European University Institute

DEPARTMENT OF ECONOMICS

Essays on Social Insurance and Allocation of Resources

Karol Mazur

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 08 November 2019

European University Institute
Department of Economics

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I, Karol Mazur, certify that I am the author of the work "Essays on Social Insurance and Allocation of Resources" I have presented for examination for the PhD thesis at the European University Institute. I also certify that this is solely my own original work, other than where I have clearly indicated, in this declaration and in the thesis, that it is the work of others.

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I confirm that chapter 4 was the result of previous study "Subsidy to Search: Welfare Consequences of Welfare Entitlement for Quitters" I undertook at the Institute for Advanced Studies in Vienna.

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Abstract

The thesis is composed of four stand-alone essays analyzing economic problems pertinent to social insurance and allocation of resources in three applications to development, education and labor. The first essay investigates co-operative patterns of farmers in rural India engaging into informal insurance and public irrigation provision. I demonstrate theoretically, empirically and quantitatively that these two margins of co-operation reinforce each other, if the irrigational infrastructure is managed by local societies. On the other hand, management by external government agencies is associated with excessive crowding-out of informal insurance.

The second essay investigates constrained efficiency of a model with educational investments subject to uninsurable dropout risk, moral hazard and an endogenous college wage premium. I show that the laissez-faire equilibrium is constrained inefficient and is characterized by under-education. To this end, I show that an optimally designed student loan program with graduation-contingent repayment rates can attain the allocative efficiency of second best.

In the third essay, I show in a simple model of lumpy educational investments that subjective pessimism over returns to education can be self-confirmed in equilibrium. This leads to two empirical implications: (i) both the degree of human capital concentration and the degree of educational investments misallocation may be increasing in the rigidity of the education system's design; and (ii) commonly pursued methods may not identify the true underlying skill distributions.

The fourth essay uses a quantitative model of labor search with unemployment insurance and voluntary quits to study welfare consequences of a policy-reform giving entitlement to workers quitting their jobs in the US. Structural results show that pursuing a generous entitlement policy for quitters may allow for significant welfare gains through improved insurance and allocation of workers. Moreover, I employ the assumption of monetary search costs and show that it can explain the empirically documented unemployed search behavior.

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This thesis contains the fruits of my scholarly journey that has taken me from my undergraduate institution in Warsaw, to Vienna, Florence, London, with an upcoming pit stop in Oxford. The way leading to these essays has been full of ups and downs and, as such, I would have never been able to arrive at this point without the efforts and support of my family, mentors, teachers, friends and peers. Although I am most likely doomed to fail, I would like to express in what follows my gratitude to all the people that made this endeavor possible and ultimately successful.

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All four chapters of this thesis have been deeply inspired and shaped by classes, seminars and office hours (with both EUI's and visiting faculty) that I had the privilege to attend. While each of these chapters contains a specific acknowledgment section, I would like to express here again my gratitude to Arpad and Ramon for spending countless hours on debating all the problems coming my way. Your terrific economic intuition and knowledge of the literature have always pushed me to learn more. Moreover, the first chapter on co-operation of rural societies in India would probably never have come into existence without my childhood summer experiences of working on my parents' farm in Komarów. From the ex-post perspective, I would like to sincerely thank them for it. Complementary to it, this chapter would have never taken its current form without thoughtful conversations with Rody Manuelli. Equally, as the last chapter of my thesis is a major extension of my research started at the Institute for Advanced Studies (IHS) in Vienna, I am indebted to the great group of peers and mentors (including my master thesis supervisor Michael Reiter) I met there for the education received.

Furthermore, in the past five years, I have been extremely fortunate to meet a great group of friends with whom we have shared all our personal and academic struggles and accomplishments. It is an understatement to say that without them I would not have been able to complete this venture. For all the delicious dinners, strong performances, arduous travels, brilliant conversations and fierce disputes, cheers go to: Adam, Alyosha, Dima, Ioan, Jason, Johannes, Josue, Leo, Nate, Tomasz and members of the Poolhouse Productions: Malte, Wiebke and our official associate Chima Boy. Likewise, I would not have become the person I am without my childhood friends, who had always been there when needed. Thank you Arnold, Banan, Dyziu, Kaczor, Kowal, Masel and Sum.

Finally, I express my deepest gratitude and love, and at the same time dedicate this thesis, to my closest family whose unconditional support has provided me with an unlimited source of inspiration, motivation and energy for finalizing this scholarly endeavor: to Karolina, our daughter Zofia Anna, my mother, father, brother and sister.

KM, August 2019, Hrubieszów, Poland

Contents

1 Sharing Risk to Avoid Tragedy in Village Economies

1.1 Introduction

Farmers are exposed to a plethora of shocks affecting their crop yields, investment decisions and, ultimately, income. Some of these shocks are idiosyncratic in their nature such as pests, human, animal, crop diseases, machine breakdowns or unemployment. For the vast majority of farmers who are located in the least- and less-developed parts of the world, self-insurance is usually very limited - either due to poverty or lack of formal credit markets. Unsurprisingly then, rural societies have developed systems of informal mutual insurance arrangements aimed at mitigating these risks (Udry, 1994; Townsend, 1994). Surprisingly, however, although repayments in such informal insurance arrangements are in principle completely voluntary, they have proved robust to excessive default rates. The main reason for this is arguably the repeated nature of interactions within these societies and the possibility of punishing deviators by shutting off their access to credit in the future.

At the same time, the major aggregate risk faced by these farmers is erratic rainfall resulting in droughts. Fortunately enough, the impact of such weather shocks can also be mitigated to some extent thanks to modern irrigation systems. Needless to say, the best outcomes can be achieved either if the degree of co-operation between affected households is high, or if enough of the necessary infrastructure is provided by central authorities. Given that investments reducing the level of aggregate risks can reduce demand for informal insurance, and that access to informal insurance may be used to elicit higher degrees of co-operation over these investments, these two institutions may be very interrelated. Investigating this cornerstone relationship is at the heart of this paper.

In particular, I extend the canonical model of risk sharing with limited commitment developed by Kehoe and Levine (1993), Kocherlakota (1996) and Ligon, Thomas and Worrall (2002) in order to study joint determination of risk sharing against idiosyncratic risks and co-operation over public investments reducing aggregate risks in the presence of limited commitment constraints. I first demonstrate theoretically that if villagers have a possibility to exclude non-cooperators (or deviators) from the local informal insurance market and the local irrigation system, the two institutions reinforce each other. On one hand, risk sharing improves the efficiency of irrigational investments through two channels. First, state-contingent risk sharing transfers reduce the degree of uninsured idiosyncratic risk distorting farmers' investment decisions by more than self-insurance (see e.g. Arrow, 1971). Second, risk sharing facilitates efficient co-operation over irrigation by providing resources for investment (and consumption) to households that may otherwise be on the verge of breaking out of the irrigational co-operation agreement.

On the other hand, co-ordinated investments into irrigational infrastructure reduce the aggregate risk affecting co-operating households, while keeping the prospect of leaving this co-operation unappealing (due to available punishments). This dynamic implies that the extent of risk sharing transfers sustainable in equilibrium is enhanced by the irrigation margin.

Importantly, however, interventions providing irrigation to rural villages may backfire if this infrastructure is owned and managed by central authorities that cannot condition access to it on other dimensions of the intra-village co-operative patterns (such as e.g. repayments on informal insurance contracts). This is precisely due to the fact such interventions provide irrigation to every household in the village (including the ones that decide to default on their credit obligations towards others) and so may excessively reduce the extent of sustainable local insurance contracts.

Using two waves of the ICRISAT panel dataset collected in 1976-1984 and 2001-2004 combined with Minor Irrigation Censuses conducted by the Indian Ministry of Water Resources, I provide empirical evidence for the mechanism studied in the case of three Indian rural societies co-operating over the two institutions.[1] In particular, I first document how the size of irrigation infrastructure available has increased and the aggregate risk faced by farmers has decreased between the two waves. Then, I show that (i) conditional on a share of government ownership of irrigation units, an increase in the share of land with access to village-owned irrigation is associated with a significant improvement in risk sharing; and that (ii) an increase in the government share in ownership of the irrigation units is associated with a reduction in risk sharing, in line with predictions

[1] Ostrom (1990) gives many examples of rural societies that both co-operated over the common resources/goods and various forms of risk sharing. For instance, Swiss villagers insured each other by dividing the village's cheese production or rebuilding houses destroyed by avalanches together. In the case of Spanish societies co-operating over irrigation, the contract on the water use also contained water sharing rules in times of droughts. In particular, the crops in most need of water were given priority. In case of the Filipino farmers, the way of dividing land was symmetrical in the sense of everyone having some land closer to and some further away from the water source; and during dry periods all the farmers collectively decided how to share the burden and assign the water rights, again with priority for the crops in most need.

of the theory presented. I also discuss further studies supporting the mechanism.

Building on these findings, I calibrate a quantitative version of the model to the setting of the three villages analyzed in order to conduct a number of experiments. First, performing the exercise of accounting for empirically documented changes in aggregate risk faced by the villagers suggests that investments into irrigation constitute a very effective way of insuring villagers' against the risk of weather fluctuations. Second, comparing models with various degrees of co-operation confirms significant reinforcing effects between the two institutions of risk sharing and irrigational co-operation, valued by villagers by up to 6% of consumption in every year. Third, although improving villagers' well-being by a simple reduction in the size of government-owned infrastructure is not possible, shifting resources from the latter to subsidies on investments made by villagers can achieve significant welfare improvements, while holding the size of overall expenditures constant. In particular, such a reform strengthens the within-village co-operation by improving both the informal insurance (achieving a reduction in consumption elasticity w.r.t. idiosyncratic shocks of up to 67%) and the efficiency of irrigational investments (achieving a reduction in variance of aggregate risk of up to 11%). As such, villagers value these reforms by up to 3% of consumption in every year.

The research question pursued in this paper is particularly important in light of the Indian government's heavy involvement in irrigation provision with the aim of improving the wellbeing of rural societies. For instance, in 2016 Indian government spending on irrigation amounted to approx. 5.5 billion USD, or 8.5% of total budget expenditures. Given the trade-off associated with central ownership and management of such investments, policy makers should carefully consider the general equilibrium effects of their actions on the welfare of rural societies. This paper seeks to guide such policies.

1.2 Literature review

The joint determination of risk sharing and irrigation provision at the heart of this paper speaks to the macro-development literature studying interactions between productivity or investment decisions and the (access to insurance against) underlying risks.[2] Buera, Kaboski and Shin (2011) show quantitatively that financial development may affect allocation of capital and entrepreneurial talent across

[2] See survey by Buera, Kaboski and Shin (2017).

tradable and non-tradable sectors and as such it may also affect the aggregate productivity. In the context of village economies, Cole, Gine and Vickery (2014), Mobarak and Rosenzweig (2012) and Karlan, Osei, Osei-Akoto and Udry (2014) conduct experiments in India and Ghana showing that access to formal insurance products induces farmers to shift production towards higher return but higher risk cash crops. Similar conclusions are found in Cai, Chen, Fang and Zhou (2015) studying the case of sow production in southwestern China. Lastly, Emerick, de Janvry, Sadoulet and Dar (2016) show that the adoption of modern flood tolerant rice varieties in rural India increases investments and co-operation through more labor-intensive planting, employment of larger farming areas and increased usage of fertilizer and credit. In what follows, I present a mechanism through which informal risk sharing may endogenously lead to productivity gains in the presence of public goods affecting productivity.

Anthropological evidence in Wade (1988) and Bardhan (2000) documents that central provision and administration of irrigation in rural villages in South India has been widespread since the 1970s and has been associated with lower social co-operation in many dimensions as compared with those provided and managed by local communities. In this paper, I develop a general equilibrium framework where these effects arise endogenously. Then, using data sources independent from the latter studies, I provide empirical evidence (i) supporting the mechanism; and (ii) allowing me to calibrate the structural model in order to quantify the impact of centralized interventions on co-operative patterns in rural communities.

This paper is also related to literature on risk sharing with limited commitment studying its interaction with other institutions affecting the value of the outside option. For instance, Abramitzky (2008) studied the influence of capital investments on collective communities living in kibbutzim. Attanasio and Rios-Rull (2000), Thomas and Worrall (2007) and Kruger and Perri (2010) examine the interaction of such insurance with other public or private insurance programs. Relating to this literature, I consider an endogenous choice of public insurance against aggregate shocks and show that if an appropriate excludability technology is available to the community, the two forms of insurance reinforce each other.

As regards the idea of joint determination of the two institutions at the heart of this paper, Morten (2018) and Meghir et al. (2019) conduct a related investigation of interaction between risk sharing and temporary migration in Indian villages. Both my own work and theirs shares the feature of one institution having an impact on the other and vice versa. Moreover, the feedback-effect of irrigation investments on consumption smoothing presented in this paper is similar to the one due to the presence of public-storage-like institutions studied in Abraham and Laczo (2018).

Finally, the theoretical mechanism studied here can be also applied to the case of fiscal unions. Thus, this paper provides theoretical and empirical underpinnings for the mechanism interactions discussed in the recent report "Reconciling risk sharing with market discipline: A constructive approach to euro area reform" by Benassy-Quere and co-authors (2018). In particular, in a parallel project, I develop a similar model to the one developed below where countries co-operate over common goods having impact on the level of aggregate productivity and show that in the presence of production or investment externalities accruing to all member states, the optimal design of such a public risk sharing institution[3] should be combined with an institution combatting the externality-related effects, such as the institutions regulating common (labor or trade) markets or, in this case, environment as e.g. the EU ETS.

Structure

This paper is organized as follows. In Section 1.3, I introduce the model environment and Section 1.4 discusses the associated allocations with various degrees of co-operation over risk sharing and/or irrigation. In Section 1.5, I present supporting empirical evidence. Based on this evidence, in Section 1.6 I calibrate the model and use it to answer the research questions stated above. Finally, Section 1.7 concludes with a policy discussion and future research outlook.

1.3 Model economy

Preferences and production technology

Consider a dynamic infinite-horizon village-economy with N ex-ante identical farming households. All households are risk averse, discounting future at the rate of β, enjoying consumption c according to utility function $u\left(c\right)$ with $u_c, -u_{cc} > 0$. All information is publicly held and common knowledge.

In any period t, farmers receive crop output according to:

$$y_{i,t} = \phi_t \cdot \theta_{i,t} \tag{1}$$

[3] See Abraham et al. (2018a) and Abraham et al. (2018b) for analysis of unemployment insurance and optimal Financial Stability Fund designs for the European Union, respectively.

The output is a function of both aggregate ϕ and idiosyncratic θ shocks realization. Moreover, (i) $\mathbf{k}_t = [k_{1,t}, \ldots, k_{N,t}]$ is a vector of village-owned irrigation investments that are co-financed by the government at the rate of s_k,[4] (ii) ω_t is the irrigation investment provided and owned by central authorities, subsidized at the rate of s_ω. Irrigational capital depreciates at the rate of δ.

More precisely, the random variable $\theta_{i,t} \in \Theta = \left(\theta^1, ..., \theta^{N_\theta}\right)$, $0 < \theta^1 < ... < \theta^{N_\theta} < \infty$ stands for idiosyncratic productivity shocks following transition matrix π_θ with a strictly positive probability for realization of all productivity levels and the two first (long-run) moments of $\mu_\theta = \mathrm{E}(\theta)$ and $\sigma_\theta^2 = \mathrm{Var}(\theta)$. These shocks should be interpreted as machine breakdowns, pests, human, animal, crop diseases or unemployment affecting the well-being of villagers in an idiosyncratic fashion.[5]

The aggregate output shocks ϕ_t are distributed according to the distribution $\Phi = \left(\phi^1, ..., \phi^{N_\phi}\right)$ with $0 < \phi^1 < ... < \phi^{N_\phi} < \infty$ and two first moments of $\mu_\phi = \mathrm{E}(\phi)$ and $\sigma^2_{\phi_{t+1}} = f^{var}\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}, \omega_{t+1}\right)$ where f^{var} is a function of irrigational investments translating the impact of weather shocks into crop output fluctuations. I assume it to be a decreasing and convex function of the current period's irrigational investments of all N households $\mathbf{k}_{t+1}$ and government ω_{t+1}.[6] These investments should be thought of as investments into irrigation infrastructure benefitting the whole community by reducing the risk of droughts (e.g. wells, canals or tanks).[7]

To be more precise, I assume that there are two Markov processes governing the aggregate risk: a "good" and a "bad" one, with the former first order stochastically dominating the latter. The transition matrices π_ϕ^G and π_ϕ^B are assumed to be symmetric with a common persistence parameter ρ_ϕ.

Given this, higher irrigational investments increase the probability of the ran-

[4] I focus only on irrigation investments and ignore investments in private production capital, machines etc. as these are usually very small among the small-holder farmers in developing countries. Nonetheless, the mechanism and results derived below generalize to a setting with a more general function of $y_{i,t} = \phi_t \cdot \theta_{i,t} \cdot \left(k_{i,t}^p\right)^\alpha$, with k^i and k^p standing for irrigation and production capital (respectively).

[5] See Townsend (1994) for a more detailed description of idiosyncratic risks faced by villagers in rural India.

[6] While it is clear that in reality such investments benefit not only the farmers pursuing these investments, the degree of benefits to others may vary depending on the exact type of investment. In order to keep things simple, this analysis assumes that the benefit of farmer i's investment to every farmer in the village is equal.

[7] Interpreted more broadly, they can also be thought of, to some extent, as constructions preventing floods (field bunds, drainage canals etc.). See Section 1.5.2 for discussion of access to irrigation among farmers in the villages analyzed empirically.

dom variable ϕ being drawn from the "good" distribution in the following way:

$$\pi_\phi\left(\phi^{t+1}\right)=\begin{cases}\pi_\phi^G\left(\phi^{t+1}\right) & with\ P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega_{t+1}\right)\\ \pi_\phi^B\left(\phi^{t+1}\right) & with\ 1-P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega_{t+1}\right)\end{cases} \tag{2}$$

where (i) $P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega_{t+1}\right)=\frac{1}{N}\sum_{j=1}^{N}\bar{P}\left(\frac{1}{1-s_k}\nu_k k_{j,t+1}+\frac{1}{N}\nu_\omega\omega_{t+1}\right)$ is a probability function with $\bar{P}(\cdot)\in[0,1]$, $\bar{P}'_{k,\omega}(\cdot)>0$, $\bar{P}''_{k,\omega}(\cdot)<0$ and ν_k,ν_ω measuring efficiency of investments into k and ω; and (ii) a variable with superscript denotes its whole history, but due to the Markov property of shocks we have that $\pi_\phi^G\left(\phi^{t+1}\right)=\pi_\phi^G\left(\phi_{t+1}|\phi_t\right)$, i.e. the realization in period $t+1$ depends only on period t realization.

Co-operation in the village

Given the nature of idiosyncratic and aggregate shocks affecting the economy, villagers have two motives for co-operation over: (i) mutual insurance against idiosyncratic shocks; and (ii) irrigational investments. To this end, this co-operation has to be individually rational at any instance as villagers are assumed to be free to walk away from such agreements. In particular, conditional on any state of the world, the associated equilibrium allocations are constrained to be such that farmers are voluntarily participating in the agreement on either risk sharing and/or irrigational investments co-operation. This means that at any instance the expected value of remaining in the agreement conditional on current state has to exceed the expected value of deviation.

As far as the benefits of such deviations are concerned, after deviating on their risk sharing or irrigational investment promises agents can:

1. consume as much of their output as they want today and in all future periods (i.e. not sharing it with others), if they co-operate on risk sharing;

2. invest into irrigation without internalizing associated positive externalities on others, if they co-operate over this margin (see below).

The first margin of deviations on risk sharing promises is well understood as it has been analyzed in the literature following Kehoe and Levine (1993), Kocherlakota (1996) and Ligon et al. (2002). In line with it, I assume that upon any deviation in an agreement involving risk sharing, the deviator will be prohibited from accessing the local credit market in all future periods. This means that from the current period onwards such a farmer would be able to rely on self-insurance only, i.e. he would be able to consume his own production and savings (in the form of depreciated capital) only.

Furthermore, I assume that the society has access to technology allowing them to exclude deviators from the part of irrigation provided and managed by other villagers (but not by government, see below). This excludability technology can be thought of as the local village council (so called panchayat) forbidding deviators to draw water from common water canals, tanks or dugwells. Moreover, it can be also thought of as a proxy for public shaming or ostracism imposed upon deviators.[8]

Additionally and as already mentioned, I assume that there exists a governmental agency with an exogenously given plan of investments into public irrigation units $\{\omega_{t+1}\}_t \geq 0$ (possibly also state-contingent) in every period t to be shared among all the N households with perfect foresight over this variable. Importantly, this centrally provided and managed irrigation comes with a significant indirect cost: it is non-excludable, meaning that the central agency is unable to condition access to their part of irrigation systems based on other private informal contracts, such as risk sharing, in the village. The reason for this is either that doing so simply does not lie in the agency's domain or it lacks information (or expertise) necessary for such a form of management.[9]

The two ways of providing irrigation discussed above imply that the deviating or non-cooperating household is able to maintain access to their own and the government-provided part of the irrigation units. This implies that irrigation provided by villagers is a club good (non-rivalrous and excludable), whereas the part provided by government is a public good (non-rivalrous and non-excludable). Mathematically, we have that in such a case the probability of realization of the

[8]As is well understood in the literature studying endogenously incomplete markets generated by the limited commitment friction, it is possible to support a continuum of equilibria with different properties depending on the exact specification of the outside option. By assuming that any deviation is punished by exclusion from both the local insurance and the village-owned part of the irrigation infrastructure, I effectively focus the analysis on equilibria with the highest supportable degree of risk sharing and most efficient public good provision. This assumption regarding the deviation on risk sharing is standard in the literature following Kehoe and Levine (1993), Kocherlakota (1996) and Ligon et al. (2002). Thus, in the case of irrigation systems being provided by villagers, assuming excludability and focusing on the best equilibrium attainable is in line with this literature. While a permanent exclusion from credit markets or irrigation infrastructure may be seen as far-fetched, these assumptions are a proxy for very strong social norms present in rural villages.

[9]See Bardhan (1993) for a discussion of these issues from both conceptual and empirical perspectives.

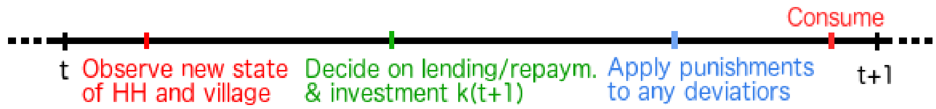


Figure 1: Timing of events

good aggregate risk distribution for agent i is equal to:

$$P\left(\frac{1}{1-s_k}\mathbf{k}^{nc}_{t+1},\omega_{t+1}\right)=\frac{1}{N}\bar{P}\left(\frac{1}{1-s_k}\nu_k k^{nc}_{i,t+1}+\frac{1}{N}\nu_\omega\omega_{t+1}\right)+\frac{N-1}{N}\sum_{j\neq i}\bar{P}\left(\frac{1}{N}\nu_\omega\omega_{t+1}\right)$$

implying that the non co-operating agents maintain access to the share $\sigma_{t+1} = \frac{\nu_\omega\omega_{t+1}}{\frac{1}{1-s_k}\nu_k\sum_{j=1}^{N}k_{j,t+1}+\nu_\omega\omega_{t+1}}$ of the village's common irrigation infrastructure.

Importantly, the subsidies for both village- and centrally-owned investments are financed with sources external to the villages. Since taxation in poor rural areas (such as the ones analyzed in this paper) is virtually non-existent, I assume in what follows that the government-owned irrigation units are fully subsidized (i.e. $s_\omega = 1$), implying no taxation motives in equilibrium.[10]

Finally, I will consider all possible benchmarks of no co-operation, joint co-operation over risk sharing and irrigation, and also co-operation separately over the former and over the latter. In the case of co-operation over irrigation investments only, I assume that households attempt to internalize the associated externalities as much as individually rational at every instance, without engaging in state-contingent risk sharing. In the alternative case, I assume that households engage in informal insurance arrangement, share all of the village-owned irrigation system, but they do not internalize irrigation externalities on others when investing.

Figure 1 summarizes the timing of events in the economy.

1.4 Allocations with various degrees of co-operation

In this section, I discuss properties of equilibria associated with various degrees of co-operation. I start with non-cooperative and first best benchmarks. Afterwards, I discuss allocations subject to limited commitment constraints where farmers co-operate over either risk sharing or irrigation (or both). Going through the all pos-

[10]More generally, the investments provided by the central agency could come e.g. at a cost of linear capital investment tax $T_{i,t}\left(k_{i,t+1}\right) = \tau_t \cdot k_{i,t+1}$ such that the government budget is balanced in every period, up to the subsidy rate s_ω: $\tau_t\sum_{i=1}^{N}k_{i,t+1} = \left(1-s_\omega\right)\omega_{t+1}$.

sible combinations will allow me to establish how the risk sharing co-operation impacts the irrigation co-operation, and vice versa.

1.4.1 Non-cooperative allocation

Given the model's description, the value of not co-operating to agent i where everyone consumes only their own production and savings (self-insurance), conditional on state $x_t = (\phi_t, \theta_{i,t}, \mathbf{k}_t^{nc}, \omega_t)$, is given by solution to the following recursive problem:

$$V_{i,t}^{nc}\left(x^t\right) = \max_{c_{i,t}^{nc}, k_{i,t+1}^{nc}} u\left(c_{i,t}^{nc}\right) + \beta \mathrm{E}_{\phi,\theta} V_{i,t}^{nc}\left(x_{t+1}|x^t\right) \tag{3}$$

subject to:

$$\left(\zeta_{i,t}^{nc}\left(x^t\right)\right) \; c_{i,t}^{nc} + k_{i,t+1}^{nc} \leq \phi_t \cdot \theta_{i,t} + (1-\delta)\, k_{i,t}^{nc} \; \forall t, x^t \tag{4}$$

where ϕ_t is drawn from the good distribution with probability $P\left(\frac{1}{1-s_k} k_{i,t}^{nc}, \omega_t\right)$ and $\left(\zeta_{i,t}^{nc}\left(x^t\right)\right)$ here (and below) stands for the Lagrange multiplier on the relevant budget or resource constraint. Furthermore, notice that each household receives the previous period's depreciated unsubsidized part of own irrigation capital.

The associated optimality conditions with respect to c and k read:

$$c: \; \zeta_{i,t}^{nc} = \; u_{i,c_t} \tag{5}$$

$$k: \; \zeta_{i,t}^{nc} = \beta \left[\frac{\partial P\left(\frac{1}{1-s_k} k_{i,t+1}^{nc}, \omega_{t+1}\right)}{\partial k_{i,t+1}^{nc}} \left\{ \mathrm{E}_\theta^G V_{i,t+1}^{nc} - \mathrm{E}_\theta^B V_{i,t+1}^{nc} \right\} \right. \tag{6}$$

$$\left. + (1-\delta) \left\{ P\left(\frac{1}{1-s_k} k_{i,t+1}^{nc}, \omega_{t+1}\right) \mathrm{E}_\theta^G \zeta_{i,t+1}^{nc} + \left(1 - P\left(\frac{1}{1-s_k} k_{i,t+1}^{nc}, \omega_{t+1}\right)\right) \mathrm{E}_\theta^B \zeta_{i,t+1}^{nc} \right\} \right]$$

where (i) I abuse notation by dropping dependance of $\zeta_{i,t}^{nc}, c_{i,t}^{nc}, k_{i,t+1}^{nc}$ on x_t (here and in what follows); and (ii) $\mathrm{E}_s^j[\cdot]$, $j \in \{B, G\}$ is an expectation operator under the "bad" or "good" aggregate risk distribution.

First of all, each household consumes what is available to them from their own crop output, net of investments made. Secondly, the irrigation investment is done at the point where the cost in terms of consuming marginally less is equalized to (i) expected value gain tomorrow weighted by the marginal increase in probability of good distribution realization, and (ii) the value of an additional unit of depreciated irrigation capital invested out of farmers' pockets (in marginal utility terms).

Moreover, since (i) the centrally owned part of the village's irrigation can be used by everyone (as opposed to each farmer having access to his private irrigation only), and that (ii) conditional on ω_{t+1} farmers can choose their optimal level of irrigation investment, we have that:

Corollary 1. *In the non-cooperative benchmark, for any $(s_k, \{\omega_t\}_t)$, an additional unit of centrally-owned irrigation strictly improves expected utility of every villager.*

Finally, any additional subsidies s_k spent on the irrigation investments pursued by villagers will also be clearly welfare improving in both the non-cooperative and all the benchmarks to follow.

1.4.2 First best allocation

For this benchmark, consider a benevolent planner maximizing utilitarian social welfare function (SWF) attaching equal Pareto weights to each household evaluated at state $x_t = \left(\phi_t, \{\theta_{i,t}\}, \mathbf{k}_t^{FB}, \omega_t\right)$:

$$V_t^{FB}\left(x^t\right) = \max_{\left\{c_{i,t}^{FB}, k_{i,t+1}^{FB}\right\}_i} \sum_{i=1}^{N} u\left(c_{i,t}^{FB}\right) + \beta \mathrm{E}_{\phi,\theta} V_{i,t}^{FB}\left(x_{t+1}|x^t\right) \tag{7}$$

The ensuing efficient benchmark allocation where the planner is free to move resources around, i.e. faces the aggregate resource constraint of:

$$\left(\zeta_t^{FB}\left(x^t\right)\right) \sum_{i=1}^{N}\left(c_{i,t}^{FB} + k_{i,t+1}^{FB}\right) \leq \phi_t \cdot \sum_{i=1}^{N}\theta_{i,t} + (1-\delta)\sum_{i=1}^{N} k_{i,t}^{FB} \ \forall t, x^t \tag{8}$$

with ϕ_t being drawn from the good distribution with probability $P\left(\frac{1}{1-s_k}\mathbf{k}_t^{FB}, \omega_t\right)$, is characterized by the following first order conditions:

$$c: \ \zeta_t^{FB} = \ u_{i,c_t} \ \Rightarrow \ \frac{u_{i,c_t}}{u_{j,c_t}} = 1 \ \forall i, j \tag{9}$$

$$k: \ \zeta_t^{FB} = \beta\left[\frac{\mathrm{d}P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}^{FB}, \omega_{t+1}\right)}{\mathrm{d}k_{i,t+1}^{FB}} \sum_{j=1}^{N}\left\{\mathrm{E}_\theta^G V_{j,t+1}^{FB} - \mathrm{E}_\theta^B V_{j,t+1}^{FB}\right\}\right. \tag{10}$$

$$\left. + (1-\delta)\left\{P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}^{FB}, \omega_{t+1}\right)\mathrm{E}_\theta^G \zeta_{t+1}^{FB} + \left(1 - P\left(\mathbf{k}_{t+1}^{FB}, \omega_{t+1}\right)\right)\mathrm{E}_\theta^B \zeta_{t+1}^{FB}\right\}\right]$$

First of all, comparing the consumption sharing rules (9) and (5) shows that in the first best with equal Pareto weights, consumption smoothing of all farmer households will be improved and, in particular, households' consumption will fluctuate only in response to endogenous adjustments in irrigation investments and aggregate fluctuations.[11]

Furthermore and importantly, the planner now internalizes all the positive externalities associated with irrigational investments. In particular, the marginal benefit in the first best irrigation capital investment rule (10) includes the impact of marginal increase in irrigational investment not only on the investing household but also on all the other households in the village.

A second insight follows from the above: since the insurance of villagers against idiosyncratic shocks is now improved, the investment distortions due to self-insurance are removed (Arrow, 1971). In particular, decisions in the non-cooperative benchmark were distorted by excessive risk, leading the self-insuring agents to smooth their (current and expected) variations in the marginal utility of consumption due to idiosyncratic shocks through inefficient adjustments in k.[12] Given all these observations and the fact that shocks are persistent, the expected continuation value of each agent $\mathrm{E}^{j}_{\cdot} V_{j,t+1}$ for both $j \in \{B, G\}$ will also increase.

1.4.3 Co-operation over irrigation and risk sharing with limited commitment

The centralized version of allocation combining both institutions where households are free to walk away from the agreement at any instance is a solution to the planner maximizing the following modified SWF:

$$V_t^{IRS}\left(x^t\right) = \max_{\left\{c_{i,t}^{IRS}, k_{i,t+1}^{IRS}\right\}_i} \sum_{i=1}^{N} \lambda_{i,t-1}^{IRS} \cdot u\left(c_{i,t}^{IRS}\right) + \beta \mathrm{E}_{\phi,\theta} V_{i,t}^{IRS}\left(x_{t+1} | x^t\right) \tag{11}$$

subject to the aggregate resource constraint equivalent of (8) and the following set of limited commitment constraints:

$$\left(\mu_i^{IRS}\left(x^t\right)\right) \ \mathrm{E_t}\left[\sum_{t'=t}^{\infty} \beta^{t'-t} u\left(c_{i,t'}^{IRS}\right)\right] \geq V_{i,t}^{nc}\left(\tilde{x}_t\right) \ \forall i, x^t \tag{12}$$

where:

1. the current state is $x_t = \left(\phi_t, \left\{\theta_{j,t}\right\}_j, \mathbf{k}_t^{IRS}, \omega_t, \left\{\lambda_{j,t-1}^{IRS}\right\}_j\right)$;

[11] This statement holds true if idiosyncratic shocks are independently distributed, as I shall assume in the quantitative Section 1.6.

[12] Notice that the excessive idiosyncratic risk may push households to increase their investments for precautionary saving reasons, potentially in the direction of internalizing the true social marginal benefits of investments. Nonetheless, this will in general not ensure socially efficient internalization of externalities (both in the non-cooperative and other benchmarks that follow below).

2. following the methodology developed by Marcet and Marimon (2019), the Pareto weights are updated according to $\lambda_{i,t}^{IRS}\left(x^t\right) = \lambda_{i,t-1}^{IRS}\left(x^{t-1}\right) + \mu_{i,t}^{IRS}\left(x^t\right)$ for $t \geq 1$ and, as in the first best benchmark, $\lambda_{i,0}^{IRS}\left(x^0\right) = 1\ \forall i$;

3. the value of the outside option is given by the value of non-cooperating, with the current state given by $\tilde{x}_t = \left(\phi_t, \theta_{i,t}, k_{i,t}^{IRS}, \omega_t\right)$. As discussed in Section 1.3, the latter means that after deviating on one's risk sharing or investment promises, the deviator will be excluded from (i) the local insurance market against idiosyncratic risk; and (ii) the benefits of investments made by other households in current and all future periods (while maintaining access to the centrally managed part of the irrigation system).

Consequently, this agreement can be characterized by the following set of optimality conditions:

$$c:\ \zeta_t^{IRS} = u_{i,c_t} \cdot \left(\lambda_{i,t-1}^{IRS} + \mu_{i,t}^{IRS}\right)\ \forall i \ \Rightarrow\ \frac{u_{i,c_t}}{u_{j,c_t}} = \frac{\lambda_{j,t-1}^{IRS} + \mu_{j,t}^{IRS}}{\lambda_{i,t-1}^{IRS} + \mu_{i,t}^{IRS}}\ \forall i \neq j \tag{13}$$

$$\begin{aligned} k:\ \zeta_t^{IRS} =& \beta \left[\frac{\mathrm{d}P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}^{IRS}, \omega_{t+1}\right)}{\mathrm{d}k_{i,t+1}^{IRS}} \sum_{j=1}^{N} \left\{ \mathrm{E}_\theta^G \lambda_{j,t+1}^{IRS} V_{j,t+1}^{IRS} - \mathrm{E}_\theta^B \lambda_{j,t+1}^{IRS} V_{j,t+1}^{IRS} \right\} \right. \\ & - \mu_{i,t}^{IRS} \frac{\mathrm{d}P\left(\frac{1}{1-s_k}k_{i,t+1}^{IRS}, \omega_{t+1}\right)}{\mathrm{d}k_{i,t+1}^{IRS}} \left\{ \mathrm{E}_\theta^G V_{i,t+1}^{nc} - \mathrm{E}_\theta^B V_{i,t+1}^{nc} \right\} \\ & + P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}^{IRS}, \omega_{t+1}\right) \mathrm{E}_\theta^G \left[\zeta_{t+1}^{IRS}(1-\delta) - \mu_{i,t+1}^{IRS} \frac{\partial V_{i,t+1}^{nc}}{\partial k_{i,t+1}} \right] \\ & \left. + \left(1 - P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}^{IRS}, \omega_{t+1}\right)\right) \mathrm{E}_\theta^B \left[\zeta_{t+1}^{IRS}(1-\delta) - \mu_{i,t+1}^{IRS} \frac{\partial V_{i,t+1}^{nc}}{\partial k_{i,t+1}} \right] \right] \end{aligned} \tag{14}$$

First of all, notice the modified consumption sharing rule: consumption in period t is determined by the Pareto weight at the beginning of the period adjusted for a potentially binding limited commitment constraint after the aggregate and idiosyncratic shocks are realized. This is done in such a way that in equilibrium no agent defaults on the contract and so that the co-operation is sustainable in long-term. Moreover, the extent of efficient risk sharing is pinned down by "how slack" are the limited commitment constraints of all the agents. In particular, the amount of resources the planner is able to transfer from agent i to others is an increasing function of distance between the household i's (inside) value of co-operation and the (outside) value of deviating in the enforcement constraint (12).

Similarly as in the first best, when deciding about irrigation investments, the planner accounts for all the benefits accruing to other farmers, up to the endogenous Pareto weights. However, as higher irrigation investments today increase the probability of ending up in the "good" aggregate state tomorrow, the planner needs to account for it by shading investments in case of binding enforcement constraints *today* for household i.[13] What is more, she also needs to take into account the negative effects the increase in irrigation capital investment of household i has on their value of the outside option through improved self-insurance (in the form of higher savings in the next period). More precisely, the planner marginally shades this choice to account for all the possible *next period's* states in which these limited commitment constraints are binding.[14] In general, these investment distortions will be larger for states when the enforcement constraints are "more binding", i.e. when the associated Lagrange multipliers have higher values.

Finally, notice that this allocation constitutes a constrained efficient benchmark where the planner is subject to the deep friction of limited commitment.

1.4.4 Co-operation over risk sharing with limited commitment

In this benchmark, I assume that villagers co-operate only over mutual insurance against idiosyncratic shocks and (for whatever reason) fail to establish co-operation over irrigation. This failure means that households do share their irrigational infrastructure with each other but fail to internalize externalities on each other. The centralized version of such an agreement is a solution to maximizing the following SWF:

$$V_t^{RS}\left(x^t\right) = \max_{\left\{c_{i,t}^{RS}, k_{i,t+1}^{RS}\right\}_i} \sum_{i=1}^{N} \lambda_{i,t-1}^{RS} \cdot u\left(c_{i,t}^{RS}\right) + \beta \mathrm{E}_{\phi,\theta} V_{i,t}^{RS}\left(x_{t+1} | x^t\right) \tag{15}$$

subject to (i) the aggregate resource constraint as in (8); (ii) the set of limited commitment constraints as in (12) with an inside value accordingly adjusted; and (iii) a constraint that the planner does not internalize the household i's externalities on others. The following FOCs characterize this benchmark:[15]

[13] The planner does not need to account for the impact of i's investments on others due to the assumed exclusion from village-owned part of irrigation.

[14] This self-insurance effect is similar to the one present in Kehoe and Perri (2002) and Ligon et al. (2002).

[15] Notice that this problem is an approximation of the full problem with moral hazard, i.e. with the presence of incentive compatibility constraints for households' investments. Solving such a

$$
\begin{aligned}
c:\ \zeta_t^{RS} =& u_{i,c_t}\cdot\left(\lambda_{i,t-1}^{RS}+\mu_{i,t}^{RS}\right)\ \forall i\ \Rightarrow\ \frac{u_{i,c_t}}{u_{j,c_t}}=\frac{\lambda_{j,t-1}^{RS}+\mu_{j,t}^{IRS}}{\lambda_{i,t-1}^{RS}+\mu_{i,t}^{IRS}}\ \forall i\neq j\\
k:\ \zeta_t^{RS} =& \beta\left[\frac{\partial P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}^{RS},\omega_{t+1}\right)}{\partial k_{i,t+1}^{RS}}\left(\mathrm{E}_\theta^G V_{i,t+1}^{RS}-\mathrm{E}_\theta^B V_{i,t+1}^{RS}\right)\right.\\
&-\mu_{i,t}^{RS}\frac{\partial P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}^{RS},\omega_{t+1}\right)}{\partial k_{i,t+1}^{RS}}\left\{\mathrm{E}_\theta^G V_{i,t+1}^{nc}-\mathrm{E}_\theta^B V_{i,t+1}^{nc}\right\}\\
&+P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}^{RS},\omega_{t+1}\right)\mathrm{E}_\theta^G\left[\zeta_{t+1}^{RS}\left(1-\delta\right)-\mu_{i,t+1}^{RS}\frac{\partial V_{i,t+1}^{nc}}{\partial k_{i,t+1}}\right]\\
&\left.+\left(1-P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}^{RS},\omega_{t+1}\right)\right)\mathrm{E}_\theta^B\left[\zeta_{t+1}^{RS}\left(1-\delta\right)-\mu_{i,t+1}^{RS}\frac{\partial V_{i,t+1}^{nc}}{\partial k_{i,t+1}}\right]\right]
\end{aligned}
$$

Based on this, we have that:

Proposition 1. *For any $(\{\omega_t\}_t, s_k)$, extending the risk sharing co-operation by co-operation over irrigational investments among villagers allows for a Pareto improvement through lower consumption fluctuations due to internalization of investment externalities and a possible increase in the extent of efficient risk sharing. Moreover, if $\forall t, x_t\ \mathbf{k}_t^{RS}(x_t)=\mathbf{k}_t^{IRS}(x_t)$, then extending the risk sharing co-operation by co-operation over irrigation does increase the extent of efficient risk sharing.*

Proof. The Pareto improvement follows from the fact that the constraint set of the RS problem is a strict subset of the constraint set in the IRS, i.e. everything achievable in the RS is achievable in the IRS, and more. Given this, the planner can replicate, and improve upon, the RS economy.

The second part follows from the fact that if $\forall t\ \mathbf{k}_t^{RS}=\mathbf{k}_t^{IRS}$, then the outside options between the RS and the IRS allocations coincide while the inside option in the IRS is strictly higher than in the RS. □

Intuitively, although internalizing externalities on others (due to extending the risk sharing co-operation by the irrigational margin) implies higher investments and as such may increase the value of the outside option (due to better

problem with public investments is cumbersome and lies beyond the scope of this paper (see Mele (2014) on how to solve such problems). As this approximation implies that the planner does not internalize the impact of risk sharing transfers on the investment incentives, it constitutes a lower bound on the true welfare attainable. Notice also that, as the "good" aggregate risk process first order stochastically dominates the "bad" one, the first order approach (FOA) in the co-operation over risk sharing with limited commitment and moral hazard would remain valid (see Rogerson, 1985).

self-insurance), the planner manages to account for it (see (14) with $\mu_{i,t}^{IRS} > 0$). In particular, she chooses investments in such a way that the net effect of reducing aggregate risk at the possible cost of a reduction in risk sharing is positive, i.e. that it leads to a Pareto improvement.

Building on the above observations and the fact that the centrally-provided investments are non-excludable driving the value of the outside option excessively high (as compared to the village-provision), it immediately follows that:

Proposition 2. *Assume that (i) the efficiency of village investments k is equal to that of government investments ω, i.e. $\nu_k = \nu_\omega$; and (ii) the villagers' investment decisions k do not change upon reform, i.e. that $\mathbf{k}^{pre-reform}\left(x^t\right) = \mathbf{k}^{post-reform}\left(x^t\right) = \mathbf{k}\left(x^t\right)$. Then, for any s_k^1, a reform eliminating centrally-owned irrigation (i.e. changing $\left\{\omega_t^1 > 0\right\}_t$ to $\left\{\omega_t^2 = 0\right\}_t$) with a compensating increase of the village-investment subsidy to $s_k^2 > s_k^1$ that is budget balanced from the perspective of government in the sense of $\sum_{j=1}^{N}\left(s_k^2 - s_k^1\right) k_{j,t}\left(x^t\right) = \omega_t^1\ \forall t$, allows for a Pareto improvement.*

Proof. The result follows because (i) $P\left(\frac{1}{1-s_k^1}\mathbf{k}_{t+1}, \omega_{t+1}^1\right) = P\left(\frac{1}{1-s_k^2}\mathbf{k}_{t+1}, 0\right)$; and (ii) the compensating increase in village-investment subsidy implies that the pre-reform value of autarky will exceed the post-reform one (i.e. $V^{nc,1} > V^{nc,2}$). The latter is warranted by (i) excludability of village-owned investments, and (ii) the assumption that k does not respond to the reform, controlling for any potential changes in self-insurance. □

The assumption about equality of investment efficiencies is needed because if ν_k was much higher than ν_ω, the reform could in principle make the outside option excessively high, leading to worse risk sharing and so a potential destruction of welfare. Nonetheless, this assumption is sufficient (and not necessary) as this effect disappears when the size of village N is large enough (as then the weight on household i's own investment in total probability $\frac{1}{N}\bar{P}\left(\frac{1}{1-s_k}\nu_k k_{i,t+1}^{nc} + \frac{1}{N}\nu_\omega \omega_{t+1}\right) + \frac{N-1}{N}\sum_{j\neq i}\bar{P}\left(\frac{1}{N}\nu_\omega\omega_{t+1}\right)$ declines. Moreover, as was the case with the Proposition 1, the second assumption is important to control for endogenous changes in self-insurance, which may affect the value of non-cooperating. Although difficult to show theoretically, notice that this assumption may be innocuous as increases in subsidies s_k are likely to reduce private investments $\mathbf{k}$. Nonetheless, in the quantitative section below I demonstrate that relaxing these two assumptions does not alter the qualitative nature of the results.

