Economics Department

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ECO No. 98/7

EUI WORKING PAPERS

EUROPEAN UNIVERSITY INSTITUTE

EUROPEAN UNIVERSITY INSTITUTE, FLORENCE
ECONOMICS DEPARTMENT

EUI Working Paper **ECO** No. 98/7

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BADIA FIESOLANA, SAN DOMENICO (FI)

Printed in Italy in April 1998
European University Institute
Badia Fiesolana
I – 50016 San Domenico (FI)
Italy

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Anna Pettini and Louis Phlips†

Abstract

This paper examines the properties of quality selection when the standard price discrimination rule is modified for redistributive purposes. A public firm is assumed to sell a private good of different qualities to people with homogeneous tastes but different incomes, in a mixed duopoly or as a monopolist: optimal marginal conditions for quality selection are compared for the different cases. We derive a generalised form of a redistributive price discrimination rule for a public monopoly. Welfare consequences and the sensibility of redistributive objectives in a partial equilibrium context are also discussed.

*We would like to thank Andrzej Baniak, Etienne Billette de Villemeur and Alessandro Petretto for very helpful comments. We are responsible for the remaining errors

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1 Introduction

Price discrimination naturally arises in the study of monopoly since a monopolist typically desires to sell additional output if it can do so without lowering the price on the units it is currently selling (Varian,[7]). Price discrimination may involve selling different varieties of the same good (horizontal product differentiation), as well as selling different qualities (vertical product differentiation). In the latter case, the monopolist exploits the differences in the ability to pay of consumers by applying a profit-maximising price-discrimination rule.

Phlips [6] applies price discrimination to the case where a monopolist sells a different quality of a good to each class of consumers. In his model, all consumers rank the different qualities in the same order, namely they have the same preferences, so that their reservation price for each quality corresponds to their ability to pay, i.e. their income. The result of his analysis is that lower income people are charged a price equal to their reservation price, while richer consumers are able to retain a positive surplus. Moreover, welfare maximising quality selection is such that the marginal evaluation of quality of richer people is equal to the marginal cost of the quality they buy, while the marginal benefit of lower income consumers for the quality chosen for them is lower than the marginal cost of it.

Suppose now that the private good to be sold at different qualities is not a car, for example, but a bed in a hospital. Quality differences between two beds in the same hospital could be represented, e.g., by the number of other beds in the same room.[1] A bed in a hospital is a private good, because it is rivalrous, excludible and has non-zero marginal cost for the marginal consumer. Nonetheless, it produces health, which is a social good since it produces externalities. This is one of the reasons why a government may desire to sell a private good and/or to regulate the market.[2] We consider the case where a government is interested in

[1]This example fits well with the case of unitary demand, as considered in Phlips's model and in the following analysis.

[2]The desirability of public provision of private goods in second best economies, i.e. when personalized lump sum transfers are not available, is today a hotly debated

providing a private good, which may or may not be already provided by the private market, for a redistributive purpose. Goods like health care or education are commonly provided by both the private and the public sector and, in general, the private sector offers better quality. The government may want everybody to have access to the service, and decides to provide it itself and target it to people who could not afford the privately provided one.

The first step of our approach is to consider a profit maximizing public firm which enters a market where there is an unregulated private monopolist. The public firm offers only the lowest quality and, by doing this, forces the private seller to adapt the price of the lowest quality to the one set by the government.[3] We call this market setting a mixed duopoly. A remark is here essential: since the private firm has to adjust to the price fixed by the public sector, strategic behavior is ruled out. The first case, that of the profit-maximising duopoly, could indeed include strategic interaction between the private and the private firm, but we use it as a benchmark of cases in which the government is not interested in price competion. The government, through the public firm, is only interested in exploring the redistributive potential of price discrimination.

Section 2 presents the model and is then divided into three cases: in cases 1 (2.1) and 2 (2.2) the public firm offers only the low quality and the private one offers both qualities. The difference between the two cases is that both firms adopt a profit maximising pricing rule in the first while, in the second, the public firm is assumed to choose a price for the quality sold such that low income people retain a positive surplus. In the third case there is only one producer, the public firm, which offers the two qualities.

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issue and has been first introduced in a paper by Guesnerie-Roberts [4] and Blackorby-Donaldson [3]. A comprehensive list of references on the theory of public provision of private goods together with optimal taxation, can be found in Balestrino [2]. Hare [5] provides a general review of the literature which invokes externalities, informational asymmetries and merit goods arguments to justify public intervention in the supply of private goods.

[3] This amounts to assuming that the price set by the public firm is lower than the price that the private monopolist would like to charge for the low quality.

Section 3 analyses some welfare implications while section 4 generalizes the case of a redistributive monopoly. A redistributive pricing rule and optimality conditions for quality selection are derived. Section 5 concludes.

