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DAY-TO-DAY MONETARY POLICY

AND THE VOLATILITY OF THE FEDERAL FUNDS INTEREST RATE

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Abstract *

We propose a model of the interbank money market with an explicit role for central bank intervention, and study how profit-maximizing behavior on the part of banks (facing periodic reserve requirements and daily shocks to liquidity) interacts with high-frequency interest rate targeting. The model delivers a number of predictions on the cyclical behavior of the federal funds rate's volatility and on its response to changes in target rates and changes in intervention procedures, such as those implemented by the Fed in 1994. We find theoretical results to be consistent with empirical patterns of interest rate volatility in the U.S. market for federal funds.

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1. Introduction

The market for federal funds, where U.S. depository institutions exchange unsecured loans of non-interest-bearing reserves, is the entry point for U.S. monetary policy in the financial sector. Through the federal funds market, liquid funds supplied by the Federal Reserve (Fed) are channeled to financial institutions and to the rest of the economy, while the interest rate that banks charge to each other in this market is linked by arbitrage activity to a broad range of longer-term financial instruments. To ensure that banks trade funds in this market at rates compatible with the target specified by the Federal Open Market Committee, the Fed's Open Market Desk intervenes daily to control the market's liquidity. In turn, the market's daily behavior conveys information to the Fed on the economy's liquidity conditions and allows the Desk to calibrate further intervention so as to maintain market conditions in line with policy targets.

The broad features of monetary policy implementation are widely studied, as are linkages between the federal funds market and markets for longer-term securities. The details of day-to-day transmission of monetary policy to short-term interest rates, however, have received limited attention in previous research. One body of work has studied the high-frequency behavior of overnight rates focussing on the micro-structure of money markets and inter-bank interactions and abstracting from the daily impact of monetary policy on banks' liquidity and on interest rates. Conversely, research on monetary policy has typically abstracted from inter-bank relationships, from the microeconomics of banks' demand for money, and from institutional details of money markets which constrain the central bank in its daily operations and depository institutions in their daily problem of liquidity management.

In this paper, we propose a dynamic equilibrium model of the inter-bank market featuring both some of the main institutional features of the U.S. market for federal funds and an explicit targeting role for the Fed. In the light of our model, we analyze empirical patterns of federal funds rate volatility, and discuss the information they convey on the style of the Fed's daily intervention and--in particular--on the effects of the changes in intervention procedures implemented by the Fed in 1994.

Volatility is indeed an important feature of daily money-market developments. Federal funds rates vary daily by amounts comparable to the Fed's typical quarter-percent target change, often by several multiples of this amount. Furthermore, the high-frequency behavior of money markets displays striking cyclical patterns: the volatility of the federal funds rate typically rises by a factor of forty during a two-week "reserve maintenance period." Of course, fluctuations of overnight rates around official targets are largely transitory, hence unlikely to be transmitted to longer-term rates. Interest rate volatility, however, affects banks' daily operations, and is felt by the Fed as undermining the credibility of its policy commitment, witness recent changes in the Fed's procedures aimed at curbing the volatility of federal funds rates. For instance, in early 1997 the Fed decided to implement its daily intervention at 10:30 a.m., to mitigate problems of market thinness at 11.30 a.m.; in July 1998 it reinstated the system of lagged reserve requirements it had abandoned in 1984, to reduce uncertainty on required reserves; most notably, in February 1994 it began to announce changes in target rates and aimed at implementing them mainly at times of FOMC meetings.

Several studies have examined the empirical link between the federal funds market's institutional features and the federal funds rate's cyclical behavior. Studies of post-1984 data include Spindt and Hoffmeister (1988), Lasser (1992), Brunner and Lown (1993), Rudebusch (1995), Griffiths and Winters (1995), Roberds et al. (1996), Hamilton (1996, 1997), and Balduzzi et al. (1997, 1998). This research has documented statistically significant (if small) predictable patterns in the federal funds rate's level, and the tendency for its volatility to display a sharply cyclical behavior. Existing models of the federal funds market, including Ho and Saunders (1985), Kopecky and Tucker (1993), and Hamilton (1996), have rationalized some of these findings, by incorporating features of the federal funds market such as risk aversion, transaction costs, lines of credit, and various reserve-accounting conventions. Since these studies do not feature an explicit role for official intervention, however, they cannot explain how these systematic interest rate patterns could survive the Fed's effort to keep rates close to their target. In particular, models without official intervention leave unexplained the tendency of the short-term rate to hover around its official target and quickly revert to it in response to shocks (see Rudebusch, 1995, and Balduzzi et al., 1997). By contrast, models such as those of Campbell (1987) and Coleman et al. (1996) have studied the effect of monetary policy on short-term interest rates, but have done so while abstracting from banks' daily liquidity management and other institutional details of the federal funds market, which lie at the heart of the Fed's operations and shape the high-frequency behavior of short-term interest rates.

The closest antecedent of the research reported here is Hamilton's (1996) work, which offers a comprehensive analysis of the federal funds market and a model, with idiosyncratic (bank-specific) shocks, aimed at rationalizing seasonal patterns in mean overnight rates in the absence of aggregate uncertainty and official intervention. We analyze patterns of interest rate volatility instead. The model we propose focuses on banks' efforts to minimize the opportunity cost of satisfying reserve requirements, by borrowing and lending in the interbank market. Since it allows for aggregate shocks to liquidity, both exogenous and intervention-induced, the model can be used to assess the effects of the Fed's day-to-day targeting effort. For simplicity, our analysis abstracts from market imperfections such as transaction costs, lines of credit, and bid-ask spreads. The resulting model allows the Fed to achieve its target exactly *on average* each day, but generates rich patterns of interest rate volatility which--our model suggests--reflect the style and stringency of the Fed's daily intervention procedures, in particular its willingness (or ability) to fully offset high-frequency liquidity shocks.

Bringing our theoretical perspective to the data, we adopt a time-series methodology similar to that of Rudebusch (1995), Balduzzi et al. (1997), and especially Hamilton (1996, 1997). In our empirical work we allow for seasonal mean effects, but focus on seasonal patterns of interest rate volatilities. Econometric analysis of 12 years of daily data confirms our model's main predictions. Among these, we find the volatility of market rates to have declined in "high-rate regimes" (i.e., when the Fed's target rate has approached the penalty rate on reserve deficiencies); and the volatility of interest rates to have declined in days around reserve settlement, in samples--such as the post-February 1994 period--where banks could hold greater confidence in the Fed's commitment to keep rates close to their current target.

2. A Model of the Federal Funds Market with Fed Intervention

We consider a competitive market populated by risk-neutral, atomistic banks of unitary total mass, subject to periodic reserve requirements. We treat time as discrete and focus on the determination of the overnight interbank rate, the "federal funds rate" r_t, in each day $t = 1, \ldots, T$

of a "reserve maintenance period" lasting T days. With no qualitative loss of generality, we set the reserve requirement at zero. Denoting bank i's reserve holdings at the end of day t by x_{it}, and its average reserve holdings by the end of the same day by $a_{it} = (x_{i1} + \ldots + x_{it})/t$, reserve requirements are then expressed by the constraint $a_{iT} \geq 0$. To better focus on the effects of periodic reserve requirements on interest rates, we assume that neither overnight nor intra-day penalties are assessed for negative reserve positions other than on a period-average basis.

The daily timing of events, summarized in Table 1, is as follows. At the beginning of each day (shortly after 10:30 a.m., in New York) the Fed alters the market's liquidity by an amount m_t, defined--in flow terms--as a change in the outstanding stock of base-money. This intervention aims at ensuring that the equilibrium interbank overnight rate lies as close as possible to its current target rate r_t^*. (We discuss in Section 3 below the possibility that this target might change across days.) In reality, the desk's intervention is configured as an auction in the repurchase-agreement (RP) market. For our purposes, however, it will suffice to treat m_t simply as a change in depository institutions' accounts at the Fed.[1]

Banks' liquidity positions may also be altered by flows independent of Fed intervention, such as Treasury payments and other flows from the non-bank sector. In our model, a first realization ν_t of a random zero-mean liquidity shock occurs each day shortly after the Fed's intervention. By this timing convention, the shock ν_t causes the market's liquidity to differ from that envisioned by the Fed when deciding its intervention m_t. Hence, ν_t is akin to a within-day forecast error on the part of the Fed.[2] After the realization of ν_t, banks may trade federal funds in the form of unsecured overnight loans. For simplicity, we follow the classic model of Poole (1968) and think of the interbank market as opening only for one instant during the day, at which time all interbank transactions are settled at a single market-clearing rate r_t.

[1] Similarly, we do not distinguish between vault cash and deposits at the Fed as separate components of the banks' stock of good funds, and refer to both of these interchangeably as "reserves" or "federal funds."

[2] In reality, the Fed has imperfect information on both within-day liquidity and banks' required reserves (which, under the almost-contemporaneous accounting regime in place between 1984 and 1998, become known to the Fed only after the end of the reserve-maintenance period). See Hamilton (1997) for discussion and empirical analysis of such targeting errors.

We denote by $\tilde{a}_{it}$ the average reserve position of bank *i* at the opening of the market in day *t*, and by $\tilde{a}_t$ its industry-wide counterpart; and we denote by b_{it} the amount borrowed (loaned, if negative) overnight by bank *i* in day *t*, and by b_t its industry-wide counterpart.[3] By definition, $\tilde{a}_{i,t} = \left((t-1)\,a_{i,t-1} + m_t + \nu_t\right)/t$. Below, we shall study in detail the characteristics of the market-clearing federal funds rate, defined as a function $r_t(\tilde{a}_t)$ of the industry's reserve position $\tilde{a}_t$ at the opening of each day's market.

After the interbank market has cleared, a second zero-mean liquidity shock ϵ_t is realized in each bank's federal funds position and banks' reserves are tallied. Hence, bank *i*'s reserve position at the end of day *t* is $a_{i,t} = \tilde{a}_{i,t} + (b_{i,t} + \epsilon_t)/t = \left((t-1)\,a_{i,t-1} + m_t + \nu_t + b_{i,t} + \epsilon_t\right)/t$. This routine repeats itself each day $t = 1, ..., T$. At the end of the maintenance period, the Fed assesses penalties for each bank, proportional to its cumulated reserve deficiency, at the penalty rate $\bar{r}$. The penalty paid by bank *i* at the end of day *T* is then $-\min\{\bar{r}\,T a_{iT}, 0\}$.

2.1. Equilibrium and targeting on settlement day. The model can be solved proceeding backwards from the last ("settlement") day *T* of the maintenance period. When the market opens on day *T*, bank *i*'s average reserve position is $\tilde{a}_{iT} = \left((T-1)\,a_{i,T-1} + m_T + \nu_T\right)/T$ and its optimization problem is straightforward. Given the market-clearing rate r_T, the bank chooses how much to lend (borrow) to minimize the expected cost of violating the reserve requirement and maximize the (possibly negative) cash flow $-r_T b_{i,T}$ generated by its overnight position. These two goals, of course, must be traded off against each other: the bank's final reserve position $\tilde{a}_{i,T} + (b_{i,T} + \epsilon_T)/T$ is random before the realization of ϵ_T and, by choosing a large $b_{i,T}$, the bank can reduce the likelihood of a reserve shortfall at the cost of carrying on its books non-interest-bearing reserves rather than overnight loans. Formally, the bank's problem is

$$\max_{b_{iT}} \; V_{i,T} \;=\; \bar{r} \int_{-\infty}^{-T\tilde{a}_{i,T} - b_{i,T}} \left[T\tilde{a}_{i,T} + b_{i,T} + \epsilon_T\right] \mathrm{d}F(\epsilon_T) \;-\; r_T b_{i,T}\,, \tag{1}$$

where $F(.)$ is the cumulative distribution function of the shock ϵ_t (assumed increasing over the

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[3] It is convenient to keep bank- and industry-level variables distinct in our notation, even though, in our representative-bank model, these variables will equal each other in equilibrium.

whole admissible range of ϵ_t). The optimal loan $b^*_{i,T}$ uniquely satisfies the first-order condition for (1),

$$F\left(-T\tilde{a}_{i,T} - b^*_{i,T}\right) = \frac{r_T}{\bar{r}} \,, \tag{2}$$

from which

$$b^*_{i,T}(\tilde{a}_{i,T}, r_T) = -T\tilde{a}_{i,T} - F^{-1}\left(\frac{r_T}{\bar{r}}\right) . \tag{3}$$

Intuitively, the bank's optimal borrowing increases with the penalty rate $\bar{r}$ and decreases with both the federal funds rate r_T and its inherited reserve position $\tilde{a}_{iT}$.

