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Macroeconomics of Distribution and Growth

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Abstract

This Chapter reviews various interactions between the distribution of income across individuals and factors of production on the one hand, and aggregate savings, investment, and macroeconomic growth on the other. Tractable models necessarily focus on specific causal channels within this complex web of interactions, and the survey is organized around a few relevant methodological insights. In a "Neoclassical" economy where all intra- and intertemporal markets exist and clear competitively, all distributional issues should be resolved before market interactions address the economic problem of allocating scarce resources efficiently, and the dynamics of income and consumption distribution have no welfare implications. Other models, recognizing that market interactions need not maximize a hypothetical representative individual's welfare, let accumulated and non-accumulated factors of production be owned by individuals with exogenously or endogenously different saving propensities, and feature interactions between the personal and functional distribution of resources and macroeconomic accumulation. Further, rates of return to savings and investments are generally heterogeneous when they are only partially (if at all) interconnected by financial markets, as is the case in overlapping generation economies, in models with binding self-financing constraints, and in models where financial market imperfections let individual consumption flows be affected by idiosyncratic uncertainty. The Chapter also reviews models where distributional tensions, far from being resolved ex ante, work their way through distortionary policies and market interactions to bear directly on both macroeconomic dynamics and income distribution. Finally, it relates theoretical insights to recent empirical work on cross-country growth dynamics and on relationships between within-country inequality and macroeconomic performance.

1 Introduction

"Macroeconomics" and "distribution" are a somewhat odd couple of words, almost an oxymoron in some contexts. While macroeconomists find it convenient to characterize economic behavior in terms of a single "representative" agent's microeconomic problems, it is only too easy to point out that relationships among aggregate variables are much more complex when individuals' objectives and/or economic circumstances are heterogeneous, as must be the case if distribution is an issue. Any attempt to model economies inhabited by millions of intrinsically different individuals would of course find it impossible to obtain results of any generality. Hence, the distinguished strands of literature that do study interactions between macroeconomic phenomena and distributional issues need to restrict appropriately the extent and character of cross-sectional heterogeneity, trading some loss of microeconomic detail for macroeconomic tractability and insights.

This chapter reviews various interactions between the distribution of income across individuals and factors of production on the one hand, and aggregate savings, investment, and macroeconomic growth on the other. It would be impossible to cover exhaustively these and related aspects of the literature here. Many insightful reviews of the subject are already available, ranging from Hahn and Matthews (1964), through the contributions collected in Asimakopoulos (1988), to the surveys of recent research offered by Bénabou (1996c) and by the papers in the January 1997 special issue of the *Journal of Economic Dynamics and Control*. In fact, distributional issues are so complex and so central to economic theories of accumulation and value determination as to call for book-length treatments (such as Roemer, 1981, Marglin, 1984, and Kurz and Salvadori, 1995 among the most recent). This necessarily limited survey makes only passing references to such deeper issues and mainly focuses on methodological aspects, with the aim of highlighting how appropriate modeling strategies make it possible to study macroeconomic dynamics without abstracting from distributional issues.

1.1 Overview

At both the aggregate and individual levels, income dynamics depend endogenously on the propensity to save rather than consume currently available resources and on the rate at which accumulation is rewarded by the economic system. In turn, the distribution of resources across individuals and across accumulated and non-accumulated factors of production may determine both the volume and the productivity of savings and investment. Tractable models necessarily focus on specific causal channels within this complex web of interactions. The survey is organized around a few such methodological insights.

The models reviewed by Section 2 study, under suitable functional form assumptions, the interaction of macroeconomic accumulation with the distribution of income, consumption, and wealth distribution when savings are invested in an integrated market. When all intra- and intertemporal markets exist and clear competitively—i.e., when the economy is "Neoclassical" for short—then savings are rewarded on the basis of their marginal productivity in a well-defined aggregate production function. In that setting, however, all distributional issues are resolved before market interactions even begin to address the economic problem of allocating scarce resources efficiently, and the dynamics of income and consumption distribution have no welfare implications.

In earlier and more recent models, by contrast, the functional distribution of aggregate income is less closely tied to efficiency considerations, and is quite relevant to both personal income distribution and aggregate accumulation. Section 2.2.1 outlines interactions between distribution and macroeconomic accumulation when accumulated and non-accumulated factors are owned by classes of individuals with different saving propensities. Not only Classical and Post-Keynesian contributions, but also many recent models of endogenous growth let factor rewards be determined by more complex mechanisms than simple allocative efficiency. If factor rewards result from imperfect market interactions and/or policy interventions, aggregate accumulation need not maximize a hypothetical representative agent's welfare even when it is driven by individually optimal saving decisions. It is then natural to explore the implications of factor-income distribution for personal income distribution and for macroeconomic outcomes. The discussion of such models in

Section 2.2.2 offers simple insights in balanced-growth situations, where factor shares are immediately relevant to the speed of economic growth and, through factor ownership, to the distribution of income and consumption across individuals.

