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DEPARTMENT OF ECONOMICS

Essays on Risk Sharing in Economic Unions

Chima Simpson-Bell

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 18 May 2020

European University Institute
Department of Economics

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Researcher declaration to accompany the submission of written work

I, Chima Simpson-Bell, certify that I am the author of the work `*Essays on Risk Sharing in Economic Unions*' I have presented for examination for the PhD thesis at the European University Institute. I also certify that this is solely my own original work, other than where I have clearly indicated, in this declaration and in the thesis, that it is the work of others.

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Statement of inclusion of previous work (if applicable):

I confirm that chapter 2 was jointly co-authored with Johannes Fleck and I contributed 50% of the work.

I confirm that chapter 3 was jointly co-authored with Alessandro Ferrari and Ramon Marimon and I contributed 33% of the work.

Signature and Date: Chima Simpson-Bell 13/5/2020

Abstract

This thesis investigates consumption insurance within economic unions from both a country and household perspective. Chapter 1 deals with the question of how an economic union like the Euro Area can support enough risk sharing, through transfer payments, to prevent the breakup of the union. I model the union as a dynamic contract between two countries. The contract captures two political restrictions which are especially relevant for the Euro Area. First, risk sharing must avoid 'permanent' transfers (including *repayments* of debt) between countries. Second, there is a requirement that countries implement policies to improve economic performance, which is subject to moral hazard. Relative to the previous literature, the specification of the reform process makes the moral hazard component of the model more powerful by allowing reform effort to have a permanent effect on the distribution of output. I then characterize the optimal transfer system, which trades off risk sharing against reform incentives. I propose an implementation of this transfer system using trading of one-period bonds with state-contingent debt restructuring. Chapter 2, which is co-authored with Johannes Fleck, deals with household earnings risk in the United States. We observe that due to differences in economic conditions across American states, and the autonomy which state governments have in implementing means tested policies, identical households may receive very different levels of earnings insurance from the government simply because they live in different states. We quantify this variation in public insurance by simulating the response of the main state and federal tax and benefit policies to earnings shocks for a prototype household, adjusting any nominal dollar amounts for purchasing power using our own measure of state living costs. We confirm that there is significant regional variation in household earnings insurance, with a large contribution coming from the design of the federal Earned Income Tax Credit. Chapter 3, which is joint work with Ramon Marimon and Alessandro Ferrari, returns somewhat to the theme of Chapter 1. We address a gap in the theoretical literature on optimal transfers in currency unions, which fails to account properly for the participation constraints imposed by each country's option to exit the union. We model a currency union as a dynamic contract between countries facing endowment risk and a nominal rigidity in the production of non-tradeable goods. The contract is constrained by each country's outside option of reclaiming its own monetary policy, which optimally eliminates the labour market distortion caused by the nominal rigidity, and defaulting on any payment obligations accumulated within the union. We find that there is still some scope for considerable risk sharing in the union, although the presence of the nominal rigidity introduces consumption risk into the stochastic steady state of the currency union.

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Contents

Chapter 1: Risk Sharing and Policy Convergence in Economic Unions

Chima Simpson-Bell

Abstract

Countries in an economic union benefit from an agreement to share fiscal risk due to enhanced macroeconomic linkages and the loss of national monetary policy. The ability to share risk is, however, constrained by a political need to limit long term transfers between countries. Risk sharing might therefore be complemented by the goal of *convergence* in national economic institutions - a long term commitment to implement best practice policies. An optimal contract between union members builds incentives for economic reform into any risk sharing agreement, by conditioning transfer payments on observed reform outcomes. In this paper, I model an economic union as a risk sharing contract between two countries, subject to (two-sided) limited enforcement and (one-sided) moral hazard. I characterize the transfer system which optimally trades off risk sharing against incentives to complete long term reforms. I then consider a mechanism for implementing the transfer payments using active management of intergovernmental liabilities.

Introduction

The leading narratives about the events which culminated in the Eurozone debt crisis identify two areas of failure in economic policy coordination. The first was the failure of individual countries to implement best practice economic policies and institutions, a form of self-insurance to improve the response to macroeconomic shocks. These policies were codified in the Stability and Growth Pact and can now be seen in the fiscal and structural reforms being proposed for countries receiving financial assistance. The second is the failure to complete, or even establish, a fiscal union, which would have enabled countries to share budgetary risks. As a counterexample, several observers have noted the intergovernmental transfer flows in the United States, where during recessions fiscally weaker states receive significant assistance from the federal government without incurring explicit liabilities (Krugman, 2013).

The package of proposals for strengthening the Euro area aims to address these failures (see for example Juncker et al. (2015)), but does so in the face of two constraints. Firstly, moral hazard in reform effort may create negative interactions between risk sharing and progress towards convergence of economic institutions. This is especially salient in the European context, where convergence - in economic policies and, eventually, living standards - has been adopted as an important objective. Secondly, there are political limits on the size and duration of transfers which countries would be willing to provide when other members of the union are underperforming.

How much risk sharing can be achieved under these constraints? This paper aims to answer this question by studying an economic union as a long term contract between two countries facing output risk. Differences in subjective discount rates and risk tolerance lead to the frontloading of consumption in one country, so that there is a tendency for cross-border liabilities to accumulate. To capture the policy design challenges outlined above, I make two key assumptions about incentives in the union:

1. **Moral hazard in reform**: The (impatient, risk averse) borrower country has access to a reform technology which improves the distribution of the economy's output endowment. By exerting effort each period, the borrower can gradually transition from a 'bad' distribution to a 'good' distribution under which high output is more likely. Moral hazard is introduced by assuming that this effort imposes a utility cost and is not fully observable, so that it is not possible to enforce a contract which directly stipulates effort levels. Consequently, any reform effort which is desired by the social planner must satisfy an incentive compatibility constraint.

2. **Limited enforcement**: Each country has access to an alternative to participating in the economic union, which imposes a set of participation constraints on the allocation. This applies equally to the (patient, risk neutral) lender country, resulting in a two-sided limited enforcement problem.

The reform technology is similar to human capital accumulation through learning-by-doing, in the sense that effort made in each period has a cumulative and permanent effect on the productivity of the economy. In the long term reform effort creates extra surplus for the union in the form of higher expected output, which can be used to alleviate participation constraints. However, the framework also produces an opposing effect whereby reforms completed inside the union raise the value of the outside option.

Due to the forward looking nature of the constraints, the contract which I construct can be solved using the recursive contract techniques in Marcet and Marimon (2019) and Mele (2014). In essence, the solution indicates that the Pareto weight of each country should increase whenever its participation constraint is binding. The increasing weight corresponds to higher relative consumption, providing the needed incentive for the country to stay in the union. In addition, the relative weight of the borrower should increase if the realization of output indicates that high reform effort was exerted in the past. This will typically happen for high output realizations. The allocation ultimately translates to a countercyclical transfer policy, such that the borrower has to make net payments when output is high and receives transfers when output is low. The dynamics of reform effort are more complex but I find that more reform effort is exerted when output is low, because the probability of exit from recession is particularly sensitive to reform.

In order to bring the insights on risk sharing closer to recent policy debates, I then explain how the system of transfers could be implemented by trading of bonds between the lender and the borrower. Bonds are a natural instrument for decentralizing the contract because, like the time-varying Pareto weights, they allow us to keep track of the effect of past commitments on the current allocation. One limitation of this approach, however, is that while the constrained efficient allocation is highly state contingent, the payments specified in standard bond contracts are fixed. I address this using results from Alvarez and Jermann (2000), Krueger et al. (2008) and Kehoe and Perri (2004) to allow restructuring of the liabilities in some states. A simulation of the resulting allocations and asset positions, calibrated to the countries which recently received financial assistance in Europe, suggests that relatively frequent adjustments in the bond positions would be required to support the risk sharing agreement. In practice, this would discourage the use of face value haircuts as an instrument for liability management, but rescheduling of principal payments or variable interest rates could be used to support enhanced risk sharing.

Related literature The closest papers to this one are Ábrahám et al. (2019) and Müller et al. (2019). Ábrahám et al. (2019) considers the design of a financial stability fund in the Eurozone using the recursive contracts framework of Marcet and Marimon (2019). An important feature of this paper is the presence of moral hazard in reform effort to improve the economy of a borrowing country. Moral hazard has important effects on the allocation in this framework but the magnitude of these effects is limited by the transitory impact of the reform technology. Müller et al. (2019) also introduces moral hazard explicitly; similar to this paper, a borrowing country is able to exert effort which increases the probability of high output in the next period. The efficient allocation is decentralized using one period GDP-linked bonds. My paper builds on this earlier work in three important aspects. First, I introduce a structural reform technology with *persistent* effects on output, which creates a much more important role for moral hazard. Second, with this alternative reform technology, I am able propose a method for calibrating the impact of reform more carefully, based on the change in expected output. Third, I incorporate debt restructuring as an instrument for implementing the risk sharing agreement.

My paper also contributes to an important literature on risk sharing in currency unions deals with optimal international transfer policies. Auclert and Rognlie (2014) show formally that in a monetary union, the incentives to share macroeconomic risks through transfers are strengthened by the loss of independent monetary policy to stabilize the economy. Farhi and Werning (2017) explores the optimal transfer system in a fiscal union with nominal rigidities. A key question for the design of currency unions is why the desired level of risk sharing cannot be achieved by developing well functioning private capital markets. This paper highlights an externality from risk sharing whereby international transfers smooth business cycles in each of the countries. Marimon et al. (2019) seeks to extend the analysis in the previous papers by explicitly accounting for the risk of exit from the union. The option to exit and regain national monetary policy places limits on the transfers which are possible within the union, but under certain conditions it is still possible to design a system of transfers which assists union members during crises. Gourinchas et al. (2018) explore the optimal enforcement of a no bailout rule in a model where default by one country in the union imposes output losses on the rest of the union. They find that incomplete enforcement may be sufficient to attain the level of fiscal discipline achieved by full enforcement, and may be desirable if it prevents immediate default due to rollover issues.

There are several papers which take a similar approach to characterizing efficient debt management, using contract theory to derive the optimal features of debt agreements between lenders and borrowers with the option to default. Atkeson (1991) studies the optimal lending contract for a sovereign borrower which cannot commit to repay, finding that the observation of capital outflows during recession arises as the solution to a moral hazard problem. Dovis (2018) finds that outcomes such as partial default and temporary exclusion from capital markets arise as equilibrium outcomes of a sovereign borrowing game. Interestingly, despite being inefficient ex post, these outcomes are found to be necessary *ex ante* to achieve the state contingency featured in the efficient allocation. Aguiar et al. (2019) consider the optimal maturity management policy for a sovereign borrower facing the risk of default. They find that the borrower should remain passive in long term debt markets, due to adverse price responses from attempting to buy back outstanding long term debt. This is consistent with the empirical observation that emerging market economies tend to shorten the maturity of their debt stocks as they approach default episodes (Broner et al., 2013).

The rest of the paper is organized as follows. In section 1, I outline the basic environment. In section 2, I discuss the constrained efficient allocation, introducing the participation constraints and the moral hazard problem. I also solve a simple version of the model to illustrate some of the main features of the constrained efficient allocation. In section 3, I consider the implementation of the efficient allocation with non-contingent bonds and state contingent debt restructuring. Section 4 contains a more complete simulation exercise, calibrated to the bailout countries in the Euro Area and estimates from the IMF on the expected output gain from fully implemented reform programmes in Greece. The behaviour of the risk sharing contract is compared to a defaultable debt economy with the same reform technology. Section 5 offers concluding comments.

1 Main Environment

This section specifies the environment in which the government borrows but cannot commit to repay. There is a small open economy, with a government that makes all consumption and borrowing decisions. The government must also allocate effort to a simple technology through which it can make permanent improvements to the distribution of the economy's random endowment.

Time is discrete and infinite, and indexed by $t = 1, 2, 3, ...$ There are three types of agent - a representative household, a benevolent domestic government and risk-neutral foreign competitive lender. There is a single tradable good; the main source of uncertainty in this economy is the realization of the domestic endowment of this good.

Preferences Sequences of consumption $\{c_t\}_{t=0}^{\infty}$ and reform effort $\{e_t\}_{t=0}^{\infty}$ are valued by the representative domestic household according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \nu(e_t)] \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor. The government (also referred to interchangeably as the borrower) is benevolent and therefore aims to maximize the utility of the representative household. The within period consumption utility function $u(.)$ is CRRA with relative risk aversion coefficient γ:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{2}$$

The effort disutility function $\nu(.)$ is continuously differentiable with $\nu'(.) \geq 0$ and $\nu''(.) \geq 0$.

The Lender The lender is risk-neutral and competitive, evaluating alternatives based on their expected values. I also assume that the lender is able to borrow and lend internationally at the risk free gross interest rate $R > 1$, and has a large endowment of the single output good.

Endowments and Reform Technology Every period the government receives an endowment of the output good. The endowment is drawn from a vector $y \in \{y_1, y_2, ..., y_N\}$ and follows a first order Markov process governed by the transition matrix $\tilde{\Pi}_t$. We use y^t to denote the history of endowment realizations $(y_0, y_1, \cdots, y_{t-1}, y_t)$. The transition matrix $\tilde{\Pi}_t$ is in turn a mixture of two sub-distributions Π_b and Π_g such that

$$\tilde{\Pi}(D_t) = w(D_t)\Pi_g + (1 - w(D_t))\Pi_b \tag{3}$$

where the weight $D \in [0, 1]$ is the outcome of past government decisions on effort and $w(D)$ satisfies $w_D(0) > 0$, $\lim_{D \to 1} w_D(D) = 0$, $w(0) = 0$ and $w(1) = 1$. Importantly, Π_b and Π_g have the same state space, so that it is not possible to distinguish between the two distributions by observing a single realization of y_t. Π_g dominates Π_b by the monotone likelihood ratio property (MLRP), which states that for any $y_1 > y_0$:

$$\frac{\Pi_g(y_1 \mid y)}{\Pi_b(y_1 \mid y)} > \frac{\Pi_g(y_0 \mid y)}{\Pi_b(y_0 \mid y)} \tag{4}$$

It will also be useful to denote the difference between the sub-distributions Π_g and Π_b by

$$\Delta_\Pi = \Pi_g - \Pi_b \tag{5}$$

Reform progress D_t evolves according to the law of motion:

$$D_{t+1} = D_t + \epsilon_{t+1} e_t \tag{6}$$

with D_0 given. The accumulation of D_t is subject to an ***i.i.d*** shock $\epsilon \in \{\underline{\epsilon}, 1\}$, where

$$Pr(\epsilon = \underline{\epsilon}) = Pr(\epsilon = 1) = \frac{1}{2} \tag{7}$$

and $\underline{\epsilon} < 1$. Crucially, the realization of ϵ is only observed by the government. In period t, the shock ϵ_t determines the level of D_t and therefore the distribution of y_t. ϵ must therefore be realized at the beginning of the period, before the remaining uncertainty is resolved.

If the government makes more reform effort over time, resulting in greater reform progress D_t, the distribution of output will change so that high output realizations are relatively more likely.[1]

I now introduce some additional notation which will be useful in later sections. Let the current state be noted by $h_t = (y_t, D_t)$ and use h^t to denote the history $(h_0, h_1, ..., h_t)$. Then the conditional probability of the transition from state h_t to h_{t+1} is

$$\begin{aligned}\tilde{\Pi}(h_{t+1} \mid h_t) &= \tilde{\Pi}(y_{t+1}, D_{t+1} \mid y_t, D_t) \\ &= \frac{1}{2}\tilde{\Pi}(y_{t+1} \mid y_t, D_t + e_t) + \frac{1}{2}\tilde{\Pi}(y_{t+1} \mid y_t, D_t + \underline{\epsilon} e_t)\end{aligned} \tag{8}$$

and the probability of the history h^{t+k} conditional on the current state h_t is

$$\tilde{\Pi}(h^{t+k} \mid h_t) = \tilde{\Pi}(h_{t+k} \mid h_{t+k-1})\tilde{\Pi}(h_{t+k-1} \mid h_{t+k-2}) \cdots \tilde{\Pi}(h_{t+1} \mid h_t) \tag{9}$$

Borrowing and Lending The government is able to borrow the output good from the lender, subject

[1]We should also note that this structure induces a positive covariance between y_t and ϵ_t, since a high realization of ϵ increases the probability of a high y. It can be shown that this covariance decreases as the stock D increases, due to the concavity of the weighting function $w(D)$.

to a commitment to make certain repayments to the lender later. At this point, I state that the **net** payment *from* the borrower *to* the lender required at time t is denoted by τ_t. This gives us the resource constraint

$$c_t + \tau_t = y_t \tag{10}$$

Information and Enforceability As stated earlier, the shock ϵ_t is only observed by the government. I also state here that the effort of the government is not observable; consequently it would not be possible to enforce a contract based directly on effort levels. However, in period t once the shock ϵ_t has been realized, I allow the reform progress D_t to be observed by all agents. Thus, the observed reform progress acts as a noisy signal of the amount of effort exerted by the government in the previous period. The most important consequence of the inability to contract directly on effort is that the effort levels stipulated in any agreement must be incentive compatible from the perspective of the borrower. This will emerge in more detail in the solution of the optimal contract.

2 Efficient Risk Sharing

In this section I solve for the efficient allocation in the environment described above, which allows the borrower to share the risk resulting from the realization of y_t with the lender. First, I will characterize the first best allocation. Then, in order to set up the full risk sharing problem, I must introduce some additional constraints on the agreement between the borrower and the lender. The purpose of these constraints is to capture the possibility that the borrower may prefer to incur a cost to repudiate its debt obligations rather than making repayments to the lender. They also limit the size of transfers from the lender to the borrower, so that the agreement is individually rational for the lender. In addition, they address the moral hazard problem resulting from the assumption that the structural reform policies are non-contractible.

2.1 First Best Allocation

The first best solves the social planner's problem for a given relative borrower Pareto weight z, with no incentive compatibility or limited enforcement constraints.

Proposition 1 The first best allocation features constant consumption for the borrower. In addition, borrower effort varies with the realization of output.

Proof Let $J(h_t)$ denote the value of the problem for the social planner. We can then write the problem as:

$$J(h_t) = \max_{\{c_t, e_t\}} \left[z\big(u(c_t) - \nu(e_t) + \beta \mathbb{E}_t V_B(h_{t+1})\big) + y_t - c_t + \frac{1}{R} \mathbb{E}_t V_L(h_{t+1}) \right] \tag{11}$$

subject to

$$D_{t+1} = D_t + \epsilon_{t+1} e_t \tag{12}$$

and

$$D_{t+1} \leq 1 \tag{13}$$

where

$$V_B(h_t) = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k [u(c(h_{t+k})) - \nu(e(h_{t+k}))]$$

and

$$V_L(h_t) = \mathbb{E}_t \sum_{k=0}^{\infty} \left(\frac{1}{R}\right)^k (y_{t+k} - c(h_{t+k}))$$

The first order condition for consumption is:

$$u_c(c_t) = \frac{1}{z} \tag{14}$$

Here we see clearly that for a given relative Pareto weight z, consumption is constant. In the first best we therefore achieve full risk sharing.

The first order condition for the effort stock D_{t+1} can be written as:

$$z\nu'(e_t) = z\beta \frac{d\mathbb{E} V_B(h_{t+1})}{dD_{t+1}} + \frac{1}{R} \frac{d\mathbb{E} V_L(h_{t+1})}{dD_{t+1}} \tag{15}$$

using the envelope theorem and the definition of the reform technology, we can write:

$$\frac{d\mathbb{E}_t V_B(h_{t+1})}{dD_{t+1}} = \frac{1}{2} \sum_{h_{t+1}|h_t} \epsilon_{t+1} w_D(D_{t+1}) \Delta_{\Pi}(y_{t+1} \mid y_t) V_B(h_{t+1}) + \frac{1}{4} \frac{\underline{\epsilon} + 1}{\underline{\epsilon}} \sum_{h_{t+1}|h_t} \tilde{\Pi}(y_{t+1} \mid y_t, D_{t+1}) \epsilon_{t+1} \nu'(e_t) \tag{16}$$

and

$$\frac{d\mathbb{E}_t V_L(h_{t+1})}{dD_{t+1}} = \frac{1}{2} \sum_{h_{t+1}|h_t} \epsilon_{t+1} w_D(D_{t+1}) \Delta_\Pi(y_{t+1} \mid y_t) V_L(h_{t+1}) \tag{17}$$

Substituting these expressions into the first order condition and rearranging, we can then write:

$$\nu'(e_t) - \frac{\beta}{4}\frac{\epsilon+1}{\epsilon}\mathbb{E}_t\epsilon_{t+1}\nu'(e_{t+1}) + \mu_t - \frac{\beta}{4}\frac{\epsilon+1}{\epsilon}\mathbb{E}_t\epsilon_{t+1}\mu_{t+1} = \tag{18}$$
$$\frac{1}{2}\sum_{h_{t+1}|h_t} \epsilon_{t+1} w_D(D_{t+1}) \Delta_\Pi(y_{t+1} \mid y_t)[\beta V_B(h_{t+1}) + \frac{1}{zR} V_L(h_{t+1})]$$

where μ_t is the Lagrange multiplier on the constraint $D_{t+1} \leq 1$. The dependence of effort on the current realization of output y_t is clear from the presence of the conditional probabilities. ■

2.2 Constrained Efficient Allocation

I now introduce the two constraints which characterize the economic union's risk sharing problem before presenting the constrained efficient allocation, which maximizes union welfare subject to these constraints.

Limited Commitment I assume that both the borrower and the lender have an outside option which provides a lower bound on the expected future utility which they must receive to remain in the risk sharing contract. In the case of indifference between honouring the agreement and leaving, I assume that both agents choose to remain in the contract.

For the borrower, the outside option involves refusing to honour any future payments to the creditor and, as a punishment, being excluded from borrowing. For simplicity, I assume that this exclusion is permanent, although it is common in the sovereign debt literature to have the borrower be forgiven each period with a certain probability; this alternative assumption would not change the results much, but it would raise the value of the outside option and make the borrower's participation constraint tighter.

Following the sovereign default literature, I also assume that when the government is excluded from borrowing it suffers an output cost $\sigma(y_t)$ in every period [2]. This is required to match the stylized fact that governments subject to default risk tend to accumulate debt when growth is high and default during recessions [3]. I use the same output cost function as Arellano (2008):

[2] We can think of these output costs as capturing a breakdown in domestic financial markets following a default episode, or the loss of access to crucial important production imputs. This second possibility is modelled explicitly in Mendoza and Yue (2012)

[3] This fact is not universally accepted as there is some evidence that defaults during periods of high output growth are

$$\sigma(y_t) = \max\{0, y_t - \Omega\mathbb{E}y_t\} \tag{19}$$

where $\Omega > 0$ is a constant. Thus for low enough realizations of output, the default cost may be zero, whereas for high realizations the cost will be positive; as a result, the government will find it more attractive to revert to autarky when the endowment is low.

The participation constraint for the borrower then takes the form:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \nu(e_t)] \geq X(h_t) \tag{20}$$

where the value of exclusion $X(h_t)$ is given by:

$$X(h_t) = \max_{e_t} u(y_t - \sigma(y_t)) - \nu(e_t) + \beta\mathbb{E}_t X(h_{t+1}) \tag{21}$$

Note that during exclusion, the borrower still has the option of exerting effort e_t to improve the distribution of output, although the return from doing so will be diminished by the presence of the default cost. This is the only choice the borrower can make while it is excluded.

I also assume that the risk neutral lender has an alternative investment opportunity with a known net present value Z. The net payments offered by the risk sharing contract must therefore offer a minimal expected net present value.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{R}\right)^t \tau_t \geq Z \tag{22}$$

For the rest of the paper I will assume that $Z = 0$, so that the sequence of transfers $\{\tau_t\}_{t=0}^{\infty}$ must at least allow the lender to break even in net present value. The requirement that the lender break even is quite strict. We will see later that when we decentralize the contract the asset position assigned to the lender will almost never be negative, due to the difference in the discount rates of the two countries. For the interpretation of the results, it may sometimes be helpful to have the lender tolerate some losses.[4].

Incentive Compatibility Given that structural reform effort undertaken by the government is not enforceable, any level of effort specified in the allocation must be incentive compatible for the borrower. Due to the MLRP assumption on the underlying distributions of y, and the concavity which I have

relatively common. Tomz and Wright (2007) examines a dataset to study the relationship between output and sovereign default over the period 1820-2004 and find a negative but relatively weak association between the borrowing country's output and default.

[4]This could reflect 'solidarity' with the borrower or the presence of a collateral cost of default imposed by the borrower on the lender (Tirole, 2015)

incorporated into the reform technology, the first order approach is justified, so that I can replace the full effort decision of the borrower with its first order condition. A necessary condition for the optimal effort choice of the borrower is given by:

$$v_e(e_t)\begin{cases} > \frac{\beta}{4}\frac{\epsilon+1}{\epsilon}\mathbb{E}_t\epsilon_{t+1}v_e(e_{t+1}) + \frac{\beta}{2}\sum_{h_{t+1}|h_t}\epsilon_{t+1}w_D(D_{t+1})\Delta_\Pi(y_{t+1}\mid y_t)V_B(h_{t+1}) & \text{if } e_t = 0 \\ < \frac{\beta}{4}\frac{\epsilon+1}{\epsilon}\mathbb{E}_t\epsilon_{t+1}v_e(e_{t+1}) + \frac{\beta}{2}\sum_{h_{t+1}|h_t}\epsilon_{t+1}w_D(D_{t+1})\Delta_\Pi(y_{t+1}\mid y_t)V_B(h_{t+1}) & \text{if } e_t = 1 \\ = \frac{\beta}{4}\frac{\epsilon+1}{\epsilon}\mathbb{E}_t\epsilon_{t+1}v_e(e_{t+1}) + \frac{\beta}{2}\sum_{h_{t+1}|h_t}\epsilon_{t+1}w_D(D_{t+1})\Delta_\Pi(y_{t+1}\mid y_t)V_B(h_{t+1}) & \text{if } e_t \in (0,1) \end{cases} \tag{23}$$

which holds with equality when $D < 1$. $V_B(h_{t+1})$ is the expected discounted utility of the borrower from period $t+1$ onwards. Incentive compatibility balances the utility cost for the borrower with the benefit of higher future consumption resulting from the improved output distribution. This benefit also includes the reduction in future effort made possible by increased current effort, which is greater when ω is higher. On the other hand, the concavity of $w(.)$ ensures that the return to effort decreases as the stock D increases.

Definition of Efficient Allocation A feasible allocation is said to be ***efficient*** if for some Pareto weights $\mu_{B,0}$ and $\mu_{L,0}$, for the government and the lender respectively, the allocation maximizes the welfare function $J(.)$, defined as:

$$J(h_0) = \mu_{B,0}\left(\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(u(c_t)-\nu(e_t))\right) + \mu_{L,0}\left(\mathbb{E}_0\sum_{t=0}^{\infty}\frac{1}{R^t}\tau_t\right)$$

subject to the resource constraint, incentive compatibility for the borrower, and the participation constraints for the borrower and the lender.

2.3 Characterization of Efficient Allocation

I now characterize the efficient risk sharing agreement more fully by deriving it as the solution of the problem of a social planner with an objective function given by $J(.)$. The social planner's problem can be written as:

$$\max_{c_t,e_t}\mu_{B,0}\left(\sum_{t=0}^{\infty}\sum_{y_t|y_0}\beta^t\tilde{\Pi}(h^t\mid h_0)(u(c(h_t))-v(e(h_t)))\right) + \mu_{L,0}\left(\sum_{t=0}^{\infty}\sum_{y_t|y_0,D_1}\left(\frac{1}{R}\right)^t\tilde{\Pi}(h^t\mid h_0)\tau(h_t)\right)$$

s.t

$$\sum_{k=0}^{\infty}\sum_{h^{t+k}|h_t}\beta^k\tilde{\Pi}(h^{t+k}\mid h_t)[u(c(h_{t+k}))-v(e(h_{t+k}))] \geq X(h_t)$$

$$v_e(e(h_t)) = \frac{\beta}{4}\frac{\epsilon+1}{\underline{\epsilon}} \sum_{h_{t+1}|h_t} \tilde{\Pi}(y_{t+1} \mid y_t)\epsilon_{t+1}v_e(e(h_{t+1})) + \frac{\beta}{2} \sum_{h_{t+1}|h_t} \epsilon_{t+1}w_D(D_{t+1})\Delta_{\Pi}(y_{t+1} \mid y_t)V_B(h_{t+1})$$

$$\sum_{k=0}^{\infty} \sum_{h^{t+k}|h_t} \left(\frac{1}{R}\right)^k \tilde{\Pi}(h^{t+k} \mid h_t)\tau(h_{t+k}) \geq 0$$

$$D_{t+1} = D_t + \epsilon_{t+1}e_t$$

$$\tau(h_t) = y_t - c(h_t)$$

By using the last two constraints to substitute for the allocation variables I can rewrite this as:

$$\max_{c_t, e_t} \mu_{B,0}\Bigg(u(c(h_t)) - v(e_t) + \beta \sum_{h_{t+1}|h_t} \tilde{\Pi}(h_{t+1} \mid h_t)V_B(h_{t+1})\Bigg) +$$
$$\mu_{L,0}\Bigg(y_t - c(h_t) + \frac{1}{R} \sum_{h_{t+1}|h_t} \tilde{\Pi}(h_{t+1} \mid h_t)V_L(h_{t+1})\Bigg)$$

s.t

$$u(c(h_t)) - v(e_t) + \beta \sum_{h_{t+1}|h_t} \tilde{\Pi}(h_{t+1} \mid h_t)V_B(h_{t+1}) \geq X(y_t, D_t)$$

$$v_e(e_t) = \frac{\beta}{4}\frac{\epsilon+1}{\underline{\epsilon}} \sum_{h_{t+1}|h_t} \tilde{\Pi}(y_{t+1} \mid y_t)\epsilon_{t+1}v_e(e_{t+1}) + \frac{\beta}{2} \sum_{h_{t+1}|h_t} \epsilon_{t+1}w_D(D_{t+1})\Delta_{\Pi}(y_{t+1} \mid y_t)V_B(h_{t+1})$$

$$y_t - c(h_t) + \frac{1}{R} \sum_{h_{t+1}|h_t} \tilde{\Pi}(h_{t+1} \mid h_t)V_L(h_{t+1}) \geq 0$$

This is a two-sided limited enforcement problem with forward-looking constraints as studied in Marcet and Marimon (2019) and Mele (2014). In Appendix A, I show how the Lagrangean version of the problem leads to the multiplier approach. The forward-looking constraints prevent the use of recursive methods to solve the problem, since one of the requirements for deriving the Bellman equation is that the feasible set is constrained only by h_t; here the set is also constrained by future choices $\{D_{t+k}\}_{k=0}^{\infty}$. However, using the results from the papers mentioned above, the social planner's problem can be rewritten as the following saddle point problem:

$$\mathcal{SP} \min_{\gamma_{B,t},\gamma_{L,t},\xi_t} \max_{c_t,e_t} \Bigg[\sum_{t=0}^{\infty} \sum_{h_t|h_0} \tilde{\Pi}(h^t \mid h_0)\beta^t \Bigg(\mu_{B,t}(u(c_t) - \nu(e_t)) - \xi_t(v_e(e_t) - \frac{\beta}{4}\frac{\underline{\epsilon}+1}{\underline{\epsilon}} \sum_{h_{t+1}|h_t} \tilde{\Pi}(y_{t+1} \mid y_t)\epsilon_{t+1} v_e(e(h_{t+1}))$$

$$+ \gamma_{B,t}\big[u(c_t) - \nu(e_t) - X(h_t)\big]\Bigg) + \sum_{t=0}^{\infty} \sum_{h_t|h_0} \tilde{\Pi}(h^t \mid h_0) \left(\frac{1}{R}\right)^t \big(\mu_{L,t+1}(y_t - c_t)\big)\Bigg]$$

$$\mu_{B,t+1} = \mu_{B,t} + \gamma_{B,t} + \xi_t w_D(D_{t+1})\frac{\underline{\epsilon}+1}{2}\frac{\Delta_\Pi(y_{t+1} \mid y_t)}{\tilde{\Pi}(y_{t+1} \mid y_t, D_{t+1})}$$

$$\mu_{L,t+1} = \mu_{L,t} + \gamma_{L,t}$$

with $\mu_{B,0}$ and $\mu_{L,0}$ given. The advantage of this representation is that the problem is now recursive; in addition to state variables y_t and D_t, we now have the time-varying multipliers $\mu_{B,t}$ and $\mu_{L,t}$ as co-state variables. These multipliers have a natural interpretation. In any period in which the participation constraint for one of the agents binds, the relative weight given to that agent by the social planner must be increased [5]. In addition, the borrower must be compensated for applying structural reform effort; this compensation must be state contingent in order to respect incentive compatibility.[6] The importance of this incentive component is increasing in $\underline{\epsilon}$, which acts as an index of the informativeness of D_t.

We can reduce the dimension of the state space using the following normalization:

$$\nu_{i,t} = \frac{\gamma_{i,t}}{\mu_{i,t}}$$

$$\tilde{\xi}_t = \frac{\xi_t}{\mu_{B,t}}$$

$$\phi(h_{t+1} \mid h_t) = \tilde{\xi}_t w_D(D_{t+1})\frac{\underline{\epsilon}+1}{2}\frac{\Delta_\Pi(y_{t+1} \mid y_t)}{\tilde{\Pi}(y_{t+1} \mid y_t, D_{t+1})}$$

$$z_t = \frac{\mu_{B,t}}{\mu_{L,t}}$$

$$z_{t+1} = \frac{1 + \nu_{b,t} + \phi(h_{t+1} \mid h_t)}{1 + \nu_{l,t}}\eta z_t$$

[5] It is not possible for the participation constraints of both agents to bind simultaneously. Since one of the agents is risk averse and the choice set of both agents inside the contract is at least as great as outside the contract, the risk sharing provided by the contract must create some surplus.

[6] This is reflected in the dependence of $\mu_{B,t+1}$ on the endowment realization y_{t+1} and the realized value of D_{t+1}, through the likelihood ratio term $\frac{\Delta_\Pi(y_{t+1}|y_t)}{\tilde{\Pi}(y_{t+1}|y_t)}$. As a result, the updated multiplier $\mu_{B,t+1}$ is a N×2 matrix. It is also important to note the timing of the updating of the multiplier $\mu_{B,t+1}$. The relevant period for the contribution of the participation constraint is period t, whereas the relevant period for the incentive component is $t+1$, where the realization of h_{t+1} is used to judge whether effort was made in period t.

where $\eta = \beta R$.

With this normalization, the co-state variable of the problem is z, the ***relative*** Pareto weight of the borrower in the social planner's problem. The saddle point problem is now characterized by the (saddle point) Bellman equation:

$$V(h,z) = \mathcal{SP} \min_{\{v_b, v_l, \xi\}} \max_{c,e} \Bigg[z \Big[(1+v_b)(u(c(h,z)) - v(e(h,z))) - v_b X(h) -$$

$$\tilde{\xi}(v_e(e(h,z)) - \frac{\beta}{4}\frac{\underline{\epsilon}+1}{\underline{\epsilon}} \sum_{h'|h} \tilde{\Pi}(y' \mid y)\epsilon' v_e(e(h',z'))) \Big] + (1+v_l)(y-c) + \frac{1+v_l}{R}\mathbb{E}V(h',z') \Bigg] \tag{24}$$

where

$$z' = \frac{1 + v_b + \phi(h' \mid h)}{1+v_l}\eta z \tag{25}$$

and

$$\phi(h' \mid h) = \tilde{\xi} w_D(D') \frac{\underline{\epsilon}+1}{2} \frac{\Delta_{\Pi}(y' \mid y)}{\tilde{\Pi}(y' \mid y, D')} \tag{26}$$

Given this formulation of the social planner's problem, the first order condition for consumption yields:

$$u_c(c(h,z)) = \frac{1}{z}\frac{1+\nu_l(h,z)}{1+\nu_b(h,z)} \tag{27}$$

According to this condition, the consumption of the borrower should be increased in periods where the borrower's participation constraints binds; conversely, it should be decreased if the lender's participation constraint is binding.

The first order condition for effort gives the following:

$$(1+\nu_b(h,z))v_e(e(h,z)) + \tilde{\xi}(h,z)\left(v_{ee}(e(h',z')) + \frac{\beta}{4}\frac{\underline{\epsilon}+1}{\underline{\epsilon}} \sum_{h'|h} \tilde{\Pi}(y' \mid y)\frac{\epsilon'}{\epsilon''} v_{ee}(e(h',z')) \right)$$

$$= \frac{1+\nu_l(h,z)}{zR}\left(\sum_{h'|h} \epsilon' w_D(D')\Delta(y' \mid y)V(h',z') + \sum_{h'|h} \tilde{\Pi}(h' \mid h)\epsilon' \frac{\partial V(h',z')}{\partial D'} + \sum_{h'|h} \epsilon' \tilde{\Pi}(h' \mid h) V_B(h',z') \frac{dz'}{dD'} \right)$$

Increasing effort imposes a utility cost on the borrower but also tightens the incentive compatibility constraint for the social planner. This brings benefits to both the borrower and the lender in the form of a greater likelihood of high output realizations and lower required reform effort in the future, as well as an additional reward for the borrower if the realization of the state in the next period indicates high reform effort. After some manipulation, this becomes

$$v_e(e(h,z))(1+\nu_b(h,z)) + \tilde{\xi}(h,z)\left(v_{ee}(e(h',z')) + \frac{\beta}{4}\left(\frac{\epsilon+1}{\underline{\epsilon}}\right)^2 \sum_{h'|h} \tilde{\Pi}(y' \mid y)\epsilon'^2 v_{ee}(e(h',z'))\right)$$
$$= \frac{1+\nu_l(h,z)}{zR}\frac{\epsilon+1}{\underline{\epsilon}}\sum_{h'|h}\epsilon' w_D(D')\Delta(y' \mid y)\left(z'V_B(h',z') + V_L(h',z')\right)$$
$$+ \frac{1+\nu_l(h,z)}{zR}\frac{\epsilon+1}{4\underline{\epsilon}}\sum_{h'|h}\Pi(\tilde{y'} \mid y)z'\left((1+\nu_b(h',z'))v_e(e(h',z'))\right.$$
$$\left.-\nu_b(h',z')v_e(e_A(h',z')) + \epsilon'\tilde{\xi}(h',z')v_{ee}(e(h',z'))\right)$$
$$+ \frac{1+\nu_l(h,z)}{2zR}\sum_{h'|h}\tilde{\Pi}(y' \mid y)\frac{\epsilon'\eta z\tilde{\xi}(h,z)}{1+\nu_l(h,z)}\frac{\epsilon+1}{2}\left[w_{DD}(D')\frac{\Delta_\Pi(y' \mid y)}{\tilde{\Pi}(y' \mid y, D')} - w_D(D')^2\frac{\Delta_\Pi(y' \mid y)}{\tilde{\Pi}(y' \mid y, D')^2}\right]V_B(h',z')$$

Then, using the incentive compatibility condition, we can rewrite this as:

$$\frac{\beta(1+\nu_b(h,z))}{4}\frac{\epsilon+1}{\underline{\epsilon}}\sum_{h'|h}\tilde{\Pi}(y' \mid y)\epsilon' v_e(e(h',z')) + \tilde{\xi}(h,z)\left(v_{ee}(e(h',z')) + \frac{\beta}{4}\left(\frac{\epsilon+1}{\underline{\epsilon}}\right)^2\sum_{h'|h}\tilde{\Pi}(y' \mid y)\epsilon'^2 v_{ee}(e(h',z'))\right)$$
$$= \beta\sum_{h'|h}\epsilon' w_D(D')\Delta(y' \mid y)\phi(h' \mid h)V_B(h',z') + \frac{1+\nu_l(h,z)}{zR}\sum_{h'|h}\tilde{\Pi}(y' \mid y)\epsilon' w_D(D')\Delta(y' \mid y)V_L(h',z')$$
$$+ \frac{1+\nu_l(h,z)}{zR}\frac{\epsilon+1}{4\underline{\epsilon}}\sum_{h'|h}\Pi(\tilde{y'} \mid y)z'\left((1+\nu_b(h',z'))v_e(e(h',z'))\right.$$
$$\left.-\nu_b(h',z')v_e(e_A(h',z')) + \epsilon'\tilde{\xi}(h',z')v_{ee}(e(h',z'))\right)$$
$$+ \frac{1}{2R}\sum_{h'|h}\tilde{\Pi}(y' \mid y)\epsilon'\eta\tilde{\xi}(h,z)\frac{\epsilon+1}{2}\left[w_{DD}(D')\frac{\Delta_\Pi(y' \mid y)}{\tilde{\Pi}(y' \mid y, D')} - w_D(D')^2\frac{\Delta_\Pi(y' \mid y)}{\tilde{\Pi}(y' \mid y, D')^2}\right]V_B(h',z')$$

Finally, using the definitions $\phi(h' \mid h) = \tilde{\xi}w_D(D')\frac{\epsilon+1}{2}\frac{\Delta_\Pi(y'|y)}{\tilde{\Pi}(y'|y,D')}$ and $z' = \frac{1+\nu_b+\phi(h'|h)}{1+\nu_l}\eta z$ we can simplify this to:

$$\frac{\beta(1+\nu_b(h,z))}{4}\frac{\epsilon+1}{\underline{\epsilon}}\sum_{h'|h}\tilde{\Pi}(y' \mid y)\epsilon' v_e(e(h',z') + \tilde{\xi}(h,z)\left(v_{ee}(e(h',z')) + \frac{\beta}{4}\left(\frac{\epsilon+1}{\underline{\epsilon}}\right)^2\sum_{h'|h}\tilde{\Pi}(y' \mid y)\epsilon'^2 v_{ee}(e(h',z'))\right)$$

$$= \frac{1+\nu_l(h,z)}{zR} \sum_{h'|h} \tilde{\Pi}(y' \mid y)\epsilon' w_D(D')\Delta(y' \mid y)V_L(h',z') + \frac{1}{2zR} \sum_{h'|h} \epsilon' \eta z \tilde{\xi}(h,z) \frac{\epsilon+1}{2} w_{DD}(D')\Delta_{\Pi}(y' \mid y)V_B(h',z')$$

$$+ \frac{\beta(\underline{\epsilon}+1)}{4\underline{\epsilon}} \sum_{h'|h} \tilde{\Pi}(y' \mid y) \left(1+\nu_b(h,z)+\phi(h' \mid h)\right) \left((1+\nu_b(h',z'))v_e(e(h',z'))\right.$$

$$\left. -\nu_b(h',z')v_e(e_A(h',z')) + \epsilon' \tilde{\xi}(h',z')v_{ee}(e(h',z'))\right)$$

The effort levels specified in the allocation must therefore satisfy both incentive compatibility and this last condition. While incentive compatibility ensures optimality from the perspective of the borrower, it is this last condition which contains the implicit punishments and rewards which force the borrower to internalize the effects of its effort decisions on the lender. By making higher output more likely, higher reform effort from the borrower increases the surplus to be shared between the agents. We can see this from the presence of the both agents' future values from the contract in the optimal effort condition. However, higher effort is also costly for the lender in the sense that the borrower must be rewarded with higher consumption when high output is realized, corresponding to lower net payments to the lender. We also see that making effort within the contract has an additional effect because, by raising D, it increases the value of the borrower's outside option $V_a(h)$. This accounts for the presence of the term $v_e(e_A(h',z'))$, the marginal disutility of reform effort exerted in autarky. There is also an anticipation effect whereby the possibility that the borrower's participation constraint may bind in the next period affects the choice of effort today, as shown by the appearance of the future multiplier ν_b'.

2.4 Numerical Exercise

I now illustrate some of the properties of the constrained efficient allocation with a simple numerical exercise, solving the model for a given set of parameter values and with a simple shock structure. I focus especially on the behaviour of the reform effort decision and the incentives for reform embedded in the contract.

Functional Forms and Parameter Values For this exercise I specify that output takes one of two values, y_L and y_H. I then use the following functional forms for the effort utility and technology:

$$v(e) = \frac{e^2}{2}$$

and

$$w(D) = 1 - (D-1)^2$$

The initial transition matrix for output Π_b, which corresponds to $D = 0$, is given by:

$$\Pi_b = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

The transition matrix Π_g, which corresponds to $D = 1$, must be chosen so that it dominates Π_b by the monotone ratio likelihood property. For this exercise I use

$$\Pi_g = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$$

Thus, once the reforms have been 'completed', the low output state becomes much less persistent, whereas the transition probabilities from the high output state are unchanged. We can therefore think of this as a reform which shortens recessions. The state space for output is given by:

$$\{y_L, y_H\} = \{0.95, 1.05\}$$

The constrained efficient allocation is solved for using policy function iteration on the first order conditions of the problem. Details of the solution method are given in Appendix B.

Policy functions: **Reform Effort** Figure 1 shows how the reform effort specified by the contract varies as a function of the relative weight z, for the two different realizations of the endowment. The reform progress is held constant at $D = 0$. The most obvious feature of the solution emerges from comparing the two panels, where we see that effort under the low endowment realization is higher than under the high realization. Due to the persistance of the endowment process, high future endowments are more likely when $y = y_H$, and thus the return to reform is relatively low. In addition, by construction the reform technology has no effect on the transition probabilities when $y = y_H$, and so the direct benefit of reform in this state comes from shortening future recessions, which is less urgent when output is high. Next we consider the shape of the schedule for a given realization of y. The black dots indicate the values of z below which the borrower's participation constraint is binding and above which the lender's participation constraint is binding. In these regions, the allocation is constant because it is constrained by the value of the outside option. Outside of these regions, we see that effort first increases with z and then decreases. The initial increase reflects the fact that motivating effort is relatively cheap when z is low and, by the optimality condition for consumption, the marginal utility of consumption is high. Moreover, when z is low, the welfare of the lender is more important to the social planner, and so the positive externality of effort for the lender has a large effect on welfare. However, as z increases, the incentive cost of effort becomes large and the benefit to the lender becomes less important, so that e decreases until the lender's participation constraint combines.[7]

[7]We should also note that if we had full enforcement on the lender's side, the fall in e as z increases would not observed. In this case the incentive cost of effort is still increasing, but since there is no threat of exit by the lender the social planner has more freedom to pay this cost.

In Figure 2 we see that optimal effort is also decreasing in the stock D . This is due to the diminishing returns to reform effort as the reform capital stock increases ($w_{DD}(D) < 0$). The effort schedule also becomes flatter as D increases, and then eventually reaches a point where the level of effort at high z is actually below the level of effort at low z. This highlights some more subtle aspects of the contract. Firstly, once the borrower's participation constraint is slack, the positive externality for the lender is the most principal determinant of the level of effort, and due to the concavity of the reform technology this externality falls very quickly as D increases. Secondly, effort also falls with D when the borrower's participation constraint is binding, but it cannot fall as fast as the rest of the effort function because it is constrained by the effort exerted by the borrower in the outside option.

This last point is more visible in Figure 3, where we compare reform effort in the contract with the first best solution and the outside option. In these figures, effort in autarky appears as a horizontal line because it is not a function of the relative weight z, and we see that when the borrower's participation constraint is binding the contract and autarky effort levels are quite similar. The output cost suffered by the borrower and the lack of borrowing opportunities make the gain from reform low when the borrower is in autarky. For both high and low output, the first best solution delivers much higher effort than the autarkic allocation and the contract, although the difference is smaller for high levels of z. It is worth noting that in the first best, optimal effort is decreasing in the relative weight. Since the first best achieves full risk sharing, the additional benefit of effort (relative to autarky) comes entirely from the externality for the lender, which corresponds to higher expected transfers from the borrower. This welfare benefit decreases smoothly as the relative weight of the lender decreases. Comparing the effort in the contract with the first best also offers another perspective on the behaviour of effort in the contract. We can see that without the incentive and participation constraints, optimal effort when z is low would be much higher. However, when the borrower can threaten to leave the contract (today or *tomorrow*), the fall in consumption which accompanies the fall in z must be partially compensated by lowering e.

Consumption In Figure 4 we see how the borrower's consumption behaves in the contract. In Section 2.1 the first best allocation featured perfect consumption insurance for a given z, which is shown by the dashed red line. We can see that *for a given* z, the contract also achieves full risk sharing when neither of the participation constraints is binding ($\nu_b = \nu_l = 0$); in other words, the borrower's consumption does not move with output for these values of z. In contrast, when one of the participation constraints is binding, there is some comovement between consumption and output. For example, when $z = 0.85$, the borrower's consumption is slightly higher when $y = y_H$. Nevertheless, the variation in consumption is still much smaller than the variation in the endowment, which is shown by the difference between the dashed grey lines in the two panels. Figure 5 shows how these consumption levels are supported by the transfer policy. Positive values of τ represent payments *from* the borrower *to* the lender, and so we see that as z increases the payments from the borrower decrease and eventually become net receipts.

The transfers also correspond to a counteryclical deficit policy for the borrower, in the sense that the borrower must make much higher payments when the endowment is high. An important feature of the transfer policy appears when the borrower's participation constraint binds while the endowment is low. In this case we see that the borrower must make a small payment to the lender. This may be counterintuitive, but we should note that even when $y = y_L$, the borrower should be willing to trade off a small amount of consumption for the benefit of more stable consumption in the future. More importantly, when we consider the dynamic behaviour of the contract, it will be clear that states like this arise from a sequence of low endowment realizations, during which the borrower will have received transfers from the lender; in this case, the borrower is therefore repaying a debt. This interpretation will be explored further in later sections.

Reform incentives We now consider how the dynamic behaviour of the contract provides incentives to complete reform over time and smooth consumption. Figure 6 shows how the borrower's relative weight is updated for different output transitions $y_t \to y_{t+1}$. In the two panels on the left hand side, the current endowment is y_L; we see here that for most values of z, the transition $y_L \to y_H$ is rewarded with an increase in the relative weight so that $z' > z$. This happens because this transition becomes much more likely when the borrower exerts reform effort. Conversely, the transition $y_L \to y_L$ results in a reduction of the relative weight, since this transition is more likely when the borrower exerts little effort. Thus, while we saw earlier that when participation constraints are slack, full risk sharing is achieved for a given z, the incentive constraint alone can force z to move over time in order to deliver the constrained efficient reform effort [8]. If we compare this with the two panels on the right hand side, we see that when $y = y_H$, this system of punishments and rewards is absent, and the two schedules for z' overlap. This results from the assumption that the reform technology has no effect on the transitions from $y = y_H$. A less obvious point is that when the borrower's participation constraint is not binding, the updated weight is always slightly below the current weight, as here z' lies below the dashed line $z' = z$. Consequently, when the incentive features of the contract are inactive, the relative weight of the borrower will gradually fall over time until the participation constraint binds. This follows from the assumption that borrower is more impatient than the lender, and will therefore frontload consumption unless this is prevented by one of the constraints. Finally, by comparing the top panels with the bottom panels of Figure 6, we can see that the incentive component of the contract becomes weaker as D increases, since z' varies less with y'. This is a result of the concavity of the reform technology.

Dynamics In order to build intutiion about the policies in the contract, I now describe how the policies respond to some simple sequences of endowment realizations. I therefore endowment series lasting 40 periods, starting with equal weighting for the borrower and the lender so that $z = 1$, and

[8] The fact that the incentive constraint interrupts consumption smoothing even when the participation constraints are slack is an important consequence of allowing reform effort to have a permanent effect, due to its accumulation in the stock D. For example, this feature does not appear in Ábrahám et al. (2019), where effort has a transient effect on the distribution of the endowment

reform progress at zero.

1. **Perpetual low output** In this case, illustrated in Figure 7, the borrower receives the low endowment in every period, which allows us to see how the contract behaves in the worst case scenario. Given the initial level of z, the country starts by receiving transfers to smooth consumption. After period five, however, it loses access to transfers, and has to make very small and decreasing positive repayments every period after that. This happens because the low output realizations are taken as signals of low effort from the borrower, and as a punishment the borrower's relative weight (and consumption) are reduced. In the long term the weight slowly increases again as the borrower completes the reform in an attempt to increase the probability of exiting the recession.

2. **Perpetual high output** Next, in Figure 8 we consider the opposite exercise and allow the borrower to receive the high endowment in every period. A striking feature of this exercise is the fall in the borrower's relative weight in the initial periods. Since high output realizations are less informative about the borrower's effort, the incentive component of the contract is inactive. As a result, the fall in the relative weight is driven by the borrower's impatience, which equates to a preference for frontloaded consumption. Eventually, the weight falls to a point where the participation constraint is binding and consumption is constant thereafter. The level of consumption is higher than in the perpetual low output case, which in the long term reflects the fact that the borrower's option to exit is more attractive when the endowment is high. We also see that overall reform progress is lower than in the previous exercise.

