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DEPARTMENT OF ECONOMICS

Essays in Macroeconomics

Viktor Marinkov

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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Abstract

This thesis contains three chapters. The first two consider deviations from rational expectations for understanding the unprecedentedly long period of a binding zero lower bound (ZLB) since the Great Recession. There I show that if agents are adaptively learning, Central Banks can use forward guidance to guide them through the novel economic environment. In the third chapter I take a more long-run structural outlook to study the interplay of skills, technologies and complementarities for understanding differences in labour market outcomes across OECD countries.

The first chapter studies the effects of forward guidance (FG) from a novel perspective. Instead of considering FG as a promise for future actions or providing better forecasting, the Central Bank (CB) in the model is giving a signal about its own reaction function. The CB uses FG as a communication device to signal a policy change. The main findings are that clear communication increases welfare compared to no communication, yet vague messages prove ineffective.

The second chapter considers the ZLB as an informational curtain for adaptively learning agents as they cannot observe the path of the interest rate. In a model I show that this results in expectations disagreement between the agents and the CB, consistent with the data. The disagreement coupled with the learning of the agents results in explosive dynamics. Forward guidance is shown to restore stability at the ZLB by preventing spurious expectational drift.

The third chapter studies the relationship between returns to skill and assortative matching. Using the PIAAC cognitive skills dataset I show that: returns to skill are systematically related to industrial sorting; high-skilled industries have more assortative matching of workers from all occupations; and more developed countries have less mismatch. I further build a model to illuminate the mechanism. I find that rich countries experience a trade-off of lower overall mismatch but higher cross-sectoral mismatch, yet due to higher search frictions poorer ones end up being more mismatched overall.

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Contents

Chapter 1

Policy Change and Forward Guidance

"By setting out how it is likely to set policy in the future, the MPC can help individuals understand how it intends to trade off the speed with which it returns inflation to the target against the scope for economic expansion. In addition, by explaining the reasons underlying that view, the MPC can also help individuals understand how it is likely to change policy in response to unanticipated developments as and when they occur."
Bank of England (2013)

1.1 Introduction

The onset of the global financial crisis in 2008 brought the policy rates of eventually all major Central Banks (CBs) to their effective lower bounds. With their primary policy instrument pinned at a sub-optimal level for the immediate future, CBs resolved to alternative measures in hope of mitigating the downturn of the crisis and contributing to a faster recovery. Among other things, Central Banks began providing information for the future path of the policy rate - a rarity in the past. Such announcements constitute what is called "Forward Guidance" and this paper proposes a novel approach to its analysis.

The literature on Forward Guidance (FG) largely agrees that the main channel of influence of FG is the information conveyed for the future path of the policy rate. There are two main classifications of FG depending on the underlying reasons for its use. The seminal work of Krugman et al. (1998) and Eggertsson and Woodford (2003) showed that promises of lower interest rates for longer can largely mitigate the

negative effects of a binding zero lower bound (ZLB) on interest rates. The stimulus comes through agents expecting low interest rates in the future (i.e. accommodative monetary policy) and higher inflation, hence cutting back less on present investment and consumption. Campbell et al. (2012) label this approach Odyssean Forward Guidance. Campbell et al. (2012) also acknowledge a more established form of FG, pursued by the Reserve Bank of New Zealand and the Riksbank in Sweden, for example. In essence, CBs engage in regular forecasts of the path of their policy rate, hence it was dubbed Delphic Forward Guidance. This type of FG may be useful to the public if the CB has better information about the state of the shocks that hit the economy.

This paper, however, considers the possibility that a Central Bank uses Forward Guidance not as a promise for future actions (Odyssean) or providing better forecasting facilities (Delphic), rather the CB is giving a signal about its own reaction function. In particular, under the ZLB if agents are unaware of a policy regime change which prolongs the ZLB, or for some reason misjudge the CB's reaction function and bias their expectations, the CB can use the expected conditions (e.g. date or thresholds) for a lift-off of the policy rate from the ZLB as a guiding tool for agents to correct their beliefs.

Indeed, statements from Central Bank officials in the recent years suggest that there is a discrepancy between the beliefs of the private sector and the CB. Yellen (2012) claims that in the last decades the policy of the Federal Reserve has become predictable to the private agents. However she continues, the recent crisis made it harder for agents to foresee policy. Bank of England (2013) clarifies that FG announcements aim to convey to the public the relative trade-off between output gap and inflation stabilization that the CB is considering. Although Delphic FG does increase the transparency of the monetary policy function, Walsh (2014) argues that in normal times when a CB has for a long time acted in a systematic manner, such forecasts have low if any informational content. Moreover, Faust and Wright (2009) find no convincing evidence of any forecasting advantage of CB over private agents. Cœuré (2013) and Fed(2011), on the other hand, reject the idea of Odyssean FG on the grounds that the CB would not like to commit itself in a time-inconsistent

manner. Also, Faust (2015) finds no overshooting relative to target in the Federal Open Market Committee's (FOMC) inflation forecasts, which is assumed to occur with Odyssean FG as in Eggertsson and Woodford (2003).

To tackle the view that under a binding ZLB agents cannot observe a policy change this paper drops the ubiquitous assumption of rational expectations (RE) and models agents as adaptive learners. The literature on adaptive learning in macroeconomics has shown that is a plausible framework of analysing agents behaviour. Evans and Honkapohja (2001) develop the theory and criteria for adaptive learners' convergence to the rational expectations equilibrium. Milani (2007) shows that an economy with adaptive learners is better able to match macroeconomic data persistence than RE models are. Finally, for a survey of experimental support for adaptive learning as a paradigm refer to Duffy (2014).

This approach allows for modelling imperfect knowledge of the CB's reaction function. Assuming that it is too costly, implausible or too restrictive that a CB can communicate its entire policy rule, FG can be used as a simple signal for a policy change. The mechanism builds on the lack of real-time observations under ZLB that adaptive learners could use for learning the new policy function. Under the ZLB and after a policy change[1] announcements of the anticipated date of departure from ZLB could potentially help the learning agents update their perceived policy function, thus making less sub-optimal decisions[2]. This mechanism is absent in RE models where agents know the true reaction function by construction. Thus, this paper strives to analyse Forward Guidance at the zero lower bound as a communication technology for the Central Bank after a policy change. It then asks the questions: *Should the Central Bank try communicate its changed reaction function to the public? What are the benefits and dangers of doing so?*

The main results are that communication does increase welfare, yet an ambiguous message may be largely inefficient. The modelling framework also captures realistic heterogeneous beliefs about the Central Bank's reaction function.

[1] After all, the ZLB is usually hit only in extreme circumstances which may well warrant a change in the policy reaction function.

[2] Naturally, a full internalization of the new policy function would lead to optimal results as later shown.

These results are supported by the evidence Engen et al. (2015) gather on private agents' expectations in the Blue Chip Economic Indicators data. First, they show that before switching to date- and state-based forward guidance in September 2011 the Federal Reserve's prior open-ended forward guidance did little to move market expectations. Later explicit announcements had a much more pronounced effect in shifting expectations of the interest rate, inflation and consumption paths. Second, they estimate private agents' perceived Taylor rule and find that during the ZLB their expectations imply a much higher reaction to economic slack in the FED's policy function. This is precisely the experiment I consider here.

The paper proceeds as follows. Section 1.2 discusses the related literature. Section 3.4 describes the model of the economy. Section 1.4 explains the expectations formation of the agents. Section 1.5 outlines the studied experiments. Section 1.6 presents the main results of the study, while Section 1.7 performs some robustness checks. Section 1.8 shows that the model can replicate some heterogeneous expectations found in the data. Finally, Section 3.5 concludes.

1.2 Literature Review

The literature on Forward Guidance largely agrees that the main channel of influence of FG is the information conveyed for the future path of the policy rate. Moreover, Woodford (2012) argues that the predominant effects of the LSAP programmes in the United States were actually achieved through the implicit signalling of the future path of interest rates contained in the announcements. In influential empirical work on the effectiveness of quantitative policies (QE) in the US Gagnon et al. (2011), Bauer and Rudebusch (2011) and Krishnamurthy and Vissing-Jorgensen (2013) find that at least 30-50% of the announcement effects of QE by the FOMC can be accounted to expectations about the federal funds rate (FFR) future path. Overall, Woodford (2012)'s suggestion is to signal information about future interest rates with a clear target as to preserve credibility and ensure that the policy is properly understood. This prescription is reminiscent of the seminal work of Eggertsson and Woodford (2003) who show that in a canonical New Keynesian (NK) model the

optimal policy in the face of binding zero lower bound is a "output-gap-weighted price level targeting". The essence of their result is the backward-looking nature of a price level targeting regime (contrary to inflation targeting regime). Then, commitment to such a policy ensures that in the future the economy will be compensated by lax monetary stance in order to make up for the inefficiency of the period under ZLB and for the price level to be brought back on target. They show that this policy is very effective at stabilizing the output gap and inflation at relatively mild costs of elevated inflation in the subsequent periods. Nonetheless, Levin et al. (2009) argue that although FG can be an effective tool against moderate natural rate shocks - "Great Moderation"-type of shock, it falls short of coping *alone* with "Great Recession"-style shocks. Levine et al. show that the large effects of FG in Eggertsson and Woodford (2003) are due to their parametrization of preferences and a two-state Markov process shock, and that more realistic values and shock processes significantly dampen the effectiveness of FG. They conclude that additional policy actions are need in times of severe crisis.

The aforementioned channel of FG has been dubbed *Odyssian Forward Guidance* by Campbell et al. (2012) in that it contains an (assumed) credible promise for keeping interest rates lower than the standard policy reaction function would imply in the initial stages of recovery. The literature analysing Odyssian FG relies heavily on rational agents who perfectly understand the economy's model. Moreover, such a policy is only credible with backward-looking monetary policy (MP) rules and not with the predominant forward-looking inflation targeting regimes of central banks, thus rendering it time-inconsistent. In fact, Del Negro et al. (2012) employ a specification for the interest rate rule of a CB in a NK model with rational expectations (RE) that features anticipated future monetary policy shocks as forward guidance:

$$i_t = \chi_\pi \pi_t + \chi_x x_t + \varepsilon_t^{MP} + \sum_{l=1}^{L} \varepsilon_{l,t-l}^{FG}$$

where π stands for inflation, x for the output gap, ε^{MP} for random monetary policy disturbances and $\varepsilon_{l,t-1}^{FG}$ for FG news shocks realized at period $t-l$ and regarding the interest rate rule at period t, i.e. Forward Guidance of l periods ahead. Del Negro

et al. find an unexpectedly large positive effect of Forward Guidance when estimated on the US economy and label this result *"the forward guidance puzzle"*. Yet, this approach may be questioned on two important grounds. First, assuming that under RE: $\mathbb{E}_t \varepsilon^{FG}_{l,t+L-l} = 0, \quad \forall\ L > l \geq 1$, i.e. agents *never* expect Forward Guidance to occur, and hence the CB can arbitrarily use these FG shocks to stimulate current demand. Second, Harrison (2014) shows that in RE NK models employing the former rule FG shocks would need to be too large in order to match the findings of Del Negro et al. (2012). In particular, Harrison (2014) shows that if agents judge policy shocks based on old experience with a statistical confidence level of 5%, they would judge that the FG shocks of Del Negro et al. (2012) are *not* a modest policy intervention, but rather a regime shift. His analysis resembles an empirical measure for assessing the Lucas' critique.

To the best of my knowledge, there are only a few papers that consider modelling Forward Guidance in an environment with deviations from the rational expectations hypothesis. Cole (2015) incorporates Del Negro et al. (2012)'s policy rule specification in a baseline NK model with agents who are adaptive learners. Overall, he concludes that FG is less effective under adaptive learning than under RE and that Central Banks should be wary of employing it if their internal models feature rational agents.

In another application of adaptive learning Mitra and Honkapohja (2015) consider Forward Guidance in the sense of incorporating the particular targets of price-level or nominal GDP targeting in agents' perceived laws of motion (PLMs) and show that these regimes perform better than an inflation targeting (IT) *without* FG. They find that without FG in any of the regimes it is not clear which of the three policies performs better. Mitra and Honkapohja (2015) use a deterministic model where the shocks are the initial conditions for the adaptive learners. Therefore, they resort to steady state (SS) learning instead of the standard recursive least squares (RLS) used in the literature. This, however, implies that agents do not have a model of the policy rule followed by the CB, but update their expectations of the gross nominal interest rate R_t through $R^e_t = R^e_{t-1} + \omega(R_{t-1} - R^e_{t-1})$, where the superscript e stands for 'expected' and ω is a learning gain parameter. This updating rule im-

plies that once the economy is at the ZLB, i.e. $R^e_{t-1} = R_{t-1} = 1$, agents will always expect that next period the ZLB is again binding - $R_t = 1$. These naive expectations implicitly suggest that agents never understand what the ZLB means and do not project its end - they simply wait for observations one period at a time. Section 1.4 outlines how this problem is solved here with agents forming expectations of the ZLB duration at every period.

The literature until now has focused on the very important convergence properties of learning models, while here I consider the merit of FG along the transition path during a binding ZLB. Evans and McGough (2018) show that a constant interest rate peg in New Keynesian models violates the Taylor principle and results in explosive learning dynamics. The temporary nature of the ZLB here, however, is not subject to such instability.

1.3 Model

The model environment in this paper is the canonical New Keynesian model with Rotemberg price-setting mechanism as in Eusepi and Preston (2010) and its micro-foundations are presented in the online appendix. The aggregate dynamics of the model, derived from the optimal decisions of households and firms, can be summarized in the following two equations:

$$x_t = \hat{\mathbb{E}}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_T - \beta \left(i_T - \pi_{T+1} \right) + \beta r^e_T \right] \tag{1.1}$$

$$\pi_t = \frac{\gamma_1 \xi}{(1-\gamma_1 \beta)} \hat{\mathbb{E}}_{t-1} \sum_{T=t}^{\infty} (\gamma_1 \beta)^{T-t} \left[(1-\gamma_1 \beta) \left(x_T + \mu_T \right) + \pi_T \right] \tag{1.2}$$

where x_t is the output gap, π_t the inflation rate, i_t the nominal interest rate. The expectation operator $\hat{\mathbb{E}}_t$ stands for the potentially non-rational expectations of the agents and, henceforth, $\mathbb{E}_t$ denotes rational expectations; r^e_t and μ_t are the exogenous natural rate and cost push shocks satisfying:

$$r^e_t = \rho_r r^e_{t-1} + \varepsilon^r_t$$

$$\mu_t = \rho_\mu \mu^e_{t-1} + \varepsilon^\mu_t$$

where $0 < \rho^r, \rho_\mu < 1$ and $\varepsilon_t^r \sim N(0, \sigma^r)$, $\varepsilon_t^\mu \sim N(0, \sigma^\mu)$ are independently and identically distributed random variables. The discount factor $0 < \beta < 1$, while $\xi > 0$ is a measure of price stickiness with $\xi \to \infty$ implying convergence to arbitrarily small costs of price adjustment (i.e. approaching fully flexible prices); and $0 < \gamma_1$ is an eigenvalue from the underlying microfoundations, where in a Calvo price adjustment it would represent the probability of not resetting the price. The parameter values are shown in Table 1.A.1 in the appendix. All variables are expressed as log-deviations from their steady state values. Thus, in steady state $x = \pi = i = r^e = \mu = 0$.

The model is closed with a monetary authority setting the nominal interest rate according to a Taylor rule subject to the zero lower bound.

$$i_t = \max\left\{i^*, \chi_\pi \hat{\mathbb{E}}_{t-1}\pi_t + \chi_x \hat{\mathbb{E}}_{t-1} x_t\right\} \tag{1.3}$$

where the policy parameters satisfy $\chi_\pi > 1$ and $\chi_x > 0$. The constant $i^* = 1 - 1/\beta < 0$ represents the effective lower bound on interest rates since, otherwise, as explained in Eggertsson and Woodford (2003) agents would choose to hold all their assets in cash. I will refer to it as the ZLB to be consistent with the arguments in the Introduction and with real world analogies.

A Taylor rule is assumed here as it has been shown to describe well the historical policy rate path of Central Banks. Nevertheless, its specification is subject to many disputes especially in such novel and uncertain times. Thus, the adoption of a rule instead of optimal policy in the model allows for changes in the reaction function of the Central Bank, which if not fully transparent, may be wrongly perceived by the public and result in unfavourable aggregate outcomes.

Finally, notice that all agents form decisions in period t based on $t-1$ information. Eusepi and Preston (2010) argue that this representation is plausible in view of the difficulty for CBs of obtaining accurate real-time data. It is also consistent, they claim, with the VAR evidence from Rotemberg and Woodford (1999) on spending and pricing decisions after a MP shock. Additionally, the rule here assumes that agents and the Central Bank have identical forecasting facilities. This implies that

the CB does not have an informational advantage about the state of the world compared to the public and the only private information it possesses is the specification of its reaction function.

1.4 Expectations

1.4.1 Expectations formation

Studying the effects of changes in policy parameters requires that agents are not fully rational as otherwise they would simply incorporate the new policy parameters in their (correct) model of the economy. In a rational expectations environment expectations are anchored by construction and no role exists for Forward Guidance in clarifying the CB policy function. To allow for a degree of bounded rationality I assume that agents form expectations through adaptive learning (ADL). In particular, they do not know the true structure of the economy and make forecasts as econometricians using simple regression models[3]. Namely, they make forecasts according to the aggregate policy functions from the minimum state-variable RE solution to the model: $x_t\left(r_t^e, \mu_t\right)$ and $\pi_t\left(r_t^e, \mu_t\right)$. Each period, as additional data becomes available, agents update the coefficients to their forecasting model. They are assumed to observe the disturbances r_t^e and μ_t and to know their autoregressive coefficients [4].

In the benchmark case of no policy change and no Forward Guidance the agents model, i.e. the "Perceived Law of Motion" (PLM), is defined as:

$$Y_t^e = \begin{bmatrix} x_t^e \\ \pi_t^e \end{bmatrix} = \Phi_{t-1}\hat{\mathbb{E}}_{t-1}z_t + e_t = \Phi_{t-1}\tilde{\phi}z_{t-1} + e_t \tag{1.4}$$

where $z_t = \begin{bmatrix} r_t^e \\ \mu_t \end{bmatrix}$, $\tilde{\phi} = \begin{bmatrix} \rho_r & 0 \\ 0 & \rho_\mu \end{bmatrix}$, and $\Phi_t = \begin{bmatrix} \phi_t^{x,r} & \phi_t^{\pi,r} \\ \phi_t^{x,\mu} & \phi_t^{\pi,\mu} \end{bmatrix}$ is a transition matrix that defines the PLM, finally e_t are iid estimation errors.

[3] Following the 'consistency principle' of Evans and Honkapohja (2001)

[4] Eusepi and Preston (2010) show that this assumption can be dispensed with and instead agents would estimate those coefficients. For simplicity, it is maintained.

Eusepi and Preston (2010) show that for determinacy and convergence to the rational expectations equilibrium (REE) it is sufficient that agents understand only the functional form of the Taylor-type monetary policy rule (1.3). Here it is exactly assumed that agents know that $i_t(x_t, \pi_t)$ is a linear function and update its coefficients as new data becomes available. The expected interest rate - i_t^e, is obtained as:

$$i_t^e = [\psi_{x,t-1} \quad \psi_{\pi,t-1}] \begin{bmatrix} x_t^e \\ \pi_t^e \end{bmatrix} = \psi'_{t-1} Y_t^e \tag{1.5}$$

To extend the mechanic adaptive learning framework described above every, period t agents are assumed to form long run expectations about $\hat{\mathbb{E}}_{t-1} x_T$, $\hat{\mathbb{E}}_{t-1} \pi_T$, $\hat{\mathbb{E}}_{t-1} i_T$ for any $T \geq t$ by iterating forward their PLMs (1.4) and (1.5). This gives them a perceived trajectory for the future of all endogenous variables. In this way in a recession with binding ZLB the agents can compute the date T^{zlb} when they expect the interest rate to be constraint for the last time, i.e. $\hat{\mathbb{E}}_{t-1} i_{T^{zlb}} \leq i^*$ and $\hat{\mathbb{E}}_{t-1} i_{T^{zlb}+1} > i^*$. In their individual decisions agents will account for this by expecting $\{i.\}_t^{T^{zlb}} = i^*$ and $\hat{\mathbb{E}}_{t-1} i_{T^{zlb}+s} > i^*$ for $s \geq 1$, thus affecting the aggregate behaviour of the economy.

At any date the CB also performs similar projections of the interest rate path into the future and obtains its own estimate of the last period of binding ZLB - T^{CB}, by using the true Taylor rule (1.3). It then communicates this to the agents and if $T^{zlb} \neq T^{CB}$, they update their perceived Taylor rule coefficients (ψ_x and ψ_π) at the end of the particular date t so that the two estimates of the terminal ZLB date coincide - $T^{zlb} = T^{CB}$. Therefore, the CB's announcements are regular and perceived as credible clarifications on its reaction function. Hence, communication is truthful, time-consistent and non-strategic similar to the practice of some inflation targeting central banks (i.e. Sveriges Riksbank and the Reserve Bank of New Zealand) and the statements from the Bank of England and the Federal Reserve from the introduction. This communication and adjustment process constitutes the nature of Forward Guidance in the paper. The next section provides the details about the policy change experiment and different scenarios of how agents interpret

FG.

The timing of expectation formation is as follows:

1. At the beginning of period t agents use information until period $t-1$, their current beliefs about the aggregate PLM (1.4) and the PLM for the interest rate (1.5) to form expectations of the endogenous variables x_T, π_T and i_T:

$$\hat{\mathbb{E}}_{t-1} Y_T = \Phi_{t-1}^{T-t+1} \tilde{\phi} z_{t-1}, \quad \forall T > t-1 \tag{1.6}$$

$$\hat{\mathbb{E}}_{t-1} i_T = \max \left\{ i^*, \psi_{t-1}' \hat{\mathbb{E}}_{t-1} Y_T \right\}, \quad \forall T > t-1 \tag{1.7}$$

From these expectations they calculated their expected terminal date for the binding ZLB:

$$T^{zlb} \text{ s.t. } \begin{cases} \mathbb{E}_{t-1} i_T = i^* \text{ for } T \leq T^{zlb} \\ \mathbb{E}_{t-1} i_T > i^* \text{ for } T > T^{zlb} \end{cases} \tag{1.8}$$

The Central Bank similarly obtains its estimate T^{CB}.

2. Based on their expectations $\hat{\mathbb{E}}_{t-1} Y_T$, $\hat{\mathbb{E}}_{t-1} i_T$ and $\hat{\mathbb{E}}_{t-1} T^{zlb}$ the agents make their decisions. These in turn feed into the aggregate demand equation (1.1), the New Keynesian Phillips curve (1.2) and the actual Taylor rule (1.3) giving rise to the actual law of motion of the economy (ALM) - Y_t and i_t.

3. At the end of period t agents observe the realisations of Y_t and i_t and update their transition matrices Φ_t according to the a recursive constant gain algorithm for the aggregate PLM:

$$\Phi_t = \Phi_{t-1} + \tau R_{t-1}^{-1} \hat{\mathbb{E}}_{t-1} z_t \left(Y_t - \hat{\mathbb{E}}_{t-1} Y_t \right) \tag{1.9}$$

$$= \Phi_{t-1} + \tau R_{t-1}^{-1} \tilde{\phi} z_{t-1} \left(Y_t - \Phi_{t-1} \tilde{\phi} z_{t-1} \right) \tag{1.10}$$

$$R_t = R_{t-1} + \tau (\hat{\mathbb{E}}_{t-1} Y_t Y_t' - R_{t-1}) \tag{1.11}$$

$$= R_{t-1} + \tau \left(\tilde{\phi} z_{t-1} \left(\tilde{\phi} z_{t-1} \right)' - R_{t-1} \right) \tag{1.12}$$

and ψ_t for the Taylor rule PLM:

(a) In the case of no binding ZLB, they update their perceived Taylor rule

by:

$$\psi_t = \psi_{t-1} + \tau Q_{t-1}^{-1} \hat{\mathbb{E}}_{t-1} Y_t \left(i_t - \psi_{t-1}' \hat{\mathbb{E}}_{t-1} Y_t \right) \tag{1.13}$$

$$= \psi_{t-1} + \tau Q_{t-1}^{-1} \Phi_{t-1} \tilde{\phi} z_{t-1} \left(i_t - \psi_{t-1}' \Phi_{t-1} \tilde{\phi} z_{t-1} \right) \tag{1.14}$$

$$Q_t = Q_{t-1} + \tau (\hat{\mathbb{E}}_{t-1} Y_t Y_t' - Q_{t-1}) \tag{1.15}$$

$$= Q_{t-1} + \tau \left(\Phi_{t-1} \tilde{\phi} z_{t-1} \left(\Phi_{t-1} \tilde{\phi} z_{t-1} \right)' - Q_{t-1} \right) \tag{1.16}$$

where R_t and Q_t are variance-covariance matrices used for weighting the deviations of the estimated parameters; τ is the gain parameter weight for new information.

(b) In the case of no Forward Guidance under ZLB, agents do not change their estimates, but only keep updating the variance-covariance matrix Q_t as new data arrives:

$$\psi_{x,t} = \psi_{x,t-1} \tag{1.17}$$

$$\psi_{\pi,t} = \psi_{\pi,t-1} \tag{1.18}$$

$$Q_t = Q_{t-1} + \tau (\hat{\mathbb{E}}_{t-1} Y_t Y_t' - Q_{t-1}) \tag{1.19}$$

$$= Q_{t-1} + \tau \left(\Phi_{t-1} \tilde{\phi} z_{t-1} \left(\Phi_{t-1} \tilde{\phi} z_{t-1} \right)' - Q_{t-1} \right) \tag{1.20}$$

(c) In the case of Forward Guidance under the ZLB, agents update their perceived Taylor coefficients so that $T^{zlb} = T^{CB}$. In particular they change linearly either ψ_x or ψ_π, or update both as described in (1.13)-(1.16) but with weight $\lambda \geq \tau$, which captures the credibility of the CB's announcement and the higher importance that it brings relative to another data point.

Evans and Honkapohja (2001) show that in the case of a decreasing gain learning - $\tau = 1/t$, the learning algorithm converges asymptotically to the least squares (LS) estimate and the REE. Under constant gain learning - τ a small constant, instead, the algorithm converges to a distribution centred around the REE. The constant weight given to new information across time in the latter allows agents to more

easily learn about structural changes in the economy. Given the emphasis here on tracking policy change over smoothing, a constant gain parameter $\tau = 0.02$ is employed which is showed to match US data well by Milani (2007) and Eusepi and Preston (2018).

Note that the system does need at least two exogenous processes driving the fundamentals (x and π), since otherwise output gap and inflation will be linear functions of the single shock and the agents will not be able to distinguish between and learn the two Taylor coefficients - ψ_x and ψ_π. Therefore, for the initial convergence to REE I keep both shocks active, yet for the policy experiment only the natural rate shock is active and causes a recession.

1.4.2 Infinite horizon

If agents were rational, the infinite sums in (1.1) and (1.2) would obey the law of iterated expectations (LIE) and can therefore be recursively represented by:

$$x_t = \mathbb{E}_{t-1} x_{t+1} - \mathbb{E}_{t-1} \left(i_t - \pi_{t+1} - r_t^e \right) \tag{1.21}$$

$$\pi_t = \xi \mathbb{E}_{t-1} x_t + \beta \mathbb{E}_{t-1} \pi_{t+1} + \mathbb{E}_{t-1} \mu_t \tag{1.22}$$

Preston (2005) and Eusepi and Preston (2010) claim that because households and firms only know their own objectives, constraints and beliefs they cannot compute aggregate probability laws. As a result, $\hat{\mathbb{E}}_t$ does not satisfy LIE and an infinite planning horizon as in (1.1) and (1.2) is required. Yet, Honkapohja et al. (2012) point out that assuming a continuum of symmetrical agents as is the case in the used NK model, one could still apply the LIE and resort to one period ahead Eurler equation learning.

I keep the infinite horizon learning for two reasons. First, it allows for incorporation of Forward Guidance into the law of motion for the aggregate variables, contrary to the one step-ahead behavioural Euler equation learning approach[5]. Second, because agents do not know the structure of the economy, they cannot foresee how the ZLB will change the ALM. This is a realistic assumption, yet taken alone it

[5]See Honkapohja et al. (2012) on the specifics of the Euler equation approach.

has the unpleasant implication that agents are oblivious of the ZLB if they simply iterate forward their PLM (1.4). Therefore, as discussed I assume that in their individual decisions agents at least account for the expected duration of the ZLB which affects immediately the realization of x_t and π_t, but they only gradually learn how the ZLB changes the aggregate ALM - the actual Φ transition matrix, and update their aggregate forecasts. This discontinuity in the perceived path of the nominal interest rate requires a long horizon learning approach since a recursive formulation would not capture the anticipated end of the ZLB.

1.5 Policy change and information structures

This section presents different scenarios of policy regimes and information possessed by the private sector regarding the current policy regime. The simulations study the effects of a 5% drop from the SS value of r^e at $t = 1$ that ex-ante brings the interest rate under ZLB for 7-8 periods. The experiments consider an increase in the reaction to the output gap χ_x from 0.1667 to 1 at $t = 2$. This is in line with the initial discussion that Central Banks wanted to communicate a changed trade-off between inflation and output gap, and the findings of Engen et al. (2015) who show that the perceived Taylor rule of private agents gradually shifted towards one with a much higher weight on output gap stabilization. The policy experiment considered is chosen to prolong the period of binding ZLB and reflect the suggestions of Central Banks that clarification regarding their reaction function is need. In particular, it is important to convey that the new reaction function weighs output gap and inflation stabilization differently from before.

