European University Institute
ECONOMICS DEPARTMENT

EUI Working Paper **ECO** No. 98/1

Binary Decision Structures and the Required Detail of Information

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Printed in Italy February 1998
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February 18, 1998

Abstract

How much does one need to know about the characteristics of heterogeneous agents in order to position them correctly in an organizational structure? This question is addressed in a project selection framework with error–prone decision–makers.

Keywords: Organizational Structure, Organizational Complexity, Information

JEL classification numbers: D70, D80

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*I am grateful to Steve Martin and Spyros Vassilakis for comments on an earlier version of this paper, and to Etienne Billette de Villemeur for stimulating conversations.

1 Introduction

It is hard to understand what the relevance is of uncertainty, diversity, and complexity for organizational design, communication, and decision–making if not the resulting difficulty in making correct choices and the accompanying likelihood of erroneous decisions.[1] Indeed, Herbert Simon has made a point of stressing that it are the limited cognitive capabilities of individual human beings that requires organizational decision–making to be multi–person decision–making. However, words like "errors", "erroneous decisions", "fallible decision–makers" and the like, are virtually absent from the literature on organizational design and communication. This may be the result of the predominantly mechanistic characterization of organizational communication as information processing which tends to focus on channels of communication and messages sent.[2] The presence of a full rationality assumption forms another obstacle to a discussion of faulty decision–making.[3] Moreover, many authors tend to use abstract phrases like organizational agents that are "affected by" or "prompted by" their environment and its characteristics, or information about threats and opportunities in the environment that "initiates" or "influences" organizational actions, by which the authors perhaps implicitly refer to the possibility of error–prone decision–makers. In any event, faulty decision–

[1] There is a large literature dealing with the effect of uncertainty, diversity, and complexity on organizational design in general and on decision–making and organizational communication in particular. See Galbraith (1973), March and Simon (1993), Thompson (1968), and many of the contributions in Jablin et al. (1987) and in Hirokawa and Poole (1986). Contributions to the principal–agent literature and the literature on pricing behaviour may serve as other examples.

[2] Some recent contributions to the economics literature on organizational design that equate communication to information processing are Radner (1993), and Bolton and Dewatripont (1994).

[3] See the discussion of the various perspectives on organizational communication in Krone et al. (1987), and in Euske and Roberts (1987). In the principal–agent literature, the optimality of the contract, and hence the absence of errors, is guaranteed by the rationality of the contract designer.

making receives explicit attention from only two small, complementary sets of papers.

Concerning the causes of systematically erroneous decisions, Donald Campbell provides various explanations based on certain human traits. With action preferred to paralysis (especially when a decision should eventually be taken), and in the presence of an associative memory, human beings show a tendency to fill gaps in messages and information that reach them, and to assimilate current information to past information on the basis of particular clues and similarities.[4] Miller provides an indeed dramatic example showing how a long distance between the original source of a message and the eventual user of the message may radically distort the content of the message:

> "A reporter was present at a hamlet burned down by the U. S. Army's 1st Air Cavalry Division in 1967. Investigation showed that the order from the division headquarters to the brigade was: "On no occasion must hamlets be burned down." The brigade radioed the battalion: "Do not burn down any hamlets unless you are absolutely convinced that the Viet Cong are in them." The battalion radioed the infantry company at the scene: "If you think there are any Viet Cong in the hamlet, burn it down." The company commander ordered his troops: "Burn down that hamlet."[5]

Although I am not sure what the reason was behind the replacement of the unconditional order "On no occasion must hamlets be burned down" by the *conditional* command "Do not burn down any hamlets unless you are absolutely convinced that the Viet Cong are in them," it seems hard to interpret as a random error. Indeed, it might be that on previous occasions the order send by division headquarters had had this conditional

[4] See Campbell (1959), pp. 341–351.

[5] Miller quoted in Huber and Daft (1987), p. 150.

structure. The current message may have been interpreted by association as a conditional command.

Campbell also notes that if a group of persons has to decide on an issue the variation in their opinion is likely to diminish as a result of communication between the members of the group. He is not specific, however, about the precise way group communication affects the variety of opinions.[6] At the time Campbell wrote, communication theory was still in its infancy, but even contemporary accounts of group communication theory pay scant attention to the interaction between communication structure and resulting errors. Randy Hirokawa and Dirk Scheerhorn (1986) do not go beyond the claim that "the social influence exerted on the group by individual members will effectively facilitate or prevent the occurrence of (...) potential sources of faulty group decision–making" (p. 76). In Gouran and Hirokawa (1986) various ways to counter erroneous inferences and decisions are listed, but the role of communication structure is not mentioned.

Indeed, the only thorough discussion of the way structure and errors interact that I am aware can be found in a series of papers by Raj Sah and Joseph Stiglitz, and by Shmuel Nitzan, Jacob Paroush et al. In either case, agents must decide whether to accept or to reject a project. An error arises when a good project is rejected or when a bad project is accepted. Sah and Stiglitz characterise an agent by a pair of probabilities capturing the likelihood with which these errors arise. They compare simple organizational architectures that differ with respect to the sequential structure of the project screening process in terms of the expected value of the projects that are eventually implemented. Nitzan, Paroush et al., on the other hand, characterise an agent by the overall probability of faulty decisions, and ignore sequential decision structures.[7]

[6] Campbell (1959), pp. 360–362

[7] Sah and Stiglitz (1985, 1986, 1988). Papers by Hendrikse (1992), Ioannides (1987), and Koh (1992, 1993a, 1993b, 1994a, 1994b) extend results obtained by Sah and Stiglitz. For the other approach see, *e.g.*, Nitzan and Paroush (1982) and Karotkin, Nitzan

In this paper I use Sah and Stiglitz' framework, and address the following two questions. Suppose that some agents are more error–prone than others, what then should be their optimal ordering within a given organizational structure? Who should be the first to evaluate a project, who should be next? Does the positioning of agents matter at all? This leads to the second question. How much does one need to know about the screening qualities of the individual agents to allocate them correctly? Suppose the ordering of agents matters. Does the ordering depend on the relative quality (who is the better agent?) or does it depend on the exact qualities of the agents under examination?

In theory, then, one can distinguish organizational structures on the basis of the detail in information necessary to find the optimal allocation of heterogeneous agents to positions within these structures. This paper classifies organizational structures on the basis of such informational requirements.

The interest in such a classification stems from a few observations. First of all, if more detail cannot be obtained, or it can be acquired but only in a distorted form and at a cost, organizations that require less detailed information for the determination of the optimal positioning of agents have a clear advantage over structures requiring more detailed information, *ceteris paribus*[8].

Secondly, with any increase in detail necessary to optimally position heterogeneous agents, errors becomes more likely. Although an organizational form requiring information that is more detailed than some other structure may perform better if the employees have been positioned correctly, its performance may be highly sensitive to errors in the positioning of agents. Indeed, these errors may cause the former to perform worse than the latter. Probably, the less detail required, the more robust a decision structure is to such errors.

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and Paroush (1988).

[8] *Ceteris paribus*, since the organization that requires more detailed information may perform better than the other when the information needed is actually available.

Thirdly, this paper suggests a novel way to think about the complexity of an organization. The notion of complexity figures prominently in the literature on the design of organizational structures. A structure is called complex if it contains many interdependent parts whose individual functioning is of importance to the overall performance of the organization. The more complex an organization the heavier the demands on its information processing capacities.[9] The analysis in the sections that follow contributes to the literature on organizational design by discussing the complexity of an organization in terms of the level of detail in information required to optimally structure error–prone agents.

Turning to the results, I show which organizational structures require no information at all about the screening capabilities of the agents. These structures are all characterized by the fact that every agent only evaluates a project if the preceding agents have all accepted the project or all preceding agents have rejected the project. The second class of structures are those that require only ordinal knowledge about the qualities of the agents. This class of structures is characterized by the fact that every agent's decision *can* be final. In other words, there is not an agent whose evaluation will always be followed by some other agent's evaluation, irrespective of the former agent's decision to accept or reject. Indeed, the mere presence of one agent whose decision will always be followed by some other agent's evaluation is enough to make ordinal information insufficient to find the optimal ordering of agents.

Section 2 describes the model in detail. Section 3 states the main propositions. Section 4 concludes. The proofs of the lemma's can be found in the appendix.

[9] See Galbraith (1973), Huber and Daft (1987), Jablin (1987), and Scott (1981) among many others.

2 The Model

In this section I rigorously characterize the main elements of the model: the project environment, the agents, different degrees of fineness of information about the screening capabilities of the agents, and the organizations.

2.1 The Project Environment

There exists a pool of projects of size 1. Projects can be either of good quality, $q = g$ (which is the case with probability α), or of bad quality, $q = b$ (which is the case with probability $1 - \alpha$). An implemented, good project gives rise to a profit X_1, while an implemented, bad project leads to a loss equal to $-X_2$.

2.2 The Agents

An agent $i \in I = \{1, \ldots, n\}$ can either accept, A, or reject, R, a project. That is, the action set D_i equals $D_i = \{A, R\}$ for every $i \in I$. Agent i is characterized by a pair of probabilities (p_i^g, p_i^b). The first element stands for the probability with which agent i accepts a good quality project, while the second represents the probability with which he accepts bad projects. This pair of probabilities captures the screening capabilities of i. I assume that an agent is fallible: some bad projects are accepted, while some good ones are rejected. Moreover, I assume that an agent is "better" than a randomizing device using a fair coin. Having projects selected by the toss of a coin means that half of the good and half of the bad projects are accepted. An agent does better, in that he accepts more than one out of two good projects, and rejects more than half of the time a bad project.[10]

[10]Defining "agent i is better than a randomizing device using a fair coin" in terms of the sign of the difference in profits generated by the addition of i or the device to some

Definition 1 *Agent i will be called* better *than a randomizing device using a fair coin if and only if $p_i^b < 1/2 < p_i^g$.*

Similarly, agent i will be called *better* than agent j if the former accepts more good projects than the latter, and rejects more bad projects than the latter.