3. **Ten period cycles** In Figures 9 and 10 we subject the borrower to alternating sequences of high and low endowments, each lasting for five periods; in Figure 9 the exercise begins with five periods of the low endowment whereas Figure 10 begins with the low endowment. As we would expect, the policy responses to these cycles combine features of the two extreme cases considered above. When the cycle begins with the low endowment, the borrower receives transfers for a few periods before having to make a small repayment, after which the endowment increases and the borrower becomes a large net payer. Alternatively, if the cycle begins with the high endowment, then the borrower is always a net payer, but the payments become very small when the endowment is low. In both cases, the relative weight (and consumption) falls in the intial periods, although in the first case this is driven by reform incentives and in the second by borrower impatience. After this the weight rises and falls with the endowment, as transitions from the low to the high endowment indicate high reform effort by the borrower. Despite the different paths taken by the economy in these two exercises, the resulting level of reform progress is almost identical.

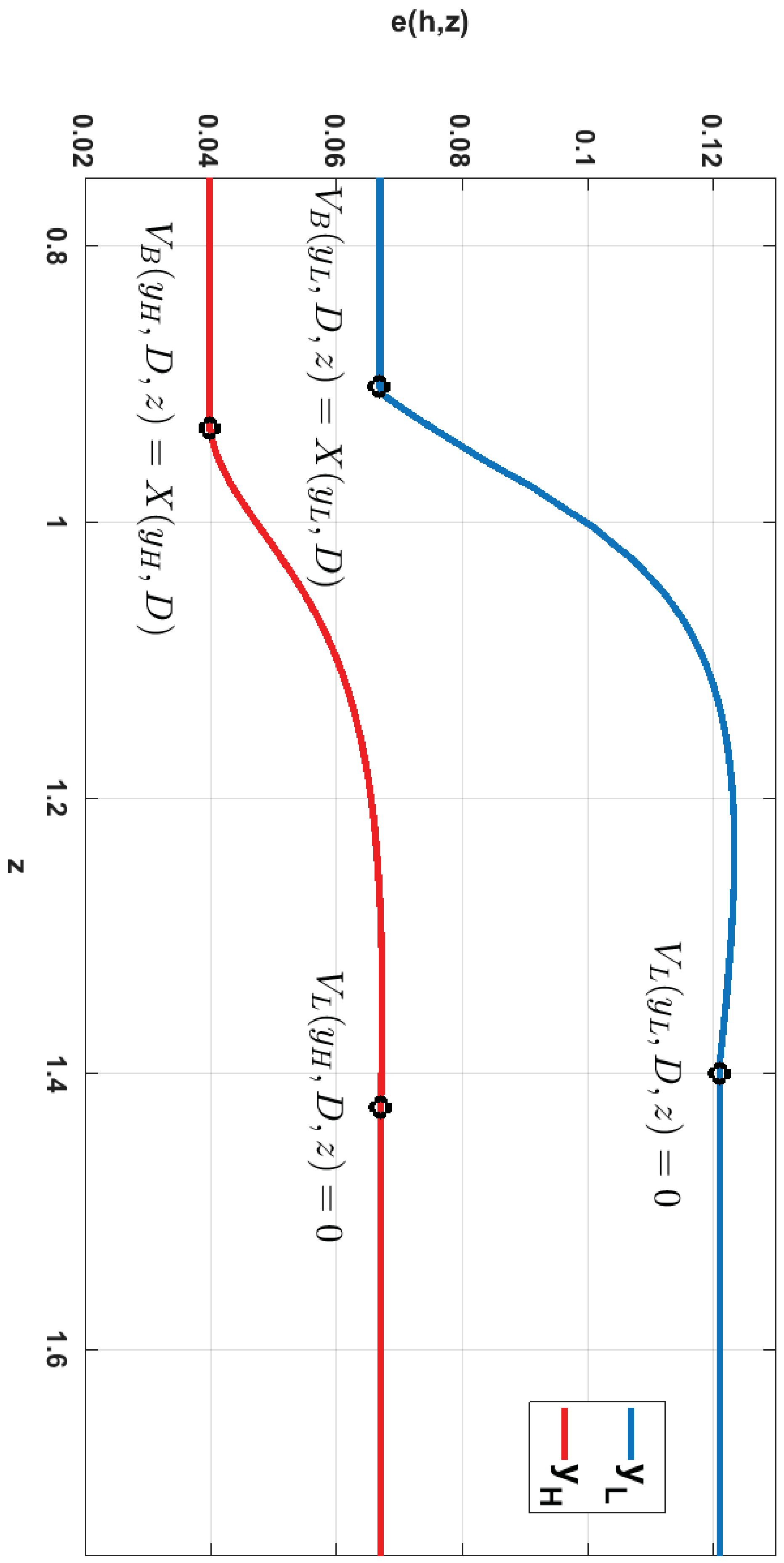

Figure 1: Optimal effort for different values of the relative borrower weight z. Low y vs High y

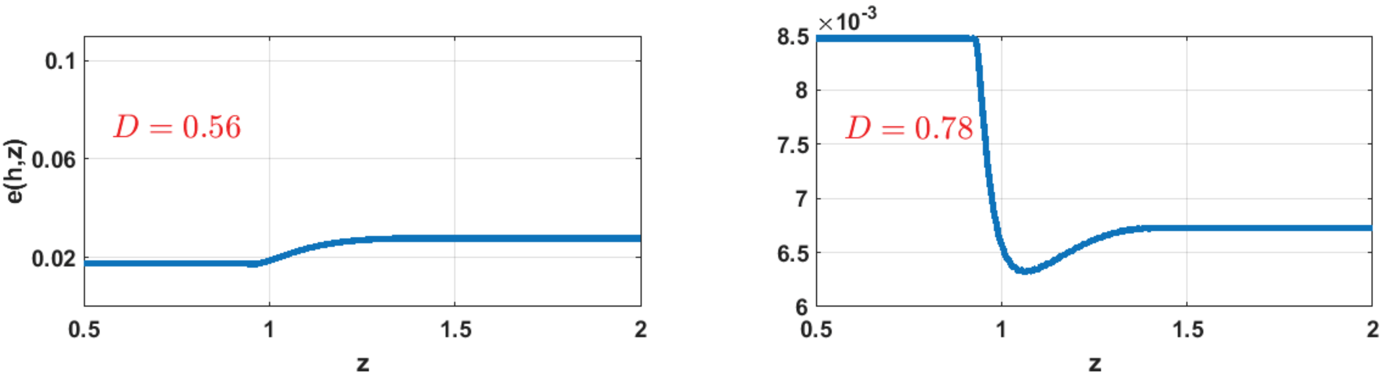

Figure 2: Optimal effort for different values of the reform progress D.

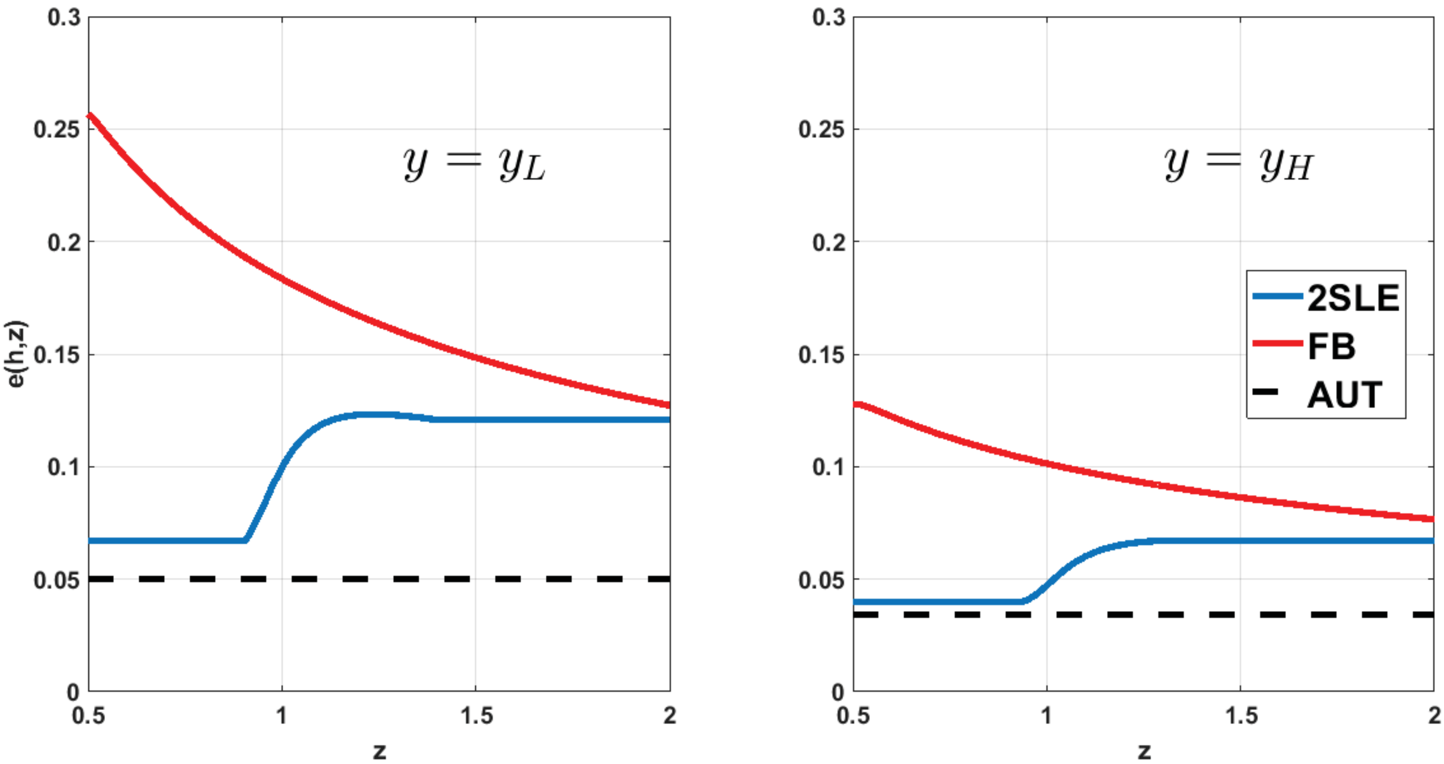

Figure 3: Comparison of the optimal effort choices in the two-sided limited enforcement (2SLE), first best (FB) and **autarky** (AUT) cases.

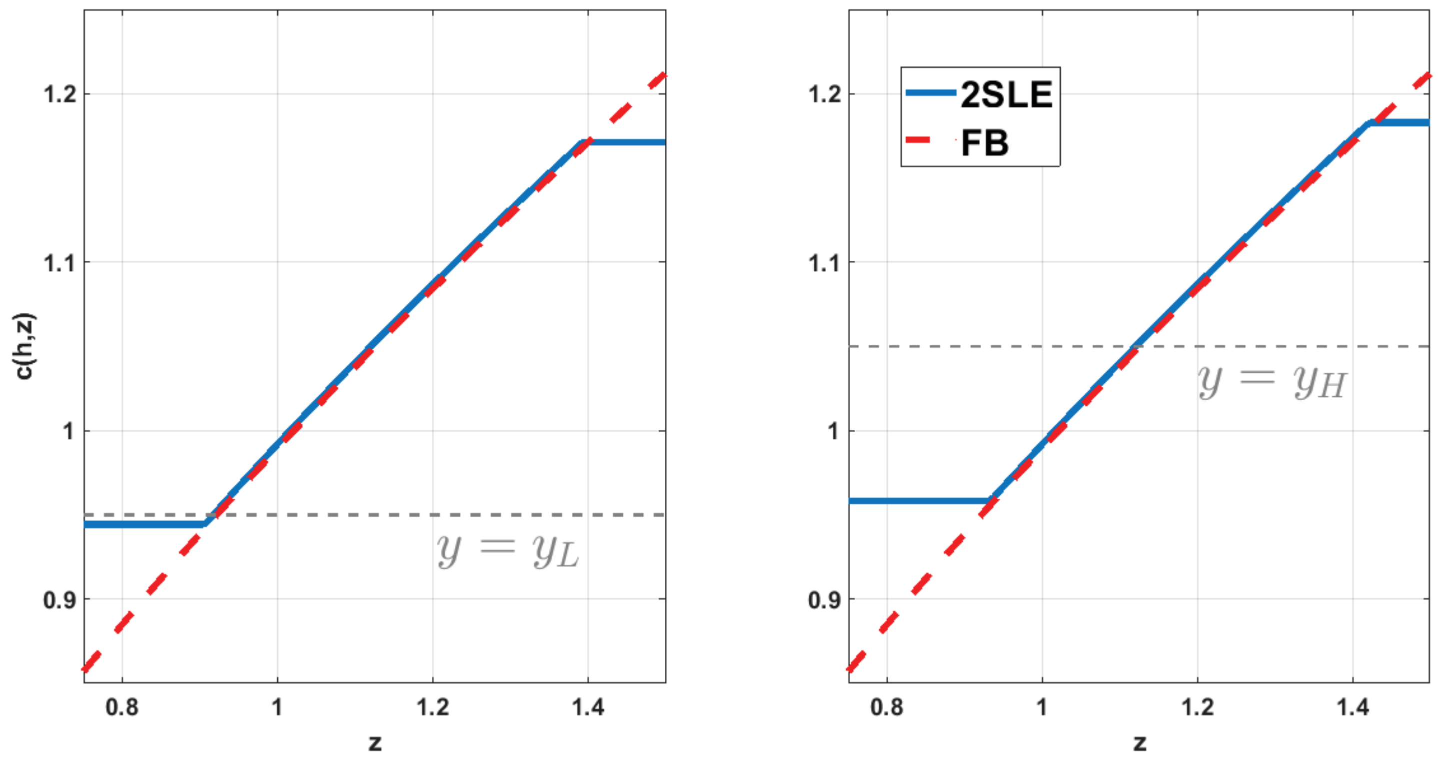

Figure 4: Constrained efficient consumption as a function of the relative weight z

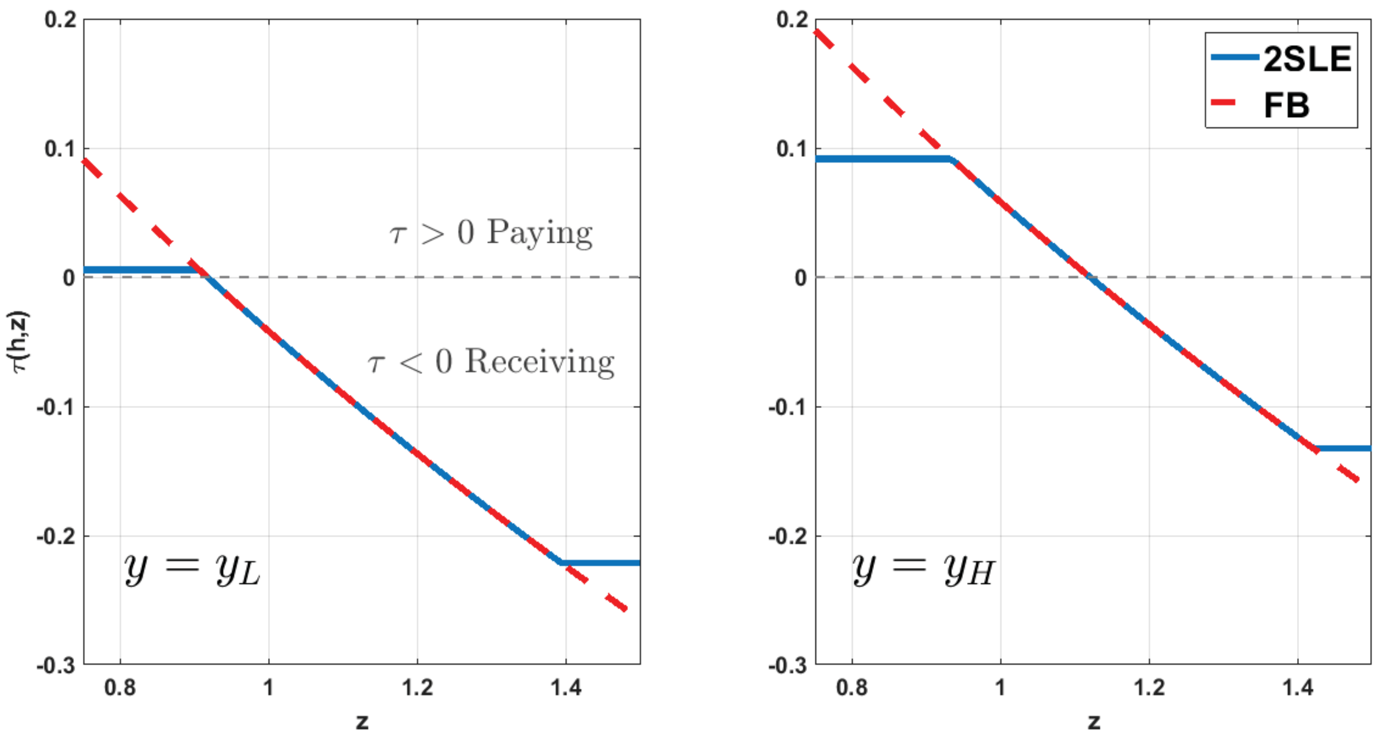

Figure 5: Transfer payments from the borrower to the lender as a function of the relative weight z

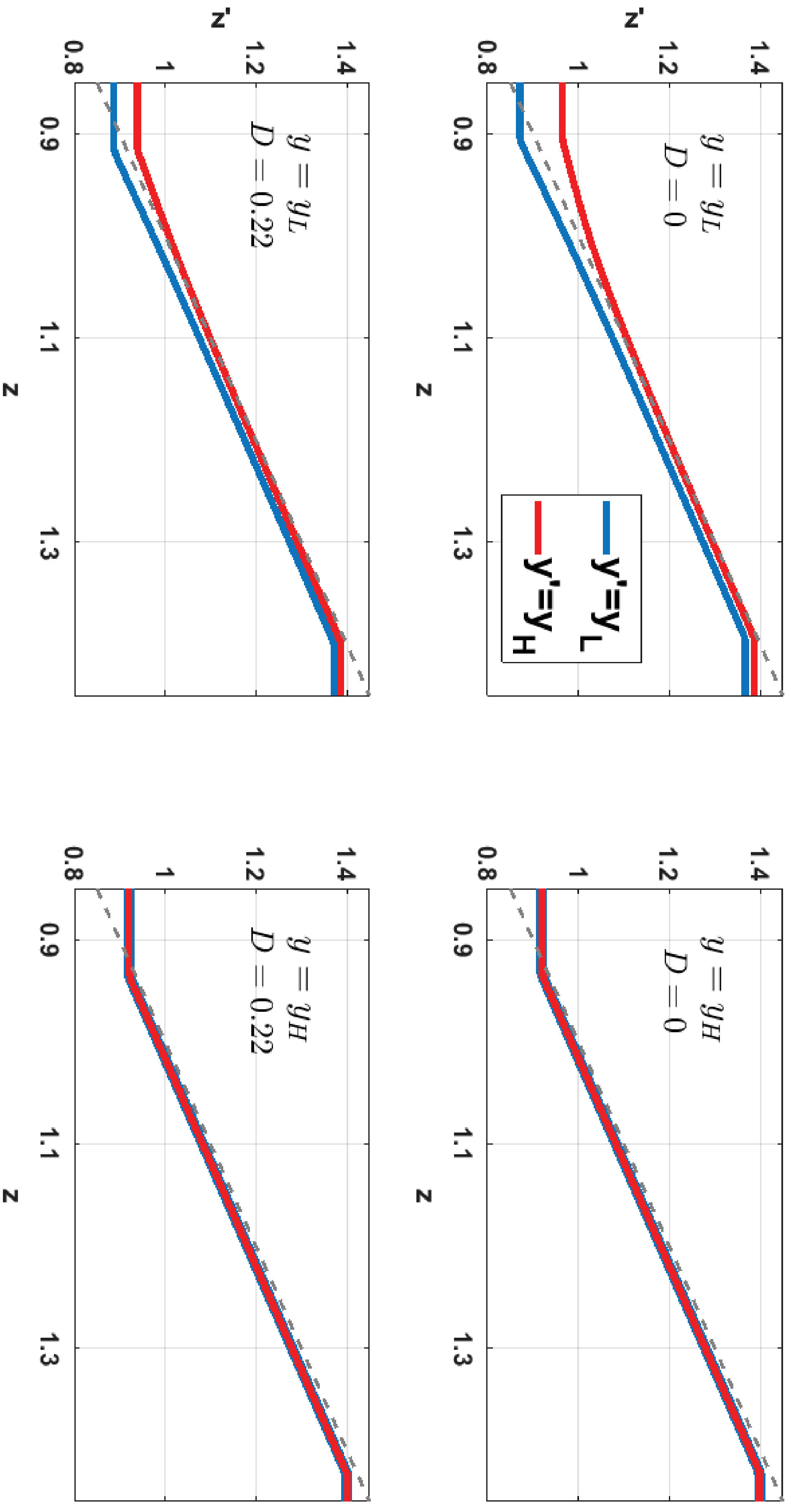

Figure 6: Updating rules for the relative weight z

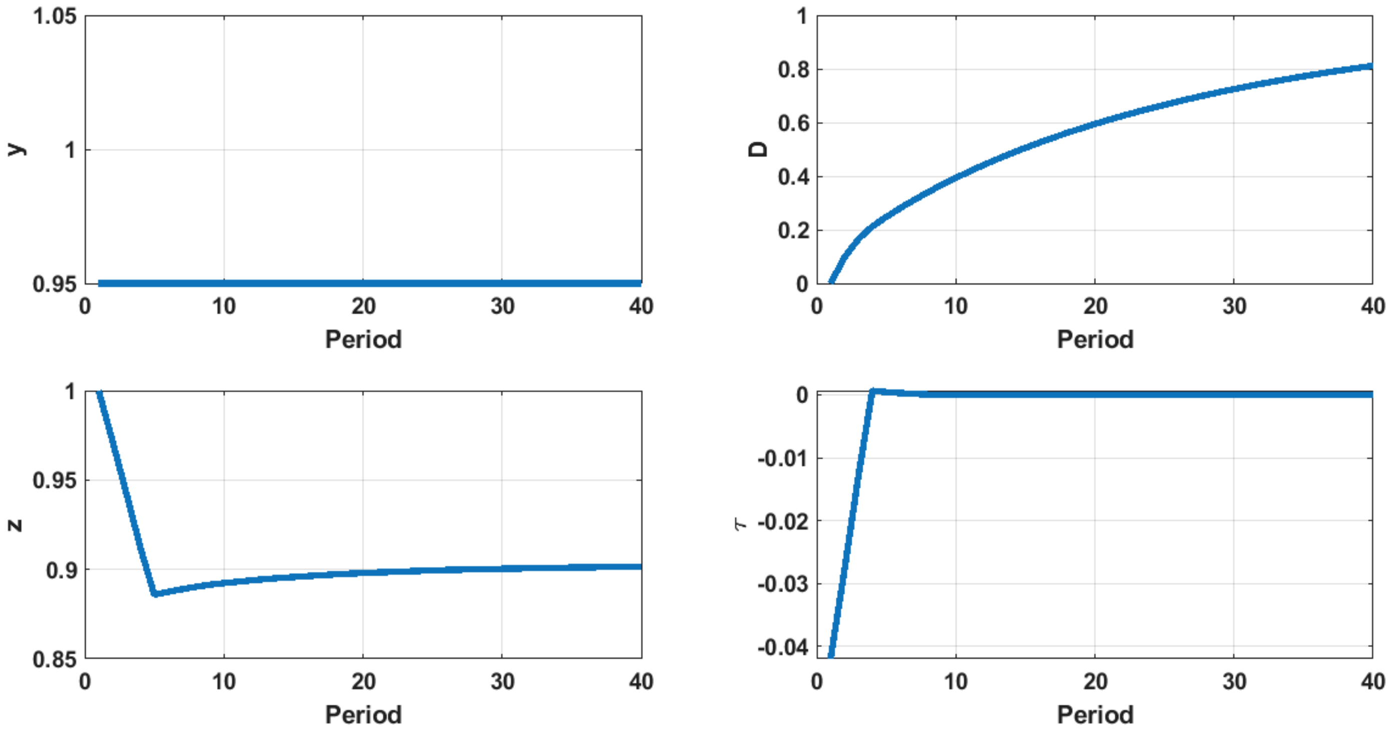

Figure 7: Perpetual low endowment

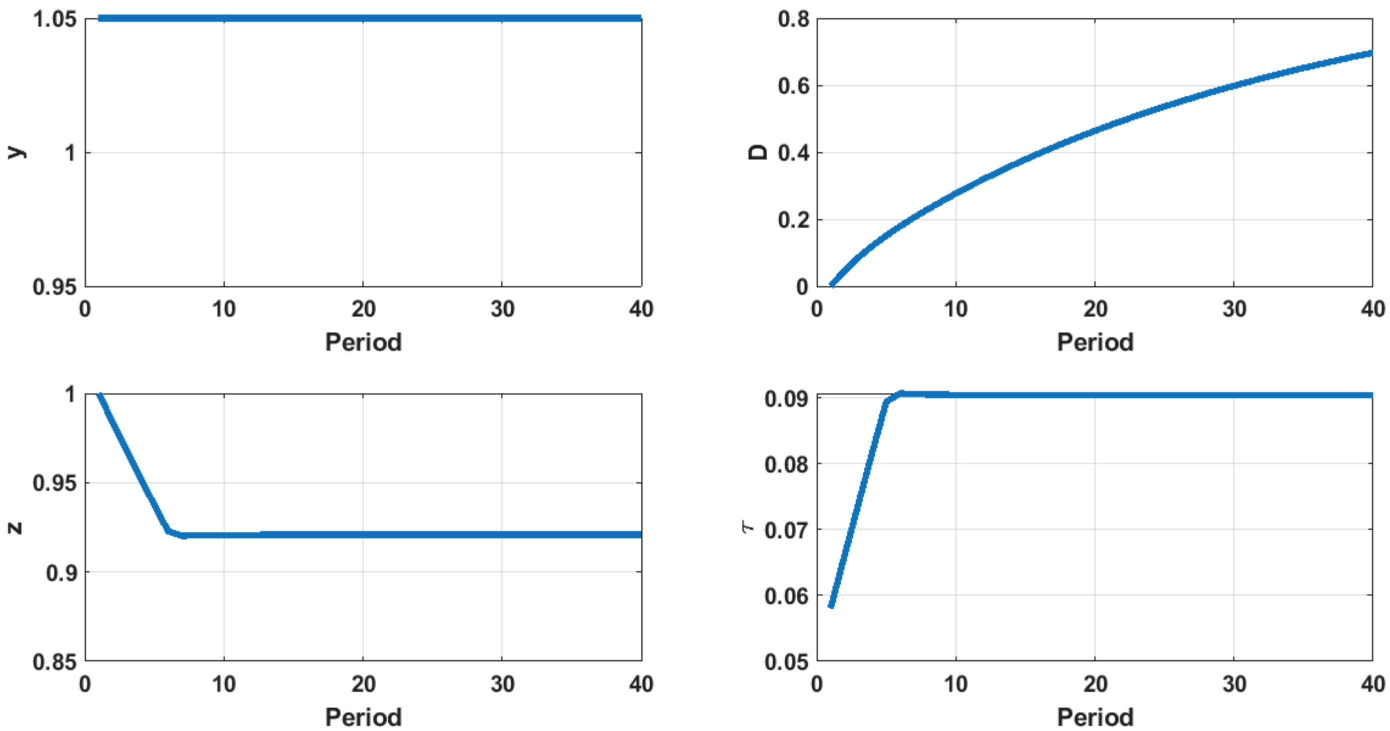

Figure 8: Perpetual high endowment

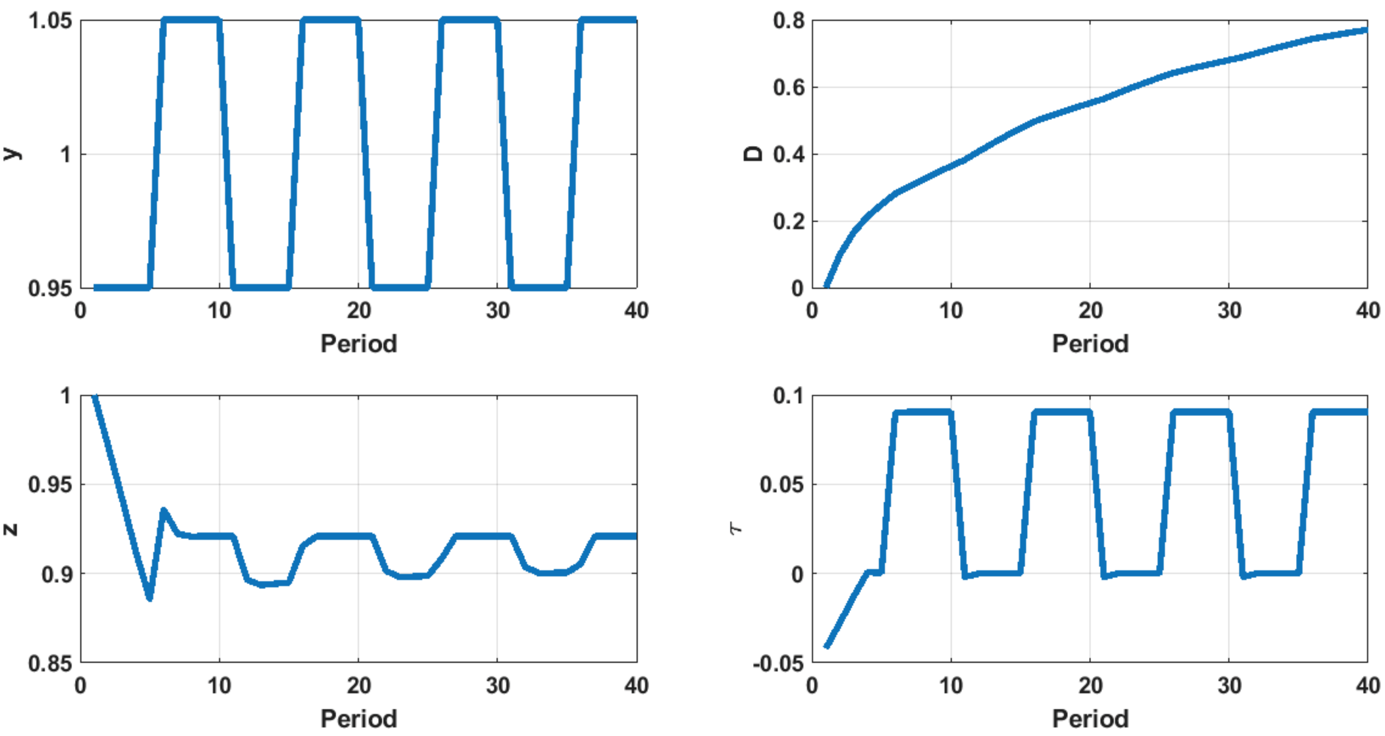

Figure 9: Ten period cycle beginning on low endowment

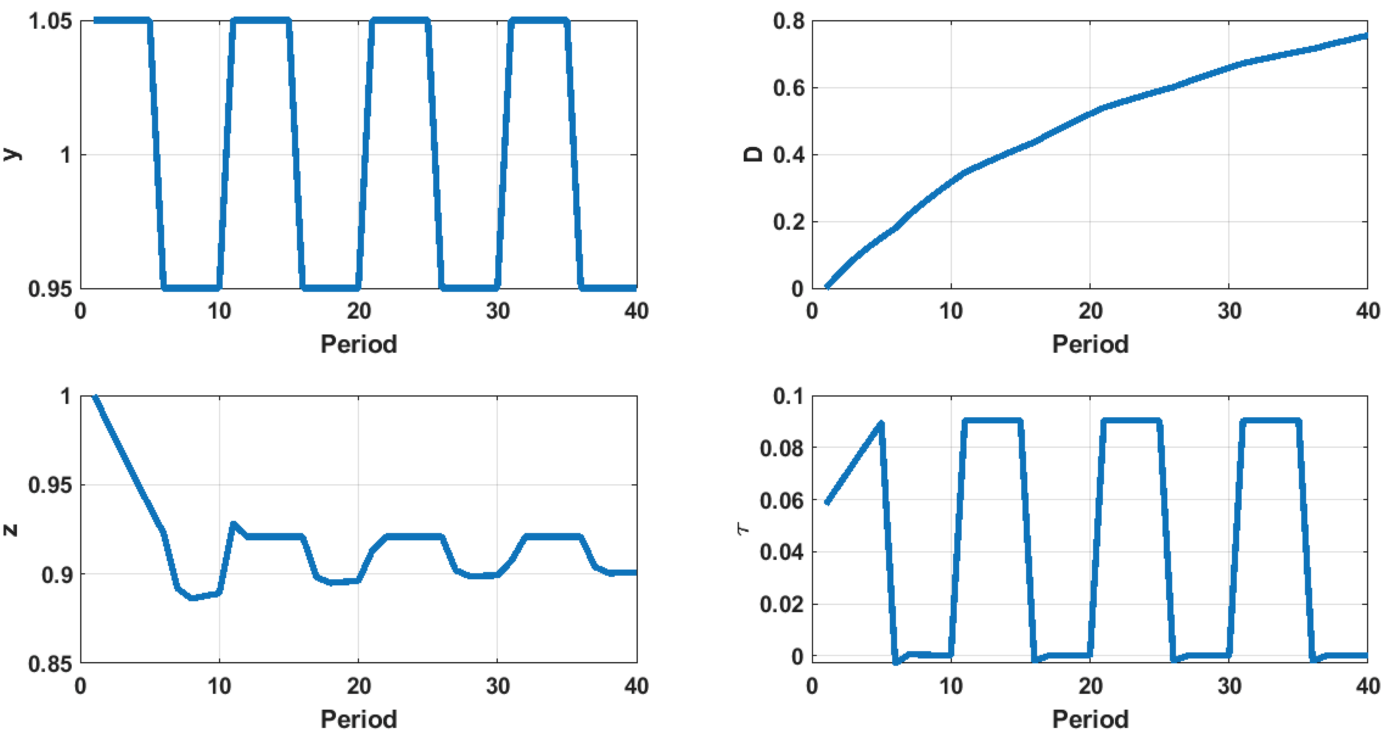

Figure 10: Ten period cycle beginning on high endowment

3 Risk Sharing as Debt Management

In the section I consider an implementation of the constrained efficient allocation using decentralized trading of non-contingent debt instruments and a mechanism for restructuring. The principal aim is to see whether the implementation can be accomplished using the types of policy instrument which are considered by proposals for debt restructuring regimes, i.e. non-contingent debt and debt restructuring.

More specifically, the problem is that the efficient allocation specifies a highly state contingent allocation. We know from previous results (e.g. Alvarez and Jermann (2000), Krueger et al. (2008)) that such an allocation can be decentralized using Arrow securities or other state contingent assets. This is the strategy which is pursued in Ábrahám et al. (2019).[9]. This paper will take a different but closely related approach by analysing an asset structure closer to that discussed in policy proposals. In effect, the state contingency will be replicated by having state dependent debt restructuring on otherwise standard one period government bonds.

An idea explored in this implementation is that conditionality can be built into the rules on debt restructuring to provide incentives to reform during default episodes. Suppose that a restructuring deal specifies that the government must implement some labour market reforms, and that if the economy grows after these reforms have been introduced, the government will be given much more time to repay its debts. In a long term contract, it is also possible to state that if the government experiences more difficulties with its debt *after* the reforms have been completed, it will be offered an even more generous restructuring as a further reward for having implemented the reforms.

3.1 Borrowing and Debt Restructuring

Each period, the borrower can issue a non-contingent bond with face value b', with a price $Q(y, b', D)$ and the lender can issue a bond with face value l' and price $Q(y, l', D)$. I consider a restructuring mechanism in which the social planner attempts to prevent the either country from defaulting completely by allowing them to decrease the burden of their liabilities in the current period through restructuring.

The system specifies a state contingent repayment rates $\theta_B(h, b) \in (0, 1)$ and $\theta_L(h, l) \in (0, 1)$ which are applied to the the outstanding principal of the bonds at the beginning of the period so that the amounts become $\theta(h, b)b$ and $\theta_L(h, l)l$. The remaining $\theta_i(.)$ of each bond then continues to be serviced as normal.

[9] On the subject of contingent debt repayments, it has often been suggested that GDP indexed debt would be useful for managing government debt burdens. A GDP linked instrument formed a small part of the package accepted by Greece's private sector creditors in its restructuring of 2012. Müller et al. (2019) note that the incomplete markets structure might be more interesting given that markets for GDP-linked debt are often absent.

The timing of the events within each period is as follows:

1. The reform shock ϵ is realized and observed only by the borrower. This then determines the level of the D_t which is observed by all agents.
2. The output shock y_t is drawn from the distribution conditional on (y_{t-1}, D_t) and observed by all agents
3. Given the realization of the output shock, the repayment rates on the liabilities of the two countries are determined. The repayment rate is exogenous from the perspective of the borrower and the lender.[10]
4. Given the realization of output and the repayment on the portfolios, the borrower and the lender then make their asset choices for the next period and the borrower makes a decision on the structural reform effort e_t

I now consider the decision problems of the borrower and the lender.

Borrower's problem The borrower maximizes utility from consumption and the effort choice subject to its budget constraint. The value function of the borrower takes the recursive form:

$$W_B(h, b) = \max_{c_b, e} \Big(u(c_b) - v(e) + \beta \mathbb{E}[W_B(h', b') \mid h] \Big) \tag{28}$$

The budget constraint is

$$c_b + \theta_B(h, b)b - \theta_L(h, l)l = y + Q(h, b')(b' - l') \tag{29}$$

The first order conditions for the asset choices determine the bond pricing equations as follows. For the choice of borrowing b we have:

$$Q(h, b') \geq \beta \mathbb{E} \frac{u_c(c_b')}{u_c(c_b)} \theta(h', b') \tag{30}$$

with equality if $b > 0$. For the choice of lending we have

$$Q(h, b') \leq \beta \mathbb{E} \frac{u_c(c_b')}{u_c(c_b)} \tag{31}$$

[10] It is possible to endogenize the repayment rates as the outcome of a dynamic game, adapting the sustainable equilibrium concept in Chari and Kehoe (1993) and Abreu (1988). I choose not to adopt this approach in order to deliver a more direct proof of the correspondence between the constrained efficient allocation and the equilibrium of the asset structure proposed here, especially with regard to the calculation of the equilibrium repayment rates.

with equality if $l > 0$ [11].

The optimal effort choice for the borrower will satisfy:

$$v_e(e_t) \begin{cases} > \frac{\beta}{4}\frac{\epsilon+1}{\epsilon}\mathbb{E}_t\epsilon_{t+1}v_e(e_{t+1}) + \frac{\beta}{2}\sum_{h_{t+1}|h_t}\epsilon_{t+1}w_D(D')\Delta_\Pi(y_{t+1} \mid y_t)W_b(h_{t+1}, b_{t+1}) & \text{if } e_t = 0 \\ < \frac{\beta}{4}\frac{\epsilon+1}{\epsilon}\mathbb{E}_t\epsilon_{t+1}v_e(e_{t+1}) + \frac{\beta}{2}\sum_{h_{t+1}|h_t}\epsilon_{t+1}w_D(D_{t+1})\Delta_\Pi(y_{t+1} \mid y_t)W_b(h_{t+1}, b_{t+1}) & \text{if } e_t = 1 \\ = \frac{\beta}{4}\frac{\epsilon+1}{\epsilon}\mathbb{E}_t\epsilon_{t+1}v_e(e_{t+1}) + \frac{\beta}{2}\sum_{h_{t+1}|h_t}\epsilon_{t+1}w_D(D_{t+1})\Delta_\Pi(y_{t+1} \mid y_t)W_b(h_{t+1}, b_{t+1}) & \text{if } e_t \in (0,1) \end{cases} \tag{32}$$

Lender's problem The lender maximizes utility from consumption subject to its budget constraint. The value function of the lender takes the recursive form:

$$W_l(h, l) = \max_{c_l, b'} \left(c_l + \frac{1}{R}\mathbb{E}[W_l(h', l') \mid y] \right)$$

The lender's budget constraint is

$$c_l + \theta_L(h, l)l - \theta_B(h, b)b = Q(h, l')(l' - b') \tag{33}$$

The bond purchasing choice of the lender gives the pricing condition:

$$Q(h, l') \geq \frac{1}{R}\mathbb{E}[\theta_b(h', b') \mid h] \tag{34}$$

with equality if $b > 0$. The bond issuance decision of the lender gives

$$Q(h, l') \leq \frac{1}{R} \tag{35}$$

with equality if $l > 0$.

Finally there are the market clearing conditions for the consumption of the two agents and the market for government bonds, which states that the bonds are in zero net supply:

$$c_l + c_b = y \tag{36}$$

$$b + l = 0 \tag{37}$$

Equilibrium Definition Given a starting level D_0 characterizing the initial distribution of output, and

[11] In practice this will not happen because of our requirement for the constrained efficient allocation that the lender's outside option $Z = 0$. $l > 0$ would require a series of (persistent) net payments from the lender to the borrower which are difficult to accommodate with this participation constraint.

initial debt positions b_0 for the borrower and l_0 for the lender, an equilibrium of the debt mechanism is a debt issuance rule $\mathbb{B}'$ for the borrower and $\mathbb{L}'$ for the lender, and a reform choice rule $\mathbb{D}$ for the borrower government, pricing rules for the government bonds $Q(.)$ and consumption rules for the borrower and the lender c_b and c_l such that

1. Given the pricing rule $Q(.)$, the government's policies $\mathbb{B}$ and $\mathbb{D}$ are the optimal policies associated with the borrower's decision problem for all (h, b)

2. Given the pricing rule $Q(.)$, the borrowing rule $\mathbb{L}$ is the optimal rule associated with the lender's decision problem for all (h, l)

3. Given the policies of the government and the lender's strategy, the bond pricing rules $Q(.)$ satisfy the pricing conditions 30, 31, 34 and 35

4. The consumption allocation rules c_b and c_l and the bond choices b and l satisfy market clearing

At this point, I also place an extra requirement on the allocations under consideration which ensures that, for appropriately defined prices, the asset positions and demands of the agents have finite values. Following the definition in Alvarez and Jermann (2000) and Kehoe and Perri (2004), I say that an allocation in our environment has *high implied interest rates* if

$$\mathbb{E}_0 \sum_{t=0}^{\infty} m_{0,t} y_t < \infty \tag{38}$$

where $m_{0,t} = m_{0,1} m_{1,2} m_{2,3} \cdots m_{t-1,t}$ and

$$m_{t,t+1} = \max \left[\beta \frac{u_c(c_{b,t+1})}{u_c(c_{b,t})}, \frac{1}{R} \right] \tag{39}$$

With this definition in place, I can now state and prove an important result for the implementation of the risk sharing agreement. The purpose of this result is to ensure that the constrained efficient allocation derived in Section 2.2 can be implemented using the decentralized trading of bonds described in this section.

Proposition 2 Any efficient risk sharing allocation with *high implied interest rates* can be sustained as an equilibrium of the bond economy with partial default .

Proof To prove this I construct the equilibrium which implements the constrained efficient allocation, broadly following the construction in Kehoe and Perri (2004). First I set the bond price:

$$Q(h, b') = \mathbb{E} \min \left[\beta \frac{u_c(c_b(h', b'))}{u_c(c_b(h, b))}, \frac{1}{R} \right] \tag{40}$$

I then set the bond repayment rates $\theta_B(h', b')$ and $\theta_L(h', l')$ as:

$$\theta_B(h', b') = \begin{cases} \eta \frac{u_c(c_b(h',b'))}{u_c(c_b(h,b))}, & \text{if } \beta \frac{u_c(c_b(h',b'))}{u_c(c_b(h,b))} \leq \frac{1}{R} \\ 1, & \text{if } \beta \frac{u_c(c_b(h',b'))}{u_c(c_b(h,b))} > \frac{1}{R} \end{cases}$$

and

$$\theta_L(h', l') = \begin{cases} \frac{1}{\eta} \frac{u_c(c_b(h,b))}{u_c(c_b(h',b'))}, & \text{if } \beta \frac{u_c(c_b(h',b'))}{u_c(c_b(h,b))} > \frac{1}{R} \\ 1, & \text{if } \beta \frac{u_c(c_b(h',b'))}{u_c(c_b(h,b))} \leq \frac{1}{R} \end{cases}$$

Note that these repayment rates always satisfy:

$$\theta_B(h', b')\theta_L(h', l') = \frac{\min\left[\beta \frac{u_c(c_b(h',b'))}{u_c(c_b(h,b))}, \frac{1}{R}\right]}{\max\left[\beta \frac{u_c(c_b(h',b'))}{u_c(c_b(h,b))}, \frac{1}{R}\right]} \tag{41}$$

For the bond position of the borrower I set:

$$b(h) - l(h) = \frac{1}{\theta_B(h, b)\theta_L(h, l)} \sum_{k=0}^{\infty} \sum_{h^{t+k}|h^t} \tilde{\Pi}(h^{t+k} \mid h_t) m(h^{t+k} \mid h_t)(y_t - c_t^b) \tag{42}$$

To pin down the gross positions b and l, I adopt the rule that when the right hand side of this equation is positive, I set $l = 0$. Conversely, when the right hand side of the equation is negative, I set $b = 0$.

I now verify that the constructed bond prices, repayment rates and positions constitute a competitive equilibrium with partial default. To see that the bond prices are part of an equilibrium, suppose that (as will be the normal case) the borrower has positive liabilities, so that $b > 0$ and $l = 0$. Then the Euler equations require that

$$Q(h, b') = \beta \mathbb{E} \frac{u_c(c_b')}{u_c(c_b)} = \frac{1}{R} \mathbb{E} \theta_B(h', b') \tag{43}$$

This condition is clearly satisfied for our proposed bond prices and repayment rates. Alternatively if the lender has positive liabilities, so that $l > 0$ and $b = 0$, then the Euler equations require that

$$Q(h, b') = \frac{1}{R} = \beta \mathbb{E} \theta_L(h', b') \frac{u_c(c_b')}{u_c(c_b)} \tag{44}$$

Again, this condition is satisfied by our chosen repayment rates and bond prices.

To verify that the asset positions are also part of an equilibrium, I write the borrower's budget constraint as

$$\theta_B(h,b)\theta_L(h,l)(b-l) = y - c_b + Q(h,b')(b'-l')$$

Iterating forward on this equation and using the transversality condition, which is

$$\lim_{t\to\infty} \mathbb{E}_t \beta^t Q(h_t, b_t) u_c(c_{b,t})(b_{t+1} - l_{t+1}) = 0$$

We can obtain:

$$b - l = \frac{1}{\theta_B(h,b)\theta_L(h,l)} \sum_{k=0}^{\infty} \sum_{h^{t+k}|h^t} \tilde{\Pi}(h^{t+k} \mid h_t) m(h^{t+k} \mid h_t)(y_t - c_t^b)$$

Finally I show that if the allocation satisfies the high interest rate condition, then it also satisfies the transversality condition. Starting from the LHS of the transversality condition, we can write

$$\lim_{t\to\infty} \mathbb{E}_t \left[\beta^t Q(h_t, b_t) u_c(c_{b,t})(b_{t+1} - l_{t+1})\right] \tag{45}$$

$$= \lim_{t\to\infty} \mathbb{E}_t \left[\beta^t u_c(c_{b,t}) \frac{\min\left[\beta \frac{u_c(c_b(h',b'))}{u_c(c_b(h,b))}, \frac{1}{R}\right]}{\theta_B(h',B')\theta_L(h',B')} \mathbb{E}_{t+1} \sum_{k=0}^{\infty} m(h^{t+k+1} \mid h_{t+1})(y_{t+k+1} - c_{t+k+1}^b)\right] \tag{46}$$

$$= \lim_{t\to\infty} \mathbb{E}_t \left[\beta^t u_c(c_{b,t}) \max\left[\beta \frac{u_c(c_b(h',b'))}{u_c(c_b(h,b))}, \frac{1}{R}\right] \mathbb{E}_{t+1} \sum_{k=0}^{\infty} m(h^{t+k+1} \mid h_{t+1})(y_{t+k+1} - c_{t+k+1}^b)\right] \tag{47}$$

$$= \frac{u_c(c_{b,0})}{\beta} \lim_{t\to\infty} \mathbb{E}_t \left[\beta^t \frac{u_c(c_{b,t})}{u_c(c_{b,0})} \mathbb{E}_{t+1} \sum_{k=0}^{\infty} m(h^{t+k+1} \mid h_t)(y_{t+k+1} - c_{t+k+1}^b)\right] \tag{48}$$

$$\leq \frac{u_c(c_{b,0})}{\beta} \lim_{t\to\infty} \mathbb{E}_t \sum_{k=0}^{\infty} m(h^{t+k+1} \mid h_0)(y_{t+k+1} - c_{t+k+1}^b) \tag{49}$$

$$\leq \frac{u_c(c_{b,0})}{\beta} \lim_{t\to\infty} \mathbb{E}_t \sum_{k=0}^{\infty} m(h^{t+k+1} \mid h_0) y_{t+k+1} \tag{50}$$

$$= 0 \tag{51}$$

Where 45 follows from the definitions of the bond price and the asset positions, 46 follows from Equation 41, the inequality in 49 follows from the definition of $m(. \mid y_0)$ and the last equality follows from the high interest rate condition.■

4 Debt Crisis Exercise

In this section I perform a more detailed numerical exercise to see how the risk sharing contract, interpreted as a debt managment policy, performs in response to a debt crisis. It is difficult to calibrate the reform stock D directly because in practice it reflects different sets of policies which (if they can be quantified) would be measured in different ways. One feasible approach would be to construct indices using structural policy indicators, such as those provided by the OECD [12]. Instead I choose to calibrate the reform technology based on the expected effect of the reforms on output.

To begin, the bad distribution Π_b is calibrated by pooling the quarterly real GDP series of the Euro Area countries which received financial assistance (Portugal, Ireland, Italy, Greece and Spain), for the period 2000-2010. I estimate the AR1 process:

$$\log y_t = \rho \log y_{t-1} + X_i + Q_t + \epsilon_t$$

where X_i is a country fixed effect for country i and Q_t is a quarter effect, since the input data are not seasonally adjusted. I then use the estimates for ρ and the error variance σ_ϵ^2. The resulting AR1 process, based on ρ and σ_ϵ^2, is discretized using the Tauchen (1986) method to form a 21 state first order Markov process for y. The good distribution Π_g is found by using an algorithm to find a transition matrix which dominates Π_b by the MLRP property; unlike the exercise in Section 2.4, reform will be allowed to affect all of the transition probabilities. The matrix Π_g is also restricted so that, in its stationary distribution, it produces a given percentage increase in mean output relative to Π_b. This increase is set to 3%[13]. IMF (2017) discusses the range of plausible increases in the level of GDP resulting from a fully implemented structural reform programme for Greece, based on several empirical studies [14]. This range is given as 3-13% and I choose the lowest value in this range. Thus, under the proposed choice of Π_g, steady state output in an economy where $D = 1$ forever is 3% higher than in an economy where $D = 0$ forever. I also set the lower value of the reform effort shock $\underline{\epsilon}$ to 0.95, which maintains the unobservability of effort but otherwise limits the role of this shock.

The remaining parameters are set to values which are common in the literature on sovereign default. The full set of parameter values chosen for this exercise is given in Table 4.

Crisis Exercise In the simulation, I calculate the allocations and the associated debt paths for 40 periods following an initial period of 'crisis'; since the endowment process is estimated at a quarterly frequency this corresponds to a ten year horizon. I define the crisis as a state in which initial output y_0 is far below its average and debt is 150% of mean output. Since the output process is chosen so that average output is 1, I choose a value of the endowment as close as possible to 0.8, corresponding to a fall of 20% in log output, and I set the initial debt level to 1.5. The initial reform capital stock D_0 is set to 0. The initial relative weight z_0 needs to be consistent with initial debt level, such that the

[12] https://www.oecd.org/economy/growth/structural-policy-indicators-database-for-economic-research.htm

[13] This procedure could be improved by using higher moments to restrict Π_g even further.

[14] It is common in discussions on structural reforms to speak of the impact on *growth*. However, the studies referred to in IMF (2017) find very little evidence of a sustained impact on the growth rate, instead identifying various percentage increases in the *level* of GDP.

Parameter		Value
Effort shock	$\underline{\epsilon}$	0.95
Subjective discount factor	β	0.96
Coefficient of relative risk aversion	γ	2
Risk free (gross) interest rate	R	1.02
Default cost parameter	ψ	0.96
Lender outside option value	Z	0
Output AR1 coefficient	ρ	0.95

Table 1: Baseline parameter values

the decentraization of the contract at (y_0, D_0, z_0) gives a debt of 1.5. This results in an initial value $z_0 = 0.7768$ I then draw y_{t+1} for each simulation period randomly from its conditional distribution given y_t and D_{t+1}. To compute impulse responses for the different economies, I take averages over 25,000 simulation paths.

4.1 Results

The impulse responses for this exercise are shown in Figure 11. As we can see from the first panel, due to the size of the fall in output and the persistence of the output process, output remains suppressed for an extended period after the initial shock. On average, output recovers to its mean under the intial distribution after 30 periods. This is an important observation for the dynamics of the other variables, which are largely driven by the fact that output is low for most of the simulation. For example, by the end of series, the reform has been almost completely implemented since D is close to 1. The fact that output is so low provides a strong incentive for reform effort over the first 30 periods. This is confirmed by the path of reform effort shown in the last panel, where we see that the reforms are strongly frontloaded in the first half of the simulation, and approach zero in the second half as the economy recovers. The reform progress also explains the fact that in last few periods, output is above 1 on average. The reform progress increases the average output of the borrower's economy from 1 to 1.03, and in the last few periods the borrower is able to enjoy the benefit of this improvement.

