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Simultaneous versus Sequential Move Structures in Principal–Agent Models *

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Abstract

We consider a model with a single principal and two agents and compare the payoff to the principal when agents choose their actions simultaneously and when they choose them sequentially. We show that when the actions are strategic complements, the principal is typically better off when moves are sequential. In particular examples, however, the principal may be worse off.

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1 Introduction

The problem of designing optimal incentive schemes when there is a single principal and many agents has been studied extensively in the literature (see for example, Holmstrom (1982) and Mookherjee (1984) in the context of moral hazard and Demski and Sappington (1984) in the case of adverse selection). The agents are, however, almost always assumed to choose their actions simultaneously. Yet in many situations agents may well have an idea of what other agents have done. For example in a production line, workers may be able to observe how hard workers at a previous stage have worked and will choose their own effort levels accordingly. This paper seeks to analyse how the principal's problem is altered when agents are able to condition their actions on the choices of other agents. In particular, we ask whether the principal is better off if agents move sequentially.

In some cases the principal may be able to determine the order of moves. In other cases the order may be determined by technology. For example, the order of car assembly is largely determined by physical and inventory considerations. On the other hand, the degree of information passed on from one stage to another may be under the control of the Principal. No information passed on corresponds to simultaneous moves, full information to sequential moves and so our model can still be applied. For example, suppose that a car needs both a high quality chassis and a high quality engine in order to be acceptable. In this case quality in the engine and chassis are strategic complements in the sense of Bulow, Geanakoplos and Klemperer (1985): more effort by one party raises the incentive for the other party to work hard. Our analysis suggests that it is better that workers at the engine stage have information on the degree of quality control at the chassis stage passed on to them.

On the other hand, when sending out a typescript for proof-reading, is it better to circulate the same copy to proof-readers in sequence or to send each proof-reader a copy and consolidate their comments? Since proof-reading qualities of different readers are strategic substitutes, that is more effort by one reader reduces the returns to increased effort by the other reader, our analysis suggests that it would be better to circulate the copies separately.[1]

In general, our analysis suggests that it is generally better to proceed sequentially when the parties' actions are strategic complements, as perhaps they are in the first example, but not if they are strategic substitutes, as in the second example.

This work builds on an earlier paper by us (Banerjee and Beggs (1989)). In that paper we analysed a simple model with two agents where the first had no direct impact on output but affected the marginal costs of the second. With simultaneous moves, the principal was unable to induce any effort by the first agent since the latter's actions had no effect on output. With sequential moves, however, if the distribution of output had shifting support, the principal could implement the first-best. Here if the first agent deviated the second would also deviate, as his marginal costs would change, and this would enable the principal to punish the first agent. This technological structure is clearly rather special and in this paper we seek to compare in a more general setting the principal's welfare under sequential moves with his welfare under simultaneous moves.

We assume that there are two agents both of whose actions have an impact on output. The support of output is assumed fixed. We assume that under sequential moves the second agent observes, with a given probability, the effort level chosen by the first agent. We show that, in general it is less costly for the principal to implement an action pair under sequential than simultaneous moves if the agents' actions are strategic complements. Intuitively, if the agents' actions are strategic complements, the second agent upon realising that the first agent has slacked will also do so. Under sequential moves, output is therefore more sensitive to the first agent's actions and the incentive problem is therefore reduced.

It is, however, possible for the principal to be worse off under sequential moves even if the agents' actions are strategic complements and we give an example of this. We give sufficient conditions for this not to be the case. These are equivalent to the usual (sufficient) conditions which ensure that it is only the constraints that the agent not wish to work less hard which

[1] In practice of course another consideration to be offset against any gains from sequential moves might be increased delay.

are binding in the sequential game.

The above logic is reversed if the agents' actions are strategic substitutes: now the second agent will tend to compensate for slack by the first agent. The principal will therefore prefer simultaneous moves.