2 The Model

Consider an economy in which a private good (q) is offered by a public firm (G) and a private firm (P). Assume a context of vertical differentiation of the product, namely a technology which allows for providing the good q at different qualities. Thus, good q can be interpreted as being the *general* good, which is converted in as many *specific* goods as the number of qualities at which it can be provided. We assume that there are only two qualities q_i ($i = 1, 2$) with $q_2 > q_1$ where q_i indicates the quality level.

The government offers the lower quality q_1 whereas the private firm offers both the low and the high quality or none of the two. Costs are continuous over the two qualities and the unitary cost of producing quality q_i for the private firm is $c^P(q_i)$, while the public sector's cost per unit in producing q_1 is $c^G(q_1)$. The unit cost of quality improvement increases with quality at a rate $\partial c^k(q_i)/\partial q_i > 0 \quad (k = G, P; i = 1 \ if \ k = G)$. Since $c^k(q_i)$ is the unit cost, total cost is $N_i^k c^k(q_i)$ (where N_i^k is the number of consumers of group i served by firm k), and marginal cost is $\partial N_i^k c^k(q_i)/\partial q_i = N_i^k \partial c^k(q_i)/\partial q_i$.

Profits of the two firms are

$$\Pi^G = N_1^G p_1 - N_1^G c^G(q_1) \tag{1}$$

$$\Pi^P = \sum_{i=1}^{2} N_i^P p_i - \sum_{i=1}^{2} N_i^P c^P(q_i). \tag{2}$$

The demand side of the market is supposed to be divided into two classes of consumers ($j = 1, 2$); both of them have the same preferences over the ranking of qualities, namely they agree on the higher quality being preferable ($R_2^j > R_1^j$ where R_1^j is the highest price that a consumer of group j is ready to pay for quality 1), but they differ in the income they have (so that the index j indicates both the class of consumers and the income level). Higher income people have a higher total willingness-to-pay (hereafter WTP) than the poorer for both qualities, i.e. $R_i^2 > R_i^1$. In addition, we assume that the individuals with larger WTP have also larger marginal WTP, so that $\partial R_i^2/\partial q_i > \partial R_i^1/\partial q_i$ and, lastly, that $\partial R_i^j/\partial q_i > 0$.

Each consumer is supposed to buy, at most, one quality and at most one unit of that quality, so that their WTP reduces to their reservation price for the unit of product they buy. Assuming that consumers are interested in buying the good is equivalent to assuming that the following participation constraint is satisfied:

$$R_i^j - p_i \geq 0 \quad \forall j, i, \tag{3}$$

where R_i^j is the reservation price of group j for quality i, and p_i is the price for quality i.

Each group j chooses quality j on the condition that no other quality gives it a higher surplus, i.e. the following self-selection constraint has to be satisfied:

$$R_j^j - p_j \geq R_i^j - p_i \quad \forall j, i, j \neq i. \tag{4}$$

If the self-selection constraint is satisfied each group is not interested in buying another quality. This self-selection constraint is a basic feature of our problem in the sense that the government wants to target the low quality to poorer people. It affects the outcomes of all the redistributive efforts of the government and is valid whether the low quality is also offered by the private firm or not.[4]

The social consumer surplus is

$$S = \sum_{i=1}^{2} N_i(R_i^i - p_i). \tag{5}$$

[4] In this respect the structure of the market or the number of suppliers is not relevant.

Let us assume that general welfare depends linearly upon social consumer surplus and the profits of the two firms

$$W = \lambda S + \Pi^G + \Pi^P, \tag{6}$$

where $0 \leq \lambda \leq 1$. When $\lambda = 1$ the maximisation of this welfare function gives the unconstrained optimum, while pure profit maximising is obtained when λ is equal to zero. This interpretation holds in the discriminating profit maximising monopoly setting. In the present analysis λ amounts to the weight assigned to social consumer surplus by the public sector.[5] (Later on λ will be indexed since its interpretation may change depending on the cases considered. The optimal quality selection will then result from maximising the welfare function under particular pricing rules for the two firms.)

We can now study three cases. In the first, the public firm offers only q_1, while the private firm offers both qualities. Each firm is interested in extracting as much surplus as possible from consumers, i.e. they select the qualities which maximise their profits. In case 2, the good is still supplied by both the government and a private firm, but the government is only interested in breaking even; the private firm offers the good at both qualities and wants to maximise its profit. In case 3, the public firm is the only producer; it offers either the low quality only or the two quality levels at prices which cover the costs. Finally, the case of introducing a redistributive device in the profit maximising pricing rule is explored.

2.1 The Initial Setting: Case 1

People are divided into two subgroups, N_1 and N_2, and the first group is served by both firms ($N_1 = N_1^G + N_1^P$). N_1^G and N_1^P are given. Nonetheless, since the relative magnitude of the two shares will be relevant to interpret our results, the whole range of possibilities, i.e. $N_1^G \leq (\geq) N_1^P$, will be discussed.

[5] λ could be interpreted as related to the marginal cost of public funds, δ as $\lambda = 1/1+\delta$. $\lambda = 1$ means $\delta = 0$. Since we are working in a partial equilibrium context with a break-even constraint, to which the public provision is subjected, the assumption $\delta = 0$ makes sense .