Definition 2 *For any two agents i and j, agent i is called* better *than agent j if and only if $p_i^g > p_j^g$ and $p_i^b < p_j^b$. That i is better than j will be denoted by $i \succ j$.*

Assuming that every agent is better than a fair coin, but still fallible then amounts to:

Assumption 1 *For every agent $i \in I$, $0 < p_i^b < 1/2 < p_i^g < 1$.*

The set I will only contain agents whose screening characteristics are ordered in the following way.

Assumption 2 *For every $i, j \in I$ either $i \succ j$ or $j \succ i$.*

The possibility of identical agents is therefore excluded. This only strengthens the results I derive below. Although the agents are ordered in this sense, this does not mean that this ordering is known. In this chapter, I distinguish three types of information concerning the screening capabilities of the agents: no information at all, ordinal information, and cardinal information. The distinction is based on the degree of fineness of information.

existing organization A is, in general, deficient as it makes the ordinal statement of i being better or not than the device dependent on cardinal knowledge of the probabilities characterizing the agents working in A. The only case in which such cardinal knowledge is not required is when agent i is better or not than the device according to definition 1. The same holds, *mutatis mutandis* for the definition of agent i being better than agent j.

Definition 3 *If there is* no information *about the screening capabilities of any agent $i \in I$, one does not know the pair (p_i^g, p_i^b) of any agent, nor can one order the agents using definition 2. Indeed, one only knows that agents are fallible, and that they are better than a fair coin.*

The other extreme in terms of richness of information about the agents is cardinal information:

Definition 4 *There is* cardinal information *about the screening capabilities of all the agents $i \in I$ if for every i the pair (p_i^g, p_i^b) is known.*

In between no information at all and cardinal information about all the agents there is the situation of ordinal information.[11] Ordinal information means information about the ordering of agents in terms of their screening qualities.

Definition 5 *There is* ordinal information *about the screening capabilities of the agents $i \in I$ if only the ordering based on assumption 2 is known, and agents are known to be fallible and to be better than a randomizing device using a fair coin.*

2.3 The organizations

An organization is characterized by its structure and by the distribution of agents over the organizational positions.

Definition 6 *An* organizational structure Σ *is a finite binary arborescence,* i.e., *an organizational structure is a finite, directed, rooted tree, in which at every node σ two edges start.*[12]

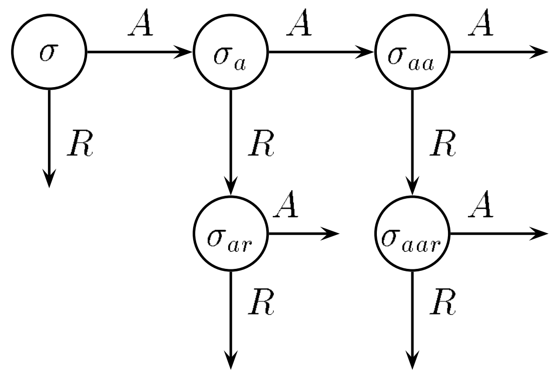

Figure 1: An organizational structure Σ

An example is the structure shown in figure 1. The nodes stand for organizational departments, bureaus, or desks, and the directed edges represent the direction of flow of evaluated projects. The labels on the edges starting at a node are associated with the actions taken at that node. Since an organizational structure is a binary arborescence, every node σ can be reached by just one, finite, ordered series of Accept and/or Reject decisions. Every node will be indexed by this series of decisions. For example, a node that is reached after an acceptance and a successive rejection, will be denoted by σ_{ar}. The root is denoted by σ. That part of the structure that starts with the node σ_{aa} is itself a structure, and will be called a sub–structure. It will be indexed by the unique series of decisions through which it can be reached. Hence, the sub–structure starting after first an acceptance and then a rejection is denoted by Σ_{AR}.

[11] I am not claiming that this is the only type of information between both extremes. Indeed, the analysis in the next section suggests that a few other degrees of fineness could be usefully introduced.

[12] The organization is a *tree*, since no project reaches one and the same desk twice. It is a *rooted* tree, since one and the same bureau is the first to evaluate every project. The tree is *directed* because projects flow just in one direction between two successive bureaus. The tree is *binary*, since at every organizational position a project can either accepted or rejected. Finally, the tree is *finite* since the number of nodes the structure contains is finite. For a discussion of graph terminology and concepts see, for example, L. R. Foulds (1992).

It will be useful to let j, l, and k stand for a finite series of A's and R's (or of a's and r's).[13] The symbol σ_j may even stand for the root σ, and analogously Σ_j may denote the whole structure Σ.

For every pair of nodes (σ_j, σ_l) let $\omega(\sigma_j, \sigma_l)$ denote the first common predecessor of σ_j and σ_l. Graphically, this is the first node that is on both the path back from σ_j to the root σ, and on the path back from σ_l to the root. The sub–structure $\Sigma(\sigma_j, \sigma_l)$ is important in the determination of the difference in profit ensuing from swapping nodes σ_j and σ_l.

Definition 7 *Let $\Sigma(\sigma_j, \sigma_l)$ be defined as the sub–structure that starts with node $\omega(\sigma_j, \sigma_l)$.*

$\Sigma(\sigma_j, \sigma_l)$ is, in some sense, the smallest sub–structure that contains both σ_j and σ_l.

Example In figure 1, $\omega(\sigma_{ar}, \sigma_{aa}) = \sigma_a$, while the sub–structure $\Sigma(\sigma_{ar}, \sigma_{aa})$ equals Σ_A.

Figure 2 shows the three different ways in which a node can be connected to its successive sub–structure(s). There are three basic building blocks that allow one to build any organizational structure. The first is a node σ_j, an edge labeled A, and a sub–structure Σ_{jA}. This building block is called a *hierarchical* connection, and will be represented by $\sigma_j \mathcal{H} \Sigma_{jA}$; the second is a node σ_j, an edge labeled R, and a sub–structure Σ_{jR}. Such a building block is called a *polyarchical* connection, and will be represented by $\sigma_j \mathcal{P} \Sigma_{jR}$; and thirdly a node σ_j, an edge labeled A, a sub–structure Σ_{jA}, an edge R, and a sub–structure Σ_{jR}. This building block is coined an *omniarchical* connection, and will be denoted by $\mathcal{O}(\sigma_j, \Sigma_{jA}, \Sigma_{jR})$. Since any sub–structure can be considered a structure itself, any organizational structure can be recursively constructed using these three building blocks. This can be done in either of two ways. Either one adds nodes at the ends

[13] Capitals will be used for (possible degenerate) sub–organizations, and small letters for single nodes.

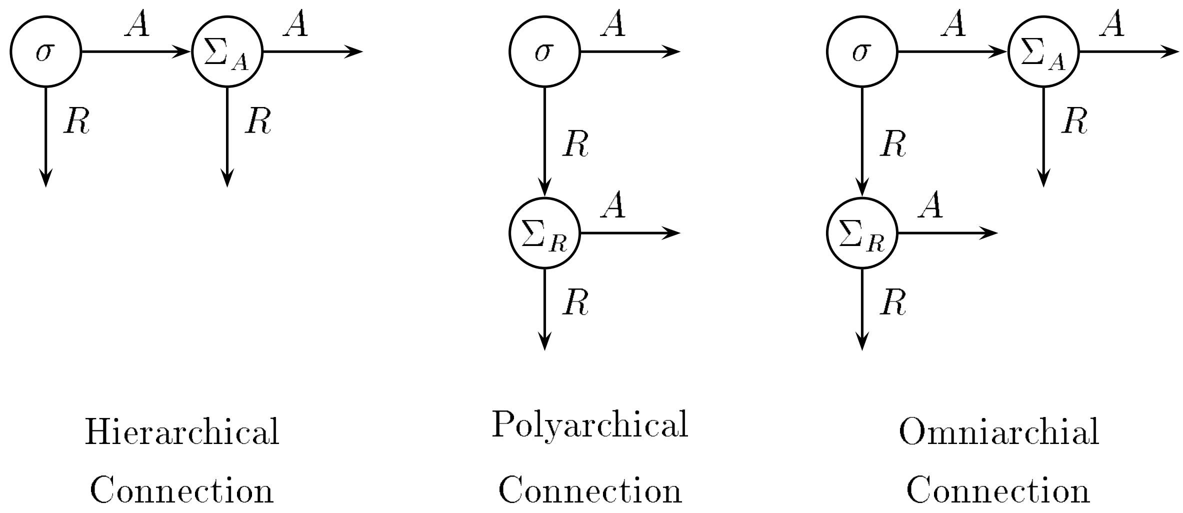

Figure 2: The three building blocks

of the arrows of nodes that are part of existing structures, or one adds nodes "in front of" existing structures, by having the arrows of the new nodes connecting to the existing structure(s). The latter way of forming structures is used in the proofs of the propositions in this chapter.

To make clear that a structure takes on a particular form, the symbol "$\stackrel{S}{=}$" is used. Hence, $\Sigma \stackrel{S}{=} \sigma\mathcal{P}\Sigma_A$ becomes a meaningful expression.

Example (continued) The structure in figure 1 is uniquely described by the expression $\Sigma \stackrel{S}{=} \sigma\mathcal{H}\mathcal{O}(\sigma_a, \sigma_{aa}\mathcal{P}\sigma_{aar}, \sigma_{ar})$.

A structure that contains only hierarchical (polyarchical) connections is called a *pure hierarchy (polyarchy)*. A *pure structure* refers to either of these. A *linear structure* may contain both hierarchical and polyarchical connections, but does certainly not have omniarchical connections. A linear structure consisting of both hierarchical and polyarchical connections is called a *mixed linear structure*. The linearity refers to the fact that the graphical representation of such a structure can be linear, with all nodes on one line, and with the label on the horizontal connection between any two nodes either R or A.

