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Herding in Delegated Portfolio Management: When is Comparative Performance Information Desirable?

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Abstract

In this paper we address the issue of investors' asset allocation decisions when they delegate portfolio management to an agent. Contrary to predictions from traditional financial theory, it is found that investors may not induce their fund manager to allocate the funds to the asset with the highest return. Instead they may wish to induce trade in a particular asset, because another manager is trading in it and despite the presence of a more profitable alternative. Doing so allows investors to write an efficiency-improving relative-performance contract. On the other hand, herding leads principals to design wage contracts strategically, resulting in more aggressive and thus less profitable trade in equilibrium. We show that investors herd in their asset allocation decision, when managers are sufficiently risk averse or when the precision of their information is low. We also show that when principals can decide whether or not to disclose information about their manager's performance, they will not do so and thus the problem of designing contracts strategically can be avoided.

Journal of Economic Literature Classification Numbers: G14, G23, D82
Keywords: Fund management, moral hazard, relative performance contracts, herding, strategic interaction

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1. Introduction

A large amount of assets on stock and bond markets are traded by professional portfolio managers who are employed by institutional investors. At the end of 1994, institutional investors held US$ 2.9 trillion in US equities, which amounts to 46.6% of the total. This compares to 44.5% at the end of 1990 and 26.7% at the end of 1970.[1]

One of the most important decisions an investor faces is the choice of assets in which to invest. Traditional financial theory like the CAPM predicts that this choice should be entirely determined by the risk return characteristics of an asset. Contrary to this prediction, we show that investors may wish to induce their respective fund managers to trade in a particular asset, because another manager is trading in it and despite the presence of a more profitable alternative. Investors may thus herd in their asset allocation decision, with the result that if there are two assets with identical characteristics, the market for one asset displays informed trade and highly efficient prices, while no informed trade and inefficient prices occur in the market for the other asset.

The driving force for our results is the moral hazard problem between the investor and the fund manager. We consider a model with two principals who each delegate the management of their portfolio to a different agent. The agency problem considered here features two instances of moral hazard. Firstly, managers need to acquire costly information to learn about the future value of an asset, where the acquired information is unknown to the principal. Secondly, a manager chooses a trading strategy which is unobservable by the principal. Before offering a wage contract to his manager, each principal chooses one of two assets in which he wishes his fund manager to trade.

When a manager is the only informed trader in a market, a manager's wage contract is solely based on individual performance. When another fund manager trades in the same market, comparative performance information (henceforth

[1] See New York Stock Exchange Fact Book (1995).

CPI) becomes available and wages can be based on relative performance. As is well known from the literature on CPI (see for example Holmstrom, 1982 and Mookherjee, 1984), this is desirable because it improves the insurance-efficiency trade-off of contracting in an agency problem with moral hazard. We show that the benefits of CPI and thus herding increase with the managers' degree of risk aversion and decrease with the precision of information about asset value.

In contrast to other treatments of herding, our results suggest that herding might not be such a bad thing after all. In our setting, herding is induced by the principals in order to mitigate the inefficiencies associated with delegation, rather than an instance of inefficiency, as for example in Scharfstein and Stein (1990). Moreover, herding increases the efficiency of prices of the asset in which agents herd, rather than reducing it as in Froot, Scharfstein and Stein (1992).

Furthermore, we identify the strategic use of comparative performance information to induce aggressive trading (high trading intensities) by fund managers as a cost of using CPI. In most of the existing literature (see Holmstrom, 1982, and Mookherjee, 1984) the use of CPI comes at no cost[2], because one agent's action does not affect the "productivity" of the other agent. In our setting, however, informed trade by one agent exerts a negative externality over the profitability of the other agent's trade.

The managers' optimal trading strategies depend on the wage contracts they receive. This allows principals to use wage contracts strategically in order to induce a more favourable type of trading behaviour. Wage contracts thus do not only serve the purpose of mitigating inefficiencies arising from delegation, but also affect the market interaction between the managers. Contracts may thus also

[2] Meyer and Vickers (1997) is an exception to this. They show that in a dynamic setting, CPI may be undesirable, because it exacerbates the ratchet effect, i.e. reduces an agent's incentives to exert effort *ex ante*, because he anticipates that a high effort level today will result in a more demanding contract tomorrow.

be used strategically, as, e.g., in Vickers (1985), where delegation is a strategic device, when two or more principals interact.[3]

When designing wage contracts principals do not take into account that a higher trading intensity of their own manager exerts a negative externality over the other manager's profitability of trade. Therefore, the equilibrium in our model features wage contracts that induce trading intensities that are above the collusive level.

In order to study the costs and benefits of comparative performance information, we also consider the case where there is only one market, but principals can *ex ante* decide whether or not to disclose information about their manager's performance. We show that in equilibrium, principals will never disclose performance information and can thus use the endogenous choice of information disclosure as a device to avoid the problems arising from strategic interaction. We characterise the set of parameters for which investors are better off when the information disclosure decision is endogenous compared to the case where they are forced to disclose this information. This problem is interesting from a regulator's point of view who may have to decide whether or not funds should be obliged to disclose performance information.

Other authors (e.g. Scharfstein and Stein, 1990, Trueman, 1994, and Zwiebel, 1995) have shown that herding among agents who are evaluated relative to their peers might result due to reputational concerns. In this paper we neglect reputational concerns and focus instead on explicit incentives. Herding, however, remains an important issue, as agents' explicit incentives are based on relative performance and hence one agent's actions do affect another agent's incentives.

[3] Kyle and Wang (1997) develop a model of strategic delegation of fund management activities. They focus on the possibility of survival of irrational agents, who are overconfident in their own forecasting ability and show that such traders can survive because they are committed to trading aggressively, thus crowding out to some extent other informed agents' trades. Their model, however, only deals with the strategic aspect of delegation and does not examine the effect of incentive contracts in such relationships.

Herding in our treatment occurs in the sense that one principal induces acquisition of a piece of information, because another agent acquires the same piece of information. This corresponds to the concept of herding as in Brennan (1990), Froot, Scharfstein and Stein (1992), and Dow and Gorton (1994).[4]

The paper closest to ours is Maug and Naik (1996). They examine the question of asset allocation in a model with only one principal agent relationship. One of two available assets is characterised by the presence of an informed (profit maximising) trader, while there is no informed trade in the other asset. They explore the design of wage contracts to the fund manager and investigate under which circumstances the principal accepts herding by the agent, despite the reduction in expected trading profits resulting from having more than one informed trader in that asset.

In contrast to our treatment, Maug and Naik assume that the choice of asset is non-contractible and thus herding may occur when it is not desired by the principal. More importantly, they assume that agents submit orders sized so as to maximise expected trading profits, rather than the agent's expected utility, given a specific wage contract. However, in order for the agency problem to be meaningful, order size must be endogenously determined by the fund manager. The trading intensity thus constitutes an additional dimension of moral hazard.

Moreover, by assuming that there is only one principal-agent pair, Maug and Naik do not capture the element of strategic interaction between the two principals, which turns out to be crucial when contracts are based on CPI.

Another novelty of our model is that we explicitly consider the impact of wage contracts on the trading strategy of the fund manager. Papers dealing with the question of optimal wage contracts for fund managers include Bhattacharya and Pfleiderer (1985), Stoughton (1993) and Heinkel and Stoughton (1994). In their models, fund managers can acquire superior information about asset values

[4] It contrasts with herding due to informational cascades as in Welch (1992), Bikhchandani, Hirshleifer and Welch (1992) and Banerjee (1992) among others.

and then either reveal the information directly through an announcement, or indirectly through the portfolio choice. Wages are then based on some measure of forecast error.

In their models the problems of direct and indirect revelation of information are isomorphic and hence it is justifiable to study the problem of an information announcement, as the equivalent of a delegated portfolio management problem. This equivalence, however, hinges on the restrictive assumption that asset prices are perfectly inelastic. If asset prices are elastic, the trading decision affects prices and therefore the economic value of the gathered information. The problem of forecasting the return on an asset and the problem of portfolio choice thus cease to be isomorphic.