An important result of the exercise is that the agreement is able to sustain an increasing level of debt following the crisis, rather than inducing a sharp consolidation in the initial periods. The long term nature of the contract allows the government to borrow efficiently against the higher expected future output, which is composed of both reversion to the mean and an increase in the mean itself. The contract therefore has a very high debt capacity compared to alternative asset structures. The dynamics of consumption are more subtle. In the inital periods the low output realizations are taken as an indication of low effort by the government, and cause a decrease in the relative weight and consumption. However, after a few periods, the reform progress and the recovery of the endowment are rewarded with higher consumption. Despite the non-monotonic path of consumption, the borrower's welfare increases smoothly over the simulation, as the negative effect of the increase in debt is outweighed by the improvement in the endowment and refom progress. We should also note while consumption varies considerably over the simulation, it does so much less than the endowment, demonstrating the risk sharing capacity of the contract.

In the fifth panel of Figure 11, we see to what extent the policy response relies on partial default to implement the constrained efficient allocation. While haircuts on the debt are negligible in the initial periods, after 8 periods they reach 4%, before decreasing gradually to 2% towards the end of the simulation. The fact that the amounts involved are relatively small is encouraging; however, the results suggest that these small haircuts would be used constantly over the course of the recovery. This partly reflects the relative impatience of the borrower, since the repayment rates are derived from the relative marginal rates of subtitution of the borrower and the lender. As we would expect, the peak in the haircuts is preceded by a peak in the interest rate spreads paid by the borrower, which reach a maximum of 4.2% in period 6. This interest rate increase is also an important determinant of the increase in debt levels during the recovery, since the 'deficits' $c_t - y_t$ are not large enough to account for the enitre difference.

Table 4.1 displays the moments for the exercise. The means of output and consumption clarify that despite the borrowing in the early periods after the crisis, on average the borrower is a net *payer* during the recovery, making a net transfer of 2.4% of mean output under the initial distribution. We also see that the volality of consumption is roughly a quarter that of output, again highlighting that the contract allows the borrower to share risk. Another feature that we should note is that the average value of the contract to the lender is equal to the average debt level. Recall that by the definition of the asset positions, when the borrower is highly indebted, the net present value of the future repayments to the lender is high, and so the contract offers a high surplus to the lender. This may run counter to our intuition about the position of the creditor, but it follows from the fact that the contract is designed so that the borrower will always (weakly) prefer delivery of the promised transfers to reneging on these commitments. The average interest spread relative to the risk free rate is 2.7%

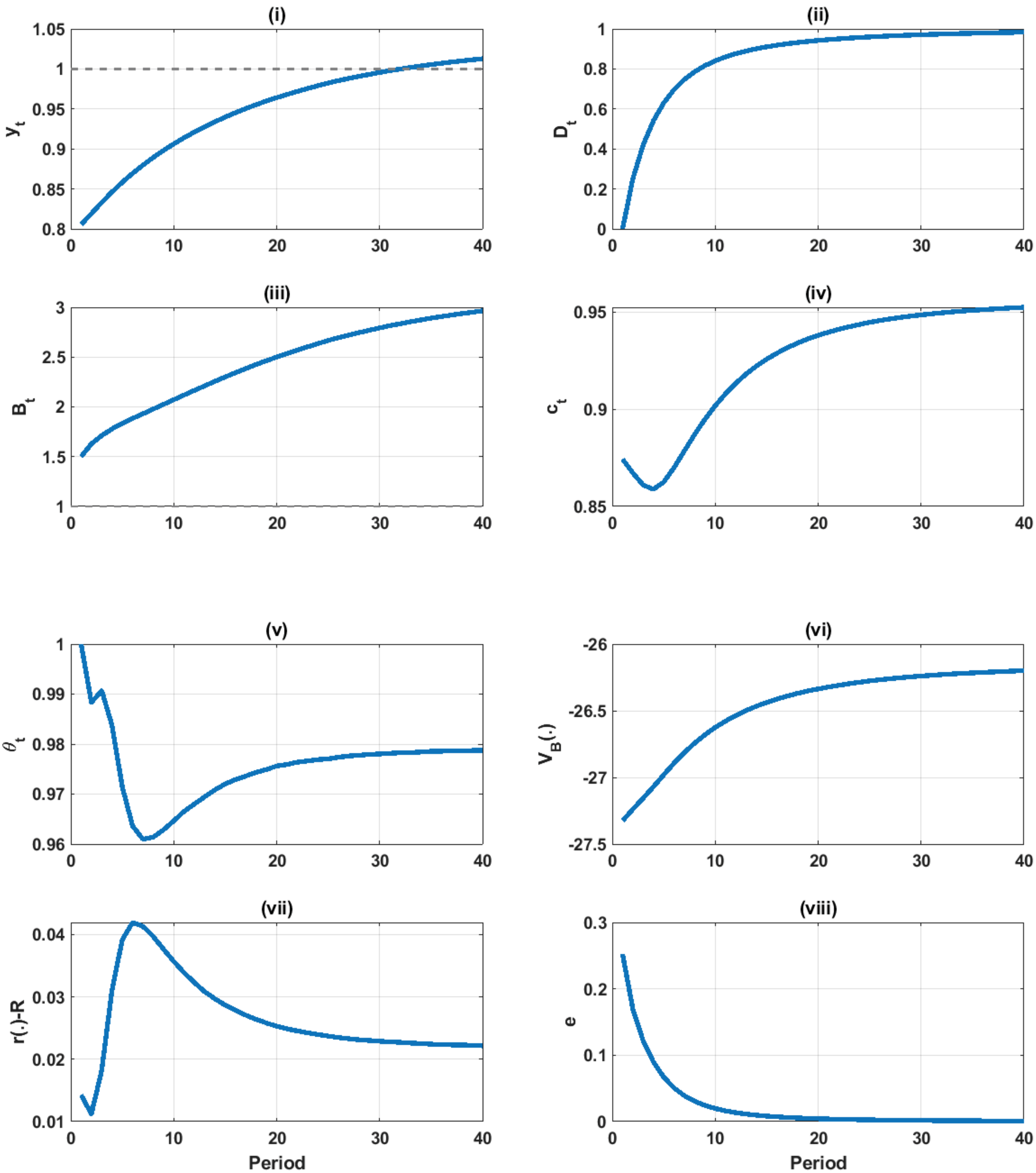

Figure 11: Impulse responses from the crisis exercise: (i) Endowment y_t, (ii) Reform progress D_t, (iii) Debt B_t, (iv) Consumption c_t, (v) Repayment rate θ_t, (vi) Borrower value V_B, (vii) Interest rate spread $r_t - R$, (viii) Reform effort e_t

	Variable		
Mean		**Variance**	
Output	y_t 0.948		y_t 0.0072
Consumption	c_t 0.924		c_t 0.0017
Transfer	τ_t 0.024		
Debt	B_t 2.43		
Relative weight	z_t 0.853		
Reform progress	D_t 0.852		
Reform effort	e_t 0.025		
Interest spread	$r_t - R$ 0.027		
Borrower Value	V_B -26.47		
Lender Value	V_L 2.43		

Table 2: Crisis Exercise Moments

5 Concluding Comments

In this paper I designed a macroeconomic risk sharing agreement for a long term economic union subject to two political constraints, chosen to reflect the policy design challenges faced by Euro Area authorities. The first is that countries in the union need to be encouraged to implement long term reform policies which improve the distribution of their output, engaging in a kind of self-insurance. The second is that transfers between countries, which are the main instrument for risk sharing, cannot be so large or persistent that payer countries would prefer to leave the agreement. This applies equally to creditors and debtors. I then considered an approach to implementing an agreement respecting these constraints using debt contracts between countries, where the required state contingent payments were replicated by allowing debt stocks to be restructured periodically.

I found that for a reasonable calibration of the model, the increase in expected output which results from a successfully implemented reform programme creates a larger surplus to be shared between the borrower and the lender. Moreover, the optimal risk sharing agreement passes a large portion of the reform benefits to the lender, to the extent that the probability of exit by the lender becomes negligible. The agreement can also sustain large liabilities between the borrowing and lending country. However, some issues arise in trying to implement the mechanism through trading of debt contracts. The asset/liability positions arising from the transfers must be adjusted quite frequently. In practice, we would therefore need to be careful about the choice of instruments used to achieve these adjustments. Manipulating the maturity structure of the debt stocks seems like a promising avenue, as well as reprofiling the interest payments on debt or rescheduling principal payments. Adjustment through partial default, on the other hand, would probably occur too frequently to be politically viable.

There are several extensions of the framework in this paper which would be useful for clarifying implementation issues. The mechanism described in this paper deals only with the *willingness to pay* problem on the side of the debtor. In the literature on sovereign default, it is also common to characterize rollover risk as resulting from the realization of a sunspot which determines the willingness of the lender to roll over maturing debts. This feature would be more appropriate for dealing with the separation between liquidity problems and solvency problems, an important element of the discussion on debt management in the Euro area. A potential criticism of the current model is that it lacks some features which would allow it to capture important macroeconomic policy issues in currency unions; in particular it lacks any pricing frictions or a role for monetary policy. These issues are addressed in Marimon et al. (2019). A problem that arises in this paper is that the option to leave the union and regain control of monetary policy is extremely powerful, and limits the extent of risk sharing which can take place within a monetary union. It could also be interesting to introduce another dimension of hidden action, whereby each country is allowed to issue debt which is managed outside the risk sharing agreement. Since the level of risk involved in trying to manage the entire stock of Euro denominated debt in the manner described in this paper would probably be unacceptable, a framework with parallel private debt markets would allow us to investigate how much sovereign debt should be managed under the risk sharing mechanism.

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A Constructing the Multipliers

In this section I explain how the time varying Pareto weights can be constructed from the social planner's problem. For simplicity, I assume that the planner only faces participation constraints in the initial period, since the main difficulty in constructing the weights comes from the contribution of the incentive compatibility constraints. I start by outlining the derivation of the borrower's incentive compatibility constraint.

Borrower's problem Borrower's utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \nu(e_t)]$$

'Policy capital' stock $D_t \in [0, 1]$ evolves according to:

$$D_{t+1} = D_t + \epsilon_{t+1} e_t$$

Transition matrix $\tilde{\Pi}_t$ of output, y_t, is given by

$$\tilde{\Pi}(D_t) = w(D_t)\Pi_g + (1 - w(D_t))\Pi_b$$

where $w(.)$ is a concave function and the distribution under Π_g dominates that under Π_b by the monotone likelihood ratio property (MLRP).

Optimal Effort Choice I define the borrower's value function $V_B(h_t)$ as

$$\begin{aligned} V_B(h_t) =& E_t \sum_{k=0}^{\infty} \beta^k [u(c_{t+k}) - \nu(e_{t+k})] \\ =& u(c_t) - \nu(e_t) + \beta \sum_{y_{t+1}|y_t} \tilde{\Pi}(h_{t+1} \mid h_t) V_B(h_{t+1}) \\ =& u(c_t) - \nu(e_t) + \beta \sum_{y_{t+1}|y_t} \tilde{\Pi}(h_{t+1} \mid h_t) V_B(h_{t+1}) \end{aligned}$$

If we ignore corner solutions, the optimal choice of e_t satisfies the first order condition:

$$\begin{aligned} v_e(e_t) =& \beta \sum_{h_{t+1}|h_t} \frac{d\tilde{\Pi}(h_{t+1} \mid h_t)}{dD_{t+1}} V_B(h_{t+1}) + \beta \sum_{y_{t+1}|y_t} \tilde{\Pi}(h_{t+1} \mid h_t) \frac{\partial V_B(h_{t+1})}{\partial D_{t+1}} \\ =& \frac{\beta}{2} \sum_{h_{t+1}|h_t} \epsilon_{t+1} w_D(D_{t+1}) \Delta_\Pi(y_{t+1} \mid y_t) V_B(h_{t+1}) + \frac{\beta}{4} \frac{\underline{\epsilon} + 1}{\underline{\epsilon}} \mathbb{E}_t \epsilon_{t+1} v_e(e_{t+1}) \end{aligned}$$

Planner's Problem Having derived the borrower's incentive compatibility condition, we can now specify the planner's problem, which also includes the risk neutral lender.

$$F(h_0) = \max_{\{c_t, D_{t+1}\}_{t=1}^{\infty}} \mu_{B,0} \sum_{t=0}^{\infty} \sum_{h_t|h_0} \Pi(h_t \mid h_0)\beta^t[u(c_t) - \nu(e_t)] + \mu_{L,0} \sum_{t=0}^{\infty} \sum_{h_t|h_0} \left(\frac{1}{R}\right)^t \Pi(h_t \mid h_0)(y_t - c_t)$$

subject to

$$D_{t+1} = D_t + \epsilon_{t+1} e_t$$

$$\nu'(e_t) = \frac{\beta}{2} \sum_{h_{t+1}|h_t} \epsilon_{t+1} w_D(D_{t+1}) \Delta_\Pi(y_{t+1} \mid y_t) V_B(h_{t+1}) + \frac{\beta}{4} \frac{\underline{\epsilon} + 1}{\underline{\epsilon}} \mathbb{E}_t \epsilon_{t+1} v_e(e_{t+1})$$

and the initial participation constraints:

$$\sum_{t=0}^{\infty} \sum_{h_t|h_0} \tilde{\Pi}(h_t \mid h_0)\beta^t[u(c_t) - \nu(e_t)] \geq V_a(h_0)$$

$$\sum_{t=0}^{\infty} \sum_{h_t|h_0} \left(\frac{1}{R}\right)^t \tilde{\Pi}(h_t \mid h_0)(y_t - c_t) \geq 0$$

Let $\beta^t \tilde{\Pi}(y_t \mid y_0)\xi_t(h_t)$, $\gamma_{b,0}$ and $\gamma_{l,0}$ be the Lagrange multipliers for the incentive compatibility, borrower participation and lender participation constraints respectively. Then we can write the Lagrangian for this problem as:

$$\begin{aligned}
&\Lambda(\gamma_{b,0}, \gamma_{l,0}, \{\xi_t, c_t, D_{t+1}\}_{t=0}^{\infty}) \\
&= \max_{\{c_t, D_{t+1}\}_{t=1}^{\infty}} \mu_{B,0} \sum_{t=0}^{\infty} \sum_{h_t|h_0} \tilde{\Pi}(h_t \mid h_0)\beta^t[u(c_t) - \nu(e_t)] + \mu_{L,0} \sum_{t=0}^{\infty} \sum_{h_t|h_0} \left(\frac{1}{R}\right)^t \tilde{\Pi}(h_t \mid h_0)(y_t - c_t) \\
&+ \gamma_{b,0} \left(\sum_{t=0}^{\infty} \sum_{h_t|h_0} \tilde{\Pi}(h_t \mid h_0)\beta^t[u(c_t) - \nu(e_t)] - V_a(h_0) \right) + \gamma_{l,0} \sum_{t=0}^{\infty} \sum_{h_t|h_0} \left(\frac{1}{R}\right)^t \tilde{\Pi}(h_t \mid h_0)(y_t - c_t) \\
&- \sum_{t=0}^{\infty} \sum_{h_t|h_0} \beta^t \tilde{\Pi}(h_t \mid h_0)\xi(h_t) \left[\nu'(e_t) - \frac{\beta}{2} \sum_{h_{t+1}|h_t} \epsilon_{t+1} w_D(D_{t+1}) \Delta_\Pi(y_{t+1} \mid y_t) V_B(h_{t+1}) - \frac{\beta}{4} \frac{\underline{\epsilon} + 1}{\underline{\epsilon}} \mathbb{E}_t \epsilon_{t+1} v_e(e_{t+1}) \right]
\end{aligned}$$

The highlighted term, which contains the value function $V_B(h_{t+1})$, can be rewritten using the definition of the value function:

$$
\begin{aligned}
&\sum_{t=0}^{\infty}\sum_{h_t|h_0}\tilde{\Pi}(h^t \mid h_0)\xi(h_t)\sum_{h_{t+1}|h_t}\frac{\beta^{t+1}}{2}\epsilon_{t+1}w_D(D_{t+1})\Delta_\Pi(y_{t+1}\mid y_t)V_B(h_{t+1})\\
&=\sum_{t=0}^{\infty}\sum_{h_t|h_0}\tilde{\Pi}(h^t \mid h_0)\sum_{h_{t+1}|h_t}\xi(h_t)\frac{\epsilon_{t+1}w_D(D_{t+1})}{2}\Delta_\Pi(y_{t+1}\mid y_t)\sum_{k=1}^{\infty}\beta^{t+k}[u(c_{t+k})-\nu(e_{t+k})]\\
&=\sum_{t=0}^{\infty}\sum_{h_t|h_0}\tilde{\Pi}(h^{t+1} \mid h_0)\sum_{h_{t+1}|h_t}\xi(h_t)\frac{\epsilon+1}{2}w_D(D_{t+1})\frac{\Delta_\Pi(y_{t+1}\mid y_t)}{\tilde{\Pi}(y_{t+1}\mid y_t,D_{t+1})}\sum_{k=1}^{\infty}\beta^{t+k}[u(c_{t+k})-\nu(e_{t+k})]
\end{aligned}
\tag{52}
$$

Then, using Abel's formula, we can write this as

$$
\sum_{t=0}^{\infty}\sum_{h_t|h_0}\left(\sum_{k=0}^{t}\xi(h_k)\frac{\epsilon+1}{2}w_D(D_{k+1})\frac{\Delta_\Pi(y_{k+1}\mid y_k)}{\tilde{\Pi}(y_{k+1}\mid y_k,D_k)}\right)\sum_{t=0}^{\infty}\beta^{t+1}\tilde{\Pi}(h^{t+1}\mid h_0)[u(c_{t+1})-\nu(e_{t+1})]
$$

We can now rewrite the original Lagrangian problem as:

$$
\begin{aligned}
&\Lambda(\gamma_{b,0},\gamma_{l,0},\{\xi_t,c_t,D_{t+1}\}_{t=0}^{\infty})\\
&=\max_{\{c_t,D_{t+1}\}_{t=1}^{\infty}}\mu_{B,0}\sum_{t=0}^{\infty}\sum_{h_t|h_0}\tilde{\Pi}(h_t\mid h_0)\beta^t[u(c_t)-\nu(e_t)]+\mu_{L,0}\sum_{t=0}^{\infty}\sum_{h_t|h_0}\left(\frac{1}{R}\right)^t\tilde{\Pi}(h_t\mid h_0)(y_t-c_t)\\
&+\gamma_{b,0}\left(\sum_{t=0}^{\infty}\sum_{h_t|h_0}\tilde{\Pi}(h_t\mid h_0)\beta^t[u(c_t)-\nu(e_t)]-V_a(h_0)\right)+\gamma_{l,0}\sum_{t=0}^{\infty}\sum_{h_t|h_0}\left(\frac{1}{R}\right)^t\tilde{\Pi}(h_t\mid h_0)(y_t-c_t)\\
&-\sum_{t=0}^{\infty}\sum_{y_t|y_0}\beta^t\tilde{\Pi}(h_t\mid h_0)\xi(h_t)\left[\nu'(e_t)-\beta\sum_{h_{t+1}|h_t}\tilde{\Pi}(h_{t+1}\mid h_t)v_e(e_{t+1})\right]\\
&+\sum_{t=0}^{\infty}\sum_{h_t|h_0}\left[\left(\sum_{k=0}^{t}\xi(h_k)\frac{\epsilon+1}{2}w_D(D_{k+1})\frac{\Delta_\Pi(y_{k+1}\mid y_k)}{\tilde{\Pi}(y_{k+1}\mid y_k,D_k)}\right)\right.\\
&\left.\times\left(\sum_{t=0}^{\infty}\beta^{t+1}\tilde{\Pi}(y^{t+1}\mid h_0)[u(c_{t+1})-\nu(e_{t+1})]\right)\right]\\
&=\max_{\{c_t,D_{t+1}\}_{t=1}^{\infty}}\sum_{t=0}^{\infty}\sum_{y_t|y_0}\mu_{B,t}\tilde{\Pi}(y_t\mid y_0)\beta^t[u(c_t)-\nu(e_t)]+\mu_{L,0}\sum_{t=0}^{\infty}\sum_{y_t|y_0}\left(\frac{1}{R}\right)^t\tilde{\Pi}(y_t\mid y_0)(y_t-c_t)\\
&+\gamma_{b,0}\left(\sum_{t=0}^{\infty}\sum_{y_t|y_0}\tilde{\Pi}(y_t\mid y_0)\beta^t[u(c_t)-\nu(e_t)]-V_a(h_0)\right)+\gamma_{l,0}\sum_{t=0}^{\infty}\sum_{y_t|y_0}\left(\frac{1}{R}\right)^t\tilde{\Pi}(y_t\mid y_0)(y_t-c_t)\\
&-\sum_{t=0}^{\infty}\sum_{y_t|y_0}\beta^t\tilde{\Pi}(y_t\mid y_0,D_0)\xi(h_t)\left[\nu'(e_t)-\beta\sum_{y_{t+1}|y_t}\tilde{\Pi}(h_{t+1}\mid h_t)v_e(e_{t+1})\right]
\end{aligned}
$$

where

$$\mu_{B,t} = \mu_{B,0} + \gamma_{b,0} + \left(\sum_{k=0}^{t-1} \xi(h_k) \frac{\underline{\epsilon}+1}{2} w_D(D_{k+1}) \frac{\Delta_{\Pi}(y_{k+1} \mid y_k)}{\tilde{\Pi}(y_{k+1} \mid y_k, D_k)} \right)$$

$$\mu_{L,1} = \mu_{L,0} + \gamma_{L,0}$$

$$\mu_{B,1} = \mu_{B,0} + \gamma_{B,0}$$

The same argument can easily be extended to the case in which there is a participation constraint in each period.

B Numerical Solution Details

Contract Allocation The first step in solving the modelling numerically involves computing the value function and policy functions for the outside option of the borrower, in which the borrower is permanently in autarky but can still accumulate the reform capital D_t. These functions are computed over discrete grids for output y and the reform capital stock D. The model is solved by value function iteration, iterating over $X(h)$ until changes in this function become small enough. For each iteration k, and the associated guess for the value function $X_k(h)$, the policy function $e_k(h)$ is updated by using the equation:

$$\begin{aligned} e'_{k+1}(h) = &\frac{\beta}{4} \frac{\underline{\epsilon}+1}{\underline{\epsilon}} \sum_{h'|h} \tilde{\Pi}(y' \mid y, D + \epsilon' e_k(h)) \epsilon' e_k(y', D + \epsilon' e_k(h)) \\ &+ \frac{\beta}{2} \sum_{h'|h} \epsilon' w_D(D + \epsilon' e_k(h)) \Delta_{\Pi}(y' \mid y) X_k(y', D + \epsilon' e_k(h)) \end{aligned}$$

Once the outside option has been solved for, the contract allocation is solved using policy function iteration. Here I solve for the value functions $V_B(h,z)$ and $V_L(h,z)$, the policy functions $e(h,z)$, $z'(h,z)$ and the Lagrange multiplier $\xi(h,z)$. Recall that the state/co-state vector for the contract is (h,z). The first step in the solution algorithm is, for each pair (h,z), to find the values of the multiplier z at which the participation constraints bind. More specifically, suppose that we have reached iteration k, and we have guesses $V_b^k(h,z)$ and $V_l^k(h,z)$ for the value functions and $D'^k(h,z)$ and $z'^k(y,d,Z)$ for the policy function for reform capital and the updated relative weight. Then for each h, we find $\underline{z}$ such that

$$\frac{\tilde{c}(\underline{z})^{1-\gamma}}{1-\gamma} + \beta \sum_{y'|y} \tilde{\Pi}(y' \mid y, D'^k(y,d,\underline{z})) V_b^k(y', D'^k(y,d,\underline{z}), z'^k(y,d,\underline{z})) = X(h)$$

to find the point at which the borrower's participation constraint binds, and $\bar{z}$ such that

$$y - \tilde{c}(\bar{z}) + \frac{1}{R} \sum_{y'|y} \tilde{\Pi}(y' \mid y, D'^k(y,d,\bar{z})) V_l^k(y', D'^k(y,d,\bar{z}), z'^k(h,\bar{z})) = 0$$

to find the point at which the lender's participation constraint binds, where

$$\tilde{c}(z) = z^{1/\gamma}$$

and

$$\tilde{\Pi}(y' \mid y, D'^k(h,z)) = w(D'^k(h,z))\Pi_g(y' \mid y) + (1 - w(D'^k(y,d,z)))\Pi_b(y' \mid y)$$

We know that outside the interval $(\underline{z}, \bar{z})$ the allocation must be constant due to the binding participation constraints, where as on this interval consumption will be given by Equation 27 with $\nu_l = \nu_b = 0$.

Updating the reform effort policy $e(h,z)$ requires several imputs, since e must be optimal both for the borrower and for the planner. In each iteration we have a guess for the value of the Lagrange multiplier on the incentive compatibility constraint ξ^k. This is used to generate $z'^{k+1}(.)$ using Equation 25. We then calculate $e^k(h,z)$ using the borrower's incentive compatibility condition. As the last step of each iteration, we update the guess $\xi^{k+1}(h,z)$ using the planner's optimal effort equation.

Decentralization The decentralization depends directly on the solution for the contract. I first solve for the bond prices $Q(h,b')$ using Equation 40, and the repayment rates $\theta(h',b')$ using the solutions $c_b(h,z)$ and $D'(h,z)$. With these in hand, we can then iterate over a recursive form of the budget constraint for the bond economy. The asset/debt choice $B'(h,z)$ is updated using

$$B'^{k+1}(h,z) = \frac{1}{\theta_B(h,z)} \left(y - c_b(h,z) + \sum_{y'|y} \tilde{\Pi}(y' \mid y, D'(h,z)) B'^k(h'(h,z), z'(h,z)) \right)$$

updating continues until the difference between $B^{k+1}()$ and $B^k()$ is sufficiently small.

Chapter 2: Public Insurance in Heterogeneous Fiscal Federations - Evidence from American Households

Johannes Fleck and Chima Simpson-Bell

Abstract

The literature on fiscal federalism usually argues that policies involving income reallocation should be administered by the highest level of government. This argument, however, neglects a uniformity constraint, which limits regional variation in its tax and welfare policies. Our paper explores the extent to which income support for poor households varies across US states due to the interaction between the federal government's uniformity constraint and regional variations in local economic conditions and the net transfer policies of state governments. Our results are based on a simulation of the combined response of federal and state net transfers to a pre-tax earnings shock. They point to large differences in the level of insurance against income shocks experienced by households with low incomes in different states.

1 Introduction

Which level of government should be responsible to design and implement policies providing income support to poor households? The overall majority of economists studying this question argue that it should be the highest level, i.e. the central or federal government. Their main argument is that tax base mobility and regional variation in fiscal capacity preclude lower levels of government from implementing such policies successfully[1]. However, the highest governmental level typically suffers from a regional uniformity constraint; because of equity considerations, federal governments have to treat citizens residing in different regions of the country alike and typically only condition their social policies on variation in nominal income and household demographics. While this limitation is an important pillar of a country's cohesion, it can have unintended implications for the efficacy of federal policies

[1] In contrast, lower levels of government are considered to have better information on preferences of local residents regarding public goods. For this reason it is generally recommended that local governments play an active role in their provision.

when regions are very heterogenous. This is particularly true for policies designed to insure households against economic hardship by providing a social safety net.

In his classic work 'Democracy in America', Alexis de Tocqueville characterizes the implications of this uniformity constraint 'a great cause of trouble and misery' in unitary nations such as his native France. Regarding the United States of America, however, he was confident that due to her federal character, her citizens would not suffer the consequences of this specific policy restriction. Yet, in recent years, given increasing economic inequality with a pronounced geographic element, the interaction of a uniform federal tax and transfer schedule and regional variation in incomes, price levels and state policies receives more and more attention by US policy makers and scientists.

Academic proposals aiming to alleviate the adverse effects of the uniformity constraint usually belong to one of two types: price indexation of federal policies and placed based policies. An example for the former are Bronchetti et al. (2017). They find that geographical variation in food prices causes severe regional differences in the real value of the federal Supplemental Nutrition Assistance Program (SNAP) benefits, which are fixed in nominal terms nationwide. Their suggestion is to reform SNAP and to index its benefits to a regional price indicator. Regarding placed based policies, Austin et al. (2018) argue that domestic migration alone can no longer dry out pockets of unemployment and poverty. They advocate for federal employment subsidizes to regions with high non-working rates, explicitly suggesting targeted, non-uniform federal policies.

In this paper, we re-examine empirically the effect of the given division of fiscal responsibilities in the US federal system on income support for poor households. We put particular emphasis on regional economic heterogeneity and our examination recognizes and accounts for two sources of regional variation in the US which may interact in undesirable ways with the federal government's uniformity constraint:

1. **Differences in economic conditions** There are large and persistent variations in price levels and income distributions between US states. In other words, the purchasing power of one US Dollar varies not only over time but also across states. This variation is illustrated in more detail in Section 2.

2. **State policy autonomy** US state governments have considerable autonomy in choosing their own budgetary policies which is reflected in both taxes and transfers to households. Section 2 provides more details on respective differences.

In order to elicit the impact of the interaction of 1. and 2. with the federal government's uniformity constraint on income support for households, we consider how the disposable income of a prototype household responds to changes in pre-tax labour earnings, and how this response changes with location. To this end, our microsimulation includes the main US means tested welfare programmes as well as income taxes and tax credits administered at the state and federal levels. In addition, we adjust our results for local living costs, allowing us to measure the impact of the regional variation in price levels and to express the changes in disposable income in comparable household consumption units. Furthermore, our simulation approach allows to separate the contributions of federal policies and state policies, and compare households with the same real income living in different states.

For the purpose of our simulation we abstract from the possibility of job loss and focus on fluctuations of disposable income due to earnings risk. There is ample evidence on the importance of household earnings risk, which can result from self-employment, shocks to productivity or performance-related bonus pay. To illustrate the ubiquity of this kind of risk, we refer to results shown in Storesletten et al. (2004) who report that for an average worker in the US, the annual standard deviation of these shocks amounts to 11,500 in 2004 US-Dollars. Daly and Valletta (2008) provide similar evidence for Germany, Great Britain and the USA.

A natural question to answer within this framework is whether the pattern of state policies mitigates the distortions caused by the uniformity of federal policies. In other words, we might expect that the redistribution motive within states would push towards more uniform support for the poor across the entire country. In addition, federal grants to state governments are designed to mitigate differences in fiscal resources; Medicaid grants to states, for instance, are more generous for poor states than they are for rich states[2]. Yet, our findings suggest that this conjecture is incorrect; we find that an earnings shock to a poor household triggers a much weaker net transfer response if the household is located in Mississippi, a poor state, than if it is located in Massachusetts, a rich state.

Related Literature The question which level of government should be assigned what kind of responsibilities is known as the *assignment problem* in the literature on fiscal federalism. Under the assumption of a benevolent social planner, the so called 'first generation' fiscal federalism (FGFF) provided the normative insight that policies involving income reallocation are best implemented by higher levels of government. Prominent examples of this view are Oates (1999), Ladd and Doolittle (1982), Inman and Rubinfeld (1996) as well as Boadway and Tremblay (2012). A more recent strand of the fiscal federalism literature, called 'second generation' fiscal federalism (SGFF) has challenged a fundamental assumption of these papers which is that subnational policy makers can be considered altruistic agents of the federal government. SGFF emphasizes the fact that subnational policy makers have their own fiscal and political interests, which may differ from those of benevolent federal planners (Oates, 2005; Weingast, 2009). As it has been widely documented that US state governments engage in considerable income reallocation (Moffitt, 2016; Baicker et al., 2010; Gordon and Cullen, 2012) – which contradicts

[2]The amount of Medicaid grants to states is determined by the federal medical assistance percentage (FMAP) which stipulates the percentage of a dollar of state spending on Medicaid which will be matched by the federal government. FMAP rates are a function of state per capita income relative to the US average, and vary from a legislated floor of 50% to a maximum of 83%

the normative conclusions of FGFF – our model captures variation in state policies and its interaction with the uniform federal tax and transfer schedule in great detail.

Albouy (2009) corroborates our emphasis on the federal uniformity constraint and provides numerical estimates for the shadow cost of the uniform income tax schedule. This paper observes that since US federal income taxes are based on nominal income, workers with the same real income pay higher taxes in high-cost areas than in low-cost areas. An empirical simulation suggests that the resulting incentives to relocate to low-cost areas have considerably lowered employment, house prices and land values in high cost areas in the long run. Conversely, Albouy (2012) finds that fiscal equalization payments to Canadian provinces, which aim to overcome differences in tax bases, impose inefficiency costs of 0.41% of income. In particular, equalization payments skew benefits towards less productive and less amenable provinces, and discourage workers from living in highly productive areas where wages are high.

There are also some papers which attempt to evaluate spatial distortions caused by the uniformity of federal taxes and transfers to households. Kaplow (1995) asks whether adjustments to the US Federal tax system to account for regional cost of living differences would promote distributional objectives, and whether such adjustments would be efficient. The discussion notes that differences in local amenities (like crime or pollution) mean that using standard cost of living indices to achieve this would be misleading, and suggests comparing wages across regions for identical occupations as a basis for measuring cost of living differences. Using a formal model, Glaeser (1998) demonstrates that the optimal indexing scheme depends on the complementarity between local amenities and other consumption goods. Based on the correlations between state price levels and Aid to Families with Dependent Children (AFDC) benefits, which were set by state authorities, this paper finds a level of implicit indexing in these benefits which is probably too high to be optimal.

Our paper also relates to the literature advocating for location-based policies in the US. As mentioned earlier, Austin et al. (2018) propose targeting of pro-employment policies towards American regions with high rates of long-term unemployment and non-employment. They document the failure of convergence of living standards across regions in recent years and present evidence that the labour supply response to public interventions is greater in more distressed areas. Bronchetti et al. (2017), also mentioned earlier, propose that adjusting SNAP benefit levels to account for geographic variation in food prices might improve healthcare and school engagement for children in low-income households. Ziliak (2016) and Hoynes and Ziliak (2018) also suggest similar adjustments to the SNAP benefits formula to alleviate food insecurity.

Our household perspective complements the existing empirical literature on risk sharing and redistribution between US states which considers changes in income at the state level. For example, Von Hagen

(1992) finds that the US federal fiscal system provides little insurance against shocks to state income. Asdrubali et al. (1996) decompose the sources of risk sharing between states into private and public insurance; while they find that the federal government does play a role in smoothing gross state products, much more insurance is provided by capital and credit markets. More recently, Rodden and Wibbels (2010) include the US in a panel of seven federations for which they find that central government grants contribute to rather than alleviating the pro-cyclicality of subnational government spending. We depart from this specific literature by thinking of the purpose of transfers in terms of household welfare as opposed to state budgets, and therefore conducting the empirical analysis at the household level.

The closest paper to ours is Hoynes and Luttmer (2011), which evaluates the welfare impact of state tax and transfer programs. In their paper, the welfare value of these programs is decomposed into a redistributive value, derived from the response to predictable income changes, and the insurance value, which responds to unexpected shocks. Grant et al. (2010) also contains an analysis related to our paper, finding a negative correlation between state levels of redistributive taxation and the standard deviation of the consumption distribution. Our research question is different from these papers in that we examine the possibility that state policies may be partly designed to address limitations in federal policies. In terms of methodology, our paper is related to Dolls et al. (2012). This paper compares the ability of the tax and transfer systems in the US and 19 European countries to provide income insurance against aggregate shocks to gross income and employment, using microsimulation of the automatic stabilizers in these countries. The results indicate a slightly stronger response of automatic stabilizers amongst EU countries than among US states.

The rest of the paper is organized as follows. In section 2 we clarify the available channels of income support in the US by outlining the main features of the American federal fiscal system, in particular highlighting evidence on the variation in state level resources and welfare policies. In Section 3 we describe the simulation which we perform to capture the responses of federal and state net transfer systems to changes in pre-tax income. In Section 4 we present our results. In Section ?? we offer concluding remarks.

2 The United States Federal System

In this section we provide evidence on the policy restrictions faced by federal policy makers which motivate our investigation. We do so by refering to the fiscal structure of the US as an exemplary federation. We present the basic flows between the federal budget, state budgets and households in figure 1.

Both the federal and the state governments raise revenue through taxes on income, consumption and property, although the composition of taxes varies widely. In figure 1, federal and state tax schedules – including tax credits – are illustrated by τ^f and τ^s, respectively. Note that households residing in both states face the same federal tax schedule. Welfare programs to targeted groups of households are provided through several funding systems, including:

- Direct transfers from the federal government to households financed from federal revenues (g^f)
- Direct transfers from state governments to households financed from state revenues (g^s)
- Transfer programs implemented and administered at the state level, using grants provided by the federal government (G^f). These can also include programs which use a mixture of funding sources, such as Medicaid, which is co-financed by state and federal revenues.

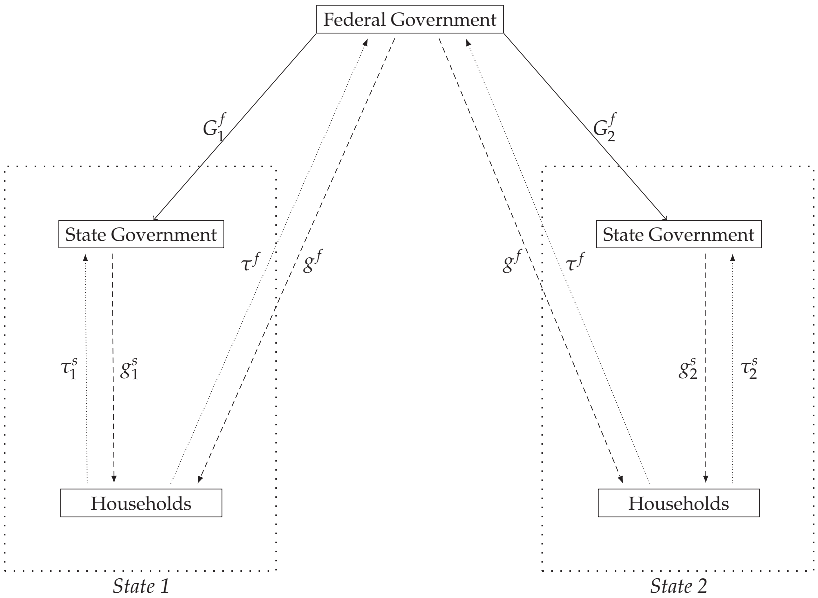

Figure 1: US fiscal flows. τ refers to taxes, g to transfers, and G to intergovernmental grants

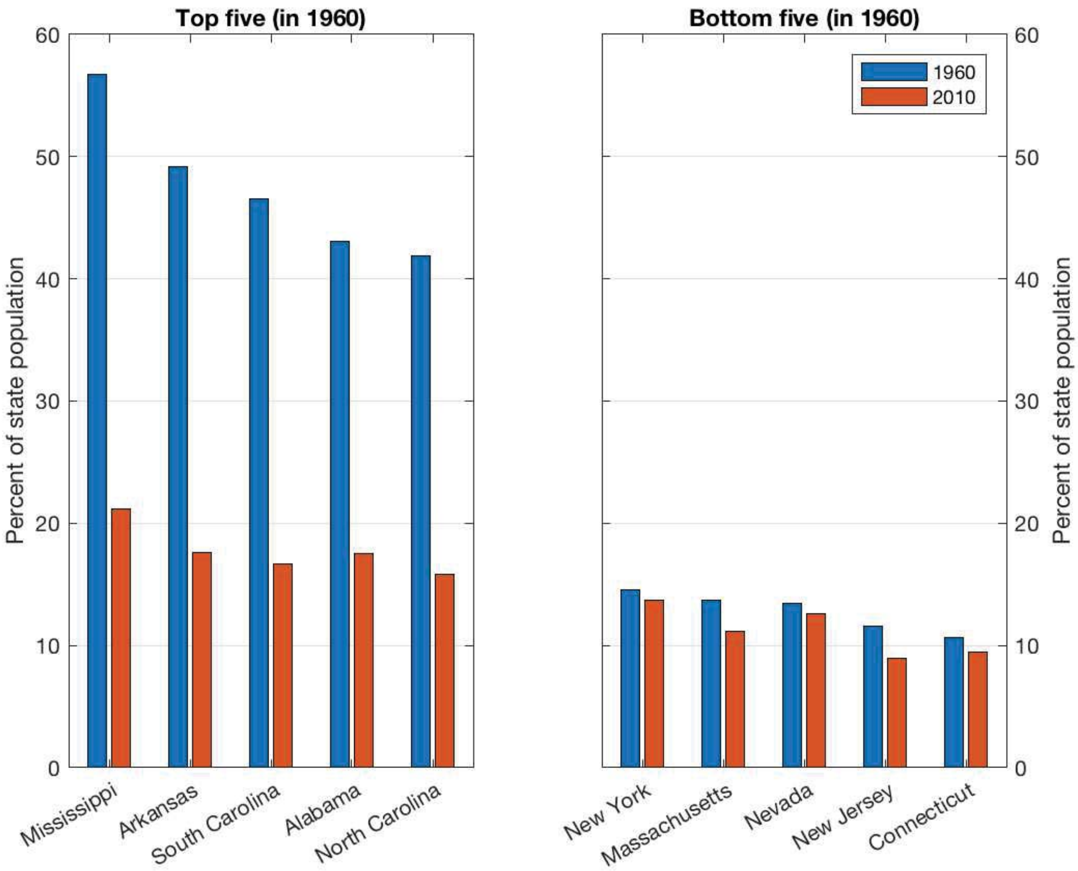

Figure 2: Percentage of families in poverty by state

Due to the uniformity constraint, the Federal government cannot condition its direct interactions with households on their state of residence. A practical implication of this restriction is that net direct transfers from the Federal government to identical households in different states will be equal in *nominal* terms. If, however, there is a welfare justification for equalizing transfers across states, it would make more sense for these transfers to be equal in *real* terms. As discussed in Ribar and Wilhelm (1999) the funding design of intergovernmental grants (matching, block, etc.) can change the effective price of spending on welfare programs. Nevertheless, as we show in the following paragraphs, the implementations of tax and transfer programs by state governments on the one hand and differences in living costs and incomes on other hand are substantial. Hence, despite targeted intergovernmental grants, it remains plausible that the combined tax and transfer system provides unequal income support across states for households with similar earnings.

Variation in Local Conditions Both the needs and the resources of the states differ because of variations in state income distributions. Figure 2 indicates the extent to which the demand for income support may vary between states, showing the proportion of households in poverty as defined by the federal poverty line in 1960 and 2010[3]; the overall incidence of (measured) poverty decreased considerably over this period, but it was still roughly twice as high in Mississippi than in a richer state like Connecticut or New Jersey in 2010. Sommeiller and Price (2014) document the variation in the prevalence of very rich taxpayers in different states; for example, the average income of the top 1% in Connecticut was $2.2 million in Connecticut in 2011, compared with $635K in Hawaii. Moreover, the states with relatively large populations of poor households tend to be those with fewer rich taxpayers. These distributional variations may limit the amount of redistribution which some of the states could achieve by themselves, suggesting a role for the federal government to reallocate resources *between* states.[4].

The variability of local economic conditions across states can also be seen in price levels. Figure 3 shows the states with the lowest and highest living costs as measured by regional price parities compared to the US average. We see that the price level faced by household in Hawaii is almost 20% higher than the national average, whereas that faced by a household in Mississippi and Alabama is almost 15% lower than the average. It is easy see that this introduces considerable variation in the welfare impact of federally administered policies, which tend to be fixed in nominal terms.

Variation in State Policies To illustrate the variation in state income tax systems, we compute a measure of state tax progressivity for each state. This measure reflects the income elasticity of net state

[3]The Federal Poverty Line is a benchmark household income level used by the US government to determine eligibility for federal aid. It does not vary between states except for Alaska and Hawaii, which are assigned higher levels than the contiguous states and the District of Columbia.

[4]In addition, the overall majority of states have self-imposed balanced budget rules which provide legislative restrictions on the accumulation of debt.

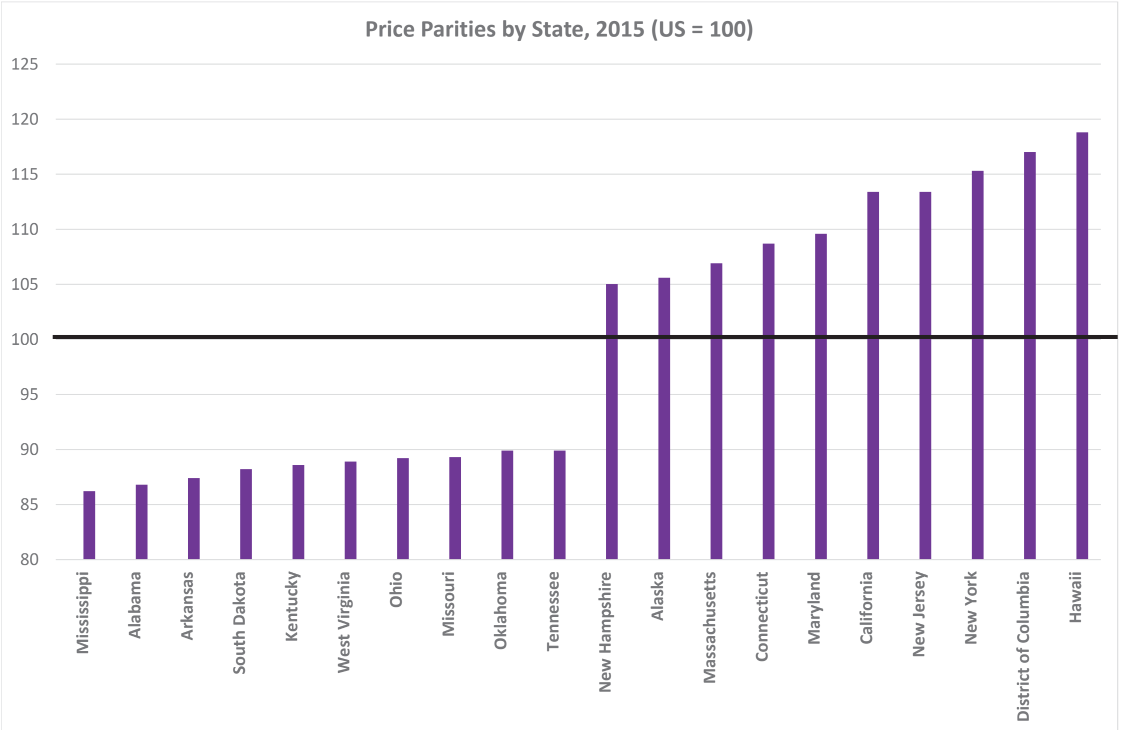

Figure 3: Regional Pricing Parities by state 2015, 10 highest and 10 lowest

collections, estimated using the constant elasticity functional form shown in Heathcote et al. (2017) to provide a good fit for net taxes at the national level. The estimates of the progressivity parameter γ for the year 2000 are shown in Figure 4. As well as state income taxes, the results also reflect the impact of deductions and state earned income tax credit systems. In general, state income tax schedules are less progressive than the federal tax, but there is large variation across states, from no income taxation (e.g. Texas), to uniform taxation (e.g. Tennessee) to more progressive systems where, for example, New York and California have a special top rate for millionaires. Regarding the income taxation of households which are considered poor Oliff et al. (2012) document that working poor families pay state income taxes in several states while a few - Alabama, Georgia, Illinois, Montana and Ohio – even levy taxes on two parent families of four earning less than 75% of the federal poverty line.

We also see considerable differences across states in the provision of welfare programs. These differences are reflected best by programs for which the federal government provides part of the funding to states and specifies some minimal policy objectives but where states are given autonomy in the implementation details. This allows the state governments to determine which services are provided, the eligibility requirements and the generosity of provision to recipients. Figure 5 shows the percentage of

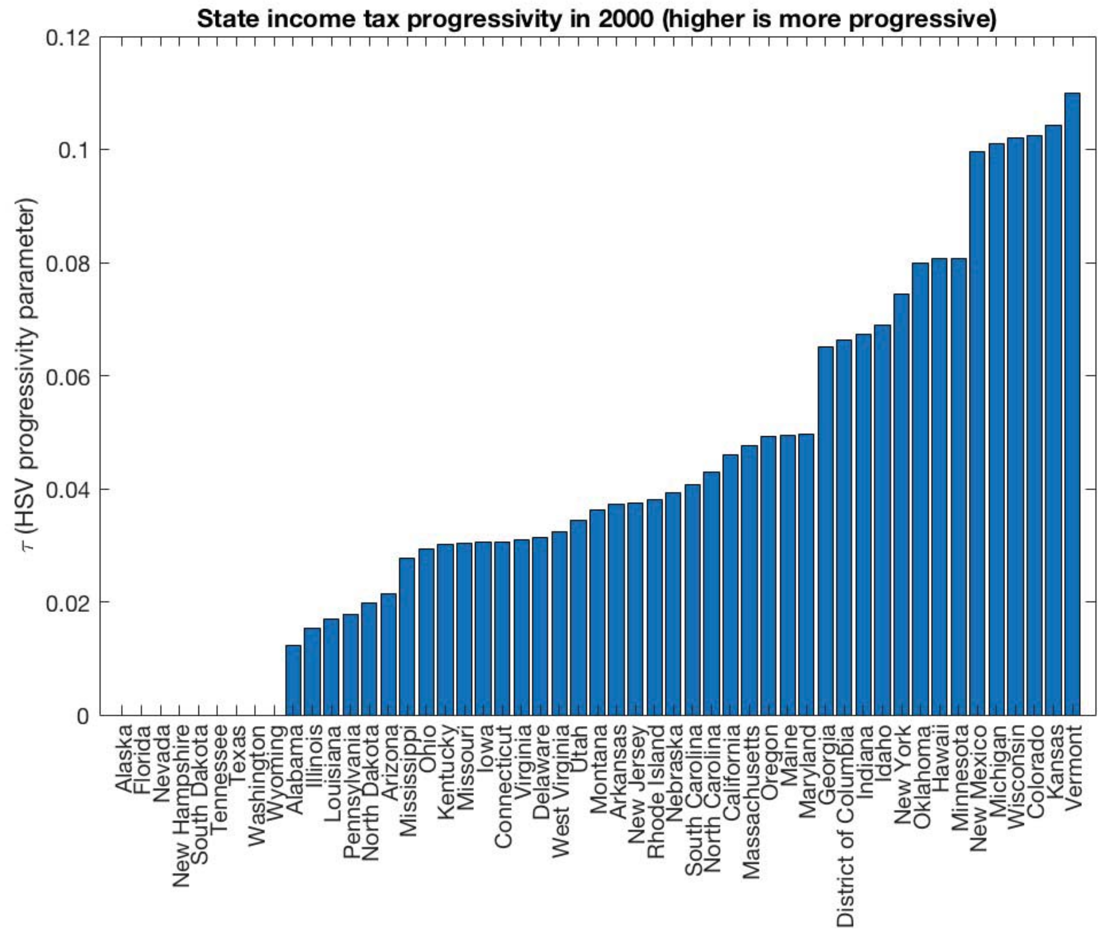

Figure 4: State tax progressivity parameters γ for HSV tax function $T(y) = y - \lambda y^{(1-\gamma)}$. $\gamma = 0$ can indicate either uniform or zero income taxation.

the population of each state receiving Temporary Assistance to Needy Families (TANF) funds, which provide cash assistance and work incentives to low income families, and the maximum monthly benefit. Figure 6 shows the recipient percentage and average monthly benefit for Medicaid[5]. In the case of Medicaid, even states which have similar proportions of recipients, such as Kentucky and New York, have widely differing benefit levels. It is also striking, however, that the availability of benefits does not seem to mirror the indicators of need – California, for example, has one of the most expansive implementations of TANF despite having one of the lowest proportions of poor households.

While in the rest of this paper, we focus on taxes levied on earnings, states also differ in the composition of their revenue sources. In 2013, the states which drew the highest proportion of their total revenues from taxes on individual income were California, Connecticut and New York with 21%, 19% and 19% respectively[6]. However, state and local governments also draw significant revenues from taxes

[5] We should note here that the Medicaid benefits are in kind in the form of medical services rather than direct cash benefits. We provide equivalent cash values here.

[6] State & Local Government Finance Data Query System. http://slfdqs.taxpolicycenter.org/pages.cfm. The Urban

on corporations, property taxes and sales taxes as well as charges for licenses and financial transactions taxes. In 2013, some states relied much more on property taxes, with New Hampshire generating 36% of its revenue from this source and Texas 22%. Nevada and Washington, in contrast, are heavily dependent on sales taxes, which provide 34% and 32% of their revenues respectively. These alternative funding sources may determine the flexibility which different states have to use income tax schedules to achieve distributional goals.