The exploited mechanism here is that agents need data points to track policy changes. Then under the ZLB when the interest rate is inelastic and no new monetary policy data is available, Forward Guidance could potentially provide them with another anchor - the date of the last expected period of ZLB, so that they can update their estimates of ψ_x and ψ_π. Next, different communication strategies by the CB and comprehensions from the public are examined. Note that since the Taylor rule is a single equation in two unknowns - χ_x and χ_π, there are in theory

an infinite number of solutions for the two parameters (ψ_x, ψ_π) for a given level of the interest rate i. The different Forward Guidance scenarios in this exercise take this into account and differ in the way that agents understand CB's announcements. Each scenario is preceded by a long simulation of an economy without a ZLB so that the transition matrices Φ and ψ have converged to the true REE with not ZLB at $t = 0$.

1. **No Change** Suppose he monetary policy reaction function does not change. The parameter values are the same as in normal times, just now the economy is subject to the ZLB.

$$\psi_{x,T} = \psi_{x,0} = \chi_{x,0}, \ \psi_{\pi,T} = \psi_{\pi,0} = \chi_{\pi,0} \ \ \forall T \geq 1$$

   This scenario is useful for observing how the learners deal with the ZLB non-linearity.

2. **Full Communication** Let the Taylor coefficient on output gap increases from $\chi_{x,0} = 0.1667$ to $\chi_{x,T} = 1$ for $T > 1$. The CB fully discloses this and people completely internalize it so that $\psi_{x,t} = \chi_{x,t}$ and $\psi_{\pi,t} = \chi_{\pi,t}$. All following models feature the exact same policy regime change. Naturally, it may be expected that the Full Communication benchmark model is with highest welfare (discussed later in Section 1.6.2).

3. **No Communication** Here the Taylor rule does change as aforementioned, yet no Forward Guidance is provided, so during the ZLB the agents keep their pre-crisis beliefs about the Taylor coefficients. Once the ZLB is over, agents gradually update their perceived Taylor coefficients as in (1.13) and (1.16).

4. **Unambiguous Forward Guidance** The Central Bank announces its expected terminal date of ZLB - T^{CB}, and agents *correctly* understand that the output gap Taylor coefficient was the one changed. They then update it linearly so that $T^{zlb} = T^{CB}$. Note that T^{CB} implies that $\hat{\mathbb{E}}^{CB}_{t-1} i_{T^{CB}} \leq i^*$ and $\hat{\mathbb{E}}^{CB}_{t-1} i_{T^{CB}+1} > i^*$. The agents, however, cannot know the exact value of the expected interest rate at T^{CB} and instead assume $i_{T^{CB}} = i^*$, which may result

it errors in their update: $\psi_{x,t} \neq \chi_{x,t}$. They update their beliefs by solving a linear equation which satisfies the FG message:

$$
\begin{aligned}
i^* &= \psi_{x,t}\hat{\mathbb{E}}_{t-1}x_{T^{CB}} + \psi_{\pi,t-1}\hat{\mathbb{E}}_{t-1}\pi_{T^{CB}} \qquad (1.23)\\
\psi_{x,t} &= \frac{i^* - \psi_{\pi,t-1}\hat{\mathbb{E}}_{t-1}\pi_{T^{CB}}}{\hat{\mathbb{E}}_{t-1}x_{T^{CB}}}\\
\psi_{x,t} &= \frac{i^* - \psi_{\pi,t-1}\begin{bmatrix}\phi^{\pi,r}_{t-1} & \phi^{\pi,\mu}_{t-1}\end{bmatrix}\tilde{\phi}^{T^{CB}-t+2}z_{t-1}}{\begin{bmatrix}\phi^{x,r}_{t-1} & \phi^{x,\mu}_{t-1}\end{bmatrix}\tilde{\phi}^{T^{CB}-t+2}z_{t-1}}
\end{aligned}
$$

5. **Confused Forward Guidance** The Central Bank announces its expected terminal date of ZLB - T^{CB}, and agents *wrongly* understand that the inflation Taylor coefficient was the one changed. They then solve for $\psi_{\pi,t}$ analogously to the Unambiguous Forward Guidance case above in (1.23).

6. **Ambiguous Forward Guidance** The Central Bank announces its expected terminal date of ZLB - T^{CB}, and agents update both $\psi_{x,t}$ and $\psi_{\pi,t}$. They use their estimated variance-covariance matrix Q_{t-1} to weigh each parameter and avoid indeterminacy.

$$
\begin{aligned}
\psi_t =& \psi_{t-1} + \lambda Q^{-1}_{t-1}\mathbb{E}_{t-1}Y_{T^{CB}}\left(i_t - \psi_{t-1}\mathbb{E}_{t-1}Y_{T^{CB}}\right) \qquad (1.24)\\
=& \psi_{t-1} + \lambda Q^{-1}_{t-1}\tilde{\phi}^{T^{CB}-t+2}z_{t-1}\left(i_t - \psi_{t-1}\Phi_{t-1}\tilde{\phi}^{T^{CB}-t+2}z_{t-1}\right)
\end{aligned}
$$

where $\lambda \geq \tau$ is the weight they put on the FG announcement. Varying λ reflects how much credence the FG has received, with higher values of λ putting more weight on FG relative to the previous beliefs. The effects of different λ's are shown in the next section.

1.6 Results

1.6.1 Impulse Response Functions

In all model scenarios the economy is initiated having converged to the Steady State. For adaptive learners this means that all variables are zero and agents' PLMs Φ and ψ coincide with the true rational expectations equilibrium in normal times - that is,

with no binding ZLB. This convergence is achieved in a training simulation sample with no ZLB. To obtain impulse response functions (IRFs), each model is simulated 5000 times[6] with a Taylor rule respecting the ZLB. The simulations are needed since adaptive learners need variability in r_t^e and μ to be able to consistently update the parameters of their PLM. Otherwise, if there were no consecutive shocks, after the initial shock experiment has died out the agents would no longer have variability in z_t to update their PLM and the values for Φ will stay constant at a wrong level. The median responses[7] of the estimated transition matrices Φ and ψ are preserved and used to reconstruct IRFs from a single natural rate shock (the same as the initial one used in the simulations):

$$\varepsilon_1^r = -0.05, \quad \varepsilon_t^r = 0 \text{ for } t \geq 2$$
$$\varepsilon_s^\mu = 0 \quad \forall s$$

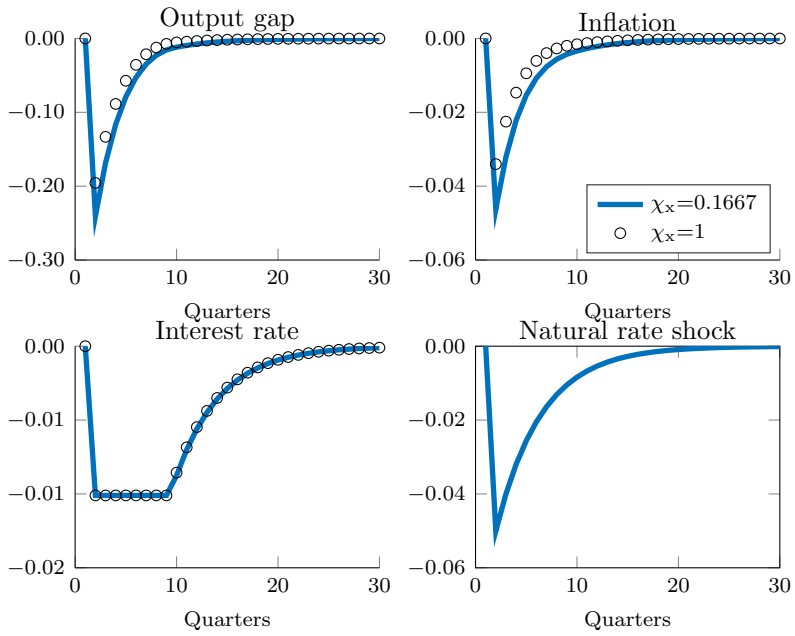
This procedure avoids any variability coming from the Monte Carlo (MC) simulations and taking the median values of output gap, inflation and the interest rate directly. An alternative approach proposed by Eusepi and Preston (2011) is for each MC draw to estimate the economy twice - once with the initial experiment shock; and once without (i.e. only with white noise). Then the IRFs will be the median of the differences between the two economies for each MC draw. This procedure gets rid of the MC variability in the final IRFs, but creates a bias in the series due to the ZLB (Eusepi and Preston (2011) do not impose a ZLB constraint). Since the experiment economy is subject to the ZLB while the control economy is always above due to the small noise shocks.

The rational expectations IRFs in presence of an occasionally binding ZLB are obtain through the algorithm 'OccBin' by Guerrieri and Iacoviello (2015). A model with such a constraint is identical to a model with two regimes - one in which the constraint is slack, and another where it is binding. Then, using an initial guess and standard perturbation techniques, iterating backwards from the expected lift-

[6] Larger number of simulations saw no gain in approximation.

[7] Note that due to the ZLB on interest rates, taking the mean responses instead would have resulted in an upward bias in the interest rate and a downward bias in the output gap on average.

Figure 1.1: Rational Expectations Impulse Responses



off date from ZLB a candidate piecewise linear solution for the binding constraint regime is obtained. The procedure is repeated until convergence and results in piecewise linear policy functions for the endogenous variables.

Rational expectations benchmark

As pointed out, non-rational expectations are required in order for a policy parameter change to prolong the ZLB. Figure 1.1 depicts how fully rational agents would behave in the two policy regimes - $\chi_x = 0.1667$ and $\chi_x = 1$. The duration of the ZLB is identical between the two economies with interest rates paths on top of each other. The more aggressive reaction to the output gap does ameliorate the downturn slightly, but the observationally equivalent interest rate paths rule out any extension of the ZLB and potential gains from Forward Guidance since the (shadow) interest rate in both economies reacts one-to-one to the demand shock. The rational agents internalise the policy structure by construction, which changes the aggregate law of motion and output and inflation, resulting in their faster recovery.

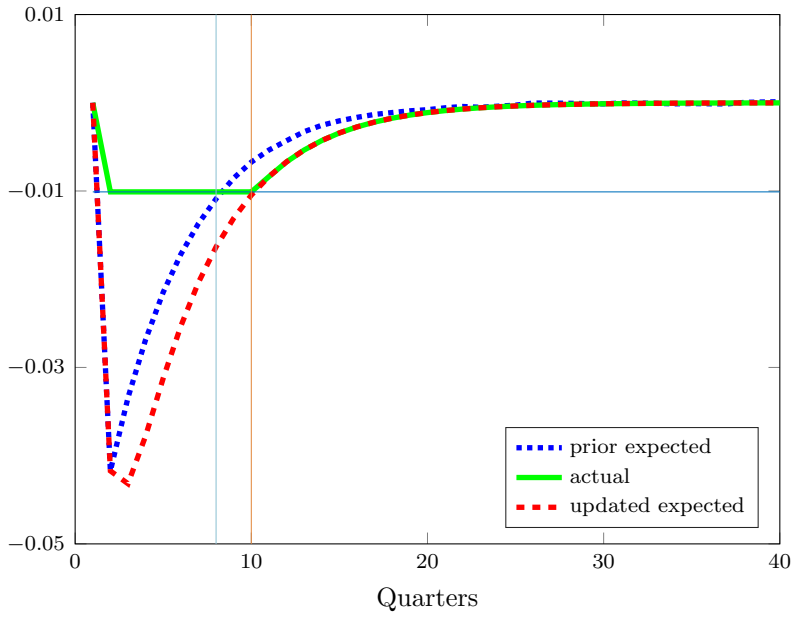
Model 1: Baseline model with no policy change

This exercise is used to illustrate the behaviour of the economy with adaptive learners and a binding ZLB. The main results are that the downturn is exacerbated, the ZLB prolonged and the aggregate transition matrix Φ changes drastically in the new environment of an inelastic interest rate. Since no policy change occurs, and under the maintained assumption of shared expectational facilities between the agents and the CB, there is no role for Forward Guidance since $T^{zlb} = T^{CB}$ at all times and the agents' expectations of the interest rate path coincides with the actual. Marinkov (2018) studies a model where the Central Bank knows the model of the economy and observes the learning agents' expectations. He shows that the ZLB causes a disagreement between the agents' and the CB's forecasts for the trajectory of the policy rate and the duration of the ZLB spell. This gives a new role for Delphic FG to align market expectations with the CB's. Here disagreement does not occur because both the agents and the CB know the Taylor rule and policy preferences remain unchanged.

Figure 1.2 shows the projected Taylor rule path under the Baseline model of no policy change and no communication. The blue dotted line is the projection for the interest rate path at the onset of the crisis (before agents realize the ZLB binds) and the left-most vertical line stands for the last ex-ante anticipated period of ZLB. As the interest rate is fixed at i^*, however, the economy lacks monetary policy's accommodative capacity, hence, the real law of motion under ZLB is different from the ex-ante converged PLM. Agents then revise their PLM for x and π, keeping the learned Taylor rule unchanged and understanding that the rate will remain zero for the implied (and updated every period) amount of time. This leads to the red dashed expectations of the interest rate path, where the right-most vertical line stands for the actual last period of ZLB (this understanding of the vertical lines is preserved throughout the paper). Because agents need to estimate how the economy will behave in such an unknown environment, they estimate Φ_T to fit closer real-time observations, thus creating a lag in the actual recovery.

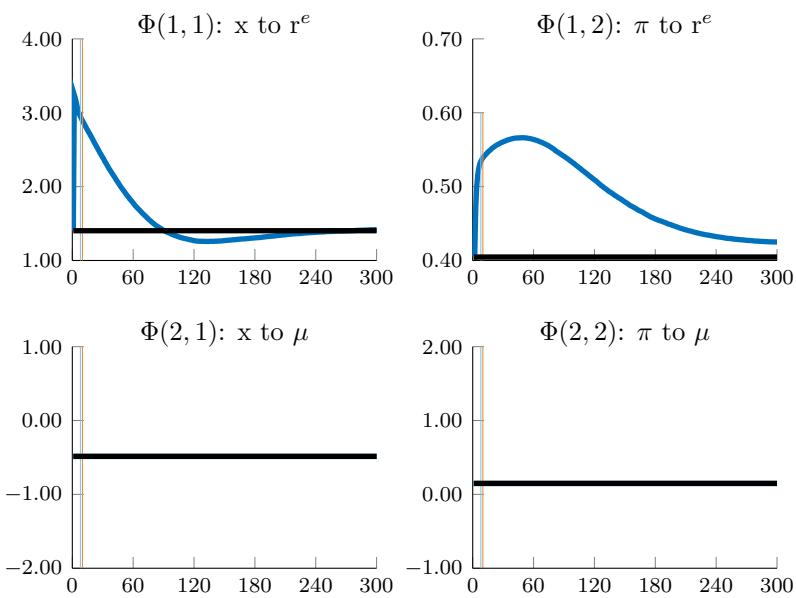
Figure 1.3 presents the four parameters of Φ_t that represent the PLM across

Figure 1.2: Interest rate paths
Model 1: Baseline, no policy change model



time. The vertical lines are as before, while the horizontal stand for the SS of each parameter. The policy function parameters with respect to the cost push shock μ stay constant as only the natural rate shock is active during in the simulations.

Figure 1.3: Aggregate transition matrix Φ
Model 1: Baseline, no policy change model

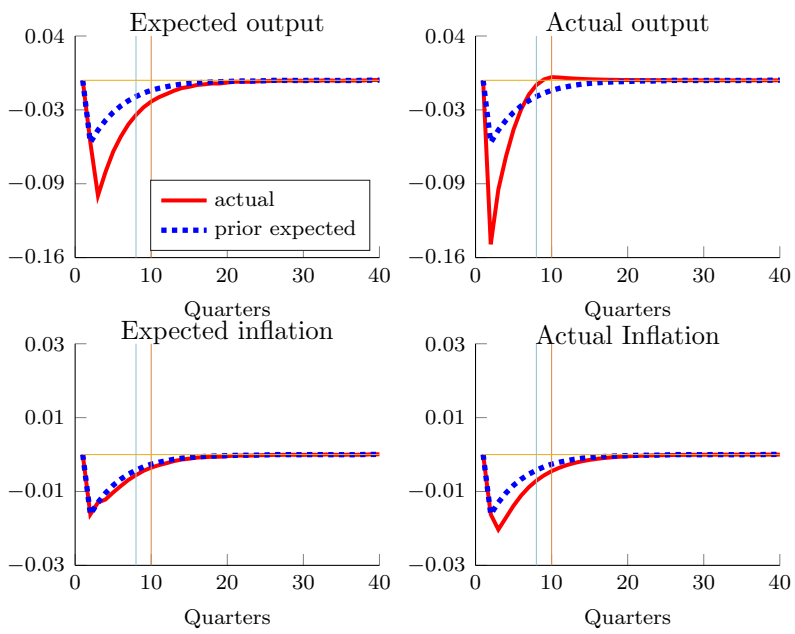


Note: the blue solid line is the value; the black horizontal line is the Steady State value

Finally, Figure 1.4 displays the reactions of expected and actual output gap

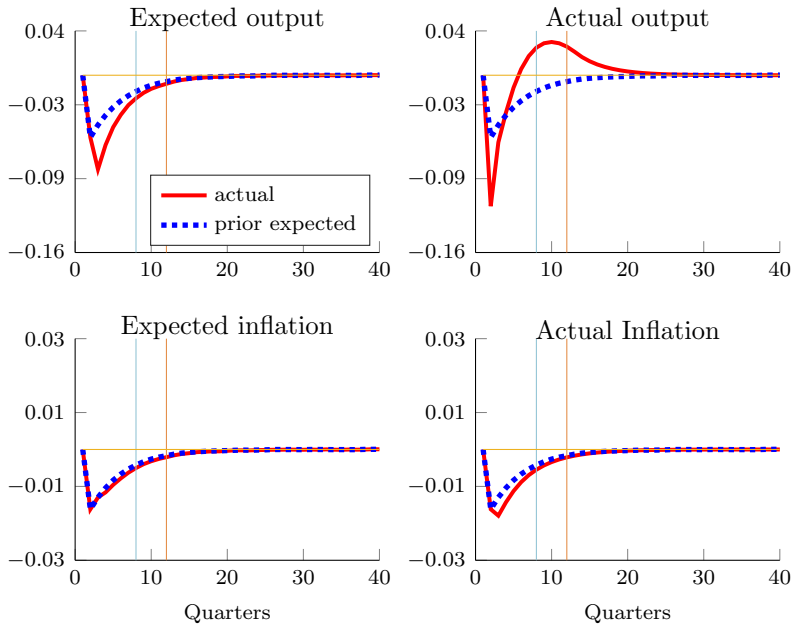
and inflation. The blue dotted lines again are the ex-ante expectations of how the crisis will affect the endogenous variables. The red dashed line, however, shows that the ex-post and actual movement of the output gap and inflation features a much prolonged crisis of large output losses and slightly bigger deflation. Again, this is due to the lack of monetary policy accommodation through elastic interest rate at the ZLB.

Figure 1.4: Output Gap and Inflation
Model 1: Baseline, no policy change model



Comparing Figures 1.1 and 1.4 shows that the recession is milder under ADL than RE. This is so because the learners do no internalize the future effects that a binding ZLB will have on future output gap and inflation, while rational agents do. The learners account for the ZLB n their individual problems ((1.1) and (1.2)), but adjust their expectations of future output gap and inflation only adaptively upon new observations. This feature is behind the inherent smoothness of learning models that Milani (2007) shows explain business cycle facts well. This difference will prove important later when constructing conditional forecasts of the endogenous variables in Section 1.8.

Figure 1.5: Output Gap and Inflation
Model 2: Full communication of policy change



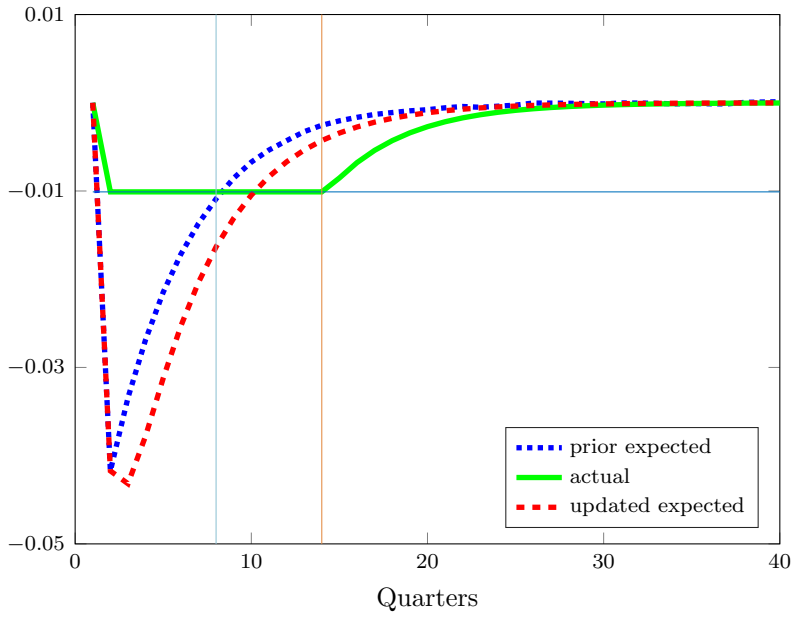
Model 2: Full Communication

With a fully internalized changed reaction to output gap - $\psi_x = \chi_x$, the model performs similarly to Model 1. The actual ZLB period is prolonged by 2 quarters due to the stronger reaction of the interest rate to negative output gap. Because the agents internalise long-run interest rate projections in their decision making, this ameliorates the downturn in output by 3%. The net effect, however, is not obvious since agents are slow to update their PLMs and the output gap overshoots its target (Figure 1.5).

Model 3: No Communication

Agents do not know that $\chi_x = 1$ and continue to believe $\psi_x = \chi_x = 0.1667$. Thus, they do not correctly foresee the last period of ZLB and expect it to be sooner (Figure 1.6). This set-up is reminiscent of the vague open-ended FG issued by the FED between 2008-2011 (see Engen et al. (2015)). This lack of internalization of the future accommodative policy leads to a harsher recession compared to Model 2. In fact, because of the adaptive nature of the economy, when agents do not internalize the stimulus from a more aggressive reaction to output gap, their expectations and

Figure 1.6: Interest rate paths
Model 3: No communication of the changed policy



the realizations of x and π are very similar to the ones from the baseline Model 1. Interestingly, the deeper recession from doveish expectations and the actual hawkish policy result in a very prolonged period of ZLB - 14 quarters, or 2 quarters more than the Full Communication case.

When the ZLB is over and agents gradually update their perceived Taylor coefficients, they entirely confuse the new policy regime - ψ_x is adjusted downwards, ψ_π upwards (Figure 1.7). Convergence to the truth occurs in the long run.

Model 4: Unambiguous Forward Guidance

Here agents correctly understand that the Central Bank's announcements signify a change in the reaction to the output gap and update their beliefs as previously outlined in Section 1.4. Before Forward Guidance, on impact of the shock the economy dips into a deep recession as in the Baseline Model 1. Later, however, as agents receive the FG messages and update their beliefs the economy recovers quickly and overshoots even more than in the Full Communication Model 2. This again leaves the net effect of the policy change unclear (Figure 1.10).

The perceived Taylor coefficient on output gap jumps as soon as the information

about the CB's reaction function has been received. The learning is not perfectly accurate as mentioned earlier and the estimates drift away from the truth in the transition back to the REE values in the long-run (Figure 1.7).

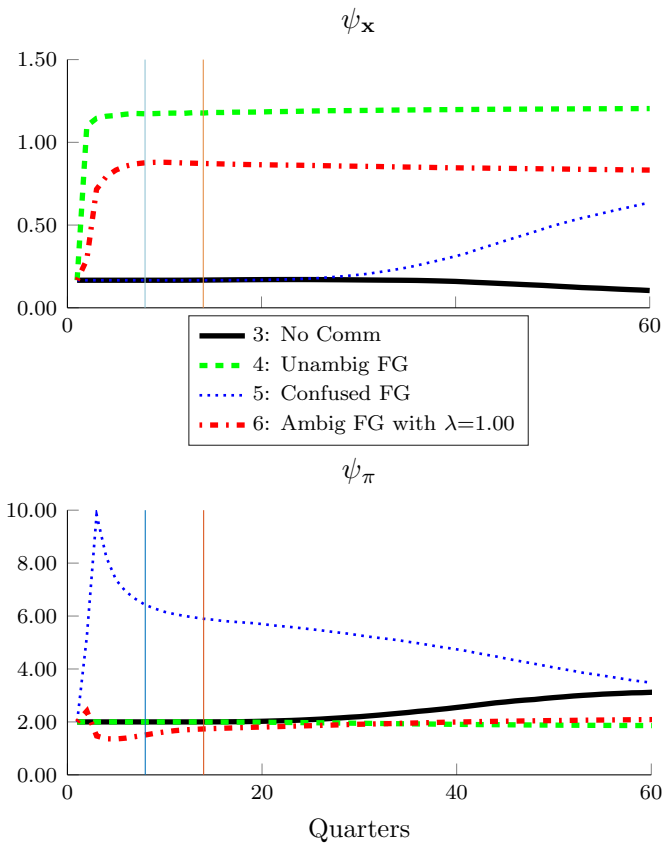
Model 5: Confused Forward Guidance

The agents wrongly update ψ_π in response to Forward Guidance. Since both output gap and inflation are below target, there still exists a ψ_π such that $T^{zlb} = T^{CB}$ and the agents manage to closely track the interest rate path, albeit with wrong Taylor coefficients (Figure 1.7). During the great recession, for example, the United States experienced low output and inflation at the same time, while the United Kingdom saw inflation floating above target. A solution for ψ_π is not guaranteed in the latter case here.

Model 6: Ambiguous Forward Guidance

Through Forward Guidance the agents are given a single anchor - $i_{T^{CB}} = i^*$. Up to now they had a single estimate (either ψ_x or ψ_π) to match the news and hence the path of the interest rates. Now, however, the agents are free to change both Taylor coefficients. To avoid indeterminacy (2 instruments for 1 anchor) they are assumed to update them according to their historical weights - the VCV matrix Q. In this way they still manage to approximate the interest rate path after the ZLB, yet their estimates of the Taylor coefficients are wrong. This happens because the Taylor rule is a single equation in two unknowns - χ_x and χ_π, and there are infinitely many combinations of the two that satisfy $\hat{\mathbb{E}}_{t-1} i_{T^{CB}} = i^*$. Figure 1.8 plots the estimated values of the Taylor rule for different update weights λ for the FG message in the recursive least squares algorithm (1.24). The darker lines stand for higher values of λ, with a maximum of $\lambda = 1$, i.e. FG is the same weight as the entire history of observations, to a minimum of $\lambda = \tau = 0.02$, i.e. FG is given the weight of a single data point. The reaction to the output gap is indeed updated upwards. But since both coefficients are changed, the reaction to inflation first jumps up and then drops below the true value. Eventually, the agents will learn the correct values, but it would take more than 300 quarters even for the model with highest reaction weight

Figure 1.7: Taylor rule coefficients



$\lambda = 1$.

The implications for output gap and inflation are shown in Figure 1.9. A weak message (low λ) results in a lower overshoot of the output gap, but also in lower inflation and output gap in the initial stages of the crisis. Due to the very steep schedule for the output gap this difference sometimes amounts to 2-3% compared to Model 4: Unambiguous FG. As the next section will show, this together with the difference in inflation which has a high welfare weight for the CB results in low welfare for the Ambiguous FG models.