In order to model the managers' behaviour on the asset market, we move away from the perfectly competitive Rational Expectation Equilibrium as in Grossman and Stiglitz (1980). Instead we consider a simple noise trader model in the spirit of Kyle (1985) which incorporates monopolistic behaviour of speculators in the presence of noise traders and a market maker who sets the price such as to break even in expectation on the trades he executes.

The paper will proceed as follows. In Section 2 we develop the basic framework. Section 3 derives equilibrium trading and price setting strategies as a function of the contracting parameters and the assets chosen by the principals. Section 4 derives the optimal linear incentive scheme under non-herding and herding, and illustrates the impact on equilibrium trading strategies. Section 5 contains the main results concerning the principals' choice of herding versus non-herding. Section 6 endogenises the choice of disclosure of performance information and illustrates when it would be desirable for principals to have that choice. Section 7 concludes. The Appendix provides the proofs.

2. The model

There are six agents in the economy: two principals P_i (i=1,2), two fund managers F_i and two market makers M_l (l=A,B). Each principal employs one fund manager, where the fund managers are assigned to a principal before the start of the game. We assume that there are many fund managers that could be hired and therefore the principal is able to extract all the surplus from the fund manager's activity.

When contracting with a manager, each principal first determines in which of two available assets l=A,B he wishes his manager to trade.[5] It is assumed to be *ex post* observable and verifiable in which market the manager traded. Each principal P_i can thus offer a contract that will force manager F_i to trade in asset l_i as determined in the contract. For notational simplicity we will not include the wage payments in case of trading in the "wrong" market and instead formulate the wage payments given the correct choice. The choice of assets becomes common knowledge among all agents and cannot be renegotiated.

Subsequently, the principals simultaneously offer a wage contract to their manager who decide whether to accept or reject it. A wage contract between principal P_i and agent F_i is a triple $C_i = \{\alpha_i, \beta_i, \gamma_i\}$, which determines wage payments w_i from principal P_i to agent F_i as

$$w_i = \alpha_i + \beta_i\pi_i - \gamma_i\pi_j \qquad i=1,2 \quad j=1,2, \quad i\neq j \qquad (1)$$

where π_i denotes agent i's realised trading profits. Let $EB_i(C_i, C_j | l_i, l_j)$ denote principal P_i's expected payoff when the principals choose contracts $\{C_i, C_j\}$, given that they have already chosen assets $\{l_i, l_j\}$.

Both fund managers have CARA utility, with the same coefficient of absolute risk aversion r:

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[5] Funds typically market themselves by referring to a particular investment objective and style. Therefore, before performance comparisons between fund managers are made, the funds are typically clustered into groups that differ through the market they invest in and their investment style. For an appraisal of the choice of the clusters that serve as benchmarks of performance comparison see Tierney and Winston (1991).

$$U_i\,(w_i,\,k) = \text{-exp}\,(\text{-}r\,(w_i - k_i))$$

Where k_i =0 if agent i does not acquire information and $k_i = c$ with $c > 0$ if he does. Agents have reservation wage W_i.

Once managers have accepted a wage contract, its terms become common knowledge to all agents and wage contracts cannot be renegotiated.[6] The managers then decide whether or not to acquire information about the value of the previously chosen asset and subsequently trade on their information. The trading strategy is chosen by each manager such that it constitutes a Nash equilibrium between traders and market maker in the trading subgame.

Each of the two assets is traded in only one market l=A,B and in each market there is one market maker M_l with whom trades can be executed. Since each asset is only traded in one market, the choice of asset in which to trade is equivalent to a choice of market and we will subsequently refer to the choice of a market.

When a manager acquires information he receives a noisy signal $\tilde{y}_l$ about asset value $\tilde{x}_l$. The *ex ante* relationship between the signal and true value is given by

$$\tilde{x}_l = \tilde{y}_l + \tilde{z}_l$$

where $\tilde{y}_l \sim N(0, V_l^y)$, $\tilde{z}_l \sim N(0, V_l^z)$. Both random variables are independent of one another and $\tilde{z}_l$ is the residual noise of asset value after information has been acquired.

Subsequently the agent can submit an order t_i for the asset to the market maker who sets the price of the asset at which he is willing to absorb all the order flow. Trading thus results in profits

[6] This corresponds to the assumption typically made in the strategic delegation literature, whereby contracts are publicly announced and cannot be secretly renegotiated. Dewatripont (1988) and Caillaud, Jullien, and Picard (1995) find precommitment effects through public announcements of contracts, even when contracts can be secretly renegotiated.

$$\tilde{\pi}_i = t_i(\tilde{x}_l - \tilde{p}_l). \quad (2)$$

Apart from the order by the informed speculator, total order flow in each asset contains a noisy component $\tilde{n}_l$, which is normally distributed with mean zero and variance V_l^n.[7] We assume that all random variables $\{\tilde{y}_A, \tilde{y}_B, \tilde{z}_A, \tilde{z}_B, \tilde{n}_A, \tilde{n}_B\}$ are independent of one another.

Market makers are assumed to be in Bertrand competition, which implies that they set prices so as to break even in expectation. Hence, the price is set such that it equals the expected value of the asset, given the information contained in total order flow. Thus, $p_l = E[x_l|T_l]$, where T_l denotes total order flow in market l. The presence of noise traders ensures that the speculators' orders will not perfectly reveal their information about asset value.

Table 1 illustrates the sequence of games that are played. Stages 1 and 2 of the game are simultaneous move games between the two principals. Stage 3 is a simultaneous move game between the two managers and the market maker.

Stage 1	Stage 2	Stage 3
Principals simultaneously choose an asset in which they wish the fund manager to invest.	Principals simultaneously choose the parameters of the wage contract, given the choice of assets made in the first stage.	Given the asset choice and the parameters of the wage contracts, agents choose their trading strategy as a Nash equilibrium in the trading game.

Table 1: The sequence of games played between the principals and the agents

Definition: An equilibrium is defined as $\{l_1^*, l_2^*, C_1^*, C_2^*, t_1^*, t_2^*, p_A^*, p_B^*\}$, such that for $i, j = 1,2$; $i \neq j$ and for $l_i, l_j \in \{A,B\}$:

(i) The price function $p_l(T_l)$ and the trading strategy $t_i(y_l)$ satisfy:

(a) $t_i^* \in \arg\max E[U_i(w_i, c)|C_i^*, C_j^*, l_i^*, l_j^*, y_l]$

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[7] The rationale for the random trading component is the presence of liquidity traders, who may have a hedging need and therefore trade in asset l.

(b) $p_l^* = E[x_l | T_l, C_i^*, C_j^*, l_i^*, l_j^*]$

(ii) The wage contracts $\{C_i^*, C_j^*\}$ solve

$$\max_{\alpha_i, \beta_i, \gamma_i} EB_i(C_i, C_j | l_i^*, l_j^*) = (1-\beta_i)E\pi_i + \gamma_i E\pi_j - \alpha_i \qquad \text{(P)}$$

s.t.

$$E[U_i(w_i, c) | C_i, C_j, l_i^*, l_j^*] \geq E[U_i(w_i, 0) | C_i, C_j, l_i^*, l_j^*] \qquad \text{(IC)}$$

$$E[U_i(w_i, c) | C_i, C_j, l_i^*, l_j^*] \geq U(W_i) \qquad \text{(PC)}$$

Denote by $EB_i(l_i, l_j) \equiv EB_i(C_i^*, C_j^* | l_i, l_j)$.

