertisements on their screens. Most of these companies incurred extensive income losses and could not survive the stock market crash in 2000 (see Section 3).

But even if lacking market expertise or following unsustainable business models, the new *dotcoms* often posed a threat to incumbents and in some cases forced them to expand and enter the online market. For instance, the appearance of many online toy retailers such as eToys, Toysmart, Toytime or Red Rocket (all of which went bankrupt after the stock market crash) forced Toys"R"Us to enter the internet market by means of a partnership with Amazon. Another well known example involves GE and the "Destroy Your Business" program launched by its CEO Jack Welch in 1999. Welch asked his managers to go through a collective exercise and think of different ways in which a new *dotcom* could destroy GE's leadership in different markets. The main idea consisted in identifying new

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[1] Although there is no consensus, a great deal of evidence suggests that technology stocks were overvalued in the late 1990s. See for instance Ofek and Richardson [2002] and Lamont and Thaler [2003].

[2] For example, the Mississippi and the South Sea bubbles involved two trading companies (the *Compagnie d'Occident* in France and the *South Sea Company* in Great Britain) that engaged in innovative financial schemes (namely the issuance of stocks to finance the acquisition of government debt); the *British railway mania* was an episode that affected the British railway industry; the *dotcom* bubble was an event concentrated on a group of internet and high-tech industries.

[3] Goldfarb and Kirsch [2008] report that between 1994 and 2001 "approximately 50,000 companies solicited venture capital to exploit the commercialization of the internet"; among these, around 500 companies had an initial public offering.

production processes or business opportunities before other companies did. As part of the "Destroy Your Business" initiative, many divisions of GE (such as GE Plastics, GE Medical Systems or GE Appliances) adopted cost-cutting programs and started providing new services through the internet.

The idea that the *dotcom* bubble was associated with a more competitive market structure is corroborated by indicators of product market competition. Figure 1 shows price-cost markups of established corporations in three industries that were at the center of the dotcom bubble: Publishing Industries (which includes software developers), Telecommunications, and Information and Data Processing Services (which includes internet publishers and web search portals); data is from COMPUSTAT for the period 1995-2005. For each panel, the red line shows the average price-cost markup for a balanced panel of firms (i.e. for all firms that were active throughout the period considered, and hence existed even before the rise of stock prices). The blue line shows a simple proxy for overvaluation in these industries – the ratio of total stock market capitalization to total sales (for all firms in the industry). A common pattern can be detected in the three industries – average markups of existing corporations decline from 1995 until the peak of the bubble in 2000/2001, and start increasing after the stock market crash. Examples of increased competition could also be found in other historical events, such as the British railway *mania* (see Section (3)).
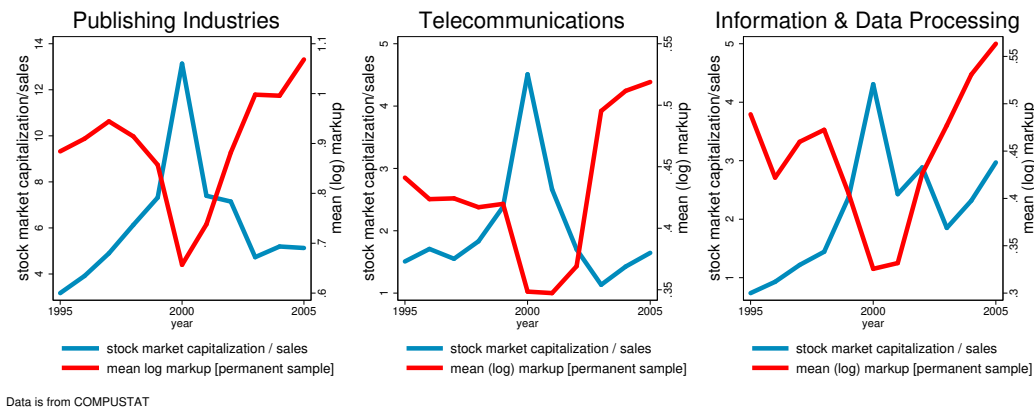
Figure 1: The *dotcom* Bubble: Average Markups of Incumbents

This figure shows aggregate price-sales ratios and average markups for three industries during 1995-2005: 'Publishing Industries' (NAICS 511), 'Telecommunications' (NAICS 517), and 'Information & Data Processing' (NAICS 518-519). The price-sales ratio is the ratio of total stock market capitalization (stock price times common shares outstanding) to total sales (COMPUSTAT item #12), constructed at the beginning of the corresponding year. Markups are the ratio of total sales (COMPUSTAT item #12) to the cost of goods sold (COMPUSTAT item #41). Average markups are constructed for a balanced panel of firms, i.e. for firms that are active throughout the 1995-2005 period.

In this article, I study the interactions between stock market bubbles and product market competition. To this end, I construct a multi-industry model featuring imperfect competition in goods markets and rational asset bubbles. The model provides two main insights. The first is that, by providing an entry or production subsidy, bubbles may encourage entry and force incumbents to cut markups. The second is that imperfect competition relaxes the conditions for the existence of rational bubbles.

The economy features a continuum of industries. In each industry, there is one productive *leader* and a potentially

large number of unproductive *followers*. The number of active followers will be determined endogenously: both the existence of a productivity gap with respect to the leader and the presence of fixed costs of production will limit the number of firms that can participate in each market. As a result, each industry may be characterized by an oligopolistic market structure, where the leader produces a suboptimal level of output and enjoys monopoly rents.

I then analyze how stock market overvaluation affects each industry's equilibrium. Overvaluation is introduced by means of rational asset bubbles, which appear attached to the value of the stocks that firms issue. I consider two different processes describing stock market sentiment. First, I assume that bubbles are exogenous at the firm level, so that each firm can issue a fixed value of overvalued stocks. In this case, overvaluation provides firms with a lump sum rent or subsidy, and can thus attract additional followers to the market. The entry of additional followers will be associated with some welfare gains (as the leader may be forced to expand and reduce his markup) and costs (through the replication of fixed production costs). The net impact on welfare can be positive when bubbles are relatively small, but will be associated with an inefficiently high level of firm entry when overvaluation is large.

Second, I assume that, instead of being fixed at the firm level, overvaluation is fixed at the industry level, so that there is a fixed value of overvalued stocks that investors want to purchase in a particular industry, irrespective of the number of active firms. Each active firm gets a fraction of the industry bubble in proportion to its market share. This process is intended to capture the fact that valuation models are often based on metrics of size (such as market shares) and not on profits.[4] I show that the appearance of a fixed industry bubble provides firms with incentives to fight for market shares and increase output, at the expense of profits. Such a process will thus feature a pro-competitive effect, which happens even when the number of active firms remains unchanged. However, when the size of the industry bubble is sufficiently large, firms may be willing to expand excessively and charge a price below their unit cost of production. The model can therefore explain the low (or even negative) profit margins exhibited by railway companies during the British *mania* of the 1840s and by internet firms at the peak of the *dotcom* bubble (Section 3).

I also discuss the impact of imperfect competition on the appearance of rational asset bubbles. From a theoretical standpoint, it is well known that rational asset bubbles can only emerge in economies in which the steady-state interest rate is lower than the growth rate.[5] In this paper, I show that imperfect competition depresses the interest in general equilibrium, hence relaxing the conditions for the existence of rational bubbles. The intuition is simple: having market power, firms restrict output and investment relative to the social optimum. As a result, both the demand for credit and the interest rate may be sufficiently depressed so that rational asset bubbles become possible even when capital accumulation is dynamically efficient.

Finally, I show that the results of the model are robust to different formulations of firms' strategic interactions

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[4]See for instance Damodaran [2006], pp. 234-235. The use of such valuation techniques is especially true in the case of young firms: they typically start with low or even negative profit margins, which makes it difficult to project *future* cash flows from *current* earnings. See the discussion in Section 2.

[5]On the one hand, for rational bubbles to exist, they must offer a return that is not lower than the interest rate. On the other hand, bubbles cannot grow faster than the economy (otherwise they can be ruled out with simple backward induction arguments).

(whereas in the main model I assume that firms play a Cournot game, in Appendix B I show that the main results and intuitions hold under Bertrand competition).

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 reviews some anecdotal evidence from two important stock market overvaluation episodes: the British *railway mania* of the 1840s and the *dotcom* bubble of the late 1990s. These episodes are reinterpreted through the lens of the theory developed in this paper. Section 4 concludes. Before proceeding, I offer a brief review of the related literature.

1.1 Related Literature

This paper is mostly related to the literature that forms the theory of rational bubbles. Different models have emphasized different aspects of asset bubbles. In very broad terms, we can divide the literature into two categories. On the one hand, there are models that view bubbles as assets whose main role is being a store of value. This is the message of the seminal contribution of Samuelson [1958] who argues that bubbles may complete intergenerational markets and provide for an efficient intertemporal allocation of consumption. Tirole [1985] makes the same point in the context of the neoclassical growth model, emphasizing a *crowding-out* effect: bubbles drive resources away from investment. However, in the model of Tirole [1985] this effect happens to be welfare-improving as it eliminates inefficient capital accumulation.[6] Being a store of value, bubbles can also be a liquidity instrument that helps firms overcome financial frictions as in Caballero and Krishnamurthy [2006], Farhi and Tirole [2012], Hirano and Yanagawa [2017], Kocherlakota [2009] or Miao and Wang [2012]. Finally, Ventura [2012] shows that bubbles can increase the return on savings in low productivity countries, thereby eliminating cross-country differences in rates of return and acting as a substitute for capital flows.

A different strand of the literature, on the other hand, has put emphasis on the appearance of new bubbles: the formation of a new pyramid scheme always provides some rent or subsidy that can have economic consequences. Within this category, we find the model of Olivier [2000] who shows that if appearing attached to R&D firms, bubbles can stimulate the invention of new goods and foster economic growth. Martin and Ventura [2012, 2016] argue that the creation of new bubbles allows credit-constrained entrepreneurs to expand borrowing and investment. Tang [2018] studies how the appearance of asset bubbles affects firm dynamics in presence of credit constraints.

In this paper, I provide a theory of how asset bubbles can be expansionary. My paper will hence be closest in spirit to the recent class of models emphasizing how asset bubbles can alleviate credit market frictions and be associated with larger investment and output (Farhi and Tirole [2012], Martin and Ventura [2012, 2016], Hirano and Yanagawa [2017], Tang [2018], Ikeda and Phan [2019]). There are however important differences. First, the focus will be on frictions in product markets, not in financial markets. Furthermore, most models featuring asset bubbles and credit market imperfections fail to explain how overvaluation may generate overinvestment, excessive entry and negative earnings. As I shall argue, these have been important aspects of the *dotcom* bubble of the late

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[6]In the presence of externalities to capital accumulation, such a crowding-out effect can be growth-impairing and welfare-reducing. This is the main message of the models of Grossman and Yanagawa [1993] and King and Ferguson [1993].

1990s, but could be also found in other historical episodes, such as the British *railway mania* of the 1840s (see section 3).

This paper also speaks to the literature describing firm and investor behavior during the British railway *mania* of the 1840s (Campbell and Turner [2010, 2015], Odlyzko [2010]) and during *dotcom* bubble of the late 1990s (such as Brunnemeier and Nagel [2004], Griffin et al. [2011], Pastor and Veronesi [2009] and Campello and Graham [2013]).

Finally, this paper is related to the vast literature studying the cyclical properties of markups, which includes contributions by Rotemberg and Saloner [1986], Chatterjee, Cooper and Ravikumar [1993], Chevalier and Scharfstein [1996] and Gilchrist et al. [2017]. By establishing a connection between product market competition and the interest rate, the model can also shed light on recent macroeconomic trends. There are signs suggesting that market power has been increasing in the US since the 1980s. For example, De Loecker and Eeckhout [2017] and Hall [2018] document an increase in price-cost markups in the US economy. Such an increase in markups has been accompanied by a decline in business dynamism, particularly evident in a secular decline in the startup rate and a greater concentration of activity and employment in larger and older firms (Decker et al. [2014]). All these trends have coincided with a persistent decline in real interest rates, which have even become negative in recent years. Even though there may be multiple forces contributing to the decline of interest rates, the model presented in this paper suggests that it can be linked to the increase in market power.

2 The Model

In this section, I describe the baseline model. It is built upon the popular overlapping generations model by Diamond [1965]. I depart however from Diamond's seminal contribution by introducing imperfect competition in goods markets. I then illustrate how rational asset bubbles (when attached to firms' values) can reduce market power. I also discuss how imperfect competition depresses the equilibrium interest rate, thus relaxing the conditions for the emergence of rational asset bubbles.

2.1 The Setup

Demographics Time is discrete and indexed by t. The economy is populated by infinitely many overlapping generations. There are two classes of individuals, the *workers* and the *entrepreneurs*. Within each group, a new mass m of agents is born every period and becomes immediately active. All active individuals are subject to a retirement shock, which occurs with constant probability δ and is independent of age. Individuals become inactive upon receiving the retirement shock and die the period after. I normalize $m = \dfrac{\delta}{1-\delta}$ so that each class has a unit mass of active members.
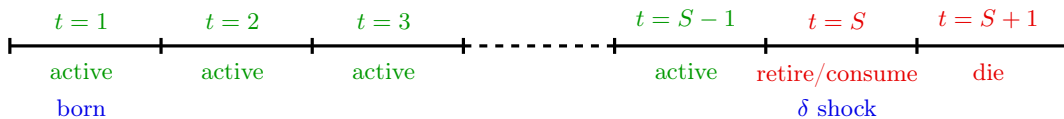
Labor Supply and Entrepreneurship   Workers are endowed with a unit labor endowment during their active life, which they supply inelastically. Labor is paid the competitive wage rate W_t.

Entrepreneurs run firms in the business sector and can make profits (as we shall see below). Firms are traded in the stock market and may contain a bubble component.

Preferences and Savings Decisions   Individuals are risk neutral and have a single consumption opportunity in their period of retirement. All active agents will therefore save the totality of their wealth. The economy contains two savings options. On the one hand, agents can buy securities in financial markets (corporate bonds and stocks). Holding a financial security between t and $t+1$ yields a gross expected return R_{t+1}. I will refer to R_{t+1} as the interest rate. On the other hand, they have access to a storage technology with return $r < 1$. Storage must be seen as an inefficient investment opportunity that may nevertheless be used in equilibrium when interest rates are low. It will impose a lower bound on the equilibrium interest rate R_{t+1}.

Since individuals face a constant retirement probability δ (independent of age), aggregate savings will consist of a fraction $1 - \delta$ of the economy's total assets A_t. The aggregate demand for securities is therefore given by

$$
D_t^S = \begin{cases}
= (1 - \delta) A_t & \text{if} \quad R_{t+1} > r \\[2em]
\in [0, (1 - \delta) A_t] & \text{if} \quad R_{t+1} = r
\end{cases}
\tag{1}
$$

This equation says that when the equilibrium interest rate R_{t+1} is above r, storage is not used and all savings will be invested in financial markets. When the interest rate R_{t+1} equals r, savers will be indifferent between purchasing financial securities and storing their assets.

Technology   There is a final good Y_t, which is a CES composite of different intermediate varieties:

$$
Y_t = \left(\int_0^1 y_{i,t}^\rho \, di \right)^{\frac{1}{\rho}}
\tag{2}
$$

where $y_{i,t}$ is the quantity of variety $i \in [0,1]$, $0 < \rho < 1$ and $\sigma \equiv \dfrac{1}{1 - \rho}$ is the elasticity of substitution. The parameter ρ measures the degree of substitutability across varieties. The final good is produced in a competitive sector and will be used as the *numeraire*.

Entrepreneurs can run one or more firms in the intermediate goods sector. In particular, entrepreneur $j \in [0,1]$ can produce variety $i \in [0,1]$ by combining capital $k_{i,t}^j$ and labor $l_{i,t}^j$ through a Cobb-Douglas technology

$$
y_{i,t}^j = \pi_i^j \left(k_{i,t}^j \right)^\alpha \left(l_{i,t}^j \right)^{1-\alpha}
\tag{3}
$$

where π_i^j is j's time-invariant idiosyncratic productivity in industry i. I will assume that productivities are dis-

tributed in a parsimonious way

$$
\pi_i^j = \begin{cases} 1 & \text{if} \quad j = i \quad \text{(leader)} \\ \\ \pi \leq 1 & \text{if} \quad j \neq i \quad \text{(followers)} \end{cases} \tag{4}
$$

Therefore, each variety $i \in [0,1]$ can be produced either with productivity $\pi_i^j = 1$ by entrepreneur $j = i$ or with productivity $\pi \leq 1$ by all the others. I will refer to entrepreneur j as the *leader* of industry $i = j$ and to all other entrepreneurs $j \neq i$ as the *followers*. Note that every entrepreneur is a leader in one industry and a follower in all the others.[7] The fact that there is only one firm with access to the most productive technology (in every industry) will create room for imperfect competition.

Labor is hired at the competitive wage W_t. The production of one unit of capital requires one unit of the final good. Furthermore, I shall assume that capital needs to be invested one period ahead and fully depreciates in production. Each unit of capital therefore costs R_t. Given these assumptions, entrepreneur j can produce good i with constant marginal cost $\dfrac{\theta_t}{\pi_i^j}$, where

$$
\theta_t \equiv \left(\frac{R_t}{\alpha} \right)^\alpha \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha}
$$

is the marginal cost function for a Cobb-Douglas technology with unit productivity. In addition to all variable costs, the production of each variety entails a fixed production cost $c_f \geq 0$ per period. Such a cost corresponds to an utility loss associated with managerial tasks.

Entrepreneurs will play a Cournot game, i.e. all firms that decide to incur the fixed cost c_f will compete via quantities. Firms' strategic decisions may however depend on financial market sentiment, in particular on the possibility of issuing bubbly stocks. Therefore, before characterizing the equilibrium in the intermediate goods sector, I will describe the workings of financial markets.

Financial Markets To finance investment, entrepreneurs may issue one period corporate bonds. Any corporate bond issued in period t must deliver a gross return R_{t+1} in period $t+1$.

Entrepreneurs can also issue stocks. Stocks are another financial instrument that can be used to raise funds in financial markets. I shall assume that, contrarily to corporate bonds, stocks do not deliver any cash flow or dividend. This assumption is made for clarity: a stock that is traded at a positive price must be a pure bubble or pyramid scheme.

Bubbles will be fully rational. This fact means that investors will purchase stocks in the expectation that their price appreciates in the future, at a rate that is not lower than the interest rate R_{t+1}. But what determines the price of a stock? To answer this equation, let B_{t+1}^j be the bubble attached to a particular firm j at time $t+1$. Such abubble includes the value of all stocks issued by firm j up to t, as well as the value of any new stock issued at

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[7]I will assume that when the leader of one industry retires, he is immediately replaced by a new entrepreneur with the same productivity.

$t + 1$. Let b_{t+1}^j represent the value of the new stocks issued at $t + 1$. I will assume that when entrepreneur j retires, his firm is liquidated and stops being traded. Therefore, any positive bubble that was attached to the firm in the past will crash. Given this assumption, we can write the time $t + 1$ bubble of firm j as

$$B_{t+1}^j = \begin{cases} \dfrac{R_{t+1}}{1 - \delta} B_t^j + b_{t+1}^j & \text{if } j \text{ is active} \\ \\ 0 & \text{if } j \text{ retires} \end{cases} \tag{5}$$

That is, when entrepreneur j retires (which happens with probability δ), the stock market value of his company becomes zero. If the entrepreneur remains active (which happens with probability $1 - \delta$), the value of the firm's stocks at $t + 1$ has two components. The first component $\left(\dfrac{R_{t+1}}{1 - \delta} B_t^j \right)$ is the $t + 1$ value of all stocks that were already traded at t. The return $\dfrac{R_{t+1}}{1 - \delta}$ compensates for the fact that with probability δ the firm could have been liquidated. The second component $\left(b_{t+1}^j \right)$ is the value of all new stocks issued at time $t + 1$. Note that equation (5) implies that

$$E_t \left\{ B_{t+1}^j \right\} = R_{t+1} B_t^j + E_t \left\{ b_{t+1}^j \right\}$$

If we iterate this equation forward, we obtain an equation determining the stock market value of firm j at time t

$$B_t^j = \lim_{k \to \infty} \frac{E_t \left\{ B_{t+k}^j \right\}}{\prod_{j=1}^k R_{t+j}} - \sum_{k=1}^\infty \frac{E_t \left\{ b_{t+k}^j \right\}}{\prod_{j=1}^k R_{t+j}} \tag{6}$$

As equation (6) makes it clear, the time t bubble of firm j can be written as the difference between two components: the expected present value of the bubble that the firm will contain in the infinite future minus the expected present value of all new stocks or bubbles that will still be issued in the future.

Note that when a firm issues a new stock or bubble b_t^j it will be effectively appropriating a rent or subsidy, which is provided by the stock market. The existence of such rent or subsidy can affect firms' entry and production decisions, as we shall see below.

2.2 Equilibrium in the Intermediate Goods Sector

The inverse demand for intermediate good i is given by

$$p_{i,t} = \left(\frac{Y_t}{y_{i,t}} \right)^{1 - \rho} \tag{7}$$

I will assume that firms compete a la Cournot: all firms that decide to enter (thus incurring the fixed cost c_f) will simultaneously announce quantities, taking the output of the other competitors as given. Furthermore, I will assume that firms make sequential entry decisions in reverse order of productivity. This assumption ensures that the

leader will produce whenever there exists an equilibrium where he can profitably do so.[8] I will start by describing the industry equilibrium when there are no bubbles.

2.2.1 Bubbleless equilibrium

If entrepreneur j decides to produce in industry i, he will choose the amount of output $y_{i,t}^j$ that maximizes his profits given the output of his competitors. Specifically, he solves

$$\max_{y_{i,t}^j} \quad \left(p_{i,t} - \frac{\theta_t}{\pi_i^j} \right) y_{i,t}^j \qquad \text{s.t.} \quad p_{i,t} = \left(\frac{Y_t}{y_{i,t}} \right)^{1-\rho}$$

$$y_{i,t} = y_{i,t}^j + y_{i,t}^{-j}$$

I will assume that the market conditions are such that entry is always profitable for the leader (such conditions will be derived explicitly below). When $n_{i,t} \geq 0$ followers also decide to enter, the industry's price and quantity are given by

$$p_{i,t} = \frac{n_{i,t} + \pi}{n_{i,t} + \rho} \frac{\theta_t}{\pi}$$

$$y_{i,t} = \left(\frac{n_{i,t} + \rho}{n_{i,t} + \pi} \frac{\pi}{\theta_t} \right)^{\frac{1}{1-\rho}} Y_t$$

(8)

and the market shares are equal to

$$s_{i,t}^L = \frac{n_{i,t} - (n_{i,t} - 1 + \rho)\pi}{(1 - \rho)(n_{i,t} + \pi)}$$

$$s_{i,t}^F = \frac{\pi - \rho}{(1 - \rho)(n_{i,t} + \pi)}$$

Inspecting the last two equations, we can see that the followers will produce a positive amount if and only if $\pi > \rho$. This requires the followers to be sufficiently productive so that their productivity disadvantage is not too large. If this condition is not satisfied, the followers never produce whenever the leader does. Throughout I will assume that $\pi > \rho$, so that any follower who decides to enter can compete against the leader.

Assumption. $\pi > \rho$

In the absence of fixed production costs ($c_f = 0$), this assumption is sufficient to guarantee an equilibrium where all firms produce ($n_{i,t} \to \infty$). In the presence of fixed production costs ($c_f > 0$), not all followers (if any) will

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[8]When the fixed cost c_f is non-negligible and only a limited set of firms can profitably produce, there can be multiple equilibria. Suppose for instance that demand and factor prices are such that any firm can profitably produce alone, but not if there is another competitor. In such a case, there are two possible equilibria in this model: a monopoly with the leader or a monopoly with one follower. Sequential entry in reverse order of productivity is a way to select a particular equilibrium (in this case, the equilibrium in which the leader is a monopolist). See Atkeson and Burstein [2008] for an identical assumption.

be able to profitably enter. Any follower that enters cannot incur a loss, which means that his production profits $\left(p_{i,t} - \frac{\theta_t}{\pi}\right) y_{i,t}^F$ cannot be lower than the fixed cost of production c_f. But how is the number of active followers determined? To answer this question, note that when there are $n_{i,t}$ followers producing, each one of them makes production profits

$$\Pi^F\left(n_{i,t}, \theta_t, Y_t\right) \equiv \left(p_{i,t} - \frac{\theta_t}{\pi}\right) y_{i,t}^F = \begin{cases} 0 & \text{if } n_{i,t} = 0 \\[2em] \frac{(\pi - \rho)^2}{1 - \rho} \left(n_{i,t} + \pi\right)^{-\frac{2-\rho}{1-\rho}} \left(\frac{n_{i,t} + \rho}{\theta_t}\pi\right)^{\frac{\rho}{1-\rho}} Y_t & \text{if } n_{i,t} > 0 \end{cases}$$

The function above can be shown to be decreasing in $n_{i,t}$ (see Appendix A.1.1 for a proof). The equilibrium number of followers must be such that (i) all followers that operate do not make a loss, (ii) but if an additional follower were to enter he would incur a loss. Denoting by $n_{i,t}^*$ the equilibrium number of followers, we have that

$$\left[\Pi^F\left(n_{i,t}^*, \theta_t, Y_t\right) - c_f\right]\left[\Pi^F\left(n_{i,t}^* + 1, \theta_t, Y_t\right) - c_f\right] \leq 0$$

Figure 2 below illustrates how the equilibrium number of followers $n_{i,t}^*$ is determined (given ρ, π and c_f, and for some fixed values of aggregate output Y_t and factor cost index θ_t). Each panel shows an equilibrium variable as a function of $n_{i,t}$. Panels (1) and (2) show the total industry output $y_{i,t}$ and the price $p_{i,t}$. Panels (3) and (5) show the output and production profits of the leader, whereas panels (4) and (6) show the same variables for each follower.