Institute-Brookings Institution Tax Policy Center. Data from U.S. Census Bureau, Annual Survey of State and Local Government Finances, Government Finances, Volume 4, and Census of Governments (2013). These figures also include the revenue of local governments.

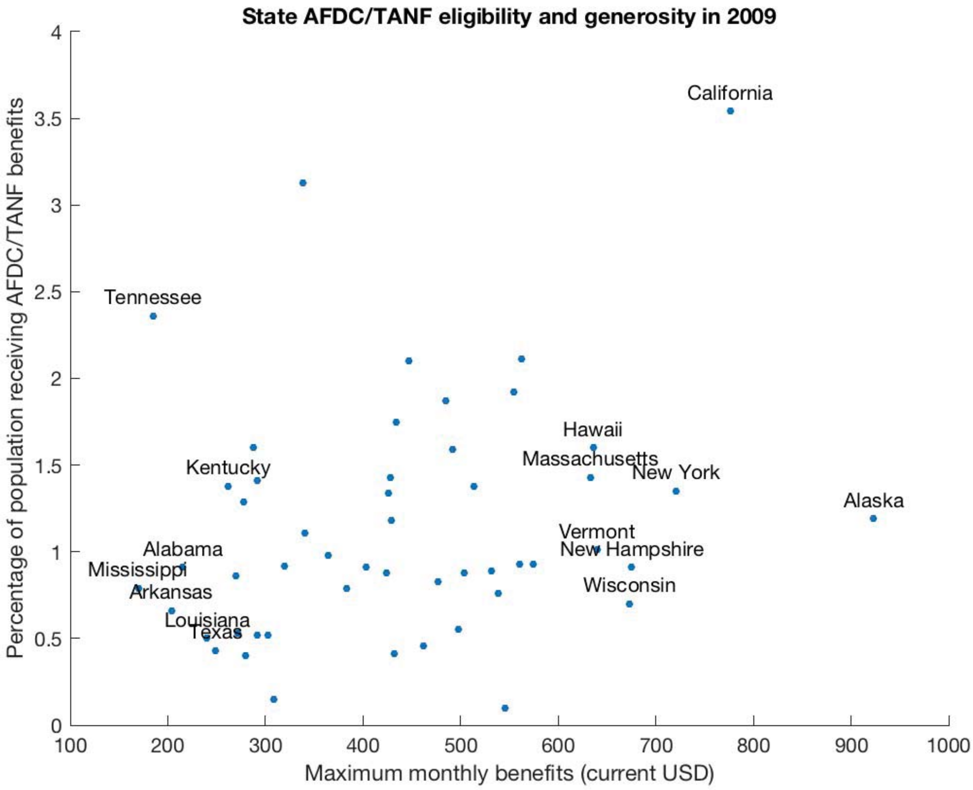

Figure 5: TANF Recipient density and benefit generosity by state in 2009

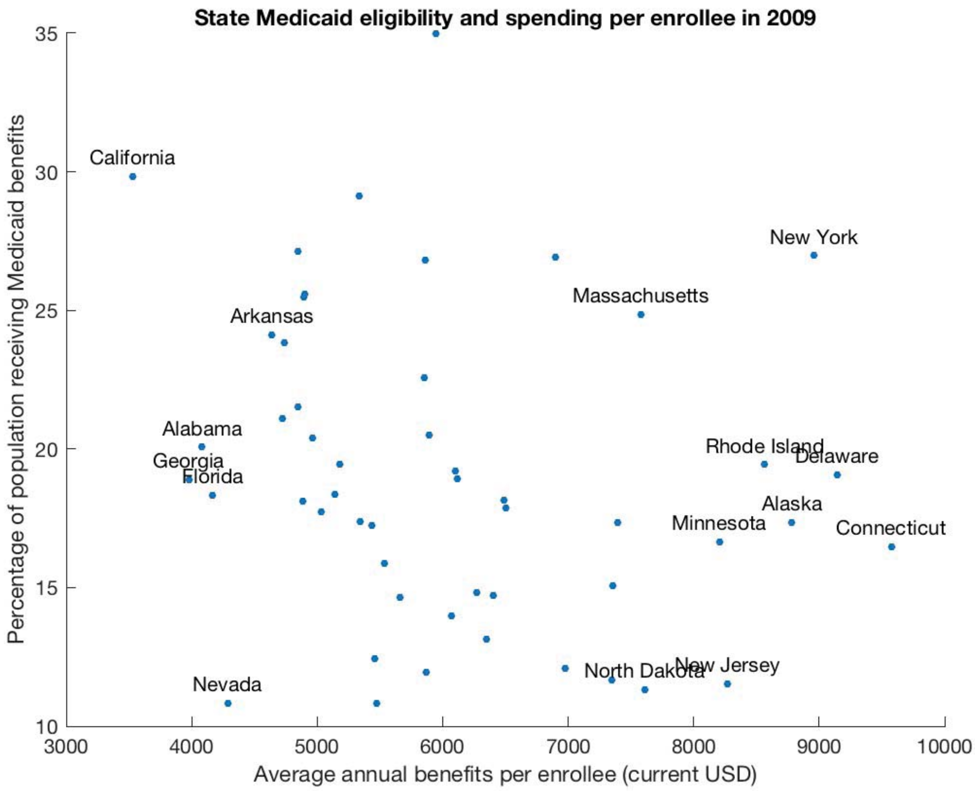

Figure 6: Medicaid Recipient density and benefit generosity by state in 2009

3 Simulation exercise

The basic idea of our simulations can be summarized as follows. For a given year, we place a prototype household in each US state with a given level of pre-tax earnings. We then shock this pre-tax income and calculate the change in ***disposable*** income which results from the response of federal and state policies. In effect, our results reflect effective marginal tax rates as presented in Maag et al. (2012), Holt and Romich (2007) or Congressional Budget Office (2012), for different years and states. In this section, we provide details on the simulation model and the experiments we employ it for. We also describe the prototype family we refer to as our household of interest, report our data sources and present the measure of insurance which we compute from the experiments.

3.1 Modelling Household Insurance

We start our investigation from the most general representation of a household's income and spending opportunities, i.e. the household budget. In any period t, the associated equation can be written as

$$w_t + a_{t-1}r = c_t + (a_t - a_{t-1}) \tag{1}$$

where w_t denotes gross (before tax and transfer) labor income, a_{t-1} is the amount of savings the household set aside in the previous period (so $a_{t-1}r$ is the net return earned on those savings) and c_t denotes consumption expenditures. Finally, $a_t - a_{t-1}$ reflects the household's change in its net asset position. In other words, if $a_t - a_{t-1} > 0$ the household increases its assets relative to the previous period, while it reduces them if $a_t - a_{t-1} < 0$. We proceed by noting that gross labor income is the sum of household transfers b_t, labor income taxes τ_t and disposable (after tax and transfer) labor income y_t. If we further index taxes and transfers by governmental level $l = \{s, f\}$, we can express gross labor income as

$$w_t = y_t + b_t^s(w_t) + b_t^f(w_t) - \tau_t^s(w_t) - \tau_t^f(w_t) \tag{2}$$

where, for the sake of exposition, we assume taxes to be always positive for now but we relax this assumption later to account for tax credits in our experiment. Note that in this exposition, we explicitly list transfers and taxes as functions of gross labor income. For reasons of notational convenience, we drop their arguments from now on.

Substituting terms, we re-write equation (1) as

$$y_t + b_t^s + b_t^f + a_{t-1}r = c_t + \tau_t^s + \tau_t^f + (a_t - a_{t-1}) \tag{3}$$

where those variables which typically represent sources of income are on the left hand side, and those which are commonly expenditure items are on the right hand side. We assume the interest rate r to be time-invariant. For the purpose of our investigation, this assumption is easily justified: we are interested in the response to an unexpected labour income shock which is specific to a single family. Hence, this shock does not affect aggregate variables such as the return on savings. Moreover, as our object of interest is a single household, it seems natural to assume that changes to its net asset position do not affect the interest rate.

To simplify equation (3), we define net transfers received from government level as $T_t^i = b_t^i - \tau_t^i$ where we allow T to take any sign. Applying this definition and reordering terms we can re-write the above equation as

$$y_t = c_t - T_t^s - T_t^f + a_t - a_{t-1}(1+r) \tag{4}$$

Next, we iterate forward by one period so the equation becomes

$$y_{t+1} = c_{t+1} - T_{t+1}^s - T_{t+1}^f + a_{t+1} - a_t(1+r) \tag{5}$$

From now on, we conceptualize the difference between labour incomes in consecutive periods as a shock. We denote this shock as $\varepsilon_t = y_{t+1} - y_t$ and use the difference operator Δ to rewrite it as $\varepsilon_t = \Delta y_t$.

Finally, we subtract item by item of equation (4) from equation (5) which yields

$$y_{t+1} - y_t = c_t - c_{t+1} - T_t^s - (-T_{t+1}^s) - T_t^f - (-T_{t+1}^f) \tag{6}$$

$$+ a_t - a_{t+1} - a_{t-1}(1+r) - (-a_t(1+r)) \tag{7}$$

Using the difference operator as defined above, this equation can be written as:

$$\varepsilon_t = \Delta c_t - \Delta T_t^s - \Delta T_t^f + \Delta a_t - (1+r)\Delta a_{t-1} \tag{8}$$

We can now discuss the channels of insurance which are available to a household facing a shock to earnings, and outline in detail which channels we explore in our experiment. We identify two types of channel: public insurance and private insurance.

1. **Public insurance** – resulting from government policies. We assume full tax compliance and benefit take up by the household and we abstract from the policies of local (i.e. municipal or county) governments.

   - Income taxes (τ) – we include state and federal tax credits in this category and thus allow taxes to be either positive or negative, where a negative tax payment corresponds to receipt of a tax credit
   - Transfers (b) – receipts from means tested welfare programmes administered by the federal and state governments. These can only take positive values.

In subsequent sections of the paper, we will refer to *net transfers* T as defined above, i.e. the difference between transfers received and taxes owed.

2. **Private insurance** – resulting from household decisions. Our experiment does not include these channels but we outline their potential sources and the modelling assumptions under which they might be restricted.

- Asset positions (a_t) or asset income (ra_t) – we think of the household as being hand-to-mouth in the sense of not having (liquid) assets which it can use to smooth consumption. The household is also not able to adjust asset *income* which we think to be exogenous from the perspective of the household.
- Change in labour decision – we assume that labour inputs are fixed before the earning shock so that the household cannot smooth the shock by adjusting hours worked.
- Migration – we treat the household is immobile, so that it cannot react to the shock by, for instance, moving to a different job in a different state.

As detailed above, our experiment is equivalent to shutting down the private channels of insurance, allowing the incidence of the earnings shock to be absorbed only by changes in net transfers. Any unsmoothed shocks are then mechanically absorbed by changes in consumption[7] As such, the insurance measures which we present can also be thought of as lower bounds on the smoothing which a household can achieve in each state. Our decision to ignore private smoothing channels is only important for our question if these channels differ systematically across states. For example, it could be that the ability to insure using private asset portfolios varies systematically between states. Overall, there is little evidence this could be a concern for the United States. In this regard, we want to mention that Hintermaier and Koeniger (2016) show that differences across states in homestead exemptions in bankruptcy procedures have small effects on welfare compared to a fully harmonized system.

Another possible objection to our analysis is that we do not account for mobility decisions of households as a response to changes in local earnings opportunities. In general, empirical evidence suggests that population adjustments between cities and states in response to local labour market shocks occur over long time horizons and are small (Blanchard and Katz, 1992; Glaeser and Gyourko, 2005). For example, Autor et al. (2013) examines the local labour market effect of increased Chinese import competition in the US; they find that the migration response is small even for regions which are highly exposed to imports. Moreover, several recent papers – examples are Kaplan and Schulhofer-Wohl (2017) and Johnson and Kleiner (2017) – have found a steady decrease of interstate migration which may be due to increasing costs of mobility (e.g. housing, occupational licenses, non-portable transfers

[7]This link between changes in consumption and changes in disposable income can also be derived by assuming that the household is liquidity constrained, i.e. wealthy hand-to-mouth using the terminology of Kaplan et al. (2014).

or pension schemes).

Regarding the type of shock and family our simulation covers, work by Kennan and Walker (2011) as well as Austin et al. (2018) shows that our assumption regarding immobility is plausible. On the one hand, Kennan and Walker (2011) found that migration decisions are mostly a response to permanent shocks to income. Moreover, they typically occur at specific stages of the individual life cycle, e.g. in the context of college attendance decisions. On the other hand, Austin et al. (2018), relate the persistence of labour market drop out rates in low wage areas to increased real estate rents in areas with high wages; they show that poor households cannot migrate to search for employment in highly productive areas as the local accommodation rates exceed their budgets. Hence, US domestic migration is more and more concentrated among workers who change jobs in high-skilled sectors. As we consider our prototype family to be working poor, abstracting from mobility decisions in the context of our experiment does therefore not omit a significant source of insurance.

Obvious omissions from this analysis are unemployment insurance and social security systems. We choose not to include these because their eligibility and benefit levels depend on the individual's history of labour market participation and social security contributions. We therefore do not see these as providing *unconditional* protection against a reduction in income; instead they function more as a (subsidized) form of self-insurance which is managed by the government. Table 22 provides a summary of the relative fiscal magnitude of the programs we include in our simulation. Regarding unemployment insurance, this table shows that public expenditures on this program are actually relatively small compared to those which we do include[8].

3.2 The Prototype Family

In order to capture correctly the changes in tax liabilities and transfer entitlements, we consider a household with fixed characteristics across years and states. Assigning specific characteristics is necessary as the sources of insurance we study depend not only on income but also on other features of household composition. For taxes, a critical determinant of liabilities and credits is the number of dependants of a tax filer and whether a couple is able to file jointly. In addition, the presence of children in the household grants specific child tax credits and deductions for childcare expenses. Mortgage expenditures and pension incomes are also deductible items.

Household characteristics are equally important for determining eligibility and generosity of the transfer programs we consider. For SNAP and AFDC/TANF the number of family members is a critical parameter. For Medicaid, a key element of the heterogeneity in eligibility across states is the extent to

[8]This does not necessarily apply to all years in the period considered in our experiment as emergency federal contributions during severe recessions can cause unemployment insurance to be quite substantial.

which children are covered. Hence, it is necessary to fix the ages of the children. For these reasons, our prototype family remains invariant throughout our microsimulation exercise with respect to relevant characteristics, listed as follows:

- A married couple with two children between ages 4 and 7
- Family income from two equal full time labour incomes
- No disabilities
- The family home is rented (no mortgage payments)
- No other family members occupy the home
- The family does not migrate as a response to changes in earnings

The advantage of our approach is that it allows us to capture differences in insurance provided by combined federal and state net transfers using a consistent benchmark. If we were to change family characteristics our results would be less informative for our research question. On the one hand, they would confound the differences we are interested in with potential differences in state government preferences over specific family types.[9] On the other hand, if we were to vary the family composition between states and years to reflect state averages, we would not be able to separate the effects of federal from state policies consistently.

3.3 The Subsistence Basket

For a given year, we locate our prototype household in a particular state and allocate a pre-tax income level which depends on the specific experiment. To convert the nominal pre-tax income into comparable consumption units, we use a subsistence expenditure basket. We calculate this subsistence expenditure as the sum of two components. The first is the minimum required level of monthly food spending as stated by the federal Thrifty Food Plan, a US government measure which specifies the minimum amount which a family of a given size needs to spend to consume a nutritious diet. We list the nominal amounts and the data sources in the appendix. The second is the average monthly rent payment for households in the same state with similar characteristics and income. We consider both of these components as fixed costs for the household, in the sense that once a shock to income is realized, the household cannot respond by forgoing these expenditures.

We obtain the conditional average monthly rent from the American Community Survey (ACS)[10] for each year and state by taking households from the state in question with similar income and demographic

[9] For now, we choose to ignore the possibility that differences in state policy arise from variation in the composition of the average household in each state. Such variation might be the result of a median voter theory on the determination of state welfare policies. We explore this issue in the discussion of our results.

[10] The ACS variable which we use for rent (RENTGRS) is 'gross monthly rental cost of the housing unit, including contract rent plus additional costs for utilities (water, electricity, gas) and fuels (oil, coal, kerosene, wood, etc.)' This makes the variable more comparable across households than net rents and, as our aim is to capture all household expenditures related to housing, this variable is ideal for our purposes.

characteristics as our prototype household. In general, we take households within a 5 percentile band of the prototype household in the state income distribution. As an illustration, if the prototype household is at the 10th percentile of California's income distribution, we take all households in California between the 5th and 15th percentiles and alsocondition on marital status and family size, as well as the number and ages of children. We then average gross monthly rents across families who reported incomes within these groups for a given year and state.[11]

3.4 Simulation

We subject the household's pre-tax income to a shock, which can be positive or negative. In our first experiment, the size of the shock will be calculated as a number of subsistence baskets, so that we can equalize the shock size in real terms across states. Our results measure the on-impact response of tax and transfers.[12] We can then compare disposable income (i.e. income adjusted for taxes paid and transfers received) at the pre-shock and post-shock levels. This exercise is repeated for all US states in different years.

Using the notation introduced earlier, for a pre-tax labor income in state s of w_0^s, the value of pre-tax income after the shock is given by[13]:

$$w_1^s = w_0^s + \varepsilon \tag{9}$$

The pre-shock and post-shock disposable incomes, y_0^s and y_1^s are then given by:

$$y_0^s = w_0^s - \tau^f(w_0^s) - \tau^s(w_0^s) + b^f(w_0^s) + b^s(w_0^s) \tag{10}$$

$$y_1^s = w_1^s - \tau^f(w_1^s) - \tau^s(w_1^s) + b^f(w_1^s) + b^s(w_1^s) \tag{11}$$

[11]Official measures of representative rents are computed in a similar way; for example, the Fair Market Rents (FMR), which are estimated by the US Department of Housing and Urban Development (HUD), correspond to the 40th percentile of the distribution of monthly rents of all units occupied by rent movers in a specified geographic area. As we are interested to estimate results for households with low incomes we condition their average rents on the income distribution as described above. In some rare cases, our collected average rents flucturate substantially from year to year. The reason for these anomalies are low number of observationss. For those state and year cells, we widen the bands of the incomes to the 2 and 18 percentiles, respectively. We then replace values which are below or above 20% of the corresponding FMR value. We report on these procedures in more detail in the appendix.

[12]Since we are simulating an idiosyncratic shock to a selected household, we also avoid any endogeneity problems by which the policy choices at the state or federal level might respond to the change in economic conditions. As discussed by Bourguignon and Spadaro (2006), this is a key advantage of using a simulation approach to evaluate the effects of policies.

[13]Here we suppress notation referring to the characteristics of the household, which will also determine eligibility for welfare programs and income tax credits. As explained above, we hold the relevant characteristics of the household fixed throughout the exercise.

3.5 Policy Calculators

In order to impute the taxes and transfers associated with different gross labor incomes in different states and years – as shown in equations (10) and (11) – we construct a simplified version of the US tax and transfer system. To cover the main income support programs, we apply a combination of previously available calculators and design a new calculator for SNAP. We impute Temporary Assistance for Needy Families (TANF, formerly Aid to Families with Dependent Children) and Medicaid benefits by employing the calculators used in the paper Hoynes and Luttmer (2011)[14]. Compared to TANF and Medicaid, the eligibility and benefit parameters of SNAP (formerly Food Stamps) are much easier to model and we design a calculator ourselves. We provide further details on the calculators, including references and data sources, in the appendix. We impute taxes by using TaxSim. This software covers both federal and state income taxes and, most importantly,includes Earned Income Tax Credits (EITC) of fiscal administrations from both levels as well as state specific child tax credits. As TaxSim is widely used for the purpose of imputing income taxes, we defer a more detailed discussion on its accuracy to the appendix. Taken together, the transfer and tax programs covered by our simulation model account for large proportion of welfare spending in the US. Their relative contributions to total expenditure are summarized for the year 2007 in figure 25 shown in the appendix.

3.6 Our Insurance Measure

For each of the experiments we consider several shock sizes ε_i, $i = 1, 2, \cdots, N$. Our measure of the insurance provided by the combined tax and transfer system in state s:

$$\chi_i^s = 1 - \frac{y_0^s - y_{1,i}^s}{\varepsilon_i} = 1 - \frac{y_0^s - y_{1,i}^s}{w_0^s - w_{1,i}^s} \tag{12}$$

Thus, if disposable income declines by the entire amount of the shock ε, the insurance measure χ will be zero, indicating that the system provides no insurance against the shock; conversely if disposable income does not decline at all, χ will be one, indicating full insurance. Intermediate values indicate partial insurance[15]. More generally, if household ***pre-tax*** income undergoes a negative shock of one subsistence expenditure basket, $1 - \chi$ is the number of baskets lost from ***disposable*** income due to changes in state and federal taxes and transfers.

After calculating the net transfer policy responses for different shock sizes, it will be useful to calculate a single measure which summarizes the level of insurance for a particular state in a given year. We do this by calculating an average of the insurance measure χ_i^s across shock sizes ε_i, weighted by the shocks themselves. We define the average insurance measure $\bar{\chi}^s$ for state s:

[14] They have been provided to us by Hilary Hoynes and Erzo Luttmer which we gratefully acknowledge.

[15] It is possible for states to have values above 1, in which case after tax transfers increase more than the amount of the shock, and negative values, which indicate that disposable income actually falls more than the value of the shock.

$$\bar{\chi}^s = \frac{\sum_{i=1}^{N} \varepsilon_i \chi_i^s}{\sum_{i=1}^{N} \varepsilon_i} \tag{13}$$

It is also informative to break down the relative contributions of the federal and state governments, which should allow us to explore the extent to which state government policies drive the differences in outcomes. For government level $l \in \{s, f\}$, in state s we define $\chi_i^{s,l}$ as

$$\chi_i^{s,l} = \frac{\tau^l(w_0^s) - \tau^l(w_{1,i}^s) + g^l(w_{1,i}^s) - g^l(w^0)}{\varepsilon_i} \tag{14}$$

which is the ratio of the change in net transfers from government level i to the income shock.

3.7 Our experiments

With this framework in place, we conduct two different experiments:

1. Give the household a pre-tax and transfer income which in each state has the same real value as the Federal Poverty Line (FPL). We find this income as follows: We compute the nominal cost of a 'national' subsistence basket using the procedure described above except that we do not condition on state of residence. Hence, the nominal cost of this basket reflects the monthly rent and food expenditures averaged across all US states. Next, we divide the FPL value for a family of four by the cost of this basket. For example, for the year 2004, this corresponds to a number of 34 baskets. Finally, we give the family a nominal pre-tax and transfer income w_0^s in each state s such that it can afford 34 baskets. We then subject this income to a shock of ε baskets. Thus, when we vary the shock size ε_i, we are considering different numbers of subsistence expenditure baskets. We vary the shock size in steps of 0.5 baskets between -5 baskets and +5 baskets.

2. Give the household a pre-tax and transfer income w_0^s which corresponds to the 10th percentile of the income distribution in each state (and year) and subject it to a shock of ε%. Thus, when we vary the shock size ε_i we are taking away (or adding) different proportions of the initial income. We vary the shock size in steps of 5% from -20% to + 20%.

The first experiment is the closest to answering our research question, which is to what extent the federal system implies differential insurance conditional on state of residence. The second experiment aims to explore the results of the first in more detail and to account for difference in states policies. As the real income of a household who is poor *relative to its state peers* differs across states, state policies to help low income households could be targeted towards households with different real incomes across states.

4 Results

In this section we present the results of our two main experiments.

Experiment 1 (Fixed Real Income) The main results of the first experiment are shown in figures 7, 8 and 9 for the year 2004. Figures 7 and 8 show the responses in different states to a negative income shock equivalent to 3.5 subsistence baskets, i.e. 10% of pre-tax number of baskets. To help build intuition, we begin by describing the results in terms of the number of consumption baskets lost from disposable income in Figure 7. This varies from a minimum of roughly 1.9 baskets in Oregon to a maximum of just over 3.2 baskets. Thus, while in states with very high insurance almost half of the negative earnings shock is absorbed by changes in net transfers, in most states almost all of the shock is reflected in disposable income. Figure 8 shows the same results in terms of our insurance measure χ_s, which varies between 0.07 (Idaho) and 0.46 (Wisconsin), with most states falling between 0.05 and 0.15.

In Figure 9 we consider a more robust measure of the level of insurance in each state by plotting the decomposition of state insurance contributions against federal contributions, averaged across the different shock sizes as described in the previous section [16]. In this plot, states with higher *total* insurance are also represented by larger circles, while the colours of the circles indicate how high living costs are in each state. In this figure a clear group emerges of states with relatively high total insurance made up of large contributions from both levels of government: these states are Wisconsin, Oregon and New York (they are also joined by the District of Columbia). Below these there is another larger group of states which have high total insurance which is mostly provided by net transfers from the Federal government. This second group includes Washington, Nevada, California, Virginia and Delaware among others.

To determine the drivers of these results we look more closely at the two sources of variation in this experiment: net transfer policies and nominal incomes. Recall that nominal incomes differ because while we give the household the same number of subsistence baskets in each state, the nominal cost of these baskets varies by state. In Figures 10 and 11 we plot SNAP and Medicaid entitlements before and after a negative earnings shocks of 3.5 subsistence baskets. In Figure 10 we see that all states experience an increase in SNAP entitlements; importantly, in no state are the SNAP entitlements exhausted after the shock. In proportional terms, those states which begin with a low level of SNAP entitlement before the shock experience the largest increase in entitlements. Thus, for states like New York, Oregon, Nevada, Washington and New Jersey, the increase in SNAP entitlements makes a large contribution to insurance from the Federal government, although as we will see, the cash amounts involved are relatively small compared to changes in tax liabilities. Medicaid does not seem to be an important insurance source for

[16] The complete set of decompositions for each state and each of the different shock sizes can be found in Appendix B

the high insurance states. In Figure 11 we see that in the vast majority of states, the household does not experience any change in its Medicaid entitlements. However, this figure does identify a small group of states which start with relatively low Medicaid entitlements and then display a very steep increase after the shock. This group includes Colorado, Mississippi, Utah, Tennessee and Texas. This strong response of Medicaid entitlements places many of these states in a middle group in Figure 9 of states with above average total insurance and a federal insurance contribution close to 0.15.

In Figures 12 and 13 we see the response of state and federal income taxes in the same experiment, noting that negative amounts refer to tax *credits*. Federal income tax policy, which at this income level refers to the EITC, is much more responsive for the high insurance states identified in Figure 9, as can be seen by their distance from the 45 degree line in Figure 12. Given the design of the EITC, this must mean that in these states the household is located slightly above or on the 'phase out' portion of the EITC schedule, so that its earnings and credit entitlements move in opposite directions; this in turn results from the fact that it has a relatively high *nominal* income because living costs in these states are high. In Figure 12 this can be seen that the fact that the response are completely ordered by the level of living costs in each state.

State income tax liabilities, on the other hand, seem to be much less responsive, with most states levying close to zero taxes on the household both before and after the shock. Oregon and New York, however, are among the states which offer the largest decrease in tax liabilities in response to the shock, which partly explains why the contributions of both state and federal government net transfers are relatively high in these states.

Overall, the results of this experiment point to the fact that households with low real incomes living in expensive states have low Federal entitlements before receiving the shock, as we would expect from the nominal uniformity of federal policies. However, Federal policies then become a very important source of insurance in response to a negative income shock for these households as they become eligible for increased SNAP benefits and EITC payments. Conversely, household with low real incomes living in cheap states, which by construction also have low nominal incomes, have already exhausted most of their entitlements before receiving the shock. They therefore do not receive much insurance in response to an income shock. There is also a small group of states like New York, Oregon and Wisconsin in which low income households receive extra insurance due to the progressivity of the state income tax schedule. In the majority of states, the household relies mostly on increases in SNAP entitlements for insurance against income shocks. A handful of states also offering increased Medicaid entitlements, which reflects the fact that they had not yet exhausted their benefits before receiving the shock.

We note that while the real income level we have chosen for this experiment is relatively low, it does not necessarily put the household at the bottom of the income distribution in each state. This is important because one possible explanation for the pattern of results which we find could be that states like California and Nevada target their welfare policies towards households with even lower incomes than the level which we investigate; conversely, in New Hampshire the household would be one of the poorest in the state, and so we would it expect it to receive higher net transfers. We explore this explanation in the next experiment.

Experiment 2 (Fixed state income percentile) Figures 14 and 15 illustrate the average insurance measure $\bar{\chi}_s$ by state for the years 2001 and 2008. We compare different years to see how stable the insurance contributions are over time. In both of these figures we see that there is much more variation across states in total insurance than we found in Experiment one, with $\bar{\chi}_s$ ranging from close to 0 to just below 0.6 in the most generous states, where the value of 0.6 indicates that more than half of the shock to earnings is absorbed by public insurance. Compared to the first experiment, some new states emerge as high insurance locations for households at the 10th percentile of the state income distribution. As well as New York, we now find that Kansas, Iowa and Wisconsin are generous states. This is shown in Figure 17 where we average $\bar{\chi}_s$ over time to derive a measure of insurance for each state over the period 2001-2008. We also see here that households at the bottom of the income distributions of Idaho, Montana, New Mexico, Texas and West Virginia received very little total insurance over this period.

Figure 16 decomposes these total insurance measures into state and federal contributions. Again, for each state the size of the circle indicates the amount of *total* insurance while the colour of the circle indicates the cost of living in that state as indicated by the price of the subsistence baskets. A key question for our analysis is whether the level of insurance provided by the state government becomes more similar when we give the household nominal incomes corresponding to the 10th percentile of each state's nominal income distribution. It is clear from the figure that this is not the case - the average state insurance contribution now ranges from 0 to 0.15, an even wider range than in the previous experiment. The variation in state insurance levels seems to be increasing in the level of federal insurance - at the maximum federal insurance contribution (close to 0.3), the state contribution varies between 0 and 0.15.

There is also no obvious pattern in the relationship between the cost of living and the level of insurance, although we do see that the states with the highest insurance from federal net transfers are predominantly cheap states. Also, it appears that in cheap states , the household enjoys less total insurance than in medium and expensive ones (as the blue circles are generally smaller). In fact, the group of states in which the household receives significant assistance from both levels of government is also larger than in the first experiment, and now includes Minnesota, Kansas, Vermont and Iowa. With the exception of Iowa, these are all medium or expensive states.

In this experiment we again have two potential sources of variation - differences in the level of state income distributions and geographical differences in net transfer policies. We try to separate the contributions of these factors by inspecting the individual policies which we simulate. An important observation is that living costs are positively correlated with the average level of nominal income in each state; thus households which live in expensive states also tend to be richer in nominal terms. As a result of this, the overall pattern of transfer responses is quite similar to that of the previous experiment. Figures 18 and 19 display the transfer response to a negative shock of 10% of earnings in the year 2004. In Figure 18 we see that in all states, the household receives an increased SNAP entitlement in response to the shock. This increase is large in percentage terms in states where the SNAP entitlement was low before the shock was received. These states tend to fall in the Expensive and Medium living cost categories. Compared to the first experiment, there are also more states which start with zero SNAP entitlement and move to a positive amount after the earnings shock, including Colorado and Maryland.

Figure 19 shows that at the income levels which we consider, the household's Medicaid benefits have already been exhausted in most states, so that they cannot respond to the earnings shock. However, there are again some more expensive/high income states where Medicaid benefits move from zero to a positive amount. In the cases of Connecticut and Illinois, the cash amounts are quite substantial, contributing around $340 and $200 to disposable income respectively.

In this experiment we find that EITC payments play a very important role for low income households in almost all states. We see this in the fact that the federal contribution levels in Figure 16 are generally higher than those in Figure 9. Figure 20 shows the response of these payments to the earnings shock. An interesting feature of this graph is that since the Federal income tax system is progressive and uniform across states, the states are ordered from left to right by the nominal income of the household at the 10th percentile of the state income distribution. The correlation between living costs and the level of nominal income is also clear in this figure. We see that there is a small number of states, like Wyoming and New Mexico, where income levels are so low that our chosen household is on or close to the plateau of the tax credit schedule; as a consequence, the credit amounts do not respond much to the negative earnings shock because the shock is small in absolute terms. In most states, however, the household experiences a large increase in its tax credit entitlement [17].

Figure 21 presents change in state income tax liabilities (or tax credit entitlements) in response to the earnings shock. Unlike in the first experiment, we see that state income tax progressivity provides insurance in most of the states. In all but a few states, the tax liability decreases; among the exceptions are New Mexico and Arizona, which provide constant tax credits to households at this income level. Amongst the other states, we can identify some like New York, Kansas, Vermont and Wisconsin which

[17] We note that for some of the richer states - Connecticut, New Hampshire and New Jersey - the response of EITC is somewhat muted compared to the rest of the sample. We are not yet able to explain why this happens. It may result from an imputation error in TaxSim or a nonlinearity in the phase out portion of the EITC schedule

provide a relatively large increase in state income tax credits after the shock; we can also identify another group, made up of areas like Maryland, Massachusetts and DC, which levy positive taxes both before and after the shock, but which reduce the tax burden by a relatively large amount after the shock. All of these states which display high tax progressivity are in the Medium and Expensive living cost groups. This partly explains the pattern which we observe in Figure 16 - a positive correlation between state living costs and total insurance.

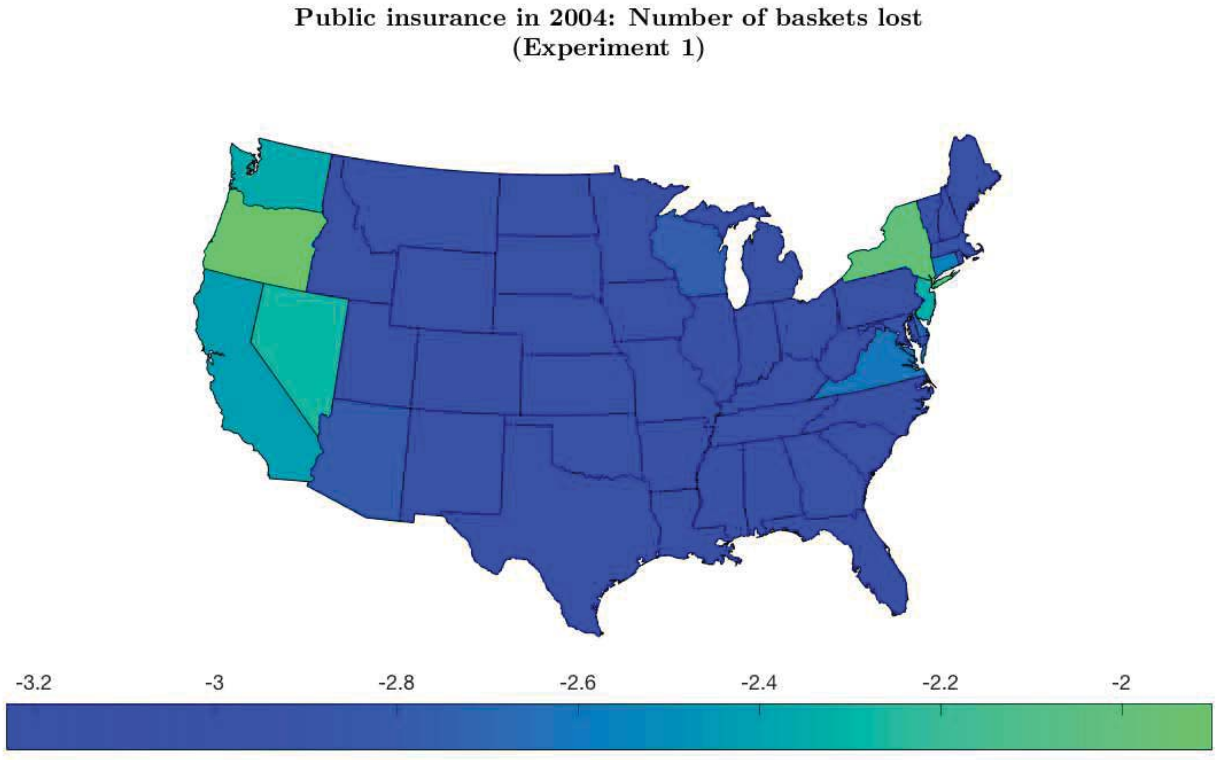

Figure 7: Experiment 1 - Baskets lost

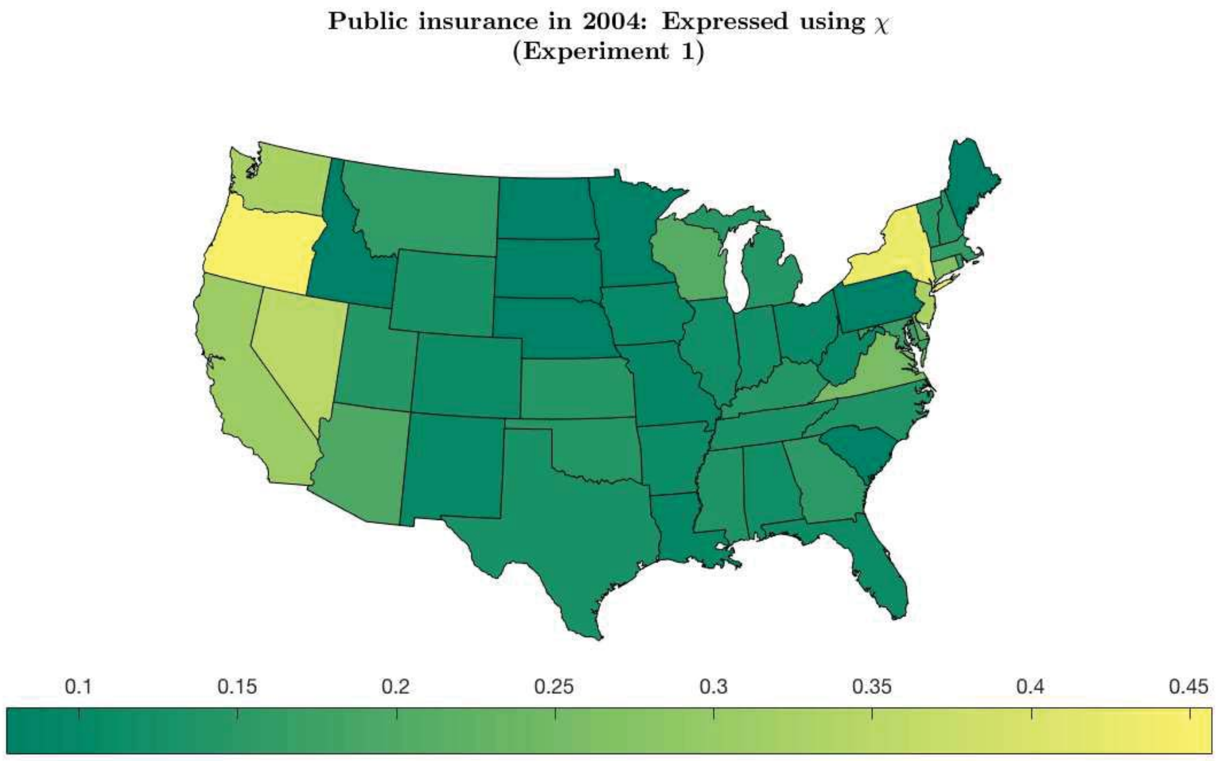

Figure 8: Experiment 1 - Insurance Measure

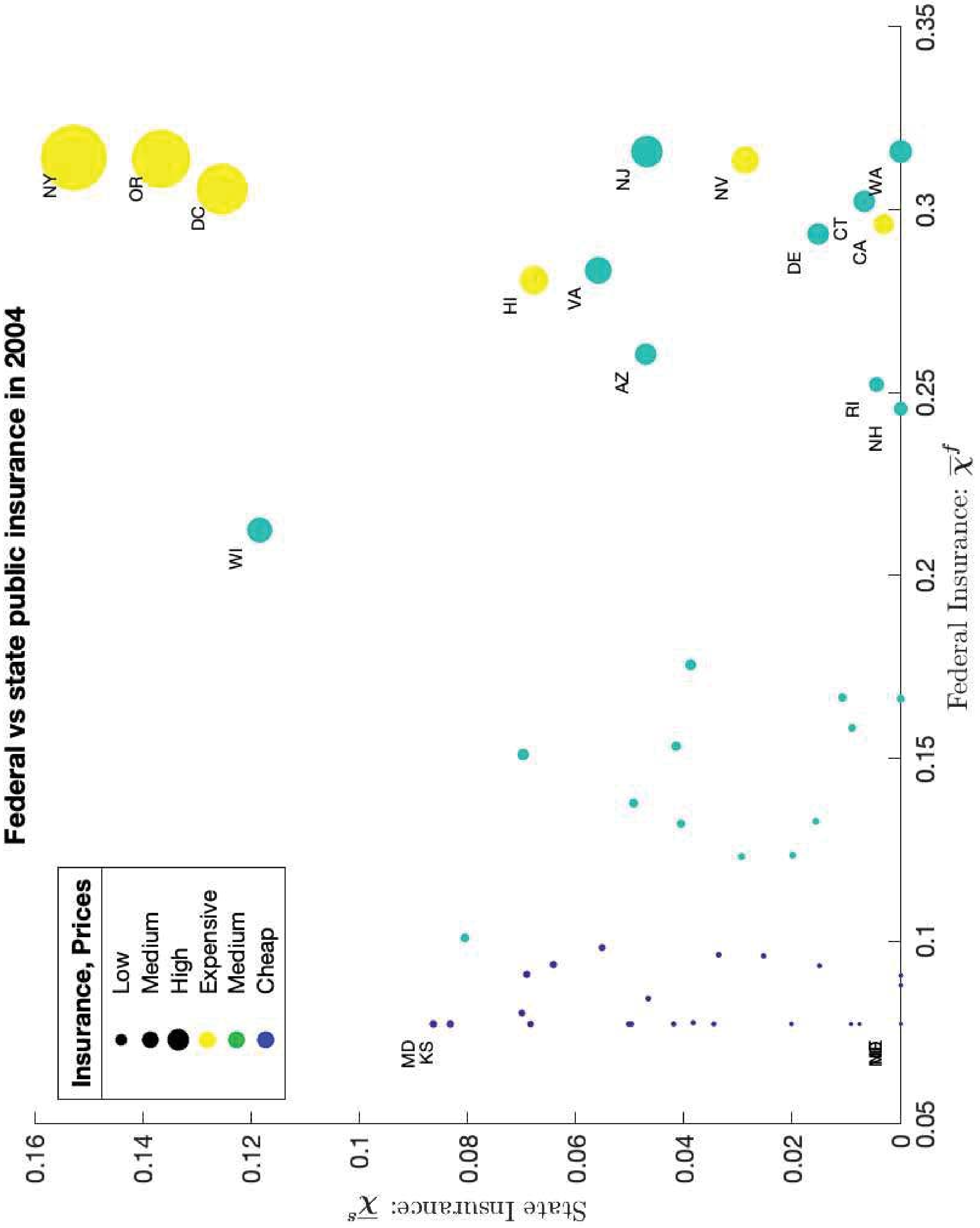

Figure 9: Experiment 1 - Relative contributions

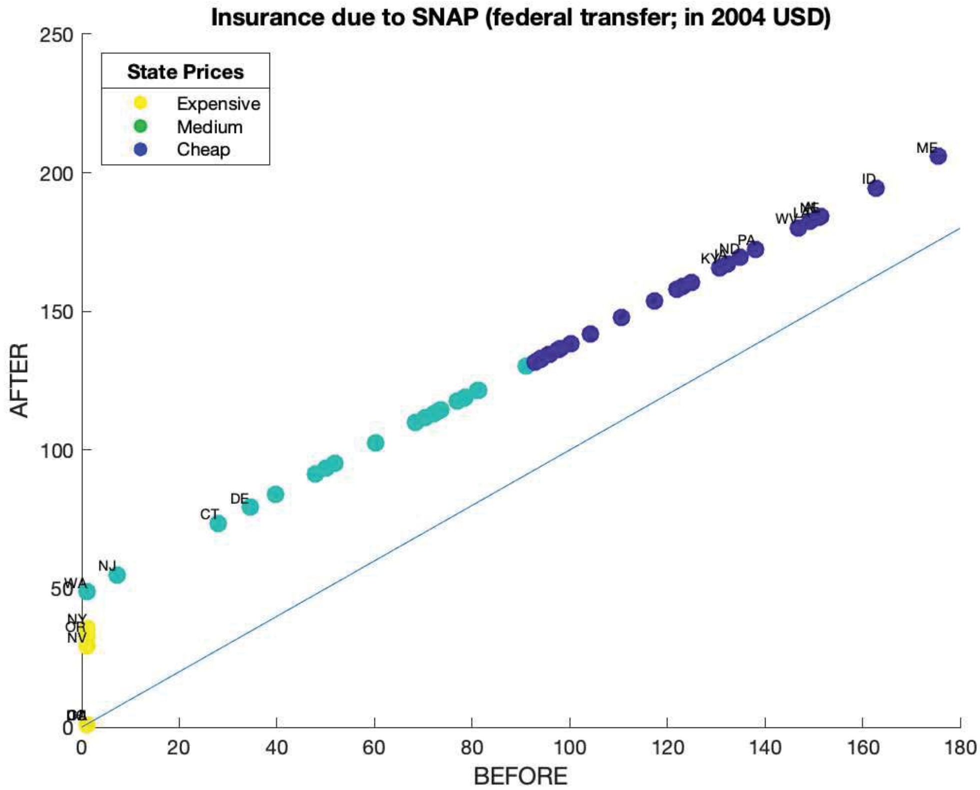

Figure 10: Experiment 1 - SNAP benefits

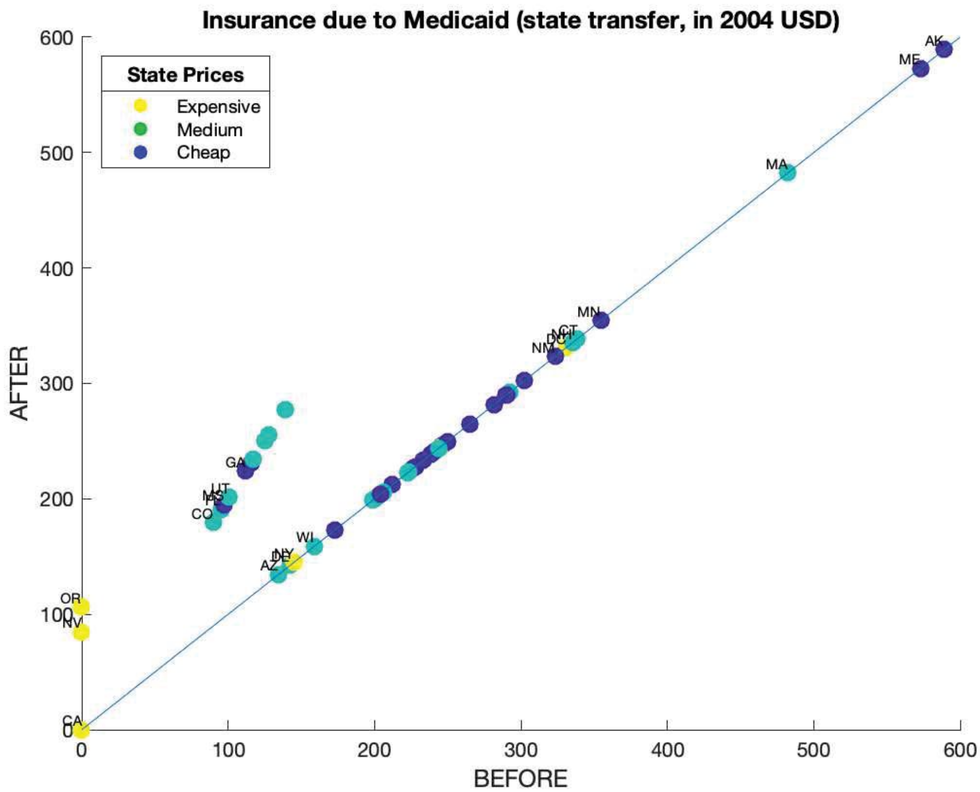

Figure 11: Experiment 1 - Medicaid Benefits

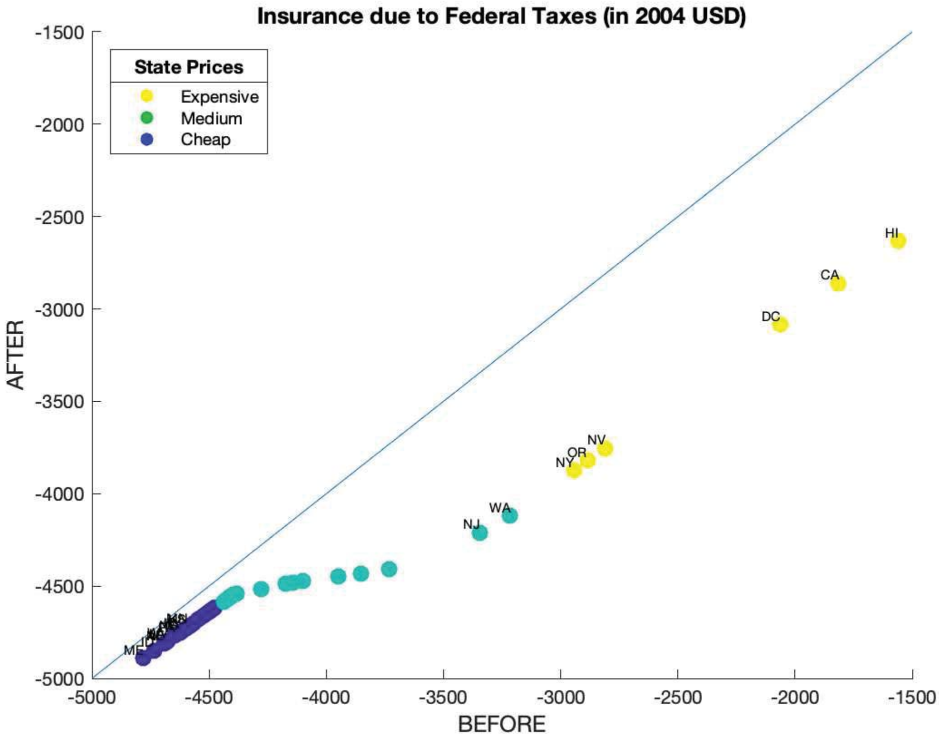

Figure 12: Experiment 1 - Federal Income Taxes

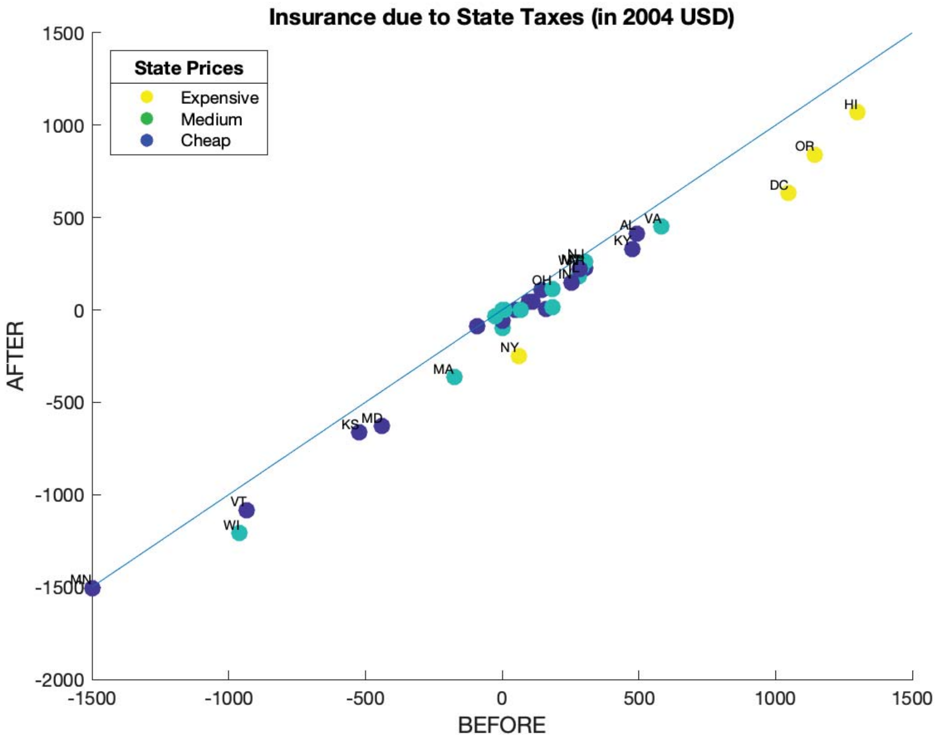

Figure 13: Experiment 1 - State Income Taxes

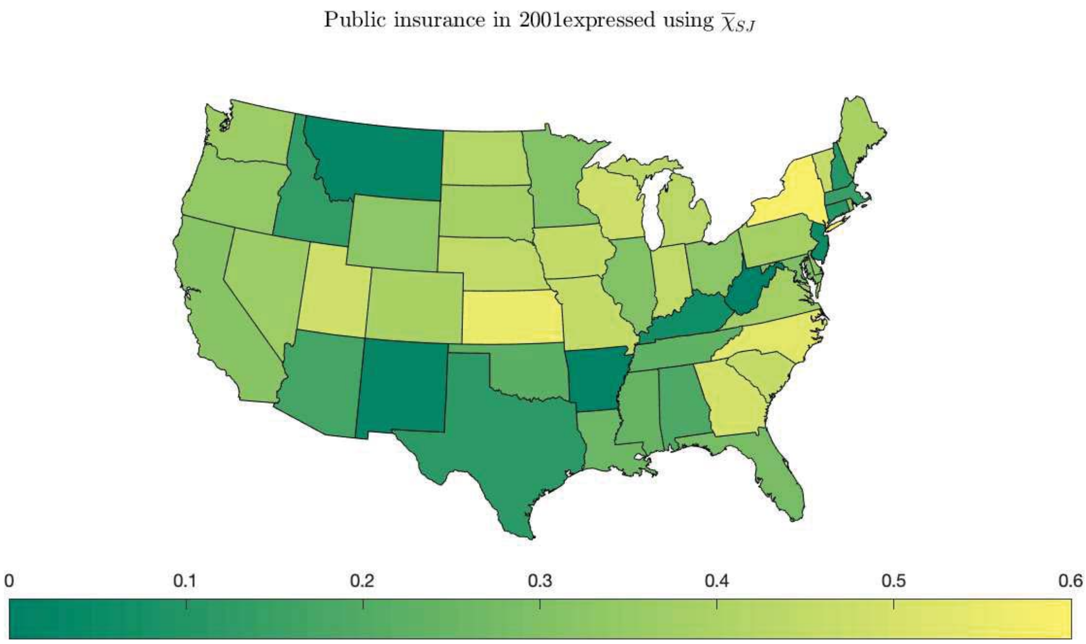

Figure 14: Experiment 2 - Insurance values, 2001

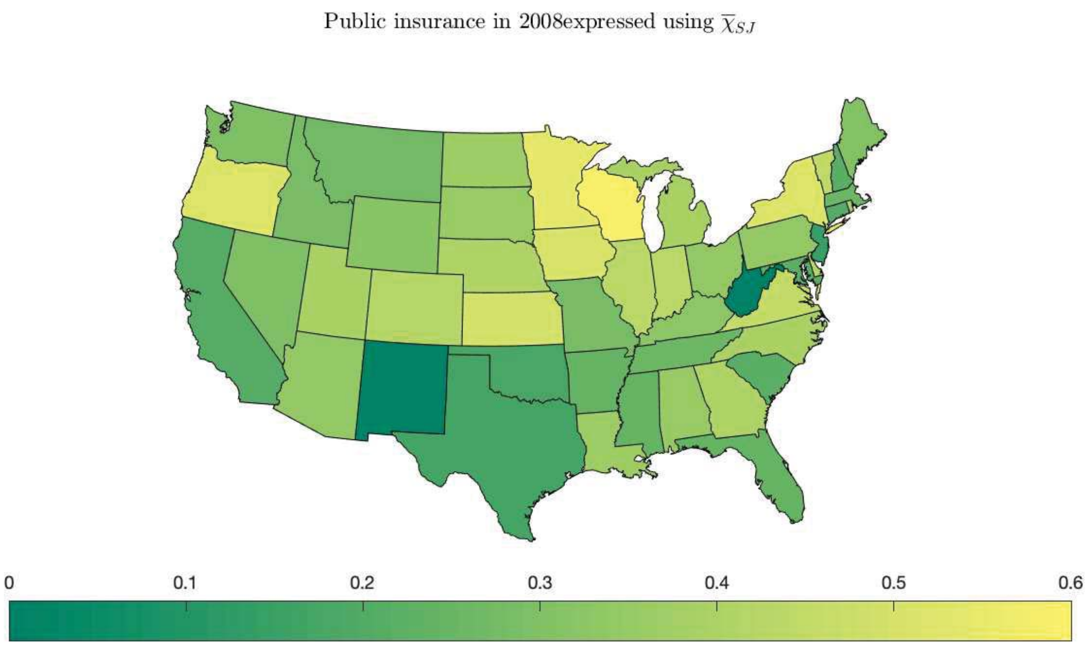

Figure 15: Experiment 2 - Insurance values, 2008

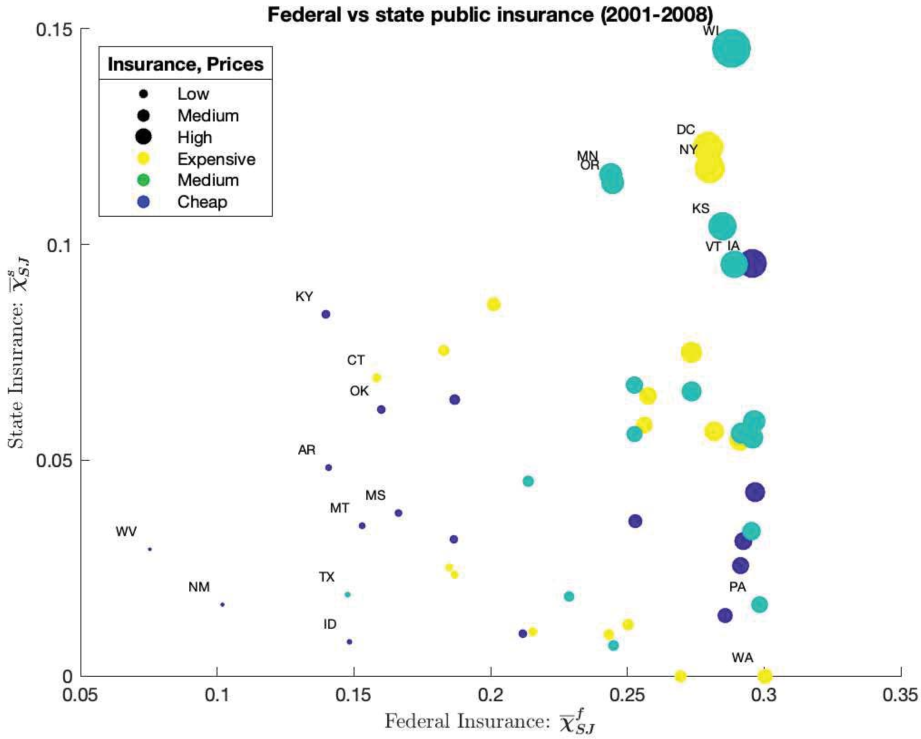

Figure 16: Federal contribution against state contribution, averages over shock size and time