Associated with every structure Σ is a probability function $p(\Sigma; q)$, which gives the probability with which a project of quality q will be accepted. Often, the notation $p(\Sigma)$, without special reference to the precise quality q, will be used instead of $p(\Sigma; q)$. Obviously, the precise probability depends on the identity of the agents located at the various nodes. Let $p(\sigma_j)$ stand for the probability that a project will be accepted at node σ_j, and $p(\Sigma_j)$ for the probability that sub–structure Σ_j accepts a project. Exploiting the fact that Σ can be recursively constructed, one can define $p(\Sigma)$ using the following recursive definition.

$$p(\Sigma_j) = \begin{cases} p(\sigma_j)p(\Sigma_{jA}) & \text{if} \quad \Sigma_j \stackrel{S}{=} \sigma_j \mathcal{H} \Sigma_{jA} \\ p(\sigma_j) + (1 - p(\sigma_j))p(\Sigma_{jR}) & \text{if} \quad \Sigma_j \stackrel{S}{=} \sigma_j \mathcal{P} \Sigma_{jR} \\ p(\sigma_j)p(\Sigma_{jA}) + (1 - p(\sigma_j))p(\Sigma_{jR}) & \text{if} \quad \Sigma_j \stackrel{S}{=} \mathcal{O}(\sigma_j, \Sigma_{jA}, \Sigma_{jR}) \end{cases} \tag{1}$$

Example (continued) Consider the example, depicted in figure 1, of $\Sigma \stackrel{S}{=} \sigma \mathcal{H} \mathcal{O}(\sigma_a, \sigma_{aa} \mathcal{P} \sigma_{aar}, \sigma_{ar})$. Its probability function equals

$$p(\Sigma) = p(\sigma) \left[p(\sigma_a) \left(p(\sigma_{aa}) + (1 - p(\sigma_{aa}))p(\sigma_{aar}) \right) + (1 - p(\sigma_r))p(\sigma_{ar}) \right] \tag{2}$$

Within a given structure, the heterogeneous agents have to be allocated to nodes in such a way as to maximize the expected profit.

Definition 8 *An* allocation $\phi : I \rightarrow \Sigma$ *is a mapping from the set of agents* I *to the set of organizational positions* Σ.[14]

An organization specifies both a structure and an allocation of agents to organizational positions.

Definition 9 *The pair* (Σ, ϕ) *is called an* organization.

[14]Note that the allocation is a mapping from agents to nodes, not from nodes to agents. This implies no limitation in the framework studied here as I exclude the possibility of identical agents. Note moreover that one needs to increase the number of agents with the number of organizational positions.

The probability that an organization (Σ, ϕ) accepts a project of quality q is denoted by $p(\Sigma, \phi; q)$. It can be calculated by substituting into the probability function $p(\Sigma; q)$ the values of p_i^q in the organizational positions according to the allocation ϕ. Take the example of structure $\Sigma \overset{S}{=} \mathcal{O}(\sigma, \sigma_a \mathcal{P} \sigma_{ar} \mathcal{H} \sigma_{ara}, \sigma_r)$, and suppose that the allocation of agents $(1,2,3,4,5)$ equals $\phi(1) = (\sigma)$, $\phi(2) = (\sigma_a)$, $\phi(3) = (\sigma_{aa})$, $\phi(4) = (\sigma_{ar})$, and $\phi(5) = (\sigma_{aar})$. The probability that organization (Σ, ϕ) accepts a project of good quality then becomes

$$p(\Sigma, \phi; g) = p_1^g \left[p_2^g (p_3^g + (1 - p_3^g) p_5^g) + (1 - p_2^g) p_4^g \right] \tag{3}$$

The expected profit of organization (Σ, ϕ) equals

$$E(\Pi; \Sigma, \phi) = \alpha X_1 p(\Sigma, \phi; g) - (1 - \alpha) X_2 p(\Sigma, \phi; b) \tag{4}$$

Let $\Phi_\Sigma^*(I)$ be the set of allocations $\phi_\Sigma^*(I)$ that maximizes expression (4) for a given structure Σ (and for a given set I). The question the propositions in the next section answer is twofold. First of all, which structures Σ allow the elements of $\Phi_\Sigma^*(I)$ to be determined using no information about the characteristics of the agents at all? And secondly, which structures Σ make ordinal information about these characteristics necessary and sufficient to find the optimal allocation.

In proving these propositions the following notation is useful. If, for a given structure Σ, the allocation of two agents to a pair of organizational positions j and l can be swapped without affecting the expected profit of the organization this will be denoted by $j \sim l$. If, for a given structure Σ the agent located at node j is better than the agent at node l this will be written as $j \succ l$. Indeed, it proves to be convenient to say that "node j is better than node l" instead of "the agent located at node j is ...". In a similar vein, it will be convenient to "swap nodes", instead of "swapping agents" at particular nodes.

The exclamation mark "!" above a symbol means "should", or, in other words, that the relationship containing some binary operator to which the exclamation mark is added is required to hold for some other condition to be satisfied. This other condition is usually the limitation of the optimizer's knowledge to ordinal information about the characteristics of the agents. For example, "$j \overset{!}{\succ} l$" should be read as "node j should be better than node l", or better still, "the agent located at node j should be better than the agent at node l". Similarly, $\Sigma \overset{!S}{=} \sigma\mathcal{H}\Sigma_A$ means that the structure Σ should be equal to $\sigma\mathcal{H}\Sigma_A$. The symbol "$\overset{O}{\Rightarrow}$" will be used in conjunction with statements containing exclamation marks: "$statement_1 \overset{O}{\Rightarrow} statement_2$", where both statements contain an exclamation mark, means that if $statement_1$ should hold, then one can deduce that $statement_2$ should hold using exclusively ordinal information.

3 The Results

This section shows that the only structures that require no information about the screening capabilities of the agents in order to find the best allocation of these agents to organizational positions are the pure hierarchy and the pure polyarchy. It is also shown that the only structures for which ordinal information is both sufficient and necessary when determining the best allocation of heterogeneous agents are linear structures. The mere presence of one omniarchical connection implies the need to use cardinal information about some agents to find the optimal allocation.

A small digression is in order. It might be true that one observes *ex post* that the level of expected profits is left unaffected after a swap of agents between two nodes. This may be the case for some specific structure and some precise values of the characteristics of all the agents involved, with the latter depending on the organizational structure. One might then be tempted to conclude that "the ordering of agents does not matter". It is not in this sense that I use here the phrase "swapping agents leaves the expected profit unaffected." With the latter I mean that one knows *ex ante* and on the basis of the structure only, that swapping agents is immaterial as far as the level of profits is concerned.

The results can be derived using binary swappings of agents and comparing the ensuing difference in profit. The difference in profit resulting from swapping (agent i at) node σ_j and (agent i' at) node σ_l will be denoted by $\Delta E(\Pi; \sigma_j, \sigma_l)$. Let me denote the probability that a project of quality q reaches node $\omega(\sigma_j, \sigma_l)$ by $p^q(\sigma \rightarrow \omega(\sigma_j, \sigma_l))$. The difference in profit can then be written as

$$\Delta E(\Pi; \sigma_j, \sigma_l) = \alpha X_1 \psi(\sigma_j, \sigma_l; g) - (1-\alpha) X_2 \psi(\sigma_j, \sigma_l; b) \tag{5}$$

where

$$\psi(\sigma_j, \sigma_l; q) := (p^q(\sigma_j) - p^q(\sigma_l)) p^q(\Sigma(\sigma_j, \sigma_l)) p^q(\sigma \rightarrow \omega(\sigma_j, \sigma_l)) \tag{6}$$

for $q \in \{g, b\}$. The dependence of $\psi(\cdot)$ on q will often remain implicit, by simply writing $\psi(\sigma_j, \sigma_l)$. The function $p^q(\Sigma(\sigma_j, \sigma_l))$ depends at most on the characteristics of agents located at nodes that are part of the substructure $\Sigma(\sigma_j, \sigma_l)$. Note that $p^q(\Sigma(\sigma_j, \sigma_l))$ is *not* equal to the probability with which some substructure accepts a project of quality q. Instead, it is merely a function of the characteristics of the agents located at nodes in the substructure $\Sigma(\sigma_j, \sigma_l)$. Remember that this substructure is the smallest substructure of Σ that contains both σ_j and σ_l.[15] The probability $p^q(\sigma \rightarrow \omega(\sigma_j, \sigma_l))$ is equal to one for $q \in \{g, b\}$ if and only if $\sigma = \omega(\sigma_j, \sigma_l)$.

Example (continued) Suppose one swaps the agents at the nodes σ_{aa} and σ_{ar}. Then $\psi(\sigma_j, \sigma_l)$ equals

$$\psi(\sigma_j, \sigma_l) = (p(\sigma_{aa}) - p(\sigma_{ar})) \left[p(\sigma_a)(1 - p(\sigma_{aar})) - (1 - p(\sigma_a))\right] p(\sigma)$$

[15] To be specific, the function $p^q(\Sigma(\sigma_j, \sigma_l))$ depends exclusively on the (agents at the) nodes on the path connecting $\omega(\sigma_j, \sigma_l)$ and σ_j, and connecting $\omega(\sigma_j, \sigma_l)$ and σ_l, and on all the successors of σ_j and of σ_l.

As noted in section 2, any structure can be built recursively by adding nodes that precede existing structures. Suppose one has written out the expression for the change in profit $\Delta E(\Pi; \sigma_j, \sigma_l)$ for some pair (σ_j, σ_l) in $\Sigma_k = \Sigma(\sigma_j, \sigma_l)$, the smallest sub–structure containing both σ_j and σ_l. Suppose one makes a new structure by adding a node that precedes the existing structure Σ_k in a polyarchical or hierarchical way. Or suppose one constructs a new structure by combining two existing structures, one of which is Σ_k, in an omniarchical way. Does such an expansion change the expression $\Delta E(\Pi; \sigma_j, \sigma_l)$? Can it change the answer to the question whether information is required to allocate σ_j and σ_l correctly? Can it affect the level of detail in information required to allocate them correctly? Can this addition affect the optimal ordering of agents at nodes σ_j and σ_l?

