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# EUI WORKING PAPER No. 89/423

Time Consistency, Discounting and the Returns to Cooperation

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European University Institute, Florence



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# EUROPEAN UNIVERSITY INSTITUTE, FLORENCE

# DEPARTMENT OF ECONOMICS



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# Time Consistency, Discounting and the Returns to Cooperation

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# Introduction

When monetary policy-makers are unwilling to precommit themselves and are not inhibited by reputational constraints, then the optimal "time consistent" policy obtained by dynamic programming may well lead to inferior outcomes, as Kydland and Prescott (1977) first pointed out. Rational expectations of future uninhibited policy optimisation can generate outcomes which are worse than would prevail if the authorities could credibly "tie their hands" at the outset.

In an application of these ideas to the open economy, Rogoff (1985) subsequently obtained the fascinating result that the optimal "time consistent" outcome available to those responsible for coordinating policy for two open economies could sometimes be improved upon by returning the decision taking to the national authorities acting individually in order to avoid the inflationary effect that coordination would have on forward looking wage setting.

In an earlier paper by two of the current authors it was found that this paradox - that coordination might not pay - was also possible in macroeconomic models with purely <u>backward-looking</u> wage/price mechanisms, provided that the foreign exchange market was forward looking (Miller and Salmon, 1985). Subsequently it was shown to be necessary that the two economies be subject to different initial rates of underlying inflation - or, more generally, that the shocks affecting national inflation be relatively uncorrelated (Miller and Salmon, 1989): for inflation shocks sufficiently common to the member countries, coordination would pay after all (see Oudiz and Sachs (1985)).

In this paper we use the same framework, with non-anticpatory wage behaviour but a flexible exchange rate set by agents with rational expectations, to see what happens when significant discounting of future welfare payoffs is present. How, if at all, does such policy myopia affect the payoff to policy coordination? For any given correlation of multinational inflation shocks, is coordination more or less likely to pay when policy makers are myopic?

It is worth emphasising that, although the focus of this paper is essentially on "time consistent" solutions satisfying Bellman's Principle of Optimality, we do not discuss the implications of myopia for "trigger" of "punishment" strategies. (For a study of the effect of discounting on the sustainability of other equilibria in a continuous time stochastic model, for example, see Currie and Levine (1985).) Nevertheless, the analysis is still fairly complicated as it involves comparing time consistent optima for different policy regimes under varying degrees of myopia.

To begin with we discuss how lack of precommitment is associated with a loss of efficiency in such models, using the one-country case for simplicity and the

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<sup>2</sup> "optimal linear feedback rule" as a basis of comparison. The formal characterisation of time consistent policy is obtained by the method proposed by Cohen and Michel (1988), and the result - that myopia increases the divergence of time consistent equilibrium from the optimal rule - is also shown diagramatically. The same techniques are then applied to determine cooperative policy in a two country framework where the problem facing policy makers is the <u>difference</u> between the rates of inflation initially prevailing. As the relevant basis of comparison is a Nash equilibrium (rather than the optimal linear rule), the latter are

comparison is a Nash equilibrium (rather than the optimal linear rule), the latter arederived as well, under an "open loop" assumption for analytical purposes and with numerical results for the "closed loop" case in addition. In either case it is found that  $\mathbb{B}$ discounting has a greater adverse effect on the Nash equilibrium than it does on the  $\subseteq$ cooperative solution. When the problem afflicting policy makers is that of a common rate of inflation, however, the analysis is a good deal simpler - with a result which tends to favour decentralised policy making (as the distortion afflicting Nash policy is reduced by discounting).

Whether, on balance, discounting makes cooperation more or less likely to pay is analysed using a stochastic interpretation of the earlier welfare results, which reveals that, for any given correlation of country specific inflation shocks, myopia makes cooperation less attractive. After summarising our conclusions, it is suggested that the analytical procedures used to obtain the various equilibria - in particular the diagrammatic characterisation we derive for time consistent solutions in a dynamic setting - may be of use independently of the particular results obtained in this application.

## 1. Time consistency, discounting and a small open economy

As indicated, we begin with the impact of discounting on time consistent policy, measured relative to the optimal rule, in a small open economy, cf Miller (1985) where the policy maker chooses real interest rates so as to minimise a discounted stream of quadratic costs arising from fluctuations in output and core inflation. Current inflation is assumed to depend on expected future monetary policy as well as past inflation, via the impact of monetary policy on the real exchange rate.

The model is summarised in Table 1. Equation (1) describes the inflationary process with a Phillips curve augmented by terms representing backward looking core inflation and movements in the real exchange rate which is taken to be a forward looking variable. Output, which is determined by demand, depends on the real interest rate and the level of competitiveness, see equation (2). The evolution of  $\overline{\Box}$ core inflation is described in equation (3) as an exponential moving average of past inflation. Instead of using  $\pi$  as the state variable, it turns out to be more convenient to use z, which is defined to measure cumulated past excess demand as described in

equation (4). The level of competitiveness is a forward looking integral of future expected real interest rate differentials, see equation (5). The discounted welfare cost to be minimised is given by (6). (As there are no stochastic disturbances in equation (2), one may alternatively minimise with respect to y, solving for r from (2), which is what we do in the Appendix.)

# Table 1 A Small Open Economy

 $i = \phi y + \pi + \sigma Dc \tag{1}$ 

 $y = -\gamma r + \delta c \tag{2}$ 

 $D\pi = \xi (i - \pi) \tag{3}$ 

$$Dz = y \quad \text{or} \quad z(t) = \int_{-\infty}^{t} y(s) ds \tag{4}$$

$$Dc = r$$
 or  $c(t) = -\int_{0}^{t} r(s) ds$  (5)

$$\min_{r} V(0) = \frac{1}{2} \int_{0}^{\infty} e^{-ut} (\beta \pi^{2} + y^{2}) dt$$
(6)

# where

- i is the rate of price inflation
- y is the log of output (measured from its "natural" rate)
- $\pi$  is core inflation
- c is the real exchange rate, competitiveness (measured as a deviation from equilibrium)
- r is the real interest rate (measured as a deviation from the constant world level)
- z is the accumulant of demand pressure
- $D = \frac{d}{dt}$  is the differential operator

E is the expectectation operator

Two different policies are derived both for the undiscounted case and then to see the effects of "myopia" - with different degrees of discounting. The first is the

"time consistent" solution which satisfies Bellman's Principle of Optimality. Given the linear quadratic structure of the problem, this "dynamic programming" solution takes the form of a linear feedback rule. (As Cohen and Michel have shown, the rule in question may also be determined by Pontryagin's Maximum Principle, so long as expectations are treated as predetermined when optimisation is carried out; and it is their approach which we adopt here.)

The very fact that, in a rational expectations context, "time consistent" decision rules are not efficient in welfare terms immediately suggests the second solution namely the pattern of linear feedback which would maximise welfare, what we refer to as the "optimal rule".