(iii) The choice of assets (l_i^*, l_j^*) satisfies:

$$EB_i(l_i^*, l_j^*) \geq EB_i(l_i, l_j^*)$$

(iv) Each principal's expected payoff in equilibrium satisfies an individual rationality constraint

$$EB_i(l_i^*, l_j^*) \geq 0.$$

To summarise, each agent chooses a trading strategy maximising his expected utility, given a price function of the market maker, given his own contract and the opponent's contract and given the choice of assets by the principals. Anticipating the managers' behaviour in the trading subgame, principals choose wage contracts so as to maximise their expected payoff (P), given the choice of assets $\{l_1, l_2\}$, where wage contracts have to satisfy the managers' participation constraints (PC) and incentive compatibility constraints (IC). Moreover, we require that the choice of assets constitutes a Nash equilibrium.

3. Equilibrium strategies in the trading subgame

In this section we solve the last stage of the game as a function of the outcome of the previous two stages. This corresponds to finding a price function and trading strategies according to definition (i). Throughout this section it is assumed that both fund managers accept the contract and that the contracts are incentive compatible, i.e. managers actually do acquire information.

There are two different cases that need to be distinguished. First, agents may be induced to get informed about and trade in different assets, which will be called the non-herding case (i.e. $l_1 \neq l_2$). Second, agents may be induced to get informed about and trade in the same asset, which will be called the herding case (i.e. $l_1 = l_2$).

3.1 Trading equilibrium under non-herding

Proposition 1: There exists a unique linear equilibrium of the trading subgame when agents get informed about and trade in different assets. Assume (w.l.o.g.) that agent 1 trades in asset *A*, while agent 2 trades in asset *B*. Then equilibrium order sizes are given by

$$t_1^N = \delta_1^N y_A \quad , \tag{3}$$

and the price setting strategy of the market maker for asset *A* is given by

$$\tilde{p}_A = \lambda_A^N \left(\tilde{t}_1 + \tilde{n}_A \right), \tag{4}$$

where δ_1^N and λ_A^N are given by equation (9) and (10) in the Appendix.

Agent 2's trading strategy and the price setting strategy by the market maker for asset *B* are given by the same formula with indices changed appropriately.

Proof see Appendix.

Properties of the trading equilibrium under non-herding

In the non herding equilibrium, the amount of trade of one agent is entirely independent of the other agent's trading decision, of the relative performance parameter γ_i as well as the characteristics of the other asset. The reason for independence is that agents have CARA utility.

Note that δ^N is an implicit function of r, β, V^y, V^z, V^n, given by substituting (10) into (9). Using the implicit function theorem it is straightforward to show that $\frac{\partial \delta^N}{\partial \beta} < 0$ and $\frac{\partial \delta^N}{\partial r} < 0$, i.e. the optimal trading intensity is a decreasing function of the incentive payment and the degree of risk aversion. From this we can also conclude that the first-best trading intensity $\delta^* \equiv \delta^N(r=0)$ is larger than the one that will be chosen by a risk averse agent whose incentive payment β is positive. This implies an agency cost due to suboptimally small trading intensities when the trading decision is delegated to a risk averse agent.

3.2 Trading equilibrium under herding

Proposition 2: There exists a unique linear equilibrium of the trading game under herding. Assume (w.l.o.g.) that both agents trade in asset *A*. The equilibrium trading strategy for agent 1 is given by

$$t_1^H = \delta_1^H y_A, \tag{5}$$

and the price setting strategy of the market maker M_A is given by

$$\tilde{p}_A = \lambda_A^H \left(\tilde{t}_1 + \tilde{t}_2 + \tilde{n}_A \right) , \tag{6}$$

with δ_1^H and λ_A^H given by equations (13) and (14) in the Appendix. Trader 2's trading intensity δ_2^H is also given by equation (13), with indices changed appropriately. Moreover, $p_B = 0$.

Proof see Appendix.

Properties of the trading equilibrium under herding

First, note that when agents herd in say asset *A*, no informed trade in asset *B* occurs and hence $p_B = 0$, i.e. the price for asset *B* contains no information about asset value.

Equation (12) in the Appendix gives trader 1's best response in trading intensity t_1 as a linear function of the opponent's trading intensity t_2. If $\gamma_1=0$, an increase in t_2 will lead to a decrease in t_1, holding λ constant. Trading intensities are strategic substitutes and the two managers interact like Cournot duopolists when determining their trading strategies. On the other hand, if $\beta_1=\gamma_1$ the trading intensity t_1 increases with t_2 and trading intensities are strategic complements.

Moreover, it can be verified easily, that in the case of perfect insurance for the managers (i.e. $\beta_1=\gamma_1$, $\beta_2=\gamma_2$), the equilibrium in the trading subgame degenerates to infinitely sized orders ($\delta_i^H = \infty$, i=1,2) and zero trading profits. Since managers anticipate the outcome of the trading subgame, they would never find costly information acquisition incentive compatible. We can therefore already conclude that optimal wage contracts under herding cannot feature perfect insurance.

4. Optimal wage contracts

We now turn to the optimisation problem each principal faces at the second stage, i.e. after a choice of assets has been made. He maximises the expected payoff from offering a contract, taking as given the contract of the other principal and the agents' actions they induce. At the stage where principals determine the parameters of the wage contract, each principal P_i faces the optimisation problem stated in definition (ii). For a pair of contracts to be an equilibrium, we require it to be a fixed point of the best response correspondence in wage contracts of each of the principals.

First, we will derive the optimal wage parameters of a contract for the cases that principals induce agents to trade in different assets. Then we analyse the case where principals induce agents to trade in the same asset.

4.1 Optimal wage contracts under non-herding

From section 3.1 we know that if agents trade in different assets, principal P_i's problem of choosing an optimal contract is independent of principal P_j's choice of contract. Hence, there is no strategic interaction between the principals when designing the wage contracts. For this case we can derive the optimal wage contract:

Proposition 3: For exp(2rc) - 1 ≤ V^y/V^z the optimal contracting parameters in the non-herding case are given by[8]

$$\alpha^N = W,$$

$$\gamma^N = 0,$$

and
$$\beta^N = \frac{u}{r}\sqrt{\frac{u\left(V^y + 2V^z\right) + \sqrt{u^2\left({V^y}^2 + 4V^zV^y\right) + 4{V^y}^2}}{2V^nV^y\left(V^y - uV^z\right)}}\,, \qquad (7)$$

where $u = e^{2rc}-1$.

If exp(2rc) - 1 > V^y/V^z there exists no contract that satisfies the agent's incentive compatibility constraint.

Proof see Appendix.

Hence, the optimal wage contract under non-herding features no relative performance component (γ^N=0), which is not surprising, given that agents' actions are independent of one another and that the performance of both managers is not correlated.

Taking the first derivative of β^N with respect to r yields $\frac{\partial \beta^N}{\partial r} > 0$, i.e. the incentive payment increases with the degree of risk aversion. This contrasts with other results in agency theory (see e.g. Milgrom and Roberts, 1992), where the optimal incentive payment decreases with the degree of risk aversion.

The intuition for this result is as follows. In our setting the incentive compatibility constraint (IC) is directly linked to the degree of risk aversion, because the agent can affect the riskiness of his wage by his trading decision. In particular, if the agent decides not to acquire information, he will optimally not

[8] Subscripts *i* and *l* are omitted as only one agent and one asset matter here.

trade at all and thereby cancel out any risk in his wage. The more risk averse an agent is, the higher the incentive payment has to be in order to induce him to take the risk that he necessarily incurs when trading.

Note moreover, that, as shown in Section 3, an agent's trading intensity is a decreasing function of β and r. Thus an increase in r not only reduces the chosen trading intensity directly, but also indirectly through an increase in the optimal incentive payment. Hence, as r increases the trading intensity moves further away from its first-best level and the principal's expected payoff decreases.

4.2 Optimal contracts under herding

Let us now turn to the contracting problem when both principals induce their managers to trade in the same asset. As discussed in Section 3.2, managers act as duopolists under herding, which gives rise to strategic interaction between principals when designing the wage contract. In particular, a principal can ensure that his agent will trade more aggressively (increase the choice of δ in the trading subgame) by increasing the relative performance parameter γ. This can be seen from the best response function (12) in the Appendix. The negative impact of large order sizes on trading profits is not internalised and hence contracts offered in equilibrium will induce trading intensities that are higher than if principals could collude when designing the wage contracts.