As far as the expression $\Delta E(\Pi; \sigma_j, \sigma_l)$ is concerned, it cannot affect $p^q(\Sigma(\sigma_j, \sigma_l))$ as this part of the expression depends exclusively on nodes in $\Sigma(\sigma_j, \sigma_l)$, a set which is left unchanged by adding a node "at the front", and, in particular, before node $\omega(\sigma_j, \sigma_l)$ of the pre–existing structure. It does affect, however, $p^q(\sigma \rightarrow \omega(\sigma_j, \sigma_l))$, where σ is the newly added node. Indeed, the probability with which a project reaches the sub–structure $\Sigma(\sigma_j, \sigma_l)$ decreases.

Saying that no information is needed to optimally order the agents located at nodes σ_j and σ_l means the same as saying that swapping the position of agents at these positions leaves the expected profit unaffected, irrespective of the characteristics of the agents. Therefore, if no information is needed $\Delta E(\Pi; \sigma_j, \sigma_l) = 0$. Since all that is known about the value of $p^q(\sigma \rightarrow \omega(\sigma_j, \sigma_l))$ is that it is equal to one (if $\sigma = \omega(\sigma_j, \sigma_l)$) or less than one (if $\sigma \neq \omega(\sigma_j, \sigma_l)$), and since $p^q(\sigma_j) \neq p^q(\sigma_l)$ for $q \in \{g, b\}$, the only way to ensure $\Delta E(\Pi; \sigma_j, \sigma_l) \overset{!}{=} 0$ is by imposing $p^q(\Sigma(\sigma_j, \sigma_l)) \overset{!}{=} 0$ for $q \in \{g, b\}$, where the latter condition should hold for any pair of characteristics of the agents involved. Moreover, this condition is clearly sufficient. Therefore, if it can be shown that no information is needed to allocate agents correctly to the pair (σ_j, σ_l) then $p^q(\Sigma(\sigma_j, \sigma_l)) \overset{!}{=} 0$ must be shown to hold for $q \in \{g, b\}$, and vice versa. If this condition can be shown to hold for a sub–structure $\Sigma_k = \Sigma(\sigma_j, \sigma_l)$, the condition can also be shown to hold for the overall structure Σ, as the set of nodes on which it depends stays the same. In short, if no information is required to correctly allocate agents to a pair of nodes in Σ_k, no information is needed to allocate a pair of agents to the same pair of nodes when this structure is merely a sub–structure in a larger structure Σ. And hence, if the ordering of a pair of nodes can be proved not to matter in Σ_k, it can be proved not to matter in Σ.

By the same token, the necessity and sufficiency of ordinal information concerning agents' characteristics for optimal allocations extends from a sub–structure $\Sigma_k = \Sigma(\sigma_j, \sigma_l)$ to a structure Σ as a whole. Lemma 1 is instrumental in this respect. It shows that the sign of $p^q(\Sigma(\sigma_j, \sigma_l))$ should be the same for $q = b$ and $q = g$ for ordinal information to be sufficient.

Lemma 1 *In any structure Σ, for ordinal information to be sufficient to determine* $\text{sign}[\Delta E(\Pi; \sigma_j, \sigma_l)]$ *the condition*

$$\text{sign}[p^g(\Sigma(\sigma_j, \sigma_l))] \stackrel{!}{=} \text{sign}[p^b(\Sigma(\sigma_j, \sigma_l))] \tag{7}$$

must hold.

This analysis shows that the characteristics of the agents preceding the node $\omega(\sigma_j, \sigma_l)$ are not relevant as to whether no information, ordinal information, or cardinal information is necessary and sufficient to correctly allocate agents to nodes σ_j and σ_l. Let me state this in the following observation as this proves useful for future reference.

Observation 1 *Whether no knowledge at all, ordinal information, or more than ordinal information is necessary and sufficient to correctly allocate heterogeneous agents to the pair of nodes (σ_j, σ_l) only depends on the (characteristics of the agents located at) nodes contained in $\Sigma(\sigma_j, \sigma_l)$. In other words, the structure preceding node $\omega(\sigma_j, \sigma_l)$ and the agents located at such nodes can be ignored for this purpose.*

One can also conclude that adding nodes to existing structures cannot decrease the required level of detail in information needed to allocate correctly *every* pair of agents.

Proposition 1 *For any finite number of nodes, and for all pairs of characteristics* (p_i^g, p_i^b), $i = 1, \ldots, n$, *the only structures in which no information is required to correctly allocate heterogeneous agents are the pure hierarchy and the pure polyarchy.*

Proof As the proposition makes a statement about the whole space of organizational structures, and since structures can be recursively defined I apply the *principle of structural induction*, which is the structural equivalent of the principle of mathematical induction. In the *basis step* one proves that the statement holds for certain basic structures with a specific number of nodes. One then supposes that the statement holds for any structure containing at most n nodes, and then proves that the statement holds for any structure containing $n+1$ nodes. This is called the *hypothesis step* or the *induction hypothesis*.[16]

($\Leftarrow$) The simplest pure hierarchy and polyarchy both contain two nodes. The probability function of a pure hierarchy with two nodes equals $p(\Sigma) = p(\sigma)p(\sigma_a)$, which is clearly independent of the ordering of the agents. The same is true for a polyarchy consisting of two agents, as its probability function amounts to $p(\Sigma) = p(\sigma) + p(\sigma_r) - p(\sigma)p(\sigma_r)$. So the implication holds for the basic forms. Now assume that the implication holds for all structures Σ containing at most n nodes. Consider the structures *(a)* $\Sigma' = \sigma\mathcal{H}\Sigma$, *(b)* $\Sigma' = \sigma\mathcal{P}\Sigma$, and *(c)* $\Sigma' = \mathcal{O}(\sigma, \Sigma_1, \Sigma_2)$. Since in *(c)* the resulting structure is not pure the implication is trivially true. In case of *(a)*, for $\Sigma' = \sigma\mathcal{H}\Sigma$ to be pure, Σ should be pure. If Σ is a pure hierarchy, so is Σ', and it is straightforward to see that no information is required to correctly allocate agents. If Σ' is a pure polyarchy, the resulting structure Σ is linear, *i.e.*, not pure, and so the implication is trivially

[16] For a formal statement of the principle see Fitting (1990).

true. The same line of reasoning can be used in case of *(b)*. This shows that any pure hierarchy or polyarchy does not require information about the agents for them to be correctly allocated.

($\Rightarrow$) Of the three basic structures, the hierarchy of two nodes, the polyarchy containing two nodes, and the omniarchy of three nodes, only the first two do not require information. Moreover, they are pure. This proves the basis step. Assume that the implication holds for all structures Σ containing at most n nodes. Consider the structures *(a)* $\Sigma' = \sigma\mathcal{H}\Sigma$, *(b)* $\Sigma' = \sigma\mathcal{P}\Sigma$, and *(c)* $\Sigma' = \mathcal{O}(\sigma, \Sigma_1, \Sigma_2)$.

In *(a)*, if no information is required for Σ' then no information is required for Σ. This follows from observation 1. So, by the induction hypothesis Σ should be pure. If Σ equals a pure polyarchy, swapping the (agents located at) positions σ and σ_a gives rise to

$$\psi(\sigma, \sigma_a) = (p(\sigma) - p(\sigma_a))p(\Sigma_{AR}) \tag{8}$$

For $p(\Sigma_{AR}) = 0$, the structure Σ_{AR} should be empty, in which case Σ' would be a pure hierarchy of two nodes. The implication applies and is true. If $p(\Sigma_{AR}) > 0$, ordinal information is required as $\sigma \overset{!}{\succ} \sigma_a$ should hold. Hence, the implication is trivially true. If Σ equals a pure hierarchy, then so is Σ'. Clearly no information is required. This proves that in case *(a)* the implication holds. By the same token, the implication holds in *(b)*.

In case of *(c)*, for no information to be required for Σ', no information should be required for Σ_i, $i = 1, 2$. By the induction hypothesis Σ_1 and Σ_2 are pure. Swapping σ and any node in, say, Σ_1 reveals that ordinal information is required. This shows that the implication is trivially true. $\square$

The second proposition characterizes the organizational structures for which ordinal information is both sufficient and necessary when determining the optimal allocation of heterogeneous agents. If ordinal information is to be sufficient, pairwise comparisons of the type, “Should the better agent be located at node j or at node l?” provide the required insight into the ordering of the agents. The series of lemmas that follows is needed in the proof of the proposition.

Lemma 2 states that if Σ contains at least two nodes, for ordinal information to be sufficient to prove that $p^q(\sigma_j) \overset{!}{>} p(\Sigma)$ for either $q = b$ or $q = g$, the first two elements of organization Σ should be hierarchically connected.

Lemma 2 *If Σ contains at least two nodes and is linear, then*

$$p^q(\sigma_j) \overset{!}{>} p^q(\Sigma) \overset{O}{\Rightarrow} \Sigma \overset{!S}{=} \sigma_j \mathcal{H} \Sigma_{jA} \tag{9}$$

for any Σ_{jA}, and either $q = b$ or $q = g$.

Lemma 3 is the analogous result for $p^q(\sigma_j) \overset{!}{<} p^q(\Sigma)$:

Lemma 3 *If Σ contains at least two nodes and is linear then*

$$p^q(\sigma_j) \overset{!}{<} p^q(\Sigma) \overset{O}{\Rightarrow} \Sigma \overset{!S}{=} \sigma_l \mathcal{P} \Sigma_{lR} \tag{10}$$

for any Σ_{jA}, and where either $q = b$ or $q = g$.

The following lemma states that if ordinal information is sufficient to show that $\sigma_j \succ \sigma_l$, then this sufficiency extends to all orderings $\sigma_j \succ \sigma_k$ where the position of σ_l and σ_k can be swapped without affecting organizational performance.