Not surprisingly, as the number of followers increases, total output $y_{i,t}$ expands (panel (1)) and the price $p_{i,t}$ declines (panel (2)). Note that the leader reacts to the entry of additional followers by increasing his level of output $y_{i,t}^L$ (panel 3). Such a decision is the outcome of a trade-off. On the one hand, by producing more, the leader can keep a high market share, but at the expense of a lower price $p_{i,t}$. On the other hand, by producing less, he will lose a larger fraction of the market, but will keep $p_{i,t}$ relatively high. The solution to such a trade-off can be shown to depend on two main parameters: the followers' productivity level π and the degree of substitutability between varieties ρ. In particular, when the followers are relatively unproductive (low π) and the degree of substitutability across varieties is high (high ρ), the leader will always expand in reaction to an increase in the equilibrium number of followers. Such a result is stated in the following lemma.[9]

Lemma 1. *(Leader's output $y_{i,t}^L$, Sufficient Condition) $y_{i,t}^L$ increases in the number of followers $n_{i,t}$ provided that*

$$\pi < \frac{1}{2 - \rho}$$

Proof. See Appendix A.1.2. ∎

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[9]Appendix A.1.2 shows examples in which the leader either reacts in a non-monotonic fashion to the entry of additional followers (first expanding and then contracting), or monotonically contracts as the number of followers increases.

Finally, panels (5) and (6) of Figure 2 show the production profits made by each type, which are a negative function of the number of active followers $n_{i,t}$. Panel (6) illustrates how the equilibrium number of followers is determined. Under the parameters chosen, if one follower were to enter, his production profits would not be enough to compensate for the fixed production cost c_f (represented by the grey dashed line). Therefore, no follower will enter in equilibrium and the industry will consist of a monopoly, where the leader is the only producer. We shall now see what happens when firms can issue bubbly stocks.
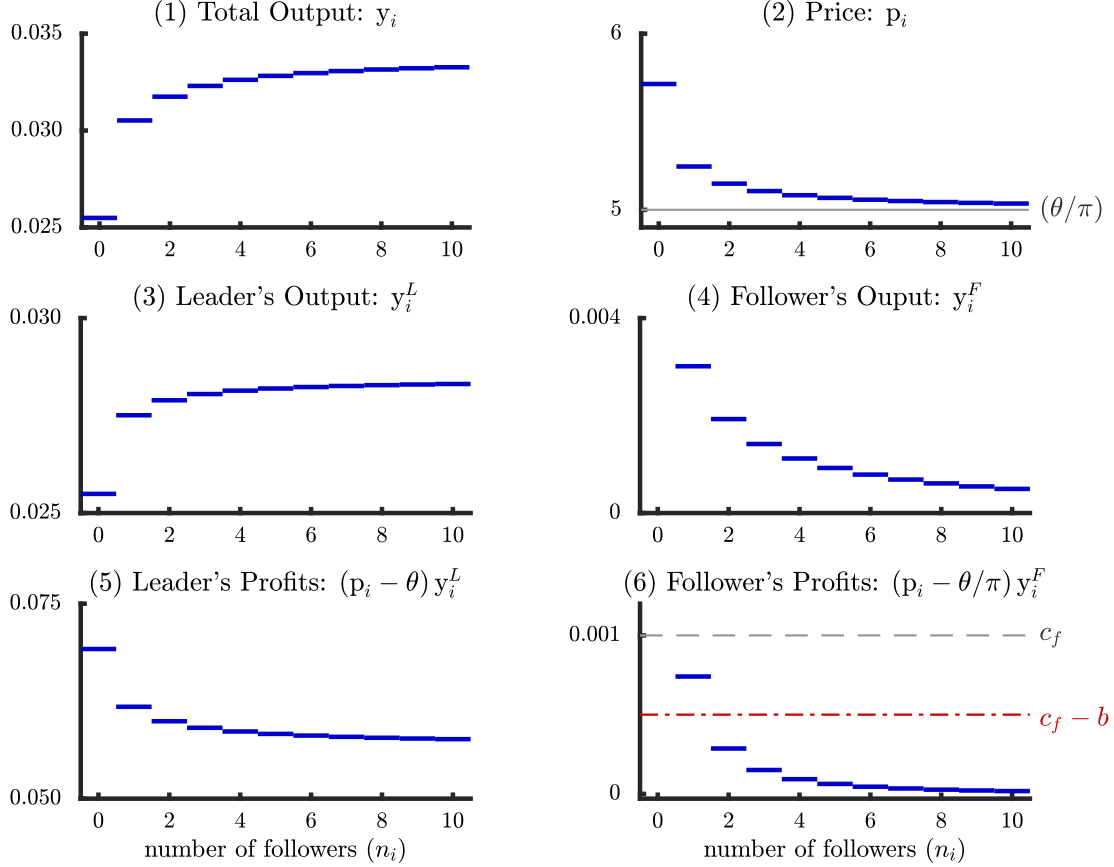
Parameters: $\rho = 0.525$, $\pi = 0.6$, $c_f = 0.001$, $Y = 1$ and $\theta = 3$

Figure 2: Industry Equilibrium

2.2.2 Bubbly equilibrium

What happens to the previous equilibrium when firms have the possibility of issuing bubbly stocks? Throughout, I will be making the assumption that firms can only issue stocks if they are active. That is, entrepreneur $j \in [0,1]$ can sell an amount $b_{i,t}^j$ of new stocks only if he enters and pays the fixed cost c_f.

I will consider two possibilities regarding investors' beliefs. First, I will assume that $b_{i,t}^j$ is fixed at the firm level (constant firm bubble). Second, I will assume that there is instead a fixed new industry bubble $b_{i,t}$ which is distributed among firms according to market shares (constant industry bubble).

Constant Firm Bubbles Suppose that firm j can issue an amount $b_{i,t}^j = b \geq 0$ of bubbly stocks. We can see the amount b as an entry subsidy: upon entering and incurring the fixed cost c_f, entrepreneurs are entitled to a rent

or subsidy that is provided by the stock market. The equilibrium number of followers will now be determined by

$$\left[\Pi^F\left(n_{i,t}^*, \theta_t, Y_t\right) - (c_f - b)\right]\left[\Pi^F\left(n_{i,t}^* + 1, \theta_t, Y_t\right) - (c_f - b)\right] \leq 0$$

Let us go back the example of Figure 2. Suppose that all active firms can issue an amount of bubbly stocks b so that $c_f - b$ takes the value represented by the red dashed line. In such a case, when there is one follower producing $(n_{i,t} = 1)$, his production profits more than compensate for the fixed cost net of the bubbly subsidy $c_f - b$. When however two followers produce $(n_{i,t} = 2)$, production profits fall short of $c_f - b$. In this example, the equilibrium will thus consist of a duopoly in which the leader and one follower produce.

Note that as the value of the firm level bubble b increases, more and more firms will be willing to enter in the market. Figure 3 represents some equilibrium variables for this industry as a function of b.

Parameters: $\rho = 0.525$, $\pi = 0.6$, $c_f = 0.001$, $Y = 1$ and $\theta = 3$
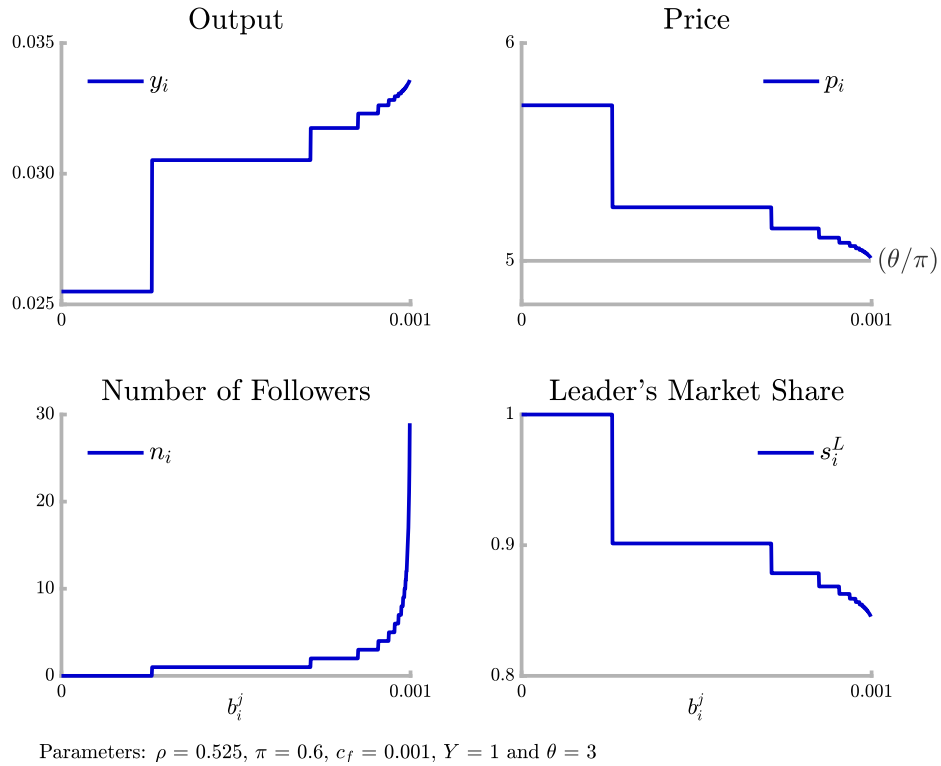
Figure 3: Industry Equilibrium with Firm Level Bubbles

When b is sufficiently low, no follower will find entry attractive and the industry remains a monopoly. However, as b rises, the followers will start entering one by one. Their entry will generate an expansion in total output $y_{i,t}$ and a decline in the good's price $p_{i,t}$.

Recall that each follower's production profits are insufficient to cover the fixed cost c_f, i.e. $\Pi^F\left(n_{i,t}, \theta_t, Y_t\right) - c_f < 0$. Therefore, by entering the market, each follower is effectively incurring a loss, which is financed by the bubbly subsidy b (panel 1 of Figure 4). The negative profits $\Pi^F\left(n_{i,t}, \theta_t, Y_t\right) - c_f$ that each follower makes can therefore be seen as the cost he needs pay to obtain the rent b.

Even though the followers will be making an operating loss, their entry will necessarily result in higher consumer

Figure 4: Industry Equilibrium with Firm Level Bubbles

welfare, as total output $y_{i,t}$ increases and the price $p_{i,t}$ decreases. To assess whether the increase in the number of followers is efficient or inefficient, we should evaluate the change in the total industry welfare or surplus

$$\Omega_{i,t} = \underbrace{\int_{p_{i,t}}^{\infty} p^{-\frac{1}{1-\rho}} Y_t \, dp}_{\text{consumer surplus}} + \underbrace{(p_{i,t} - \theta_t) \, y_{i,t}^L - c_f + n_{i,t} \left[\left(p_{i,t} - \frac{\theta_t}{\pi} \right) y_{i,t}^F - c_f \right]}_{\text{producer surplus}}$$

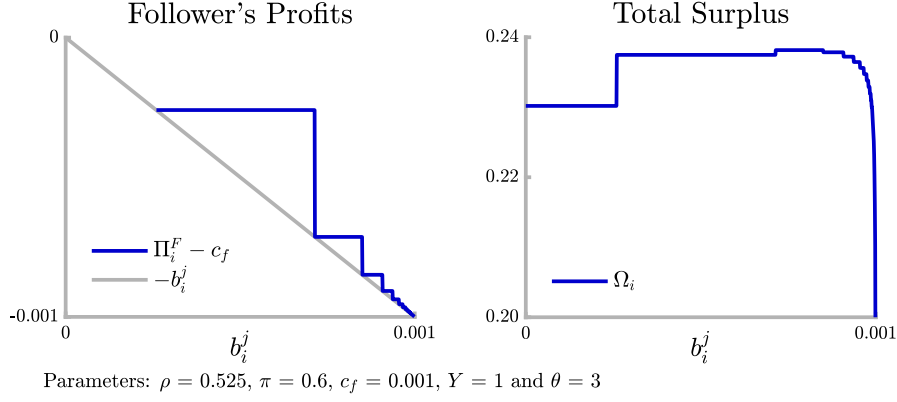This is a measure of economic efficiency and ignores the private return stemming from the issuance of bubbly stocks. The second panel of Figure 4 represents the total industry surplus as a function of the firm-level bubble subsidy b. Recall that as b increases, the number of active followers $n_{i,t}$ increases and the price $p_{i,t}$ decreases. This fact results necessarily in higher consumer welfare (the first term in the expression above), but in a lower producer surplus (the second term). When b is small (so that there are few followers producing), the increase in consumer welfare exceeds the decrease in producer surplus, so that the total industry welfare increases. Note that such positive impact on consumer welfare may come from two channels – (i) the additional output that each follower brings to the market and (ii) the reaction of the leader (who, as we have seen above, will produce more output in response to an increase in the number of followers). As the bubble b becomes large and more firms produce, the increase in consumer welfare resulting from an increase in b is outweighed by the reduction in producer surplus and total welfare declines. Given the parameters chosen, the total industry welfare (absent the bubble rents) is maximized when two followers produce.

So far, I have assumed that bubbles are fixed at the firm level. Despite being the standard assumption made in the rational bubbles literature, this hypothesis can be problematic in the current context. First, it seems unrealistic to think that a firm can sell a fixed amount of new stocks $b > 0$ even when producing nothing. Second, as $b \to c_f$, all followers would be willing to enter $(n_{i,t}^* \to \infty)$ and the value of all new bubbles being started would be infinite – an obvious impossibility.

Having these observations in mind, I consider a different process for stock market sentiment. In particular, I will assume that there is a constant industry bubble that is split among firms according to their market shares.

Constant Industry Bubbles   Suppose that, instead of emerging at the individual firm level, bubbles emerge at the industry level. In particular, assume that (i) there is a bubble with size $b_{i,t} > 0$ appearing in industry i at time t and that (ii) each entrepreneur gets a fraction of this industry bubble that is equal to his market share. According to this formulation, investors' total demand for stocks in industry i exceeds the industry's fundamental value by a fixed amount $b_{i,t}$. Furthermore, this industry bubble is distributed across firms according to their market shares, so that larger firms also get a larger share in the bubble.

This process captures one aspect of financial markets – namely the fact that valuation models are often based on multiples of revenues or market shares and not on profits.[10] The use of such valuation techniques is especially true in the case of young firms: they typically start with low or even negative profit margins, which makes it difficult to project *future* cash flows from *current* earnings. For instance, Hong, Stein and Yu [2007] provide detailed evidence that equity analysts offering valuations for Amazon in the 1997-1999 period tended to emphasize its growth path (in terms of sales) and highly disregarded operating margins. A well-known consequence of such valuation methods is that they induce firms to boost revenues or market shares, thus disregarding profit margins (Aghion and Stein [2008]). Indeed, as noted in the context of the recent Silicon Valley boom: "*With valuations based on multiples of revenue, there's ample incentive to race for growth, even at the cost of low or even negative gross margins. The many taxi apps and instant delivery services competing for attention, for example, are facing huge pressure to cut prices in the hope of outlasting the competition*".[11]

Taking this process into account, we must reformulate the firms' problem:

$$
\max_{y_{i,t}^j} \quad \left(p_{i,t} - \frac{\theta_t}{\pi_i^j} \right) y_{i,t}^j + \frac{y_{i,t}^j}{y_{i,t}} b_{i,t} \qquad \text{s.t.} \quad p_{i,t} = \left(\frac{Y_t}{y_{i,t}} \right)^{1-\rho}
$$

$$
y_{i,t} = y_{i,t}^j + y_{i,t}^{-j}
$$

This yields a first order condition for firm j

$$
p_{i,t} \left[1 - (1-\rho) s_{i,t}^j \right] + \left(1 - s_{i,t}^j \right) \frac{b_{i,t}}{y_{i,t}} = \frac{\theta_t}{\pi_i^j} \tag{9}
$$

It can be shown that the industry price that is obtained under this problem satisfies

$$
p_{i,t} = \frac{1}{n_{i,t} + \rho} \left[\theta_t \left(1 + \frac{n_{i,t}}{\pi} \right) - n_{i,t} \frac{b_{i,t}}{y_{i,t}} \right] \tag{10}
$$

Equation (10) establishes a negative relationship between the industry price $p_{i,t}$ and the size of the industry bubble $b_{i,t}$ for any positive number of followers $n_{i,t} > 0$. Indeed, it immediately follows from the previous equation that when no follower operates (i.e. $n_{i,t} = 0$), changes in the industry bubble $b_{i,t}$ have no consequence on the industry

---

[10]See for instance the textbook by Damodaran [2006], pp. 234-235.

[11]"Dotcom history is not yet repeating itself but it is starting to rhyme" (03/12/2015), Financial Times

equilibrium. The intuition is simple: when the leader is a monopolist, he has a constant market share $s_{i,t}^L = 1$ and always appropriates the entire industry bubble $b_{i,t}$. However, if at least one follower produces, the appearance of an industry bubble will increase firms' desired market shares. As a result, firms will compete more aggressively and increase output. The following proposition summarizes the main results of this process (see Appendix A.2.3 for a proof).

Proposition 1. *Suppose that the number of followers in industry i is constant and equal to $n_{i,t} \geq 1$. In such a case, as the industry bubble $b_{i,t}$ increases*

1. *All firms increase output ($\uparrow y_{i,t}^j \ \forall j$)*

2. *The leader loses market share ($\downarrow s_{i,t}^L$)*

The appearance of an industry bubble $b_{i,t}$ hence leads to an increase in total output even when the number of firms remains fixed (as long as there is at least one follower producing). A corollary of this fact is that bubbles can be expansionary even when fixed costs are negligible ($c_f = 0$) and the number of followers is infinity ($n_{i,t} \to \infty$). To see this fact, note that when we take the limit of (10) as $n_{i,t} \to \infty$ we obtain

$$p_{i,t} = \left(\frac{\theta_t}{\pi} - \frac{b_{i,t}}{y_{i,t}} \right)$$

In such a case, when there is no bubble, the price equals the followers' marginal cost $\dfrac{\theta_t}{\pi}$. However, as an industry bubble appears, firms will fight for market shares and hence compete more aggressively. As a consequence, output will grow and the price will fall short of the followers' marginal cost of production.

Proposition 1 pertained to an infinitesimal change in the industry bubble $b_{i,t}$, holding the number of followers $n_{i,t}$ fixed. However, as $b_{i,t}$ grows the number of followers deciding to enter will also increase. Figure 5 below shows some equilibrium variables as a function of the industry bubble process described in this section (all parameter values as the same chosen for the previous pictures).

Note that, as they fight for market shares, firms may even find it optimal to charge a price below their marginal cost of production. Indeed, as shown in Figure 5, for sufficiently large values of the industry bubble b_i, the price will fall short of the leader's marginal cost θ_t .

Figure 6 shows how the profits of the followers vary with the industry level bubble b_i as well as the total industry welfare Ω_i.[12] The total industry welfare exhibits the inverted-U shape that we have also identified under the constant firm-level bubble process.[13] In Appendix A.2.4, I provide a direct comparison between the constant

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[12]It is interesting to note that, contrarily to the constant firm bubble process analyzed above, the profits of the followers do not decrease monotonically on the total bubble size. In particular, whenever the industry bubble increases enough to trigger the entry of a new follower, each individual follower may increase his profits. This fact happens because of a reallocation of activity and profits from the leader to the followers.

[13]Note that now a larger industry bubble b_i may be associated with a reduction in total industry welfare even when there are no

Output

Price

Number of Followers

Leader's Market Share

Parameters: $\rho = 0.525$, $\pi = 0.6$, $c_f = 0.001$, $Y = 1$ and $\theta = 3$
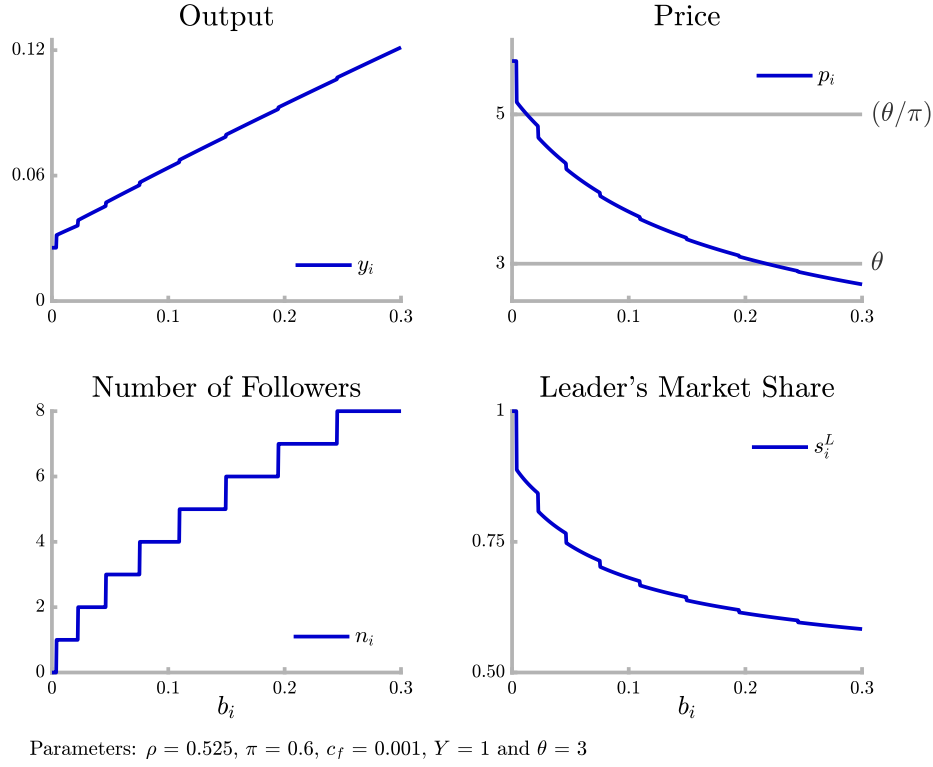
Figure 5: Industry Equilibrium with the Industry Bubble

Follower's Profits

Total Surplus

Parameters: $\rho = 0.525$, $\pi = 0.6$, $c_f = 0.001$, $Y = 1$ and $\theta = 3$
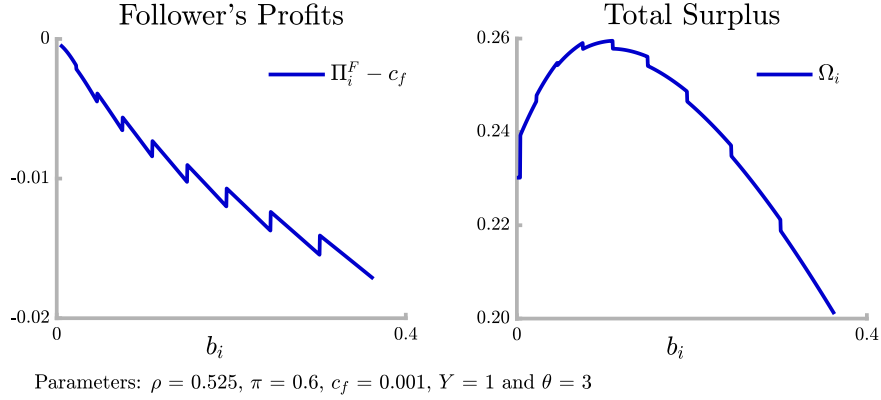
Figure 6: Industry Equilibrium with the Industry Bubble

firm bubble and the constant industry bubble processes. Having described the equilibrium of a particular industry, we can now solve for aggregate variables.

2.3   General Equilibrium

In this section, I characterize the economy in general equilibrium. I start by focusing on a static equilibrium, in which I describe aggregate output and factor prices for a given capital stock K_t (the state variable). I then characterize the equilibrium dynamics with and without bubbles.

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changes in the extensive margin. This fact can be explained by the reallocation of market shares from the leader to the followers (Proposition 1) and by the fact that firms may find it optimal to charge a price below their marginal cost of production.

2.3.1 Static Equilibrium

This economy can feature both symmetric and asymmetric equilibria. In a symmetric equilibrium, all industries are identical and have the same number of followers. In an asymmetric equilibrium, on the other hand, different industries may have a different number of followers. I first characterize a symmetric equilibrium.