We begin with the optimal time consistent policy, which is derived using the methodology of Cohen and Michel as in Miller (1985), (for the convenience of the reader the algebraic derivation is provided in Appendix 1). The issues involved are Treader the algebraic derivation is provided in Appendix 1). The issues involved at the readily accessible in diagramatic form, as we now proceed to show. The upshot of the formal analysis is that the time consistent policy is determined by three differentiat form, as we now proceed to show. The upshot of the equations  $\begin{bmatrix}
Dz \\
Dc \\
Dm
\end{bmatrix} = \begin{bmatrix}
0 & 0 & -1 \\
0 & \frac{\delta}{\gamma} & \frac{1}{\gamma} \\
-\phi h & -\sigma h & u
\end{bmatrix} \begin{bmatrix}
z \\
c \\
m
\end{bmatrix}$ where  $h = \beta\xi^2(\phi + \sigma\theta)$ where c is the forward looking real exchange rate m is the (current value) Pontryagin shadow price for z z is the contribution of demand pressure to inflation
and the system is assumed to lie on the stable manifold. Since both
"competitiveness" and the "shadow price" are jump variables the manifold is simply a vector, so c= $\theta z$  and m= $\psi z$  where these parameters are defined by the eigensystem  $\begin{bmatrix}
0 & 0 & -1 \\
0 & \frac{\delta}{\gamma} & \frac{1}{\gamma} \\
-\phi h & -\sigma h & u
\end{bmatrix} \begin{bmatrix}
1 \\
\theta \\
\psi
\end{bmatrix}$ which is used to construct Figure 1.
Private sector expectations as to the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate are represented by the eigensystem of the real exchange rate ar readily accessible in diagramatic form, as we now proceed to show. The upshot of the

0	0	-1]	[1]		1	
0	8/2	1/2	θ	$=\lambda_s$	θ	
_φh	-oh	u	Ψ_	$=\lambda_s$	Ψ	

Private sector expectations as to the real exchange rate are represented by the parameter  $\theta$  measured in absolute value along the horizontal axis of Figure 1, where high values of  $|\theta|$  are associated with expectations of sustained high real interest rates and an uncompetitive level of the exchange rate. What level of interest rates iemay be rational to expect depends on the nature of policy feedback. The strength of policy feedback is represented by the parameter  $\psi$  measured on the vertical axis of the figure. As is evident from the first row of the eigensystem, this parameter is in fact

5

equal to the absolute value of the stable root so the higher is  $\psi$ , the faster inflation will be squeezed out of the system.

# [Insert figure 1 here]

The link between policy feedback and private sector expectations appears in the figure as the schedule OX. This relationship (which is obtained from the second row of the eigensystem as  $|\theta|=1/(\gamma+\delta\psi^{-1})$ ) slopes upwards to the right since stronger (monetary) policy feedback implies high interest rates and a high exchange rate. (Note that the equation describing OX does not involve the rate of discount used in determining the strength of policy adjustment.)

How the rate of discount affects the choice of policy feedback is implicit in the third equation we can derive from the eigensystem (equation (a11) of the appendix). However, to see what is involved, it is easiest to start with the undiscounted case. To find the time consistent equilibrium, following the method of Cohen and Michel, one uses this third equation of the eigensystem to derive the policy "reaction function" giving the choice of feedback which minimises welfare costs <u>conditional</u> on the state of private sector expectations. This is labelled AB in the figure and is the locus of points where the iso-cost curves are vertical. By setting u=0 in equation (a11) one obtains  $\psi = \xi (\phi + \sigma \theta) \beta^{-1/2}$  as the expression for AB.

Since E lies on AB and also satisfies the rational expectations restriction, OX, it is, in fact, the <u>time consistent equilibrium</u> which we seek. At this point, by construction, the expectations the authority take for granted when choosing their policy feedback, are precisely what the private sector will associate with the policy feedback selected.

The "inefficiency" of this equilibrium is evident because, by construction, the schedule OX must cross an iso-cost curve (not shown) passing vertically through E. This implies that a stronger response (less policy "lethergy") would reduce integrated welfare costs. The point R where the schedule OX touches the lowest cost contour defines the "optimal linear feedback rule" (mathematically derived in the appendix). This rule is not, of course, time consistent so would need to be sustained by some system of reward or punishment. We do not discuss this here, but use it nevertheless as a bechmark with which the time consistent equilibrium may be compared.

What happens to each of these equilibria, and the difference between them, when future costs are positively discounted? For the time consistent equilibrium the effect depends on how the policy reaction function shifts when, for example, the discount rate u=u'>0 in equation (a11). Graphically the result is shown by the shift from AB to A'B; i.e. there is no movement of the horozontal intercept, but the function becomes convex and intersects the vertical axis at a lower level. Thus the

field of discounting is to reduce the strength of policy feedback, with the greatest for discounting is to reduce the strength of policy feedback, with the greatest for speed of adjustment that would be observed in a closed economy, for which 6 is irrelevant.) Since future inflation is reduced by incurring costs in the present, it seems intuitively reasonable that applying a discount factor discourages current scriftice. With no change in OX, the time consistent equilibrium moves from E to E. The effects on the optimal rule are indicated in the figure by a "discounted" isocost schedule tangent to OX at R. For similar reasons, the optimal linear feedback rule falls too: but the reduction of the feedback coefficient is less pronounced, as the figure shows. I Insert figure 2 here ] Insert figure 2 here ] Increases without limit. We find, however, that the tangency point in the left thangende approaches the point shown as L in the limit, so that the feedback coefficient is not but at many of the show as L in the limit, so that the feedback coefficient is not but at many of the set of the optimal limit, we not a point shown as L in the limit, so that the feedback coefficient is not but and the set of the optimal limit is on the unit is on the top of the discourt for the feedback vanishes as the discourt factor of the optimal time.

panel approaches the point shown as L in the limit, so that the feedback coefficient and the case of the optimal rule,  $\psi$ , does not vanish but approaches  $\psi_I$  in the limit as uIItends to infinity. The Author(

Table 2. provides some numerical results for the effect of discounting on these two policies with parameter values  $\beta = \xi = \phi = 1$ ,  $\delta = \gamma = 1/2$ ,  $\sigma = 1/10$ .

# Table 2.

### Feedback Coefficients for the Small Open Economy

Discount	Optimal	Time Consistent
rate	Rule	Rule
0.0	0.95116	0.90499
0.1	0.90735	0.85896
0.5	0.75828	0.70102
1.0	0.62244	0.55469
2.0	0.46092	0.37610
00	0.14788	0.00000

To conclude, therefore, we find that increasing policy myopia (i.e. raising u, the discount factor) induces more policy lethargy and that it affects the dynamic

programming solution more strongly. The gap between the feedback coefficients widens (approaching the value  $\psi_L$  in the limit). So, in this sense, we may say that discounting exacerbates the inefficiency of time consistent policy relative to the optimal rule. How it compares with a Nash equilibrium in a two country model is the subject of the next section.

# 2. The two country case.

To discuss issues of policy coordination, we consider the case of two symmetric open economies. The model we use for this purpose, published earlier in Miller and Salmon (1985), is a straightforward development from the one country case, which treats as endogenous foreign interest rates and income, see Table 3. (Variables pertaining to the foreign country are labelled with an asterisk.)