While each bank views itself as able to borrow or lend arbitrarily large amounts of funds at r_T, the market must clear with no net borrowing or lending. Aggregating (2) over the unitary measure of banks and setting total borrowing at zero (i.e., setting $b^*_{i,T}=b^*_T=0$ and $\tilde{a}_T=\tilde{a}_{i,T}$) yields the market-clearing rate

$$r_T(\tilde{a}_T) = \bar{r}F(-T\,\tilde{a}_T) \ . \tag{4}$$

Intuitively, the equilibrium rate prevailing on day *T* is a probability-weighted average of the possible marginal values of funds *after* the realization of ϵ_T, i.e. the penalty rate $\bar{r}$ (incurred by banks when the realization of ϵ_T leaves them with negative reserves) and zero (the value to banks of unremunerated excess reserves). To rule out arbitrage profits by risk-neutral banks, therefore, the rate at which funds may be traded among banks *before* the realization of ϵ_T must equal $\bar{r}$ times the probability of a reserve deficiency by day's end. By (4), this probability is a decreasing function of the industry's inherited reserve position $\tilde{a}_T$.

Consider next the Fed's problem of selecting, earlier in day *T*, the amount of reserves m_T to be injected into (or drained from) the market. Since the Fed intervenes before obtaining access to all information relevant to the market's clearing, the equilibrium rate for day *T* does not generally coincide with the target rate: once the Fed has left the market, the realization of ν_T offers new information as to the likelihood of penalties ultimately being imposed, and thus alters the rate at which banks are willing to trade funds. If intervention aims at minimizing the expected deviation of the market-clearing rate from the target rate r^*_T, then the Fed will choose m_T so that

$$\bar{r}\,\mathrm{E}\Big[\,F\Big(-(T-1)\,a_{T-1}-m_T-\nu_T\Big)\Big] = r_T^* \;, \tag{5}$$

where the expectation is taken over the probability distribution of ν_T.

2.2. Non-settlement days equilibrium. As long as reserves are held for the sole purpose of satisfying periodic requirements, a no-arbitrage logic similar to that holding on day *T* applies on previous days as well. On any given day, banks willingly hold reserves only if the opportunity cost of doing so (the overnight interbank rate) equals, in expected discounted terms, the cost of satisfying reserve requirements by borrowing on day *T*. For instance, an atomistic bank could take its chances and hold negative reserves on all days but the last one, earning interest on the amount loaned. This strategy may entail payment of penalties at *T*, but enables the bank to meet its requirements at little (or no) cost, if the likelihood of eventual excess liquidity is sufficiently high. For this strategy to yield no expected profit in equilibrium, overnight rates must equal the expected penalty on reserve deficiencies on *all* days of the maintenance period: banks would arbitrage away day-to-day differences in interest rates, by trying to meet reserve requirements--thus bidding up rates--on days when interest rates are relatively low, and vice versa. Thus, as noted by Campbell (1987) and others, equilibrium interest rates cannot be expected to differ between days in the same maintenance period, i.e., they must follow a martingale.

Formally, banks' profit-maximizing strategy can be studied by solving a standard dynamic programming problem, details of which we provide in Appendix A. The appendix, in particular, verifies that (even in our model with official intervention) an atomistic, price-taking bank's dynamic optimality conditions are satisfied only if the equilibrium interest rate on each *t* equals its (discounted) expected value on day *t*+1, that is, $r_t = \mathrm{E}_t\big[r_{t+1}\big]/(1+r_t)$. Since the approximation is negligible for realistic values of r_t, we simplify the exposition by assuming that banks do not discount cash flows within each biweekly period. Then, recursively,

$$r_t \;=\; \mathrm{E}_t\big[r_{t+1}\big] \;=\; \mathrm{E}_t\big[r_T\big]\,. \tag{6}$$

Violations of equation (6) are well documented in the literature, and our own work in Section 4 confirms and expands the existing evidence: U.S. federal funds rates feature small, but statistically significant departures from the martingale property (6). Such predictable changes may well be consistent with market equilibrium in the presence of transaction costs, inter-bank credit limits, daily overdraft penalties, and other obstacles to the intra-period perfect arbitrage activity which supports the argument above. The main point we develop in our paper, however, is that systematic patterns in interest rate volatility should arise even when equation (6) holds, and that these patterns reflect the interaction of banks' profit-maximizing behavior with the Fed's intervention procedures. To illustrate this point, it is helpful first to analyze two opposite extreme cases: the case where no official intervention takes place ($m_t \equiv 0$ for all t), and the case where the Fed intervenes to offset all liquidity shocks, so as to achieve a constant interest rate target.

2.3. Interest rates and liquidity shocks without intervention. Consider first how the volatility of overnight rates would evolve within each maintenance period without Fed intervention. In this case, the evolution of r_t reflects banks' daily updating of forecasts of their day-T reserve position, a_T. From (6) and (3), the market-clearing rate on day t is

$$r_t \;=\; \mathrm{E}_t\big[r_T\big] \;=\; \bar{r}\,F_t\big(\tilde{a}_T\big)\,, \tag{7}$$

where $F_t(\,.\,)$ is the cumulative distribution function of settlement-day reserves, $\tilde{a}_T$, based on information available at market-clearing time on day t.

A direct implication of the first equality in (7) is that the level of the interest rate should become more volatile through each maintenance period. To see this, note that both r_t and r_{t-1} are rational forecasts of the same settlement-day interest rate. Since information accruing between t-1 and t must be uncorrelated with that available at t-1, then $\mathrm{cov}(r_t - r_{t-1}, r_{t-1}) =$ $\mathrm{cov}\big(\mathrm{E}_t\big[r_T\big] - \mathrm{E}_{t-1}\big[r_T\big], \mathrm{E}_{t-1}\big[r_T\big]\big) = 0$, and $\mathrm{var}(r_t) \equiv \mathrm{var}(r_t - r_{t-1} + r_{t-1}) = \mathrm{var}(r_t - r_{t-1}) + \mathrm{var}(r_{t-1}) >$ $\mathrm{var}(r_{t-1})$, with strict inequality reflecting interest rate variability between t and t-1.

To explore the implications of the second equality in equation (7), write

$$\tilde{a}_T = \frac{1}{T}\left((t-1)\,a_{t-1} + \sum_{i=t}^{T-1} (\nu_i + \epsilon_i) + \nu_T \right) = \left[\frac{1}{T}\big((t-1)\,a_{t-1} + \nu_t\big)\right] + \frac{1}{T}\sum_{i=t}^{T-1} (\epsilon_i + \nu_{i+1}), \tag{8}$$

and note that the term in the square brackets is proportional to average reserves at the time when r_t is determined, $\tilde{a}_t = ((t-1)a_{t-1} + \nu_t)/t$, while the remaining terms are random on the basis of information available at that time. By (7) and (8), overnight rates are linked to current reserve levels and to the probability distribution of further reserve shocks during the maintenance period. This link pins down the equilibrium price/quantity schedules $r_t = r_t(\tilde{a}_t)$, for $t = 1, \ldots, T$.

To develop intuition on the nature of these schedules and on their implications for cyclical volatility patterns, let liquidity shocks be normal, independently and identically distributed (as in Angeloni and Prati, 1996). With $\nu_t \sim \mathrm{N}(0, \sigma_\nu^2)$ and $\epsilon_t \sim \mathrm{N}(0, \sigma_\epsilon^2)$,

$$\sum_{i=t}^{T-1} (\epsilon_i + \nu_{i+1}) \sim \mathrm{N}\Big(0, (T-t)(\sigma_\epsilon^2 + \sigma_\nu^2)\Big), \text{ for all } t<T. \tag{9}$$

As of information available at market-clearing time on days $t<T$, $\tilde{a}_T$ is then a normally distributed random variable with mean $t\tilde{a}_t/T$ and variance $(\sigma_\epsilon^2 + \sigma_\nu^2)(T-t)/T^2$. Hence, by (7), the equilibrium rate for day t is

$$r_t = \bar{r}\,\Phi\left(\frac{t}{\sqrt{(T-t)(\sigma_\nu^2 + \sigma_\epsilon^2)}}\, \tilde{a}_t \right), \tag{10}$$

where $\Phi(.)$ is the cumulative standard normal distribution.

This expression is plotted in Figure 1 as a function of $\tilde{a}_t$, for t= 1,... ,10. For each day t, the equilibrium schedules $r_t(\tilde{a}_t)$ slope downward: higher reserve levels depress equilibrium interest rates, by reducing the likelihood of penalties on reserve deficiencies on settlement day. The slope of the schedules $r_t(\tilde{a}_t)$ depends on both calendar time and reserve stocks: as t approaches T, uncertainty about settlement-day reserves is gradually resolved, and the current reserve position becomes an increasingly precise signal of the likelihood of eventual penalties. Future random events become increasingly less relevant to the reserve position at settlement time than the current reserve position, and the $r_t(\tilde{a}_t)$ schedules become increasingly steep.[4]

[4] Of course, the schedules $r_t(\tilde{a}_t)$ steepen also because they plot interest rates as a function of *average* reserves $\tilde{a}_t$, whose changes map into increasingly larger changes in *cumulative* reserve, $t\tilde{a}_t$, as t approaches T (the impact increases by the factor (t+1)/t between day t and day t+1). This effect, however, is exactly offset by the fact that the response of $\tilde{a}_t$ to a contemporaneous shock declines at the rate t/(t+1) from day t to day t+1. Thus, the choice of state variable

This pattern of increasingly-steep curves causes interest rates to become more responsive, on average, to similar-sized liquidity shocks as t approaches T. To study this pattern formally, it is useful to study first the behavior of interest rate *levels* and then that of their daily *changes*. By the time interest rates are determined on day t the market has received t independent realizations of the liquidity shock ν, and t-1 of the shock ϵ. Hence, cumulative reserves $t\tilde{a}_t$ (the fundamental determinant of the equilibrium rate) are distributed normally, with variance $t\sigma_\nu^2+(t-1)\sigma_\epsilon^2$. The variance of the interest rate level in day t is then

$$\mathrm{var}(r_t) = \int_{-\infty}^{\infty}\left[\bar{r}\Phi\left(\frac{x}{\sqrt{(T-t)(\sigma_\nu^2+\sigma_\epsilon^2)}}\right)\right]^2\phi\left(\frac{x}{\sqrt{t\sigma_\nu^2+(t-1)\sigma_\epsilon^2}}\right)\mathrm{d}x-\left(\frac{\bar{r}}{2}\right)^2, \tag{11}$$

where $\phi(.)$ is the standard normal density function and we use $\mathrm{E}\left[r_T\right]=\bar{r}/2$. Of course, our example conforms to the martingale property, so that the volatility of interest rate levels must rise throughout the maintenance period. The variance of (uncorrelated) interest rate changes also rises throughout the period, i.e., the variance of interest rate levels increases more than proportionally with t. Intuitively, as t approaches T, a liquidity shock causing a change in $\tilde{a}_t$ is increasingly likely to cause a corresponding change in $\tilde{a}_T$, thus causing an ever stronger response in r_t. At the opposite end, changes in $\tilde{a}_1$ contain little information on likely changes in day-T reserves, so that day-1 liquidity shocks cause only very small changes in r_1. In the limiting case where $T\rightarrow\infty$, $\tilde{a}_1$ would contain *no* information on $\tilde{a}_T$, and r_1 would simply equal the unconditional expectation $\mathrm{E}\left[r_T\right]$ of the settlement-day interest rate for all values of $\tilde{a}_1$.

The rate of increase in first-difference volatilities depends on the relative variances of the shocks realized before and after market clearing on each day.[5] The former ($\sigma_\nu^2=0.2$) is much smaller than the latter ($\sigma_\epsilon^2=2.0$) in the example of the Figure (so that, if the Fed did intervene, it could do so using most of the information relevant to the day's equilibrium rate). The rate of

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(average reserves $\tilde{a}_t$, or cumulative reserves $t\tilde{a}_t$) is completely inconsequential for our model's solution.