Symmetric Equilibrium   Suppose all industries are identical and have one leader and $n_t \geq 0$ followers. In such a case, each variety will be characterized by the same level of output $y_{i,t} = y_t = Y_t$ and hence the same price $p_{i,t} = 1$. Given a fixed aggregate labor supply $L_t = 1$, we can write aggregate output Y_t as a function of the aggregate capital stock as

$$Y_t = \varphi(n_t) \; K_t^\alpha$$

The term $\varphi(n_t)$ can be seen as a measure of aggregate TFP and is equal to

$$\varphi(n_t) = \frac{\pi(1-\rho)(n_t + \pi)}{\pi[(2-\pi)n_t + (1-\rho)\pi] - \rho n_t}$$

It can be shown to be a negative function of n_t when $\pi < 1$ - that is, when the followers are less productive than the leader, the higher n_t, the lower is aggregate TFP. The following lemma summarizes the behavior of $\varphi(n_t)$.

Lemma 2. *(Aggregate TFP) Let $\varphi(n_t)$ denote aggregate TFP in a symmetric equilibrium in which all industries have one leader and n_t followers. We have that*

1. $\varphi(n_t + 1) < \varphi(n_t)$ if and only if $\pi < 1$

2. $\varphi(0) = 1$

3. $\lim_{n_t \to \infty} \varphi(n_t) = \dfrac{\pi(1-\rho)}{\pi(2-\pi) - \rho}$

Note that aggregate TFP is always above π. Recall that, even when there are infinitely many followers ($n_t \to \infty$), the leaders always have a non-negligible market share (provided that $\pi < 1$).

Although the aggregate labor supply is fixed and equal to one, the aggregate supply of capital will depend on the interest rate. To describe the law of motion of this economy, it is therefore necessary to determine factor prices. Recall that in a symmetric equilibrium, all varieties will have the same price $p_{i,t} = 1$. We can hence use (8) to determine the aggregate factor cost index

$$\theta(n_t) = \frac{n_t + \rho}{n_t + \pi}\pi$$

The factor cost index can be shown to increase in the number of active followers (n_t) as stated in the following lemma.

Lemma 3. *(Factor Cost Index) Let $\theta(n_t)$ denote the factor cost index in a symmetric equilibrium with n_t followers*

in every industry. We have that

$$\theta(n_t + 1) > \theta(n_t)$$

Lemma 3 states an important result of the model – an increase in the number of active firms will always be associated with higher factor costs. This result is intuitive – the higher is the number of firms operating in every industry, the higher is the level of competition and so are factor demand and factor costs. We can also determine the aggregate factor and profit shares. Let $\sigma(n_t)$ denote the aggregate factor share, i.e. the ratio of aggregate labor and capital payments to total output

$$\sigma(n_t) := \frac{W_t + R_t K_t}{Y_t}$$

Note that the aggregate profit share (exclusive of fixed production costs) is given by $1 - \sigma(n_t)$. It is easy to see that the aggregate factor share $\sigma(n_t)$ satisfies

$$\sigma(n_t) = \frac{\theta(n_t)}{\varphi(n_t)}$$

Since $\theta(n_t)$ increases in n_t and $\varphi(n_t)$ decreases in n_t, it immediately follows that $\sigma(n_t)$ is a positive function of the number of followers n_t. This is again an intuitive result – as the number of firms increases in every industry, competition becomes more intense, so that the profit share decreases and the factor share increases. The following lemma summarizes the behavior of $\sigma(n_t)$.

Lemma 4. *(Aggregate Factor Share) Let $\sigma(n_t)$ denote the aggregate factor share in a symmetric equilibrium in which there are n_t followers per industry. We have that*

1. $\sigma(n_t + 1) > \sigma(n_t)$

2. $\sigma(0) = \rho$

3. $\lim_{n_t \to \infty} \sigma(n_t) = \dfrac{\pi(2 - \pi) - \rho}{1 - \rho}$

A corollary of the above lemma is that the minimum factor share is obtained when all sectors are a monopoly. In such a case, we have that $\sigma(n_t) = \rho$, implying a profit share (exclusive of fixed costs) equal to $1 - \rho$.[14] Second, the aggregate factor share is always lower than one, provided that $\pi < 1$. In other words, if the leaders have a productivity advantage over the followers, we have $\sigma(n_t) < 1$ even when $n_t \to \infty$. Note that when there are infinitely many followers, the price of each variety will coincide with their marginal cost of production, but will still be above the marginal cost of the leaders. Note that $\sigma(n_t) = 1$ is only obtained as $n_t \to \infty$ and when $\pi = 1$. In such a case, we achieve a situation of perfect competition – there are infinitely many identical firms, all of which make zero production profits.

---

[14]This is a well-known result: the profit share in an equilibrium in which all industries are a monopoly is equal to the inverse of the elasticity of substitution, which in this model is given by $\dfrac{1}{1 - \rho}$.

Having defined the aggregate factor share, we can determine factor prices

$$W_t = (1 - \alpha) \, \sigma \, (n_t) \, \varphi \, (n_t) \, K_t^{\alpha}$$

$$(11)$$

$$R_t = \alpha \, \sigma \, (n_t) \, \varphi \, (n_t) \, K_t^{\alpha - 1}$$

The two equations above show again an important result of the model. Recall that $\sigma \, (n_t) \, \varphi \, (n_t) = \theta \, (n_t)$ and that $\theta \, (n_t)$ increases in n_t. Therefore, for a given capital stock K_t, the higher is the number of followers that operate in each industry, the higher are factor prices W_t and R_t. Such a positive relationship between the number of active firms and factor prices will be crucial to understand how the appearance of stock market bubbles can sustain a larger output in general equilibrium.

As equation (11) also makes it clear, the presence of imperfect competition - i.e. the fact that $\sigma \, (n_t) < 1$ - creates a wedge between factor prices and marginal products. For instance, the interest rate is lower than the marginal product of capital $\dfrac{\partial Y_t}{\partial K_t}$ whenever $\sigma \, (n_t) < 1$

$$R_t = \alpha \, \sigma \, (n_t) \, \varphi \, (n_t) \, K_t^{\alpha - 1} < \alpha \, \varphi \, (n_t) \, K_t^{\alpha - 1} = \frac{\partial Y_t}{\partial K_t}$$

As we shall see below, this fact means that the existence of market power will relax the conditions for the existence of rational asset bubbles.

To conclude, we must define the conditions under which a symmetric equilibrium with $n_t \geq 0$ followers is possible. Note that for such an equilibrium to be possible: (i) the aggregate capital stock must be sufficiently large so that none of the active firms makes a loss but (ii) aggregate capital cannot be too large, so that no additional follower has incentives to enter in any industry. These two conditions define a range of values under which the aggregate capital stock must fall for every $n_t \geq 0$, as stated in the next lemma.

Lemma 5. *(Symmetric Equilibrium) A symmetric equilibrium in which one leader and $n_t \geq 0$ followers produce in every industry is possible provided that*

$$K_t \in \begin{cases} \left[\left(\dfrac{c_f}{1 - \rho} \right)^{\frac{1}{\alpha}}, \overline{K} \, (0) \right] & \text{if } n_t = 0 \\[4mm] \left[\underline{K} \, (n_t), \overline{K} \, (n_t) \right] & \text{if } n_t \geq 1 \end{cases}$$

where the functions $\underline{K} \, (n_t)$ and $\overline{K} \, (n_t)$ are increasing in n_t and satisfy $\underline{K} \, (n_t) < \overline{K} \, (n_t) < \underline{K} \, (n_t + 1)$.

Proof. See Appendix A.3.3. ∎

Asymmetric Equilibria Whenever $K_t \in \left[\overline{K}(n_t), \underline{K}(n_t+1)\right]$ for some $n_t \geq 1$, a symmetric equilibrium with $n_t \geq 1$ followers will not be possible. In such a case, the capital stock is too large to be consistent with $n_t \geq 1$ followers per industry, but too low to sustain the existence of $n_t + 1$ followers. The equilibrium in this context will feature an asymmetry across industries – a fraction λ_t of all industries will have $n_t + 1$ followers and a fraction $1 - \lambda_t$ have n_t followers. The number λ_t of industries with $n_t + 1$ followers must be such that, in all these industries, followers do not make a loss. In other words, the fraction of industries with $n_t + 1$ followers is pinned down by a zero profit condition

$$\Pi^F(n_t + 1, \theta_t, Y_t) = c_f$$

Aggregate TFP $\varphi(n_t, \lambda_t)$, the factor cost $\theta(n_t, \lambda_t)$ and factor share $\sigma(n_t, \lambda_t)$ are all defined in Appendix A.3.4. Aggregate TFP $\varphi(n_t, \lambda_t)$ can be shown to be a negative function of both n_t and λ_t, whereas $\theta(n_t, \lambda_t)$ and $\sigma(n_t, \lambda_t)$ increase in both n_t and λ_t.[15]

Some Aggregate Variables Figure 7 shows aggregate output as a function of the aggregate capital stock. The full lines represent aggregate output under a symmetric equilibrium in which all industries have the same number of followers n_t. The dashed lines represent the *transition* regions corresponding to asymmetric equilibria. Output is not globally concave on the aggregate capital stock because, as mentioned above, aggregate TFP decreases in the number of active followers (Lemma 2).

Figure 8 shows the equilibrium interest rate as a function of the aggregate capital stock. Note that within the transition regions $\left[\overline{K}(0), \underline{K}(1)\right]$ and $\left[\overline{K}(1), \underline{K}(2)\right]$, the interest rate R_t increases in the aggregate capital stock K_t. This result can be understood from equation (11). As the number of followers increases in every industry (for instance from $n = 0$ to $n = 1$), competition gets more intense and both $\sigma(\cdot)$ and $\theta(\cdot)$ increase. Such an increase in factor shares (and factor costs) may be sufficiently strong to offset the existence of decreasing returns to capital.[16]

We have fully characterized the static equilibrium of this economy – how aggregate output and factor prices are determined give the state variable K_t. We will now characterize the dynamics of K_t.

2.3.2 Equilibrium Dynamics

Equilibrium in the credit market is obtained by equating aggregate savings $(1 - \delta)(Y_t + r \cdot S_{t-1})$ to total capital formation K_{t+1}, storage S_t and bubbly stocks B_t.

$$\underbrace{(1 - \delta)(Y_t + r \cdot S_{t-1})}_{\text{savings}} = \underbrace{K_{t+1}}_{\text{capital}} + \underbrace{S_t}_{\text{storage}} + \underbrace{B_t}_{\text{bubbles}} \tag{12}$$

---

[15]Appendix A.3.5 also characterizes the solution of the model when $K_t \leq \left(\frac{c_f}{1 - \rho}\right)^{\frac{1}{\alpha}}$.

[16]When decreasing returns are high, the interest rate can be monotonically decreasing in the aggregate capital stock. See Figure 17 in Appendix A.3.6 for an example.

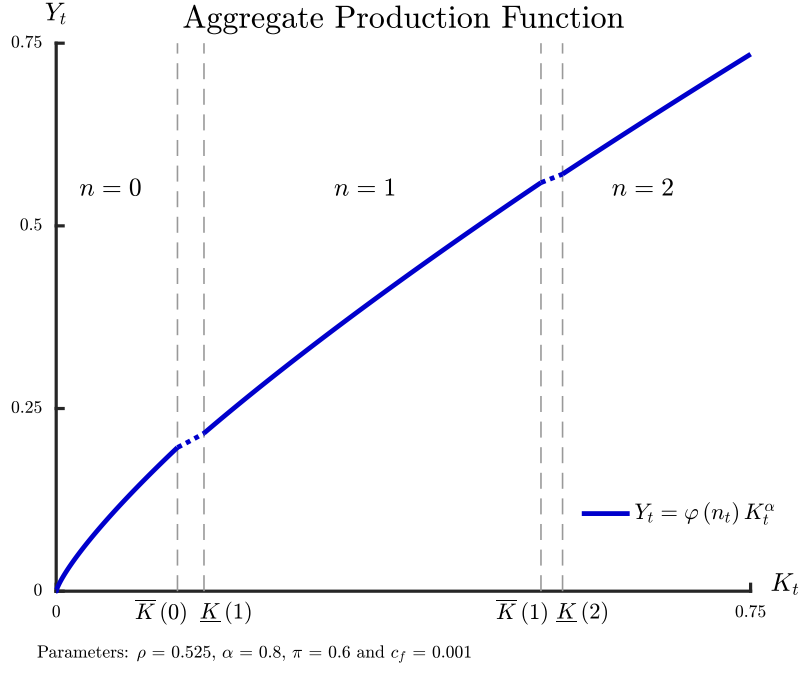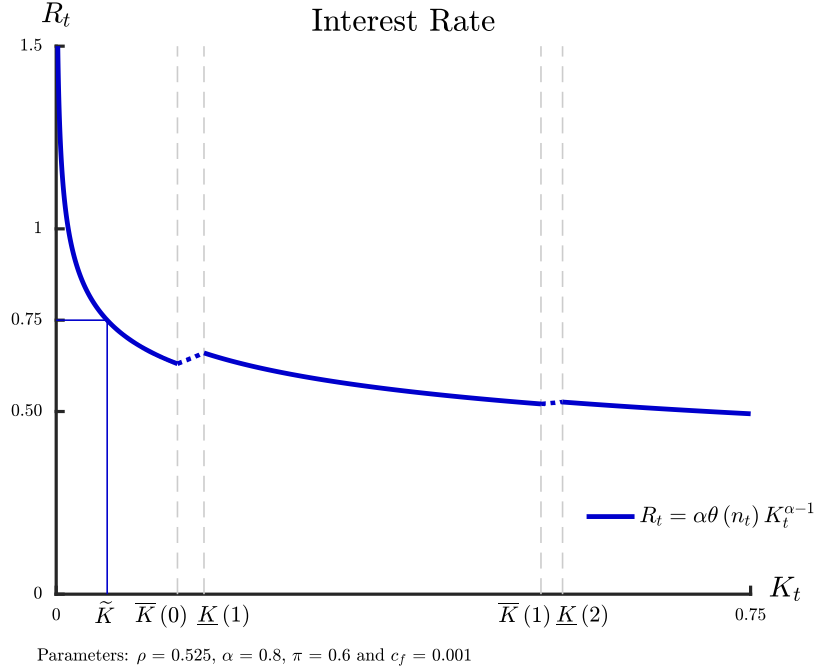Figure 7: Aggregate Production Function

Figure 8: Interest Rate

When $R_{t+1} > r$, the return on stocks and corporate bonds dominates the one on storage and $S_t = 0$. Therefore, storage will only be built when $R_{t+1} = r$. We hence have that

$$
S_t \begin{cases} = 0 & \text{if} \quad R_{t+1} > r \\ \\ \in [0, (1 - \delta)(Y_t + r \cdot S_{t-1})] & \text{if} \quad R_{t+1} = r \end{cases} \tag{13}
$$

A corollary of the previous two equations is that asset bubbles can only be expansionary when $R_{t+1} = r$. In such a case, when a bubble B_t appears/grows, it will crowd out storage S_t and can potentially increase capital K_{t+1}. When $R_{t+1} > r$, storage is not used and the appearance of a bubble B_t will necessarily crowd out capital K_{t+1}.

Finally, we can determine the law of motion of the aggregate bubble B_t as

$$B_t = R_t \cdot B_{t-1} + \underbrace{\int_i \sum_{j \in \mathcal{H}_{it}} b_{it}^j}_{\text{new bubbles}} \tag{14}$$

b_{it}^j is the new bubbly stocks issued by firm j in industry i and $\mathcal{H}_{it}$ is the set of active firms in industry i. This equation says that the value of all bubbles issued in the past B_{t-1} must provide an average return that is equal to interest rate R_t. Note that B_{t-1} hides heterogeneity across individuals stocks. Recall from equation (5) that the stocks of firms that are liquidated provide a zero return, whereas stocks of continuing firms grow at rate $\dfrac{R_t}{1-\delta}$.

Let us understand the dynamics of the model with the help of a diagram. I will consider an example in which the economy starts with no storage ($S_{t-1} = 0$) and in which there is no bubble ($B_t = 0$). In such a case, the aggregate capital stock at $t + 1$ is given by

$$K_{t+1} = \begin{cases} (1-\delta)\,\varphi(n_t)\,K_t^\alpha & \text{if } K_t \leq \left\{ \dfrac{1}{(1-\delta)\,\varphi(n_t)} \left[\dfrac{\theta(n_{t+1})\,\alpha}{r} \right]^{\frac{1}{1-\alpha}} \right\}^{\frac{1}{\alpha}} \\[3ex] \left[\dfrac{\alpha\,\theta(n_{t+1})}{r} \right]^{\frac{1}{1-\alpha}} & \text{if } K_t > \left\{ \dfrac{1}{(1-\delta)\,\varphi(n_t)} \left[\dfrac{\theta(n_{t+1})\,\alpha}{r} \right]^{\frac{1}{1-\alpha}} \right\}^{\frac{1}{\alpha}} \end{cases} \tag{15}$$

The law of motion has two regions. When the current capital stock is low enough, all savings $(1-\delta)\,\varphi(n_t)\,K_t^\alpha$ will be converted into capital; the resulting interest rate $\alpha\,\theta(n_{t+1})\,K_{t+1}^{\alpha-1}$ will not be lower than the return on storage r. When the current capital stock is sufficiently high, not all savings can be converted into capital; the capital stock is such that the resulting interest rate $\alpha\,\theta(n_{t+1})\,K_{t+1}^{\alpha-1}$ is equal to the return on storage r. Note that the number of active followers n_{t+1}, and hence the factor cost index $\theta(n_{t+1})$, are a function of K_{t+1} and shall be determined according to Lemma 5.[17]

I pick parameter values $(\rho, \alpha, \pi, c_f, \delta)$ and plot the law of motion above for two values for the return on storage: $r = 0$ (storage is never built) and $r = 0.75$ (storage may be built). The laws of motion are represented in the two panels of Figure 9 below.

Let us start with the case in which $r = 0$, so that storage is never built (panel A). In such a case, aggregate savings (which correspond to a fraction $1 - \delta$ of total output) are always converted into capital and the economy exhibits a unique steady-state K_{SS}^1. As we can see in Figure 9, $K_{SS}^1 \in \left[\underline{K}(1), \overline{K}(1) \right]$, which means that at the steady-state all industries are a duopoly with one leader and one follower.

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[17]Note that the above law of motion may not always pin down a unique value of K_{t+1}. In other words, there can be multiple equilibria: the same value of K_t can be consistent with two or more values of K_{t+1}. The intuition for such a result is explain below. See Appendix A.3.7 for an example.
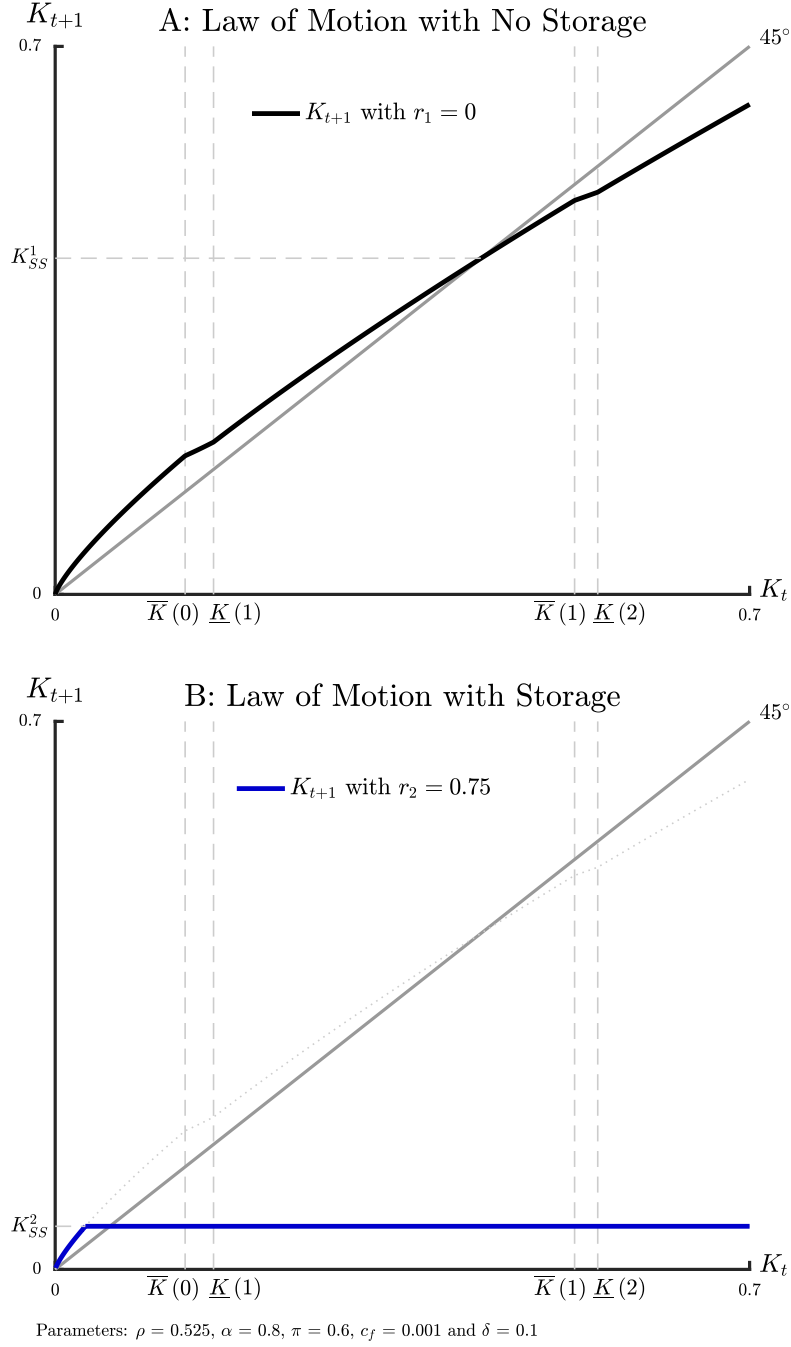
Figure 9: Law of Motion

Suppose now that the return on storage is positive and equal to $r = 0.75$ (panel B). Such a return will impose a lower bound on the equilibrium interest rate: when $R_{t+1} < 0.75$ investors will not purchase corporate bonds nor stocks and will store the totality of their savings. This means capital stock cannot surpass the level $\widetilde{K}$ indicated in Figure 8 above. As a result, the economy achieves now a lower steady-state $K_{SS}^2 < K_{SS}^1$. The new steady-state satisfies $K_{SS}^2 \in \left[\underline{K}(0), \overline{K}(0)\right]$, which means that all industries consist of a monopoly where the leader is the only producer.[18]

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[18]There is one aspect that is worth mentioning. As we can seen in Figure 8, there is a unique value of the capital stock $\widetilde{K}$ for which $R_t = r$. When aggregate savings are below $\widetilde{K}$, all savings are converted into capital; when aggregate savings are below $\widetilde{K}$, some savings

Having understood the dynamics of the economy without bubbles, we can now describe the consequences of rational asset bubbles in general equilibrium. Recall that when $r = 0$, all savings are converted into capital accumulation. Therefore, in that case, when a bubble appears, it will necessarily crowd out investment and be contractionary. The more interesting example happens when storage is built, for in that case bubbles may crowd-out storage (and not investment) and be potentially expansionary. Suppose then that we have $r = 0.75$ and that the economy is at its steady-state (where storage is built). Recall that, given the particular parameters chosen, in this steady-state all industries are a monopoly where the leader is the only producer. Suppose now that firms can issue a certain amount b of bubbly stocks. Let us start assuming that b is a constant firm level bubble. If b is arbitrarily small ($b \to 0$), the aggregate equilibrium remains unchanged: firms will have their entry subsidized by a small amount b, but such a value will be insufficient to compensate for the fixed production cost c_f. However, if b is sufficiently large, it will allow one follower to entry in each industry. Indeed, the minimum bubble that allows n followers to simultaneously enter in all industries, in an equilibrium where storage is built, is given by

$$\underline{b}(n, r) := c_f - \frac{1}{1-\rho} \left(\frac{\pi - \rho}{n + \pi} \right)^2 \varphi(n) \left[\frac{\alpha}{r} \theta(n) \right]^{\frac{\alpha}{1-\alpha}}$$

Figure 10 below shows how the issuance of bubbly stocks affects the law of motion. Assume that the economy starts at the initial steady-state K_{SS}, where all industries consist of a monopoly where the leader is the only producer. Suppose now that firms can issue an amount $b_1 \in [\underline{b}(1, r), \underline{b}(2, r))$ of bubbly stock every period. In such a case, the law of motion will have a new horizontal segment at $K'_{SS} > K^2_{SS}$. The new capital stock is defined by

$$\theta(1) \cdot \alpha \cdot \left(K'_{SS} \right)^{\alpha - 1} = r$$

This condition helps us understand how the issuance of bubbly stocks can sustain an expansion in general equilibrium. As firms can issue an amount of bubble stock $b_1 \in [\underline{b}(1, r), \underline{b}(2, r))$, one follower will be able to enter in every industry. This fact will translate into a higher demand for capital and, because capital supply is infinitely elastic when storage is built, a new equilibrium capital stock $K'_{SS} > K_{SS}$ with the same interest rate r.[19] As the amount of bubbly stocks that firms can issue increases further to $b_2 \in [\underline{b}(2, r), \underline{b}(3, r))$, there will exist two followers in every industry. The demand for capital is now even larger and an aggregate equilibrium with storage is now consistent with a larger capital stock $K''_{SS} > K'_{SS}$.