If both firms maximise their profits, they will try to extract as much surplus as possible from each class of buyers. The pricing rule resulting from this objective subject to the self-selection and the participation constraints is

$$\begin{aligned} p_1^G(= p_1^P = p_1) &= R_1^1 \\ p_2^P(= p_2) &= R_2^2 - R_1^2 + p_1. \end{aligned} \tag{7}$$

Prices are completely determined by the objective we attribute to the sellers, so that these are the equilibrium prices. Indeed, these are the highest possible prices that satisfy the self-selection and participation constraints. Since prices for the homogeneous low quality have to be equal while there is only one price for the high quality, they can be simply labelled p_1 and p_2.

Lemma 1 *The pricing rule (7) is valid whether the low quality is offered by a monopolist or by the government and a private firm.*

Proof. Both public firm and the private one are assumed to set the prices in order to extract as much surplus as possible from consumers. Thus, the highest possible prices satisfying both participation and self-selection constraints are (7). Since quality 1 provided by the two firms is assumed to be homogeneous, they face the same reservation price R_1^1. Consequently, p_1^G cannot differ from p_1^P because all people of class N_1 would otherwise choose the quality offered at the lower price. *q.e.d.*

Social consumer surplus and social profits are expressed as

$$S = N_1(R_1^1 - p_1) + N_2(R_2^2 - p_2) \tag{8}$$

$$\begin{aligned} \Pi = \Pi^G + \Pi^P &= N_1^G p_1 + N_1^P p_1 + N_2 p_2 \\ &-N_1^G c^G(q_1) - N_1^P c^P(q_1) - N_2 c^P(q_2). \end{aligned} \tag{9}$$

Profits differ when costs are different. $\Pi^G = \Pi^P$ only if technology is homogeneous, i.e. when costs for producing the same quality are equal, and if $N_1^G = N_1^P$.

General welfare is the weighted sum

$$\begin{aligned} W &= \lambda_1 S + \Pi \\ &= \lambda_1 \left[N_1(R_1^1 - p_1) + N_2(R_2^2 - p_2) \right] + \\ &\quad (N_1^G + N_1^P)p_1 + N_2 p_2 - N_1^G c^G(q_1) \\ &\quad - N_1^P c^P(q_1) - N_2 c^P(q_2), \end{aligned} \tag{10}$$

where $0 \leq \lambda_1 \leq 1$. Substituting the pricing rule into (10) we obtain

$$\begin{aligned} W = {} & N_2[R_2^2 - (1-\lambda_1)(R_1^2 - R_1^1) - c^P(q_2)] + \\ & N_1^G[R_1^1 - c^G(q_1)] + N_1^P[R_1^1 - c^P(q_1)]. \end{aligned} \tag{11}$$

Maximisation of (11) with respect to the qualities leads to

$$\begin{aligned} \frac{\partial R_1^1}{\partial q_1} = {} & \left[\frac{N_1^G}{N_1 + N_2(1-\lambda_1)} \right] \frac{\partial c^G(q_1)}{\partial q_1} + \left[\frac{N_1^P}{N_1 + N_2(1-\lambda_1)} \right] \frac{\partial c^P(q_1)}{\partial q_1} + \\ & \left[\frac{N_2(1-\lambda_1)}{N_1 + N_2(1-\lambda_1)} \right] \frac{\partial R_1^2}{\partial q_1} \end{aligned} \tag{12}$$

$$\frac{\partial R_2^2}{\partial q_2} = \frac{\partial c^P(q_2)}{\partial q_2}, \tag{13}$$

which is analogous to the result in Phlips [6]. The only special feature is that there are two derivatives for the unit cost, one for the public firm and the other for the public one. Equations (12) and (13) show that marginal WTP is equal to per capita marginal cost only for the highest quality. For group 1 marginal WTP is a weighted average of marginal costs and the marginal value placed on quality 1 by group 2.

It is worth noting that the difference between R_1^1 and the marginal cost of quality 1 in a discriminating monopoly is null when $\lambda_1 = 1$. It is in fact the unconstrained optimum. The same is true here, when marginal cost is measured as the weighted average

$$\left(N_1^G \frac{\partial c^G}{\partial q_1} + N_2^P \frac{\partial c^P}{\partial q_1}\right) / N_1$$

About quality selection we can state the following.

Proposition 2 *Let* $\partial c^k(q_i)/\partial q_i > 0$. $[q_{1D}^* \mid 0 \leq \lambda_1 < 1] < [q_{1D}^* \mid \lambda_1 = 1]$

where q_{1D}^* is optimal quality 1 obtained in a discriminating duopoly. Provided $0 \leq \lambda_1 < 1$, optimal quality selection is such that the quality offered to the lower income group is lower than at the unconstrained optimum. Indeed, at a constrained optimum, quality 1 decreases with marginal WTP of group 2 for quality 1 ($\partial R_1^2/\partial q_1$).