In some examples, as noted above, the move order may be determined by physical constraints or at least very costly for the principal to alter. An alternative interpretation of our results is to think of the move order as fixed but with the principal determining the probability with which the second agent observes the first agent's action. We show that that it is always better for the principal for the second agent to have a small probability of observing the first agent's action than to have no such chance (which is equivalent to agents moving simultaneously). For the reasons outlined above, however, the principal may be worse off if this probability is large.

Demski and Sappington (1984) and Sappington and Demski (1983) study a model where two agents simultaneously take observable actions but have private information. If the information of the agents is correlated then the principal can use the actions of one agent to make inferences about the information of the other agent and designs incentives accordingly. Our model is one of moral hazard rather than adverse selection but is similar in spirit in that with sequential moves the agent who moves second has information about the actions of the first agent.

It should be noted that our work is distinct from that on monitoring in agencies. In those models, the supervisor has no productive role but may observe the output of the worker (see for example Tirole (1990) for a survey of this work). In our model, both agents have a productive role and the monitoring occurs implicitly, via the action choice of the second agent. That is we focus on the gains from sequential production in a model where there is no cheap talk and all the agents' actions have productive implications. Hence our analysis complements papers in the literature which focus on the monitoring aspects of hierarchies and provides reasons for organising production sequentially, even in the absence of explicit monitoring and reporting mechanisms.

Section 2 sets out the model. Section 3 presents out main results. Section 4 discusses some further aspects of our results. Section 5 concludes.

2 The Model

There is a single risk-neutral principal who wishes to maximise his expected revenue net of payments to the agents. There are two agents, 1 and 2. Agent 1 can take a finite number of actions $\alpha_1 < ... < \alpha_K$ and agent 2 can take actions $\beta_1 < ... < \beta_L$. The actions may be thought of as effort levels. There are n possible outcomes with monetary value $x_1 < ... < x_s < ... < x_n$ to the principal. If the agents take actions α and β respectively then the probability of outcome s occurring is $p_s(\alpha, \beta)$. The principal cannot observe the actions taken by the agents.

Agent 1 has utility function $U^1(I^1) - \alpha$, where I^1 is the monetary payment made by the principal and α is the action taken by agent 1. Similarly Agent 2 has utility function $U^2(I^2) - \beta$. Given the (standard) assumption of additive separability of utility in income and effort, the assumption that the cost of effort is linear in effort is made without loss of generality as one can always choose non-linear units so that this is true. We will allow there to be a lower limit, $\underline{I}_i$ (which may be minus infinity), to the payments that can be made to an agent, but will assume that income falling to this level yields him arbitrarily low utility, so that we do not need to worry about bankruptcy constraints binding (this assumption is commonly made, see for example Grossman and Hart (1983)). Apart from this, the following assumptions are standard and require no comment:

Assumption 1 *U^i is continuous, strictly increasing and strictly concave and $U^i(I)$ tends to minus infinity as I tends to $\underline{I}_i$, $i = 1, 2$.*

Assumption 2 *Agents 1 and 2 have reservation utilities $\underline{U}_1$ and $\underline{U}_2$ respectively.*

We will also assume

Assumption 3 *$\sum_{s=k}^{n} p_s(\alpha, \beta)$ is increasing in α and β for each k.*

This simply says that working harder raises the probability of higher outcomes in the sense of first-order stochastic dominance. Again, this seems innocuous.

Assumptions 1–3 will be in force throughout the analysis. We will also need the following standard technical assumptions for some of our results:

Assumption 4 *$(\sum_{s=k}^{n}\{p_s(\alpha_{i+1},\beta)-p(\alpha_i,\beta)\})/(\alpha_{i+1}-\alpha_i)$ is decreasing in i for all k and similarly for agent 2.*

Assumption 5 *$p_s(\alpha',\beta)/p_s(\alpha,\beta)$ is increasing in s for $\alpha' > \alpha$ and $p_s(\alpha,\beta')/p_s(\alpha,\beta)$ is increasing in s for $\beta' > \beta$.*

These are standard sufficient conditions for the optimal payment scheme to be increasing in output and no upwards constraints to be binding[2] in the simultaneous case. The first condition essentially states that the cumulative distribution function of output is a convex function of each agent's effort. The second says that high output is relatively more likely to be observed if effort is high. This implies Assumption 3. With two outcomes it is in fact equivalent to Assumption 3 but is in general stronger than it. Both these conditions are standard, though strong, conditions (see, for example, Grossman and Hart (1983)).