Before proceeding, note that the economy admits one steady-state where (i) storage is built and (ii) each active

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are converted into storage. This fact explains when the law of motion in panel B of Figure 9 is initially concave and then flat. However, for different values of r, there could be multiple values of the capital stock $\widetilde{K}$ for which $R_t = r$. In such a case, the law of motion will feature more than one flat region and there will be equilibrium multiplicity (and perhaps multiple steady-states). See Figure 18 in Appendix A.3.7 for an example.

[19]In other words, despite the existence of decreasing returns to capital, a higher capital stock $K'_{SS} > K_{SS}$ can be consistent with the same interest rate $R_{t+1} = r$ because of more intense competition.

firm issues an amount of bubbly stock b in every industry and period if

$$(1-\delta)\underbrace{\varphi(n)\left[\frac{\alpha\theta(n)}{r}\right]^{\frac{\alpha}{1-\alpha}}}_{Y_{SS}}-\underbrace{\left[\frac{\alpha\theta(n)}{r}\right]^{\frac{1}{1-\alpha}}}_{K_{SS}}-\underbrace{\frac{(1+n)b}{1-r}}_{B_{SS}}\geq 0 \tag{16}$$

i.e. if a fraction $(1-\delta)$ of aggregate output Y_{SS} is greater than the sum of the aggregate steady-state capital stock K_{SS} and the aggregate steady-state bubble B_{SS}, so that storage is used. Appendix A.4.2 characterizes the constant industry bubble process in general equilibrium.

Parameters: $\alpha=0.8$, $\rho=0.525$, $\pi=0.6$, $c_f=0.001$, $\delta=0.1$ and $r=0.75$
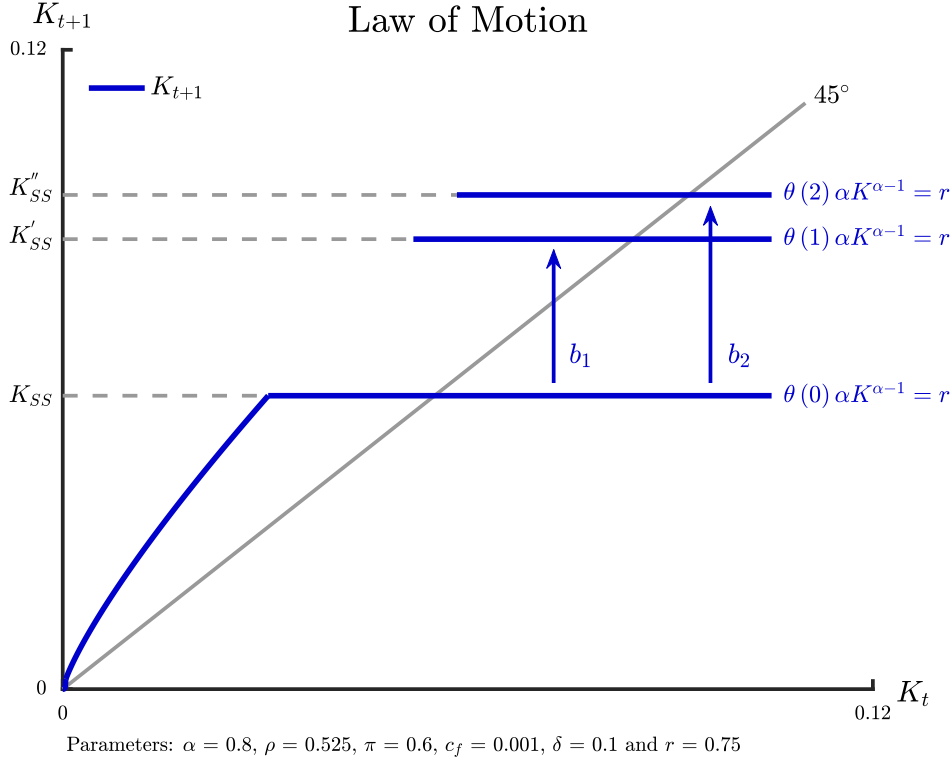
Figure 10: Law of Motion with Bubbles

The appearance of asset bubbles can lead to a reallocation of resources from storage to investment. But is such reallocation efficient or can it be inefficient? Under what conditions? I shall now address these questions.

2.3.3 The Steady-State Interest Rate, Dynamic Inefficiency and Underinvestment

Rational asset bubbles can emerge when the steady-state interest rate is below the economy's growth rate. As shown by Tirole [1985], this condition is satisfied in the standard OLG model if and only if the economy features excessive capital accumulation, so that a reduction in investment can raise consumption (i.e. if the capital accumulation is *dynamically inefficient*). In other words, interest rates are depressed in the standard OLG model if and only if the aggregate return to investment is low. If such equivalence were to hold in my model, bubbles could never lead to an efficient increase in investment. Note however that I depart from the world of Tirole [1985] by assuming imperfect competition in product markets. In this section, I derive the conditions for the existence of rational asset bubbles and ask whether the equivalence of Tirole [1985] is - or is not - verified in my model.

Let us start by characterizing the steady-state of the bubbleless equilibrium. Note that in a steady-state that features a symmetric equilibrium with n^* per industry, the interest rate R^* and the aggregate capital stock K^* are given by

$$R^* = \max\left\{\frac{\alpha\,\sigma\,(n^*)}{1-\delta}, r\right\}$$

$$K^* = \min\left\{[(1-\delta)\,\varphi\,(n^*)]^{\frac{1}{1-\alpha}}, \left[\frac{\alpha\,\theta\,(n^*)}{r}\right]^{\frac{1}{1-\alpha}}\right\}$$

To understand the previous expression, note that the existence of storage imposes a lower bound r on the equilibrium interest rate, which is associated with an upper bound $\left[\dfrac{\alpha\,\theta\,(n^*)}{r}\right]^{\frac{1}{1-\alpha}}$ on the equilibrium capital stock. The number of followers n^* will depend on the value of the fixed production cost c_f. The following proposition states the conditions for the existence of a steady-state with n^* followers per industry (with and without storage).

Proposition 2. *(Steady-State) The economy features a steady-state with n^* followers per industry*

1. **(Storage)** *in which storage is built, provided that*

$$c_f \in [\underline{\mathfrak{c}}^{ss}\,(n^*,r)\,, \overline{\mathfrak{c}}^{ss}\,(n^*,r)]$$

and that

$$\alpha\,\sigma\,(n^*) < (1-\delta)\,r$$

2. **(No Storage)** *in which storage is not built, provided that*

$$c_f \in [\underline{\mathfrak{c}}\,(n^*,r)\,, \overline{\mathfrak{c}}\,(n^*,r)]$$

and that

$$\alpha\,\sigma\,(n^*) > (1-\delta)\,r$$

The thresholds $\{\underline{\mathfrak{c}}^{ss}\,(n^*,r)\,, \overline{\mathfrak{c}}^{ss}\,(n^*,r)\}$ and $\{\underline{\mathfrak{c}}\,(n^*,r)\,, \overline{\mathfrak{c}}\,(n^*,r)\}$ are defined in Appendix A.3.8.

Proof. See Appendix A.3.8. ∎

Intuitively, storage will be used whenever the capital share in production α or the aggregate factor share $\sigma\,(n^*)$ are low (so that the equilibrium interest rate is depressed for any given capital stock K^*), when the savings rate $1-\delta$ is high (so that capital is relatively abundant) or when the return on storage r is high.[20]

When are rational asset bubbles possible in this economy? The (gross) interest rate will be below the (gross)

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[20]Note that the economy can feature at most one steady-state with no storage (which lies in the concave segment of the law of motion defined in (15)). Such a steady-state can however coexist with one (or more) steady-states with storage (see footnote 18 and Figure 18 in Appendix A.3.7 for an example).

growth rate if and only if

$$\frac{\alpha \, \sigma \, (n^*)}{1 - \delta} < 1$$

Note that this is the condition for the existence of rational asset bubbles.

Proposition 3. *(Possibility of Rational Asset Bubbles) Suppose that the economy features one steady-state with n^* followers. Rational asset bubbles can emerge if*

$$1 - \delta > \alpha \, \sigma \, (n^*)$$

In the particular case of a steady-state where all industries are a monopoly where the leader is the only producer $(n^* = 0)$, the above condition becomes $1 - \delta > \alpha \, \rho$.

Under what conditions is capital accumulation dynamically inefficient (i.e. the economy overaccumulates capital)? To answer this question, note that the steady-state capital accumulation is dynamically inefficient if the marginal product of capital is below its marginal cost of production

$$\left. \frac{\partial Y}{\partial K} \right|_{K=K^*} < 1$$

If the steady-state K^* is such that all savings are converted into capital, such a condition is verified when $1 - \delta > \alpha$ (i.e. the savings rate $1 - \delta$ exceeds the capital share in production α). If, on the other hand, storage is also used, the condition becomes $\sigma \, (n^*) > r$ (i.e. the factor share $\sigma \, (n^*)$ is greater than the return on storage r). This result is stated in the following proposition

Proposition 4. *(Overaccumulation of Capital) Suppose that the economy features one steady-state with n^* followers.*

1. If storage is not used in such a steady-state, capital accumulation is dynamically inefficient when

$$1 - \delta > \alpha$$

2. If storage is used in such a steady-state, capital accumulation is dynamically inefficient when

$$\sigma \, (n^*) > r$$

How do Propositions 3 and 4 compare? Note that when storage is used, the steady-state interest rate is necessarily below one, so that the condition in Proposition 3 is trivially satisfied. When storage is not used, 3 and 4 coincide if and only if $\sigma \, (n^*) = 1$, i.e. when there is perfect competition. Recall that the aggregate factor share is equal to one in the limit case where $c_f = 0$ (so that there are infinitely many firms producing) and $\pi = 1$ (so that the leader does not have a productivity advantage over the followers). In such a limit case, the model can be seen as a standard OLG model with perfect markets, so that the result of Tirole [1985] holds – rational asset bubbles

are possible if and only if the economy overaccumulates capital.

Finally, we shall ask under which conditions the economy features underinvestment. Note that underinvestment will arise whenever (i) storage is built despite (ii) there being no overaccumulation of capital (i.e. capital accumulation is dynamically efficient at the margin). This region is of particular interest because, in such a case, the emergence of asset bubbles can potentially lead to an efficient increase in capital accumulation. Proposition 5 below states the conditions under which the economy features a steady-state with underinvestment.

Proposition 5. *(Underinvestment) Suppose that the economy features one steady-state with n^* followers. Such a steady-state features underinvestment if*

$$\sigma\left(n^*\right) < \min\left\{\frac{1-\delta}{\alpha}r, r\right\}$$

Proof. First note that storage is used in a steady-state provided that $\frac{\alpha\,\sigma\left(n^*\right)}{1-\delta} < r$. Second, note that steady-state capital accumulation is dynamically efficient if $\alpha\,\varphi\left(n^*\right)\left(K^*\right)^{\alpha-1} > 1$. In a steady-state where storage is used, the capital stock is equal to $K^* = \left[\frac{\alpha\sigma\left(n^*\right)\varphi\left(n^*\right)}{r}\right]^{\frac{1}{1-\alpha}}$, so that this condition becomes $\sigma\left(n^*\right) < r$. ∎

This condition says that the economy features underinvestment if the steady-state factor share $\sigma\left(n^*\right)$ is low and the return on storage r is high. Note that the particular case where the steady-state consists of a symmetric monopoly across all industries ($n^* = 0$), the condition is $\rho < \min\left\{\frac{1-\delta}{\alpha}r, r\right\}$.

When the economy is in a steady-state in which the condition of Proposition 5 is satisfied, the appearance of asset bubbles can be associated with an efficient increase in investment. However, note that although such a condition is necessary, it is not sufficient. The reason is that Propositions 4 and 5 consider a marginal increase in the capital stock holding aggregate TFP, and the set of active firms, fixed. However, the emergence of asset bubbles will necessarily imply a reduction in aggregate TFP – either because the number of followers increases or because the existing followers gain market share (Proposition 1 of constant firm level bubble). Furthermore, as more firms enter, there is a necessary waste of resources associated with the duplication of fixed production costs. Proposition 5 would give a necessary and sufficient condition for asset bubbles to generate an efficient increase in investment only if the set of active firms and their market shares did not change after the appearance of a bubble. This is will happen for instance under Bertrand competition, as we shall see in Appendix B.

To assess whether asset bubbles lead or not to an efficient expansion, we should look at the variation in aggregate net output

$$\widehat{Y}_t = Y_t - \left[K_t + \left(1 + n_t\right)c_f\right]$$

i.e. output net of the stock of capital employed in production (given that I assume full depreciation) and the value of all fixed costs incurred by all active firms (all leaders and all n_t followers).[21]

Figure 11 below takes the same parameters considered above and represents some equilibrium variables as a

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[21]Recall that I have assumed no disutility from labor.

function of the firm-level bubble b. b ranges from zero to the maximum firm-level bubble consistent with a steady-state where storage is used, as given by (16). As we can see in the first panel, as b increases, more followers will be able to enter in each industry. As shown in the second panel, both aggregate output Y and total aggregate costs $K_t + (1 + n_t) c_f$ increase. The expansion will be efficient whenever aggregate output Y expands faster than total aggregate costs $K_t + (1 + n_t) c_f$. As we can see in the last panel, as the number of active followers increases from $n = 0$ to $n = 1$, net output increases. However, further increases in the number of followers will be associated with a decline in net aggregate output. Net output is therefore maximized when all industries consist of a duopoly with the leader and one follower.
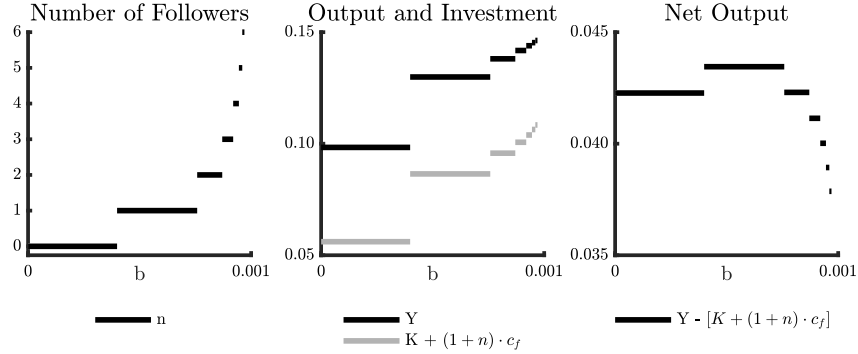
Parameters: $\alpha = 0.8$, $\rho = 0.525$, $\pi = 0.6$, $c_f = 0.001$, $\delta = 0.1$ and $r = 0.75$

Figure 11: Aggregate Efficiency under a Constant Firm Level Bubble

To understand such a non-monotonic relationship, recall that the entry of a follower brings about benefits and costs. On the positive side, the entry of each follower poses a threat to the leader (the most efficient producer), who may be forced to expand. On the negative side, the entry of a follower will necessarily entail a waste of resources, namely the duplication of fixed production costs (already incurred by the leader). Appendix A.4.2 characterizes the steady-state under a constant industry bubble.

3   Competition in Famous Bubbly Episodes

Stock market boom/bust episodes are recurring phenomena in financial history. Famous examples include the Mississippi and the South Sea bubbles of 1720, the British *railway mania* of the 1840s or more recently the *dotcom bubble* of the late 1990s. In this section, I provide a brief description of two of these episodes - the British *railway mania* of the 1840s and the *dotcom* bubble of the late 1990s - and discuss how they can be reinterpreted in light of the theory developed above.[22]

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[22]In spite of its relatively short duration, the emergence of the *South Sea Bubble* of 1720 also seems to have resulted in larger entry and competition in the British financial industry. As the prices of the *South Sea Company* soared, several joint stock companies started to adopt competing financial schemes in the London stock market. The capital attracted by some of these companies - in particular the recently founded *Royal Assurance Company* and the *London Assurance Company* - posed a direct threat to the *South Sea Company*, which was forced to seek political support from the British Parliament to preserve its *status quo*. The process culminated in the Bubble Act of June 1720, which forbade the creation of new unauthorized joint-stock companies (see Garber [1990], Neal [1990] and Harris

3.1 The British Railway *Mania* of the 1840s

British Railway Share Price Index

Figure 12: British Railway Share Price Index, 1843-1850
(Source: Campbell and Turner [2015])

The mid 1840s was a period of fast economic growth in Britain: favorable weather conditions (resulting in abundant harvests), together with historically low interest rates made Britain's GDP grow at an average of rate of 4.6% between 1843 and 1845. It was within this environment that a collective enthusiasm about railways emerged. Contrarily to the majority of other countries, where the construction of railway lines was essentially a public investment, the expansion of the British railway system was financed by private companies and individuals. This widespread excitement attracted many new investors to the stock market and triggered a boom in stock prices: between January of 1843 and October of 1845, the share prices of railway companies increased by more than 100% (Campbell and Turner [2010]).[23] At the same time, investment shot up: total investment in new railway lines authorized by the British Parliament rose by an average of £4 million per year prior to 1843, to £60 million in 1845 and £132 million in 1846 (Haacke [2004]).[24] Even though not all investments granted parliamentary authorization would ever materialize, total capital formation by railway companies reached £30 million in 1846 and £44 million and in 1847, which represented 5.2% and 7.3% of the British GDP respectively. By comparison, during the *dotcom* bubble of the late 1990s, total US investment in technological industries reached a maximum of 2.8% of GDP in the year 2000.[25] Given the magnitude of these investments, the British railway mania has been referred to as "arguably

[1994]).

[23]Individual investors financing railway projects around this time include famous scientists, intellectuals and politicians such as Charles Darwin, Charles Babbage, John Stuart Mill or Benjamin Disraeli (Odlyzko [2010]).

[24]Despite being private investment, the construction of new railway lines required parliamentary authorization. This happened because they often involved processes of land expropriation (Odlyzko [2010]).

[25]Data is from the Bureau of Economic Analysis. The industries considered include Computer and Electronic Products, Publishing Industries, Broadcasting and Telecommunications and Information and Data Processing Services.

the greatest bubble in history".[26]

Such collective enthusiasm would however cease in the middle of the decade. A recession in 1845, associated with the failure of the potato crop in Ireland, led many people to fear times of famine and scarcity. At the same time, the escalation of construction costs resulted in substantial calls for capital from railway shareholders.[27] Several projects ended up being less profitable than expected. Many commentators and newspapers (such as the recently founded *The Economist*) also started raising concerns about the potentially negative effects of such large-scale railway investments. As a result, the share prices of railway companies started to decline and between October 1845 and December 1850 the total stock market capitalization of railway companies decreased by 67% (Campbell and Turner [2010]).

The deteriorating performance of railway companies was ultimately related to an environment dominated by intense competition and, in some cases, overinvestment. Not only new lines opened in relatively unprofitable regions (serving sparsely populated areas) but there were also obvious examples of duplication of railway lines. Situations of line duplication were described (and sometimes harshly criticized) by many contemporary authors. One example, which is described in Cotterill [1849], is the railway line that connected Shrewsbury to Stafford, which opened in 1849 and was in operation until 1966. It was run by The Shropshire Union Railways and Canal Company, founded in 1846:

> "*The Shropshire Union Railway is another instance of the baneful principle [of competition]. It is a line from Shrewsbury to Stafford, joining the Trent Valley; and there being no intermediate traffic, the expenditure of 6 or 700,000l to effect this junction, appears prima facie to be lavish; because, if the Shrewsbury people wish to go to London, there is the Shrewsbury and Birmingham Railway, accommodating at the same time an immense intervening population. If the Shrewsbury people are desirous of moving north, the Shrewsbury and Chester, a line long since in operation, would give ample accommodation. The Shropshire Union to Stafford would therefore appear to be unnecessary and useless. But it is the fruit of competition.*"

Another example involving the duplication of railway lines was the connection between Birmingham and Wolverhampton, described in Martin [1849, p.37]. In 1846, the two cities were already connected by the *Grand Junction Railway* (and by water through the *Birmingham Canal*). Still, two other companies - the *London and North Western Railway* and the *Great Western Company* - were granted authorization to build two additional lines between the two cities:

> "*Three years ago, the district between Birmingham and Wolverhampton possessed a double communication for its traffic (...) by means of the Birmingham Canal and the Grand Junction Railway, each connecting the two towns. Additional Railway accommodation was, however, supposed to be desirable,*

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[26] *The Economist*, "The Beauty of Bubbles", 2008/12/18.

[27] *The Economist*, "The Railway Crisis - its Cause and its Cure", 1848/10/21.

and two Companies presented their rival plans to a Committee of the House of Commons for selection.
Both Railways are now in the course of formation, traversing a highly valuable and thickly peopled district
in parallel lines (at some points nearly touching each other)*, and each intended to terminate in separate*
stations in the centres of the two towns. At least four millions of money will thus be unprofitably sunk,
in order that three lines of railway and one canal may afford a redundant accommodation to a tract some
fourteen miles in length."

This example makes the author conclude that "*Monopoly has an ill sound: but, unless it can be proved to be incapable*
of regulation, we must prefer even monopoly to competition run mad."

The idea that the British railway *mania* was associated with an environment of increased competition is corroborated by indicators of market power. For instance, in their study of competition during the railway *mania*, Campbell and Turner [2015] found that the fraction of lines that enjoyed absolute monopoly fell from 72% in 1843 to 32% in 1850. Furthermore, the *per mile* profits of established companies (i.e. existing in 1843) fell from £1,811 to £1,231 (by 32%). Despite the lower profitability, and confirming some of the anecdotes described above, incumbent companies expanded their capacity quite dramatically: between 1843 and 1850, the milage operated by the average incumbent company grew from 36 to 153 miles.

Why did railway companies expand so quickly? What was behind "*competition run mad*", to use the words of Martin [1849, p.37]? Although different factors may have contributed to the expansion of the British railway system during the 1840s (such as a political environment highly favorable to free markets and competition), these events can be rationalized by the model presented in this paper.[28] As investors perceived railway stocks to be good financial assets (whose price was likely to appreciate in the future), vast amounts of money were poured into the British railway industry. Such high demand for railway shares may have then opened the door to the appearance of new companies and lines that were not profitable from an operating point of view. That the *mania* was a time characterized by positive sentiment and speculation in railway companies is confirmed by several contemporaneous writers. For instance, keeping his critical view on the events, Martin [1849, p.40] observes that

> "*Men and women, high and low, rich and poor, entered the destructive road of which the gates were*
> *so widely opened by the Legislature, in the expectation that all could suddenly become rich; the result to*
> *many was, that the rich were impoverished, and persons without a shilling rose on their ruin. Shopkeepers*
> *augmented their expenditure by hundreds, brokers and share speculators by thousands; 332 new schemes*
> *were brought before the public down to the 30th September, 1845, involving capital to the enormous sum*
> *of £270,959,000 of which £23,057,492 would have to be deposited with the Accountant-General before*
> *Parliament would receive application for the Acts*"

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[28]The political environment in Britain at this time was highly favorable to a private market for railways. This contrasted with other countries were governments subsidized the construction of railway line or regulated tariffs (Martin [1849, p.26]). Furthermore, there was a widespread agreement about the necessity of promoting competition between railway companies to prevent monopolies. This explains for instance why the British parliament approved many railway schemes that constituted duplication of existing lines.

Seen in this way, the expansion of the British railway system may have been commanded (at least in part) by financial market sentiment. The idea that investor sentiment may drive firms' expansion at the expense of profit margins, and ultimately provide a subsidy to consumers, was a central message of the model presented in this paper. As noted by Jackman [1916, p. 602], "*although many of the railways were not profitable to their owners in yielding large financial returns they may still have been* **beneficial to the public** *in providing for the necessities and conveniences of traffic*".
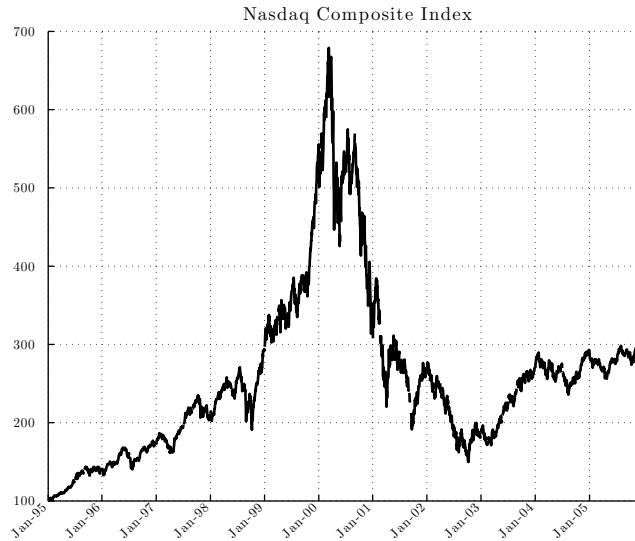
3.2 The *Dotcom* Bubble of the Late 1990s

Figure 13: The NASDAQ Composite Index, 1995-2005

Another famous stock market boom and crash would take place in the United States one century and a half later. Associated with the appearance of the internet and in a period marked by low interest rates, the NASDAQ index increased by more than 560% between January 1995 and March 2000 (Figure 13). However, as in the British railway *mania* of the 1840, such widespread enthusiasm would also cease. Concerns about the persistently negative profitability of the new internet firms and the fact that some were running out of cash marked a turning point in market sentiment. An article published in *Barron's* magazine in March 2000 sounded the alarm: "*An exclusive study conducted for Barron's by the Internet stock evaluation firm Pegasus Research International indicates that at least 51 'Net firms will burn through their cash within the next 12 months. This amounts to a quarter of the 207 companies included in our study.*" And it added "*It's no secret that most Internet companies continue to be money-burners. Of the companies in the Pegasus survey, 74% had negative cash flows. For many, there seems to be little realistic hope of profits in the near term.*" A natural question therefore emerged: "*When will the Internet Bubble burst?*".[29] The downturn would start that very same month: between March 2000 and October 2002, the NASDAQ index decreased by 77%.