Proof.
Compare the case where $\lambda_1 = 1$ with that in which $\lambda_1 = 0$. For $\lambda_1 = 1$,

$$\begin{aligned} \frac{\partial R_1^1}{\partial q_1} &= \frac{N_1^G}{N_1} \frac{\partial c^G(q_1)}{\partial q_1} + \frac{N_1^P}{N_1} \frac{\partial c^P(q_1)}{\partial q_1} \\ &= \frac{1}{N_1}(MC_1^G + MC_1^P) \end{aligned} \tag{14}$$

For $\lambda_1 = 0$,

$$\frac{\partial R_1^1}{\partial q_1} = \frac{MC_1^G + MC_1^P}{N_1 + N_2} + \frac{N_2}{N_1 + N_2} \frac{\partial R_1^2}{\partial q_1} \tag{15}$$

Assume R_i^i to be linearly dependent of q_1 so that $\partial R_i^i/\partial q_i$ is a given constant with respect to q and the left hand sides of both (14) and (15) turn out to be equalized to the same value, a. Let $(\partial R_1^2/\partial q_1)N_2 = b$ (> 0 by assumption). Equations (14) and (15) are satisfied respectively for a value q^* and $\bar{q}$ of quality 1 (where q stands for q_1 until the end of this proof). The relevant comparison is therefore when $q^* > (<)\bar{q}$. In other words, we are interested in the sign of Δq in $\bar{q} = q^* + \Delta q$. Take

$$\frac{MC^G(q^*) + MC^P(q^*)}{N_1} = \frac{MC^G(\bar{q}) + MC^P(\bar{q})}{N_1 + N_2} + \frac{b}{N_1 + N_2} = a \tag{16}$$

and, for simplicity, let $[MC^G(q^*)+MC^P(q^*)] = f(q^*)$ and $[MC^G(\bar{q})+MC^P(\bar{q})] = f(\bar{q})$; (16) becomes

$$\frac{f(q^*)}{N_1} = a \tag{17}$$

$$\frac{f(\bar{q})}{N_1+N_2} + \frac{b}{N_1+N_2} = a. \tag{18}$$

Compare now (16) and (17) in which the two terms of (16) have been solved for a. $f(q^*)$ is always greater than $f(\bar{q})$ since $\partial R_1^1/\partial q_1$ is less than $\partial R_1^2/\partial q_1$ and costs are monotonically increasing in q. *q.e.d.*

The intuition behind Proposition (2) is that optimal quality 1 is the higher, the greater is the importance given to social consumer surplus when maximizing general welfare. If no weight is attributed to consumer surplus and profits only are maximised, optimal quality 1 is at the lowest possible level. This is due to the presence of the WTP of consumers of class 2 for quality 1 which appears when $\lambda_1 < 1$.

Another comparison involves the difference, if any, between optimal qualities in a discriminating monopoly, as in Phlips [6], and that selected in the duopoly we have been analysing. Not surprisingly, optimal quality 2 is the same since quality 2 is still produced by one firm only. As for quality 1, compare expressions (14) and (15) with the corresponding expressions

$$\frac{\partial R_1^1}{\partial q_1} = \frac{\partial c^M(q_1)}{\partial q_1}, \text{for } \lambda_1 = 1 \tag{19}$$

$$\frac{\partial R_1^1}{\partial q_1} = \frac{N_1}{N_1+N_2}\frac{\partial c^M(q_1)}{\partial q_1} + \frac{N_2}{N_1+N_2}\frac{\partial R_1^2}{\partial q_1}, \text{for } \lambda_1 = 0 \tag{20}$$

for a discriminating monopoly.

Proposition 3 *With the pricing rule (7), $q_{1D}^* > (<) q_{1M}^*$ if $MC^G(q_1) + MC^P(q_1) < (>) MC^M(q_1)$, $\forall \lambda_1 : 0 \leq \lambda_1 \leq 1$.*

$MC^M(q_1)$ is the monopolist's marginal cost of quality 1 and q_{1M}^* optimal quality 1 selected in a discriminating monopoly.

According to Proposition 3, when each firm in a discriminating duopoly maximises its profits, optimal quality 1 to be selected is higher (smaller) than in a discriminating monopoly if the sum of the duopolists' marginal costs of producing q_1 is smaller (higher) than monopolist's marginal cost for quality 1. Results of this section will serve as benchmarks.

2.2 Introducing a Redistributive Objective: Case 2

Assume now that the government is interested in giving the low quality to lower income people at a price below their reservation price. We assume that the price set by the public sector is either equal to the unit cost of low quality (eq.(21)), which allows the public sector to break even, or equal to the marginal cost of low quality (eq.(29)). In both cases the government sets the price according to its own costs, and the private firm has to follow the price set by the government for low quality.