Lemma 4 *Consider any three nodes σ_j, σ_l, and σ_k that are part of a linear structure Σ, and suppose that ordinal information is sufficient to show that $\sigma_j \succ \sigma_l$ and $\sigma_l \sim \sigma_k$ hold. Then ordinal information is sufficient to show that $\sigma_j \succ \sigma_k$ holds.*

Lemma 5 states that ordinal information is not sufficient to prove that one single node σ_l is better or worse than a linear structure Σ.

Lemma 5 *Ordinal information is not sufficient to prove that*

$$\mathrm{sign}[p^g(\sigma_l) - p^g(\Sigma)] \stackrel{!}{=} -\mathrm{sign}[p^b(\sigma_l) - p^b(\Sigma)] \tag{11}$$

holds in a linear structure Σ.

However, it cannot be shown either, using exclusively ordinal information, that the opposite holds:

Lemma 6 *Ordinal information is not sufficient to prove that*

$$\mathrm{sign}[p^g(\sigma_l) - p^g(\Sigma)] \stackrel{!}{=} \mathrm{sign}[p^b(\sigma_l) - p^b(\Sigma)] \tag{12}$$

holds in a linear structure Σ.

The lemmas 7 and 8 state that if the two structures Σ_j and Σ_l are characterized by the same connection between the first node and the subsequent sub–structure, ordinal information is not enough to show $\mathrm{sign}[p^g(\Sigma_j) - p^g(\Sigma_l)] \stackrel{!}{=} \mathrm{sign}[p^b(\Sigma_j) - p^b(\Sigma_l)]$.

Lemma 7 *Assume that* $\Sigma_j \stackrel{S}{=} \sigma_j \mathcal{H} \Sigma_{jA}$ *and* $\Sigma_l \stackrel{S}{=} \sigma_l \mathcal{H} \Sigma_{lA}$. *Then ordinal information information is not sufficient to show* $\mathrm{sign}[p^g(\Sigma_j) - p^g(\Sigma_l)] \stackrel{!}{=} \mathrm{sign}[p^b(\Sigma_j) - p^b(\Sigma_l)]$.

Lemma 8 *Assume that* $\Sigma_j \stackrel{S}{=} \sigma_j \mathcal{P} \Sigma_{jR}$ *and* $\Sigma_l \stackrel{S}{=} \sigma_l \mathcal{P} \Sigma_{lR}$. *Then ordinal information information is not sufficient to show* $\mathrm{sign}[p^g(\Sigma_j) - p^g(\Sigma_l)] \stackrel{!}{=} \mathrm{sign}[p^b(\Sigma_j) - p^b(\Sigma_l)]$.

In lemma 1 it was shown that $\mathrm{sign}[p^g(\Sigma(\sigma_j, \sigma_l))] \stackrel{!}{=} \mathrm{sign}[p^b(\Sigma(\sigma_j, \sigma_l))]$ for ordinal information to be sufficient. Imagine that one reduces the difference between any pair of characteristics $p^q(\sigma_k)$ and $p^q(\sigma_{k'})$, for all

$\sigma_k, \sigma_{k'} \in \Sigma(\sigma_j, \sigma_l)$, while maintaining the ordering of these agents. Clearly, if in the limit of no difference at all between any agents the function $p^q(\cdot)$ takes on opposite signs for $q = b$ and for $q = g$, then $p^q(\cdot)$ will also assume opposite sign for any ordering of the agents as long as their characteristics are almost identical. This is made precise in the following lemma for a special case:

Lemma 9 *If $p^g(\Sigma(\sigma_j, \sigma_l)) \to x$ for $p^g(\sigma_k) \to 1$ for all $\sigma_k \in \Sigma(\sigma_j, \sigma_l)$, and if $p^b(\Sigma(\sigma_j, \sigma_l)) \to y$ for $p^b(\sigma_k) \to 0$ for all $\sigma_k \in \Sigma(\sigma_j, \sigma_l)$, and where* $\text{sign}[x] = -\text{sign}[y]$*, then ordinal information is not sufficient to determine the optimal ordering of σ_j and σ_l.*

Note carefully that the lemma does not state that x should equal 1 and that y should equal 0. This does not have to hold as $p^q(\Sigma(\sigma_j, \sigma_l))$ is *not* the probability of acceptance of $\Sigma(\sigma_j, \sigma_l)$. I am now able to prove the second proposition of this chapter:

Proposition 2 *For any number of nodes, and for any pair of characteristics (p_1^g, p_i^b) of the agents, $i = 1, \ldots, n$, the only structures for which ordinal information is both necessary and sufficient are structures that are linear.*

Proof ($\Leftarrow$) First the basis step. Consider the two simplest linear structures, $\sigma\mathcal{H}\sigma_a\mathcal{P}\sigma_{ar}$ and $\sigma\mathcal{P}\sigma_r\mathcal{H}\sigma_{ra}$. It is straightforward to see that ordinal information is necessary and sufficient. This proves the basis step. Suppose that the implication holds for all structures Σ containing at most n nodes, and consider *(a)* $\Sigma' = \sigma\mathcal{H}\Sigma$, *(b)* $\Sigma' = \sigma\mathcal{P}\Sigma$, and *(c)* $\Sigma' = \mathcal{O}(\sigma, \Sigma_1, \Sigma_2)$. Note that in case *(c)* the structure is not linear, and therefore the implication is trivially true.

In *(a)*, if $\Sigma' = \sigma\mathcal{H}\Sigma$ is linear then Σ should be pure or linear. If Σ is a pure hierarchy then so is Σ', and so the implication does not apply. If Σ is a pure polyarchy, then Σ' is linear. Write $\Sigma' = \sigma\mathcal{H}\sigma_a\mathcal{P}\Sigma_{AR}$, with Σ_{AR}

a pure polyarchy. Therefore, $\sigma_a \sim \sigma_{ar} \sim \cdots \sim \sigma_{ar\ldots r}$. Moreover, $\sigma \overset{!}{\succ} \sigma_a$ and thus, by lemma 4, $\sigma \overset{!}{\succ} \sigma_{ar}, \ldots, \sigma_{ar\cdots r}$ etc. That is, ordinal information is necessary and sufficient.

If, on the other hand Σ is linear, then, by the induction step, ordinal information is necessary and sufficient for Σ to be correctly organized. So, by obervation 1, ordinal information is necessary for structure Σ'. Take any node $\sigma_j \in \Sigma$. Then either *(i)* $\Sigma_j = \sigma_j \mathcal{H} \Sigma_{jA}$ or σ_j is the final node, or *(ii)* $\Sigma_j = \sigma_j \mathcal{P} \Sigma_{jR}$.

In *(i)*, if all links between σ and σ_j are hierarchical (or if $\sigma_j = \sigma_a$), then $\sigma \sim \sigma_j$. In the contrary case, $\sigma \overset{!}{\succ} \sigma_j$.

In *(ii)*, $\sigma \overset{!}{\succ} \sigma_r$.

($\Rightarrow$) Of all the structures that contain at most three nodes, the implication holds: if ordinal information is necessary and sufficient, then the structure is linear. This is the basis step. Now suppose that the implication holds for all structures with at most n nodes, and consider *(a)* $\Sigma' = \sigma \mathcal{H} \Sigma$, *(b)* $\Sigma' = \sigma \mathcal{P} \Sigma$, and *(c)* $\Sigma' = \mathcal{O}(\sigma, \Sigma_1, \Sigma_2)$.

In case *(a)*, if ordinal information is necessary and sufficient for $\Sigma' = \sigma \mathcal{H} \Sigma$ then ordinal information must be sufficient (and perhaps be necessary) for Σ. This follows from observation 1. So, Σ is pure or linear by the induction step.

If Σ is a pure hierarchy, then Σ' is a pure hierarchy and therefore, by proposition 1 no information is required. If Σ is a pure polyarchy, then Σ' is linear. If Σ is linear, then Σ' is linear. This proves the correctness of the implication in case of *(a)*.

Case *(b)* can be solved in the same fashion.

Finally case *(c)*. Suppose ordinal information is necessary and sufficient. Then, by observation 1, ordinal information should be sufficient for Σ_1 and Σ_2. Therefore, by the induction step, Σ_1 and Σ_2 are linear or pure.

I split this case up in four sub–cases: *(i)* $\Sigma' \overset{S}{=} \mathcal{O}(\sigma, \sigma_a \mathcal{H} \Sigma_{AA}, \sigma_r \mathcal{P} \Sigma_{RR})$, *(ii)* $\Sigma' \overset{S}{=} \mathcal{O}(\sigma, \sigma_a \mathcal{P} \Sigma_{AR}, \sigma_r \mathcal{P} \Sigma_{RR})$, *(iii)* $\Sigma' \overset{S}{=} \mathcal{O}(\sigma, \sigma_a \mathcal{H} \Sigma_{AA}, \sigma_r \mathcal{H} \Sigma_{RA})$, and *(iv)* $\Sigma' \overset{S}{=} \mathcal{O}(\sigma, \sigma_a \mathcal{P} \Sigma_{AR}, \sigma_r \mathcal{H} \Sigma_{RA})$.

(i) Consider swapping the nodes σ_a and σ_r, and in particular

$$\psi(\sigma_a, \sigma_r) = (p(\sigma_a) - p(\sigma_r))p(\Sigma(\sigma_a, \sigma_r)) \tag{13}$$

with

$$p(\Sigma(\sigma_a, \sigma_r)) = p(\sigma)p(\Sigma_{AA}) + (1 - p(\sigma))p(\Sigma_{RR}) - (1 - p(\sigma)) \tag{14}$$

If $p(\sigma_j) \rightarrow 1$ for all $\sigma_j \in \Sigma(\sigma_a, \sigma_r)$, then $p(\Sigma(\sigma_a, \sigma_r)) \rightarrow 1$, while if $p(\sigma_j) \rightarrow 0$ for all σ_j, then $p(\Sigma(\sigma_a, \sigma_r)) \rightarrow -1$. Then, by lemma 1 and 9, ordinal information is not sufficient.