[5] In fact, if banks faced no post-market uncertainty on their reserve position on day T then r_T would almost always equal either 0 or $\bar{r}$. This would increase the relative volatility of r_T-r_{T-1}, as liquidity shocks could swing rates from the middle of their range (where they are most likely to be found, as a result of the averaging of independent shocks) to either extreme.

increase in interest-rate variances is then shallow enough that the standard deviation of the rate's daily changes remains essentially constant. As we shall see, these patterns contrast sharply with those displayed by U.S. data and with those predicted by a model with an active role for the central bank.

2.4. The role of official targeting. Like earlier microeconomic models of the inter-bank market, the one outlined in the previous subsection does not feature an explicit role for monetary policy. However, official intervention combines with exogenous liquidity shocks to determine banks' reserve positions and--through the schedule $r_t(\tilde{a}_t)$ linking the market's liquidity to interest rates--the equilibrium rate of interest. To capture these effects, we extend the logic underlying intervention in day T and suppose that, each day t=1,...,T, the Fed intervenes to provide just enough liquidity for banks to trade funds, on average, at r_t^*. That is, the Fed chooses m_t so that

$$\mathrm{E}\left[r_t\left(\tilde{a}_t\right)\right] = \mathrm{E}\left[r_t\left((t-1)\,a_{t-1} + m_t + \nu_t\right)\right] = r_t^* \;, \qquad (12)$$

where the expectation is taken over realizations of ν_t, the shock realized between the Fed's intervention and the market-clearing time.

One might be tempted to model daily official intervention simply as a movement along the reverse-S demand functions displayed in Figure 1 above. However, the shape of the schedules $r_t(\tilde{a}_t)$ depends crucially on banks' expectations of future liquidity shocks, inclusive of "official" shocks m_t. As banks rationally forecast the Fed's actions, it would be incorrect to introduce official intervention while treating the market's reaction to it as an invariant feature of the model.

This point is most clearly illustrated by assuming the Fed to defend a constant target r_0^* from day 1 to day T. Then, by (6), $r_t = \mathrm{E}_t\left[r_T\right] = r_0^*$ for all $t = 1, \ldots, T-1$: throughout the maintenance period, banks are unwilling to trade funds at a rate different from r_0^*. Hence, the rate expected to prevail on settlement day holds exactly in all prior days, irrespective of the market's reserve position. The martingale process followed by equilibrium rates has no innovations, the reverse-S shaped schedules of Figure 1 are flat at r_0^* on days $t = 1, \ldots, T-1$, and neither the interest rate level, nor its daily change, display any volatility through day T-1.

Figure 2 illustrates this degenerate configuration of the model, and points to another implication of official intervention in the forward-looking market under study. Since the Fed must intervene before the realization of ν_T, the equilibrium rate for day T generally differs from r_0^*. Hence, the volatility of interest rates is wholly clustered on settlement day. This pattern emerges if the Fed offsets liquidity shocks daily. Interestingly, however, it also emerges if the Fed spreads desired intervention over the remaining portion of the maintenance period (see Feinman, 1993, for evidence of such behavior by the Fed's Open Market Desk) or it engages in a once-for-all operation on day T. Even if liquidity shocks are not offset daily, banks let their reserve account work as a buffer, counting on the Fed's offsetting action by settlement day.

3. Models of imperfect day-to-day targeting

Not surprisingly, neither of the volatility patterns predicted by the previous extreme cases corresponds to that displayed by historical U.S. federal funds data. The case of Section 2.4, where intervention achieves its target on average and volatility is clustered on settlement day, does capture some important features of the federal funds market. Federal funds rates, for instance, are much more likely to hover around official targets than elsewhere, though--of course--not as closely as predicted by this model. As documented by Hamilton (1997) and others, and confirmed by the summary evidence of Figure 3 (see in particular, the outlier-insensitive statistics displayed in the figure), federal funds rates also display a clear heteroskedastic pattern, recording sharply higher volatility on settlement days than on previous days.

In contrast with our model of Section 2.4, however, interest rate volatility is far from zero on previous days. It is essentially constant, and even falls slightly, in the early days of a typical reserve-maintenance period; it then rises gradually and substantially in the last few days. This suggests that banks do not expect the Fed to always provide liquidity infinitely elastically at the current target rate, and that some of the features of the model without intervention of Section 2.3 need to be captured by a realistic model of the federal funds market.

In general, the Fed may accommodate liquidity shocks incompletely either because it is *unable* to supply funds elastically at the current target rate--for instance, because market rigidities hamper its ability to intervene on any given day--or because it is *unwilling* to do so--for instance, because it would rather let liquidity shocks be partly absorbed by changes in target interest rates. In what follows, we study these channels of transmission of liquidity shocks and examine their predictions for the behavior of interest rate volatility--in particular, predictions on the likely response of interest rate volatilities to the Fed's adoption in 1994 of a procedure of publicly announcing target rates, and changing these mainly at times of FOMC meetings.[6] This policy shift suggests that, if cyclical patterns of interest rate volatility do reflect the Fed's intervention style as our model predicts, then differences in the behavior of federal funds rates' volatility should be apparent by comparing data from the pre-1994 and post-1994 regimes.

3.1. Limits to RP Operations. Historically, the Fed has found it difficult to implement repurchase agreement (RP) operations of unusual size in the federal funds market, for reasons that are well understood by both the central bank and market participants. When undertaking an RP with the Fed, banks must have on hand sufficient collateral (i.e., Treasury securities) to cover their open position. Banks may change their holdings of collateral as part of their portfolio management activity. This, however, is costly, and simply cannot be done as fast as would be required by federal funds market transactions: for purposes of daily liquidity management, the collateral which banks can use as a counterpart to the Fed's RPs is largely constrained by past decisions.[7] Indeed, the market for Treasury securities is relatively thin at the time when the Fed intervention has traditionally taken place (Fleming, 1997, finds that late-morning volume averages only half of its 8:30 a.m. peak), a problem which the Fed has tried to mitigate by shifting its intervention from 11:30 a.m to 10:30 a.m. in early 1997.

The Fed may also face difficulties when attempting to withdraw liquidity from the market

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[6] By contrast, until February 1994 target changes were a constant threat, occurring about once every two maintenance periods, on average (Rudebusch, 1995, and Balduzzi et al., 1997, 1998).

[7] Note that from the viewpoint of accommodating Fed intervention, a bank's ability to adjust its holdings of securities by trading with other banks is irrelevant, for it does not alter the stock of securities held by the banking system as a whole. The relevant constraint is banks' ability to alter their portfolio of Treasury securities--the counterpart in Fed's RP*s*--vis-à-vis their non-bank customers.

by “reverse RP” operations. These difficulties largely stem from the Fed’s own procedures, in particular from its unwillingness to extend discount credit to banks which have undertaken a reverse in recent days.[8] Finally, even when these constraints do not bind, the Fed typically prefers to avoid open-market operations of unusual size: on the one hand, the Fed fears that unusually large operations may destabilize bond markets; on the other hand, the Fed is reluctant to undertake large operations that might need to be reversed in response to new information on reserve flows because (at least until February 1994) such reversals could provide mixed signals of its policy stance.

A simple way to study the effects of limits to the Fed’s ability (or willingness) to conduct RP (or reverse RP) operations is to constrain m_t to lie in the range $[-\bar{m}, \bar{m}]$, a parameterization that embeds our previous case of no Fed intervention (see Section 2.3), as the special case with $\bar{m}=0$, and the case of unconstrained Fed intervention (see Section 2.4) as that with $\bar{m}\rightarrow\infty$.

Even before discussing the solution of our model when $m_t \in [-\bar{m}, \bar{m}]$, it may be intuitively clear why limits to Fed intervention should imply a rise in interest rate volatility *before* the end of the maintenance period--thus capturing features of the case without Fed intervention in a model with interest rate targeting. Since the equilibrium rate depends on the cumulative intervention expected over the rest of the maintenance period, banks can expect liquidity shocks realized early in the period to be offset by a series of relatively small interventions in the following days. Shocks occurring late in the period, however, have a stronger impact on equilibrium interest rates, because offsetting them fully over the shorter remaining portion of the maintenance period is more likely to require large (and, by assumption, impossible) interventions on a daily basis. As information relevant to the Fed’s ultimate inability to hit its target is increasingly likely to reach the market as settlement approaches, the variance of

[8] As noted by Stigum (1990), quoting a Federal Reserve official, “*The market is often incapable of handling a large amount--either because on the repo side they lack collateral or because on the reverse side we have exhausted the supply of banks that want to do reverses. «Banks who do reverses with us are not as welcome at the discount window as they would be if they did not. So banks are reluctant to do reverses because they fear the money market might tighten and they might have to come into the discount window. The rationale for this policy is that a bank should not borrow from us money that they have in fact lent us.»*” We discuss below the extent to which this characterization may be still relevant today.

interest rate innovations increases as $t \to T$.

To explore the implications of this model in detail, we assign specific functional forms to the shocks' probability distributions (our qualitative results are robust to such details). Appendix B outlines the numerical procedure we use to solve a parameterized version of the model, and Figure 4 illustrates the solution for the case where $T = 10$, $\bar{r} = 0.04$, $r_0^* = 0.02$, and the shocks ϵ_t and ν_t are uniformly distributed with variances $\sigma_\epsilon^2 = 0.2$ and $\sigma_\nu^2 = 2$.

In the upper panel of Figure 4, the steepest curve is that for day 10, and it has the linear form of the cumulative distribution function of a uniform random variable. The curves for days $t = 9, 8, \ldots, 1$ are progressively flatter, for the same reasons discussed in Section 2.4: banks recognize that a reserve imbalance carried in days closer to settlement day is more likely to translate into an end-period imbalance of the same sign. This causes market-clearing rates to deviate more from the current target rate in response to a change in $\tilde{a}_t$ as $t \to T$.

The lower panel of Figure 4 plots the implied pattern of volatility of interest rate levels and daily changes, by day of the maintenance period. (The volatility pattern is a sample statistic from a Monte Carlo simulation of the model with 10,000 iterations.) The sharp dichotomy in volatility between the first *T*-1 days and settlement day studied in Section 2.4 (where the Fed was assumed to supply liquidity infinitely elastically to banks) gives way to a smoother pattern of interest rate volatilities. Since shocks to banks' reserves have a larger impact on interest rates--and are less likely to be offset by Fed intervention--when the reverse-S shapes in the main panel of Figure 4 are steeper, the volatility of market interest rates is higher towards the end of the period. By contrast, early in the period, confidence in the Fed's (at least partial) commitment to target the overnight rate yields substantial stability of market rates.

Our model also predicts that structural changes in patterns of interest rate volatility should be observed in response to changes in the Fed's operating procedures. To illustrate this point, Figure 5 plots variance profiles corresponding to three different limits to Fed intervention, $\bar{m} = 2$, $\bar{m} = 3$, and $\bar{m} = 5$. Since this parameter indexes the Fed's willingness (or ability) to undertake large operations for the purpose of enforcing its target rate, a higher value of $\bar{m}$ (i.e., stronger commitment by the Fed to the target rate) implies a pattern of volatility more similar to that of Section 2.4 (where all the volatility is clustered in day *T* and the volatility on day *T* itself is lower) than to that of Section 2.3 (where the volatility rises gradually from day 1 to day *T*). Thus,

Figure 5 delivers a clear qualitative message and an interesting testable hypothesis: the greater is the confidence with which banks view the Fed's commitment to intervene in the market to enforce the target rate, the lower should the volatility of interest rates be on settlement day, and the flatter should the profile of volatilities be on non-settlement days.