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[29]Jack Willoughby, "Burning Up; Warning: Internet companies are running out of cash - fast", *Barron's*, March 20, 2000

Behind the poor performance of so many *dotcom* firms was a search for rapid growth involving aggressive commercial practices - such as extremely low penetration prices, overspending in advertising and excess capacity - and which resulted in low levels of profitability or even extensive losses.[30] For instance, many new companies offered their services at unprofitably low prices or even for free. This was, for instance, common among delivery companies. *Kozmo.com* and *UrbanFetch* were two such examples – offering one-hour delivery services of books, videos, food and other goods totally for free. Many products would even be sold at a discount, gifts were sometimes included and tips were not accepted. None of them survived the stock market crash in 2000. The online music industry also observed many of these practices, with companies such as *CDNow.com*, *Riffage.com* or *Napster* offering downloads or peer-to-peer sharing of music for free.[31] Another example is the software company SunMicrosystems, which decided to enter the office suite market (largely dominated by Microsoft Office) with a software that was made available completely for free (this example is reviewed in more detail below). The pressure for growth was in some cases so high that some companies would actually pay customers to use their services. One well-known example is the advertising company *AllAdvantage.com* (launched in 1999), which has made famous the slogan "*Get Paid to Surf the Web*". Users of *AllAdvantage.com* needed to download a viewbar that displayed advertisements at the bottom of their screens and would be paid $0.5 per each hour logged. Furthermore, members could also invite friends (without any limit) and would receive an additional $0.1 for every hour that person was active. In the first quarter of the year 2000 (which coincided with the peak of the bubble) *AllAdvantage.com* paid a total of $40 million to its members, leading to a loss of $66 million. It also did not survive the market crash and ceased its operations in that same year. Companies that engaged in similar practices include Spedia, Click-Rebates, Jotter Technologies, Radiofreecash and Adsavers.com (Haacke [2004]).

These business strategies were often justified by a first-mover advantage type of argument – most internet businesses were understood to be natural monopolies, where only one firm could ultimately survive. Hence the search for rapid growth and the "*get big fast*" or "*get large or get lost*" mottos. However, it is important to note that such extreme commercial practices were also incited by financial markets. As already mentioned in Section 1, the fact that valuation metrics were often focused on revenue targets or market shares created incentives for rapid growth at the expense of profits (Aghion and Stein [2008]). Indeed, venture capitalists and company executives explicitly admitted their strategies were influenced by financial market sentiment. For instance, Michael Moritz - founder of Sequoia Capital, a venture capital firm that was an initial funder of Yahoo! - admitted in an interview that "*The world was rewarding us for raising $250 million and penalizing [us for] raising $25 million. Daring to be great overweighted being cautious*".[32] In a similar vein, eToys' founder and CEO Toby Lenk admitted that "*It was the whole land-grab mentality. Grow, grow, grow. Grab market share and worry about the rest later. When you're in that cycle, and less capable people are doing I.P.O.'s, it's like an arms race. If you turn down the gun and put*

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[30]See Haacke [2004] and Razi, Siddiqui and Tarn [2004] for an analysis of some of the biggest *dotcom* failures.

[31]See Haacke [2004], p.91 and Razi, Siddiqui and Tarn [2004]

[32]See Haacke [2004], p.108

it on the table, all you're doing is letting other people pick it up and shoot you. I made the decisions and I take full responsibility. But there were a lot of amazing forces at work."[33] Like many other *dotcoms*, eToys would not survive the stock market crash in 2000. Toby Lenk recognizes that the attempt to grow too fast was one of the main reasons behind the failure of eToys: *"We had the capacity for $500 million in revenue but came to a stop at $200 million. That's hard to survive"*.

It is therefore interesting to note that, as in the British railway *mania* 150 years before, the NASDAQ boom of the late 1990s was also associated with rising competitive pressures in product markets, and with situations of excessive investment and low (or even negative) profit margins that became unsustainable once market sentiment reversed. As argued by Varian: "*the driving force behind the rise and fall of the Nasdaq was simple competition.* [...] *in 1999 there was no fundamental scarcity of new business models for dot-coms. The result was an intensely competitive environment, where it has been extremely difficult to make money.*"[34]

However, even if lacking market expertise and in many cases investing beyond reasonable levels, many of the new companies posed a competitive threat to incumbents. I next review some examples.

Sun Microsystems and Microsoft   One significant example in this category is the one involving SunMicrosystems and Microsoft, which is described in Varian [2003]. Back in 1999 when the *dotcom* bubble was about to reach its peak, Sun Microsystems decided to enter the office suite market, which was largely dominated by Microsoft Office. It decided to launch a new office suite called StarOffice and to make it available for free. Besides releasing the software at zero price, SunMicrosystems also promised to make its source code, file formats, and protocols free. This move was seen at that time as a clear attack on Microsoft's dominant position in the market: *"Many in the industry view Sun's move as a direct assault on Microsoft's second most lucrative monopoly"*.[35] However, Sun would be severely hit by the stock market crash (its stock price plunged from $63.4 in 8/31/2000 to $3.28 in 11/12/2002).

The threats posed by companies such as SunMicrosystems were recognized by Microsoft in its annual reports. For instance, the 2000 report states that "*Rapid change, uncertainty due to new and emerging technologies, and fierce competition characterize the software industry, which means that Microsoft's market position is always at risk. "Open source" software [...] are current examples of the rapid pace of change and intensifying competition. [...] Competing operating systems, platforms, and products may gain popularity with customers, computer manufacturers, and developers, reducing Microsoft's future revenue*" [Annual Report, 2000, p. 16].

Microsoft also anticipated the necessity to reduce the price of some of its products*: "The competitive factors described above may require Microsoft to lower product prices to meet competition, reducing the Company's net income*" [Annual Report, 2000, p. 17]; and to increase its R&D expenditure significantly *"It is anticipated that investments in research and development will increase over historical spending levels [...] Significant revenue from*

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[33]"How to Lose $850 Million – And Not Really Care" (06/09/2002), retrieved from http://www.nytimes.com/2002/06/09/magazine/how-to-lose-850-million-and-not-really-care.html

[34]Hal Varian, "Economic Scene: Comparing Nasdaq and Tulips Unfair to Flowers", The New York Times, 2001/02/08.

[35]Joe Barr, "Is Sun's StarOffice a Microsoft Killer?", 10/08/1999, CNN.com.

these product opportunities may not be achieved for a number of years, if at all" [Annual Report, 2000, p. 16].

eToys and Toys"R"Us The retail market for toys experienced considerable action in the late 1990s. Several firms such as eToys, Toysmart, Toytime and Red Rocket appeared as online toy retailers, but went bankrupt in the years 2000 and 2001 as stock prices started to decline. The case of eToys was particularly impressive: it was established in 1997, had its IPO in 1999 and in the same year reached a market capitalization of 8 billion dollars. This value was 33% larger than that of the market leader Toys"R"Us, a well-known company, much larger in terms of size and profitability (see Table 1).

Firm	Market Value	Sales	Profits
Toys "R" Us	$ 6 billion	$ 11,200 million	$ 376 million
eToys	$ 8 billion	$ 30 million	$ -28.6 million

Table 1: Sales and profits refer to the fiscal year 1998, whereas market value refers to 1999

Despite their short existence, the newly founded companies posed a serious competitive threat to Toys"R"Us, which was forced to enter the online market. After a series of unsuccessful experiences with its own website (toysrus.com), it then started a 10-year partnership with Amazon.com in the year 2000. According to the agreement Toys"R"Us was to be Amazon's exclusive supplier of toys, games and baby products.

The case of eToys is presented by Shiller [2000] as an example of a clear market inefficiency: it reached a market capitalization greater than the purportedly more efficient firm (Toys"R"Us), but went bankrupt immediately after. But even if one agrees that eToys lacked expertise in the toy market and was a relatively inefficient firm, the above conclusion is still unwarranted. It crucially ignores the fact that Toys"R"Us was forced to enter the online market (and hence to expand) as a strategic response to the entry of eToys and all the other competitors. Seen in this way, the bubble attached to eToys had a positive side effect: it increased competition and forced the market leader to expand.

GE and the "Destroy Your Business" Strategy The strategic reaction of Toys"R"Us was common among many large, well-established corporations. One well-known example is the "Destroy Your Business" program launched by GE's CEO Jack Welch in 1999. Welch asked all GE's managers to think of possible ways in which Internet startups could challenge their market leadership in different businesses and to adopt effective strategies to avoid such scenarios. The process was focused on adopting the necessary innovations before a new *dotcom* company appeared and took advantage of such an opportunity. For instance, GE Plastics (a specialized supplier of plastics, established in 1973 as a division of GE), decided to enter the online market in 1997. As part of the "Destroy Your Business" program, GE Plastics e-commerce manager Gerry Podesta and his team decide to equip their website with new tools and functionalities. They got inspiration from car manufacturers' websites, which were developing

configuration tools that allowed consumers to customize their cars. A similar scheme was then introduced in the website GE Plastics, allowing potential customers (such as engineers from manufacturing plants) to design their products online, indicating different materials that could be used, their characteristics and cost.

We can also mention the example of several other GE divisions, such as GE Transportation, GE Power Systems, GE Appliances or GE Medical Systems. GE Medical Systems - a manufacturer of diagnostic imaging systems such as CAT scanners and mammography equipment - launched a platform called iCenter as part of the "Destroy Your Business" initiative. This was an online system designed to monitor GE customers' equipment, collect data and provide each customer with information on his relative performance and suggestions on how to improve it. GE Appliances also started using the internet to sell its products. Appliances were traditionally sold through retail stores, but GE feared that such a model could be challenged with the emergence of new internet retailers (which could give preference to appliances from alternative brands). It then developed a point-of-sale system placed in traditional retail stores where customers could make online orders. Customers could also schedule an appointment to have the items delivered and installed at their convenience. This way, consumers would benefit from the advice of retailers while the goods would be sent directly into their hands (allowing stores to have reduced inventories). In 2000 GE Appliances reported that 45% of its sales took place on the internet.[36]

4   Conclusion

Financial history shows that stock market boom/bust episodes are often an industry phenomenon that can be accompanied by significant changes in the market structure. Motivated by this observation, this paper developed a framework to think about the interactions between asset bubbles and product market competition. At the heart of the model is the idea that asset bubbles may sometimes reduce barriers to entry and force firms to expand, to the ultimate benefit of consumers. An interesting aspect of the theory is that asset bubbles may force (productive) market leaders to expand even when they are attached to potential (unproductive) competitors. This observation helps us think about different real-world questions. For instance, how will a large company react to a bubble on its stock prices? Will Apple lower the price of its *iPhones* if investors suddenly become excited about the company and its market value doubles? This paper suggests that it will probably not. Instead, Apple will more likely expand and cut its profit margins in the presence of a generalized boom in which potential competitors (perhaps smaller and less innovative) can also get overvalued. In such a case, as barriers to entry decrease, Apple may be forced to expand in order to preserve its market share. Although subject to each reader's own assessment, I believe this view is not totally unreasonable.

The model developed in this paper also gives us a novel perspective on famous stock market overvaluation episodes. For instance, it may explain why British railway companies duplicated some of their own lines during the

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[36]"E-Business Strategies: Scenario Planning" (07/18/2000), retrieved from https://www.computerworld.com.au/article/84638/e-business_strategies_scenario_planning

1840s *mania* or why large corporations (such as GE) had incentives to quickly adapt their businesses to the internet in the late 1990s. Furthermore, it provides a simple rationale for the low and negative profitability levels reported by internet firms at the peak of the *dotcom* bubble. Rather than the realization of a negative technology shock (as argued by Pastor and Veronesi [2009]) this paper suggests that such income losses may have been a rational reaction to an environment characterized by high stock prices. This view seems indeed to receive support from the anecdotal evidence reviewed in Section 3.

I conclude by pointing to some avenues for future research. The first one is about the relationship between bubbles and moral hazard. A central tenet of this model is that, despite being attached to unproductive firms, asset bubbles can nevertheless improve the workings of good markets and be welfare-improving. One may however argue that bubbles can have the opposite effect: overvaluation can subsidize bad projects or firms, which may impair the workings of both product and financial markets. For instance, in the *dotcom* bubble of the late 1990s we can find many examples of inexperienced firms offering poor services to consumers (such as online retailers failing to make deliveries on time) or even situations of fraud (such as the manipulation of income statements).[37] Can asset bubbles exacerbate moral hazard problems and have a negative impact on consumers' or investors' welfare? I believe these are interesting issues that should be explored in future theoretical work.

Secondly, by making a connection between the degree of competition in product markets and the interest rate, this paper may also shed light on recent US macroeconomic trends. The last four decades of US history have been characterized by both a steady decline in real interest rates and an increase in market power, evident from an increase in markups (De Loecker and Eeckhout [2017]) and measures of industry concentration (Autor et al. [2017]). Although there may be different forces contributing to the interest rate decline, this model suggests that it can be connected to the increase of market power. I believe that a serious assessment of this hypothesis is an important avenue for future research.

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[37]For instance, the telecommunications company Worldcom used fraudulent accounting techniques to artificially increase its earnings during the *dotcom* bubble. Examples of fraud could also be found in the South Sea Bubble (Garber [1990]).

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A Proofs

A.1 Industry Equilibrium

A.1.1 Profits

Given aggregate output Y_t, a factor cost index θ_t and $n_{i,t}$ followers, the leader makes production profits

$$\Pi^L\left(n_{i,t},\theta_t,Y_t\right) \equiv \left(p_{i,t}-\theta_t\right)y_{i,t}^L = \left[\frac{n_{i,t}-\left(n_{i,t}-1+\rho\right)\pi}{n_{i,t}+\pi}\right]^2\left(\frac{n_{i,t}+\rho\,\pi}{n_{i,t}+\pi\,\theta_t}\right)^{\frac{\rho}{1-\rho}}\frac{Y_t}{1-\rho}$$

and each follower

$$\Pi^F\left(n_{i,t},\theta_t,Y_t\right) \equiv \left(p_{i,t}-\frac{\theta_t}{\pi}\right)y_{i,t}^F = \left(\frac{\pi-\rho}{n_{i,t}+\pi}\right)^2\left(\frac{n_{i,t}+\rho\,\pi}{n_{i,t}+\pi\,\theta_t}\right)^{\frac{\rho}{1-\rho}}\frac{Y_t}{1-\rho}$$

It is easy to show that $\Pi^F\left(n_{i,t},\theta_t,Y_t\right)$ is decreasing in $n_{i,t}$

$$\frac{\partial\Pi^F\left(n_{i,t},\theta_t,Y_t\right)}{\partial n_{i,t}}<0$$

$$\Leftrightarrow\quad 2\left(\frac{\pi-\rho}{n_{i,t}+\pi}\right)\frac{-\left(\pi-\rho\right)}{\left(n_{i,t}+\pi\right)^2}\left(\frac{n_{i,t}+\rho\,\pi}{n_{i,t}+\pi\,\theta_t}\right)^{\frac{\rho}{1-\rho}}+\frac{\rho}{1-\rho}\left(\frac{\pi-\rho}{n_{i,t}+\pi}\right)^2\left(\frac{n_{i,t}+\rho\,\pi}{n_{i,t}+\pi\,\theta_t}\right)^{\frac{\rho}{1-\rho}-1}\frac{\pi}{\theta_t}\frac{n_{i,t}+\pi-\left(n_{i,t}+\rho\right)}{\left(n_{i,t}+\pi\right)^2}<0$$

$$\Leftrightarrow\quad -2\left(\pi-\rho\right)+\frac{\rho}{1-\rho}\left(\frac{\pi-\rho}{n_{i,t}+\pi}\right)\left(\frac{n_{i,t}+\rho\,\pi}{n_{i,t}+\pi\,\theta_t}\right)^{-1}\frac{\pi}{\theta_t}\left(\pi-\rho\right)<0$$

$$\Leftrightarrow\quad -2+\frac{\rho}{1-\rho}\left(\frac{\pi-\rho}{n_{i,t}+\rho}\right)<0$$

$$\Leftrightarrow\quad \left(\pi-\rho\right)\rho<2\left(1-\rho\right)\left(n_{i,t}+\rho\right)$$

which is always satisfied given than $\pi-\rho<1-\rho$ and $\rho<n_{i,t}+\rho$.

A.1.2 The Output of the Leader

When there are $n_{i,t}$ followers, the leader chooses an output level that is equal to

$$y_{i,t}^L = s_{i,t}^L y_{i,t}$$

$$= \frac{n_{i,t} - (n_{i,t} - 1 + \rho)\,\pi}{(1-\rho)\,(n_{i,t} + \pi)} \left(\frac{n_{i,t} + \rho}{\theta_t} \frac{\pi}{n_{i,t} + \pi} \right)^{\frac{1}{1-\rho}} Y_t$$

$$= \frac{n_{i,t}\,(1-\pi) + (1-\rho)\,\pi}{n_{i,t} + \pi} \left(\frac{n_{i,t} + \rho}{n_{i,t} + \pi} \right)^{\frac{1}{1-\rho}} \left(\frac{\pi}{\theta_t} \right)^{\frac{1}{1-\rho}} \frac{Y_t}{1-\rho}$$

I want to find a sufficient condition under which the above expression always increases in the number of followers $n_{i,t} \geq 0$. Although $n_{i,t}$ is discrete, it will be easy to take it as a continuous variable and compute its partial derivative with respect to $n_{i,t}$.

$$\frac{\partial y_{i,t}^L}{\partial n_{i,t}} > 0$$

$$\Leftrightarrow \quad \frac{(1-\pi)\,(n_{i,t} + \pi) - [n_{i,t}\,(1-\pi) + (1-\rho)\,\pi]}{(n_{i,t} + \pi)^2} \left(\frac{n_{i,t} + \rho}{n_{i,t} + \pi} \right)^{\frac{1}{1-\rho}} + \frac{n_{i,t}\,(1-\pi) + (1-\rho)\,\pi}{n_{i,t} + \pi} \frac{1}{1-\rho} \left(\frac{n_{i,t} + \rho}{n_{i,t} + \pi} \right)^{\frac{1}{1-\rho} - 1} \frac{\pi - \rho}{(n_{i,t} + \pi)^2} >$$

$$\Leftrightarrow \quad (1-\pi)\,(n_{i,t} + \pi) - [n_{i,t}\,(1-\pi) + (1-\rho)\,\pi] + \frac{n_{i,t}\,(1-\pi) + (1-\rho)\,\pi}{n_{i,t} + \rho} \frac{\pi - \rho}{1-\rho} > 0$$

$$\Leftrightarrow \quad (1-\pi)\,(n_{i,t} + \pi) > [n_{i,t}\,(1-\pi) + (1-\rho)\,\pi] \left(1 - \frac{1}{n_{i,t} + \rho} \frac{\pi - \rho}{1-\rho} \right)$$

$$\Leftrightarrow \quad n_{i,t} + \pi > \left(n_{i,t} + \frac{1-\rho}{1-\pi}\pi \right) \left(1 - \frac{1}{n_{i,t} + \rho} \frac{\pi - \rho}{1-\rho} \right)$$

$$\Leftrightarrow \quad n_{i,t} + \pi > n_{i,t} - \frac{n_{i,t}}{n_{i,t} + \rho} \frac{\pi - \rho}{1-\rho} + \frac{1-\rho}{1-\pi}\pi - \frac{1}{n_{i,t} + \rho} \frac{\pi - \rho}{1-\pi}\pi$$

$$\Leftrightarrow \quad \pi > -\frac{n_{i,t}}{n_{i,t}+\rho}\frac{\pi-\rho}{1-\rho} + \frac{1-\rho}{1-\pi}\pi - \frac{1}{n_{i,t}+\rho}\frac{\pi-\rho}{1-\pi}\pi$$

$$\Leftrightarrow \quad \pi\left(1 - \frac{1-\rho}{1-\pi}\right) > -\frac{\pi-\rho}{n_{i,t}+\rho}\left(\frac{n_{i,t}}{1-\rho} + \frac{\pi}{1-\pi}\right)$$

$$\Leftrightarrow \quad \pi\left(\frac{1-\pi-1+\rho}{1-\pi}\right) > -\frac{\pi-\rho}{n_{i,t}+\rho}\left(\frac{n_{i,t}}{1-\rho} + \frac{\pi}{1-\pi}\right)$$

$$\Leftrightarrow \quad \pi\left(\frac{\rho-\pi}{1-\pi}\right) > -\frac{\pi-\rho}{n_{i,t}+\rho}\left(\frac{n_{i,t}}{1-\rho} + \frac{\pi}{1-\pi}\right)$$

$$\Leftrightarrow \quad \frac{n_{i,t}}{1-\rho} + \frac{\pi}{1-\pi} > \frac{\pi}{1-\pi}\left(n_{i,t}+\rho\right)$$

$$\Leftrightarrow \quad \frac{\pi}{1-\pi}\left(1-\rho\right) > n_{i,t}\left(\frac{\pi}{1-\pi} - \frac{1}{1-\rho}\right)$$

The above condition is always satisfied provided that

$$\frac{\pi}{1-\pi} - \frac{1}{1-\rho} < 0$$

$$\Leftrightarrow \quad \frac{\pi}{1-\pi} < \frac{1}{1-\rho}$$

$$\Leftrightarrow \quad \pi\left(1-\rho\right) < 1 - \pi$$

$$\Leftrightarrow \quad \pi\left(2-\rho\right) < 1$$

$$\Leftrightarrow \quad \pi < \frac{1}{2-\rho}$$

When this condition is not satisfied, the leader may find it optimal to reduce his equilibrium level of output when the number of followers increases.

Inverted-U shape   Given the parameters chosen below, the leader chooses a higher level output when the number of followers increases from $n_{i,t} = 0$ to $n_{i,t} = 1$; as the number of followers increases further, the leader finds it optimal to contract (see panel (3)).