The models reviewed in Section 3 recognize that rates of return to savings and investments are generally heterogeneous when they are only partially (if at all) interconnected by financial markets in potentially integrated macroeconomies. Under certainty, the scope of financial markets is limited by finite planning horizons in the overlapping-generations models considered in Section 3.1, and by self-financing constraints in models discussed in Section 3.2. If the rate of return on individual investment is inversely related to wealth levels, then inequality tends to disappear over time and reduces the efficiency of investment. If instead the large investments made by rich self-financing individuals have relatively high rates of return, then inequality persists and widens as a subset of individuals cannot escape poverty traps, and unequal wealth distributions are associated with higher aggregate returns to investment.

Section 3.3 reviews how idiosyncratic uncertainty may affect the dynamics of income distribution and of aggregate income. A complete set of competitive financial markets would again make it straightforward to study aggregate dynamics on a representative-individual basis, and deny any macroeconomic relevance to resource distribution across agents. While financial markets can be perfect and complete in only one way, they can and do fall short of that ideal in many different ways. In the models considered by Section 3.3.1, returns to individual investment are subject to idiosyncratic uncertainty which might, but need not, be eliminated by pooling risk in an integrated financial market. Section 3.3.2 discusses the impact of financial market imperfection for savings, growth, and distribution in the complementary polar case where all individual asset portfolios yield the same constant return, but non-accumulated income and consumption flows are subject to uninsurable shocks and lead individuals to engage in precautionary savings.

Once theoretical mechanisms are identified that link distribution to growth and growth to distribution, it is natural to study how an economy's characteristics may endogenously determine both distribution and growth. Section 4 reviews models where distributional tensions, far from being resolved *ex ante*, work their way through distortionary policies and market interactions to bear directly on both macroeconomic dynamics and income distribution. Section 5 reviews empirical work on the cross-sectional dynamics of different countries' aggregate production and income, and on the relationship between within-country inequality, savings, and growth. A brief final section mentions related issues, neglected here for lack of space.

1.2 Preliminaries

The models reviewed below can be organized around a simple accounting framework. At the level of an individual (or a family), a discrete time dynamic budget constraint reads

$$\Delta k = y - c, \text{ or } \Delta k = rk + wl - c, \tag{1}$$

where c denotes a period's consumption flow, and the contemporaneous income flow y accrues from l units of non-accumulated factors of production, each rewarded at rate w, and k units of accumulated factors ('wealth'), each yielding r units of income.[1] As in most of the relevant literature, the factor l will be dubbed 'labor' in what follows, and its level and dynamics (if any) will be treated as exogenous in all models reviewed below to better focus on the role of the accumulated factor k. This might include human capital and knowledge as well as physical and financial assets and evolves endogenously, as in (1), on the basis of individual decisions to save rather than consume a portion of income.

Any or all of the variables in (1) may bear a time index, and may be random in models with uncertainty. To address distributional issues, it is of course necessary to let consumption, income, and their determinants be heterogeneous across individuals. Heterogeneous income and consumption levels may in general reflect different (k^i, l^i) basket of factors owned by individuals indexed by i, and/or different reward rates r^i

[1] In (1) the reward rates r and w associate an income flow over discrete time periods to the factor stocks owned at a point in time; wealth, capital, income, and consumption are measured in the same units at all points in time. Continuous-time specifications more often yield elegant closed-form solutions, and avoid the need to specify whether stocks are measured at the beginning or the end of each period. Empirical aspects and the role of uncertainty, however, are discussed more easily in a discrete-time framework.

and w^i for factors which, while measured in similar units, are owned by different individuals.

To address macroeconomic issues, let upper case letters denote the aggregate counterpart of the corresponding lower-case letter, so that for example

$$Y \equiv \int_{\mathcal{N}} y^i dP(i), \tag{2}$$

where $\mathcal{N}$ is the set of individuals in the aggregate economy of interest and $P(\cdot)$, with $\int_{\mathcal{N}} dP(i) = 1$, assigns weights to subsets of $\mathcal{N}$.[2] The set $\mathcal{N}$ may not be fixed, but its variation over time need not be made explicit unless population growth, finite lives, or immigration have a role in the phenomena of interest.

Heterogeneity of the non-accumulated income flow wl may be accounted for by differences in w and/or l across individuals. To better focus on endogenous accumulation dynamics, the models reviewed below take l as exogenously given. Hence, little generality is lost by treating it as a homogeneous factor, and

$$L = \int_{\mathcal{N}} l^i dP(i) \tag{3}$$

denotes the amount of non-accumulated factors available to the aggregate economy. Since the relative price of c and Δk is unitary in (1), aggregating wealth as in

$$K \equiv \int_{\mathcal{N}} k^i dP(i) \tag{4}$$

measures the aggregate stock K in terms of foregone consumption. The definitions in (2), (3), and (4) readily yield a standard aggregate counterpart of (1),

$$\Delta K = RK + WL - C = Y - C. \tag{5}$$

Two points of interpretation deserve to be noted as to the relationship between the definition of K and its economic interpretation as "aggregate

[2] If $\mathcal{N}$ has n elements, then the weight function $P(i) = 1/n$ defines Y as the arithmetic mean of individual income levels y. The general notation in the text, where the relative size or weight $P(A)$ of a set $A \subset \mathcal{N}$ of individuals can be arbitrarily small, conveniently lets the idiosyncratic uncertainty introduced in Section 3.3 average to zero in the aggregate.