In the first case we consider, the agents choose their actions simultaneously. The principal can offer the agents payments $I^1(x)$ and $I^2(x)$ conditional on the level of output x. If both agents accept these contracts they then choose which actions to take. If either rejects, they receive their reservation utilities. It will be assumed that the actions the agents take form a Nash equilibrium. If there is more than one Nash equilibrium for the given incentive scheme then we will initially assume that the principal can choose which equilibrium the agents play. It may be the case (see Mookherjee (1984)) that the optimal incentive scheme has multiple equilibria, one of which the agents prefer but is worse for the principal. In Section 4 we consider the implications of imposing the restriction that an action pair can only be implemented by an incentive scheme if there is no other equilibrium which both agents strictly prefer. We show that this restriction does not affect our basic comparison between simultaneous and sequential move structures. The principal's objective is to choose the incentive schemes and actions taken to maximise his expected profit.

In the second case we consider, there is a probability $q(> 0)$ that the second agent observes which action the first has taken. In this case, the

[2] That is one can neglect the constraints that the agents should not wish to work harder than the principal wishes.

second agent is assumed to take a best response to the first agent's action. If he does not observe the action taken, he believes that the first agent has taken his equilibrium action. As before, the principal proposes incentive schemes and if these are accepted the agents choose actions. We assume that these form a sub-game perfect equilibrium and that if there are several equilibria, the principal can choose which one they play. As before, the principal's aim is to maximise his expected payoff.

This monitoring technology is rather crude in that the second agent either observes perfectly the first agent's actions or observes nothing. A more sophisticated approach would allow different signals to be received depending on the actions taken. This would, however, be substantially more complicated as one would have to specify what beliefs the agent should have about which action had been taken if he received a signal that cannot occur in equilibrium (a signal that is compatible with the equilibrium action must lead to the belief that the equilibrium action has been taken with probability one). The issue of what constitute reasonable beliefs would involve refinements of sequential equilibrium and would take us away from our main interests.

Note that q is assumed to be independent of the actions taken. One interpretation would be that there is a given probability that agent 1's work is inspected and the results passed on to agent 2 (but not to the principal). For most of our analysis the reader will not lose much by taking q to be one (perfect observability).

3 Results

In this section, we present our principal results. Section 3.1 presents our main result on when it is cheaper to implement an action pair with sequential moves. Section 3.2 looks at the special case of two outcomes, for which one can obtain slightly stronger results. Section 3.3 discusses the case when the intensity of monitoring can be varied. Section 3.4 examines the case when agents' actions are strategic substitutes rather strategic complements.

3.1 The Main Result

In this sub-section we present our main result on when it is cheaper to implement an action pair with sequential moves. The key assumption for our analysis is

Assumption 6 *$(\sum_{s=k}^{n}\{p_s(\alpha',\beta)-p_s(\alpha,\beta)\})/(\alpha'-\alpha)$ is increasing in β if $\alpha'>\alpha$ for all k. Similarly for player 2.*

In words this assumption says that the harder agent 2 works, an increase in effort by agent 1 produces a greater increase in the probability of higher outcomes for the same increase in cost. It implies that, provided an agent is paid more for high output, an agent will work harder the harder the other agent works, *i.e.* the agents' actions are strategic complements in the sense of Bulow, Geanakoplos and Klemperer (1985) (see also Vives (1990) and Milgrom and Roberts (1990) for further discussion of strategic complementarities).