# Table 3: The two country model

 $E[Dc] = r - r^*$ 

Static Equations:

1)Aggregate Demand	$y = -\gamma r + \delta c + \eta y^*$	$y^* = -\gamma r^* - \delta c + \eta y$
2)Phillips Curve	$i = \phi y + \sigma Dc + \pi$	$i^* = \phi y^* - \sigma Dc + \pi^*$
3)Core Inflation	$\pi = \xi \phi z + \xi \sigma c$	$\pi^* = \xi \phi z^* - \xi \sigma c$

Home Country

**Dynamic Equations:** 

4)Accumulation Dz = y

5)Arbitrage

6)

**Policy Objectives:** 

Loss functions 
$$\min_{y} V(0) = \frac{1}{2} \int_{0}^{\infty} e^{-ut} (\beta \pi^{2} + y^{2}) dt$$
  $\min_{y^{*}} V^{*}(0) = \frac{1}{2} \int_{0}^{\infty} e^{-ut} (\beta \pi^{*2} + y^{*2}) dt$ 

7) Current Value Hamiltonians H =1/2( $\beta\pi^2$  + y<sup>2</sup>) +m<sub>z</sub>y +m<sub>z</sub>\*y\*; H\* =1/2( $\beta\pi^{*2}$  + y\*<sup>2</sup>) +m\*<sub>z</sub>\*y\* +m\*<sub>z</sub>y

# Definitions of variables :

i rate of change of consumer price index, inflation

 $\pi$  'core' inflation

y output (in logs) measured from the natural rate

z integral of past output

**Overseas** Country

 $Dz^* = v^*$ 

On the other hand, the system determining global averages is like a closed economy, $\Box$ where the policy problem is to squeeze global inflation out of the system without reliance on the exchange rate mechanism.

The fact that the policy problem can be decomposed in this way and the contrast between the conduct of Nash and coordinated policy has been discussed formally in Miller and Salmon (1989). Here we focus on the effects of discounting. We begin with the system of international differences which resembles most closely the treatment of the previous section, and then turn to the global averages.

### (i) International Differences

The problem analysed here is essentially that of minimising the sum of welfare costs for the two countries suffering from non-zero rates of core inflation of the same absolute value but of opposite sign. The time consistent cooperative solution can once again be characterised by a third order differential equation (equation (a20)), which together with the stability restriction permits a geometric representation.

As a consequence, we may repeat the apparatus already developed in Figure 1, with only minor parameter changes which reflect the fact that the policy coordinator takes account of welfare costs in both countries. Figure 3 describes the situation in the two country case. The schedule showing how rational exchange rate expectations should depend upon the coefficient of feedback (again labelled OX) is now  $|\theta| = (1+\eta)/(\gamma+2\delta\chi^{-1})$  (where  $\chi$  is the cooeficient of feedback). The policy reaction function for the undiscounted case labelled AG has the same vertical intercept as the small open economy but the horizontal intercept is only half as large compare AG and AB in Figure 3. As before the intersection of OX and AG defines the time consistent equilibrium at C, the optimal linear feedback rule is found at R, and the effects of discounting are displayed in Figure 4.

Figure 3 here.

Rather than comparing these two equilibria for coordinated policy, it seems more natural to compare the dynamic programming solution achieved by <u>coordinated</u> policy with that which results from a <u>Nash policy game</u> - where each policy maker separately minimises its welfare costs taking as given both the exchange rate expectations ( $\theta$ ) and the policy of the other, and to see how myopia affects the comparison.

First consider the case of no discounting. To find the dynamic programming solution to the Nash game involves finding a policy reaction function for such uncoordinated policy. On the simplifying assumption of <u>open loop</u> Nash (OLN) behaviour, we find that the relevant reaction function is simply the geometric average of AG and AB. So the Nash equilibrium is at point N where the coefficient of policy response is larger than for coordinated policy arrived at by dynamic programming methods. Since the Nash equilibrium has lower welfare costs this is a case where coordination does not pay (see Miller and Salmon, 1989).

Now consider how "policy myopia" affects these equilibria. Although the Nash reaction function is no longer a geometric average of the coordinated and small open economy reaction functions, its general qualitative features are unaltered by discounting, and in particular it continues to have the same vertical interecpt as the coordinated reaction function. As in the previous section, discounting has the effect of reducing this intercept towards zero. We therefore find that the strength of policy response falls monotonically becoming vanishingly small as the discount factor rises without limit. This is illustrated in the left hand panel of Figure 4 while the effect of discounting on policy feedback is traced out in the right hand panel of that figure. Thus while policy myopia makes the global policy maker increasingly lethargic (relative to the optimal rule), it has an even greater effect on uncoordinated policy makers.

# Figure 4 here.

Table 4 provides some numerical results for the effect of discounting on the feedback coefficients for the various equilibria (with parameter values  $\beta = \xi = \phi = 1$ ,  $\delta = \gamma = 1/2$ ,  $\sigma = 1/10$ ,  $\eta = 1/3$ ). As the table shows discounting has the same qualitative effect in the more attractive - but analytically less tractable - <u>closed loop</u> Nash (CLN) equilibrium between two countries.

We have seen that discounting reduces the distortion induced by cooperation when it comes to removing inflation differentials by monetary policy. The implication might appear to be that coordination is therefore more likely to pay relative to Nash policy the higher is the discount factor. But it would be premature to draw any such general conclusion before considering how discounting affects the comparison between coordinated and Nash policy in the face of "global" inflation, as we do in the next section.

		Table 4		
1	Feedback Coeff	icients for the Di	fferences System	n
Discount	Coordin	nated	OLN	CLN
rate	Optimal	TC		
0.0	0.9384	0.842009	0.877143	0.876343
0.1	0.8988	0.799234	0.832965	0.832228
0.5	0.7652	0.653869	0.681849	0.681360
1.0	0.6448	0.520694	0.542072	0.541813
2.0	0.5016	0.358152	0.370566	0.370499
∞	0.2078	0.000000	0.000000	0.000000
Global Average	<u>s</u>			
While it may	v he true that o	oordinated polic	w is more inoffi	cient in the face of
			e better able to c	

common to all countries will be recognised as such, and the response will be a "global" increase in interest rates leaving exchange rates unchanged. In the case of Nash policy however, each individual country may be tempted to believe that its own inflation shock can be handled by tightening national monetary policy, raising interest rates and so the exchange rate. The "fallacy of composition" involved here will only emerge when the country discovers, in the event, that its partners also raised their interest rates to the same extent, leaving exchange rates unchanged.

The formal evidence that uncoordinated policy involves such a fallacy is the presence of  $\theta$  (a parameter relating to exchange rates) in the equation system describing global averages arising from Nash policy (see equation (a26)). The globa coordinator, knowing that there are no exchange rate changes in prospect will go for a larger increase in interest rates, and a more rapid deflation, which does in fact generate lower welfare costs to the partner countries. In Figure 5 we indicate at point2 C the stronger feedback implemented by coordinated policy while N shows the weaker Nash response and the curves trace the impact of policy myopia. (Table 5 provides some numerical results using the same parameter values as for Table 4.)

# Figure 5 here.

The impact of discounting on coordinated policy is of course to slow down the speed of response (why cause so much pain now for benefits that come later if you care less for the future), as was already clear in the previous two sections from the shift of the vertical intercept of the "policy reaction functions". But the effect on the Nash response is more subtle.

It is true that, as the diagram shows, that the strength of policy feedback falls; but it falls less than for the coordinated policy, so reducing the distortion attributable to the fallacy of composition. The reason for this is apparent from Figure 4 in the previous section. It can be seen from there that raising the discount rate reduces the parameter  $\theta$  in the Nash equilibrium. But this is precisely the parameter whose effect is to slow the Nash response to global shocks below the coordinated optimum. So higher discounting reduces the distortion associated with Nash policy.