capital." First, as the individual-level budget constraint (1) features net income flows, so does (5), hence the aggregate Y flow is obtained subtracting capital depreciation from every period's gross output flow. Second, it may or may not be possible to give a meaningful economic interpretation to R, the aggregate rate of return on past accumulation. In the models discussed in Section 2 below, all units of each factor are rewarded at the same rate; then, $r = R$, $w = W$, and income distribution straightforwardly depends on factor-ownership patterns and on R and W themselves. In the more complex and realistic models reviewed in Section 3, however, unit factor incomes are heterogeneous across individuals. At the same time as it introduces additional channels of interaction between distribution and macroeconomic dynamics, such heterogeneity also makes it difficult to give an economic interpretation to aggregate factor supplies and remuneration rates. As a matter of accounting, the conventions introduced in (1-5) define R and W as weighted (by factor ownership) averages of their heterogeneous microeconomic counterparts,

$$R = \int_{\mathcal{N}} r^i \frac{k^i}{K} dP(i), \qquad W = \int_{\mathcal{N}} w^i \frac{l^i}{L} dP(i). \tag{6}$$

These aggregate factor prices, however, are ambiguously related to both income distribution and macroeconomic developments. In particular, the "capital stock" K as defined in (4) may not be the argument of an aggregate production function when not only the economy's aggregate wealth, but also its distribution influence the size of the aggregate production flow, as is the case in the models considered by Section 3 below.

2 Aggregate accumulation and distribution

The models reviewed in this Section assume away all uncertainty and rely on economy-wide factor markets to ensure that all units of k and l are always rewarded at the same rate. This relatively simple setting isolates a specific set of interactions between factor remuneration and aggregate dynamics on the one hand, which depend on each other through well-defined production and savings functions; and personal income distribution on the other hand, which is readily determined by the remuneration of aggregate factor stocks and by the size and composition of individual factor bundles.

2.1 Neoclassical allocation and accumulation

Models where income distribution is determined in complete and competitive markets are familiar to most readers, and their benign neglect of distributional issues prepares the ground for more articulate models below. If firms employ units of accumulated and non accumulated factors in concave production functions $f(\cdot,\cdot)$, and take as given the prices at which factors can be rented from families, then the first order conditions

$$\frac{\partial f(k,l)}{\partial k} = r, \quad \frac{\partial f(k,l)}{\partial l} = w \tag{7}$$

are necessary and sufficient for profit maximization. The factor prices r and w might in general depend on the identity of the agents concerned, and the production function $f(\cdot)$ could itself be heterogeneous across firms. If at least one of the factors can be allocated across firms so as to arbitrage marginal productivity differentials, however, all units of each factor must be rewarded at the same rate, hence $w = W$ and $r = R$. Since the two factors' marginal productivities are equal in all of their possible uses, the equilibrium allocation maximizes the aggregate production flow obtained from a given stock of the two factors, and defines an aggregate production as a function $F(K,L)$ of aggregate capital and labor. In the general case where firms' technologies are heterogeneous, the form of the aggregate production function depends on that of firm-level production functions and on the distribution of fixed factors across firms. If all firm-level functions have constant returns to scale, however, so does the aggregate production function, and aggregate factor-income flows coincide with total net output, since

$$F(K,L) = \frac{\partial F(K,L)}{\partial L}L + \frac{\partial F(K,L)}{\partial K}K = WL + RK \tag{8}$$

for $F(K,L)$ a linearly homogeneous function.[3] Decreasing returns to scale at the firm level can be accomodated by including fixed factors in

[3] By the accounting conventions of Section 1.2, both firm-level and aggregate production functions are defined net of capital depreciation. This has no implications for the simple argument above, since the net production function is concave and has constant returns to scale if the gross production function does and, as is commonly assumed, a fixed portion of capital in use depreciates within each period.

the list of (potentially) variable factors, and the rents accruing to them in aggregate income.

The level and dynamics of income flows accruing to each individual or family indexed by i are determined by the amounts k^i and l^i of factors it brings to the market, and by their prices R and W . For this survey's purposes, it will be convenient to treat each l^i as an exogenously given and (for simplicity) constant quantity.[4] A constant-returns production structure and competitive markets make it possible to disaggregate income not only across individuals, but also across factors:

$$Y = F(K, L) = \int_{\mathcal{N}} y^i dP(i) = R \int_{\mathcal{N}} k^i dP(i) + W \int_{\mathcal{N}} l^i dP(i). \quad (9)$$

2.1.1 Saving propensities and the dynamics of distribution.

In the macroeconomic accumulation relationship

$$\Delta K = F(K, L) - C = \int_{\mathcal{N}} \left(y^i - c^i\right) dP(i), \quad (10)$$

the personal distribution of income is directly relevant to aggregate savings if individual consumption depends nonlinearly on individual income and/or wealth. If, for example, poorer individuals have a higher marginal propensity to consume than richer ones, then more equal income distributions are associated with a smaller ΔK and slower aggregate accumulation.[5] The potential relevance of consumption-function nonlinearities is obvious, but hard to make precise in the absence of precise theoretical foundations. It is insightful to focus for the moment on the case where (10) *can* be written in terms of aggregate capital and income

[4]In more general models where the non-accumulated factor is identified with labor, its individual and aggregate supply should depend endogenously on current and expected wage rates, on financial wealth, and on the structure of preferences. For a discussion of how models of labor/leisure choices may yield analytically convenient and realistic aggregate models under appropriate simplifying assumptions, see Rebelo (1991), and his references.