1.6.2 Welfare Analysis

Figure 1.10 plots the relative output gap and inflation dynamics in all the different models, respectively. Table **??** presents the welfare losses of each scenario. The

Figure 1.8: Taylor rule coefficients
Model 6: Ambiguous Forward Guidance

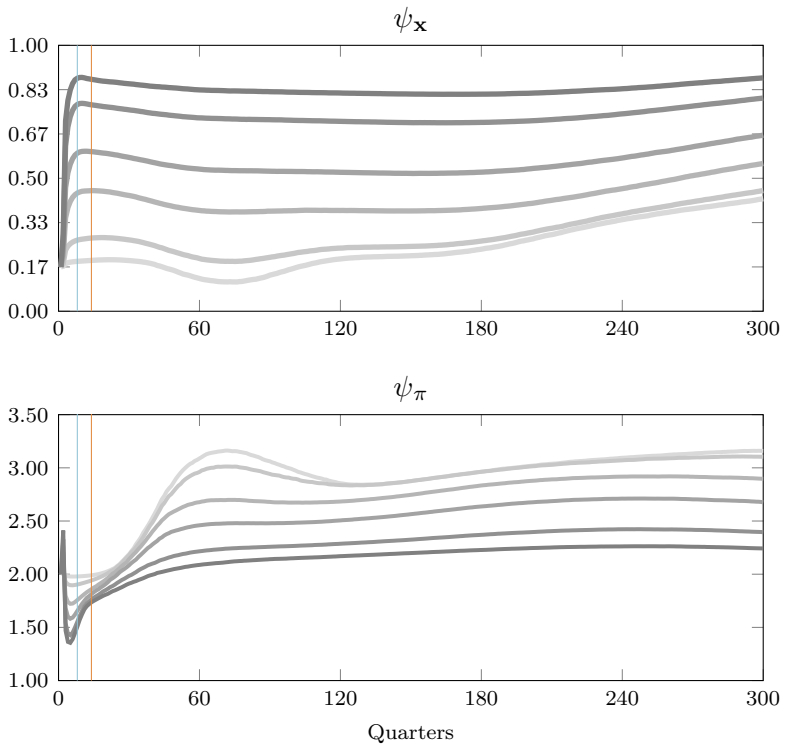
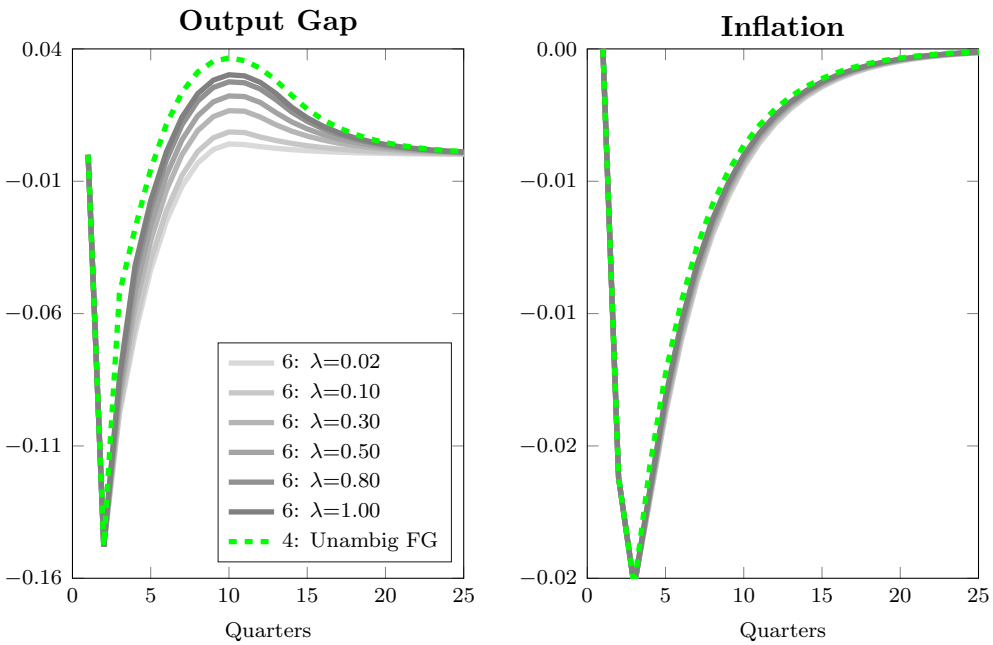


Figure 1.9: Paths for the Output Gap and Inflation
Model 6: Ambiguous Forward Guidance



welfare loss function is:

$$\min \mathbb{E}_{t-1} \sum_{T=t}^{\infty} \beta^T \left(\pi_T^2 + \lambda_x x_T^2\right) \tag{1.25}$$

which is used by Eusepi and Preston (2010) in the derivation of the Taylor coefficients of the Baseline scenario. Naturally, the Full Communication model performs best followed by the Unambiguous FG model and Confused FG. These models outperform both the Baseline and the No Communication model with respect to both output gap and inflation. Therefore, in the former models the initial boost of output gap and inflation after the policy change are more than enough to offset the negative effect of the output gap overshooting in later periods. Interestingly, even all the cases of Ambiguous FG outperform the No Communication Model 3. Thus, if a Central Bank was considering a policy change similar to the one described here, it would be better off communicating this to the agents through Forward Guidance. Of course, this result holds under the assumption of common expectational facility between the CB and the agents. Hence, the agents understand the nature of FG as clarifying a policy change and do not mistake it for a pessimistic prognosis (Delphic FG)[8].

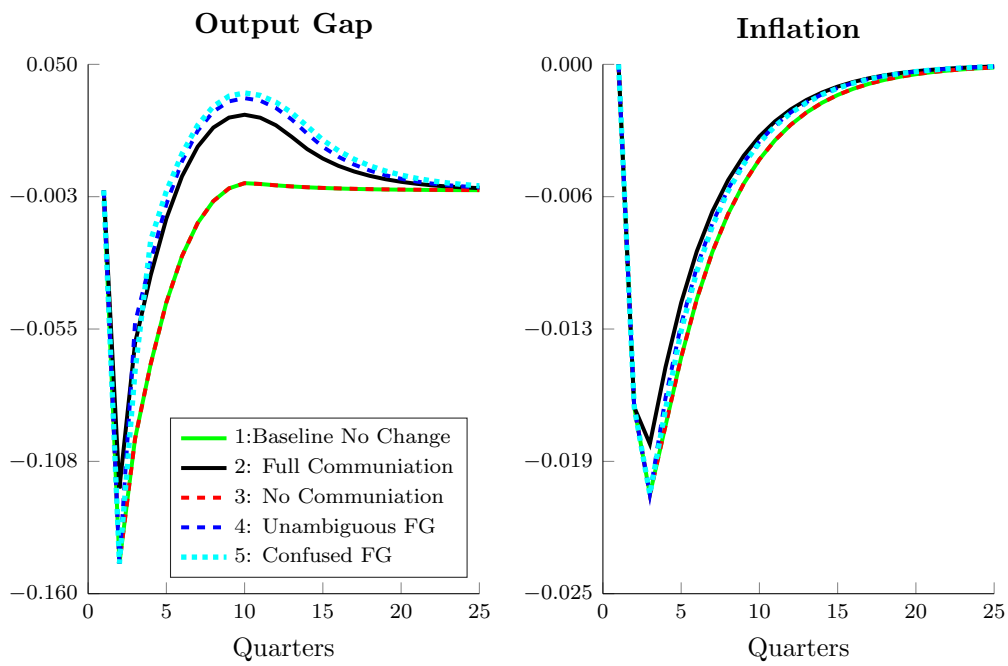
Table 1.1: Welfare Losses

	Model 1 No change	Model 2 Full Comm.	Model 3 No Comm.	Model 4 Unambig. FG	Model 5 Confused FG	Odyssean FG FG shocks
total	165.5712	121.6338	165.5686	145.2381	150.2124	150.7157
$\sum x^2$	3891.0823	2368.4768	3891.1160	3328.9318	3666.5521	3183.6913
$\sum \pi^2$	146.1158	109.7914	146.1131	128.5935	131.8797	134.7973
	Model 6: Ambiguous FG					
	$\lambda = 0.02$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.8$	$\lambda = 1$
total	165.1382	163.6390	160.8773	159.0180	156.9034	155.8127
$\sum x^2$	3857.9714	3758.1448	3624.6171	3588.5302	3581.0165	3588.7283
$\sum \pi^2$	145.8483	144.8483	142.7543	141.0754	138.9983	137.8691

$\times 10^{-6}$

[8] See Woodford (2012) and Walsh (2014) for further commentary.

Figure 1.10: Output gaps across models



1.7 Robustness exercises

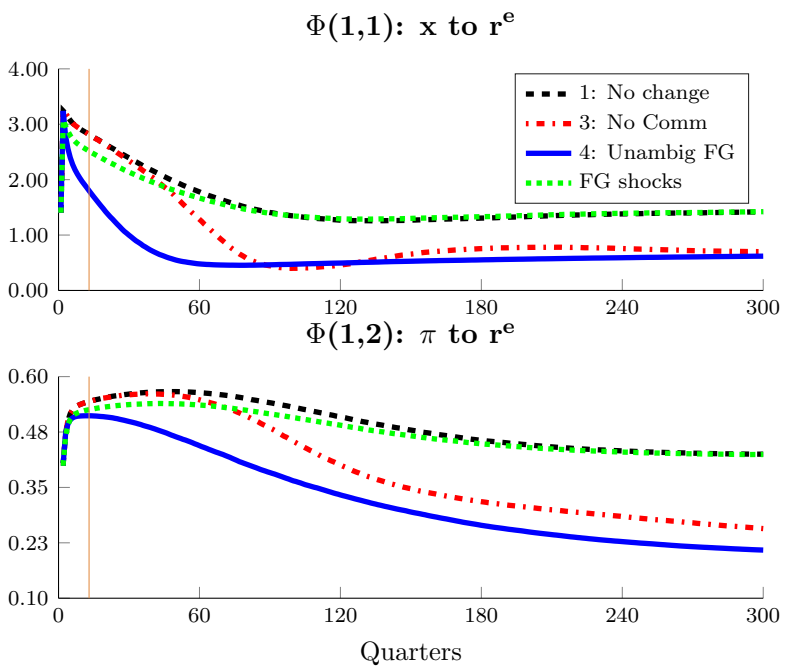
Figure 1.10 shows that the economies of Model 1: No change and Model 3: No Communication behave almost identically despite the different policy undertaken by the CB. This happens because during the ZLB uninformed agents would not know that a policy change has occurred and the economy will be identical to the baseline. Later, the different Taylor rule is only gradually learned (Figure 1.6) and the aggregate PLM is updated accordingly (Figure 1.11). Therefore, in the long run agents will learn the new parameters of the economy and future shocks would see the two economies diverge.

As aforementioned, an alternative specification of Forward Guidance widely used in the literature is through news shocks as in Del Negro et al. (2012). The interest rate rule then is:

$$i_t = \chi_\pi \pi_t + \chi_x x_t + \sum_{l=1}^{L} \varepsilon_{l,t-l}^{FG}$$

where $\varepsilon_{l,t-l}^{FG}$ are publicly known FG shocks at time $t-l$ that keep the interest rate at the ZLB at time t despite the standard Taylor rule, $i_t = \chi_\pi \pi_t + \chi_x x_t$, calling for a raise. This is an example of an Odyssean FG. Viewed through the policy

Figure 1.11: Aggregate Perceived Laws of Motion



exercises of Section 1.5 this approach corresponds to an economy where the true Taylor coefficients remain unchanged (Model 1: No change), but the interest rate remains at the ZLB as if the policy had changed and people understood it (Model 2: Full Communication). In particular, an economy featuring χ_x^{M1}, χ_π^{M1} and full sequence $\left\{T^{zlb} = T^{CB}\right\}^{M2}$, where superscripts stand for the model number from Section 1.5. Figure 1.12 plots the output gaps and inflation for the cases of Full Communication, No Communication and Odyssean FG. Both variables perform best under Model 2: Full Communication, while the Odyssean FG slightly outperforms Model 3: No Communication[9]. It is interesting to see that the Forward Guidance puzzle documented by Del Negro et al. (2012) is gone with ADL rather than RE agents with the current policy change. As discussed above, this is due to the adaptive updating of expectations for the learners while the RE agents internalize news about the future in the present. Related to this, the literature has solved the counterfactual strong stimulative effects of FG in RE models (FG puzzle) by introducing discounting tin the Euler equation (see Campbell et al. (2012)).

Figure 1.12, moreover, shows schedules from hybrid economies which contain the

[9]According to Table **??**, the Odyssean FG setup is better than providing no information, but is inferior to the other models of a Taylor coefficient change and FG.

Taylor coefficients and ZLB series from Model 2: Full Communication coupled with the aggregate PLM - Φ matrix, from Model 3: No Communication and Odyssean FG shocks model, respectively. The purpose of this exercise is to decompose the effects from the policy experiment on the endogenous variables coming from changes in the aggregate PLM and the perceived Taylor rule. With the crossing the schedules for output gap shift up and resemble the overshooting nascent for the Forward Guidance models (i.e. Models 2, 4 and 5). The ones for inflation, however, are completely identical (the red lines coincide with the blue and are thus hidden). This occurs because, as discussed, the stimulus in this economy comes mainly from the interest rate channel, which only directly appears in the aggregate demand equation (1.1), while only $\hat{\mathbb{E}}_{t-1}x_T = [\phi_T^{x,r}\phi_T^{x,\mu}]\hat{\mathbb{E}}_{t-1}z_T$ enters the New Keynesian Phillips Curve (1.2) and the Φ matrix is kept unchanged as in Model 3 and Odyssean FG[10]. Albeit not perfect, this decomposition illustrates that the bulk of the difference between the different models does indeed come from the particular policy change and the information possessed by the private sector. The future discounted policy behaviour in the ALM results in an overshoot. This is similar to the transmission channel of Odyssean FG but does not affect inflation which depends on the slowly adjusting expectations of the learners. This explains the lack of the FG puzzle present in the RE model of Del Negro et al. (2012). These results show that the length of the ZLB period is of secondary importance and strengthen the case for clear informative communication provided by the Central Bank for changes in its reaction function which brings the bulk of the stimulus.

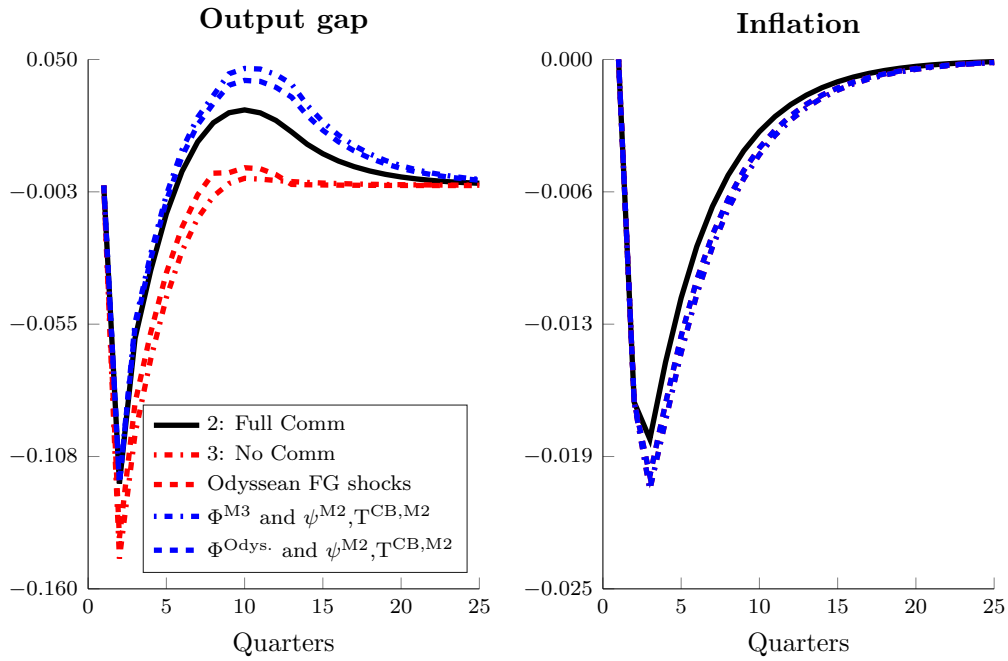
1.8 Heterogeneous expectations

The different models can be interpreted as agents having heterogeneous beliefs but mass measures of zero and hence not affecting the aggregate law of motion[11]. In particular, agents may have different understandings of what the CB really means

[10]The reversed crossings: with Φ^{M2} and Taylor coefficients from Model 3 and Odyssean FG also resemble closely the economies from which the Taylor coefficients series are taken. This is not plotted in Figure 1.12 for less clutter.

[11]One could argue that professional forecasters are also a small portion of the economy, for example.

Figure 1.12: Crossing models' Φ and ψ
Output gaps and Inflation



when it uses Forward Guidance. Some will very well understand, some will update one parameter with some noise and some will link the announcement to historical trends and update both coefficients.

A similar idea with agents with heterogeneous beliefs about the meaning of Forward Guidance is proposed by Andrade et al. (2019). In their model the heterogeneity between agents comes from some believing that FG is Odyssean - therefore, expansionary; and others perceiving it as Delphic, or pessimistic and contractionary. Andrade et al. (2019) show that in this way they can generate realistic heterogeneous expectations of output gap and inflation, while keeping short run interest rate expectations fairly homogeneous. The authors also show evidence that this is the observed pattern in the data from the Great Recession.

In the setup here agents short run interest rate expectations are also very homogeneous - that is, they agree on the duration of the ZLB that the CB announces. But their estimated Taylor coefficients are very diverse (Figure 1.7) as well as their updated transition matrices Φ (Figure 1.11). Thus, that their expectations of output gap and inflation, which depend on Φ, are also diverse, despite the agreement on the interest rate path.

Figure 1.A.1 in the appendix plots the total disagreement between the lowest and highest projections for the future of the three endogenous variables across all models. The one year ahead forecasts show a significantly higher disagreement than the 2 year ones. This is in line with Andrade et al. (2019) and the data. Additionally, Andrade et al. (2019) find that the disagreement about output gap and inflation forecasts relative to the interest rate path disagreement increased in the data at time when FG was used. Accordingly, Figure 1.A.2 presents the relative forecast disagreements. The observed pattern of high relative disagreement following FG is preserved. In fact, the magnitude of relative output disagreement is similar to the one in the data (Figure 1.A.3). Inflation responsiveness to policy change and FG in the model, however, is weak. This is also evident in Figure 1.10 above as inflation varies much less across models as does the output gap. This occurs since agents incorporate FG in their decision making by having an implied projection path for the interest rates and the period under ZLB. As adaptive learners they cannot know how a policy will affect the aggregate law of motion of the economy - Φ, and hence even though they understand how the ZLB will affect them individually, they are unaware of the consequences this will have on the economy - that is, they keep updating $\hat{\mathbb{E}}_{t-1}x_t$ and $\hat{\mathbb{E}}_{t-1}\pi_t$ gradually as in normal times. Forward Guidance decreases the real interest rate here mainly through the nominal interest rate channel and not through higher expected future inflation as in Eggertsson and Woodford (2003).

Marinkov (2018) introduces the lagged policy rate as a state variable which improves the interest rate's transmission channel to inflation expectations of the learners. Nonetheless, the adaptive learning framework is much less forward looking than rational expectations and alternative specifications with more forward-looking behaviour are an important area for future research.

1.9 Conclusion

This paper studied the effects of Forward Guidance (FG) from a novel perspective. Instead of considering FG as a promise for future actions (Odyssean) or providing better forecasting facilities (Delphic), the Central Bank in the model is giving a

signal about its own reaction function. This was shown to be in line with what Central Banks have tried to communicate during the crisis.

To evaluate FG the paper assumes that agents are non-rational adaptive learners. The Central Bank then uses FG as a communication device to signal a policy change in the Taylor rule to the agents. The mechanism builds on the lack of real-time observations under ZLB that adaptive learners could otherwise use for learning the new policy function.

The main findings are that clear communication increases welfare compared to no communication. Nevertheless, if the message has been too vague and agents view it through the lens of past observations, providing Forward Guidance is barely superior than giving no signals at all. Forward Guidance at the ZLB, however, is shown to create a persistent drift in agents' Perceived Law of Motion which might cause instability and welfare losses due to future shock realisations. This is an important and novel insight and is left for future research.

Finally, the model is able to replicate some features of the data on private forecasts during the zero lower bound and Forward Guidance episodes.

Appendix

1.A Appendix

Figure 1.A.1: Heterogeneous expectations

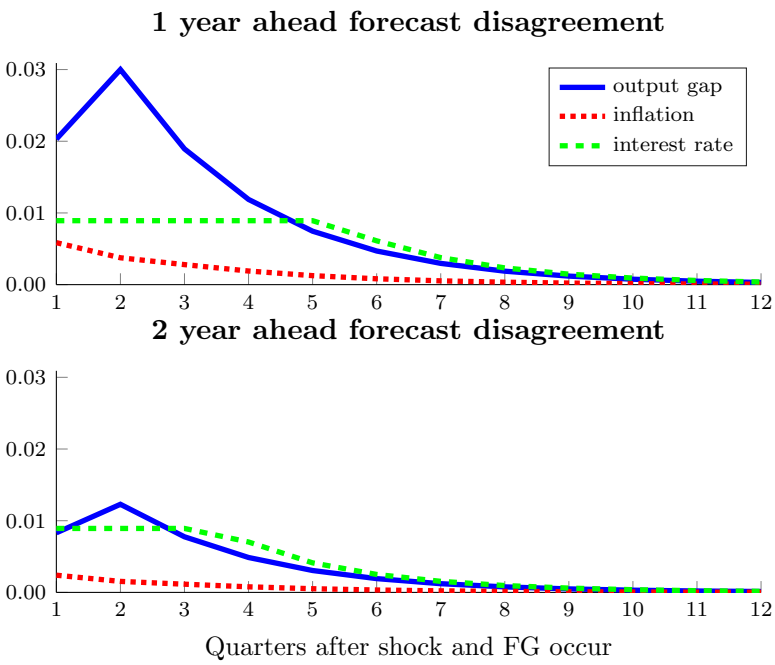


Figure 1.A.2: Relative Forecast Disagreements

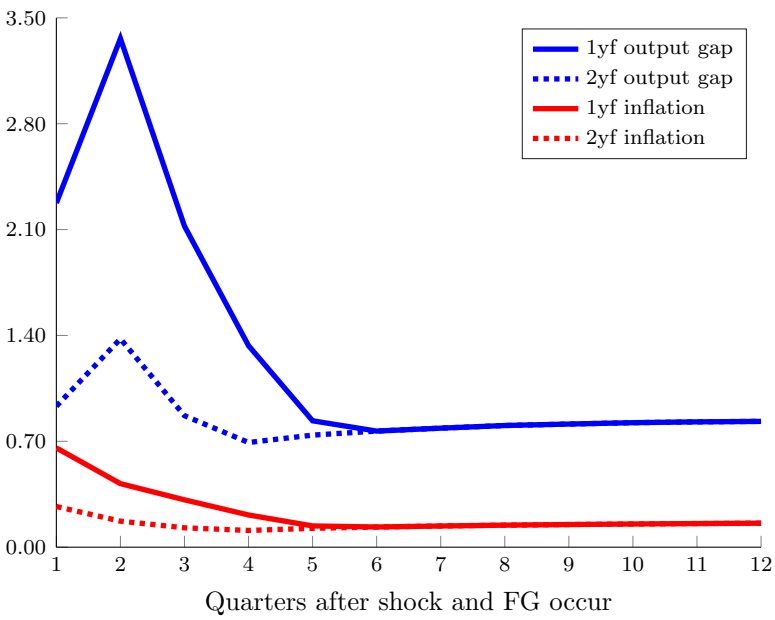
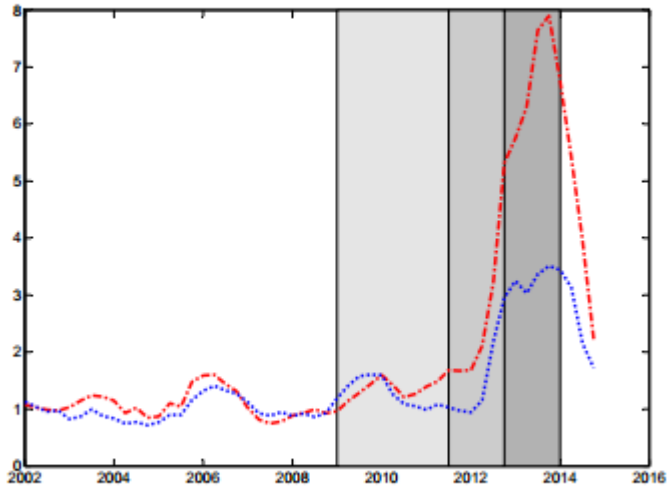
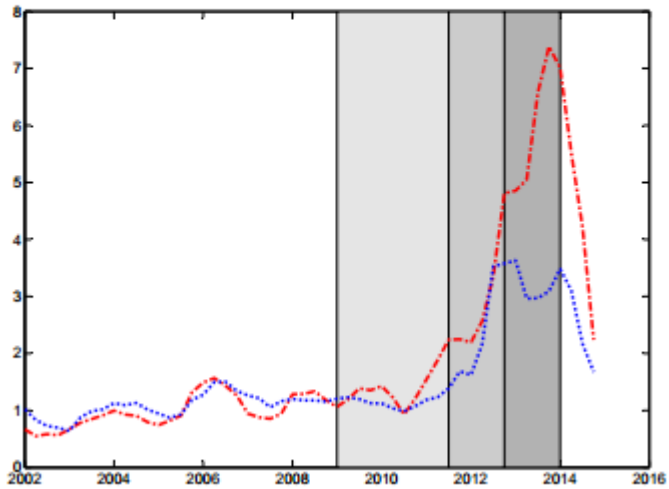


Figure 1.A.3: Relative Forecast Disagreements in Data

(a) Consumption



(b) Inflation



The figure provides the ratio of disagreement on 1-year (red line) and 2-year (blue line) ahead consumption growth and inflation to disagreement on 1-year and 2-year the short-term interest rates. Disagreements are measured as a moving average over the last 4 quarters of the 75/25 inter-quantile range in the distribution of corresponding individual mean point forecasts. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "fixed-date" forward guidance and the "state-contingent" forward guidance.
Source: Andrade et al. (2019)

Table 1.A.1: Parameter values

Param	Value	Source
β	0.99	Eusepi and Preston (2010)
ξ	0.06	-——-
λ_X	0.005	-——-
ψ_π	2	-——-
ψ_x	$\frac{\lambda_x \psi_\pi}{\xi} = 1.667$	-——-
γ_1	0.7866	-——-
τ	0.02	Cole (2015)
ρ_r	0.8	
ρ_μ	0.8	
σ_r	0.006	
σ_r	0.006	

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Chapter 2

Communication at the Zero Lower Bound: The Case for Forward Guidance

2.1 Introduction

The unprecedented length of the period when interest rates were limited by the zero lower bound after the Great Recession has spurred a large literature trying to understand the behaviour of the economy in such novel circumstances. This phenomenon was also accompanied by unconventional monetary policy instruments to which Central Banks resorted once the interest rates were no longer flexible.

This paper focuses on one of these instruments - namely, forward guidance, and strives to provide a structural justification for its use. The literature on forward guidance (FG) largely agrees that the main channel of influence of FG is the information conveyed for the future path of the policy rate. There are two main classifications of FG depending on the underlying reasons for its use. The seminal work of Krugman et al. (1998) and Eggertsson and Woodford (2003) showed that promises of lower interest rates for longer can largely mitigate the negative effects of a binding zero lower bound (ZLB) on interest rates. The stimulus comes through agents expecting low interest rates in the future (i.e. accommodative monetary policy) and higher inflation, hence cutting back less on present investment and consumption. Campbell et al. (2012) label this approach Odyssean Forward Guidance. Campbell et al. (2012) also acknowledge a more established form of FG, pursued by the Reserve Bank of New Zealand and the Riksbank in Sweden, for example. In essence, CBs engage in regular forecasts of the path of their policy rate, hence it was

dubbed Delphic Forward Guidance. This type of FG may be useful to the public if the CB has better information about the state of the shocks that hit the economy. Moreover, in Marinkov (2018) I propose another function of forward guidance as a communication strategy for policy change. There, the ZLB acts as an informational curtain for adaptive learners who fail to perceive a potential policy change as the policy rate is bound by the ZLB. Then, forward guidance is a useful tool in helping them learn the new policy regime through announcing future lift-off dates[1].

Here I built on this previous work but pursue a more fundamental reason for FG. Instead of considering a policy regime change, I show that the non-linearity introduced by ZLB itself acts as a regime change for adaptive learners and this creates disagreement between their policy rate forecasts and the Central Bank's forecasts, who knows the precise structure of the economy. Therefore, FG acts as a helping hand for learners to update their perceived law of motion of the economy under the ZLB regime. Such information revelation about the structure of the economy is akin to Delphic forward guidance. Although empirically supported by Campbell et al. (2017), these authors and others[2] only incorporate Odyssean FG through anticipated monetary policy shocks in their models and do not study theoretically or numerically the effects and nature of Delphic FG. The model here allows for Delphic FG by showing a channel which could explain the observed policy rate forecast disagreement in the data between central banks and the private sector.

The main message is that the zero lower bound calls for a necessary increase in transparency and communication by the Central Bank at the ZLB because it acts both as a regime change and an information curtain preventing agents from correctly adjusting their expectations about the path of the interest rate. First, forward guidance is shown to have a welfare-improving effect by helping the agents update their expectations even in the absence of interest rate observations. The benefit is not negligible, but neither is it huge, so no forward guidance puzzle is present. Second,

[1] Marinkov (2018) explores various communication and interpretation schemes for the FG signal. Wrong interpretation or small weights of the signal are shown to still be marginally over no communication at all. The stimulative effects of a prolonged ZLB duration are modest and no forward guidance puzzle is present.

[2] see Eggertsson and Woodford (2003), Del Negro et al. (2012), Campbell et al. (2012), Ben Zeev et al. (2017) among others

forward guidance helps prevent an expectational drift due to agents expecting an earlier lift-off from the ZLB. This is numerically shown to improve the stability of the system by keeping it tighter within the basin of convergence to the rational expectations equilibrium. This is a novel result which complements prior work on the stability implications of monetary policy in learning models (see Evans and Honkapohja (2003)). In the simple model this communication is achieved through forward guidance, yet in reality a combination of FG and asset purchases might be needed to achieve the necessary shift in expectations. For instance, Campbell et al. (2017) and Andrade et al. (2019) show that FG was successful at shifting short-term expectations but quantitative easing was more adept at affecting the longer end of expectations.

The paper proceeds as follows. Section 2.2 provides evidence for the disagreement between the Central Bank and private agents at the ZLB. Section 3.4 presents the model, while Section 2.4 studies the effect of forward guidance. Finally, Section 2.5 concludes and discusses future work.