Lemma 2 in the Appendix states the incentive compatibility constraint for agents in the case of herding. With the use of Lemma 2 the programme in definition (ii) can be solved numerically, which yields the unique and symmetric equilibrium of the wage contracting game.

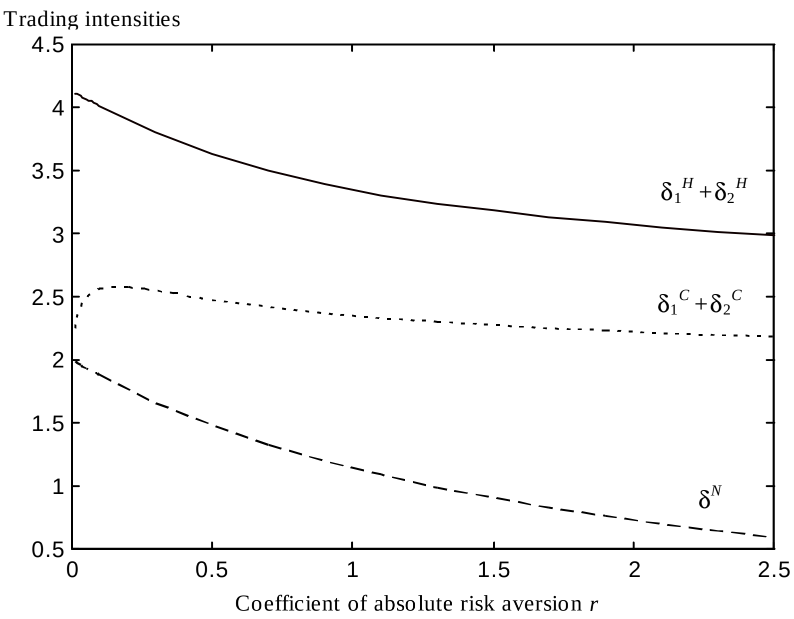

Figure 1: Shows the equilibrium trading intensities as a function of the coefficient of absolute risk aversion under optimal wage contracts in the case of herding (solid line), the case of herding with collusion among principals (dotted line) and under non-herding (dashed line). The first best total trading intensity is δ=2. For r>0 the trading intensity under non-herding is always below the first best level. The trading intensity under herding is always above the first best level and also above the collusive level. The parameter values are c=0.1, V^n =2, V^y =0.5, V^z =1.5.

Figure 1 shows the equilibrium trading intensities as a function of the coefficient of absolute risk aversion for the case of herding (solid line), herding with collusion among principals (dotted line) and non-herding (dashed line). When managers herd, the total trading intensity is above the first best level (in this example at $\delta_1+\delta_2$ =2) and above the collusive level, which illustrates the effect of strategic interaction among principals on the equilibrium trading intensities. Moreover, total trading intensity under herding is a decreasing function of the degree of risk aversion. As agents become more risk averse it becomes more costly to induce them to trade aggressively, which mitigates the strategic interaction problem.

On the other hand, as shown in section 3.1, when managers do not herd, the optimal trading intensity is always below the first-best level and decreasing in the coefficient of risk aversion.

5. The choice of herding versus non-herding

In this section we characterise the conditions under which it is a Nash equilibrium for principals to induce managers to herd or not to herd. This corresponds to the first stage of the game (definition (iii)), where principals choose an asset for their manager to trade in. When making their choice, principals take their opponents choice as given and anticipate the actions induced in the two subsequent stages of the game.

For any choice of $\{l_1, l_2\}$, principals receive the expected payoff as characterised in the previous sections. Payoffs as a function of asset choice can thus be summarised in the following payoff matrix

	$l_2=A$	$l_2=B$
$l_1=A$	$EB_1(A,A)$, $EB_2(A,A)$	$EB_1(A,B)$, $EB_2(B,A)$
$l_1=B$	$EB_1(B,A)$, $EB_2(A,B)$	$EB_1(B,B)$, $EB_2(B,B)$

Table 2: This table provides the payoff matrix for the choice of asset of each principal. Since the wage contracts are designed after the choice of asset becomes common knowledge to all players, the expected payoffs are given as the optimal payoffs from herding/non-herding as characterised in the previous sections.

To highlight the importance of the insurance motive for herding, suppose in what follows that both assets have identical characteristics, i.e. $V_A^y = V_B^y$, $V_A^z = V_B^z$, $V_A^n = V_B^n$. In this case the motive for herding will not be that one asset is inherently more profitable than another and therefore both principals prefer to induce trading in that same asset.

An equilibrium will feature herding in one of the two assets if and only if $EB_1(A,A) \geq EB_1(B,A)$ and $EB_2(A,A) \geq EB_2(A,B)$. By symmetry these conditions will either both be violated or both be satisfied. Therefore, if $EB_1(A,A) \geq$

$EB_1(B,A)$ the equilibrium displays herding (in either asset) and non-herding otherwise.

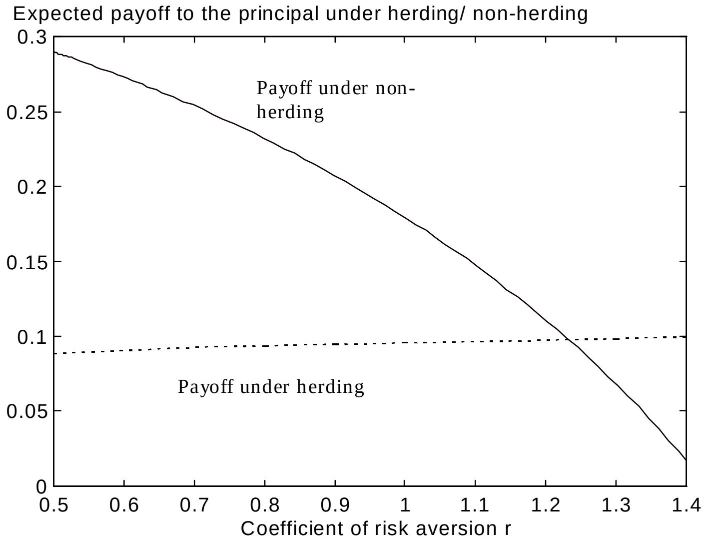

Figure 2: Plots the expected payoff to a principal under herding (dotted line) and non-herding (solid line). The parameters are c=0.09, V^y =0.5, V^z =1.5, V^n =1.5. Expected payoff under optimal herding contracts is an increasing function of the degree of risk aversion over some range of r. An increase in r mitigates the detrimental effect of strategic interaction between the principals.

We find that the expected payoff to the principal in the herding case is an increasing function of the coefficient of risk aversion, for r not too large. In Figure 2 expected payoff at the optimal contract is plotted for different levels of risk aversion.[9] It can be seen that in a region where the coefficient of absolute risk aversion is not too high, the expected payoff to the principal is increasing in the degree of risk aversion.[10] This result contrasts with other results in agency theory (see Milgrom and Roberts) and with our result in the non-herding case, where the agency cost increases with the degree of risk aversion, due to the fact that the insurance-efficiency trade-off worsens as the agent becomes more risk averse.

[9] In this and all the following simulations, parameters are chosen such that the investors' individual rationality constraints, given in definition (iv), are satisfied.

[10] Once the degree of risk aversion increases beyond a certain level, expected payoffs fall, because the increasingly negative impact of the insurance need on the efficiency of the contract will dominate the effect of "too large" order sizes.

The intuition for this result is simple. As discussed above, both principals face the problem of strategically designing the wage contracts of their agents such that they are induced to trade more aggressively than would be optimal if both principals could collude. As agents become more risk averse, it becomes more costly to induce managers to trade aggressively. Therefore, the problem of submitting "too large" orders is mitigated when managers become more risk averse. On the other hand, as shown in section 4.1, an increase in the degree of risk aversion is costly in the case where CPI is not available. Therefore herding is a Nash equilibrium in the choice of assets for sufficiently high values of r.