The Pareto improvement is possible due to the fact that lowering the size of centrally provided irrigation releases resources that can be channeled into investments by villagers which have the advantage of being excludable and thus not increasing the value of the outside option as much as the centrally owned ones. In general, this may be reflected either in improved consumption insurance against idiosyncratic shocks or a reduction in the latter compensated by an increase in the level of consumption. In Section 1.5.4, I provide empirical evidence from three Indian villages supporting these predictions, i.e. that (i) more village-provided irrigation improves risk sharing, and (ii) higher government-ownership is associated with crowding out of risk sharing.

Finally, notice that a related and interesting question is whether it is possible to achieve a Pareto improvement with an *uncompensated* reduction in ω. This may in principle be possible since the expected marginal utility of consumption in the outside option (of being self-insured only but with access to a household's own and centrally-owned part of irrigation) is going to necessarily be higher than within co-operation. Given this, a marginal reduction in ω may in fact reduce the outside value by much more than the inside value, leading to a potential Pareto improvement. I investigate this issue in the quantitative section below.

1.4.5 Co-operation over irrigation with limited commitment

For this benchmark, I assume that villagers co-operate over irrigational investments, but not over risk sharing. In this case, the centralized version of the agreement is a solution to maximizing SWF:

$$V_t^I\left(x^t\right) = \max_{\left\{c_{i,t}^I, k_{i,t+1}^I\right\}_i} \sum_{i=1}^{N} \lambda_{i,t-1}^I \cdot u\left(c_{i,t}^I\right) + \beta \mathrm{E}_{\phi,\theta} V_{i,t}^I\left(x_{t+1}|x^t\right) \tag{16}$$

subject to (i) the set of budget constraints as in (4); and (ii) the set of limited commitment constraints as in (12) with an inside value adjusted accordingly.

In this case, the allocation can be characterized by the following optimality conditions:

$$c:\ \zeta^I_{i,t} = u_{i,c_t} \cdot \left(\lambda^I_{i,t-1} + \mu^I_{i,t}\right)\ \forall i\ \Rightarrow\ \frac{u_{i,c_t}}{u_{j,c_t}} = \frac{\zeta^I_{i,t}}{\zeta^I_{j,t}} \cdot \frac{\lambda^I_{j,t-1} + \mu^I_{j,t}}{\lambda^I_{i,t-1} + \mu^I_{i,t}}\ \forall i \neq j \tag{17}$$

$$\begin{aligned} k:\ \zeta^I_{i,t} = \beta \Bigg[& \frac{\mathrm{d}P\left(\frac{1}{1-s_k}\mathbf{k}^I_{t+1}, \omega_{t+1}\right)}{\mathrm{d}k^I_{i,t+1}} \sum_{j=1}^{N} \left(\mathrm{E}^G_\theta \lambda^I_{j,t+1} V^I_{j,t+1} - \mathrm{E}^B_\theta \lambda^I_{j,t+1} V^I_{j,t+1}\right) \\ & - \mu^I_{i,t} \frac{\mathrm{d}P\left(\frac{1}{1-s_k}k^I_{i,t+1}, \omega_{t+1}\right)}{\mathrm{d}k^I_{i,t+1}} \left\{\mathrm{E}^G_\theta V^{nc}_{i,t+1} - \mathrm{E}^B_\theta V^{nc}_{i,t+1}\right\} \\ & + P\left(\frac{1}{1-s_k}\mathbf{k}^I_{t+1}, \omega_{t+1}\right) \mathrm{E}^G_\theta \left[\zeta^I_{i,t+1}(1-\delta) - \mu^I_{i,t+1} \frac{\partial V^{nc}_{i,t+1}}{\partial k^I_{i,t+1}}\right] \\ & + \left(1 - P\left(\frac{1}{1-s_k}\mathbf{k}^I_{t+1}, \omega_{t+1}\right)\right) \mathrm{E}^B_\theta \left[\zeta^I_{i,t+1}(1-\delta) - \mu^I_{i,t+1} \frac{\partial V^{nc}_{i,t+1}}{\partial k^I_{i,t+1}}\right] \Bigg] \end{aligned} \tag{18}$$

Notice that in this case, although she cannot do so directly, the planner with a utilitarian SWF still wants to insure households against idiosyncratic shocks. Thus, she will do so through inefficient assignment of investments. In particular, a household that is subject to a low idiosyncratic shock will be indirectly insured by reducing their irrigation investments, which will be partly balanced by increased investments on the side of households enjoying high productivity. This will lead to: (i) an increase in the consumption of low productivity households; and (ii) maintenance of their continuation value at a relatively high level due to higher investments made by others who are currently better-off. This inefficient insurance mechanism suggests that in the presence of limited commitment constraints, risk sharing facilitates voluntary co-operation over irrigation: it provides resources to "critical" households with low productivity that would otherwise find themselves on the verge of defaulting, allowing them to increase their consumption by undercutting their investments to socially suboptimal levels. Formally:

Proposition 3. *For any $(\{\omega_t\}_t, s_k)$, extending the co-operation over irrigational investments by risk sharing allows for a Pareto improvement through lower overall consumption fluctuations and an increase in the extent of efficient risk sharing. This is achieved through (i) removing investment distortions due to self-insurance; and (ii) if $\forall t, x_t\ \mathbf{k}^I_t(x_t) = \mathbf{k}^{IRS}_t(x_t)$, through relaxing limited commitment constraints.*

Proof. Pareto improvement follows due to the fact that the constraint set of the co-operation over irrigation is a strict subset of the constraint set in IRS, i.e. everything achievable in co-operation over irrigation is also achievable in IRS, and more. Increase in the extent of risk sharing follows trivially by construction.

Then, comparing FOCs (18) and (14) shows that risk sharing removes distortions to investment decisions due to self-insurance. Secondly, if $\forall t, x_t$ $\mathbf{k}_t^I(x_t) = \mathbf{k}_t^{IRS}(x_t)$, the value of the outside option $V_{i,t}^{nc}$ is the same for both the irrigation and joint co-operation, whereas the value of the inside option associated with joint co-operation $V_{i,t}^{IRS}$ strictly exceeds that associated with irrigation co-operation only $V_{i,t}^{I}$. This means that $\forall t, x_t$ the limited commitment constraints are more slack in the joint co-operation. The latter in conjunction with the fact that risk sharing co-operation introduces state-contingent assets imply better risk sharing. □

1.5 Empirical evidence

The purpose of this section is to provide empirical evidence supporting the causal mechanism at the heart of this paper. Moreover, this evidence will be used later in Section 1.6 to calibrate the quantitative model used for optimal policy analysis.

1.5.1 Data description

For what follows, I use two waves of the ICRISAT's Village Level Studies (VLS) panel data set on households living in rural South India. Following the literature, I focus analysis on 3 villages: Aurepalle, Shirapur and Kanzara. The first wave contains monthly records starting from July 1975 until the end of 1984,[16] and the second wave is a yearly panel covering the years 2001-2004.[17] Each village sample was divided into 4 groups of 10 households each representing families in four classes: laborers, small-, medium- and large-farmers.

The VLS records data on production, labor supply, assets, price of goods, rainfall, monetary and non-monetary transaction, household size, age and education. Townsend (1994) gives a detailed description of the data. Thus, here I discuss only the issues specific to this paper.

First of all, I aggregate the monthly data in the first wave to yearly so that the two waves constitute comparable data sets and use years 1976-1984.[18] I drop

[16]The first wave of the ICRISAT's dataset is one of the most commonly used datasets in the macro-development literature, see e.g. Townsend (1994), Ligon et al. (2002), Mazzocco and Saini (2012), Laczo (2015) or Abraham and Laczo (2018).

[17]Morten (2018) also uses the 2001-2004 wave of the dataset to study joint determination of risk sharing and migration.

[18]Strictly speaking, the data in the first wave of ICRISAT is collected approximately every month. This is approximate since the frequency of the interviews varies and the dates of the

laborer households and, in case of the first wave, households with fewer than 80 data points. Moreover, I exclude outliers by trimming the data at the top and bottom percentiles. This implies that I am left with 86 and 180 households in the first and second waves respectively.

The household income data in the first wave is constructed as a sum of farm and non-farm income, profits from agriculture, livestock, capital and interest income (net of remittances). In the second wave, this variable is recorded directly by ICRISAT and is a sum of similar accounts: income from farm and non-farm work, profits from agriculture, migration, caste occupations, livestock, capital and interest income (net of remittances).

For the analysis of consumption smoothing, apart from data on household income, I need further data on consumption expenditures and household demographics. The non-durable consumption in both waves is equal to the sum of expenditures on milled grain, oil, animal products, fruits and vegetables, and on other non-durable goods such as electricity, water charges, cooking fuels for household use, and expenses for domestic work. In order to control for the household's age and size composition, the consumption and income variables are transformed into per-capita using the age-gender weight as in Townsend (1994).[19] Finally, both the income and consumption variables are deflated to the 1975 price level using the consumer price index for agricultural laborers published by the Labour Bureau of India. Table 1 reports summary statistics of the main variables.

Providing empirical support for the mechanism of irrigation crowding-in or crowding-out risk sharing requires data on weather, the implied aggregate (post-irrigation) risk, the size of irrigation available in each village and its ownership / management structure. Since the second wave of the ICRISAT panel does not contain rainfall data, I use the rainfall records in each province available through the Indian government's OGD platform.[20] As far as aggregate risk is concerned, as already mentioned, I will control for it in regressions with village-time fixed effects, and otherwise (in a non-regression based analysis) I will proxy it with variance of mean village income. The data on actually irrigated and irrigable land

interviews differ across households. In order to overcome this problem, I follow Mazzocco and Saini (2012) and for each interview that covers two months I compute the percentage of days that belongs to each month. Then, I assign the corresponding data point to each month using this percentage.

[19]In particular, it is computed by adding the following numbers: for adult males, 1.0; for adult females, 0.9; for males aged 13-18, 0.94; for females aged 13-18, 0.83; for children aged 7-12, 0.67; for children aged 4-6, 0.52; for Toddlers 1-3, 0.32; and for infants 0.05.

[20]This data is available under https://www.data.gov.in

	1976-1984		2001-2004	
Variable	Average	Std. Dev.	Average	Std. Dev.
Household size	7.99	3.43	4.82	2.03
log Cons. (real, per cap.)	6.94	0.951	7.06	0.472
log Income (real, per cap.)	7.17	0.838	7.54	0.706
Age-gender weight	6.57	2.85	4.27	1.61
Number of HHs	86	-	180	-
Observations	734	-	690	-
% of small farm HHs	34%	-	47%	-
% of medium farm HHs	32%	-	37%	-
% of large farm HHs	34%	-	16%	-

Table 1: Data summary statistics

comes from the ICRISAT panel. Unfortunately it does not record the associated management structures. To address this issue, I use the 1st, 3rd and 4th Minor Irrigation Censuses conducted in 1986-1987, 2000-2001 and 2006-2007.[21] This census contains district-level data on the number, ownership-management structure and the irrigated- and irrigable-area of irrigation units such as dugwells, tanks, deep and shallow tubewells, check dams or diversion channels. Given this data limitation, the analysis below effectively assumes that the three villages analyzed are representative in each district to which they belong.

1.5.2 Aggregate risk and irrigation in ICRISAT villages

Weather risk

I start by documenting that the weather risk in all three villages has increased over time. In particular, Table 2 shows that between the two waves: (i) the quantity of annual rainfall has decreased; and (ii) it has become more erratic (with the exception of Aurepalle where the standardized variance has decreased somewhat). For robustness reasons, I also include a longer time frame covering 9 years (as in the first wave) between 1996 and 2004 which confirms these results.

As a cross-check, I make use of the survey component of the second wave of the ICRISAT data where in 2001 households were asked about their perceptions of changes in rainfall characteristics between the end of the first and the beginning of the second wave. In line with results in Table 2, virtually all the households in three villages answered that: (i) the quantity of rainfall and the number of rainy

[21] This data is available under http://micensus.gov.in

	1976-1984		2001-2004		1996-2004	
	mean	$\frac{std.dev.}{mean}$	*mean*	$\frac{std.dev.}{mean}$	*mean*	$\frac{std.dev.}{mean}$
Aurepalle	1020.80	0.250	829.45	0.166	910.60	0.153
Kanzara	911.47	0.104	831.87	0.160	896.88	0.163
Shirapur	773.26	0.235	619.22	0.347	714.63	0.237

Table 2: Rainfall in ICRISAT villages

Note: Rainfall in Aurepalle is measured as rainfall in district Telangana, Kanzara - Madhya Maharashtra and Shirapur - Matathwada. Mean and standard deviation of rainfall are computed for the annual rainfall in each district for specified periods.

days have decreased; (ii) the distribution of rainfall has become highly erratic; and (iii) monsoons tend to arrive later than usual.[22]

Irrigation development and exposure to weather risk

The overall access of irrigation systems in all three villages analyzed has on average increased between the 1970/80s and 2000s. This can be seen in Figure 2, presenting the data from the two waves on households' (i) actually irrigated land; and (ii) irrigable land (defined as area that would be irrigated in normal times).[23] While the use of irrigation has increased over the two waves, these changes have been characterized by significant variations in both waves. Obviously, this dynamic may reflect changes in both the demand (e.g. due to changes in weather conditions) or supply (e.g. due to breakdowns of some of the infrastructure, collapsing wells etc.). While identification of the sources of these variations is not at the center of this paper, these will be used below to provide evidence on interaction between irrigation and risk sharing.

Columns 1-3 of Table 3 summarize Figure 2 and show that between the two waves the share of irrigated land has increased in all three villages, with a much higher relative growth in Kanzara and Shirapur. At the same time, columns 4 and 5 of Table 3 show that the variance of mean village income has decreased by 66%-

[22]Subjective reasons given by households for these changes were: changes in nature/monsoon patterns, deforestation and pollution.

[23]Importantly, while the data on irrigated land is recorded in both waves for every year, the data on irrigable land is available only for the years 2001-2004 and 1985 (the end of the first wave). In order to address the missing data problem in the case of irrigable land, I impute the data in the first wave by fitting in a linear long-run trend of irrigable land growth rate between 1985 and 2004. Obviously, this imputation procedure does not affect the recorded data on irrigable land in the second wave.

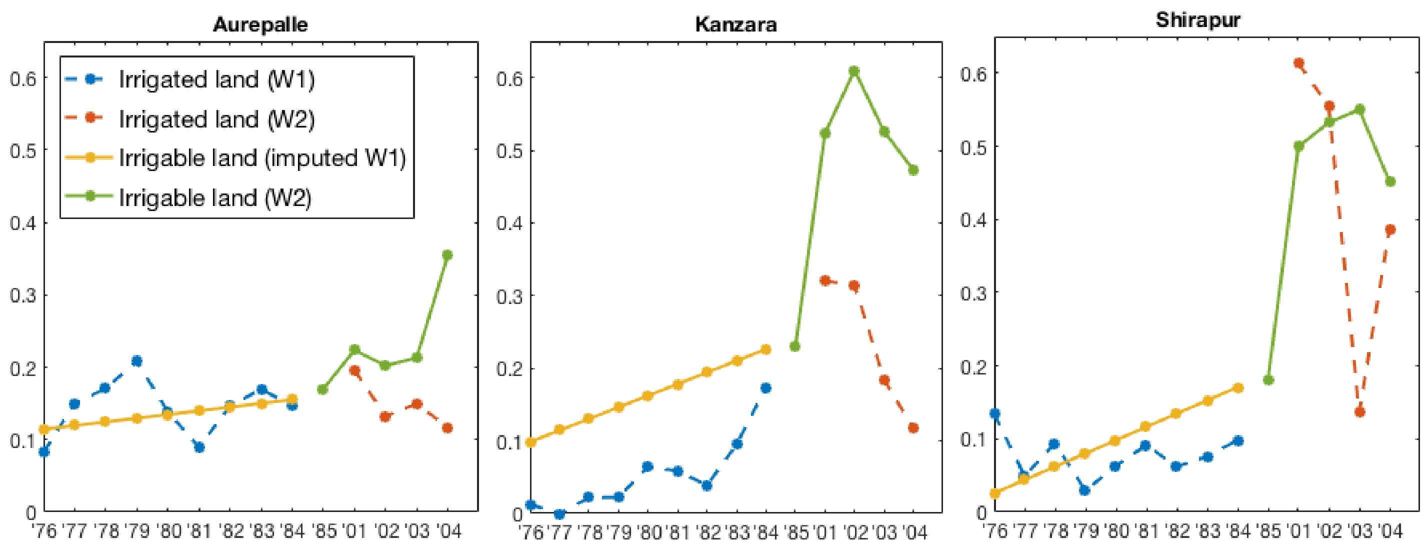


Figure 2: Irrigation in the ICRISAT villages

Note: Irrigated and irrigable land are defined as the shares of total land owned in a village recorded by ICRISAT that has been either actually irrigated, or reported as 'irrigable in normal times'. The data on irrigable land in wave 1 is imputed as it is missing in the ICRISAT dataset. See Footnote 23 for details on this imputation. The data used for constructing the time series on both the irrigated and irrigable land come from a full sample of the first and second waves of the ICRISAT panel.

	irr_{W1}	irr_{W2}	g_{irr}	var^y_{W1}	var^y_{W2}	g_{var}
A.	13.9%	19.8%	42%	0.067	0.013	-80.5%
K.	10.8%	37.9%	250%	0.069	0.023	-66.6%
S.	8.7%	46.6%	435%	0.054	0.018	-66.6%

Table 3: Irrigable land and "post-irrigation" aggregate risk in two waves

Note: A./K./S. stand for Aurepalle/Kanzara/Shirapur. Variable *irr* is defined as an average of mean irrigable and irrigated land in each wave, i.e. $irr_i = \frac{1}{2}\left(\bar{irrigable}_i + \bar{irrigated}_i\right),\ i \in \{W_1, W_2\}$; var^y denotes variance of log mean village income in corresponding waves; g_{irr} and g_{var} stand for the growth rates of irrigable land and variance of log mean village income.

80%, depending on the village.[24][25] Given the high magnitude of these numbers, a natural follow-up question is how much of this change is due to improvements in irrigation.[26] Arguably, the second most important factor (after irrigation) reducing aggregate risk faced by villagers are potential changes in crop trade prices.

Furthermore, Evenson and Gollin (2003) call the 1960-2000 period a "Green Revolution", characterized by large increases in both the levels and stability of crop yields worldwide. Apart from irrigation, they list advancements in the following factors that have contributed to these developments: use of fertilizers or pesticides, mechanization and modern crop varieties. While most of these factors affect farmers' idiosyncratic risk or the level of their crop yields,[27] they may still have an effect on the computed measure of aggregate risk.

Thus, I will use the quantitative model in Section 1.6 below to measure how

[24]Notice that the number of households in the second wave of the panel significantly exceeds its counterpart in the first wave. In order to avoid biasing the computations of this variance by differences in sample sizes, I divide the second wave of the sample in each village randomly into a number of subgroups that are a multiple of the first wave sample sizes. Then, I compute mean variance income of each of these subgroups in each village and compute the total village variance as a weighted (by subgroup measures) average variance. I repeat this procedure 1000 times and compute the final variance as a mean over these simulations.

[25]See also Kukal and Irmak (2018) for evidence on the mechanism studied in case of the Great Plains in the US. In line with results above, they show that irrigated fields, apart from having significantly higher yields, were characterized by a significantly lower yield variability as compared to similar fields without irrigation.

[26]Notice that reverse causality (i.e. that e.g. lower aggregate risk causes higher irrigation investments) is arguably not confounding the interpretation of results in this case.

[27]By destroying insects affecting particular crops, pesticides mostly reduce the level of idiosyncratic risk affecting farmers having different crops (Carlson, 1989; Villano and Fleming, 2006). On the other hand, fertilizers increase fertility of land, and so their main role is to increase the yield. Similarly, mechanization mostly increases work efficiency and so the yield. Lastly, increased use of modern high yielding varieties (the adoption of which has mostly been possible in places with reliable rainfall or irrigation (Evenson and Gollin, 2003)) is associated with both increases in yield and reductions in variability.

much of the documented reduction in variance is due to the increased usage of irrigation.

Access to irrigation within villages

One of the main assumptions of the model is that, upon agreeing to co-operate on irrigation, all the households share the benefits of irrigation equally. While farmers may obviously install irrigation units that are completely private in nature (e.g. private wells on own land), it is usually much more economically sensible to invest in common large water tanks, canals, shared water pumps etc. This is seen particularly often in dry and semi-arid regions of south India (the regions of the three ICRISAT villages studied in this paper), where the motives for co-operation are particularly strong and so the local communities organize informal bodies aimed at co-ordinating the village-wide irrigation strategies (see e.g. Wade, 1988 and Bardhan, 2000).

In reality, however, it need not be the case that households share access to irrigation equally, even if it is largely a public good. This can occur for a variety of reasons, the most basic being due to differences in demand for irrigation stemming from differential crop choices and associated crop farming requirements. Alternatively, if using irrigation is associated with additional water charges, it may be the case that only the wealthier households with larger landholdings can afford irrigation. To this end, the first row in Figure 3 ranks households in each village according to their total owned land between 1976-1984 and shows that access to irrigation has to a large extent been shared by everyone in all three villages.

However, Indian rural areas are characterized by strong organization along caste lines. For instance, individuals from a given caste are usually not allowed to get married to a person from another caste. Moreover, the consequences of caste systems also extend to the economic and political functioning of the relevant areas. For instance, Munshi and Rosenzweig (2016) show that informal risk insurance networks organized along caste lines[28] have significant implications for rural households' migration decisions and associated (mis)allocation of labor.[29]

[28]The argument that the relevant risk-sharing unit in Aurepalle, Kanzara and Shirapur is the caste (and not village) is made by Mazzocco and Saini (2012). However, a comment by Shrinivas and Fafchamps (2018) somewhat weakens the case for it.

[29]Furthermore, Munshi and Rosenzweig (2013) argue that caste networks enable local communities to elect the most competent representatives, overcoming political commitment problems. In terms of career choices, Munshi and Rosenzweig (2006) show that males from some castes (as compared with females) are much more likely to attend traditional, local language schools leading to traditional occupations (which have been becoming less profitable in recent decades).

More relatedly however, Anderson (2011) studies markets for water irrigation in northern Indian villages and finds evidence for discrimination of lower-castes in access to irrigation in villages dominated by upper-castes. Although the focus of this paper is on villages in the south of India, this discrimination may well also be present in the case of Aurepalle, Kanzara and Shirapur. To this end, the second and third rows in Figure 3 break down the same shares as above along two different measures of caste ranks in each village.[30] Consistent across these two measures of caste rank and with the work of Anderson (2011), I find some evidence consistent with discrimination in access to irrigation for lower caste households. As this observation may have important implications for any policy recommendations below, I return to it in the conclusion.

1.5.3 Imperfect insurance

Next I turn to study the patterns of insurance against idiosyncratic shocks in the ICRISAT villages. First, I perform a standard test of consumption smoothing documenting that risk sharing in the villages is incomplete. Secondly, I delve deeper into the nature of idiosyncratic risks affecting villagers by studying their persistence and volatility.

Consumption smoothing test

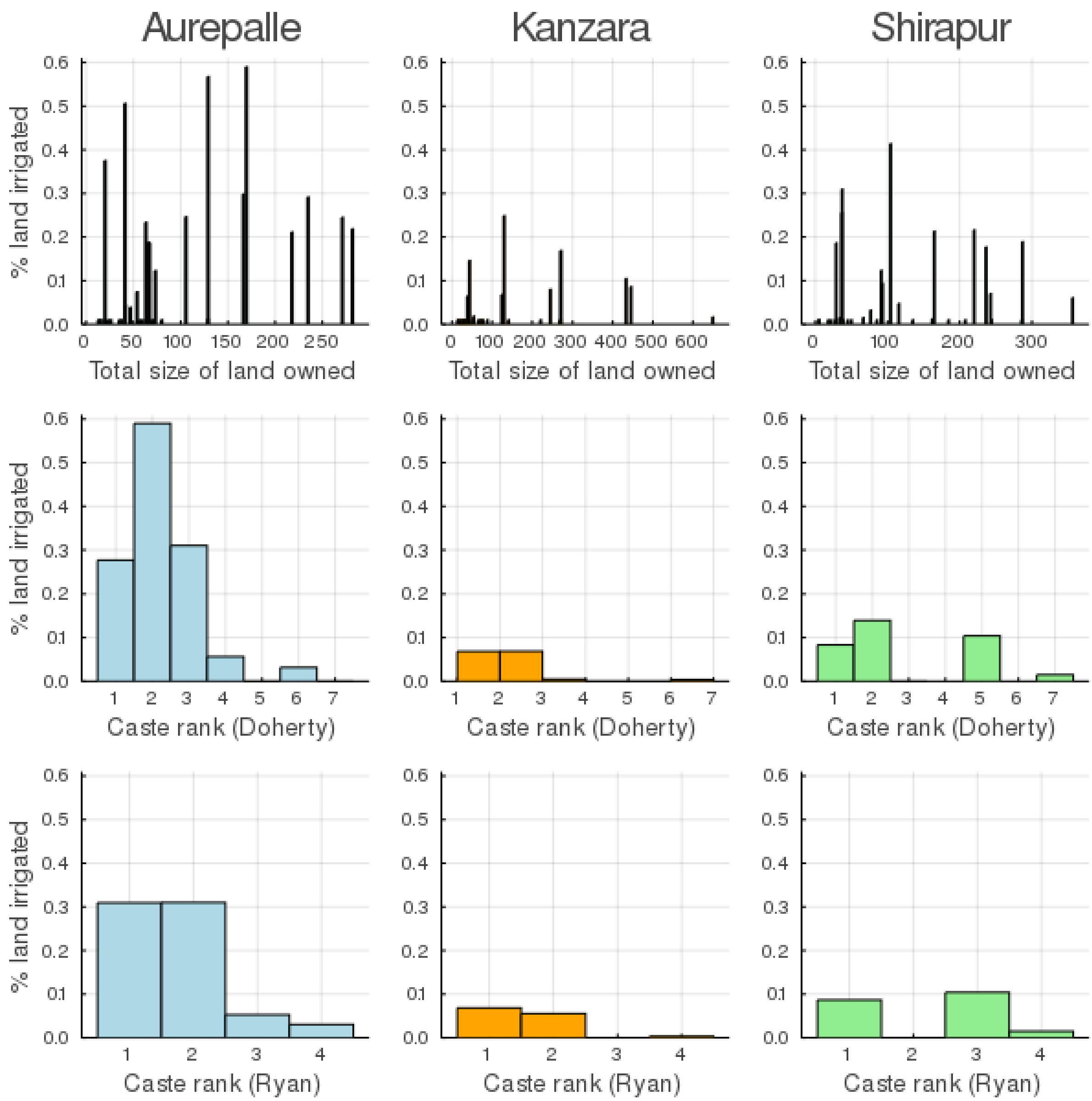
Along the lines of literature following Townsend (1994), I test the null hypothesis of perfect consumption smoothing by running the following regression:

$$\log\left(cons_{i,t}\right) = \alpha + \beta \log\left(inc_{i,t}\right) + \tilde{\beta}_i + \gamma_{v,t} + \epsilon_{i,t} \tag{19}$$

where *cons* is per capita expenditure of households on non-durable consumption (excluding savings), *inc* is their income net of remittances, $\tilde{\beta}_i$ is a household fixed effect, γ_{vt} is a village-year fixed effect that captures the total resources available to the village at time t (i.e. it controls for aggregate fluctuations in the village) and errors ϵ_{ivt} are clustered at the village-year level.[31]

[30] Note that these graphs do not capture differences in land size across castes. Moreover, datapoints-households with no irrigated land are marked as 1% irrigated share datapoints; and the 0% points imply that there is no household in the data with a given landholding size.

[31] Unfortunately, due to the smaller sample size, I am not able to run (19) with household fixed effects in the second wave. To address this issue, I instead include demographic controls of household and landholding size. Nonetheless, the estimates from a pooled sample are obviously with household fixed effects.



Note: Total size of land owned is a sum of owned land between 1976-1984. Datapoints-households with no irrigated land are marked as 1% irrigated share datapoints; and the 0% points imply that there is no household in the data with a given landholding size. Caste ranks come from the ICRISAT dataset. The caste rank by V. S. Doherty is based on the social, religious, and economic standing of a caste in the village. The caste rank by J. G. Ryan is based on an inspection of descriptive data on occupation and socioeconomic condition of individual castes.

Figure 3: Access to irrigation in the ICRISAT villages (wave 1) according to households' landholdings and caste

Dep var: cons. expenditures	1976-1984	2001-2004	Pooled
Income in all villages	0.258***	0.280***	0.292***
	(0.052)	(0.40)	(0.035)
in Aurepalle	0.223**	0.250**	0.326***
	(0.090)	(0.065)	(0.063)
in Kanzara	0.340***	0.246**	0.194***
	(0.083)	(0.051)	(0.072)
in Shirapur	0.257***	0.303**	0.318***
	(0.065)	0.0698	(0.053)
Village-time FE	Yes	Yes	Yes
Household FE	Yes	No	Yes
R^2 (3 villages)	0.7532	0.3651	0.7313
Number of obs. (3 villages)	734	690	1395

Table 4: Consumption smoothing in ICRISAT villages in two waves

Note: Table presents coefficient β from running regression $\log(cons_{i,t}) = \alpha + \beta\log(inc_{i,t}) + \tilde{\beta}_i + \gamma_{v,t} + \epsilon_{i,t}$ for the first wave and pooled sample; and $\log(cons_{i,t}) = \alpha + \beta\log(inc_{i,t}) + \tilde{\beta}X_{i,t} + \gamma_{v,t} + \epsilon_{i,t}$ for the second wave. Standard errors provided in brackets. Errors are clustered at the village-year level. The difference in sample size between individual waves and the pooled dataset is due to trimming (at the top and bottom percentiles) the data after matching of the two (untrimmed) waves is done.

Table 4 presents the results of running regression (19) for all 3 villages, both separating and pooling the two waves of ICRISAT panel dataset.[32] We see that while in both waves the test rejects perfect insurance, the shocks to income are relatively well insured with the best insurance against idiosyncratic shocks in Aurepalle (in the first wave) and in Kanzara (in the second wave). In line with the informal insurance mechanism studied in this paper, Mazzocco and Saini (2012) document that the real per-capita transfers given and received by households in the first wave amount on average to 28.3% and 21.1% of non-durable expenditures in Aurepalle; 8.9% and 15.9% in Kanzara; and 16% and 21% in Shirapur.

1.5.4 Interaction between risk sharing and irrigation provision

One part of the theory developed above suggests that, due to differential impact on the outside option, government interventions aimed at providing irrigation infrastructure in village economies lead to excessive crowding out of local risk

[32]Notice that the obtained elasticities may differ from results obtained in other papers using ICRISAT data due to a different specification of the econometric test ((19) is similar to the one used by Morten, 2018; see Townsend, 1994, Mazzoco and Saini, 2012 or Abraham and Laczo, 2018 for other specifications) or focusing the analysis on households with positive land holdings.

Village (district)	metric	Gov. share '86/'87	Gov. share '00/'01	Gov. share '06/'07
Aurepalle (Mahbubnagar)	irrigable	31.28%	8.9%	6.93%
	irrigated	39.39%	6.86%	3.90%
Kanzara (Akola)	irrigable	39.93%	20.6%	0.07%
	irrigated	34.89%	20.82%	0.07%
Shirapur (Solapur)	irrigable	3.48%	0.40%	2.88%
	irrigated	2.98%	0.80%	2.87%

Table 5: Government share in ownership of irrigation units

Note: Table presents the shares of government ownership in irrigation structures in the districts of Mahbubnagar, Akola and Solapur in the first (1986-1987), third (2000-2001) and fourth (2006-2007) Minor Irrigation Censuses. The irrigable metric is defined as Culturable Command Area ("The area which can be irrigated from a scheme and is fit for cultivation."); and the irrigated metric is defined as Irrigation Potential Utilised ("The gross area actually irrigated during reference year out of the gross proposed area to be irrigated by the scheme during the year.") - see http://micensus.gov.in for more details.

sharing as compared with provision by villagers (Proposition 2). In what follows, using two only indirectly dependent measures of irrigation infrastructure size in the ICRISAT villages, I provide evidence for this mechanism. In particular, I show that (i) irrigation improves risk sharing, conditional on the share of irrigation infrastructure being governed by central authorities; and that (ii) an increase in the share of irrigation infrastructure owned by government is associated with a reduction in risk sharing.[33]

For the proxy of irrigation infrastructure installed in the villages, I use the above discussed ICRISAT data presented in Figure 2. Moreover, and as mentioned previously, I use the Minor Irrigation Census to establish the share of infrastructure units owned and administered by central authorities. To this end, I construct the following variable measuring it:

$$gov = \sum_i share\ of\ gov.\ owned\ units_i \cdot \frac{irrigation\ area_i}{\sum_j irrigation\ area_j} \tag{20}$$

where (i) the sums are over indices i, j indicating the type of the infrastructure unit (such as dugwell, tank, deep or shallow tubewell, check dam or diversion channel); (ii) $share\ of\ gov.\ owned\ units$ is defined as $\frac{No.\ of\ gov.-owned\ units}{No.\ of\ gov.-owned+priv.-owned\ units}$ for a given unit type i; and (iii) $irrigation\ area$ is either the irrigable or actually irrigated area covered by the given irrigation unit type i.

The data is presented in Table 5, where we observe that the districts of Au-

[33]See the later part of this subsection for discussion of empirical literature documenting mechanisms similar to this and the other element of the theory suggesting that risk sharing improves co-operation over irrigation.

repalle and Kanzara have experienced significant privatization of irrigation infrastructure from 31-40% of government ownership in 1980s down to 0-7% in late 2000s. On the other hand, in Shirapur these rates have always been low, at levels between 1-3%. In order to exploit this variation and investigate its interaction with risk sharing, I combine this data with pooled two waves of the ICRISAT panel[34] and run the following regressions:

$$\begin{aligned}\log\left(cons_{i,t}\right) = \alpha &+ \beta_1 \log\left(inc_{i,t}\right) + \beta_2 \log\left(inc_{i,t}\right) \cdot irr_{v,t} \\ &+ \beta_3 \log\left(inc_{i,t}\right) \cdot irr_{v,t} \cdot gov_{v,w} + \beta_4 \log\left(inc_{i,t}\right) \cdot gov_{v,w} \\ &+ \beta_5 irr_{v,t} \cdot gov_{v,w} + \beta_6 irr_{v,t} + \beta_7 gov_{v,w} + \tilde{\beta}_i + \gamma_{v,t} + \epsilon_{i,t}\end{aligned} \tag{21}$$

where $gov_{v,w}$ is the measure of government ownership share (20) in village v during wave w (1 or 2), $\tilde{\beta}_i$ is the household fixed effect, $\gamma_{v,t}$ is the village-time fixed affect and $irr_{v,t}$ stands for the share of irrigated or irrigable land in village v at time t.

Table 6 with regression results supports the theory outlined above by documenting that (i) an increase in the share of land with access to irrigation is associated with a significant improvement in risk sharing; and that (ii) conditional on the share of land with access to irrigation, an increase in the government ownership of the irrigation units is associated with a significant reduction in risk sharing. These conclusions are consistent for both measures of irrigation (irrigated and irrigable land) with the exception of comparing irrigated land across the 1st and 3rd censuses, where the coefficient on the income-irrigation-government share interaction maintains the right sign but is statistically insignificant.

While the evidence presented here should obviously not be interpreted as causal, it is indicative of a significant empirical interaction between the two institutions along the lines of the theory outlined above. Nonetheless, the correlational results suggest that for all three villages pooled, a one standard deviation increase in total irrigation (equal to approx. 75% of its mean) while holding the absolute government-ownership constant (reflecting an increase in the village-owned irrigation) is associated with crowding-in of risk sharing equivalent to an average reduction in the elasticity of consumption of 13.6%.[35] Similarly, holding total irri-

[34]When pooling the two waves, I match the households that have not split over the 16 years gap between the two waves. Households that have split over time are treated as independent from the first wave.

[35]Notice that the ICRISAT data does not break down the irrigation data according to ownership structure. Thus, in order to measure the benefit to risk sharing from increasing village-owned

Dep var: cons. expend.	(irrigated 1+3)	(irrigable 1+3)	(irrigated 1+4)	(irrigable 1+4)
income	0.393***	0.412***	0.407***	0.413***
	(0.089)	(0.065)	(0.087)	(0.067)
income·irrigation	-0.406*	-0.507***	-0.548**	-0.690***
	(0.229)	(0.106)	(0.213)	(0.161)
income·irrig.·gov share	0.465	4.300*	5.321**	9.633***
	(1.367)	(3.471)	(2.584)	(3.378)
income·gov share	-0.089	-0.752*	-0.536	-1.349***
	(0.326)	(0.394)	(0.370)	(0.469)
Village-time FE	Yes	Yes	Yes	Yes
Household FE	Yes	Yes	Yes	Yes
R^2	0.7335	0.7353	0.7359	0.7372
Number of observations	1395	1395	1395	1395

Table 6: Empirical interaction of risk sharing with irrigation and its ownership structure

Note: Table presents the coefficients $\beta_1, \beta_2, \beta_3$ and β_4 of running regression (21) for both waves of the panel pooled, with $irr_{v,t}$ standing for the share of irrigated or irrigable land in village v at time t; regressions (1+3) are on the government share data from the 1st (1986-1987) and 3rd (2000-2001) censuses; and (1+4) on the 1st and 4th (2006-2007) censuses. Errors are clustered at the village-year level.

gation fixed at its mean level, a one standard deviation increase in the government share (equal to approx. 100% of its mean) is associated with crowding-out of risk sharing equivalent to an average increase in the elasticity of consumption of approx. 37%.[36]

Existing evidence in the literature cited above provides further support for the modeling assumption of local irrigation ownership and management allowing for stronger punishments helping to elicit better co-operation, as opposed to

irrigation, I linearly extrapolate what would be the government-ownership rate upon a corresponding increase in village-owned irrigation. Furthermore, this decrease in elasticity amounts to 17.4%, 14.9%, 12.8% and 9.2% for the 1st and 3rd census with irrigated land, the 1st and 3rd census with irrigable land, the 1st and 4th census with irrigated land and the 1st and 4th census with irrigable land, respectively. For instance, in the case of "irrigated 1+3" regression, I compute this change as $\Delta\epsilon = \frac{0.393-0.406\cdot irr_2+0.465\cdot gov_2\cdot irr_2-0.0891\cdot gov_2}{0.393-0.406\cdot irr+0.465\cdot gov\cdot irr-0.0891\cdot gov} - 1$, where $irr = 17.65\%$, $irr_2 = 33.1\%$, $gov = 17.43\%$ and $gov_2 = 9.23\%$ (representing the means of respective variables in the first and second wave of the dataset, see Figure 2 and Table 5).

[36]More precisely, this increase in elasticity amounts to 84.5%, 17.9% and 45.4% for the 1st and 3rd census with irrigable land, the 1st and 4th census with irrigated land and the 1st and 4th census with irrigable land, respectively. Note that these results are very large partly due to huge variation in government ownership data resulting in 1 standard deviation of gov_t ranging between 86% and 110% of its mean. Unsurprisingly, given the statistically insignificant coefficient on the double-interaction term "irrigated 1+3" regression, there is no change in elasticity for the 1st and 3rd census with irrigated land.

centralized provision and management. In particular, Wade (1988) conducts a study of 31 irrigated villages in Andhra Pradesh (the state of Aurepalle) between the 1970s and 80s. Among other findings, he points out that in many villages access to irrigation water has been regulated partly by the local community and partly by the state Irrigation Department (Wade, 1988, p.7, 72). More importantly however, Wade (1988, p. 14, 73) finds that the latter has largely been unable to efficiently regulate this access vis a vis the local community (which succeeded in eliciting much better co-operation through social ostracism or imposing local norms).

Similar conclusions are found in a different set of South Indian villages by Bardhan (2000, p. 849, 852). He studies 48 irrigated villages and notes that in the 1990s on average around half of irrigation units were funded and administered by the central government's Public Works Department, and the other half by the local community. Furthermore, he documents higher degrees of social co-operation in cases where communities have more autonomy in self-regulating the distribution of water as opposed to ones where this is mostly handled by the Public Works Department.[37]

Finally, recall that the theory also suggests that risk sharing facilitates and increases efficiency of co-operation over irrigational investments. While providing evidence for this using the ICRISAT data is much harder, several papers in the literature find that reductions in associated risk stimulate agricultural investments. For instance, Cole, Gine and Vickery (2014), Mobarak and Rosenzweig (2012) and Karlan, Osei, Osei-Akoto and Udry (2014) conduct experiments in India and Ghana showing that access to formal insurance products induces farmers to shift production towards higher-return but higher risk cash crops. Similarly, Cai, Chen, Fang and Zhou (2015) document that the promotion of formal financial products providing insurance against the risk of infection-related deaths of sows increases production. Lastly, Emerick, de Janvry, Sadoulet and Dar (2016) show that the adoption of modern flood tolerant rice varieties in rural India increases investments and co-operation through more labor-intensive planting, employment of larger farming areas and increased usage of fertilizers and credit.

[37]See also Bardhan (1993) for an overview of literature arguing for mechanisms along these lines.

1.6 Quantitative analysis

Given empirical evidence consistent with the theory presented, I proceed to estimate the structural parameters of the model. The main purpose of this exercise is: (i) to externally validate the model; (ii) to provide independent support for the interaction between risk sharing and investments in public irrigation systems, predicted by the theory; (iii) to measure the extent to which irrigation development can account for the reduction in aggregate risk documented in Table 3; and (iv) to conduct a number of counterfactual experiments. The latter aim to measure the size of reinforcing effects between risk sharing and public irrigation systems and economic losses due to centralized ownership of irrigation.

In what follows, I employ the approximation method commonly used in the literature, reducing the computational complexity of solving for optimal decisions of N-households to solving for decisions of one household with the other representing the rest of the village. See Appendix A for details on the numerical algorithm and recursive formulation of FOCs used for solving the model.

1.6.1 Calibration

Estimating the structural parameters of the model involves choosing parameters such that moments generated by the model in equilibrium capture the variations captured in the first wave of the ICRISAT panel data. This task is non-trivial as the income process, risk sharing and investment decisions are all endogenous and depend on actions of every other household in the village. Identification is based mostly on (i) the crowding-in effects due to interaction between risk sharing and irrigation provision documented in Section 1.5; and (ii) evidence in Morten (2018) concerning the impact of negative rainfall shocks on household income in the ICRISAT villages (see Figure 8 in Appendix C).

Since the calibration is performed on the first wave of the dataset, I set the population size equal to the one of the empirical samples, i.e. to $N = 29$ in Aurepalle, $N = 30$ in Kanzara and $N = 27$ in Shirapur.[38] Moreover, this approach implies that the assumed period length equals one year.

In what follows, I describe the simulated method of moments approach which aims to capture the empirically documented variations. Notice that since the equilibrium of the dynamic model is complex, in some cases one parameter may affect

[38]This follows the practice common in the literature on rural risk-sharing applying similar approximations, see e.g. by Ligon et al. (2002), Laczo (2015) and Bold and Broer (2018).

many targeted moments.

Preferences and investment technology

Utility function and discounting. I assume that agents value consumption according to the log-utility function, i.e. $u(c) = \log(c)$. The time preference parameter β will be used to target elasticity of consumption w.r.t. income documented in Table 4.

Depreciation. I pin down the depreciation parameter δ using the Indian Ministry of Statistics data on national accounts on input and output of crop sector in the country for the years 2011-2017. Based on these, I assume that the depreciation rate in all three villages is 4.5%.

Irrigation and government. First of all, in order to save on the number of state variables, I assume that government-provided irrigation is constant over time, i.e. $\omega_t = \omega \ \forall t$. I choose ω such that the average share of irrigation infrastructure owned by government[39] $\hat{GS} = \frac{\nu_\omega \omega}{\nu_\omega \omega_2 + \nu_k \frac{1}{T} \sum_{t=0}^{T} \sum_j k_{j,t}}$ in the simulated economy matches the evidence of $\hat{GS} \in \{35.33\%, 37.41\%, 3.23\%\}$ reported in Table 5.

As mentioned above, since smallholder farmers in rural areas of developing countries usually face no taxation, I assume that the government-owned irrigation is subsidized at the rate of 100% (implying that $\tau_t(x_t) = 0 \ \forall t, x_t$). Furthermore, I pin down the subsidy parameter for village-owned irrigation s_k using data from the Minor Irrigation Census. Based on this, I assume that the village-owned infrastructure investments in Aurepalle, Kanzara and Shirapur are co-financed at the rates of 58.12%, 10.40% and 5.44%. See Appendix B for details on estimating this subsidy rate.