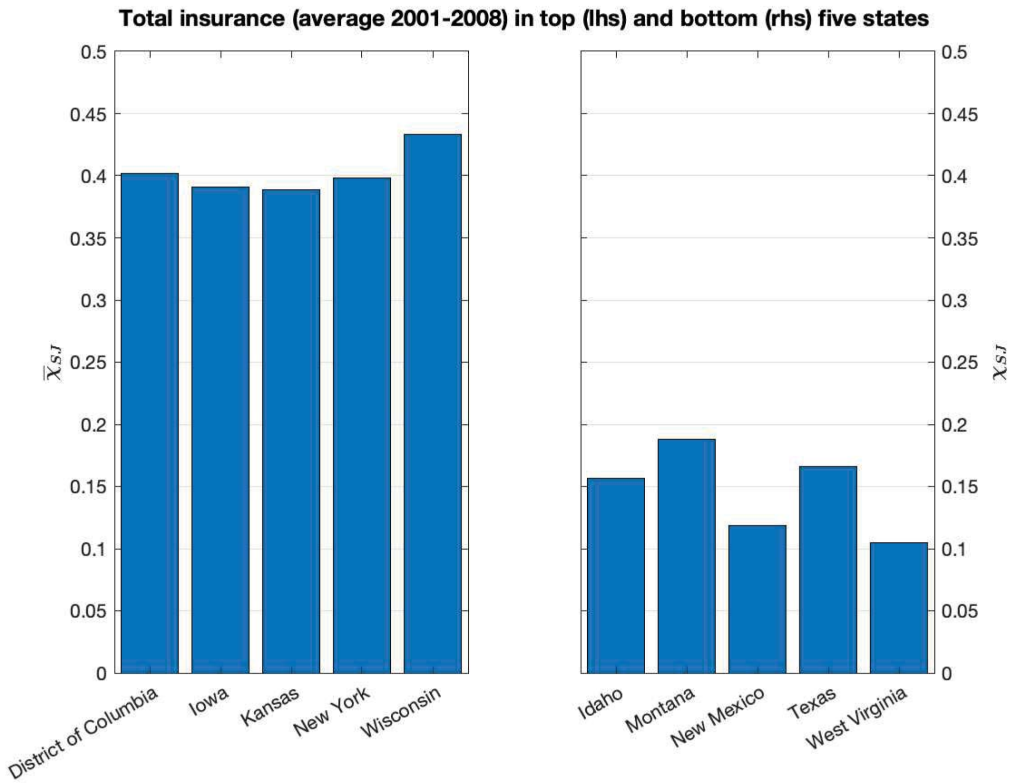

Figure 17: Overall insurance, highest and lowest insurance states

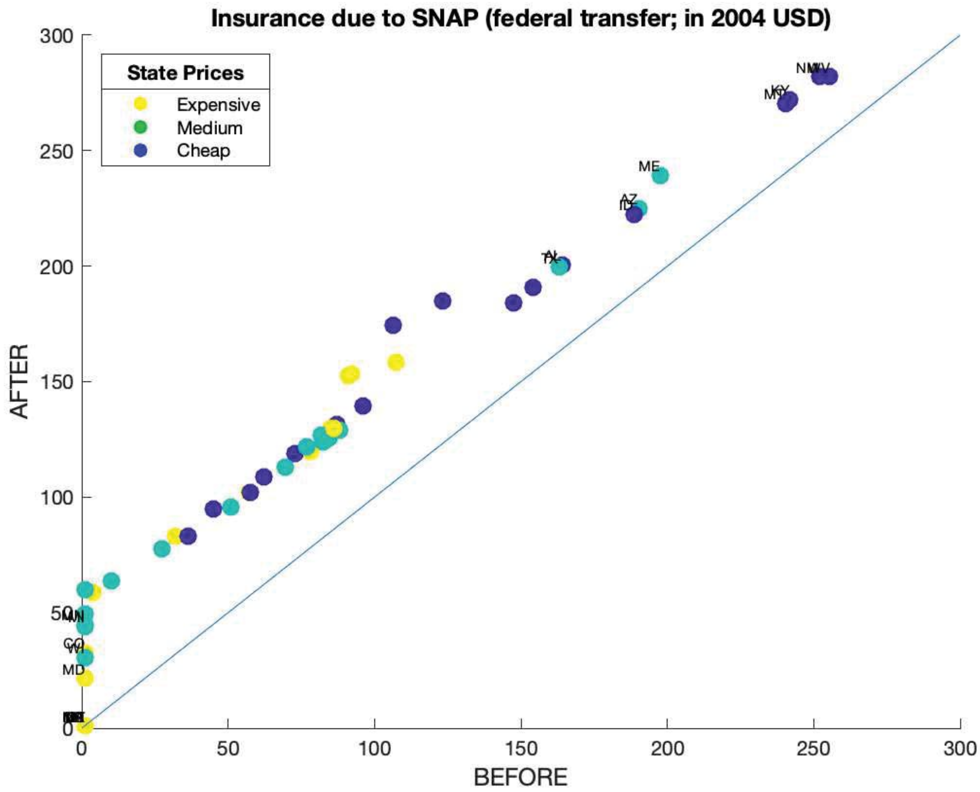

Figure 18: Experiment 2 SNAP benefits for a negative earnings shock of 10%

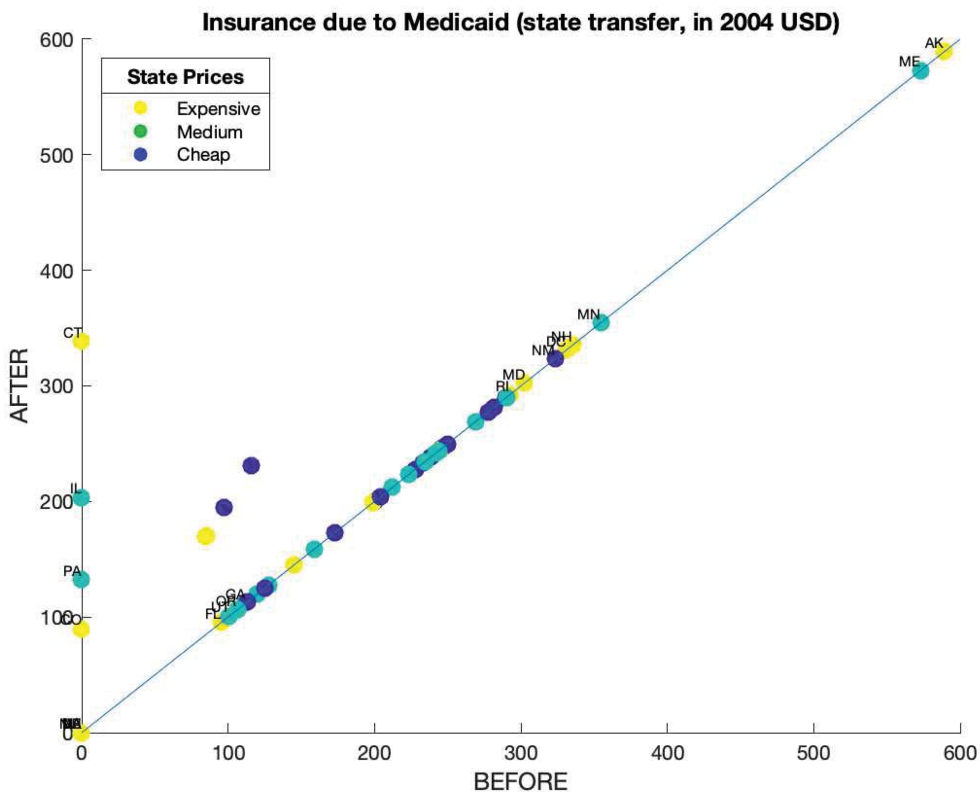

Figure 19: Experiment 2 Medicaid benefits for a negative earnings shock of 10%

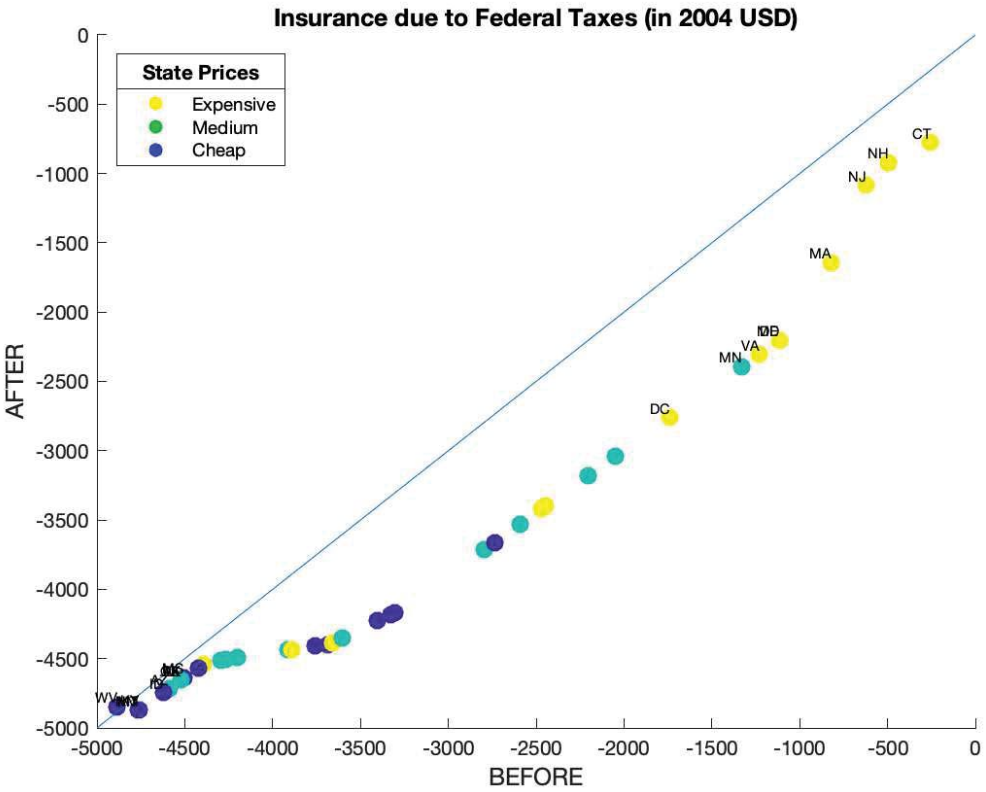

Figure 20: Experiment 2 Federal Income tax/credits for a negative earnings shock of 10%

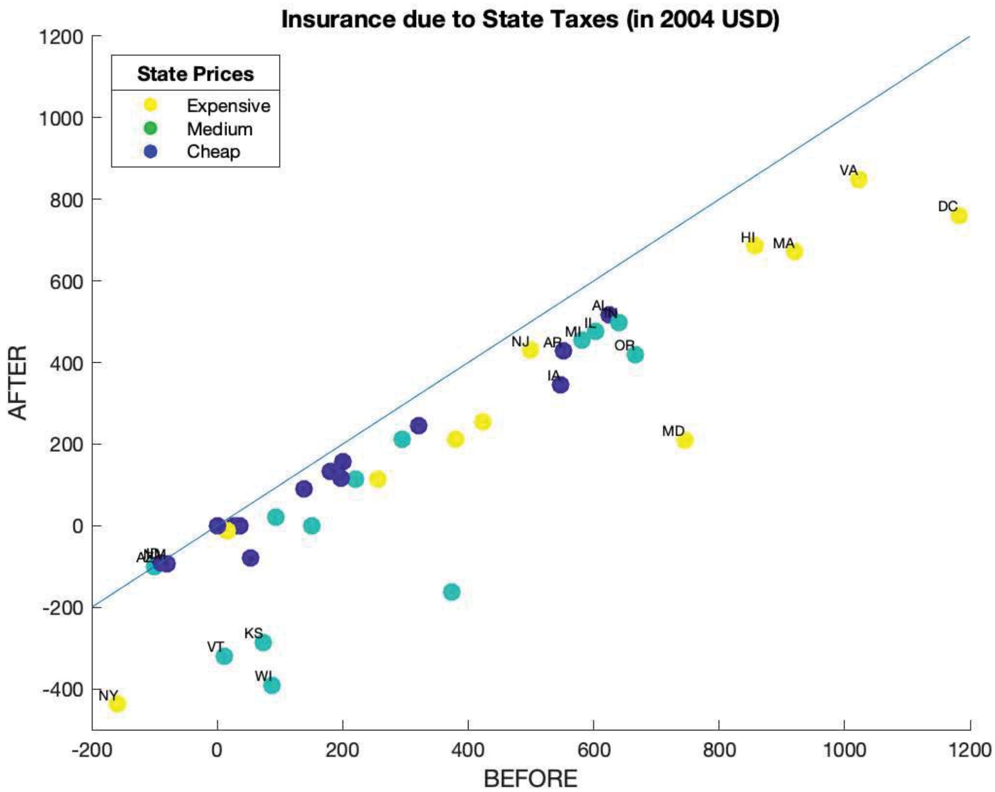

Figure 21: Experiment 2 State income tax/credits for a negative earnings shock of 10%

5 Conclusion

In this paper we present a framework for comparing the level of earnings insurance received by a prototype household, with two adults and two children, living in different US states; our focus is on insurance provided by federal and state income support policies. We do this by assembling a set of calculators for the main means tested tax and transfer programmes in the US, at the state and federal government levels; these calculators tell us the benefit entitlements and tax liabilities corresponding to a given amount of pre-tax income earned by our prototype household. One of the contributions of our work is to construct calculators which did not exist for some programmes. We can then simulate the response of these policies to changes in pre-tax earnings. We also construct measures of the cost of living for each of the US states so that we can correct nominal dollar amounts for purchasing power differences across states.

While the benchmark of uniform earnings insurance for households with the same composition and real income may be a strong one, the extent of geographical variation which we find is difficult to explain. One of the sources is well documented - federal tax and welfare programmes are designed based on household nominal incomes, even if these nominal amounts have different real values based on the cost of living in each state. We are able to quantify these (unintended) disparities using our measures of living costs in each state. This decomposition results in the finding that federal assistance is more responsive to negative earnings shocks for households with *high* nominal incomes, even if they have the same purchasing power as households with lower nominal incomes living in cheaper states. The households with lower nominal incomes tend to have exhausted their welfare entitlements in our experiments, so that they cannot receive any additional inflows in response to a negative earnings shock. The shape of the Earned Income Tax Credit schedule plays a particularly strong role in this mechanism.

Moreover, in addition to these mechanical differences in federal support, we also find considerable variation in the implementation of state level tax and transfer policies. State governments differ, for example, in whether they levy income taxes on low income households or pay them tax credits. Importantly, we do not find any evidence that these policies are calibrated to counteract the disparities in insurance arising from the design of federal policies. In sum, for a given level of real income, households which live in more expensive states tend to receive more earnings insurance.

An important extension of this work would be to identify factors which drive the variation in insurance which we see across US states. There are two main candidates which we find promising. The first is differences in state political preferences. It is possible that the pattern of state government policies which we see, especially relating to welfare and income taxes, may reflect different preferences over redistribution and the size of government. To the extent that these preferences correspond to voting behaviour, we would expect to see different results for voting in presidential elections and the composition of state legislatures for states where households receive high earnings insurance compared to states where insurance is lower. The second factor of interest is differences in the size of a typical earnings shock in different states, which may correspond to industrial composition of employment. As documented in Caliendo et al. (2014), the sectoral compositions of the regional economies in the USA are very heterogeneous; it is possible that the resulting exposures to different economic sectors mean that a typical pre-tax income shock is very different from one state to another. If for example, earnings in a specific state have low volatility, then it would make sense for the policies in that state to provide relatively little earnings insurance. An exercise based on this reasoning would also allow us to speak more about welfare differences across households in different states. However, we suspect that the available household level data is not granular enough for to allow for serious estimations of earnings volatility in every US state.

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A Appendix

A.1 Welfare Programs

Selected US governmental policies insuring income risk	Expenditures (2007 USD)		Imputation
FEDERAL	total (bn)	recipient/month	
Supplemental Nutrition Assistance Program (SNAP, 'Food Stamps')	30	96	calculator (own brew)
Earned Income Tax Credit (EITC)	49	165	federal tax (TaxSim)
STATE			
Medicaid	329	482	calculator (H. Hoynes)
Temporary Assistance for Needy Families (TANF, formerly AFDC)	12	234	calculator (H. Hoynes)
State Earned Income Tax Credit (SEITC)	NA	NA	state tax (TaxSim)
Unemployment insurance (UI)	*32*	*354*	*(not considered)*

Figure 22: Transfer programs included in simulation

Imputations

A.2 AFDC/TANF and Medicaid

Hoynes and Luttmer (2011) determined eligibility and benefits of the transfer programs "Temporary Assistance for Needy Families" (TANF; "Aid to Families with Dependent Children", AFDC, until 1996) and Medicaid. We use the same calculators and we cordially thank Hilary Hoynes for sharing them with us. The appendix of her paper provides details on the calculators so we just briefly describe their main features.

AFDC/TANF The benefit formula is given as

benefit = maximum benefit - benefit reduction rate $\times$ *(earnings - earnings disregard) - unearned income*

The state specific regulations regarding eligibility and generosity of this program materialize through differences in the earnings disregard, the benefit reduction rate and the maximum benefit. State policy makers enjoyed much less freedom to adjust program parameters prior to the introduction of the AFDC waivers and TANF. For years corresponding to this period the AFDC calculator uses "the most generous tax and disregards for all calculations" while, for years after 1996, the TANF calculator "does not take into account lifetime time limits or work requirements". Finally, the calculators assume a uniform take-up rate of 100%. Regarding the quality of the imputations produced by the calculators, Hoynes and Luttmer (2011) state that their calculations compare favorably with administrative data and other studies (see the appendix of their paper for details).

Medicaid Prior to 1987, eligibility for AFDC leads to mandatory eligibility for Medicaid. To capture the state determined Medicaid expansions in later years, Hoynes and Luttmer (2011) by include state and year specific eligibility parameters for child age and family income thresholds[18]) The benefits are established from administrative data on average expenditures per adult and child by state and year. Regarding take-up rate, the calculator assumes 100% if eligibility arises through AFDC while the take-up rate for eligible children varies by year as given in other sources.

A.3 SNAP (Food Stamps)

Since we could not find a calculator to determine eligibility to the "Supplemental Nutrition Assistance Program" (SNAP, formerly "Food Stamps") and to impute benefits we followed Hoynes and Luttmer (2011) in constructing our own calculator. Our main reference to design its elements was Moffitt (2016), the chapter by Hoynes and Schanzenbach (2015) in particular, as well as the comprehensive summaries and benchmark imputations presented in Hoynes et al. (2014) and Tremblay (1994). We also consulted Aussenberg (2014), Wilde (2001) and Hanson and Andrews (2009) to familiarize ourselves with details on the SNAP definition of net income as presented in section A.3.2 and its interaction with other transfer programs.

A.3.1 Eligibility

In general, as SNAP is a federal program, the importance of state parameters for eligibility (and generosity) is minor. In fact, they mostly result in marginally different definitions of countable assets in the means test. SNAP regulations define the unit for which eligibility needs to be established as consisting of all household members "who purchase and prepare food together". In concrete terms, any household has to meet three criteria to be considered eligible:

1. Gross monthly income has to be below or equal to 130% of the Federal Poverty Level (FPL).[19]

2. Net income (income after specified deductions, see section A.3.2) has to be below or equal to 100% of the FPL.

3. Countable assets may not exceed a certain amount.

Our calculator accounts for 1 and 2 but does not consider 3. This is because we have not been able to find the asset limits in current nominal US Dollars for the different years and family sizes as well as a comprehensive definitions of countable assets. As mentioned above, there are minor differences across states in this respect.

[18] Pregnancy eligibility is also accounted for by the calculator.

[19] While the term FPL is used frequently, this measure actually refers to the 'Poverty Guidelines' (PG). They are published as current US Dollar amounts for varying family sizes each year in the Federal Register by the Department of Health and Human Services (HHS). Note that different PGs apply for Alaska and Hawaii which accounts for the higher cost of living in these two states.

A.3.2 Benefits

Following the information provided in our references, the SNAP benefit formula is given as

SNAP benefit = maximum benefit – benefit reduction rate $\times$ *net income*

Maximum Benefit We capture maximum benefits by making use of the fact the SNAP is designed to cover monthly food expenditures of families with different sizes as established by the Thrifty Food Plan (TFP)[20]. Hence, to obtain the maximum benefit data for various family sizes and years, we collected data on the current US Dollar amounts corresponding to the TFP.[21] As this information is not available in a consolidated database for the years we study, we combined information from several sources for different time periods:

- **1976 to 1995** We use data provided by Castner (2000) (Table B3). Note that current US Dollar amounts are only available for even years until 1990. For uneven years before, we use the average value of the preceding and following year.
- **1996 to 2003** We use data information from the "Supplemental Nutrition Assistance Program Quality Control Data". They are included in the quality control reports published by the U.S. Department of Agriculture's (USDA) Food and Consumer Service (FCS), which is administering SNAP (and already administered the program when it was called Food Stamp Program).[22]
- **from 2004** We use data from the Cost of Living Adjustment (COLA) database. The data are provided by the USDA's Food and Nutrition Service.[23]

As a consistency check, we compare the values from the different sources for years in which they overlap. We find that between 1990 and 2000, the data from Castner (2000) and the USDA are identical, while from 2000 to 2005, the USDA and COLA data are the same. Hence, we have confidence that the maximum benefits are correctly specified in our calculator.

Net Income Following the official program definitions, we establish SNAP net income as

[20]"Benefits are tied to the cost of a "market basked of foods which if prepared and consumed at home, would provide a complete, nutritious diet at minimal cost", the so-called Thrifty Food Plan, (...)." Moffitt (2016), page 226. The Thrifty Food Plan (TFP) measures the average monthly cost of a healthy meal plan for different family sizes. It is computed by the US Department of Agriculture and a key policy measure in setting nutritional cost standards.

[21]Note that Congress can choose to increase maximum benefits above the TFP level during economic downturns. For example, this was one element of the American Recovery and Reinvestment Act of 2009. Our calculator accounts for this temporary policy change.

[22]Mathematica Policy Research was contracted to produce the reports and datasets. Both are available at https://host76.mathematica-mpr.com/fns/ See appendix C of the technical documentation for program parameters such as the maximum benefit, income screen etc.

[23]See https://www.fns.usda.gov/snap/cost-living-adjustment-cola-information

	cash pre-tax income	(1)
–	standard deduction	(2)
–	20% deduction of earned income	(3)
–	excess shelter cost deduction	(4)
–	deduction for childcare costs associated with working and training	(5)
–	medical cost deduction for elderly and disabled	(6)
=	net income	(7)

As our prototype household does not meet the criteria captured in (4), (5) and (6), our calculator does not consider them. For (2), we could not find the data for different years and family sizes so we omit this deduction. However, the calculator carefully considers the fact that SNAP regulations define cash pre-tax income listed in (1) to exclude in-kind benefits and tax credits. In other words, (1) does not include Medicaid, state and federal earned income as well as child tax credits. It does include include cash transfers. While disbursements of social security, disability income and unemployment insurance would meet this criterion, they are not relevant due to the specification of our prototype household. What matters for our household are AFDC/TANF transfers which our SNAP calculator adds to cash pre-tax income.

Benefit Reduction Rate While the official SNAP benefit reduction rate is 0.3, Hoynes and Schanzenbach (2015) argue that the rate which applies in practice is below this statutory value because of the deductions to net income described above. Another source of variation of the benefit reduction rate is described in Hanson and Andrews (2009). They show that, from a household perspective, the SNAP benefit reduction rate is subject to interaction with other welfare programs such as AFDC/TANF. As the benefits of these programs vary by state and year, the SNAP benefit reduction rate is likely to vary across states and years as well. To account for this issue, we simulate our model with two different benefit reduction rates (0.3 and 0.15). However, our results are robust as the quantitative changes induced by this variation are minor. This is because eligibility is not affected by the benefit reduction rate (see section A.3.1) and because those households which receive SNAP benefits have very low values of net income.

A.3.3 Take-up rates

The USDA publishes annual reports titled "Estimates of State Supplemental Nutrition Assistance Program Participation Rates". These reports document that participation rates vary considerably across states and years. It has been pointed out that these differences are partly associated with asymmetric state business cycle movements and other state and local policies such as school lunch and emergency food programs. Moreover, participation rates also depend on the amounts of collectable benefits. As we are interested to study a household which is comparable across years and states (our prototype family), we do not account for differences in take up rates in our SNAP calculator. On the one hand, this is because we aim to measure the maximum amount of transfers available to households and not those actually collected. On the other hand, we do not want to capture outcomes which are plausibly linked to specific state (and year) effects to keep our results focused on comparability.

A.4 Taxes

We use TaxSim to obtain federal and state liabilities for different years and states. TaxSim provides federal taxes since 1960 and state taxes since 1977. Importantly, it includes state and federal earned income tax credits and accounts for different state rules on deductibility of federal taxes as well as child care tax credits. Moreover, it allows to account for household characteristics such as number of children which are relevant determinants of a family's actual total tax burden. We use the income data we obtained from the data (see above) corresponding to poor and rich families in the different years and states. Moreover, we account for transfer incomes imputed by our calculators described above, i.e. we make sure to use the best estimate for taxable income.

How accurate are our federal and state tax imputations? We first note that TaxSim is the almost exclusive tool used for this purpose so our imputations are no worse than those of the vast majority of other contributions. Second, since we cannot observe the tax data we are interested in (see above discussion), we have to rely on imputation. Hence, the only benchmark for comparison are alternative tax calculators. To the best of our knowledge, there are two other candidates: The tax calculator developed and maintained by Bakija (2017) and the Urban Institute's Transfer Income Model (TRIM).[24] To judge if any of these alternatives is strictly superior to TaxSim for the purpose of our project, we present below a succinct summary of Wheaton and Stevens (2016) who conduct a detailed comparison of all three tax calculators.

Federal Taxes For federal income taxes (table 2A), there are only negligible differences between the three tax calculators. All three are either equally close to or far from the target defined by administrative tax data. Differences between them are always in the ballpark of two to three percentage points.[25] This impression is confirmed by the more detailed comparisons presented in table 2B and applies even more to tax credits (table 2C) where the three alternatives produce virtually identical results.

State Taxes Regarding state income taxes (tables A4 and 4B) TRIM performs consistently worse than TaxSim and Bakija. While they are close in terms of meeting the target, it appears that TaxSim is marginally superior. For state earned income tax credits, all three are lining up closely but TaxSim seems again a marginal winner based on the summary evaluation presented in the final rows of table 4C.

[24] See here http://trim.urban.org/T3Welcome.php

[25] The only exception is the alternative minimum tax where TRIM performs better than TaxSim and Bakija by eight and five percentage points respectively.

As Wheaton and Stevens (2016) demonstrate the relative performance of the three tax calculators also depends on the source of the tax variables. However even for different inputs (Census or TRIM tax variables), the variation between them remains minor. Since we are using our own input variables – which are different from the ones used by Wheaton and Stevens (2016) – we conclude that conditional on our inputs the variation across the different calculators is likely to be small and that TaxSim is overall the best choice for our project. Therefore, we think the imputation procedure of federal and state taxes is the best we can achieve as we have no reason to believe that any of the other tools would give more accurate results.

A.5 Data inputs

As inputs to our simulation model, we obtain data on households in different years and states from the Integrated Public Use Microdata Series (IPUMS, Ruggles et al. (2017)) USA dataset. It provides cross-sectional variables on households in different states and years (dating back as early as 1850). Since we only have state taxes since 1977, we choose 1980 as the first year of our analysis. IPUMS assembles information from several sources, such as the decennial censuses (for years 1980 and 1990) and the American Community Survey (ACS; annual since 2000). Importantly, variable codes and labels are harmonized across years and data sources so that they consistently contain the same information.

A.5.1 Household Income

We use total annual family income[26] as the income variable of households and classify households as poor or rich based on the 10% and 90% percentiles of the corresponding state and year distributions. This measure for income comprises current USD amounts of annual "total pre-tax money income earned by one's family from all sources". Since this variable sums the incomes of all family members who are related to the head it excludes incomes of family members who are not related to the head. To check if this aspect makes a quantitative difference, we also use a personal income variable[27] which "reports income earned from wages or a person's own business or farm for the previous year" and sum it for all family members (related to the head or not) by using the family relationship variables. The results are virtually identical.

[26] IPUMS variable is "FTOTINC"

[27] "INCEARN"

B Complete Results of Main Experiments

The tables for the state and federal government decompositions of the two experiments in the text of the paper are presented in the next pages in the following order:

1. Experiment 1: Federal insurance against negative shocks
2. Experiment 1: Federal insurance against positive shocks
3. Experiment 1: State insurance against negative shocks
4. Experiment 1: State insurance against positive shocks
5. Experiment 2: Federal insurance against positive and negative shocks
6. Experiment 2: State insurance against positive and negative shocks

	-0.5	-1	-1.5	-2	-2.5	-3	-3.5	-4	-4.5	-5
Alabama	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Alaska	0.0771	0.0773	0.0772	0.0773	0.0772	0.0773	0.0772	0.0773	0.0772	0.0773
Arizona	0.2785	0.2531	0.2442	0.2212	0.1925	0.1732	0.1595	0.1493	0.1412	0.1349
Arkansas	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
California	0.3095	0.3102	0.3104	0.3105	0.3105	0.3106	0.3104	0.3105	0.3105	0.3114
Colorado	0.0772	0.0774	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Connecticut	0.3265	0.3273	0.3271	0.3153	0.2978	0.2860	0.2646	0.2411	0.2229	0.2084
Delaware	0.3263	0.3271	0.3128	0.2915	0.2786	0.2570	0.2314	0.2121	0.1971	0.1851
District of Columbia	0.3107	0.3107	0.3107	0.3107	0.3107	0.3107	0.3107	0.3107	0.3126	0.3140
Florida	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Georgia	0.0774	0.0774	0.0774	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Hawaii	0.3108	0.3108	0.3108	0.3104	0.3105	0.3106	0.3106	0.3106	0.3106	0.3106
Idaho	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0772	0.0772	0.0772	0.0772
Illinois	0.0772	0.0774	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Indiana	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Iowa	0.0771	0.0773	0.0772	0.0773	0.0773	0.0772	0.0773	0.0773	0.0773	0.0773
Kansas	0.0770	0.0772	0.0773	0.0772	0.0772	0.0773	0.0772	0.0772	0.0773	0.0773
Kentucky	0.0772	0.0774	0.0774	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Louisiana	0.0773	0.0773	0.0773	0.0773	0.0772	0.0772	0.0772	0.0773	0.0773	0.0773
Maine	0.0771	0.0771	0.0772	0.0772	0.0773	0.0772	0.0772	0.0773	0.0772	0.0772
Maryland	0.0773	0.0773	0.0773	0.0774	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Massachusetts	0.0771	0.0772	0.0773	0.0772	0.0773	0.0772	0.0773	0.0773	0.0773	0.0773
Michigan	0.0774	0.0772	0.0773	0.0772	0.0773	0.0772	0.0773	0.0773	0.0773	0.0773
Minnesota	0.0775	0.0773	0.0772	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Mississippi	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Missouri	0.0772	0.0774	0.0773	0.0773	0.0772	0.0773	0.0773	0.0773	0.0773	0.0773
Montana	0.1061	0.0917	0.0868	0.0845	0.0831	0.0821	0.0814	0.0809	0.0805	0.0802
Nebraska	0.0772	0.0772	0.0772	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Nevada	0.3102	0.3102	0.3107	0.3143	0.3168	0.3187	0.3199	0.3208	0.3214	0.3221
New Hampshire	0.2274	0.2274	0.2204	0.1846	0.1631	0.1488	0.1386	0.1310	0.1250	0.1202
New Jersey	0.3278	0.3279	0.3273	0.3275	0.3272	0.3273	0.3191	0.3076	0.2987	0.2787
New Mexico	0.0775	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
New York	0.3119	0.3112	0.3157	0.3186	0.3203	0.3214	0.3222	0.3228	0.3233	0.3152
North Carolina	0.0773	0.0773	0.0772	0.0772	0.0772	0.0773	0.0773	0.0773	0.0773	0.0772
North Dakota	0.0771	0.0771	0.0773	0.0772	0.0772	0.0773	0.0773	0.0772	0.0773	0.0773
Ohio	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Oklahoma	0.0775	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Oregon	0.3099	0.3106	0.3127	0.3166	0.3186	0.3201	0.3210	0.3219	0.3224	0.3183
Pennsylvania	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Rhode Island	0.2417	0.2341	0.2319	0.1988	0.1745	0.1583	0.1467	0.1380	0.1313	0.1259
South Carolina	0.0774	0.0774	0.0772	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
South Dakota	0.0771	0.0772	0.0772	0.0772	0.0773	0.0772	0.0773	0.0773	0.0772	0.0773
Tennessee	0.0771	0.0771	0.0773	0.0772	0.0772	0.0773	0.0773	0.0772	0.0773	0.0773
Texas	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Utah	0.0771	0.0773	0.0772	0.0773	0.0772	0.0773	0.0772	0.0773	0.0772	0.0773
Vermont	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Virginia	0.3268	0.3166	0.2867	0.2718	0.2544	0.2249	0.2038	0.1879	0.1757	0.1658
Washington	0.3248	0.3263	0.3263	0.3267	0.3269	0.3268	0.3269	0.3185	0.3083	0.3002
West Virginia	0.0772	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Wisconsin	0.2270	0.1928	0.1543	0.1350	0.1235	0.1158	0.1103	0.1061	0.1029	0.1004
Wyoming	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0772	0.0772	0.0772

	5	4.5	4	3.5	3	2.5	2	1.5	1	0.5
Alabama	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Alaska	0.1633	0.1452	0.1297	0.1159	0.0972	0.0773	0.0773	0.0773	0.0773	0.0775
Arizona	0.3232	0.3245	0.3262	0.3273	0.3272	0.3271	0.3274	0.3272	0.3270	0.3278
Arkansas	0.0891	0.0772	0.0772	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
California	0.2182	0.2314	0.2478	0.2689	0.2971	0.3108	0.3108	0.3108	0.3108	0.3108
Colorado	0.2572	0.2495	0.2398	0.2274	0.2105	0.1872	0.1706	0.1519	0.1137	0.0772
Connecticut	0.3176	0.3183	0.3193	0.3204	0.3222	0.3247	0.3274	0.3276	0.3274	0.3282
Delaware	0.3193	0.3204	0.3215	0.3232	0.3252	0.3273	0.3275	0.3274	0.3279	0.3279
District of Columbia	0.2571	0.2746	0.2964	0.3105	0.3105	0.3104	0.3107	0.3107	0.3107	0.3107
Florida	0.2850	0.2802	0.2744	0.2668	0.2567	0.2426	0.2215	0.2061	0.1955	0.1636
Georgia	0.1656	0.1476	0.1314	0.1176	0.0995	0.0772	0.0772	0.0773	0.0772	0.0770
Hawaii	0.1804	0.1893	0.2004	0.2148	0.2339	0.2607	0.3008	0.3108	0.3108	0.3108
Idaho	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Illinois	0.2562	0.2483	0.2385	0.2258	0.2086	0.1849	0.1689	0.1495	0.1107	0.0772
Indiana	0.1326	0.1204	0.1071	0.0900	0.0773	0.0773	0.0772	0.0772	0.0771	0.0773
Iowa	0.0773	0.0773	0.0773	0.0773	0.0772	0.0773	0.0773	0.0772	0.0773	0.0771
Kansas	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0774	0.0774
Kentucky	0.0773	0.0773	0.0773	0.0773	0.0772	0.0772	0.0772	0.0772	0.0772	0.0772
Louisiana	0.0773	0.0773	0.0772	0.0772	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Maine	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0774	0.0773	0.0775
Maryland	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0772	0.0772
Massachusetts	0.1977	0.1834	0.1653	0.1447	0.1310	0.1117	0.0830	0.0773	0.0772	0.0774
Michigan	0.1754	0.1585	0.1387	0.1260	0.1090	0.0855	0.0772	0.0773	0.0772	0.0774
Minnesota	0.0773	0.0773	0.0772	0.0773	0.0772	0.0773	0.0773	0.0772	0.0773	0.0771
Mississippi	0.1895	0.1741	0.1550	0.1378	0.1230	0.1021	0.0772	0.0772	0.0771	0.0770
Missouri	0.1828	0.1668	0.1468	0.1324	0.1164	0.0943	0.0773	0.0773	0.0772	0.0772
Montana	0.2978	0.2946	0.2904	0.2853	0.2782	0.2683	0.2538	0.2290	0.2277	0.2277
Nebraska	0.0772	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0772	0.0772	0.0772
Nevada	0.3106	0.3107	0.3106	0.3106	0.3107	0.3105	0.3106	0.3107	0.3109	0.3102
New Hampshire	0.3229	0.3242	0.3253	0.3250	0.3246	0.3243	0.3236	0.3223	0.3196	0.3118
New Jersey	0.3121	0.3122	0.3125	0.3127	0.3130	0.3136	0.3142	0.3152	0.3180	0.3249
New Mexico	0.1741	0.1572	0.1377	0.1250	0.1078	0.0840	0.0772	0.0772	0.0773	0.0771
New York	0.3105	0.3105	0.3105	0.3105	0.3105	0.3105	0.3105	0.3105	0.3105	0.3104
North Carolina	0.2638	0.2567	0.2478	0.2365	0.2213	0.2004	0.1803	0.1645	0.1331	0.0773
North Dakota	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0776
Ohio	0.2438	0.2345	0.2228	0.2079	0.1880	0.1659	0.1506	0.1250	0.0771	0.0773
Oklahoma	0.2782	0.2726	0.2658	0.2572	0.2456	0.2290	0.2045	0.1929	0.1753	0.1237
Oregon	0.3106	0.3107	0.3106	0.3107	0.3106	0.3107	0.3106	0.3108	0.3106	0.3113
Pennsylvania	0.0772	0.0773	0.0773	0.0773	0.0773	0.0772	0.0772	0.0772	0.0773	0.0773
Rhode Island	0.3239	0.3253	0.3271	0.3273	0.3272	0.3271	0.3274	0.3273	0.3270	0.3278
South Carolina	0.2923	0.2883	0.2836	0.2773	0.2689	0.2572	0.2400	0.2205	0.2169	0.2063
South Dakota	0.1531	0.1340	0.1222	0.1073	0.0873	0.0773	0.0773	0.0773	0.0774	0.0774
Tennessee	0.1839	0.1678	0.1479	0.1331	0.1173	0.0954	0.0773	0.0773	0.0773	0.0775
Texas	0.2926	0.2889	0.2841	0.2779	0.2696	0.2583	0.2410	0.2213	0.2182	0.2088
Utah	0.2805	0.2752	0.2689	0.2604	0.2494	0.2337	0.2105	0.1973	0.1827	0.1375
Vermont	0.1109	0.0979	0.0818	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773
Virginia	0.3208	0.3218	0.3233	0.3251	0.3274	0.3271	0.3272	0.3274	0.3268	0.3268
Washington	0.3107	0.3106	0.3108	0.3107	0.3106	0.3108	0.3108	0.3106	0.3111	0.3111
West Virginia	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0772	0.0772	0.0772	0.0772
Wisconsin	0.3116	0.3102	0.3081	0.3054	0.3018	0.2965	0.2889	0.2762	0.2508	0.2282
Wyoming	0.2442	0.2349	0.2234	0.2085	0.1887	0.1665	0.1513	0.1259	0.0773	0.0773

	-0.5	-1	-1.5	-2	-2.5	-3	-3.5	-4	-4.5	-5
Alabama	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400
Alaska	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Arizona	0.0000	0.1339	0.0893	0.0670	0.0536	0.0446	0.0383	0.0335	0.0698	0.0628
Arkansas	0.0360	0.0360	0.0361	0.0361	0.0361	0.0361	0.0361	0.0361	0.0361	0.0361
California	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0221	0.0197	0.0177
Colorado	0.0000	0.0000	0.0000	0.0000	0.0518	0.0432	0.0370	0.0324	0.0288	0.0259
Connecticut	0.0060	0.0055	0.0050	0.0045	0.0040	0.0035	0.0030	0.0027	0.0024	0.0021
Delaware	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
District of Columbia	0.1277	0.1277	0.1277	0.1277	0.1277	0.1277	0.1263	0.1233	0.1210	0.1192
Florida	0.0000	0.0000	0.0000	0.0000	0.0000	0.0452	0.0388	0.0339	0.0301	0.0271
Georgia	0.0301	0.2002	0.1435	0.1151	0.0981	0.0867	0.0773	0.0796	0.0730	0.0677
Hawaii	0.0680	0.0680	0.0680	0.0680	0.0680	0.0673	0.0669	0.0665	0.0662	0.0660
Idaho	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Illinois	0.0405	0.0406	0.0406	0.0406	0.0405	0.0405	0.0405	0.0405	0.0406	0.0405
Indiana	0.0467	0.0467	0.0467	0.0467	0.0466	0.0466	0.0466	0.0466	0.0466	0.0467
Iowa	0.0585	0.0587	0.0587	0.0546	0.0437	0.0364	0.0312	0.0273	0.0243	0.0219
Kansas	0.0663	0.0665	0.0666	0.0665	0.0666	0.0666	0.0665	0.0666	0.0666	0.0666
Kentucky	0.0570	0.1345	0.1045	0.0890	0.0797	0.0735	0.0691	0.0657	0.0631	0.0611
Louisiana	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
Maine	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Maryland	0.0896	0.0896	0.0896	0.0898	0.0897	0.0893	0.0883	0.0875	0.0868	0.0853
Massachusetts	0.0803	0.0805	0.0806	0.0805	0.0805	0.0805	0.0805	0.0805	0.0805	0.0805
Michigan	0.0663	0.0661	0.0661	0.0661	0.0661	0.0661	0.0661	0.0661	0.0661	0.0661
Minnesota	0.0222	0.0111	0.0074	0.0055	0.0044	0.0037	0.0032	-0.0119	-0.0327	-0.0494
Mississippi	0.0300	0.1760	0.1273	0.1030	0.0870	0.0725	0.0622	0.0544	0.0483	0.0435
Missouri	0.0250	0.0250	0.0250	0.0242	0.0233	0.0228	0.0224	0.0221	0.0219	0.0214
Montana	0.0240	0.0240	0.0240	0.0240	0.0240	0.0891	0.0798	0.0729	0.0674	0.0631
Nebraska	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Nevada	0.1952	0.0976	0.0651	0.0488	0.0390	0.0325	0.0279	0.0244	0.0217	0.0195
New Hampshire	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
New Jersey	0.0175	0.0175	0.0175	0.0168	0.0163	0.0159	0.0156	0.0154	0.0153	0.0151
New Mexico	0.0000	0.0628	0.0452	0.0364	0.0311	0.0276	0.0251	0.0232	0.0217	0.0206
New York	0.1036	0.1034	0.1033	0.1033	0.1032	0.1032	0.1032	0.1032	0.1032	0.1032
North Carolina	0.0252	0.0126	0.0084	0.0063	0.0784	0.0653	0.0560	0.0490	0.0436	0.0392
North Dakota	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Ohio	0.0297	0.0297	0.0297	0.0297	0.0297	0.0292	0.0284	0.0279	0.0274	0.0270
Oklahoma	0.0708	0.0706	0.0705	0.0706	0.0706	0.0693	0.0680	0.0671	0.0664	0.0658
Oregon	0.3468	0.2238	0.1826	0.1622	0.1498	0.1416	0.1357	0.1313	0.1279	0.1252
Pennsylvania	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Rhode Island	0.0045	0.0045	0.0045	0.0045	0.0042	0.0040	0.0038	0.0036	0.0035	0.0034
South Carolina	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
South Dakota	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Tennessee	0.0000	0.1736	0.1157	0.0868	0.0694	0.0579	0.0496	0.0434	0.0386	0.0347
Texas	0.0000	0.0000	0.0000	0.0000	0.0000	0.0553	0.0474	0.0415	0.0368	0.0332
Utah	0.0329	0.0330	0.0308	0.0288	0.0277	0.0746	0.0673	0.0596	0.0530	0.0477
Vermont	0.0674	0.0674	0.0674	0.0674	0.0674	0.0674	0.0674	0.0674	0.0674	0.1184
Virginia	0.0499	0.0501	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
Washington	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
West Virginia	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300
Wisconsin	0.1305	0.1174	0.1084	0.1039	0.1013	0.0995	0.0982	0.0973	0.0965	0.0960
Wyoming	0.0000	0.0000	0.0000	0.0909	0.0727	0.0606	0.0520	0.0455	0.0404	0.0364