# Table 5 Feedback Coefficients for the Averages System

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Discount rate	Coordinated TC	OLN	CLN
0.0	1.000000	0.958500	0.957608
0.1	0.951249	0.911308	0.910486
0.5	0.780776	0.747356	0.746807
1.0	0.618034	0.592314	0.592022
2.0	0.414214	0.399402	0.399327
~	0.000000	0.000000	0.000000

# 3. Policy Myopia and Policy Coordination.

Before turning to normative considerations, we briefly summarise the positive results of the previous section where the effect of increased discounting on the strength of policy feedback was calculated, contrasting coordinated control with a decentralised Nash alternative, (and decomposing the problem into "differences" and "averages" as usual).

Where initial inflation rates differ and the impact of differential monetary policy on the exchange rate is only too evident, it was noted that coordinated policy is excessively lethargic, due to a "time consistency" bias; but this bias, measured relative to the Nash alternative fell as discounting increased. This suggests that discounting would, ceteris paribus, make coordination more attractive. For common inflation shocks however, it is the "Nash policy" which tended to be excessively lethargic - due to a "fallacy of composition". This bias also is reduced, relative to the coordinated alternative, as discounting increases. *Ceteris paribus*, this would make Nash policy more acceptable. The answer to the question we address in this paper - how discounting affects when coordination pays - must obviously depend on the weight attached to these two different factors. Specifically, where initial inflation differences are paramount then discounting works in favour of coordination, and vice versa.

such a conclusion may seem rather unsatisfactory, depending as it does on specific initial conditions, but this is simply a reflection of the deterministic nature of the analysis so far. In an earlier paper, Miller and Salmon (1989), it was shown that there exists a natural stochastic interpretation of these initial inflation correspondent rates of core inflation which differ, but sum to zero, correspond to perfect negative correlation. If initial conditions of the deterministic problem can only be described as some combination of averages and differences, then stochastic analysis will require a  $\overline{\omega}$ correlation lying between these two extremes. (The case where one country starts with positive core inflation and the other starts with zero inflation provides a simple  $\overline{\omega}$ illustration: the inflation difference is that prevailing in the first country while the average is half that; and the stochastic analogue is zero correlation.)

Using some results of Levine and Currie (1987a), it was shown that the expected welfare costs from the stochastic problem could be expressed as the weighted sum of those arising from two deterministic control problems (of simple averages and pure differences) and that the latter can also be used to compute the critical correlation coefficient where coordination and Nash policy would deliver the same expected welfare results. **Jniversity** 

For the convenience of the reader the principal steps are repeated in Appendix 3. Briefly what they involve is first equating the welfare costs under deterministic control to solve for those initial conditions which make the two regimes equally attractive and then computing the analogous correlation coefficient p\*. Coordination would pay for correlation greater than the critical value so calculated (i.e. for  $p > p^*$ ).

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In order to see now the critical correlation coefficient is affected. II, when simply observe how the critical correlation coefficient is affected. II, when discounting increases,  $\rho^*$  were to fall this would imply that the reduction of the time consistency problem affecting policy coordination was sufficiently important that turns to the numerical results given in Table 6 (obtained using the same parameter values as above), one finds that the critical correlation coefficient increases with the discount factor. This must mean that the effect of discounting on the lethargy of Nash policy is more important, so enhances the relative attraction of decentralised policy in this economic model. In other words, as policy makers become more myopic the returns to cooperation are reduced.

This result is indicated graphically in Figure 6, where the critical correlation coefficient, p\*, is shown as an increasing function of u, the discount factor. As the figure shows, the range of correlation where coordination does not pay, indicated by the shading underneath the curve, increases with discounting.

The Critical Correlation Coefficient					
Discount	ρ*				
rate	OLN	CLN			
0.0	-	-			
0.1	0.633999	0.615767			

0.5

1.0

2.0

8

# Table 6 ent

0.722979

0.805647

0.902507

1.000000

0.711481

0.800488

0.901028

1.000000

### Conclusion.

In open economy macromodels with sluggish wage/price behaviour and rational exchange rate expectations, the fact that coordination may not pay can be attributed to the time consistency bias which affects a policy coordinator (who is not inhibited by preannouncement or concern for reputation) more than it does Nash players. In the continuous time model studied here, the bias this induces is towards excessive policy lethargy: the presumption that future policy coordinators will fight inflation differentials with real interest differentials changes the exchange rate and reduces the immediate problem to be solved by the current coordinator.

On first examining a small open economy, we find that discounting (naturally enough) increases such lethargy; the feedback coefficient falls absolutely and also relative to the "optimal linear feedback rule". But the relevant comparison for a coordinated policy maker is not presumably an optimal rule (which would require precommitment or punishment to be sustained) but time consistent solutions for a Nash game. This comparison of regimes was made first with initial inflation "differences" and then with inflation rates in partner countries beginning at the same level. In the former case it was found that discounting increases the lethargy the Nash policy makers even more than it did for the coordinated policy maker: which implies that discounting might be good for coordination. In the latter case, the roles were reversed; as discounting reduces the fallacy of composition affecting decentralised policy.

The balance between these two elements was found to move against policy coordination as discounting increased. This was shown via a stochastic interpretation which allows one to compute a critical correlation between country specific inflation shocks which balances the advantage and disadvantage of coordination. This critical correlation rose with discounting. For any given pattern of correlation of international inflation shocks we thus conclude that coordination is less likely to pay the higher the rate of discounting, on the assumption that both equilibria are obtained via dynamic programming.

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# Appendix 1: The mathematics of the small open economy case

# The Time Consistent Solution

The model consisting of equations (1) to (5) may be reduced through the substitution of (1) into (3) to yield a relation for the level of core inflation as

$$\pi = \xi \phi z + \xi \sigma c \tag{a1}$$

Then using the restriction that  $c=\theta z$  this becomes,

 $\pi = \xi(\phi + \sigma\theta)z \tag{a2}$ 

Following Cohen and Michel (1988) the time consitent solution may then be found using Pontryagin's maximum principle with equation (4), (Dz = y), representing the only state constraint. Given the linear mapping restricting the exchange rate in a fixed relationship to the state variable, with  $\theta$  yet to be determined, there is no need to set up a costate variable for c. In effect there is no further independent dynamics in c to be taken into account in solving the dynamic optimisation problem for the optimal policy. The discounted loss function is given as equation (6) of the text and as discussed above this may either be minimised with respect to the level of real interest rates directly or alternatively, given equation (2), with respect to output.

So the optimisation problem then becomes,

 $\min_{y} V(0) = \frac{1}{2} \int_{0}^{\infty} e^{-ut} (\beta \pi^{2} + y^{2}) dt$ 

subject to (4), with  $c = \theta z$ , and (a2)

The current value Hamiltonian may be written,

 $H = \frac{1}{2}(\beta\pi^2 + y^2) + my$  where m(t) is the current value costate variable.