We will also consider the implication of the opposite assumption, that the actions are strategic substitutes, so that the harder one agent works, the less hard the other wishes to work:

Assumption 7 *$(\sum_{s=k}^{n}\{p_s(\alpha',\beta)-p_s(\alpha,\beta)\})/(\alpha'-\alpha)$ is decreasing in β if $\alpha'>\alpha$ for all k. Similarly for player 2.*

In order to analyse the problem we will find it convenient to follow Grossman and Hart (1983)'s approach to the principal-agent problem. That is, for every pair of actions we will compute the least cost means (if any) of implementing them. To this end, it is more convenient to think of the principal choosing utilities to pay the agents if certain outcomes are revealed and inverting these to find the equivalent monetary payments, rather than working with money directly. Let $V^i(u)$ be the inverse of agent $i's$ utility function. Let $EU^1(\alpha,\beta)=\sum_s p_s(\alpha,\beta)u_s^1-\alpha$ denote the expected utility obtained by agent 1 if he takes action α and agent 2 takes action β and the payment scheme from the principal gives utility u_s^1 in state i to the agent. For convenience explicit reference to the payment scheme is suppressed. Define EU^2 similarly. In order to implement the action pair (α,β) at minimum cost if actions are chosen simultaneously the principal must solve the following program:

(P1)

$$\min_{u_1^1,\dots,u_n^1,u_1^2,\dots,u_n^2} \sum_s p_s(\alpha,\beta)[V^1(u_s^1)+V^2(u_s^2)]$$

s.t.

$$EU^1(\alpha,\beta) \geq \underline{U}^1 \qquad \text{(PC1)}$$
$$EU^2(\alpha,\beta) \geq \underline{U}^2 \qquad \text{(PC2)}$$
$$EU^1(\alpha,\beta) \geq EU^1(\alpha',\beta) \qquad \forall\alpha' \qquad \text{(IC1)}$$
$$EU^2(\alpha,\beta) \geq EU^2(\alpha,\beta') \qquad \forall\beta' \qquad \text{(IC2)}$$

(PC1) and (PC2) are the participation constraints and guarantee that each agent receives at least his reservation utility. (IC1) and (IC2) are the incentive constraints and simply require that (α, β) forms a Nash equilibrium. If the constraints cannot be satisfied then that particular action pair cannot be implemented.

When the two agents take their actions sequentially, the principal must take into account the fact that the second agent can condition his action on the first agent's, if he observes it. Instead of a single action the principal can now specify a reaction function, $R(.)$, for agent 2. To implement the pair (α, β) at minimum cost the principal must solve the problem:

(P2)

$$\min_{u_1^1,\dots,u_n^1,u_H^2,\dots,u_n^2,R(.)} \sum_s p_s(\alpha,\beta)[V^1(u_s^1)+V^2(u_s^2)]$$

s.t.

$$R(\alpha) = \beta$$
$$EU^1(\alpha,\beta) \geq \underline{U}_1 \qquad \text{(PC1)}$$
$$EU^2(\alpha,\beta) \geq \underline{U}_2 \qquad \text{(PC2)}$$
$$\alpha \in \text{argmax}_{\alpha'}\, qEU^1(\alpha', R(\alpha')) + (1-q)EU^1(\alpha',\beta) \qquad \text{(IC1}'\text{)}$$
$$R(\alpha') \in \text{argmax}_{\beta'}\, EU^2(\alpha',\beta') \qquad \forall\alpha' \qquad \text{(IC2}'\text{)}$$

The first condition simply requires that R specifies that β be chosen by agent 2 in response to α. (PC1) and (PC2) are unchanged from before. The new incentive constraint (IC1′) reflects the fact that with probability q the second agent will observe the action taken by the first agent and will take response $R(\alpha')$ to it, while with probability $1-q$ he will observe nothing, believe that the first agent has not deviated, and take action β. (IC2′) requires that $R(.)$ specify a best response for the second agent to every possible action by the first agent, that is there is a sub-game perfect equilibrium.

To see why this sequential move structure may be better for the principal, consider the optimal responses of agent 2 to agent 1's actions. If agent 1 works less hard, then under sequential moves agent 2 may observe this and, if Assumption 6 holds, has an incentive to work less hard himself. As a result, the outcome is worse than under simultaneous moves when only agent 1 deviates. The incentive for agent 1 to deviate is therefore less under sequential moves.