Stochastic processes

Both the idiosyncratic and the aggregate risk processes are estimated using indirect inference and are modeled as symmetric two-state Markov processes with shock levels $\{\theta_H, \theta_L, \phi_H^B, \phi_H^G, \phi_L^B, \phi_L^G\}$ and persistence parameters $\{\rho_\theta, \rho_\phi\}$, respectively.

Aggregate risk. First, for the impact of investments on the probability of the

[39] For T large enough and ν_k, ν_ω standing for effective return rates to an extra unit of investing in village- or centrally-owned irrigation.

"good" distribution realization, I make a parametric assumption of

$$P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega\right)=1-\frac{1}{N}\sum_{j=1}^{N}\exp\left(-\nu_k\frac{1}{1-s_k}k_{j,t+1}-\nu_\omega\frac{1}{N}\omega\right)$$

with ν_k,ν_ω being scaling/investment efficiency parameters. The latter two parameters affect both the size of crowding-in/out effects and, by affecting the measure of mass attached to the good and bad distributions, the equilibrium distribution of aggregate shocks. Thus, I first choose ν_k to match the observation that a 75% increase in irrigation owned by villagers (holding the government part constant), generates a reduction in elasticity of consumption of 13.6% (in line with the mechanism discussed in Proposition 2 and the empirical evidence in Section 1.5.4). In order to engineer these changes in irrigation capital, I re-solve the model for a new equilibrium with new consumption insurance transfers (respecting the new limited commitment constraints) using the same parameters but a modified probability function of $P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega\right)=1-\frac{1}{N}\sum_{j=1}^{N}\exp\left(-\nu_k\left(1+g\right)\frac{1}{1-s_k}k_{j,t+1}-\nu_\omega\frac{1}{N}\omega\right)$, where $(1+g)$ is used to generate a variation of increasing village-owned irrigation while keeping the government-owned part constant (the variation described in Section 1.5.4).[40]

Then, I normalize the average level of good aggregate shock to 1, i.e. I assume that $\phi_H^G+\phi_L^G=2$. Furthermore, I assume that irrigation reduces only the downside risk without affecting the upside, i.e. that $\phi_L^G>\phi_L^B$ and $\phi_H^G=\phi_H^B$. Then, I pin down the values of aggregate output shock levels $\left\{\phi_L^B,\phi_L^G\right\}$, the persistence parameter ρ_ϕ and the private investment efficiency parameter ν_k using empirical evidence in Morten (2018, see Figure 8 in Appendix C). She combines the first wave of the ICRISAT panel with the University of Delaware precipitation database and shows that a bottom 10%/20%/50% negative rainfall shock occurs in 14%/28%/49% of periods and reduces household's log income by 0.923 / 0.231

[40] In particular, I generate this variation of increasing the total size of irrigation by 75% (1 standard deviation in the data) while holding absolute government ownership constant (reflecting an increase in the village-owned irrigation) by using parameter g obtained from the following equation:

$$\left(1+g\right)\frac{1}{T}\frac{1}{1-s_k}\sum_{t=0}^{T}\sum_{j}k_{j,t}+\omega=1.75\cdot\left(\frac{1}{T}\frac{1}{1-s_k}\sum_{t=0}^{T}\sum_{j}k_{j,t}+\omega\right) \quad (22)$$

with $\mathbf{k}_t$ being the vector of simulated (baseline) optimal capital investment decisions and T being large enough. Interpreted literally, the changes through $1+g$ can be thought of as changes in private investment subsidies. Interpreted more broadly, they can be thought of as a proxy for income growth that has occurred between the two waves of the panel.

	Aurepalle	Kanzara	Shirapur
Π_θ	0.546**	0.100*	0.525***
	(0.165)	(0.050)	(0.0543)
$\bar{sd}\left(e_{i,t}\right)$	0.413	0.325	0.555

Table 7: Persistence and volatility of idiosyncratic risk in ICRISAT villages (1976-1984)

Note: $\bar{sd}\left(e_{i,t}\right)$ stands for mean standard deviation of idiosyncratic risk $e_{i,t}$ taken over all the village households. Errors are clustered at the village-year level.

/ 0.104.[41] Based on this, I first choose the value of parameters ρ_ϕ and ν_k so that in simulations the long-run probability of the ϕ_L^G, ϕ_L^B shocks' occurrence is 28% and 21%. Second, I choose the level parameters $\left\{\phi_L^B, \phi_L^G\right\}$ so that the coefficients from running regression $\log\left(inc_{i,t}\right) = \alpha + \beta_1 \cdot 1_{\phi_L^B} + \beta_2 \cdot 1_{\phi_L^G} + \epsilon_{i,t}$ on the simulated data replicate the empirical evidence of the bad low shock reducing log income by 0.577 (mean over the 10% and 20% bottom shocks in Figure 8) and the good low shock reducing income by 0.104 (i.e. that $\beta_1 = -0.577$ and $\beta_1 = -0.104$).

Idiosyncratic risk. I estimate parameters of the idiosyncratic risk process following the methodology proposed by Storesletten et al. (2004). In particular, I use the following decomposition of household i's real per-capita earnings in the first wave of the data:

$$\log\left(inc_{i,t}\right) = \alpha_0 + \tilde{\beta}_i + \gamma_{v,t} + \tilde{\epsilon}_{i,t} \tag{23}$$

where $\tilde{\beta}_i$ is a household fixed effect and γ_{vt} stands for village-time fixed effects capturing the aggregate risk. Consequently, $\tilde{\epsilon}_{i,t}$ is an indirect measure of households' idiosyncratic risk. In order to allow for some persistence of shocks, I model the idiosyncratic risk component as an AR(1) process:

$$\tilde{\epsilon}_{i,t} = \Pi_\theta \tilde{\epsilon}_{i,t-1} + e_{i,t} \tag{24}$$

I compute volatility of idiosyncratic risk $\bar{sd}\left(e_{i,t}\right)$ as a mean of idiosyncratic risk's standard deviation taken over all the village households. Table 7 shows the results of this estimation.

Secondly, I assume that (i) the idiosyncratic shock levels add up to 1 (i.e.

[41]Morten (2018) defines the negative aggregate shock as a rainfall event falling below the 20th percentile of the 1900-2008 rainfall distribution. The probability of shock occurrence differs from the estimated long-run density since the regression is done on a subperiod of the first wave of the panel (1975-1984).

Parameter	Aurepalle	Kanzara	Shirapur	Source/target
Preferences & investment technology				
Time preference β	0.84	0.685	0.795	Cons. smoothing in Table 4
Depreciation rate δ	4.5%			Indian Min. of Statistics
Gov.-owned irrig. ω	2.8	0.75	0.093	Minor Irr. Census/Gov. share in Table 5
Subsidy rate s_k on k	58.12%	10.40%	5.44%	Minor Irrigation Census, Table 12
Subsidy rate s_ω on ω	100%			Modelling choice
Shock processes				
Idiosyn. shock θ_L	0.265	0.34	0.225	Std. dev. in Table 7
Idiosyn. shock persist. ρ_θ	0.77	0.55	0.765	Persistence in Table 7
Scaling parameter ν_ω	2.65	7.05	7.33	Crowding-in of risk sharing, Table 6
Scaling parameter ν_k	5.32	16.72	17.00	Morten (2018): good neg. shock prob.
Low $B-$Aggr. shock ϕ_L^B	0.58			Morten (2018): bad neg. shock impact
Low $G-$Aggr. shock: ϕ_L^G	0.95			Morten (2018): good neg. shock impact
High Aggr. shock $\phi_H^B = \phi_H^G$	1.05			Normalization $\phi_H^G + \phi_L^G = 2$
Aggr. shock persistence ρ_ϕ	0.76			Morten (2018): bad neg. shock prob.
Numerical parameters				
Min/max $k-$grid points	0.01 / 0.6			modelling choice
# $k\ (= k_1 \cdot k_2)$ $-$grid points	100			modelling choice
Min/max Pareto grid points	0.25 / 4.0			modelling choice
# Pareto weight grid points	9			modelling choice

Table 8: Parameter values in the quantitative model

$\theta_H + \theta_L = 1$), and (ii) that they are independently distributed among the village's population.[42]

Given the above, I impose the moment restrictions for regressions from the simulated model to replicate their empirical counterparts in Table 7. In particular, I choose parameters ρ_θ and θ_L such that (i) the standard deviation of $e_{i,t}$, and (ii) the persistence of idiosyncratic shocks Π_θ; in $\tilde{\epsilon}_{i,t} = \Pi_\theta \tilde{\epsilon}_{i,t-1} + e_{i,t}$ (where $\tilde{\epsilon}_{i,t}$ is the residual from $\log(y_{i,t}) = \alpha_0 + \phi_t + \tilde{\epsilon}_{i,t}$) matches the empirical evidence provided.

Table 8 summarizes calibration parameters chosen for all the three villages.

1.6.2 Model evaluation

In this section, I discuss results of solving the model economy of joint co-operation over risk sharing and irrigation with limited commitment. I first discuss the fit of the model to the data. Then, to better understand the mechanism, I discuss optimal household consumption and investment policy functions. Finally, I present simulation paths of the benchmark model compared with the first best case.

[42]Given that the solution to the N-household case is approximated, the latter statement implies that the income process of the rest of the village is given by $y_{RoV,t} = \phi_t \cdot \frac{1}{2}$.

Calibration results

Table 11 evaluates the precision of the calibration. To generate the respective moments, I simulate the model for 10,000 periods, with the first 100 being discarded.

Most of the target moments are reasonably well matched by the quantitative model, with the highest estimate precision in the case of Shirapur. The largest deviation from the empirical moment is in the crowding-in of risk sharing in Aurepalle.

Interestingly, notice that the calibrated parameters are s.t. $\nu_\omega < \nu_k$, implying that government-owned investments are less efficient than the investments made by villagers themselves (in pure investment return terms, i.e. even without accounting for the impact on local co-operative patterns). This finding may reflect the superior knowledge of local society on how and where such investments should be conducted. Moreover, notice that these efficiency parameters are much larger in Kanzara and Shirapur. This is due to the fact that as the government-owned irrigation ω and/or investment subsidy rate s_k are much lower in these villages, matching the (common across the three villages) moment of long-run shock occurrence probability from Morten (2018) requires settings these numbers to much higher values.

Note that although the estimated values of β may seem to be relatively low, they should not be seen as invalid because they are implied by the data. The main moment driving the value of this parameter is the relatively high correlation between consumption and income in the data.[43]

Finally and importantly, the last row of Table 9 shows that the model is also able to generate crowding-out of risk sharing.[44] However, its magnitude is signif-

[43]Moreover, these estimates are in the ballpark of other papers' estimates of time preferences for the same villages, e.g. 0.7-0.95 in Ligon et al. (2002) and 0.65-0.8 in Morten (2018).

[44]Recall that the empirical evidence suggests that doubling the government share (while holding the total size of irrigation constant, as measured by $1+g$) is associated with a 37% increase in the elasticity of consumption. I generate this variation using the calibrated parameters and resolving the model for a new equilibrium with ω increased to appropriate value ω_2, holding the size of total irrigation constant. I solve for parameters g and ω_2 from the following system of equations:

$$\hat{GS} = \frac{\nu_\omega \omega_2}{\nu_\omega \omega_2 + \nu_k (1+g) \frac{1}{T} \frac{1}{1-s_k} \sum_{t=0}^{T} \sum_j k_{j,t}} \tag{25}$$

$$\nu_k (1+g) \frac{1}{T} \frac{1}{1-s_k} \sum_{t=0}^{T} \sum_j k_{j,t} + \nu_\omega \omega_2 = (1+\hat{g}) \cdot \left(\nu_k \frac{1}{T} \frac{1}{1-s_k} \sum_{t=0}^{T} \sum_j k_{j,t} + \nu_\omega \omega \right) \tag{26}$$

where the simulated path of private irrigation capital $\mathbf{k}_t$ is taken from the benchmark calibrated model, and I set $(1+\hat{g}) = 1$ and $\hat{GS} \in 2 \cdot \{35.33\%, 37.41\%, 3.23\%\}$ for Aurepalle, Kanzara and

Moment	Aurepalle		Kanzara		Shirapur	
	Data	Model	Data	Model	Data	Model
Cons. elasticity	0.223	0.230	0.340	0.343	0.257	0.254
Gov. share in irrigation	35.33%	34.63%	37.41%	39.14%	3.23%	3.65%
Variance of θ process	0.413	0.427	0.325	0.329	0.555	0.526
Persistence of θ process	0.546	0.545	0.100	0.116	0.525	0.525
Crowding-in of risk sharing	13.6%	42%	13.6%	29.7%	13.6%	18.4%
Income-rainfall reg. coeff. ϕ_L^B	-0.577	-0.593	-0.577	-0.590	-0.577	-0.577
Income-rainfall reg. coeff. ϕ_L^G	-0.104	-0.107	-0.104	-0.094	-0.104	-0.102
Long-run prob. of ϕ_L^B	28%	27.7%	28%	31.5%	28%	30%
Long-run prob. of ϕ_L^G	21%	22.2%	21%	17.7%	21%	17.2%
Not targeted						
Crowding-out of risk sharing	37%	147%	37%	71%	37%	69.7%

Table 9: Moments in the data versus generated by the quantitative model

Note: Crowding-in (-out) of risk sharing is measured as a reduction (increase) in the elasticity of consumption w.r.t. idiosyncratic shocks (in percentage terms).

icantly higher than the empirical evidence presented in Section 1.5.4 (especially in the case of Aurepalle). Nonetheless, given that this moment was not targeted, this finding effectively constitutes a test of the model's external validity. Similarly, the model generates low (pre-subsidy) investment rates with mean investment amounting to only 8.6%, 3.24% and 8.4% of mean consumption in Aurepalle, Kanzara and Shirapur (see Table 10), in line with findings in the empirical literature studying poor rural areas of developing countries.[45]

Policy functions

The main outcome of solving the quantitative model are the associated optimal policy functions. These will be used to simulate the model below and to derive the main welfare statistics. Figure 4 shows optimal first best (dashed lines) and limited commitment (solid) policies for an individual household i and household RoV representing the rest of the village (red and blue lines, respectively). The plots are for two current productivity states of $\phi = \phi_H^G \left(= \phi_H^B\right)$ and (i) $\theta = \theta_L$ for household i (and $\theta = 0.5 \cdot (\theta_L + \theta_H)$ for the representative household); and (ii) $\theta = \theta_H$ for household i. In both cases, households are assumed to hold the same capital stock of $k = 0.5 \cdot (k_{min} + k_{max})$.

The first row presents optimal policies for updating multipliers, where a lower

Shirapur.

[45]See e.g. Bandiera et al. (2017) and Banerjee et al. (2015).

Pareto weight implies a higher weight on household i. Notice that for the high idiosyncratic productivity state, the planner is able to enforce higher insurance transfers from the RoV-household to household i, and at the same time, she cannot transfer as much to the RoV as in the low productivity state scenario. These dynamics reflect the higher bargaining position of the household i, when it is more productive.

In any case, both agents are willing to forgo some of the current consumption (by accepting lower relative weights). This reflects the insurance value and efficiency gains associated with the contract: agent 1 is ready to maintain cooperation within the contract even with high weight on the other agent, in expectation that in some future period they will be worse-off and will benefit from insurance transfers from others.

In terms of consumption policies, observe first the monotonicity of first best policies along the changes in the relative Pareto weight. Second, the optimal consumption policies follow the first best rules up to the points where the enforcement constraints become binding. For instance, in case of θ_L, the planner cannot increase the consumption of the rest of the village above approx. 1.0 (as the current Pareto weight increases above approx. 1.8), since at this point the limited commitment constraint of agent i becomes binding.

Although similar logic applies to the optimal investment rules, these differ from the first best rules due to the occasionally binding enforcement constraints. First, recall that when assigning optimal investment decisions, the planner needs to shade these decisions due the fact that an additional unit of investment will both (i) improve the self-insurance of the investing household in the next period, and (ii) improve their next period expected aggregate state realization, driving the value of deviating up (see the FOC (14)).

Second, notice that for high (low) relative Pareto weights the investment level under limited commitment is above (below) the first best one. This dynamic is due to the impact of binding limited commitment constraints on the rate at which the investment externalities are internalized (best seen in the FOC (14) used for computations, described in Appendix A). In particular, the fact that for household i's relatively high Pareto weights (below 1) the limited commitment constraints of the household representing the rest of the village tends to be binding (which is obviously not the case in the first best), implies that the rate of externalities' internalization exceeds the efficient one. Nonetheless, the average (over Pareto weights) investment level of a household with low (high) θ tends to be below (above) the first best rule assignment.

Figure 9 in Appendix D shows a similar graph of optimal decisions for high and low aggregate productivities ϕ from the Bad distribution, holding idiosyncratic productivity θ constant. While all other conclusions remain unchanged, it shows that households invest more in periods of low aggregate productivity. This dynamic is due to the fact that (i) both the Good and Bad aggregate risk Markov processes are persistent and symmetric, and that (ii) the high value of productivity is equal across the two distributions. These imply that the marginal benefit of investing is much higher in low rainfall/ϕ states, as these are the states when it makes a large difference whether the productivity is drawn from the Bad or Good distribution.

Finally, value functions reflect all the observations mentioned above with the difference being that their curvature over Pareto weights in the economy with limited commitment is much lower as compared to the first best. This is due to the fact that in the latter the planner has to respect the limited commitment constraints and as such cannot redistribute as freely as in the first best.

Simulation results

Figure 5 presents results of simulating household i's decisions in the first best (with constant and symmetric Pareto weight) and limited commitment benchmarks approximated to the N-household case in Aurepalle (see Appendix A for description of the algorithm).[46] Firstly, notice how the Pareto weights are adjusted along the simulations: due to high assumed persistence of θ, as household i spends more and more time with high idiosyncratic productivity, the relative Pareto weight decreases (recall, again, that a higher relative Pareto weight implies lower weight on household i). Importantly, the asymmetry in relative Pareto weights comes about as this policy function comes from the 2-household model with one household representing the rest of the village. Since the approximation method implies that the latter household does not face any idiosyncratic risk, it effectively has a much higher relative bargaining power.

Furthermore, we observe lower consumption fluctuations in the first best benchmark as compared to the model with limited commitment.[47] In particular, since

[46] In order to ensure comparability, I feed in the same stochastic process into the two benchmarks.

[47] In particular, for the case of Aurepalle, the elasticity of consumption w.r.t. idiosyncratic shocks amounts to 0.230 and to 0.03 in the limited commitment and first best cases (respectively).

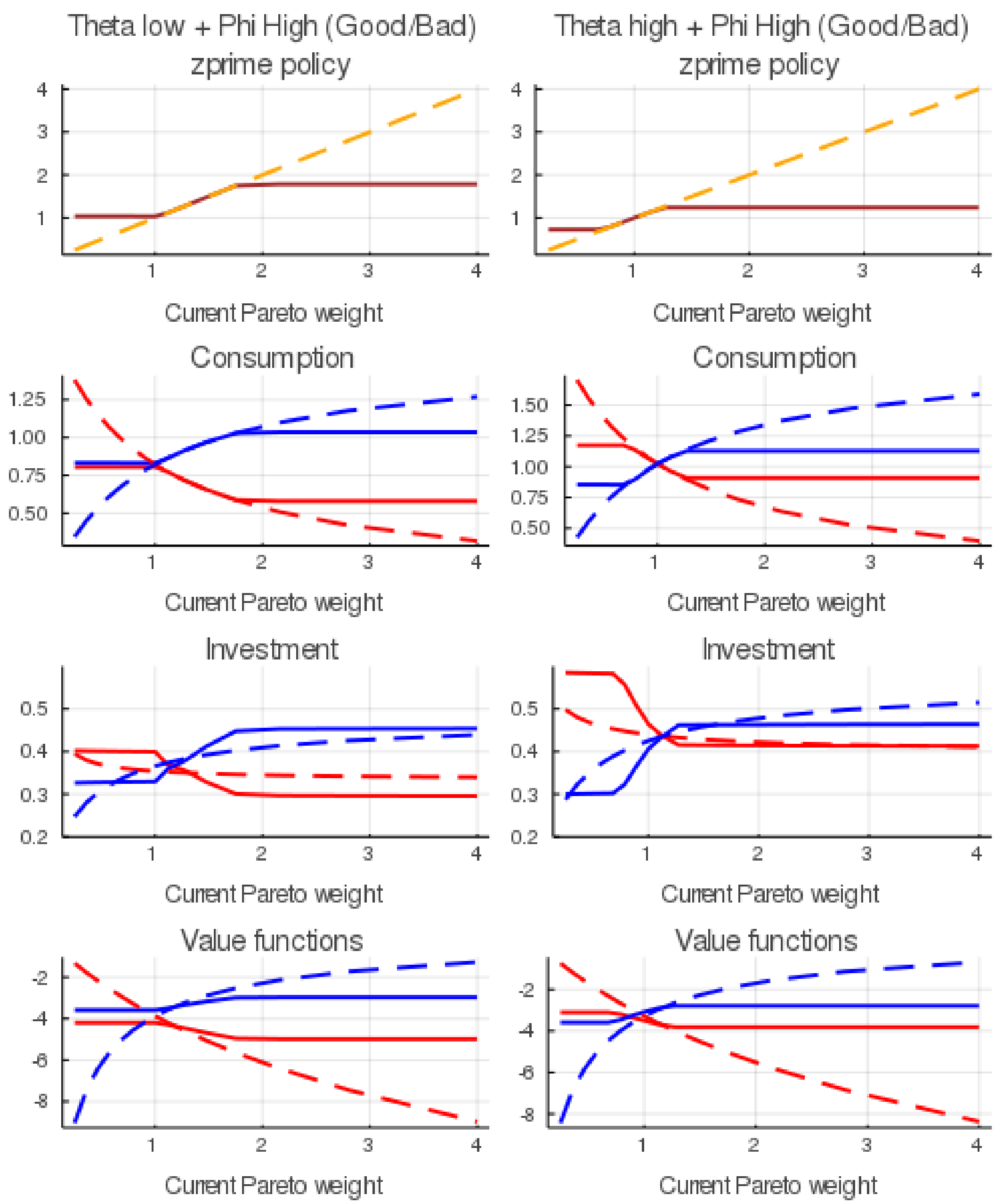


Figure 4: Optimal policy functions: household *i* vs household representing the rest of the village

Note: Red plots represent optimal policy functions of household *i* and blue ones - of the household representing the rest of the village. Solid plots stand for optimal policies in the first best model, and dashed ones - for the ones in the model with limited commitment.

the enforcement issues are not present in the latter, in first best each household receives the constant shre of $\frac{1}{N}$ in every period (with the associated relative Pareto weight always equal to 1). On the other hand, in the limited commitment case the share of total consumption assigned to household i follows closely the realization of its idiosyncratic productivity θ. For instance, consumption of household i exceeds the first best assignment in periods of high idiosyncratic productivity, when their relative Pareto weight increases.

In terms of optimal (pre-subsidy) investment decisions, we observe two interesting patterns. First, for similar reasons as in the discussion of policy functions above, conditional on idiosyncratic productivity θ, households invest more in periods of low aggregate productivity $\phi \in \{\phi_L^B, \phi_L^G\}$ (as compared to $\phi = \phi_H^B = \phi_H^G$). Second, since in the limited commitment model the degree of uninsured idiosyncratic risk is strictly higher than in the first best benchmark, so the precautionary saving motive is stronger in the former implying higher average investment in the model with limited commitment.

1.6.3 Counterfactual analysis

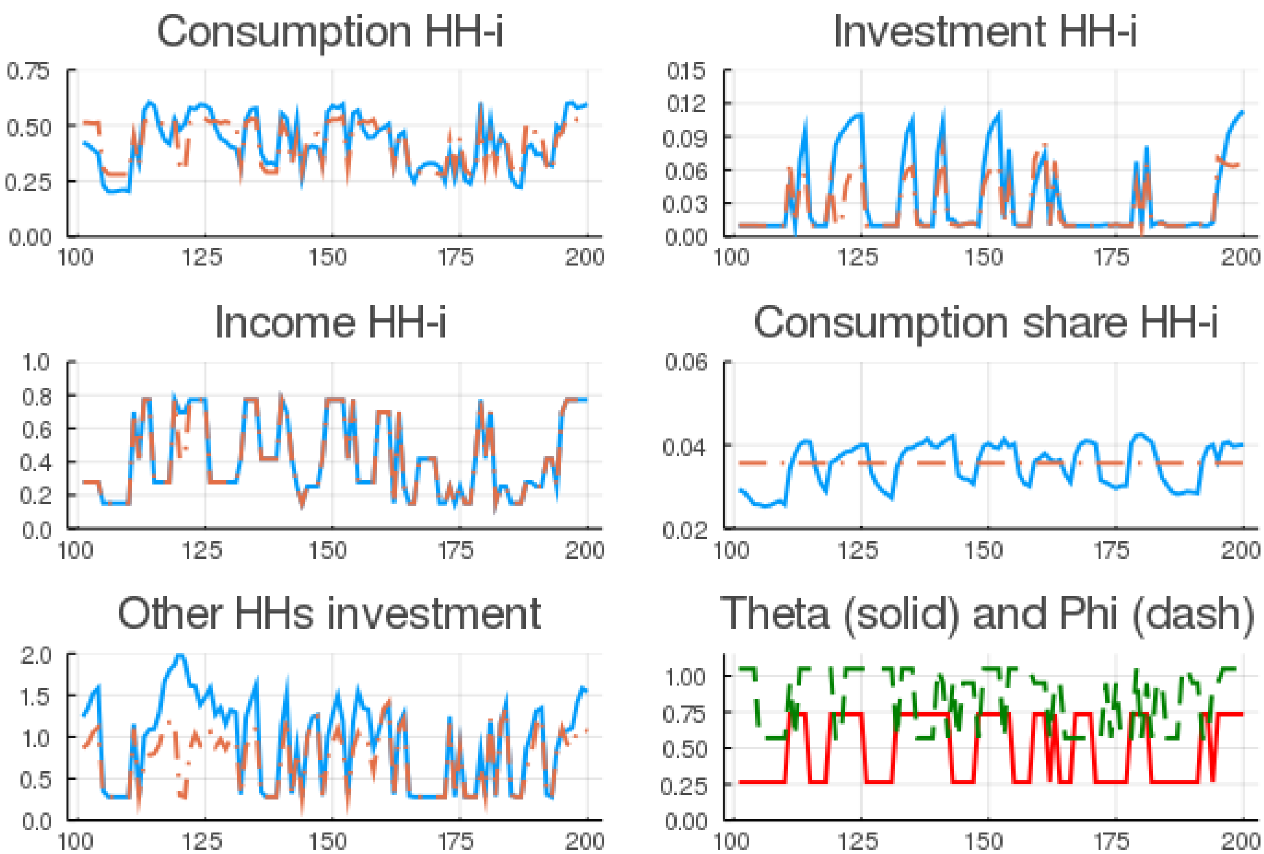
In this section, I conduct a number of counterfactual experiments. In particular, I first measure the extent to which irrigation development can account for the reduction in consumption fluctuations due to aggregate risk documented in Table 3. Then, I compute welfare gains associated with moving from partial co-operation (over risk sharing or irrigation only) to the joint one. Finally, I analyze (i) the effects of reducing the size of centrally-owned irrigational investments; and (ii) potential welfare gains attainable by replacing centrally-owned irrigation with a compensating increase in the co-financing rate for the village-owned investments.

In order to compare different policies in what follows, I will use a standard measure of lifetime consumption equivalent changes in welfare. It shows by how much households' consumption should change in every period such that their expected utility in the pre-reform scenario is equal to their expected utility associated with a given policy reform. Given the assumption of log-utility, this measure is expressed as:

$$W = \exp\left((1-\beta)\left(V^1 - V^0\right)\right) \tag{27}$$

where V^0 and V^1 stand for the approximated[48] value functions pre- and post-

[48] I approximate these value functions as an average of discounted sums of utilities along shorter



Note: Income stands for household's output $y = \phi \cdot \theta$. Consumption share is defined as $\frac{c_{i,t}^{approx}(x_{i,t})}{c_{tot,t}(x_{i,t})}$. See Appendix A for details on computing these simulation paths.

Figure 5: Model simulations: first best (dashed) and limited commitment (solid)

reform. Thus, W greater (lower) than 1 indicates that post-reform households' consumption needs to be decreased (increased) in every period by $|W - 1| \cdot 100\%$ in order to keep the household indifferent w.r.t. the pre-reform status quo, indicating a welfare improvement (deterioration).

Irrigation development and accounting for changes in aggregate risk

Figure 6 shows the relationship between changes in irrigation use and in the variance of the aggregate shock ϕ faced by farmers in the ICRISAT villages (see the note below Figure 6 for details on how it is measured). Importantly, since I perform this exercise on the baseline calibration, I hold the weather risk constant at the 1976-1984 level. We observe that as the size of irrigation investment increases, the aggregate risk drops at a declining rate. These rates of change are different for each village and stem from differences in investment rates (amounting to 8.6% of mean consumption in Aurepalle, 3.24% in Kanzara and 8.4% in Shirapur, see Table 10).

More importantly though, in the case of Aurepalle, we observe that the increase in total irrigation of 42% documented in Table 3 generates a reduction in aggregate risk of approx. 25%, accounting for approximately $31.13\%(= \frac{25}{80} \cdot 100\%)$ of the reduction in aggregate risk documented above. In the case of Kanzara, results suggest that the 250% growth in irrigation implies a 63% drop in aggregate risk, accounting for approx. $95\%(= \frac{63}{66} \cdot 100\%)$ of the documented drop in aggregate risk. Finally, the empirical drop in aggregate risk in Shirapur of 66% is somewhat below the reduction of approx. 86.8% implied by the quantitative model.

These findings mean that irrigation constitutes an effective insurance against aggregate risk. In Aurepalle, where the irrigation growth has been modest relative to Kanzara and Shirapur, a large chunk of the residual decline in aggregate risk seems to be due to other factors mentioned in Section 1.5, such as lower crop trade price variation, increased use of fertilizers, pesticides, high yielding crop varieties or improved mechanization. In Kanzara, the much larger increase in irrigation accounts for most of the empirical evidence. Last but not least, the fact that the documented drop in Shirapur's aggregate risk is well below the drop generated by the quantitative model may suggest e.g. that the weather or the crop price risk faced by local farmers has significantly increased between the two waves of the

sub-paths of simulations. In particular, I simulate the model for 10,000 periods and compute this average using discounted sums for 100 short paths of 100 periods length each.

ICRISAT panel.

Interaction between risk sharing and irrigational investments

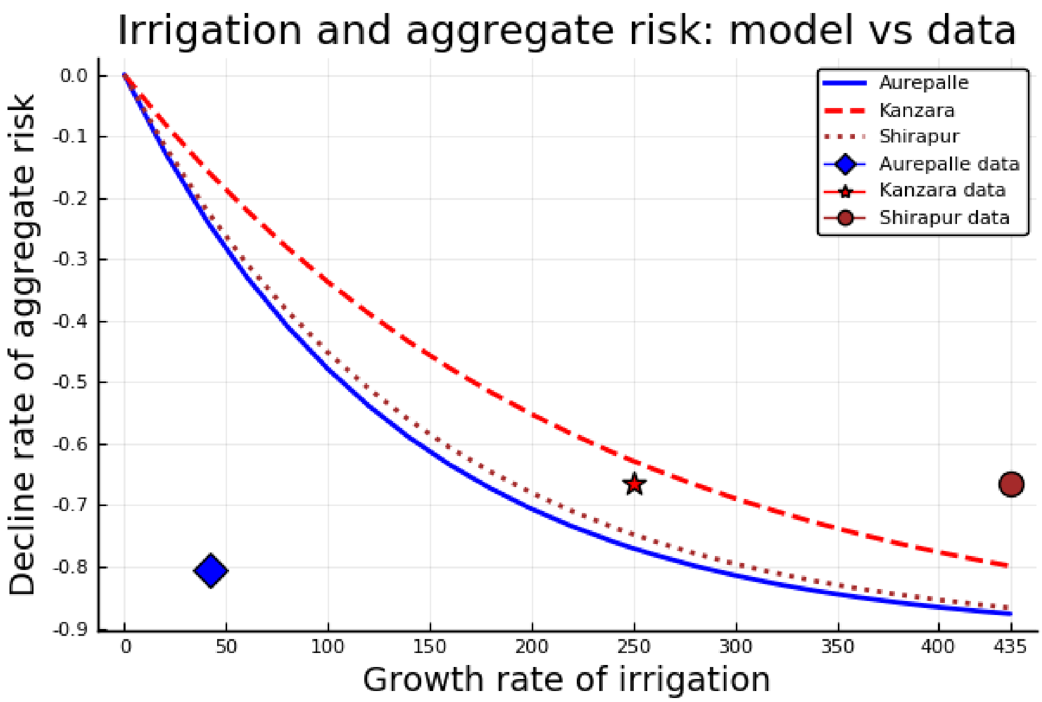
Table 10 presents key welfare statistics from models of co-operation over both risk sharing and irrigation ("I+RS"), irrigation only ("I") and risk sharing only[49] ("RS"), with the same calibrated parameters across the models. In line with the theory presented in Section 1.4, it lends quantitative support to mutual reinforcement between the two institutions of co-operation over risk sharing and irrigational investments.

On one hand, we can see that households co-operating over irrigation are ready to give up between 2.9% (in Kanzara) and 6.2% (in Shirapur) of their consumption in every period in order to maintain co-operation over risk sharing. This welfare improvement comes mostly due to the improved insurance against idiosyncratic shocks. This is manifested by a 63%-73% reduction in the elasticity of consumption (w.r.t. idiosyncratic income shocks) upon introducing risk sharing, depending on the village. This finding confirms that in rural settings informal risk sharing significantly dominates self-insurance.

Moreover, as introducing risk sharing reduces precautionary saving motives, the mean irrigational investment drops by 28%-58%, depending on the village. This implies a slight drop in mean consumption of 0.6%-3%, compensated by a 43%-70% reduction in its overall variance. For the same reasons, the mean aggregate productivity drops by 1%-3.7%, with an associated 5%-30% increase in its variance. In general, the importance of risk sharing for villagers increases with the variance of persistence of idiosyncratic shocks (compare Kanzara with the least volatile and persistent θ-process to the other two villages).

On the other hand, households co-operating over risk sharing are willing to reduce their consumption by 0.05% in Aurepalle and Kanzara, and 1.6% in Shirapur

[49] Notice that the manner of modeling risk sharing co-operation may lead to the lower-bound estimates. It is made for analytical and computational convenience, as by assuming that risk sharing agents do not share irrigation with each other, I would effectively idiosyncratize the aggregate risk of each household. In such a case, the analysis could lead to ambiguous results as it would become difficult to disentangle how much risk sharing against idiosyncratic shocks is due to θ-shocks and how much due to ϕ-shocks. An alternative and computationally tractable approach would involve solving for the risk sharing only allocation by assuming that the aggregate risk weight-probability function for each household takes for the argument average village investments, i.e. assuming that $P\left(\pi^{\phi}=\pi^{G}\right)=P\left(\frac{1}{1-s_k}\frac{1}{N}\sum_i k_{i,t+1},\omega_{t+1}\right)$. This approach could consequently lead to an upper-bound on the true underlying effects.



Note: Figure shows the empirical findings in Table 3 (points) and a relationship between growth rate of irrigation and decline rate of aggregate risk (lines). Changes in aggregate risk are measured as $\Delta \text{var}(\phi; g) = \frac{P\left(\frac{1+g}{1-s_k}\bar{\mathbf{k}},\omega'\right)\left[0.5\cdot\left(\left(\phi_L^G\right)^2+\left(\phi_H^G\right)^2\right)-\left(0.5\cdot\left(\phi_L^G+\phi_H^G\right)\right)^2\right]+\left(1-P\left(\frac{1+g}{1-s_k}\bar{\mathbf{k}},\omega'\right)\right)\left[0.5\cdot\left(\left(\phi_L^B\right)^2+\left(\phi_H^B\right)^2\right)-\left(0.5\cdot\left(\phi_L^B+\phi_H^B\right)\right)^2\right]}{P\left(\frac{1}{1-s_k}\bar{\mathbf{k}},\omega\right)\left[0.5\cdot\left(\left(\phi_L^G\right)^2+\left(\phi_H^G\right)^2\right)-\left(0.5\cdot\left(\phi_L^G+\phi_H^G\right)\right)^2\right]+\left(1-P\left(\frac{1}{1-s_k}\bar{\mathbf{k}},\omega\right)\right)\left[0.5\cdot\left(\left(\phi_L^B\right)^2+\left(\phi_H^B\right)^2\right)-\left(0.5\cdot\left(\phi_L^B+\phi_H^B\right)\right)^2\right]} - 1$, where (i) g is the growth rate of irrigation capital; (ii) ω' is computed using (25) s.t. at every point the government share equals $\hat{GS} \in \{6.65\%, 10.39\%, 1.74\%\}$ in Aurepalle, Kanzara and Shirapur (means over the 3rd and 4th census data in Table 5) with $\omega' = \omega$ at the 0% growth rate of irrigation; (iii) the unconditional probability that the Markov chain is in state L/H equals 0.5 due to the assumption of chains being symmetric; and (iv) $\bar{\mathbf{k}}$ denotes mean of the simulated investment decisions in the baseline model.

Figure 6: Impact of irrigation development on aggregate risk in ICRISAT villages: model vs data

Statistic	Aurepalle			Kanzara			Shirapur		
	I+RS	I	RS	I+RS	I	RS	I+RS	I	RS
Cons.-equiv. welfare	1	0.958	0.995	1	0.971	0.995	1	0.942	0.984
Elasticity of cons.	0.230	0.858	0.287	0.343	0.934	0.368	0.254	0.861	0.401
Mean consumption	0.449	0.461	0.449	0.438	0.441	0.437	0.445	0.459	0.449
Var. of consumption	0.013	0.042	0.015	0.016	0.028	0.017	0.017	0.057	0.027
Mean aggr. prod. ϕ	0.900	0.924	0.900	0.883	0.891	0.882	0.889	0.924	0.896
Var. of aggr. prod. ϕ	0.041	0.033	0.041	0.044	0.042	0.044	0.043	0.033	0.041
Mean irrig. invest. k	0.039	0.084	0.043	0.015	0.021	0.014	0.037	0.088	0.053
Var. of irrig. invest. k	0.0014	0.0063	0.0020	0.0001	0.0003	0.0001	0.001	0.007	0.003

Table 10: Comparison of key statistics across models of various degrees of co-operation.

in every period in order to maintain co-operation over irrigation.[50] In particular, we observe that irrigation co-operation significantly improves informal insurance against idiosyncratic shocks as evidenced by a reduction in the elasticity of consumption of 19.9% in Aurepalle, 6.8% in Kanzara and 36.7% in Shirapur.

In general, irrigation co-operation becomes more important as government involvement gets smaller (3.23% in Shirapur vs 35.33%-37.41% in Aurepalle and Kanzara). However, notice that the exact effect on consumption, aggregate productivity and investment metrics differs between the villages due to the interplay of different government's irrigation ownership shares with the general equilibrium effects working through the limited commitment constraints.

Reducing size of centrally-owned irrigation

Figure 7 presents the evolution of the key welfare statistics[51] in the three villages upon changing the ω parameter from 100% to 0% of its calibrated value. The first and intuitive observation is that the effects of reducing the size of government-owned irrigation become smaller as its initial share declines (with the largest share in Aurepalle, followed by Kanzara and Shirapur).

Secondly, as the centrally-owned irrigation is progressively removed, the variance of aggregate shocks ϕ increases by up to 25% in Aurepalle. Relatedly, the mean value of ϕ declines by up to 6%, followed one-to-one by mean consumption. These welfare-reducing effects are compensated by improvements in risk sharing. In particular, we observe very large reduction in consumption elasticity w.r.t. idiosyncratic income shocks, up to a 66.6% in Aurepalle, 59.3% in Kanzara

[50] See Footnote 49.

[51] These statistics are the same as used in Table 10.

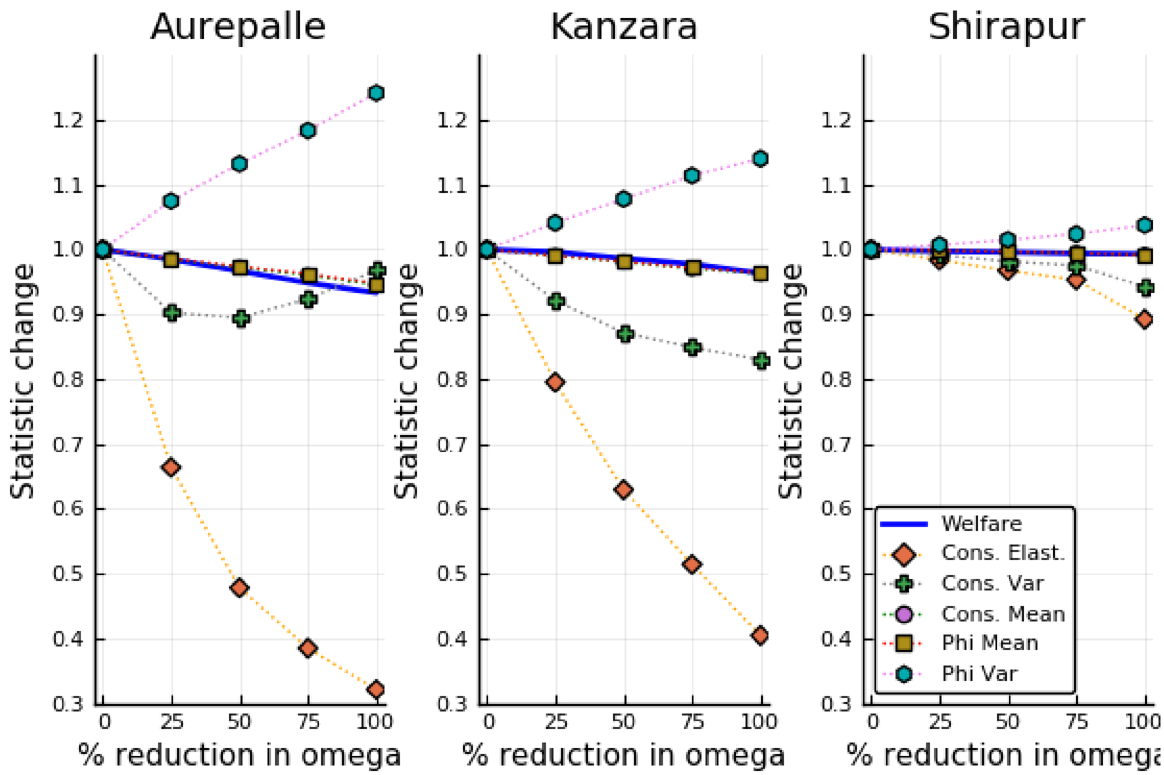


Figure 7: Change in welfare statistics upon reducing ω

and 10.5% in Shirapur. The overall variance of consumption decreases by up to 11.4% in Aurepalle (at a 50% reduction in ω), 16.7% in Kanzara and 5.4% in Shirapur.

Nonetheless, the overall welfare effects are clearly negative, with households in Aurepalle valuing ω at the equivalent of up to 6.7% of consumption in every period, in Kanzara - 3.4% and in Shirapur - 0.7%. These dynamics suggest that although a simple removal of government-owned investments may improve local co-operation in village economies (as measured by risk sharing), such reforms may not be able to generate budget savings without destroying welfare of these societies as the associated losses due to removal of resources are first order.

Shifting resources from centrally- to village-owned irrigation

While improving welfare by simply reducing the size of government-owned irrigation seems impossible, Proposition 2 above says that we can achieve a Pareto improvement by redirecting funds spent on the centrally-owned irrigation to subsidies for privately done investments. Table 11 presents results of conducting such reforms in the three villages, supporting the theory developed above.

Statistic	Aurepalle		Kanzara		Shirapur	
	Status-quo	Optimal	Status-quo	Optimal	Status-quo	Optimal
Policy ω, s_k	2.8; 58.12%	0; 85%	0.75; 10.40%	0; 65%	0.093; 5.44%	0; 15%
Cons.-equiv. welfare	1	1.013	1	1.030	1	1.005
Elasticity of cons.	0.230	0.074	0.326	0.258	0.254	0.230
Mean cons.	0.449	0.452	0.441	0.451	0.438	0.438
Var. of cons.	0.013	0.009	0.016	0.014	0.017	0.016
Mean aggr. prod. ϕ	0.900	0.905	0.879	0.900	0.884	0.884
Var. of aggr. prod. ϕ	0.041	0.039	0.044	0.039	0.437	0.440
Mean irrig. invest. k	0.039	0.028	0.015	0.014	0.036	0.032
Var. of irrig. invest. k	0.0014	0.0005	0.0001	0.00009	0.0013	0.0009

Table 11: Comparison of key statistics upon compensated replacement of government-owned irrigation ω.

In general, the largest welfare gains are attainable in Kanzara, where the status-quo (i) subsidy rate is relatively low, (ii) government share is high, and (iii) the transmission of idiosyncratic shocks to consumption is large. More precisely, Kanzara's villagers are willing to pay up to 2.9% of every period's consumption in order to implement the reform of fully eliminating the government irrigation ω and using the resources released to increase the subsidy rate s_k from 10.40% to 65%. The respective welfare gains in Aurepalle and Shirapur are smaller given higher initial subsidies s_k or lower government involvement there, and amount to 1.3% and 0.5% of consumption in every period.