[5]Stiglitz (1969) discusses the implications of nonlinear consumption functions. The empirical relevance of the idea that a more equal distribution of permanent income should be associated with higher aggregate consumption is explored by Blinder (1974). Its theoretical implications are studied in more detail by Bourguignon (1981).

only, regardless of how the same variables are distributed across individuals. If at the individual level savings $y^i - c^i$ depend linearly on y^i and k^i, i.e. if

$$c^i = \bar{c} + \hat{c}\, y^i + \tilde{c} k^i \tag{11}$$

where the $\bar{c}$, $\hat{c}$, and $\tilde{c}$ are constant parameters, then

$$\Delta K = (1 - \hat{c})Y - \tilde{c}\, K - \bar{c}, \tag{12}$$

at the aggregate level, regardless of income and wealth heterogeneity.

While a linear consumption function in the form (11) lets aggregate savings be independent of distribution, the converse need not be true. The evolution over time of individual income and wealth depends endogenously on the parameters of individual savings functions, on the character of market interactions, and on the resulting aggregate accumulation dynamics. Following Stiglitz (1969), consider the implications of (11) for the dynamics of income and wealth distribution dynamics in a neoclassical economy where factor markets assign the same income to all units of each factor and all individuals own the same amount $\bar{l} = L$ of the non-accumulated factor, so that all income and consumption inequality is due to heterogeneous wealth levels.[6] Using (11) in (1), the dynamics of individual i's wealth obey

$$\Delta k^i = (1 - \hat{c})y^i - \tilde{c}k^i - \bar{c} = (1 - \hat{c})\left(Rk^i + W\bar{l}\right) - \tilde{c}k^i - \bar{c}. \tag{13}$$

In an economy where R, W, and $\bar{l}$ are the same for all individuals, the individual wealth level k is the only possible source of income and consumption heterogeneity. Suppressing the i index on k^i and normalizing (13) by individual wealth k, the dynamic evolution of any such heterogeneity is driven by wealth accumulation according to

$$\frac{\Delta k}{k} = (1 - \hat{c})R - \tilde{c} + \frac{(1 - \hat{c})W\bar{l} - \bar{c}}{k}.$$

Heterogeneity tends to be eliminated and the economy's distribution converges towards equality if higher levels of wealth (and income and consumption) are associated to slower rates of accumulation, or if

$$(1 - \hat{c})W\bar{l} > \bar{c}. \tag{14}$$

[6] If l^i is permanently different across i, wealth accumulation tends to different asymptotic values and reinforces income inequality. Stiglitz (1969, Section 6) discusses such phenomena in the case where $\bar{c}$ is still the same for all individuals.

If $\hat{c} = 1$ and $\bar{c} = 0$, both terms in (14) are identically zero, and wealth inequality remains forever constant in proportional terms. In this case, as in the models discussed in Section 2.2.1, all wage income is consumed, while savings are positive (as long as $\tilde{c} < 0$) and proportional to wealth. Condition (14) has straightforward implications for other familiar macroeconomic models which do rely on special cases of (11), i.e., on easily aggregated linear consumption functions. The textbook Solow (1956a) growth model assumes that savings are a constant fraction of income flows: with $\bar{c} = 0$ and $\hat{c} < 1$, condition (14) is satisfied as long as $W\bar{l} > 0$. Thus, a constant average savings propensity unambiguously tends to equalize wealth, income, and consumption across individuals.

The tendency towards equality would be even stronger if $\bar{c} < 0$, i.e., if the average savings rate was higher for poorer individuals, but it may be more appealing to assume that richer agents have a higher average propensity to save or bequeath wealth, i.e., that $\bar{c} > 0$ in (11), as in textbook Keynesian macroeconomic models.[7] If $\bar{c}$ is so large as to violate the inequality in (14), wealthier agents save a larger proportion of their income and, for given R and W, wealth inequality tends to increase over time. In a neoclassical economy, however, factor prices depend endogenously on aggregate accumulation. The rate of return $R = \partial F(K, L)/\partial K$ is a decreasing function of K and, as shown by Stiglitz (1969), this exerts a further equalizing force. If the aggregate economy converges to a stable equilibrium, in fact, net returns to accumulation tend to vanish, and the distributional impact of heterogeneous wealth levels and saving rates weakens over time. Formally, aggregate accumulation obeys

$$\Delta K = Y - C = (1 - \hat{c})F(K, L) - \tilde{c}K - \bar{c}, \qquad (15)$$

and the aggregate economy approaches a stable steady state where $\Delta K = 0$ only if a larger stock of capital is associated with a smaller (and possibly negative) rate of accumulation in its neighborhood.[8] Using the stability

[7] A positive correlation between income levels and savings propensity can be rationalized in that and other contexts by consumption smoothing in the face of income fluctuations (see also Section 3.3.2 below). In a long-run framework of analysis, however, cross-sectional relationships between savings rates and income levels are not as easy to document by hard evidence as introspection and casual empiricism might indicate (see, e.g., Williamson 1991, p.71, and his references).