The first order conditions for a minimum are then given by,

$H_y = 0$	$\Rightarrow$ y = -m	(a3)
$H_z = um-Dr$	$m \Rightarrow Dm = um -\beta\xi(\phi + \sigma\theta)\pi$	(a4)
H <sub>m</sub> = Dz	$\Rightarrow$ Dz = y	(a5)

Then given Dc = r, and from (2)  $r = -\frac{1}{\gamma}y + \frac{\delta}{\gamma}c$ , we find that under the optimal tme consistent policy the path for the real exchange rate will be governed by,

$$Dc = \frac{1}{\gamma}m + \frac{\delta}{\gamma}c$$

12  $Dc = \frac{1}{\gamma}m + \frac{\delta}{\gamma}c$ Collecting together the dynamics of the economy under this time consistent policy and substituting expression (a1) for  $\pi$  into (a4) we find a third order system of differential equations,  $\begin{bmatrix}Dz\\Dc\\Dm\end{bmatrix} = \begin{bmatrix} 0 & 0 & -1\\0 & \delta'_{\gamma} & 1'_{\gamma}\\0 & -\phi h & -\sigma h & u \end{bmatrix} \begin{bmatrix} z\\c\\m\end{bmatrix}$ where  $h = \beta\xi^2(\phi + \sigma\theta)$ (a7) The normalised eigenvectors that govern these dynamics are then given by,  $\begin{bmatrix} 0 & \delta\\-\phi h & -\sigma h & u \end{bmatrix} \begin{bmatrix} 1\\\theta\\\psi\end{bmatrix} = \lambda \begin{bmatrix} 1\\\theta\\\psi\end{bmatrix}$ which provides explicit expressions that must be satisfied by the "expectations" principle  $\mu = \lambda_s$   $|\theta| = \frac{1}{\pi + \lambda m^{-1}}$ 

$$\begin{bmatrix} Dz \\ Dc \\ Dm \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & \frac{\delta}{\gamma} & \frac{1}{\gamma} \\ -\phi h & -\sigma h & u \end{bmatrix} \begin{bmatrix} z \\ c \\ m \end{bmatrix} \qquad \text{where } h = \beta \xi^2 (\phi + \sigma \theta)$$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & \frac{\delta}{\gamma} & \frac{1}{\gamma} \\ -\phi h & -\sigma h & u \end{bmatrix} \begin{bmatrix} 1 \\ \theta \\ \psi \end{bmatrix} = \lambda_s \begin{bmatrix} 1 \\ \theta \\ \psi \end{bmatrix}$$

$$\psi = -\lambda_{s}$$
$$|\theta| = \frac{1}{\gamma + \delta\psi^{-1}}$$
$$\psi^{2} + u\psi - \beta\xi^{2}(\phi + \sigma\theta)^{2} = 0$$

Expressions (a10) and (a11) then form the basis of the schedules drawn in Figure 1.

(a10) (a1)

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b) The optimal linear rule

The problem in this case is to find that value of the feedback coefficient,  $\psi$ , that minimises V(0) without the time consistency restriction  $c = \theta z$ , but which lies on the mapping (a10) describing the private sector's choice of  $\theta$ . Hence we need to solve

$$\frac{\mathrm{d}V(0)}{\mathrm{d}\psi} = \frac{\partial V(0)}{\partial \psi} + \frac{\partial V(0)}{\partial \theta} \frac{\partial \theta}{\partial \psi} = 0$$

Since

$$V(0) = \frac{1}{2} \int_{0}^{\infty} e^{-ut} (\beta \pi^{2} + y^{2}) dt \quad \text{and} \quad \pi(t) = e^{\lambda_{s} t} \pi(0) \quad \text{and} \quad y(t) = e^{\lambda_{s} t} y(0)$$

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we have

$$V(0) = \frac{1}{2} \int_{0}^{\infty} e^{(2\lambda_{*} - u)t} (\beta \pi^{2}(0) + y^{2}(0)) dt$$

or

$$V(0) = \frac{1}{2} \left[ \left\{ \beta \pi^2(0) + y^2(0) \right\} \frac{e^{(2\lambda_s - u)t}}{(2\lambda_s - u)} \right]_0^{\infty} \right]$$

which leads to

$$V(0) = \frac{\beta \pi^2(0) + y^2(0)}{2(2\psi + u)}.$$
 (a13)

Given the relationships implied by the model we have

 $\pi(0) = \xi(\phi + \sigma \theta) z(0) \quad \text{and} \quad y(0) = -\psi z(0)$ 

which enables us to write the welfare loss as

$$V(0) = \frac{\beta \xi^2 (\phi + \sigma \theta)^2 + \psi^2}{2(2\psi + u)} z^2(0)$$
 (a14)

Minimising this expression with respect to  $\psi$  (taking account of the dependence of  $\theta$  on  $\psi$ ) leads to a polynomial in  $\psi$ , given below as (a15).

$$\left[\psi(u+2\psi)-\beta\xi^{2}(\phi+\sigma\theta)^{2}-\psi^{2}\right](\delta+\gamma\psi)^{2}-\sigma\beta\xi^{2}\delta(\phi+\sigma\theta)(u+2\psi)=0$$
(a15)

Only one root of this polynomial, lying in the positive  $(\psi, |\theta|)$  quadrant indicated in figure 1, is economically meaningful and hence there is no ambiguity in finding the optimal rule although it is often easier to find this value numerically by simulation. It can however be seen from (a15) that the value  $\psi=0$  is not a solution to the polynomial, even when u is infinite, and hence the limit point for the optimal linear feedback coefficient deviates from the time consistent solution which has a zero limit point as the discount rate is increased.

# Appendix 2: The mathematics of the two country case

# a) The time consistent solutions

The model relevant for the two country case is found in Table 2 of the text and is a straightforward extension of the small open economy model used above. In this case we have two time consistent solutions to find , we start with the derivation of the Cooperative equilibrium and next consider the Nash solution. Notice that we derive the open loop solutions below simply for analytic tractability and because strictly the figures above are constucted from these solutions. Numerical calculations derived from the Closed loop solutions are reported in the main body of the paper and confirm that the results carry over to this case.

### i) The Cooperative Equilibrium

The loss functions for the individual governments are given in Table 2 and we assume that the cooperative policy is derived by taking a simple average of these two loss functions. So the problem of deriving the cooperative policy becomes one of;

$$\min_{y,y^*} \frac{1}{2} (V(0) + V^*(0))$$

subject to the constraints of the model and the time consistency restriction which, given the symmetry in the model, may now be written as  $c = \theta(z - z^*)$  leaving just the two state variables z and  $z^*$ . The current value Hamiltonian then becomes

$$H = \frac{1}{4} (\beta \pi^2 + y^2 + \beta \pi^{*2} + y^{*2}) + m_z^c y + m_{z^*}^c y^*$$

where  $m_z^c$  and  $m_{z^*}^c$  represent the current value costate variables associated with the state variables z and z\* respectively. Applying Pontryagin's Maximum principle once again we find the following set of first order conditions ;

$H_y = 0 \implies y/2 = -m_z^c$	(a17)i
$H_{y^*} = 0 \implies y^*/2 = -m_{z^*}^c$	ii
$H_m = 0 \implies Dz = y$	iii
$H_{m^*} = 0 \implies Dz^* = y^*$	iv
$H_z = um_z^c - Dm_z^c \implies Dm_z^c = -1/2\beta\xi(\phi + \sigma\theta)\pi + 1/2\beta\xi\sigma\theta\pi^* + um_z^c$	v
$H_{z^*} = um_{z^*}^c - Dm_{z^*}^c \Rightarrow Dm_{z^*}^c = 1/2\beta\xi\sigma\theta\pi - 1/2\beta\xi(\phi+\sigma\theta)\pi^* + um_{z^*}^c$	vi

(a15)

(a16)