Let the pricing rule be

$$\begin{aligned} p_1 &= c^G(q_1) \\ p_2 &= R_2^2 - R_1^2 + p_1. \end{aligned} \tag{21}$$

Substituting this rule into general welfare, we have

$$\begin{aligned} W &= \lambda_2 S + \Pi^G + \Pi^P \\ &= \lambda_2[N_1(R_1^1 - c^G(q_1)) + N_2(R_1^2 - c^G(q_1))] + \\ &\quad N_1^P[c^G(q_1) - c^P(q_1)] + N_2(R_2^2 - R_1^2) + N_2 c^G(q_1) - N_2 c^P(q_2) \end{aligned} \tag{22}$$

where $0 < \lambda_2 \leq 1$ and $c^G(q_1) \neq c^P(q_1)$. The weight assigned to social consumer surplus within the expression for social welfare cannot be assumed to be zero, in the current and following sections: it would be contradictory to assign no importance to consumer surplus in a problem where the specific purpose of one of the two firms is to leave a positive surplus to lower income people.

Optimal quality selection is given by substituting the pricing rule (21) into equation (22), and maximising the latter with respect to the qualities.

Resulting optimal qualities at the constrained optimum are such that

$$\frac{\partial R_1^1}{\partial q_1} = \left[1 - \frac{N_2}{N_1}\frac{(1-\lambda_2)}{\lambda_2}\right]\frac{\partial c^G(q_1)}{\partial q_1} + \left[\frac{N_2}{N_1}\frac{(1-\lambda_2)}{\lambda_2}\right]\frac{\partial R_1^2(q_1)}{\partial q_1} +$$
$$\frac{1}{\lambda_2}\frac{N_1^P}{N_1}\left(\frac{\partial c^P(q_1)}{\partial q_1} - \frac{\partial c^G(q_1)}{\partial q_1}\right) \tag{23}$$
$$\frac{\partial R_2^2}{\partial q_2} = \frac{\partial c^P(q_2)}{\partial q_2}. \tag{24}$$

At the unconstrained optimum (i.e. when $\lambda_2 = 1$)

$$\frac{\partial R_1^1}{\partial q_1} = \frac{N_1^G}{N_1}\frac{\partial c^G(q_1)}{\partial q_1} + \frac{N_1^P}{N_1}\frac{\partial c^P(q_1)}{\partial q_1} \tag{25}$$
$$\frac{\partial R_2^2}{\partial q_2} = \frac{\partial c^P(q_2)}{\partial q_2}. \tag{26}$$

Proposition 4 *When* $\lambda_2 = 1$, $[q_{1D}^* \mid (21)] = [q_{1D}^* \mid (7)]$.

Proof.
Equations (25) and (26) can be written as

$$\frac{\partial R_1^1}{\partial q_1} = \frac{1}{N_1}(MC_1^G + MC_1^P) \tag{27}$$
$$\frac{\partial R_2^2}{\partial q_2} = \frac{1}{N_1^P}MC_2^P, \tag{28}$$

where MC_i^k denotes marginal cost of firm k for quality i. Equation (27) is the same as (14).

When $\lambda_2 = 1$, optimal quality 1 in a profit-maximising discriminating duopoly is the same as that selected in a duopoly in which the price of quality 1 is set equal to the unit cost of producing the same quality. In other words, it is possible to leave a positive surplus to lower income people and, at the same time, let them get the same quality as in a profit maximising duopoly. The explanation of this result is that welfare maximization is left unaltered by the changes in the pricing rule from (7) to (21): when social consumer surplus and social profits are equally weighted in the expression for general welfare ($\lambda_2 = \lambda_1 = 1$), the positive surplus left to consumers of class 1 is compensated by the zero profit given to the public firm. This is exactly what the public firm aims at.

The open question is whether unit cost of the private firm for quality 1 are higher or smaller than that of the public firm. Indeed, general welfare is lower when $c_1^P < c_1^G$ because the private firm sells quality 1 at a price lower than the cost of it (the opposite holds when $c_1^P > c_1^G$). This is clear by looking at the third term of the right hand side of (23).

Now let the pricing rule be

$$p_1 = \frac{\partial N_1^G c^G(q_1)}{\partial q_1} \qquad (29)$$

$$p_2 = R_2^2 - R_1^2 + p_1$$

where $[\partial N_1 c^G(q_1)]/(q_1) \neq [\partial N_1^P c^P(q_1)]/(\partial q_1)$.

General welfare is

$$\begin{aligned} W &= \lambda_2 S + \Pi^G + \Pi^P = \\ &= \lambda_2 N_1 \left[R_1^1 - N_1^G \frac{\partial c^G(q_1)}{\partial q_1} \right] + \lambda_2 N_2 \left(R_1^2 - \frac{\partial c^G(q_1)}{\partial q_1} \right) + \\ &\quad N_1^G \left[N_1^G \frac{\partial c^G(q_1)}{\partial q_1} - c^G(q_1) \right] + N_1^P \left[N_1^G \frac{\partial c^G(q_1)}{\partial q_1} - c^P(q_1) \right] + \\ &\quad N_2 \left[R_2^2 - R_1^2 + N_1 \frac{\partial c^G(q_1)}{\partial q_1} - c^P(q_2) \right]. \end{aligned} \qquad (30)$$