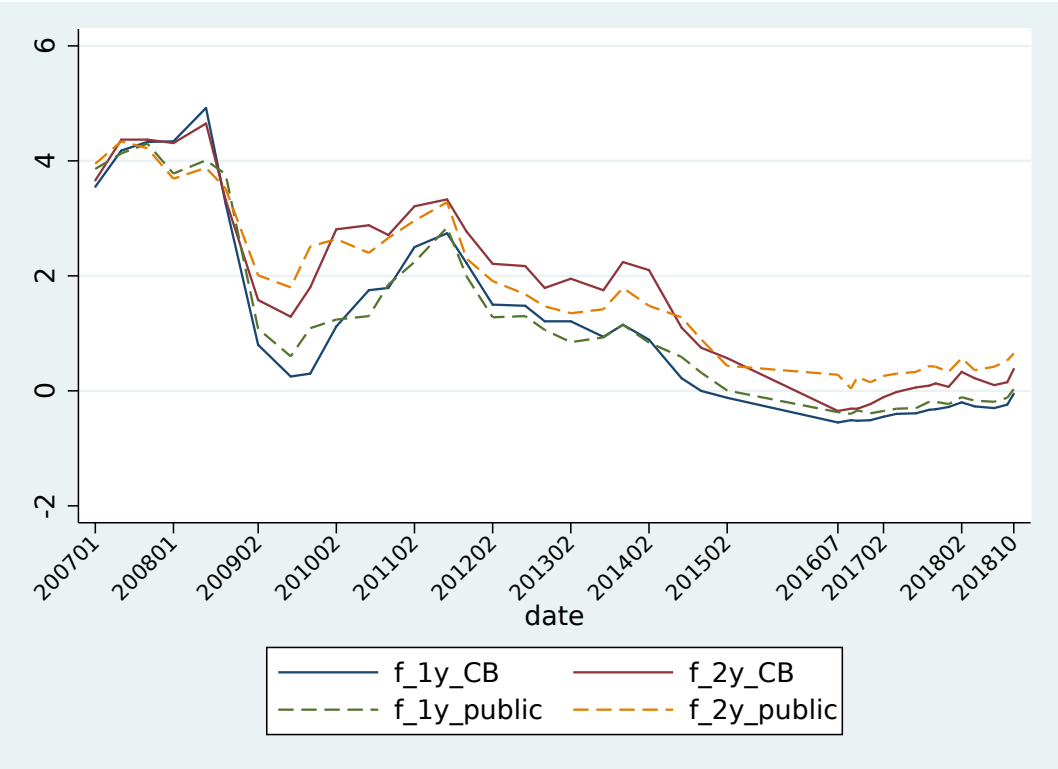
2.2 Motivation

The Great Recession and the followed long spell of binding ZLB were unprecedented events that caught the public by surprise. Andrade et al. (2019) show that this lead to very high levels of disagreement by historical standards among private forecasters. Additionally, agents often expected earlier lift-off than the Central Bank but this could be due to policy changes (Marinkov, 2018; Engen et al., 2015). To disentangle the disagreement between the CB and the private agents both their forecasts are needed. Among major central banks the Swedish Riksbank is one of the few who publish internal consensus interest rate forecasts along with private market forecasts. They began releasing their internal forecasts in the 2007 issue of their Monetary Policy Report.

Figure 2.2.1 plots the 1-year-ahead and 2-year-ahead repo rate forecasts for both the Riksbank (solid lines) and the public (dashed lines). As expected, they are not too disparate from one another, yet there are two important features of data. First,

whenever interest rates are expected to be binding to some lower bound, the private forecasts are always supportive of an earlier lift-off than the Riksbank's. Second, Sweden is a special case among developed economies because it dipped twice to the zero lower bound (ZLB), thus it provides more comparable data above and below the ZLB and allows for testing the theory that the ZLB causes disagreement between the CB and the agents.

Figure 2.2.1: Forecasts of Swedish repo rate



Source: Riksbank's Monetary Policy Report 2007-2018

To quantify the disagreement between the agents and the Riksbank Table 2.2.1 computes the difference between the forecasts of the Bank and those of the market. The measure is set up such that a positive disagreement means that the Riksbank expects higher future repo rate than the market. The data is split in two regimes - Low and High, where Low is classified as expected 1-year-ahead repo rate to be smaller than 0.25, and High - to be larger than 0.25. The table shows the classification according to future expected repo rates by both Riksbank and the market. Further robustness classifications on horizons and cut-offs are performed in Table 2.A.1 in the Appendix.

Table 2.2.1: 1-year-ahead disagreement on Swedish repo rate

Based on private agents' expected 1-year-ahead repo rate					
	count	mean	se(mean)	min	max
Low	16	-.1135	.0085	-.18	-.05
High	24	.0107	.0755	-.79	.91

Based on Riksbank's expected 1-year-ahead repo rate					
	count	mean	se(mean)	min	max
Low	18	-.1391	.0192	-.37	-.05
High	21	.0616	.0804	-.79	.91

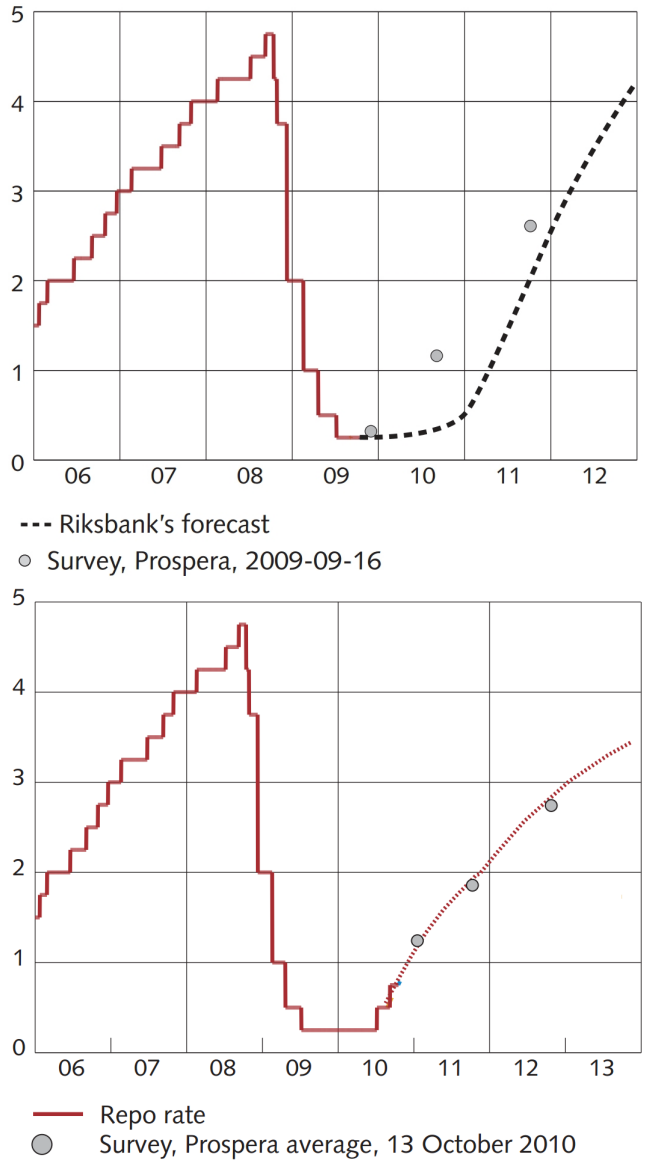
Note: 'High' and 'Low' states refer to 1-year-ahead expected repo rate above or below 0.25, respectively. The first block defines 'High' and 'Low' based on private agent's expectations and the second - on Riksbank's forecast.
Source: Riksbank Monetary Policy Report (2007-2018)

Table 2.2.1 shows that regardless of the classification private agents expect an earlier lift-off than the CB (negative and significant average disagreement) when the economy is a Low regime of near zero interest rates. Moreover, the High regime of normal times exhibits no systematic forecast bias for either party. As a case in point, Figure 2.2.2 shows that during the first ZLB spell in 2009 agents expected a higher interest rate path than the Bank, but already a year later when the interest rate left the ZLB expectations aligned perfectly.

It is worth noting that disagreement between the Riksbank and the market continued throughout the ZLB spell. This is an unexpected fact because of Riksbank's open and explicit interest rate forecasts which one would expect are one of the most transparent and informative means of CB communication. Perhaps, the market did not put a high enough weight on their routine announcements while the unprecedented forward guidance by the Federal Reserve and the Bank of England among others had a notable effect on market expectations as shown by Engen et al. (2015), Andrade et al. (2019) and Campbell et al. (2017). See Marinkov (2018) on the implications of imprecise or unconvincing forward guidance in a model with learning agents.

Finally, a similar study of disagreement is not possible for the USA because the Federal Reserve does not publish its internal consensus forecasts. Yet, Figure 2.2.3 shows the average expectations of professional forecasters in the US. It is seen that the period of explicit date- and state-contingent forward guidance (2011-2013) saw

Figure 2.2.2: Forecasts of Swedish repo rate 2009-2010



Source: Riksbank's Monetary Policy Report 2009-2010

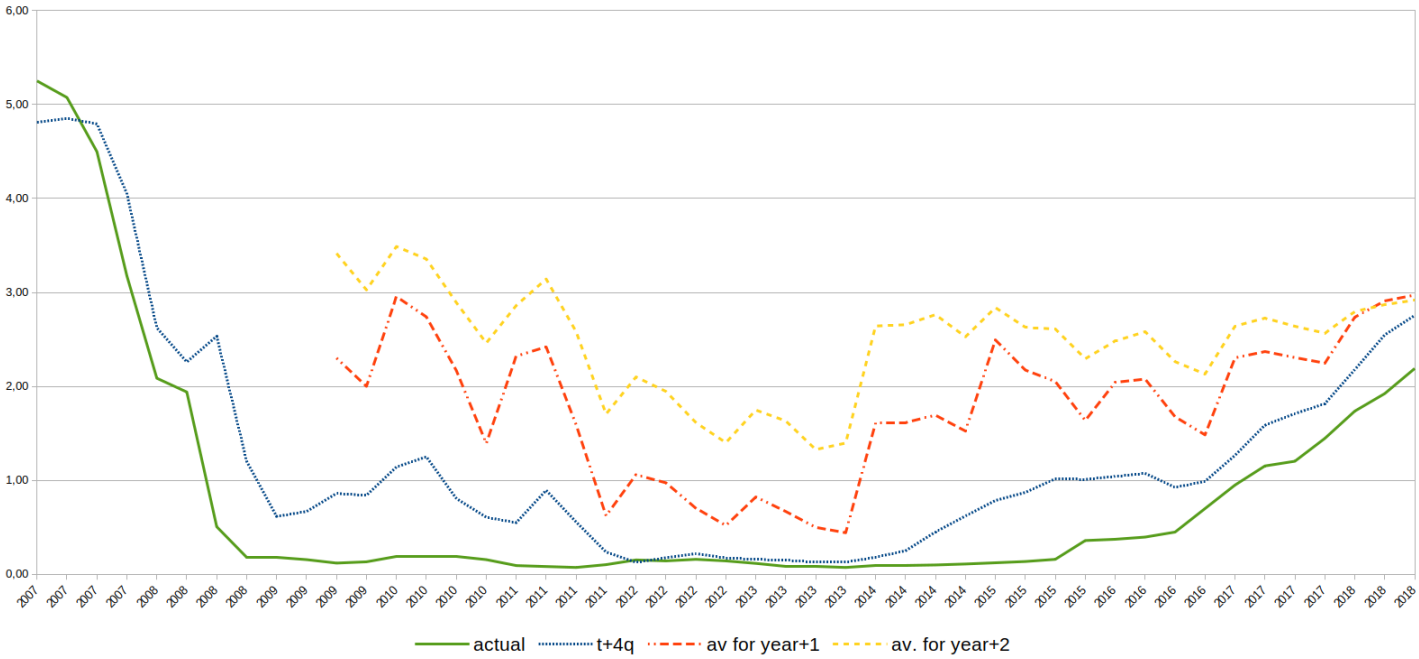
market expectations converging closely to what ended up being the actual rate. Yet, before that period and even after it around 2015 market expectations were higher than what the T-bill rate ended up being. This is indirect evidence of a similar pattern as observed in Sweden above.

Abstracting from disagreement, Campbell et al. (2012) find that future monetary policy tightening lowers unemployment expectations and increases inflation expectations in the US, contrary to the predictions of New Keynesian models. They interpret this finding as evidence for successfully communicated Delphic forward guidance by the FOMC. Campbell et al. (2017) study empirically the hypothesis that FOMC's meeting announcements carried Delphic forward guidance. They classify private information of the FED by the difference between the Greenbook forecasts on inflation, GDP growth and unemployment rate and Bluechip survey of private forecasters' expectations. They find that the four-quarter ahead futures contract rate is statistically positively correlated with policy makers' forecast of future GDP growth being higher than the market expects (and lower for unemployment). This is evidence that the committee's private information about the future of the economy was transmitted through the FOMC's announcements - supporting a Delphic forward guidance interpretation.

2.3 Model

To explain why disagreement between the Central Bank and the private agents arises at the zero lower bound I build a simple New Keynesian model featuring adaptive learning as an expectation formation framework for the agents. Marinkov (2018) outlines in detail the difference between rational and learning agents and how each model interacts with the non-linearity of an occasionally binding ZLB. Notable differences in the current paper are the reduced form knowledge of the Taylor rule by the agents and the presence of the lagged interest rate as a state variable. As will later become clear the first assumption eliminates the simultaneity in determining the output gap, inflation and the policy rate, while the second makes the learners' forecasts of output gap and inflation more responsive to the ZLB - a necessity pointed

Figure 2.2.3: Private Forecasts of US T-bill rate



Source: Survey of Professional Forecasters

out in Marinkov (2018).

2.3.1 Rational Expectations

The model environment is the canonical New Keynesian model with a representative consumer and monopolistically competitive firms subject to Calvo pricing. As extensively discussed in Woodford (2003), under rational expectations (RE) the linearised aggregate economy can be summarized by the following two equations:

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t\right) \tag{2.1}$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \tag{2.2}$$

with shock processes

$$r_t = \rho_r r_{t-1} + \varepsilon_t^r, \quad \varepsilon_t^r \sim N(0, \sigma_r^2) \tag{2.3}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \quad \varepsilon_t^u \sim N(0, \sigma_u^2) \tag{2.4}$$

where x_t is the current output gap, defined as the difference between output and its natural rate in an economy with fully flexible prices; π_t denotes the inflation rate; i_t the nominal interest rate; β is the discount factor; σ is the elasticity of inter-temporal substitution of consumption; and κ is a convolution of structural parameters. All endogenous variables are expressed as log-deviations from their steady state values. Thus, in steady state $x = \pi = i = 0$. Finally, r_t and u_t stand for exogenous natural rate and cost-push shocks, respectively, and follow $AR(1)$ processes.

The model is closed with a Taylor rule subject to the zero lower bound (ZLB).

$$i_t = \max\{i^*, \delta i_{t-1} + (1-\delta)(\chi_\pi \pi_t + \chi_x x_t)\} \tag{2.5}$$

where the reaction parameters satisfy $\chi_\pi > 1$ and $\chi_x > 0$, and the interest rate smoothing $\delta \in (0,1)$. The constant $i^* = 1 - 1/\beta < 0$ represents the effective lower bound on interest rates since, otherwise, as explained in Eggertsson and Woodford (2003) agents would choose to hold all their assets in cash. I will refer to it as the ZLB to be consistent with the arguments in the Introduction and with real world analogies.

2.3.2 Expectations formation

The specification of expectations employed is adaptive learning (ADL). In particular, agents do not know the true structure of the economy and make forecasts as econometricians using simple regression models[3]. Namely, they make forecasts according to the aggregate policy functions from the minimum state-variable RE solution to the model:

$$\underset{3\times1}{Y_t} \equiv \begin{bmatrix} x \\ \pi \\ i \end{bmatrix}_t = \underset{3\times3}{\Gamma} \begin{bmatrix} u_t \\ r_t \\ i_{t-1} \end{bmatrix} \equiv \Gamma Z_t \tag{2.6}$$

where due to the smoothing in the Taylor rule, the lagged interest rate becomes a

[3]Following the 'consistency principle' of Evans and Honkapohja (2001)

state variable[4].

Adaptive learning is a linear updating procedure, yet the ZLB creates a non-linearity in the expectations for the path of the interest rate, because agents must understand it cannot be realised below i^*. To get around this issue I model the agents as forming expectations about the shadow interest rate and then applying the ZLB to their expectations. However, they use realised rather than shadow prices when forming expectations of x_t and π_t. The shadow rate is needed during a period of binding ZLB such that agents could form lift-off expectations consistent with the known policy prior to the ZLB. If they were to use $i_{-1} = i^*$ as a basis for expectations for $t = 0, 1...$, they would have an upward bias in their projected paths for the interest rate because the ZLB i^* is higher than the shadow rate at $t = -1$. Hence, as described below, I assume that above the ZLB agents rely solely on realised prices. When the ZLB binds, on the other hand, due to a lack of exact observable data on the policy rate, they rely on their shadow rate projections for the full path of realisations of i_t. Thus, even though the use of a shadow rate complicates the notation, this dichotomy is necessary for more realistic and sophisticated expectations. In this sense the imperfect knowledge of the agents here is conservative.

The agents have similar forecasting models to (2.6), as shown below. But each period as additional data becomes available, they update the coefficients to their perceived transition matrix Φ_t following a recursive constant gain algorithm. They are assumed to observe the disturbances r_t and u_t and to know their autoregressive coefficients [5].

Adaptive Learning

Denote by $S_t \equiv \begin{bmatrix} u_t & r_t & s_{t-1} \end{bmatrix}'$ the state variables vector where s_t is the shadow interest rate. Note that above the ZLB the Taylor rule (2.5) implies that the actual and shadow interest rates coincide - that is $i_t = s_t$ if $s_t > i^*$, while s_t could go below the ZLB and then $i_t = i^*$. This distinction is vital for the correct formulation of

[4]Note that (2.6) represents the solutions of the model under RE without a ZLB. If the ZLB is respected, when binding the solution of the model will be piece-wise linear featuring a sequence of different policy transformations Γ^i for every period i when the ZLB is binding.

[5]Eusepi and Preston (2010) show that this assumption can be dispensed with and instead agents would estimate those coefficients. For simplicity, it is maintained.

expectations of the agents because output gap and inflation are determined by actual prices (i.e. by the actual interest rate i_t). As mentioned above, on the other hand, the trajectory for the interest rate is determined by the shadow rate since otherwise the policy smoothing in the Taylor rule would create artificial upward drift in the interest rate due to the ZLB being an inefficiently high rate last period - $i_{t-1} = i^*$ but $s_{t-1} < i^*$.

Then, just like in the rational expectations solution in (2.6) the learning agents use the state variables vector and a transition matrix to forecast the endogenous variables vector $\begin{bmatrix} x_t & \pi_t & s_t \end{bmatrix}'$. Unlike RE, however, they do not know the correct transition matrix Γ from (2.6) and instead use their perceived 3-by-3 transition matrix Φ_{t-1} from the end of period $t-1$. Remember that the RE state variables vector with actual prices is $Z_t \equiv \begin{bmatrix} u_t & r_t & i_{t-1} \end{bmatrix}'$. Given the discussion above I assume agents use Z_t to form expectations of the output gap and inflation, but they use S_t to forecast the interest rate as follows:

$$\begin{aligned} \hat{\mathbb{E}}_t \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} &= \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}_{t-1} Z_t \\ \hat{\mathbb{E}}_t s_t &= \phi_{3,t-1} S_t \\ \hat{\mathbb{E}}_t i_{t+j} &= \max\left\{ i^*,\ \hat{\mathbb{E}}_t s_{t+j} \right\}, \qquad j \geq 0 \end{aligned} \tag{2.7}$$

where $\hat{\mathbb{E}}$ is the expectations operator for the learners and $\phi_{n,t}$ is the nth row of their perceived 3-by-3 transition matrix Φ_t. Agents update this perceived law of motion (LOM) by a recursive constant gain algorithm using the discrepancies between their expectations of endogenous variables $\hat{\mathbb{E}}Y_t$ and the actual realizations Y_t. They weigh this discrepancy by the historical variance-covariance matrix R_{t-1} of the endogenous variables and use the weighted forecast discrepancy for error correction. Each error correction term is given a constant gain weight τ against their prior beliefs from $t-1$ [6]. Finally, they update the VCV matrix R_t in a similar fashion.

[6] Note that here I assume constant gain learning instead of the decreasing gain learning used in Evans and Honkapohja (2001). The reason is that the former is more useful for tracking regime changes, while the latter is useful for studying asymptotic convergence properties of learning models to their RE counterparts. Given the current emphasis on the ZLB, tracking is a necessary feature of the model.

$$\begin{aligned}
\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}_t &= \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}_{t-1} + \tau R_{t-1}^{-1} Z_t \left(\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} - \hat{\mathbb{E}}_t \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} \right)' \\
\phi_{3,t} &= \phi_{3,t-1} + \tau R_t^{-1} S_t (i_t - \hat{\mathbb{E}}_t i_t) \\
R_t &= R_{t-1} + \tau (Z_t Z_t' - R_{t-1})
\end{aligned} \tag{2.8}$$

2.3.3 Bounded Rationality and the Actual Law of Motion

Replacing RE with ADL means that the structural equations of the economy (2.1)-(2.2) need to be modified accordingly. For a related class of models Preston (2005) and Eusepi and Preston (2016) argue that under ADL aggregate expectations $\hat{\mathbb{E}}_t$ are an average of the expectations of heterogeneous households and firms who know only their own objectives, constraints and beliefs and cannot compute aggregate probability laws, i.e. cannot obtain model-consistent expectations like RE. Thus, agents act rationally when it comes to their own objective functions but unlike rational agents fail to anticipate the aggregate laws of motion and resort to econometric learning as in section 2.3.2. A representative agent occurs when a symmetric equilibrium is assumed in which although everyone's problem is identical, no individual is aware of that and as a result the representative agent cannot compute aggregate probability laws. This breaks the law of iterated expectations (LIE) for the operator $\hat{\mathbb{E}}$, and hence the recursion from which the aggregate demand (2.1) and Phillips curve (2.2) equations are derived. These two equations under ADL and $\hat{\mathbb{E}}$ then depend on a long horizon expectations reading:

$$x_t = \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_{T+1} - \frac{1}{\sigma} (i_T - \pi_{T+1} - r_T) \right] \tag{2.9}$$

$$\pi_t = \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-1} \left[\kappa (x_T + u_T) + (1-\alpha)\beta \pi_{T+1} \right] \tag{2.10}$$

where $\hat{\mathbb{E}}_t$ again stands for the expectations of the adaptive learners and α is the Calvo probability of not being able to reset prices. I will refer to these two equations as the actual law of motion (ALM) of the economy.

Yet, Honkapohja et al. (2012) point out that assuming a continuum of sym-

metrical agents as is the case in the used NK model, one could still apply the LIE and resort to one period ahead Euler equation learning. I keep the infinite horizon learning for two reasons.

First, it allows for incorporation of FG as information about future policy rates into the law of motion for the aggregate variables, contrary to the Euler equation learning approach. Second, because agents do not know the structure of the economy, they cannot foresee how the ZLB will change the actual law of motion of the system. Take equation (2.1) and apply $\hat{\mathbb{E}}$ instead of the RE operation like the Euler equation learning approach advocated by Honkapohja et al. (2012) would prescribe. Now, if the ZLB is expected to be binding for a few periods ahead, its effect should come through expectations of the output gap ($\hat{\mathbb{E}}_t x_{t+1}$) and inflation ($\hat{\mathbb{E}}_t \pi_{t+1}$). But these are only gradually updated (as described in section 2.3.2), implying that although agents respect the ZLB in their forecasts for the interest rate, they are completely oblivious of its future effects on inflation and the output gap when only the Euler equation (2.1) is used. In contrast, suppose that agents expect $t = T^{ZLB}$ as the last period of binding ZLB. Then in the long horizon approach (2.9) they could set $\hat{\mathbb{E}}_t i_{t+s} = i^*$ for all $t + s \leq T^{ZLB}$. Then, the expected duration of the ZLB has an effect on the realisations of the output gap both through the current and future binding periods, which in turn is reflected on future inflation as in (2.10). Thus, the economy driven by the learners features minimal deviations from the rational expectations economy which are reflected only in the recursively updated $\hat{\mathbb{E}}_{t+s} x_{t+s+j}$ and $\hat{\mathbb{E}}_{t+s} \pi_{t+s+j}$ for $s \geq 1$, $j \geq 0$.

Disagreement between the CB and the learning agents

Suppose the economy exists for a long enough period with no extreme shocks that bring it to the ZLB. Then, following the forecast and updating procedures from section 2.3.2 the learning agents converged to the RE solution of the model in (2.6). This implies that at some period $t - s$ the perceived transition matrix has converge to the actual one - $\Phi_{t-s} = \Gamma$. Therefore, the agents have fully learned the model with no binding ZLB. The period of the Great Moderation is a useful analogy for this scenario.

Now, suppose the economy is hit by a demand shock ε_r at period t which brings the interest rate to the ZLB for at least 2 periods. Since the agents respect the ZLB in their expectations they know that today the interest rate will be at the ZLB - $\hat{\mathbb{E}}i_t = i^*$. Hence, from (2.8) the error correction term for the interest rate's law of motion is zero and no updating occurs - $\phi_{3,t} = \phi_{3,t-1}$. On the other hand, their perceived LOMs for output gap and inflation ($\phi_{1,t-1}$ and $\phi_{2,t-1}$) are the first and second rows of the transition matrix for a world with no binding ZLB ($\Phi_{t-1} = \Gamma$). A model prescribed by Γ is characterised by a fully flexible Taylor rule which accommodates demand shocks. This, however, is no longer true with a binding ZLB which locks the interest rate at an inefficiently high level i^*. Therefore the agents' forecasts for time t will be based on $t-1$ beliefs of the Great Moderation and will be too optimistic. At the end of period t they will observe the realisations and update their expectations as in (2.8). Overall, during the expected period of the ZLB the agents will not update their perceived Taylor rule for the shadow rate but will update their beliefs for the laws of motion of output gap and inflation.

I assume the Central Bank knows the ALM of the model (2.9) and (2.10) and observes agents' expectations $\hat{\mathbb{E}}Y_t$ for all endogenous variables[7]. Upon observing agents' expectations the CB plugs them into the ALM equations (2.9) and (2.10) and obtains model-consistent forecasts. Given its projections for output gap and inflation it uses the Taylor rule (2.5) and forms projections for the shadow rate. Because the CB's shadow rate forecasts are based on constantly updated $\hat{\mathbb{E}}_t$ expectations through the ALM, it is better able to anticipate the trajectory of the interest rate than the agents, who due to their fulfilled expectations of a binding ZLB in the immediate future fail to adjust the law of motion for the interest rate ($\phi_{3,t+s} = \phi_{3,t-1}$ if $\hat{\mathbb{E}}_{t+s-1}i_{t+s} = i^*$) and expect an earlier lift-off. Thus, they gradually update their output gap and inflation expectations, but the binding ZLB prevents them from understanding how the new regime changes the dynamics of the Taylor rule even in the absence of an explicit policy change. The only source of change in the system is the ZLB which affects the propagation of the state variables Z_t to the endogenous

[7]Considering the vast amounts of information collected and processed by central banks as well as their sophisticated forecast models this does seem like a realistic assumption.

variables Y_t

Proposition 1. *Suppose the economy is brought to the zero lower bound after a period of convergence to a rational expectations model with no binding ZLB. Then, the mechanics described above result in a disagreement between the agents and the Central Bank about the future path of the interest rate even in the absence of any policy change. Namely, the agents expect an earlier lift-off from the ZLB than the Central Bank.*

2.3.4 Forward guidance

Henceforth, I assume that in order to correct the disagreement about the future path of interest rates the CB uses forward guidance (FG) by truthfully revealing its expected lift-off date during every period of a binding ZLB. Next I describe how forecasts are made and the considered experiments of the use of FG. The next section presents simulations for each experiment and discusses their implications and effectiveness.

Forecasting

Every period the agents form long-run expectations $\hat{\mathbb{E}}_t \{x_j, \pi_j, i_j, s_j\}_{j=t}^{\infty}$ as outlined in section 2.3.2. This allows them to estimate the last period of binding ZLB defined as:

$$T^{ag} \text{ such that } \begin{cases} \hat{\mathbb{E}}_t i_{T^{ag}} = i^* \\ \hat{\mathbb{E}}_t i_{T^{ag}+1} > i^* \end{cases} \tag{2.11}$$

The Central Bank is assumed to have rational model-consistent expectations, but no choice variable and to truthfully reveal its expectations, thus abstracting from strategic behaviour. It observes agents' expectations ($\hat{\mathbb{E}}_t Y_{t+j},\ j > 0$ and T^{ag}) and uses them to form expectations according to the structural equations of the model (2.9)-(2.10). Then it sets its instrument i_t according to the Taylor rule (2.5) and in a similar fashion to (2.11) obtains its expectation of the last period of binding ZLB - T^{cb}. As per Proposition 1, we would have $T^{cb} > T^{ag}$, because agents'

expectations adjust to reflect the new regime[8] brought by the ZLB only gradually through observations. This disagreement about the path of the interest rate is the rationale for FG.