This consideration, and the historical evolution of the federal funds market and of the Fed's intervention style, point to predictions on the behavior of federal funds rates over "early" and "late" samples. In recent years specialized dealers, rather than banks, have participated in the Fed's RP auctions and have played an important role in the interbank market. These dealers are endowed with significant stocks of Treasury securities and are generally unconcerned with the Fed's retaliatory behavior at the discount window following a reverse. The Fed's recent shift to an earlier intervention time is also deemed by its officials to have improved the RP market's ability to accommodate interventions. Most interestingly, after switching to a procedure of publicly announced targets in February 1994, the Fed has had less reason to be concerned with large operations as confusing signals of its policy stance. In the stylized parameterization of our model, these considerations suggest that the more recent period might be characterized by a less binding constraint $\left[-\bar{m}, \bar{m}\right]$ than applicable to the late-1980s or early-1990s period, yielding the prediction discussed above for the behavior of interest rate volatilities.

A further testable implication of our model is illustrated by Figure 6, which plots the solution and variance profiles for two different levels of the target rate in relation to the penalty rate. The model predicts that a higher *level* of the target rate r_t^* (relative to the penalty rate $\bar{r}$) should be associated with a lower *volatility* of federal funds rates. Why this should be the case is apparent in the top panel of Figure 6: the higher is the target rate relative to the penalty rate $\bar{r}$, the flatter are the demand curves around the target level of reserves, as more and more of the fluctuations of r_t in response to shocks are truncated by banks' arbitrage at the margin $\bar{r}$.[9] In turn, flatter demand curves translate into a more inelastic response of interest rates to such shocks, and hence into a less volatile behavior of interest rates.

[9] Symmetrical considerations apply to the lower half of the rate's fluctuation band. However, we focus on the upper half of the band, as the U.S. effective federal funds rate has never fallen below half the penalty rate during the past 12 years.

3.2. Liquidity shocks and target changes. The fact that banks face a persistent risk of changes in target rates provides a different--though qualitatively similar, in its effects--channel for liquidity shocks to be partly absorbed through changes in market rates. A stylized analysis of this channel can be developed by extending our treatment of Section 2.4, which assumed the Fed to intervene to provide just enough liquidity for a *given* target rate to prevail, on average, in the market during the day. We now allow for changes in the Fed's target and link these changes to shocks affecting the non-bank sector's demand for liquidity as well as to shocks with no immediate effect on banks' liquidity. A simple way to parameterize this link is the following.

Let the Fed's reaction function be described by a general model, $r_t^*\left(\mathbf{Y}_{t-1}\right)$, linking the current target rate r_t^* to a set of macro-economic variables $\mathbf{Y}_{t-1}$. Let $\mathbf{Y}_t \equiv \{\{\epsilon_t\},\{\nu_t\},\{\eta_t\}\}$, and let us distinguish between variables that affect banks' liquidity (the shocks ϵ_t and ν_t defined above), from a residual variable η_t, which captures factors affecting the Fed's target rate decisions but not banks' liquidity. (The brackets {.} indicate that $\mathbf{Y}_t$ may include the whole history of that variable.) Let these two sets of variables enter separately in the Fed's reaction function, as in

$$r_t^* = \psi\left(\{\eta_{t-1}\},\Delta r^*\right) + \varphi\left(\{\epsilon_{t-1}\},\{\nu_{t-1}\},\Delta r^*\right), \tag{13}$$

where Δr_t^* is the standard step for target rate changes (e.g., Δr^*=¼ percent). Now define

$$\begin{aligned}\psi\left(\{\eta_{t-1}\},\Delta r^*\right) &= \left\{\ldots,\ -\Delta r^*,\ 0,\ \Delta r^*, ...\right\}\\ &\text{if}\ \ \eta_{t-1} \in \left\{\ldots, [-2\Delta r^*, -\Delta r^*),\ [-\Delta r^*, \Delta r^*),\ [\Delta r^*, 2\Delta r^*), \ldots\right\},\end{aligned} \tag{14}$$

where η_t is a zero-mean, i.i.d. shock. Thus, r_t^* is defined as a random spread $\psi(\,.\,)$ around $\varphi(\,.\,)$, such that "large" shocks η_t cause a target-rate change, while "small" shocks η_t leave the target unchanged. The distinction between "large" and "small" shocks depends on the step Δr^*: the larger is Δr^*, the less likely are the shocks η_t to cause a target rate change, and vice versa.

Next, define $\varphi\left(\{\epsilon_{t-1}\},\{\nu_{t-1}\},\Delta r^*\right)$ as the solution for φ of

$$\min_{\varphi\in\{\mathbf{r}^*\}} abs\left[\varphi - \mathrm{E}_t\left[r_t\left(\frac{(t-1)a_{t-1}+\nu_t}{t}\right)\right]\right], \tag{15}$$

where **{r*}** is the set of feasible target rates, **{r*}**$=\{0, \Delta r*, 2\Delta r^*, ..., \bar{r}-\Delta r^*, \bar{r}\}$, and expectations are taken over realizations of ν_t. According to (15), $\varphi(.)$ is the closest rate, among those in the admissible grid $\{\mathbf{r}^*\}$, to that which the Fed would expect to prevail if it were not to intervene at all in day *t* (so that $m_t \equiv 0$, and $\tilde{a}_t \equiv [(t-1)a_{t-1} + m_t + \nu_t]/t = [(t-1)a_{t-1} + \nu_t]/t$).

This specification of $\varphi(.)$ allows us to parameterize by Δr^* the correlation of liquidity shocks with target rate shifts, and thus the extent to which these shocks are accommodated by an adjustment of reserves or by an adjustment in interest rates. For instance, when $\Delta r^* \to 0$, the Fed *never intervenes*: as in the simple model of Section 2.3, market (and target) rates adjust daily in response to *all* liquidity shocks, however small. When $\Delta r^* \to \infty$ instead, the Fed *never adjusts its target:* as in Section 2.4, it provides liquidity elastically at a fixed target rate r_0^*.

In fact, the Fed does adjust its target rate infrequently and in discrete steps, as implied by the stylized model proposed here for positive, finite values of Δr^*.[10] Hence, one would not be surprised to find that it is in the case where $0 < \Delta r^* < \infty$ that the model delivers the most realistic patterns of interest rate volatility, qualitatively similar to those apparent from historical U.S. data and discussed at the beginning of this Section.[11]

Figure 6 corroborates this claim by plotting the variance profile implied by this section's model and three target-rate steps ($\Delta r^* = 1, 15, 25$ basis points). There is a clear qualitative analogy--which extends to all our main results--between the predictions of a model with constraints on intervention, and the predictions of a model with unconstrained intervention, but occasional target rate changes.[12] In fact, both models imply that exogenous liquidity shocks will

[10] See Bernanke and Blinder (1992), Brunner (1994), Hamilton and Jorda (1997), and Bernanke and Mihov (1998) for empirical models of the Fed's behavior, and Goodfriend (1991), for a general discussion.

[11] Our exposition and empirical work focus on a strict interpretation of the notion of "target changes." However, similar results would emerge under a broader interpretation of this notion, encompassing all cases where the Fed chooses to offset liquidity shocks only partially and to allow market rates to deviate from their target. For instance, similar patterns in volatility would arise in a model capturing the Open Market Desk's habit (see Akhtar, 1997) of accommodating reserve shocks only "nearly completely" in days when the Fed is highly uncertain of the market's reserve needs, and in days with exceptional reserve pressure (especially settlement days).

[12] For instance, the model of this section features a relationship between interest rate volatility and (endogenous) current target rates qualitatively similar to that discussed in Section 3.1.

spill more often into market rates: whenever the Fed is bound by $\bar{m}$ (or $-\bar{m}$), the market ends up trading at a higher (lower) rate than in an unconstrained world--reflecting the increased likelihood of a liquidity imbalance by period-end--just as it would if the Fed had chosen to alter its target rate in response to shocks that tighten (respectively, loosen) the market's liquidity. Whether this situation is the result of deliberate or constrained Fed behavior, it is of little importance to banks. In particular, the model suggests that a stronger commitment of the Fed to the current target rate (captured here by a larger value of the target-rate step Δr^*) should, once again, imply a pattern of volatility more similar to that of Section 2.4 than to that of Section 2.3. Empirical comparison of data from the pre-February 1994 and post-February 1994 regimes, conducted in the next section, supports this prediction.

4. Time-series Properties of the U.S. Federal Funds Rate

4.1. Data and summary evidence. Our sample includes daily data from January 1, 1986, to June 4, 1997, for a total--excluding weekends and holidays--of 2,866 observations. The data, obtained from the Federal Reserve Bank of New York, include effective (transaction-weighted) and target federal funds rates.[13] Our sample extends considerably beyond those of previous studies of the federal funds market. These typically include data through the early 1990s, thus preventing analysis of the procedural changes implemented by the Fed in 1994; and they only occasionally include data on target rates, which were not announced by the Fed until 1994 (exceptions include Rudebusch, 1995, Balduzzi et al., 1997, and Hamilton and Jorda, 1997). Our analysis required us to identify FOMC dates, which we drew from the *Federal Reserve Bulletin*.

Inspection of the data revealed many realizations of the (annualized) daily change of the federal funds rate of half-percent or more (i.e., multiples of the Fed's typical quarter-percent change in target rates), as well as a number of large outliers. Closer inspection also revealed both time-persistence in the volatility of daily rate changes and systematic volatility patterns. These appeared to reflect both maintenance-period effects and other calendar effects: settlement days,

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[13] Occasionally, a range of about a quarter-percent-point in size was specified as a target, rather than a point value. In these cases, we took the midpoint of the range as the target rate.

days at the end of years and quarters, and days preceding and following holidays, featured especially large interest rate changes. Indeed, Figure 3 is suggestive both of systematic volatility patterns over the maintenance period, and of substantial differences between raw and outlier-robust statistics. This informal evidence suggests that our hypothesis-testing should rely on a rich empirical model, allowing for special calendar effects, conditional heteroskedasticity and other effects on both the mean and volatility of interest rates.

4.2. The empirical model. Our empirical model is similar in structure to that proposed by Hamilton (1996). Our general strategy was to model fairly accurately the empirical behavior of the federal funds rate (in particular, by modeling deviations from martingale behavior), even though our theoretical model only aims at reproducing the main patterns in volatility apparent in the data. This effort is complementary to Hamilton's, who identifies a variety of systematic patterns in both the level and volatility of interest rates, but only analyzes theoretically the former in a model with deterministic interest rates.

We specify the empirical model of the federal funds rate as

$$r_t = \mu_t + \sigma_t v_t , \tag{16}$$

where v_t is a mean-zero, unit variance, i.i.d. error term, and μ_t and σ_t are functions of t (through a series of calendar effects), lagged interest rates, the differential between the penalty and the target rate, variables capturing FOMC meetings and target-rate changes, and lagged values of σ_t.

For all days of the maintenance period after the first, we model the conditional mean of the interest rate, $\mu_t = \mathrm{E}\left[r_t\right]$, as the sum of the previous day's rate r_{t-1} (which the martingale hypothesis suggests should be the only relevant variable known at time *t*-1) and assorted calendar effects (to account for failures of the martingale hypothesis). To minimize the risk that estimated differences in variances between samples could merely capture level-shift effects of changes in target rates, we included among the determinants of mean interest rates a series of target rate changes (taking a value equal to the change in target on days when a change was implemented, and zero otherwise):

$$\mu_t = r_{t-1} + \delta_{s_t} + \kappa^{/} k_t + \iota (r_t^{*} - r_{t-1}^{*}) , \tag{17}$$

where $s_t \in \{2,...,10\}$ indicates the day of the maintenance period associated with observation t, δ_{s_t} is a constant specific to day s_t of the maintenance period, and k_t is a set of zero-one dummies defining days before and after holidays, end-of-quarter and end-of-year days.[14]

In the presence of binding reserve-carryover limits, r_t need not follow a martingale across maintenance periods. Hence, we follow Hamilton and model the conditional mean on the first day of the maintenance period by an auto-regressive model, which we estimated as a function of the changes in the federal funds rate between day 8 and day 9 and between day 9 and day 10 of the previous maintenance period:

$$\mu_t = r_{t-1} + \phi_1(r_{t-1} - r_{t-2}) + \phi_2(r_{t-2} - r_{t-3}) + \delta_1 + \kappa' k_t + \iota(r_t^* - r_{t-1}^*). \qquad (18)$$

We model the variance of the federal funds rate, $\sigma_t^2 = E\left[(r_t - \mu_t)^2\right]$, as a function of day-of-maintenance-period effects, ξ_{s_t}; calendar effects, h_t; the number of nontrading days between trading days t-1 and t, N_t; and the target rate as a proportion of the penalty rate, z_t. The vector h_t includes end-of-year and end-of-quarter dummies, as well as two additional dummies: a first dummy for the 1986-1987 period, during which the Fed did not implement a strict interest-targeting procedure (see Meulendyke, 1988) and which should presumably be associated with higher volatility; and a second dummy (also used by Hamilton, 1997) for the maintenance period from 1/10/1991 to 2/6/1991, which immediately followed the reform in reserve requirements of beginning-1991 and during which volatility was also extraordinarily high. We also include among the determinants of the variance of federal funds rates a dummy variable taking a value of one only on days in which a target change was implemented, and zero otherwise.