The expressions for optimal quality selection (when the price for low quality is set equal to its marginal cost) are

$$\frac{\partial R_1^1}{\partial q_1} = \frac{1}{N_1}\left[N_1^G \frac{\partial^2 c^G(q_1)}{\partial q_1^2} - \frac{1}{\lambda_2} N_1^P \frac{\partial^2 c^P(q_1)}{\partial q_1^2}\right] + \frac{N_2}{N_1}\frac{\partial R_1^2}{\partial q_1}\left(\frac{1-\lambda_2}{\lambda_2}\right) \quad (31)$$

$$\frac{\partial R_2^2}{\partial q_2} = \frac{\partial c^P(q_2)}{\partial q_2}. \quad (32)$$

where the total number of individuals is normalised to one ($N_1 + N_2 = 1$).

Socially optimal quality 1 from pricing rules at equations (21) and (29) depends on the functional form of costs. When, for example, a cost function based on a Cobb-Douglas technology is assumed

$[q_{1D}^* \mid (21)] > [q_{1D}^* \mid (29)]$.

The unconstrained optima are as in (25) – (26)and (27) – (28).

2.3 The Private Sector Drops Out: Case 3

Imagine now that the public firm produces the two qualities and that there is no private firm. The public firm chooses among different options. In the first and second option (33) and (36) it chooses to sell both qualities at prices respectively equal to the unit cost and the marginal cost of the corresponding quality. In the third case, the public firm chooses a pricing rule which allows for positive profits in selling the higher quality, while it keeps the price for low quality equal to its unit cost.

The pricing rule is

$$p_1 = c^G(q_1) \quad (33)$$
$$p_2 = c^G(q_2),$$

so that there is no price discrimination. Profits are zero, and general welfare reduces to $W = S$ since maximising S or $\lambda_3 S$ amounts to the same. Only unconstrained welfare maximisation is relevant and optimal quality selection obeys

$$\frac{\partial R_1^1}{\partial q_1} = \frac{\partial c^G(q_1)}{\partial q_1} \quad (34)$$

$$\frac{\partial R_2^2}{\partial q_2} = \frac{\partial c^G(q_2)}{\partial q_2}. \quad (35)$$

If, instead, the pricing rule is

$$p_1 = MC_1 = \frac{N_1 \partial c^G(q_1)}{\partial q_1} \tag{36}$$
$$p_2 = MC_2 = \frac{N_2 \partial c^G(q_2)}{\partial q_2},$$

positive profits are possible, since $(\partial c^G(q_i)/\partial q_i) > 0$, so that it makes sense to define social welfare as

$$\begin{aligned} W = {} & \lambda_3 S + \Pi^G \\ & \lambda_3 N_1(R_1^1 - \frac{N_1 \partial c^G(q_1)}{\partial q_1}) + \lambda_3 N_2(R_2^2 - \frac{N_2 \partial c^G(q_2)}{\partial q_2}) + \\ & N_1 \left[\frac{N_1 \partial c^G(q_1)}{\partial q_1} - c^G(q_1)\right] + N_2 \left[\frac{N_2 \partial c^G(q_2)}{\partial q_2} - c^G(q_2)\right], \end{aligned} \tag{37}$$

where $0 < \lambda_3 \leq 1$. Optimal quality selection is such that

$$\frac{\partial R_1^1}{\partial q_1} = \frac{1}{\lambda_3} \frac{\partial c^G(q_1)}{\partial q_1} - \left[\frac{N_1(1-\lambda_3)}{\lambda_3}\right] \frac{\partial^2 c^G(q_1)}{\partial q_1^2} \tag{38}$$
$$\frac{\partial R_2^2}{\partial q_2} = \frac{1}{\lambda_3} \frac{\partial c^G(q_2)}{\partial q_2} - \left[\frac{N_2(1-\lambda_3)}{\lambda_3}\right] \frac{\partial^2 c^G(q_2)}{\partial q_2^2}, \tag{39}$$

while the unconstrained optimum again implies (34) and (35). Putting prices equal to unit costs or marginal costs leads to the same unconstrained optimum. Note the appearance of second order derivatives in the constrained optima.

A remark is in order. When both prices are set equal to marginal cost of the corresponding quality, self-selection constraints are not necessarily binding since the number of people of classes 1 and 2 are not given any more. Marginal costs are affected by this, since they depend on the number of people choosing a given quality.

Third, suppose a redistributive objective is applied to the price for low quality only. The pricing rule is then as defined in equation (21), so that the participation and self-selection constraints are satisfied and the rich will not buy the lower quality.