Of course, expected payoff to the principal depends also on the other parameters, namely the variance of noise trade, the variance of the signal and the variance of the asset value *ex ante*. For a given *ex ante* variance of asset value, consider an increase in the variance of the signal received by the traders. An increase in V^y for constant V^y+V^z, corresponds to an increase in the information content of the signal.[11] A more informative signal means not only higher expected trading profits, but also a reduction in residual risk. This suggests that the insurance need and hence the case for herding, is larger when the signal precision is low.

Figure 3 shows the set of parameters (r, V^y) for which herding is a Nash equilibrium in the choice of assets. In this simulation V^y+V^z is constant and hence an increase in V^y corresponds to an increase in the information content of the signal. For a given level of risk aversion, an increase in signal precision reduces the residual risk associated with trading and hence reduces the manager's insurance need. Correspondingly, herding will only occur for a low level of signal precision. Similarly, an increase in the coefficient of absolute risk aversion raises the manager's insurance need and reduces the strategic interaction problem between the principals. This increases the expected payoff to the principal of

[11] The informativeness of the signal can be measured as $V^y/(V^y+V^z)$, which is the regression coefficient of y on x. It is linearly increasing in V^y, for V^y+V^z=constant.

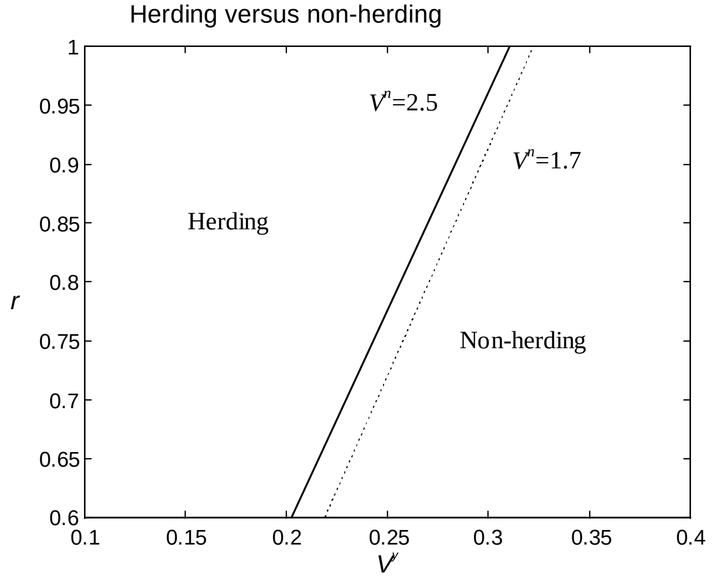

Figure 3: Illustrates the equilibrium choice of herding versus the non-herding decision. The parameters are c=0.09, V^y+V^z=1.5. The solid line is the region of indifference between herding and non-herding for V^n =2.5 and the dotted line for V^n =1.7. For a given level of risk aversion, an increase in V^y (increase in precision of the signal) reduces the insurance need and increases the payoff under non-herding relative to herding. Similarly, an increase in the coefficient of risk aversion increases the insurance need and mitigates the problem of strategic interaction amongst principals when choosing the wage parameters.

using CPI that becomes available under herding. Hence, herding is a Nash equilibrium for high levels of risk aversion.

Moreover, we observe that the region of parameters for which herding occurs decreases with an increase in the variance of noise trade. This seems counterintuitive, given that an increase in the variance of noise trade increases the execution risk for the trader (i.e. the riskiness of the clearing price after having submitted an order). On the other hand, an increase in the variance of noise trade leads to a flattening of the best-response functions (12) of each trader's trading intensities. In the contracting stage of the game, flatter best response functions in trading intensities lead to an exacerbation of the strategic interaction problem between the principals, which ultimately leads to a reduction in the expected payoff from herding relative to the payoff under non-herding.[12] *Ceteris paribus*,

[12] The actual expected payoff from herding increases with the variance of noise trade, because trading profits are an increasing function of the level of noise trade in the market.

we would therefore expect to see herding in markets with lower levels of noise trade.

6. Strategic non-disclosure of performance information

In this section we address the question of how the inefficiency of setting the contracting parameters at values that induce too high trading intensities can be mitigated. We will therefore focus on the case where there is only one asset and both agents trade in that asset, i.e. the first stage of the game, when principals choose an asset is deleted. Instead we will introduce another first stage of the game, in which principals can choose simultaneously whether or not to release information about their manager's performance, once trading profits are realised. The information disclosure decision is taken, given optimal contracting and trading strategies in the subsequent stages of the game. The decision is assumed to be irreversible and becomes common knowledge before contracting parameters are chosen.

The equilibrium of the trading subgame is still given by Proposition 2, except that $\gamma_i \equiv 0$, if principal P_j does not disclose performance information.

Proposition 4: The unique Nash equilibrium in the information disclosure game is for both principals not to disclose performance information.

Proof: Note from equation (12) in the Appendix (best response function in trading intensities), that an increase in γ_i results in an increase in δ_i, holding λ constant. Since *j*'s choice not to release information results in $\gamma_i \equiv 0$, the trading intensity of *j*'s opponent will be lower, when CPI is not available to him. A lower δ_i, however, makes principal *j* better off, regardless of whether or not his opponent releases information. Hence, it cannot be part of any Nash equilibrium for a principal to release information about his manager's performance.

q.e.d.

If both principals decide not to release performance information, they are constrained to offer individual performance contracts. Although each manager's equilibrium choice of action still depends on the opponent's wage parameters, there is now a unique β that makes the incentive compatibility constraint binding. By omitting CPI, principals can commit to reducing the number of degrees of freedom in the contract by one, which avoids the detrimental effect of strategic interaction. At the same time it removes the insurance gain from offering relative performance contracts.

Now compare the effect on principals' expected payoffs when they play the information disclosure game (and hence no information is released), to the case where information is always released. This amounts to a comparison of the benefits of using CPI (increase in the insurance-efficiency trade-off in contracting) and the losses of using this information, which are manifested in the strategic interaction problem between the principals at the contracting stage.

An increase in the variance of noise trade increases trading profits and therefore expected payoff to the principal in either case. As discussed in the previous section, an increase in the variance of noise trade exacerbates the strategic interaction problem between the principals, because the traders' best-response functions in trading intensities become flatter. As illustrated in Figure 4, the loss due to offering aggressive wage contracts outweighs the gains of using CPI, when the variance of noise trade is high.

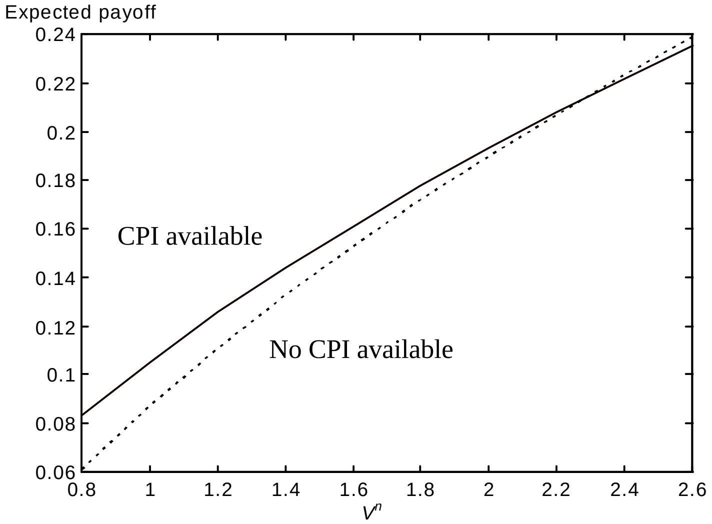

Figure 4: The solid line plots expected payoff to the principal when performance information is released and contracting parameters are set such that they constitute an equilibrium according to Section 4. The dotted line shows expected payoff when CPI is not available. For high levels of the variance of noise trade it becomes more profitable for principals to omit relative performance information. Parameter values are c=0.1, r=1, V^y=V^z=1.