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collecting these together we find the dynamic evolution of the two economies under cooperative policy is determined by the fifth order differential equation system

$$\cdot \begin{bmatrix} Dz \\ Dz^{*} \\ Dc \\ Dm_{z}^{c} \\ Dm_{z}^{c} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2\gamma^{-1}\delta & 2\gamma^{-1}(1+\eta) & -2\gamma^{-1}(1+\eta) \\ g_{1} & g_{2} & 0 & u & 0 \\ g_{2} & g_{1} & 0 & 0 & u \end{bmatrix} \begin{bmatrix} z \\ z^{*} \\ c \\ m_{z}^{c} \\ m_{z}^{c} \end{bmatrix}$$
(a18)  
where  $g_{1} = -1/2\beta\xi^{2}[(\phi + \sigma\theta)^{2} + (\sigma\theta)^{2}]$  and  $g_{2} = \beta\xi^{2}\sigma\theta(\phi + \sigma\theta).$ 

A substantial simplification is now available given the assumption of symmetry in the two economies that enables us to transform this fifth order system by defining new state variables as the averages and differences of the original variables. This transformation of the states allows us to effectively diagonalise the original dynamics into two subsystems each involving one stable root. Expressions (a19) and (a20) below then give the global averages and the international differences,

Averages 
$$\begin{bmatrix} Dz_a \\ Dm_a^c \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -\frac{1}{2}\beta\xi^2\phi^2 & u \end{bmatrix} \begin{bmatrix} z_a \\ m_a^c \end{bmatrix}$$
 (a19)

$$\underbrace{\text{Differences}}_{\text{Dr}_{d}} \begin{bmatrix} Dz_{d} \\ Dc \\ Dm_{d}^{c} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 2\gamma^{-1}\delta & 2\gamma^{-1}(1+\eta) \\ -\frac{1}{2}\phi h^{*} & -\sigma h^{*} & u \end{bmatrix} \begin{bmatrix} z_{d} \\ c \\ m_{d}^{c} \end{bmatrix}$$
(a20)

where  $h^* = \beta \xi^2 (\phi + 2\sigma \theta)$ 

Following the same form of analysis as discussed above for a small open economy we can find explicit relations linking the policy feedback parameter, $\psi$ , and the expectations parameter  $\theta$ . Taking the eigenvector system corresponding to the stable root in (a20) we find

$$\begin{bmatrix} 0 & 0 & -2 \\ 0 & 2\gamma^{-1}\delta & 2\gamma^{-1}(1+\eta) \\ -\frac{1}{2}\phi h^* & -\sigma h^* & u \end{bmatrix} \begin{bmatrix} 1 \\ \theta \\ \psi \end{bmatrix} = \lambda_s \begin{bmatrix} 1 \\ \theta \\ \psi \end{bmatrix}$$
(a21)

so we find from the first line of (a21) that

European

$$-2\psi = \lambda_c$$

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For the sake of comparison with the Nash solution derived below we define  $\chi$  as the policy feedback coefficient where  $\chi = |\lambda_s| = 2\psi$ . This follows from the first order conditions that  $y_d = -2m_d$  and  $m_d = \psi z_d$  and hence  $\chi$  is an appropriate measure  $\alpha$ the policy feedback of output in response to inflation. The second row of (a20) then yields,

$$2\gamma^{-1}\delta\theta + \gamma^{-1}(1+\eta)\chi = \chi\theta \tag{a23}$$

which leads to an explicit expression, (a24), for  $\theta$ , the coefficient that determines the private sector's expectations exactly as in (a10) for the small open economy.

$$\partial J = \frac{1+\eta}{\gamma+2\delta\chi^{-1}}$$

Finally the third row of the eigensystem (a21) yields the following polynomial from which the correct root may be selected to determine  $\chi$  in terms of  $\theta$ . (a25)

$$\chi^2 + u\chi -\beta\xi^2(\phi+2\sigma\theta)^2 = 0$$

# ii) The Nash equilibrium

1

The analysis in this case follows a very similar path to the cooperative solution given above. We now have two dynamic optimisation problems that must be solved simultaneously under the Nash assumption that the other government's policy path (in this open loop analysis) is taken as given together with the time consistency restriction used above. The two current value Hamiltonians are given in (7) of Table 2 and once again Pontryagin's Maximum Principle may be applied to yield a set of first order conditions. Similarly these may be reduced using averages and differences to yield the two subsystems each of which is again driven by a single stable root.

Averages 
$$\begin{bmatrix} Dz_a \\ Dm_a \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -\phi h & u \end{bmatrix} \begin{bmatrix} z_a \\ m_a \end{bmatrix}$$

$$\frac{\text{Differences}}{\text{Dm}_{d}} \begin{bmatrix} Dz_{d} \\ Dc \\ Dm_{d} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 2\gamma^{-1}\delta & \gamma^{-1}(1+\eta) \\ -\phi h & -2\sigma h & u \end{bmatrix} \begin{bmatrix} z_{d} \\ c \\ m_{d} \end{bmatrix}$$

(a26)

(a27)

(a24)

(a22)

where  $h = \beta \xi^2 (\phi + \sigma \theta)$ 

Again concentrating on the differences subsystem we can derive the three relations we need to construct Figure 3. The eigensystem corresponding to the stable root is given by

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 2\gamma^{-1}\delta & \gamma^{-1}(1+\eta) \\ -\phi h & -2\sigma h & u \end{bmatrix} \begin{bmatrix} 1 \\ \theta \\ \psi \end{bmatrix} = \lambda_s \begin{bmatrix} 1 \\ \theta \\ \psi \end{bmatrix}$$
(a28)

From the first row we now find

 $-\psi = \lambda_{\rm s} \tag{a29}$ 

(Notice in this Nash solution that the policy feedback coefficient  $\chi$  is defined as  $\chi = |\lambda_s| = \psi$  since in this case  $y_d = -m_d$  where  $m_d = \psi z_d$ .) The second row returns exactly the same relation between the policy feedback coefficient  $\chi$  and  $\theta$  as given in the Cooperative solution above, i.e.

$$|\theta| = \frac{1+\eta}{\gamma+2\delta\chi^{-1}}$$

finally the third row delivers

$$\chi^{2} + u\chi -\beta\xi^{2}(\phi + \sigma\theta)(\phi + 2\sigma\theta) = 0$$
 (a30)

# b) The Optimal Linear rule

The derivation of the optimal linear rule in the two country case is similar to the small open economy case outlined in Appendix. Welfare costs are now given by

$$V_{d}(0) = \frac{\beta \xi^{2} (\phi + 2\sigma \theta)^{2} + \chi^{2}}{2(2\chi + u)} z_{d}^{2}(0)$$

The minimisation of this with respect to  $\chi$  (taking account of the dependence of  $\theta$  on  $\chi$  given by equation (a24)) gives the following polynomial in  $\chi$ 

$$\left[\chi(u+2\chi)-\beta\xi^{2}(\phi+2\sigma\theta)^{2}-\psi^{2}\right](2\delta+\gamma\chi)^{2}-4\sigma\beta\xi^{2}\delta(\phi+2\sigma\theta)(u+2\chi)=0$$

Again there is only one positive root and this is illustrated in figure 2. As with the small open economy  $\chi=0$  is not a solution to the polynomial even when u is infinite and hence the limit point for the optimal linear feedback coefficient deviates from the time consistent cooperative and Nash solutions which have zero limit points.