The welfare improvement takes place as switching to the more efficient local village management of irrigation infrastructure leaves farmers with more resources, and at the same time, improves patterns of local co-operation. To this end, the reform reduces the elasticity of consumption by 67%, 20.9% and 9.4% in Aurepalle, Kanzara and Shirapur (respectively). Notice also that although the reform reduces precautionary saving motives, and so reduces average levels of investment, it directs funds to villagers who have an advantage over government in terms of investment efficiency. Thus, the reform also increases the mean and reduces the variance of aggregate productivity ϕ.

1.7 Conclusion

The economic literature has well-documented the prevalent use of informal insurance contracts in rural areas of developing countries. Since the functioning of these contracts often relies on the idiosyncratic nature of shocks affecting the income of population, their ability to provide insurance against aggregate shocks is very limited. However, given the very high prevalence of agriculture in these areas, villagers can significantly mitigate the effects of aggregate shocks due to erratic rainfall through co-ordinated irrigation investments. This paper studies how these two margins of co-operation interact with each other.

Theoretically, I perform this analysis through the lenses of a limited commitment model characterized by joint determination of risk sharing against idiosyncratic shocks and co-operation over irrigation investments reducing impact of aggregate shocks. The analysis suggests that the two institutions are complementary to each other, if the society can regulate access to the irrigation and as such punish deviators by excluding them from it. In such a case, improved value of irrigation's public good within the co-operation increases the extent of efficient risk sharing. On the other hand, by reducing the degree of the society's idiosyncratic risk, the risk sharing institution improves the allocation of investments. However, if exclusion from irrigation is not feasible, as I argue is the case when irrigational structures are owned and managed by central authorities, improvements in its provision may lead to excessive crowding-out of local risk sharing due to excessive improvements in the value of defaulting on insurance credit promises.

I provide empirical support for the theory developed using the ICRISAT's Village Level Studies panel data set on three South Indian villages. In particular, I find that increases in total size of irrigation through adding new village-owned infrastructures are associated with significant crowding-in of risk sharing (as measured by reductions in consumption elasticity w.r.t. idiosyncratic shocks), and that increases in the government share of villages' irrigation are associated with significant crowding-out.

Building on these findings, I calibrate a quantitative version of the model to the setting of three villages analyzed in order to conduct a number of experiments. First, performing the exercise of accounting for empirically documented changes in aggregate risk faced by the villagers suggests that investments in irrigation constitute a very effective way of insuring villagers against the risk of weather fluctuations. Second, comparing models with various degrees of co-operation confirms significant reinforcing effects between the two institutions of risk sharing and irrigation co-operation. Third, although improving villagers' well-being by a simple reduction in the size of government-owned infrastructure is not possible, shifting resources from the latter to subsidies on investments made by villagers can achieve significant welfare improvements, while holding the size of overall expenditures constant. In particular, such a reform strengthens the within-village co-operation by improving both the informal insurance and the efficiency of irrigation investments.

Given the experimental results shown, policy recommendations may seem obvious. In a way, the evidence from the Minor Irrigation Census (Table 5) shows that such practices of increasing privatization of irrigation systems have already been followed to some extent, at least in the case of villages analyzed here. However, policy makers need to bear in mind that traditional functioning of Indian rural societies built around castes may imply that such reforms may come at the unwanted cost of redistributing welfare gains towards the more privileged castes. Thus, it may be necessary to establish appropriate institutions aimed at facilitating fair access to irrigation. However, it is important to stress that any type of traditional institutions enforcing such fair access may harm the superior local management of irrigation systems. To this end, it could be worthwhile to consider different types of interventions supporting inter-caste co-operation within villages, especially given that Anderson (2011) finds that the potential gains of removing discrimination in access to irrigation are enormous. Findings of this paper shed some light on possible sources of these gains.

In terms of avenues for further research, notice that in the above I have assumed away any type of private information frictions, shown by Ligon (1998) to be potentially relevant in the Indian villages analyzed here.[52] Staying within the context of village economies, the prime example of an investment margin where private information might be especially important is fertilizer use. For analysis of the latter in conjunction with risk sharing, see Pietrobon (2018). In this case, providing more village insurance induces farmers to reduce their private effort and expenses on fertilizers. Thus, it might be worthwhile to investigate this private information friction jointly with village risk sharing and co-operation over irrigation investments. Similarly, given the exposed importance of interaction between the two institutions on allocation of resources in village economies, it may be particularly interesting to investigate the implications of this relationship for growth of rural regions, such as the villages analyzed here.

Finally, the mechanism studied here can be also applied to settings other than rural ones. One example of such an application may be to the case of economic unions such as the EU or the USA co-operating over risk sharing and common goods, such as pollution abatement. This application is already on my research

[52]Similarly, a more recent paper by Ligon and Schechter (2018) shows in the context of Paraguayan villages that considering both frictions at the same time might be important.

agenda.

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1.8 Appendix A: Recursive formulation and solution algorithm

In order to save on complexity of solving for the N-agent co-operation equilibrium, I follow the commonly used approximation of solving for co-operation between one household and a household representing the rest of the village (used e.g. in Ligon et al., 2002; Laczo, 2015; Morten, 2018; Bold and Broer, 2018) extended for public irrigational investments. In particular, I consider a problem of individual i that (i) chooses capital investment $k_{i,t}$ affecting the aggregate risk of all N households; and (ii) co-operates with agent $-i$ representing the rest of the $N-1$ households living in the village. The use of this representative agent, assumed to have the same preferences as all village members and to receive a productivity shock equal to the average across $N-1$ villagers, implicitly assumes that the rest of the village can (i) share the idiosyncratic risk; and (ii) internalize investment externalities as in the first-best allocation. Consequently, the vector of outside options' values is pinned down by consumption of individual and village-average net endowments in every period (given by $\phi_t\left(k_{i,t},\omega_t\right)\cdot\theta_{i,t}-k_{i,t+1}$ and $\phi_t\left(\mathbf{k}_t,\omega_t\right)\cdot\left(\frac{\theta_L+\theta_H}{2}\right)-k_{-i,t+1}$, respectively).

Before describing the numerical algorithm, I show first how to rewrite the model's first order conditions in a recursive way allowing for a numerical solution. Since the steps are similar to the ones in Kehoe and Perri (2002), I skip the lengthy derivations. The only equation that has to be rewritten, is the intertemporal Euler equation. In particular, it can be shown that for the case of irrigation and risk sharing co-operation it is equivalent to (the other cases are adjusted as in Section 1.4):

$$
\begin{aligned}
U_{i,c_t} = & \beta \Bigg[\frac{\mathrm{d}P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega_{t+1}\right)}{\mathrm{d}k_{i,t+1}} \Bigg\{ \sum \pi^G\left(\theta',\phi'|\theta,\phi\right)\left[\frac{V_{i,t+1}}{1-v_{i,t+1}}+\frac{z'_t\cdot V_{-i,t+1}}{1-v_{-i,t+1}}\right] \\
& \qquad - \sum \pi^B\left(\theta',\phi'|\theta,\phi\right)\left[\frac{V_{i,t+1}}{1-v_{i,t+1}}+\frac{z'_t\cdot V_{-i,t+1}}{1-v_{-i,t+1}}\right] \Bigg\} \\
& - \frac{\mathrm{d}P\left(\frac{1}{1-s_k}k_{i,t+1},\omega_{t+1}\right)}{\mathrm{d}k_{i,t+1}} \sum \pi^G\left(\theta',\phi'|\theta,\phi\right) v_{i,t} V^{nc}_{i,t+1} \\
& + \frac{\mathrm{d}P\left(\frac{1}{1-s_k}k_{i,t+1},\omega_{t+1}\right)}{\mathrm{d}k_{i,t+1}} \sum \pi^B\left(\theta',\phi'|\theta,\phi\right) v_{i,t} V^{nc}_{i,t+1} \\
& + P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega_{t+1}\right) \sum \pi^G\left(\theta',\phi'|\theta,\phi\right) U_{i,c_{t+1}} \frac{1-\delta}{1-v_{i,t+1}} \\
& - P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega_{t+1}\right) \sum \pi^G\left(\theta',\phi'|\theta,\phi\right) \frac{v_{i,t+1}}{1-v_{i,t+1}} \frac{\partial V^{nc}_{i,t+1}}{\partial k_{i,t+1}} \\
& + \left(1-P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega_{t+1}\right)\right) \sum \pi^B\left(\theta',\phi'|\theta,\phi\right) U_{i,c_{t+1}} \frac{1-\delta}{1-v_{i,t+1}} \\
& - \left(1-P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega_{t+1}\right)\right) \sum \pi^B\left(\theta',\phi'|\theta,\phi\right) \frac{v_{i,t+1}}{1-v_{i,t+1}} \frac{\partial V^{nc}_{i,t+1}}{\partial k_{i,t+1}} \Bigg]
\end{aligned}
$$

where $z'\left(x^t\right) = \frac{\lambda_{-i}\left(x^t\right)+\mu_{-i,i}\left(x^t\right)}{\lambda_i\left(x^t\right)+\mu_{i,i}\left(x^t\right)}$ and $v_i\left(x^t\right) = \frac{\mu_i\left(x^t\right)}{\lambda_i\left(x^t\right)}$.

Secondly, the approximation method implies that (i) a transfer of $c_{i,t}$ resources for consumption of household i leaves $c_{-i,t} = \left(y_{i,t}+y_{-i,t}-c_{i,t}-k_{-i,t}-k_{i,t}\right)$ for consumption by the household representing the rest of the village; and that (ii) investments $\left(k_{i,t+1},k_{-i,t+1}\right)$ of agents $(i,-i)$ imply

$$
\begin{aligned}
P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1},\omega_{t+1}\right) = & 1-\frac{1}{N}\exp\left(-\nu_k\frac{1}{1-s_k}k_{i,t+1}-\nu_\omega\frac{1}{N}\omega_{t+1}\right) \\
& -\frac{N-1}{N}\exp\left(-\nu_k\frac{1}{1-s_k}k_{-i,t+1}-\nu_\omega\frac{1}{N}\omega_{t+1}\right)
\end{aligned}
$$

.

Given the recursive formulation, the model is solved using a policy function iteration method. Let $x_t = \left(z_t,k_{i,t},k_{-i,t},\phi_t,\{\theta_{i,t}\}\right)$ be the state variable with the understanding that z_t, $k_{i,t}$ and $k_{-i,t}$ are inherited from the previous period. Given this, I define a discrete grid G on the state space. Define also value functions for each agent i:

$$V_{i,t}\left(x_t\right) = u\left(c_{i,t}\left(x_t\right)\right) + \beta\left[P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}, \omega_{t+1}\right)\sum_{s_{t+1}}\pi^G\left(x_{t+1}|x_t\right)V_{i,t}\left(x_{t+1}\right)\right.$$
$$\left. + \left(1 - P\left(\frac{1}{1-s_k}\mathbf{k}_{t+1}, \omega_{t+1}\right)\right)\sum_{s_{t+1}}\pi^B\left(x_{t+1}|x_t\right)V_{i,t}\left(x_{t+1}\right)\right]$$

I proceed by guessing policy functions $\left\{c_i^0(g), k_i^0(g), z'^0(g), v_i^0(g), V_i^0(g)\right\}$ $\forall g \in G$. Given these, I update the guess in the following way. Suppose we are on the n-th iteration of updating the vector of unknown functions, and suppose we are at point q in the grid. First assume neither enforcement constraint binds. Thus, immediately $v_i(q) = 0$ and $z'^0(q) = z$. (At the same time, this assumption corresponds to solving for the first best allocation.) Given the guesses, we iterate over the Euler equations of both agents over all the grid points with consumption shared according to z.[53] This gives an update of the consumption policy function for both agents. We compute consumption policy of the rest of the village as a residual from the resource constraint. Given these updated policy functions, I update the value functions and repeat the algorithm until the optimal consumption policy function converges.[54]

Now, we check whether any of the limited commitment constraints binds. If not, we proceed to the next grid point. Otherwise, I will proceed by assuming that only one of the enforcement constraints binds, i.e. either that of the household or of the rest of the village.

If e.g. the limited commitment constraint of agent i binds, we are solving for $\left\{c_i(g), k_i(g), z'(g), v_1^0(g)\right\}$ $\forall g \in G$. First, we use the old guesses of c and k' in order to compute total wealth and total consumption. Then, we use the binding enforcement constraint (still with old guesses) to solve for new relative Pareto weight z' (and so also v) at the given grid point. Given the new relative Pareto weight, we can solve for c_i and c_{-i} using the consumption sharing rule.

Then, we use the Euler equations of both agents to solve for investment deci-

[53]In particular, when solving for optimal decisions of agent i, I use his Euler equation with old guess of agent $-i$'s decisions in order to solve for optimal investment (and, by resource constraint and given the z, consumption) of agent i. Then, I proceed accordingly using the Euler equation of agent $-i$.

[54]Notice also, that in order to solve for the value of autarky it is enough to solve the model using above described algorithm and (i) imposing separate budget constraints, and (ii) assuming that agents ignore their externalities on others.

sions at the given grid point. However, since now $v_i(g) > 0$ (binding enforcement constraint), we need to take care of the derivatives of autarky value functions in period $t+1$ w.r.t. current capital. To arrive at the latter, differentiate the value function of each agent in autarky in period t w.r.t. k, use the envelope condition (guaranteeing that $\frac{\partial k_{i,t+1}}{\partial k_{i,t}} = 0$) and iterate one period forward. E.g. in case of agent $i$we arrive at:

$$\frac{\partial V^{nc}_{i,t+1}(\tilde{x}_{t+1})}{\partial k^{coop}_{i,t+1}} = \left(c^{nc}_{i,t+1}\right)^{-\sigma}\left(\phi_{t+1}\left(k^{coop}_{i,t+1}, \omega_{t+1}\right)\theta_{i,t+1} + (1-\delta)\right)$$

With these derivatives and old guesses in hand, we use agents' Euler equations to solve for new optimal investment decisions (similar to above). We check the sup-norm of the change in the consumption policy rule, and keep updating until the latter is very small.

I finalize the approximation by constructing the panel of village consumption and investment data. In order to do so, I simulate the model N times for long enough where in each period tI proceed according to the following steps:

1. I assign the optimal new multiplier $z^{\prime,approx}_{i,t} = z^{\prime}_t(x_{i,t})\ \forall i \in \{1, ..., N\}$, where state is given by $x_{i,t} = \left(\phi_t, \theta_{i,t}, \frac{1}{2}(\theta_L + \theta_H), k^{approx}_{i,t}, k^{approx}_{-i,t}, z^{approx}_{i,t}\right)\ \forall i \in \{1, ..., N\}$

2. I then compute approximate consumption rules $c^{approx}_{i,t}(x_{i,t})\ \forall i \in \{1, ..., N\}$ according to:[55]

$$c^{approx}_{i,t}(x_{i,t}) = c_{tot,t} \cdot \frac{\left[\frac{\frac{1}{z_{i,t}}\left(1+\tilde{\mu}_{i,t}\right)}{\sum_{j\neq i}\frac{1}{z_{j,t}}\left(1+\tilde{\mu}_{j,t}\right)}\right]^{\frac{1}{\sigma}}}{1+\left[\frac{\frac{1}{z_{i,t}}\left(1+\tilde{\mu}_{i,t}\right)}{\sum_{j\neq i}\frac{1}{z_{j,t}}\left(1+\tilde{\mu}_{j,t}\right)}\right]^{\frac{1}{\sigma}}} \tag{28}$$

   where (i) I use the duality between the Promised Utility and Marcet and Marimon's multiplier approach implying that $z' = z \cdot \frac{1+\tilde{\mu}_{-i}}{1+\tilde{\mu}_i}$ (with $\tilde{\mu}$ being the Lagrange multiplier on the promise keeping constraint in the dual problem); and (ii) the total consumption is derived from the resource constraint and is given by $c_{tot,t} = \phi_t\left(\theta_{i,t} + \frac{N-1}{2}[\theta_H + \theta_L]\right) + (1-\delta)\left(k^{approx}_{i,t} + (N-1)k^{approx}_{-i,t}\right) - \left(k^{approx}_{i,t+1} + (N-1)k^{approx}_{-i,t+1}\right)$.

[55]See equation (20) in Ligon et al. (2002) and Bold and Broer (2018, p. 8-9) for details on deriving (28).

3. I assign investment rules $k^{approx}_{i,t+1}(x_{i,t})\ \forall i \in \{1,...,N\}$ using the investment rules from the *exact* solution for a given state $x_{i,t}$ and multiplier $z'_{i,t}(x_{i,t})$. Given these, I impute the approximate investments by the rest of the village as $k^{approx}_{-i,t+1}(x_{-i,t}) = \frac{1}{N-1}\sum_{j\neq i} k^{approx}_{j,t+1}(x_{j,t})\ \forall i \in \{1,...,N\}$.

4. Having computed all the objects of interest, I proceed to the next period $t+1$ with the state given by $x_{i,t+1} = \left(\phi_{t+1}, \theta_{i,t+1}, \frac{1}{2}(\theta_L + \theta_H), k^{approx}_{i,t+1}, k^{approx}_{-i,t+1}, z^{approx}_{i,t+1}\right)$ $\forall i \in \{1,...,N\}$, where:

   (a) $k^{approx}_{-i,t+1} = \frac{1}{N-1}\sum_{j\neq i} k^{approx}_{j,t+1}$,

   (b) $z^{approx}_{i,t+1} = z^{\prime,approx}_{i,t}$,

   (c) ϕ_{t+1} is drawn from the good distribution with probability $P\left(\frac{1}{1-s_k}\mathbf{k}^{approx}_{t+1}, \omega_{t+1}\right)$ and with $1 - P\left(\frac{1}{1-s_k}\mathbf{k}^{approx}_{t+1}, \omega_{t+1}\right)$ from the bad one.

1.9 Appendix B: Estimation of irrigation subsidies

The following data comes from the 3rd Minor Irrigation Census conducted in the years 2000-2001. The data on sources of financing is unfortunately not available in earlier censuses and thus I assume that the subsidy rates estimated using the 2000-2001 census hold in the first wave of the ICRISAT panel.

The subsidies are computed using the following formula:

$$sub_v = \sum_i share\ total_{i,v} \cdot share\ subsidized_{i,v} \cdot \frac{irrigation\ area_{i,v}}{\sum_j irrigation\ area_{j,v}} \tag{29}$$

where (i) sub_v denotes subsidy for private investments in village v, (ii) i is the index for types of irrigation units (as in the construction of the data in Table 5), (iii) $share\ total_i$ stands for the share of village-owned units in the total number of irrigation units in village v; (iv) $share\ subsidized_i$ is the share of the subsidy financed units in the total number of subsidy, private and loan financed units; and (v) the "irrigation area" metric is the same as the one for "irrigable-" and "irrigated land" (as in construction of the data in Table 5).

There are two important caveats to the construction of this variable. First, some of the irrigation units recorded in the Minor Irrigation Census are financed by "some loan and some subsidy". In such a case, I assume that half of such units have been financed by loan (private sources) and the other half by subsidy. Second, the census does not contain information on sources of finance for

Village (district)	metric	Subsidy rate
Aurepalle (Mahbubnagar)	irrigable	54.21%
	irrigated	62.04%
Kanzara (Akola)	irrigable	9.04%
	irrigated	11.70%
Shirapur (Solapur)	irrigable	5.56%
	irrigated	5.32%

Table 12: Subsidy rates on village-owned irrigation in ICRISAT villages

Note: Table presents the subsidy rates of irrigation structures in districts of Mahbubnagar, Akola and Solapur in the third (2000-2001) Minor Irrigation Censuses. The irrigable metric is defined as Culturable Command Area ("The area which can be irrigated from a scheme and is fit for cultivation."); and the irrigated metric is defined as Irrigation Potential Utilised ("The gross area actually irrigated during reference year out of the gross proposed area to be irrigated by the scheme during the year.") - see http://micensus.gov.in for more details.

village-owned units *separately* (i.e. it only contains information on sources of finance for the aggregate number of units). Thus, I impute the latter as $share\ total_{i,v} \cdot share\ subsidized_{i,v}$.

For calibration, I assume the mean values of irrigated- and irrigable land subsidy rate estimates (see Table 8).

1.10 Appendix C: Impact of aggregate shocks on income in ICRISAT villages

Figures 6 presents the evidence from Morten (2018) on the impact of weather shocks on households' log income. Data used comes from the first (1975-1984) wave of the ICRISAT panel and the University of Delaware precipitation database. Negative aggregate weather shock is defined as a rainfall event falling below the 20th percentile of the 1900-2008 long-run rainfall distribution.

Dep. variable: Log Income	(1) b/se	(2) b/se	(3) b/se	(4) b/se
Number days monsoon late	-0.009***			
	(0.001)			
Bottom 10% shock		-0.923***		
		(0.103)		
Bottom 20% shock			-0.231***	
			(0.064)	
Bottom 50% shock				-0.104**
				(0.050)
Household FE	Yes	No	No	No
Long run prob. shock		0.14	0.28	0.49
R-squared	0.606	0.625	0.591	0.586
Number observations	931	931	931	931

Notes: OLS regressions using VLS1 (1975-1984). Rainfall shocks computed using the distribution of rainfall 1900-2008 from the University of Delaware precipitation database, and these thresholds applied to the ICRISAT colllected rainfall for 1975-1984. Monsoon start date is computed as the first day with more than 20 mm of rain after June 1, following Rosenzweig and Binswanger (1993).

Figure 8: Impact of aggregate weather shocks on income (from Morten, 2018)

1.11 Appendix D: Additional policy functions

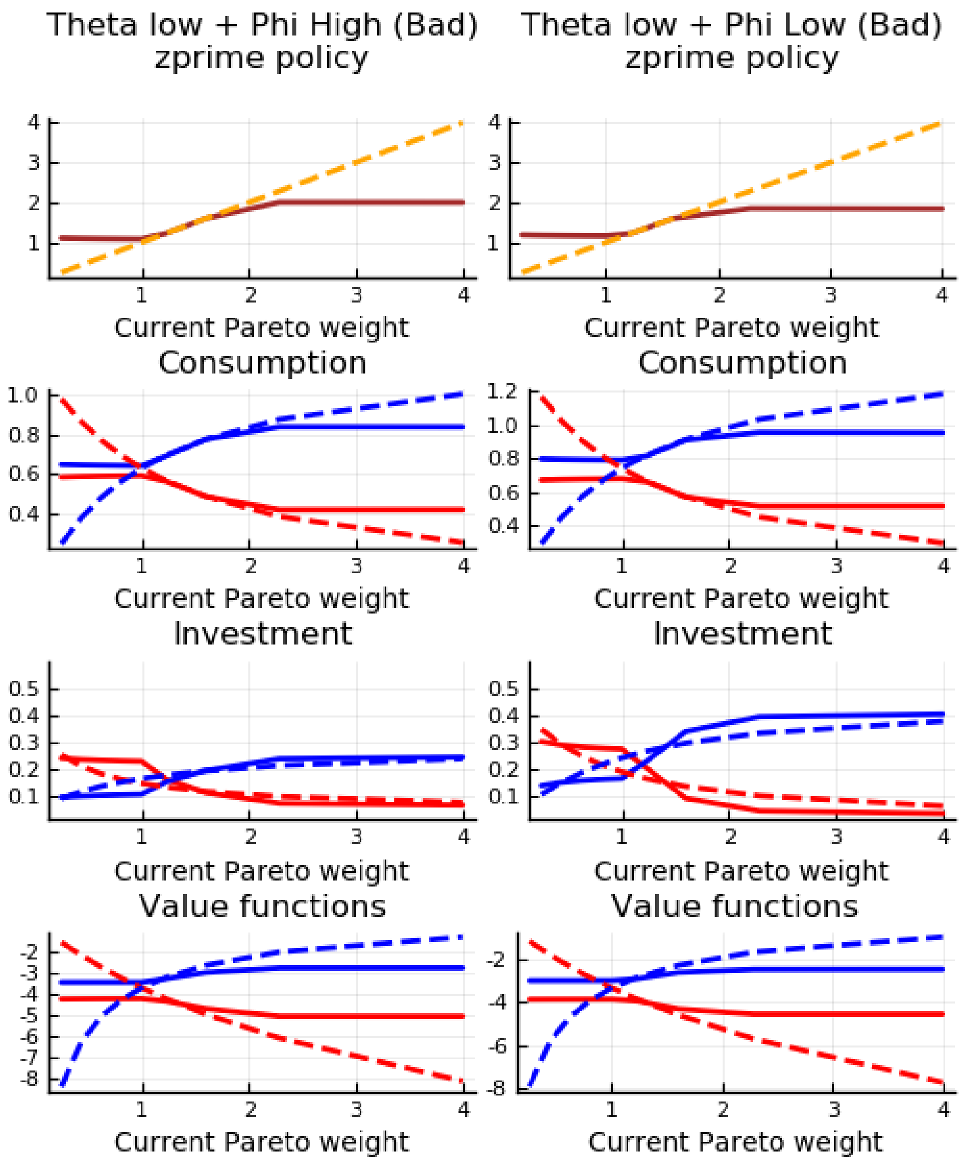

Figure 9: Optimal policy functions: household i vs household representing the rest of the village

Note: Red plots represent optimal policy functions of household i and blue ones - of the household representing the rest of the village. Solid plots stand for optimal policies in the first best model, and dashed ones - for the ones in the model with limited commitment.

2 Student Loans with Risky Graduation and College Wage Premium

2.1 Introduction

Over the past decades, student loan programs have become an ever more important means of financing higher education in the US, with the level of outstanding student debt in 2019 reaching USD 1.6 trillion (making it the second largest debt category in the US after house mortgages).[56][57] To large extent, the latter number reflects very high overall costs of pursuing higher education in the US. For this reason, policy makers see the student loan programs as tools allowing for increasing equality of opportunity for students from disadvantaged backgrounds.[58] At the same time however, enrolling into a college may be very risky: Long (2018) reports that the overall completion rate of full-time students in four- and two-year institutions in the year 2016 amounted to only 49.1% and 38.6%, respectively. In light of such high drop out rates, a natural question is what are the distributional and efficiency consequences of student loans, do they work as intended and, if not, how could they be optimally designed? Investigating these issues is at the center of this paper.

I begin by investigating theoretical properties of a dynamic human capital investment model with ex-ante homogenous students exerting private effort in order to increase their probability of graduating from college. The latter event cannot be insured and as such generates market incompleteness. In other words, students are subject to uninsurable college graduation shocks, which result in two types of workers: those with college- and those with high school education. Moreover, these two types of workers are assumed to be imperfectly substitutable when employed by firms (in line with the evidence in Katz and Murphy, 1992, Ciccone and Peri, 2005 and Malmberg, 2018), giving rise to endogenous college wage premium. I first study constrained efficiency of educational choices in the competitive allocation, with the constraint on efficiency coming from miss-

[56] According to the Federal Reserve data:
https://www.federalreserve.gov/releases/g19/HIST/cc_hist_memo_levels.html

[57] The idea of income-contingent student loans has arguably been first proposed and discussed by Milton Friedman (1955).

[58] At the same time however, around 63% of the outstanding debt is held by students with parental income in the third and fourth highest quartiles. This is according to calculations of the Urban Institute using the Survey of Consumer Finances: https://www.urban.org/urban-wire/which-households-hold-most-student-debt

ing insurance markets for graduation shocks.[59] Given the latter and the presence of general equilibrium effects working through the college wage premium, the laissez-faire decisions turn out to be constrained inefficient and characterized by under-education. In particular, had every student exerted more effort, the college wage premium would become smaller and as such provide better (from the ex-ante point of view) insurance for everyone. Thus, the model endogenously generates motives for some form of government intervention.

As mentioned above, one form of such government intervention in the US is the provision of income-contingent student loans. In line with observed policies, I show that an appropriately designed student loan program which is self-financed and provides insurance against the risk of college drop-out, can achieve the allocative efficiency of a second best benchmark where the insurance markets are complete. Importantly, this implementation has to account for the associated efficiency-equity trade-off generated by moral hazard due to the educational decisions being made at a privately optimal level. In particular, on the one hand insurance against college failure improves consumption smoothing across their lifetimes, but on the other it destroys incentives for students to invest more effort in their education.

Related literature. The analysis in this paper relies on an extended (by the dynamic and risky human capital accumulation decision) version of the "canonical model" in Goldin and Katz (2008). This extension gives rise to the general equilibrium effects associated with college wage premium similar to the ones demonstrated by Stiglitz (1982). The theoretical novelty of this paper is its studying the interaction of these effects with incomplete markets.

The latter combination implies that educational decisions in competitive equilibrium are constrained inefficient and are characterized by under-investment. This finding is related to Davila et al. (2012) who study a neoclassical growth model rendering capital investment decisions constrained inefficient due to the general equilibrium effects (or pecuniary externalities, as they called them) associated with the interest rate (which is taken as given by agents). Moreover, I abstract from physical capital in order to clearly expose another important source of externalities due to the students' wage taking behavior. Furthermore, the interaction between education and capital accumulation for constrained efficiency of competitive equilibria has been investigated by Gottardi, Kajii and Nakajima

[59] See Diamond (1967) for seminal work on the constrained efficiency approach.

(2015) and Park (2018).[60] Both of these papers study a version of the Davila et al. (2012) environment with risky human capital accumulation. Differently to the approach here, in their frameworks workers are perfectly substitutable between the college/high-skilled and non-college/low-skilled sectors. By analyzing the sectoral composition of workers, I characterize a novel channel of inefficiencies.

The result of under-education is in line with broad empirical literature (for an early contribution see e.g. Friedman and Kuznets, 1954) documenting that returns to human capital are excessively high as compared to the returns on physical capital, arguably for the precise reason that capital market imperfections limit investments in education. While comparison of human and physical capital investments is not at the center of the analysis, I show that some of the risk associated with human capital is endogenous to aggregate educational decisions and as such can be mitigated by 'collective' actions benefitting everyone, without introducing any direct forms of insurance.

Building on these theoretical findings, I show that the second best allocation can be implemented with an income-contingent student loan system.[61] To this end, Gary-Bobo and Trannoy (2015) and Findeisen and Sachs (2016) both study dynamic environments of one-shot risky college education where students possess private information about their ability and effort exerted. They show that a Pareto optimal allocation can be implemented using an integrated tax and student loan system with income contingent repayment rates. Similarly, Stantcheva (2017) studies optimal policies with human capital accumulation over the whole life-cycle of workers and also finds that the second-best allocations can be implemented with the combination of income-tax enhanced by income-contingent loans. The approach in this paper differs due to the presence of demand for skills leading to endogenous skill premium.

The second best implementation with student loans is naturally related to the literature studying borrowing constraints in the context of education. To this end, Lochner and Monge-Naranjo (2012) review this literature and conclude that in recent years credit constraints have become an important determinant of educational outcomes among the youth. On the other hand, Keane and Wolpin (2001) and Johnson (2013) estimate structural models of higher education and find that

[60] See also Mayr (2018) for analysis of optimal policy in an environment with entrepreneurial talent where competitive equilibrium is similarly constrained inefficient.

[61] See Lochner and Monge-Naranjo (2016) for a recent review of the literature with some optimal policy recommendations. Their work ignores the issues related to the endogeneity of the college wage premium analyzed in this paper.

relaxing borrowing constraints has only a modest impact on educational attainment. The latter findings can be very well generated by the uninsured risk associated with pursuing college education. In this paper, I analyze the insurance role of income-contingent student loans.

Furthermore, there is a broader literature studying tax and subsidy policies in relation to education and its impact on earnings inequalities, see e.g. Abbott, Gallipoli, Meghir and Violante (2018), Benabou (2002), Bovenberg and Jacobs (2005), Heathcote, Storesletten, and Violante (2017), Krueger and Ludwig (2013, 2016) and Vardishvili and Wang (2019). Most of these papers allow for general equilibrium effects of government policies on relative factor prices. In this paper, I take a closer look at the general equilibrium impact on college wage premium and the associated insurance role of student loans.

Structure. This paper is organized as follows. In the next section, I outline the two period model with ex-ante homogenous agents. In Section 2.3, I define and characterize the competitive equilibrium. In Section 2.4, I show how to decentralize the constrained efficient benchmark using a student loan system characterized by income contingent repayments. Section 2.5 concludes. Appendix B develops the quantitative model with the associated calibration strategy being outlined in Appendix C.

2.2 Model economy

The economy has a continuum of ex-ante identical risk averse student-workers with a unit measure. Each agent is maximizing her expected utility. They live for two periods, value consumption and dislike educational effort according to functions $u(c)$ and $v(e)$ with both satisfying $u'(c), -u''(c), v'(e), v''(e) > 0$ and being continuous.

In the first period, agents have just finished high school and face a single decision on whether to pursue higher education and how much effort to exert on this. In order to do so, they need to (i) cover the tuition fee F, for which they need to borrow against their future income; and (ii) exert educational effort e improving their probability of graduating. More precisely, the human capital accumulation process depends on effort in the following way:

$$E[\theta_i w_i] = \begin{cases} \theta_H w_H & \textit{with probability } p(e) \\ \theta_L w_L & \textit{with probability } 1 - p(e) \end{cases} \tag{30}$$

where θ_H, θ_L are the skill levels associated with each level of education, $p(e)$ is the graduation probability function satisfying $p'(e),\ -p''(e) > 0$; and w_H, w_L are the wage rates offered by competitive firms (see next section).

Education shock is realized at the beginning of the second period. This pins down workers' income, which they use to consume and repay their student debt together with interest accruing at the exogenously given rate of r. Normalizing time preference to 1, agents wish to maximize the following quantity:

$$-v(e) + p(e)\, u(c_H) + (1 - p(e))\, u(c_L)$$

Subject to the budget constraint of:

$$c_i + (1+r)\, F \cdot 1_{e>0} \leq \theta_i w_i \ \forall i \in \{H, L\} \tag{31}$$

where $1_{e>0}$ is the indicator function taking value equal to 1 if the student decides to go to college, i.e. when the effort supplied is positive. Moreover, the setup implicitly assumes that the borrowing constraints are such that agents can always finance their tuition fee F.

Finally, notice the assumed incomplete structure of capital markets. In particular, students do not have access to any instruments that would allow them to hedge the college graduation risk. One example of such an instrument could be a long-term contract between a student and a firm making their income conditional on their graduation outcome. With this in mind, the assumed market incompleteness can be micro-founded by limited commitment on students' side. In particular, in a world with such long-term contracts, any student after successful graduation would have incentives to default on their contract and go to work in another firm offering a market wage rate. This mechanism may well lead to collapse of such markets in equilibrium.

Production sector

The production sector is perfectly competitive. Moreover, I consider equilibria where all firms are identical, and so I study the problem of a representative firm. Importantly, in order to produce, firms need to employ both types of the high- ($\hat{N}_H$) and low-skilled ($\hat{N}_L$) workers. Strictly speaking, the elasticity of substitution between the college and non-college workers is assumed to be less than infinity.

In particular, firms produce using the technology $Y = \hat{N_H}^{\rho}\hat{N_L}^{1-\rho}$.[62]

The objective of firm managers is to maximize their profits given by $\Pi = \hat{N_H}^{\rho}\hat{N_L}^{1-\rho} - \hat{N}_H w_H - \hat{N}_L w_L$, where the wage rates are given by the marginal products of the relevant worker type.

2.3 Competitive equilibrium

In this Section, I first define the competitive equilibrium, discuss the associated dynamics and then discuss its welfare properties. In particular, I show that the competitive equilibrium is constrained inefficient.

2.3.1 Dynamics of the competitive equilibrium

I now formally define the relevant competitive equilibrium concept:

Definition 1. Given the economy in Section 2, the associated competitive equilibrium with rational expectations is defined as decisions (c_H, c_L, e) and prices (w_H, w_L) such that:

1. Agents maximize their expected life-time utility:

$$\max_{e,c_H,c_L} -v(e) + p(e)\,u(c_H) + (1 - p(e))\,u(c_L) \tag{32}$$

   subject to budget constraints:

$$c_i + (1+r)\,F \cdot 1_{e>0} \leq \theta_i w_i \;\forall i \in \{H, L\} \tag{33}$$

2. Firms maximize their profits while remunerating workers at the rate of their marginal product: $\max_{\hat{N}_H,\hat{N}_L} \hat{N_H}^{\rho}\hat{N_L}^{1-\rho} - w_H\hat{N}_H - w_L\hat{N}_L$.

3. The labor market clears: $\hat{N}_H = p(e)\,\theta_H$ and $\hat{N}_L = \theta_L(1 - p(e))$.

Since the problem is nicely concave, typically the ensuing equilibrium will be unique. In order to analyze issues related to student borrowing, let us assume that:

[62]The Cobb-Douglas assumption is without loss of generality and is made to simplify analysis and exposition. Strictly speaking, it implies that the intertemporal elasticity of substitution between worker types equals 1, not far from empirical estimates in Katz and Murphy (1992), Ciccone and Peri (2005) and Malmberg (2018) ranging between 1.3 and 1.5.

Assumption 1. The cost of attending college F is such that in equilibrium effort supplied is positive, i.e. $e > 0$.

Now, the equilibrium dynamics are characterized by the following first order condition:

$$e: \; v'(e) = p'(e)\left(u(c_H) - u(c_L)\right) \tag{34}$$

This equation equalizes the marginal cost of educational investment with the expected benefit. As such, it also reveals the associated moral hazard problem: too much of any potential insurance offered to students against graduation shocks destroys their incentives for exerting educational effort.

As far as the firm sector is concerned, the wage rates w_H, w_L are given by:

$$w_H = MPL_{\hat{N}_H} = \rho\left[p(e)\,\theta_H\right]^{\rho-1}\left[\theta_L\left(1-p(e)\right)\right]^{1-\rho} \tag{35}$$

$$w_L = MPL_{\hat{N}_L} = (1-\rho)\left[p(e)\,\theta_H\right]^{\rho}\left[\theta_L\left(1-p(e)\right)\right]^{-\rho} \tag{36}$$

Notice that since the graduation probability is a smooth function, in equilibrium every student decides to go to college and chooses the same level of educational effort $e > 0$.

2.3.2 Constrained inefficiency of the competitive equilibrium

In this section, I show that the competitive equilibrium is generally inefficient. To focus the discussion, I define the relevant concept of constrained efficiency following Davila et al. (2012).

Definition 2. The competitive equilibrium (c_H, c_L, e, w_L, w_H) is said to be *constrained efficient* if there exists no decision $\hat{e}$ and the implied equilibrium wages $(\hat{w_L}, \hat{w_H})$ such that:

$$\begin{aligned} &-v(\hat{e}) + p(\hat{e})\,u\left(-(1+r)F + \theta_H \hat{w_H}\right) + (1-p(\hat{e}))\,u\left(-(1+r)F + \theta_L \hat{w_L}\right) \\ &> -v(e) + p(e)\,u\left(-(1+r)F + \theta_H w_H\right) + (1-p(e))\,u\left(-(1+r)F + \theta_L w_L\right) \end{aligned}$$

In other words, the equilibrium is said to be constrained efficient if there is no level of educational effort $\hat{e}$ such that, given the associated competitive wages, the expected life-time utility of agents is higher than under competitive equilibrium. In particular, the constraint on efficiency is due to a lack of well functioning capital markets (or, equivalently, due to the planner having access to no redistributive instruments).

In order to show the constrained inefficiency of the competitive equilibrium, consider the impact of a small variation de of the level of educational effort exerted by everyone. Differentiating the objective function (32), I obtain:

$$\mathrm{d}U = -v'(e)\,\mathrm{d}e + p'(e)\left(u(c_H) - u(c_L)\right)\mathrm{d}e + p(e)\,u'(c_H)\,\mathrm{d}c_H + (1 - p(e))\,u'(c_L)\,\mathrm{d}c_L \tag{37}$$

where $\mathrm{d}c_H = \theta_H \mathrm{d}w_H$ and $\mathrm{d}c_L = \theta_L \mathrm{d}w_L$. The effects of an increase in educational effort e on the wage rates w_H, w_L are:

$$\frac{\mathrm{d}w_H}{\mathrm{d}e} = \rho(\rho - 1)\left[p(e)\,\theta_H\right]^{\rho-2}\left[(1 - p(e))\,\theta_L\right]^{-\rho} p'(e)\,\theta_H < 0 \tag{38}$$

$$\frac{\mathrm{d}w_L}{\mathrm{d}e} = \rho(1 - \rho)\left[p(e)\,\theta_H\right]^{\rho-1}\left[(1 - p(I))\,\theta_L\right]^{-\rho-1} \theta_H p'(e) > 0 \tag{39}$$

These dynamics reflect the production technology's imperfect substitution between the college and non-college workers. As higher supply of the high-skilled workers implies lower supply of the low-skilled, the prices for both groups have to adjust in the direction reflecting changes in supply and demand.

Using the individual FOC (34) in (37), I arrive at:

$$\mathrm{d}U = p(e)\,u'(c_H)\,\theta_H \mathrm{d}w_H + (1 - p(e))\,u'(c_L)\,\theta_L \mathrm{d}w_L \tag{40}$$

If (40) turns out to be different from zero, the competitive equilibrium is constrained inefficient. The steps below confirm that it is indeed strictly greater than zero:

$$p(e)\,u'(c_H)\,\theta_H \frac{\mathrm{d}w_H}{\mathrm{d}e} > -(1-p(e))\,u'(c_L)\,\theta_L \frac{\mathrm{d}w_L}{\mathrm{d}e}$$

$$\Longleftrightarrow$$

$$\frac{-p(e)}{1-p(e)} \cdot \frac{u'(c_H)}{u'(c_L)} > \frac{\theta_L \frac{\mathrm{d}w_L}{\mathrm{d}e}}{\theta_H \frac{\mathrm{d}w_H}{\mathrm{d}e}}$$

$$\Longleftrightarrow$$

$$\frac{p(e)}{1-p(e)} \cdot \frac{u'(c_H)}{u'(c_L)} < \frac{p(e)}{1-p(e)}$$

$$\Longleftrightarrow$$

$$\frac{u'(c_H)}{u'(c_L)} < 1 \tag{41}$$

where in the second step I have used wage setting conditions in both sectors.

Clearly, as the marginal utility of the high types is necessarily always lower than that one of the low types, the inequality (41) holds always true. This means that:

Theorem 1. *The competitive allocation is constrained inefficient and is characterized by under-education.*

In other words, due to the pecuniary externalities, social welfare can be enhanced by a uniform increase in educational investment. Intuitively, by reducing the gap between $\theta_H w_H$ and $\theta_L w_L$, a uniform increase in educational effort provides better insurance against the risk of failing to graduate.

Interestingly, as the initial wealth goes to infinity and the wage share in the second period's income becomes marginal (as does the uninsurable graduation risk), I have $\lim_{\omega\to\infty} \frac{u'(c_H)}{u'(c_L)} = 1$ implying that in such case the competitive allocation converges to constrained efficiency.

Finally, it is important to stress again that the constrained inefficiency arises due to the lack of long-term financial contracts between students and firms, e.g. due to human capital not constituting a good form of collateral. Indeed, applying insights of Prescott and Townsend (1984) to this environment suggests that if students were able to sign such binding and observable contracts, the constrained efficiency of the competitive equilibrium would be restored.[63]

[63]Moreover, the above analysis has ignored the possibility of self-insurance through adjust-

2.4 Second best allocation and student loan programs

In this section, I first study properties of the second best allocation where planner (i) can redistribute resources across agents; (ii) internalizes the endogenous response of the college wage premium, and (iii) has to respect students' incentive compatibility constraints. After this, I show that the second best benchmark can be implemented as a competitive equilibrium with an optimally designed program of student loans with graduation-contingent repayment rates.

2.4.1 Second best benchmark

The planning problem just described reads:

Problem 1. Second best allocation is a solution to:

$$\max_{c_H,c_L,e} -v\left(e\right)+p\left(e\right)u\left(c_H\right)+\left(1-p\left(e\right)\right)u\left(c_L\right) \tag{42}$$

s.t.

$$\begin{aligned} p\left(e\right)c_H+\left(1-p\left(e\right)\right)c_L &\leq p\left(e\right)\theta_H w_H+\left(1-p\left(e\right)\right)\theta_L w_L-\left(1+r\right)F \\ e &= \text{argmax}_{\tilde{e}}\left\{-v\left(\tilde{e}\right)\ +p\left(\tilde{e}\right)u\left(c_H\right)+\left(1-p\left(\tilde{e}\right)\right)u\left(c_L\right)\right\} \end{aligned}$$

It is well understood that the second constraint corresponds to a continuum of inequality constraints (Rogerson, 1985). However, if Problem 1 is concave, I can replace it by a necessary and sufficient first order condition:

$$v'\left(e\right)\geq p'\left(e\right)\left(u\left(c_H\right)-u\left(c_L\right)\right) \tag{43}$$

which will hold with equality whenever $e>0$ (warranted by Assumption 1). It is easy to see the sufficiency of (43) since:

$$-v''\left(e\right)+p''\left(e\right)\left(u\left(c_H\right)-u\left(c_L\right)\right)\leq 0\ \forall e \tag{44}$$

This is due to (i) the effort disutility $v\left(e\right)$ and graduation probability $p\left(e\right)$ functions being strictly convex and concave (respectively), and (ii) the requirement

ments in labour hours provided. In principle, this mechanism could improve consumption smoothing at the cost of worse labor supply smoothing. Nonetheless, the empirical evidence in Appendix B shows that the college wage premium is large enough suggesting that the drop-out risk remains empirically large. Therefore, considering endogenous labor supply should not alter the qualitative nature of any of the conclusions.

that the equilibrium effort choice has to be individually rational, i.e. in equilibrium $c_H \geq c_L$ has to hold. Given that the first order approach is valid, the following holds:

Proposition 4. *The second best benchmark is characterized by the following consumption sharing and effort assignment conditions:*

$$\frac{u'(c_H)}{u'(c_L)} = \frac{p(e)}{1-p(e)} \cdot \frac{1-p(e)-\psi p'(e)}{p(e)+\psi p'(e)} \tag{45}$$

$$\mu \begin{bmatrix} p(e)\,\theta_H \frac{\partial w_H}{\partial e} + (1-p(e))\,\theta_L \frac{\partial w_L}{\partial e} \\ +p'(e)\left[(\theta_H w_H - c_H) - (\theta_L w_L - c_L)\right] \end{bmatrix} = -\psi\left[v''(e) - p''(e)\left(u(c_H) - u(c_L)\right)\right] \tag{46}$$

where ψ and μ denote the Lagrange multipliers on the incentive compatibility and resource constraints, respectively.