For our main result we need some preliminary lemmas. By upwards constraints for, say, agent 1 we mean incentive constraints of the form 'α_i is preferred to α_{i+j}, $j > 0$', i.e. agent 1 prefer not to work harder. Downwards constraints are of the same form with $j < 0$, that is agent 1 prefer not to work less hard.

Lemma 1 *If Assumptions 4 and 5 hold for agent j, then the optimal payment scheme under simultaneous moves has payment increasing with effort (u^j_s is increasing with s) and no upwards constraints bind.*

In the case of sequential moves, it is harder to find clean sufficient conditions for the upwards constraints not to bind because the probability that a given outcome occurs depends on the reaction function of the second agent, $R(.)$. In the case of continuous actions it is possible to find some rather ugly primitive conditions in terms of third derivatives guaranteeing Assumption 8.[3] These are however not very informative and we prefer to make the following direct assumption:

[3]See the working paper version of this paper, Banerjee and Beggs (1994).

Assumption 8 *Assumptions 4 and 5 hold with $p_s(\alpha, \beta)$ replaced by $(1 - q)p_s(\alpha, \beta) + qp_s(\alpha, R(\alpha))$, all s, for player 1.*

This directly implies

Lemma 2 *Under Assumption 8, no upwards constraints are binding under sequential moves.*

Lemma 3 *If Assumption 6 holds for agent 2 and his optimal payment scheme is increasing (u_s^2 is increasing in s), then $R(.)$ can be chosen to be increasing in s.*

Proof: See appendix

Note that optimal reactions need not be unique, hence the qualification 'can be chosen'.

Proposition 1 *Let (α, β) be an action pair implementable under simultaneous moves. Provided Lemmas 1 to 3 hold, then (α, β) can be implemented more cheaply under sequential moves if agent 1 moves first and agent 2 is given the same payment scheme as is optimal under simultaneous moves.*

Proof: See Appendix

The intuition for this result is that outlined above. Since agent 2's reaction function is increasing, agent 1 will obtain lower payoffs if he works less hard under sequential moves (by Lemma 1) than simultaneous moves. This therefore relaxes all downwards constraints on agent 1. By Lemmas 1 and 2, the upwards constraints do not bind in either problem so the constraints on agent 1 have been relaxed and those on agent 2 are the same, hence the principal is better off.

Remark 1 *It should be noted that Assumptions 4, 5 and 8 are only used in the proofs of Lemmas 1 and 2. Proposition 1 would therefore continue to hold if they were replaced by other, possibly weaker, assumptions provided Lemmas 1 and 2 continue to hold, that is the payment scheme is increasing under simultaneous moves and no upwards constraints are binding in either problem. Assumption 6, however, is pivotal for our analysis.*

Remark 2 *The conditions given above are only sufficient for sequential moves to be superior. In particular one might wonder if one could not do* *better by changing agent 2's payoffs. When there are two outcomes, this is not the case: in the next sub-section we show that there it is optimal to give agent 2 the same payoffs under sequential moves. Furthermore in Section 4.1, we give an example where even though Assumption 6 holds, the principal is worse off because Assumption 8 fails to hold and upwards constraints become binding.*

3.2 The Case of Two Outcomes

The analysis above can be illustrated in the case of two outcomes. In fact one can obtain a tighter result than in the general case. Namely, we will show that if an outcome can be implemented under both sequential and simultaneous moves it can be implemented more cheaply under sequential moves.

We label the two outcomes L and H rather than 1 and 2 for convenience and let $p(\alpha, \beta)$ be the probability of outcome H.

Proposition 2 *Under Assumption 6 if an outcome can be implemented under both simultaneous moves and sequential moves, it can be implemented more cheaply under sequential moves.*

Proof: See Appendix.