We also introduce "Exponential GARCH" effects (Nelson, 1991) allowing for asymmetric effects of lagged innovations on each day's log variance. Following Nelson, the nontrading day variable N_t enters our specification in the form $\log(1+\gamma N_t)$, so that the γ parameter indexes the extent to which each previous nontrading day increases the variance of day

[14] See Allen and Saunders (1992) for evidence on banks' tendency to engage in temporary window-dressing operations in correspondence to quarter-end and year-end reporting dates.

t.[15] To test the prediction that the variance of the federal funds rate should depend on the differential between the penalty and the target rate, we include the difference between the penalty and the target rate (divided by the penalty rate), z_t, among the determinants of the rate's variance. The EGARCH(1,1) model we estimate allows for persistent deviations of the (log of) the conditional variance from $\xi_{s_t} + \omega' h_t + \zeta z_t + \log(1+\gamma N_t)$, its unconditional expected value.[16]

The resulting model for the variance of federal funds rate changes is

$$\log(\sigma_t^2) - \xi_{s_t} - \omega' h_t - \zeta z_t - \log(1+\gamma N_t) = \lambda\left[\log(\sigma_{t-1}^2) - \xi_{s_{t-1}} - \omega' h_{t-1} - \zeta z_{t-1} - \log(1+\gamma N_{t-1})\right] + \alpha\,|v_{t-1}| + \theta\, v_{t-1}\ . \quad (19)$$

One of the main implications of our theoretical analysis is that the time profile of interest rate volatility should depend on the Fed's inclination to accommodate liquidity shocks. To test this hypothesis we split our sample into three sub-samples: a first sub-sample including all maintenance periods prior to February 1994 during which no FOMC meeting was scheduled (hereafter, "pre-1994 sample"); a second sub-sample including all post-February 1994 periods during which no FOMC meeting was scheduled (hereafter, "post-1994 sample"); and a third sub-sample including all maintenance periods (pre- and post-February 1994) during which an FOMC meeting was scheduled (hereafter, "FOMC sample").[17] This split is suggested by the Fed's change in target-setting procedures in February 1994, whereby changes in target rates have been both publicly announced and implemented almost exclusively at times of FOMC meetings. Hence, a target rate change in the post-1994 sample should be regarded as less likely than in the pre-1994 sample or in the FOMC sample. According to our model, this should imply a lower

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[15] If, for example, γ=0.1 and there are two nontrading days between trading day t-1 and trading day t, then σ_t^2 is 20% higher than usual.

[16] Standard ARCH tests did not reveal residual conditional heteroskedasticity. We also explored an EGARCH (2,2) model, but the coefficients associated with the second lag were statistically insignificant.

[17] We also estimated a model with four sub-samples, constructed by splitting the FOMC sample into pre- and post-1994 sub-samples. The coefficients in these latter samples were found to be insignificantly different from each other.

and flatter profile of interest rate volatilities as a function of the maintenance period. To test this hypothesis, we include in the model of the variance three sets (one for each sub-sample) of 10 day-of-maintenance-period dummies. To ease comparison of volatility profiles across sub-samples, our empirical model parameterizes the coefficients of dummies for days 2-10 as deviations from the coefficient of day 1 in each sub-sample.

Finally, we assume a *t*-distribution for the innovations v_t, and obtain maximum likelihood estimates of the parameters--including the number of degrees of freedom in the *t*-distribution--by numerical optimization. This specification allows us to match well both the fat tails and the concentration of small changes found in the empirical distribution of interest rates.[18] (Hamilton, 1996 and 1997, captures the same features by a mixture of normal distributions for the innovations.) To circumvent convergence problems induced by the non-differentiability of the EGARCH variance at the origin, we used a twice-differentiable approximation to the absolute-value function $|v_t|$.[19]

4.3. Results. Table 2 and Figure 8 summarize the estimation results. First, consider estimates of special calendar and EGARCH effects, which we introduce to clean the data of effects we do not model theoretically. The federal funds rate declines on the last working day of the year; the variance on that day, and the two days before and after it, is estimated to be 18 times larger than on a typical day (Table 2b). The rate tends to increase considerably on the last business day of quarters 1, 2, and 3, and to fall the day after, with a variance 9 times larger than on a typical day. The rate tends to decline on days preceding a holiday and to increase in the following days. The

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[18] Confirming this property, a quantile-quantile plot of the empirical distribution of estimates of v_t against a randomly generated *t*-distribution (with the estimated degrees of freedom) was very close to a straight line. To verify the robustness of our results to the assumed distribution of the errors v_t, we re-estimated the model under a Generalized Error Distribution (GED) obtaining very similar results. In this case, the estimated value of the tail-thickness parameter of the GED was equal to 0.994 and insignificantly different from 1, suggesting that--within this class of distributions--a fat-tailed double exponential captures the empirical distribution of the data well.

[19] Specifically, we followed Andersen and Lund (1997, p. 351) in setting $|v_t|=|v_t|$ for $|v_t|\geq\pi/2K$, and $|v_t|=(\pi/2-\cos(K\,v_t))/K$ for $|v_t|<\pi/2K$. We set K=20, but any large value of K would ensure a very close and twice-differentiable approximation to $|v_t|$.

additional contribution of each nontrading day to the variance of the first following trading day is estimated to be almost 70 percent. The variance is also marginally higher in the 1986-87 period, when the Fed did not follow a strict interest-targeting procedure (Table 2c). As in Hamilton (1997), we find strongly significant dummies for the period of unusual volatility at the beginning of 1991, which surrounded a reform in reserve requirements. Our estimates of the EGARCH parameters are strongly significant and comparable to those estimated by Hamilton, suggesting persistence in the volatility of the underlying liquidity-shock process. The estimated number of degrees of freedom of the Student-t distribution is insignificantly different from 3, implying a very fat-tailed error distribution. The significance of the day-of-maintenance-period dummies k_t (Table 2a) implies a rejection of the martingale hypothesis, confirming evidence uncovered by Hamilton (1996), Balduzzi et al. (1997), and other related studies, and explained by Hamilton (1996) as the result of transaction costs, line of credit, and other imperfections in the federal funds market.

Next, consider estimates of maintenance-period effects on the (log of the unconditional) variance of federal funds rates (Table 2a and Figure 8). Figure 8 plots the estimated profile of the standard deviation of federal funds rates in each subsample, expressed as a ratio of the estimated standard deviation on day 1. In all three subsamples, volatility reaches its minimum between the second and third day of the maintenance period. The unusually high volatility between the first and second day probably reflects carry-over effects: in the early day of each maintenance period, banks often need to unwind positions opened to satisfy reserve requirements in the previous period.

Patterns of interest rate volatility are otherwise consistent with our model's predictions. The volatility of interest-rate changes increases through the rest of the period. The estimated variance is extraordinarily high on the last day of the maintenance period (in the pre-1994 sample, it is 42 times larger than its low on day 3, and 8 times larger than its value on day 9). Most interestingly, the post-1994 volatility profile is closer to the one that could be expected with perfect Fed targeting than the other two volatility profiles. A Wald test strongly rejects the null hypothesis of equality between same-day-of-maintenance-period pre-94 and post-94 coefficients.[20] A similar test accepts, instead, the hypothesis of equality between pre-94 and FOMC coefficients.[21] This evidence confirms our theoretical prediction that seasonal patterns in volatility should be more pronounced when the public's confidence in the Fed's commitment to the current target is greater, i.e, in periods when intervention takes place in a deeper, dealer-based market, and in periods when targets are transparently announced and altered mainly on the occasion of FOMC meetings. We also estimate a higher target rate (with respect to the penalty rate) to decrease the variance of the federal funds rate. This effect is strongly significant and robust across empirical specifications. According to our estimates, an increase in the target rate from ½ the penalty rate to ¾ the penalty rate would reduce the variance of the federal funds rate by more than 30 percent.

5. Concluding Remarks

We study interactions between the Fed's dynamic targeting activity and banks' forward-looking reserve-demand behavior, in a microeconomic model of the federal fund market which goes some way towards bridging the methodological gap between textbook analysis of monetary policy and the micro-analysis of money markets. The market's equilibrium, represented by a set of S-shaped relationships between interest rates and banks' reserves, features the typical heteroskedastic behavior of interest rates over the maintenance period and links it to the character of the Fed's targeting procedures. The model suggests that patterns of interest rate volatility should reflect the confidence with which market participants view the Fed's commitment to target interest rates. Our analysis of data from the U.S. market for federal funds confirms this theoretical prediction: transparent targeting and the tendency to change target rates only on the occasion of FOMC meetings since 1994 have been associated with lower interest rate volatility on and immediately before settlement days than during the pre-reform period.

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[20] The Wald test of the 9 relevant restrictions yields a χ^2 of 25.24 and a p-value of 0.0027. In particular, coefficients of the post-1994 dummies tend to be significantly smaller than those of the pre-1994 (or FOMC period) dummies towards the end of the maintenance period.

[21] The Wald test of the 9 relevant restrictions yields a χ^2 of 5.59 and a p-value of 0.800.

Our theoretical model is narrowly focused on interest rate developments within maintenance periods and is, of course, quite stylized. For instance, while we study the effects of Fed intervention on interest rates, we do not rationalize the Fed's behavior from first principles. Nor do we claim that mechanical time aggregation of our high-frequency model would yield a macroeconomic model of the type encountered in standard quarterly or annual analyses of monetary policy: clearly, many of the high-frequency shocks featured in our model may be smoothed out by time averaging, and both the market's information structure and the nature of exogenous shocks are likely to differ at different frequencies. Also, while our model incorporates key institutional details of the interbank market, it simplifies the banks' liquidity-management problem in several important respects. For instance, we do not account for other (non-interbank) channels a bank may utilize to borrow and lend funds, and we abstract from non-pecuniary penalties a bank faces when incurring a reserve deficiency, such as more intense Fed scrutiny, rationing of future borrowing, and--most subtly--signals to competing banks of the bank's possible financial weakness. Accounting for these features would require a much more complex theoretical model than that we study here, and estimation using a time-varying (state-contingent) cost of reserve deficiencies (see Peristiani, 1998, for a related discussion).

Despite these limitations, our model features on a daily basis the same concern for the Fed's role in interest rate determination as macroeconomic models of monetary policy, and should prove useful as a step toward further research on the role of monetary policy in money markets. Careful modeling of market imperfections, which previous studies have shown useful to explain systematic deviations of interest rates from their benchmark martingale behavior, should have high priority in further research. Yet we find it interesting that no market imperfection needs to be invoked to explain the main high-frequency patterns in the volatility of federal funds interest rates: these patterns emerge naturally from the interaction of banks' profit-maximizing behavior and the Fed's daily intervention procedure, and offer useful information as to the latter's character on a day-to-day basis.

Appendix A: Recursive solution of banks' liquidity-management problem

From (1)-(4), bank *i*'s optimized profit at the opening of the market on day *T* is

$$V_{iT}^{*}(\tilde{a}_{iT}, r_T) = \bar{r} \int_{-\infty}^{F^{-1}(r_T/\bar{r})} \epsilon_T f(\epsilon_T) \mathrm{d}\epsilon_T + r_T T \tilde{a}_{iT} , \tag{A1}$$

defined as a function of the bank's average reserve position up to that point, $\tilde{a}_{iT}$, and of the interest rate r_T which the bank takes as given.