Proof. See Appendix A. □

As far as the consumption sharing rule is concerned, notice that since $p'(e) > 0$ and the incentive compatibility constraint (43) will be binding in equilibrium (by Assumption 1, implying $\psi > 0$), the presence of moral hazard pushes the allocation of consumption bundles c_H, c_L further away (since $\frac{1-p(e)-\psi p'(e)}{p(e)+\psi p'(e)} > \frac{1-p(e)}{p(e)}$) in order to elicit the efficient level of effort. Clearly, students are strictly better-off in the second best than in the competitive allocation, either due to a higher level of consumption [64] or better consumption smoothing (or both).

Finally, notice that if effort had been perfectly contractible (or if moral hazard was absent, corresponding to the case of $\psi = 0$), consumption would be perfectly smoothed across graduation states, as in the first best benchmark with a utilitarian social welfare function and equal Pareto weights.

2.4.2 Student loans

So far I have only considered a direct assignment of consumption and incentive-compatible effort levels by the planner. Now, I explore implementation of this

[64] It is straightforward to show that output maximization requires that the supply of high skilled workers equals their share in production function, i.e. $p(e) = \rho$. This effect is internalized by the planner (and not by students), and as such the planner can achieve the efficient level of output.

allocation with income-contingent student loans. Intuitively, providing insurance might be possible through charging the successful college-graduates higher interest rates compared to drop-outs, in a way that balances the loan repayments in the aggregate. This is precisely the logic on which Lochner and Monge-Naranjo (2016) base their suggestion that the optimal student loan design has to be such that:

1. loans are fully repaid in expectation;
2. they provide insurance against failure to graduate or find the right job (e.g. through income-contingent repayments);
3. they provide incentives for students to work hard (given the distortion coming from the insurance).

In the context of this paper, full repayment amounts to the following condition:

$$1 + r = p(e)(1 + r_H) + (1 - p(e))(1 + r_L) \tag{47}$$

I firstly characterize the dynamics associated with competitive equilibrium upon introducing a student loan system with graduation-contingent repayment rates:

Proposition 5. *A competitive equilibrium with a student loan program characterized by graduation-contingent repayment rates of r_H if $\theta_i = \theta^H$ and r_L if $\theta_i = \theta^L$ is characterized by the following conditions:*

$$\frac{u'\left(c_H^{SL}\right)}{u'\left(c_L^{SL}\right)} = \frac{1 - p\left(e^{SL}\right)}{p\left(e^{SL}\right)} \cdot \frac{\mu_H}{\mu_L} \tag{48}$$

$$v'\left(e^{SL}\right) = p'\left(e^{SL}\right)\left[u\left(c_H^{SL}\right) - u\left(c_L^{SL}\right)\right] \tag{49}$$

where μ_L and μ_H denote the Lagrange multipliers on budget constraints of the low and high type agents, respectively.

Proof. See Appendix A. □

The difference between this allocation and the competitive equilibrium with non-contingent repayment rates studied above lies in the levels of consumptions, which here are given by $c_H^{SL} = \theta_i w_i^{SL} - (1 + r_i) F$ and, in the latter, by $c_H = \theta_i w_i - (1 + r) F$.

Given the above, I can state the main student loan implementation result:

Proposition 6. *A student loan program with the following graduation-contingent repayment rates implements the second best benchmark and satisfies the full repayment condition (47):*

$$r_H(e) = \frac{\theta_H w_H(e) - c_H^{SB}(e)}{F} - 1$$
$$r_L(e) = \frac{\theta_L w_L(e) - c_L^{SB}(e)}{F} - 1$$

where $c_i^{SB}(e)$, $i \in \{H, L\}$ *solve:*

$$c_H^{SB} = u'^{-1}\left(u'\left(\frac{p(e)\,\theta_H w_H + (1-p(e))\,\theta_L w_L - (1+r)\,F - p(e)\,c_H^{SB}}{1-p(e)}\right)\right.$$
$$\left.\cdot\frac{p(e)}{1-p(e)}\cdot\frac{1-p(e)-\psi p'(e)}{p(e)+\psi p'(e)}\right)$$
$$c_L^{SB} = u'^{-1}\left(u'\left(\frac{p(e)\,\theta_H w_H + (1-p(e))\,\theta_L w_L - (1+r)\,F - (1-p(e))\,c_L^{SB}}{p(e)}\right)\right.$$
$$\left.\cdot\frac{1-p(e)}{p(e)}\cdot\frac{p(e)+\psi p'(e)}{1-p(e)-\psi p'(e)}\right)$$

Furthermore, the repayment rates satisfy $r_H(e) > r_L(e)$.

Proof. See Appendix A. □

The implementation result comes from using income-contingent repayment rates in order to match second best consumption allocations for a given level of effort (which, due to the incentive compatibility constraint, is pinned down in the same way in laissez-faire and second best allocations). Furthermore, and as expected, the positive difference between high and low repayment rates constitutes the market-based insurance against college drop-outs. Finally, observe that the gap between the high and low repayment rates shrinks as the level of tuition fees F increases.

Upon assuming functional forms for utility functions, I get the following corollary:

Corollary 2. *If the utility function is CRRA $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$:*

$$r_H(e) = \frac{1}{F}\left(\theta_H w_H(e) - \frac{\left[p(e)\theta_H w_H + (1-p(e))\theta_L w_L - (1+r)F\right]\left(\frac{p(e)}{1-p(e)} \cdot \frac{1-p(e)-\psi p'(e)}{p(e)+\psi p'(e)}\right)^{-\frac{1}{\sigma}}}{1+\frac{p(e)}{1-p(e)}\left(\frac{p(e)}{1-p(e)} \cdot \frac{1-p(e)-\psi p'(e)}{p(e)+\psi p'(e)}\right)^{-\frac{1}{\sigma}}}\right) - 1$$

$$r_L(e) = \frac{1}{F}\left(\theta_L w_L(e) - \frac{\left[p(e)\theta_H w_H + (1-p(e))\theta_L w_L - (1+r)F\right]\left(\frac{1-p(e)}{p(e)} \cdot \frac{p(e)+\psi p'(e)}{1-p(e)-\psi p'(e)}\right)^{-\frac{1}{\sigma}}}{1+\frac{1-p(e)}{p(e)}\left(\frac{1-p(e)}{p(e)} \cdot \frac{p(e)+\psi p'(e)}{1-p(e)-\psi p'(e)}\right)^{-\frac{1}{\sigma}}}\right) - 1$$

If the utility function is CARA $u(c) = 1 - \exp(-\rho c)$:

$$\begin{aligned} r_H(e) &= \frac{1}{F}\Big[\theta_H w_H(e) - p(e)\theta_H w_H + (1-p(e))\theta_L w_L - (1+r)F \\ &\quad + \frac{1-p(e)}{\rho}\log\left(\frac{p(e)}{1-p(e)} \cdot \frac{1-p(e)-\psi p'(e)}{p(e)+\psi p'(e)}\right)\Big] - 1 \\ r_L(e) &= \frac{1}{F}\Big[\theta_L w_L(e) - p(e)\theta_H w_H + (1-p(e))\theta_L w_L - (1+r)F \\ &\quad + \frac{p(e)}{\rho}\log\left(\frac{1-p(e)}{p(e)} \cdot \frac{p(e)+\psi p'(e)}{1-p(e)-\psi p'(e)}\right)\Big] - 1 \end{aligned}$$

Finally, notice that since the second best benchmark constitutes the upper bound on attainable welfare for allocations subject to moral hazard and no direct redistribution, the competitive allocation is strictly Pareto dominated by the (second best) optimal student loan benchmark.

2.5 Conclusion

Results derived in this paper show that in an incomplete market economy, educational decisions are inherently inefficient due to the (endogenous) risk manifested by the college wage premium. In particular, in an environment with ex-ante homogenous agents, I show that this uninsured risk leads to under-investment in human capital. Consequently, this finding calls for some corrective measures. One form of such intervention is the introduction of a student loan program with income contingency, which is shown to constitute a potentially powerful tool for policy makers interested in providing insurance against dropping out from college. Moreover, I show that these financial instruments are characterized by an inherently inbuilt efficiency-equity trade-off: the higher the degree of insurance provided, the lower the educational effort of students becomes.

At this point, I would like to draw the reader's attention to the link between the constrained inefficiency and the second best implementation results. On one hand, the latter improves the allocation because of the (assumed) government's power to "complete" the markets by offering different repayment rates conditional on education and labor market experiences. In practice, enforcement of these contracts is facilitated by the ability of governments to e.g. garnish wages, impose additional taxes or withhold tax returns.

On the other, the finding that the endogeneity of college wage premium is a source of indirect insurance against the college graduation risk suggests that the need for direct insurance via student loans (or other programs) is lower than in a world with an exogenous premium. This is true since by reducing the risk of attending college, student loans incentivize more education, driving the risk associated with the college wage premium down.

In Appendices B and C, I proceed by constructing and describing the calibration strategy of a rich life-cycle OLG education economy with student loans including relevant heterogeneities w.r.t. learning abilities and initial wealth endowments. In the future, I plan to use this model to investigate effects of income-contingency on college enrollment and resulting welfare and income inequalities: to first evaluate the current income-based repayment system in the US; and to quantify the welfare gains of pursuing optimal student loan policies. Second, the model can be used to investigate how much the optimal policy would differ if the endogenous response of the college wage premium was ignored by policy makers.

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2.6 Appendix A: Proofs

Proposition 4. The second best benchmark is characterized by the following consumption sharing and effort assignment conditions:

$$\frac{u'(c_H)}{u'(c_L)} = \frac{p(e)}{1-p(e)} \cdot \frac{1-p(e)-\psi p'(e)}{p(e)+\psi p'(e)}$$

$$\mu \left[\begin{array}{c} p(e)\,\theta_H \frac{\partial w_H}{\partial e} + (1-p(e))\,\theta_L \frac{\partial w_L}{\partial e} \\ +p'(e)\left[(\theta_H w_H - c_H) - (\theta_L w_L - c_L)\right] \end{array} \right] = -\psi\left[v''(e) - p''(e)\left(u(c_H) - u(c_L)\right)\right]$$

where ψ and μ denote the Lagrange multipliers on the incentive compatibility and resource constraints, respectively.

Proof. The second best maximization problem reads:

$$\max_{c_H, c_L, e} - v(e) + p(e)\,u(c_H) + (1-p(e))\,u(c_L)$$

s.t.

$$(\mu)\;\; p(e)\,c_H + (1-p(e))\,c_L \leq p(e)\,\theta_H w_H + (1-p(e))\,\theta_L w_L - (1+r)\,F$$

$$(\psi)\;\; v'(e) \geq p'(e)\left(u(c_H) - u(c_L)\right)$$

The FOCs w.r.t. c_H, c_L and e read:

$$c_H:\;\; p(e)\,u'(c_H) + \mu + \psi p'(e)\,u'(c_H) = 0$$

$$c_L:\;\; (1-p(e))\,u'(c_L) + \mu - \psi p'(e)\,u'(c_L) = 0$$

$$\begin{aligned} e:\; & -v'(e) + p'(e)\left[u(c_H) - u(c_L)\right] \\ & + \mu\left[p'(e)\left((\theta_H w_H - c_H) - (\theta_L w_L - c_L)\right) + p(e)\,\theta_H \frac{\partial w_H}{\partial e} + (1-p(e))\,\theta_L \frac{\partial w_L}{\partial e}\right] \\ & + \psi\left[v''(e) - p''(e)\left(u(c_H) - u(c_L)\right)\right] = 0 \end{aligned}$$

Using the incentive compatibility in the FOC for e and dividing both c_H and c_L conditions by each other yields the result. □

Proposition 5. A competitive equilibrium with a student loan program characterized by graduation-contingent repayment rates is characterized by the following conditions:

$$\frac{u'(c_H)}{u'(c_L)} = \frac{1-p(e)}{p(e)} \cdot \frac{\mu_H}{\mu_L}$$
$$v'(e) = p'(e)\left[u(c_H) - u(c_L)\right]$$

where μ_L and μ_H denotes the Lagrange multipliers on budget constraints of the low and high type agents, respectively.

Proof. Each agent solves the following problem:

$$\max_{c_H, c_L, e} - v(e) + p(e)\,u(c_H) + (1-p(e))\,u(c_L)$$

s.t.

$$(\mu_H)\;\; c_H \leq \theta_H w_H - (1+r_H)\,F$$
$$(\mu_L)\;\; c_L \leq \theta_L w_L - (1+r_L)\,F$$

The FOCs w.r.t. consumptions are:

$$c_H:\;\; p(e)\,u'(c_H) + \mu_H = 0$$
$$c_L:\;\; (1-p(e))\,u'(c_L) + \mu_L = 0$$

The resulting FOC w.r.t. e is:

$$v'(e) - p'(e)\left[u(c_H) - u(c_L)\right] = 0$$

□

Proposition 6. Student loan program with the following graduation-contingent repayment rates implements the second best benchmark and satisfies the full repayment condition (47):

$$r_H(e) = \frac{\theta_H w_H(e) - c_H^{SB}(e)}{F} - 1$$
$$r_L(e) = \frac{\theta_L w_L(e) - c_L^{SB}(e)}{F} - 1$$

where $c_i^{SB}(e)\,,\ i \in \{H, L\}$ solve:

$$c_H^{SB} = u'^{-1}\left(u'\left(\frac{p(e)\,\theta_H w_H + (1-p(e))\,\theta_L w_L - (1+r)\,F - p(e)\,c_H^{SB}}{1-p(e)}\right)\right.$$
$$\left.\cdot\frac{p(e)}{1-p(e)} \cdot \frac{1-p(e)-\psi p'(e)}{p(e)+\psi p'(e)}\right)$$
$$c_L^{SB} = u'^{-1}\left(u'\left(\theta_H w_H + \theta_L w_L - (1+r)\,F - c_L^{SB}\right) \cdot \frac{1-p(e)}{p(e)} \cdot \frac{p(e)+\psi p'(e)}{1-p(e)-\psi p'(e)}\right)$$

Furthermore, the repayment rates satisfy $r_H(e) > r_L(e)$.

Proof. The optimal repayment rates can be found by equating $c_i^{SB} \stackrel{!}{=} c_i^{SL} = \theta_i w_i(e) - (1 + r_i(e))\,F,\ i \in \{H, L\}$ where the implicit expressions for $c_H^{SB}(e)$ and $c_L^{SB}(e)$ come from the optimal consumption sharing rule:

$$\frac{u'\left(c_H^{SB}\right)}{u'\left(c_L^{SB}\right)} = \frac{p(e)}{1-p(e)} \cdot \frac{1-p(e)-\psi p'(e)}{p(e)+\psi p'(e)}$$

and where consumption bundles are taken from the resource constraint

$$p(e)\,c_H^{SB} + (1-p(e))\,c_L^{SB} = p(e)\,\theta_H w_H + (1-p(e))\,\theta_L w_L - (1+r)\,F$$

Then, since effort is chosen according to the same FOC in both second best and student loan allocations (and is a function of consumption bundles only), I automatically have $e^{SL} = e^{SB}$ and also $w_i^{SB} \stackrel{!}{=} w_i^{SL},\ i \in \{H, L\}$, proving the decentralization.

Now, given the latter, full repayment follows since:

$$p(e)\,r^H + (1-p(e))\,r^L = p(e)\,\frac{\theta_H w_H(e) - c_H(e)}{F} + (1-p(e))\,\frac{\theta_L w_L(e) - c_L(e)}{F} - 1$$
$$= r$$

Finally, to see that $r_H(e) > r_L(e)$, notice that:

$$
\begin{aligned}
r_L(e) &< r_H(e) \\
&\Longleftrightarrow \\
\frac{\theta_L w_L^{SB}(e) - c_L^{SB}(e)}{F} - 1 &< \frac{\theta_H w_H^{SB}(e) - c_H^{SB}(e)}{F} - 1 \\
&\Longleftrightarrow \\
\theta_L w_L^{SB}(e) - \theta_H w_H^{SB}(e) &< c_L^{SB}(e) - c_H^{SB}(e) \\
&\Longleftrightarrow \\
\theta_L w_L^{SB}(e) - \theta_H w_H^{SB}(e) &< \frac{1}{1-p(e)} \Big[p(e)\,\theta_H w_H^{SB}(e) + (1-p(e))\,\theta_L w_L^{SB}(e) \\
&\quad - (1+r)\,F - p(e)\,c_H^{SB}(e) \Big] - c_H^{SB}(e) \\
&\Longleftrightarrow \\
c_H^{SB}(e) &< \theta_H w_H^{SB}(e) - (1+r)\,F
\end{aligned}
$$

which holds always due to the assumed objective function with preference for insurance. □

2.7 Appendix B: Quantitative model

For the quantitative analysis, I consider an overlapping generations model consisting of students-workers that are heterogenous with respect to age, wealth, learning ability, education and labor productivity. Firms produce final goods by hiring labor and capital on competitive spot markets. The government operates education subsidies, pension system and the tax system financing it. Moreover, students have access to graduation-contingent student loans. Decisions of individuals differ depending on the phase in which they find themselves. In the first period of life, after having drawn their learning ability and initial wealth endowment, each individual decides whether to attend college. College education takes two periods and graduation from it is risky, with more effort and monetary investment translating into (i) a higher graduation probability, and (ii) a higher human capital stock. Apart from this, individuals make consumption and labor supply decisions in each period of their working life. Finally, workers retire and live on capital income and pension benefits. I describe details of the model environment in what follows.

Demographics

In each period t, the economy is populated by J overlapping generations indexed by $j = 1, 2, ..., J$, where J denotes the maximum age. Individuals survive from age j to $j+1$ with Poisson probability ψ_{j+1}. For simplicity, I assume that the survival rate before retirement is equal to one; agents face death hazard once they retire, i.e., $\phi_j = 0$ for $j < j_r$ and $\phi_j \in (0, 1]$ for $j \geq j_r$, where j_r denotes the retirement age. I normalize the size of every cohort entering the economy in each period to $\frac{1}{J}$. To facilitate aggregation later on, I define m_j as the size of population of age cohort j relative to the youngest cohort alive in the current period:

$$m_j \equiv \Pi_{i=0}^{j-1} \psi_i$$

Firms

Firms hire labor and capital on competitive spot markets to produce a final good. Firms employ workers for two types of tasks: low- and high-skilled ones ($s_f \in \{H, L\}$). I index the "human capital category" of workers by $s \in \{cg, cd, n\}$, where the college-graduates ($s = cg$) work as high skilled workers, and those who dropped out of college ($s = cd$) or have not attended it ($s = n$) - as low skilled ones. Within each skill type, labor is perfectly substitutable across ages; but across skill types, labor is imperfectly substitutable.[65] Let $L_{t,H}$ and $L_{t,L}$ denote aggregate labor in terms of efficiency units of the skilled and unskilled workers, respectively. Given this, aggregate labor across skill types is given by:

$$L_t \equiv \left(L_{t,H}^{\zeta} + L_{t,L}^{\zeta} \right)^{1/\zeta} \tag{50}$$

with the elasticity of substitution between skilled and unskilled labor being given by $\frac{1}{1-\zeta}$.

I assume existence of a representative firm producing according to standard Cobb-Douglas production function $Y_t = K_t^{\alpha} L_t^{1-\alpha}$. The representative firm takes the wage rates of skilled and unskilled labor, $w_{t,H}$ and $w_{t,L}$, and the interest rate r_t as given.

[65] See Katz and Murphy (1992), Ciccone and Perri (2005) and Malmberg (2018).

Endowments, labor productivity and preferences

Newborn students are heterogeneous in their learning ability a (constant over lifetime) and initial endowment ω drawn from truncated normal distributions F_a and F_ω with supports on the unit interval $(0, 1)$. Furthermore, students also differ w.r.t. their parents' education status $s_p \in \{cg, cd, n\}$. I assume positive correlations ρ_s and ρ_ω between (i) s_p and a; and (ii) s_p and ω. In addition, every individual is endowed with one unit of labor in each period of their life.

There are three elements to labor productivity $h_{s,j}(e, I)$ for each generation j and skill type $s \in \{cg, cd, n\}$. First, post-education productivity is given by $\theta(e, I)$, where $\theta(\cdot)$ is increasing and concave in effort e and investment I and is constant for each period of life j (see the "College Education" section below). Furthermore, the life-cycle productivity profile is reflected by deterministic changes in the age- and skill-specific variable $\epsilon_{s,j}$. Finally, labor productivity is also subject to stochastic shocks $\eta_{s,j}$ that evolve according to a Markov process π_{η_s}. Mathematically:

$$h_{s,j}(e, I) = \begin{cases} \epsilon_{s,j} \cdot \exp\left(\theta_s(e, I) + \eta_{s,j}\right), & if\ j < j_r \\ 0 & o.w. \end{cases}$$

Individuals have preferences over streams of consumption c_j, leisure l_j and education effort e. For those not in college $l_j = 1 - n_j$, where n_j denotes labor supply. The leisure of college students amounts to $l_j = 1 - n_j - \xi(a)$, where $\xi(a)$ is the time cost of college (decreasing and concave in ability a). More precisely, student's preferences are given by:

$$u(c_1, l_1) - v(e) + \beta \mathrm{E} \sum_{j=2}^{J} \beta^{j-1} \left(\Pi_{i=0}^{j} \psi_i\right) u\left(c_j, l_j\right)$$

where (i) the effort disutility function satisfies $v'(e), -v''(e) > 0$, and (ii) $1_{college}$ denotes the indicator function taking value of 1 for individuals deciding to attend the college. Notice that the education effort takes a toll in forms of both reducing the agent's time endowment and mental disutility.[66]

College education

At age one, after having drawn her learning ability a and education-contingent initial wealth ω, an individual decides whether to attend college. Enrolling into

[66] See Mazur (2016) for analysis of differences between separable and non-separable disutility costs of effort in the context of unemployment insurance and labor search.

college is associated with a tuition fee F and the outcome of education is realized in period 2. Individuals which choose not to attend college are characterized by a human capital stock of $a \cdot \underline{\theta}$. On the other hand, students graduate with probability $p(e, I; a)$, which (i) is increasing and concave in both educational effort e and supplementary (above F) monetary investment I; and (ii) exhibits complementarity between effort, monetary investments and ability. As mentioned above, apart form improving the probability of graduation, higher effort e and monetary investment I increase the return to having a college degree in terms of improving the post-education productivity $\theta_s(e, I)$ at each working period $j \in [2, ..., j_r]$. Strictly speaking, I assume the following human capital accumulation technology:

$$\theta_s(e, I) = \begin{cases} \theta_{cg}(e, I) = a \cdot \left(\underline{\theta} + e^{\chi_\theta} I^{1-\chi_\theta}\right) & with\ p(e, I),\ if\ 1_{college} = 1 \\ \theta_{cd}(e, I) = a \cdot \left(\underline{\theta} + \lambda_\theta \cdot e^{\chi_\theta} I^{1-\chi_\theta}\right) & with\ 1 - p(e, I),\ if\ 1_{college} = 1 \\ \theta_n(e, I) = a \cdot \underline{\theta} & if\ 1_{college} = 0 \end{cases} \tag{51}$$

Notice, in particular, that the difference in (51) between college graduates and drop outs is expressed with the fraction $1 - \lambda_\theta$ of additional human capital for successful college graduates. While dropouts are assumed to work in the same unskilled sector with non-college workers, the additional human capital for college drop outs will account for the empirically documented gap in earnings between no college and some college workers (see Appendix C for details on calibration).

For the graduation probability function, I make the parametric assumption of:

$$p(e, I; a) = \frac{a \cdot e^{\chi_p} I^{1-\chi_p}}{1 + e^{\chi_p} I^{1-\chi_p}} \tag{52}$$

The optimal resource cost of attending college $F + I$ is endogenously determined by students. As indicated above, college education is also associated with (i) a time cost $\xi(a) \in (0, 1)$, that depends on learning ability; and (ii) disutility $v(e)$. Furthermore, college dropouts bear only share λ_{F+I} of the total cost $F + I$.

Finally, students have also access to government subsidies awarded based on merit and initial endowment. I denote by $z_{a,\omega}$ the fraction of monetary investment $F + I$ borne by government for individual (a, ω). Upon deciding about going to college, financially constrained individuals can work part-time and/or take out student loans subject to a borrowing constraint (see the next Subsection).

Financial market structure and borrowing constraints

Financial markets are incomplete in that there is no insurance against idiosyncratic labor productivity shocks and mortality risks. Individuals can self-insure against those risks by saving in risk-free deposits d with associated return r_t.

Borrowing b is allowed only using the student loans for financing college education. The student loan program's repayment rates are contingent on students' graduation outcomes, i.e. students from period $j \geq 2$ repay their loans at the rate of $q_s,\ s \in \{cg, cd\}$. Notice that if the program is designed such that $q_{cd} < q_{cg}$, then the student loan program brings the market structure closer to complete by providing insurance against the drop-out risk and effectively cross-subsidizing the unsuccessful students.

Furthermore, I assume that (i) workers have to make non-negative repayments of the debt in all future periods; and (ii) the student debt is repaid by the time of retirement. Taken together, these assumptions imply the following borrowing constraints:

$$\begin{aligned} \underline{\mathrm{A}}_1 &= (1 - z_{a,\omega})\,(F + I) \\ \underline{\mathrm{A}}_{j,s} &= \max\left\{0, (1+q_s)\, b_{j,s}\right\},\ j \in [2, ..., j_r - 1]\ \textit{and}\ s \in \{cg, cd\} \\ \underline{\mathrm{A}}_{j_r,s} &= 0 \end{aligned} \tag{53}$$

where notice that I assume that student loan can be taken up to the full amount of college expenses.

In order to ensure that the student loans program is self-financed in aggregate, I impose the following condition guaranteeing equivalence between the new borrowing by students and the repayments by elder ones:

$$\int\int_{\mathbb{X}(1,c)} \underbrace{b_1'(a,\omega)}_{\textit{new loans}} m_1^{col} \mathrm{d}a\mathrm{d}\omega = \sum_{s\in\{cg,cd\}} \sum_{j=2}^{j_r} \int\int_{\mathbb{X}(j,s)} \underbrace{\left[b_{j,s}'(a,d) - (1+q_s)\, b_{j,s}(a,d)\right]}_{\textit{old loan repayments}} \cdot m_j(s,\epsilon_s)\,\mathrm{d}a\mathrm{d}d \tag{54}$$

where $\mathbb{X}(j,s)$ denotes the subset of the state space corresponding to age j students/workers with decision/college outcome s (see the formal Competitive Equilibrium definition below), m_1^{col} denotes the relative mass of newborns attending the college and $b_1(a,\omega)$ denotes their respective borrowing decision.

Finally, notice that although dependence of the borrowing constraint in period

1 on agents' actions may in principle complicate the numerical solution, I derive the following result guaranteeing that all of the education expenses will be covered through student loans (facilitating the computations):

Conjecture 1. *If the student loan repayment rate upon dropping out is lower than upon successful graduation, i.e.* $q_{cd} < q_{cg}$*, every college enrollee strictly prefers to finance all of college expenses with student loans and use their own wealth for consumption or saving, i.e.* $b_1(a, \omega | s = c) = -\underline{A}_1$.

Government

The government runs a pay-as-you-go pension system: the government collects contributions from current workers and distributes the revenues directly to current pensioners. Moreover, the government also collects tax for the purpose of financing education subsidies. More precisely, in every period t, current workers of both skill types contribute a fraction τ of their labor income to the tax proceeds, and (i) current students receive their subsidies (see above); and (ii) current retirees receive a pension benefit that is proportional to their average life-time income: $pen_t(s, \theta_s) = \kappa w_{t,s_f} \bar{L}_t(s, \theta_s)$, where $\bar{L}_t(s, \theta_s)$ is the average labor supply, in efficiency unit terms, of working age cohort with characteristics (s, θ_s). The budget constraint of the tax system then reads:

$$\sum \tau w_{t,s_f} L_{t,s_f} = \sum_{s \in \{cg, cd, n\}} \sum_{j=j_r}^{J} \int_\theta pen_t(s, \theta_s) \, m_j(s, \theta_s) \, \mathrm{d}\theta + Z_t \tag{55}$$

where (i) $Z_t = z \int_\theta (p(e, I) + (1 - p(e, I)) \lambda_{F+I}) (F + I(a, \omega)) \, m_1^{col}(\theta) \, \mathrm{d}\theta$ denotes the total spending on education subsidies with the subsidy rate of z; and (ii) $m_j(s, \epsilon_s)$ is the relative size of age cohort j that falls into the skill category s and has a human capital level of θ.

Finally, the government collects accidental bequests and redistributes them to the generation that has just entered the economy. To come up with a plausible calibration of the initial wealth distribution, I assume that the government fills the gap between accidental bequests and actual transfers in each period.

Recursive life-cycle formulation

In this subsection, I set out precisely the recursive problem of individuals' over their life-cycle. For what follows, denote the current state by $(j, a, s, d, b, \epsilon, \eta)$, where $j \in \{1,...,J\}$ denotes age, $a \in (0,1)$ inborn ability, $s \in \{c,n\}$ is the schooling decision in period 1 and $s \in \{cg, cd, n\}$ is schooling outcome from period 2 onwards, $d \geq 0$ is the current balance of student's deposit, $-\underline{A}_{j,s} \leq b \leq 0$ is the current balance on the student loan, ϵ is the current state in worker's life-time productivity profile and η is the current realization of stochastic productivity component. Notice also that state s defines the type of worker's job skill-category $s_f \in \{H, L\}$.

Age $j = 1$*:* Before the college decision is made, each student draws its learning ability a and initial wealth ω. Given these, individuals decide whether to attend college, which is captured with the following indicator function:

$$1_s(a,\omega) = \begin{cases} 1 & if\ W(a,c,\omega,b,\epsilon) > W(a,n,\omega,b,\epsilon) \\ 0 & o.w. \end{cases} \tag{56}$$

where

$$W(a,s,\omega,b,\epsilon) \equiv \sum_{\eta'} \pi_{\eta_n}(\eta)\, V(1,a,s,\omega,b,\epsilon,\eta) \tag{57}$$

is the expected period-1 value of life-time utility of an agent with college decision s, learning ability a and an initial wealth endowment of ω. Notice that the initial idiosyncratic productivity η is drawn from distribution for unskilled workers. Then, I assume that upon a successful college graduation, skilled workers redraw the idiosyncratic shock η from the distribution for skilled workers and so thereafter the stochastic productivity component evolves over time according to $\pi_{\eta_s}(\eta'|\eta)$.

Given its initial wealth ω, college decision s, human capital stock θ and its initial draw of stochastic productivity η, every individual then chooses consumption and labor supply so as to maximize its expected present value of life-time utility. Formulated recursively, each individual solves the following Bellman equation:

$$\begin{aligned} V(1,a,s,\omega,0,\epsilon,\eta) = \max\nolimits_{c,n,d',b',e,I} \Big\{ & u\left(c, 1 - 1_{college} \cdot \xi(a) - n\right) - v(e) \\ & + \beta \sum_{\eta'} \pi_{\eta_s}(\eta'|\eta) \left[\left(1 - 1_{college}\right) \cdot V\left(2,a,n,d',0,\epsilon,\eta\right) \right. \\ & + 1_{college} \cdot \left(p(e,I) \cdot V\left(2,a,cg,d',b',\epsilon,\eta\right) \right. \\ & \left. \left. + (1 - p(e,I)) \cdot V\left(2,a,cd,d',b',\epsilon,\eta\right) \right) \right] \Big\} \end{aligned}$$

subject to:

$$c + d' + b' + 1_s\,(1-z)\,(F+I) \leq (1+r_t)\,\omega + (1-\tau)\,w_{t,s_f} h_{s,1} l + Tr$$
$$b' \geq -\underline{A}_1$$
$$d' \geq 0$$

Age $j \in \{2, ..., j_r - 1\}$*:* While decisions at age one may differ for college and non-college individuals, decisions during the working life are pretty standard: given the current state $(j, a, s, d, b, \epsilon, \eta)$, each agent chooses consumption and labor supply so as to maximize its present value of utility. The associated Bellman equation reads:

$$V\,(j, a, s, d, b, \epsilon, \eta) = \max_{c,n,d',b'} \left\{ u\,(c, 1-n) + \beta \sum_{\eta'} \pi_{\eta_s}\left(\eta'|\eta\right) V\left(j+1, a, s, d', b', \epsilon, \eta\right) \right\}$$

subject to:

$$\begin{aligned} c + d' + b' + &\leq (1+r_t)\,d + 1_s \cdot (1+q_s)\,b \\ &\quad + 1_{s=cd} \cdot (1-\lambda_{I+F})\,(1-z)\,(F+I) + (1-\tau)\,w_{t,s_f} h_{s,j} l + Tr \\ b' &\geq -\underline{A}_{j,s} \\ d' &\geq 0 \end{aligned}$$

where recall that the non-college individuals have no student debt, i.e. $b' = 0$ if $s = n$; and the term $1_{s=cd} \cdot (1-\lambda_\theta)\,(F+I)$ is due to the assumption that college droupouts bear only fraction λ_{F+I} of total education costs.

Decisions at age $j \in \{j_r, ..., J\}$*:* After retirement, workers' labor productivity drops to zero, and they live on savings and pension benefits. The associated Bellman equation is given by:

$$V\,(j, a, s, d, b, \epsilon, \eta) = \max_{c,n,d'} \left\{ u\,(c, 1-n) + \beta \sum_{\eta'} \pi_{\eta_s}\left(\eta'|\eta\right) V\,(j+1, a, s, d, b, \epsilon, \eta) \right\}$$

subject to:

$$
\begin{aligned}
c + d' + &\leq (1 + r_t)\, d + (1 + q_s)\, b + pen_t\,(s, \theta) + Tr \\
d' &\geq 0
\end{aligned}
$$

where, as student debt is assumed to be repaid by the time of retirement, I have $b' = 0$ for $j \geq j_r$ and $b = 0$ necessarily for $j \geq j_r + 1$.

Competitive equilibrium

To define the competitive equilibrium of the economy, let us introduce some additional notation. In particular, let $\mathcal{E} = [0, 1]$, $\mathcal{J} = \{1, ..., J\}$, $\mathcal{S}_1 = \{c, n\}$, $\mathcal{S}_j = \{cg, cd, n\}$, $\mathcal{D} = \mathbb{R}$, $\mathcal{F} = \mathbb{R}$ and $\mathcal{H} = \mathbb{R}$ denote the state space for ability a, age j, education choice and outcome s, wealth d, human capital level θ and the stochastic productivity component η. Given these, let Σ denote the Borel σ-algebra defined on the product space $\mathbb{X} = \mathcal{E} \times \mathcal{J} \times \mathcal{S}_1 \times \mathcal{S}_j \times \mathcal{D} \times \mathcal{F} \times \mathcal{H}$. As for any $X \in \mathbb{X}$ a measure $\phi\,(X)$ can now be properly defined, I proceed to the definition of equilibrium:

Definition 3. A stationary recursive competitive equilibrium is a collection of: (i) decision rules of individuals $\{1_s\,(a, \omega)\,, e\,(a, \omega)\,, I\,(a, \omega)\,, c/n/d'/b'\,(j, a, s, d, b, \epsilon, \eta)\}$; (ii) aggregate capital and labor $\{K_t, L_{t,H}, L_{t,L}\}$; (iii) value functions $V\,(j, a, s, d, b, \epsilon, \eta)$; (iv) government policies $\{\tau_p, \tau_l\,(y)\,, pen\,(s, \theta)\,, \kappa_s, z, Tr\}$; (v) prices $\{r_t, w_{t,H}, w_{t,L}\}$; and (vi) a vector of measures ϕ, such that:

1. The decision rules of individuals solve their respective life-cycle problems, and $V\,(j, a, s, d, b, \epsilon, \eta)$ is the associated value function.

2. Aggregate capital and labor inputs $\{K_t, L_{t,H}, L_{t,L}\}$ solve the representative firm's profits maximization problem, which is fully characterized by the following first order conditions:

$$
\begin{aligned}
r_t &= \alpha k_t^{\alpha - 1} - \delta \\
w_{t,H} &= (1 - \alpha) \left(\frac{K_t}{L_t}\right)^{\alpha} \left(\frac{L_t}{L_{t,H}}\right)^{1-\zeta} \\
w_{t,L} &= (1 - \alpha) \left(\frac{K_t}{L_t}\right)^{\alpha} \left(\frac{L_t}{L_{t,L}}\right)^{1-\zeta}
\end{aligned}
$$

and the college wage premium is defined as:

$$\frac{w_{t,H}}{w_{t,L}} = \left(\frac{L_{t,L}}{L_{t,H}}\right)^{1-\zeta}$$

3. The labor market for each skill type $s_f \in \{H, L\}$ clears:

$$L_{t,H} = \sum_{j=2}^{j_r-1} \int \int \int_{\mathbb{X}(j,cg)} h_{cg,j}\left(\theta,\eta\right) l\left(j,a,cg,d,b,\epsilon,\eta\right) \phi\left(j,a,cg,d,b,\epsilon,\eta\right) \mathrm{d}d\mathrm{d}\theta\mathrm{d}\eta$$

$$\begin{aligned} L_{t,L} = & \sum_{s\in\{c,n\}} \int \int \int_{\mathbb{X}(1,s)} h_{s,1}\left(\theta,\eta\right) l\left(1,a,s,d,0,\epsilon,\eta\right) \phi\left(1,a,s,d,0,\epsilon,\eta\right) \mathrm{d}d\mathrm{d}\theta\mathrm{d}\eta \\ & + \sum_{s\in\{cd,n\}} \sum_{j=2}^{j_r-1} \int \int \int_{\mathbb{X}(j,s)} h_{s,j}\left(\theta,\eta\right) l\left(j,a,s,d,b,\epsilon,\eta\right) \phi\left(j,a,s,d,b,\epsilon,\eta\right) \mathrm{d}d\mathrm{d}\theta\mathrm{d}\eta \end{aligned}$$

where $\mathbb{X}\left(j,s\right)$ is the subset of the state space $\mathbb{X}$ corresponding to age j and skill type s.

4. The capital market clears:

$$K_{t+1} + B_{t+1} = D_{t+1}$$

where B_{t+1} is the supply of government bonds, and:

$$D_{t+1} = \sum_{s\in\{cd,n\}} \sum_{j=1}^{J} \int \int \int_{\mathbb{X}(j,s)} d'\left(j,a,s,d,b,\epsilon,\eta\right) \phi\left(j,a,s,d,b,\epsilon,\eta\right) \mathrm{d}d\mathrm{d}\theta\mathrm{d}\eta$$

5. The good market clears:

$$Y_t = C_t + F_t + I_t + Inv_t$$

where the aggregate consumption and education spendings equal:

$$C_t = \sum_{s\in\{c,n\}} \int\int\int_{\mathbb{X}(1,s)} c\,(1,a,s,d,0,\epsilon,\eta)\,\phi\,(1,a,s,d,0,\epsilon,\eta)\,\mathrm{d}d\mathrm{d}\theta\mathrm{d}\eta$$

$$+\sum_{j=2}^{J}\sum_{s\in\{cd,cg,n\}} \int\int\int_{\mathbb{X}(j,s)} c\,(j,a,s,d,b,\epsilon,\eta)\,\phi\,(j,a,s,d,b,\epsilon,\eta)\,\mathrm{d}d\mathrm{d}\theta\mathrm{d}\eta$$

$$F_t + I_t = \int\int\int_{\mathbb{X}(1,c)} \left(p\,(e,I) + (1-p\,(e,I))\,\lambda_{F+I}\right) I\,(1,a,c,\omega_c,0,\epsilon,\eta)$$

$$\cdot\,\phi\,(1,a,c,\omega_c,0,\epsilon,\eta)\,\mathrm{d}d\mathrm{d}\theta\mathrm{d}\eta$$

$$+F\int\int\int_{\mathbb{X}(1,c)} \left(p\,(e,I) + (1-p\,(e,I))\,\lambda_{F+I}\right)\phi\,(1,a,c,\omega_c,0,\epsilon,\eta)\,\mathrm{d}d\mathrm{d}\theta\mathrm{d}\eta$$

and $Inv_t = \delta K_t$ is gross investment.

6. The government budget constraints hold:

$$\sum \tau w_{t,s_f} L_{t,s_f} = \sum_{s\in\{cg,cd,n\}}\sum_{j=j_r}^{J}\int_\theta pen_t\,(s,\theta_s)\,m_j\,(s,\theta_s)\,\mathrm{d}\theta + Z_t$$

$$(1+r_t)\,D_{b,t} + B_{t+1} = (1+r_t)\,B_t + (1+r_t)\,D_{init,t} + Tr_t$$

where Z_t is the aggregate amount of resources spent on education subsidies:

$$Z_t = z\int_D\int_{\mathcal{A}} 1_s\,(a,d)\left(p\,(e,I) + (1-p\,(e,I))\,\lambda_{I+F}\right) I\,(1,a,c,d,0,\epsilon,\eta)$$

$$\cdot\,\phi\,(1,a,c,d,0,\epsilon,\eta)\,\mathrm{d}d\mathrm{d}a$$

$$+zF\int_D\int_{\mathcal{A}} 1_s\,(a,d)\left(p\,(e,I) + (1-p\,(e,I))\,\lambda_{I+F}\right)\phi\,(1,a,c,d,0,\epsilon,\eta)\,\mathrm{d}d\mathrm{d}a$$

$D_{b,t}$ denotes accidental bequests:

$$D_{b,t} = \sum_{j=j_r-1}^{J}\sum_{s\in\{cd,cg,n\}}\int\int\int_{\mathbb{X}(j,s)} d'\,(j,a,s,d,b,\epsilon,\eta)\left(1-\psi_{j+1}\right)$$

$$\cdot\,\phi\,(j,a,s,d,b,\epsilon,\eta)\,\mathrm{d}d\mathrm{d}\theta\mathrm{d}\eta$$

and $D_{init,t}$ is the aggregate wealth transfer to the newly arrived generation:

$$D_{init,t} = \sum_{s\in\{c,n\}}\int_{\mathcal{D}} d\phi\,(1,a,s,d,0,\epsilon,\eta)\,f\,(d)\,\mathrm{d}d$$

where $f(d)$ is the distribution from which initial wealth is drawn.

7. The student loan program is self-financed in the aggregate, i.e. (**??**) holds.

8. Individual behaviors are consistent with aggregate behavior: measure ϕ is a fixed point of $\phi(X) = \Pi(X, \phi)$, for any $X \in \mathbb{X}$, where $\Pi(X, \cdot)$ is the transition function generated by the decision rules of individuals, the process of exogenous states, and the survival probabilities.

9. All aggregate variables are constant over time.

2.8 Appendix C: calibration

This section discusses parameter choices. I calibrate the model to the US economy. The majority of parameters are either estimated directly from the data or calibrated internally by matching certain aggregate moments in the US data. The rest of the parameters are taken from the literature.

Demographics

New generations enter the economy at the age of 18. A period in the model corresponds to four years and so it takes one period to complete college. Everyone retires at the age of 66 and the maximum age is 94. Survival probabilities $\{\psi_j\}$ are computed from actuarial life tables for the full time male workers in the US. The reference year is 2011.

Preferences

I consider a CRRA instantaneous utility function of the following form:

$$u\left(c_j, 1 - 1_{college} \cdot \xi(a) - n_j\right) = \frac{\left[c^{\nu}\left(1 - 1_{college} \cdot \xi(a) - n_j\right)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1 - \frac{1}{\gamma}} \tag{58}$$

where ν is a taste parameter for consumption and $\frac{1}{\gamma}$ is a risk aversion parameter. These two parameters jointly determine (i) the average labor supply, (ii) the intertemporal elasticity of substitution of consumption, and (iii) the Frisch labor supply elasticity. I set γ to 0.25 and choose ν such that workers supply on average one-third of their time time endowment.[67] The subjective discount factor β is used to target a capital-output ratio of around 2.4, which falls in the range commonly used in the literature.