(ii) The analysis is based on the expression

$$\psi(\sigma_a, \sigma_r) = (p(\sigma_a) - p(\sigma_r))p(\Sigma(\sigma_a, \sigma_r))$$

with

$$p(\Sigma(\sigma_a, \sigma_r)) = p(\sigma)(1 - p(\Sigma_{AR})) - (1 - p(\sigma))(1 - p(\Sigma_{RR})) \tag{15}$$

If $p(\sigma_j) \rightarrow 0$ for all $\sigma_j \in \Sigma(\sigma_a, \sigma_r)$, then $p(\Sigma(\sigma_a, \sigma_r)) \rightarrow -1$, and therefore

$$\operatorname{sign}[p^q(\Sigma(\sigma_a, \sigma_r))] \overset{!}{=} -1 \tag{16}$$

for $q \in \{g, b\}$. Let me first look at the good projects. If $p^g(\Sigma_{AR}) < p^g(\Sigma_{RR})$ then $p^g(\Sigma(\sigma_a, \sigma_r)) > 0$, which violates condition (16). So the remaining case is $p^g(\Sigma_{AR}) \overset{!}{>} p^g(\Sigma_{RR})$. From

$$\psi(\sigma, \sigma_a) = (p(\sigma) - p(\sigma_a))(\Sigma(\sigma, \sigma_a)) \tag{17}$$

with

$$p(\Sigma(\sigma, \sigma_a)) = p(\Sigma_{AR}) - (p(\sigma_r) + (1 - p(\sigma_r))p(\Sigma_{RR})) \tag{18}$$

one can deduce that $\Sigma_{AR} \overset{S}{\neq} \sigma_{ar}\mathcal{P}\Sigma_{ARR}$ by lemma 8. Hence, of the three possible orderings of $p^g(\sigma)$, $p^g(\Sigma_{AR})$, and $p^g(\Sigma_{RR})$ satisfying $p^g(\Sigma_{RR}) \overset{!}{<} p^g(\Sigma_{AR})$, namely $p^g(\sigma) \overset{!}{<} p^g(\Sigma_{RR}) \overset{!}{<} p(\Sigma_{AR})$, $p^g(\Sigma_{RR}) \overset{!}{<} p^g(\sigma) \overset{!}{<} p^g(\Sigma_{AR})$, and $p^g(\Sigma_{RR}) \overset{!}{<} p^g(\Sigma_{AR}) \overset{!}{<} p^g(\sigma)$, only the latter does not require $\Sigma_{AR} \overset{!S}{=} \sigma_{ar}\mathcal{P}\Sigma_{ARR}$. Instead, this ordering requires $\Sigma_{AR} \overset{!S}{=} \sigma_{ar}\mathcal{H}\Sigma_{ARA}$, and $\Sigma_{RR} \overset{!S}{=} \sigma_{rr}\mathcal{H}\Sigma_{RRA}$. The latter inequality should hold for otherwise one cannot show using ordinal information only that $p^g(\Sigma_{RR}) \overset{!}{<} p^g(\Sigma_{AR})$

On the other hand, for the bad quality projects, one cannot impose $p^b(\Sigma_{AR}) \overset{!}{>} p^b(\Sigma_{RR})$, as this, in combination with $p^g(\Sigma_{AR}) \overset{!}{>} p^g(\Sigma_{RR})$, and with the restriction on the structures of Σ_{AR} and Σ_{RR} would require more than ordinal information by lemma 7. But, imposing $p^b(\Sigma_{AR}) \overset{!}{<} p^b(\Sigma_{RR})$ on its own is not enough to show $p^b(\Sigma(\sigma, \sigma_a)) < 0$, and so a restriction of the type $p^b(\sigma) \overset{!}{>} p^b(\Sigma_{AR})$ would be needed. However, any restriction of this type, in conjunction with the restriction $p^g(\Sigma_{RR}) \overset{!}{<} p^g(\Sigma_{AR}) \overset{!}{<} p^g(\sigma)$, would lead to the insufficiency of ordinal information, either by lemma 5 or 6. That is, in case *(ii)* more than ordinal information is needed.

(iii) From $\psi(\sigma, \sigma_r) = (p(\sigma) - p(\sigma_r))(\Sigma(\sigma, \sigma_r))$ with

$$p(\Sigma(\sigma, \sigma_r)) = p(\sigma_a)p(\Sigma_{AA}) - p(\Sigma_{AR}) \tag{19}$$

in combination with lemma 7 it follows that $\Sigma_{AR} \overset{!S}{\neq} \sigma_{ar}\mathcal{H}\Sigma_{ARA}$. From $\psi(\sigma_a, \sigma_r)$

$$\psi(\sigma_a, \sigma_r) = (p(\sigma_a) - p(\sigma_r))p(\Sigma(\sigma_a, \sigma_r)) \tag{20}$$

with

$$p(\Sigma(\sigma_a, \sigma_r)) = p(\sigma)p(\Sigma_{AA}) - (1 - p(\sigma))p(\Sigma_{AR}) \tag{21}$$

If $p(\sigma_j) \to 1$ for all $\sigma_j \in \Sigma(\sigma_a, \sigma_r)$, then $p(\Sigma(\sigma_a, \sigma_r)) \to 1$, and therefore, from lemma 1 it follows that

$$\text{sign}[p^q(\Sigma(\sigma_a, \sigma_r))] \overset{!}{=} + \tag{22}$$

for both $q = b$ and $q = g$. In particular, $p^b(\Sigma(\sigma_a, \sigma_r)) \overset{!}{>} 0$, which requires at least $p^b(\Sigma_{AA}) \overset{!}{>} p^b(\Sigma_{RA})$, and some restriction on $p^b(\sigma)$. Since $\Sigma_{RA} \overset{!S}{\neq} \sigma_{ra}\mathcal{H}\Sigma_{RAA}$, the only possible restriction is $p^b(\Sigma_{AA}) \overset{!}{>} p^b(\Sigma_{RA}) \overset{!}{>} p^b(\sigma)$. The same line of reasoning as used in *(ii)* leads to the conclusion that ordinal information is not sufficient either in this case.

(iv) Once again, the analysis is based on the expression $p(\Sigma(\sigma_a, \sigma_r))$:

$$\psi(\sigma_a, \sigma_r) = (p(\sigma_a) - p(\sigma_r)) \left[p(\sigma) - (p(\sigma)p(\Sigma_{AR}) + (1 - p(\sigma))p(\Sigma_{RA}))\right] \tag{23}$$

If $p(\sigma) \overset{!}{>} \max(p(\Sigma_{AR}), p(\Sigma_{RA}))$ for $q \in \{g, b\}$, then both $p^g(\sigma) \overset{!}{>} p^g(\Sigma_{AR})$ and $p^b(\sigma) \overset{!}{>} p^b(\Sigma_{AR})$ should hold. Lemma 6 shows that more than ordinal information is required to prove these statements jointly. By the same token, $p(\sigma) \overset{!}{<} \min(p(\Sigma_{AR}), p(\Sigma_{RA}))$ cannot be proved using ordinal information only. Hence,

$$p^q(\sigma) \in \left(\min\left(p^q(\Sigma_{AR}), p^q(\Sigma_{RA})\right), \max\left(p^q(\Sigma_{AR}), p^q(\Sigma_{RA})\right)\right)$$

for both $q = b$ and $q = g$ is the only remaining possibility. In view of lemmas 5 and 6, this gives rise to two possibilities:

(a) $p^g(\sigma) \overset{!}{\in} (p^g(\Sigma_{AR}), p^g(\Sigma_{RA}))$ and $p^b(\sigma) \overset{!}{\in} \left(p^b(\Sigma_{RA}), p^b(\Sigma_{AR})\right)$. The former condition implies $\Sigma_{AR} \overset{!S}{=} \sigma_{ar}\mathcal{H}\Sigma_{ARA}$ (see lemma 2), while the latter requires $\Sigma_{AR} \overset{!S}{=} \sigma_{ar}\mathcal{P}\Sigma_{ARR}$ (see lemma 2). These conditions cannot be satisfied at the same time.

(b) $p^g(\sigma) \overset{!}{\in} (p^g(\Sigma_{RA}), p^g(\Sigma_{AR}))$ and $p^b(\sigma) \overset{!}{\in} \left(p^b(\Sigma_{AR}), p^b(\Sigma_{RA})\right)$. The same conflicting requirements concerning the structure of Σ_{RA} as under *(a)* show the insufficiency of ordinal information. $\square$

It should be clear from the proof, and in particular from the first part, that there is a very simple procedure that ensures a correct allocation of agents to nodes in a linear (pure or mixed) structure.

Corollary 1 *Consider any linear structure Σ (eiher pure or mixed). Then a procedure to correctly allocate heterogeneous agents to the nodes $\sigma_j \in \Sigma$ is to position the best agent at the first node, the second best at the second, etc., until the last position has been filled.*

Note that this procedure holds for *any* linear structure Σ, independent of the particular arrangement of the individual nodes. Correctly allocating agents to positions in the presence of omniarchical relationships is much more complicated, and is beyond the scope of this chapter.

4 Conclusion

To err is human. The literature on organizational design, however, largely foregoes a detailed analysis of the importance this proverbial truth may have on the empirical studies it contains and on the recommendations it proposes. This chapter has related the presence of fallible agents to the detail of information required to position such agents correctly in organizational communication structures. This classification of information induces a typology of structures characterized by increasing levels of complexity.

Sah and Stiglitz conjectured that the relative simplicity of observed organizational structures reflects their alleged robustness to changes in the environment.[17] This chapter suggests another, complementary explanation. It is the organizational designer's limited knowledge about the characteristics of the employees in combination with the costs of obtaining such information that prohibits her from designing complex and theoretically superior structures. This type of argument is similar to the one proposed in the literature on contract design. The limited cognitive capabilities and knowledge of the contract designer are put forward as

[17] See Sah and Stiglitz (1988, p. 467).

reasons why real world contracts are extremely simple when compared to the optimal contracts derived in the literature.