Social welfare is

$$\begin{aligned} W &= \lambda_3 S + \Pi^G \\ &= \lambda_3 N_1 (R_1^1 - c^G(q_1)) + \lambda_3 N_2 (R_1^2 - c^G(q_1)) \\ &\quad + N_2 (R_2^2 - R_1^2 + c^G(q_1)) - N_2 c^G(q_2). \end{aligned} \tag{40}$$

where, again, $0 < \lambda_3 \leq 1$, and

$$\frac{\partial R_1^1}{\partial q_1} = \left[1 - \frac{N_2}{N_1}\frac{(1-\lambda_3)}{\lambda_3}\right]\frac{\partial c^G(q_1)}{\partial q_1} + \left[\frac{N_2}{N_1}\frac{(1-\lambda_3)}{\lambda_3}\right]\frac{\partial R_1^2}{\partial q_1} \tag{41}$$

$$\frac{\partial R_2^2}{\partial q_2} = \frac{\partial c^G(q_2)}{\partial q_2}. \tag{42}$$

When $\lambda_3 = 1$, we obtain again (34) and (35): the unconstrained optimum is the same when the two qualities are priced at unit cost or when only the low quality is priced at unit cost.

3 Some Remarks on General Welfare and Redistribution

The previous section analyses quality changes when the price rule is modified by the introduction of a redistributive objective. What can we say in terms of general welfare resulting from the examined changes in the pricing rule? In other words, when poorer people are left a positive surplus, who pays for it?

In general, the possibility of leaving poorer people a positive surplus is payed by the firms. In the case where low quality price is set equal to unit cost, the positive surplus left to consumers of class 1 is directly translated into zero profits for the public firm. A similar mechanism applies to every other redistributive modification of the pricing rule. This is due to the linearity of the social welfare function in its arguments. Had that function been non-linear, results could have been different.

To what extent can one analyse a redistributive objective in a partial equilibrium context, such as ours? It is well known that the approach in which consumers and producers surpluses are taken as a measure of social benefits implicitly assumes that individual preferences are represented by a quasi-linear utility function. This kind of utility function implies a constant marginal utility of income and, as a consequence, zero income effects. A constant marginal utility of income means that one dollar given to a poor has the same impact on his welfare as one given to a rich. The linearity of the social welfare function maintains this property. In this environment, redistribution of income and of welfare are equivalent.

4 A general case of redistributive monopoly

This section will generalize the case of a discriminating monopoly when a redistributive objective is introduced. Assume that the government applies the profit maximising pricing rule but with the following redistributive device: the price for low quality is lessened by a positive amount, K, which is added to the price for high quality. K is assumed to be a function of the two qualities to be specified later. The self-selection and participation constraints are satisfied, so that richer people are prevented from choosing the low quality, and at the same time poorer people are allowed to retain a positive surplus.

Proposition 5 $K(q_1, q_2)$ *modifies the profit maximising pricing rule as follows:*

$$\begin{aligned} p_1 &= R_1^1 - K(q_1, q_2) \\ p_2 &= R_2^2 - R_1^2 + R_1^1 + K(q_1, q_2). \end{aligned} \tag{43}$$

Proof.[6]

The participation and self-selection constraints are

$$R_1^1 \geq p_1 \; R_2^2 \geq p_2$$
$$R_1^1 - p_1 \geq R_2^1 - p_2 \; R_2^2 - p_2 \geq R_1^2 - p_1.$$

[6]The proof of this proposition follows the derivation of the non-linear pricing rule done by Varian[7].

and introducing $K(q_1, q_2)$ (=K during the following) they become respectively

$$p_1 \leq R_1^1 - K \quad (44)$$
$$p_2 \leq R_2^2 + K \quad (45)$$
$$p_1 \leq R_1^1 - 2K - R_2^1 + p_2 \quad (46)$$
$$p_2 \leq R_2^2 + 2K - R_1^2 + p_1. \quad (47)$$

The monopolist wants to select the largest possible p_1 and p_2. Thus, just one of the two equations (44) and (45) as well as of (46) and (47) will be binding.

About p_2. Suppose that (45) is binding, i.e. $p_2 = R_2^2 + K$. From (47) we get $R_1^2 \leq p_1 + K$. Combining the assumption by which $R_1^2 > R_1^1$ with 47, we have $R_1^2 > R_1^1 < p_1 + K$, which contradicts (44). Consequently, (47) is binding.

About p_1. If (46) was binding, $R_2^2 - R_1^2 = R_2^1 - R_1^1$ which is by definition

$$\int_{q_1}^{q^2} \frac{\partial R_i^1}{\partial t} dt = \int_{q_1}^{q^2} \frac{\partial R_i^2}{\partial t} dt \quad (48)$$

which violates the assumption that $\partial R_i^2 / \partial q_i > \partial R_i^1 / \partial q_i$. Thus, (44) is binding. *q.e.d.*

General welfare is

$$\begin{aligned} W &= \lambda_4 S + \Pi^G = \\ &= -N_1 \lambda_4 K(q_1, q_2) + \lambda_4 N_2 [R_1^2 - R_1^1 + K(q_1, q_2)] + \\ &\quad N_1 [R_1^1 - K(q_1, q_2) - c(q_1)] + \\ &\quad N_2 [R_2^2 - R_1^2 + R_1^1 + K(q_1, q_2) - c(q_2)] \end{aligned} \quad (49)$$