# **Appendix 3: Stochastic interpretation**

In a deterministic context the payoff to coordination depends on how big the initial disturbance to the average of core inflation is compared to the initial disturbance to the difference in core inflation. For coordination to pay the square of the ratio of differences to averages must be less than a critical value which is derived in this appendix. It is also shown that there is a direct correspondence between the differnces/averages ratio of initial shocks in a deterministic context and the correlation coefficient between shocks in a stochastic context. The critical ratio European University Institute. derived in the former provides a critical correlation for the latter.

The first stage is to disaggregate the total welfare cost into averages and differences as follows

$$W = \frac{1}{2}V + \frac{1}{2}V^*$$
$$= \frac{1}{2}\int_{0}^{\infty} e^{-ut} (\beta \pi_{a}^{2} + y_{a}^{2}) dt + \frac{1}{8}\int_{0}^{\infty} e^{-ut} (\beta \pi_{d}^{2} + y_{d}^{2}) dt$$

This can be explicitly solved in terms of the stable roots to yield

$$W = -\frac{1}{2(2\lambda_{a} - u)}(\beta\pi_{a}^{2}(0) + y_{a}^{2}(0)) - \frac{1}{8(2\lambda_{d} - u)}(\beta\pi_{d}^{2}(0) + y_{d}^{2}(0))$$

But since the model implies that

$$y_{a}(0) = \lambda_{a} z_{a}(0) \qquad \pi_{a}(0) = \xi \phi z_{a}(0)$$
$$y_{d}(0) = \lambda_{d} z_{d}(0) \qquad \pi_{d}(0) = \xi (\phi + 2\sigma \theta) z_{d}(0)$$

this can be written as

$$W = -\frac{\beta^2 \xi^2 \phi^2 + \lambda_a^2}{2(2\lambda_a - u)} z_a^2(0) - \frac{\beta^2 \xi^2 (\phi + 2\sigma\theta)^2 + \lambda_d^2}{8(2\lambda_d - u)} z_d^2(0)$$

or

$$W = k_{a} z_{a}^{2}(0) + k_{d} z_{d}^{2}(0)$$

Clearly  $k_a$  and  $k_d$  depend on the parameters of the model and the strategic assumption characterising the setting of policy. Thus for coordinated policy welfare costs are

$$W^{c} = k^{c}_{*} z^{2}_{*}(0) + k^{c}_{d} z^{2}_{d}(0)$$

and for Nash policy welfare costs are

$$W^{N} = k_{a}^{N} z_{a}^{2}(0) + k_{d}^{N} z_{d}^{2}(0)$$

Coordination pays when W<sup>C</sup> > W<sup>N</sup> or when

$$\frac{z_{d}^{2}(0)}{z_{a}^{2}(0)} < \frac{k_{a}^{N} - k_{a}^{C}}{k_{d}^{C} - k_{d}^{N}}$$

 $\alpha^{*} = \frac{k_{a}^{N} - k_{a}^{N}}{k_{d}^{C} - k_{d}^{N}}$  is the critical differences/averages ratio mentioned above.

Moving to a stochastic model requires replacing equations (2) of the model with

 $\begin{array}{l} \operatorname{idt} = d(\operatorname{domestic price level}) = \varphi y dt + p dt + \sigma E(dc) + db \\ \operatorname{i^*dt} = d(\operatorname{foreign price level}) = \varphi y^* dt + p^* dt - \sigma E(dc^*) + db^* \end{array}$ 

and equations (4) of the model with

 $dz = ydt + \phi^{-1}db$  and  $dz^* = y^*dt + \phi^{-1}db^*$ 

where b and b\* are Brownian motion processes characterised by a variancecovariance matrix  $\Sigma = v \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ . The optimisation problem becomes one of minimising the <u>expected</u> discounted welfare costs subject to the modied model. (Note the discount rate must be strictly positive to prevent integrated welfare costs from diverging.)

Levine and Currie (1987) have shown that "if the welfare (cost) of the deterministic problem is written as W=f(Z(0)) then the corresponding welfare loss for the stochastic problem can be written as  $E(W)=f(u^{-1}\Sigma)$ " where  $Z(0)=\begin{bmatrix} z(0)\\ z'(0)\end{bmatrix} \begin{bmatrix} z(0) & z^*(0) \end{bmatrix}$ .

It is evident from this result that a stochastic problem with a given correlation coefficient is equivalent to a weighted sum of two deterministic problems with linearly independent initial displacements and weights determined by the correlation coefficient, p. To find the required deterministic problems one diagonalises the covariance matrix thus

$$u^{-1}\sum = u^{-1} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} = \frac{1}{2u} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1+\rho & 0 \\ 0 & 1-\rho \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
$$= \left(\frac{1+\rho}{2u}\right) \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} + \left(\frac{1-\rho}{2u}\right) \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}$$
$$= Z_1(0) + Z_2(0)$$

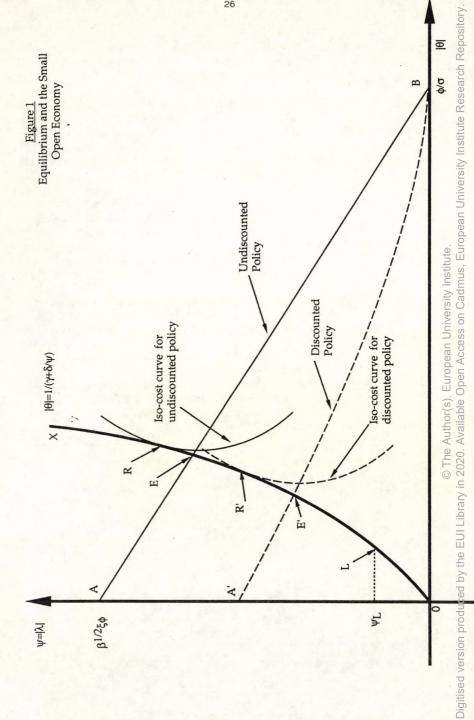
We note that the first deterministic data set corresponds to a matching displacement with  $z_a^2(0) = \frac{1+\rho}{2u}$  and the second data set has a displacement of opposite sign with  $z_{4}^{2}(0) = \frac{2(1-\rho)}{2}$ . Substituting these values for initial average and difference European University Institute. 11 displacements into the expression for  $\alpha^*$  and rearranging yields the following expression for the critical correlation coefficient

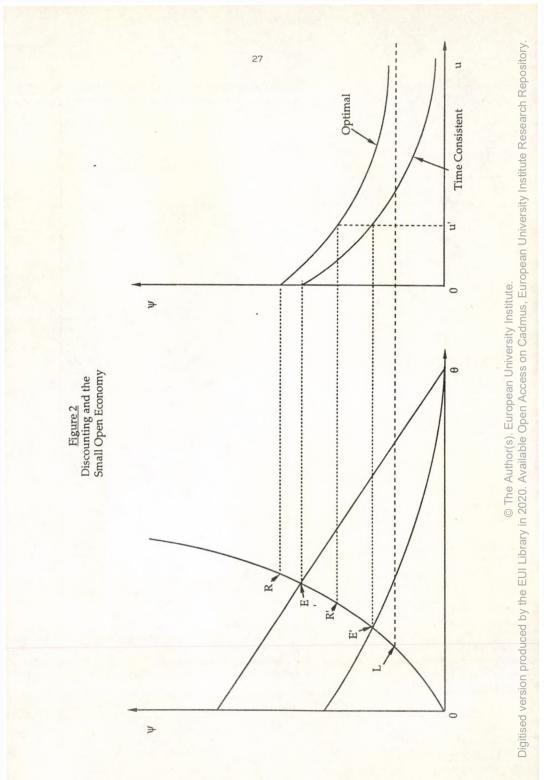
$$\rho^{*} = \frac{4 - \alpha^{*}}{4 + \alpha^{*}}$$