	5	4.5	4	3.5	3	2.5	2	1.5	1	0.5
Alabama	0.0471	0.0467	0.0463	0.0458	0.0451	0.0441	0.0427	0.0402	0.0400	0.0400
Alaska	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Arizona	0.0601	0.0636	0.0680	0.0736	0.0811	0.0000	0.0000	0.0000	0.0000	0.0000
Arkansas	0.0437	0.0434	0.0431	0.0426	0.0420	0.0412	0.0399	0.0377	0.0361	0.0361
California	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Colorado	0.0244	0.0219	0.0189	0.0150	0.0098	0.0025	0.0000	0.0000	0.0000	0.0000
Connecticut	0.0128	0.0111	0.0104	0.0099	0.0094	0.0089	0.0084	0.0079	0.0074	0.0070
Delaware	0.0739	0.0768	0.0341	0.0322	0.0295	0.0258	0.0203	0.0110	0.0000	0.0000
District of Columbia	0.1143	0.1186	0.1241	0.1276	0.1276	0.1276	0.1277	0.1277	0.1277	0.1277
Florida	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Georgia	0.0439	0.0432	0.0424	0.0413	0.0400	0.0400	0.0400	0.0400	0.0400	0.0398
Hawaii	0.0680	0.0680	0.0680	0.0680	0.0680	0.0680	0.0680	0.0680	0.0680	0.0680
Idaho	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Illinois	0.0405	0.0405	0.0405	0.0405	0.0405	0.0405	0.0405	0.0405	0.0405	0.0405
Indiana	0.0466	0.0466	0.0466	0.0466	0.0466	0.0466	0.0466	0.0466	0.0465	0.0467
Iowa	0.0587	0.0587	0.0587	0.0587	0.0587	0.0587	0.0587	0.0587	0.0587	0.0585
Kansas	0.1058	0.1101	0.1156	0.1226	0.1319	0.1450	0.0666	0.0666	0.0667	0.0667
Kentucky	0.0570	0.0570	0.0570	0.0570	0.0570	0.0570	0.0570	0.0570	0.0570	0.0570
Louisiana	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
Maine	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Maryland	0.0831	0.0831	0.0832	0.0833	0.0835	0.0836	0.0839	0.0844	0.0854	0.0883
Massachusetts	0.0805	0.0806	0.0806	0.0805	0.0806	0.0805	0.0806	0.0806	0.0805	0.0807
Michigan	0.0535	0.0550	0.0570	0.0594	0.0628	0.0661	0.0661	0.0661	0.0661	0.0663
Minnesota	0.1030	0.1030	0.1030	0.1030	0.1029	0.1030	0.1030	0.1029	0.1030	0.1027
Mississippi	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0299	0.0298
Missouri	0.0293	0.0287	0.0283	0.0281	0.0278	0.0273	0.0267	0.0256	0.0250	0.0250
Montana	0.0281	0.0280	0.0280	0.0280	0.0280	0.0280	0.0280	0.0280	0.0281	0.0280
Nebraska	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Nevada	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
New Hampshire	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
New Jersey	0.0720	0.0780	0.0856	0.0953	0.1083	0.1265	0.1537	0.0175	0.0175	0.0174
New Mexico	0.0031	0.0016	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
New York	0.1369	0.1407	0.1454	0.1514	0.1595	0.1707	0.1876	0.2158	0.2722	0.4412
North Carolina	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600
North Dakota	0.0382	0.0425	0.0478	0.0546	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Ohio	0.0303	0.0297	0.0297	0.0297	0.0297	0.0297	0.0297	0.0297	0.0297	0.0298
Oklahoma	0.0705	0.0705	0.0705	0.0705	0.0705	0.0705	0.0705	0.0705	0.0704	0.0704
Oregon	0.1005	0.1006	0.1005	0.1006	0.1005	0.1006	0.1005	0.1006	0.1005	0.1008
Pennsylvania	0.0451	0.0501	0.0564	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Rhode Island	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045
South Carolina	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
South Dakota	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Tennessee	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Texas	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Utah	0.0422	0.0411	0.0398	0.0387	0.0382	0.0374	0.0363	0.0343	0.0331	0.0331
Vermont	0.0674	0.0674	0.0674	0.0674	0.0674	0.0674	0.0674	0.0674	0.0674	0.0674
Virginia	0.1042	0.1102	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0499	0.0499
Washington	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
West Virginia	0.0738	0.0379	0.0376	0.0373	0.0368	0.0362	0.0352	0.0336	0.0304	0.0300
Wisconsin	0.1348	0.1331	0.1321	0.1315	0.1308	0.1307	0.1307	0.1308	0.1309	0.1312
Wyoming	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

	-0.2	-0.15	-0.1	-0.05	0.05	0.1	0.15	0.2
Alabama	0.1944	0.2381	0.1349	0.1135	0.1679	0.1886	0.1987	0.2597
Alaska	0.3179	0.3199	0.2756	0.3160	0.0869	0.3234	0.2776	0.2400
Arizona	0.2066	0.2746	0.1326	0.0891	0.2067	0.2662	0.2581	0.2757
Arkansas	0.1273	0.1453	0.1309	0.2761	0.0656	0.0732	0.0877	0.2206
California	0.2753	0.2737	0.1393	0.1858	0.1667	0.2242	0.2087	0.2485
Colorado	0.2687	0.3025	0.2053	0.3047	0.3074	0.3148	0.3088	0.3161
Connecticut	0.1517	0.1578	0.1438	0.1466	0.1753	0.1493	0.1647	0.1765
Delaware	0.2548	0.1875	0.2369	0.2221	0.2835	0.2494	0.2965	0.3181
District of Columbia	0.2853	0.3167	0.1846	0.2717	0.3140	0.2928	0.3104	0.2611
Florida	0.2642	0.2733	0.1300	0.2227	0.2407	0.2776	0.2645	0.2740
Georgia	0.3127	0.3155	0.1663	0.2232	0.2903	0.2930	0.2888	0.2990
Hawaii	0.3119	0.2995	0.1698	0.2783	0.1926	0.1771	0.3109	0.3209
Idaho	0.2106	0.1259	0.1098	0.0798	0.1639	0.1217	0.1027	0.2734
Illinois	0.2468	0.2763	0.2684	0.2969	0.3193	0.2973	0.3112	0.3192
Indiana	0.2953	0.3176	0.2080	0.3132	0.2904	0.3199	0.3088	0.3188
Iowa	0.2840	0.3105	0.2204	0.2985	0.3141	0.3111	0.3108	0.3162
Kansas	0.3124	0.3079	0.1894	0.2813	0.2754	0.3064	0.2931	0.3119
Kentucky	0.1273	0.1284	0.1273	0.0707	0.0985	0.1845	0.1637	0.2187
Louisiana	0.2254	0.1532	0.1273	0.1081	0.1871	0.1845	0.2020	0.3060
Maine	0.3138	0.3206	0.1524	0.2211	0.2023	0.1605	0.1617	0.2983
Maryland	0.2057	0.2326	0.2567	0.2252	0.1708	0.1543	0.1709	0.1922
Massachusetts	0.1601	0.1633	0.1641	0.1870	0.2063	0.1910	0.1786	0.2129
Michigan	0.2897	0.3045	0.2679	0.2895	0.3053	0.3150	0.2964	0.2980
Minnesota	0.2021	0.2001	0.1980	0.2425	0.3043	0.2673	0.2420	0.2964
Mississippi	0.2308	0.2099	0.1364	0.1228	0.0961	0.1528	0.1378	0.2447
Missouri	0.3154	0.3182	0.2080	0.2552	0.1958	0.2369	0.2235	0.2698
Montana	0.1307	0.1282	0.1267	0.0698	0.2068	0.1910	0.1153	0.2568
Nebraska	0.3162	0.3215	0.2179	0.2941	0.2901	0.3083	0.3070	0.3189
Nevada	0.3123	0.3101	0.1568	0.1828	0.1397	0.3064	0.2938	0.2990
New Hampshire	0.1749	0.1766	0.1633	0.1539	0.1815	0.1671	0.2087	0.2525
New Jersey	0.1648	0.2124	0.2030	0.1598	0.1830	0.1759	0.1851	0.2119
New Mexico	0.1575	0.1457	0.1175	0.0697	0.0619	0.0421	0.0877	0.1343
New York	0.3137	0.3202	0.1800	0.2823	0.2711	0.2791	0.2938	0.3011
North Carolina	0.3144	0.2742	0.1579	0.2559	0.2246	0.2252	0.2751	0.2932
North Dakota	0.3124	0.3050	0.2327	0.2699	0.2908	0.3150	0.2977	0.3155
Ohio	0.2699	0.2971	0.2696	0.2818	0.3102	0.3179	0.3088	0.3073
Oklahoma	0.1999	0.2446	0.1273	0.1081	0.0791	0.1339	0.1851	0.2021
Oregon	0.1920	0.3075	0.1363	0.2059	0.2947	0.2808	0.2305	0.3106
Pennsylvania	0.3135	0.3194	0.2066	0.3125	0.3060	0.3085	0.3060	0.3134
Rhode Island	0.2479	0.3073	0.2544	0.3111	0.2526	0.2855	0.2965	0.2979
South Carolina	0.3132	0.2017	0.2044	0.2817	0.2023	0.2641	0.2214	0.2721
South Dakota	0.2951	0.3178	0.2463	0.3170	0.2924	0.3158	0.2305	0.3169
Tennessee	0.2603	0.2412	0.1391	0.1691	0.1848	0.2257	0.1987	0.2749
Texas	0.1919	0.1769	0.1328	0.1134	0.0984	0.1116	0.1298	0.2261
Utah	0.3035	0.2921	0.1376	0.2112	0.2549	0.2525	0.2693	0.2998
Vermont	0.2713	0.2744	0.2368	0.2919	0.3213	0.3094	0.3111	0.2979
Virginia	0.2665	0.2887	0.2680	0.2304	0.3095	0.2817	0.2485	0.2956
Washington	0.3184	0.3210	0.2662	0.2574	0.3152	0.3034	0.3097	0.3110
West Virginia	0.1273	0.0686	0.1128	0.0204	0.0397	0.0243	0.0702	0.1387
Wisconsin	0.2472	0.2726	0.2671	0.2878	0.3110	0.3084	0.2965	0.3148
Wyoming	0.2886	0.2932	0.2178	0.2975	0.2593	0.3103	0.3023	0.3159

	-0.2	-0.15	-0.1	-0.05	0.05	0.1	0.15	0.2
Alabama	0.0472	0.0490	0.0645	0.0699	0.0654	0.0664	0.0676	0.0832
Alaska	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Arizona	0.0254	0.0480	0.0303	0.0241	0.0588	0.0707	0.0425	0.0607
Arkansas	0.0383	0.0365	0.0401	0.0461	0.0362	0.0336	0.0872	0.0675
California	0.0106	0.0113	0.0116	0.0106	0.0109	0.0145	0.0000	0.0121
Colorado	0.0632	0.0553	0.0471	0.0570	0.0540	0.0585	0.0425	0.0594
Connecticut	0.0521	0.0637	0.0664	0.0717	0.0700	0.0885	0.0473	0.0927
Delaware	0.0507	0.0480	0.0560	0.0521	0.0573	0.0712	0.0616	0.0683
District of Columbia	0.1060	0.1147	0.1128	0.1133	0.1391	0.1318	0.1382	0.1245
Florida	0.0099	0.0097	0.0110	0.0115	0.0118	0.0110	0.0000	0.0123
Georgia	0.0629	0.0713	0.0599	0.0595	0.0708	0.0719	0.0574	0.0746
Hawaii	0.0693	0.0623	0.0626	0.0635	0.0638	0.0638	0.0652	0.0692
Idaho	0.0031	0.0000	0.0000	0.0000	0.0037	0.0222	0.0000	0.0344
Illinois	0.0492	0.0510	0.0590	0.0589	0.0670	0.0574	0.0405	0.0672
Indiana	0.0557	0.0571	0.0549	0.0703	0.0466	0.0742	0.0466	0.0667
Iowa	0.0744	0.0950	0.1037	0.0958	0.1048	0.1061	0.0752	0.1091
Kansas	0.0886	0.1049	0.1088	0.1110	0.1070	0.1091	0.0962	0.1078
Kentucky	0.0550	0.0546	0.0582	0.0512	0.0955	0.1206	0.1214	0.1138
Louisiana	0.0367	0.0320	0.0238	0.0215	0.0287	0.0300	0.0334	0.0472
Maine	0.0173	0.0879	0.0077	0.0069	0.0044	0.0013	0.0013	0.0203
Maryland	0.0858	0.0915	0.0989	0.0947	0.0861	0.0810	0.0713	0.0794
Massachusetts	0.0618	0.0585	0.0696	0.0855	0.0879	0.0885	0.0578	0.0936
Michigan	0.0567	0.0566	0.0563	0.0660	0.0468	0.0511	0.0412	0.0667
Minnesota	0.0916	0.0862	0.0946	0.1182	0.1467	0.1298	0.1202	0.1434
Mississippi	0.0315	0.0378	0.0387	0.0389	0.0379	0.0457	0.0273	0.0446
Missouri	0.0428	0.0409	0.0368	0.0337	0.0301	0.0335	0.0332	0.0355
Montana	0.0390	0.0329	0.0329	0.0263	0.0421	0.0423	0.0203	0.0430
Nebraska	0.0346	0.0340	0.0368	0.0360	0.0367	0.0553	0.0531	0.0549
Nevada	0.0091	0.0119	0.0141	0.0106	0.0176	0.0147	0.0000	0.0165
New Hampshire	0.0269	0.0349	0.0256	0.0335	0.0236	0.0354	0.0000	0.0208
New Jersey	0.0175	0.0233	0.0238	0.0175	0.0175	0.0175	0.0312	0.0396
New Mexico	0.0164	0.0151	0.0088	0.0089	0.0109	0.0111	0.0290	0.0322
New York	0.1078	0.1179	0.1255	0.1246	0.1205	0.1156	0.1018	0.1286
North Carolina	0.0746	0.0699	0.0733	0.0650	0.0579	0.0665	0.0590	0.0727
North Dakota	0.0293	0.0318	0.0322	0.0275	0.0351	0.0378	0.0171	0.0386
Ohio	0.0359	0.0360	0.0355	0.0332	0.0334	0.0328	0.0315	0.0294
Oklahoma	0.0522	0.0671	0.0646	0.0646	0.0600	0.0659	0.0658	0.0546
Oregon	0.1184	0.1160	0.1139	0.1135	0.1183	0.1139	0.1005	0.1204
Pennsylvania	0.0135	0.0178	0.0153	0.0179	0.0210	0.0228	0.0000	0.0241
Rhode Island	0.0586	0.0477	0.0591	0.0469	0.0621	0.0613	0.0565	0.0608
South Carolina	0.0445	0.0000	0.0098	0.0022	0.0000	0.0001	0.0000	0.0000
South Dakota	0.0282	0.0358	0.0280	0.0379	0.0219	0.0258	0.0000	0.0274
Tennessee	0.0000	0.0000	0.0107	0.0147	0.0161	0.0208	0.0000	0.0163
Texas	0.0190	0.0215	0.0219	0.0213	0.0229	0.0218	0.0000	0.0222
Utah	0.0718	0.0689	0.0462	0.0482	0.0577	0.0494	0.0418	0.0642
Vermont	0.0874	0.0868	0.0991	0.0944	0.1019	0.0983	0.1019	0.0928
Virginia	0.0586	0.0596	0.0701	0.0560	0.0717	0.1008	0.0792	0.1044
Washington	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
West Virginia	0.0300	0.0300	0.0303	0.0300	0.0313	0.0313	0.0219	0.0296
Wisconsin	0.1231	0.1350	0.1435	0.1439	0.1517	0.1537	0.1536	0.1587
Wyoming	0.0151	0.0095	0.0165	0.0179	0.0155	0.0240	0.0000	0.0142

Chapter 3: Fiscal and Currency Union with Default and Exit

Alessandro Ferrari , Ramon Marimon and Chima Simpson-Bell

Abstract

Countries which share a common currency potentially have strong incentives to share macroeconomic risks through a system of transfers to compensate for the loss of national monetary policy. However, the option to leave the currency union and regain national monetary policy can place severe limits on the size and persistence of transfers which are feasible inside the union. In this paper, we derive the optimal transfer policy for a currency union as a dynamic contract subject to enforcement constraints, whereby each country has the option to unpeg from the common currency, with or without default on existing obligations. Our analysis shows that the lack of independent monetary policy, or an equivalent independent policy instrument, limits the extent of risk-sharing within a currency union; nevertheless, in the latter, the optimal state-contingent transfer policy take as given the optimal monetary policy as to implement a constrained efficient allocation that minimises the losses of the monetary union. At the steady state welfare is lower than in a fiscal union with independent monetary policies. Nevertheless, in our simulations, the macroeconomic stabilisation effects and the social values achieved, under the two different union regimes, are quantitatively almost the same.

1 Introduction

In a federal state, with a single currency, states share risks through the federal budget (automatic stabilisers) and other risk-sharing fiscal policies. Furthermore, well-functioning, and integrated, markets – in particular, financial markets – also provide insurance against local shocks and can help to circumvent local nominal rigidities, leaving little role for an independent monetary policy. However, in a monetary union – such as the European EMU – the federal fiscal risk-sharing instruments are missing and markets may not be developed and integrated enough to provide the necessary private risk-sharing. Therefore, independent monetary policy may still have potential value. This point is made formally in

Auclert and Rognlie (2014) and Farhi and Werning (2017) who derive optimal risk-sharing policies in a setting with nominal rigidities. Nevertheless, they do not account for two characteristic aspects of unions: in a union of sovereign countries there is limited enforcement. – exit is always an option even if, as in the case of Brexit, it can be a costly option –, but a union is a long-term partnership where mutually beneficial policies bind countries together, deterring them from exiting. Risk-sharing policies can play this role.

In fact, in the euro crisis the threat of exit from the Euro Area, and defaulting on payment obligations, has triggered sovereign debt spreads of Greece and other countries. Bayer et al. (2018) provide evidence that market participants even attached positive probability to Germany and France's exit from the common currency during the crisis. Any risk-sharing within a currency union is therefore subject to participation constraints, for both borrowing and lending countries. This point has been made by Abraham et al. (2019) who characterise constrained-efficient risk-sharing contracts as a self-enforcing mechanism within a union which is subject to limited enforcement constraints. Nevertheless, since they model a fiscal – not a monetary – union, the loss (or possible gain, if exiting) of an independent monetary policy plays no role in their analysis.

Our paper integrates these two earlier approaches by analysing long-term risk-sharing contracts as self-enforcing mechanisms in currency unions, taking into account that in monetary unions exit can take two forms. Union members can exit the union to regain control of monetary policy. For example, Sweden or Poland are full members of the European Union who persist in keeping their currencies. Alternatively, union members can exit the union to renege on their obligations; in particular, default on their debts. Default, or partial default, does not necessarily imply exit from the union (e.g. defaulting states in United States, Greece in 2012) but, as a union's 'participation constraint' , the relevant case is when the possibility of default is associated with exit.

We model the union as two identical countries facing a simple nominal rigidity which creates a stabilisation role for monetary policy. There is no aggregate risk, meaning that country-risks are fully negatively correlated. We then derive the optimal history dependent transfer policies as a long-term dynamic contract subject to participation constraints. Under these constraints, the contract must improve upon an outside option in which each country has independent monetary policy, allowing it to eliminate the distortion caused by the nominal rigidity, and can borrow and lend using defaultable one-period bonds. Due to the forward looking nature of the participation constraints, we are able to characterize the constrained efficient allocation using the recursive contract solution techniques developed in Marcet and Marimon (2019). Effectively, the contract, as a social contract, gives more weight to a country whenever its participation constraint binds.

We compare the performance of the currency union against two benchmarks: an optimal fiscal union in which the nominal rigidity does is fully eliminated by independent monetary policy, and a two good version of the defaultable debt economy in Arellano (2008). We start by characterizing a number of results regarding the comparison between the fiscal union and the currency union with fiscal transfers.

We show that if the fiscal transfers are able to achieve full risk-sharing then the two unions are identical. This result is a version of what the literature has labeled the "risk-sharing miracle". This result stems from the ability of a common currency to stabilize both economies when full risk-sharing is achieved on the fiscal side.

We show that when the planner cannot attain full risk-sharing the fiscal union is strictly better than the currency union. This comes from the deadweight loss that the common monetary policy entails. Such loss shrinks the production possibility frontier, thereby reducing the maximum value attainable by a planner in a currency union.

We also show that optimal common monetary policy is designed to minimize the deadweight loss and that the planner allocation in this economy is still constrained efficient.

We then simulate our economy to study three main features of our model. First we ask whether these kind of contracts are feasible. Secondly, if they are, we ask what is the optimal design of fiscal transfers in terms of size and cyclicality. Thirdly, we investigate how costly is the deadweight loss stemming from the common monetary policy.

In our simulations we find that the fiscal and the currency union are close to identical. We attribute this result to the ability of the optimal policies in the currency union to produce very small deadweight losses. Secondly, in most of our parametrizations, the steady states feature partially state dependent consumption, meaning that the limited enforcement constraints prevent the central planner from achieving full risk-sharing.

As we have mentioned, our work is close to Auclert and Rognlie (2014) and Farhi and Werning (2017) and, therefore, to Hoddenbagh and Dmitriev (2017) who takes a similar approach. We build on this work by considering the participation constraints implied by the option of unpegging from the common currency and , taking it a step forward, by deriving constrained-efficient recursive policies in a monetary union with equally patient countries. We also build on Abraham et al. (2019), although, in contrast with our work, they assume – as the sovereign literature does, to match observed levels of debt – that the 'debtor country' is impatient while the 'lender country' is risk-neutral, in fact, the latter acts as a *financial stability fund*[1]. In this respect, our work is also related to the extensive sovereign debt literature; for example, Gourinchas et al. (2019) solve for the optimal application of a no-bailout rule, finding that less than full enforcement can be sufficient to prevent a fiscally weak country from engaging in risky borrowing.

In our analysis we take the entry or formation decision for the union as given, and focus on the possibility of a breakup, but there is also a literature which considers union formation incentives. In particular, Cooper and Kempf (2003) investigate the conditions under which countries will be able to cooperate to realize the gains from entering a monetary union. Cooper et al. (2008) examines the

[1]Their contracts also account for moral-hazard constraints: the 'debtor country' can reduce its risk profile with non-contractable effort. We abstract from this feature.

conditions under which a central authority in a multi-region economy will find it optimal to take on the obligations of regional governments.

After solving for the constrained efficient allocation in our framework, we also propose an approach to implementing the net payments within the union through trading of state contingent debt contracts. For this we rely on Kehoe and Perri (2004) and Alvarez and Jermann (2000) which demonstrate how constrained efficient allocations with limited commitment can be decentralised using trading of securities. A technical contribution of our paper is that it features a unique feedback between the constrained efficient allocation and its decentralisation. The asset positions calculated in the decentralisation also determine the liabilities which each country carries into its outside option, and these outside options in turn determine the constrained-efficient allocation (recall that in our framework, exit does not imply default). This creates an interesting fixed point problem which we are able to solve numerically.

The rest of the paper is organised as follows. In Section 2 we describe the basic two-goods open economy with monopolistic competition in the non-tradeable goods sector and a nominal friction. We describe the contracts which make up the fiscal and currency unions, and the outside options available to each country. We then characterize the constrained efficient allocations of the union contracts and the associated implementation using state contingent debt. In this section we provide the main theoretical results. In Section 3 we display the policy functions in the different economies and simulate their responses to a debt crisis. We also study the behaviour of the stochastic steady states.Section 4 shows discusses the results under different parametrization of the model. Section 6 offers concluding comments.

2 Model

We start by modelling a fiscal union between two countries. The two countries have endowments of tradable goods and produce non-tradeables. Differently from the existing literature, we model the two countries as symmetric in terms of risk aversion and patience. Agents can partake in risk-sharing through a long term contract subject to participation constraints, where the outside option is defined by an Arellano economy. Namely, countries can opt out of the risk-sharing contract and borrow through non-state contingent bonds from a risk neutral lender. When in the Arellano economy agents can default on their debt subject to an output cost and temporary exclusion from financial markets. A key feature of this economy is that if a country leaves the contract but does not default, it starts with a stock of liabilities equal to the present discounted value of the promised transfers in the fiscal union.

Secondly, we extend the setup to accommodate a currency union as a long term risk-sharing contract. In this context nontradables producers are subject to staggered prices nominal rigidities. In this setup the outside option allows countries to move to an Arellano economy with independent monetary policy. When countries leave the currency area they can again pay their previous obligations or default on them. Previous work focuses on optimal risk-sharing schemes within a monetary union without accounting for the incentives of the countries to leave the union.

2.1 Environment

Two infinitely lived countries $i \in \{1, 2\}$. Time is discrete. Each period a country receives a random endowment of an identical, freely tradeable good Y_T^i. This is the only source of uncertainty in the model. As there is no aggregate uncertainty the country specific endowments are fully negatively correlated; i.e. for all $t \geq 0$, $Y_{T,t}^1 + Y_{T,t}^2 = Y_T$. Uncertainty is described by a finite state Markov process $\{s_t\}$ with elements $s_t \in \mathcal{S}$ and transition matrix Π – in fact, $s_t = Y_{T,t}^1$. Given this Markov structure, the relevant exogenous state in this environment will be the vector (s_{t-1}, s_t)[2].

Preferences over consumption of tradeable goods C_T, non-tradeable goods C_{NT} and labour supply N are given by the utility function:

$$U_i = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left(\frac{(C_{T,t+k})^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{(C_{NT,t+k})^{1-\gamma}}{1-\gamma} - \frac{N_{t+k}^{1+\phi}}{1+\phi} \right) \right) \tag{1}$$

All goods are perishable, non-tradeable goods must be consumed in the country in which they are produced, and labour is immobile between countries.

Non-tradeable goods are produced by each country using a technology which is linear in labour input. In each country there is a continuum of firms $j \in [0, 1]$ which produce output according to:

$$Y_{NT}^{ij} = N_{ij} \tag{2}$$

The consumer's utility value from consuming each of the varieties j is given by the CES aggregator:

$$C_{NT}^i = \left(\int_{j=0}^{1} (C_{NT}^{ij})^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where $\epsilon > 1$ is the elasticity of substitution between varieties. Consumption of each variety must equal output, i.e. $C_{NT}^{ij} = Y_{NT}^{ij}$, and labour market clearing implies that $N_i = \int_j N_{ij}$.

We assume that production of non tradeables is subsidised at $1/\epsilon$ to erase the monopoly quantity

[2] With an abuse of notation, we also denote (s_{t-1}, s_t) by s, making this explicit when needed.

friction. Given its price P_{NT}^{ij}, the producer of variety j satisfies demand C^{ij} by hiring N^{ij} workers and earns profits

$$\Pi^{ij} = (P_{NT}^{ij} - (1 + \tau_L^i)W^i)N^{ij} \tag{4}$$

Where τ_L^i is the government labor subsidy. Profits are distributed to households.

2.2 Fiscal Union

We model the fiscal union as a long term contract. In this setup countries receive state contingent net transfers of the tradeable good $\tau^i(s)$ from each other to absorb the risk associated with the realization of the tradeable goods shock. In this sense, the contract can also be interpreted as a fiscal union. Country i's consumption of the tradeable good is then

$$C_T^i(s) = Y_T^i(s) + \tau^i(s) \tag{5}$$

Countries remain in the contract as long as they do not choose to leave the fiscal union. Leaving the union results in the loss of the state contingent transfers.

2.2.1 Planner's problem

The planner arranges transfers within the union subject to each country's outside option:

$$\max_{\{C_{T,i}, C_{NT,i}, N_i\}} \sum_{i=1,2} \mu_i E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_{T,t})^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{(C_{NT,t})^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \right) \tag{6}$$

s.t.

$$C_T^i(s) = Y_T^i(s) + \tau^i(s) \tag{7}$$

$$\sum_{i=1,2} \tau^i(s) = 0 \tag{8}$$

$$\sum_{k=0}^{\infty} \beta^k \left(\frac{(C_{T,t+k})^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{(C_{NT,t+k})^{1-\gamma}}{1-\gamma} - \frac{N_{t+k}^{1+\phi}}{1+\phi} \right) \right) \geq V_i^o(s, B) \tag{9}$$

$$Y_{NT}^{ij} = N_{ij} \tag{10}$$

$$C_{NT}^i = \left(\int_{j=0}^1 (C_{NT}^{ij})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{11}$$

$$N_i = \int_j N_{ij} \tag{12}$$

Debt B is defined as the stock of liabilities that a country would have outside the contract. We define this stock as the net present value of the promised transfers inside the contract. The decision of leaving the contract is irreversible. We assume that if a country leaves the contract it refinances its debt with a competitive outside lender borrowing at a risk free rate r. In other words, the outside option to the fiscal union is an Arellano type economy. Formally, the stock of liabilities that a country carries outside the contract is defined as

$$B_{i,t} = \mathbb{E}_t \sum_{s=t}^{\infty} q_{t,s}(Y_{i,s} - c_{i,s}) \tag{13}$$

Where $q_{t,s} = \prod_{s=t}^{k} q_{s,s+1} \;\; q_{t-1,t} \equiv \max_j \left\{ \beta \frac{U_j'^{t}}{U_j'^{t-1}} \right\}$. This assignment of liabilities will be made clearer when we outline the decentralization of the contract which describes the union. Note that the planner does not internalize the effect of within contract transfers on the value of the outside option.

2.2.2 Outside options

In each period, each country has the option of defaulting on its payments within the union, and choosing to leave the fiscal union. Thus, when it is inside the contract, country i faces a choice over the actions $\{SR, LR, LD\}$, i.e. stay in the fiscal union and repay transfer commitments, leave the contract and honour payments, and leave and default. We assume that defaulting on payments triggers temporary exclusion from financial markets so that the country can no longer trade bonds for a stochastic number of periods. Following Arellano (2008), a country in autarky also suffers an output cost on its endowment of the tradeable good, $\chi(Y_T^i)$; this output cost is chosen to ensure that a country is more likely to consider default when its endowment of the tradeable good is low.

We can write the decision problem of each country outside the contract in a recursive form. The value of leaving the contract takes the following form:

$$V_i^o(s,B) = \max_{LR,LD}\{V_i^{LR}(s,B), V_{LD}^i(s)\} \tag{14}$$

Namely, the agent can choose whether to repay the promised transfers or default on its obligations. If the country defaults it is temporarily relegated to financial autarky and faces a proportional output cost in terms of tradable endowment.

$$V_i^{LD}(s) = \max_{C_{NT,i},N_i} \frac{(Y_{T,i}-\chi(Y_{T,i}))^{1-\gamma}}{1-\gamma} + \alpha\left(\frac{C_{NT}^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}\right) + \beta\mathbb{E}\left[\theta V_i^o(s',0) + (1-\theta)V_i^{LD}(s')\right] \tag{15}$$

Where θ is the probability with which the country financial markets exclusion is terminated and $\chi(E_T)$ is the output cost of financial autarky which, for a constant parameter ψ, takes the form:

$$\chi(Y_T) = \begin{cases} Y_T - \bar{Y}_T, & \text{for } Y_T \geq \bar{Y} \\ 0, & \text{for } Y_T < \bar{Y}_T \end{cases}, \text{ where } \bar{Y}_T = \psi\mathbb{E}Y_T$$

If the country regains access to financial markets, it does so with zero outstanding liabilities. If the country has left the risk-sharing agreement but opted not to default on its obligations, then the value of the problem is

$$V_i^{LR}(s,B) = \max_{C_{T,i},C_{NT,i},N_i,B_i'} \frac{C_{T,i}^{1-\gamma}}{1-\gamma} + \alpha\left(\frac{C_{NT}^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}\right) + \beta\mathbb{E}V_i^o(s',B_i') \tag{16}$$

The budget constraints in each case are below. Where the price of tradables is the numeraire.

If the country leaves and repays then it has access to the non contingent one period bond as saving technology:

$$C_T^i(s) + P_{NT}^i(s)C_{NT}^i(s) + B_i \leq Y_T^i(s) + W^i(s)N^i(s) + \Pi^i(s) + B_i'Q(s,B_i') \tag{17}$$

We omit the production subsidy since it is rebated to households and cancels out with increased profits. The subsidy is such that output reaches its efficient level. One can think of a government taxing profits and subsidising production. This will exactly cancel out in the household budget constraint as profits are rebated. Where $Q(s, B'_i)$ is the bond price set between the country outside the contract and the competitive lender. The bond price is given by

$$Q(s, B'_i) = \frac{1}{r}\mathbb{E}_t(1 - D(s', B'_i))$$

Where $D(s', B'_i)$ is the decision to default on debt in the next period.

If the country leaves and default on past liabilities then it has no saving technology and is subject to a per period output cost:

$$C^i_T(s) + P^i_{NT}(s)C^i_{NT}(s) \leq Y^i_T(s) - \chi(Y^i_T(s)) + W^i(s)N^i(s) + \Pi^i(s) \tag{18}$$

2.3 Currency Union

Next we study a nominal version of the model to evaluate how the to design a transfer system in a currency union. In order to account for the money in this economy we assume that non-tradeable producers are subject to staggered prices.

2.3.1 Price setting for non-tradeables

Producers of non-tradeable goods face a rigidity in their price setting decisions. We incorporate this by assuming that at the beginning of each period firms must make their pricing decision before the realisation of the tradeable goods endowments $Y_{T,i}$, and wages cannot be conditional on this realisation[3].

Given its price P^{ij}_{NT}, the producer of variety j satisfies demand C^{ij} by hiring N^{ij} workers and earns profits

$$\Pi^{ij} = (P^{ij}_{NT} - (1 + \tau^i_L)W^i)N^{ij} \tag{19}$$

which are distributed to households.

[3]Note that in our framework having independent *state-contingent labour taxes* will play the same role as having an independent monetary policy.

For convenience, define the labour wedge $\kappa(s)$ as

$$\kappa^i(s) = 1 - \frac{U^i_N(s)}{U^i_{NT}(s)} = 1 - {C^i_{NT}}^{\gamma+\phi}(s) \tag{20}$$

Such wedge arises due to the lack of state-contingent non-tradeables prices.

The non-tradeable good price is predetermined, hence producers maximize the expected profits across states. The optimal price setting rule is characterized in the following Lemma:

Lemma 1 (Optimal Price Setting). *The optimal price for non-tradeables producers is:*

$$P_{NT} = \frac{\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma} W(s)}{\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma}}$$

Such pricing implies zero expected labor wedge

$$\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma} \kappa(s) = 0. \tag{21}$$

Proof. See Appendix A. ■

Lemma 1 shows that, in the absence of uncertainty, the optimal price is equal to the wage. Recall that this result is obtained by levying a labor subsidy that undoes the monopoly markup. Similarly, in absence of uncertainty, the labor wedge is equal to zero. This result can be understood as the lack of state-contingency being inconsequential when the state is constant.

We model the currency union as a long term contract. Countries within the currency union face the same price for tradeable goods, so that:

$$P^1_T(s) = P^2_T(s) \tag{22}$$

The implicit assumption is that the fixed nominal exchange rate within the currency union is 1.

Countries remain in the contract as long as they do not choose to unpeg from the common currency, or default on the net payments specified by the contract. In this case the Ramsey planner is subject to pricing frictions and different outside options.

2.3.2 Outside options

In each period, each country has the option of defaulting on its payments within the union, and choosing to unpeg from the common currency and regain control of its monetary policy. We assume that either defaulting or unpegging implies abandoning the common currency. Thus, when it is inside the contract, country i faces a choice over the actions $\{PR, UR, UD\}$, i.e. maintain the peg and repay transfer commitments, unpeg and honour payments, and unpeg and default. We assume that defaulting on payments triggers temporary exclusion from financial markets so that the country can no longer trade bonds.

We also assume that the cost of unpegging from the common currency is that the country can only trade non-state contingent bonds B, limiting its consumption smoothing ability. The repayment commitments which remain are still denominated in the common currency. Unpegging is also an irreversible decision.

We can write the decision problem of each country outside the contract in a recursive form. Suppose that country i has already both defaulted on its debt and unpegged from the common currency. Its value function $V_i^{UD}(s)$ can then be written as:

$$V_i^{UD}(s) = \max_{C_{NT,i}, N_i} \frac{(Y_{T,i} - \chi(Y_{T,i}))^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{C_{NT}^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi} \right) + \beta \mathbb{E} \left[\theta V_U^i(s', 0) + (1-\theta) V_i^{UD}(s') \right] \tag{23}$$

Suppose now that country i has unpegged but not defaulted yet. Its value function $V_i^U(s)$ can be written as:

$$V_i^U(s, B) = \max_{P,U} \{ V_i^{UR}(s, B), V_i^{UD}(s) \} \tag{24}$$

where V_{UR}, the value of maintaining repaying the contractual obligations once the country has unpegged, is given by

$$V_i^{UR}(s, B) = \max_{C_{T,i}, C_{NT,i}, N_i, B_i'} \frac{(C_{T,i})^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{(C_{NT})^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi} \right) + \beta \mathbb{E} V_U^i(s', B') \tag{25}$$

Finally, the outside option of a country that is still inside the currency union contract is given by the option value of, just unpegging, or defaulting and unpegging at the same time:

$$V_i^o(s,B) = \max\{V_i^{UR}(s,B), V_i^{UD}(s)\} \tag{26}$$

The budget constraints in each case are below. In each case the constraint is written in the common currency of the union (Euros). If the country unpegs from the common currency, $\epsilon^i(s)$ is the number of units of i's currency per Euro. Note that the price of tradables is always in Euros.

Unpeg without defaulting:

$$P_T^i(s)C_T^i(s) + \frac{P_{NT}^i(s)C_{NT}^i(s)}{\epsilon^i(s)} + B_i' \leq P_T^i(s)Y_T^i(s) + \frac{W^i(s)N^i(s)}{\epsilon^i(s)} + \frac{\Pi^i(s)}{\epsilon^i(s)} + B_i'\frac{Q(s,B_i')}{\epsilon^i(s)} \tag{27}$$

Unpeg and default:

$$P_T^i(s)C_T^i(s) + \frac{P_{NT}^i(s)C_{NT}^i(s)}{\epsilon^i(s)} \leq P_T^i(s)(Y_T^i(s) - \chi(Y_T^i(s))) + \frac{W^i(s)N^i(s)}{\epsilon^i(s)} + \frac{\Pi^i(s)}{\epsilon^i(s)} \tag{28}$$

2.3.3 Monetary Policy

We follow Farhi and Werning (2017) and Auclert and Rognlie (2014) in the definition of monetary policy. Within the currency union, due to the underlying price rigidity, demand externalities arise, generating a wedge between the private and social value of risk-sharing. Monetary policy optimally sets the union wide weighted wedge to zero.

Outside the monetary union, monetary policy is independent and country specific wedges are optimally equal to zero. This implies a relatively increased value of the outside option, compared to the fiscal union setup due to the independent monetary policy outside the currency area.

By the intratemporal first order condition the labour wedge is equal to zero in absence of nominal rigidities (given the subsidy to production). In presence of nominal rigidities the price of non tradeables is not equal to the wage implying that $\kappa^i(s) \neq 0$. Independent monetary policy equates the country specific labour wedge to zero.

We assume that monetary policy inside the currency union equates the weighted average labour wedge to zero. The weights are symmetric and equal to 1/2.

The following Lemma characterizes optimal monetary policy.

Lemma 2 (Optimal Monetary Policy). *Optimal independent monetary policy implies*

$$\kappa^i(s) = 0, \quad \forall s$$

Optimal monetary policy in a currency union implies

$$\sum_{i=1,2} {C_{NT}^{i}}^{1-\gamma} \kappa^{i}(s) = 0, \quad \forall s$$

Proof. See Appendix A ■

This implies that the $\kappa^i(s) \neq 0$, for $i = 1, 2$, due to asymmetry of the shock process.

2.4 The Union

In this section we describe how to rewrite the problem as a saddle point. We characterize the setup for the currency union since it is more general. The problem for the fiscal union with two independent monetary authorities is identical up to the presence of pricing frictions. We model the currency union with optimal transfers as a long term contract. This contract is subject to two sided limited commitment, whereby both countries can renege on the contract and switch to one of the outside options. The optimal contract is the solution to the following problem:

$$\max_{\{C_{T,i}(s^t), C_{NT,i}(s^t), N_i(s^t)\}_{i=1,2}} \sum_{i=1,2} \mu_{i,0} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{T,i}(s^t)^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{C_{NT,i}(s^t)^{1-\gamma}}{1-\gamma} - \frac{N_i(s^t)^{1+\phi}}{1+\phi} \right) \right)$$

s.t.

$$\sum_{i=1,2} (P_T(s^t) C_T^i(s^t) + P_{NT,i} C_{NT}^i(s^t)) \leq \sum_{i=1,2} (P_T(s) Y_T^i(s) + W_i(s) N_i(s) + \Pi^i(s)) \tag{29}$$

$$\sum_{i=1,2} C_T^i(s^t) = \sum_{i=1,2} Y_T^i(s^t) \tag{30}$$

$$\mathbb{E}_t \sum_{r=t}^{\infty} \beta^r \left(\frac{C_{T,i}(s^r)^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{C_{NT,i}(s^r)^{1-\gamma}}{1-\gamma} - \frac{N_i(s^r)^{1+\phi}}{1+\phi} \right) \right) \geq V_i^o(s_t, B) \tag{31}$$

It is known from Marcet and Marimon (2019) that this problem can be rewritten as the saddle point problem:

$$\mathcal{SP} \min_{\{\lambda_{i,t}\}_{i=1,2}} \max_{\{C_{T,i}, C_{NT,i}, N_i\}_{i=1,2}} \sum_{i=1,2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\mu_{i,t} \left(\frac{C_{T,i}(s^t)^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{C_{NT,i}(s^t)^{1-\gamma}}{1-\gamma} - \frac{N_i(s^t)^{1+\phi}}{1+\phi} \right) \right) + \lambda_{i,t} \left(\frac{C_{T,i}(s^t)^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{C_{NT,i}(s^t)^{1-\gamma}}{1-\gamma} - \frac{N_i(s^t)^{1+\phi}}{1+\phi} \right) - V_i^o(s_t, B_t) \right) \right] \tag{32}$$

$$\mu_{i,t+1} = \mu_{i,t} + \lambda_{i,t} \tag{33}$$

Here $\lambda_{i,t}$ is the Lagrange multiplier of country i's participation constraint (PC). We now also have a ***co-state*** variable $\mu_{i,t}$ which effectively keeps track of the cost of keeping each agent inside the contract. We can then make use of a normalization which will reduce the dimension of the state space in the final problem. First we define the ***relative weight*** z_t of country 1 as

$$z_t = \frac{\mu_{1,t}}{\mu_{2,t}} \tag{34}$$

Then we rescale each country's Lagrange multiplier as follows:

$$\nu_{i,t} = \frac{\gamma_{i,t}}{\mu_{i,t}} \tag{35}$$

We can now derive a new equation of motion for the relative weight z_{t+1}:

$$z_{t+1} = z_t \frac{1+\nu_{1,t}}{1+\nu_{2,t}} \tag{36}$$

After this normalization, the state/co-state vector is (s, z) and the **saddle point Bellman equation** can be written as

$$\Omega(s,z) = \mathcal{SP} \min_{\{\nu_i\}_{i=1,2}} \max_{\{C_{T,i}, C_{NT,i}, N_i\}_{i=1,2}} z \left((1+\nu_1)(\frac{C_{T,1}(s,z)^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{C_{NT,1}(s,z)^{1-\gamma}}{1-\gamma} - \frac{N_1^{1+\phi}}{1+\phi} \right) - \nu_1 V_1^o(s,B) \right)$$

$$+(1+\nu_2)\left(\frac{C_{T,1}(s,z)^{1-\gamma}}{1-\gamma}+\alpha(\frac{C_{NT,2}(s,z)^{1-\gamma}}{1-\gamma}-\frac{N_2(s,z)^{1+\phi}}{1+\phi}\right)-\nu_2 V_2^o(s,B)+(1+\nu_2)\beta E\Omega(s',z') \tag{37}$$

$$z'=z\frac{1+\nu_1}{1+\nu_2} \tag{38}$$

$$\sum_{i=1,2}(P_T(s,z)C_T^i(s,z)+P_{NT,i}C_{NT}^i(s,z))\leq\sum_{i=1,2}(P_T(s,z)Y_T^i(s,z)+W_i(s,z)N_i(s,z)+\Pi^i(s,z)) \tag{39}$$

$$\sum_{i=1,2}C_T^i(s^t)=\sum_{i=1,2}Y_T^i(s^t) \tag{40}$$

The policies in the union are given by the first order conditions of this problem. For tradeable goods consumption these are:

$$\frac{z(1+\nu_1)}{C_{T,1}(s,z)^{\gamma}}=\zeta(s,z)P_T(s,z) \tag{41}$$

$$\frac{1+\nu_2}{C_{T,2}(s,z)^{\gamma}}=\zeta(s,z)P_T(s,z) \tag{42}$$

Where $\zeta(s)$ is the multiplier on the resource constraint. From this we can derive the relative tradeables consumption of the two countries as:

$$\frac{C_{T,1}}{C_{T,2}}=\left(\frac{z(1+\nu_1)}{1+\nu_2}\right)^{\frac{1}{\gamma}}=(z')^{\frac{1}{\gamma}} \tag{43}$$

It follows that each country's consumption of the tradeable good is:

$$C_{T,1}(s,z)=\frac{(z')^{\frac{1}{\gamma}}}{1+(z')^{\frac{1}{\gamma}}}\sum_{i=1,2}Y_T^i(s^t) \tag{44}$$

and

$$C_{T,2}(s,z) = \frac{1}{1+(z')^{\frac{1}{\gamma}}} \sum_{i=1,2} Y_T^i(s^t) \tag{45}$$

The conditions for the non-tradeables consumption and labour supply of country i are then:

$$C_{NT,i}(s,z) = \left(\alpha \frac{P_T(s,z)}{P_{NT,i}(s,z)}\right)^{\frac{1}{\gamma}} C_{T,i}(s,z) \tag{46}$$

and

$$N_i = C_{NT,i}(s,z) \tag{47}$$

Furthermore the Union's value function takes the form:

$$\Omega^U(s,z) = zV_1^U(s,z) + V_2^U(s,z) \tag{48}$$

for $U = F, M$, depending on whether it refers to a Fiscal Union with two independent monetary authorities or to a Monetary Union[4].

2.4.1 Decentralization with Endogenous Borrowing Limits

We now show how the contract allocation can be decentralized as a competitive equilibrium with trading of state contingent debt contracts and borrowing constraints.

We will be interested in union allocations for which the present value, at the correctly defined prices, is finite. We say that an allocation has *high implied interest rates* if

$$E_0 \sum_{t=0}^{\infty} q(s^t, z_t \mid s_0, z_0)(Y_{1,t} + Y_{2,t}) < \infty \tag{49}$$

where

$$q(s_{t+1}, z_{t+1} \mid s_t, z_t) = \max_i \beta \left(\frac{C_{T,i}(s_{t+1}, z_{t+1})}{C_{T,i}(s_t, z_t)}\right)^{-\gamma} \tag{50}$$

[4]In $\Omega^M(s,z)$, s denotes (s_{-1}, s).

and $q(s^{t+k}, z_{t+k} \mid s_t, z_t) = \prod_{n=0}^{k-1} q(s_{t+n+1}, z_{t+n+1} \mid s_{t+n}, z_{t+n})$.

Country Problem Each country i has access to a one period state contingent debt contract $B_i(s) = \{b_i(s' \mid s)\}_{s'}$, which denotes the amount of the tradeable good which country i promises to deliver in the state s'. In addition, let the price of a unit of an Arrow security which pays in state s' be $q(s' \mid s)$. Then the value of the debt contract is $\sum_{s'} q(s' \mid s) b_i(s' \mid s)$. Country i solves the following problem:

$$\omega(b_i, s) = \max_{\{C_T, C_{NT}, N, B_i(s)\}} \frac{C_{T,i}^{1-\gamma}}{1-\gamma} + \alpha \left(\frac{C_{NT,i}^{1-\gamma}}{1-\gamma} - \frac{N_i^{1+\phi}}{1+\phi} \right) + \beta E[\omega(b_i', s') \mid s]$$

subject to

$$\begin{aligned} C_T^i(s) + P_{NT,i} C_{NT}^i(s) + b_i(s) \leq \\ Y_T^i(s) + W_i(s) N_i(s) + \Pi_i(s) + \sum_{s' \mid s} q(s' \mid s) b_i(s' \mid s)) \end{aligned} \tag{51}$$

and

$$b_i(s' \mid s) \leq \bar{B}_i(s') \tag{52}$$

Where $\bar{B}(s')$ is a state contingent **endogenous borrowing limit**.

Definition 1 (Equilibrium). *A competitive equilibrium with borrowing limits is a collection of borrowing limits $\{\bar{B}(s)\}$ and initial debt positions $\{b_i(s_0)\}$, together with an allocation $\{C_{T,i}(s), C_{NT,i}(s), N_i(s)\}$, state contingent debt contracts $\{B_i'(s)\}$, goods prices $\{P_T(s), P_{NT}(s), W_i(s)\}$ and asset prices $q(s' \mid s)$ such that $\{C_{T,i}(s), C_{NT,i}(s), N_i(s)\}$ solves country i's decision problem, markets clear and the resource constraint holds.*

The consumption and asset choice decisions give us the Euler equation:

$$q(s' \mid s) \geq \beta \pi(s' \mid s) \left(\frac{C_T^i(s', b')}{C_T^i(s, b)} \right)^{-\gamma} \tag{53}$$

A competitive equilibrium with borrowing limits therefore satisfies this equation and the transversality conditions:

$$\lim_{t\to\infty} E_t \beta^t q(s^{t+1} \mid s_t) C_T^i(s_t, b_i(s_t))^{-\gamma} b_i(s_{t+1}) = 0 \tag{54}$$

Proposition 1 (Decentralized Equilibrium). *Any union allocation with high implied interest rates can be decentralized as a competitive equilibrium with endogenous borrowing limits.*

Proof. See Appendix A ■

With this implementation of the union allocations, we are now able to specify the liabilities generated by each country's participation in the union. In any given state, these same liability levels would also need to be financed outside of the union if one of the countries chose to exit (which, in equilibrium, never happens). Given that in the outside option, each country can decide to default completely on its debt, an obvious question is whether there is any case in which a participation constraint binds and the constrained country's preferred outside option is to continue repaying its debts. In the full currency union, this involves a complex comparison of the value of independent monetary policy with the value of enhanced risk-sharing in the contract. In the fiscal union, however, where there is no nominal friction, we are able to show that there is no case in which a country is indifferent between remaining in the union and the alternative of leaving and continuing to repay its debt.

Proposition 2 (Optimal Exit in Fiscal Unions). *In the fiscal union with two independent monetary authorities, whenever the participation constraint is binding for country i, $V_i^{LD}(s) > V_i^{LR}(s, B)$.*

Proof. See Appendix A ■

The next proposition formalises a different aspect of the two problems: if the currency union is able to achieve full risk-sharing, then it will attain the same value as the fiscal union.

Proposition 3 (Risk-sharing Miracle). *If in the steady state the currency union attains full risk-sharing, i.e. $(C_T^1(s)/C_T^2(s))^{-\gamma} = \bar{c}$, $\forall s$, then the common monetary policy is able to stabilize both economies at once.*

Proof. See Appendix A ■

Similarly to Auclert and Rognlie (2014) when countries attain full risk-sharing, stabilizing one economy through common monetary policy also puts the other country's labour wedge to zero. This result carries important consequences for the type of steady state that may arise in this model, and, in particular, for the comparison between fiscal and currency unions in full risk-sharing steady states.

The following definition describes the two types of steady state which can emerge in the monetary and fiscal unions.

Definition 2 (Steady States).

a) A steady state with perfect risk-sharing is a path in which for some k, the relative weight z_t is constant for all $t > k$.

b) A steady state with imperfect risk-sharing is a path in which for some k, the relative weight $z_t \in \{\underline{z}, \ldots, \bar{z}\}$ (i.e. it is in the discrete support of the ergodic distribution), for all $t > k$, where $\bar{z} > \underline{z}$, $\underline{z} = \min_{s \in S \times S}\{z : V_1^U(s,z) = V_1^o(s,B)\}$ and $\bar{z} = \max_{s \in S \times S}\{z : V_2^U(s,z) = V_2^o(s,B)\}$.

Given these definitions, an immediate corollary of Proposition 3 is that whenever full risk sharing is achieved, the values of the fiscal and currency unions coincide.

Corollary 1 (Values in Full Risk-sharing Steady States). *In a constant weight steady state*

$$V_i^M(s,z) = V_i^F(s,z), \quad i = 1, 2.$$

Proof. See Appendix A ■

In this economy full risk-sharing always characterizes the constrained efficient allocation. The implication of Corollary 1 is that if such allocation can be attained, then the currency union delivers the same level of utility as the fiscal union. This is a direct consequence of the risk-sharing miracle.

Another equivalence result can be obtained by focusing on periods in which any participation constraint binds in an imperfect risk-sharing steady steady state.

Proposition 4 (Values with Binding Constraints). *If the optimal choice in the outside option economy is to leave and default on outstanding liabilities, then, whenever the participation constraint binds for country i, the two values coincide. Formally, if*

$$V_i^o(s,B) = V_{UD}^i(s),$$

then

$$V_i^F(s, \underline{z}(s)) = V_i^M(s, \underline{z}(s)) = V_i^o(s,B).$$

Where $\underline{z}(s)$ denotes the relative weight at which the participation constraint binds in state s.

Proof. See Appendix A ■

The result in Proposition 4 states that if it is optimal to default upon leaving the contract, then, as both the fiscal and the currency union start from the same level of liability (namely zero liabilities) then the values of the outside option is identical in the two cases. This, further implies that whenever the participation constraint binds, the value inside the contract coincides with the outside option value.

Corollary 1 and Proposition 4 provide two equivalence results of the two contracts. These special cases characterise the contingencies in which we find that such equivalence will hold true. In general, and in particular in any steady state with non constant weights, the fiscal union provides higher value than the currency union. Theorem 1 formalises this result.

Theorem 1 (Steady States with Imperfect Risk-sharing). *In steady states with imperfect risk-sharing*

$$V^F(s, z) > V^M(s, z).$$

Proof. See Appendix A ■

Theorem 1 states that whenever the steady state features a cycle the value of the fiscal union is strictly larger than the one of the currency union. This result follows from the previous discussion on the inability of monetary policy to stabilize both economies at once when consumption is not constant. The lack of independent monetary policy implies that the non-tradeable side of both countries is not at the first best level, as the labour wedge is non-zero. As the outside options of the two economies are the same (for given states) the fiscal union can always replicate the tradeable allocation of the currency union without incurring in the welfare loss generated by suboptimal non-tradeable consumption.

So far we have characterised our results in the case in which the social planner picks the relative Pareto weights and the monetary authority always uses constant weights. We now consider the case in which we endow the common central bank with the planner's Pareto weights and ask whether this can be welfare improving. Proposition 5 formalises the result in this case.