Underlying such disparate reaction is a trade-off faced by the leader. By expanding, the leader can keep a high market share, but at the expense of a lower price $p_{i,t}$. By contracting, he will lose a larger fraction of the market, but will keep $p_{i,t}$ relatively high. When the number of followers is low and $p_{i,t}$ is still high, the benefits of keeping a large market share are high, and the leader ends up producing more. As the number of followers increases and $p_{i,t}$ becomes sufficiently low, the leader prefers to contract to avoid a further drop in $p_{i,t}$.
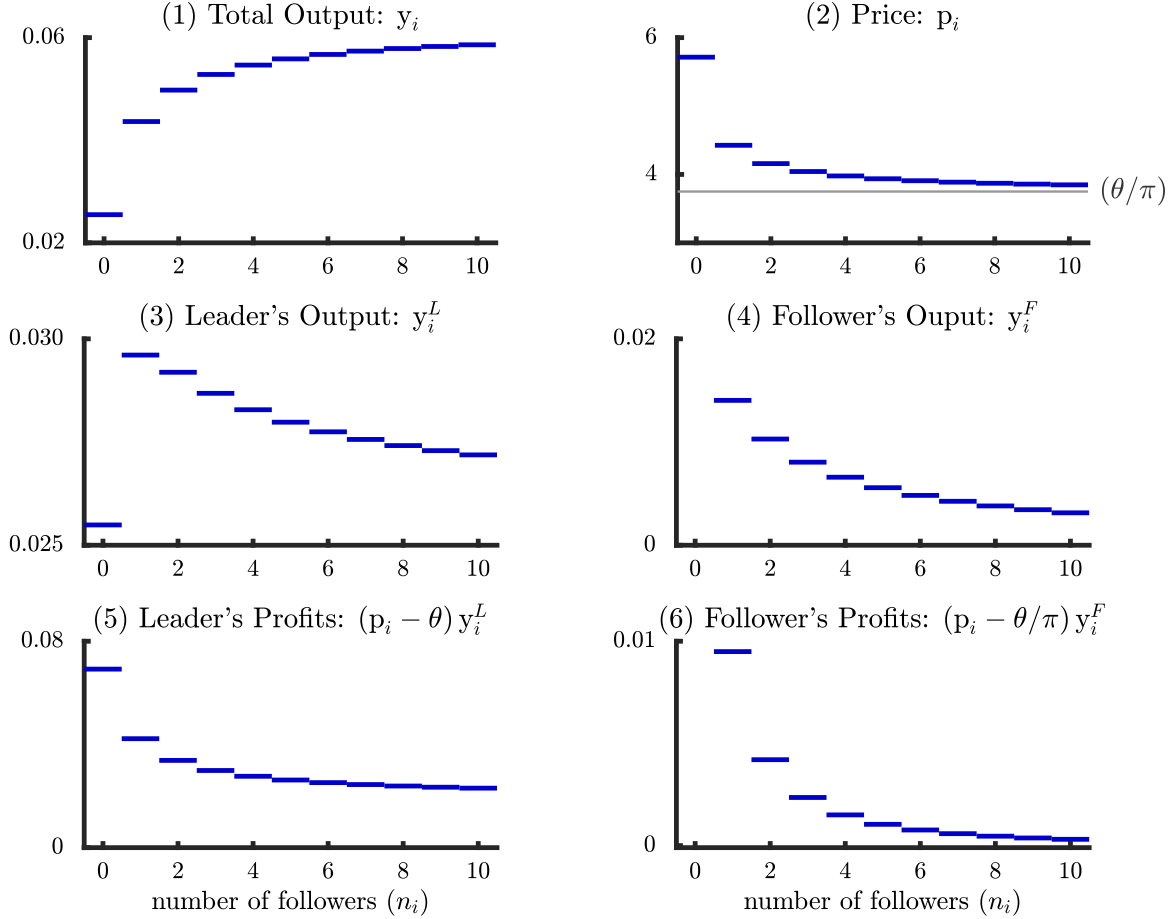
Parameters: $\rho = 0.525$, $\pi = 0.8$, $Y = 1$ and $\theta = 3$

Figure 14: Industry Equilibrium

Decreasing Output   Given the parameters chosen below, the leader always shrinks in reaction to an increase in the number of followers (see panel (3)).
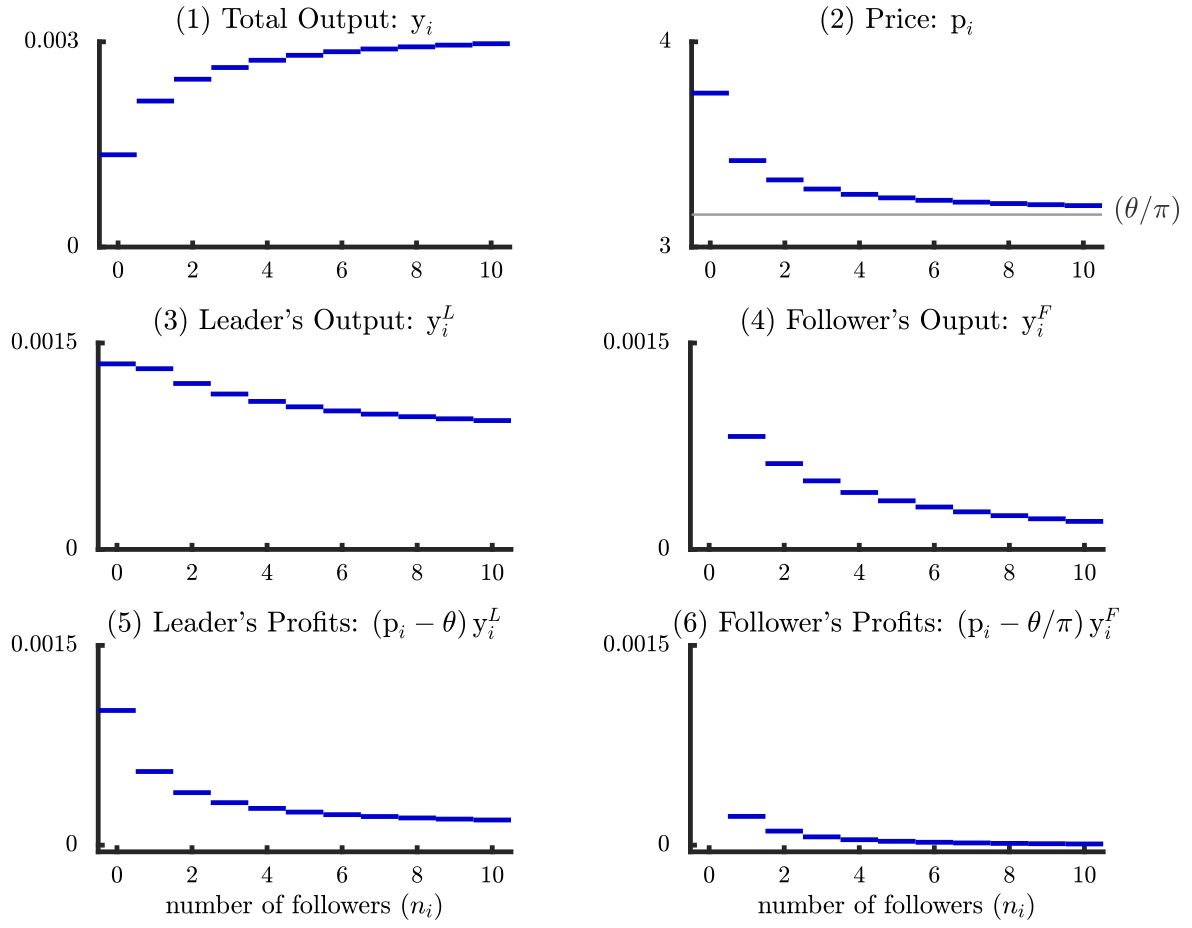

Parameters: $\rho = 0.8$, $\pi = 0.95$, $Y = 1$ and $\theta = 3$

Figure 15: Industry Equilibrium

A.2 The Industry Bubble

Under the industry bubble process studied in section 2.2 firms solve

$$\max_{y_{i,t}^j} \left[\left(1 + \theta^j\right) p_{i,t} - \frac{\theta_t}{\pi_i^j} \right] y_{i,t}^j + \frac{y_{i,t}^j}{y_{i,t}} b_i \qquad \text{s.t.} \quad p_{i,t} = \left(\frac{Y_t}{y_{i,t}}\right)^{1-\rho}$$

$$y_{i,t} = y_{i,t}^j + y_{i,t}^{-j}$$

The solution to this problem yields individual best response functions

$$\left(\frac{Y_t}{y_{i,t}}\right)^{1-\rho} \left[1 - (1-\rho) \frac{y_{i,t}^j}{y_{i,t}} \right] = \frac{\theta_t}{\pi_i^j} - \frac{y_{i,t} - y_{i,t}^j}{y_{i,t}^2} b_{i,t}$$

A.2.1 Fixed Number of Followers

When there are $n_{i,t} \geq 0$ followers, the industry price is characterized by

$$p_{i,t} = \frac{1}{n_{i,t} + \rho} \left[\theta_t \left(1 + \frac{n_{i,t}}{\pi}\right) - n_{i,t} \frac{b_{i,t}}{y_{i,t}} \right]$$

and aggregate output is given by

$$y_{i,t} \, \theta_t \left(1 + \frac{n_{i,t}}{\pi}\right) - (n_{i,t} + \rho) \, Y_t^{1-\rho} \, y_{i,t}^\rho = n_{i,t} \, b_{i,t}$$

From this equation we have that

$$\frac{\partial y_{i,t}}{\partial b_{i,t}} = \frac{1}{n_{i,t}} \left[\theta_t \left(1 + \frac{n_{i,t}}{\pi}\right) - \rho \left(n + \rho\right) p_{i,t} \right]$$

Furthermore, the leader's market share is equal to

$$s_{i,t}^L = \frac{p_{i,t} - \theta_t + \dfrac{b_{i,t}}{y_{i,t}}}{(1-\rho) \, p_{i,t} + \dfrac{b_{i,t}}{y_{i,t}}}$$

A.2.2 Determining the Number of Followers

Suppose there are $\tilde{n}_{i,t} \geq 0$ followers in the bubbleless equilibrium. Let $\underline{b}(n_{i,t})$ denote the minimum industry bubble leading to the existence of $n_{i,t} > \tilde{n}_{i,t}$ followers. Such a bubble is implicitly defined by the following two equations

$$y_{i,t}\,\theta_t\left(1 + \frac{n_{i,t}}{\pi}\right) - (n_{i,t} + \rho)\,Y_t^{1-\rho}\,y_{i,t}^\rho = n_{i,t}\,\underline{b}(n_{i,t})$$

$$\underbrace{\frac{1}{n_{i,t}}\frac{\theta_t - \rho\left(\dfrac{Y_t}{y_{i,t}}\right)^{1-\rho}}{(1-\rho)\left(\dfrac{Y_t}{y_{i,t}}\right)^{1-\rho} + \dfrac{b_{i,t}}{y_{i,t}}}}_{s_{i,t}^F}\,\underbrace{\left\{\left[\left(\frac{Y_t}{y_{i,t}}\right)^{1-\rho} - \frac{\theta_t}{\pi}\right]y_{i,t} + \underline{b}(n_{i,t})\right\}}_{\left(p_{i,t} - \frac{\theta_t}{\pi}\right)y_{i,t} + \underline{b}(n_{i,t})} - c_f = 0$$

The first equation defines total industry output as a function of the bubble $\underline{b}(n_{i,t})$, for a given number $n_{i,t}$ of followers. The second equation states that the production profits of any follower, plus his share on $\underline{b}(n_{i,t})$, should exactly compensate for the fixed cost c_f.

A.2.3 Proof of Proposition 1

To prove Proposition 1 I will show that, for the bubble process described before and given a fixed number of followers $n_{i,t}$ we have that

1. $\dfrac{\partial y_{i,t}}{\partial b_{i,t}} > 0$

2. $\dfrac{\partial y_{i,t}^L}{\partial b_{i,t}} > 0$

3. $\dfrac{\partial s_{i,t}^L}{\partial b_{i,t}} < 0$

Points 2 and 3 above imply that all firms expand (if the leader expands while losing market share, it must be the case that the followers also expand). Point 1 will hence follow immediately. It will however be convenient to start the proof by showing the first point.

1. $\dfrac{\partial y_{i,t}}{\partial b_{i,t}} > 0$

First note that when there is at least one follower, the price will be strictly lower than the leader's monopoly price, i.e. $p_{i,t} < \dfrac{\theta_t}{\rho}$. In such a case we have that

$$
\begin{aligned}
\frac{\partial y_{i,t}}{\partial b_{i,t}} &= \frac{1}{n_{i,t}} \left[\theta_t \left(1 + \frac{n_{i,t}}{\pi} \right) - \rho \left(n + \rho \right) p_{i,t} \right] \\[2mm]
&> \frac{1}{n_{i,t}} \left[\theta_t \left(1 + \frac{n_{i,t}}{\pi} \right) - \rho \left(n + \rho \right) \frac{\theta_t}{\rho} \right] \\[2mm]
&= \frac{1}{n_{i,t}} \left[\theta_t \left(1 + \frac{n_{i,t}}{\pi} \right) - \left(n + \rho \right) \theta_t \right] \\[2mm]
&= \frac{1}{n_{i,t}} \left[n_{i,t} \theta_t \left(\frac{1}{\pi} - 1 \right) + \left(1 - \rho \right) \theta_t \right] \\[2mm]
&> 0 \quad \text{QED}
\end{aligned}
$$

Therefore, given a fixed number of followers $n_{i,t} \geq 1$, as the industry bubble $b_{i,t}$ increases, total output $y_{i,t}$ increases.

2. $\dfrac{\partial y_{i,t}^L}{\partial b_{i,t}} > 0$

We can write the leader's market share as

$$s_{i,t}^L = \frac{n_{i,t}\theta_t\left(\dfrac{1}{\pi} - 1\right) + \theta_t - \rho p_{i,t}}{\theta_t\left(\dfrac{n_{i,t}}{\pi} + 1\right) - \rho p_{i,t}\left(n_{i,t} + 1\right)}$$

We can therefore write the leader's output as

$$y_{i,t}^L = y_{i,t}\frac{n_{i,t}\theta_t\left(\dfrac{1}{\pi} - 1\right) + \theta_t - \rho p_{i,t}}{\theta_t\left(\dfrac{n_{i,t}}{\pi} + 1\right) - \rho p_{i,t}\left(n_{i,t} + 1\right)} \equiv y_{i,t}\frac{\mathrm{num}_{i,t}}{\mathrm{den}_{i,t}}$$

Note that we can write the derivative of the price $p_{i,t}$ with respect to the industry bubble $b_{i,t}$ as

$$\frac{\partial p_{i,t}}{\partial b_{i,t}} = -\left(1 - \rho\right)\frac{p_{i,t}}{y_{i,t}}\frac{\partial y_{i,t}}{\partial b_{i,t}}$$

Therefore, we have that

$$
\begin{aligned}
\frac{\partial y_{i,t}^L}{\partial b_{i,t}} &= \frac{\partial y_{i,t}}{\partial b_{i,t}}\frac{\mathrm{num}_{i,t}}{\mathrm{den}_{i,t}} + y_{i,t}\frac{\rho(1-\rho)\dfrac{p_{i,t}}{y_{i,t}}\dfrac{\partial y_{i,t}}{\partial b_{i,t}}\mathrm{den}_{i,t} - \rho(n_{i,t}+1)(1-\rho)\dfrac{p_{i,t}}{y_{i,t}}\dfrac{\partial y_{i,t}}{\partial b_{i,t}}\mathrm{num}_{i,t}}{\mathrm{den}_{i,t}^2}\\[2mm]
&= \frac{\partial y_{i,t}}{\partial b_{i,t}}\frac{\mathrm{num}_{i,t}}{\mathrm{den}_{i,t}} + \frac{\rho(1-\rho)p_{i,t}\dfrac{\partial y_{i,t}}{\partial b_{i,t}}\mathrm{den}_{i,t} - \rho(n_{i,t}+1)(1-\rho)p_{i,t}\dfrac{\partial y_{i,t}}{\partial b_{i,t}}\mathrm{num}_{i,t}}{\mathrm{den}_{i,t}^2}\\[2mm]
&= \frac{1}{\mathrm{den}_{i,t}}\frac{\partial y_{i,t}}{\partial b_{i,t}}\left[n_{i,t}\theta_t\left(\frac{1}{\pi} - 1\right) + \theta_t - \rho p_{i,t} + \rho(1-\rho)p_{i,t} - \rho(n_{i,t}+1)(1-\rho)p_{i,t}\frac{n_{i,t}\theta_t\left(\dfrac{1}{\pi} - 1\right) + \theta_t - \rho p_{i,t}}{\theta_t\left(\dfrac{n_{i,t}}{\pi} + 1\right) - \rho p_{i,t}\left(n_{i,t} + 1\right)}\right]\\[2mm]
&= \frac{1}{\mathrm{den}_{i,t}}\frac{\partial y_{i,t}}{\partial b_{i,t}}\left[n_{i,t}\theta_t\left(\frac{1}{\pi} - 1\right) + \theta_t - \rho p_{i,t} + \rho(1-\rho)p_{i,t} - \frac{\rho(n_{i,t}+1)(1-\rho)p_{i,t}}{\theta_t\left(\dfrac{n_{i,t}}{\pi} + 1\right) - \rho p_{i,t}\left(n_{i,t} + 1\right)}\left[n_{i,t}\theta_t\left(\frac{1}{\pi} - 1\right) + \theta_t - \rho p_{i,t}\right]\right]
\end{aligned}
$$

$$\frac{1}{\mathrm{den}_{i,t}}\frac{\partial y_{i,t}}{\partial b_{i,t}}\left\{\left[n_{i,t}\theta_t\left(\frac{1}{\pi} - 1\right) + \theta_t - \rho p_{i,t}\right]\left[1 - \frac{\rho(n_{i,t}+1)(1-\rho)p_{i,t}}{\theta_t\left(\dfrac{n_{i,t}}{\pi} + 1\right) - \rho p_{i,t}\left(n_{i,t} + 1\right)}\right] + \rho p_{i,t}(1-\rho)\right\}$$

Note that

$$\mathrm{den}_{i,t} = \theta_t\left(\frac{n_{i,t}}{\pi} + 1\right) - \rho p_{i,t}\left(n_{i,t} + 1\right) = n_{i,t}\left(\frac{\theta_t}{\pi} - \rho p_{i,t}\right) + \left(\theta_t - \rho p_{i,t}\right) > 0$$

Furthermore, we already know from above that

$$\frac{\partial y_{i,t}}{\partial b_{i,t}} > 0$$

Therefore, it suffices to show that

$$\left[n_{i,t}\theta_t \left(\frac{1}{\pi} - 1 \right) + \theta_t - \rho p_{i,t} \right] \left[1 - \frac{\rho \left(n_{i,t} + 1 \right) \left(1 - \rho \right) p_{i,t}}{\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - \rho p_{i,t} \left(n_{i,t} + 1 \right)} \right] + \rho p_{i,t} \left(1 - \rho \right) > 0$$

$$\Leftrightarrow \left[\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - \rho p_{i,t} \left(n_{i,t} + 1 \right) - \rho \left(n_{i,t} + 1 \right) \left(1 - \rho \right) p_{i,t} \right] + \rho p_{i,t} \left(1 - \rho \right) \frac{\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - \rho p_{i,t} \left(n_{i,t} + 1 \right)}{n_{i,t}\theta_t \left(\frac{1}{\pi} - 1 \right) + \theta_t - \rho p_{i,t}} > 0$$

$$\Leftrightarrow \left[\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - \rho p_{i,t} \left(n_{i,t} + 1 \right) \left(2 - \rho \right) \right] + \rho p_{i,t} \left(1 - \rho \right) \frac{\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - n_{i,t}\rho p_{i,t} - \rho p_{i,t}}{\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - n_{i,t}\theta_t - \rho p_{i,t}} > 0$$

Since $\rho p_{i,t} < \theta_t$, the above condition is implied by

$$\left[\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - \rho p_{i,t} \left(n_{i,t} + 1 \right) \left(2 - \rho \right) \right] + \rho p_{i,t} \left(1 - \rho \right) \frac{\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - {\color{red} n_{i,t}\theta_t } - \rho p_{i,t}}{\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - n_{i,t}\theta_t - \rho p_{i,t}} > 0$$

$$\Leftrightarrow \left[\theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - \rho p_{i,t} \left(n_{i,t} + 1 \right) \left(2 - \rho \right) \right] + \rho p_{i,t} \left(1 - \rho \right) > 0$$

$$\Leftrightarrow \theta_t \left(\frac{n_{i,t}}{\pi} + 1 \right) - \rho p_{i,t} \left(2n_{i,t} + 2 - \rho n_{i,t} - \rho - 1 + \rho \right) > 0$$

$$\Leftrightarrow \frac{\theta_t}{\rho p_{i,t}} \left(\frac{n_{i,t}}{\pi} + 1 \right) - \left[n_{i,t} \left(2 - \rho \right) + 1 \right] > 0$$

Note that we have that

$$p_{i,t} \quad \leq \frac{n_{i,t} + \pi}{\left(n_{i,t} + \rho \right) \pi} \theta_t$$

$$\frac{\theta_t}{\rho p_{i,t}} \quad > \frac{\left(n_{i,t} + \rho \right) \pi}{\rho \left(n_{i,t} + \pi \right)}$$

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Therefore, the previous condition is implied by

$$\frac{(n_{i,t} + \rho)\pi}{\rho(n_{i,t} + \pi)}\left(\frac{n_{i,t}}{\pi} + 1\right) - [n_{i,t}(2 - \rho) + 1] > 0$$

$$\Leftrightarrow \quad \frac{n_{i,t} + \rho}{\rho(n_{i,t} + \pi)}(n_{i,t} + \pi) - [n_{i,t}(2 - \rho) + 1] > 0$$

$$\Leftrightarrow \quad n_{i,t} + \rho - \rho[n_{i,t}(2 - \rho) + 1] > 0$$

$$\Leftrightarrow \quad n_{i,t} - \rho n_{i,t}(2 - \rho) > 0$$

$$\Leftrightarrow \quad n_{i,t}[1 - \rho(2 - \rho)] > 0$$

The above condition is always true provided that $\rho < 1$.

3. $\dfrac{\partial s_{i,t}^L}{\partial b_{i,t}} < 0$

We have that

$$\frac{\partial s_{i,t}^L}{\partial b_{i,t}} = \frac{\rho\left(1-\rho\right)\dfrac{p_{i,t}}{y_{i,t}}\dfrac{\partial y_{i,t}}{\partial b_{i,t}}\mathrm{den}_{i,t} - \rho\left(n_{i,t}+1\right)\left(1-\rho\right)\dfrac{p_{i,t}}{y_{i,t}}\dfrac{\partial y_{i,t}}{\partial b_{i,t}}\mathrm{num}_{i,t}}{\mathrm{den}_{i,t}^2}$$

Therefore, we need to show that

$$\frac{\rho\left(1-\rho\right)\dfrac{p_{i,t}}{y_{i,t}}\dfrac{\partial y_{i,t}}{\partial b_{i,t}}\mathrm{den}_{i,t} - \rho\left(n_{i,t}+1\right)\left(1-\rho\right)\dfrac{p_{i,t}}{y_{i,t}}\dfrac{\partial y_{i,t}}{\partial b_{i,t}}\mathrm{num}_{i,t}}{\mathrm{den}_{i,t}^2} < 0$$

$$\Leftrightarrow \quad \mathrm{den}_{i,t} - \left(n_{i,t}+1\right)\mathrm{num}_{i,t} < 0$$

$$\Leftrightarrow \quad \theta_t\left(\frac{n_{i,t}}{\pi}+1\right) - \rho p_{i,t}\left(n_{i,t}+1\right) - \left(n_{i,t}+1\right)\left[n_{i,t}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{i,t}\right] < 0$$

$$\Leftrightarrow \quad \theta_t\left(\frac{n_{i,t}}{\pi}+1\right) - \left(n_{i,t}+1\right)\left[n_{i,t}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t\right] < 0$$

$$\Leftrightarrow \quad \theta_t\left(\frac{n_{i,t}}{\pi}+1\right) - \left(n_{i,t}+1\right)\left[\theta_t\left(\frac{n_{i,t}}{\pi}+1\right)-n_{i,t}\theta_t\right] < 0$$

$$\Leftrightarrow \quad -n_{i,t}\theta_t\left(\frac{n_{i,t}}{\pi}+1\right) + \left(n_{i,t}+1\right)n_{i,t}\theta_t < 0$$

$$\Leftrightarrow \quad n_{i,t}\theta_t\left[n_{i,t}-\frac{n_{i,t}}{\pi}\right] < 0$$

which is always satisfied provided that $\pi < 1$.

A.2.4 A Comparison Between The Two Processes

To conclude this section, I make a brief comparison between the two bubble processes considered above. Lemma 6 below states two main results.

Lemma 6. *Suppose there are two industries, A and B; firms in industry A can issue stocks according to the constant firm level bubble process, whereas firms in industry B issue stocks according to the constant industry bubble. We have that*

1. if $\sum_j^{n_A} b_A^j = \sum_j^{n_B} b_B^j$, then $n_A \geq n_B$

2. if $n^A = n^B \geq 1$, then $y^A < y^B$

The first point of the above lemma says that, if the two industries have the same aggregate industry bubble b_i, then industry A must have at least as many followers as industry B. In other words, the size of the aggregate industry bubble b_i leading to the entry of the n-th follower is lower under the constant firm level bubble process. To understand this result, suppose that each industry is a monopoly under the bubbleless equilibrium and let $\underline{b}^F$ be the minimum firm level bubble leading to the entry of the first follower in industry A (so that the aggregate industry bubble is equal to $2\,\underline{b}^F$)

$$\left(p_i - \frac{\theta}{\pi}\right) y^F < c_f - \underline{b}^F$$

Now suppose that there is a firm level bubble with size $2\,\underline{b}^F$ in industry B. Clearly, the leader in industry B will produce more than the leader in industry A, implying that the follower in industry B needs to produce more than that of industry A in order to appropriate a bubble with size $\underline{b}^F$. However, as the two firms in industry B produce more, the follower will make lower production profits and will therefore be unwilling to enter the market with an industry bubble with size $2\,\underline{b}^F$

$$\left(\widetilde{p}_i - \frac{\theta}{\pi}\right) \widetilde{y}^F < c_f - \frac{\widetilde{y}^F}{\widetilde{y}^L + \widetilde{y}^F} 2\underline{b}^F$$

The second point says that, when the two industries have the same number of followers, the output in industry B will be strictly larger. This fact can be seen from a direct comparison between equations (8) and (10) and is a consequence of the *fight-for-market-shares* effect emphasized above.