Experiments

At period $t = 1$ the economy is in its RE equilibrium above the ZLB. Then a large persistent natural rate shock (ε_2^r - see Table 2.B.1), pushes it to the ZLB. Both the agents and the CB anticipate a lift-off date according to the described procedures above. Whenever forecasts disagree, there is scope for forward guidance. Three cases of such CB communication are considered. In all cases where communication occurs, the CB is assumed to release its beliefs truthfully, abstracting from strategic behaviour.

1. **Baseline no FG** - the agents expect a lift-off at T^{ag} and are surprised by the continuing ZLB. They gradually update their beliefs by comparing s_t and i_t.

2. **FG as the length of the ZLB spell** - the CB releases T^{cb} and if different from T^{ag}, the agents adopt it outright in their expectations. This is reflected in the aggregate demand equation (2.9). Note that in this case the law of motion for the interest rate is not updated, so even at lift-off date (T^{cb}) there might be some disagreement between the CB and the agents.

3. **FG interpreted by adjusting** $\hat{\mathbb{E}}_t s_t = \boldsymbol{\phi_{3,t-1}} \begin{bmatrix} u_t \\ r_t \\ s_{t-1} \end{bmatrix}$

$$\phi'_{3,t-1} = \phi_{3,t-1} + \lambda R_{t-1}^{-1} S_{T^{cb}} \left(i^* - \hat{\mathbb{E}}_t i_{T^{cb}} \right) \tag{2.12}$$

   - Equation (2.12) shows that now when the CB announces T^{cb} the agents try to adjust their perceived LOM for the shadow rate such that as of today their expectations for date T^{cb} are for $\hat{\mathbb{E}}_t i_t = i^*$.

[8] as manifested through the transition matrix Φ_t.

- here $\lambda = \tau$ gives weight to FG announcements as 1 quarter worth of data. Variation in λ can proxy how credible or well understood FG is.

2.4 Experiments

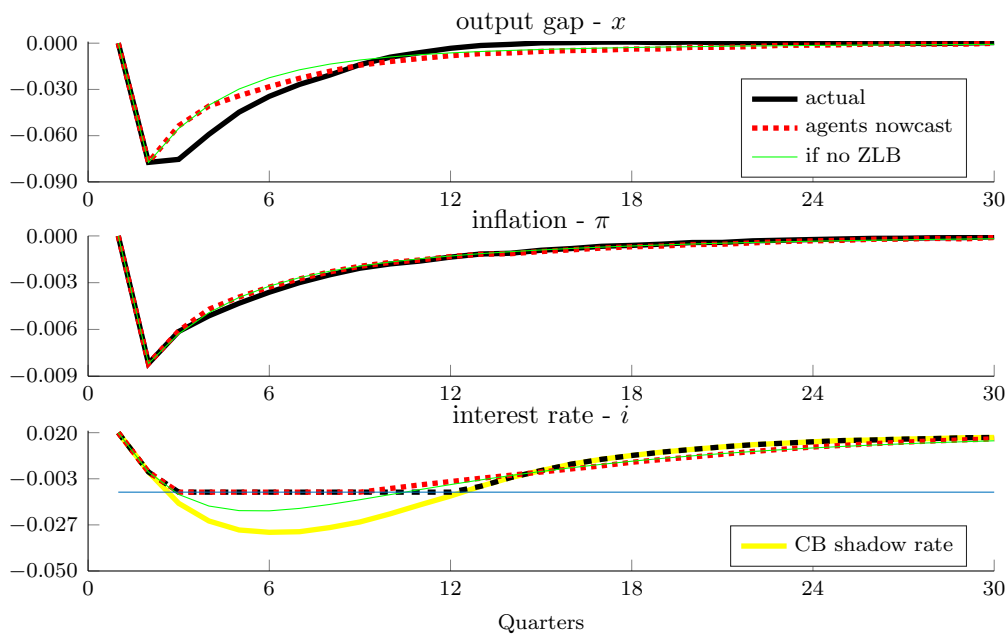
This section presents the conducted experiments and results. Throughout, I use the parameter and shock values from Table 2.B.1 in the appendix 2.B.1. Impulse responses are calculated as point-wise median from 5000 simulations of random iid ε_r and ε_u shocks. This is done in order to provide enough variability for the learners to update their perceived transition matrix. The zero lower bound is respected throughout and the only commonality between simulations is the negative natural rate shock at period $t = 2$.

2.4.1 No forward guidance

Figure 2.4.1 below shows the impulse response functions (IRFs) of x_t, π_t and i_t (both expected and realised) to an -8% natural rate shock in period 2, which results in a prolonged period of binding ZLB. The bold black line shows the actual end-of-period realisations of the endogenous variables, while the dashed red line is the beginning-of-period expectations of the agents. Both expectations and realisations are, as expected, below the schedules which would have occurred was there not ZLB constraint. Moreover, as explained in previous sections, agents' expectations of future output gap and inflation only change with observations even if they understand what the ZLB means for the path of the interest rate. Thus, initially they expect a faster recovery, yet since the ZLB changes the economy's response to shocks, the actual output gap and inflation turn out to be lower. The constant gain learning results in a quick updating of beliefs and convergence of the dotted and solid lines.

Note that without FG the agents' projected shadow rate will be identical to the hypothetical one if no lower bound constraint existed (green thin solid line). Figure 2.4.2 below zooms in on the end of the ZLB spell to highlight the disagreement about the lift off date between the agents and the CB. Even in this parsimonious model disagreement does occur and it is around 150 basis points at period 9 when

Figure 2.4.1: no FG - IRFs



the agents expect lift-off next period. A richer model featuring more persistence (e.g. habit formation or price indexation) as used by central banks today is likely to produce even larger disagreements. Finally, Figure 2.4.3 plots the expected duration of the ZLB of both agents and the CB when asked at every period. Disagreement persists with agents consistently expecting a 3-4 quarters shorter ZLB duration than the more informed CB.

2.4.2 "Period" forward guidance

Now suppose whenever disagreement occurs at the beginning of a period (as in Figure 2.4.3), the CB announces T^{cb} and the agents outright adopt it without changing their perception of the law of motion for the interest rate. Naturally, now the expected durations of the ZLB coincide throughout (Figure 2.B.1 in appendix 2.B.2). This situation is akin to the framework of forward guidance as anticipated shocks by Del Negro et al. (2012). Agents understand the length of the ZLB spell will be different but do not update their perceived LOM of the interest rate. Notably the agents' perceived LOM during the ZLB is misspecified but since no updating has occurred, it is in fact the correct one upon exit from the ZLB.

Figure 2.4.2: no FG - interest rate paths

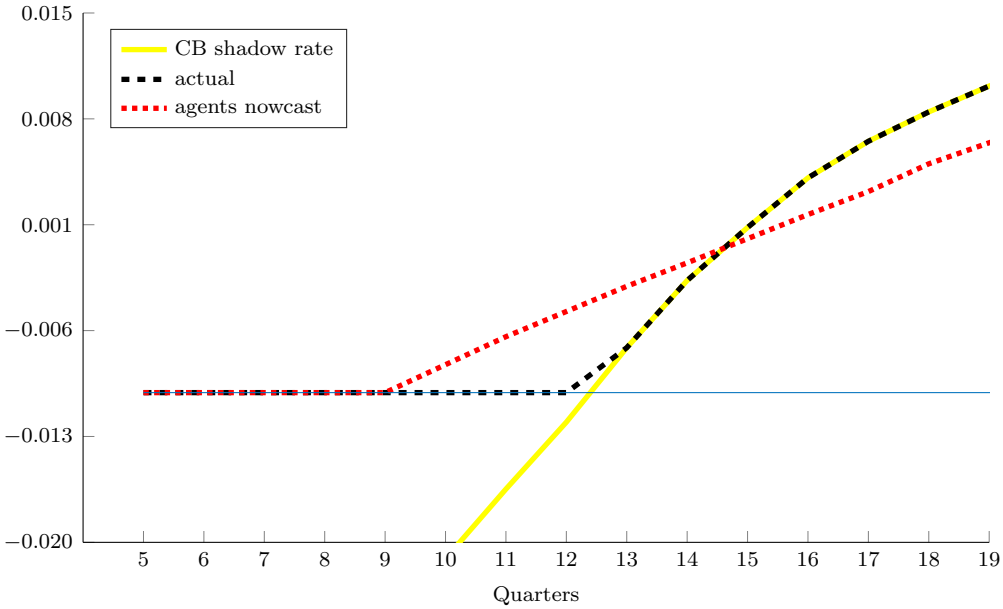
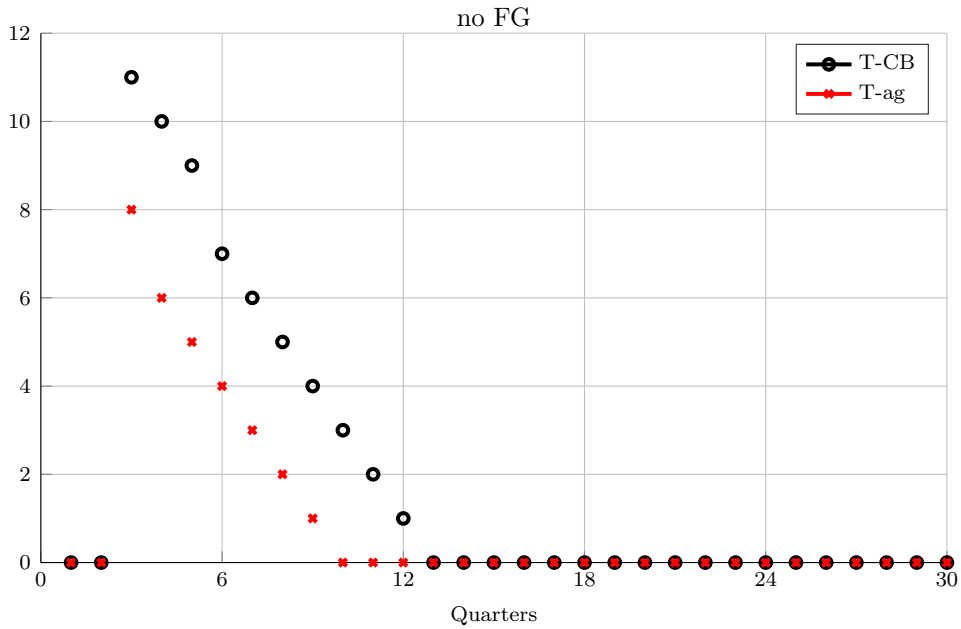


Figure 2.4.3: no FG - Anticipated duration of ZLB



2.4.3 Update from forward guidance

Such smooth transfer of information as above is not very likely in practice. In this scenario the CB again announces T^{cb} but instead of directly adopting it, the agents use their usual learning procedure aiming to adjust their expectations for the interest rate at time T^{cb} ($\hat{\mathbb{E}}_t i_{T^{cb}}$) to equal i^*. Note that this communication scheme resembles the conditional FG that CBs have implemented in practice.

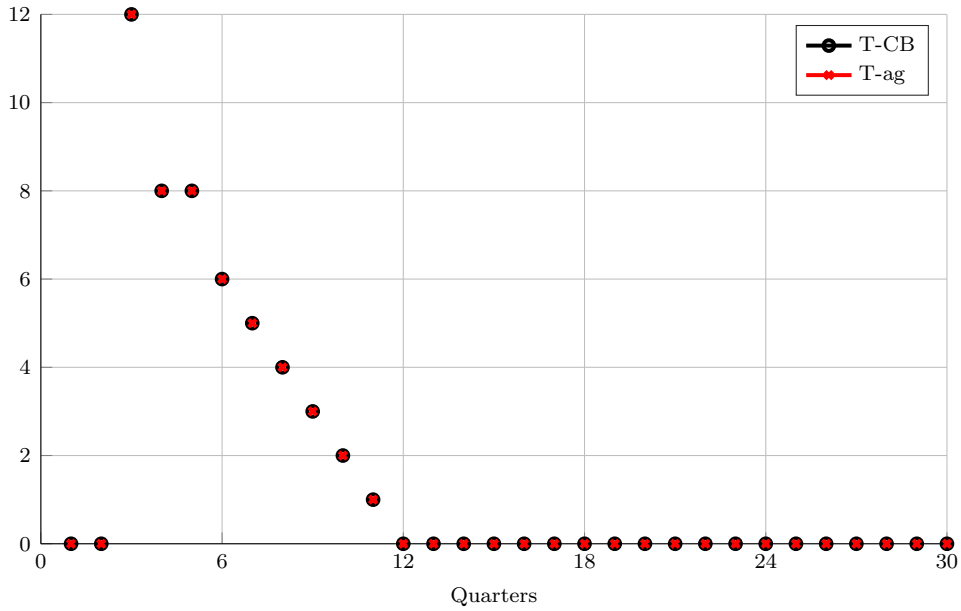
There are two differences with this learning step compared to their usual updating. First, it regards further than 1 period ahead forecasts. Second, the learning gain (λ) here can be varied to emulate the credibility of the message released by the CB. Here it is assumed that $\lambda = \tau = 0.02$, or the agents view CB's FG announcements as just another data point. See Marinkov (2018) for comparisons of the effects of different λ's. The learning for FG announcements (2.12) is restated below.

$$\phi_3' = \phi_{3,t-1} + \lambda R_{t-1}^{-1} S_{T^{cb}} \left(i^* - \hat{\mathbb{E}}_t i_{T^{cb}} \right)$$

A benefit of the "learning FG" scenario is that it could be beneficial in cases of earlier or delayed lift-offs than announced due to future shocks. Agents could better anticipate those if they have updated their perceived LOM for the interest rate. A potential downside compared to the "period" FG above is that this communication causes an expectational change in the perceived law of motion of the agents which might threaten the stability of the system.

Figure 2.4.4 shows the corresponding anticipated ZLB durations. Given that the agents solve a linear problem in order to match the announced lift-off date (2.12), it is no surprise that their perceived duration of the ZLB coincides with the CB's announcement. Notice that in period 4 the common perceived duration drops below the value of the no communication case in Figure 2.4.3. This happens because of the feedback of the updated long-run agents' expectations from (2.12) into the ALM (2.9) and (2.10).

Figure 2.4.4: "Learning FG" - Anticipated duration of ZLB



2.4.4 Welfare comparisons

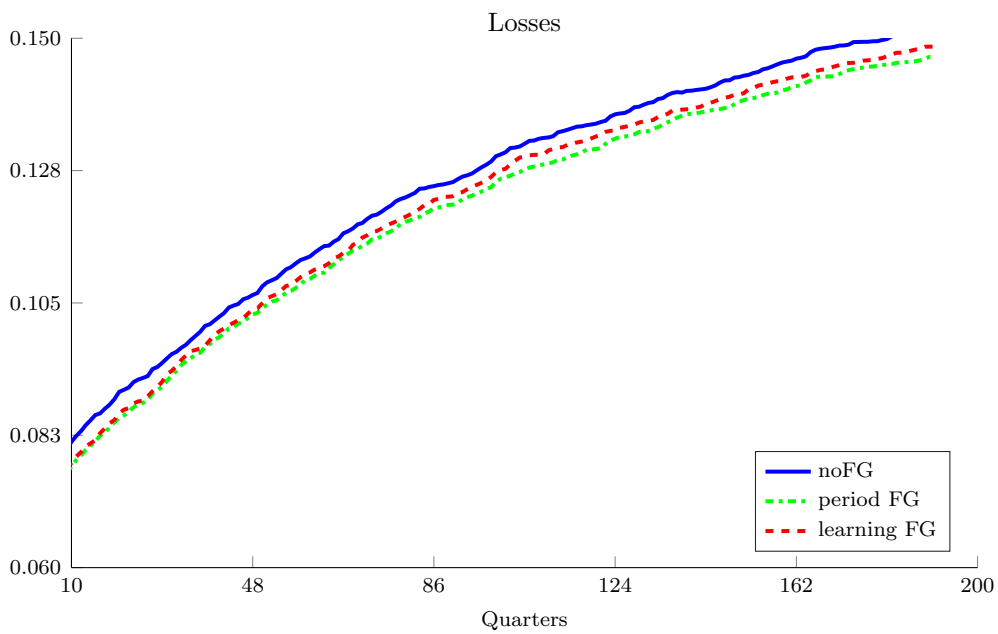
Figure 2.4.5 plots the cumulative welfare loss associated with the cases for forward guidance described above. It is computed through a standard central bank welfare loss function: $L_t = L_{t-1} + \beta^{t-1}\left(\pi_t^2 + 0.5x_t^2\right)$. Naturally, period forward guidance has the best welfare outcome since it results in full agreement and in contrast to the learning forward guidance it does not create any expectational drift from the announcements. Thus, after lift-off agents still hold their pre-crisis beliefs about the law of motion of the interest rate, which are in fact the correct ones for the case of above the ZLB. Although this is welfare improving, the gains are marginal and no forward guidance puzzle is present.

2.4.5 Beliefs' drift and Stability

This section discusses the underlying updating of beliefs in the three experiments. Figure 2.4.6 shows the drifts in the elements of the transition matrix Φ_t mapping states into expectations of endogenous variables. Although in the long-run these converge back to their equilibrium values under RE[9], they exhibit a prolonged drift

[9]Due to constant gain learning instead of decreasing gain learning they converge to a distribution centered around their RE values (Evans and Honkapohja, 2001)

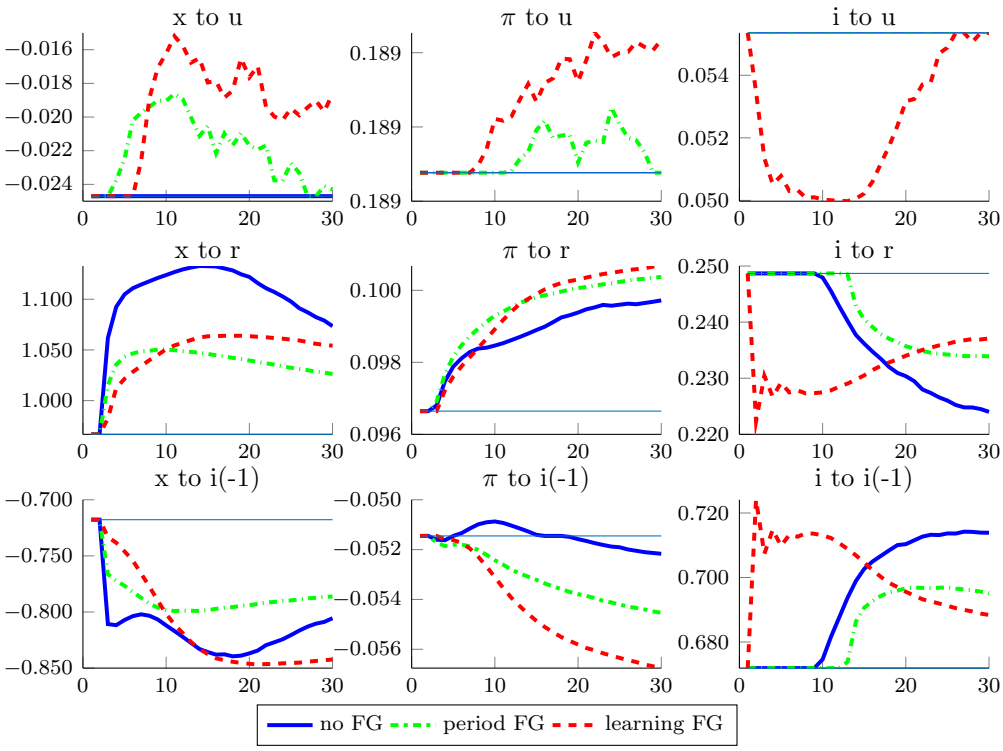
Figure 2.4.5: Welfare losses



away that lasts much longer after the ZLB is no longer binding. Regarding the welfare of the economy in the presence of future volatility this may be important in richer models or if shocks had larger variances such that future binding ZLB periods were more likely. Note that it is also consistent with the findings in section 2.2 where private expectations in Sweden remained higher than the CB's even in the end of the second ZLB spell.

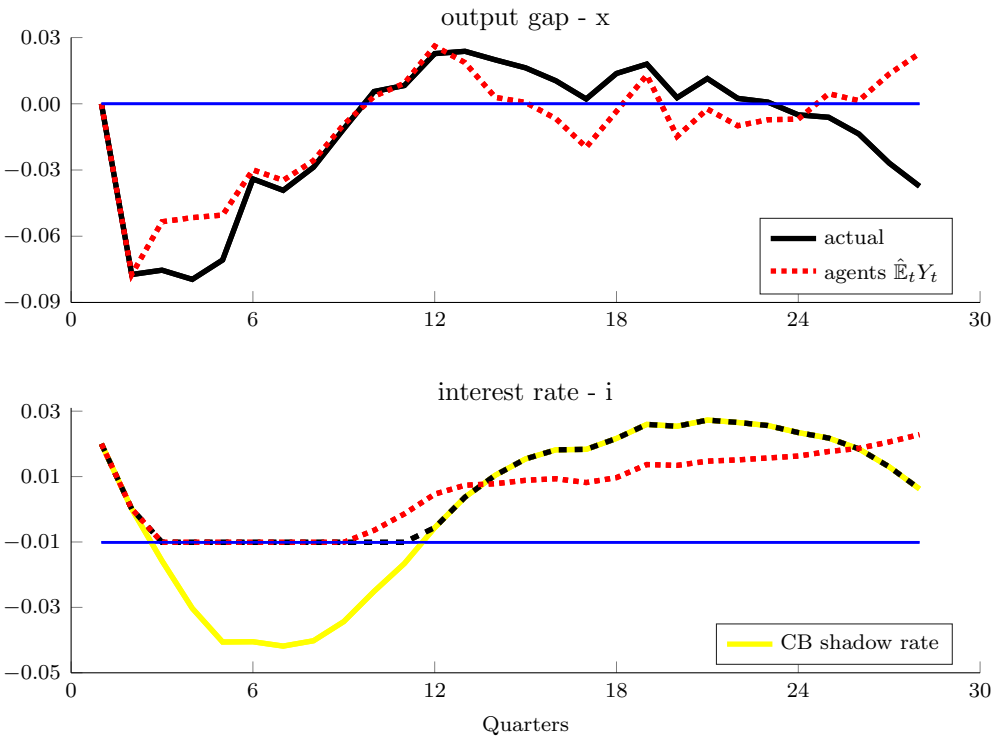
Importantly, the drift is especially dangerous in the case of no communication. As already established, the agents expect an earlier lift-off than the CB. After their last anticipated period of ZLB - T^{ag}, they expect an interest rate above the ZLB but observe it still remains at the ZLB. This causes them to increase their perceived persistence of the policy rate through their learning algorithm (2.8). All figures above are median outcomes from Monte Carlo simulations. Nonetheless, during these simulations I find that over 10,000 draws and 500 periods over half of the draws end up in instability due to perceived unit root in the law of motion of the interest rate. Figure 2.4.7 plots the impulse responses of an identical economy as in the no communication case but it allows for moderate future shocks after the initial period. The familiar disagreement about the lift-off date and the severity of the crisis are still present. This time, however a sequence of very small negative demand

Figure 2.4.6: Drift in perceived transition matrix Φ



shocks in period 12 push the economy beyond the bounds of stability. As established above, after their expected lift-off date (period 9) the agents observe a still binding ZLB which causes them to increase their perceived persistence of the interest rate. Iterated in their medium-run expectations in the ALM (2.9), this creates a boom in the economy around period 12. The Central Bank increases the interest rate in order to tame the boom, but this creases more disagreement in the interest rate forecasts with the agents. Given the small negative shocks at period 12 and the increasing policy rate, the agents again are lead to belief that the interest rate depends more on its past value rather than shocks. This again affects the medium to long-run expectations of the agents who now (around period 18) expect very high interest rates in the future, thus causing the economy to experience a recession. The CB, following its Taylor rule, quickly lowers interest rates, thus creating yet another big disagreement between with the agents. This causes even higher perceived persistence of the interest rate until around period 30 it surpasses 1 (unit root) and renders the economy explosive. The trajectory of the policy rate disagreement and the continual drift towards a unit root of the perceived persistence of the interest rate are depicted

Figure 2.4.7: no FG - single simulation IRFs



in Figure 2.4.8.

The learning literature has long established that the stability of the economy is greatly improved by a CB which reacts not to actual data as assumed here, but to the expectations of the agents (see Evans and Honkapohja (2003)). This is a remarkable analytical result in environments with no regime changes such as a zero lower bound. Here, I numerically make the case that FG can greatly improve the stability of a system with occasionally binding ZLB even when the CB reacts to contemporaneous data. The reason for this is that the communication provided by the CB provides a workaround for the unobservable shadow rate to the agents who adjust their expectations. This helps minimize the initial expectational drift caused by the ZLB period and keeps the economy tighter within the basin of convergence, which greatly improves its stability.

Forward guidance in both of its iterations considered above has a stabilizing effect on the economy by keeping expectational drift at bay, thus preserving stability. Figure 2.4.8 shows on the first row the disagreement between the agents and the

CB for the interest rate nowcast and on the second the AR(1) persistence in the perceived law of motion of the interest rate in the same Monte Carlo draw as in Figure 2.4.7 [10]. Although the two FG schemes exhibit some disagreement after the lift-off date, it is very contained and does not cause big drifts in the perceived persistence of the interest rate. The case of no communication, however, shows that disagreement keeps growing even after the lift-off and this is fuelled by an upward drifting perceived interest rate persistence. Once the perceived interest rate reaches 1, the system becomes explosive due to the long-horizon expectations in (2.9).

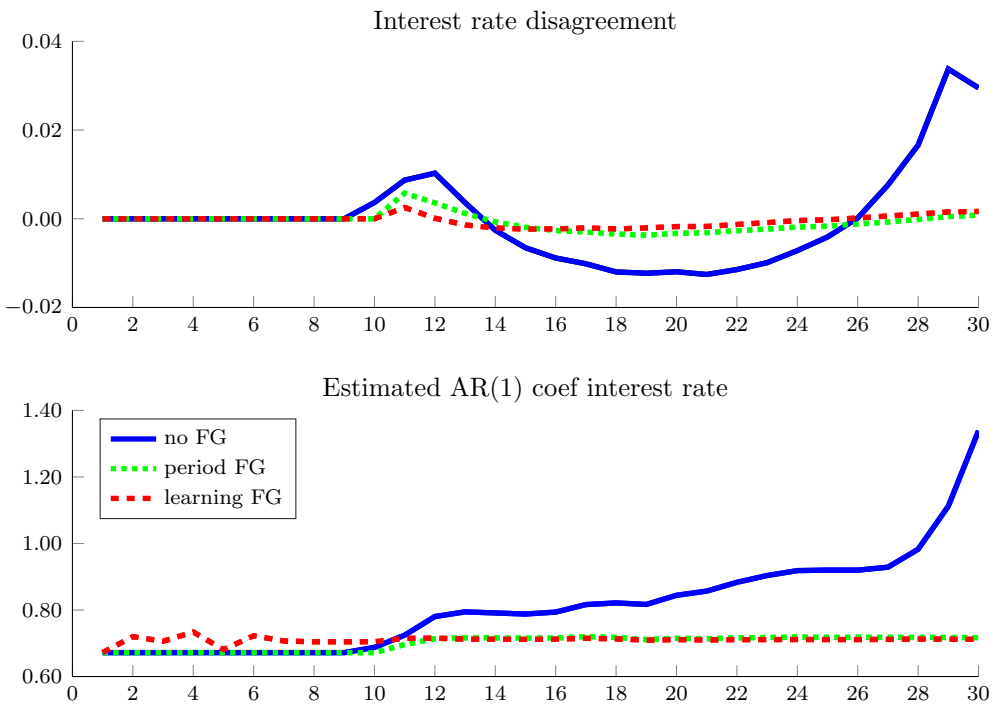
Thus, the last result of the paper is the forward guidance can be used at the ZLB to restore stability to the system. This is so because if no communication is issued, the learners will wrongly think the prolonged ZLB reflects higher persistence in the interest rate. Their updating quickly leads them to believe there is a unit root in the interest rate's law of motion since it does not react to shocks (the shadow rate does, but it is unobserved). When this happens, the economy becomes unstable. Forward guidance prevents this spurious drift in expectations and preserves the stability of the economy.

2.5 Discussion

This paper shows that the zero lower bound calls for a necessary increase in transparency and communication by the Central Bank because the ZLB kink distorts private agents' expectations of the trajectory of the policy rate. The private agents' and Central Bank's expectations diverge because the Bank is better able to understand the effects of the new ZLB regime on the aggregate law of motion of the economy. In particular, a binding ZLB causes private agents to expect and earlier lift-off than the CB does. In the simple model communication is achieved through forward guidance, yet in reality a combination of FG and asset purchases might be needed to achieve the necessary shift in expectations. The discrepancy is not negligible, but neither is it huge, so no forward guidance puzzle is present. Importantly, forward guidance can be used as a stabilizing tool to ensure stability at the ZLB by

[10] Figure 2.B.2 in the Appendix shows how an economy with the same sequence of shocks as in Figure 2.4.7 but with period FG preserves its stability has suffers less volatility.

Figure 2.4.8: Interest rate disagreement ($\hat{\mathbb{E}}_{ag} - \mathbb{E}_{cb}$) and perceived AR(1) persistence of interest rate



preventing spurious expectational drift.