Proposition 5 (Monetary Policy with Planner Weights). *If the central bank of a currency area adopts the relative Pareto weights of the planner then risk sharing decreases. This is paired with an increased inefficiency of the non-tradeables consumption.*

Proof. See Appendix A

■

The intuition behind this result can be understood by noting that the relative Pareto weight of a country increases as its participation constraint binds. In addition, the participation constraint will bind as the country's exogenous state improves because, for a given relative weight, the improvement in the state increases the outside option and reduces the country's surplus in the contract. In the context in which the central bank adopts the same weights as the social planner, an increase of the relative weight also implies that the country's wedge is targeted more intensely by the monetary authority. As a result, both the monetary policy and the transfer policy favour the country which has been relatively fortunate.

We conclude this section with a result on the welfare properties of the currency union allocation.

Proposition 6 (Constrained Efficient Currency Unions). *The optimal allocation in a currency union is constrained efficient.*

Proof. See Appendix A

■

In the next section we solve the model numerically to study whether the contracts are feasible (positive surplus), what is the structure of the optimal transfers and whether the optimal policies in a currency union can make up for the deadweight loss due to the lack of independent monetary policy.

3 Quantitative Results

In this section we describe the algorithm used to solve the model, the parameterisation and the numerical results for both the real and the nominal setup.

3.1 Solution Algorithm and Parameters

The solution algorithm first requires solving for the value functions and policy functions of the outside option, which is an Arellano economy with two goods. The Arellano economy is solved by value function iteration, following the algorithm in Arellano (2008), adjusted to allow updating of the bond pricing schedule in each iteration. This gives us the consumption of tradeables and the borrowing choices in terms of tradeables. Since monetary policy is independent in the outside option, so that non-tradeable production is always at the first best level, we simply set $C_{NT} = 1$ and $N = 1$ for all states; this is also true for the non-tradeable allocation in the fiscal union contract, where there is no nominal rigidity.

The contract allocations are solved for using policy function iteration. We start with an initial guess for the value functions of the contract, and the liabilities. At each iteration k, for a given assignment of liabilities $B_k(y, z)$ and a guess for the value functions $V_k(y, z)$, we find the value of relative weight z at which the participation constraint binds in each endowment state y. Using the symmetry of the environment, we can then calculate an interval $(\underline{z}(y), \bar{z}(y))$ within which the participation constraints do not bind; outside of this range, the allocations are constant due to the binding participation constraints.

Once we have updated the allocation, we can update the implied liabilities using a recursive version of the budget constraint, as in Equation 77. This allows us to update the assignment of the outside option values $V_i^o(y, B(y, z))$; any values of $B(y, z)$ which lie outside the grid used to solve the Arellano economy are calculated using cubic spline interpolation (or extrapolation if required). We then continue onto the next iteration $k + 1$, by again finding the binding values of z. We continue iterating until the changes in the value function for the contract and the liabilities function $B(y, z)$ are sufficiently small.

Description	Parameter	Value
Openness parameter	α	1
Discount Factor	β	0.95
Utility Curvature	γ	2
Labour Elasticity	ϕ	3
Risk Free Rate	r	1.02
Reinclusion Probability	θ	0.17
Default Output Cost	ψ	0.96
Endowment AR1 parameter	ρ	0.9
Endowment Shock Variance	σ_y^2	0.01

Table 1: Baseline parameter Values

This completes the solution algorithm for the fiscal union. For the currency union, there are two additional steps in each iteration, to calculate the relative prices $\frac{P_{1,NT}}{P_{2,NT}}$ and $\frac{\epsilon(s)}{P_{1,NT}}$. The exact expressions needed for calculating these relative prices can be found in Auclert and Rognlie (2014), although we have adjusted them to allow for a more flexible specification of risk aversion. Both of these prices are required to calculate the non-tradeable allocation variables C_{NT} and N.

The parameter values used for all exercises are shown in Table 1. These values have not been calibrated but have been chosen to lie within ranges which are common in the macroeconomic literature. An exception in this regard is the relative risk aversion parameter γ, which at 5 would be considered high. Under our current solution algorithm, it becomes extremely difficult to achieve convergence of the currency union contract solution for a lower value of γ, without introducing aggregate risk into the model. For the sake of simplicity, we have chosen to retain the higher value of γ, rather than introduce an additional state variable into the model.

The Markov process for the tradeable endowment y is produced by discretizing an AR1 process with persistence and volatility parameters ρ and σ_y^2, as given in Table 1. We discretize the process using the Rouwenhorst method, which achieves better performance for near unit root persistence. For all of the economies we use a 5 state Markov chain. We report the full transition matrix in equation 82 in the Appendix.

3.2 The Outside Option

We begin our discussion of the results by first describing the behaviour of the defaultable debt economy which is the outside option to remaining in the union contract.

Figure 1 displays the behaviour of financial variables in the outside option economy. We plot the current account and stock of assets against different levels of current liabilities. Figure 1a shows the evolution of net borrowing. To the left of the zero on the horizontal axis countries have stocks of assets. The optimal policy here suggests that countries run a current account deficit when they are in the lowest endowment realization to smooth consumption. When they are in the highest endowment realization, countries run a current account surplus and increase the stock of assets. Moving rightward, countries have positive debt. Since default is costly and priced in by the lender, countries tend to run current account surpluses to deleverage and reduce the cost of borrowing. The steep declines in debt correspond to default episodes. The general deleveraging pattern is evident in Figure 1b since the lines slopes are less than 1, meaning that tomorrow's debt is lower than the current liability stock.

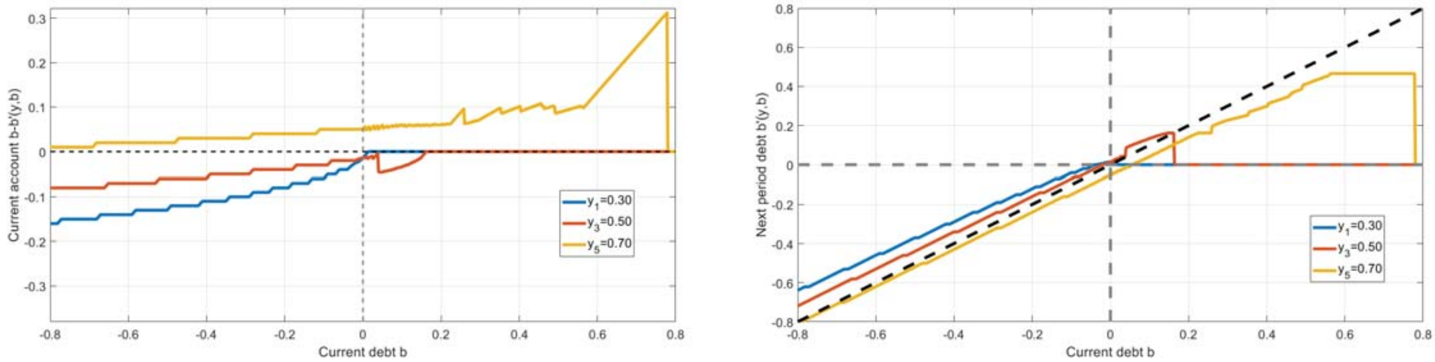
In Section 2.3.2 we outlined the full set of choices available to each country when considering whether or not to remain in the fiscal or currency union. Figure 1 also gives some information about the preference ordering of these outside options. Suppose the country is in the 3rd highest endowment state $y_3 = 0.5$, represented by the red lines in Figure 1. In Figure 1b, we see that for a current debt level below 0.135, the debt choice B' is non-zero (with an exception around $B = 0.02$ where the country optimally chooses to deleverage slightly to $B' = 0$), meaning that for these debt levels the country continues to participate in financial markets. However, for any current debt level above 0.135, the country's debt in the next period collapses to zero because it defaults. Using the notation of Section 2.3.2, this tells us that when $y = 0.5$ and $B \leq 0.135$, $V_i^{LR}(s, B) \geq V_i^{LD}(s, B)$, so the country chooses to continue servicing its debt; conversely, when $y = 0.5$ and $B > 0.135$, $V_i^{LR}(s, B) < V_i^{LD}(s, B)$, and so the country chooses to default on its existing liabilities.

Furthermore, these preference orderings tell us about the off-equilibrium behaviour of each country in the case that it decides to leave the union (recall that in equilibrium this will never happen because the contracts are designed so that the participation constraints are always satisfied). Consider again the case where country i's current endowment is $y_3 = 0.5$, but now assume that is inside the fiscal union. If its current liabilities inside the union are 0.1, for example, then it considers the choice between remaining in the fiscal union, and leaving the union but continuing to service the liabilities which it has accumulated. On the other hand, if it has liabilities of 0.2 (or any amount greater than 0.135), then it instead considers the choice between remaining inside the fiscal union and leaving the union and immediately defaulting on these liabilities.

In the results which follow, we will see that in the former case, where liabilities are relatively low, the country always *strictly* prefers to remain in the union. This result holds *a fortiori* for the case where the country has accumulated assets within the union. Importantly, we also find that whenever the participation constraint binds, so that the country is indifferent between staying in the union and leaving, its liabilities inside the union are always so large that if it were to leave the union, it would immediately default. These findings hold for both the fiscal and the currency union.

3.3 Fiscal Union

We start the description of the quantitative results of the fiscal union model by characterizing the optimal policies inside the risk-sharing contract.



(a) Current Accounts Outside Option Economy

(b) Debt Law of Motion Outside Option Economy

Figure 1: Outside Option Economy policis

In what follows we plot the policy functions inside the dynamic contract as a function of the relative weight z. We plot all policies for different endowment realizations[5].

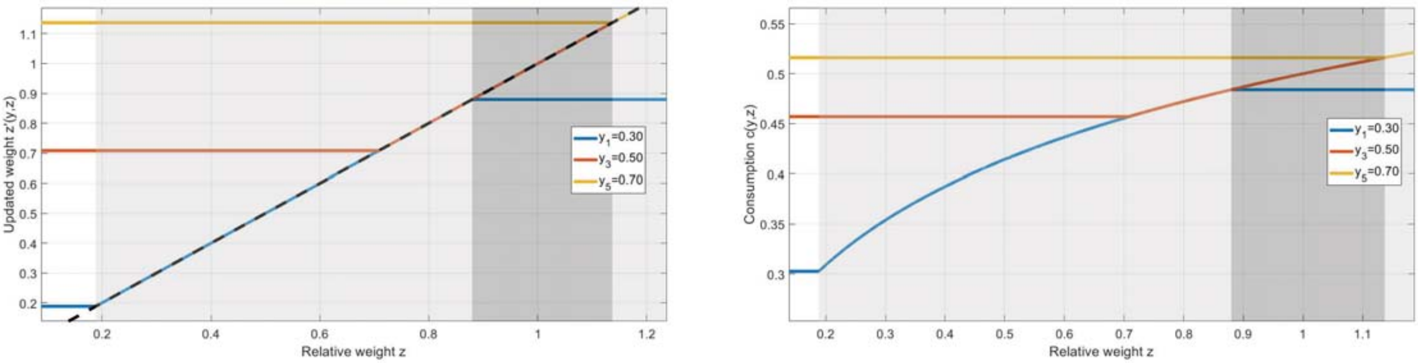
Figure 2 shows the relative weights and consumption policies. The dark grey shaded area represents the set of weights that characterize the ergodic distribution of the contract. This is the set in which the weights will lie and fluctuate in the steady state. The lighter grey shaded area represents the basins of attraction of the ergodic set. All graphs contain the lowest, the median and the highest realization of the tradeable endowment.

Figure 2a shows the optimal relative weight policy. Every line corresponds to a specific realization of the endowment. In every line flat regions represent areas in which one of the participation constraints is binding. The flat region to the left is where the participation constraint of country 1 is binding, the flat region on the right shows where the participation constraint of country 2 binds. Since the relative weight describes the consumption allocation of country 1 relative to country 2, in the left area of the graph the relative weight is too low, meaning that country 1 is receiving too little consumption, hence the country is against its participation constraint. Conversely, as one moves rightward, there is a flat portion of the line where the relative weight is too high and country 2's participation constraint is binding. The regions in which the optimal weight coincide with the 45 degree line are areas where neither participation constraint binds, hence the weight is constant across periods.

Figure 2b displays the consumption allocation. When neither country wants to leave the contract the future relative weight is equal to the ratio of marginal utility of tradeable consumption between the two countries. Hence the graph shows that consumption tracks the current relative weight in the same areas where the weight is not updated. Concavity is inherited by preferences.

Figure 2 already shows that this contract features an imperfect risk-sharing steady state. Using Definition 2 it is visible that as there is no set of weights in which neither the particiation constraint of

[5]Recall that in this setting there is no aggregate risk, meaning that when country 1 is in a high endowment state, country 2 is in a low endowment one.



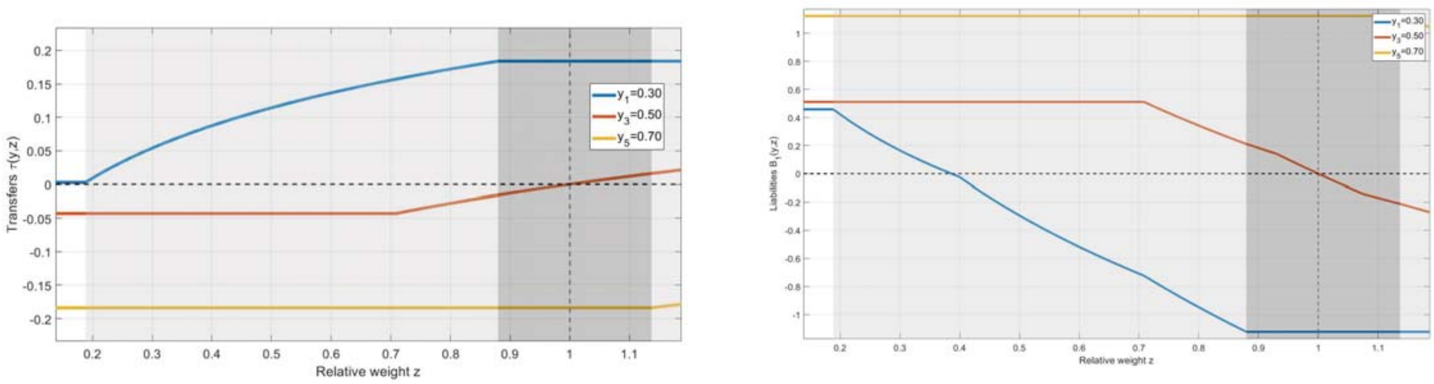
(a) z' (b) c

Figure 2: Outside Option Economy policies

the high endowment country nor the one of the low is binding, the weights must fluctuate as the state changes. Graphically this can be seen by observing, in Figure 2a, that the flat region to the right on the lowest endowment relative weight lies to the left of the region where the PC stops binding for the high endowment. This steady state will then not be able to attain full risk-sharing. In particular in the steady state the following path will occur: suppose we start with both countries at the middle endowment and a relative weight of 1. At this level of weight, given the state, neither participation constrain binds. Suppose now that country 1 moves to the highest endowment level (hence country 2 moves to the lowest). At a relative weight of 1, in the highest endowment state, country 1's participation constraint binds (this can be seen on the yellow line). The planner will then increase the relative weight next period to the minimum level to make country 1 indifferent between the contract and the outside option. This weight is the rightmost point on the dark grey area, namely where the PC is barely binding in the highest endowment state. As long as the states do not change both countries' PCs are slack. Suppose now that the state changes and country 1 moves to the lowest endowment state (hence country 2 moves to the highest). At these relative weights country 2's participation constraint binds and the planner will increase the weight till the PC is slack again. These dynamics define the imperfect risk-sharing steady state.

The next two graphs in Figure 3 show the key policies inside the contract. Figure 3a displays the optimal transfer policy. The contract features optimally large countercyclical transfers between the countries. When countries are at symmetric endowment realizations and neither participation constraint binds, the transfers range between -18% and 18% of the total tradeable endowment. They are as large as 2/3 of the endowment for lower realizations.

Figure 3b shows the liabilities positions. Countries have higher stocks of debt whenever they have a low relative weight. One feature of the contract is that, since the countries are symmetric and given the persistence of the endowment, a high realization today implies future surpluses in expected terms. This, in turn, generates positive stocks of liabilities today. This feature however is not true for any level of the relative Pareto weight. This result is common to other similar models of dynamic contracts



(a) τ (b) b

Figure 3: Fiscal Union policies

(see Abraham et al., 2019). The key difference is our setting is that the debt position can take both signs (i.e. assets or liabilities) for both countries. This feature stems from the symmetry of risk aversion and impatience in our model. This result can also be interpreted as countries with better endowment realizations being able to absorb larger stocks of debt.

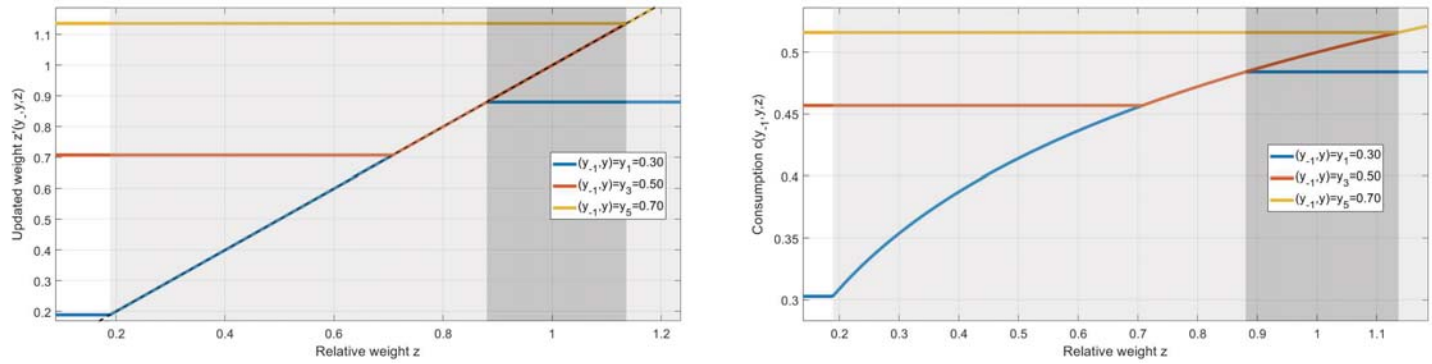
Finally we discuss the steady states of this economy. The fiscal union features an imperfect risk-sharing steady states. The dark shaded area represents the ergodic set. This range of relative weights has the property that if an economy starts (say, in Period 0) with a relative weight inside this set it will always stay there. This set has a basin of attraction, both from the left and from the right, such that if an economy starts inside the basin it will eventually converge to the ergodic set. This basin of attraction is represented by the light grey areas in the graphs.

3.4 Monetary Union

In this section we describe the results for the currency union model. In this setup non-tradeables producers face staggered prices friction. In the outside option economy countries have independent monetary policy and close the labour wedge.

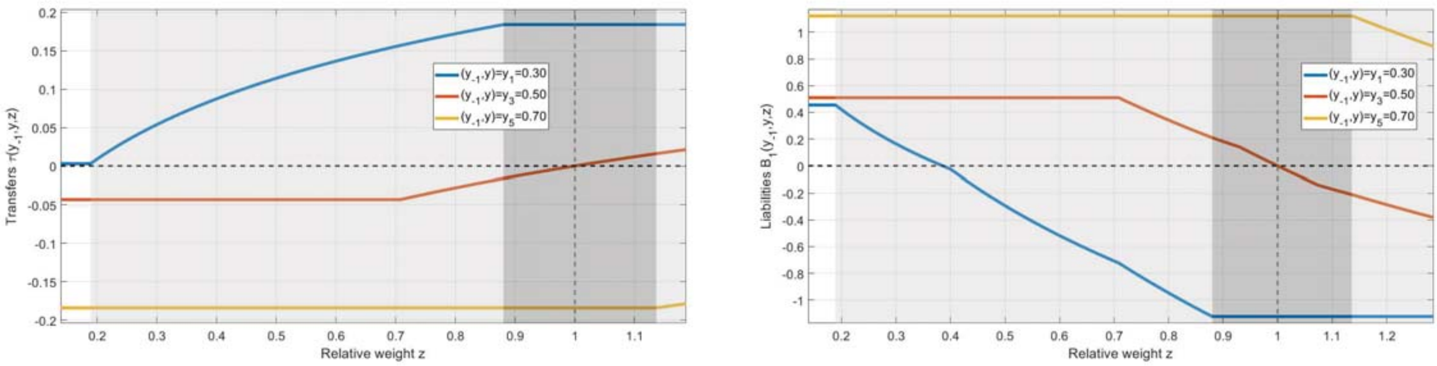
One important feature of this model is that the outside option value is identical to the one in the real version of the model since monetary policy eliminates nominal rigidities entirely. However, inside the contract, countries face lower surplus since the economy is not producing at the efficient level. This is a direct consequence of Corollary 1 and Theorem 1. From Theorem 1 the currency union can never yield a higher value than the fiscal union in imperfect risk-sharing steady states. From Corollary 1 they can be at most equal in constant weights steady states. As the fiscal union features imperfect risk-sharing it is never possible for the currency union to attain the same value of the problem.

Surprisingly, the contract is qualitatively identical to the one described in the previous section. In Figure 4 we plot the future relative weight and consumption as a function of the current relative weight



(a) z' (b) c

Figure 4: Monetary Union policies



(a) τ (b) b

Figure 5: Monetary Union policies

z.

Figure 4a displays the law of motion of the relative weight. All lines feature a flat region on the left where the country's participation constraint binds, a sloped part where it coincides with the 45 degree line and flat region on the right where the other country's participation constraint is binding. The weights are updated upward whenever the country's PC binds, downward when the other country threatens to leave the contract and they remain constant when neither is against the outside option. At first inspection, the path in the ergodic set resembles the one in the fiscal union. This feature will be extensively discussed later in the paper.

Figure 4b plots the consumption allocations for different levels of endowments. Consumption closely tracks the relative weight behaviour, as in the fiscal union setting.

The economy features countercyclical optimal transfer of the tradeable endowment. Their size is numerical identical to the ones in the fiscal union. The same holds for the stock of liabilities, displayed in Figure 5b.

The outside option economy behaves identically to the real model outside the fiscal union. Hence the behaviour of current accounts and the debt law of motion can be seen in Figure 1.

It is important to notice that while the optimal policies in the two economies representing the outside options of the contract are the same the starting levels outside are not. To see this, recall that the stock of liabilities of a country leaving the union is given by the outstanding set of promises to the other country. As the transfer policies inside the fiscal and currency unions could be different, so would be the liabilities inside the contract. Hence the starting stock of debt upon breakup could be different. In other words, conditional on a given level of b the policy for b' is the same in the two economies. However, in the same state inside the contract, upon leaving, the countries could start with different levels of b.

Exactly as in the fiscal union, this economy features an imperfect risk-sharing steady state. Hence the planner is unable to attain full risk-sharing.

3.5 Comparison of the Contracts

In this section we compare the optimal policies in the two contracts. From the previous discussion, we see that the two contracts seem to have very similar values and policies despite the fact that the currency union cannot achieve the optimal allocation of non-tradeable goods. In Section 2, we showed formally that in some special cases (for example, when the participation constraint is binding), the currency union attains the same value as the fiscal union; the numerical results, however, seem to suggest that the similarity is more general. How is this possible? The answer lies in a difference in the behaviour of the optimal transfers in the currency union which enables the planner to compensate partially for the labour wedge.

We start by recalling that the full state space for the economy is (y_{t-1}, y_t, z_t). We can then distinguish between two cases. In the first, between period $t-1$ and t, the endowment of tradeables goods does not change, i.e. $y_{t-1} = y_t$. In the second, the endowment does change between periods so that $y_{t-1} \neq y_t$. If the endowment is quite persistent, as we tend to assume, then in period $t-1$ agents' expectations will place a large probability mass on the first case, in which the endowment does not change. In particular, the pricing decisions of the non-tradeable good producers will put a large weight on this outcome. As a result, if the endowment does not change between periods, the labour wedge in the currency union will be relatively small, whereas if does change, it will be larger; in fact, the larger the transition $y_{t-1} \to y_t$, the large the labour wedge in period t.

This explains why in the comparisons considered so far, where the realization of y is held constant, the currency union behaves similarly to the fiscal union. If instead we consider transitions where the endowment changes between periods, we see that the transfer policy in the currency union is more complex that that of the fiscal union.

Figure 6 shows consumption in the two contracts when $y_t = y_1$ and the country's participation con-

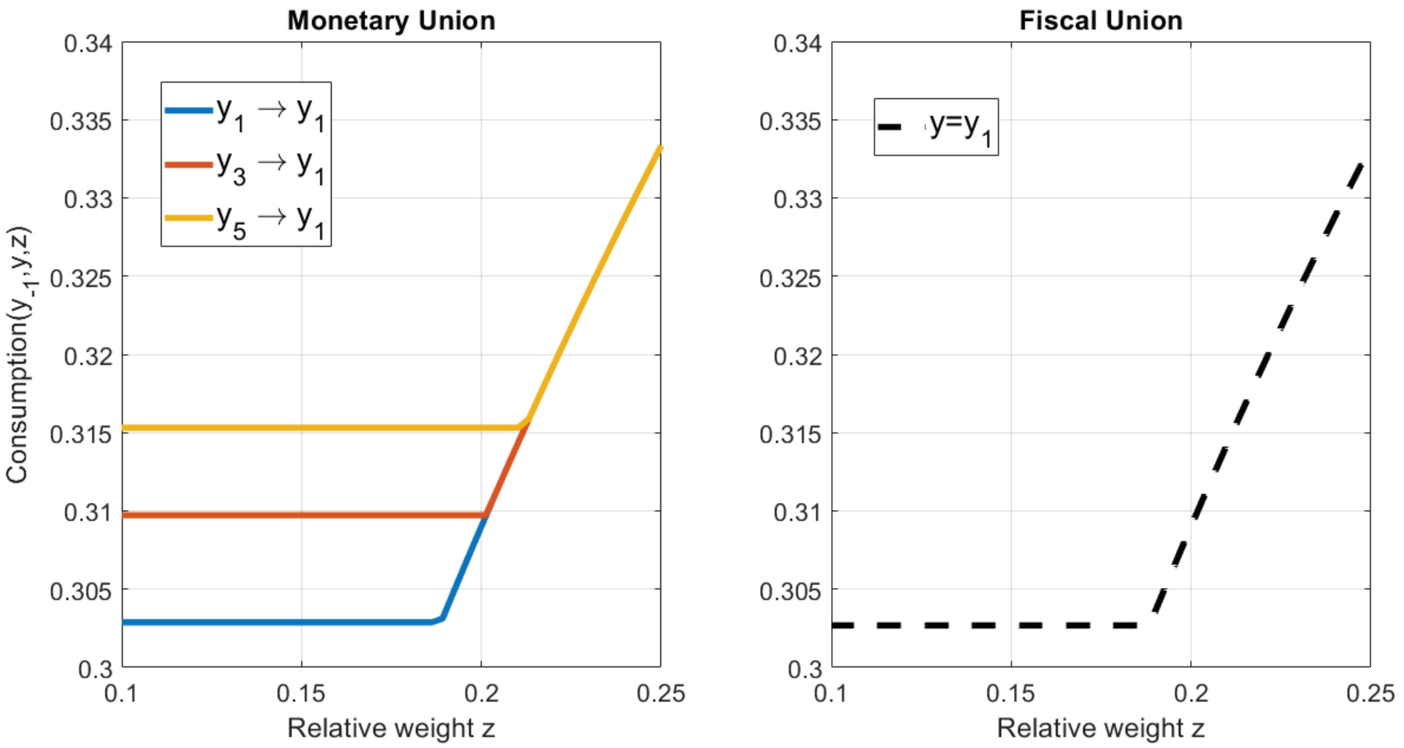


Figure 6: Consumption adjustment in the contracts

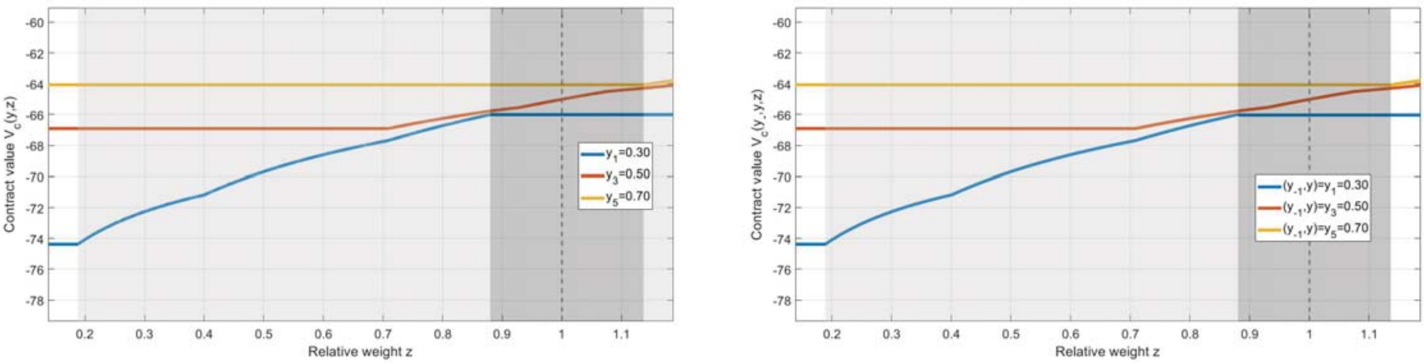
straint is binding. We see that in the monetary union, when the constraint binds, the level of consumption depends not only on y_t but also on y_{t-1}. Moreover, when the economy arrives at y_1 from a higher previous endowment, is receives high current tradeables consumption. This higher consumption compensates for the fact that when the transition is large (say $y_5 \to y_1$), the labour wedge is also large; the additional consumption is therefore needed to keep the country in the monetary union. In contrast, in the fiscal union, monetary policy completely eliminates the wedge; as a consequence consumption does not need to be conditioned on y_{t-1} in this way.

We should reiterate at this point that the extra adjustment of transfers in the monetary union is not enough to completely undo the deadweight loss from having joint monetary policy. We showed formally that the value of the fiscal union is always weakly higher than that of the currency union. However, under certain conditions, the transfer policy in currency union can make the overall loss very small.

3.5.1 Steady States

Before discuss the features of the steady states in detail, it is worth describing in more depth the set of weights defining the basin of attraction of the ergodic sets of the two contracts.

In Figures 16 and 17 we plot, for every endowment realization, the set of weights in which the participation constraints do not bind. The ergodic sets are defined by the upper bound of the lowest realization



(a) Fiscal Union (b) Monetary Union

Figure 7: Values of the Contracts

of output and the lower bound of the highest realization. This area is shaded in grey.

The reciprocal bounds, namely the lowerbound for the lowest endowment and the upperbound of the highest endowment, define the the basin of attraction. In order words one can read off the graphs the set of starting weights that will produce convergence to the ergodic set.

We start the discussion on the features of the stochastic steady states by providing one simulation to exemplify the dynamics. We start by simulating one history of endowments for 100 periods. We then plot the optimal policies of a country in the fiscal union, one in the currency union and one in the defaultable debt economy.

We start the contracts with a relative weight of 1 in the median endowment state. As $z = 1$ is the center of the ergodic set in both unions the economy will permanently remain in such set. We then sample 100 period and plot the simulation policies.

The path of the endowment and consumption is plotted in Figure 8. In the left panel, showing the endowment history, the red line shows the path for the contracts economies, while the black line for the defaultable debt one. The vertical black lines show episodes of default and financial market exclusion for the outside option economy. The endowment history is the same, though recall that when output is sufficiently large and the defaultable debt economy is in a period of exclusion from financial markets, it pays a fraction of endowment as a default cost. Hence the small deviations between the two paths when the outside option economy has default episodes. We denote periods of financial autarky, following a default, as a dot at the top of the graph, while periods of financial market access as dots at the bottom of the graph.

The left panel of Figure 8 shows the behaviour of the consumption of tradeables. The two contracts provide the same level of consumption (red and blue lines). The planner is able to smooth close to all of the fluctuations in the endowment. The jumps in consumption are given by updating in the relative weight, following a binding participation constraint for one of the two countries. Finally the defaultable debt economy shows high volatility in consumption as there is limited possibility to insure against the

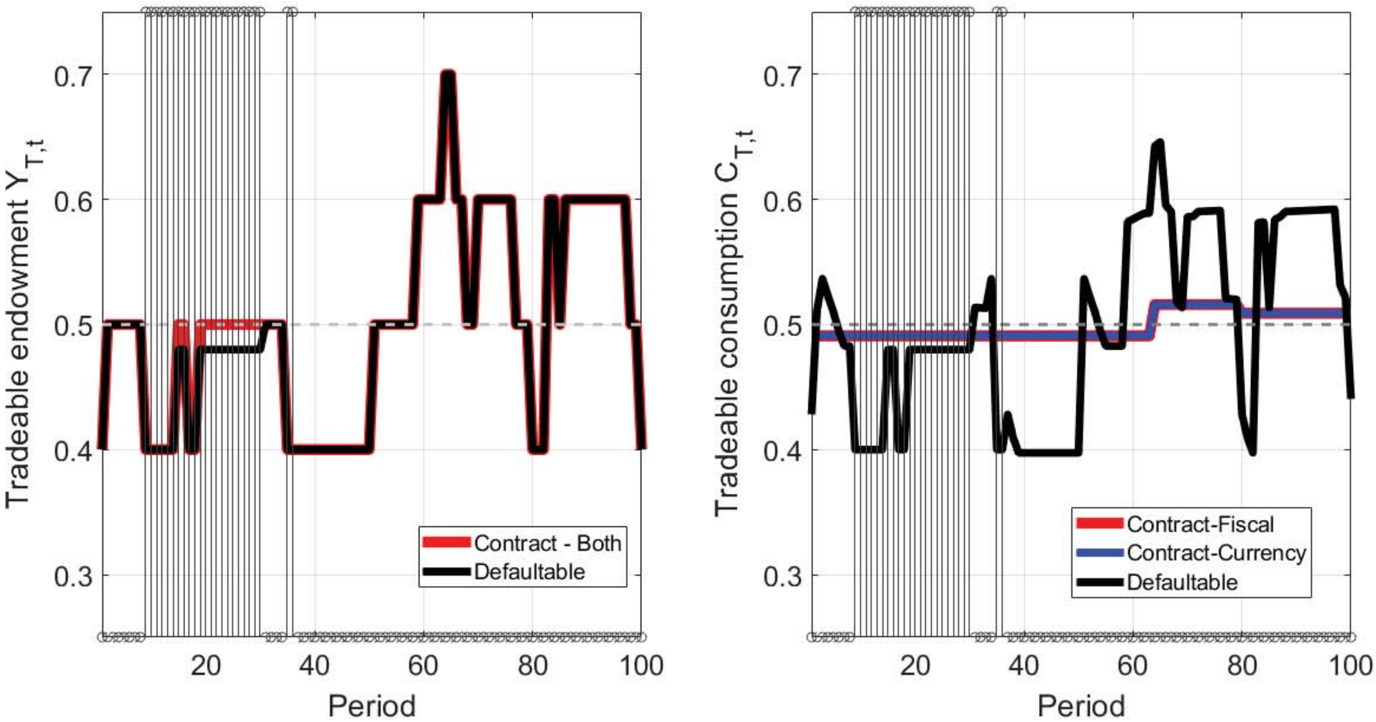


Figure 8: Steady State Endowment and Consumption

idiosyncratic risk.

Figure 9 shows the behaviour of the relative weight and some financial variables of these economies. The top left panel shows the behaviour of the relative weight, which is numerically identical for the two contracts. The transfer policy, in the top left panel, shows that the transfers are countercyclical and large, relatively to the endowment. Secondly it shows that, as the relative weight is stable in the first half of this history, consumption of tradeables does not change. This implies that the transfers absorb the entire difference between the constant consumption and the varying endowment. Together with the contract transfers we plot the current account balance of the outside option economy. The defaultable debt economy behaves very differently. At the beginning of this history the endowment realization is low and the economy has some assets. Once these assets are used to smooth consumption and the country starts accumulating debt, as the endowment drop, the country defaults. The country is excluded from financial markets for an extended period of time, hence the zero debt and current accounts. As the economy is reincluded in financial markets and borrows the endowment drop again and the country defaults again. Subsequently the country enjoys of sustainable borrowing and high endowment though it is still unable to absorb the large variations in the endowment and consumption is quite volatile. Lastly, the bottom right panel, shows the behaviour of the risk spread. In this graph is clear how the external lender prices in default before it happens by increasing the interest rate charge on the defaultable debt.

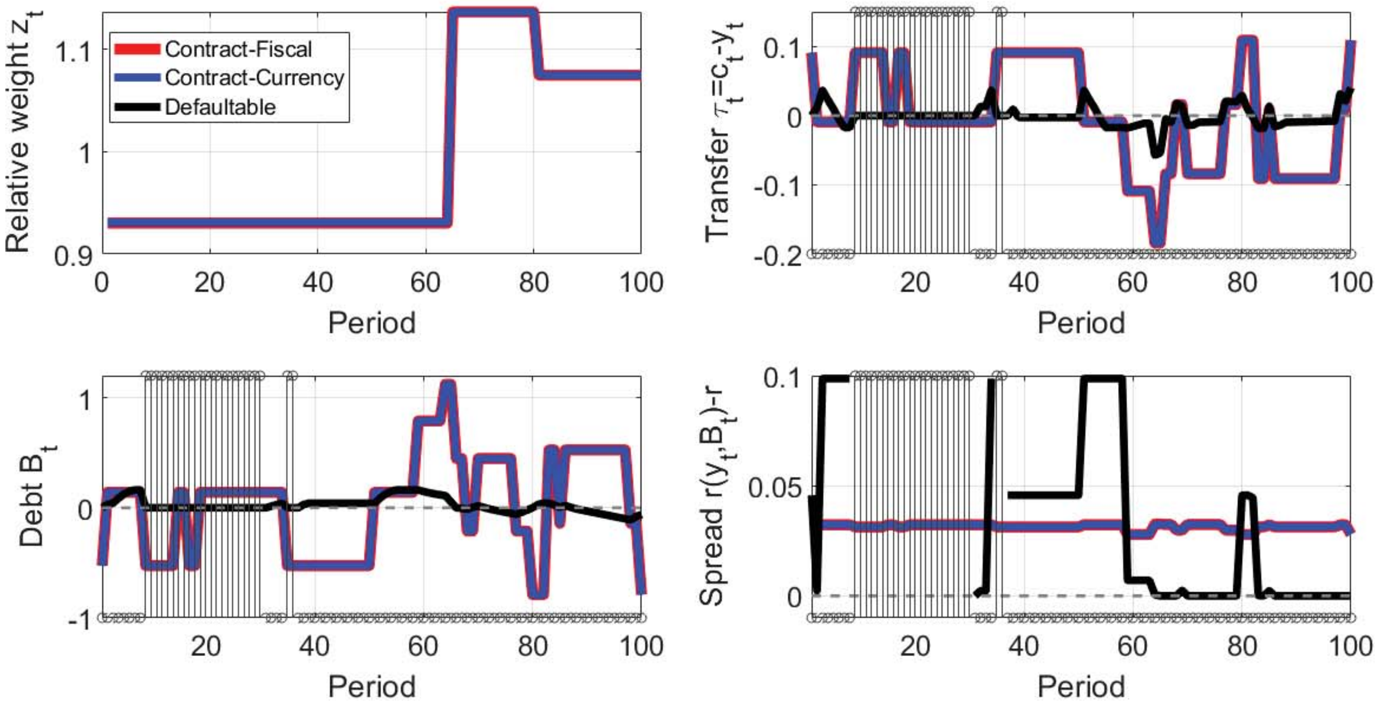


Figure 9: Steady State Optimal Policies

Table 2 shows some key moments of the economies in steady state. These moments are computed by averaging 50000 simulations in the steady state.

As expected the defaultable debt economy provides less consumption smoothing than the two contracts with a a consumption volatility 7 times higher. Secondly, consumption is lower in the outside option than in the contract due to default episodes in which the endowment is reduced. Thirdly, the two contracts deliver the same policies, which large (10% of GDP) countercyclical fiscal transfers and approximately the same values.

The risk-sharing value of the agreements is evidenced by the correlation between consumption and the endowment. The two contracts significantly reduce this comovement though such correlation is still positive. This stems from periods in which the weights change procyclically, for example when a country moves to high endowment and this makes the PC bind, implying an upward revision of its relative weight.

An important point of comparison between the fiscal and the currency union is in the row labelled $Pr(PCbdinding)$. This represents the fraction of periods in which any participation constraint is binding in this agreement. As discussed above, the fiscal planner in a currency union is implementing transfers that depend on y and y_{-1}. Particularly the planner is rewarding the country with the larger transition through higher tradeables consumption. This implies that the steady state path is fluctuates in narrower bands in the currency union. As such, there exists a set of pairs of endowment transitions

	Outside-Defaultable Debt	Contract-Fiscal	Contract-Currency
Mean			
Y_t	0.4975	0.4996	0.4996
$C_{T,t}$	0.4973	0.5	0.5
GDP_t	0.754	0.75	0.75
$\mid \tau_t \mid$	0.013	0.075	0.075
B_t	-0.001	-0.001	-0.002
z_t	-	1.005	1.005
$V(y, b/z)$	-66.832	-65.031	-65.032
$Pr(PCbinding)$	-	0.029	0.029
Standard deviation			
$\sigma(c_{T,t})$	0.094	0.013	0.013
$\sigma(Y_t)$	0.099	0.1	0.1
Correlation			
$\rho(C_{T,t}, Y_t)$	0.983	0.5	0.501
$\rho(\tau_t, Y_t)$	-0.379	-0.993	-0.993

Table 2: Steady State Moments

in which a participation constraint would be binding in the fiscal union but it is not in the monetary union. This yields a lower probability of a binding PC in the currency union. In this case, however, we find that the difference is negligible numerically.

The higher risk-sharing capacity of the contracts is also visible in the maximum amount of liabilities that countries can have inside the agreements.

In the defaultable debt economy countries are unable to borrow due to the high likelihood of default. Inside the unions can accumulate liabilities. .

The graphs in Figure 10 carry one additional set of information. Looking at the red lines in the top graphs, the maximum amount of liabilities describe when the country would optimally default. A country leaving the risk-sharing contract with some stock of liabilities in some given endowment state would default on its obligations if debt was higher than the red line. The line depicts the maximum debt that the country would optimally repay.

In summary, the two contracts behave identically numerically. They both yield higher risk-sharing than the outside option, thereby producing higher values for the problem.

3.5.2 Crisis Simulation

In this section we describe the economies after a crisis event. Inside the contract we define a crisis state as a country having the lowest endowment realization and having a binding participation constraint. Outside we define the crisis as the lowest endowment realization and having a stock of debt such that

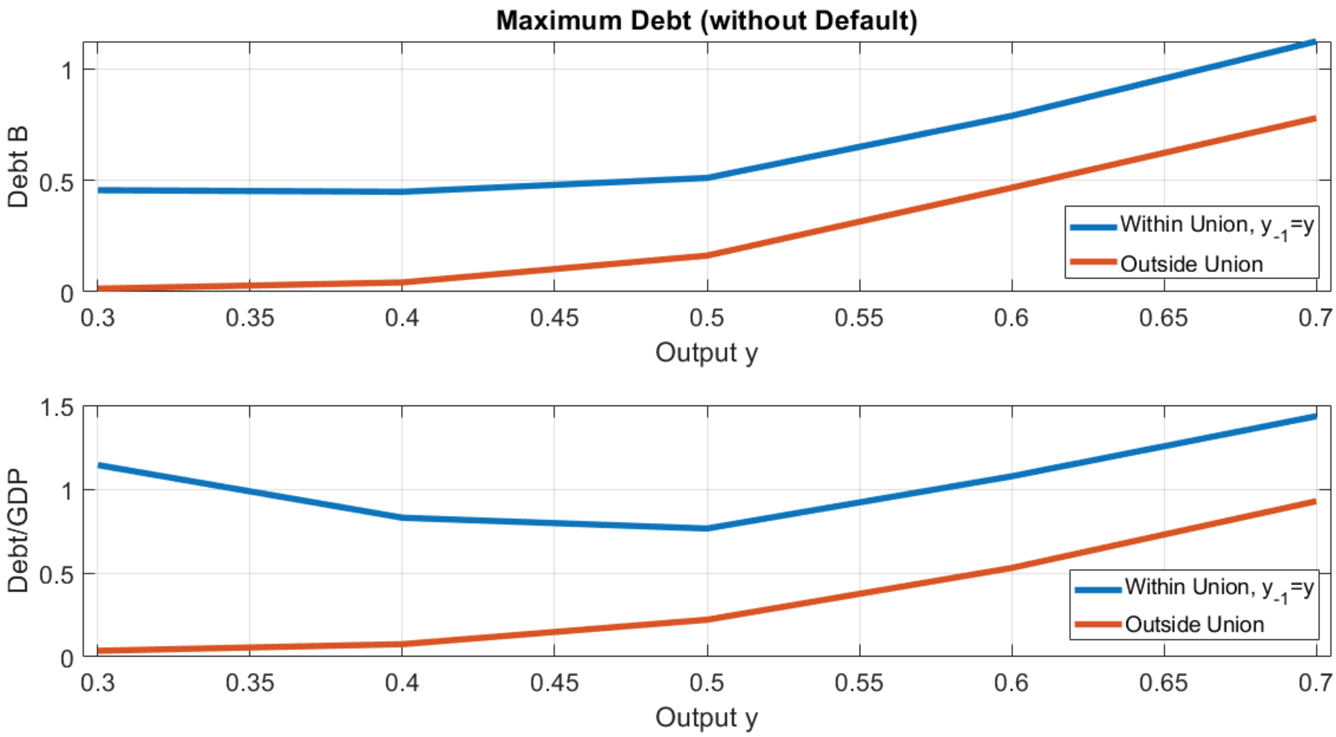


Figure 10: Maximum Debt

the country is indifferent between repaying and defaulting.

We start by showing the result for a single simulation over 100 periods and comparing the behaviour of the fiscal union, the currency union and the defaultable debt economy. Recall that in the outside option the nominal and real economy coincide since monetary policy eliminates pricing frictions.

Figure 11 shows the history of endowment realizations and the consumption of tradeables in the three scenarios: in the defaultable debt economy, in the fiscal union and in the currency union.

The right panel shows the consumption of tradeables over the first 100 periods after the crisis. The black line displays the path for the economy in the outside option. Consumption closely tracks the endowment state, showing limited scope for consumption smoothing through defaultable debt. In the outside option economy default occurs a number of times after the initial crisis before the economy manages to stabilize during a period of above average output realizations, before defaulting again towards the end of the simulation. We also see that the country pays the default cost when the endowment is high enough during exclusion, as evidenced by the difference in the endowments between the outside option and contract economies in the first panel. The two contracts behave identically, as we would expect from the policy functions: consumption increases relatively soon after the crisis and remains flat for many periods, before increasing again in response to high endowments. Since the simulation starts with the lowest endowment and relative weight, we only observe increases in consumption as the endowment reverts to its mean and the relative weight moves towards one.

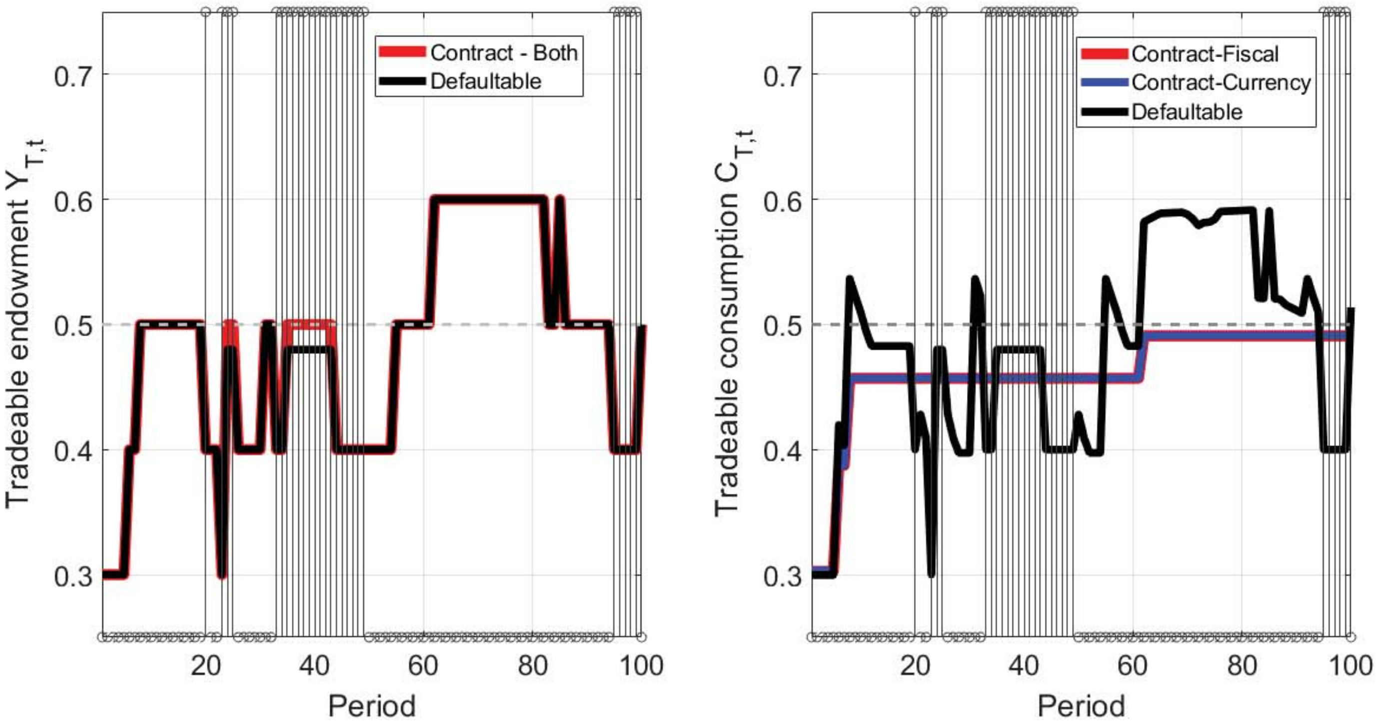


Figure 11: Endowment and Consumption

In Figure 12 we plot the endowment, transfers, debt and the interest rate spread. The defaultable debt economy shows that when the country defaults and is temporarily excluded from financial markets, it has zero debt and zero net borrowing (which we compare with transfers in the contract). In the path of the interest rate spreads, default periods can be seen by noting that there is no spread (i.e. no debt is being traded and so there is no bond price to quote). As we would expect, spreads rise as the country approaches a default episode, reflecting the increase probability of default. After a long period of exclusion, the country regains access to financial markets in the middle of the simulation, and begins to accumulate a small amount of debt before starting to save, through current account surplus, as it experiences high endowment realizations. During this saving period the interest rate spread is zero. At the end of the simulation output falls again, spreads rise as the country borrows in an attempt to smooth consumption, and eventually the economy defaults again.

Inside the risk-sharing contracts the paths of debt and transfers are the same. At the beginning of the crisis, the country is so indebted that it can no longer borrow and so it receives zero transfers; however, the stock of debt which it is able to accumulate is much higher than in the defaultable debt economy. The liabilities of the country are then reduced in response to a sharp fall in output. In this case, the fall in output it so large that the country expects to receive transfers in the near future; this corresponds to a net asset position. For the rest of the simulation, the country accumulates debt when output falls, and repays it when it rises, in order to smooth consumption. The interest spreads are lower and more stable inside the contract, and are actually negative immediately after the crisis.