[8] When $\bar{c} > 0$ then the economy also features an unstable steady state, where

condition $\partial(\Delta K)/\partial K < 0$ in (15) yields

$$(1-\hat{c})\frac{\partial F(L,K)}{\partial K} - \tilde{c} < 0,$$

thus $(1-\hat{c})R - \tilde{c} < 0$ in the neighborhood of a stable steady state where $(1-\hat{c})\left(RK + W\bar{l}\right) - \tilde{c}K - \bar{c} = 0$. Since these two relationships imply (14), aggregate convergence to a stable steady state is accompanied by cross-sectional convergence of individual wealth levels.[9]

2.1.2 Optimal savings and the distribution of consumption.

The *ad hoc* consumption functions considered above usefully highlight mechanic interactions between distribution and macroeconomic growth. For the purpose of assessing welfare implications, however, deeper determinants of savings behavior must be taken into account. Accordingly, let individuals aim at maximizing a standard objective function in the form

$$V(\{c_{t+s}\}) = \sum_{s=0}^{T}\left(\frac{1}{1+\rho}\right)^{s} U(c_{t+s}), \tag{16}$$

and let the time horizon T (which may be infinite), the rate of time preference $\rho \geq 0$, and the increasing and concave period utility function $U(\cdot)$ be the same across individuals. Since savings yield the rate of return R_{t+1} between periods t and $t+1$, the optimal consumption path satisfies

production is absorbed by consumption even though returns to accumulation are high. Stiglitz (1969) notes that growth is associated with increasing inequality in the neighborhood of such a steady state, and that relatively poor individuals may decumulate wealth indefinitely (or, more realistically, until the budget constraint that is left implicit by ad hoc consumption functions becomes binding) even as the aggregate economy converges to its stable steady state. Bourguignon (1981) discusses the resulting "inegalitarian" steady state where a destitute class has no wealth.

[9] The stability condition is satisfied if the net rate of return R is negative and/or the propensity to consume out of wealth $\tilde{c}$ is positive. The net income flow R generated by each unit of capital can be negative if capital depreciates, and a positive $\tilde{c}$ also reflects of capital depreciation in the standard Solow growth model, where the average and marginal savings propensity is constant for *gross* rather than net income, to imply that the aggregate accumulation equation reads $\Delta K = (1-\hat{c})[Y + \delta K] - \delta K = (1-\hat{c})Y - \hat{c}\delta K$, or $\tilde{c} = \hat{c}\delta$ in the notation of (15).

first order conditions in the form

$$U'(c_t) = \frac{1+R_{t+1}}{1+\rho} U'(c_{t+1}), \tag{17}$$

for all $t < T$, and an appropriate transversality condition at the end of the planning horizon. In the above framework of analysis, aggregate savings were independent of resource distribution if, and only if, consumption is a linear function of income and wealth. In an optimizing setting, similarly straightforward aggregation obtains if preferences are "quasi-homothetic."[10] As a simple example, let the marginal utility function be

$$U'(c) = (c - \bar{c})^{-\sigma} \tag{18}$$

where $0 < \sigma$. Using (18) in (17) yields

$$c_{t+1} = (1 - \xi_{t+1})\,\bar{c} + \xi_{t+1} c_t, \text{ where } \xi_{t+1} \equiv \left(\frac{1+R_{t+1}}{1+\rho}\right)^{1/\sigma} \tag{19}$$

depends on R_{t+1} but is homogeneous across individuals who have the same utility function and discount rate, and earn the same rate of return on their savings. Dividing (19) through by each individual's consumption level,

$$\frac{c_{t+1}}{c_t} = (1 - \xi_{t+1})\,\frac{\bar{c}}{c_t} + \xi_{t+1}, \tag{20}$$

establishes that higher current consumption is associated with faster or slower consumption growth depending on whether $\bar{c} \gtreqless 0$, and on whether $\xi_{t+1} \gtreqless 1$. Since the relationship (19) between individual consumption levels in adjoining periods is linear, a similar relationship holds at the aggregate level as well:

$$C_{t+1} = (1 - \xi_{t+1})\,\bar{c} + \xi_{t+1} C_t. \tag{21}$$

Hence, aggregate consumption is steady if and only if $\xi_{t+1} = 1$, which by (20) also implies that $c_{t+1} = c_t$ (all individual consumption levels are

[10] For this class of preferences, also known as "extended Bergson" or "hyperbolic absolute risk aversion," marginal utility is proportional to a power of a linear function of consumption. Not only the constant relative risk aversion specifications considered here, but also other common specifications of utility (including constant absolute risk aversion and quadratic ones) belong to this class; see Merton (1971), Chatterjee (1994), and their references.