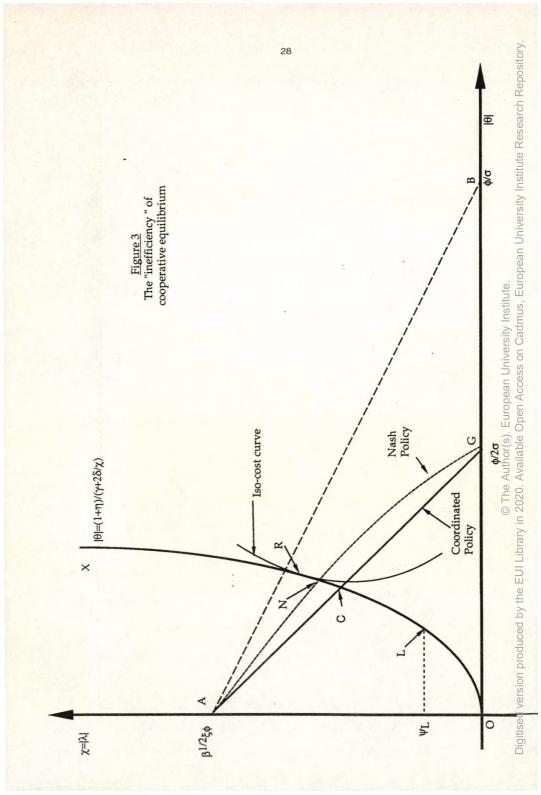
and this is used to calculate the results given in table 6.

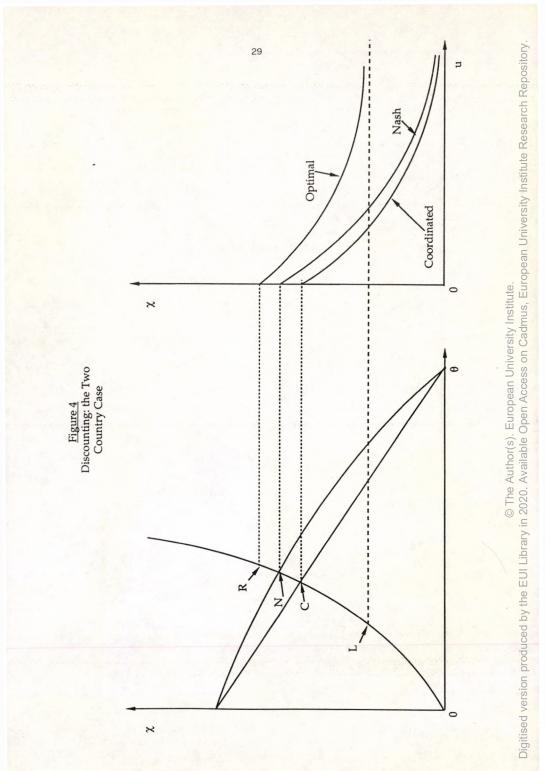
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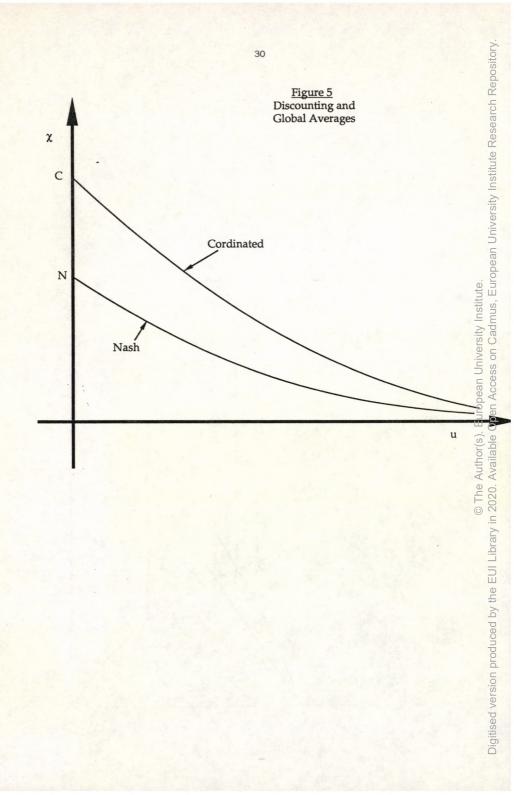
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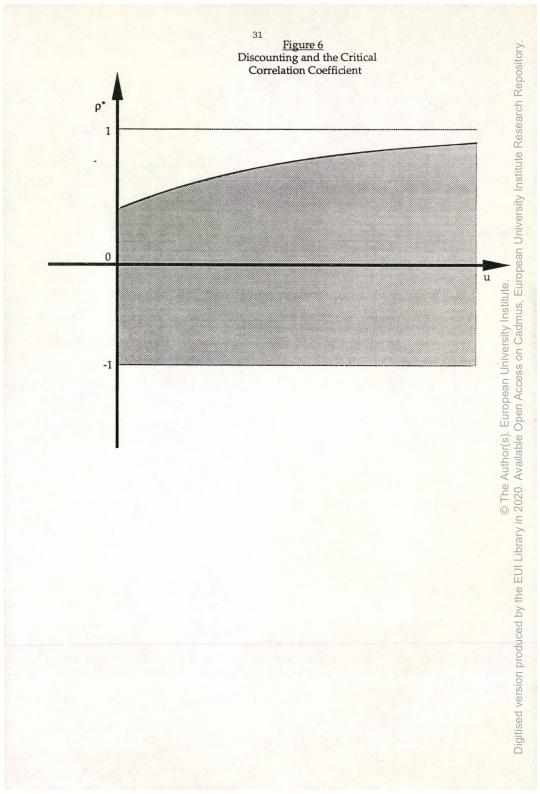


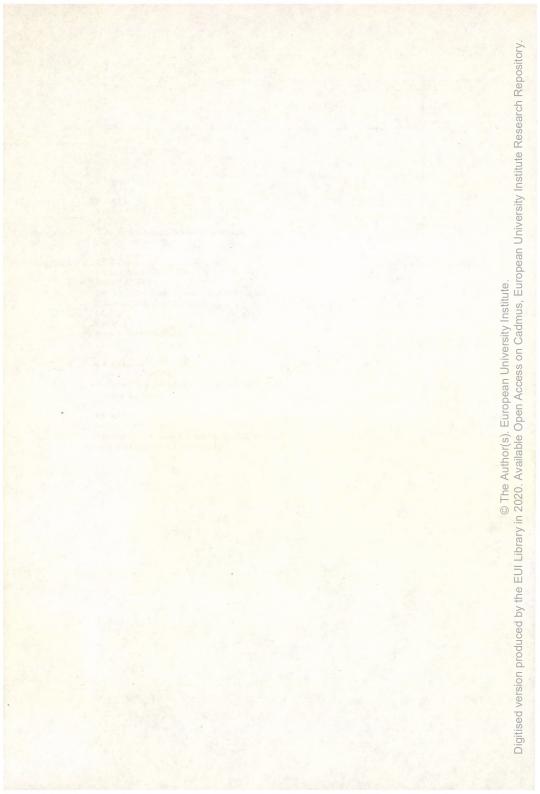












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