Figure 16 compares three equilibrium variables (the number of followers, total industry output and total industry welfare) in the two industries as a function of the total industry bubble b_i. The first panel illustrates the first point of Lemma 6. The second panel shows that, even when industry B features a lower number of followers, its output can still be larger than that of industry A. Such a fact can be explained by the second point of Lemma 6. Note that the maximum level of welfare achieved by industry B is larger than that of industry A. This fact, though not general, can again be linked to the second point of Lemma 6 – industry B can achieve the perfect competition level of output with a fewer number of firms and hence a smaller waste of resources than industry A.[38]

---

[38]To see why the maximum level of welfare achieved by industry B is not always larger than that of industry A, take the limit case in which $c_f = 0$ and $\pi = 1$ so that the bubbleless equilibrium features perfect competition. In such a case, the appearance of a constant firm level bubble in industry A will have no impact on welfare. However, as a constant firm level bubble appears in industry B, welfare
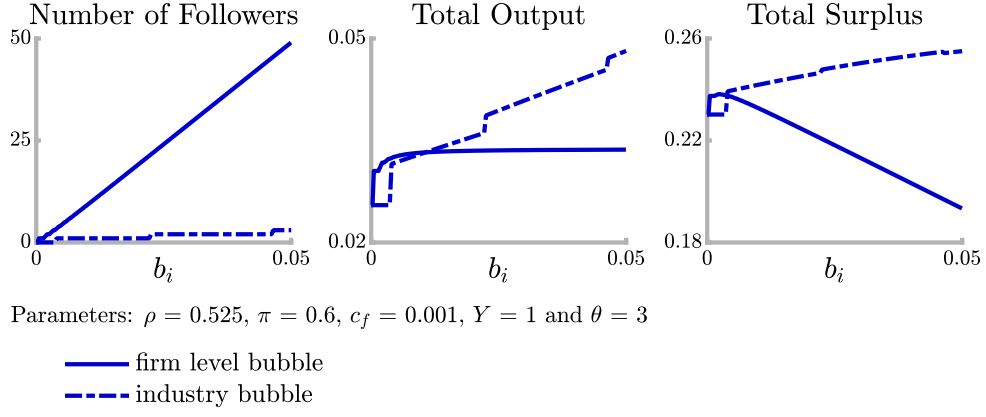
Parameters: $\rho = 0.525$, $\pi = 0.6$, $c_f = 0.001$, $Y = 1$ and $\theta = 3$

——— firm level bubble
- - - - industry bubble

Figure 16: Constant Firm Level Bubble versus Constant Industry Level Bubble

A.3 General Equilibrium with No Bubbles

A.3.1 Aggregate TFP

Note that we can write aggregate TFP as

$$\varphi\left(n_t\right) = \frac{\pi\left(1-\rho\right)n_t + \left(1-\rho\right)\pi^2}{\left[\pi\left(2-\pi\right)-\rho\right]n_t + \left(1-\rho\right)\pi^2}$$

Note that

$$\pi\left(1-\rho\right) < \pi\left(2-\pi\right)-\rho \Leftrightarrow \rho\left(1-\pi\right) < \pi\left(1-\pi\right)$$

which is always satisfied given the assumption that $\rho < \pi$. It then follows immediately that $\varphi\left(n_t\right)$ is decreasing in n_t.

A.3.2 Aggregate Factor Share

Note that we can write the aggregate factor share as

$$\sigma\left(n_t\right) = \frac{\pi\left(1-\rho\right)n_t + \left(1-\rho\right)\pi^2}{\left[\pi\left(2-\pi\right)-\rho\right]n_t + \left(1-\rho\right)\pi^2}$$

Note that

$$\pi\left(1-\rho\right) < \pi\left(2-\pi\right)-\rho \Leftrightarrow \rho\left(1-\pi\right) < \pi\left(1-\pi\right)$$

which is always satisfied given the assumption that $\rho < \pi$. It then follows immediately that $\varphi\left(n_t\right)$ is decreasing in n_t.

---

will necessarily decrease, since firms will charge a price below their marginal cost of production.

A.3.3 Number of Followers

Suppose that there are n_t followers in every industry, except in industry i (which has $n_{i,t}$ followers). Since the size of each individual industry is negligible, we have $\theta_t = \dfrac{n_t + \rho}{n_t + \pi}\pi$ and $Y_t = \varphi(n_t) K_t^\alpha$. Each follower in this industry i will therefore make profits

$$\Pi^F(n_{i,t}, n_t) = \left(\frac{\pi - \rho}{n_{i,t} + \pi}\right)^2 \left(\frac{n_{i,t} + \rho}{n_{i,t} + \pi}\frac{n_t + \pi}{n_t + \rho}\right)^{\frac{\rho}{1-\rho}} \frac{\varphi(n_t) K_t^\alpha}{1 - \rho}$$

For a symmetric equilibrium with n_t followers to be possible, we first need that when $n_{i,t} = n_t$, no follower makes a loss in industry i

$$\Pi^F(\textcolor{red}{n_t}, n_t) \geq c_f$$

$$\Leftrightarrow \quad \left(\frac{\pi - \rho}{n_t + \pi}\right)^2 \frac{\varphi(n_t) K_t^\alpha}{1 - \rho} \geq c_f$$

$$\Leftrightarrow \quad K_t \geq \underbrace{\left[\frac{(1 - \rho) c_f}{\varphi(n_t)}\left(\frac{n_t + \pi}{\pi - \rho}\right)^2\right]^{\frac{1}{\alpha}}}_{\equiv \underline{K}(n_t)}$$

Second, we need that if an additional follower were to enter industry i (i.e. $n_{i,t} = n_t + 1$), he could make no gain

$$\Pi^F(\textcolor{red}{n_t + 1}, n_t) \leq c_f$$

$$\Leftrightarrow \quad \left(\frac{\pi - \rho}{n_t + 1 + \pi}\right)^2 \left(\frac{n_t + 1 + \rho}{n_t + 1 + \pi}\frac{n_t + \pi}{n_t + \rho}\right)^{\frac{\rho}{1-\rho}} \frac{\varphi(n_t) K_t^\alpha}{1 - \rho} \leq c_f$$

$$\Leftrightarrow \quad K_t \leq \underbrace{\left[\frac{(1 - \rho) c_f}{\varphi(n_t)}\left(\frac{n_t + 1 + \pi}{\pi - \rho}\right)^2 \left(\frac{n_t + 1 + \pi}{n_t + 1 + \rho}\frac{n_t + \rho}{n_t + \pi}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1}{\alpha}}}_{\equiv \overline{K}(n_t)}$$

We hence have that a symmetric equilibrium in which one leader and $n_t \geq 0$ followers produce in every industry is possible provided that

$$K_t \in \begin{cases} \left[\left(\dfrac{c_f}{1 - \rho}\right)^{\frac{1}{\alpha}}, \overline{K}(0)\right] & \text{if } n_t = 0 \\[4mm] \left[\underline{K}(n_t), \overline{K}(n_t)\right] & \text{if } n_t \geq 1 \end{cases}$$

Let us start by analyzing the case in which $n_t \geq 1$. A symmetric equilibrium in which one leader and $n_t \geq 1$ followers produce is possible if $\underline{K}(n_t) \leq K_t \leq \overline{K}(n_t)$. The first inequality requires aggregate capital to be sufficiently large, so that the existing $n_t \geq 1$ followers do not make a loss. The threshold $\underline{K}(n_t)$ defines the aggregate stock of capital at which, in a symmetric equilibrium with one leader and n_t followers, every follower makes exactly zero

gains (i.e. his production profits equal the fixed cost c_f). The second inequality requires aggregate capital not to be too large, so that there are no profitable entry opportunities for any additional follower. The threshold $\overline{K}(n_t)$ defines the aggregate stock of capital at which, in a symmetric equilibrium with one leader and n_t followers, an additional follower in one industry would make exactly zero gains.

Let us now examine the case in which $n_t = 0$. A symmetric equilibrium with one leader and zero followers in every industry will be possible if $\left(\dfrac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}} \leq K_t \leq \overline{K}(0)$. The first inequality guarantees that, in a symmetric equilibrium in which only the leaders produce, no leader makes a loss. The threshold $\left(\dfrac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}$ is the minimum aggregate stock of capital at which all leaders can produce together without making a loss. The second inequality requires that, in such an equilibrium, no follower can make a gain upon entering the market.

A.3.4 Asymmetric Equilibria

Let λ_t be the fraction of all industries with $n_t + 1$ followers. The remaining $1 - \lambda_t$ industries have n_t followers.

Aggregate TFP is given by

$$\varphi\left(n_t, \lambda_t\right) = \frac{\pi\left(1 - \rho\right)\left(n_t + \pi\right)\left[\left(1 - \lambda_t\right)\left(\dfrac{n_t + \rho}{n_t + \pi}\right)^{\frac{\rho}{1-\rho}} + \lambda_t\left(\dfrac{n_t + 1 + \rho}{n_t + 1 + \pi}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1}{\rho}}}{\left(1 - \lambda_t\right)\left(\dfrac{n_t + \rho}{n_t + \pi}\right)^{\frac{1}{1-\rho}}\left\{n_t\left[\pi\left(2 - \pi\right) - \rho\right] + \left(1 - \rho\right)\pi^2\right\} + \lambda_t\left(\dfrac{n_t + 1 + \rho}{n_t + 1 + \pi}\right)^{\frac{1}{1-\rho}}\left\{\left(n_t + 1\right)\left[\pi\left(2 - \pi\right) - \rho\right] + \left(1 - \rho\right)\pi^2\right\}}$$

The aggregate factor cost index is

$$\theta\left(n_t, \lambda_t\right) = \left[\left(1 - \lambda_t\right)\left(\frac{n_t + \rho}{n_t + \pi}\right)^{\frac{\rho}{1-\rho}} + \lambda_t\left(\frac{n_t + 1 + \rho}{n_t + 1 + \pi}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}}$$

The aggregate factor share is equal to

$$\sigma\left(n_t, \lambda_t\right) = \frac{\theta\left(n_t, \lambda_t\right)}{\varphi\left(n_t, \lambda_t\right)}$$

A.3.5   Transition to Full Monopoly

Suppose that the economy is characterized by a monopoly in every industry. In such a case, the profit share (exclusive of fixed costs) is equal to $1-\rho$, meaning that each leader makes total profits that are equal to $(1-\rho)\,K_t^\alpha$. However, such an equilibrium is only feasible if

$$(1-\rho)\,K_t^\alpha - c_f \geq 0$$

$$\Leftrightarrow \quad K_t \geq \left(\frac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}$$

for otherwise each leader would incur a loss. If the capital stock is relatively small, so that the previous condition is not satisfied, a monopoly is not feasible in all industries. In such a case, production will only take place in a measure $M_t \in (0,1)$ of all industries. Aggregate output is equal to

$$Y_t = M_t^{\frac{1}{\rho}} y_t$$

And each industry's price and output are given by

$$p_t = \quad M_t^{\frac{1-\rho}{\rho}}$$

$$y_t = \quad M_t^{-1} K_t^\alpha$$

We hence have that

$$Y_t = M_t^{\frac{1-\rho}{\rho}} K_t^\alpha$$

The number of active industries is pinned down by the no profit condition

$$(1-\rho)\,p_t y_t = c_f$$

$$\Leftrightarrow \quad (1-\rho)\,M_t^{\frac{1-\rho}{\rho}} M_t^{-1} K_t^\alpha = c_f$$

$$\Leftrightarrow \quad (1-\rho)\,M_t^{\frac{1-2\rho}{\rho}} K_t^\alpha = c_f$$

$$\Leftrightarrow \quad M_t = \left(\frac{c_f}{1-\rho} K_t^\alpha\right)^{\frac{\rho}{2\rho-1}}$$

Note that $M_t \leq 1$ requires that

$$2\rho - 1 \geq 0$$

$$\Leftrightarrow \quad \rho \geq \frac{1}{2}$$

Aggregate output is hence given by

$$Y_t = \left(\frac{1-\rho}{c_f}\right)^{\frac{1-\rho}{2\rho-1}} K_t^{\frac{\alpha\rho}{2\rho-1}}$$

In other words, when $\rho \geq \frac{1}{2}$ and $K_t \leq \left(\frac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}$, the economy operates under monopolistic competition. In When $\rho \geq \frac{1}{2}$ and $K_t \leq \left(\frac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}$ an equilibrium is not possible.

Let us understand this result. Suppose that $K_t \leq \left(\frac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}$. If $\rho \geq \frac{1}{2}$, the degree of substitutability across varieties is high. This means that there are weak decreasing returns and each industry will be relatively large. Industries can break even when there are few active industries ($M_t \leq 1$)

If on the other hand $\rho < \frac{1}{2}$, the degree of substitutability is low. This means that there are strong decreasing returns and each industry will be relatively small. In such a case, each individual industry can break even only when there is a large number of active industries ($M_t > 1$). Given that each industry is small, a large number of industries is necessary for aggregate output to be high.

A.3.6   Interest Rate

When there are strong decreasing returns to scale (low α), the equilibrium interest rate can monotonically decrease in the aggregate capital stock.
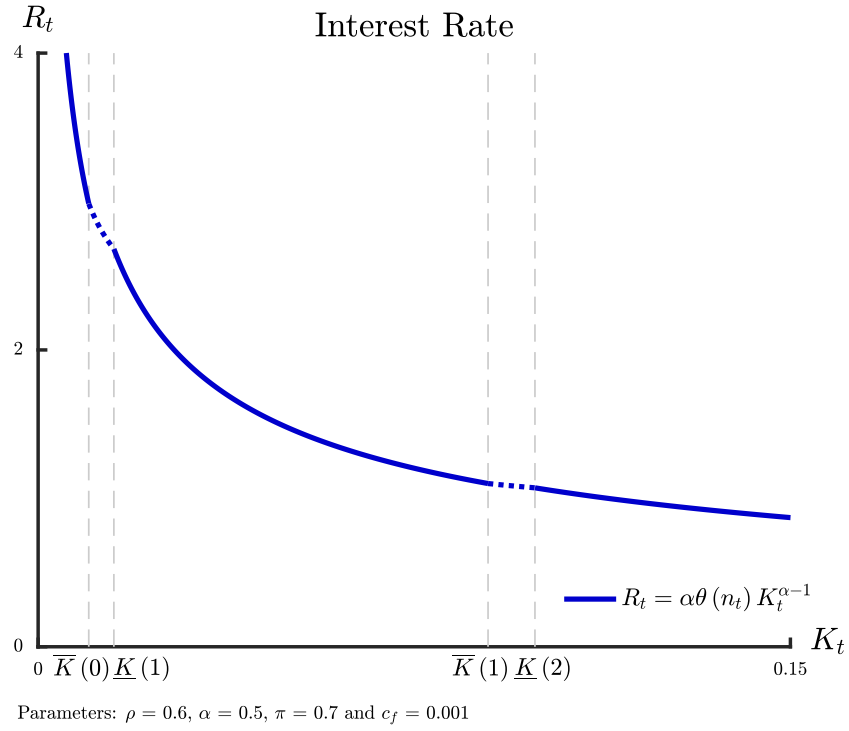

Parameters: $\rho = 0.6$, $\alpha = 0.5$, $\pi = 0.7$ and $c_f = 0.001$

Figure 17: Interest Rate

A.3.7 Multiple Equilibria with No Bubbles

Parameters: $\rho = 0.4$, $\alpha = 0.8$, $\pi = 0.7$, $c_f = 0.01$ and $\delta = 0.1$

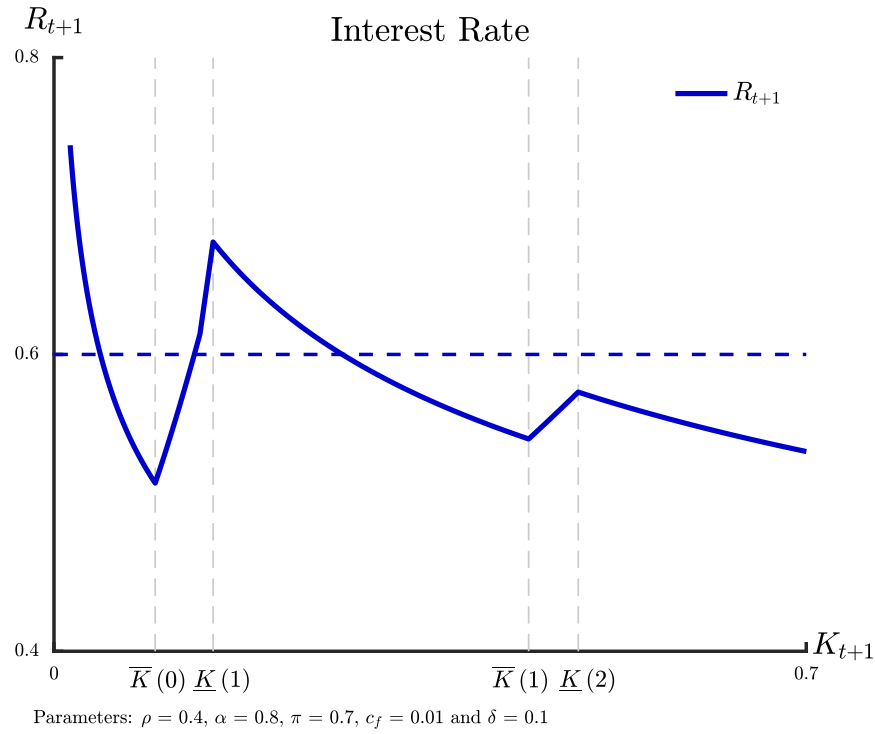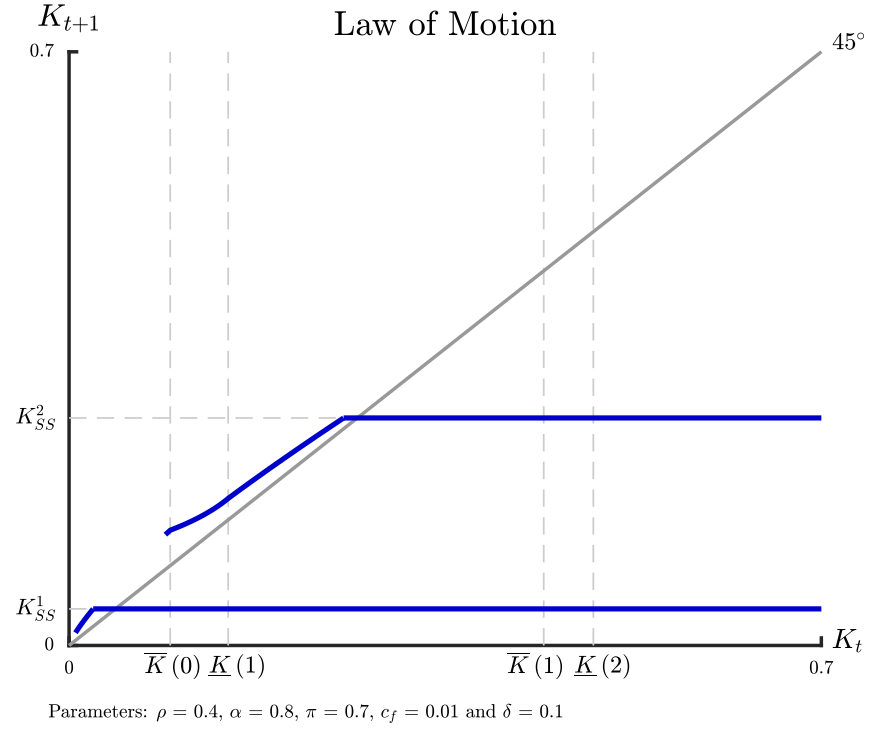Parameters: $\rho = 0.4$, $\alpha = 0.8$, $\pi = 0.7$, $c_f = 0.01$ and $\delta = 0.1$

Figure 18: Multiple Equilibria with No Bubbles

A.3.8 Steady-State

No Storage Note that in a steady-state where storage is built and there are n followers in all industries, aggregate output is equal to

$$Y = \varphi(n)\left[(1-\delta)\varphi(n)\right]^{\frac{\alpha}{1-\alpha}}$$

In order to have a symmetric equilibrium where n followers produce in every industry we need to have

$$c_f \in \left[\underline{c}(n^*,r), \overline{c}(n^*,r)\right]$$

where

$$\underline{c}(n,r) = \begin{cases} \rho(1-\delta)^{\frac{\alpha}{1-\alpha}} & \text{if } n = 0 \\[2em] \left(\dfrac{\pi-\rho}{n+\pi}\right)^2 \dfrac{\varphi(n)\left[(1-\delta)\varphi(n)\right]^{\frac{\alpha}{1-\alpha}}}{1-\rho} & \text{if } n \geq 1 \end{cases}$$

and

$$\overline{c}(n,r) = \left(\frac{\pi-\rho}{n+1+\pi}\right)^2 \left(\frac{n+1+\rho}{n+1+\pi}\frac{n+\pi}{n+\rho}\right)^{\frac{\rho}{1-\rho}} \frac{\varphi(n)\left[(1-\delta)\varphi(n)\right]^{\frac{\alpha}{1-\alpha}}}{1-\rho}$$

Furthermore, we need that

$$R > r$$

$$\Leftrightarrow \quad \alpha\sigma(n)\varphi(n)K^{\alpha-1} > r$$

$$\Leftrightarrow \quad \alpha\sigma(n)\varphi(n)\left[(1-\delta)\varphi(n)\right]^{\frac{\alpha-1}{1-\alpha}} > r$$

$$\Leftrightarrow \quad \alpha\sigma(n) > (1-\delta)r$$

Storage Note that in a steady-state where storage is built and there are n followers in all industries, aggregate output is equal to

$$Y = \varphi(n)\left[\frac{\alpha\sigma(n)\varphi(n)}{r}\right]^{\frac{\alpha}{1-\alpha}}$$

In order to have a symmetric equilibrium where n followers produce in every industry we need to have

$$c_f \in \left[\underline{c}^{\text{ss}}(n^*,r), \overline{c}^{\text{ss}}(n^*,r)\right]$$

where

$$
\underline{\mathfrak{C}}^{\text{ss}} =
\begin{cases}
\rho \left(\dfrac{\alpha \rho}{r} \right)^{\frac{\alpha}{1-\alpha}} & \text{if } n = 0 \\[3em]
\left(\dfrac{\pi - \rho}{n + \pi} \right)^2 \dfrac{\varphi(n) \left[\dfrac{\alpha \sigma(n) \varphi(n)}{r} \right]^{\frac{\alpha}{1-\alpha}}}{1 - \rho} & \text{if } n \geq 1
\end{cases}
$$

and

$$
\overline{\mathfrak{C}}^{\text{ss}}(n, r) = \left(\frac{\pi - \rho}{n + 1 + \pi} \right)^2 \left(\frac{n + 1 + \rho \; n + \pi}{n + 1 + \pi \; n + \rho} \right)^{\frac{\rho}{1-\rho}} \frac{\varphi(n) \left[\dfrac{\alpha \sigma(n) \varphi(n)}{r} \right]^{\frac{\alpha}{1-\alpha}}}{1 - \rho}
$$

Furthermore, we need that

$$
(1 - \delta) Y > K
$$

$$
\Leftrightarrow \quad (1 - \delta) \varphi(n) \left[\frac{\alpha \sigma(n) \varphi(n)}{r} \right]^{\frac{\alpha}{1-\alpha}} > \left[\frac{\alpha \sigma(n) \varphi(n)}{r} \right]^{\frac{1}{1-\alpha}}
$$

$$
\Leftrightarrow \quad (1 - \delta) \varphi(n) > \frac{\alpha \sigma(n) \varphi(n)}{r}
$$

$$
\Leftrightarrow \quad (1 - \delta) r > \alpha \sigma(n)
$$

A.4 General Equilibrium with Bubbles

A.4.1 The Firm Level Bubble in General Equilibrium

Bubbles can lead to larger aggregate output even in the absence of productivity differences ($\pi = 1$), as the entry of new firms forces incumbents to expand.
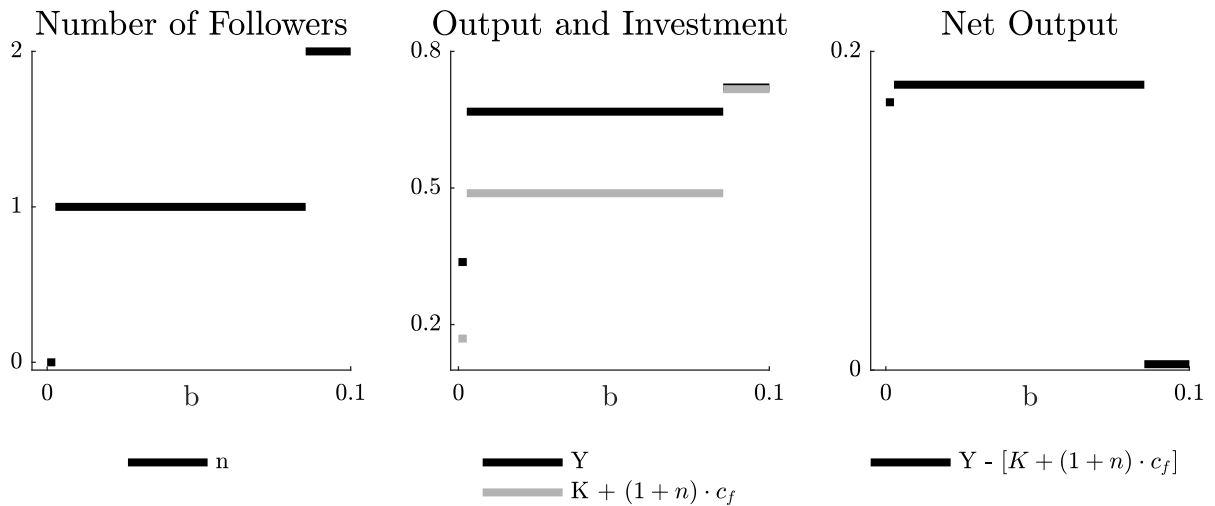

Parameters: $\alpha = 0.225$, $\rho = 0.05$, $\pi = 1$, $c_f = 0.16125$, $\delta = 0.001$ and $r = 0.475$

Figure 19: Aggregate Efficiency: No Productivity Differences

A.4.2 The Industry Bubble in General Equilibrium

As demonstrated in Appendix A.2.3, s_t^L decreases with b (irrespective of aggregate variables). Furthermore, we can show that there is a negative relationship between the factor cost index θ_t and the leaders' market share s_t^L. To see this, note that in a symmetric equilibrium with $p_{i,t} = 1$, equation (9) implies that

$$s_t^L = \frac{n_t\left(\frac{\theta_t}{\pi} - \theta_t\right) + \theta_t - \rho}{n_t\left(\frac{\theta_t}{\pi} - \rho\right) + \theta_t - \rho}$$

We can rearrange this expression, to write θ_t as a function of s_t^L

$$\theta_t = \frac{\rho\left[(n_t + 1)\, s_t^L - 1\right]}{\left(\frac{n_t}{\pi} + 1\right) s_t^L - n_t\left(\frac{1}{\pi} - 1\right) - 1}$$

It is easy to see that, in general equilibrium, the factor cost index θ_t decreases with s_t^L

$$\frac{\partial \theta_t}{\partial s_t^L} < 0$$

$$\Leftrightarrow \quad \frac{\rho\,(n_t + 1)\left[\left(\frac{n_t}{\pi} + 1\right) s_t^L - n_t\left(\frac{1}{\pi} - 1\right) - 1\right] - \left(\frac{n_t}{\pi} + 1\right)\rho\left[(n_t + 1)\, s_t^L - 1\right]}{\left[\left(\frac{n_t}{\pi} + 1\right) s_t^L - n_t\left(\frac{1}{\pi} - 1\right) - 1\right]^2}$$

$$\Leftrightarrow \quad (n_t + 1)\left[\left(\frac{n_t}{\pi} + 1\right) s_t^L - n_t\left(\frac{1}{\pi} - 1\right) - 1\right] - \left(\frac{n_t}{\pi} + 1\right)\left[(n_t + 1)\, s_t^L - 1\right] < 0$$

$$\Leftrightarrow \quad (n_t + 1)\left[\left(\frac{n_t}{\pi} + 1\right) s_t^L - n_t\left(\frac{1}{\pi} - 1\right) - 1 - \left(\frac{n_t}{\pi} + 1\right) s_t^L\right] + \left(\frac{n_t}{\pi} + 1\right) < 0$$

$$\Leftrightarrow \quad (n_t + 1)\left[-n_t\left(\frac{1}{\pi} - 1\right) - 1\right] + \left(\frac{n_t}{\pi} + 1\right) < 0$$

$$\Leftrightarrow \quad (n_t + 1)\left(-\frac{n_t}{\pi} + n_t - 1\right) + \left(\frac{n_t}{\pi} + 1\right) < 0$$

$$\Leftrightarrow \quad n_t\left(-\frac{n_t}{\pi} + n_t - 1\right) + n_t < 0$$

$$\Leftrightarrow \quad n_t^2\left(1 - \frac{1}{\pi}\right) < 0 \quad \checkmark$$

To understand this result, note that s_t^L can be seen as a measure of market power. The higher is s_t^L, the higher are average profit margins and hence the lower are factor shares. This fact translates into a lower factor price index θ_t.