constant). To close the model, note that $\xi_{t+1} \gtreqless 1$ hinges on whether $R \gtreqless \rho$, recall that $R = \partial F(K, L)/\partial K$ by (7) in a neoclassical model, and consider the macroeconomic dynamics implied by (21) and (5). If the macroeconomy is growing towards the steady state (i.e., $R > \rho$) and the required consumption level $\bar{c}$ is positive, then higher consumption levels are associated with faster consumption growth in the individual condition (19), and consumption inequality increases over time.[11]

The distribution of consumption and its dynamic evolution, however, have little economic significance in the "neoclassical" setting we are considering. On the one hand, the same functional form assumptions that make it possible to characterize aggregate dynamics as in (21) imply that the speed of aggregate growth depends only on aggregate variables, not on their distribution across individuals. If preferences lend themselves nicely to aggregation, as in (18), then macroeconomic dynamics can be interpreted in terms of representative-agent savings choices even as the economy features persistent and variable heterogeneity of individual consumption paths.

On the other hand, the dynamics of consumption distribution have no substantive welfare implications: if all individuals' savings earn the same rate of return, relative welfare remains constant over time even though, as in (19), relative consumption levels may diverge or converge. To see this, consider that equations in the form (17) hold for all individuals, and take ratios of their left- and right-hand sides for different individuals: for any $i, j \in \mathcal{N}$ and all t we may write

$$\frac{U'(c_t^i)}{U'(c_t^j)} = \frac{U'(c_{t+1}^i)}{U'(c_{t+1}^j)} \equiv \frac{\omega^j}{\omega^i} \tag{22}$$

where ω^i differs from ω^j if individuals i and j enjoy different consumption flows, but neither ω^i nor ω^j depend on time. It is straightforward to show that conditions in the form (22) are necessary and sufficient for maximization of a weighted sum of individual welfare functionals in the form (16) under an aggregate resource constraint. Formally, the market allocation of the neoclassical economy under consideration solves the

[11] This result, and symmetric ones under other combinations of assumptions, are derived by Chatterjee (1994).

social planning problem

$$\begin{array}{ll} \max & \int_{\mathcal{N}} \omega^i V(\{c_t^i\}) dP(i) \\ \text{s.t.} & \int_{\mathcal{N}} c_t^i dP(i) \leq F(K_t, L) + K_t - K_{t+1}, \ \forall t \geq 0, \end{array}$$

and the dynamics of consumption inequality (if any) are just a byproduct of efficient once-and-for-all allocation of a maximized welfare "pie." In the context of the simple example above, convergence or divergence of cross-sectional consumption rates in the $\bar{c} \neq 0$ case reflects the fact that individuals who are relatively privileged in the initial allocation must remain so in all future periods. Since the elasticity of marginal utility is not constant when $\bar{c} \neq 0$, the market allocation adjusts relative consumption levels so as to keep marginal utilities aligned as in (22), and preserve the relative welfare weights of different individuals. Less tractable and more general specifications of preferences may not allow straightforward aggregation as in (21). Regardless of whether aggregate accumulation dynamics may be interpreted on a representative-individual basis, however, when intertemporal markets clear competitively then dynamic changes in the distribution of consumption and income flows across individuals have no implications for their welfare, which depends only on their initial endowment of factors of production.

2.2 Factor income distribution and growth

In a market economy, each individual's entitlement to a portion of aggregate output is based on factor ownership, and the distribution of income and consumption flows is determined by the size and composition of each individual's bundle of factors. When competitive economic interactions yield an efficient allocation, initial factor endowments and equilibrium factor rewards determine each individual's income entitlements, and the welfare weights ω^i in the equivalent social planning problem characterized above. In reality, of course, the distribution of income across *factors* of production need not always reflect efficiency considerations, and the literature has often studied it in different frameworks of analysis.

To focus on factor-income distribution in the simple two-factor setting of the derivations above, let γ denote the fraction of consumable income Y that is paid to owners of L, the non-accumulated factor of production. The remaining $(1-\gamma)$ fraction of aggregate resources is paid to owners of accumulated factors of production (i.e., past foregone consumption), and the two factors are remunerated according to

$$W = \gamma \frac{Y}{L}, \qquad R = (1-\gamma)\frac{Y}{K}, \tag{23}$$

where all quantities and the factor shares themselves may in general be variable over time. In the neoclassical economy characterized above, aggregate output Y is efficiently produced from available factors according to the production function $F(K, L)$, and W and R coincide with the aggregate marginal productivities of the two factors of production. The shares γ and $1-\gamma$ of the two factors are uniquely determined by the aggregate K/L ratio under constant returns, and are constant if the production function has the Cobb-Douglas form $Y = L^{\gamma}K^{1-\gamma}$. For this and more general convex functional forms, the reward rate R decreases over time in a growing economy where an increasing capital stock K is associated with a declining output/capital ratio Y/K for given L.