This can be understood by considering Figure 1. Suppose agent 1 has three actions α_1, α_2 and α_3 and agent 2 has actions β_1 to β_3. Suppose that the prinicipal wishes to implement the action pair (α_2, β_2). With simultaneous moves, the utility pairs (u_L^1, u_H^2) lying on or above the line PC1 satisfy the participation constraint. In order that he prefer α_2 to α_1, the utility pair must lie above the line $\alpha_2 \sim \alpha_2$. In order that he prefer α_2 to α_3, the utility pair must lie below the line $\alpha_2 \sim \alpha_3$. Note that in the two-outcome case, only the difference $u_H^1 - u_L^1$ affects the incentive constraints and so $\alpha_1 \sim \alpha_2$ and $\alpha_2 \sim \alpha_3$ are parallel lines. Assumption 4 guarantees that $\alpha_2 \sim \alpha_3$ lies above $\alpha_1 \sim \alpha_2$, so the problem is feasible and the optimal choice for the principal is to give the agent as close to perfect insurance (the points on the 45° line) as possible, that is at the intersection of PC1 and $\alpha_1 \sim \alpha_2$, that is the downwards incentive constraint binds and the participation constraint binds.

With simultaneous moves, agent 1 takes agent 2's action β_2 as given. With sequential moves, however, agent 2 may respond to decreased effort by working less hard himself. As a result, $u^1_H - u^1_L$ can be lowered to induce a given amount of effort. Both incentive constraints are therefore lower (and (PC1) is unchanged). Provided that $\alpha_1 \sim \alpha_2$ continues to lie below $\alpha_2 \sim \alpha_3$, that is the upwards and downwards incentive constraints are compatible, (α_2, β_2) can still be implemented. Since the intersection of PC1 and $\alpha_1 \sim \alpha_2$ is closer to the 45° line the principal is better off as he can offer better insurance.

It follows that in the two-outcome case, the upwards and downwards constraints are only compatible if the upwards constraints do not bind, hence the stronger result. This need not however be true with more than two outcomes.

As part of the proof of Proposition 2, it is shown that

Lemma 4 *The optimal payments to agent 2 to implement a given action pair are the same with simultaneous and sequential moves.*

Proof: See Appendix.

The reason for this is that in the simultaneous game, the difference between agent two's payments in two outcomes $u^2_H - u^2_L$ is chosen as small as possible as is consistent with making β a best response to α, so this cannot be lowered.[4] Raising $u^2_H - u^2_L$ simply makes upward deviation in response to more effort from agent 1 more attractive and so simply tightens his upwards constraint.

This is perhaps a somewhat surprising result as one might expect the principal to choose agent 2's payments to make maximum use of his ability to monitor agent 1. It turns out that, in the two-outcome case, the optimal simultaneous payments are also optimal for inducing monitoring.

3.3 Monitoring Intensity

So far we have emphasised the comparison between pure simultaneous and pure sequential moves. In some examples, it may be more plausible to

[4]Note that only the difference $u^2_H - u^2_L$ affects the incentive constraints. The level of the payments is determined by the participation constraints.

think of the move order as determined by physical considerations but the principal may be able to influence the probability that the second agent is able to observe the first agent's action (q). In this case, a small amount of monitoring by the second agent will always benefit the principal. If q is small (but positive) then agent 1's upwards constraints will still not be binding under sequential moves if they do not bind under simultaneous moves, hence

Proposition 3 *For small enough q, an action pair can be implemented more cheaply with sequential moves if no upwards constraint for agent 1 is binding with simultaneous moves (and Lemma 3 holds).*

On the other hand if q is large, this may not hold so too much knowledge may be a dangerous thing.

3.4 Strategic Substitutes

In the case of strategic substitutes, the analysis above is reversed. Under strategic substitutes, if agent 1 works less hard then agent 2 to some extent compensates by working harder, so slacking is more attractive for agent 2. One therefore has (proof in Appendix)

Proposition 4 *Let (α, β) be an action pair implementable under simultaneous moves. Assume that*

a. *Agent 2's reaction function $R(.)$ cannot be chosen to be increasing when given his optimal simultaneous move payoffs.*

b. *The optimal payment scheme for agent 1 is increasing under simultaneous and sequential moves (that is u_s^1 is increasing in s)*

c. *No upwards constraints are binding under simultaneous moves for agent 1.*

Then it is more expensive to implement (α, β) under sequential moves if agent 1 moves first and agent 2 is given the same payment scheme as is optimal under sequential moves.