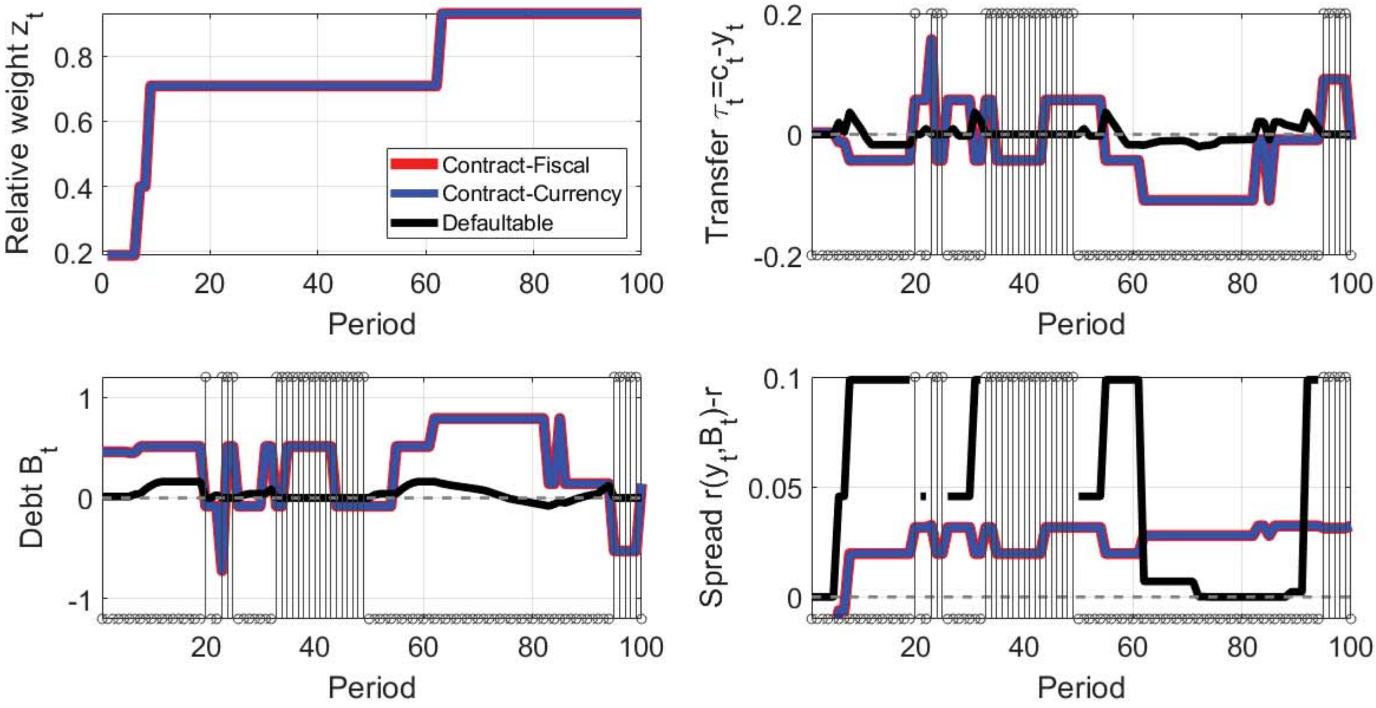


Figure 12: Financial Variables

Next we look at the average behaviour of the contracts in response to a crisis episode. We do so by averaging across 25000 crisis simulations. Figure 13 shows the average response of consumption in the fiscal and currency union. The dashed lines represent the interquartile range.

Table 3 shows the main moments of some key outcomes of the simulations from the perspective of the country in crisis (recall that if one country is in crisis, the other must be experiencing a boom). As the economy starts in a recession and the endowment state is persistent the average endowment is .48, lower than its unconditional average of 0.5. In the economy with defaultable debt the average endowment is further decreased by the default cost. Consumption, conversely, is at its highest in the defaultable debt economy. The same ranking however holds for consumption volatility and its correlation with the endowment state. The average absolute value of transfers (current accounts) is much smaller in the defaultable debt economy compared to the unions, reflecting the reduced borrowing capacity outside the contract. The stocks of liabilities are quite different inside and outside the contract. In the outside option, the country on average has a small amount of debt, roughly one eighth of the level inside the contracts.

Finally, transfers are largely countercyclical, particularly inside the risk-sharing contracts. Countercyclicality is stronger in the union, which explains the much greater stabilization of output displayed in the simulations.

In the next two figures we plot the impulse responses of the tradeable goods in the three economies, as

	Outside-Defaultable Debt	Contract-Fiscal	Contract-Currency
Mean			
Y_t	0.478	0.480	0.480
$C_{T,t}$	0.478	0.471	0.471
GDP	0.716	0.696	0.696
$\mid \tau_t \mid$	0.011	0.0652	0.0652
B_t	0.023	0.177	0.176
z_t	-	0.839	0.840
$V(y, b/z)$	-67.541	-66.392	-66.391
$Pr(PCbinding)$	-	0.057	0.066
Standard deviation			
$\sigma(c_{T,t})$	0.1	0.052	0.052
$\sigma(Y_t)$	0.105	0.0105	0.105
Correlation			
$\rho(C_{T,t}, Y_t)$	0.987	0.617	0.617
$\rho(\tau_t, Y_t)$	-0.351	-0.872	-0.872

Table 3: Crisis Moments

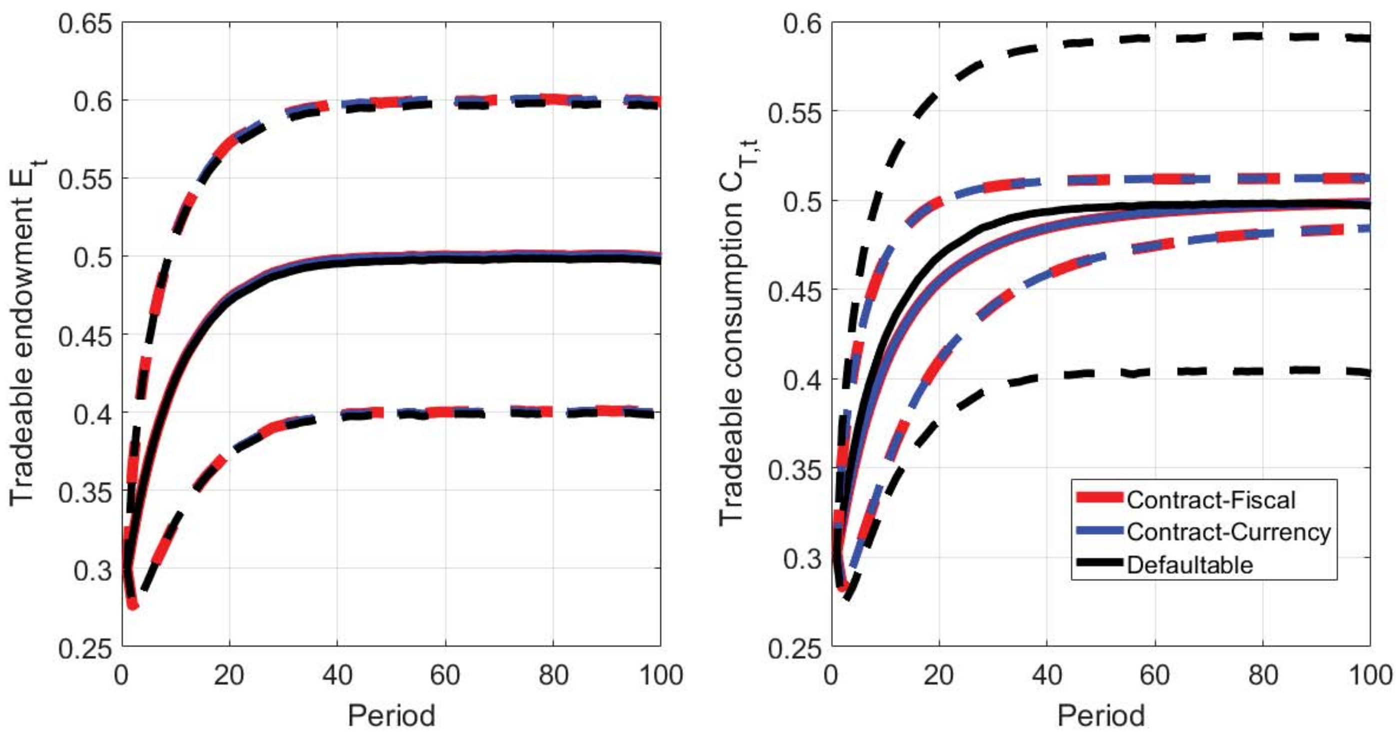


Figure 13: Tradeables Impulse Response After Crisis

well as the relative weight of the crisis country in each of the contracts. The solid lines represent the average paths of the variables whereas the dashed lines represent paths one standard deviation away from the mean. In the right hand panel of Figure 13, we see that after 100 periods, the average level of consumption is roughly the same in all three economies, and close to the mean level of the tradeable goods endowment. After the crisis, consumption also tends to recover faster in the defaultable debt economy. However, the dashed lines tell us that consumption is much more volatile outside the union than it is inside.

Figure 14 shows the average path of the transfers, the stock of debt and interest rate spreads in the fiscal and currency unions compared to the defaultable debt economy outside the contract. We see that on average transfers are close to zero in the defaultable debt economy, reflecting an inability to borrow, whereas in the union the crisis country initially makes net payments to the other country. The fact that the country in the union makes net payments in the aftermath of the crisis may be counterintuitive. However, as we can see in the top left panel of 14, the country begins the crisis with a very low relative weight, which corresponds to low consumption. Along any history which leads to this crisis state, the country will have been able to borrow large amounts to smooth consumption, an option which would not have been available outside the union.

The paths of liabilities are also very different for in the contracts, compared to the outside option. In the contracts, the economy begins the crisis with a large stock of debt, which it gradually repays over the course of the simulation. The defaultable debt economy, on the other hand, tends to spend the periods after crisis with zero net liabilities because it frequently defaults when it enters a crisis and subsequently spends some periods in financial autarky.

In the bottom right panel of Figure 14 we see the average path of the interest rate spreads which correspond to these movements in liabilities. The bold black line, which shows the median spreads for the defaultable debt economy [6], is calculated only for those states in which the economy does not default, since if it does default no debt is traded and there is no interest rate. We therefore see that if the economy does not default immediately during the crisis, it faces elevated interest spreads due to the high probability of default in the future. After this, the tradeables endowment reverts to its mean level, where default is less likely, and we see that spreads are volatile but typically close to zero. In contrast, in the union contracts we see *negative* spreads before the country gradually recovers from the crisis and its outstanding liabilities trade with a stable positive spread. The initial negative spreads are an artefact of the lack of aggregate risk in the union. While one country is in crisis, the other is experiencing a boom, and is therefore extremely willing to hold assets against the likelihood that its endowment (and tradeable consumption) will fall in the near future [7].

[6]For the paths of interest rate spreads, we face the problem that in some states spreads in the defaultable debt economy jump to extremely high levels, which inhibits the convergence of the standard deviation and the average paths across simulation. We therefore plot the median and interquartile range for the defaultable debt economy, since these statistics are more robust to outliers.

[7]See Appendix A for the relationship between the (implicit) interest rates on the liabilities with the contracts and the

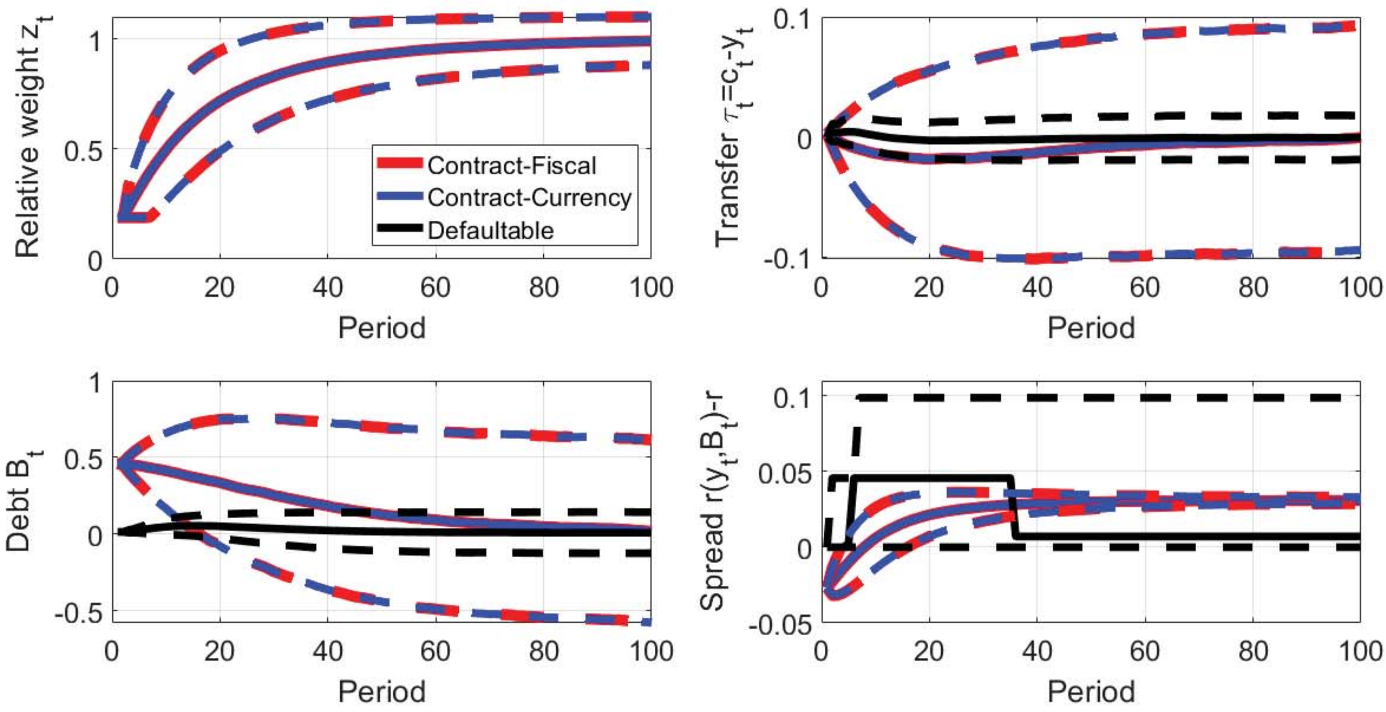


Figure 14: Financial Variables

Finally, in the top left panel of Figures 14 we see the impulse responses of the relative weight of the crisis country in the two contracts. During the crisis, the country receives the lowest level of tradeables endowment, and is therefore willing to accept a very low relative weight because its outside option is also very unattractive. As the country's endowment reverts to its mean however, the initial level of z is too low to satisfy the country's participation constraint, and so the relative weight is driven upwards to keep the country inside the contract, until the weight reaches one. We should recall that, due to the imperfect risk-sharing in the steady state, while the impulse response for z exhibits a smooth path, actual changes in the relative weight take place through discrete jumps (as seen, for example in Figure 12), due to the discrete set of values at which the participation constraint binds for different realizations of the endowment.

4 Robustness Checks

In this section we provide the steady state moments for three alternative calibrations of our model. In the first two we change the parameter governing the risk aversion, which allows us to alter how agents value risk-sharing. In the final one we revert to the risk aversion of the baseline parameter set ($\gamma = 2$), and instead reduce the persistence of the endowment process.

marginal rates of substitutions for tradebles consumption in the two countries

	Outside-Defaultable Debt	Contract-Fiscal	Contract-Currency
Mean			
Y_t	0.498	0.5	0.5
$C_{T,t}$	0.502	0.5	0.5
GDP_t	0.637	0.625	0.625
$\mid \tau_t \mid$	0.021	0.076	0.076
B_t	-0.201	-0.002	-0.002
z_t	-	1.001	1.001
$V(y, b/z)$	-59.303	-55.018	-55.021
$Pr(PCbinding)$	-	0.0093	0.0093
Standard deviation			
$\sigma(c_{T,t})$	0.088	0.005	0.005
$\sigma(Y_t)$	0.1	0.1	0.1
Correlation			
$\rho(C_{T,t}, Y_t)$	0.954	0.364	0.364
$\rho(\tau_t, Y_t)$	-0.508	-0.999	-0.999

Table 4: Moments: $\gamma = 3$

Tables 4 and 5 show the steady state moments for the same model discussed above but with the risk aversion parameter equal to 3 and 4, respectively.

Starting from Table 4, in the outside option economy agents show a higher level of steady state assets. This driven by the higher precautionary motif, which also, through positive interest rate on the assets, allows the country to consume more than the endowment on average.

The contracts show again similar values, with a marginally bigger difference in values between the fiscal and the currency union. As agents value smooth consumption more than in the previous simulation, the planner optimally reduces the variance of consumption by increasing the countercyclicality of transfers and increasing their average size by about 1.5%.

Table 5 provides a very different picture. The outside option economy increases the steady state level of assets compared to the previous economies, which significantly increases the average consumption of tradeables due to returns on the stock of assets.

The contracts are now very different from before. The risk aversion is large enough that the limited enforcement friction has little bite, allowing the planner to achieve full risk-sharing. In these economies the steady states feature a constant relative weight. As full risk-sharing is achieved this economy falls into the case described in Proposition 3. We therefore observe that, as there is no deadweight loss, the fiscal and currency union can attain exactly the same allocation.

Finally we consider economies with much lower persistence in the endowment process, where the AR1 parameter ρ is reduced from 0.9 to 0.5. As shown in Table 6, this parameter choice also delivers a constant weight steady state. However, the mechanism is slightly different. Since output reverts to

	Outside-Defaultable Debt	Contract-Fiscal	Contract-Currency
Mean			
Y_t	0.5	0.5	0.5
$C_{T,t}$	0.51	0.5	0.5
GDP_t	0.577	0.563	0.563
$\mid \tau_t \mid$	0.031	0.075	0.075
B_t	-0.509	-0.002	-0.002
z_t	-	1	1
$V(y, b/z)$	-73.654	-65	-65
$Pr(PCbinding)$	-	0	0
Standard deviation			
$\sigma(c_{T,t})$	0.083	0	0
$\sigma(Y_t)$	0.1	0.1	0.1
Correlation			
$\rho(C_{T,t}, Y_t)$	0.916	0	0.
$\rho(\tau_t, Y_t)$	-0.583	-1	-1

Table 5: Moments: $\gamma = 4$

	Outside-Defaultable Debt	Contract-Fiscal	Contract-Currency
Mean			
Y_t	0.5	0.5	0.5
$C_{T,t}$	0.502	0.5	0.5
GDP_t	0.755	0.75	0.75
$\mid \tau_t \mid$	0.048	0.077	0.077
B_t	0	-0.001	-0.001
z_t	-	1	1
$V(y, b/z)$	-65.594	-65	-65
$Pr(PCbinding)$	-	0	0
Standard deviation			
$\sigma(c_{T,t})$	0.061	0	0
$\sigma(Y_t)$	0.1	0.1	0.1
Correlation			
$\rho(C_{T,t}, Y_t)$	0.807	0	0
$\rho(\tau_t, Y_t)$	-0.813	-1	-1

Table 6: Moments: $\rho = 0.5$

the mean more quickly with lower persistence, a country currently receiving a high endowment faces a more similar future endowment stream to a country with a low endowment. The sets of relative weights which will satisfy both countries is therefore more similar, and actually overlaps. Compared to the baseline parameter set, the experience of the defaultable debt economy is much improved when the persistence of output is lower. In particular, since periods of low output are shorter on average, the economy is more able to borrow against higher future income, and the consumption smoothing which it can achieve is higher; the volatility of consumption is reduced by about one third compared to the baseline.

5 Productivity shocks

In this section we extend the model to include productivity shocks in the non-tradeable sector. The combination of endowment shocks and separable homothetic preferences lies behind the equivalence result presented in the previous section. The intuition is that the cost of losing independent monetary policy is proportional to the variance of consumption and is zero when consumption is perfectly smoothed across states. As fiscal policy is able to fully or almost fully smooth consumption we find that a common currency carries zero to little cost relative to independent monetary policy.

In this section we propose an extension in which non-tradeable production is subject to stochastic productivity. The goal is to check how large the welfare losses from the common currency are in presence of non-insurable variations in consumption.

Formally equation 2 becomes

$$Y_{NT}^{ij}(s) = A_i(s) N_{ij} \tag{55}$$

Equation 20, which defines the labour wedge, now becomes

$$\kappa^i(s) = 1 - \frac{1}{A_i(s)} \frac{U_N^i(s)}{U_{NT}^i(s)} = 1 - \frac{1}{A_i(s)} {C_{NT}^i}^{\gamma+\phi}(s) \tag{56}$$

and Equation 47 which gives the labour supply is now

$$N_i = \frac{1}{A_i(s)} C_{NT,i}(s,z) \tag{57}$$

Where s now denotes a two variable state vector which includes the endowment and the productivity realization. The rest of the model can be read from the previous section where everything is now

	Outside-Defaultable Debt	Contract-Fiscal	Contract-Currency
Mean			
Y_t	0.5	0.5	-
A_t	1	1	-
$C_{T,t}$	0.47	0.5	-
$\mid \tau_t \mid$	0.014	0.075	-
B_t	-0.03	-0.003	-
z_t	-	1	-
$V(y, b(z))$	-71.08	-65.46	-
$Pr(PCbinding)$	-	0.039	-
Standard deviation			
$\sigma(c_{T,t})$	0.089	0.013	-
$\sigma(Y_t)$	0.1	0.1	-
$\sigma(A_t)$	0.1	0.1	-
Correlation			
$\rho(C_{T,t}, Y_t)$	0.99	0.49	-
$\rho(\tau_t, Y_t)$	0.685	-0.01	-
$\rho(C_{T,t}, A_t)$	-0.069	0.022	-
$\rho(\tau_t, A_t)$	-0.056	-0.007	-

Table 7: Moments: $\gamma = 2$

contingent of both state variables.

Tables 7 provide the key steady state moments for the economies parametrized with risk aversion coefficient of $\gamma = 2$. Recall that in the benchmark economy, with $\gamma = 2$, we found that both the fiscal and the currency union had a steady state cycle and that their values were almost identical quantitatively. The addition of stochastic productivity breaks this negligible difference in steady state values. Starting from the currency union, we find that the contract has no surplus, hence a common currency joint with common fiscal policy cannot be sustained. As discussed above, a single monetary authority is ill-equipped to smooth the variations coming from both the endowment and the productivity shocks for both countries. At the same time in the outside option economy the independent central bank can maintain the non-tradeable side of the economy at first best levels. The fiscal union, similarly to the benchmark case, displays a steady state cycle. Consumption behaves as in the benchmark economy, even displaying the same level of volatility. The lower value of the contract is given by the higher volatility of the non-tradeable side of the economy, which now responds to the fluctuations in productivity.

To further analyse the properties of this economy we report the same steady state moments when agents are more risk-averse ($\gamma = 4$). Recall that under this parameterization the benchmark model achieved full risk sharing in both the fiscal and the currency union contracts. In the presence of productivity shocks we find that the fiscal union still achieves full risk sharing. The currency union, however, retains

	Outside-Defaultable Debt	Contract-Fiscal	Contract-Currency
Mean			
Y_t	0.5	0.5	0.5
A_t	1	1	1
$C_{T,t}$	0.5	0.5	0.4996
$\mid \tau_t \mid$	0.014	0.075	0.069
B_t	-0.025	-0.006	0
z_t	-	1	1
$V(y, b(z))$	-80.65	-65.42	-66.72
$Pr(PCbinding)$	-	0	0.035
Standard deviation			
$\sigma(c_{T,t})$	0.094	0	0.031
$\sigma(Y_t)$	0.1	0.1	0.1
$\sigma(A_t)$	0.1	0.1	0.1
Correlation			
$\rho(C_{T,t}, Y_t)$	0.983	0	0.629
$\rho(\tau_t, Y_t)$	0.821	-0.012	-0.008
$\rho(C_{T,t}, A_t)$	0.005	0	0.128
$\rho(\tau_t, A_t)$	-0.015	-0.003	0.006

Table 8: Moments: $\gamma = 4$

steady state fluctuations in tradeable consumption. This can be explained by the smaller surplus of this contract, as the outside option provides further smoothing possibilities via independent monetary policy.

Lastly, we study how these economies behave for different levels of variance of productivity shocks. When the variance $\sigma_p = 0$, we obtain our benchmark model. Figure 15 shows the values of the problem against the productivity shocks variance. Recall that the currency union with $\gamma = 2$ has no surplus. A number of features are worth discussion. First, when full risk sharing is achieved (fiscal union with $\gamma = 4$) the value of the problem is monotonically decreasing in the variance of productivity. By full risk sharing the economy is at first best, this implies that the only residual variation is given by the optimal non-tradeable consumption plan inheriting volatility from the productivity process. As this volatility increases with the variance of the shock, by risk aversion, the value decreases. Secondly, we cannot state a similar result for the cases in which full risk sharing is not achieved. When the economy still has non zero variance in tradeable consumption we see that the value of the contract can be locally increasing in the variance of the productivity shocks. This result is likely to be due to the behaviour of the outside option economy. We observe that increasing the variance of productivity has non-monotonic effects on the likelihood of default in the defaultable debt economy, due to the opposing effects of an increased precautionary savings motive and a higher probability of low productivity realizations. This in turn causes non-monotonic changes in the value of the outside option and the value of the contract, since

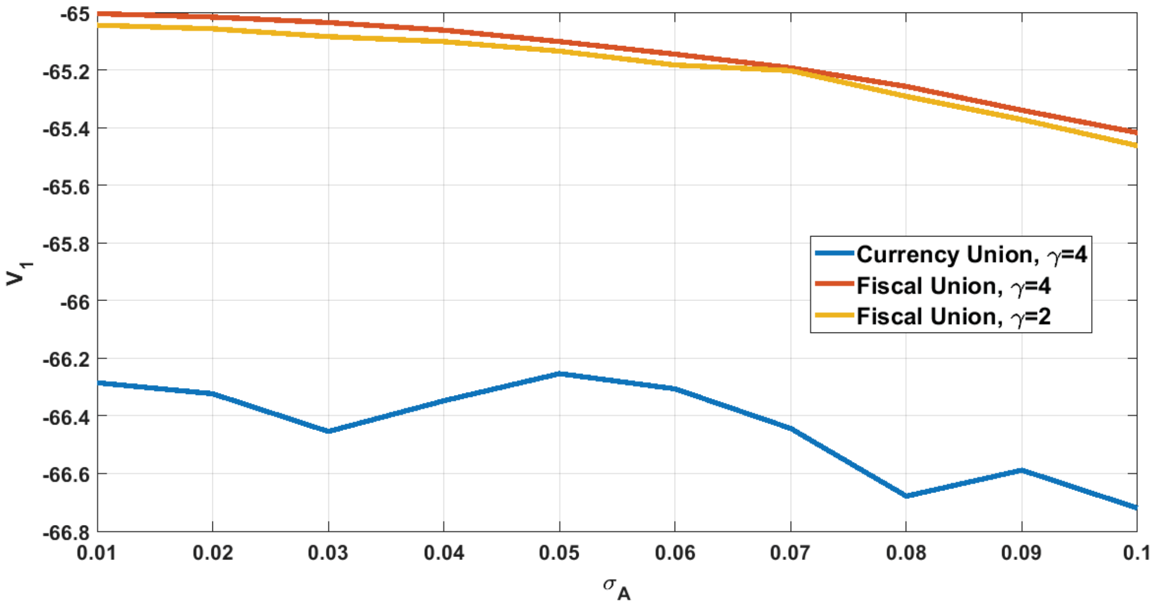


Figure 15: Steady State Values

the movements in steady state consumption are driven by the outside option.

6 Conclusions

In this paper we develop a model of fiscal and currency unions as recursive contracts. We lay down a framework in which two symmetric, equally patient, risk-averse countries face idiosyncratic risk on their tradeables endowment. There is no aggregate risk since the risks are fully negatively correlated. They partake in a risk-sharing agreement subject to a participation constraint. In this constraint the outside option is defined by an Arellano (2008) type economy, in which countries can borrow and default on a risk-neutral lender. Inside the agreement they are able to set up state contingent transfers to reduce consumption volatility. We show that a fiscal union with two independent monetary authorities manages to achieve considerable consumption smoothing.

The fiscal union with two independent monetary authorities has one more policy instrument than the currency union and, therefore, it achieves a higher value. The role of independent monetary policy is to close the labor wedge resulting from the pricing rigidities faced by non-tradeables producers. In a currency union the lack of independent monetary policy implies that the economy is producing at a suboptimal level since a single monetary policy cannot simultaneously close the wedges of both countries. Therefore, the possibility of having an independent monetary policy outside the union makes this institutional design relatively more attractive. We show that, indeed, at the steady state, the monetary union cannot do better than the fiscal union with independent monetary policies. Nevertheless, we quantitatively find that an optimal design of state-dependent transfers, taking as given the optimal monetary policy of the currency union, can compensate almost all the losses of loosing monetary independence.

We provide a characterization of the optimal cross country transfers. We show that the optimal policy requires large countercyclical transfers as a device to smooth consumption. In addition, since in the currency union larger changes in the endowment of tradeables result in large labour wedges, the optimal transfers should be higher after large transitions. It is this extra adjustment in transfers which partially closes the gap between the currency union and the fiscal union. In our simulations, where idiosyncratic risk is significant, the monetary union risk-sharing agreement also allows a significantly higher debt capacity than the defaultable debt economy. Neither the fiscal union nor the monetary union achieves full risk-sharing but they are both able to reduce the volatility of consumption by about 4/5, compared to the defaultable debt economy. However, significant cyclicality of consumption remains.

A number of extensions of this paper would be of interest. An extension of this model in which countries are not symmetric, particularly with respect to the average size of their output, would allow us to analyse situations closer to real world experiences such as the Euro Area. In the same spirit, an extension allowing for aggregate uncertainty would be of interest. Lastly, a further interesting addition would be that of a fiscal externality. This would allow the analysis of cases like the Greek debt crisis, where Greek debt was being held by German banks.

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Appendix

A Proofs

Proof of Lemma 1. Non-tradeable goods producers maximize the expected profits across states, inheriting the households' nominal discount factor $1/\epsilon(s)C_T(s)^{-\gamma}$. Firms maximize

$$\Pi(p) = \sum_s \pi(s|s_{-1})\frac{1}{\epsilon(s)C_T(s)^{-\gamma}}\left[(p-(1+\tau_L)W(s))\left(\frac{p}{P_{NT}(s)}\right)^{-\frac{\varepsilon}{\gamma}}\left(\frac{\alpha\epsilon(s)}{P_{NT}(s)}\right)^{\frac{1}{\gamma}}C_T(s)\right] \tag{58}$$

The first order condition with respect to the price p is

$$\frac{\partial\Pi(p)}{\partial p} : \alpha^{\frac{1}{\gamma}}\sum_s \pi(s|s_{-1})p^{-\frac{\varepsilon}{\gamma}}\epsilon(s)^{\frac{1-\gamma}{\gamma}}P_{NT}(s)^{\frac{\varepsilon-1}{\gamma}}C_T(s)^{1-\gamma}\left[1-\frac{\varepsilon}{\gamma}(p-(1+\tau_L)W(s))p^{-1}\right]=0 \tag{59}$$

Using $p = P_{NT}(s) = P_{NT}$, $\forall s$, this condition becomes

$$\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}}P_{NT}^{-\frac{1}{\gamma}}C_T(s)^{1-\gamma}\left[1-\frac{\varepsilon}{\gamma}\left(1-(1+\tau_L)\frac{W(s)}{P_{NT}}\right)\right]=0 \tag{60}$$

Which yields

$$P_{NT} = \frac{\varepsilon}{\varepsilon - \gamma}(1+\tau_L)\frac{\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma} W(s)}{\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma}} \tag{61}$$

Using the labor subsidy $(1+\tau_L) = \frac{\varepsilon-\gamma}{\varepsilon}$, it simplifies to the first statement in Lemma 1

$$P_{NT} = \frac{\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma} W(s)}{\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma}} \tag{62}$$

To obtain the second statement, notice that, using the definition of the labor wedge and the household first order condition, one has

$$\frac{W(s)}{P_{NT}} = 1 - \kappa(s) \tag{63}$$

Hence, the optimal non-tradeable good price implies

$$\sum_s \pi(s|s_{-1})\epsilon(s)^{\frac{1-\gamma}{\gamma}} C_T(s)^{1-\gamma}\kappa(s) = 0 \tag{64}$$

Which completes the proof.

■

Proof of Lemma 2. The goal of the central bank is to maximize agents' welfare by means of the exchange rate ϵ. The exchange rate in this setting is equivalent to the price of the tradeable good P_T . Recall the following relationships from the household's first order conditions: $C_T^{-\gamma} = \frac{\alpha P_T}{P_{NT}} C_{NT}^{-\gamma}$. Using $\epsilon = P_T$ and inverting the previous relationship one gets $\frac{\partial C_{NT}}{\partial \epsilon} = \frac{1}{\gamma}\frac{C_{NT}}{\epsilon}$. Finally, recall that by labor market clearing $C_{NT} = N$. As monetary policy does not carry intertemporal effects, the central banks maximizes the contemporenous stream of utility:

$$v(\epsilon) = \frac{C_T^{1-\gamma}}{1-\gamma} + \frac{C_{NT}^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}. \tag{65}$$

Maximizing with respect to the exchange rate

$$\frac{\partial v(\epsilon)}{\partial \epsilon} : \frac{C_{NT}}{\gamma \epsilon}[C_{NT}^{-\gamma} - C_{NT}^{\phi}] = 0 \tag{66}$$

Recalling the definition of the labor wedge

$$\kappa^i(s) = 1 - \frac{U_N^i(s)}{U_{NT}^i(s)} = 1 - C_{NT}^{\gamma+\phi}(s) \tag{67}$$

Then optimal monetary policy implies setting

$$k^i(s) = 0 \tag{68}$$

Which proves the first part of the lemma.

In a currency union, the monetary authority maximizes the weighted sum of the welfare of member states. Assuming equal weighing implies maximizing

$$v(\epsilon) = \frac{1}{2}v^1(\epsilon) + \frac{1}{2}v^2(\epsilon) \tag{69}$$

Maximizing with respect to the exchange rates yields

$$\frac{\partial v(\epsilon)}{\partial \epsilon} : C_{NT}^1[{C_{NT}^1}^{-\gamma} - {C_{NT}^1}^{\phi}] + C_{NT}^2[{C_{NT}^2}^{-\gamma} - {C_{NT}^2}^{\phi}] = 0 \tag{70}$$

Using the definition of the labor wedge, optimal monetary policy implies

$$\sum_{i=1,2} {C_{NT}^i}^{1-\gamma} \kappa^i(s) = 0, \quad \forall s$$

■

Proof of Proposition 1. We prove the proposition by constructing the competitive equilibrium which corresponds to the union allocation.

It will be convenient to have the following notation for the marginal rates of substitution of tradeable goods:

$$q(s', z' \mid s, z) = \max_i \beta \left(\frac{C_{T,i}(s', z')}{C_{T,i}(s, z)} \right)^{-\gamma} \tag{71}$$

We can now set the price of an Arrow security in this economy as

$$Q(s' \mid s) = \pi(s' \mid s) q(s', z' \mid s, z) \tag{72}$$

These Arrow prices clearly satisfy the Euler equation, with equality for the country which has the highest marginal rate of substitution. The value of the state contingent debt contract in state s is then

$$\sum_{s'|s} Q(s' \mid s) d(s' \mid s) = Eq(s', z' \mid s, z) d(s' \mid s) \tag{73}$$

We can derive from the equation of motion for z' that

$$\frac{z''}{z'} = \frac{1 + \nu_1(s', z')}{1 + \nu_2(s', z')} \tag{74}$$

And from the solution to the union contract we know that

$$\left(\frac{C_{T,2}(s, z)}{C_{T,1}(s, z)} \right)^{-\gamma} = z' \tag{75}$$

Thus we can write

$$\begin{aligned} \frac{z''}{z'} &= \frac{1 + \nu_1(s', z')}{1 + \nu_2(s', z')} \\ &= \left(\frac{C_{T,2}(s', z')}{C_{T,2}(s, z)} \right)^{-\gamma} \Bigg/ \left(\frac{C_{T,1}(s', z')}{C_{T,1}(s, z)} \right)^{-\gamma} \end{aligned} \tag{76}$$

From this expression we can see that the maximum marginal rate of substitution will be attained by the country which is unconstrained ($\nu_i = 0$) in state (s', z').

For the current debt position of each country, we write the budget constraint of country i as

$$b_i(s) = Y_T^i(s) - C_T^i(s,b) + \sum_{s'|s} Q(s' \mid s) b_i(s' \mid s) \tag{77}$$

and iterate forward on this equation and apply the transversality condition to obtain

$$b_{i,t} = \mathbb{E}_t \sum_{k=0}^{\infty} q(s^{t+k} \mid s_t)(Y_{i,t+k} - c_{i,t+k}) \tag{78}$$

where

$$q(s^{t+k} \mid s_t) = \prod_{n=0}^{k-1} q(s_{t+n+1} \mid s_{t+n}) \tag{79}$$

It should be clear from this definition of the debt position and the resource constraint that

$$B_1(s) = -B_2(s) \tag{80}$$

so that asset markets clear in every state. We set the initial debt positions as $b_{i,0} = \mathbb{E}_0 \sum_t^\infty q_{0,t}(Y_{i,t} - c_{i,t})$. We then choose borrowing constraints which are ***not too tight*** in the sense of Alvarez and Jermann (2000) so that

$$\omega(s, \bar{B}_i(s)) = V_o^i(s, \bar{B}_i(s)) \tag{81}$$

By definition, we will then have $b_i(s) = \bar{B}_i(s)$ whenever country i's participation constraint is binding.

To complete the proof we must show that an allocation which has a high implied interest rate also satisfies the transversality condition:

$$\lim_{t\to\infty} \mathbb{E}_t \beta^t q(s^{t+1} \mid s_t) C_T^i(s_t, b_i(s_t))^{-\gamma} b_i(s_{t+1})$$
$$= \lim_{t\to\infty} \mathbb{E}_t \left[\beta^t C_T^i(s_t, b_i(s_t))^{-\gamma} \max_i \beta \left(\frac{C_{T,i}(s_{t+1}, b_i(s_{t+1}))}{C_{T,i}(s_t, b_i(s_t))} \right)^{-\gamma} \right.$$

$$\times\mathbb{E}_{t+1}\sum_{k=0}^{\infty}q(s^{t+k+1}\mid s_{t+1})(Y_{i,t+k+1}-C_{T,i}(s_{t+k+1},b_i(s_{t+k+1})))\Bigg]$$

$$=\lim_{t\to\infty}\sum_{s_{t+1}|s_t}\pi(s_{t+1}\mid s_t)\left[\beta^t C_T^i(s_t,b_i(s_t))^{-\gamma}\max_i\beta\left(\frac{C_{T,i}(s_{t+1},b_i(s_{t+1}))}{C_{T,i}(s_t,b_i(s_t))}\right)^{-\gamma}\right.$$

$$\left.\times\mathbb{E}_{t+1}\sum_{k=0}^{\infty}q(s^{t+k+1}\mid s_{t+1})(Y_{i,t+k+1}-C_{T,i}(s_{t+k+1},b_i(s_{t+k+1})))\right]$$

$$=\lim_{t\to\infty}\sum_{s_{t+1}|s_t}\beta^t C_T^i(s_t,b_i(s_t))^{-\gamma}\mathbb{E}_{t+1}\sum_{k=0}^{\infty}q(s^{t+k+1}\mid s_t)(Y_{i,t+k+1}-C_{T,i}(s_{t+k+1},b_i(s_{t+k+1})))$$

$$=\lim_{t\to\infty}\sum_{s_{t+1}|s_t}\beta^t C_T^i(s_0,b_i(s_0))^{-\gamma}\frac{C_T^i(s_t,b_i(s_t))^{-\gamma}}{C_T^i(s_0,b_i(s_0))^{-\gamma}}\mathbb{E}_{t+1}\sum_{k=0}^{\infty}q(s^{t+k+1}\mid s_t)(Y_{i,t+k+1}-C_{T,i}(s_{t+k+1},b_i(s_{t+k+1})))$$

$$\leq C_T^i(s_0,b_i(s_0))^{-\gamma}\lim_{t\to\infty}\mathbb{E}_{t+1}\sum_{k=0}^{\infty}q(s^{t+k+1}\mid s_t)(Y_{i,t+k+1}-C_{T,i}(s_{t+k+1},b_i(s_{t+k+1})))$$

$$\leq C_T^i(s_0,b_i(s_0))^{-\gamma}\lim_{t\to\infty}\mathbb{E}_{t+1}\sum_{k=0}^{\infty}q(s^{t+k+1}\mid s_0)(Y_{1,t+k+1}+Y_{2,t+k+1})$$

$$=0$$

Where the last equality follows from the high implied interest rate condition in Equation 49. ■

Proof of Proposition 2. We prove this by contradiction. Assume that country i has a binding participation constraint, so that $\lambda_i > 0$.

Recall that

$$V_i^o(s,B)=\max_{LR,LD}\{V_i^{LR}(s,B),V_{LD}^i(s)\}$$

We have shown in Proposition 1 that the union allocation can be decentralized as a competitive equilibrium with state contingent debt and endogenous borrowing constraints. Recall that $\omega(b_i,s)$ is the value of the problem in the decentralized equilibrium. If the participation constraint binds ($\lambda_i>0$), it must be that

$$V_i^o(s,B)=\omega(b_i,s)$$

Recall that

$$B_{it}=\mathbb{E}_t\sum_{s=t}^{\infty}q_{t,s}(Y_{i,s}-c_{i,s})=\mathbb{E}_t\sum_{k=0}^{\infty}q(s^{t+k}\mid s_t)(Y_{i,t+k}-c_{i,t+k})=b_{i,t}$$

i.e. the face value of the debt in the outside option is the appropriately discounted value of the net payments in the decentralized economy.

In the outside option and in the decentralized economy, the agents maximize the same objective function under different constraints. The budget constraint in the case of exiting and repaying the liabilities is

$$C_T^i(s) + P_{NT}^i(s)C_{NT}^i(s) + B_i \leq Y_T^i(s) + W^i(s)N^i(s) + \Pi^i(s) + B_i'Q(s, B_i')$$

whereas in the decentralization of the union allocation it is

$$C_T^i(s) + P_{NT,i}C_{NT}^i(s) + b_i(s) \leq Y_T^i(s) + W_i(s)N_i(s) + \Pi_i(s) + \sum_{s'|s} q(s' \mid s)b_i(s' \mid s)$$

In the latter, the country is also subject to an endogenous borrowing limit, which we have specified in such a way that it is never binding if the participation constraint is slack. In addition, when the participation constraint binds, the country's liabilities are exactly equal to the borrowing limit. The borrowing limit therefore does not change the allocation.

Comparing the two budget constraints above, it is clear that the allocation in the outside option in case of repayment can always be exactly replicated in the decentralized fiscal union, since the state contingent debt can replicate any payments delivered by non state contingent bonds.

Hence, by optimality, it can never be that the value of the problem is higher in the case of leaving and repaying than in the decentralized fiscal union. Formally, $V_i^{LR}(s, B) < \omega(b_i, s)\ \forall s, B$.

This implies that if the participation constraint binds, it must be that $V_i^o(s, B) = V_{UD}^i(s) > \omega(b_i, s) \geq V_i^{LR}(s, B)$. In other words, it can never be that the participation constraint binds and the country would like to exit and *not* default. ■

Proof of Proposition 3. Using the definition of optimal non-tradeable prices, imposing full risk-sharing and taking the ratio of the non-tradeable prices in the two countries, we obtain

$$\frac{P_{NT}^1}{P_{NT}^2} = \left(\frac{C_T^1}{C_T^2}\right)^{\gamma} = \bar{c}^{-\gamma}$$

We show that imposing a zero wedge condition in one country immediately implies a zero wedge in the other. If country 1 has no labor wedge, $\kappa^1(s) = 0$, then

$$1 = \left(\frac{\alpha\epsilon}{P_{NT}^1}\right)^{\frac{1}{\gamma}} C_T^1$$

Substituting in the relative prices and the relative consumption as a function of the constant $\bar{c}$

$$1 = (\alpha\epsilon)^{\frac{1}{\gamma}} \bar{c}^{\frac{1}{\gamma}} {P_{NT}^2}^{-\frac{1}{\gamma}} \bar{c}^{-\frac{1}{\gamma}} C_T^2 = \left(\frac{\alpha\epsilon}{P_{NT}^2}\right)^{\frac{1}{\gamma}} C_T^2$$

Which implies $\kappa^2(s) = 0$ and completes the proof ■

Proof of Corollary 1. A full risk-sharing steady state is a steady state in which relative weights are constant. As a consequence tradeable consumption is constant and marginal utilities are equal to some constant number. In this case Proposition 3 applies and the common monetary policy has no cost as countries attain the optimum on the non-tradeables side of the economy. Conditioning on the current states s, z the economy has the same level of tradeable consumption and the (optimal) non-tradeable and labour supply. As this is a steady state the continuation values are also identical, which proves that the value of the two programs coincide. ■

Proof of Proposition 4. Conditional on the optimal choice in the outside option being default, the value of the outside option is independent of the current relative weight as it is independent of the stock of liabilities.

Furthermore, whenever a participation constraint binds, the country's relative weight is increased exactly of the amount that makes it indifference between the contract and the outside option. This implies $V_i^F(s,z) = V_i^M(s,z) = V_i^o(s,B)$. As the outside option value $V_i^o(s,B) = V_{UD}^i(s)$ is independent of the relative weight it must be the same for the fiscal and the currency union. Hence the statement of the proposition

$$V_i^F(s,z) = V_i^M(s,z) = V_i^o(s,B) = V_{UD}^i(s).$$

■

Proof of Theorem 1. We start by showing that in a monetary union consumption fluctuates in narrower bands whenever the steady state features non constant consumption. By Lemma 2 in a fiscal union the wedge is zero for both countries in every period, while it is non-zero for both countries in a currency union. We also know that, other things equal, the value of the problem decreases as the wedge moves away from zero

$$\frac{\partial \Omega^M(s,z)}{\partial |\kappa|} < 0, \quad \frac{\partial \Omega^F(s,z)}{\partial \kappa}|_{\kappa=0} = 0,$$

in fact, when $\kappa \neq 0$, $\frac{\partial V_i^M(s,z)}{\partial |\kappa|} < 0$ for $i = 1$ and 2, since domestic labor and consumption of non-tradeables is distorted in both economies. However, regarding tradeables, what is important is how the wedge affects limited enforcement constraints. Recall that if $\nu_i > 0$ then $\nu_j = 0$, $j \neq i$. Without loss of generality assume that $\nu_2 = 0$ and $\nu_1 > 0$. The value of the Lagrange multiplier ν_1 is given by

$$\frac{\partial \Omega(s,z)}{\partial V_1^M}|_{V_1^M = V_1^0} = \nu(s,z)$$

By concavity of $\Omega(s,z)$ it must be that

$$\nu_1(s,z)|_{\kappa(s)\neq 0} > \nu_1(s,z)|_{\kappa(s)=0}$$

Recall that $z' = z\frac{1+\nu_1}{1+\nu_2}$, therefore, it must be that

$$z'(s,z)|_{\kappa(s)\neq 0} > z'(s,z)|_{\kappa(s)=0}$$

This implies that for any given z, if a PC binds, next period z' will be larger in currency unions than in fiscal unions. By the definition of steady states with imperfect risk-sharing it must be that consumption fluctuates in *broader bands* in currency unions than in fiscal unions.

Per se such higher volatility of consumption decreases the value of the problem. Furthermore, in currency unions, this is always paired with suboptimal non-tradeables. Hence in a steady state (s,z) the value of the currency union is lower than the value of the fiscal union with independent monetary policies.

■

Proof of Proposition 6. The common currency monetary policy objective function is such that it minimizes the deadweight loss. Such minimized deadweight loss defines the set of feasible allocations in a monetary union. Conditioning on this restricted feasible allocation set the transfer policy solves the planner problem, thereby picking the efficient allocation in the constrained set. ■

Proof of Proposition 5. A central bank using the planner's relative Pareto weight maximizes

$$v(\epsilon) = zv^1(\epsilon) + v^2(\epsilon)$$

This results in the following first order condition:

$$z{C_{NT}^1}^{1-\gamma}\kappa^1(s) + {C_{NT}^2}^{1-\gamma}\kappa^2(s) = 0, \quad \forall s$$

Without loss of generality, assume that $z < 1$. This also implies that $C_{NT}^1 < C_{NT}^2$. Comparing this monetary policy rule with the one of a central banks that weighs equally the two countries:

$$C_{NT}^{1\ 1-\gamma}\kappa^1(s) + C_{NT}^{2\ 1-\gamma}\kappa^2(s) = 0, \quad \forall s,$$

country 1 will have a larger wedge as it carries less weight in the first order condition.

Following similar lines as the proof of the previous theorem, as country 1 has a larger wedge, if there is surplus in the contract, it will be rewarded with a larger z' for all current z in which the PC binds.

As in the theorem this implies a higher level of consumption fluctuations and a higher wedge, particularly so for the agent with high marginal utility.

■

B Quantitative Model

In section 3, we produce a 5 state Markov process for the stochastic endowment y of the tradeable good in each country. We do this by discretizing an AR1 process with persistence parameter $\rho = 0.9$ and shock variance $\sigma_y^2 = 0.01$, using the Rouwenhorst method. The transition matrix for this Markov process is:

$$\pi = \begin{pmatrix} 0.8145 & 0.1715 & 0.0135 & 0.0005 & 0 \\ 0.0429 & 0.8213 & 0.1290 & 0.0068 & 0.0001 \\ 0.0023 & 0.0860 & 0.8235 & 0.0860 & 0.0023 \\ 0.0001 & 0.0068 & 0.1290 & 0.8213 & 0.0429 \\ 0 & 0.0005 & 0.0135 & 0.1715 & 0.8145 \end{pmatrix} \tag{82}$$

The following graphs show, for each level of the tradeable endowment y, the interval $[\underline{z}(y), \bar{z}(y)]$ within which the participation constraints are satisfied. They therefore accompany the discussions in Section 3 on the ergodic sets for z in each contract and the basins of attraction for these ergodic sets.

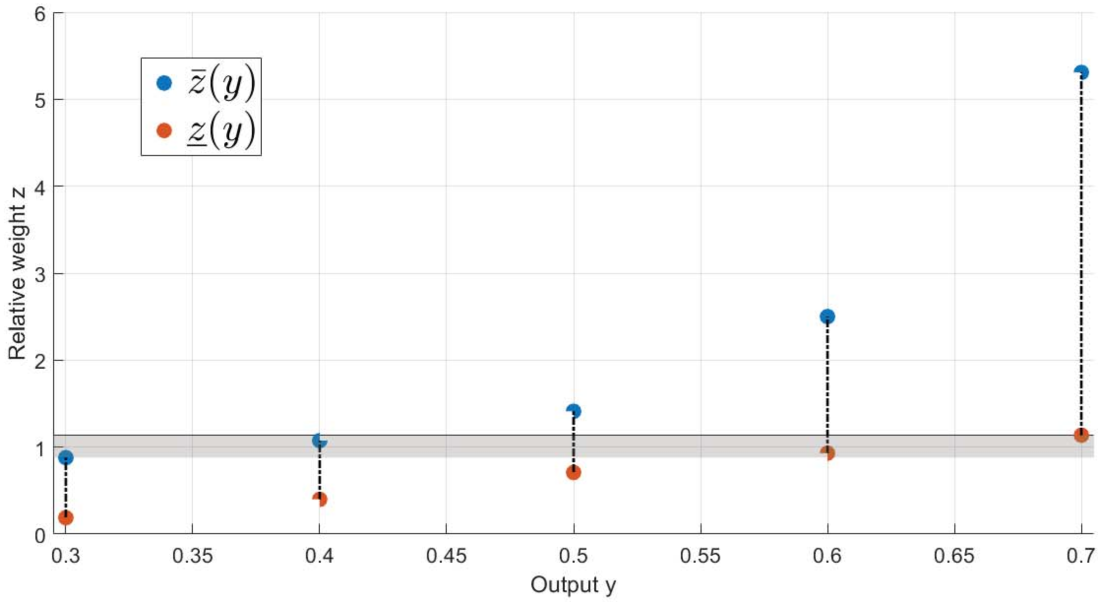

Figure 16: Relative Weights Bounds in Fiscal Unions

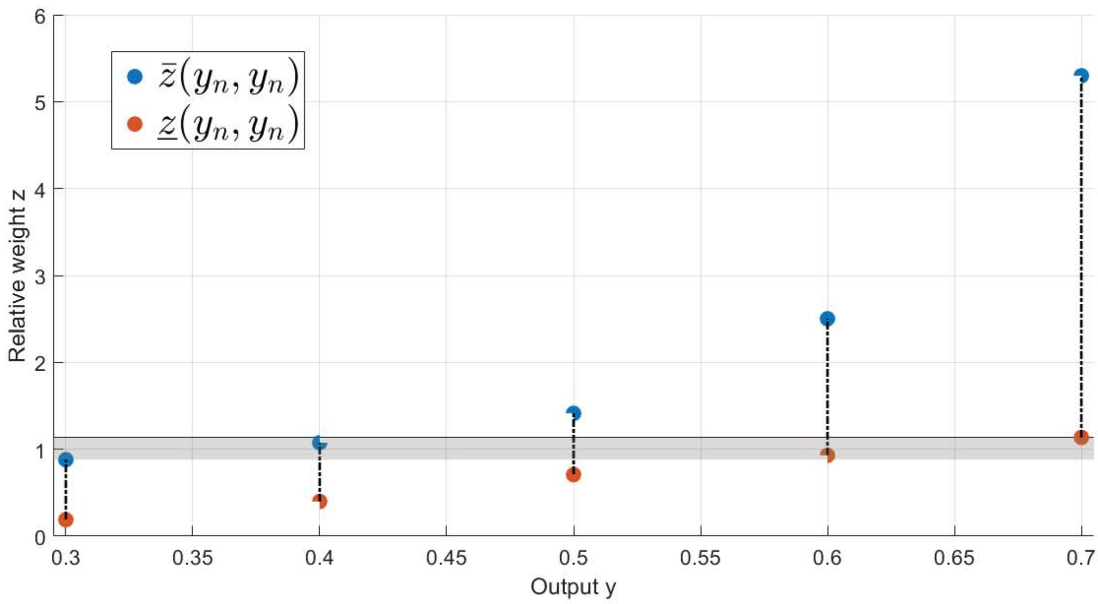

Figure 17: Relative Weights Bounds in Currency Unions