On the other hand, an increase in the degree of risk aversion, increases the benefit of using CPI and mitigates the strategic interaction problem. In Figure 5 the expected payoff under optimal contracts is plotted for the case when CPI is available (solid line) and when it is not available (dotted line). Expected payoff increases with the degree of risk aversion when CPI is available and decreases when CPI is not available.

We can therefore conclude that for high levels of noise trade and low levels of risk aversion, principals are better off not using CPI. As shown in Proposition 4, one credible way to achieve this is to endogenise the choice of information disclosure. This of course, raises the normative question of whether or not a regulating authority should leave this choice to the principals. Our results show that CPI increases competition between funds through an increase in equilibrium trading intensities and hence also the information content of prices. A welfare analysis is beyond the scope of this paper, but our model proposes a framework for study of the interdependence of information disclosure, competition among traders and informativeness of asset prices. [13]

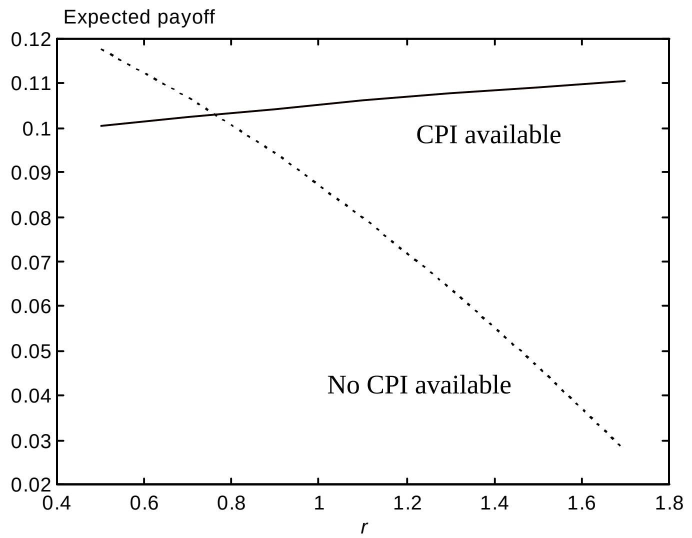

Figure 5: The solid line shows expected payoff to the principal when CPI is available as a function of the degree of risk aversion. The dotted line shows expected payoff when CPI is not made available. As the degree of risk aversion increases, the externality problem under CPI is mitigated and the insurance benefit of CPI increases. Hence, omitting CPI is only beneficial for low degrees of risk aversion. Parameter values are c=0.1, $V^n = V^y = V^z = 1$.

7. Conclusion

In the preceding study we explored the contracting problem between a risk averse fund manager and a principal and how this contracting problem can give rise to herding of investors' asset allocation decisions. In our treatment, fund managers have discretion over two sets of actions, both of which are non-contractible. Firstly, a fund manager decides whether or not to acquire costly information about the value of an asset. Secondly, he chooses the trading intensity, which determines his order size as a function of his private information about asset value.

[13] A welfare analysis would require endogenous noise traders, which could be modelled as rational agents with a hedging need. For a model with noise trade due to rational hedgers, see Spiegel and Subrahmanyam (1992).

In order to model this last instance of moral hazard, we use a market maker model similar to Kyle (1985). When designing a wage contract, a principal has to take into account that his agent's actions have to be implementable as a Nash equilibrium in the trading subgame between a market maker, his own manager and, possibly, another fund manager.

By inducing their managers to trade in the same asset, principals are enabled to use comparative performance information to design relative wage contracts. CPI has the benefit of improving the insurance efficiency trade-off of the wage contract, while introducing a detrimental element of strategic interaction between the principals. The latter arises because principals cannot commit to not offering a wage contract to their manager that induces him to trade aggressively.

We show that principals may nonetheless induce their managers to trade in the same market (herding), when their degree of risk aversion is high, or when the precision of the signal for asset value is low. Furthermore, we show that the problem of strategic interaction between principals can be overcome by endogenising the choice of information disclosure of a fund manager's performance. Principals can thus be made better off when their managers are not very risk averse or when the level of noise trade in a market is high. In both these cases strategic interaction between principals is particularly important.

Appendix

Proof of Proposition 1:

First, we have to find the profits from trading amounts t_1 and t_2. From (2) we can write

$$\tilde{\pi}_1 = t_1(y_A + \tilde{z}_A - \tilde{p}_A) = t_1(y_A + \tilde{z}_A - \lambda_A(t_1 + \tilde{n}_A))$$
and
$$\tilde{\pi}_2 = \tilde{t}_2(\tilde{y}_B + \tilde{z}_B - \tilde{p}_B) = \tilde{t}_2(\tilde{y}_B + \tilde{z}_B - \lambda_B(\tilde{t}_2 + \tilde{n}_B)) \qquad (8)$$

where λ_A and λ_B are the price setting parameters by the market maker. They are the coefficients by which the total order flow is multiplied to yield prices.

Note that manager 1 can only observe y_A but not y_B, which is why in manager 2's profits, y_B and t_2 enter as random variables, while t_1 is non-random, given y_A .

The optimal amount of trade is the solution to

$$\max_{t_1} E\left[-\exp\left(-r\left(\alpha + \beta t_1(y_A + \tilde{z}_A - \lambda_A(t_1 + \tilde{n}_A)) - \gamma\tilde{t}_2(\tilde{y}_B + \tilde{z}_B - \lambda_B(\tilde{t}_2 + \tilde{n}_B)) - c\right)\right)\right]$$

Note that the random variables x_B, n_A and n_B are independent, which allows us to rewrite expected utility as

$$EU = E\left[-\exp\left(-r\left(\alpha + \beta t_1(y_A + \tilde{z}_A - \lambda_A(t_1 + \tilde{n}_A))\right)\right)\right]E\left[\exp\left(-r\left(-\gamma\tilde{t}_2(\tilde{x}_B - \lambda_B(\tilde{t}_2 + \tilde{n}_B)) - c\right)\right)\right]$$

The second expectations term is constant in t_1, which allows us to treat it as a constant for the maximisation problem. This is a special feature of CARA utility and simplifies the analysis, because we can now analyse the certainty equivalent of expected utility.

Thus, t_1 is the solution to

$$\max_{t_1} \quad CE = \alpha - c + \beta t_1 y_A - \beta\lambda_A t_1^2 - r/2 * (\beta t_1)^2 (V_A^z + \lambda_A^2 V_A^n)$$

The first-order condition of this optimisation problem is

$$\beta y_A - 2\beta\lambda_A t_1 - r\beta^2 t_1 (V_A^z + \lambda_A^2 V_A^n) = 0$$

Which yields the solution

$$t_1 = \frac{y_A}{2\lambda_A + r\beta\left(V_A^z + \lambda_A^2 V_A^n\right)}$$

Thus $t_1^N = \delta_1^N y_A$

with $$\delta_1^N = \frac{1}{2\lambda_A^N + r\beta_1\left(V_A^z + \lambda_A^{N^2} V_A^n\right)} \qquad (9)$$

This proves the first part of the proposition.

For the following derivation of the price setting strategy, the subscripts for asset and trader are suppressed, since only one trader and one asset matter. The market maker sets price equal to expected value of the asset conditional on order flow, given his knowledge of the contracting parameters and knowing that only one informed trader submits an order in his market.

$$\tilde{p} = E\left(\tilde{x}\middle|\tilde{t} + \tilde{n}\right) = \frac{Cov\left(\tilde{y} + \tilde{z}, \delta\tilde{y} + \tilde{n}\right)}{Var\left(\delta\tilde{y} + \tilde{n}\right)}\left(\delta\tilde{y} + \tilde{n}\right) \equiv \lambda\left(\delta\tilde{y} + \tilde{n}\right)$$

Since asset value and noise trade are independent,

$$\lambda = \frac{\delta V^y}{\delta^2 V^y + V^n}.$$

The price setting strategy of the market maker for asset *A* is thus given by:

$$\tilde{p}_A = \lambda_A^N\left(\tilde{t}_1 + \tilde{n}_A\right),$$

where

$$\lambda_A^N = \frac{\delta_1^N V_A^y}{\delta_1^{N^2} V_A^y + V_A^n} \qquad (10)$$

q.e.d.