Note that a sufficient condition for (a) is (proof in Appendix)[5]

[5]Assumptions 4, 5 and 8 are certainly sufficient for (b) and (c) as in Lemmas 1 and 2.

Lemma 5 *If Assumption 7 holds strictly and agent 2's optimal payment scheme is increasing under simultaneous moves, then $R(.)$ cannot be chosen to be increasing.*

The phrase 'Assumption 7 holds strictly' means that the ratio in Assumption 7 is strictly increasing. This is simply used to deal with the problem of multiple best responses. The intuition is clear: working less hard is now more attractive and so by (c), this tightens a binding constraint, so the principal is worse off. [6,7]

4 Discussion

This discusses briefly some further issues in our model.

4.1 Counter-Example

So far we have simply examined the costs of implementing an arbitrary action pair. Even if the optimal simultaneous action pair cannot be implemented with sequential moves, it is possible that the optimal sequential pair yields a higher payoff to the principal. We therefore now present an example in which the principal is strictly worse off with sequential moves no matter which agent moves first.

Each agent has four actions. There are two outcomes. The probabilities of the high outcome occurring are shown in the matrix in Figure 2. Agent 1 chooses the row, agent 2 the column. The costs of choosing the actions are are $0, 1, 2.2$ and 2.32 for agent 1 and $0, 1, 2.16$ and 2.35 for agent 2. We assume that $q = 1$. It is straightforward to check that Assumptions 1–6, are satisfied.

We assume that both agents are risk-neutral. This violates Assumption 1 but it is easy to see that the counter-example remains valid if the utility

[6] In this case, it does not matter whether the upwards constraints bind under sequential moves.

[7] Note that if Assumptions 4 and 5 hold standard arguments, see for example Grossman and Hart (1983), show that any action pair is implementable with simultaneous moves, so moving to sequential cannot make the principal better off.

functions are made slightly concave. The advantage of working with risk-neutral agents is that is possible to solve the principal's problem very easily.

Under the above assumptions, any action pair can be implemented under simultaneous moves. Moreover this can be done at first-best cost: since agents only care about their expected payments, moving them away from the 45° line to give them incentives to choose the correct actions imposes no distortion. It is easy to check that if $6\frac{2}{3} < x_H - x_L < 6.675$ then the unique first-best pair is (1,1).

If we can show that (1,1) cannot be implemented under sequential moves regardless of which agent moves first, then the principal must be worse off, since (1,1) is the first-best point and therefore whatever else he implements must be worse. This is indeed the case.

In order to show that (1,1) cannot be implemented with sequential moves it must be shown that the incentive constraints of whichever agent moves first cannot be satisfied. This is straightforward and the details are omitted. The key observation is to use Lemma 4, which shows that it may be assumed that whichever player moves second is given the lowest payoffs that make action 1 a best response to action 1.

Note finally that in the simultaneous game with payment levels $u_H^1 - u_L^1 = u_H^2 - u_L^2 = 5$, the only equilibria are (1,1) and (0,0) and so (1,1) is indeed the Pareto dominant equilibrium for the agents. The considerations of the next sub-section therefore do not arise.

4.2 Uniqueness of Equilibrium

One potential caveat to our results is that they ignore the possibility of multiple equilibria. If there are multiple equilibria and the agents strictly prefer one equilibrium to the one the principal prefers, then one might argue that they will choose the former and so the characterisation of implementability is incomplete. Nevertheless, we will show that Proposition 1 remains valid even when one imposes the requirement that the agents not play an equilibrium that is strictly Pareto-dominated.