The role of factor-income distribution in determining individual savings, income distribution, and aggregate accumulation was far from explicit in the above derivations. A relationship between the factor composition of income and saving propensity is implicit in the linear specification (11): since inserting $y \equiv W\bar{l} + Rk$ in it yields

$$c = \bar{c} + \hat{c}(W\bar{l} + Rk) + \tilde{c}k = \bar{c} + \hat{c}\,W\bar{l} + (\hat{c}R + \tilde{c})\,k, \tag{24}$$

the propensity to consume out of wealth (or out of accumulated income, Rk) does generally depend on the rate of return R and differs from that relevant to non-accumulated income flows. Factor-income shares and the unit incomes they imply through (23) also have a subdued role in the determination of optimal savings. A higher rate of return R on savings makes it optimal to plan faster consumption growth, but also lets it be financed by a smaller volume of savings, and the net effect on savings depends on the balance of these substitution and income effects.[12] The complex and ambiguous role of R and W in individual and aggregate

[12]For the quasi-homothetic preferences considered above, the marginal propensity to consume available resources is independent of their level and is a decreasing function of R if $\sigma < 1$, while income effects are stronger than substitution effectis if $\sigma >$

optimal savings decisions, and the fact that factor incomes are viewed as a byproduct of efficient market allocation of available resources, lead "neoclassical" models of personal savings to pay little attention to issues of factor-income distribution.

2.2.1 Savings, accumulation, and classes

A long and distinguished stream of earlier models, by contrast, viewed factor ownership as an essential determinant of individual savings behavior.[13] The logic of such models is simply stated. As above, let individuals be entitled to portions of the economy's aggregate income flow on the basis of factor ownership and, for simplicity, let consumption and income consist of a single, homogeneous good.[14] The owner of each unit of capital receives a return R and, as in (23), each unit of the non-reproducible factor L entitles its owner to W units of income. The factor(s) denoted by L may include land and other natural resources as well as labor. It is unnecessary to disaggregate L along such lines, however, if none of the income flows accruing to non-reproducible factors are saved. Accordingly, let both land-owning "rentiers" and "workers" consume all of their income, while the propensity to save is positive for owners of reproducible factors of production, or "capitalists." Such simple income-source-based characterizations of savings behavior is natural if sharp income-source heterogeneity across hereditary class lines has stylized-fact status, as it probably did in the early 19th century, and its implications are consistent

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1. Under complete markets, consumable resources include the present discounted value of future non-accumulated income flows, or "human wealth," and factor-income distribution affects individual saving decisions through such wealth effects as well as (ambiguously) through income and substitution effects.

[13] This section draws on Bertola (1993,1994b,c). Space constraints make it impossible to survey properly a literature that spans Physiocratic tableaus, Ricardian theory, and Post-Keynesian growth models. Asimakopoulos (1988) offers a more extensive review of this material. Levine (1988) discusses Marxian theories of the distribution of surplus (the portion of net income in excess of what is necessary to reproduce the economy's capital and labor force).

[14] The models implicit in the work of Ricardo did feature multiple goods, and in particular a distinction between luxuries and basic consumption goods. The relationship of simpler Post-Keynesian single-good macromodels to Ricardian theory is discussed in, e.g., Kaldor (1956, Section I) and Pasinetti (1960, footnote 24).

with the assumed association between wealth and further accumulation: an individual can come to own more accumulated wealth than others because his past savings propensity was relatively high, and the Classical assumption amounts to a presumption that such heterogeneity (whatever its source) persists over time.[15] At the theoretical level, the assumption that workers never save any portion of their resources may be rationalized by the Classical notion of a "natural" wage rate which barely suffices to let the labor force subsist and reproduce but leaves no room for savings.[16] Once savings behavior is assumed to depend on income sources, aggregate accumulation is straightforwardly related to resource distribution. If $(1 - s^p)$ denote the portion of capitalists' income which is consumed, and capital depreciation is ruled out for simplicity, the aggregate capital stock evolves according to

$$\Delta K = s^p R K = s^p (1 - \gamma) Y, \tag{25}$$

where (23) is used to express R in terms of capital's factor share $(1 - \gamma)$. Since the savings propensity s^p is viewed as an exogenous parameter, the income share γ of non-accumulated factors of production has a crucial role in determining the economy's accumulation rate. In turn, factor shares depend on the economy's dynamic behavior through the law of diminishing returns. As more capital is accumulated, relatively scarce

[15] A preference-based class structure can also feature a non-zero propensity to save out of labor income. Pasinetti (1962) lets a relatively low (but strictly positive) savings propensity apply to both the accumulated and non-accumulated income flows accruing to individuals belonging to the working class, and shows that the exact value of their propensity to save is irrelevant in the long run as long as it is lower than the aggregate one. Samuelson and Modigliani (1966) argue that it is more realistic and insightful to attach different savings propensities to income sources rather than recipients. In his comments, Kaldor (1966) acknowledges that high savings propensity "attaches to profits as such, not to capitalists as such" (p.310).