In earlier days, bank *i*'s Bellman equation for period *t* is

$$V_{it}^{*} = \max_{b_{it}} -r_t b_{it} + \frac{1}{1+r_t} \mathrm{E}_t\left[V_{i,t+1}^{*}\left(\tilde{a}_{i,t+1}, r_{t+1}\right)\right] , \tag{A2}$$

with expectation taken over realizations of ϵ_t and ν_{t+1}, taking into account that, by the Fed's intervention policy, $m_{t+1} = m_{t+1}^{*}(a_t)$. The solution to this problem, the function $b_t^{*}(\tilde{a}_{it}, \tilde{a}_t, r_t)$, yields the equilibrium rate for period *t*, $r_t(\tilde{a}_t)$, upon setting $\tilde{a}_{it} = \tilde{a}_t$ and then $b_t^{*}(\tilde{a}_t, \tilde{a}_t, r_t) = 0$.

Now, assume that V_{it}^{*} can be written as

$$V_{it}^{*} = \max_{b_{it}} -r_t b_{it} + \frac{\mathrm{E}_t\left[r_{t+1}\right]}{1+r_t}[t\tilde{a}_{it} + b_{it}] + g_t(\tilde{a}_t) , \tag{A3}$$

where $g_t(.)$ is a function of $\tilde{a}_t$ alone. Then $V_{i,t-1}^{*}$ can be written as

$$V_{i,t-1}^{*} = \max_{b_{i,t-1}} -r_{t-1} b_{i,t-1} + \frac{\mathrm{E}_{t-1}\left[r_t\right]}{1+r_{t-1}}[(t-1)\tilde{a}_{i,t-1} + b_{i,t-1}] + g_{t-1}(\tilde{a}_{t-1}) , \tag{A4}$$

where $g_{t-1}(.)$ is a function of $\tilde{a}_{t-1}$ alone.

To prove this claim, we first verify that $V_{i,T-1}^{*}$ can be written in the form of (A3). To do so, first rewrite (A1) as

$$V_{iT}^{*} = h_T(r_T) + r_T T \tilde{a}_{iT} , \tag{A5}$$

where $h_T(r_T) \equiv \bar{r} \int_{-\infty}^{F^{-1}(r_T/\bar{r})} \epsilon_T f(\epsilon_T) \mathrm{d}\epsilon_T$. Substitute $\tilde{a}_{iT} = \frac{(T-1)\tilde{a}_{i,T-1} + b_{i,T-1} + \epsilon_{T-1} + m_T + \nu_T}{T}$ into (A5) and the result into (A2), evaluated at $t = T-1$. Write $m_T = m_T^{*}(a_{T-1})$, with $a_{T-1} = \tilde{a}_{T-1} + \frac{\epsilon_{T-1}}{T-1}$, take expectation over ϵ_{T-1} and ν_T; and rearrange terms, to obtain (A3) for $t = T-1$.

Next, proceed by recursion. Let (A3) hold at *t*. The first-order condition for (A3) with respect to b_{it} is

$$\frac{\partial}{\partial b_{it}}\left[b_{it}\left(-r_t + \frac{\mathrm{E}_t\left[r_{t+1}\right]}{1+r_t} \right) \right] = 0 \tag{A6}$$

which yields an interior optimum for b_{it} only if $r_t = \frac{\mathrm{E}_t\left[r_{t+1}\right]}{1+r_t}$. This condition (the "martingale property" referred to in the text) defines r_t as a function $r_t(\tilde{a}_t)$ of $\tilde{a}_t$, and can be substituted into (A3) to yield

$$V_{it}^{*} = r_t(\tilde{a}_t)\, t\tilde{a}_{it} + g_t(\tilde{a}_t) \ , \tag{A7}$$

where the key fact is that (A7) is linear in $\tilde{a}_{it}$. Now, lag (A2) once; substituting (A7) into this, yields

$$V_{i,t-1}^{*} = \max_{b_{i,t-1}} \; - r_{t-1} b_{i,t-1} + \frac{1}{1+r_{t-1}} \mathrm{E}_{t-1}\left[r_t(\tilde{a}_t)\, t\tilde{a}_{it} + g_t(\tilde{a}_t) \right] . \tag{A8}$$

Replace the transition equations for $\tilde{a}_t$ and $\tilde{a}_{T-1}$, $\tilde{a}_{it} = \frac{(t-1)\tilde{a}_{i,t-1} + b_{i,t-1} + \epsilon_{t-1} + m_t + \nu_t}{t}$ and $\tilde{a}_t = \frac{(t-1)\tilde{a}_{t-1} + \epsilon_{t-1} + m_t + \nu_t}{t}$ into (A7), as well as $m_t = m_t^{*}(\tilde{a}_{t-1} + \frac{\epsilon_{t-1}}{t-1})$. Finally, take the expectation with respect to ϵ_{t-1} and ν_t and rearrange terms, to obtain (A4).

Note that the argument in this Appendix is essentially unchanged if the function $m_t = m_t^{*}(\tilde{a}_{t-1}, \eta_t)$ depends on a stochastic shift factor η_t, as assumed in the model of Section 3.2. In this case, expectations as of each *t*-1 must be taken with respect to η_t as well as with respect to ϵ_{t-1} and ν_t.

Appendix B: Solution method for the models with imperfect targeting

We describe the solution for the model with target changes of Section 3.2. The solution for the model with RP limits of Section 3.1 is identical, except that η_t is set identically to zero, r_t^{*} is set identically to a constant r_0^{*} , and $m_t(a_{t-1})$ is truncated to the range $[-\bar{m}, \bar{m}]$ for all *t*.

We let the shocks ν_t, ϵ_t, and η_t be distributed as uniform random variables. We then solve the model backward from *T*, using the analytical solution for day *T* as a terminal condition.

To solve the model of Section 3.2, we begin by discretizing the functional $r_T(\tilde{a}_T)$ over a one-dimensional grid $\{\tilde{a}_T\}$, the functional $V_{iT}^*(\tilde{a}_{iT}, r_T(\tilde{a}_T))$ over a two-dimensional grid $\{\tilde{a}_T, \tilde{a}_{iT}\}$, and the functional $m_T^*(a_{T-1}, \eta_{T-1}, \Delta r^*)$ over a two-dimensional grid $\{a_{T-1}, \eta_{T-1}\}$. Stepping back to *T*-1, we compute $\mathrm{E}_{T-1}\left[V_{i,T}^*\left(\tilde{a}_{i,T}, r_T(\tilde{a}_T)\right)\right]$, the banks' expected value function for period *T*, by taking expectations over ϵ_{T-1}, η_{T-1}, and ν_T, and interpolating over the discretized state-space to obtain $\mathrm{E}_{T-1}\left[V_{i,T}^*\left(\tilde{a}_{i,T}, r_T(\tilde{a}_T)\right)\right]$, for each realization of ϵ_{T-1}, η_{T-1}, and ν_T, from the known grid values of $V_{iT}^*(\tilde{a}_{iT}, r_T(\tilde{a}_T))$, $r_T(\tilde{a}_T)$, and $m_T^*(a_{T-1}, \eta_{T-1}, \Delta r^*)$.

One can easily check that the model's assumptions guarantee a unique solution for the equilibrium rate r_t, since banks' optimal policy would entail indefinitely large borrowing or lending positions whenever r_t violates (A6). An iterative algorithm then solves for the market equilibrium at *T*-1, by forming an initial guess for r_{T-1}; solving banks' problem over choices of $b_{i,T-1}$ (which, in the absence of transactions costs, will generally take extreme values); and adjusting the current guess for r_{T-1} (upward, if banks' demand for federal funds is positive; downward, if negative), until the guessed value for r_{T-1} does not change significantly between iterations. Given $r_{T-1}(\tilde{a}_{T-1})$, the Fed's targeting problem for day *T*-1 is then solved, conditional on a_{T-2} and η_{T-2}, by calculating the value of m_{T-1} for which $\mathrm{E}_{T-1}\left[r_{T-1}(\tilde{a}_{T-1})\right] = r_{T-1}^*$. The functional $m_{T-1}^*(a_{T-2}, \eta_{T-2}, \Delta r^*)$ is then discretized over the two-dimensional grid for $\{a_{T-2}, \eta_{T-2}\}$, and so on--iteratively--for $t = T-2, T-3, .., 1$.

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Table 1. Timing of the Model in Day *t*

- The Fed injects or withdraws m_t to target r_t^*
- A first liquidity shock ν_t is realized
- Given mid-day average reserve balances $\tilde{a}_{it}$, banks borrow b_{it} overnight at a rate r_t
- A second liquidity shock ϵ_t is realized
- End-of-day average reserve balances a_{it} are computed
- If $t = T$, penalties on reserve deficiencies are imposed at the rate $\bar{r}$

Table 2a. EGARCH(1,1) maximum likelihood estimates with Student-t error distribution
(standard errors in parenthesis)

	mean **parameters**	***variance*** **parameters (as difference from Day 1 estimated coefficient)**		
	full sample	pre-1994 (no FOMC periods) sub-sample	post-1994 (no FOMC periods) sub-sample	FOMC periods sub-sample
	Day-of-the-Maintenance-Period Effects			
First Thursday	0.021 (0.007)	-3.950 (0.287)	-3.765 (0.379)	-3.765 (0.324)
First Friday	-0.057 (0.005)	-0.138 (0.230)	-0.759 (0.327)	-0.354 (0.291)
First Monday	0.058 (0.005)	-1.277 (0.300)	-1.926 (0.378)	-1.297 (0.334)
First Tuesday	-0.051 (0.004)	-1.048 (0.244)	-1.133 (0.354)	-1.062 (0.295)
First Wednesday	-0.031 (0.004)	-1.118 (0.233)	-0.931 (0.373)	-0.787 (0.299)
Second Thursday	0.014 (0.004)	-0.508 (0.243)	-1.488 (0.384)	-0.699 (0.305)
Second Friday	-0.040 (0.004)	-0.671 (0.245)	-1.389 (0.391)	-0.543 (0.316)
Second Monday	0.081 (0.006)	0.424 (0.310)	-1.830 (0.415)	-0.794 (0.351)
Second Tuesday	-0.060 (0.007)	0.413 (0.236)	-0.011 (0.364)	0.095 (0.295)
Second Wednesday	0.143 (0.018)	2.463 (0.203)	1.707 (0.308)	2.311 (0.227)

Table 2b. EGARCH(1,1) maximum likelihood estimates with Student-t error distribution
(standard errors in parenthesis)

	***mean* parameters**	***variance* parameters**
	End-of-year Effects	
t is the last day of the year	-0.223 (0.076)	
t is the end-of-year, one of the previous two days, or one of the following two days		2.865 (0.333)
	End-of-quarter 1, 2, or 3 Effects	
t is the last day of quarter	0.172 (0.038)	
t is one day after the end of the quarter	-0.180 (0.042)	
t is the end-of-quarter, the previous day, or the following day		2.219 (0.192)
	Holiday Effects	
t precedes a 1-day holiday	-0.029 (0.017)	
t precedes a 3-day holiday	-0.020 (0.008)	
t follows a 1-day holiday	0.056 (0.018)	
t follows a 3-day holiday	0.203 (0.013)	
fraction by which each previous nontrading day increases the variance of day t		0.694 (0.209)

Table 2c. EGARCH(1,1) maximum likelihood estimates with Student-t error distribution
(standard errors in parenthesis)

	mean **parameters**	***variance*** **parameters**
	Other Calendar Effects	
t is before 1/1/1988		0.276 (0.156)
t is between 1/10/1991 and 2/6/1991		3.641 (0.635)
	Other Effects	
fraction of day 10 change reversed in day 1	-0.831 (0.021)	
fraction of day 9 change reversed in day 1	-0.542 (0.028)	
day t change when target rate is changed by 1 on the same day	0.428 (0.051)	
t is the day target changes		0.821 (0.262)
effect of (Penalty-Target) over Penalty		1.478 (0.464)
	Exponential GARCH Effects	
effect of t-1 log variance on today's log variance		0.617 (0.041)
effect of *absolute* value of t-1 innovation on today's log variance		0.687 (0.069)
effect of *positive* t-1 innovation on today's log variance		0.254 (0.042)
	Tail index of Student-t distribution	
degrees of freedom of Student t-distribution		3.022 (0.221)