In fact, provided neither agent is taking their least-cost actions, the optimal simultaneous incentive scheme always induces multiple equilibria if the agents actions are strategic complements.[8] In particular, if (α, β) is the desired action pair, then there is always an equilibrium in which both players take lower actions.[9] Nevertheless, this does not seem problematic as (α, β) Pareto dominates any lower equilibrium.[10] It seems natural therefore to suppose that the agents will coordinate on the Pareto dominant equilibrium.

There is no guarantee, however, that there are not equilibria of the simultaneous game in which both players work harder than (α, β). It follows from Milgrom and Roberts (1990)[11] that the game between the agents has a largest equilibrium, in the sense that each player's action is as high as his action in any other equilibrium, and that this equilibrium is Pareto dominant. If this equilibrium is not (α, β) then it could be argued that the agents will deviate to it since it yields them higher payoffs.

Imposing the restriction that the equilibrium played not be strictly Pareto dominated by another equilibrium may therefore restrict the set of feasible actions. Ma (1988) shows that allowing players to send messages to the principal allows any action pair to be implemented uniquely at no extra cost in the model of Mookherjee (1984). A similar problem arises in the model of Demski and Sappington (1984) mentioned in the introduction and Ma, Moore and Turnbull (1988) propose a mechanism to eliminate it. Nevertheless, these mechanisms have some unappealing features (for example Ma's mechanism relies on having an unbounded message space[12]) and we do not consider them here. The simple mechanisms of the form considered here seem more natural and intuitive than those produced by message exchange.[13]

[8] If Assumptions 4 and 5 hold.

[9] A proof of this can be found in the Appendix.

[10] This follows from Theorem 7 of Milgrom and Roberts (1990).

[11] See Theorems 5 and 7

[12] In the context of Ma, Moore and Turnbull (1988), Mookherjee and Reichelstein (1990) show that one can use finite mechanisms if one only requires that the desired equilibrium be the unique pure strategy equilibrium of the game, so it may be possible to apply these results to Ma's model. Nevertheless, it is not clear why mixed strategy equilibria should be ignored.

[13] Moore (1992) is an excellent survey on implementation and discusses the extent to which one can find appealing mechanisms to guarantee unique implementation.

The situation with regard to uniqueness is somewhat different with sequential moves. The first agent can choose whichever point on agent 2's reaction function is optimal for him and so there can be no point which strictly Pareto dominates the equilibrium point for both players. If an action pair can be implemented, the requirement that the equilibrium action pair not be strictly Pareto dominated by another action pair therefore imposes no extra restriction.

Since imposing the restriction that an equilibrium not be strictly Pareto-dominated by another equilibrium reduces the set of implementable outcomes under simultaneous moves but does not affect the set of outcomes under sequential moves, it follows that Proposition 1 remains valid with this stricter interpretation of implementability. Furthermore the case for sequential moves is strengthened, since one can write down examples in which an action pair can be implemented with sequential moves but is always strictly Pareto-dominated by another equilibrium with simultaneous moves. On the other hand, the example in the previous sub-section shows that an action pair may be the Pareto dominant equilibrium with simultaneous moves but unimplementable with sequential moves, so moving to sequential moves may still be worse even when one takes into account the problem of multiple equilibria.

4.3 Other Remarks

If an action pair can be implemented with agents moving in either order, which is not always the case, one might ask what determines the optimal move order. In general, this question is hard to answer since it depends upon the agents' utility functions and reservation price level as well as the probability functions. It also depends on the precise action pair to be implemented. Intuitively, one should put the agent whose action is most sensitive to the other agent's action moving second, since then the monitoring value is greatest, but one also needs to consider which agent's incentive constraint it is more valuable to relax.

5 Conclusions

This paper has demonstrated the importance of taking into account the precise move structure when studying problems with multiple agents. In particular, we showed that under reasonable conditions the principal would prefer to have agents choosing their actions sequentially rather than simultaneously if their actions are strategic complements.

A natural generalisation would be to allow more than two agents by having both more levels in the "hierarchy" and more than one agent in each level. This would be straightforward, although our sufficient conditions would become rather more cumbersome.