Proof of Proposition 2:

Agent 1 receives the following wage as a function of his own and agent 2's trading strategy.

$$\tilde{w}_1 = \alpha_1 + (\beta_1 t_1 - \gamma_1 t_2)(y_A + z_A - \tilde{p}_A) \qquad (11)$$

where

$$\tilde{p}_A = \lambda_A (t_1 + t_2 + \tilde{n}_A).$$

Given this, agent 1 faces the following optimisation problem:

$$\max_{t_1} E\left[-\exp\left(-r\left(\alpha_1 + (\beta_1 t_1 - \gamma_1 t_2)(y_A + \tilde{z}_A - \lambda_A (t_1 + t_2 + \tilde{n}_A)) - c\right)\right)\right]$$

Note, that here t_2 is not a random variable, because in equilibrium agent 1 knows agent 2's trading strategy and the signal he received. Again we can use the certainty equivalent of utility to find the optimal trading strategy.

$$CE = \alpha_1 - c + (\beta_1 t_1 - \gamma_1 t_2)(y_A - \lambda_A(t_1 + t_2)) - r/2 * (\beta_1 t_1 - \gamma_1 t_2)^2 (V_A^z + \lambda_A^2 V_A^n)$$

Taking the first-order condition and solving for t_1 yields

$$t_1 = \frac{\beta_1 y_A - t_2\left(\lambda_A(\beta_1 - \gamma_1) - r\beta_1\gamma_1\left(V_A^z + \lambda_A^2 V_A^n\right)\right)}{2\beta_1\lambda_A + r\beta_1^2\left(V_A^z + \lambda_A^2 V_A^n\right)} \qquad (12)$$

Since agent 2 has the same utility function as agent 1, his choice of strategy is given by (12) with appropriately modified indices. Substituting t_2 in (12) by this formula into (12) and solving for t_1 yields the result in Proposition 2, with a trading intensity parameter given by

$$\delta_1^H = \frac{\lambda_A^H \beta_2(\beta_1 + \gamma_1) + r\beta_1\beta_2(\beta_2 + \gamma_1)B}{\left(2\beta_1\lambda_A^H + r\beta_1^2 B\right)\left(2\beta_2\lambda_A^H + r\beta_2^2 B\right) - \left(\lambda_A^H(\beta_1 - \gamma_1) - r\beta_1\gamma_1 B\right)\left(\lambda_A^H(\beta_2 - \gamma_2) - r\beta_2\gamma_2 B\right)} \qquad (13)$$

where $$B = \left(V_A^z + \lambda_A^{H\,2} V_A^n\right).$$

δ_2^H is given by (13) with indeces changed appropriately.

As before the coefficient on order flow that determines prices, is the regression coefficient of asset value on observed order flow:

$$\tilde{p} = E\left(\tilde{x} \middle| \tilde{t}_1 + \tilde{t}_2 + \tilde{n}\right) = \frac{Cov\left(\tilde{x}, (\delta_1 + \delta_2)\tilde{y} + \tilde{n}\right)}{Var\left((\delta_1 + \delta_2)\tilde{y} + \tilde{n}\right)}\left((\delta_1 + \delta_2)\tilde{y} + \tilde{n}\right) \equiv \lambda\left((\delta_1 + \delta_2)\tilde{y} + \tilde{n}\right)$$

hence, $$\lambda_A^H = \frac{\left(\delta_1^H + \delta_2^H\right)V_A^y}{\left(\delta_1^H + \delta_2^H\right)^2 V_A^y + V_A^n} \qquad (14)$$

q.e.d.

For the proof of Proposition 3 and Lemma 2, we need to calculate the expectation of exponential utility when wage is distributed as a quadratic function of normally distributed random variables. To this end we use Lemma 1, which gives a formula to calculate this expectation. A similar lemma and proof can be found for example in Bray (1981).

Lemma* 1:* Let **u** be an m dimensional vector of normally distributed random variables with variance-covariance matrix Σ. Wage *w* is a quadratic function of **u**, α is the non-random part of wage and *c* the cost of information acquisition. Expected utility is then given by

$$EU = -\left(|\Sigma||\mathbf{A}|\right)^{-\frac{1}{2}} \exp\left(-r(\alpha - c)\right)$$

where **A** is given by

$$r(w(\mathbf{u}) - c) + \tfrac{1}{2}\mathbf{u}'\Sigma^{-1}\mathbf{u} = 1/2\ \mathbf{u}'\mathbf{A}\mathbf{u} + r(\alpha - c).$$

Proof: Expected utility can be written as

$$EU = -\frac{1}{(2\pi)^{m/2}}|\Sigma|^{-1/2}\int_{\Re^m}\exp\left(-K\right)d\mathbf{u} \qquad (15)$$

and $K = r(w(\mathbf{u}) - c) + \frac{1}{2}\mathbf{u}'\Sigma^{-1}\mathbf{u}$ (16)

This simply stems from multiplying the utility function with the density function for multivariate normally distributed random variables.

The next step is to rearrange K such that it is possible to carry out the integration. Thus, define $\mathbf{A}$ such that

$K = 1/2\ \mathbf{u}'\mathbf{A}\mathbf{u} + r(\alpha\text{-}c)$,

Next we carry out the following transformation

$\mathbf{A} = \mathbf{B}\mathbf{B}'$.

Then we substitute $\mathbf{u}$ in expected utility (15) by $\mathbf{q} = \mathbf{B}\mathbf{u}'$

This yields

$$\int_{\Re^m} \exp(-K)d\mathbf{u} = \int_{\Re^m} \exp\left(-\tfrac{1}{2}\mathbf{u}'\mathbf{A}\mathbf{u} - r(\alpha - c)\right)d\mathbf{u}$$
$$= |\mathbf{A}|^{-\frac{1}{2}} \int_{\Re^m} \exp\left(-\tfrac{1}{2}\mathbf{q}'\mathbf{q} - r(\alpha - c)\right)d\mathbf{q} = (2\pi)^{m/2}|\mathbf{A}|^{-\frac{1}{2}}\exp\left(-r(\alpha - c)\right) \tag{17}$$

A sufficient condition for the convergence of the integral is that the matrix $\mathbf{A}$ is positive definite.

q.e.d.

Proof of Proposition 3:

Suppose w.l.o.g. that principal P_1 induces $l_1 = A$. From (9) we can see that the agent's trading strategy is independent of the other agent's actions. Hence, the contracting problem between principal and agent i is independent from that of principal and agent j ($i \neq j$). Moreover, γ does not enter δ^N as an argument and trading profits of the agents are independently distributed. Hence, there is no gain from relative performance contracts and $\gamma_1^N = 0$.

In order to evaluate the incentive compatibility constraint (IC) we need to calculate the expected utility of the agent under a given contract, taking into account his subsequently chosen trading strategy.

Agent F_1's *ex ante* (i.e. before observing y_A) wage is a non-normally distributed random variable

$$\tilde{w}_1 = \alpha_1 + \beta_1 \delta_1^N \tilde{y}_A \left(\tilde{x}_A - \lambda_A^N \left(\delta_1^N \tilde{y}_A + \tilde{n}_A \right) \right)$$

Under a given contract and equilibrium in the trading game, the agent's expected utility can be calculated with the help of Lemma 2:

$$EU^N = -\left(|\Sigma_A| |\mathbf{A_N}| \right)^{-\frac{1}{2}} \exp\left(-r(\alpha_1 - c) \right) \tag{18}$$

where

$$\mathbf{A_N} = \begin{bmatrix} \frac{1}{V_A^y} + 2r\beta_1\delta_1(1-\delta_1\lambda_A) & r\beta_1\delta_1 & -r\beta_1\delta_1\lambda_A \\ r\beta_1\delta_1 & \frac{1}{V_A^z} & 0 \\ -r\beta_1\delta_1\lambda_A & 0 & \frac{1}{V_A^n} \end{bmatrix}$$

and

$$\Sigma_A = \begin{pmatrix} V_A^y & 0 & 0 \\ 0 & V_A^z & 0 \\ 0 & 0 & V_A^n \end{pmatrix}$$

Moreover, δ_1 and λ_A are given from Proposition 1.