Although reference was made to the possible differences in robustness of structures I have not dealt with this issue in this chapter. One might want to know which organizational building blocks are robust, and how their robustness is related to its internal structure and the distribution of employees. The simplicity of the agents' binary decision problem as modelled in this chapter may provide an ideal playground to gain insight into these matters. Relatedly, but from an empirical angle, one might like to gain understanding of the robustness of commonly observed communication structures.

Appendix: Proofs

In this appendix, the proofs of the lemmas are given.

Proof of lemma 1 Suppose one wants to know whether $\Delta E(\Pi;\sigma_j,\sigma_l) > 0$. If

$$\Delta E(\Pi;\sigma_j,\sigma_l) = \alpha X_1 \psi(\sigma_j,\sigma_l;g) - (1-\alpha) X_2 \psi(\sigma_j,\sigma_l;b) > 0 \qquad \text{(A.2)}$$

then the agents located at node σ_j and at σ_l are well positioned, while if the difference is negative, the position of the agents should be swapped. Equation (A.2) is equivalent to

$$\frac{(p^g(\sigma_j)-p^g(\sigma_l))}{(p^b(\sigma_j)-p^b(\sigma_l))}\frac{p^g(\Sigma(\sigma_j,\sigma_l))}{p^b(\Sigma(\sigma_j,\sigma_l))}\frac{p^g(\sigma\rightarrow\omega(\sigma_j,\sigma_l))}{p^b(\sigma\rightarrow\omega(\sigma_j,\sigma_l))} > \frac{1-\alpha}{\alpha}\frac{X_2}{X_1} \qquad \text{(A.3)}$$

for $\psi(\sigma_j,\sigma_l;b) > 0$ and

$$\frac{(p^g(\sigma_j)-p^g(\sigma_l))}{(p^b(\sigma_j)-p^b(\sigma_l))}\frac{p^g(\Sigma(\sigma_j,\sigma_l))}{p^b(\Sigma(\sigma_j,\sigma_l))}\frac{p^g(\sigma\rightarrow\omega(\sigma_j,\sigma_l))}{p^b(\sigma\rightarrow\omega(\sigma_j,\sigma_l))} < \frac{1-\alpha}{\alpha}\frac{X_2}{X_1} \qquad \text{(A.4)}$$

for $\psi(\sigma_j,\sigma_l;b) < 0$.

I show that if one is in possession of ordinal information only, equation (A.3) cannot be shown to hold, while equation (A.4) can be shown to hold if and only if the condition in equation (7) is satisfied. Given the limitation to ordinal information, the values of the constituent parts on the LHS of equations (A.3) and (A.4) are not known. Consider first equation (A.3). Since the right hand side is larger than zero, $(p^g(\sigma_j) - p^g(\sigma_l))p^g(\Sigma(\sigma_j, \sigma_l))p^g(\sigma \rightarrow \omega(\sigma_j, \sigma_l)) \overset{!}{>} 0$ should hold.

The intuition behind the proof is to fix a certain allocation of agents to nodes, and to vary the values of the agents' characteristics, while respecting the ordering of the agents. Remember that the word "ordering" refers to the ordering of the screening qualities of agents at various nodes ("agent located at node σ_j is better than agent located at σ_l"). Suppose a certain allocation ϕ applies, inducing an ordering of agents over organizational nodes, and conduct the following mental experiment. Keep the value of $(p^b(\sigma_j) - p^b(\sigma_l))$ and of $p^b(\Sigma(\sigma_j, \sigma_l))$ fixed, while reducing the difference between $p^g(\sigma_j)$ and $p^g(\sigma_l)$, without violating the ordering of σ_j and σ_l. Since $\Sigma(\sigma_j, \sigma_l)$ contains just a finite number of nodes, $0 \leq |p^g(\Sigma(\sigma_j, \sigma_l))| < M_g$. Therefore,

$$\frac{(p^g(\sigma_j) - p^g(\sigma_l))}{(p^b(\sigma_j) - p^b(\sigma_l))} \frac{p^g(\Sigma(\sigma_j, \sigma_l))}{p^b(\Sigma(\sigma_j, \sigma_l))} \frac{p^g(\sigma \rightarrow \omega(\sigma_j, \sigma_l))}{p^b(\sigma \rightarrow \omega(\sigma_j, \sigma_l))} \rightarrow 0 \tag{A.5}$$

for $p^g(\sigma_j) \rightarrow p^g(\sigma_l)$. Similarily, it can be shown that

$$\frac{(p^g(\sigma_j) - p^g(\sigma_l))}{(p^b(\sigma_j) - p^b(\sigma_l))} \frac{p^g(\Sigma(\sigma_j, \sigma_l))}{p^b(\Sigma(\sigma_j, \sigma_l))} \frac{p^g(\sigma \rightarrow \omega(\sigma_j, \sigma_l))}{p^b(\sigma \rightarrow \omega(\sigma_j, \sigma_l))} \rightarrow \infty \tag{A.6}$$

for $p^b(\sigma_j) \rightarrow p^b(\sigma_l)$ without violating the ordering of the agents. That is, one and the same ordering of agents can give rise to any positive number. Hence, ordinal information is not enough to show that equation (A.3) holds. For the same reason, ordinal information is not sufficient in case of equation (A.4) if $(p^g(\sigma_j) - p^g(\sigma_l))p^g(\Sigma(\sigma_j, \sigma_l))p^g(\sigma \rightarrow \omega(\sigma_j, \sigma_l)) < 0$.

Indeed, the only possibility when ordinal information may be sufficient is when

$$\frac{(p^g(\sigma_j) - p^g(\sigma_l))}{(p^b(\sigma_j) - p^b(\sigma_l))} \frac{p^g(\Sigma(\sigma_j, \sigma_l))}{p^b(\Sigma(\sigma_j, \sigma_l))} \frac{p^g(\sigma \to \omega(\sigma_j, \sigma_l))}{p^b(\sigma \to \omega(\sigma_j, \sigma_l))} \overset{!}{<} 0 \qquad \text{(A.7)}$$

for $(p^b(\sigma_j) - p^b(\sigma_l))p^b(\Sigma(\sigma_j, \sigma_l))p^b(\sigma \to \omega(\sigma_j, \sigma_l)) \overset{!}{<} 0$. Because of assumption 2, the expression

$$\frac{p^g(\sigma_j) - p^g(\sigma_l)}{p^b(\sigma_j) - p^b(\sigma_l)} < 0$$

holds for every pair of agents, while

$$\frac{p^g(\sigma \to \omega(\sigma_j, \sigma_l))}{p^b(\sigma \to \omega(\sigma_j, \sigma_l))} > 0$$

and therefore, in the light of equation (A.7), for ordinal information to be sufficient

$$\text{sign}[p^g(\Sigma(\sigma_j, \sigma_l))] \overset{!}{=} \text{sign}[p^b(\Sigma(\sigma_j, \sigma_l))] \qquad \text{(A.8)}$$

should hold, which is condition (7). □

Proof of lemma 2 Consider $p^g(\sigma_j) \overset{!}{>} p^g(\Sigma)$ (the same line of reasoning applies for $q = b$). Then, either *(i)* $\Sigma \overset{S}{=} \sigma_l \mathcal{P} \Sigma_{jR}$ or *(ii)* $\Sigma \overset{S}{=} \sigma_l \mathcal{H} \Sigma_{jA}$ as Σ is linear. In case *(i)*, $p^g(\sigma_j) \overset{!}{>} p^g(\Sigma) = p^g(\sigma_l) + (1 - p^g(\sigma_l))p^g(\Sigma_{lR})$. However, for $p^g(\sigma_j) - p^g(\sigma_l)$ sufficiently small, and for *any* ordering of σ_j, σ_l, and the nodes of Σ_{lR}, $p^g(\sigma_j) < p^g(\sigma_l) + (1 - p^g(\sigma_l))p^g(\Sigma_{lR})$ holds. Hence, ordinal information is not sufficient. In case *(ii)*, $p^g(\Sigma) = p^g(\sigma_l)p^g(\Sigma_l)$. Hence, if $p^g(\sigma_j) > p^g(\sigma_l)$ then $p^g(\sigma_j) > p^g(\Sigma)$. That is, ordinal information can be sufficient. □

Proof of lemma 3 The proof used for lemma 2 also applies, *mutatis mutandis*, for this lemma. □

Proof of lemma 4 By proposition 1 if $\sigma_l \sim \sigma_k$ then either only hierarchical or only polyarchical building blocks are used in connecting nodes σ_l

and σ_k. Therefore $\sigma_k \in \Sigma(\sigma_j, \sigma_l)$. Since ordinal information is sufficient to show that $\sigma_j \overset{!}{\succ} \sigma_l$, $p^q(\Sigma(\sigma_j, \sigma_l)) \overset{!}{>} 0$ can be shown to hold using ordinal information only. Since $\sigma_l \sim \sigma_k$, $\sigma_l \in \Sigma(\sigma_j, \sigma_k)$, and $p(\Sigma(\sigma_j, \sigma_k)) = p(\Sigma(\sigma_j, \sigma_l))$. Hence, $\text{sign}[p^q(\Sigma(\sigma_j, \sigma_k))] = \text{sign}[p^q(\Sigma(\sigma_j, \sigma_l))]$, and so $\sigma_j \overset{!}{\succ} \sigma_k$. $\square$

Proof of lemma 5 Suppose $p^g(\sigma_l) \overset{!}{>} p^g(\Sigma_j)$ and $p^b(\sigma_l) \overset{!}{<} p^b(\Sigma_j)$ (the same line of reasoning that follows can be applied to the opposite case $p^g(\sigma_l) \overset{!}{<} p^g(\Sigma_j)$ and $p^b(\sigma_l) \overset{!}{>} p^b(\Sigma_j)$). Then, from lemma 2, $\Sigma_j \overset{!S}{=} \sigma_j \mathcal{H} \Sigma_{jA}$, while lemma 3 shows that $\Sigma_j \overset{!S}{=} \sigma_j \mathcal{P} \Sigma_{jA}$. This is a contradiction. $\square$

Proof of lemma 6: I discuss the case where *(i)* $p^g(\sigma_l) \overset{!}{>} p^g(\Sigma)$ and $p^b(\sigma_l) \overset{!}{>} p^b(\Sigma)$, and the case where *(ii)* $p^g(\sigma_l) \overset{!}{<} p^g(\Sigma)$ and $p^b(\sigma_l) \overset{!}{<} p^b(\Sigma)$ in turn.