We hence have that, as the industry bubble b increases, the leaders lose market share and factor prices increase.

Proposition 6. *In a symmetric equilibrium in which all industries have n_t followers, as the industry bubble b*

increases, we have that

1. the leaders lose market share, i.e. $\dfrac{\partial s_t^L}{\partial b} < 0$

2. the factor cost index increases, i.e. $\dfrac{\partial \theta_t}{\partial b} > 0$

I now characterize the reaction of aggregate output Y_t to a change in the industry bubble b. In a symmetric equilibrium in which all industries have n_t followers and an industry bubble with size b, aggregate output is implicitly defined by

$$Y_t \left[\theta_t \left(1 + \frac{n_t}{\pi} \right) - (n_t + \rho) \right] - n_t b = 0$$

Recall that the factor cost index θ_t increases with b. Hence, it is not clear from the previous equation whether Y_t increases or decreases with b. If θ_t reacts negligibly to a change in b $\left(\dfrac{\partial \theta_t}{\partial b} \to 0 \right)$, then Y_t will increase with b $\left(\dfrac{\partial Y_t}{\partial b} > 0 \right)$. To derive some further intuitions, note that when storage is used, the factor cost index can be written as

$$\theta_t = \left\{ \frac{Y_t}{\pi} \left[1 - s_t^L (1 - \pi) \right] \right\}^{\frac{1-\alpha}{\alpha}} \frac{r}{\alpha} \tag{17}$$

We therefore have that

$$Y_t \underbrace{\left[\left\{ \frac{Y_t}{\pi} \left[1 - s_t^L (1 - \pi) \right] \right\}^{\frac{1-\alpha}{\alpha}} \frac{r}{\alpha} \left(1 + \frac{n_t}{\pi} \right) - (n_t + \rho) \right] - n_t b = 0}_{F\left(Y_t, s_t^L, b \right)}$$

We have that

$$\frac{\partial Y_t}{\partial b} > 0$$

$$\Leftrightarrow \quad -\frac{\frac{\partial F(\cdot)}{\partial b}}{\frac{\partial F(\cdot)}{\partial Y_t}} > 0$$

$$\Leftrightarrow \quad \frac{\partial F(\cdot)}{\partial b} < 0$$

$$\Leftrightarrow \quad Y_t \frac{r}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} \left\{\frac{Y_t}{\pi}\left[1 - s_t^L(1-\pi)\right]\right\}^{\frac{1-\alpha}{\alpha}-1} \left[-\frac{Y_t}{\pi}(1-\pi)\right] \frac{\partial s_t^L}{\partial b} - n_t < 0$$

$$\Leftrightarrow \quad Y_t \frac{r}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} \left(\frac{Y_t}{\varphi_t}\right)^{\frac{1-\alpha}{\alpha}-1} \frac{Y_t}{\pi}(1-\pi) \frac{\partial s_t^L}{\partial b} + n_t > 0$$

$$\Leftrightarrow \quad \varphi_t Y_t \frac{r}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} \left(\frac{Y_t}{\varphi_t}\right)^{\frac{1-\alpha}{\alpha}} \frac{1-\pi}{\pi} \frac{\partial s_t^L}{\partial b} + n_t > 0$$

$$\Leftrightarrow \quad \varphi_t Y_t \frac{\alpha \varphi_t K_t^{\alpha-1}}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} (K_t^\alpha)^{\frac{1-\alpha}{\alpha}} \frac{1-\pi}{\pi} \frac{\partial s_t^L}{\partial b} + n_t > 0$$

$$\Leftrightarrow \quad \varphi_t Y_t \frac{\alpha \sigma_t \varphi_t K_t^{\alpha-1}}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} K_t^{1-\alpha} \frac{1-\pi}{\pi} \frac{\partial s_t^L}{\partial b} + n_t > 0$$

$$\Leftrightarrow \quad \varphi_t \theta_t Y_t \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} \frac{1-\pi}{\pi} \underbrace{\frac{\partial s_t^L}{\partial b}}_{<0} + n_t > 0$$

There are different ways to interpret the expression above. One possible interpretation is that we have $\frac{\partial Y_t}{\partial b} > 0$ provided that $\frac{\partial s_t^L}{\partial b}$ is not too negative.[39] This fact is equivalent to requiring that the decline in aggregate TFP resulting from an increase in the followers' market shares is not too high.

Another possible interpretation is that we have $\frac{\partial Y_t}{\partial b} < 0$ when $\frac{1-\alpha}{\alpha}$ is sufficiently high (so that the capital elasticity α is low). Recall that the factor cost index θ_t always increases with b – a higher industry bubble makes firms compete more aggressively and demand more factors of production. When storage is used, capital is infinitely elastic and the interest rate is fixed. In such a case, only the labor price W_t rises after an increase in b. The extent to which W_t rises depends however on the labor elasticity $1-\alpha$ - the higher $1-\alpha$, the larger the increase in W_t. Indeed, if $1-\alpha$ is sufficiently large, W_t may increase so much (after an increase b) that firms can be discouraged from increasing output.

---

[39]Indeed, we would always observe $\frac{\partial Y_t}{\partial b} > 0$ if $\frac{\partial s_t^L}{\partial b} = 0$.

Steady-State   We can express the aggregate capital stock in a steady-state as

$$K = \left(\frac{Y}{\varphi}\right)^{\frac{1}{\alpha}}$$

Recall that

$$\varphi = \frac{\pi}{1 - s^L (1 - \pi)}$$

We hence have that

$$\frac{\partial K}{\partial b} = \frac{1}{\alpha} K^{1-\alpha} \left[-\frac{1-\pi}{\pi} Y \frac{\partial s^L}{\partial b} + \frac{1 - s^L (1 - \pi)}{\pi} \frac{\partial Y}{\partial b} \right]$$

The industry bubble will lead to an efficient expansion provided that

$$\frac{\partial Y}{\partial b} > \frac{\partial K}{\partial b}$$

which is equivalent to

$$\frac{\partial Y}{\partial b} \left(\alpha \varphi K^{\alpha-1} - 1\right) > \frac{1-\pi}{1 - s^L (1 - \pi)} Y \left(-\frac{\partial s^L}{\partial b}\right) \tag{18}$$

This expression defines the conditions under which an increase in the industry bubble b leads to an efficient expansion (for a fixed number of followers in every industry). To derive some intuition, let us focus on a steady-state where capital accumulation is dynamically efficient. The condition for dynamic efficiency can be written as

$$\alpha \varphi K^{\alpha-1} > 1$$

Recall that $\frac{\partial s^L}{\partial b} < 0$. Therefore, the requiring that $\frac{\partial Y}{\partial b} > 0$ is not sufficient to guarantee that $\frac{\partial Y}{\partial b} > \frac{\partial K}{\partial b}$. In words, even if capital accumulation is dynamically efficient, an economic expansion triggered by an increase in b may not always be desirable. Let us understand this result. The condition for dynamic efficiency means that the return to investment exceeds its cost in the current steady-state. This fact implies that, *holding aggregate TFP constant,* if we increase investment by ε, output increases by some amount $\kappa > \varepsilon$. However, as the industry bubble b increases, the leaders lose market share and *aggregate TFP declines.* Therefore, an increase in output resulting from an increase in b comes at the cost of a less efficient allocation of resources, and hence a higher cost of investment. Condition (18) above states that, for the increase in b to lead to an efficient expansion, the increase in output $\frac{\partial Y}{\partial b}$ needs to be sufficiently large to compensate for the loss in the leaders' market shares, given by $-\frac{\partial s^L}{\partial b}$.

Figure 20 characterizes the constant industry bubble in a steady-state with storage. Given the particular parameters chosen, the maximum constant industry bubble that the economy can sustain, in a steady-state with storage, is such that all industries are a duopoly with one leader and one follower.
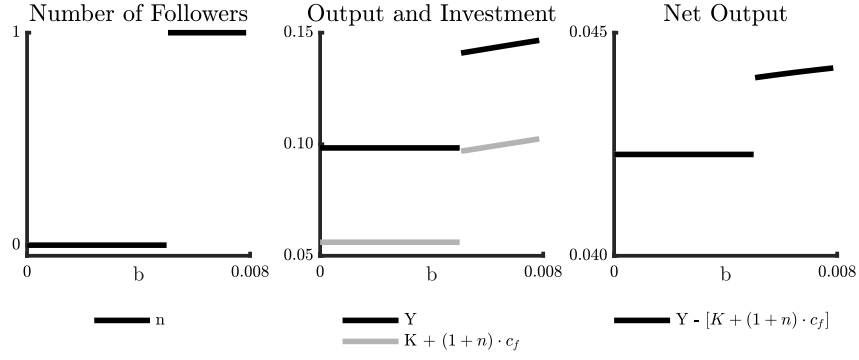
Parameters: $\alpha = 0.8$, $\rho = 0.525$, $\pi = 0.6$, $c_f = 0.001$, $\delta = 0.1$ and $r = 0.75$

Figure 20: Aggregate Efficiency under a Constant Industry Bubble

B The Model with Bertrand Competition

In this section I characterize the model under an alternative market structure: I will assume that firms engage in price competition (Bertrand), instead of competition via quantities (Cournot).

B.1 Industry Equilibrium

Assume now that there are no fixed costs of production ($c_f = 0$), but that demographics, technology and financial markets are otherwise identical to the framework described in Section 2.1. In particular, each firm can still produce with constant marginal cost $\dfrac{\theta_t}{\pi_i^j}$ where

$$
\pi_i^j = \begin{cases} 1 & \text{if} \quad j = i \quad \text{(leader)} \\[2em] \pi \leq 1 & \text{if} \quad j \neq i \quad \text{(followers)} \end{cases}
$$

As before, I shall assume that $\pi > \rho$.

Bubbleless Equilibrium In the bubbleless equilibrium, the leader will set a price equal to the followers' marginal cost (limit pricing). We hence have that

$$
p_{i,t} = \frac{\theta_t}{\pi}
$$

$$
y_{i,t} = \left(\frac{\pi}{\theta_t}\right)^{\frac{1}{1-\rho}} Y_t
$$

(19)

As we can see, the leader will charge a markup $\dfrac{1}{\pi}$ over his marginal cost θ_t. Such a markup decreases in the followers' productivity level π. Note that in the limit case where $\pi = 1$, the equilibrium of this industry features perfect competition. π can therefore be seen as a measure of competition.

Constant Firm Level Bubbles If firms can issue a constant amount of stock b, there is no impact on the previous equilibrium. The leader will still set a price equal to the marginal cost of the followers, so that the equilibrium is still described by equations (19).

Constant Industry Bubbles Suppose now that there is a constant industry bubble equal to b_i, which is distributed according to market shares. Will the leader still produce the quantity given by (19)? The answer is no. To see this, note that for any industry output level $y_{i,t+1}$ such that

$$\frac{\theta_t}{\pi} < p_{i,t} + \frac{1}{y_{i,t}}b_i$$

the followers can profitably enter. The reason that their marginal cost of production is still $\dfrac{\theta_t}{\pi}$, but they now get an additional return of $\dfrac{1}{y_{i,t}}b_{i,t+1}$ per each unit that they sell. Therefore, to prevent the entry of the followers, the leader must set a limit price lower than the followers' marginal cost so that the above condition holds with equality. In this case, the leader will set a limit price that is implicitly defined by

$$p_{i,t} = \frac{\theta_t}{\pi} - \frac{1}{y_{i,t}}b_i \qquad (20)$$

It is easy to prove that the price implicitly defined by (20) is decreasing in the industry bubble b_i.

Proposition 7. *The price charged by the leader under Bertrand competition decreases in the size of the industry bubble b_i.*

Proof. It suffices to show that $y_{i,t}$ increases in b_i. We can write the equilibrium condition as

$$\frac{\theta}{\pi}y_i = y_i^\rho \cdot Y^{1-\rho} + b_i$$

Define

$$\Xi(x) = \frac{\theta}{\pi}x - x^\rho \cdot Y^{1-\rho}$$

Ξ is increasing in b_i. Moreover, we have that

$$\Xi'(x) > 0 \Leftrightarrow \frac{\theta}{\pi} - \rho x^{\rho-1}Y^{1-\rho} > 0 \Leftrightarrow \frac{\theta}{\pi} > \rho x^{\rho-1}Y^{1-\rho} > 0 \Leftrightarrow \theta > \pi\rho x^{\rho-1}Y^{1-\rho} \Leftrightarrow x > \left(\frac{\pi\rho}{\theta}\right)^{\frac{1}{1-\rho}}Y$$

Finally, note that

$$x\,|_{b=0} = \left(\frac{\pi}{\theta}\right)^{\frac{1}{1-\rho}}Y > \left(\frac{\pi\rho}{\theta}\right)^{\frac{1}{1-\rho}}Y$$

∎

As before, as b_i becomes sufficiently large, the leader may even set a price below his marginal cost θ_t. Note however that, contrarily to the model with Cournot competition, under Bertrand competition the leader is always

the only producer.

So far I have assumed that all potential market participants (the leader and the followers) can appropriate a fraction of the constant industry bubble (according to market shares). A slight modification to the previous assumption offers however an interesting insight. Suppose that there is still a potential industry bubble but that, contrarily to what I assumed before, b_i can only be shared among the followers (still in proportion to their market shares). The assumption that the leader cannot issue overvalued stocks may be realistic in a world in which investors value change or novelty (and hence penalize incumbents). Suppose then that there is a potential industry bubble b_i and that each individual firm can appropriate a fraction

$$s_b^L \quad = 0$$

$$s_b^{j,F} \quad = \begin{cases} 0 & \text{if } y_i^F = 0 \\\\ \dfrac{y_i^{j,F}}{y_i^F} b_i & \text{if } y_i^F > 0 \end{cases}$$

Under this alternative assumption, the industry price is still given by equation (20). Output will however be produced by either the leader or the followers, depending on the size of b_i. When the size of the potential bubble b_i is sufficiently small, so that the price implied by (20) is not lower than θ_t, the leader will still be the only producer. Note however that, because the followers are inactive, in such a region no bubble actually materializes. In that case, the industry is characterized by a latent bubble – the threat that the followers can issue overvalued stocks (whenever they have a non-negligible market share) will force the leader to set a lower limit price and produce a larger output level. Note that the leader can set a limit price not lower than his marginal unit cost θ_t provided that

$$b_i \leq \bar{b}_i := \theta^{-\frac{\rho}{1-\rho}} Y_t \frac{1-\pi}{\pi}$$

When $b_i = \bar{b}_i$, the leader produces the perfect competition benchmark level of output, so that the maximum level of welfare is obtained. When $b_i > \bar{b}_i$, the leader cannot produce (for otherwise he would incur a loss). In such a case the followers become active, for that reason, the industry bubble materializes. Figure 21 shows some equilibrium variables as a function of the total industry potential bubble b_i.

B.2   General Equilibrium

Bubbleless Equilibrium   Under Bertrand competition, the leader is the only producer in a bubbleless equilibrium. All industries will hence behave identically and share the same price $p_{i,t} = 1$. In such a case, aggregate TFP is constant and equal to $\varphi_t = 1$. Total output Y_t will hence be a simple function of the aggregate capital stock K_t

$$Y_t = K_t^\alpha$$

Figure 21: The Latent Bubble
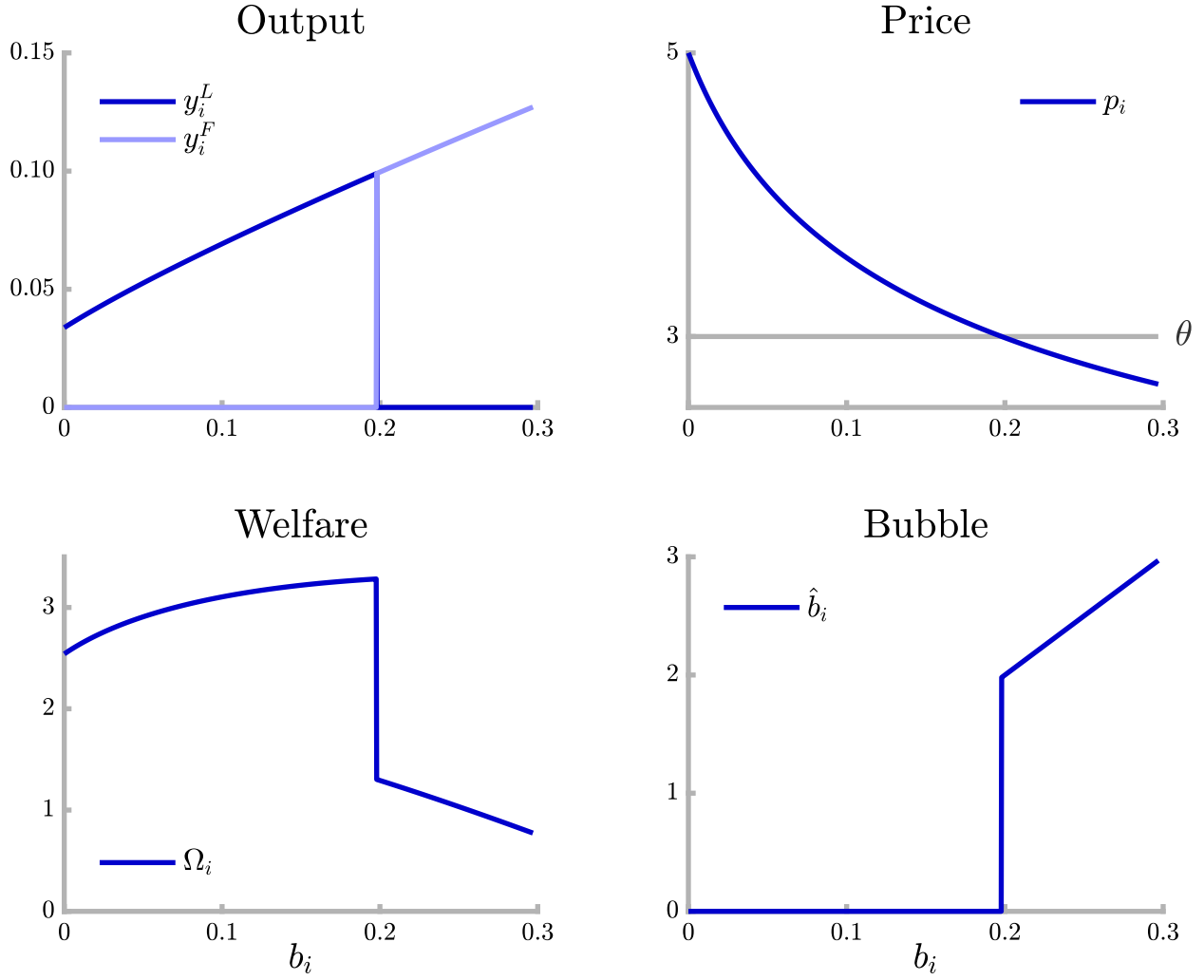
Parameters: $\rho = 0.525$, $\pi = 0.6$, $Y = 1$ and $\theta = 3$

The aggregate cost index θ_t will coincide with the aggregate factor share, $\theta_t = \sigma_t$. We have that

$$\sigma_t = \pi$$

The aggregate factor share is equal to the inverse of the leaders' markup $\dfrac{1}{\pi}$. Note that perfect competition is achieved when $\pi = 1$, so that the leader has no productivity advantage over the followers; in such a case there is a unit factor share $\sigma_t = 1$.

Factor prices will be equal to

$$W_t = \quad (1 - \alpha)\, \pi\, K_t^\alpha$$

$$R_t = \quad \alpha\, \pi\, K_t^{\alpha - 1}$$

Note that factor prices depend positively on the followers productivity π which, as we have just seen, corresponds

to the aggregate factor share. The existence of market power creates as before a wedge between factor prices and their aggregate marginal products. For instance, the interest rate will be below the marginal product of capital whenever $\pi < 1$

$$R_t = \alpha\,\pi\,K_t^{\alpha-1} < \alpha\,K_t^{\alpha-1} = \frac{\partial Y_t}{\partial K_t}$$

The equilibrium dynamics will take a simple form. In particular, the economy is characterized by a law of motion

$$K_{t+1} = \begin{cases} (1-\delta)\,K_t^\alpha & \text{if } K_t \leq \left[\frac{1}{(1-\delta)}\left(\frac{\alpha\,\pi}{r}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\alpha}} \\[3mm] \left(\frac{\alpha\,\pi}{r}\right)^{\frac{1}{1-\alpha}} & \text{if } K_t > \left[\frac{1}{(1-\delta)}\left(\frac{\alpha\,\pi}{r}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\alpha}} \end{cases}$$

This law of motion uniquely pins down the value of K_{t+1} for any given value of K_t. When the current capital stock is low enough, all savings $(1-\delta)\,K_t^\alpha$ will be converted into capital; the resulting interest rate $\alpha\,\pi\,K_{t+1}^{\alpha-1}$ will not be lower than the return on storage r. When the current capital stock is sufficiently high, not all savings can be converted into capital; the capital stock is such that the resulting interest rate $\alpha\,\pi\,K_{t+1}^{\alpha-1}$ is equal to the return on storage r. This economy will converge to a steady-state characterized by

$$K^* = \min\left\{(1-\delta)^{\frac{1}{1-\alpha}}, \left(\frac{\alpha\,\pi}{r}\right)^{\frac{1}{1-\alpha}}\right\}$$

$$R^* = \max\left\{\frac{\alpha\,\pi}{1-\delta}, r\right\}$$

Note that Propositions 3 to 5 still hold. We can however make use of the equilibrium result $\sigma^* = \pi$ and rewrite them as an explicit function of the model parameters.

Proposition 8. *(Possibility of Rational Asset Bubbles) Rational asset bubbles are possible if*

$$1 - \delta > \pi\alpha$$

Proposition 9. *(Overaccumulation of Capital)*

   1. If storage is not used in such a steady-state, capital accumulation is dynamically inefficient when

$$1 - \delta > \alpha$$

   2. If storage is used in such a steady-state, capital accumulation is dynamically inefficient when

$$\pi > r$$

Proposition 10. *(Underinvestment) The economy features underinvestment in its steady-state if*

$$\pi < \min\left\{\frac{1-\delta}{\alpha}r, r\right\}$$

The economy features underinvestment when the aggregate factor share π is low or the return on storage r is high.