Optimal qualities correspond to the condition that

$$\frac{\partial R_1^1}{\partial q_1} = \left[\frac{N_1}{N_2(1-\lambda_4) + N_1} \right] \frac{\partial c(q_1)}{\partial q_1} + \left[\frac{N_2(1-\lambda_4)}{N_2(1-\lambda_4) + N_1} \right] \frac{\partial R_1^2}{\partial q_1} +$$

$$\left[\frac{(1+\lambda_4)(N_1-N_2)}{N_2(1-\lambda_4)+N_1}\right]\frac{\partial K(q_1,q_2)}{\partial q_1} \tag{50}$$

$$\frac{\partial R_2^2}{\partial q_2}=\frac{\partial c(q_2)}{\partial q_2}+\left[\frac{(N_1-N_2)}{N_2}(1+\lambda_4)\right]\frac{\partial K(q_1,q_2)}{\partial q_2}. \tag{51}$$

When $\lambda_4 = 1$ the last two expressions are

$$\frac{\partial R_1^1}{\partial q_1}=\frac{\partial c(q_1)}{\partial q_1}+\left[\frac{2(N_1-N_2)}{N_1}\right]\frac{\partial K(q_1,q_2)}{\partial q_1} \tag{52}$$

$$\frac{\partial R_2^2}{\partial q_2}=\frac{\partial c(q_2)}{\partial q_2}+\left[\frac{2(N_1-N_2)}{N_2}\right]\frac{\partial K(q_1,q_2)}{\partial q_2}. \tag{53}$$

Let q^*_{iR} and q^*_{iM} be the optimal quality resulting from respectively a redistributive monopoly and a (standard) discriminating monopoly. Equations (52) and (53) lead to the following

Proposition 6 *Let* $\lambda_1 = \lambda_4 = 1$ *and* $\partial c(q_i)/\partial q_i > 0$.
(A) $q^*_{iR} > q^*_{iM}$ *if* $N_1 > (<) N_2$ *and* $(\partial K(q_1,q_2)/\partial q_i) > (<) 0$.
(B) $q^*_{iR} < q^*_{iM}$ *when* $(N_1 - N_2)$ *and* $(\partial K(q_1,q_2)/\partial q_i)$ *have opposite sign.*

Proof.
Along the lines of the proof of Proposition 2.

It can immediately be observed that the introduction of a positive device makes the number of people of the two classes relevant for the optimal quality selection. The special case in which the two classes of customers are of equal size, is such that lower people are left a positive surplus but everybody gets the same quality as in the case of a redistributive monopoly. If $N_1 \neq N_2$, the rules of the last proposition apply.

Proposition 6 has several implications. First, it is straightforward that optimal qualities are the same in the two kinds of monopoly when either $K(q_1, q_2) = 0$ or its derivative with respect to quality is equal to zero. Moreover, $K(q_1, q_2)$ must have a non-zero response to quality variations for the present case to be meaningful.

Second, the number of people of the two classes is a given constant, whereas $K(q_1, q_2)$ is set by the monopolist; thus, the monopolist has the following possible choices. If the number of low income people is greater than the number of high income people, than the only opportunity to give the two classes a quality higher than in a discriminating monopoly (q_M) is to make $K(q_1, q_2)$ increasing in q_1. i.e. he has to charge a lower price to N_1 and a higher price to N_2. When $N_1 < N_2$, on the other hand, the two classes can be offered a quality higher than q_M, if $K(q_1, q_2)$ decreases with q_1. In other words, $K(q_1, q_2)$ acts as a redistributive device because it allows consumers of class 1 to retain a positive surplus which is taken away from consumers of class 2 . The second class of consumers, in turn, accept the burden imposed to them as long as the quality they are offered is higher than q_M.

5 Concluding Comments

In this paper we have extended the profit-maximizing pricing rule of a discriminating monopoly to a discriminating mixed duopoly, where a vertically differentiated private good is offered by both a public and a private firm. The public firm is supposed to produce only the first quality of the good, while the private one produces the two qualities, and discriminates the price charged to different categories of consumers in order to extract as much surplus as possible from them. A first result of the paper is that the non-linear pricing rule of a profit maximising monopolist is extendable to a mixed duopoly. In fact, a government may decide to act as a monopolist or to enter the market where a private firm is already selling the good and regulate it. Optimal quality in a discriminating duopoly is higher or lower than that in a discriminating monopoly according to the relation between the costs of the monopolist and the duopolists. The public firm is then supposed to experiment different modifications of the pricing rule, in order to let lower income people retain a positive surplus.

Optimal quality selection in a profit maximizing discriminating duopoly is equal to that in a duopoly in which the public firm sets the price of the low quality equal to its unit cost. The interesting feature of this result is that poorer people are left a positive surplus.

We also analyse the case of a public monopoly, in order to be able to compare optimal quality selection resulting from a standard discriminating monopoly, with a situation in which the monopolist is the public sector. This section shows that optimal quality selected can be higher for both income groups, whatever the number of rich and poor consumers is, provided the monopolist sets the redistributive variable in the appropriate way.

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