Avenues for future work include allowing for Central Bank learning and considering optimal policy. A different expectation formation in the form of rational inattention also has the potential to explain disagreement and the effectiveness of forward guidance [11].

[11] Note that this is similar to varying the weight on CB announcements λ studied in Marinkov (2018)

Appendix

2.A Data

2.A.1 Robustness policy rate forecasts disagreement - Swedish Riksbank

Table 2.A.1 performs robustness checks on disagreement between the Riksbank and private agents. As in Table 2.2.1 in Low regimes agents expect on average higher interest rates than the CB (disagreement is negative and significant). In High states there is no significant disagreement between the CB and the agents at 3-months and 1-year forecast horizons. Interestingly, there is some evidence that at 2-year forecast horizons the Riksbank expect higher interest rates than the agents (disagreement is positive and significant). This might be due to better long-run forecasting abilities of the Central Bank or it might reflect a private agents' perception of more past-dependent policy compared to what the CB claims.

Table 2.A.1: Disagreement on Swedish repo rate

Based on private agents' expected 1-year-ahead repo rate				
	$\mathbb{E}_t i_{t+1y} \lesseqgtr 0.25$		$\mathbb{E}_t i_{t+1y} \lesseqgtr 0.75$	
	mean	se(mean)	mean	se(mean)
dis_3m_low	-.011	.0051	-.0426	.018
dis_1y_low	-.1135	.0085	-.1502	.0213
dis_2y_low	-.3111	.0414	-.306	.038
dis_3m_high	-.0376	.0423	-.0128	.0458
dis_1y_high	.0107	.0755	.0616	.0804
dis_2y_high	.1539	.0807	.2157	.0826

Based on Riksbank's expected 1-year-ahead repo rate				
	$\mathbb{E}_t i_{t+1y} \lesseqgtr 0.25$		$\mathbb{E}_t i_{t+1y} \lesseqgtr 0.75$	
	mean	se(mean)	mean	se(mean)
dis_3m_low	-.0322	.0155	-.041	.0172
dis_1y_low	-.1391	.0192	-.1822	.0378
dis_2y_low	-.2947	.0384	-.3262	.0414
dis_3m_high	-.0128	.0458	-.013	.0482
dis_1y_high	.0616	.0804	.1042	.0717
dis_2y_high	.2157	.0826	.262	.072

2.A.2 Estimating the Taylor rule

I use US data to estimate the following Taylor rule with policy smoothing:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\chi_\pi \pi_t + \chi_x x_t)$$

...

2.B Model

2.B.1 Calibration

Table 2.B.1: Calibration

parameter	value	source
α	0.75	sticky prices last for 3 quarters
β	0.99	implying 4.1 % annual rate of return
κ	0.024	Woodford(2003)
σ	3	implying IES of $\frac{1}{3}$
ρ_r	0.9	arbitrary
ρ_u	0.4	irrelevant
ρ_i	0.85	consistent with staff estimates
σ_r, σ_u	0.015	only for welfare loss calculations
ε_2^r	-0.07	a "Great Recession" shock
τ	0.02	standard in learning lit; robust to changes

2.B.2 Figures

Figure 2.B.1: "Period FG" - Anticipated duration of ZLB

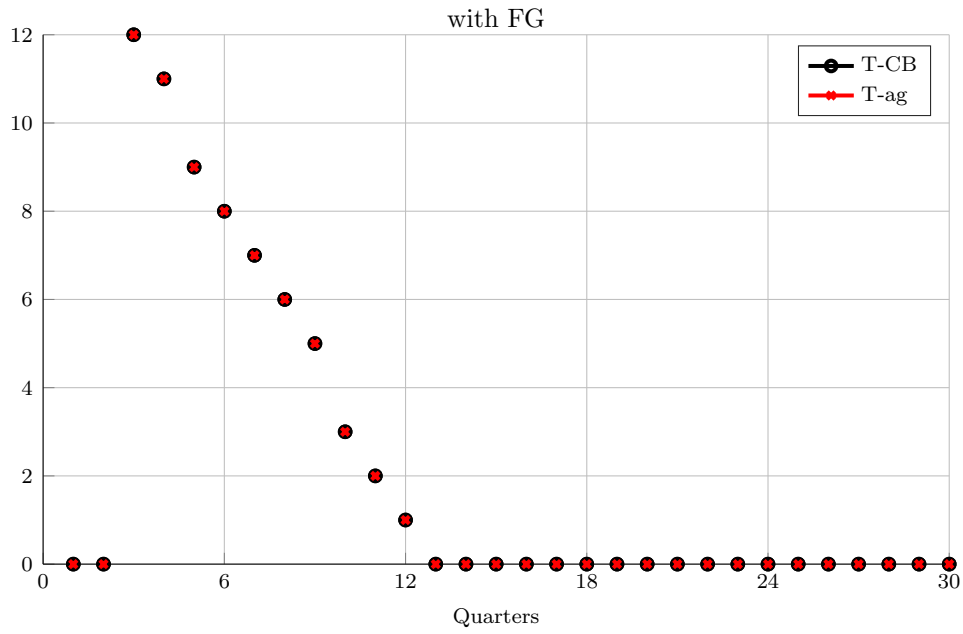
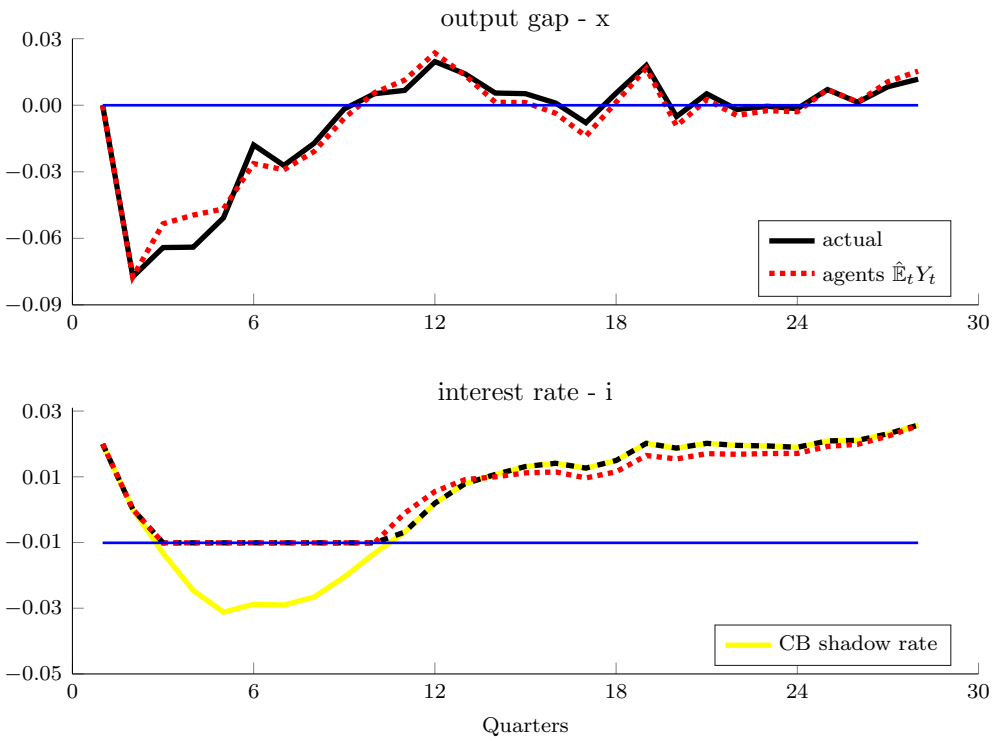


Figure 2.B.2: period FG - single simulation IRFs



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Chapter 3

Returns to skill and Industrial Sorting

3.1 Introduction

In this paper I analyse the sorting of workers to industries, studying whether strong positive assortative forces prevail in the data as they do in our theories, and whether the patterns differ across countries. Studying the sorting of workers to jobs according to their skills is important as the exact allocation of skills in production has large efficiency implications depending on whether there are complementarities or substitutes between and within skills and technologies. The strength of assortative forces in a country also affects the returns to skill and, as a result, the wage inequality.

The precise study of assortativeness often relies on matched employer-employee datasets, which although becoming gradually more accessible are still hard to obtain and limited to a few individual countries and industries. Here I use a novel dataset on workers' skills in OECD countries, which allows me to define new measures of assortativeness and examine its differences across countries. I find that assortative forces differ along the skill ladder of industries such that top-skilled industries have stronger assortative allocations of workers. Moreover, I show evidence that the development of a country is associated with stronger assortative forces and less mismatch. Industrial development towards the frontier in turn is shown to be systematically related to returns to skill - a novel finding which motivates the theoretical model.

Technological change is at the heart of the increase in sorting observed in recent decades[1]. Here I focus on the importance of complementarities and returns to skill

[1]Section 3.2 reviews the relevant literature. See Buera et al. (2018) and Håkanson et al. (2015), among others.

rather than skill-biased technological change. I build a model which illuminates the differences in returns to skill across sectors and countries as a driving force behind the levels of assortativeness and mismatch within and across industries. The model extends the frictionless 1-to-1 matching model of Becker (1973) and the frictional model of Eeckhout and Kircher (2011), and incorporates two sectors to which agents can sort on top of their matching in teams. Due to the presence of search costs randomly matched team members might choose to stay together even if a second-stage match would team them differently. The degree of sectoral cross-match is shown to be decreasing in the difference of returns to skill between the two sectors. Intuitively, if workers have a lot to gain by moving to the correct sector, they are more likely to re-match despite the search cost. Interestingly, in the model rich countries are found to have lower within sectoral matching bands, but larger cross-sectoral mismatch for a given search cost. This is caused by the overall higher returns to skill in rich countries but also smaller differences between sectors. To explain the uncovered higher cross-mismatch in poorer countries in the data, the model implies that poor countries must exhibit higher and perhaps additional types of labour frictions than rich ones.

This paper contributes to the empirical literature by showing evidence of the connection between returns to skill and the sorting of workers across countries. The model developed, on the other hand, shows the mechanism though which returns to skill drive sorting and identifies a trade-off between tighter within-industry matches versus higher cross-industry mismatch present in richer countries.

The paper is organised as follows. Section 3.2 discusses the relevant literature regarding returns to skill and assortativeness in the labour market. Section 3.3.1 studies the relationship between returns to skill and industrial development towards the frontier, while Section 3.3.2 analyses assortativeness across industries and countries. Section 3.4 develops a model of frictional matching and sorting, calibrates it and presents numerical results about the nature of assortativeness in rich and poor countries. Finally, Section 3.5 concludes.

3.2 Literature Review

Studying sorting has been historically difficult due to the requirements for detailed micro-level data on worker abilities. Earlier attempts relied on measures of segregation of similar workers at a firm or plant level. Examples include Kramarz et al. (1996) who find that segregation of workers has increased in France between the late 1980s and early 1990s and Kremer and Maskin (1996) who show the same for the UK and US. Moreover, Dunne et al. (2004) test the theoretical hypothesis of Caselli (1999) and Kremer and Maskin (1996) that differential rates of technological adoption across plants results in increased sorting of workers and wage dispersion. For the US they show that the between-plant wage and productivity dispersions have been growing over the prior decades and are a growing part of the rising total wage dispersion.

Relying on segregation measures suffers from the possibility that skill-biased technological progress, international trade and specialisation can cause wage dispersion without changing the underlying sorting patters of workers. Following the seminal methodology of Abowd et al. (2002), later studies have attempted to study sorting as imputed from the more common but still not commonplace matched employer-employee wage data sets. Bagger et al. (2013) analyse a full population Danish matched employer-employee dataset panel for 1980-2006. They study the correlation between worker and firm fixed effects which can be interpreted as "wage sorting". Their panel data allows them to study the evolution of this correlation and they show it has been steadily increasing throughout most of the sample period. They further show that this increase is exclusively driven by changes in the covariance of the two fixed effects and that it is not caused by compositional changes in the labour force in terms of education, age, and gender. Finally, they show that the rise in wage sorting explains 41% of the increased wage inequality in Denmark. Similarly, Card et al. (2013) find stronger assortative wage matching in West Germany between 1985 and 2009.

Nonetheless, Eeckhout and Kircher (2011) argue that using wage data alone it is impossible to determine the sign of the sorting due to the non-monotonicity of

wages in the firm's type. Intuitively, wages for a given worker are higher when matched with a correspondingly ranked firm because in a lower productivity firm the marginal product of the worker is lower, while a higher productivity firm would only match with a lower ranked worker if he or she accepts a pay cut.

Taking all these considerations into account, Håkanson et al. (2015) show that higher sorting and stronger complementarities help explain the rise in wage inequality, and confirm that inferring sorting from wage distributions may indeed be misleading. The authors study how the sorting of workers to firms has changed over time through the use of direct cognitive and non-cognitive skill measures linked to firm level data in the entire Swedish private sector. They decompose the variance of both types of skills into within and between firm and industry components. Then, they document that between 1986 and 2008 within firm variance has decreased and between firm variance has increased - all pointing to an increase in sorting[2]. Interestingly, the lower overall within firm variance has been largely driven by stronger sorting at the firm level rather than the structural move towards industries with inherently higher sorting. Lastly, the authors show that more high skill-intensive firms exhibit more positive assortative matching.

This discussion illustrates the importance of having direct measures of workers' ability when studying their sorting and matching patterns. The Programme for the International Assessment of Adult Competencies (PIAAC) is a novel dataset that measures the cognitive abilities of workers in a large set of OECD countries. Although available only at the 2-digit industry and 2-digit occupation levels, these data allow for direct comparison of the nature of workers' sorting in different countries without imputing it from wage data as previously discussed. Using the self-reported mismatch component of PIAAC McGowan and Andrews (2015) study skill and qualifications mismatch in 19 OECD countries. They find that higher mismatch of both skills and qualifications is associated with lower firm productivity and lower allocative efficiency. The former stems from the standard complementarities and positive assortative matching argument, while the latter might be explained by the restric-

[2]Håkanson et al. (2015) also use bootstrap techniques to confirm that the sorting in the data is orders of magnitude higher than what a random matching of workers to firms would imply.

tions that mismatch poses on the growth of high-productivity firms. For instance, they show that mismatch explains around 20% of the difference in allocative efficiency between the US and Spain or Italy. Similarly, Pellizzari and Fichen (2017) develop a richer methodology for defining upper and lower proficiency bounds for well-matched workers using PIAAC and find that on average 75% of workers are well-matched, yet this masks a large heterogeneity across countries.

The sorting of workers according to skill is directly affected by the returns to skill in different occupations, industries and countries. That is, high-skilled workers are likely to sort to jobs where their skills are rewarded the most. These forces in turn affect not only sorting but also wage inequality in a country. A few papers have used the PIAAC dataset to study the returns to skill and wage inequality at a granular level. Broecke et al. (2017) "use full distribution accounting techniques ... to decompose cross-country differences in inequality into differences in skills prices, on the one hand, and skills endowments, on the other", also accounting for supply and demand conditions. They find that skill prices are far more important than skill endowments in explaining cross-country differences in wage inequality. Moreover, they show that market forces contribute just as much as labour market institutions to the difference of wage inequality relative to the USA of the other participating countries - namely 25%.

Taking advantage of the harmonised cross-country cognitive scores in PIAAC, Hanushek et al. (2015) show that higher numeracy, literacy, and problem-solving scores are all systematically positively related to higher wages in all participating countries. However, the authors find large differences in returns to skills in countries - from 28% in the USA to only 12% in the Nordic countries, Italy and the Czech Republic, and call for better understanding of the relationship between individual skills, labour market outcomes and productivity. In a follow-up work, Hanushek et al. (2017) use the second round of PIAAC containing more countries to uncover even larger cross-country differences in returns to skill. They also find that prior GDP growth is strongly positively correlated with returns to skill, which they argue might be a sign of high-skilled workers coping better with change.

Exploring further the richness of PIAAC, Grundke et al. (2017a) document that

the dispersion of skills among countries is higher *within* industries than *across* industries. This suggests that the relative skill requirements of industries are broadly comparable across countries. They also point out that high-skilled services (e.g. Finance and Insurance) have the highest average worker skills. The authors show that higher cognitive skills are robustly related to higher productivity at the industry level beyond the effects of educational attainment. Through factor analysis they further extract "task-based" skills from the questionnaire portion of PIAAC and find that the so-called "ICT skills" is the only factor consistently positively correlated with productivity in all industries, but is especially prominent in services and high-skilled occupations. This finding might be explained by the importance of skill-biased technological change for productivity growth[3], related to the finding of Hanushek et al. (2017) above. Finally, cognitive skills are found to have a strong positive correlation with global value chains integration of an industry[4]. The top of the skills distribution is much more strongly and significantly correlated with productivity or trade, suggestive of assortative matching between workers, firms and industries.

The present paper contributes to the literature on mismatch, complementarities and sorting by studying both empirically and theoretically the sorting of workers to industries and comparing the assortativeness of the labour market outcomes across countries. Thus, it bridges the micro-level labour literature with the macro-level structural and technological change literatures.

3.3 Empirical

The Programme for the International Assessment of Adult Competencies (PIAAC) developed by the OECD conducts the Survey of Adult Skills in over 40 countries [5]

[3]See Berman et al. (1998), Acemoglu (1998), Caselli (1999), Acemoglu and Autor (2011) and Buera et al. (2018) and references therein among others.

[4]In a follow-up study Grundke et al. (2017b) explore the theory of Ohnsorge and Trefler (2007) that bundles of skills affect comparative advantage and find supportive evidence.

[5]Round 1 (2008-2013): Australia, Austria, Belgium (Flanders), Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Japan, Korea, Netherlands, Norway, Poland, Russian Federation, Slovak Republic, Spain, Sweden, United Kingdom (England and Northern Ireland), United States
Round 2 (2012-2016): Chile, Greece, Indonesia, Israel, Lithuania, New Zealand, Singapore, Slovenia, Turkey

and evaluates workers' proficiency in literacy, numeracy and problem solving[6]. The measures of literacy and numeracy are very closely correlated, while the problem solving indicator has a much smaller sample and is based on computer usage which differs across countries. For instance, Pellizzari and Fichen (2017) show that 90% of workers who are well-matched in literacy are also well-matched in numeracy, pointing to a substantial overlap in the two measures; while Broecke et al. (2017) find that the correlation between numeracy and literacy is 0.91 and their main findings hold for both measures (see Section 3.2). Thus, following Hanushek et al. (2015) and Broecke et al. (2017) I will rely on the measure of numeracy to represent workers' skills[7].

Apart from workers' scores, the PIAAC dataset also records their two digit ISCO occupation and two digit ISIC industry of employment and in some cases wages. Henceforth, I will use "sectors" for broad classifications such as agriculture, manufacturing and services and "industries" for more granular classifications (e.g. at 2 digits); I will also use scores and skills interchangeably. To avoid any industrial bias due to natural resources or institutions, I omit agricultural, energy and governmental industries in my analysis. Appendix ?? shows which industries are selected and how they are labelled across the three broad economic sectors - manufacturing, low-skilled services and high-skilled services. Note that although the numerical score is naturally an incomplete measure of a person's characteristics, Håkanson et al. (2015) have shown that sorting at the industry level is much more important for similar cognitive measures than for non-cognitive ones, and increasing over time for both.

3.3.1 Structural change

Buera et al. (2018) show that a country's development as measured by GDP per capita is associated with a structural move towards high-skill intensive industries and an increased skill premium. They label this phenomenon skill-biased structural change. Their finding is robust across countries but it relies on a very rough mea-

Round 3 (2016-2019): Ecuador, Hungary, Kazakhstan, Mexico, Peru, United States

[6]See Kankaraš et al. (2016) for a comprehensive survey of descriptive statistics in PIAAC.

[7]The definition of *numeracy* in PIAAC is: "ability to access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life."

sure of skill - namely, college versus non-college educated people. The secular move towards more skill-intensive industries raises the question whether the same industries are expanding in all countries and what features of their production technology affect the skill premium. Understanding the nature of the expanding skill-intensive industries will allow us to study returns to skill and wage inequality even within skill groups of workers.

To study this, I use the detailed PIAAC skills data to explore more granular structural features of development related to the sorting of workers to industries according to their skill level. I draw on the trade literature and construct an industrial rank correlation of skills for each country[8]. I rank industries by their average scores in each country and compare their ordering relative to the USA using a Spearman rank regression. Theoretical rank correlations vary between a perfect match of 1 and a polar opposite of -1. Column (e) in Table 3.3.1 shows that richer countries have higher skill rank correlations, i.e. their industrial structure is more similar to the US[9]. This confirms the findings of Sampson (2016) who finds that richer countries have more similar *wage* rank correlations to the US in manufacturing industries - a finding he attributes to differential costs of capital between countries. Table 3.3.1 complements his work by showing that the pattern holds even when skills are precisely measured, not proxied by wages, and the services sector is also included.

The evolving ranking of industries by skills might also have implications for the returns to skill in countries. Since workers sort across industries based on their wages, a more similar sorting of workers (that is - a higher rank correlation) would suggest a more similar return to skills between countries. Sampson (2016) finds no such evidence when using school attainment as a proxy for returns to skill and concludes that differential returns to skill are not related to wage rank reversals. The PIAAC dataset, however, enables the direct estimation of cross-country returns to skill. I obtain the returns to skill from a Mincerian regression of PPP-adjusted wages

[8]See Sampson (2016) for an application of rank correlations and assignment reversals to comparative advantage and wage inequality.

[9]Table 3.A.2 in Appendix 3.A shows the equivalent regression for occupational rank correlations. The relationship between development and occupational rankings is less robust suggesting that occupations, unlike industries, might be driven by other national or idiosyncratic factors - e.g. education and training system, preferences, tradition, etc. Expectedly, returns to skill are still strongly correlated with *occupational* rank correlations rather than sectoral.

and numerical scores following Hanushek et al. (2017) and include it as a regressor in Table 3.3.1. All regressions net out the countries' dispersion of skills defined as the ratio of the 90-th to 10-th percentile of numerical scores, thus focusing on the incremental importance of skills rather than distributional difference. Columns (a) - (d) show a robust positive correlation between returns to skill and industrial rank correlations regardless of the measure of development controlled for. This suggests that the higher rank correlations of countries are to a large extent driven by industries with high output elasticities to skill, thus resulting in higher returns to skills. This is yet another piece of evidence showing that the industries which tend to develop at the technological frontier are those where skills are rewarded the most. This could be interpreted as more efficient allocation of skill in richer countries - a hypothesis to which we return in the next section. This finding is in accordance with the evidence shown by Buera et al. (2018) but in addition to their work shows that the structural transformation has implications for the returns to skill (and hence wage inequality) along the entire skill distribution rather than merely between college and non-college graduates.

The higher returns to skills in countries with rank correlations closer to the US is confirmed also from augmenting the Mincerian regression to include interaction terms of individuals' numeracy scores and their country's rank correlation. Table 3.3.2 shows that the positive relationship holds in the economy as a whole but also in different broad sectors such as those encompassing above or below median skilled industries as well as the traditional split of industries into manufacturing and services sectors. Table 3.3.2 shows that a country's rank correlation has better explanatory power for its returns to skill than its GDP per capita does. The regressions partial out the mean and dispersion of skills as well as their interactions with the numeracy score, thus abstracting from distributional skill differences across countries.

Overall, we have seen that development is accompanied by a systematic move towards more similar industrial skill ranking. This industrial ordering is shown to be closely related to higher returns to skill beyond what individual or distributional country characteristics would imply. The results are consistent with those of Broecke et al. (2017) who show that skill prices are much more important for wage inequality

Table 3.3.1: Industrial Rank correlations

	a	b	c	d	e
Returns to skill	0.24**	0.30**	0.17**	0.26**	
	(0.08)	(0.09)	(0.08)	(0.09)	
Lgdppc	0.15**				0.21**
	(0.05)				(0.07)
Lckpc		0.09**			
		(0.03)			
Lhc			0.58***		
			(0.14)		
Mean VApc				0.07**	
				(0.03)	
Observations	25	25	25	23	25
Adjusted R^2	0.360	0.319	0.426	0.361	0.157

Robust standard errors in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$
Note: Lgdppc, Lckpc and Lhc refer to log deviations of GDP per capita, capital per capita and human capital relative to the US as measured by the Penn World Table. Mean VApc is the average value added per capita of at 4-digit industries from the UNIDO's INDSTAT4 dataset. To control for countries' different skill ditributions all regressions partial out the 90-th to 10-th percentile skill in each country. The US is not included in regressions with rank correlations because its rank correlation is trivially 1.

Table 3.3.2: Returns to skill and rank correlations

	all sectors	above median	below median	manufacturing	services
Numeracy score	0.22***	0.21***	0.18***	0.19***	0.23***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
x Lgdppc	-0.05	-0.08	-0.04	-0.00	-0.07
	(0.06)	(0.06)	(0.05)	(0.06)	(0.07)
x Rank correlation	0.16**	0.14*	0.17**	0.25**	0.14*
	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)
Country FE	YES	YES	YES	YES	YES
Controls with interactions	YES	YES	YES	YES	YES
Observations	25391	10339	14603	8592	16799
Adjusted R^2	0.22	0.21	0.19	0.21	0.23

Robust standard errors in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$
Note: All regressions include personal controls as in (**??**), GDP per capita and skill distributional controls and interaction terms. Country variables are partialed out and weights are normalised so countries have equal total weight of 1.

than skill endowments within a country. These findings suggest that the sorting of workers to industries is systematically related to the differences in returns to skill across countries. The next section, therefore, delves deeper into the differences in sorting and skill returns across industries and across countries.

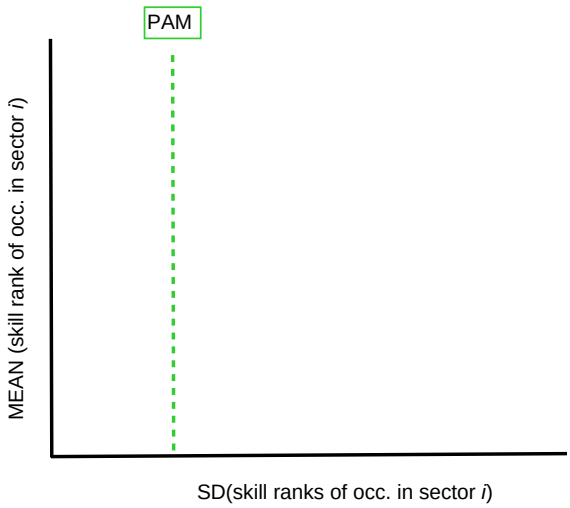
3.3.2 Measures of Assortativeness

This section studies the strength of the sorting forces across countries and broad sectors. First, it develops and motivates a metric which compares industries on their assortativeness. Second, it tests the matching between managers and non-managers for differences in assortativeness across countries.

To study the assortativeness of the sorting pattern of workers to industries, I will compare the average type of workers against the standard deviation of types within industries. To understand why this is a useful metric, suppose we are in a frictionless matching world populated by workers with heterogeneous skills. Define an industry (or a firm representative of an industry) as a team of N workers. Then, if all teams consist of equally skilled workers, we say the sorting exhibits perfect positive assortative matching (PAM). In this case the dispersion of scores *within* each team is zero, while their average scores are all different and ordered. If we were to plot all teams' average scores against their dispersion of workers scores, we would get a perfectly vertical line as in Figure 3.3.1 at zero dispersion. Building on the seminal work of Becker (1973), Eeckhout and Kircher (2011) show that with complementarity in production between workers and search frictions the matching will exhibit imperfect PAM. Each worker will have matching bands for potential partners rather than matching only with like-skilled workers. Importantly, the matching bands have constant width across skill ladder [10]. With dense enough population the resulting matching will translate in a vertical region of constant thickness in Figure 3.3.1 at some dispersion bigger than zero. Thus, teams will again be ordered in their average ranks, and at each average level of scores there is a constant variability in scores dispersion.

[10] Acceptance bands are parallel to the matching function only in the special case of $\theta = 1$ considered by the authors.

Figure 3.3.1: Theoretical PAM



Any deviations from such a pattern would imply deviations from PAM[11]. To study this I construct a similar measure using the PIAAC dataset. To abstract from the skill distributions differences across countries I standardize workers' scores to mean 0 and standard deviation 1 within each country. Due to the nature of production industries hire occupations in different intensities. Occupations, however, differ in their average levels of skills. What we are interested in is whether the best professionals of all occupations are matched together in an industry resembling PAM[12], or whether the sorting deviates from the theoretical first-best. Thus, to avoid compositional problems arising from using workers' skills to classify an industry, I proceed in three steps.

First, within each occupation across all industries I order workers according to their standardized score and give industries an occupation-specific rank depending on the average skill of professionals they hire of every given occupation. Since not all occupations are hired in all industries the rank queues differ across occupations. To deal with this I normalise the industries' ranks within each occupation such that, for example, the highest scoring industry in skills of economists, lawyers, managers and so on get a rank 1 for the specific occupation, while the lowest ranked get rank

[11]Note that in the one-to-one matching case perfect negative assortative matching (NAM) is represented by a flat horizontal line in Figure 3.3.1. In teams of more than two NAM is not well defined.

[12]E.g. in the most skilled industry the best economists are matched with the best lawyers, managers, etc.

0[13]. Second, I compute the average and standard deviations of normalised ranks of the hired occupations within each industry. Thus, if an industry hires the most skilled professionals of each occupation it uses, it will have an average rank of 1 and standard deviation of ranks 0. The use of ranks instead of scores avoids the issues associated with industries using a different number and type of occupations by measuring relative occupational ranking instead of overall numerical score. Thus, the measurement is closest to the theoretical counterpart of assortativeness.