Production technology

The aggregate production function is of Cobb-Douglas form. The capital share α is set to 0.33. I set the elasticity of substitution between skilled labor and unskilled labor $\frac{1}{1-\zeta}$ to 1.5 – in accordance with the estimate of Katz and Murphy (1992), Ciccone and Peri (2005) and Malmberg (2018). In addition, I set the annual depreciation rate δ to 7.55%, as in Krueger and Ludwig (2016).

Labor productivity

Recall that the labor productivity of workers with education outcome $s \in \{cg, cd, n\}$ and of age j is given by:

$$h_{s,j}(e, I) = \begin{cases} \epsilon_{s,j} \cdot \exp\left(\theta_s(e, I) + \eta_{s,j}\right), & if\ j < j_r \\ 0 & o.w. \end{cases}$$

For calibration of the endogenous part of the process - $\theta_s(e, I)$ - see Subsection "Education process" below. In order to estimate the deterministic life-cycle component $\epsilon_{s,j}$ and the stochastic component $\eta_{s,j}$, I run the following cross-sectional regression on the PSID data:

$$\ln w_j = f\left(X_j; x\right) + \tilde{w}_j \tag{59}$$

where $f\left(X_j; x\right)$ is a function of age and education capturing the life-cycle productivity profile with X_j being a vector of observables including education dummies and a cubic polynomial in age, and $\tilde{w}_j$ is a residual term.

I estimate $\epsilon_{s,j}$ by indirect inference method, i.e. by matching the coefficients on the age polynomial from regression (59) in analogous regression on the data generated by the model. Furthermore, I normalize $\epsilon_{s,j}$'s of both types going and not going to college s.t. $\epsilon_{1,n} = 1$.

[67] These preferences imply that the coefficient of relative risk aversion equals $\nu\left(\frac{1}{\gamma} - 1\right) + 1$ and a Frisch elasticity of labor supply of $\frac{1-\mu\left(1-\frac{1}{\gamma}\right)}{\frac{1}{\gamma}} \cdot \frac{n}{1-n}$.

	AR(1)		Markov Chain		HH FE
	ρ	σ^2_η	p_s	η_s	σ^2_η
College graduate	0.963	0.011	0.982	$\{-0.041, 0.041\}$	0.048
College dropout	–	–	–	–	–
No college	0.926	0.019	0.963	$\{-0.050, 0.050\}$	0.065

Table 13: Estimates for the labor productivity process

Then, I model the stochastic component $\eta_{s,j}$ as a two-state, education-specific Markov chain taking two values $\{-\eta_s, \eta_s\}$ with the following transition matrix:

$$\pi_{\eta_s} = \begin{bmatrix} p_s & 1-p_s \\ 1-p_s & p_s \end{bmatrix}$$

I estimate this Markov process using the residual term $\tilde{w}_j$. In particular, within each education group I model it as an AR(1):

$$\begin{aligned} \log\tilde{w}_{i,t} &= \alpha_i + z_{i,t} \\ z_{i,t} &= \rho z_{i,t-1} + \eta_{i,t} \end{aligned}$$

where α_i is an individual fixed effect that I assume to be normally distributed with cross-sectional variance σ^2_α. Table 1 summarizes the results of estimation.

Education process

Calibrating returns to and costs of education require specifying parameters of the tuition fee F (standing for the basic fee that can be supplemented with I); the human capital accumulation process $\theta_s(e, I)$; the graduation probability function $p(e, I; a) = \frac{a \cdot e^{\chi_p} I^{1-\chi_p}}{1+e^{\chi_p} I^{1-\chi_p}}$; and the education effort disutility function $v(e) = e^{\chi_e}$. I focus on private four-year colleges as the data on these is more likely to reflect the true resource cost.

Recall that the human capital accumulation process is given by:

$$\theta_s(e, I) = \begin{cases} \theta_{cg}(e, I) = a \cdot \left(\underline{\theta} + e^{\chi_\theta} I^{1-\chi_\theta}\right) & with\ p(e, I),\ if\ 1_{college} = 1 \\ \theta_{cd}(e, I) = a \cdot \left(\underline{\theta} + \lambda_\theta \cdot e^{\chi_\theta} I^{1-\chi_\theta}\right) & with\ 1 - p(e, I),\ if\ 1_{college} = 1 \\ \theta_n(e, I) = a \cdot \underline{\theta} & if\ 1_{college} = 0 \end{cases}$$

First, I normalize $\underline{\theta}$=1. Second, I calibrate the level of basic tuition fee F using the evidence in Trends in College Pricing (2018, Figure 2, p.11). In the data for the year 2016, the bottom 25th percentile of students enrolling into private four-year colleges spends on average USD 25,000 on tuition and fees. Moreover, in 2016 the nominal US GDP per capita amounted to ca. USD 57,000. Thus, I calibrate F so that in equilibrium $\frac{F}{Y} = \frac{25,00}{4 \cdot 57,000} \approx 10.96\%$.

The college wage premium, graduate shares, the level of (supplementary) education investment I and the level of students' indebtedness are pinned down jointly by parameters governing the $\theta_s(e, I)$, $p(e, I)$ and $v(e)$ functions. Thus, I use $\lambda_{F+I}, \chi_\theta, \chi_e$ and χ_p jointly to match:

1. evidence in Valletta (2016) from CPS documenting that in 2000s the average real wage of workers with college education has been approx. 72% higher than that of high school graduates;

2. median supplementary spending on college education $\bar{I}$ such that the median total spending on college education $F + \bar{I}$ in 2016 amounts to USD 36,000, i.e. $\frac{F+\bar{I}}{4 \cdot Y} \approx 15.78\%$ (Trends in College Pricing, 2018, Figure 2, p.11);

3. 27% of population having college education, 35% being college dropouts and 37% having high school education only (Valletta, 2016);

4. average student debt amount owed by dropouts of approx. USD 14,000, or $\frac{14,000}{4 \cdot 57,000} \approx 6.14\%$ of output (Brown, 2017).

Furthermore, Valletta (2016) documents also that in 2000s the premium of workers with some college (taken as college dropouts for the purpose of this paper) over high school graduates has amounted to approx. 20%. I use parameter λ_θ for matching this number.

Finally, I pin down the amount of government-financed education subsidies using the evidence in Trends in College Pricing (2018, Figure 8 and 9, p. 17-18). According to the latter, the average net (after grant aid and tax benefits) and gross prices of "tuition and fees" at private (non-profit) four-year institutions USD 14,000 and 34,500. Thus, I assume that the subsidy rate amounts to $z = \frac{20,500}{34,500} \approx 59.4\%$. This implies that for the benchmark calibration I assume the fraction of educational costs borne by the government to be the same for all individuals, regardless of their learning ability and financial situation. I consider need- and merit-based grants and scholarships later on as extensions. Table 2 summarizes the results of estimation.

θ	λ_θ	χ_θ	χ_p	χ_e	F
1	-	-	-	-	-

Table 14: Estimates for the human capital accumulation technology

Wealth distribution

Heterogeneity in initial wealth in this paper is intended to capture the family income effects on college attendance and college quality choice. Ionescu (2009) pins down the initial asset distributions using the Survey of Consumer Finances (1983) and the High School and Beyond (1980) datasets on individuals aged 18-20 (SCF) and below 18 (HSB) deciding about going to college. Importantly, the latter dataset contains data on expected family contribution for college, where Ionescu (2009) finds that "the expected family contribution in the HB data is not different across groups of students that eventually enroll or not in college." Ultimately, she arrives at the mean of USD 23,100 and a standard deviation of USD 32,415 in 1984 constant dollars. I choose parameters μ_{F_ω} and $\sigma^2_{F_\omega}$ such that the model generated endogenous distribution of initial wealth corresponds to these findings, relative to the economy's output Y.

Learning ability distribution and college time costs

In calibrating ability distribution and college time costs, I follow mostly Krueger and Ludwig (2016). In particular, the time requirement for attending class and studying in college is given by the linear function:

$$\xi\left(a\right) = 1 - a$$

so that children with lowest ability face prohibitively large time costs of going to college, $(e = 0) = 1$.

Moreover, newly born agents draw their ability from a distribution $\pi_{s_p}\left(a\right)$ that depends on the education level of their parents and follows a normal distribution with parameters μ_{a,s_p} and σ_a which are discretized to 10 values, $e \in \{e_1 = 0, ..., e_{10} = 1\}$, I choose parental education specific means μ_{s_p} to match college completion rates of students by parental education levels, and choose the variance σ_{a,s_p} such that the probability mass of the original normal distributions located in the unit interval $[0, 1]$ is 90% on average over the three groups of s_p.

To obtain college completion rates of students by parental education I turn to the National Education Longitudinal Study (NELS:88). I compute the percent of individuals from this nationally representative sample who were first surveyed as eighth-graders in the spring of 1988, that by 2000 had obtained at least a Bachelors degree, conditional on the highest education level of their parents. I identify $s_p = cg$ in the model with the highest education of a parent being at least a Bachelors degree (obtained by 1992), and with $s_p = cd$ with parents who have high school and some college education. I find that for students with parents in the $s_p = cg$ category 63.3% have completed a Bachelors degree, in $s_p = cd$ - 30% and in $s_p = n$ - 22%. Although in the model these shares are endogenously determined, they are mainly driven by the values for the education specific means μ_{s_p}.

Government policy

Consistent with the current social security configuration, pension benefits are set to be 45% of the average income of each skill group, i.e., $\kappa_s = 45\%$. Payroll tax rate τ is then set to balance the spending on pensions and education subsidies.

Student loans

The baseline calibration is meant at evaluating the current income-based repayment (IBR) component of student loans in the US. It entails usually[68] a monthly payment of 10%-15% of borrowers monthly gross income (if it makes the borrower better-off than repaying at the market rate) and a full discharge of the loan after 20-25 years. Importantly, IBRs are not a default solution in the US. Moreover, transforming one's student loan into an income-based one is associated with significant administrative efforts and as such is seen as one of the major reasons why the usage of IBRs is relatively small (see Barr, Chapman, Dearden and Dynarski (2019) for discussion of these issues). In what follows, I will abstract from this and so will proceed as if the system of IBRs was a default component of the student loan design in the US.

In order to implement this numerically, I impose the repayments of college dropouts to be $b'_{j,s}(a,d) - (1+r_t)\, b_{j,s}(a,d) = 12.5\% \cdot w_{t,s_f} h_{s,j} l$ for the maximum of 20 years (unless repaid earlier). Notice that this solution may imply that college dropouts do not fully repay their loans, forcing the government to cover the missing part by taxing working population.

[68]Source: http://www.ibrinfo.org/existingidr.vp.html

As outlined in the modelling section above, in the counterfactual exercises I will allow for elastic repayments through endogenous choice of b' at the two repayment rates of $q_{cd} < q_{cg}$. Notice that this solution should dominate the fixed repayment rate policy, if I properly account for the amount of non-repaid loans that need to be covered through labor taxation.

3 A Note on Pessimism in Education and its Economic Consequences

3.1 Introduction

Beliefs about returns to education constitute perhaps the most important determinant of educational investment decisions. The very nature of these investments being rare and taking a lot of effort, time and money to deliver a positive net payoff, suggests that these investments may be lumpy (see more on this below). As such, we cannot expect people to get their decisions right by trial-and-error or by making marginal adjustments.

Unsurprisingly then, the empirical literature in economics of education has seen a recent surge in studies documenting biases in students' and/or parental beliefs about returns to and costs of college education, see e.g. Hoxby and Turner (2015), Wiswall and Zafar (2015b), Belfield, Boneva, Rauh and Shaw (2016), Boneva and Rauh (2017), Boneva and Rauh (2018), Bleemer and Zafar (2018) or Dizon-Ross (2019). These biases may arise for a variety of reasons ranging from a lack of role models in the neighbourhood, unawareness about available options, scholarships, subsidies, or preferential loans, to simply being generated by media reports of students burdened by their loans.

A common conclusion flowing from this literature is firstly that these beliefs are important for decisions about the level and direction of educational investments pursued. Secondly, these beliefs are oftentimes severely downwardly biased, i.e. pessimistic. Finally, this pessimism is much more often found among poorer or less-educated households. Building on these conclusions, the literature advocates for and demonstrates the effectiveness of interventions removing these informational frictions with the ultimate goal of improving the equality of opportunity and allocation of talent (see e.g. Hoxby and Turner, 2013; Pallais, 2015; Wiswall and Zafar, 2015a,b; Alan and Ertac, 2017 or Dynarski, Libassi, Michelmore and Owen, 2018).

In this note, combining the aforementioned insights, I show that in the presence of uncertainty about the true returns to lumpy educational investments, pessimistic beliefs may persist in the long-run due to being self-confirmed in equilibrium (in the sense of Fudenberg and Levine, 1993). In other words, I show that while optimistic agents will be able to find out through their labor market outcomes that their beliefs about returns were upwardly biased, this will not be true of pessimistic market participants since in equilibrium they will receive labor income confirming their pessimistic priors.

The general idea behind education being a lumpy investment is that it cannot be varied by small amounts. On the one hand, actual years of schooling are bunched around years associated with the completion of primary, secondary or higher education. Similarly, the pay-offs from pursuing different professions are often concentrated at different levels. On the other hand, when making a school choice, families need to take into account not only the differences in quality, but also the earnings foregone and the differences in the level of tuition fees and living costs due to different geographical locations.[69]Unsurprisingly then, the ensuing belief system about the returns and costs of educational investments will very likely take the form of a step function (not necessarily the "correct" one, as the above mentioned literature finds), mimicking the real world constraints.

Building on this simple theoretical model, I argue that the lumpy nature of educational investments and the persistence of biased beliefs have important empirical implications. Firstly, the lumpy nature of educational investments implies that countries with more rigid institutional setups will be characterized by (i) a higher degree of human capital concentration; (ii) a higher degree of human capital investments misallocation; and (iii) larger discrepancies between the observed human capital and the underlying initial ability distributions. Secondly, the self-confirming nature of pessimism implies that the true underlying skill distributions cannot be identified without having identified the underlying beliefs of students or their parents. I discuss these issues in the concluding Section 3.3. In Appendix A, I discuss further theoretical implications for long-run economic growth and political voting patterns. Then, in Appendix B, I describe possible circumstances conducive to the persistence of such beliefs in dynamic environments with overlapping generations, social learning, peer effects or costly information acquisition.

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[69]Strictly speaking, the lumpy investment profile may arise due to non-convex education adjustment costs. Card (2001) reviews the literature on returns to schooling and offers some reconciliatory explanations for seemingly contrary conclusions arrived at in this literature. Similarly to the approach followed here, one of these explanations (p. 1156) is based on the discrete differences in education costs and institutional features constraining the optimal choice of educational attainment. Moreover, Heckman, Lochner and Todd (2008) provide estimates of low returns to college attendance vs. high returns to college completion - consistent with the step function returns assumed here.

3.2 Model economy ...

3.2.1 ... with rational expectations

Consider a decision problem of a risk averse agent-student. She lives for two periods, discounts the future at the rate of β and values consumption according to the utility function $u(c)$ with $u'(c), -u''(c) > 0$ and the Inada condition at 0. Therefore, she wishes to maximize the quantity:

$$u(c_1) - v(e) + \beta \mathrm{E} u(c_2)$$

where E is the rational expectations operator, $e \in [0,1]$ stands for educational (mental) effort and $v(e)$ represents the associated disutility satisfying $v'(e), v''(e) > 0$ and $v(0) = 0$.

The student starts her life in period 1 with initial endowment ω, which can be thought of as a family transfer or potential earnings during her youth. She is just about to make a decision about the level of her investment in education (think of choosing the whole path of education, from high school education to the finishing of a Ph.D.). Specifically, given the wealth available, she decides on [1] her consumption in the current period (c_1), [2] saving or borrowing b at the risk-free interest rate r (exogenously given); and [3] educational investment in the form of physical effort e and associated lump sum[70] education cost $F \geq 0$ (e.g. tuition fee). Thus, the student faces the following budget constraint in period 1:

$$c_1 + b + 1_{e>0} \cdot F \leq \omega \tag{60}$$

Furthermore, assume that the saving/borrowing decision b satisfies $b \geq B$, where $B < 0$ is a natural borrowing limit.[71] Motivated by issues discussed in the introduction, the return $\theta(e)$ from investing effort e in education is an $N-$step function:

$$\theta(e) = \begin{cases} \underline{\theta} & if\ e \in [0, q_1) \\ \theta(q_1) & if\ e \in [q_1, q_2) \\ ... & ... \\ \theta(q_N) & if\ e \in [q_N, 1] \end{cases} \tag{61}$$

where $\underline{\theta} > 0$ stands for some minimum level of human capital attainable without pursuing any further education.

[70]This is without loss of generality, modeling educational costs as an increasing function of total effort would not change any of the main results below.

[71]Notice that due to the assumed Inada condition, the borrowing constraint will never bind.

Then, in the second period, after the student graduates, she collects her savings with the incurred interest $(1+r)\,b$ and the wage income of $w \cdot \theta(e)$, where w is the (exogenous[72]) market wage rate. Thus, the budget constraint in period 2 reads:

$$c_2 \leq (1+r)\,b + w \cdot \theta(e) \tag{62}$$

Equilibrium

In equilibrium, the student's optimal decisions will be made according to:

$$b^* : u'\left(c_1^*\right) = (1+r)\,\beta \mathrm{E} u'\left(c_2^*\right) \tag{63}$$

$$e^* : \; u\left(c_1^* \mid e_i^*\right) - v\left(e_i^*\right) + \beta \mathrm{E} u\left(c_2^* \mid e_i^*\right) \geq u\left(c_1 \mid e_j\right) - v\left(e_j\right) + \beta \mathrm{E} u\left(c_2 \mid e_j\right) \;\; \forall j \neq i \tag{64}$$

The savings decision b is made according to a standard intertemporal Euler condition involving comparing the marginal loss in consumption today and the expected marginal gain in consumption tomorrow (in marginal utility terms). On the other hand, the optimal education decision e involves exercising educational effort at a level $e^* \in \{0, q_1, ..., q_N\}$ maximizing the students' (expected) life-time utility.

3.2.2 ... with subjective beliefs

Consider now a student with a following subjective belief system:

Assumption 2. Subjective beliefs are assumed to be as follows:

- A pessimistic student's belief $\mathrm{E}^S(\theta(e))$ is that $\theta(e)$ is a step function as in (61) with $N^P < N$ steps s.t. $\mathrm{E}^S(\theta(q_i)) = \theta(q_i)\,,\ i = 1, ..., N^P$.
- An optimistic student's belief $\mathrm{E}^S(\theta(e))$ is that $\theta(e)$ is a step function as in(61) with $N^O > N$ steps s.t. $\mathrm{E}^S(\theta(q_i)) = \theta(q_i)$ for some $N' \leq N$ steps and $\mathrm{E}^S(\theta(q_i)) > \theta(q_i)$ for the remaining $N^O - N'$ steps.

Figure 10 presents an example of lumpy objective, subjective pessimistic and subjective optimistic beliefs on returns to education. The objective process involves four steps, while the pessimistic one involves three. This can be thought of as returns associated with pursuing high school, undergraduate or graduate education, where the pessimist does not believe in any returns from completing a 2 year

[72]Theoretically, wages are completely pinned down by a linear constant returns to scale technology. I thus ignore the impact of changes in supply on the wages associated with schooling.

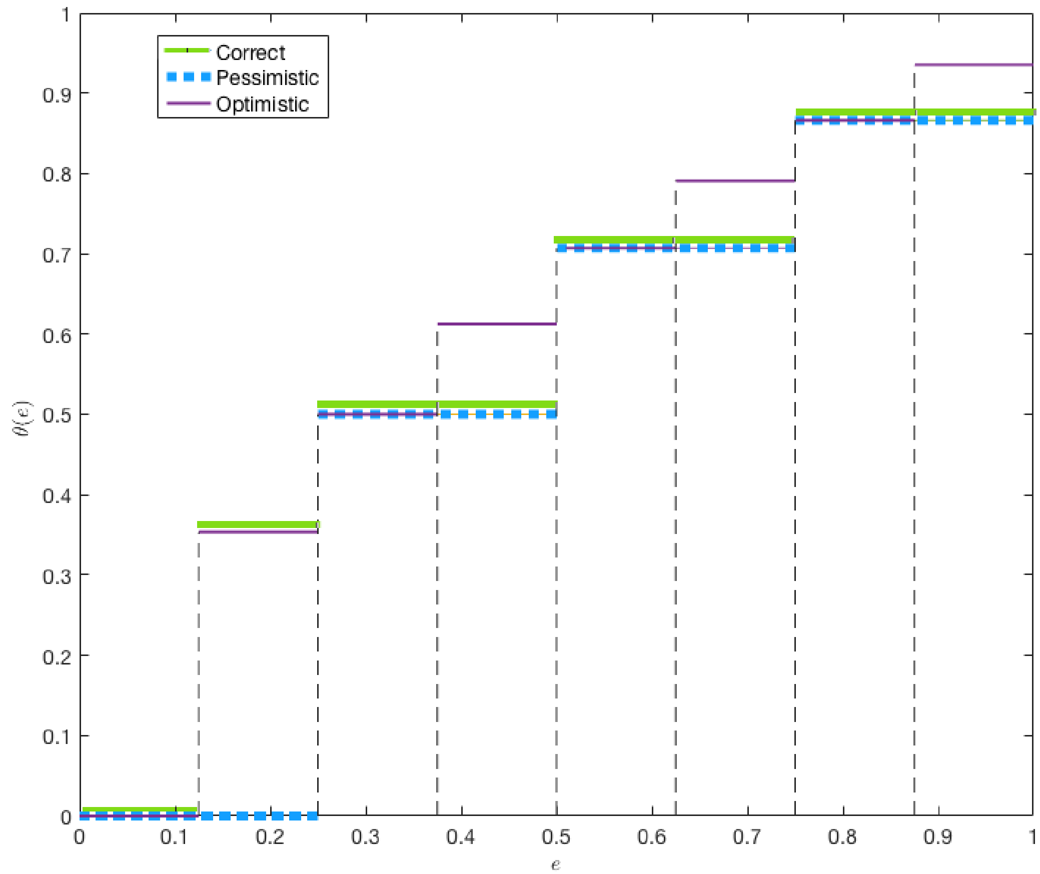

Figure 10: Example of objective, pessimistic and optimistic beliefs on returns to education

community college education (for reasons discussed in the introduction). On the other hand, the optimist believes in 8 different levels of educational investments with some of these not being as productive as believed in.

Finally and importantly, such a lumpy educational investment profile could arise endogenously as an optimal decision made by students facing uncertainty about the true return function and costly information acquisition along the lines of rational inattention literature. In such a case, lumpy educational investment would be optimal even if the optimal profile without rational inattention were continuous. For more on this, see the work of Matejka and Sims (2011), Matejka (2015) and Ellison and Macaulay (2017).

Equilibrium

Now, I want to argue that the competitive equilibrium associated with subjective beliefs $\mathrm{E}^S(\theta(e))$ (as in the Assumption 2) is self-confirming in the sense of Fudenberg and Levine (1993). To make this clear, I first spell out the notion of self-confirming equilibrium (SCE) in the setting outlined.

Definition 4. The SCE is defined as decisions (c_1^*, c_2^*, b^*, e^*) and the student's subjective belief $\mathrm{E}^S(\theta(e))$ on returns to education such that given the numbers $\{r, w\}$:

1. (Maximization) Agent maximizes her expected life-time utility:

$$\max_{c_1^*, c_2^*, b^*, e^*} u(c_1) - v(e) + \beta \mathrm{E}^S u(c_2) \tag{65}$$

   subject to the natural borrowing constraint[73] $b \geq B$ and budget constraints:

   (a) in $t = 1$: $c_1 + b + 1_{e>0} \cdot F \leq \omega$;

   (b) in $t = 2$: $c_2 \leq (1+r)\, b + w \cdot \theta(e)$

2. (Self-confirmation) Agents' beliefs on returns to education are confirmed in equilibrium, i.e. $\mathrm{E}^S(\theta(e^*)) = \theta(e^*)$.

3. (Experimentation) Agents can realize that their beliefs are wrong by deviating from the equilibrium strategies, i.e. choosing $e' = e^* \pm \epsilon$ produces a return of $\theta(e') \neq \mathrm{E}^S(\theta(e'))$.

The equilibrium choices of borrowing/saving and educational effort are characterized by conditions similar to (4)-(5) with subjective expectation operators:

$$b^* : u'(c_1^*) = (1+r)\, \beta \mathrm{E}^S u'(c_2^*) \tag{66}$$

$$e^* : \ u(c_1^* \mid e_i^*) - v(e_i^*) + \beta \mathrm{E}^S u(c_2^* \mid e_i^*) \geq u(c_1 \mid e_j) - v(e_j) + \beta \mathrm{E}^S u(c_2 \mid e_j) \ \ \forall j \neq i \tag{67}$$

In terms of the example in Figure 10, the optimal decision of a pessimistic student will be to choose e at one of the levels of 0, 0.25, 0.5 or 0.75. Notice that the above discussion implies that:

Proposition 7. *The allocation associated with pessimistic subjective beliefs as in Assumption 2 constitutes a self-confirmed equilibrium.*

[73] Strictly speaking, the associated natural borrowing limit is also a function of the subjective and objective beliefs. As it is still the case that it will not be binding in equilibrium, this fact does not matter for further analysis.

In particular, because the subjective and objective beliefs coincide at the points of discontinuity, the beliefs will be confirmed in equilibrium (i.e. $\mathrm{E}^S(\theta(e)) = \theta(e)$).

Furthermore, had the student experimented by pursuing an off-equilibrium educational investment decision, she could find out that her subjective beliefs were wrong. Returning to the example in Figure 10, assuming parameter values such that that the optimal decision with pessimistic beliefs is to invest at the level of $e^* = 0.25$, an investment of $e' = 0.25 - \epsilon$ would produce a return of $\theta(e') \neq \mathrm{E}^S(\theta(e'))$ refuting the subjective belief. If education was to be taken again (e.g. at a higher level) or the student was to have offspring facing a similar schooling decision problem in the future, this could incentivize her to learn more about the true returns to education (see the Appendix B for discussion of some possible learning algorithms).

Notice that a pessimistic subjective belief system could in principle result in students choosing education at levels both above or below the ones with rational expectations. Nonetheless, I refer to such beliefs as pessimistic due to the very fact that such a decision would be due to underestimating returns to education at lower levels and as such would lead to a strictly lower lifetime utility.

In case of an optimistic student, however, the equilibrium labor income realization upon choosing an overly optimistic level of educational investment (e.g. the level of $e = 0.625$ in Figure 10), will reveal the student's optimistic bias. However, this will not be the case for pessimists as in equilibrium they will receive labor income confirming their prior.

3.3 Conclusion: empirical implications

Institutional design and allocation of human capital

Design of education systems varies tremendously from country to country. This is especially the case in Europe, where in some regions, such as e.g. eastern Europe, France, the Mediterreanean and Scandinavian countries, primary and secondary education is mostly universal, without much specialization at early stages. On the other hand, institutional design in countries such as e.g. Austria, Germany, Lithuania, Switzerland or the Netherlands is characterized by highly developed apprenticeship systems streaming their pupils into particular occupations at a very early stage. Similarly, while in some countries the quality of higher education is fairly comparable across most of the public universities (e.g. in Germany), in others there is a small set of elite universities that strictly dominate the rest of the higher education system (e.g. in France or the UK).

Needless to say, these differences in "discreteness" - i.e. how easy it is to reach a higher level of qualification, or to reverse one's previously chosen education path - may have serious implications for the allocation of human capital. To this end, let us re-interpet Figure 10 as presenting education systems in three countries: a country with a least "discrete" education system ("optimistic" profile in Figure 10), a country with a most "discrete" system ("pessimistic" profile in Figure 10), and a country in-between ("correct" profile in Figure 10). With this interpretation in mind and assuming that the distribution of initial ability is continuous and the same across countries,[74] the model outlined above suggests that the more "discrete" the institutional design of a country's education system, the more it will be characterized by: (i) a higher degree of human capital concentration; and (ii) larger discrepancies between the observed human capital and the underlying initial ability distributions. Notice that this result holds regardless of the correctness of students' beliefs (as long as they are non-degenerate).

Secondly, let us re-interpret Figure 10 as presenting: the education system in a country with a less "discrete" education system ("optimistic" profile in Figure 10); a country with a more "discrete" one ("pessimistic" profile in Figure 10); and a common subjective belief about returns to education ("correct" profile in Figure 10). In such a case, the model suggests that the degree of human capital *investments* misallocation should be significantly higher in countries with more rigid (or "discrete") education systems. Moreover, self-improvement of these misallocations may be particularly difficult to achieve due to the self-confirming nature of pessimistic biases.

Identification of underlying skill distributions

As a second implication, the self-confirming nature of beliefs combined with an endogenous human capital accumulation process has a direct implication for academic research. Following the insights of Saez (2001), it is a common practice nowadays in public finance to infer true underlying skills from observed income distributions. The results above imply that without pinning down the beliefs of relevant parts of population affecting their human capital accumulation decisions, the true underlying skill distribution cannot be identified (but rather its lower bound).

[74]This assumption is arguably sensible in the case of European countries where populations have been interacting and mixing with each other for many centuries.

Therefore, for instance, it seems like a worthwhile endeavour to verify the optimal policy prescriptions derived in the literature when the downwardly biased beliefs are properly accounted for. While this is obviously not to say that we should tax or subsidize people differently based on their past mistakes, it might be worthwhile to investigate the changes to optimal policy prescriptions and the size of social welfare losses due to a combination of the two forces.

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3.4 Appendix A: Theoretical implications

Growth

Constraints on human capital accumulation imposed by pessimistic beliefs may have important implications for long-run economic growth. This is especially so in light of a dynamic complementarity between human capital accumulation and firms' technology adoption (or R&D) demonstrated by Lloyd-Ellis and Roberts (2002) and Stokey (2017). On one hand, without continued technological progress workers would not find it profitable to acquire new skills. On the other, in the face of a shortage of appropriate skills required for operating new technologies, firms would reduce their technological investments. As a direct implication, pessimism would slow down the rate of the economy's long-run growth rate.

Political economy

Piketty (1995), Alesina and Angeletos (2005) and Benabou and Tirole (2006) develop political economy models where workers face the identification problem of distinguishing the impact of effort and luck on the income received. This leads some dynasties to believe that their outcomes are mostly due to the luck component, and some that it is more due to their effort. As a consequence, these dynasties have endogenous preferences for more or less redistributive policies (e.g. on public education provision) which feed back into their effort decisions leading to a set of self-confirmed equilibria with different efficiency and inequality levels. Given that students' and parents' expectations are an important factor determining educational investments (Wiswall and Zafar, 2016; Bleemer and Zafar, 2018; Hastings, Neilson and Zimmerman, 2018; Belfield et al. 2019), and that the latter have significant impact on the level and variance of labor income (see e.g. Meghir and Pistaferri, 2004), the combination of misinformation about returns to education with the just discussed political economy consequences may lead not only to implementation of sub-optimal education policies, but also to a sub-optimal design of the general welfare system at large.

3.5 Appendix B: Robustness

In what follows, I firstly discuss robustness of the self-confirmed equilibrium result to deviations from the Assumption 2; and secondly I discuss broadly some dynamic environments in which the self-confirmed beliefs may well survive in spite of students having ways to learn the truth. Obviously, the purpose of this exercise is certainly not to argue that such beliefs will necessarily persist in the long-run, but rather to complement the applied literature by pointing out the critical groups[75] and contingencies where such informational interventions may be particularly needed.

Robustness to assumptions on subjective beliefs

An alternative way of defining pessimism is to consider a subjective profile of returns that is strictly below the objective. With pessimism defined in such a way, the student will always be positively surprised by her returns to education, breaking the result of self-confirmation. To this end, notice that in the above the student was assumed to have a degenerate unitary belief system assigning probability 1 to one particular education return function. However, it might be more realistic to assume that students' prior on returns to educational investments consists of a distribution of return functions with corresponding subjective probabilities.[76] In such a case, the self-confirmed equilibrium becomes obviously more robust as with non-degenerate beliefs the educational outcome would have to coincide with only one of the return functions from the distribution of prior subjective expectations.

Robustness to introducing learning algorithms

Consider first a version of the baseline model extended by overlapping generations living for two periods each. In such an environment, one can think of multiple cross-generational information transmission mechanisms from parents (or from neighbourhood role models) to young offspring. One example could be social learning through observed market outcomes in a richer environment with endogenized wage rates[77] and agents differing w.r.t. their inborn ability and parental wealth. Interestingly, if we assume the inborn ability to affect the returns to educational investments and its distribution Ω to be unobservable (to students

[75] Chetty and Hendren (2017a, b) identify such critical groups and regions in the US by showing that the quality of a neighborhood and county where children grow up has causal impact on their college attendance and future earnings, among others.

[76] A source of such a diffused prior could be e.g. uncertainty about one's innate ability or imperfect information about market outcomes of similar individuals.

[77] One way of generating wage rates that are endogenous to supply of skill in the economy could be by assuming existence of competitive firms that have to employ e.g. two types of workers (high- and low-skilled) that are imperfectly substitutable.

themselves), then conditional on the realized distribution of θ's and vector of market wage income $` \cdot \mathbf{w}$, every agent with a wrong subjective belief can come up with a wrong prior $\tilde{\Omega}$ that is consistent with her subjective belief. In particular, after observing market outcomes that are inconsistent with her subjective belief, a pessimistic student may revise her prior belief on the inborn ability distribution in a way that supports the self-confirming equilibrium. The same can be said about social learning from the realized human capital levels θ: if a student is surprised by high accomplishments of her friends, she may similarly revise her prior beliefs about the friend's inborn skill in a way that supports her own pessimism.

Peer effects, defined as the impact of others educational effort on a student's own human capital level (i.e. a model with $\frac{\partial \theta_i(\mathrm{e})}{\partial e_j} \neq 0$, $i \neq j$ and $\mathbf{e}$ being a vector of population's efforts), constitute another candidate mechanism for breaking the self-confirmed pessimism. In principle, going to college with a high quality student body may enable some individuals to achieve a career they initially did not even think of. However, this need not necessarily be true if the underlying returns to education are a step function and the peer externalities are weak enough, failing to push the pessimistic student to the next notch of human capital level. Clearly, in such a case pessimism could persist again.

Finally, students may also have an opportunity to acquire information about returns to education but processing it can be costly, as in models of limited attention with discrete choices (such as educational investments) by Matejka and McKey (2015). If education is the only choice variable whose outcomes are uncertain, a model with agents differing w.r.t. wealth should lead the poorer students to acquire more information as their marginal utility of consumption will be much higher in expectation. In such a case, the pessimism should disappear over time (at least on the side of poor households).

However, it is arguably more realistic to consider environments with a trade-off in allocation of attention between consumption and education margins. This could be thought of as uncertainty about product prices in shops, uncertainty about the choice of a travel agency offering the best value for money holidays, or simply a difference between consumption preferences in the short- and long-run. In such a case, the households would have to allocate their attention optimally between these two margins. Since the variation in consumption price would be much more important for the poorer households, they would allocate less attention to information acquisition on the true returns to education, as compared with richer households. In such a case, pessimism would not only persist but could also become deeper relative to the wealthier (and potentially more optimistic) part of the population.

4 Can Welfare Abuse be Welfare Improving?

4.1 Introduction

In the late 1970s the labor markets in the US and Europe began to diverge and these differences are profoundly visible until today. Unsurprisingly, this contrasting evolution has attracted interest of many economists. Among many topics related to it, arguably the most attention has been devoted to unemployment insurance (UI) systems (vide e.g. the research program ran by Ljungqvist and Sargent). In this paper, I model one particular aspect of UI which differs strikingly between the two continents: the benefit entitlements for workers quitting jobs voluntarily. While in the US no quitter is eligible for receiving unemployment benefits[78], the entitlement policy in Europe is generally more generous and usually allows for payment of benefits in such cases subject to some sanctions. The exact requirements and sanctions have been described by Venn (2012). In general, there is a fixed work experience (or rather a social security contribution) requirement which is the same for both fired workers and quitters - usually it varies between 6 to 18 months of employment within the last 12-36 months preceding unemployment. On top of it, in order to discourage quitting, there are sanctions[79] in form of payment suspensions: in Lithuania and Slovakia there are no such sanctions, in Denmark there is a 3-week sanction, in Austria - 4; in Belgium - 7; in Sweden - 9; in Germany - 12. Nevertheless, there are also European countries not paying out the benefits for voluntarily unemployed, like Estonia, Italy or the Netherlands. To the best of my knowledge, there is no research analyzing the welfare effects of these policy choices. This paper is trying to fill this gap.

To this extent, I construct and calibrate to the US labor market a job search model where fired workers are eligible for time-limited UI and ask what is the optimal entitlement policy for voluntarily unemployed, i.e. whether such quits should be punished by no UI entitlement or, if not, for how long should such workers be employed before being awarded UI entitlement. In order to pick the best policy I perform a social welfare analysis. This is a natural approach as it requires a consistent accounting for both benefits (such as more time and resources available for job search) and adverse incentive effects of the UI (such as work-

[78] Some states in the US allow quitters to apply for benefits if backed with a "good cause". Nevertheless, as Venn (2012) reports, most (including e.g. the seven largest states where almost 50% of the U.S. population lives) do disqualify all the voluntary quits.

[79] These sanctions are often not executed if the employer does not contest worker's UI claim.

ers being more picky, generating possibly higher unemployment rate and consequently higher tax rate to finance the welfare system).

In particular, when an *ex-ante* homogenous worker becomes unemployed she consumes her savings or unemployment benefits (defined as the replacement ratio tied to her most recent wage). Furthermore, in order to find a job, she exercises costly search effort. The search effort is random meaning that although the expected wage offer is increasing in amount of the search effort exercised, some unemployed are luckier and receive higher wage offers than others. Consequently, workers set optimally their reservation wages and reject all the wage offers below it. Moreover, some employed workers find it optimal to behave opportunistically and quit their jobs. Thus, workers become *ex-post* heterogenous with respect to their employment status, wage income received and savings.

The latter means that there are some jobs in the economy which workers enter solely in order to build up their saving accounts and (if the policy allows for it) regain eligibility for the benefits, quit the job short after and search for a better one thereafter. I refer to this opportunistic behavior as a welfare abuse since if the search effort was perfectly observable, workers would clearly exercise a higher effort and so in such a case the policy of entitlement for quitters might be unnecessary. Consequently, this moral hazard related behavior results in an excessive use of the welfare system and thus a higher tax burden on employed workers.

In fact, there is evidence that workers do behave as predicted by the model employed in this paper. First of all, although quitters in the US are not entitled to the UI, on average 10% of unemployed workers are job leavers (according to the CPS data set). Moreover, Christofides and McKenna (1996) studied data from Canadian Longitudinal Labour Market Activity Survey for 1986/87 and found a significant increase in the job separation probability in the week right after a worker satisfies unemployment benefit eligibility. This finding was later confirmed by Green and Riddell (1997) and Baker and Rea (1998) who studied the same data for the year 1990. Similarly, Jurajda (2002) studied the US labor market and found that entitlement for unemployment insurance significantly increases the probability of a lay-off. Importantly, these studies do not look explicitly at voluntary quits. Nevertheless, given that we should not always blindly believe in a dichotomy between lay-offs and voluntarily quits (as discussed for example by Feldstein (1976)), it is surely possible for many quitters to pass themselves off as being fired. However, it also seems very reasonable that there is still a significant share of quits due to personal reasons of the employees (especially in labor markets where quitters receive benefits). In what follows, I am modeling the latter phenomenon where there is a clear distinction between the two groups.

Furthermore, in the model presented below workers behave opportunistically in order to improve upon the match quality. Indeed, Tatsiramos (2009) presents empirical evidence for the role of unemployment insurance in correcting the misallocations in labor markets: he finds that for workers entitled to receiving benefits the subsequent employment spells are longer and that this relationship is more profound in countries with relatively more generous welfare systems.

Results suggest that, in spite of the associated fiscal costs in form of a higher unemployment rate and so a higher tax rate, the optimal policy is characterized by entitlement to UI for quitters. In particular, pursuing a generous entitlement policy leads to long run welfare gains equivalent to 4.38% of life time consumption. Importantly, these results should be robust to the possibility of quitters passing themselves off as being laid-off, as surely not every worker is able to do this and as firing a worker is associated with non-negligible firing costs (for example in the US the unemployment insurance tax is experience rated). The intuition for the result is two-fold. Firstly, as already mentioned, the policy allows for average match quality improvement. It does so by reducing the income risk associated with quitting a job in order to look for a better one. Secondly, it extracts many long-term unemployed into employment by increasing the non-wage value of low paid jobs and so by lowering the reservation wage of those workers.

Interestingly, the results of the model also suggest that the policy studied here may be a force pushing characteristics of the US labor market towards the European one. Firstly, following the optimal policy generates a higher unemployment rate. This is due to the fact that next to fired workers, the policy increases the mass of voluntarily unemployed ones. Secondly, it reduces both the pre- and after-tax income inequality (i.e. there is no efficiency-equity trade-off). This is due to two effects induced by the entitlement policy: (i) a significant reduction in mass of unemployed on social assistance; and (ii) an increase in the budget balancing tax rate bringing the income of employed individuals closer to the income of unemployed. Thirdly, the average match quality post-reform is higher, in line with evidence in Manacorda and Petrongolo (1999) that the labor market mismatch has grown much faster in the US as compared to Europe.

Moreover, I investigate the assumption of monetary (or non-separable) search costs which is mostly ignored in the literature. Results show that this assumption is able to generate the empirically documented spike in search effort at the benefit exhaustion. Furthermore, it also generates an initial decrease in search effort at the beginning of unemployment spell - in line with the recent evidence in Faberman and Kudlyak (2016). On the other hand, as is already acknowledged in the literature (see e.g. Krueger and Mueller (2010)), the usually employed in the literature assumption of separable search costs does not deliver such features. Significantly, as opposed to the latter assumption, increasing generosity of unemployment benefits in the model with monetary search costs does not necessarily decrease the search effort. Finally, the model generates important testable implications about differences in search effort and reservation wage behavior, and so in labor market outcomes for similar workers differing only with respect to their financial wealth.

My paper builds on a long literature of unemployment insurance. While the most common rationale for the payment of unemployment benefits is to provide risk averse workers with income insurance allowing for consumption smoothing, there is also a smaller strand of research work starting with Burdett (1979) which does not see the unemployment insurance solely as a serious distortion but rather argues for the role of insurance as a subsidy to search. In this literature the role of unemployment insurance is not only to give unemployed the time and resources to find a job but also to find the *right* one, i.e. it allows the workers to improve upon the quality of matches in labor markets. In this paper, I argue for a similar role of unemployment insurance.

While searching for reasons of labor markets divergence, Ljungqvist and Sargent (1998) argued that although in times of low micro-economic labor volatility the presence of unemployment insurance system has moderate impact on the unemployment rate, the systems which are relatively more generous may have a much more profound effect on the number of unemployed in times of high turbulence. In a more recent contribution, Kitao, Ljungqvist and Sargent (2015) explain this divergence with higher minimum wages in Europe and human capital depreciation during unemployment. On the other hand, Marimon and Zilibotti (1999) used a model with both heterogenous workers and firms, search frictions and skilled-biased technological change coupled with the assumption of complementarity between capital and capital-specific-skills to show that the differences in generosity of unemployment systems may account for the observed discrepancies between the US and European labor markets. In particular, they showed that although upon the technology-specific shock the economy with more generous unemployment welfare system has a higher unemployment rate, it is characterized by a higher quality of matches, i.e. a higher growth of productivity per worker and a relatively lower wage inequality - a result complementary to the one in Ljungqvist and Sargent (1998). In this work, I identify a concrete real world policy which may be a channel of effects similar to these described in Marimon and Zilibotti (1999).

As I assume perfect distinction between lay-offs and quits, this paper is complementary to Hopenhayn and Nicolini (2009) where they derived an optimal unemployment insurance design under assumption that principal cannot distinguish quits from lay-offs. This assumption generates an opportunistic worker behavior similar to the one imputed in my paper. Their conclusion is that under the latter assumption the optimal contract involves conditioning of the benefit eligibility on worker's employment history.

Furthermore, the result of no efficiency-equity trade-off derived in this model is analogous to the one reached by Acemoglu and Shimer (1997) in a general equilibrium search setup. In their model, unemployment insurance induces risk averse workers to seek higher paying jobs with higher unemployment risk and so also induces firms to invest into higher paying technologies. In other words, the unemployment insurance increases both the output and improves the risk sharing.

Finally, Aguiar, Hurst and Karabarbounis (2013) and other authors mentioned therein argued that modeling consumption and leisure in a non-separable way is important for explaining a wide variety of macroeconomic observations in business cycle models. Thus, the differences in the worker behavior generated by the two assumptions are complementary to this literature and suggest that the same is also true when it comes to explaining workers' search behavior.

This paper is organized as follows. The next section outlines the theoretical model together with measures used for evaluating the policy experiments. In Section 4.3, I calibrate the model to the US labor market. Section 4.4 discusses the results. Finally, Section 4.5 concludes.

4.2 Model economy

The economy consists of a continuum of *ex-ante* identical, risk-averse, infinitely-lived agents with measure normalized to 1. Workers have access to risk-less saving accounts and UI system. Time is discrete and in every period an unemployed worker receives with some probability a wage draw. This probability depends on the amount of random search effort chosen by the worker. After the draw, she has to decide whether to accept the job or not. Employed workers make a decision about quitting or staying on the job.