FIGURE 1: No official intervention (normally distributed shocks)

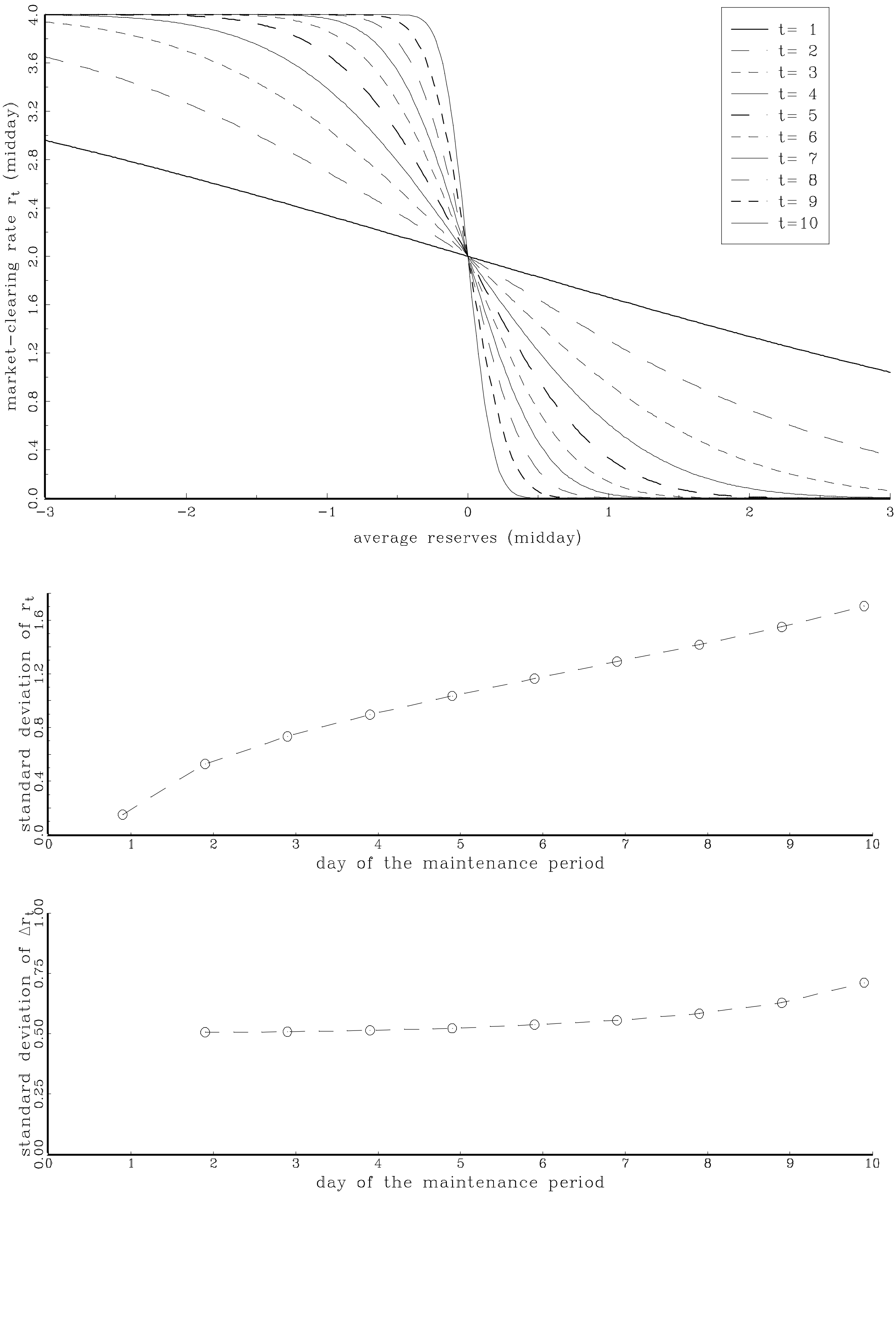

FIGURE 2: Fed intervention to enforce target

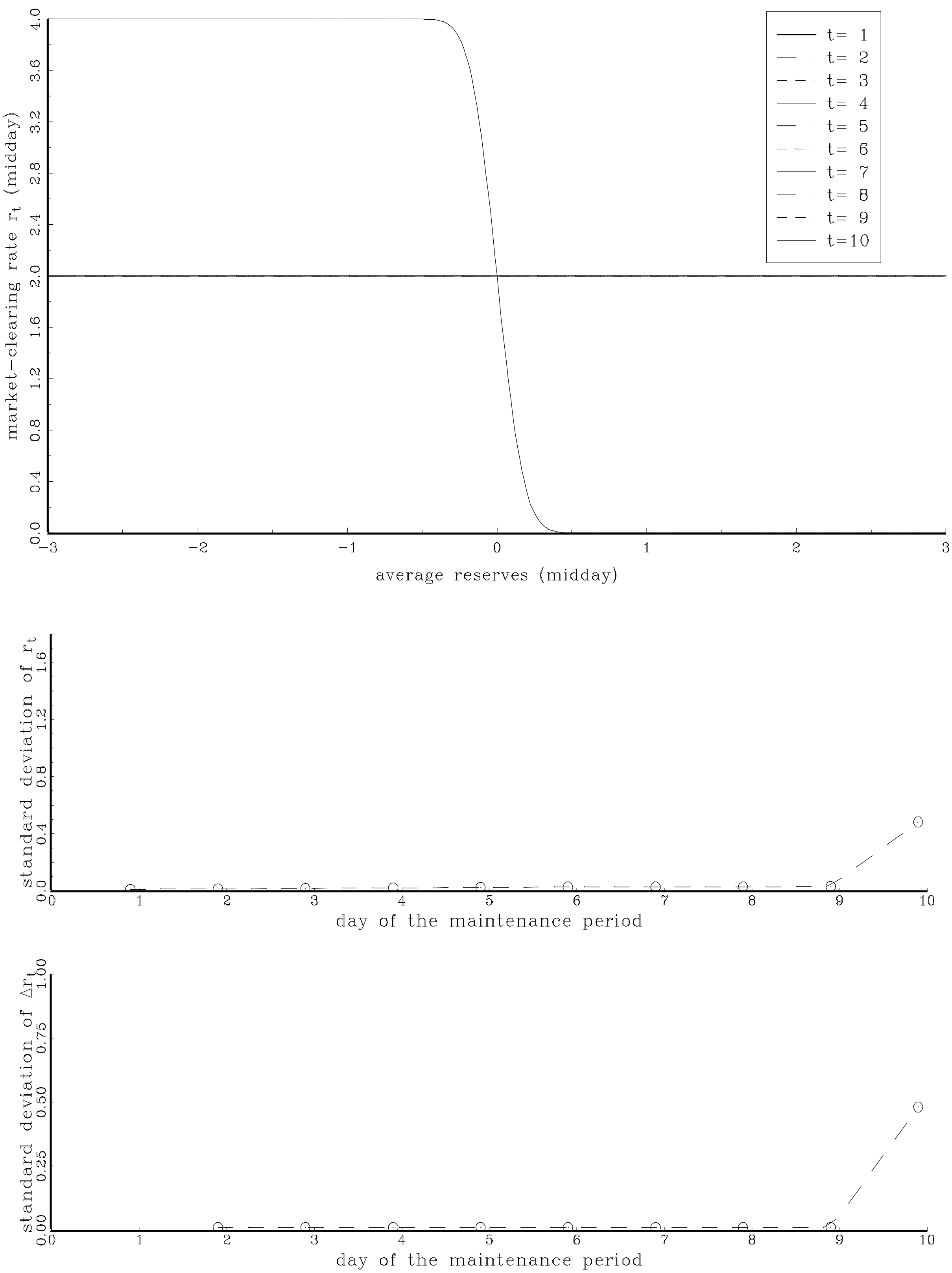

FIGURE 3: Daily changes of the Federal Funds rate:
Volatility measures by day of the maintenance period