(i) From lemma 2, $\Sigma \overset{!S}{=} \sigma_j \mathcal{H} \Sigma_{jA}$, implying that

$$p^q(\sigma_l) \overset{!}{>} p^q(\Sigma) \;\overset{Q}{\Rightarrow}\; \begin{cases} p^g(\sigma_l) \overset{!}{>} p^g(\sigma_j) \quad \wedge \quad p^b(\sigma_l) \overset{!}{>} p^b(\Sigma_{jA}) \quad \text{or} \\ p^b(\sigma_l) \overset{!}{>} p^b(\sigma_j) \quad \wedge \quad p^g(\sigma_l) \overset{!}{>} p^g(\Sigma_{jA}) \end{cases} \tag{A.9}$$

where Σ_{jA} is either *(a)* a degenerate (sub–)organization equal to $\Sigma_{jA} = \sigma_{ja}$, or *(b)* a proper (sub–)organization containing at least two nodes. In case *(a)*, the conditions in equation (A.9) can be rewritten as

$$p^q(\sigma_l) \overset{!}{>} p^q(\Sigma) \;\overset{Q}{\Rightarrow}\; \begin{cases} \sigma_l \overset{!}{\succ} \sigma_j \quad \wedge \quad \sigma_{ja} \overset{!}{\succ} \sigma_l \quad \text{or} \\ \sigma_j \overset{!}{\succ} \sigma_l \quad \wedge \quad \sigma_l \overset{!}{\succ} \sigma_{ja} \end{cases} \tag{A.10}$$

However, since $\Sigma \overset{!S}{=} \sigma_j \mathcal{H} \sigma_{ja}$, $\sigma_j \sim \sigma_{ja}$ holds, and thereore, by lemma 4 either $\sigma_l \overset{!}{\succ} \sigma_j \sim \sigma_{ja}$ or $\sigma_j \sim \sigma_{ja} \overset{!}{\succ} \sigma_l$, which violates either condition mentioned in equation (A.10). This shows that both $p^g(\sigma_l) \overset{!}{>} p^g(\Sigma)$ and $p^b(\sigma_l) \overset{!}{>} p^b(\Sigma)$ cannot be shown using ordinal information only when $\Sigma_{jA} = \sigma_{ja}$.

In case *(b)*, $p^q(\sigma_l) \overset{!}{>} p^q(\Sigma_{jA}) \overset{Q}{\Rightarrow} \Sigma_{jA} \overset{!S}{=} \sigma_{ja}\mathcal{H}\Sigma_{jAA}$. Since $\Sigma \overset{!S}{=} \sigma_j\mathcal{H}\sigma_{ja}\mathcal{H}\Sigma_{jAA}$, $\sigma_j \sim \sigma_{ja}$ should hold. Consider the top line of condition A.9 first. Since $\sigma_j \sim \sigma_{ja}$ should hold, the condition can be rewritten as $\sigma_l \overset{!}{\succ} \sigma_j \sim \sigma_{ja} \quad \wedge \quad p^b(\sigma_l) \overset{!}{>} p^b(\Sigma_{jA})$. The latter part boils down to $p^b(\sigma_l) \overset{!}{>} p^b(\Sigma_{jA}) = p^b(\sigma_{ja})p^b(\Sigma_{jAA})$, which, together with $p^b(\sigma_l) \overset{!}{<} p^b(\sigma_{ja})$, implies $p^b(\sigma_l) \overset{!}{>} p^b(\Sigma_{jAA})$. That is, Σ_{AA} should be hierarchically structured, and one enters an infinite regress. However, structures are assumed to be finite. Therefore, more than ordinal information is required. The proof of the insufficiency of ordinal information in case of the condition at the bottom line of (A.9) proceeds in the same fashion.

(ii) From lemma 3, $\Sigma \overset{!S}{=} \sigma_a\mathcal{P}\Sigma_{jR}$, implying that

$$p^q(\sigma_l) \overset{!}{<} p^q(\Sigma) \overset{Q}{\Rightarrow} \begin{cases} p^g(\sigma_l) \overset{!}{<} p^g(\sigma_j) \quad \wedge \quad p^b(\sigma_l) \overset{!}{<} p^b(\Sigma_{jR}) \quad \text{or} \\ p^b(\sigma_l) \overset{!}{<} p^b(\sigma_j) \quad \wedge \quad p^g(\sigma_l) \overset{!}{<} p^g(\Sigma_{jR}) \end{cases} \tag{A.11}$$

The same way of reasoning as under *(i)* applies. Hence, ordinal information is not sufficient to show that both $p^g(\sigma_l) < p^g(\Sigma)$ and $p^b(\sigma_l) < p^b(\Sigma)$ hold. □

Proof of lemma 7 I discuss the case $\text{sign}[p^g(\Sigma_j) - p^g(\Sigma_l)] \overset{!}{=} \text{sign}[p^b(\Sigma_j) - p^b(\Sigma_l)] = +$ (This implies no limitation as one can freely interchange the structures Σ_j and Σ_l). Since $\Sigma_j \overset{S}{=} \sigma_j\mathcal{H}\Sigma_{jA}$ and $\Sigma_l \overset{S}{=} \sigma_l\mathcal{H}\Sigma_{lA}$, $p^g(\Sigma_j) \overset{!}{>} p^g(\Sigma_l)$ equals $p^g(\sigma_j)p^g(\Sigma_{jA}) \overset{!}{>} p^g(\sigma_l)p^g(\Sigma_{lA})$, while $p^b(\Sigma_j) \overset{!}{>} p^b(\Sigma_k)$ amounts to $p^b(\sigma_j)p^b(\Sigma_{jA}) \overset{!}{>} p^b(\sigma_l)p^b(\Sigma_{lA})$. For both these conditions to hold either of the following set of conditions must hold:

$$\begin{cases} p^g(\sigma_j) \overset{!}{>} p^g(\sigma_l) \\ p^g(\Sigma_{jA}) \overset{!}{>} p^g(\Sigma_{lA}) \\ p^b(\sigma_j) \overset{!}{>} p^b(\Sigma_{lA}) \\ p^b(\Sigma_{jA}) \overset{!}{>} p^b(\sigma_l) \end{cases} \tag{A.12}$$

or

$$
\begin{cases}
p^b(\sigma_j) \overset{!}{>} p^b(\sigma_l) \\
p^b(\Sigma_{jA}) \overset{!}{>} p^b(\Sigma_{lA}) \\
p^g(\sigma_j) \overset{!}{>} p^g(\Sigma_{lA}) \\
p^g(\Sigma_{jA}) \overset{!}{>} p^g(\sigma_l)
\end{cases}
\tag{A.13}
$$

In case of condition (A.12) it follows from $p^b(\sigma_j) \overset{!}{>} p^b(\Sigma_{lA})$ that $\Sigma_{lA} \overset{!S}{=} \sigma_{la}\mathcal{H}\Sigma_{lAA}$, which in turn implies that $p^b(\sigma_j) \overset{!}{>} p^b(\sigma_{la})p^b(\Sigma_{lAA})$ and $\sigma_l \sim \sigma_{la}$. Moreover, since $p^g(\sigma_j) \overset{!}{>} p^g(\sigma_l)$, $\sigma_j \overset{!}{\succ} \sigma_l$ holds. Therefore $\sigma_j \overset{!}{\succ} \sigma_l \sim \sigma_{la}$ or $p^b(\sigma_{la}) \overset{!}{>} p^b(\sigma_j)$. The latter implication together with $p^b(\sigma_j) \overset{!}{>} p^b(\sigma_{la})p^b(\Sigma_{lAA})$ shows that $p^b(\sigma_j) \overset{!}{>} p^b(\Sigma_{lAA})$ should hold. That is $\Sigma_{lAA} \overset{!S}{=} \sigma_{laa}\mathcal{H}\Sigma_{lAAA}$, and one enters an infinite regress. This proves that ordinal information is insufficient, since the structures are finite. In case of condition (A.12) one enters an infinite regress for the same reason. □

Proof of lemma 8 The structure of the conditions that should hold for $\text{sign}[p^g(\Sigma_j) - p^g(\Sigma_l)] \overset{!}{=} \text{sign}[p^b(\Sigma_j) - p^b(\Sigma_l)] = +$ are identical to the structure of the conditions of lemma 7. One can now derive an infinite regress by showing that Σ_j should be an infinite pure polyarchy, which is impossible given the limitation to finite structures. □

Proof of lemma 9 Since the function $p(\Sigma(\sigma_j, \sigma_l))$ is continuous in its arguments, *any* ordering of the nodes $\sigma_k \in \Sigma(\sigma_j, \sigma_l)$, with values $p^g(\sigma_k)$ sufficiently close to 1 satisfies $\text{sign}[p^g(\Sigma(\sigma_j, \sigma_l))] = \text{sign}[x]$.

Similarily, *any* ordering of the nodes $\sigma_k \in \Sigma(\sigma_j, \sigma_l)$, with values $p^b(\sigma_k)$ sufficiently close to 0 satisfies $\text{sign}[p^b(\Sigma(\sigma_j, \sigma_l))] = \text{sign}[y]$. That is, one and the same ordering can give rise to opposite signs of $p(\Sigma(\sigma_j, \sigma_l))$. Then, by lemma 1, ordinal information is not sufficient. □

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