Furthermore, because of the particular form of matrix $\mathbf{A_N}$, a necessary and sufficient condition for $\mathbf{A_N}$ to be positive semidefinite is $|\mathbf{A_N}|>0$.

The participation constraint (PC) can thus be written as

$$-\left(|\Sigma_A| |\mathbf{A_N}| \right)^{-\frac{1}{2}} \exp\left(-r(\alpha_1 - c) \right) \geq -\exp(-rW_1) \tag{19}$$

Moreover, using (18) the incentive compatibility constraint (IC) can be written as

$$-\left(|\Sigma_A| |\mathbf{A_N}| \right)^{-\frac{1}{2}} \exp\left(-r(\alpha_1 - c) \right) \geq -\exp(-r\alpha_1) \tag{20}$$

Substituting the binding inequality (20) into (19) yields

$\alpha_1 \geq W_1$.

Since α_1 cancels out in (20), the optimal choice of α_1 makes (19) binding. Hence, $\alpha_1^N = W_1$.

In order to calculate the optimal β_1 rewrite (20) as

$$|\Sigma_A||\mathbf{A_N}| \geq \exp(2rc)$$

Calculating $|\Sigma_A||\mathbf{A_N}|$ yields

$$1+2rV^y\beta_1\delta_1(1-\lambda_A\delta_1) - r^2\beta_1{}^2\delta_1{}^2(V^z+\lambda_A{}^2V^n)V^y \geq \exp(2rc) \qquad (21)$$

Suppose $|\mathbf{A_N}| < 0$. In that case (19) could never be satisfied. Hence, every contract that satisfies (19) features $|\mathbf{A_N}|>0$ and therefore the formula in Lemma 2 can be applied.

Substituting (9) into (21) and rearranging the terms yields

$$\beta_1\delta_1 \geq \frac{\exp(2rc)-1}{rV_A^y} \qquad (22)$$

Now calculate the principal's expected payoff, by first calculating expected trading profits

$$\tilde{\pi}_1{}^N = \delta_1^N \tilde{y}_A\left(\tilde{x}_A - \lambda_A^N\left(\delta_1^N\tilde{y}_A + \tilde{n}_A\right)\right)$$

the expected value of which is (23)

$$E\pi_1{}^N = \delta_1^N\left(1 - \frac{\delta_1^{N2}V_A^y}{\delta_1^{N2}V_A^y + V_A^n}\right)V_A^y = \frac{V_A^nV_A^y}{\delta_1^{N2}V_A^y + V_A^n}\delta_1^N$$

Thus, using (23) and (P) we can write

$$EB_1{}^N = (1-\beta_1)\frac{V_A^nV_A^y}{\delta_1^{N2}V_A^y + V_A^n}\delta_1^N - \alpha_1$$

which is a decreasing function in β_1. Hence, the optimal β_1 will be chosen such that (22) is binding.

Substituting (10) into (9) and (9) into the binding (22) yields after some simplifications

$$\beta_1^4\left(\frac{V_A^n}{V_A^y}\right)^2 - \beta_1^2\frac{V_A^n}{V_A^y}ra^3\frac{V_A^y + 2V_A^z}{1-raV_A^z} - a^4\frac{1+raV_A^z}{1-raV_A^z} = 0 \qquad (24)$$

where $a \equiv \dfrac{\exp(2rc) - 1}{rV_A^y}$

Solving the quartic equation (24) for β_1 yields one positive real root, given by (7) if $\exp(2rc)-1 \leq V^y/V^z$. Otherwise no real root exists, which means that no β exists that satisfies the incentive compatibility constraint.

q.e.d.

***Lemma* 2**: The incentive compatibility constraint (IC) for agent F_1 in the case of herding can be written as[14]

$$1 + 2rV^y(\beta_1\delta_1 - \gamma_1\delta_2)(1 - \lambda(\delta_1 + \delta_2)) - r^2(\beta_1\delta_1 - \gamma_1\delta_2)^2(V^z + \lambda^2 V^n)V^y \geq$$
$$\exp(2rc)\left[1 - 2rV^y\gamma_1\delta_2(1 - \lambda\delta_2) - r^2\gamma_1^2\delta_2^2(V^z + \lambda^2 V^n)V^y\right]$$

where δ_1, δ_2, λ are given by (13) and (14).

***Proof*:**

The incentive compatibility constraint can be written as

$$EU^H \geq EU^{H \to NI} \tag{25}$$

where the LHS of (25) denotes expected utility under information acquisition and accordingly optimal trading. The RHS denotes expected utility when no information is acquired. The best trading strategy in that case is not to trade at all, since this minimises the riskiness of wage.

Agent 1's wage is

$$\tilde{w}_1 = \alpha_1 + (\beta_1\delta_1 - \gamma_1\delta_2)\tilde{y}\left(\tilde{x} - \lambda\left((\delta_1 + \delta_2)\tilde{y} + \tilde{n}\right)\right) \tag{26}$$

With the help of Lemma 2 we can write expected utility of agent 1 under information acquisition as

[14] Subscripts for the asset are omitted, as only one asset is relevant here.

$$EU^{H} = -\left(\left|\Sigma_{\mathrm{H}}\right|\left|\mathbf{A}_{\mathbf{H}}\right|\right)^{-\frac{1}{2}} \exp\left(-r(\alpha_1 - c)\right) \tag{27}$$

with

$$\mathbf{A}_{\mathbf{H}} = \begin{bmatrix} \frac{1}{V^y} + 2r(\beta_1\delta_1 - \gamma_1\delta_2)(1-(\delta_1+\delta_2)\lambda) & r(\beta_1\delta_1 - \gamma_1\delta_2) & -r(\beta_1\delta_1 - \gamma_1\delta_2)\lambda \\ r(\beta_1\delta_1 - \gamma_1\delta_2) & \frac{1}{V^z} & 0 \\ -r(\beta_1\delta_1 - \gamma_1\delta_2)\lambda & 0 & \frac{1}{V^n} \end{bmatrix} \tag{28}$$

and $$\Sigma_{\mathrm{H}} = \begin{bmatrix} V^y & 0 & 0 \\ 0 & V^z & 0 \\ 0 & 0 & V^n \end{bmatrix}$$

Next, we derive expected utility for an agent who deviates from a herding equilibrium by not acquiring information at all. Agent 2's profits are affected by agent 1's decision not to acquire information and not to trade. This is because total order flow changes as agent 1 ceases to trade, which in turn affects prices. Expected utility can be derived straightforwardly by setting $\delta_1=0$ in (28).

$$EU^{H\rightarrow NI} = -\left(\left|\Sigma_{H}\right|\left|\mathbf{A}_{\mathbf{H\rightarrow NI}}\right|\right)^{-\frac{1}{2}} \exp\left(-r\alpha_1\right)$$

with

$$\mathbf{A}_{\mathbf{H\rightarrow NI}} = \begin{bmatrix} \frac{1}{V^y} - 2r\gamma_1\delta_2(1-\delta_2\lambda) & -r\gamma_1\delta_2 & r\gamma_1\delta_2\lambda \\ -r\gamma_1\delta_2 & \frac{1}{V^z} & 0 \\ r\gamma_1\delta_2\lambda & 0 & \frac{1}{V^n} \end{bmatrix} \tag{29}$$

Calculation of the determinants and rearranging of the inequality (25) yield the desired result. *q.e.d.*

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