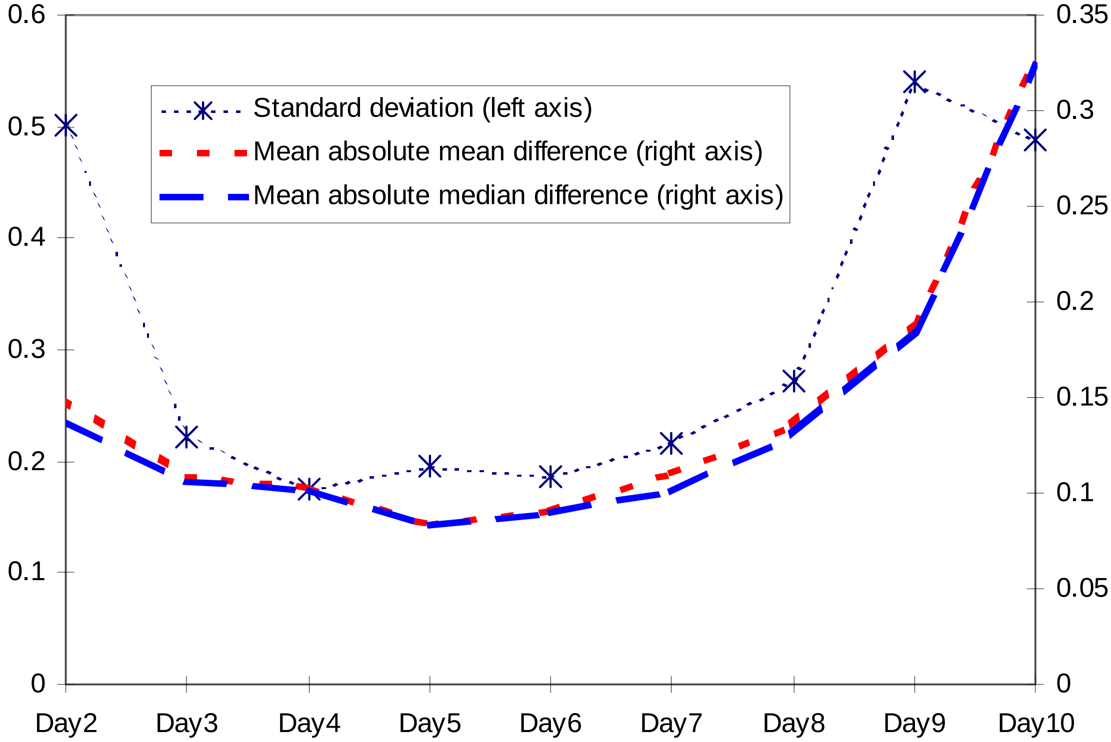

FIGURE 4: Official intervention with RP limits

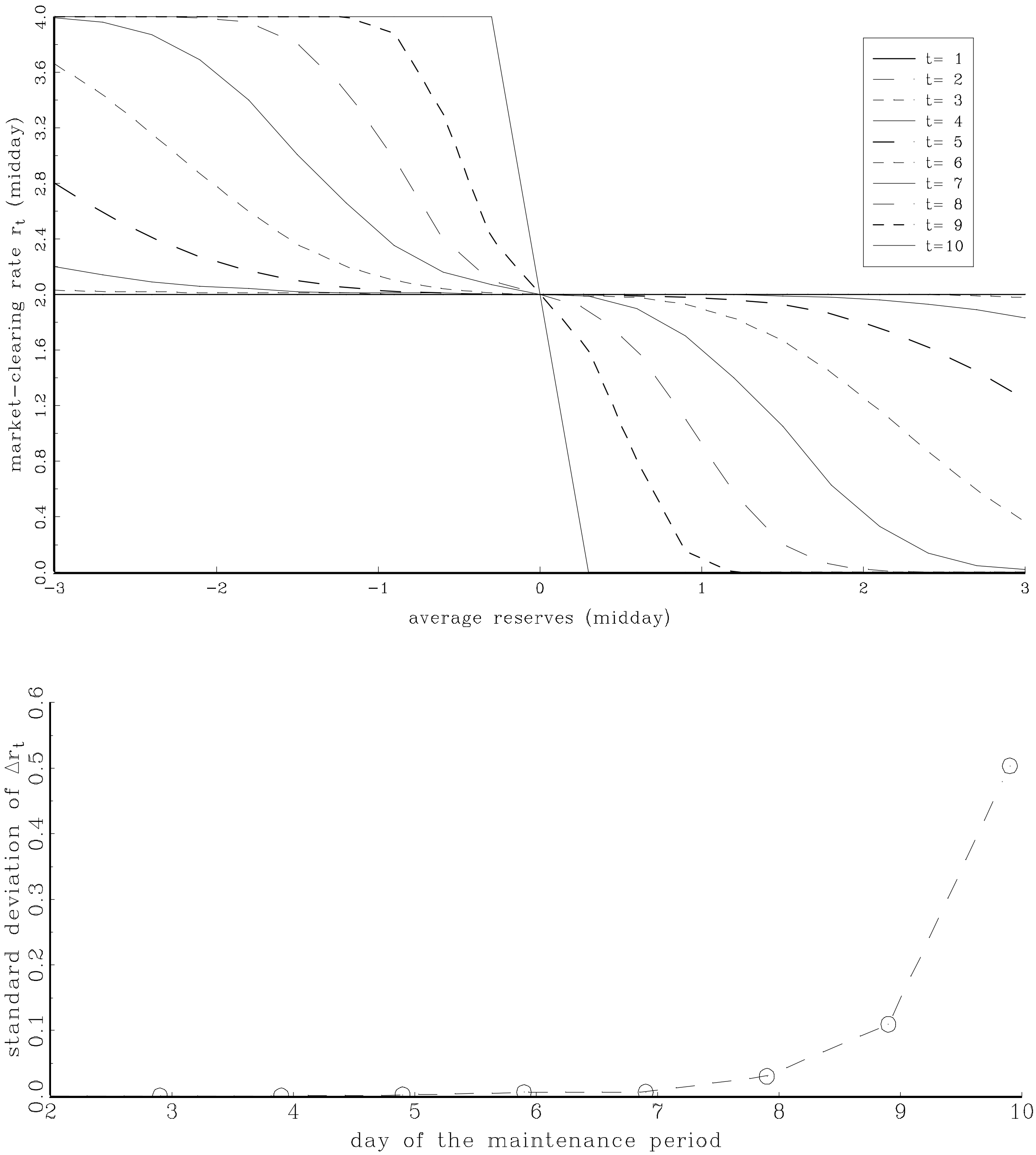

FIGURE 5: Different RP limits on official intervention

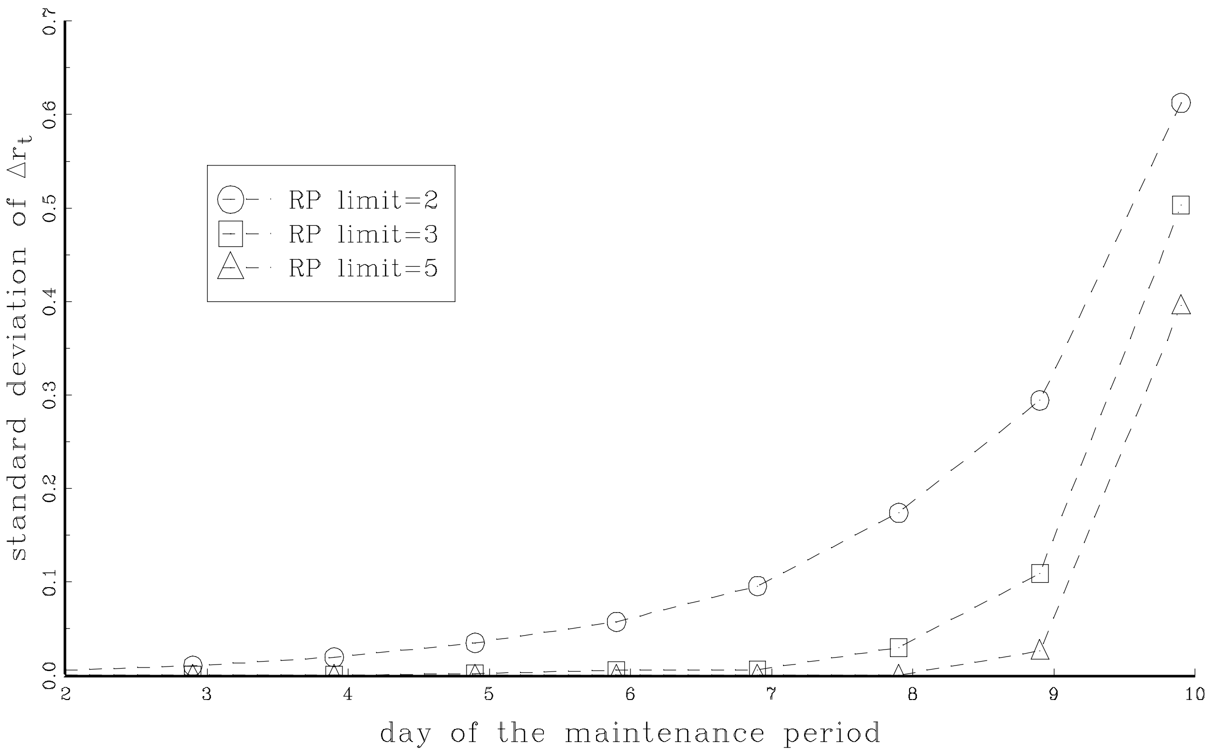

FIGURE 6: Effect of higher target rate

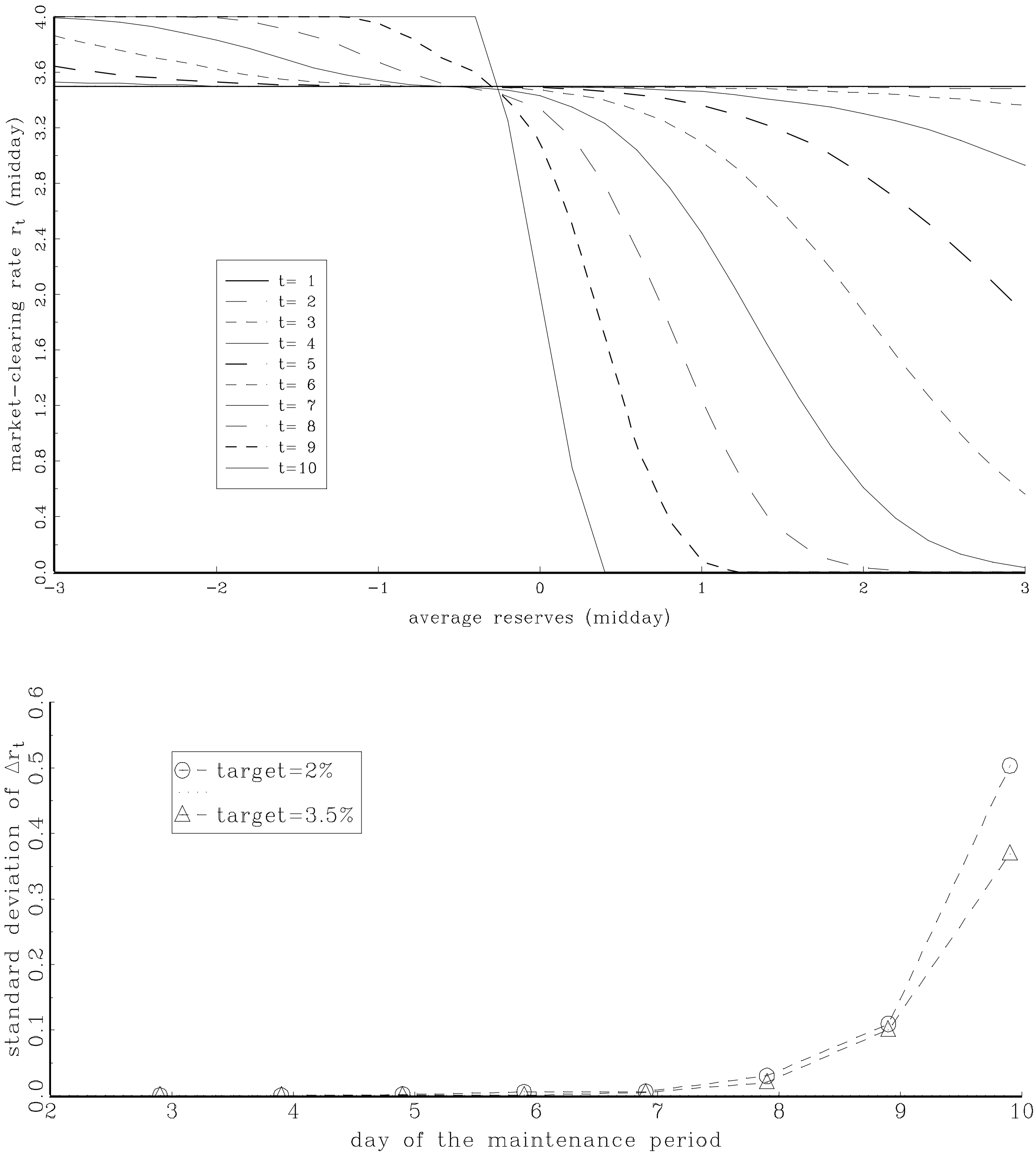

FIGURE 7: Different parameterizations of the target-change process

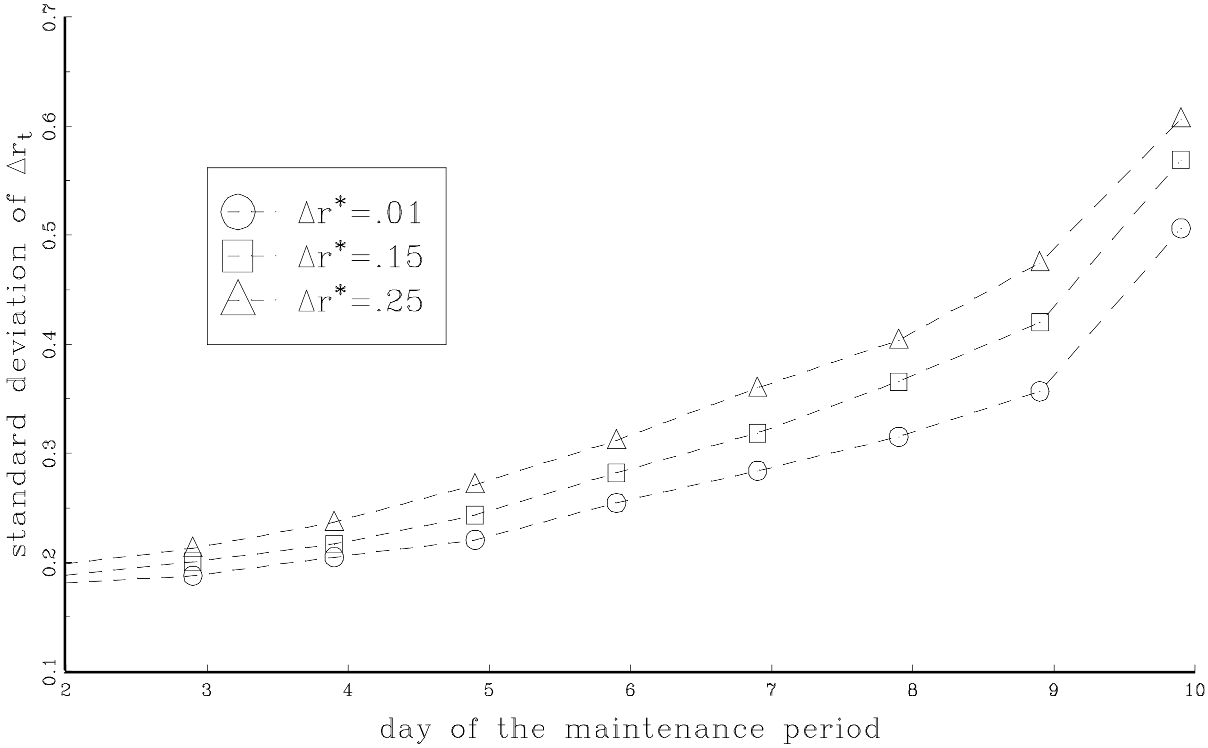

Figure 8: EGARCH estimates of the standard deviation of the federal funds rate (expressed as a ratio of estimated Day 1 standard deviation)