4.2.1 Workers

Working does not yield disutility. Any worker can be either unemployed or employed and is maximizing her discounted life-time utility with respect to (1) savings a, the level of unobservable random search effort required to find the job q (when unemployed only, i.e. there is no on the job search - see the discussion of this assumption in Section 4.4) and then, if she draws a wage offer from some distribution, whether to accept it; or (2) savings a and the decision about staying on the job or quitting it in order to search for another. When employed she faces a risk of an exogenous separation happening at the Poisson rate σ.

Workers have a common instantaneous CRRA utility function $U(x) = \frac{x^{1-\theta}}{1-\theta}$. When unemployed they exercise search effort q which is subject to convex costs given by $g(q) = \alpha q^{\zeta}$, where $q \in [0,1]$. The mapping from worker's effort to the effective transition probability is governed by an identity function $f(q) = q$. Importantly, searching is unobservable giving rise to standard moral hazard of suboptimal search effort.

In each period an unemployed individual faces a stochastic employment opportunity: either she is offered a job opportunity for wage w or not. There are n different wage offers that the worker may draw and their support is on the $[0,1]$ interval. Denote this wage distribution by F. To explain this heterogeneity in wages, just think of n different technologies and of many firms having access solely to one of them. Also, I assume that the wage rate received by an employed worker is constant over time. Finally, monitoring of job applicants is impossible and therefore a worker who rejects a work opportunity continues to receive unemployment benefits according to her benefit payment path (for description of UI see Section 4.2.2).

Apart from the UI, workers have access to risk-less saving accounts with a constant rate of return r. Considering this channel is important as it allows workers to self-insure against the state of unemployment and so it may have some first-order effects on the worker's search and offer acceptance behavior (and so on all the other equilibrium outcomes).[80]

Importantly, I assume that the cost of search effort is in terms of consumption,

[80] See e.g. Abdulkadiroglu, Kuruscu and Sahin (2002) for discussion of the role of self-insurance for welfare conclusions.

i.e. worker's utility function is of the form $U(c - g(q))$. In this model, the role of unemployment benefits is not only to insure workers against the state of unemployment but also to provide them with a subsidy to search so that they can find the *right* job. Notice that this formulation introduces wealth and substitution effects between consumption and search effort. In particular, workers 1) increase their search effort if the wealth effect dominates; or 2) reduce it (in order to smooth consumption) if the substitution effect dominates. The wealth effect is especially strong right before the benefit exhaustion. At that moment workers want to avoid falling into the badly paid state of social assistance implying a significant increase in probability of a long-term unemployment lock-in (see Section 4.4 for a detailed discussion).

Moreover, this assumption is realistic in two ways. First of all, searching for a job costs money as well as time or physical effort. Any unemployed worker that wants to find a job has to buy and maintain a suit, travel for an interview, send out applications on nice paper or get some professional training. Secondly, many consumption expenditures such as a comfortable car or a home computer with internet access are complementary to job search. Such a modeling assumption was employed solely for tractability with CARA utility by Werning (2001) and also by Shimer and Werning (2007).

Significantly, the mechanics generated by the assumption of monetary search costs employed here are supported by the empirical evidence documented by Blau and Robins (1990), Wadsworth (1991) and more recently by Krueger and Mueller (2014) who found that unemployed workers eligible for unemployment compensation search more actively than those not eligible. Similarly, as already mentioned, the assumption generates a spike in search effort at the benefit exhaustion. Nevertheless, there are results speaking against the monetary search cost as for example Jones (1989) or Krueger and Mueller (2010) who find that higher benefits reduce the time devoted to search among benefit recipients. However, with unemployment benefits defined as a replacement ratio, such a behavior could stem from the fact that workers receiving higher benefits (and so having worked in better jobs) find themselves in a more favorable search environment not requiring that much of a time investment (e.g. due to knowing well connected people).

Finally, note that typically in the literature search effort is modeled using the separable utility function of the form $U(c) - g(q)$, which can be interpreted in terms of physical effort. Since there is no research on the degree of substitutability of money and time devoted to job search (in the extreme think of wealthy individuals employing headhunters), both should be seen as reasonable modeling assumptions. Therefore, for comparability with other papers and exposition of the dynamics induced by the two assumptions, I also investigate all the results in the model with separable search cost. As I discuss in Sections 4.3 and 4.4, such a model is not able to match the time profile of search effort documented empirically. In particular, it cannot generate search effort spike at the benefit exhaustion. On the other hand, the monetary cost of search can fully explain this behavior.

4.2.2 Unemployment system

The design of the unemployment insurance system together with search frictions and exogenous separations are the source of *ex-post* heterogeneity among the workers. It is financed with linear taxes raised by the government running a balanced budget. The tax distorts the decision about the search effort chosen. Any unemployed worker whose match was separated exogenously qualifies for benefits. Note that below I calibrate the model to the exogenous separation probability of 0.1 quarterly. This implies that in expectation workers are fired once in 2.5 years. This work experience satisfies labor attachment requirement in virtually every country.

The worker who decides to quit the job voluntarily is eligible for benefits if she had worked in her last job for at least $\hat{T}$ periods (policy experiment parameter); otherwise she receives the social assistance income i_{SA}. The latter value is also financed by the government and pins down the value of worker's outside option. Any unemployed and eligible worker receives the value of $b = \gamma w$ for T periods, where w is the worker's most recent wage. From period T onwards, she receives the income of i_{SA} until she finds a new job.

4.2.3 Recursive formulation

Given Sections 4.2.1 and 4.2.2, each worker's current state can be captured with a vector $s = (a, t, w, x)$ of four state variables:

1. Amount of savings a in worker's account.
2. Worker's most recent[81] wage $w \in \{w_1, \ldots, w_n\}$.
3. Worker's current employment status x: if employed $x = \mathrm{e}$, or if unemployed $x = \mathrm{u}$.

[81] This state variable also captures the current wage of employed workers.

4. Time t spent in current stage. Notice that for unemployed workers after the drop in the benefit schedule from high to low, i.e. in period $T+1$, the environment becomes stationary due to the benefit schedule being constant from that point onwards. A similar argument applies to the employed workers. Therefore, $t \in \left\{1, 2, \ldots, \max\left\{\hat{T}, T+1\right\}\right\}$. In particular, if a worker has been employed for $t < \hat{T}$ periods and decides to quit, she immediately jumps from the state (a, t, w, e) to the low benefit state $(a, T+1, w, u)$. However, if a match is separated exogenously or endogenously after at least $\hat{T}$ periods on the job, then the unemployed worker is entitled to benefits, i.e. she lands in the state $(a, 1, w, u)$.

Therefore, the following Bellman equations hold for unemployed and employed workers:

$$
\begin{aligned}
V_u\left(a,t,w\right) &= \max_{\mathrm{a}',\mathrm{q}}\left\{U\left(b\left(t,w\right)+\left(1+r\right)a-a'-g\left(q\right)\right)\right\} \\
&+ \max_{\mathrm{a}',\mathrm{q}}\left\{f\left(q\right)\beta\sum_{w'}\max\left\{0, V_e\left(a',1,w'\right)-V_u\left(a',t',w\right)\right\}\mathbb{P}\left(w'\right)\right. \\
&\left. +\beta V_u\left(a',t',w\right)\right\}
\end{aligned} \tag{68}
$$

where $t' = \min\left\{t+1, T+1\right\}$.

$$
\begin{aligned}
V_e\left(a,t,w\right) &= \max_{\mathrm{a}'}\left\{U\left(\left(1-\tau\right)w+\left(1+r\right)a-a'\right)\right\} \\
&+\max_{\mathrm{a}'}\left\{\beta\left(\sigma V_u(a',1,w)+\left(1-\sigma\right)\max\left\{V_u(a',t'',w), V_e(a',t^{\dagger},w)\right\}\right)\right\}
\end{aligned} \tag{69}
$$

where $t'' = \begin{cases} 1 & \text{if } t = \hat{T} \\ T+1 & \text{if } t < \hat{T} \end{cases}$, $t^{\dagger} = \min\left\{t+1, \hat{T}\right\}$.

Finally, notice that given the setup, the model possesses the *reservation wage property*. Given the design of the unemployment system, the reservation wage (just as the effort exerted q) depends on the three state variables: savings a, length of unemployment spell t and last wage w.

Proposition (reservation wage property): Consider a worker exercising search effort to find a job in the market described in Section 4.2.1 and facing unemployment system described in Section 4.2.2. Then the optimal job search strategy of such a worker has a reservation wage characterization conditional upon worker's current state: the worker will accept a job if and only if the wage draw w' is weakly greater than her reservation wage, i.e. $w' \geq \bar{w}(a,t,w)$.

Proof. See the Appendix A.

4.2.4 Steady state equilibrium and government

In the steady state, the measure of workers in each of the states is constant over time. Let D_u and D_e be the cross sectional distributions over the states of all the (un)employed workers in the economy and let $d_u(a,t,w)$ and $d_e(a,t,w)$ be the mass of (un)employed currently in a given state with $\sum_{a,t,w}(d_u(a,t,w)+d_e(a,t,w)) = 1$, i.e. the two are the associated probability mass functions. Steady state is characterized by an invariant cross sectional distributions D^* (and probabilities d^*) such that $D^*\Gamma = D^*$, where Γ is the Markov transition matrix defined precisely in the Appendix E.

Moreover, revenue and expenditures of the government have to be balanced in each period. Thus, I close the model with the following condition:

$$\left[\tau\sum_{a}\sum_{t=1}^{\hat{T}}\sum_{w} wd_e^*(a,t,w)\right] = \left[\gamma\sum_{a}\sum_{t=1}^{T}\sum_{w} wd_u^*(a,t,w) + i_{SA}\sum_{a}\sum_{w} d_u^*(a,T+1,w)\right] \tag{70}$$

Equation (70) equalizes the government revenue (equal to the taxable portion of the income of employed workers) with the expenditure of the government (equal to the measure of unemployed receiving unemployment benefits and social assistance income multiplied by the expenditure).

4.2.5 Welfare, inequality and unemployment duration measures

In order to rank each policy choice $\hat{T}$ given a budget balancing tax rate τ, I use a standard measure of life time consumption equivalent changes in the aggregate welfare. It shows by how much the agents' average consumption should change such that the utilitarian welfare in the no entitlement for quitters scenario is equal to the utilitarian welfare associated with a given policy reform. Mathematically:

$$Welfare = \left(\frac{\sum_{a,t,w,x}\lambda_1(a,t,w,x)\,v_1(a,t,w,x)}{\sum_{a,t,w,x}\lambda_0(a,t,w,x)\,v_0(a,t,w,x)}\right)^{\frac{1}{1-\theta}} \tag{71}$$

where θ is the coefficient of risk aversion, λ_i is the steady state distribution of agents and v_i is the associated value function with $i=0$ standing for the status quo of no entitlement for quitters and $i = 1$ standing for a given policy reform.[82]

Furthermore, in order to measure (pre- and after-tax) income inequality associated with each policy, I use the Gini coefficient given by:

$$Inequality = 1 - \frac{\Sigma_{i=1}^{n} \mathbb{P}\left(y_i\right)\left(S_{i-1} + S_i\right)}{S_n} \tag{72}$$

where y_i denotes (pre- or after-tax) income, $S_i = \Sigma_{j=1}^{i} \mathbb{P}\left(y_j\right) y_j$, $S_0 = 0$ and $y_i < y_{i+1}$. A coefficient of 0 means perfect equality.

Finally, I propose the following measure of expected mean unemployment duration:

$$UD = \sum_{a} \sum_{t=1}^{T+1} \sum_{w} \left(t + \sum_{\hat{t}=t+1}^{\infty} \left(\hat{t} \prod_{\tilde{t}=t}^{\hat{t}-1} \mathbb{P}\left(w' < \bar{w}\left(a, \min\left\{\tilde{t}, T+1\right\}, w\right)\right)\right)\right) d_u^*\left(a, t, w\right)$$

i.e. the weighted (by mass of unemployed workers in given states) average of expected unemployment duration. Each worker's expected unemployment duration is given by the sum of products of possible unemployment periods t and corresponding probabilities of moving into them.

4.3 Calibration

I calibrate the model to properties of the existing UI system in the US. The calibration of the non-separable model described below is later on referred to as the *baseline* calibration. I assume a monthly periodicity. The assumed coefficient of the relative risk aversion is $\theta = 3$. For the exogenous separation probability, I choose a quarterly value of $\sigma = 0.1$ from the Job Openings and Labor Turnover Survey as in Hall and Milgrom (2008). Moreover, since the model employed here abstracts from capital, I target the yearly interest of 2%. This is a half of what is commonly

[82]Note that due to the functional form of the utility function in the separable search costs model, this metric is no longer valid. The adjusted one reads instead:

$$Welfare = \left(\frac{\sum_{a,t,w,x} \lambda_1\left(a,t,w,x\right) v_1\left(a,t,w,x\right) + \sum_{a,t,w,x} \lambda_0\left(a,t,w,x\right) v_0^S\left(a,t,w,x\right)}{\sum_{a,t,w,x} \lambda_0\left(a,t,w,x\right) v_0^C\left(a,t,w,x\right)} \right)^{\frac{1}{1-\theta}}$$

where $v_0^S\left(s\right) = -g\left(q\left(s\right)\right) \cdot 1_{x=u} + \beta \sum_{s'} \Pi\left(s'|s\right) v_0^S\left(s'\right)$ (the search cost component of the value function before the reform), $v_0^C\left(s\right) = u\left(c\left(s\right)\right) + \beta \sum_{s'} \Pi\left(s'|s\right) v_0^C\left(s'\right)$ (the consumption component) and c, q are the optimal decisions.

assumed for calibration of macro models with capital (see e.g. McGrattan and Prescott (2005)).

The discount factor is assumed to be $\beta = 0.918$. The value is chosen to match the evidence on savings of unemployed in Gruber (2001) and Chetty (2008) documenting that 17% of workers entering unemployment report zero gross financial wealth. However, upon excluding unsecured debt, i.e. looking at workers' net financial wealth, this number grows to 50% and moreover many workers below have negative positions. Since unemployment is often an unforeseen event, the second number may be also very relevant for the analysis. Therefore, I aim to replicate the observation of a number between 17% and 50% of newly unemployed workers with or close to zero savings. Importantly, note that although the value of β may seem to be relatively low, it should not be seen as invalid because it is implied by the data. The saving grid consists of 60 equidistant points from $[0, 1.1]$.

w_i	0.0297	0.0383	0.0493	0.0634	0.0815	0.1046	0.1343	0.1724
$\mathbb{P}(w_i)$	0.0022	0.0068	0.0182	0.0407	0.0758	0.1181	0.1541	0.1683

0.2212	0.2840	0.3647	0.4686	0.6026	0.7758	1.0000
0.1541	0.1181	0.0758	0.0407	0.0182	0.0068	0.0022

Table 15: Wage offer distribution (pre-tax)

As far as it concerns the *wage offer* distribution in the US, Hall and Mueller (2015) approximate it as log-normal, and more importantly they conclude that its standard deviation is 0.28. However, once they also account for non-wage values, the dispersion goes up to 0.43. Thus, I pick an average of the two numbers and so I assume the wage offer distribution F to be log-normally distributed with dispersion parameter $\rho = 0.35$ and $\mu = -1.7581$ (s.t. the support is on $[0, 1]$). Also, I assume that there are $n = 15$ different wages in the market. Noteworthy, if we think about some of workers in the economy as e.g. managers and waiters, the assumed variance parameter is very large. In real life such workers are facing different wage distributions with a much smaller dispersion. Nevertheless, the model accounts for this phenomenon to some extent by 1) the mechanism of reservation wages; and 2) the effective restrictions on possible search effort. The resulting *wage offer* distribution is presented in Table 15 and the implied equilibrium *wage* distribution is presented below in Figure 16. Importantly, the associated with the *wage* distribution "mean-min wage ratio" (as proposed by Hornstein et al. (2011)) amounts to 2.71 (i.e. $\frac{0.2840}{0.1046}$). This number falls between 2.54 (i.e. $\frac{\$38,428}{\$15,080}$) and 2.79 (i.e. $\frac{\$42,068}{\$15,080}$) which are the corresponding median-min wage ratios[83] in the US between 2009 and 2015 (computed using the CPS data and the minimum wage rate of USD 7.25 per hour as introduced in 2009).

Following the review of the UI system in the US by Pavoni and Violante (2007) and Moffit (2002), I take $\gamma = 0.6$, $T = 6$ and $i_{SA} = 0.15 \cdot \bar{w}$. The first number translates into a replacement ratio of 60% of workers' last wage, the second number implies the UI entitlement for a maximum of half a year and then being followed by the social assistance income i_{SA} set to 15% of the median wage in the steady state economy. To pin it down, I take first the average social assistance income in the year 2000 comprising mostly of the Temporary Cash Assistance for Needy Families (TANF) and Food Stamps and amounting to approximately USD 879 for a family of three (as documented by Moffit (2002)). After accounting for the family size and the median[84] personal income in the US in the year 2000 (as recorded in the CPS), we end up with the social assistance income amounting to 12.8% (i.e. $\frac{879}{3 \cdot 2300}$) of the median monthly wage income. I choose a number slightly higher in order to allow for some increasing returns to scale within a family. Finally, the baseline calibration to the US implies that quitters are entitled solely to the social assistance income and, in particular, not to the UI. However, in Section 4.4, I will look for an optimal value of the re-entitlement parameter $\hat{T}$.

For the search effort, I choose the grid to consist of $m = 30$ equidistant points from the $[0, 1]$ interval. More importantly, in the case of monetary search costs, parameters (α, ζ) of the cost function g are such that: (i) the baseline model under no-entitlement policy replicates the mean long-run US unemployment rate since 1930s equal to 7.10%; and (ii) the shape of a 'representative unemployed' worker's search effort along the unemployment path resembles the one documented by Krueger and Mueller (2010) for UI eligible (shown in Figure 11.a). By 'representative unemployed', I mean here a 7-period average (weighted by steady states measures of workers in each saving and wage categories) behavior simulation of a newly unemployed worker following her optimal actions conditional on not having found employment. [85]

[83] The bias stemming from using the mean-min wage ratio would be too big because of the very rich in the US.

[84] The bias stemming from using the average would be too big because of the very rich in the US.

[85] The cross sectional unemployed worker behavior would amount to looking at the average

Note that in their work (and as is most common in the literature), the search effort is measured in job search minutes. Since effort in this paper is in monetary units and there is no empirical research on this subject, I aim to match the relative magnitudes of the search effort in different stages of unemployment for eligible workers. Thus, I look for parameters that give growth and decline rates of the model-imputed search effort for the representative unemployed worker between the trough before the benefit exhaustion, the peak at the benefit exhaustion and the trough one month after the exhaustion which are approximately equal to the counterpart ratios in Krueger and Mueller (2010) for the weeks 14, 25/26 and 30. The parameters chosen for the search cost function are $\alpha = 0.092$ and $\zeta = 1.09$.

Another evidence on the job search behavior comes from Faberman and Kudlyak (2016). When combined with the one in Krueger and Mueller (2010), it suggests that the unemployed search behavior until the benefit exhaustion may in fact be U-shaped, and followed by a decline after. In particular, they study relationship between search intensity and search duration using a micro data set containing information on a number of applications job seekers send to vacancies posted on an online job search website and corresponding unemployment spell lengths. Figure 11.b presents their main findings: the number of applications sent initially declines steadily and then starts increasing slowly over time. However, once they control for fixed job seeker characteristics, the number of applications sent monotonically declines over time. At this point two comments are due. Firstly, their data come from the period of Great Recession when the UI benefits were extended to 99 weeks and so it is inapt to capture the spike at benefit exhaustion. Secondly, the fact that number of applications sent declines does not necessarily imply the same about the search effort. The latter is true as when workers apply for ever more jobs, the pool of (relevant) vacancies becomes ever smaller. Notwithstanding, the initial decline in search effort is still very plausible. This is especially likely given that the data for the first 3 months in Krueger and Mueller (2010) contains a lot of noise, and so authors report only average estimates.

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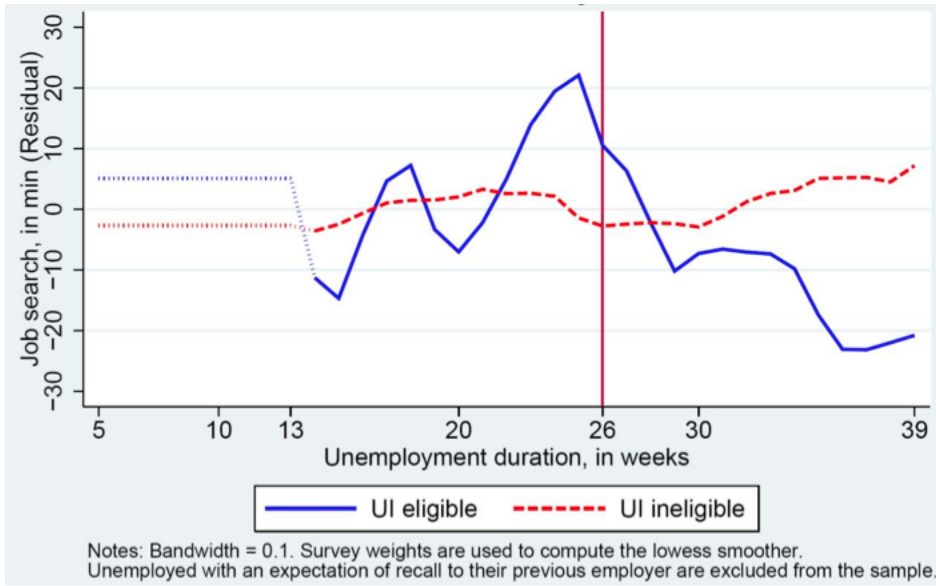
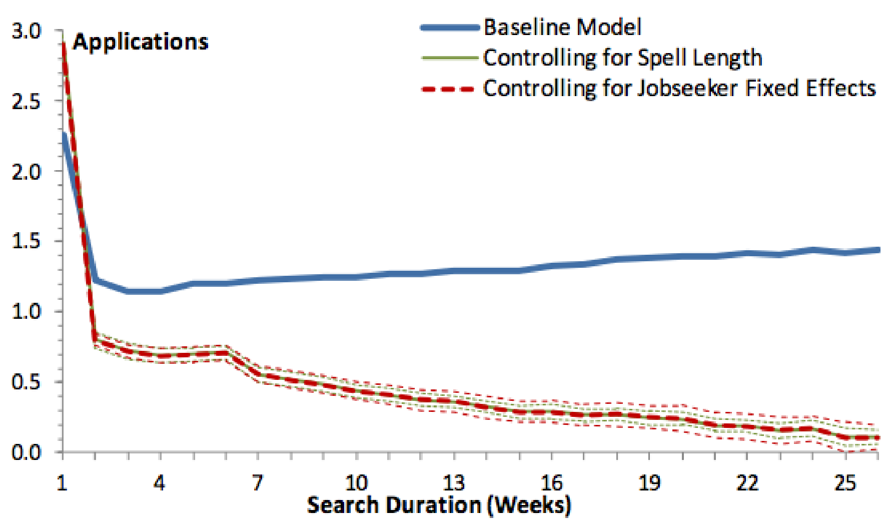
behavior of all the unemployed workers weighted by their measures in the steady state. I use the construct of the 'representative unemployed' in order to control for composition effects in the cross section.



(a) Job search (in minutes) by unemployment duration (source: Krueger and Mueller (2010))
Note: To compute the ratios for calibration, one should add the regression mean of 32.1 to the residuals. Due to noisiness of the data in the first 14 weeks the Figure shows average time allocated to search, from week 14 on the Figure presents LOWESS-smoothed data based on residuals after removing searchers' individual characteristics such as age, sex, etc.



(b) Job search (number of applications sent) by unemployment duration (source: Faberman and Kudlyak (2016))
Note: Dashed lines represent 95% confidence intervals.

Figure 11: Evidence on search effort

parameter	interpretation	value (NS / S)	source/target
β	discount factor	0.918 / 0.94	target savings
r	interest rate	0.1652%	2% yearly interest rate
θ	risk aversion coefficient	3	modeling choice
σ	exogenous separations rate	$\frac{1}{10\cdot 3}$	JOLTS/Hall and Milgrom (2008)
γ	replacement ratio for UI	0.6	UI in the US, Pavoni and Violante (2007)
T	periods of UI entitlement	6	UI in the US, Pavoni and Violante (2007)
i_{SA}	social assistance income	$0.15 \cdot \bar{w}$	SA in the US, Moffit (2002)
n	number of wages	15	modeling choice
m	search effort grid density parameter	30	modeling choice
a_M	savings grid density parameter	60	modeling choice
(a_{min}, a_{max})	savings grid bounds	(0, 1.1)	modeling choice
F	distribution of wage offers	log-normal	Hall and Mueller (2015)
μ	dist. log-scale parameter	-1.7581	support of F on $[0, 1]$
ϱ	dist. shape parameter	0.35	Hall and Mueller (2015)
α	search cost f-n g parameter 1	0.092 / 15.22	target U and search profile
ζ	search cost f-n g parameter 2	1.09 / 2.65	target U and search profile

Notation: NS - non-separable utility, S - separable utility.

Table 16: Parameter values in the model

However, I do not use the evidence from Faberman and Kudlyak (2016) directly for calibration since (i) it is difficult to combine this search effort measure with the other one in Krueger and Mueller (2010); and (ii) for the purpose of this paper the search effort around the benefit exhaustion is much more relevant. Nevertheless, I show in Section 4.4 that the model with monetary search costs and savings generates the discussed U-shape behavior.

On the other hand, for the model with separable search costs, I pick different values for parameters. This is due to the fact that workers are no more facing search effort constraints and so behave differently. Consequently, I assume $\beta = 0.94$ and the search costs to be much higher by choosing $\alpha = 15.22$ and $\zeta = 2.65$. Table 16 summarizes the calibration.

Finally, Table 17 presents the targeted empirical moments and the ones implied by the two models tested. The model with monetary search costs matches the empirical moments pretty well. In particular, although in the steady state there are close to nil newly unemployed workers without any savings, there is 23% and 50% of population with savings below 0.09 and 0.19, respectively. The latter two numbers translate into only 32% and 67% of the steady state mean monthly gross wage income. The only moment somewhat off the target is the growth rate of

Moment	q growth rate	q decline rate	U Rate	% U liq. constr.
Empirical	$\frac{54.1}{19.1} \approx 2.83$	$\frac{54.1}{22.1} \approx 2.44$	7.10%	17% / 50%
Non-Separable Model	1.56	2.53	7.19%	23% / 50%
Separable Model	1.26	1.00	9.29%	42% / 79%

Note: Table presents approximate moments of the worker search effort documented in Krueger and Mueller (2010), of the US long-term mean unemployment rate and of the percentage of newly unemployed workers who are (close to) liquidity constrained documented in Gruber (2001) and Chetty (2008) and the corresponding implied moments in both versions of the model.
Notation: U Rate - Unemployment Rate; % U liq. constr. - share of liquidity constrained among all newly unemployed workers.

Table 17: Empirical and implied moments

search effort from the trough when still on UI till the benefit exhaustion.

On the other hand, the separable model performs relatively poorly. Firstly, it is not able to match the target of 7.10% unemployment rate.[86] Secondly, 42.83% of newly unemployed are with zero savings. While the latter number is close to the upper bound evidence, having 79% of newly unemployed with savings below 0.09 (and 82% below 0.19) is significantly off the empirical evidence. The reason for the latter is a low β which effectively governs not only the saving moment, but also the unemployment rate: the more impatient are the workers, the less picky they are about the job offers. In other words, the chosen value of β constitutes a compromise between missing the unemployment rate target and overshooting the savings target. Finally, the worker search behavior does not match the empirical evidence: although the search effort does grow over the unemployment spell, it remains at a constant level after the benefit exhaustion (in line with previous findings in the literature). Section 4.4.1.2 provides a more detailed discussion of unemployed worker behavior shedding some more light on why the second model fails.

[86] I have also tried to calibrate the separable search costs model with a richer mapping of the worker's effort to the effective transition probability given by $f(q) = \xi (1 - \exp(-\chi q))$. Nevertheless, the model's unemployment rate has not gone down significantly in spite of the two additional free parameters.

4.4 Results

In this section I discuss the results from solving the calibrated model for different re-entitlement policy experiments. In Section 4.4.1, I discuss the worker behavior. Section 4.4.2 presents the equilibrium properties of the economy under different policy settings. In particular, I discuss the potential welfare gains associated with following the entitlement policy. Section 4.4.3 discusses the feature of omitted on the job search. Finally, Section 4.4.4 discusses some of differing features between the European and US labor markets in light of the results discussed here.

Importantly, many of the main conclusions in the separable search cost model are qualitatively the same. If this is not the case, I discuss these differences in the due place. Appendix B contains all the twin figures for the separable search costs model.

Additionally, as a robustness check, Appendix C contains quasi comparative statics results, i.e. a comparison of the model predictions when one of parameters changes. For description of the numerical method used for solving the model see Appendix D.

4.4.1 Worker behavior

Figures 12 and 13 present the behavior of representative unemployed and cross sectional (across all the wage categories and maximum and minimum saving categories) employed workers (respectively) in the model with monetary search costs and both no entitlement for quitters (NE) and with UI entitlement after 1 month on the job ($\hat{T} = 1$). In particular, they show 1) workers' search behavior, current saving positions, reservation wage and consumption levels; and 2) quit or stay on the job decisions. The first part of this Section focuses only on the baseline policy of NE. The differences between the two policies are discussed in Section 4.4.1.2. At the end, in Section 4.4.1.3, I discuss some testable implications of the model.

Worker behavior under no entitlement for quitters

Figure 12 with search effort behavior shows that the baseline model replicates very well not only the empirical evidence of the spike at benefit exhaustion but also generates a plausible U-shaped path (as discussed in Section 4.3) of the search effort.[87] In particular, as workers enter the first period of unemployment they

[87]Thus, if the "U-shape conjecture" is true, then the monetary search costs model rationalizes the worker search behavior.

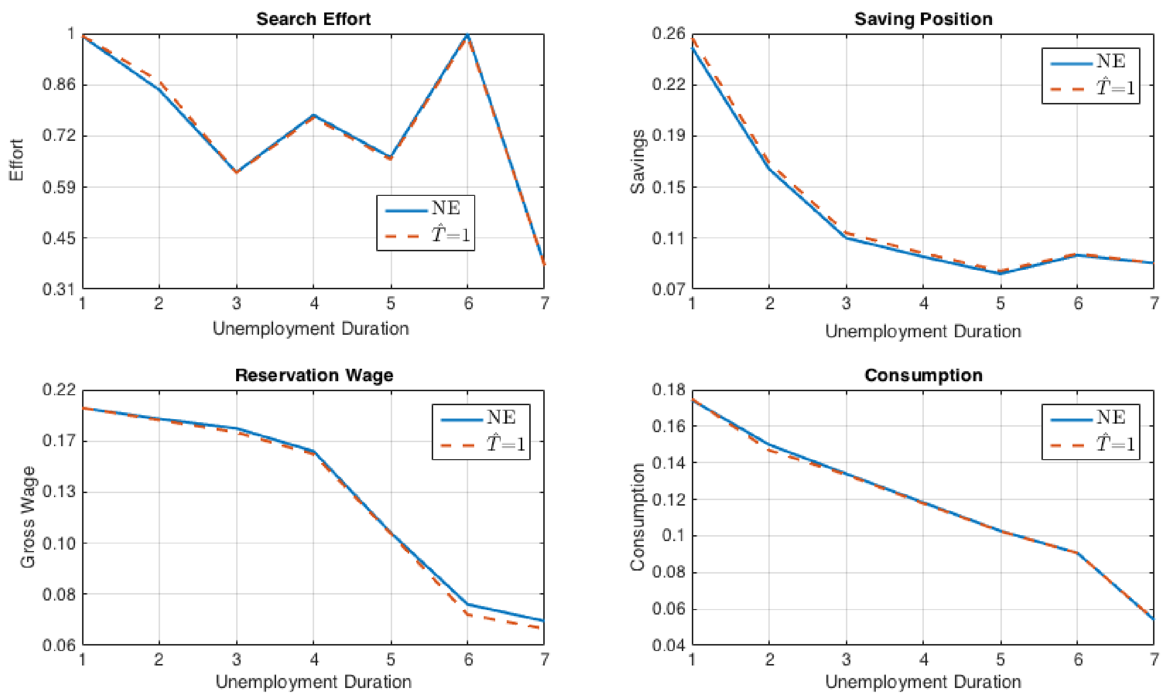
have quite some savings, and so the income effect dominates: for a given probability of finding a good job, the cost in terms of marginal utilities is relatively low, and so workers want to invest a lot of their resources into finding a good job. However, as workers deplete their savings, the substitution effect starts dominating: exercising extra search effort by agents becomes more expensive in terms of marginal utility. Therefore, we get the decreasing effort in the first 3 months.

Note that between the months 3 and 5 there is a little spike in search effort. The increase in effort comes about because of workers getting closer to the benefit exhaustion. Then, the decrease is a consequence of increased savings in the month 5 (note the increase in *saving position* in the month 6) as a precautionary measure against falling into the social assistance.

The spike at the benefit exhaustion in the last month of benefit entitlement is a consequence of workers trying hard to avoid falling into the social assistance state which may imply kind of a long term unemployment lock-in and so having to agree on a much worse job offer than before the exhaustion. The latter is also reflected by the steady decrease in the reservation wage (behavior akin to discouragement). In other words, as workers approach the period of benefit exhaustion the income effect dominates (the return in form of finding a job jointly with avoiding the social assistance state in the next period is very high). After the benefit exhaustion, search effort drops significantly as workers' disposable income gets much smaller.

Finally, plots in Figure 12 speak in favor of dynamics generated by the model. Firstly, the decline of reservation wage over the spell of unemployment is in line with empirical evidence documented in Krueger and Mueller (2014). Secondly, the wealth holdings decline steadily along the unemployment spell - as documented in Gruber (2001). Thirdly, the increase in search effort and decrease in reservation wage imply together an empirically documented spike in unemployment exit rate. However, there is still an ongoing debate in the literature whether this spike is due to a spike in the re-employment probability or rather only due to workers moving out of the labor force (see e.g. discussion in Card, Chetty and Weber (2007)).

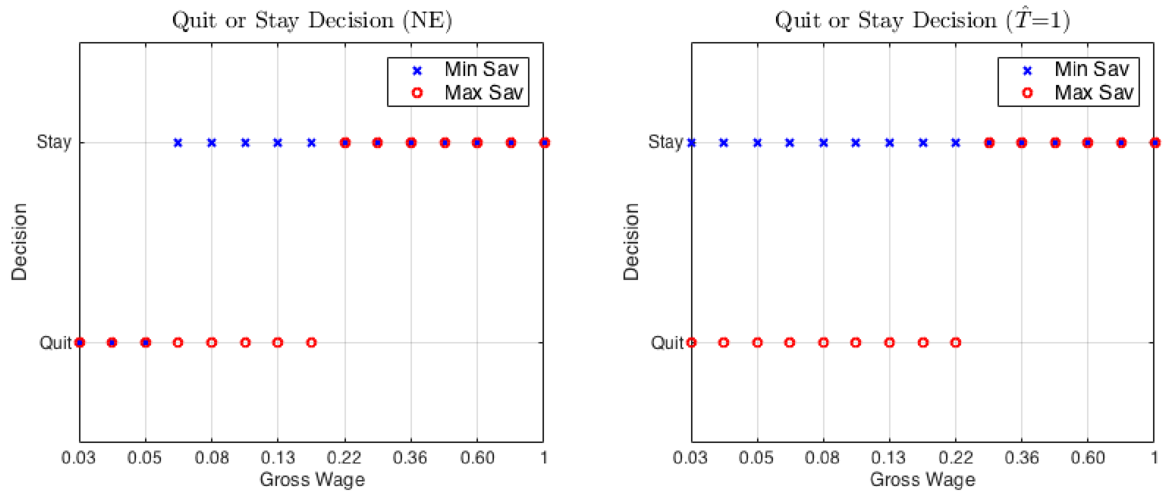
On the other hand, the worker behavior in separable search costs model fails to match the empirical evidence (see Figure 18). The search effort neither drops at the benefit exhaustion nor is U-shaped. Furthermore, the implied savings behavior is also counterfactual as savings decline over the first 4 months and then increase steadily. These dynamics are due to the fact that in this model search costs do not enter the budget constraint and so workers face no trade-off between exercising search effort and accumulating savings against the social assistance state.



Notation: NE - no-entitlement, $\hat{T} = 1$ - re-entitlement after 1 period on the job.
Figure 12: Representative unemployed worker behavior with monetary search costs

Furthermore, Figure 13 presents the cross sectional behavior of employed workers across all the 15 wage categories and minimum and maximum savings. In the model with no entitlement for quitters, the workers with minimum savings quit all the jobs with $w \leq 0.05$. On the other hand, the workers with maximum savings are much more picky and quit all the jobs with $w < 0.22$. This means that the model with no entitlement for quitters is (realistically) characterized by a positive mass of people leaving jobs: workers with little or no savings find it optimal to enter some jobs for a number of periods in order to build up their saving accounts and then go back to unemployment and use savings in order to find a better match. In fact, in the steady state of the model with no entitlement, the quit rate[88] amounts to 0.68%. For comparison, the quit rate in the separable search costs model amounts to only 0.1%.

[88]Quit rate is defined as the number of quits during the month as a percent of total employment.



Notation: 1 stands for staying on the job, 0 for quitting; Min (Max) Sav - behavior of a worker with minimum (maximum) savings conditional on her current wage; NE - no-entitlement, $\hat{T} = 1$ - re-entitlement after 1 period on the job.

Figure 13: Employed worker behavior with the monetary search costs

According to the JOLTS and CPS data for the years[89] 2000-2015, the average quit rate in the US amounted to 1.84%. However, since some quits reflect job-to-job transitions, this number constitutes an upper bound for comparison with the model-generated quit rate. In particular, between the years 1980-2015, quitters constituted on average 10.72% of the unemployed.[90] Thus, the 'adjusted' quit rate for this period amounts to 0.73% (i.e. $10.72\% \cdot \frac{6.42\%}{93.58\%}$).

Summing up, even tough under the baseline policy quitters are not entitled to UI, some workers still decide to leave their jobs in order to find a better one and this is especially true at the lower end of wage distribution. Moreover, the data and the model-dynamics give strong support to the mechanism presented in this paper.

Worker behavior under entitlement for quitters

I begin with discussion of the employed worker behavior and then move to the case of unemployed ones.

The right panel in Figure 13 shows that workers with minimum savings stay now in all types of jobs. Note that this is obviously not to say that they keep working until exogenously separated. Similarly as before, they stay on these jobs

[89] The relatively short period taken is due to data availability.

[90] This number does not include unemployed workers from temporary jobs or on temporary lay-offs.

for some periods in order to accumulate savings and then quit in order to find a better job and collect their UI. I refer to this kind of worker behavior as being opportunistic since now workers use some lower paid jobs not only to accumulate savings but also to receive the welfare payments. Consequently, under the new policy the quit rate goes up to 1.59% from 0.68%.

Furthermore, the model with separable search costs generates quite different conclusions about the employed worker behavior (see Figure 19). In particular, upon the policy reform, both workers with minimum and maximum savings stay in fewer jobs than before the reform. As search costs are non-monetary, workers do not spend as much time in their jobs in order to accumulate savings and so quit those just after the re-entitlement. The quit rate goes up now to 1.91% from 0.1%.

On the other hand, the representative unemployed worker behavior in the case of $\hat{T} = 1$ is not affected much (see Figure 12). This should not be surprising given that the policy is aimed at the employed workers who would like to quit and so affects directly the value of being employed, and only indirectly the value of being unemployed. Nevertheless, the new policy lowers the reservation wages before the benefit exhaustion. This means that the new policy extracts some workers from social assistance by making the low paying jobs more attractive due to the additional re-entitlement value.

However, since the discussion above was about a representative unemployed worker (constructed across all the states), there may still be some workers who are affected more significantly. To this end, Figure 14 presents the behavior of a (truncated-) representative unemployed conditional on[91] recent wage $w = 0.17$ and assets $a \in [0.35, 1.1]$. Post-reform workers from this group enter unemployment wealthier and so are able to search and consume more. The higher stock of savings comes from a better (on average) employment history - see the discussion in Section 4.4.2. Finally, notice the interesting crossing of reservation wage functions. In the first 4 months of unemployment the post-reform workers from this group are more picky as they have quit their jobs exactly in order to find a better one. However, once the unemployment spell gets longer, they want to avoid the long-term unemployment and so are willing to take up some less paying jobs.

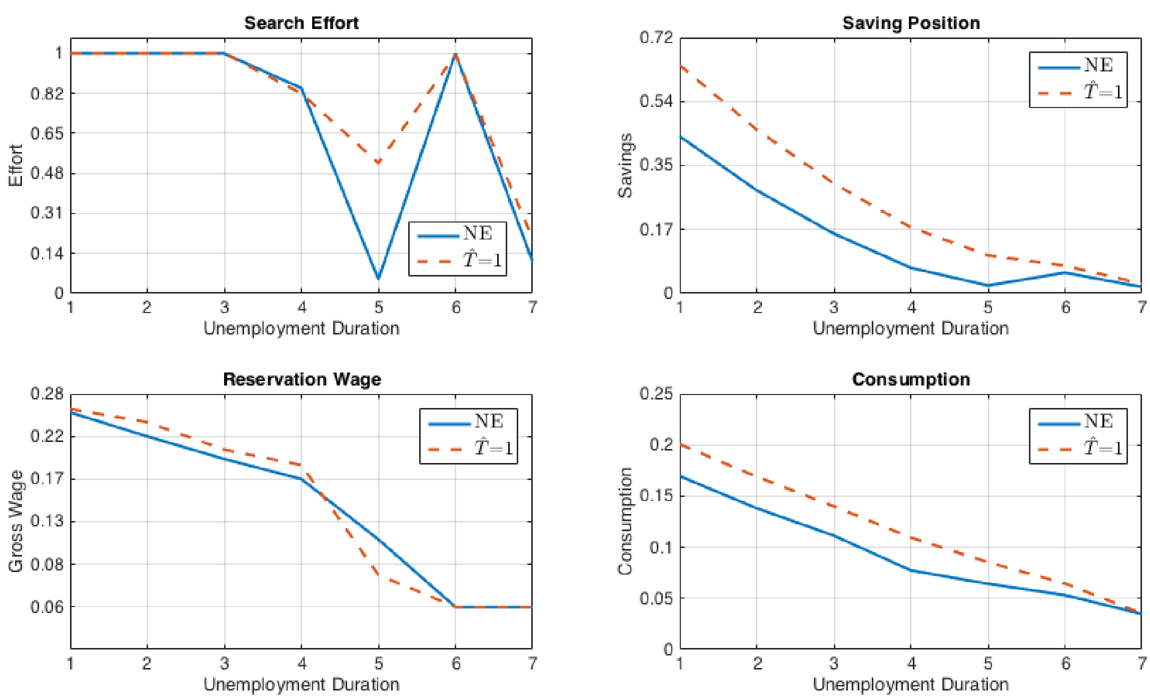
Notice that in the model with separable search costs the reform affects the representative unemployed by much more[92] (see Figure 18). Since the value of

[91]See Section 4.4.2 below for explanation on how this group has been identified.

[92]For this reason I do not plot the twin of Figure 14 for the separable search costs model.

the (easy to find) low paying jobs in the post-reform economy is increased (which translates into the gap between the values of being unemployed and employed becoming smaller), the workers do not find it optimal to search as hard as before and so the search profile is shifted down.

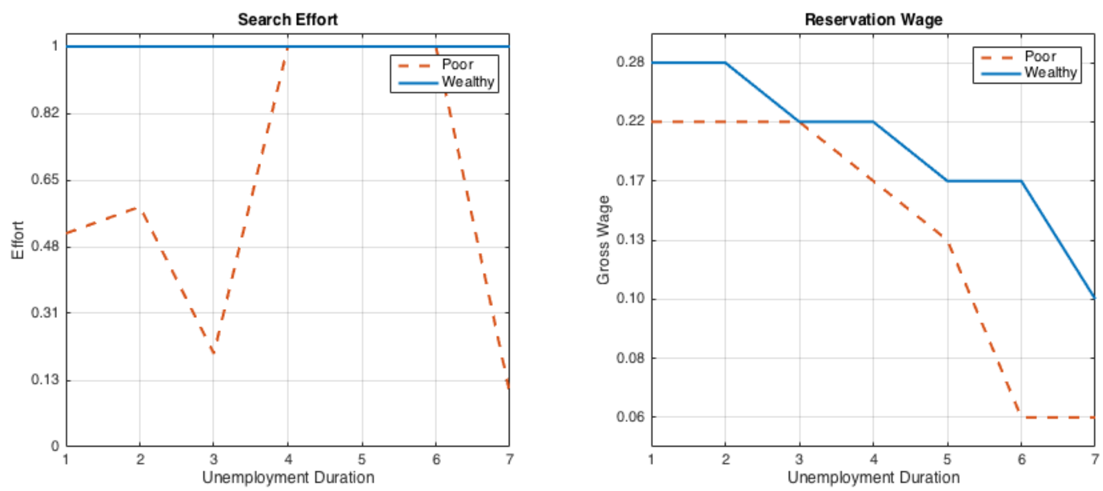
Finally, note that the introduction of entitlement for quitters is a form of increasing the overall UI generosity. Thus, the results on search behavior suggest that an increase in the UI generosity need not necessarily lead to a decrease in search effort (as was usually found in the literature). I further expand on this point in the Appendix C.