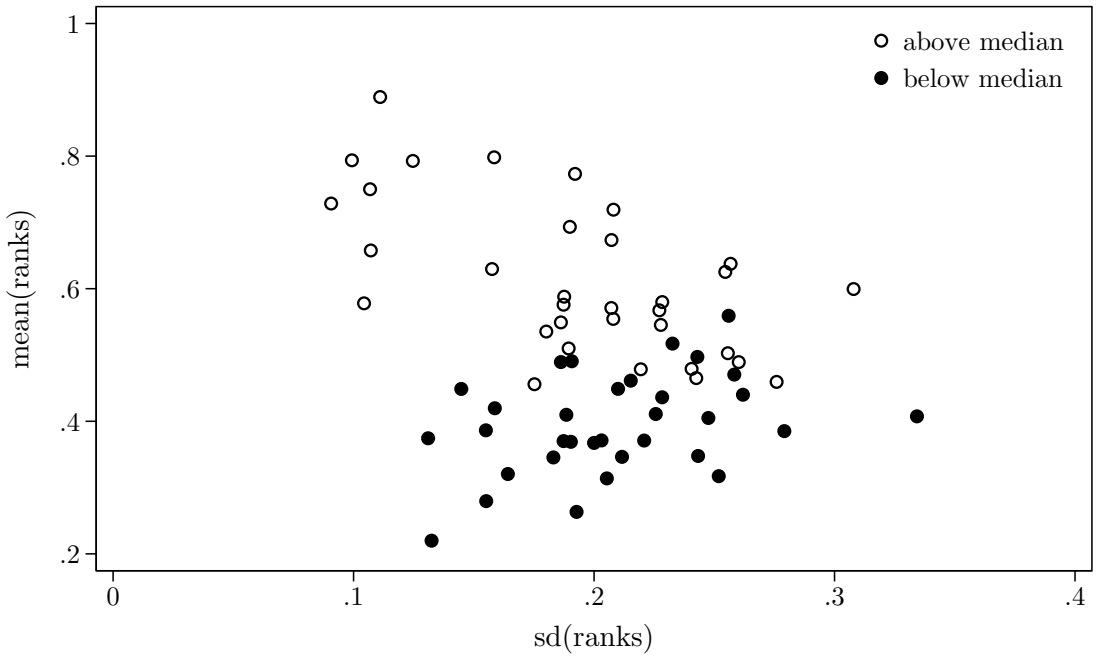
Figure 3.3.2 plots the mean skill rank of occupations in an industry against their standard deviation of occupational ranks. Here all countries are pooled because insufficient observations are available for individual countries at the 2-digit industry by 2-digit occupation level. Rankings are based on standardized scores as described above. Overall, the plot is reminiscent of the theoretical analogue of Figure 3.3.1 in that the dispersion of occupational ranks is fairly stable as mean ranks increase even if dispersion differs across industries unlike the stylised theoretical result. A distinct feature of the data, however, is that high-skilled industries (displayed in circles and defined as above median average occupational skill rank) tend to have lower dispersion of ranks, suggesting tighter acceptance regions for mismatched pairs and overall stronger assortativeness. Highest dispersion of ranks is observed in industries around the average mean rank. This feature of the data could be explained considering that in the presence of search costs it is the average skill levels which exhibit highest match-acceptance regions overlap. This futher translates in highest cross-sectoral mismatch and hence heightened dispersion of occupational ranks. This idea is further explained by the model in Section 3.4.

Thus, two main conclusions arise. First, perfect PAM does not hold across all industries as in the theoretical plots (the vertical line at 0 in Figure 3.3.1) since rank dispersion is significant. This motivates the introduction of search costs à la Eeckhout and Kircher (2011) in the model of Section 3.4. Second, assortative forces

[13]The normalisation works as follows. Suppose the order of industries for a given occupational ranking is such that rank 1 is the industry hiring the most skilled professionals of the given occupation and every following rank $(2...N)$ has decreasing average skilled workers from the same occupation. Let the ranking queue for each occupation be q_o. Then, ranks per occupation are standardised such as 1 is the highest and 0 the lowest by: $r_{norm} = 1 - \frac{r-1}{q_o-1} \in [0,1]$, where the normalised ranks r_{norm} are the ones used in the empirical estimations.

tend to be stronger in more skilled industries and there is highest cross-sectoral mismatch at average skill levels.

Figure 3.3.2: Sorting in the pooled sample



Note: Above and Below median are groups of industries which are respectively above and below the median average occupational skill rank in the pooled sample.

There are a few potential features of the data, however, which can be driving this pattern instead of it being due to assortative forces. First, one might imagine that all industries need managers (a relatively high-skilled profession), while only some use low-skilled labour, hence the negative relationship between the mean and dispersion. Remember, however, that Figure 3.3.2 is based on ranks within a profession, rather than numerical scores. Thus, the measure is not subject to the aforementioned caveat because it asks whether the managers employed in the low-skill intensive industry are one of the best or worst *managers*. Second, it could be that the pattern above is a result of specialisation of industries. In particular, it could be the case that the higher skilled industries are more specialised in their occupations and outsource many of the less-skilled or uncompetitive tasks[14]. Alternatively, the nature of production of

[14]Håkanson et al. (2015) note that although firms in Sweden between 1986 and 2008 gradually become more specialised in the type of workers they hire, the majority of the variance in skill

some industries could be such that they hire very diversely skilled occupations, while other hire very like-skilled ones. Even though the ranks measure is not susceptible to average skill differences between occupations, it could still be the case that such disparities of occupational types might affect the relative ranks of employees from different occupations.

To inspect the latter set of concerns Table 3.3.3 shows the regression results by broad sectors while controlling for the number of occupations in each industry to account for any systematic differences between industries with many or only a few occupations. Manufacturing and low-skilled services seems to exhibit no relationship between average ranks and assortativeness. The main result from the regressions is that services, and in particular high-skilled ones, are driving the pattern of stronger assortative forces in higher-skilled industries consistent with Figure 3.3.2. When industries are separated in above and below the median US industry skill it is again the high-skilled industries which have more tightly matched workers despite the control for number of occupations. Table 3.3.4 shows that the distributions of the number of occupations by industry are comparable, with low-skilled services having slightly higher average number of occupations.

Table 3.3.3: Mean rank by broad sectors in pooled sample

	all	M	S	L	H	> US med	≤ US med
SD ranks	-1.12***	0.06	-1.50***	0.05	-1.71***	-1.51***	0.75
	(-3.48)	(0.10)	(-3.72)	(0.06)	(-4.37)	(-4.82)	(1.68)
Number of occupations	-0.01***	-0.00	-0.01**	-0.01	-0.01	-0.01**	-0.00
	(-3.61)	(-0.65)	(-3.40)	(-1.52)	(-1.57)	(-2.29)	(-0.81)
Constant	0.92***	0.52**	1.03***	0.58**	1.05***	1.05***	0.31**
	(9.83)	(2.90)	(9.46)	(2.60)	(8.93)	(10.04)	(2.54)
Observations	66	27	39	20	19	32	34
Adjusted R^2	0.220	-0.056	0.344	0.037	0.497	0.424	0.090

t statistics in parentheses
Note: The broad sector abbreviations are as follows: M - Manufacturing, S - Service, L - Low-skilled service, H - High-skilled services, > US med and ≤ US med - industries with average skills above and below the median US industry.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$

between education groups is within firms. This means that firms in all industries employ workers from educational groups with very different average cognitive skills. This finding is even more important when aggregated at the industry level as in the PIAAC dataset.

Table 3.3.4: Number of occupations per industry by broad sectors in pooled sample

	count	mean	sd
M	27	19.63	5.18
H	19	17.32	5.51
L	20	21.50	7.72
Total	66	19.53	6.26

To circumvent the issue of insufficient observations by industry-occupation-country cells but still study cross country comparisons, I split countries in two samples and pool countries' observations within the sample. Motivated by the findings of the importance of industrial rank correlations for the returns to skill in Section 3.3.1 I segregate countries in above- and below-median rank correlations. I name the lower group "Club 0" and the higher "Club 1"[15]. Given the high correlation of development statistics and rank correlations in Table 3.3.1, henceforth I will also call them the poor and rich clubs, respectively. Table 3.3.5 shows the equivalent regressions for each country group. Again, the manufacturing and low-skilled industries exhibit no correlation between average ranks and standard deviation of ranks in either club and are therefore omitted. What is evident from the table is that high-skilled industries exhibit the familiar pattern of tighter matching (i.e. lower dispersion of ranks) at higher average skills. Nonetheless, richer countries exhibit a much steeper slope, suggestive of even stronger assortative forces at the top. Visual plots similar to Figure 3.3.2 are presented in Appendix 3.B for each club and again confirm that high-skilled industries have a similar consistent slope in both clubs, while low-skilled ones show no or even reverse relation. Again, the dispersion of ranks seems to be highest in the average-skilled industries suggestive of high cross-industry mismatch as discussed above and further in Section 3.4[16]. Table 3.A.1 in Appendix 3.A shows similar regressions while controlling for the dispersion of occupational types. There for each occupation I compute the average skill in the full country club sample. Then for each industry I compute the average type and dispersion of types of occupations by their sample average skills. This allows us to see whether some industries hire

[15]Table 3.A.3 in Appendix 3.A shows the countries by club.

[16]There we also see that ranks dispersion is overall higher in the poorer club. This is a finding to which we return in Section 3.4.2.

more or less homogenously skilled occupations and if this is related to their average type. The significance of the ranks mean-standard deviation relationship remains intact in both specifications, confirming that the assortativeness pattern is a robust feature beyond occupational specialisation and types.

Table 3.3.5: Mean rank by broad sectors and country clubs

	Club 0			Club 1		
	all	below median	above median	all	below median	above median
SD ranks	-0.29	0.87*	-1.28**	-1.28***	0.43	-1.65***
	(-0.85)	(1.87)	(-3.33)	(-3.73)	(0.92)	(-5.12)
Number of occupations	-0.01**	-0.00	-0.01**	-0.01**	-0.00	-0.01**
	(-2.55)	(-0.88)	(-2.20)	(-2.98)	(-0.45)	(-2.66)
Constant	0.69***	0.33**	1.00***	0.94***	0.34**	1.13***
	(7.17)	(2.74)	(8.36)	(8.82)	(2.42)	(9.86)
Observations	55	30	25	59	32	27
Adjusted R^2	0.078	0.094	0.336	0.211	-0.015	0.484

t statistics in parentheses
Note: Above and below median skill industries are defined according to the median-skilled industry in the US.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$

So far, we have established that higher-skilled industries tend to have workers of more similar skill ranks from all occupation, which we interpret as stronger assortativeness. Another intuitive measure of assortativeness is to compare the rankings of two types of occupations within industries and see whether they follow a similar ranking. To this end I compute the average ranks of managers[17] and those of non-managers in a manner identical to before, and study their correlation. Perfect PAM would be present if the correlation between the ranks of two groups of occupations across industries is equal to 1 and explains the entire variance. Naturally, and as seen so far, the data is much richer and features search frictions which reduces the explained variance. Importantly, comparisons across broad sectors (consisting of above and below median industries, for example) is not valid here because any cross-sectoral mismatch will result in steeper slopes (for instance, when a low rank manager ends up in the above median sector). Table 3.3.6 shows the only valid regressions of the full economy by country clubs. We see that rich countries have

[17]Managers consist of occupations of ISOC 2-digit scores of 11, 12, 13 and 14.

stronger assortative matching shown by their higher R^2 as well as steeper matching functions which are closer to the theoretical PAM slope of 1. This corresponds to lower mismatch regions for richer countries.

Table 3.3.6: Managerial rank to non-managerial rank, all industries

	Club 0	Club 1
Non-managerial rank	0.74**	0.87***
	(0.25)	(0.18)
Constant	0.13	0.07
	(0.13)	(0.09)
Observations	53	59
Adjusted R^2	0.125	0.273

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$

In summary, in this section we have established that industries differ in their assortativeness. First, high-skilled industries tend to have very closely matched workers by ranks from all occupations. Second, middle-skilled industries tend to have highest dispersion of ranks which could be explained by higher cross-industry mismatch due to labour frictions. At average skills workers are more likely to end up in a over- or under-qualified industry than are workers at the ends of the skill ranking. Moreover, we established that Club 1 countries have more assortative matching of managers to non-manager workers. These are all features of the data which will be studied in the theoretical model in the next section emphasising the importance of returns to skill for the assortativeness of the matching within and across industries.

3.4 Model

The model is an extension of the frictional 1-to-1 matching models (frictionless - Becker (1973), and frictional - Eeckhout and Kircher (2011)) incorporating two sectors to which heterogeneous workers sort according to their wage. A firm is a team of one manager and one worker operating under the technology of one of the sectors. The model is static, consisting of two stages. In the first stage, managers and workers meet randomly to form a team. They can decide to stay together or

pay a search cost and re-match to their optimal partner in the second stage. To abstract from consumer preferences the model is kept in partial equilibrium where the relative prices and productivities of the two sectors are taken as given[18].

Workers have skill $x \in [1, 2]$, while managers can have skills y above 2. Similar to Sampson (2014) who assumes that firms choose their technology subject to a R&D cost, I assume firms first employ their managers y subject to a sector-specific salary and then look for workers x. Although at odds with the normalised ranks between 0 and 1 in the previous section, this approach brings the benefits of analytical tractability as it allows us to solve analytically for the matching function $x = \mu(y)$. An interpretation of this setup is to assume managers are much more mobile and come from an exogenous world supply which is unaffected by any single country, while workers are mostly a locally determined factor. The alternative approach would be to also restrict the managers' skill support between 1 and 2. Then the matching function will depend on the assumed distributions of each type over the support and there will be no analytical solution like in the model of Grossman et al. (2017).

There is free entry of firms in either sector which in equilibrium ensures zero profits for all firms. A firm in sector i hiring worker x and manager y pays them respectively salaries $w_i(x)$ and $z_i(y)$. Working together the two produce $f_i(x, y) = A_i(xy)^{\theta_i}$, where A_i is an industry productivity shifter. As aforementioned for analytical tractability the salary of the managers in a given industry is given exogenously as $z_i(y) = p_i \delta_i y^{2\theta_i}$, where the elasticity of the salary function was chosen to ensure that the industrial matching function $x = \mu_i(y)$ is linear in y. The functional form is motivated by the Mincerian equation which postulates that wages are log-linear in skill. Lastly, p_i is the price deflator for industry i. Given all of this the profits of the firm are given as follows and due to free entry are equal to zero:

$$\Pi_i(x, y) = p_i f_i(x, y) - w_i(x) - z_i(y) = 0 \tag{3.1}$$

$$p_i A_i (xy)^{\theta_i} - w_i(x) - p_i \delta_i y^{2\theta_i} = 0 \tag{3.2}$$

[18]Note that Grossman et al. (2017) adopt a similar strategy.

The first-order condition with respect to y yields the matching function:

$$x = \left(\frac{2\delta_i}{A_i}\right)^{\frac{1}{\theta_i}} y \tag{3.3}$$

Note that the empirical equivalent to this object is the regression coefficient between the rank of managers to non-managers' rank in Table 3.3.6.

Using the zero profit condition, matching function and managers' salary schedule we can pin down the workers' wage:

$$w_i(x) = p_i A_i (xy)^{\theta_i} - p_i \delta_i y^{2\theta_i} \tag{3.4}$$

then plugging in for the matching function from (3.3) and the manager's salary $z_i(y) = p_i \delta_i y^{2\theta_i}$ we get the equilibrium worker x wage for industry i in the frictionless economy:

$$w_i^*(x) = \frac{p_i A_i^2}{4\delta_i} x^{2\theta_i} \tag{3.5}$$

The model exhibits labour market frictions in terms of a search cost. Following Eeckhout and Kircher (2011) there are two stages of the matching. In the first stage managers and workers meet randomly, then they can choose to stay together or pay a search cost $p_i c$ and match frictionlessly in the second stage which is described as above. Thus, in the second period, once matched frictionlessly, a worker x will certainly receive $w_i^*(x) - p_i c$, while a manager $\boxed{y \text{ - } z_i(y) - p_i c}$. A match surplus in the first stage is defined as:

$$p_i f_i(x, y) - (w_i^*(x) - p_i c) - (z_i(y) - p_i c) \tag{3.6}$$

Assuming that this match surplus is split by Nash bargaining with equal weights, a

first-period match will be retained only if the net match surplus is positive

$$p_i f_i(x,y) - (w_i^*(x) - p_i c) - (z_i(y) - p_i c) \geq 0 \tag{3.7}$$

$$p_i f_i(x,y) - w_i^*(x) - z_i(y) \geq -2cp_i \tag{3.8}$$

$$p_i A_i (xy)_i^{\theta} - \frac{p_i A_i^2}{4\delta_i} x^{2\theta_i} - p_i \delta_i y^{2\theta_i} \geq -2cp_i \tag{3.9}$$

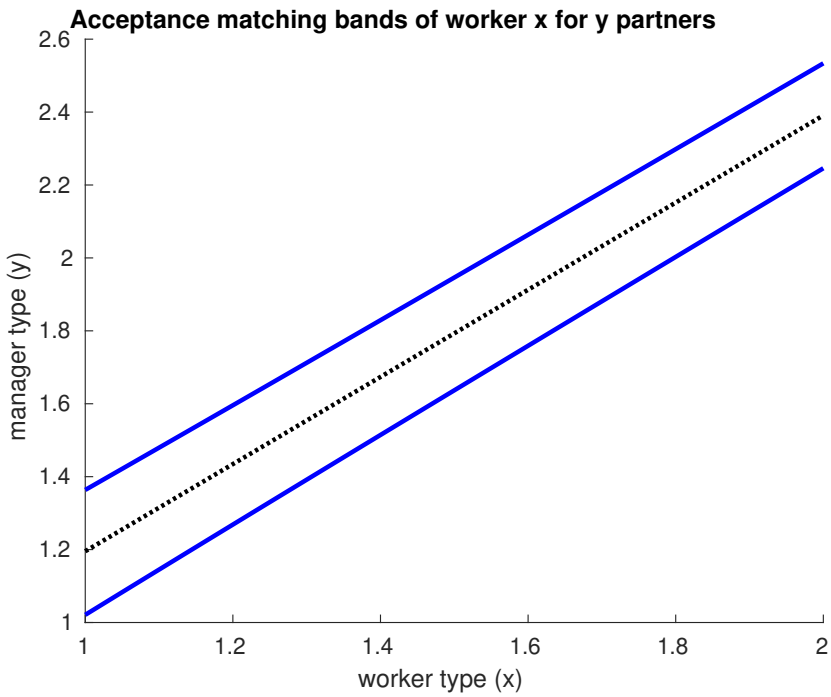
Consider for now that a pair chooses whether to stay together in industry i or jump to stage 2 and match frictionlessly again in industry i. This scenario is equivalent to the one considered in Eeckhout and Kircher (2011). Note, however, that given the existence of only one sector and a fixed support for the skill of the managers, the matching function in their model is always $x = y$. As discussed in Section 3.3.2 the assortativeness and hence the matching functions differ across high- and low-skilled sectors and later will define the two-sector equilibrium in our model.

This same-sector frictional matching problem presents a quadratic inequality in (3.9) which has a solution with acceptance bands for partners. For a given worker type x the acceptance bands for a manager partner are:

$$B_{ii}(x) = \left\{ \left(\frac{A_i}{2\delta_i} x^{\theta_i} - \sqrt{\frac{2c}{\delta_i}} \right)^{\frac{1}{\theta_i}}, \left(\frac{A_i}{2\delta_i} x^{\theta_i} + \sqrt{\frac{2c}{\delta_i}} \right)^{\frac{1}{\theta_i}} \right\} \tag{3.10}$$

That is, a worker x would be willing to stay with a manager y in period 1 if and only if $y \in B_{ii}(x)$. Notice that in the frictional case $c = 0$ the acceptance region collapses to the matching function as in (3.3) $y = \left(\frac{A_i}{2\delta_i}\right)^{\frac{1}{\theta_i}} x$ or $x = \left(\frac{2\delta_i}{A_i}\right)^{\frac{1}{\theta_i}} y$. Thus, the existence of search costs results in deviations from perfect PAM with moderate mismatch within a sector. Figure 3.4.1 illustrates the acceptance region for $\theta_i = 1.25$. Note that the matching bands are narrowing as x grows, resulting in more assortative matching. If $\theta_i = 1$, then the bands are linear and parallel to the frictionless matching dotted line throughout, while the case of $\theta_i < 1$ exhibits widening acceptance region at higher x.

Figure 3.4.1



3.4.1 Two sectors

The empirical results in Section 3.3.1 show that the returns to skill are tightly linked to the skill order of industries. Moreover, Section 3.3.2 illustrated that high-skilled industries tend to have tighter matching bands and overall more assortative matching of workers. To study the mechanisms connecting returns to skill, industrial skill and assortativeness this section augments the model with two sectors. Taking industrial output prices (p_i) and productivities (A_i) as given workers and managers endogenously choose to which sectors to sort, where sectors differ in their prices, productivities and, importantly, returns to skill. Although rank correlations are not well defined in a two-sector world, below we argue that cross-sectoral mismatch is a proxy for industrial rank correlations as analysed in Section 3.3.1.

To ensure that both sectors exist, we will look for a Roy-type threshold equilibrium in the frictionless case such that workers with skill $x > x^*$ sort to the high-skilled sector, those with $x < x^*$ sort to the low-skilled sector and at $x = x^*$ workers are indifferent between working in either sector. To achieve this we need the two wage schedules to have a single-crossing at a point $x^* > 1$.

Assumption 1. *Call the high-skilled sector H and the low-skilled one L. Then let $\theta_H > \theta_L$, so that the high-skilled has higher returns to skill.*

Assumption 1 is consistent with our findings that the high-skilled sector has higher returns to skill in Table 3.3.2. This higher elasticity of output with respect to skills implies steeper wages in sector H than sector L and ensure a threshold equilibrium. We can then find x^* as follows:

$$w_H^*(x^*) = w_L^*(x^*) \tag{3.11}$$

$$\frac{p_H A_H^2}{4\delta_H} x^{*2\theta_H} = \frac{p_L A_L^2}{4\delta_L} x^{*2\theta_L} \tag{3.12}$$

$$\Rightarrow x^* = \left(\frac{p_L A_L^2}{p_H A_H^2} \frac{\delta_H}{\delta_L} \right)^{\frac{1}{2(\theta_H - \theta_L)}} \tag{3.13}$$

Note that if $\theta_H = \theta_L$, then x^* is undetermined since the wage schedules are parallel and a threshold equilibrium does not exist. An analogous cut-off exists for managers too:

$$z_H(y^*) = z_L(y^*) \tag{3.14}$$

$$p_H \delta_H y^{*2\theta_H} = p_L \delta_L y^{*2\theta_L} \tag{3.15}$$

$$\Rightarrow y^* = \left(\frac{p_L \delta_L}{p_H \delta_H} \right)^{\frac{1}{2(\theta_H - \theta_L)}} \tag{3.16}$$

The bands for matching pairs on either side of the cut-off are identical as in the single-sector model in (3.10). However, with the introduction of two sectors there could be cross-matching. In particular, a match in the first stage might be between partners who in the second frictionless stage would end up being in different industries and hence subject to different production technologies and wages. This requires that additionally to solving for the same-sector matching bands we need to compute bands for cross-matched pairs who prefer to stay cross-matched with partners of the other sector than enter the second stage and search again.

I keep the convention that agent y as the manager determines the used technology. If $y > y^*$, then the technology used will be of sector H, otherwise it will be of sector L. Another convention is that the first letter in a matching pair type is for

the worker (x) and the second for the manager (y), so a LH match is one between a L worker ($x < x^*$) and a H manager ($y > y^*$).

Assumption 2. *Managers determine the technology used in cross-sector mismatched pairs.*

Consider first the LH match case. The pair uses H technology due to the manager. The condition for sustaining the first stage match is analogous to the derivation of (3.7). That is, the net surplus of the first-stage match over the outside option of a frictionless second-stage match must be positive in order to sustain the first-stage matching. Here, however, in the second stage the worker will be sorted to sector L and gets $w_L^*(x) - p_L c$, while the manager remains in sector H and gets $z_H(y) - p_H c$.

$$p_H f_H(x, y) - (w_L^*(x) - p_L c) - (z_H(y) - p_H c) \geq 0 \tag{3.17}$$

$$p_H A_H (xy)^{\theta_H} - \left(\frac{p_L A_L^2}{4\delta_L} x^{2\theta_L} - c(p_L + p_H) \right) - p_H \delta_H y^{2\theta_H} \geq 0 \tag{3.18}$$

Notice that this inequality is quadratic in y^{θ_H} but not in x^{θ_L} for $\theta_H \neq \theta_L$, hence it lacks an analytical solution for the acceptance bands of managers for their worker partners. The acceptance region of workers for manager partners is given by:

$$B_{LH}(x): \tag{3.19}$$

$$\left(\frac{A_H}{2\delta_H} x^{\theta_H} \pm \frac{1}{2\delta_H p_H} \sqrt{x^{2\theta_L} p_H p_L A_L^2 \frac{\delta_H}{\delta_L} \left(x^{2(\theta_H - \theta_L)} \frac{A_H^2 p_H}{\delta_H} \frac{\delta_L}{A_L^2 p_L} - 1 \right) + 4c(p_L + p_H)\delta_H p_H} \right)^{\frac{1}{\theta_H}}$$

Similarly, the HL case requires positive net first-stage surplus to avoid re-matching. Note that here the worker sorts to sector H in the frictionless case and gets $w_H^*(x) - p_H c$, while the manager remains in sector L with net salary $z_L(y) - p_L c$.

$$p_L f_L(x, y) - (w_H^*(x) - p_H c) - (z_L(y) - p_L c) \geq 0 \tag{3.20}$$

$$p_L A_L (xy)^{\theta_L} - \left(\frac{p_H A_H^2}{4\delta_H} x^{2\theta_H} - c(p_L + p_H) \right) - p_L \delta_L y^{2\theta_L} \geq 0 \tag{3.21}$$

Analogous to before this results in the acceptance region $B_{HL}(x)$:

$$B_{HL}(x): \tag{3.22}$$

$$\left(\frac{A_L}{2\delta_L}x^{\theta_L} \pm \frac{1}{2\delta_L p_L}\sqrt{x^{2\theta_L}p_L^2A_L^2\left(1 - x^{2(\theta_H-\theta_L)}\frac{A_H^2p_H}{\delta_H}\frac{\delta_L}{A_L^2p_L}\right) + 4c(p_L+p_H)\delta_L p_L}\right)^{\frac{1}{\theta_L}}$$

Again, as in the LH case the acceptance bands of manager y for worker partners x lack an analytical solution.

In contrast to the same-sector match where the absence of search costs collapsed the acceptance region to the frictionless matching function, here in the absence of search costs the match should be suboptimal and not constitute an equilibrium. Intuitively, since x and y are complements in production, the output of a pair is maximized when assortatively matched[19]. Thus, if there were no search costs $c = 0$, workers and managers can costlessly re-match optimally in a second stage which guarantees them highest income. Thus, no member of a cross-matched pair optimally chooses to remain cross-matched.

Proposition 2. *In the absence of search costs ($c = 0$) no cross-matched pairs choose to stay together and the two sectors are perfectly segregated with managers $y > y^*$ and workers $x > x^*$ sorting to sector H, while those of lower skills sort to sector L. That is, the two cross-matching outcomes LH and HL are suboptimal and do not constitute an equilibrium.*

Proof. Consider first the LH match case. By assumption the worker here will get matched in the lower sector in the frictionless economy, so it must be that $x < x^*$ and $w_L(x) > w_H(x)$. From (3.5) the latter implies $x^{2(\theta_H-\theta_L)}\frac{A_H^2p_H}{\delta_H}\frac{\delta_L}{A_L^2p_L} < 1$ and hence the parentheses in the square root of equation (3.19) are negative, violating such a match.

Turning to the HL case - it must be the case that $x > x^*$ and $w_L(x) < w_H(x)$. Therefore $x^{2(\theta_H-\theta_L)}\frac{A_H^2p_H}{\delta_H}\frac{\delta_L}{A_L^2p_L} > 1$ and this time the parentheses in equation (3.22) are negative, violating the match.

[19]This is a standard result in the positive assortative matching literature. See Becker (1973) and Eeckhout and Kircher (2011) among others.

Therefore, if $c = 0$, neither of the LH or HL matches can be sustained and they are always strictly dominated by the same-sector matches LL and HH. □

3.4.2 Calibration and numerical solution

We can use the model to analyse how the level of matching assortativeness in different countries is affected by differential returns to skill. In particular, not only can we derive the width of the same-sector matching bands as in (3.10), but we can also study the degree of cross-sectoral mismatch as defined by the cross-matching LH and HL bands in (3.19) and (3.22), respectively. Tighter matching bands imply stronger assortative matching since the skill of one of the partners has more explanatory power for the skill of the other (see Table 3.3.6 for the empirical equivalent). A higher level of cross-matching, on the other hand, implies less skill-segregation between sectors and hence a lower average skill difference between the sectors. In a multi-sector scenario this could change the ranking of sectors by average skills and hence represents the rank correlation reversals identified in Section 3.3.1.

This section calibrates the model for the two country clubs and numerically shows the same-sector and cross-sector matching bands. Since the search costs are unobservable, they will be imputed from the data to match the level of mismatch both within and across sectors in the two countries.

To simplify the calibration and focus on the role of returns to skill I assume that the managers' share of output are constant among all country clubs j and sectors i ($\frac{\delta_{ij}}{A_{ij}} = \bar{\delta}$), while the search costs (c_j) are country club specific.

Assumption 3. *Managers' share of output is the same for all broad sectors and all country clubs - $\frac{\delta_{ij}}{A_{ij}} = \bar{\delta}$. Search costs are the same for H and L sectors in a given country club.*

Calibrating the model requires determining the values of 10 parameters per country club. Note that the choice of A and p is equivalent to a choice of units[20] and what matters for the results of the model are the revenue productivity ratios $\frac{p_{Hj}A_{Hj}}{p_{Lj}A_{Lj}} = 1$ for each country club j. Thus, I normalise $A_{H0} = A_{H1}$, which implies $\delta_{H0} = \delta_{H1} = \bar{\delta}$.

[20]For instance, the search costs c would scale with A and p.

Next, I normalise $p_{H1} = p_{L1} = p_{H0} = 1$ which leaves p_{L0} for calibration.

Buera et al. (2018) show a robust positive correlation between log GDP per capita and the relative price of the high-skilled to low-skilled industries. Given the normalisation of the relative prices of the two broad sectors to 1 in the rich club of countries, using the average log GDP per capita for each club and the estimated slope by Buera et al. (2018) yields a relative price of the high-skilled to low-skilled sector in club 0 of $\frac{p_{H0}}{p_{L0}} = 1.15 \Rightarrow p_{L0} = 1.15$.

Turning to the calibration of the sorting cut-off x^*, note that it is a theoretical object in the frictionless equilibrium and does not exist in the data which features many occupations, search costs and other important worker characteristics beyond numerical score. As discussed in Section 3.3.2 there is a lot of cross-sectoral mismatch in the data which results in large overlap of numerical scores between sectors at middle of the skill distribution. Table 3.4.1 summarises the distributions of average occupational ranks across industries within broad sectors. The overlap of average occupational ranks is much higher in Club 0 evident by the smaller difference in average occupational ranks between the H and L sectors. Figures 3.B.3 and 3.B.4 in Appendix 3.B plot the kernel densities by country club and sector to illustrate the overlaps.

Table 3.4.1: Descriptive statistics - occupational ranks by sector

	Club 0		Club 1	
	mean	sd	mean	sd
L	0.47	0.11	0.41	0.10
H	0.56	0.13	0.60	0.14

Note: H and L sectors include industries above and below the median-skilled industry in the US.

Since I define the high- and low-skilled sectors as above and below the median skill, respectively, it is natural that the sorting cut-off for workers is $x^* = 1.5$. I keep this value for both country clubs.

The elasticity of the wage with respect to skill - $2\theta_{ij}$ from (3.5), is directly

estimated through a Mincerian regression featuring interaction terms for all 4 combinations of country clubs (0 and 1) and broad sectors (H and L). The numerical scores in the pooled sample were normalised to the interval $[0, 1]$ for coherence with the model's worker skill support of length 1. Table 3.4.2 shows the base returns to this normalised skill in club 0 sector L and the additional returns beyond it in other sectors.

Table 3.4.2: Returns to skill by country club and sector

	log wage
Numeracy score	
(0 L)	0.711***
	(0.0355)
x 0 H	0.229***
	(0.0187)
x 1 L	0.426***
	(0.0177)
x 1 H	0.503***
	(0.0189)
Occupation FE	YES
Observations	24942
Adjusted R^2	0.422

Standard errors in parentheses

Robust standard errors in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$
Note: Regression includes personal controls as in (**??**), GDP per capita, rank correlations and skill distributional controls and interaction terms. Occupational 2-digit FE are partialed out and weights are normalised so countries have equal total weight of 1. Sectors H and L are aggregations of industries, respectively above and below the US median-skilled industry.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table implies that $\theta_{L0} = \frac{0.711}{2} = 0.356$, $\theta_{H0} = \frac{0.711+0.229}{2} = 0.47$, $\theta_{L1} = \frac{0.711+0.426}{2} = 0.569$ and $\theta_{H1} = \frac{0.711+0.503}{2} = 0.607$. The high-skilled sector features higher returns to skill and the returns to skill in Club 1 are higher than those in Club 0 in both sectors - both findings are consistent with the result from Table 3.3.2. Lastly, a notable feature of the data is the bigger difference between the sectoral returns to skill in Club 0, which will help determine the overall search costs in Club 0 below. This is again supported by Table 3.3.2 where higher rank correlations translate to higher returns to skill more steeply in the low-skilled sector.

The only remaining structural parameters are A_{L0} and A_{L1} which we can obtain from the wage indifference condition at x^* from (3.13). Thus, in each country club

from (3.13) we get $A_L = x^{*2(\theta_H - \theta_L)} A_H \frac{p_H}{p_L}$ where we have made use of Assumption 3 which implies $\frac{A_L \delta_H}{A_H \delta_L} = 1$. Plugging in for the calibrated values implies $A_{L0} = 0.954$ and $A_{L1} = 1.032$. This calibration implies that the productivity of the low-skilled sector has grown faster with GDP per capita from Club 0 to Club 1 than has the productivity of the high-skilled sector. Note that this is consistent with the findings of Buera et al. (2018) who show much faster TFP growth in the low skill-intensive sector, which they attribute to technological advances in the goods industries.

Table 3.4.3 summarizes all calibrated and estimated values.

Table 3.4.3: Calibration

	Club 0	Club 1	Source
A_H	1	1	normalisation
A_L	0.954	1.032	implied from wage crossing at x^*
δ_H	0.45	0.45	assumed $\bar{\delta} = 0.45$
δ_L	0.43	0.469	implied from A_L and $\bar{\delta}$
θ_H	0.47	0.607	estimated in Table 3.4.2
θ_L	0.356	0.569	estimated in Table 3.4.2
p_H	1	1	normalised
p_L	1.15	1	Buera et al. (2018) for Club 0; normalised for Club 1
x^*	1.5	1.5	by sectoral segregation at median skill

To calibrate the search cost c I will target the goodness of fit in the manager to non-manager matching regressions in each country club from Table 3.3.6. The R-squared imply the following ratios of unexplained relative variance (SSE to SST[21]): $\frac{SSE_0}{SST_0} = 0.8584$ and $\frac{SSE_1}{SST_1} = 0.7141$, which correspond to relative standard errors of 0.9265 and 0.8450. Thus, Club 0 has roughly 10% larger standard errors than Club 1. Tee 95% of the distribution of standard errors in Club 1 is 0.33. To back out the implied search cost I target the radius of the within-sector matching bands at the sectoral cut-off $x^* = 1.5$ in the above median sector in Club 1 to be equal to the

[21] where SSE = Sum of Squared Errors; SST = Sum of Squared Total; $R^2 = 1 - \frac{SSE}{SST}$

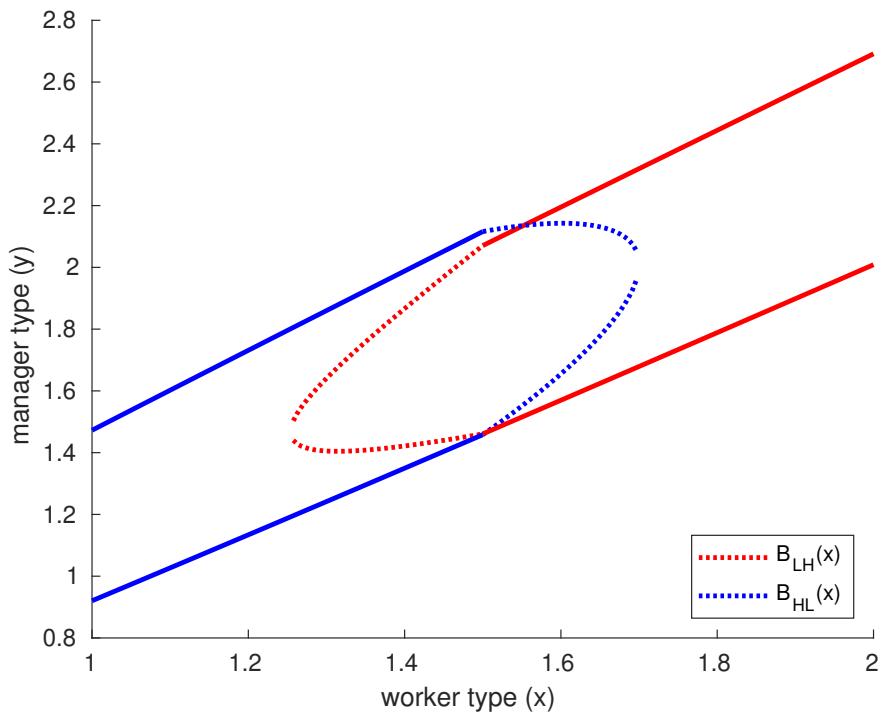
95th percentile of standard errors. That is:

$$\left(\frac{A_H}{2\delta_{H1}}x^{*\theta_{H1}} + \sqrt{\frac{2c_1}{\delta_{H1}}}\right)^{\frac{1}{\theta_{H1}}} = \underbrace{0.33}_{\text{standard error}} + \underbrace{\frac{A_H}{2\delta_{H1}}^{\frac{1}{\theta_{H1}}} x^*}_{\text{match acc. to matching function}}$$

Solving for c_1 using the calibration in Table 3.4.3 yields $c_1 = 0.005$. For Club 0 to have 10% wider matching bands at x^* in the above median sector it must be that $c_0 = 0.00433$.

Using the above calibration and search costs Figure 3.4.2 plots the acceptance regions of worker x for manager partners y in country club 16. Below $x^* = 1.5$ workers sort to sector L because $w_L(x) > w_H(x)$ and vice versa above x^*. Nonetheless, the dotted lines indicate that there are some workers around the threshold x^* which would accept a match with the opposite sector's managers. These are precisely the $B_{LH}(x)$ and $B_{HL}(x)$ cross-matching acceptance bands from (3.19) and (3.22).

Figure 3.4.2: Club 1:
Acceptance regions of worker w for a manager partner y



Note also that the matching regions are slightly tighter in the high-skilled sector due to its higher elasticity to skill ($\theta_{H1} > \theta_{L1}$). This is consistent with our empirical findings of the relationship between average ranks and the dispersion of ranks in

Figure 3.3.2 and Table 3.3.5, as well as Table 3.3.4 where we saw that high-skilled industries have much smaller dispersion of occupational skill ranks corresponding to the width of the acceptance regions in the model.

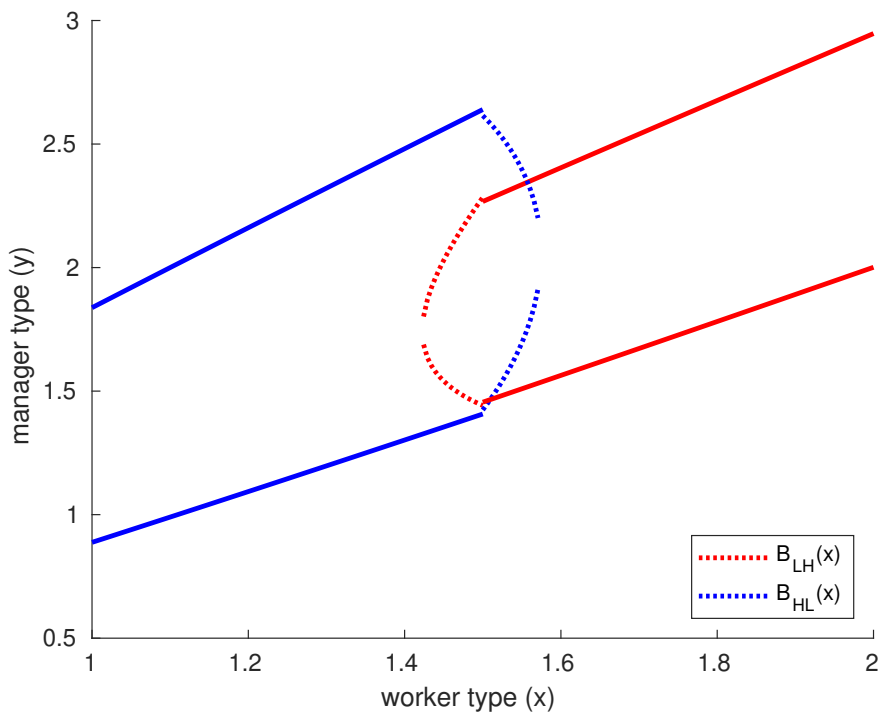
Figure 3.4.3 shows the solution for Club 0 using the calibrated search cost of $c_0 = 0.00433$. Immediately, a few interesting results come up. First, as targeted for the H sector, the matching bands of the both sectors are wider than in the richer club, implying noisier within-sector matching. This lower assortativeness in Club 0 is due to its lower values of θ and is consistent with the lower R^2 in the managerial to non-managerial rank matching in Table 3.3.6. The difference in returns to skill is strong enough that even if search costs are higher in the richer club ($c_1 > c_0$), the assortativeness is still lower in the poor set of countries. Intuitively, the lower returns to skill mean that agents have less to gain from paying the search cost and matching optimally in the second stage. Thus, a wider region of the first-period matches are sustained in equilibrium. Second, the acceptance regions for the L sector are much wider than those of the H sector compared to the difference between the two in Club 1. This bears the same explanation and is due to the higher difference $\theta_{H0} - \theta_{L0} > \theta_{H1} - \theta_{L1}$, that is - workers in sector L in Club 0 are least picky about their partners as they have very little to gain from matching optimally in the second stage.

Interestingly, the cross-matching regions ($B_{LH}(x)$ and $B_{HL}(x)$) appear smaller in Club 0 than Club 1. This is due again to the larger gap in returns to skill in Club 0. In particular, the returns to skill in Club 1 are very comparable between the sectors so cross-matched pairs face a relatively small gain from re-matching to their optimal sector. In Club 0, however, the difference in returns to skill is more pronounced and, for instance, H managers would require too high compensation from L workers in a cross-matched pair such that the workers would prefer to re-match .

Nonetheless, remember that earlier in Table 3.4.1 and the companion kernel densities (3.B.3 and 3.B.4 in Appendix 3.B) we showed that the data suggests higher cross-sectoral mismatch in Club 0. Moreover, as argued in Section 3.3.1 lower rank correlations may be interpreted as a result of larger cross-mismatch and by the definition of our country clubs based on rank correlations, it is expected

that Club 0 should have higher cross-mismatch. To explain the larger observed cross-mismatch, it must be that Club 0 has stronger labour frictions. Intuitively, the higher frictions counteract the high difference in returns to skill in Club 0. In particular, cross-mismatched workers have a stronger incentive to re-match in Club 0 as discussed above. However, if the search costs are much higher as well, this will dampen the gains from re-matching caused by high returns to skill difference and in equilibrium there will be more cross-matching. Increasing the search cost c_0, however, would augment not only the cross-matching bands but also the within sector ones. The latter were calibrated as 10% wider in the H sector than in Club 1 and any higher costs would violate this empirical finding.

Figure 3.4.3: Club 0:
Acceptance regions of worker w for a manager partner y



What this numerical exercise shows us is that higher returns to skill in rich countries contribute to tighter matching bands and hence more assortative matching in both broad sectors. Nonetheless, the returns to skill difference between the two sectors is lower in richer countries which facilitates more cross-sectoral mismatch as workers and managers have less to gain from matching to their theoretically optimal sector. To explain the higher level of mismatch found in the PIAAC dataset

for poorer countries, it must be that poorer countries have higher and perhaps additional labour market frictions to rich ones.

Overall, the model matches well the overall within- and cross-sector mismatch in rich countries. It also illustrates how lower returns to skill in poorer countries contribute to higher mismatch for any given search costs. Importantly, the model illustrates a novel trade-off for assortativeness in multi-sector models. Namely, the development of countries is associated not only with higher returns to skill but also lower differences in these returns across sectors. The improvement in returns to skill leads to lower overall mismatch both across and within sectors. The convergence of returns across sectors, however, leads to higher cross-sectoral mismatch which counteracts the allocative gains from overall higher returns to skill. Despite bearing merit in accounting for the empirical matching bands, the calibrated model is at odds with the data by predicting that richer countries have higher cross-sectoral mismatch. Accounting for this trade-off stands as an important area for future research with potential explanations relating to differential search costs by sectors not only countries and introducing additional types of labour market frictions (say, retraining costs of changing sectors).

3.5 Concluding remarks

This paper studies the relationship between returns to skill and assortative matching. It first shows evidence from the novel PIAAC dataset of workers' cognitive scores that returns to skill are closely related to industrial progress towards the frontier in OECD countries. It then establishes that high-skilled industries exhibit strongest assortative forces in all countries by developing a novel empirical measure. A major contribution of the paper is the study of assortativeness across countries. It finds that richer countries have stronger assortative forces and less cross-sectoral mismatch. This is attributed to the higher estimated returns to skill in richer countries and the mechanism is illustrated through a matching model.

The model builds on the frictional 1-to-1 matching model of Eeckhout and Kircher (2011) and incorporates two sectors to which workers can sort. Due to

the presence of search costs randomly matched team members might choose to stay together even if a second-stage match would sort them optimally. The degree of sectoral cross-match is shown to be decreasing in the difference of returns to skill between the two sectors. Intuitively, if workers have a lot to gain by moving to the correct sector, they are more likely to re-match. Interestingly, rich countries are found to have lower within sectoral matching bands, but larger cross-sectoral mismatch for a given search cost. This is caused by the overall higher returns to skill in rich countries but also smaller differences between sectors. To explain the uncovered higher cross-mismatch in poorer countries in the data, the model implies that poor countries must exhibit higher and perhaps additional types of labour frictions than rich ones. Studying the source of higher labour frictions in poorer countries stands as an important area for future research. Potential explanations might include differential search costs by sector not only country and explicit retraining costs for workers when changing sectors.

Appendix

3.A Tables

Table 3.A.1: Mean rank by broad sectors and country clubs - occupational type controls

	Club 0		
	H	above median	below median
SD occupational scores	-0.279	-0.443	-0.508
	(-0.80)	(-1.59)	(-1.49)
SD ranks	-1.305**	-0.886**	0.395
	(-4.17)	(-3.32)	(1.38)
Constant	0.934***	0.933***	0.526***
	(9.15)	(10.20)	(4.15)
Observations	16	28	30
Adjusted R^2	0.589	0.396	0.043

	Club 1		
	H	above median	below median
SD occupational scores	-0.73*	-0.69**	-0.20
	(-1.80)	(-2.63)	(-0.83)
SD ranks	-2.08***	-1.28***	0.42
	(-4.17)	(-4.27)	(1.13)
Constant	1.24***	1.13***	0.36**
	(7.00)	(11.27)	(2.55)
Observations	17	29	29
Adjusted R^2	0.505	0.537	0.019

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$

Table 3.A.2: Occupational Rank correlations

	a	b	c	d	e
Returns to skill	0.12*	0.16**	0.10	0.15*	
	(0.07)	(0.08)	(0.08)	(0.08)	
Lgdppc	0.09**				0.12**
	(0.03)				(0.04)
Lckpc		0.06**			
		(0.02)			
Lhc			0.25**		
			(0.11)		
Mean VApc				0.02	
				(0.02)	
Observations	25	25	25	23	25
Adjusted R^2	0.173	0.155	0.139	0.121	0.082

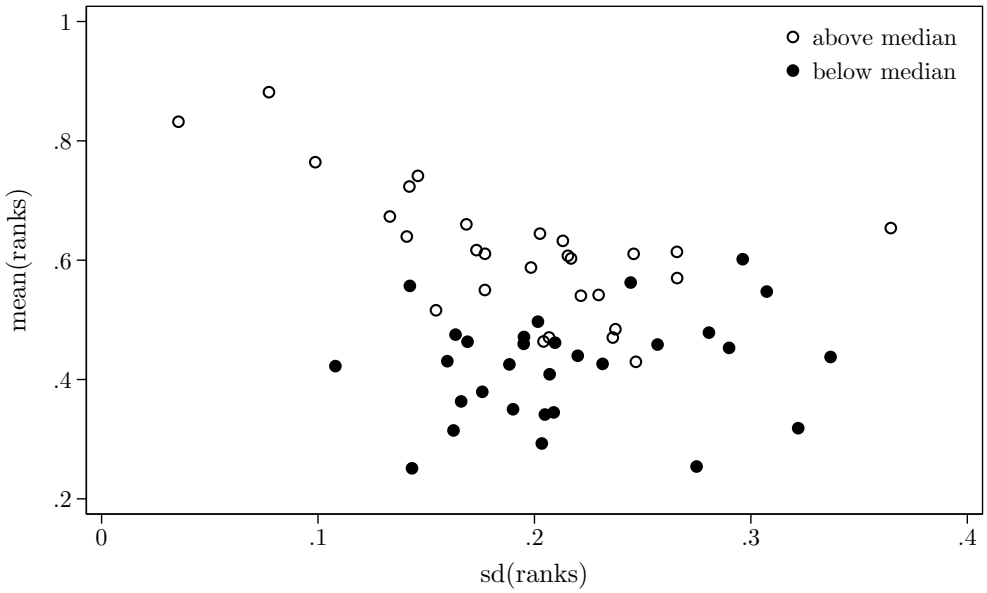
Robust standard errors in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$
Note: Lgdppc, Lckpc and Lhc refer to log deviations of GDP per capita, capital per capita and human capital relative to the US as measured by the Penn World Table. Mean VApc is the average value added per capita of at 4-digit industries from the UNIDO's INDSTAT4 dataset. To control for countries' different skill ditributions all regressions partial out the 90-th to 10-th percentile skill in each country. The US is not included in regressions with rank correlations because its rank correlation is trivially 1.

Table 3.A.3: Country clubs - above and below median industrial rank correlation

Club 0	Club 1
Chile	Austria
Cyprus	Belgium
Czech Republic	Canada
Estonia	Denmark
France	Finland
Greece	Germany
Ireland	Israel
Italy	Japan
Lithuania	Korea
New Zealand	Netherlands
Norway	Singapore
Poland	Slovenia
Slovakia	Sweden
Spain	United Kingdom
Turkey	United States

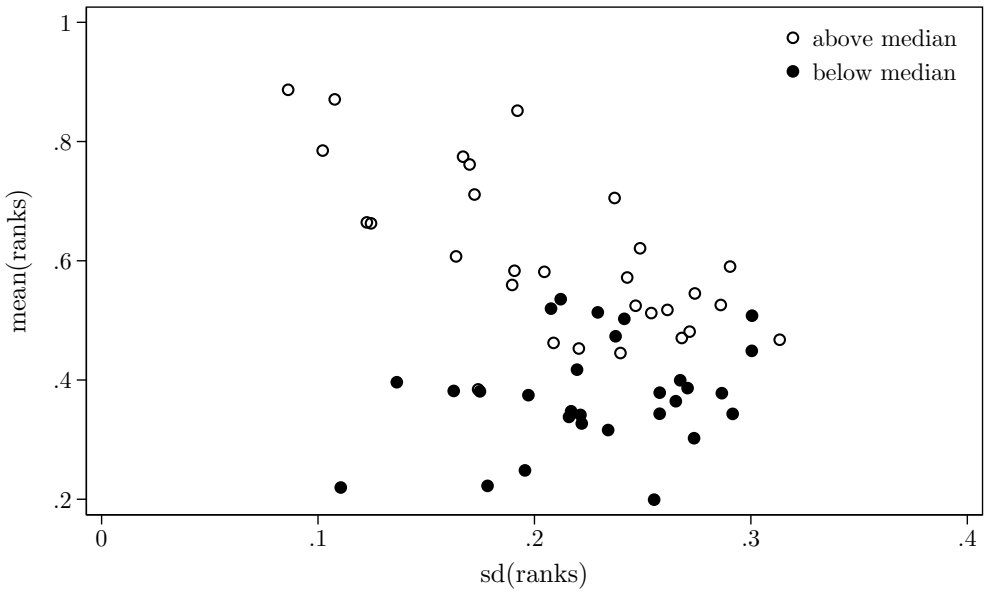
3.B Figures

Figure 3.B.1: Sorting in Club 0



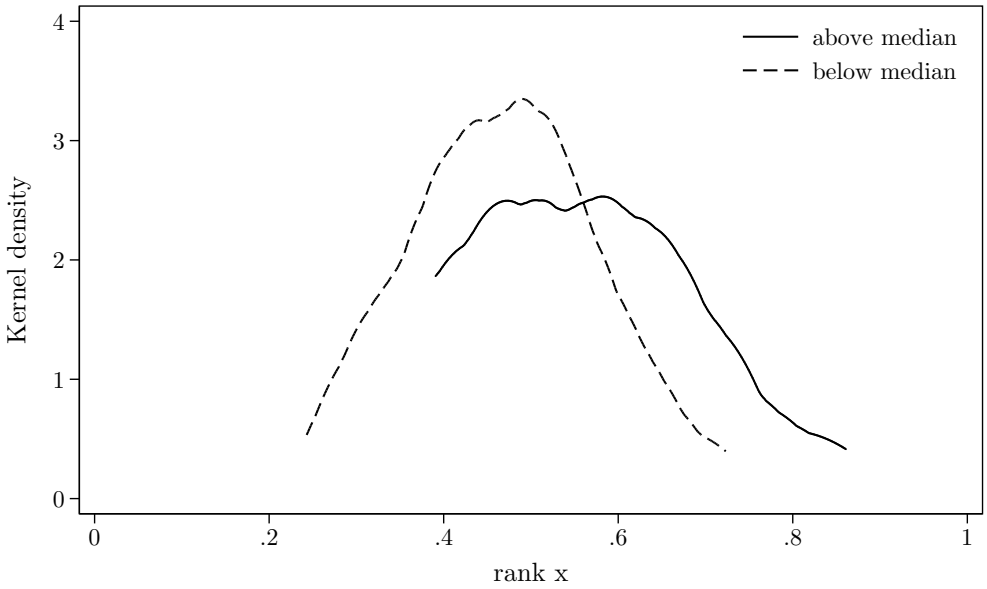
Note: Above and Below median are groups of industries which are respectively above and below the median average occupational skill rank in the pooled sample.

Figure 3.B.2: Sorting in Club 1



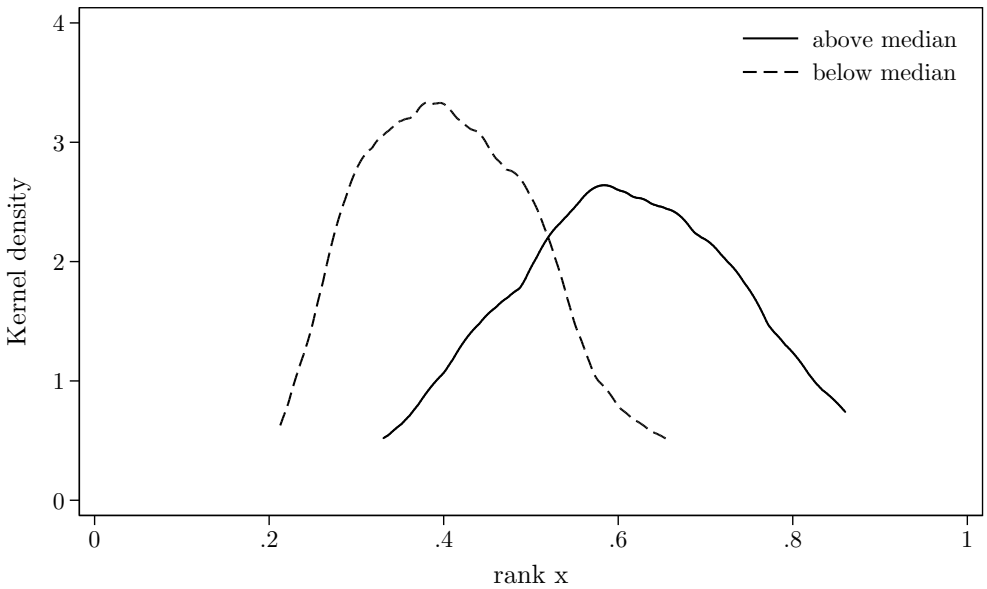
Note: Above and Below median are groups of industries which are respectively above and below the median average occupational skill rank in the pooled sample.

Figure 3.B.3: Kernel density of mean occupational ranks in Club 0



Note: Above and Below median are groups of industries which are respectively above and below the median average occupational skill rank in the US.

Figure 3.B.4: Kernel density of mean occupational ranks in Club 1



Note: Above and Below median are groups of industries which are respectively above and below the median average occupational skill rank in the US.

3.C Data

The PIAAC dataset provides final weights for each individual which are meant to correct national "sample data for bias resulting from survey errors such as sampling error, nonresponse error or noncoverage error" (Perry et al., 2017). These weights are required for an unbiased estimation on a national level. When pooling countries together I normalise the individual final weight such that each country has a cumulative weight of 1 (see Hanushek et al. (2015)). The PIAAC final weights, however, are not appropriate in weighting occupation-industry cells against each other since the weights are calculated on individual demographic and geographic characteristics vis-à-vis the national, and not on individual professional characteristics such as occupation or industry. In these cases I assign equal weight to every worker in the occupation-industry-country cell. Occupations in an industry are then weighted by their number of workers.

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