Notation: NE - no-entitlement, $\hat{T} = 1$ - re-entitlement after 1 period on the job.
Figure 14: Representative unemployed worker behavior with $w = 0.17$, $a \in [0.35, 1.1]$ and monetary search costs

Worker behavior and financial wealth

Notice, however, that the above dynamics of unemployed workers were weighted averaged across the wage and saving positions. To this end, Figure 15 presents search and reservation wage behavior of poor (with 0 savings) and wealthy (with maximum savings) worker with the recent wage $w = 0.28$ in the pre-reform economy. It can be immediately seen that the worker with less savings 1) is effectively searching much less; and 2) is less picky about the job offers. Since workers in the model are *ex-ante* identical, these observations deliver important implications that can be tested empirically. Are similar people differing only with respect to their financial wealth searching with different intensity? Do they differ with respect to their reservation wages? And so finally: are they more likely to end up in better paying jobs?



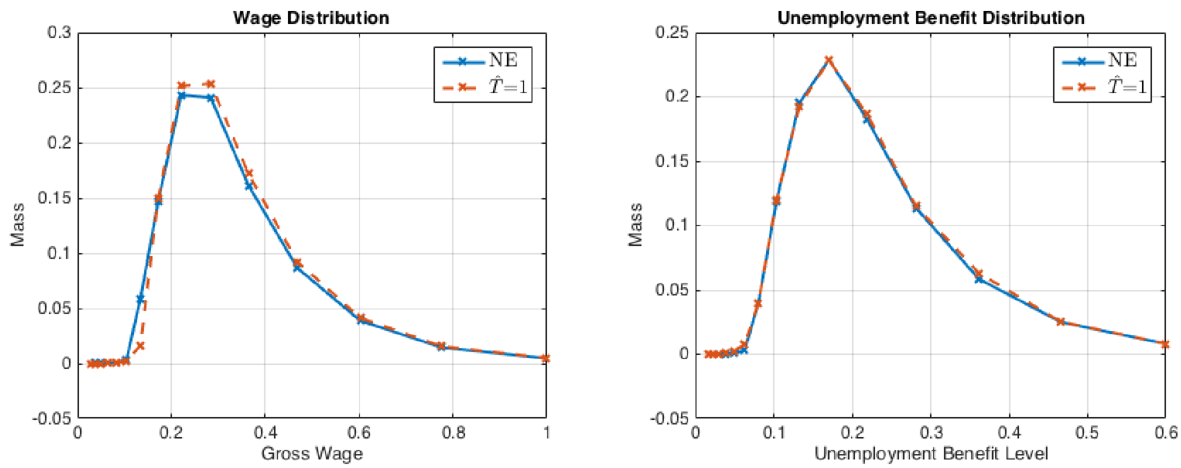
Notation: Poor - unemployed worker with $w = 0.28$ and $a = 0$; Wealthy - unemployed worker with $w = 0.28$ and $a = 1.1$.
Figure 15: Behavior of poor and wealthy unemployed workers with monetary search costs and no entitlement

Although the separable model delivers qualitatively the same dynamics of the reservation wage, it differs with respect to the search behavior (see Figure 20). Now, since workers can adjust their search effort arbitrarily, the poor worker consumes less and so has more incentives to search harder in order to get back to employment.

4.4.2 Employment, welfare and inequality

In this section, I discuss first the distributional implications of the policy reform. Then, I move on to analyzing key equilibrium statistics of the pre- and post-reform economies like the associated welfare, unemployment and inequality rates.[93] In particular, I show that following the policy of entitlement for quitters is associated with significant welfare gains and reduction in income inequality.

[93]Based on the computer results for the employed parameter values, the equilibria presented here are the unique ones.



Notation: NE - no-entitlement; $\hat{T} = 1$ - re-entitlement after 1 period on the job; Mass - a share of workers in a given category out of all employed or unemployed, respectively.

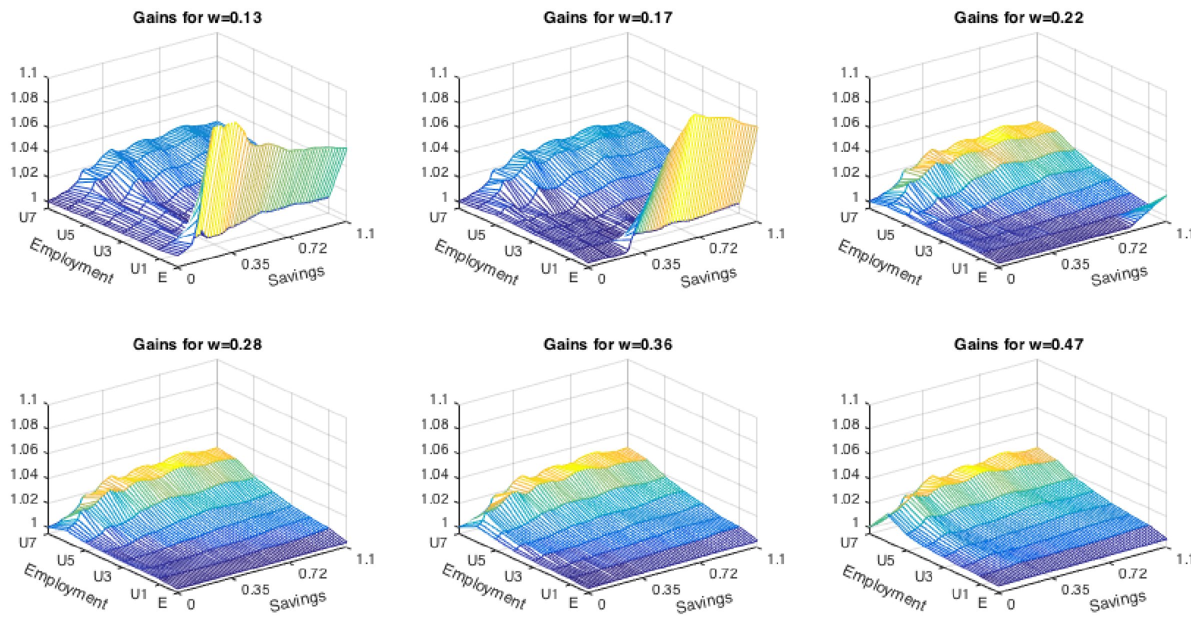
Figure 16: Distribution of workers' income with monetary search costs

First of all, Figure 16 presents the steady state distributions of workers' income in the two economies with NE and $\hat{T} = 1$. The first plot with wages shows that the reform allows workers to improve upon the average quality of their match. In particular, before the reform there were 5.88% of workers in jobs paying less than $w = 0.13$ and only 1.79% post-reform. Note that the changes in distribution of unemployment benefits do not follow the ones in wage distribution very closely. This is because under the entitlement policy $\hat{T} = 1$ there are more voluntary quits in the lower wage categories, bringing the benefit distributions for NE and $\hat{T} = 1$ close to each other for wages below 0.2.

Consequently, as the above discussions suggest, the new policy affects mostly workers at the lower end of the wage distribution. Since these workers have a relatively high marginal utility from consumption, the welfare impact of the policy reform on this group can be potentially very large. This is confirmed in Figure 17 which presents the state-by-state breakdown of welfare gains[94] associated with the new policy for six wage categories in which there are at least 3% of employed (these plots cover 90% of employed in the economy with no entitlement). In particular, we observe that the highest welfare gains equivalent to between 6% and 10% of life-time consumption are for the employed workers (employment state E) with $w \in \{0.13, 0.17\}$ and some positive savings. These gains reflect the insurance value of the policy reform: as the associated risk with being unemployed

[94]Mathematically, the state-by-state welfare gains are given by the formula $Welfare\left(a,t,w,x\right) = \left(\frac{v_1(a,t,w,x)}{v_0(a,t,w,x)}\right)^{\frac{1}{1-\theta}}$.

becomes lower, workers in low wage categories are now more likely to quit in order to search for a better match.



Notation: Employment state E stands for being currently employed; employment states U1 - U7 stand for being either on UI (between 1 and 6) or on social assistance (state 7).

Figure 17: Breakdown of welfare gains with monetary search costs

On the other hand, there are also significant welfare gains for unemployed workers in all the wage categories depicted. These gains are especially high at between 2.5% and 3% around the state of social assistance (state U7). This reflects the additional non-wage value of low-paid jobs which workers are now more likely to accept in the post-reform economy (recall the worker behavior discussed above). Moreover, note that: 1) employed workers with higher wages do not gain much as they are would not want to quit anyway; and 2) virtually no one is loosing from the reform (discussion below explains why).

All the welfare conclusions in case of the model with separable search costs are similar (see Figure 22). The only difference is that the welfare gains are spread more uniformly. This is due to the fact that the optimal post-reform search effort is shifted down (for reasons discussed above), which consequently reduces the search costs of all the unemployed workers.

Table 18 summarizes key statistics of the equilibrium unemployment rate, budget balancing tax rate, welfare, income inequality and unemployment duration for both models with non-separable and separable utility functions. First of all, notice that following the policy of entitlement for quitters is associated with a higher unemployment rate and so with a higher budget balancing tax rate. Nevertheless, in both cases of non-separable and separable search effort cost assumptions there is an (overall) welfare improvement associated with following the entitlement policy ($\hat{T} \in \{1, 2, 3, 6\}$) as compared to the actual US policy of no entitlement (NE). In particular, the policy of entitlement for quitters after having worked for at least 1 month ($\hat{T} = 1$) results in the highest long-run welfare gains equivalent to a 4.38% and 3.69% increase in the life-time consumption, for non-separable and separable models respectively. Consequently, the benefits of providing search subsidy to workers (in form of insurance for employed and extraction of long term unemployed into employment) outweigh the costs generated by the moral hazard of rejecting or quitting jobs (in form of a higher tax rate needed to finance UI spending given higher unemployment rate).

$\hat{T}$	τ (%)		U Rate (%)		C.E. Welfare (%)		Income Inequality (%)				UD (months)	
	NS	S	NS	S	NS	S	NS		S		NS	S
							pre-tax	after-tax	pre-tax	after-tax		
1	5.84	8.07	7.57	11.18	104.38	103.69	22.82	22.69	20.63	20.33	4.03	6.44
2	5.81	8.01	7.55	11.01	104.35	103.02	22.81	22.69	20.66	20.36	4.05	6.40
3	5.79	7.96	7.54	10.92	104.31	102.84	22.82	22.70	20.68	20.39	4.06	6.40
6	5.74	7.85	7.50	10.70	104.26	102.65	22.83	22.70	20.71	20.44	4.08	6.37
NE	5.65	7.41	7.19	9.29	100	100	23.72	23.61	20.68	20.48	4.21	6.18

Notation: C.E. Welfare - life-time consumption-equivalent welfare increase; U Rate - Unemployment Rate; UD - unemployment duration, NS - results for a model with non-separable utility assumption, S - with separable utility, $\hat{T}$ - required number of months on the job in order to get eligibility for benefits after quitting, NE - no entitlement for quitters policy.

Table 18: Main results of the model

To understand the underlying trade-off better, note first that the required increase of 0.19 p.p in the budget balancing tax rate is of a very small magnitude. Thus, as the currently employed workers barely notice the change in the tax rate, there is virtually no one loosing from the policy reform and (as discussed above) there are significant gains for the workers in low wage categories and for the long-term unemployed.[95] This small increase in the tax rate is due to two factors: 1)

[95]Moreover, the policy reform also serves as an indirect insurance for workers in high wage categories leading to some gains off-setting the negative impact of the tax increase. This is true as every worker in the economy faces a positive probability of becoming unemployed and being

a relatively small increase in the unemployment rate due to a relatively small increase in the quit rate (change from 0.68% to 1.59%); and 2) an improvement of average match quality leading to a higher average tax revenue per worker.

Interestingly, under both assumptions of separability and non-separability the model kills two birds with one stone: the re-entitlement policy leads not only to an increase in efficiency but also in (both pre- and after-tax) equity. First of all, although unemployment rate goes up, the mass of long-term unemployed with social assistance income declines sixfold. Secondly, due to the higher tax rate, the after-tax income of employed individuals gets closer to the income of unemployed.

Similarly, the optimal policy is characterized by $\hat{T} = 1$, as opposed to $\hat{T} > 1$, exactly due to the effects on long-term unemployment. As the parameter goes up, the non-wage value of low-paid jobs is reduced and so the reservation wage of workers unemployed for 5-7 periods increases (see Figure 12 with reservation wage and note that NE corresponds to $\hat{T} = \infty$). Consequently, there are more workers ending up poor (with high losses in marginal utility terms) in long-term unemployment which they cannot escape easily.

Moreover, as the long-term unemployed workers in the post-reform economy become less picky about wage offers, the unemployment duration goes down. Notice that this is not true in the separable search costs model: although also here the reservation wage is lower under the policy $\hat{T} = 1$ as compared to NE, the level of the post-reform search effort is shifted down (as explained above in Section 4.4.1.2) leading to opposite conclusions.

Notice that the model delivers realistic dynamics in terms of other untargeted moments, again speaking in favor of the robustness of results. First of all, the implied unemployment duration of 4.2 months is just in line with the average mean unemployment duration in the FRED dataset for the years 1980-2015.

Secondly, the implied budget balancing tax rate of 5.65% is not too unrealistic. Although the average unemployment tax rate in the United States varies (depending on the state) from 0.05% to 2%, as reported by Henchman (2011), these tax rates are in many cases too low as during many recessions some of the unemployment insurance trust funds became insolvent due to too low fund reserves and increased unemployment caused by economic downturn.

Finally, the model generates substantial income inequality. Nevertheless, due

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unlucky in receiving low wage offers. In such a case, a worker with previously high wage may end up in long-term unemployment where she would benefit from the reform.

to the simplifying assumption of worker *ex-ante* homogeneity, the implied after-tax income inequality of 23.61% is 14 p.p. below the corresponding Gini index for the US (according to the OECD).

4.4.3 On the job search

The model (and so the results above) abstracted from an empirically important feature of the on the job search. Fallick and Fleischman (2004) constructed a dataset based on the CPS from February and March 1997 and 1999 and report that out of all the employed people around 4.4% engaged in active on the job search. After two months, out of those actively searching around 11.3% reported having a new employer and around 5.6% reported being unemployed.

Moreover, up to 80% of job to job changers in their sample were not actively searching on the job. Such transitions may be due to various reasons such as 1) some of the workers having a very short (and so unrecorded in the data) period of unemployment in-between, 2) due to poaching, or 3) due to having found a job through their networks. The first case is covered by the mechanism employed in this paper. Nonetheless, a waitress working in a restaurant may be very likely poached by another restaurant or get information about a position in a friend's restaurant with a *slightly* higher wage. Obviously, my model is not suited for the analysis of such phenomena. However, given the discrete nature of the wage grid employed in this paper, one can think about each wage as representing some particular job category. With this interpretation in mind, it seems plausible to think that a waitress may be better-off leaving her job and her current network in order to focus her search effort on finding a new job (ideally a much better paid one) as her current network may be unlikely to provide her with such an opportunity.

Nevertheless, it is easy to see that a standard on the job search extension of the model would preserve qualitative nature of the results if the difference between the costs of on- and off-the-job search is large enough. In fact, as has been documented by Krueger and Muller (2010), the on the job search costs are strictly greater than searching while unemployed: just think about how much time a waitress or a management consultant working for 60+ hours per week may be willing to devote for the on the job search. However, if we think of searching being costly in terms of money as opposed to time, one should not forget that going to an interview may require the worker to take one day off (or even more in case of remote destinations or a series of interviews for more qualified jobs).

Consequently, apart from the lost output due to worker's absence, it seems also possible that an active on the job search may trigger e.g. lower worker output or loss of employer's trust. This phenomenon could be captured by a higher firing rate while doing on the job search translating into higher wage income risk and so into higher monetary search costs. Interestingly, such cases of firings upon the loss of trust have been documented[96] in interviews ran by Bewley (1999).

Importantly, the data supports the mechanism at the core of this paper. The variance of the aforementioned quit rate in the US is significant: the quit rate in government sector amounted to only 0.7% while in retail sector it was already 2.9% and in food services even 4%. Unsurprisingly then, as mentioned in the literature review, there is some empirical analysis documenting existence of the type of opportunistic worker behavior analyzed here.

Finally, given that the model-implied quit rate of 0.68% is already slightly below the one in the data, introducing the on the job search would reduce it even further and so lead to unclear conclusions. Thus, assuming away the on the job search should not be seen as critical for the results obtained.

4.4.4 Empirical observations vs the model

The labor markets in Continental Western Europe and the United States have been at odds in many features for many years so far. First of all, since 1980s the unemployment rate for the EU-15 countries has been persistently[97] higher than in the US by 1% to 4.5%, depending on the time period. Secondly, since mid-1980s income inequality in the US has risen much faster than in Europe and since then has been persistently higher. Thirdly, Manacorda and Petrongolo (1999) show that although the unemployment rate is a more pressing problem in the Continental Europe, its labor market mismatch levels have not increased as much as in the US. Finally, the increase in European unemployment rate has been accompanied by decreasing rates of exit from unemployment resulting in longer duration of unemployment spells and increase in the number of long-term unemployed.

As documented by Venn (2012), the unemployment insurance systems with and without benefit entitlement for workers quitting voluntarily are characteristic for many countries in Europe and the US, respectively. The model presented

[96] These cases might be partially captured by the 5.6% of active on the job searchers being unemployed in the dataset of Fallick and Fleischman.

[97] The only exception was the unemployment rate in 2010 when the two got close to each other for short period of time but then diverged again.

above abstracts from many important factors that could be equally likely to contribute to the differences in the US and European labor markets and focuses solely on entitlement to benefits for quitters in order to find whether it may explain at least some of the empirical evidence.

As I have shown, the model is able to somehow reconcile the first three observations. The mechanism employed in the model leads to a higher unemployment rate. Also, following the benefit entitlement policy for workers quitting jobs voluntarily (Europe) is associated with 1) a lower income inequality, and 2) a higher match quality, as compared to the case of US status-quo. Nevertheless, although the associated with the optimal policy unemployment duration is higher in the separable search costs model, it is lower in the non-separable one.

Importantly, mind that the model has been calibrated solely to the US labor market which obviously has different fundamentals than the European. Moreover, characteristics of the latter differ a lot between individual countries. With this being said, the effects exposed by the model may very well be exacerbated by other institutional differences among the continents such as e.g. longer UI duration, more unionization or more stringent Employment Protection Legislation in the Continental Europe. In other words, this paper is identifying the direction of effects driving the economy associated with particular policies and does not aim at trying to investigate whether the differences in policies may account for the observed labor market discrepancies. Given this and other important economic factors not included in the model, the prevalent in Europe policy of paying benefits after voluntary quits may account for some but obviously not all of the observed differences in unemployment rates, match quality and income inequality.

Finally, there is a vast literature discussing reasons for the observed difference in characteristics of the US and European labor markets. A good review of possible explanations is provided by Bertola and Ichino (1995). Thomas Sargent and Lars Ljungqvist ran a major research program that aimed at identifying the reasons of these differences. Their theory is that generous European welfare system combined with a permanent change in the microeconomic labor conditions led to a sustained and high unemployment rate in Europe. Although the model presented here also attributes the observed discrepancies to generous welfare systems, it points exactly at one particular policy which may be partially responsible for the observed divergence of labor markets. Moreover, it also shows that economists should not only investigate the reasons of these observations, but also look at their consequences (e.g. in welfare terms). It might well be the case that the higher unemployment rate in Europe does not necessarily represent a huge waste of human resources and welfare, but to the contrary allows for a better allocation. To fully address this question, economists need more comprehensive models taking relevant general equilibrium effects into account.

4.5 Concluding discussion

In this paper, I study a labor model with random search, unemployment insurance, savings, voluntary quits and various labor attachment requirements. In particular, I look for the optimal unemployment re-entitlement policy for quitters. In order to do this, I embark upon the method accounting for all the benefits and adverse effects generated by the policy, i.e. the social welfare analysis.

The model is calibrated to the US labor market where quitters are not entitled to UI. The results raise the question about the observed quitter-related UI policies differing from country to country and suggest that they may be rather ad hoc and may be a source of welfare inefficiencies. In particular, upon following the generous entitlement policy the implied welfare, unemployment rate and match quality are higher and both the pre- and after-tax income inequality are lower than in the case of no-entitlement for quitters. Interestingly, given that in Europe quitters are often eligible for UI, the model with its results identifies a concrete policy that could partially explain the divergence of the US and European labor markets.

The intuition for the results is two-fold. Firstly, the policy is insuring the income risk associated with a voluntary quit and although it increases the mass of quitters (and so the required tax rate to fund the UI), it improves the allocation of workers in the economy. Secondly, by increasing the non-wage value of low paid jobs, the policy lowers reservation wages of long term unemployed and so extracts some of them back into employment.

I find the results in this paper complementary to the literature discussed. First of all, worker's opportunism was found in Hopenhayn and Nicolini (2009). I show that allowing for this opportunistic and seemingly inefficient behavior of workers quitting jobs in order to find a better one may lead to significant long run welfare gains. Furthermore, I investigate the consequences of the unexplored assumption of monetary search costs. It turns out that this assumption is capable of generating search behavior in line with the one documented in the empirical literature. What is more, the model generates testable implications about differences in search behavior, and so in labor market outcomes for similar workers differing only with respect to their financial wealth. Thirdly, this paper fills an important gap in the UI literature by analyzing welfare consequences of entitlement to benefits for quitters. Finally, the results presented here hint at the need of new direction of economic research concentrated on (i) studying fully optimal UI design taking into account the possibility of endogenous quits (given how powerful this mechanism seems to be); (ii) the relevance of monetary costs and substitutability between time and money for job search and (iii) the welfare consequences of unemployment and thus, among others, of the divergence of the US and European labor markets.

Importantly, the model performs well not only in directions in which it has been calibrated but also others like implying realistic UI tax rates, unemployment duration, significant income inequality and generating realistic reservation wage behavior and endogenous quits after spending some time on the job (even without entitlement for quitters). The model lends itself easily to extensions like more involved and realistic eligibility criteria (for example labor attachment requirement for fired workers), sanctions (suspension periods for quitters), monitoring (penalties for insufficient search effort) and endogenizing wage distribution.

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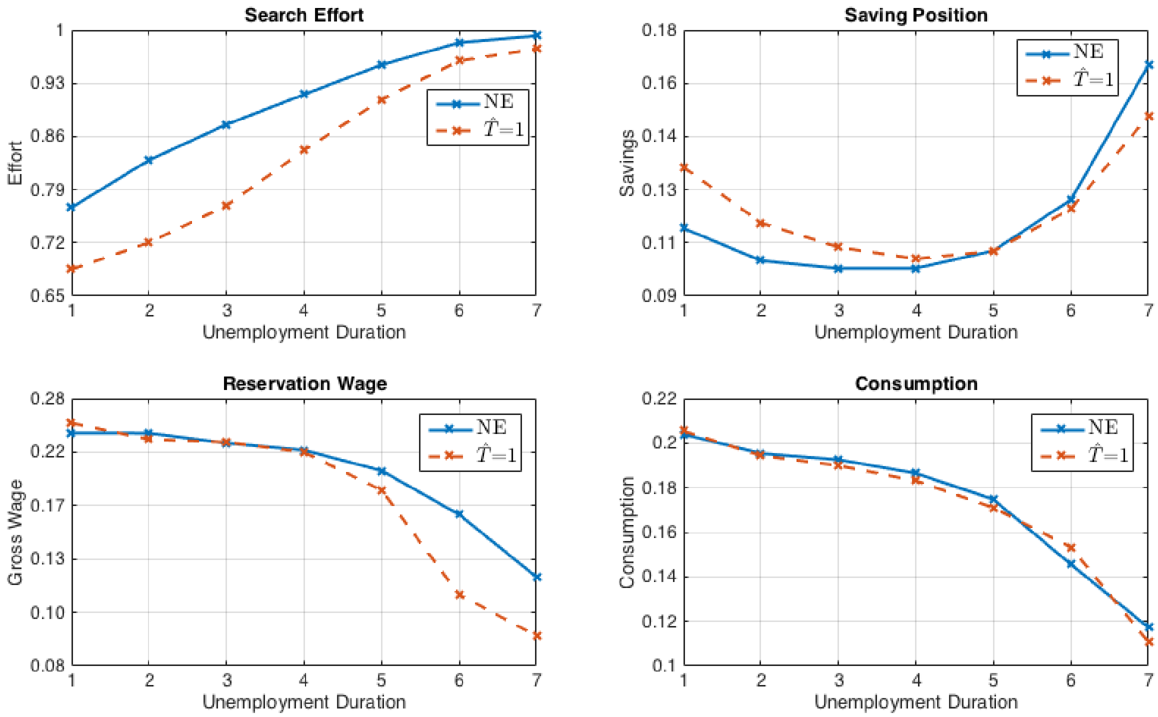
4.6 Appendix A: Proof of the reservation wage property

Existence of such a reservation wage $\bar{w}(a,t,w)$ follows by the fact that being unemployed is associated with a continuation value which (given the distribution on wage draws) entails expectation about the wage draws in the future. If the wage draw today is low, then the value of declining it and being unemployed until tomorrow may yield greater value to the worker as the draw tomorrow is higher in expectation. Thus, it is optimal for the worker to decline such a wage offer. Note that, since I consider only benefits which are strictly smaller than recent wage and there is no disutility of working, there always exists a wage w in the support of wage distribution which the worker is willing to accept. By continuity there exists at least one wage at which the worker is indifferent between accepting and rejecting a job offer. Let me denote this value by $\bar{w}\left(a,t,w\right)$.

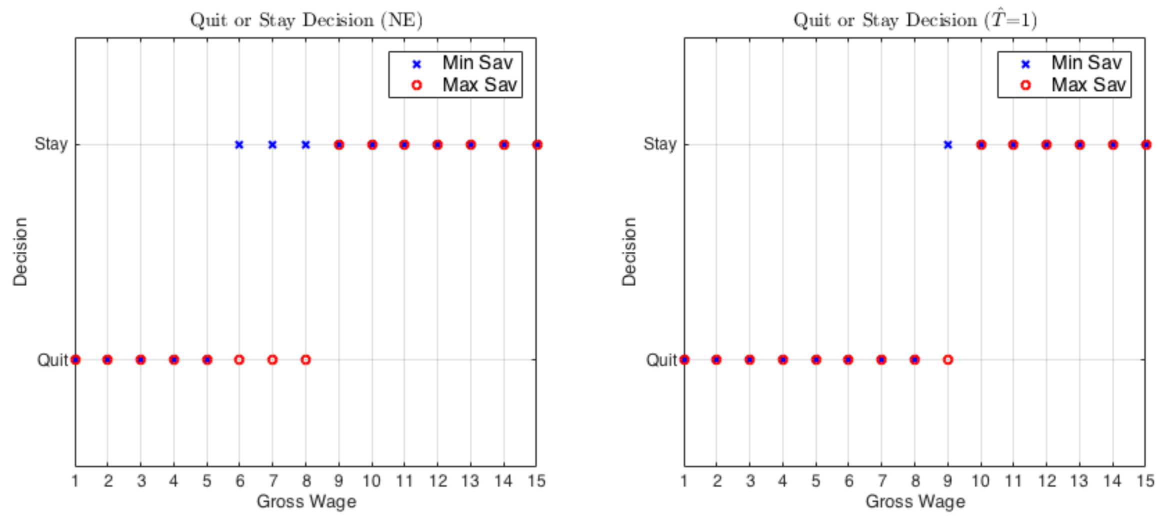
It remains to show that the gain from accepting a job is monotonic in the offered wage, and thus for $w' \geq \bar{w}$ it holds that $V_e\left(a,1,w'\right) \geq V_e\left(a,1,\bar{w}\left(a,t,w\right)\right) = V_u\left(a,t,w\right)$ and $V_e\left(a,1,w'\right) < V_u\left(a,t,\bar{w}(a,t,w)\right)$ for $w' < \bar{w}$.

Note that due to the UI design, the value of being unemployed is clearly monotone in wage, i.e. $V_u\left(a,1,w'\right) \geq V_u\left(a,1,w\right)$. By this and the fact that once accepted the wage is constant over the employment time until separated, it follows that the value of being employed V_e, which includes the value of being unemployed at some point in the future, is also monotone in wage, i.e. $V_e\left(a,1,w'\right) \geq V_e\left(a,1,w\right)$ if and only if $w' > w$. Therefore $V_e\left(a,1,w'\right) \geq V_e\left(a,1,\bar{w}\left(a,t,w\right)\right) = V_u\left(a,t,\bar{w}\left(t,w\right)\right)$ for $w' \geq w$ and conversely $V_e\left(a,1,w'\right) < V_u\left(a,t,\bar{w}\left(a,t,w\right)\right)$ for $w' < \bar{w}$, where the monotonicity and definition of the reservation wage is used. This establishes the reservation property.

4.7 Appendix B: Figures with results of the separable utility model

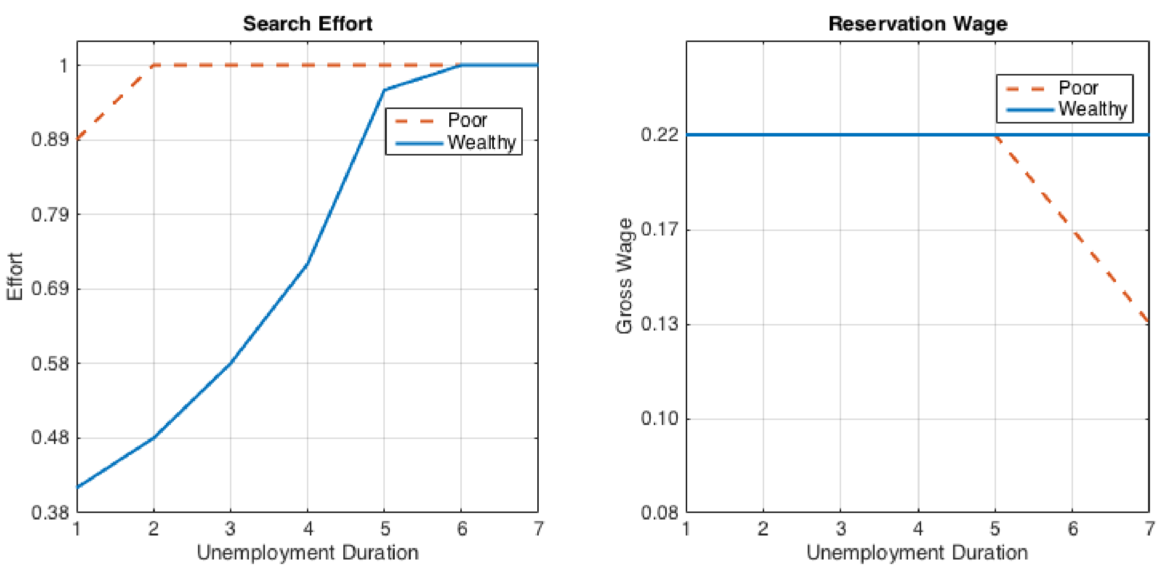


Notation: NE - no-entitlement, $\hat{T} = 1$ - re-entitlement after 1 period on the job.
Figure 18: Representative unemployed worker behavior with separable search costs



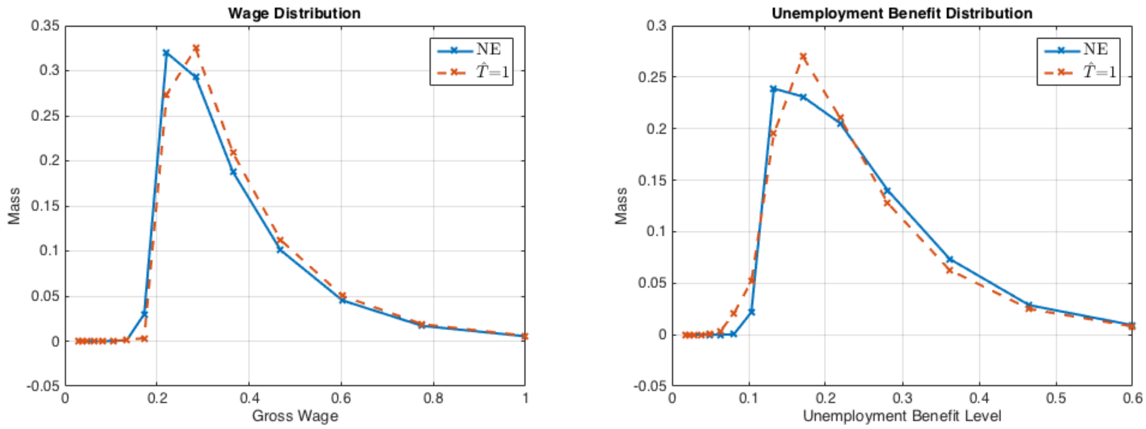
Notation: 1 stands for staying on the job, 0 for quitting; Min (Max) Sav - behavior of a worker with minimum (maximum) savings conditional on her current wage; NE - no-entitlement, $\hat{T} = 1$ - re-entitlement after 1 period on the job.

Figure 19: Employed worker behavior with separable search costs



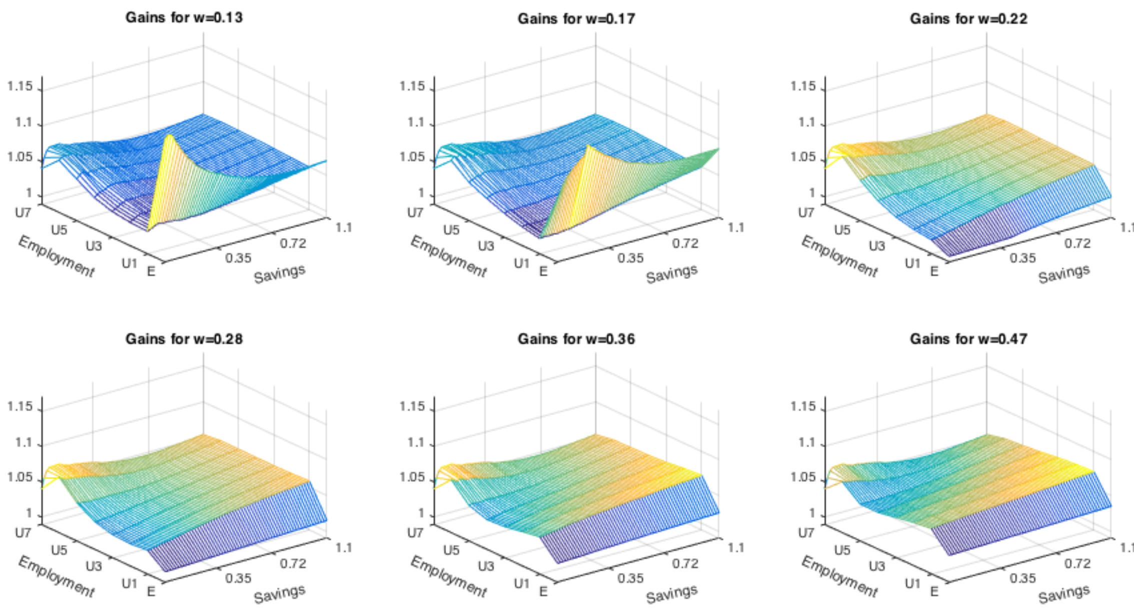
Notation: Poor - representative unemployed worker with $w = 0.28$ and $a = 0$; Wealthy - representative unemployed worker with $w = 0.28$ and $a = 1.1$.

Figure 20: Behavior of representative poor and rich unemployed workers with separable search costs and no entitlement



Notation: NE - no-entitlement; $\hat{T} = 1$ - re-entitlement after 1 period on the job; Mass - a share of workers in a given category out of all employed or unemployed, respectively.

Figure 21: Distribution of workers' income with separable search costs



Notation: Employment state E stands for being currently employed; employment states U1 - U7 stand for being either on UI (between 1 and 6) or on social assistance (state 7).

Figure 22: Breakdown of welfare gains with separable search costs

4.8 Appendix C: Scenario comparison

In this Appendix, I perform a quasi comparative statics exercise in the model with monetary search costs. In order to do so, I solve the model for different values of a parameter of interest while holding everything else constant. Note that the purpose of this exercise is to perform a robustness check of the model's behavior and, in particular, not to find a fully optimal UI schedule (which goes beyond the scope of this paper). Importantly, due to very long computational time required, I solve here the re-calibrated model[98] with hand-to-mouth workers. Table 19 provides results of this exercise, i.e. the qualitative total effects of changes in the level of replacement ratio and the length of unemployment benefits payments on reservation wages, search effort, unemployment rate, tax rate, welfare and inequality.

Consider first the effect of an increase in the replacement ratio[99] γ. As unemployed workers receive higher benefits, they become more picky about the jobs they are willing to accept - firstly due to the outside option having a relatively higher value, and secondly due to having *possibility* of exercising higher search effort and effectively facing a better wage distribution.

Furthermore, the average search effort goes up. The main reason for it is the assumed here monetary cost of search - as more resources are available to search-constrained workers, they take advantage of it in order to escape the unemployment state and the prospect of falling into the unemployment lock-in after benefit exhaustion. However, once the tax rate needed to finance the reform but its effect is less significant. This comparative statics finding is to the contrary of standard findings in literature as e.g. in Shavell and Weiss (1979), where with a separable utility there was no binding search constraint and so higher benefits reduced the gap between the values of employment and unemployment and thus lowered the search effort. The separable utility version of my model confirms its robustness by replicating this standard observation (not shown in the table).

Although the average search effort increases, the reservation wage increase dominates and so the unemployment rate goes up. This implies a higher tax rate needed to balance the government budget. This implies that the welfare will be hump-shaped: when γ is too high, the welfare improving effects associated with better the match quality are dominated by the welfare abuse.

Similarly, the inequality is also hump-shaped: for γ small enough, the increase in the replacement ratio and the tax rate bring the income from unemployment benefits closer to the wage income and so the inequality in the economy decreases. However, once γ is large enough, although more workers access the top-paying jobs, the distribution of employees across lower wage categories does not change

[98]This alternative calibration is obviously targeting the same moments (apart from the saving moments) as the model with savings. For space saving reasons, I do not discuss its details here.

[99]Obviously, I assume that $\gamma \in [0, 1]$.

	$d\bar{w}$	dq	dU	$d\tau$	$dWelfare$	$dInequality$
$d\gamma$	+	+	+	+	?	?
dT	+	-	+	+	?	?

Note: Table shows scenario comparison of total changes in key statistics on impact of change in parameters

Table 19: Scenario comparison

much leading to a net increase in inequality.

Consequently, the welfare is maximized at $\gamma = 78\%$. The associated unemployment rate increases from 7.10% to 8.62%, the tax rate almost doubles going up from 4.09% to 8.01% and the Gini coefficient goes down from 17.55% and 17.41% to 16.43% and 16.33%, respectively for pre- and after-tax.

As the length of the benefit entitlement period is increased, workers stick longer to their initial reservation wage (which is the highest over the unemployment spell) and so the average reservation wage goes up. Moreover, given that workers expect to receive the UI for longer, the average search effort declines. As a result, the unemployment rate increases. For similar reasons as above, the tax rate on employed workers increases. However, the effect on inequality is ambiguous. For T low enough, the relationship between inequality and this parameter is negative - higher tax rate on employed reduces net wage income and redistributes it to the unemployed. However, once T is large enough this relationship is no longer clear for reasons similar to the ones discussed above.

Similarly as above, the impact on inequality is hump-shaped. As the entitlement gets longer, the policy saves more people from entering the social assistance state and so the inequality is reduced. However, above some high value of T these effects become much smaller and so further increases in T start increasing the inequality.

Consequently, the welfare is maximized with UI entitlement for 1 year. The associated unemployment rate increases from 7.10% to 10.09%, the tax rate goes up from 4.09% to 7.26%, the Gini coefficient goes down from 17.55% and 17.41% to 16.53% and 16.24%, respectively for pre- and after-tax.

4.9 Appendix D: Computation method

The model is solved numerically for a steady state equilibrium. In order to do this, I use an iterative method of successive approximations. First a policy parameter

$\hat{T}$ is chosen and a tax rate τ is guessed. Given the two, value function iteration is used to solve the functional equations (68) and (69) for optimal choices of consumption, saving, search effort, reservation wages and quit decisions. Next, the invariant distribution G^* is computed using a transition matrix Γ for given optimal decisions. Finally, the invariant distribution is used to evaluate the government budget balance. If it is significantly different from zero, a bisection method is used to bracket the root and the steps described are repeated until an equilibrium is found.

In order to solve for optimal decision rules, I use the standard technique of dynamic programming for infinite horizon case. The first step involves discretizing the action space by choosing a grid of feasible search efforts and saving decisions: it is chosen sufficiently fine such that adding more grid points does not affect the results. Thus, given the description of the model above, the whole model is discretized. Then, optimal decision rules for each state are computed by starting with an initial approximation of the value function in (69) and computes the right hand side of it in order to obtain a subsequent approximation. This procedure is repeated until convergence of the value function is achieved.

4.10 Appendix E: Markov transition function

Given the model, the worker's employment opportunities state, s, follows a $n \times a_M \times \left(\hat{T} + T + 1\right)$-state Markov chain. If $s = \{u, a, t, w_i\}$ $i \in \{1, 2, ..., n\}$, she remains unemployed for that period, receives unemployment benefit according to the benefit payment path (pinned down by t and w_i), has savings a and may receive a wage offer w' after setting the optimal search effort q. After having received a wage offer she makes a decision about her reservation wage $\bar{w}$ and thus pins down her transition probabilities to states tomorrow. Furthermore, she can be employed on a job with the one of the n possible wages: if $s = \{e, a, t, w_i\}$ she is still employed and has worked for a wage w_i for t periods so far. While on the job, based on comparison of V_u and V_e, the worker makes a decision about quitting the job or staying in it which determines her transition probabilities to other states in the next period.

Therefore, the transition function for the employment opportunities state given worker's decision function is a $\left[a_M \cdot n \cdot \left(\hat{T} + T + 1\right)\right] \times \left[a_M \cdot n \cdot \left(\hat{T} + T + 1\right)\right]$ matrix $\Gamma = \left[\Gamma_{ij}\right]$, where $i, j \in \left\{1, 2, ..., a_M \cdot n \cdot \left(\hat{T} + T + 1\right)\right\}$.

For instance, $\Gamma_{\hat{T}+1, 2\left(\hat{T}+T+1\right)+1} = \mathbb{P}\left\{s_{t+1} = \{e, 0, 1, w_3\} \mid s_t = \{u, 0, 1, w_1\}\right\}$ is the transition probability to employment with wage w_3 and 0 assets conditional on being unemployed for 1 period, receiving unemployment benefits tied to the most recent wage w_1 and having 0 assets. As there is no on the job search, the effort exerted will only affect probabilities of transition from unemployment to employment. Figure 23 presents example of transition probabilities for unemployed and employed workers.

State	$\{u,a',t',w_i\}$	$\{e,a',t^\dagger,w_1\}$	$\cdots$	$\{e,a',t^\dagger,w_n\}$
$\{u,a,t,w_i\}$	$1-f(q_t)\sum_{w\geq\bar{w}}\mathbb{P}(w)$	$\begin{cases} f(q_t)\,\mathbb{P}(w_1) & w_1 \geq \bar{w} \\ 0 & otherwise \end{cases}$	$\cdots$	$\begin{cases} f(q_t)\,\mathbb{P}(w_n) & w_n \geq \bar{w} \\ 0 & otherwise \end{cases}$
$\{e,a,t,w_1\}$	$\begin{cases} 1 & V_u(a',t',w_1) > V_e(a',t^\dagger,w_1) \\ \sigma & otherwise \end{cases}$	$\begin{cases} 0 & V_u(a',t',w_1) > V_e(a',t^\dagger,w_1) \\ 1-\sigma & otherwise \end{cases}$	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\{e,a,t,w_n\}$	$\begin{cases} 1 & V_u(a',t',w_n) > V_e(a',t^\dagger,w_n) \\ \sigma & otherwise \end{cases}$	0	$\cdots$	$\begin{cases} 0 & V_u(a',t',w_n) > V_e(a',t^\dagger,w_n) \\ 1-\sigma & otherwise \end{cases}$

Note: Each row contains probability of transition from current row-state to a possible column-state. Notice that the agents make a decision about the state a' so all the transition probabilities are conditional on the decision a' being optimal in a given state.

Notation: $\{e,a,t,w_i\}$ - employed for t periods at wage w_i with savings a, $\{u,a,t,w_i\}$ - unemployed for t periods so far, having the most recent wage w_i and savings a. Also:

$$t^\dagger = \begin{cases} \min\{\hat{T}, t+1\} & \text{if previously } x = e \\ 1 & \text{if previously } x = u \end{cases} \quad t' = \begin{cases} 1 & \text{if previously } x = e \text{ and } t = \hat{T} \\ T+1 & \text{if previously } x = e \text{ and } t < \hat{T} \\ \min\{T+1, t+1\} & \text{if previously } x = u \end{cases}$$

Figure 23: Transition function for workers in states $\{u,a,t,w_i\}$ and $\{e,a,t,w_j\}$