[16] A subsistence approach to wage determinaton makes it natural for Classical theories to suppose that wage payments precede production flows. Thus, wages are a portion of the economy's working capital, and the notion of "organic" composition of capital plays an important role in Marxian studies of factor-income distribution (see Roemer, 1981, for a critical review and formal results in this field). For simplicity—and consistently with Marglin (1984), Kaldor (1956), Sraffa (1960)—the timing of wage outlays is the same as that of accumulated-factor income flows in this section's equations.

non-accumulated factors can command an increasing share of aggregate output if they are rewarded according to marginal productivity and earn inframarginal *rents* on intensive or extensive margins. A wage level higher than the "subsistence" one that would let the labor force reproduce itself may lead to faster population growth, implying that no rents accrue to labor in the long run (Pasinetti, 1960; Casarosa, 1982).[17] No such mechanism restrains the inframarginal rents paid to factors in fixed supply ("land"). As an ever larger share of the economy's resources is paid to landowners with low propensity to save, the economy tends to settle into a stationary state where capitalists' savings and investment just suffice to reproduce the existing capital stock. The capitalists' savings propensity determines not only the distribution but also the growth rate and ultimate level of aggregate income.

The idea that decreasing returns and increasing rents would prevent capitalists' savings from endlessly fueling accumulation could be acceptable to nineteenth-century economists who had not experienced prolonged periods of economic development. Later, long-run growth at approximately constant rates achieved "stylized fact" status (Kaldor, 1961), and theories of accumulation needed to account for technological progress. Given a constant stock of L (or in per-capita terms), constant proportional output growth and a constant savings rate are consistent with each other if the capital/output ratio is constant, and an index of technological efficiency enters the production function in L-augmenting fashion as in

$$F_t(K,L) = \tilde{F}(K, A_t L). \tag{26}$$

Denoting $A_{t+1}/A_t \equiv \bar{\theta}$, it is straightforward to derive the Kaldor (1956) link between growth and income distribution. If capitalists save a given portion s^p of their income flow and other agents' savings are negligible, then along a balanced growth path where $K_{t+1}/K_t = A_{t+1}/A_t = \bar{\theta}$ equation (25) implies $R = \left(\bar{\theta} - 1\right)/s^p$ and, in light of (23),

$$\bar{\theta} = 1 + s^p(1-\gamma)\frac{Y}{K}. \tag{27}$$

[17] Through Malthusian population dynamics, labor is an accumulated factor of sorts in a Classical economy: while no part of workers' income is saved in the form of capital, faster population growth when wages exceed subsistence levels does contribute to extend the economy's production possibilities. The role of endogenous population dynamics in modern growth models is surveyed by Nerlove and Raut (1997).

This tight relationship between the propensity to save of capitalists, the economy's rate of balanced growth, and aggregate share of accumulated factors of production is given a precise causal interpretation by post-Keynesian theorists. If not only $\bar{\theta}$, but also the output/capital ratio Y/K is viewed as an exogenously given parameter, the capitalists' savings propensity s^p is consistent with only one value of γ, and hence determines the unique balanced-growth configuration of income distribution.

2.2.2 Factor shares in long-run growth

The neoclassical approach to the same issues lets factor incomes be determined by efficient market interactions and, as noted above, has looser implications for the relationship between factor income distribution and aggregate accumulation. Decreasing returns to accumulation, however, still play a role. Competitive determination of factor incomes is well defined only if the neoclassical aggregate production function has constant returns to K and L together. This implies that accumulation of K encounters *decreasing returns* if it is not accompanied by a proportional increase in L, and makes it hard for savings to sustain long-run growth even when they accrue from all income rather than from profits only. In fact, the growth rate of total production can be written

$$\frac{\Delta Y}{Y} \approx \frac{\partial F(K,L)}{\partial K}\frac{\Delta K}{Y} + \frac{\partial F(K,L)L}{\partial L}\frac{1}{Y}\frac{\Delta L}{L} \tag{28}$$

by a Taylor approximation (which would be exact in continuous time). On the right-hand side of (28), the net savings rate $\Delta K/Y$ is multiplied by the marginal productivity of capital, $\partial F(K,L)/\partial K$, which under constant returns is a function of the capital/labor ratio only, and decreasing in it. In the canonical Cobb-Douglas specification, for example, $F(K,L) = K^{\gamma}L^{1-\gamma}$ with $\gamma < 1$, implies $\partial F(K,L)/\partial K = \gamma\,(K/L)^{\gamma-1}$. Thus, as capital intensity increases the economy's savings propensity is an ever less important driving force of growth. If, as in the Cobb-Douglas case,

$$\lim_{K\to\infty}\frac{\partial F(K,L)}{\partial K} = \lim_{K\to\infty}\frac{\partial F(K/L,1)}{\partial\,(K/L)} \leq 0 \tag{29}$$

(with strict inquality if capital depreciates), then only the second term on the right-hand side of (28)—i.e., the competitive share of L times its proportional growth rate—remains relevant as capital intensity increases. Hence, the economy tends to settle in a steady state where aggregate income growth depends on the exogenous growth $\Delta L/L$ of non-accumulated factors in (28), and is accompanied rather than generated by endogenous factor accumulation.

Even under constant returns, the limit in (29) may be strictly positive. If, for example, $F(K,L) = K^\alpha L^{1-\alpha} + BK$ with $0 < \alpha < 1$, then

$$\frac{\partial F(K,L)}{\partial K} = \alpha \left(\frac{K}{L}\right)^{1-\alpha} + B \tag{30}$$

tends to $A > 0$ and, if this limit is larger than the rate of time preference, the economy features unceasing accumulation driven growth, as was already recognized by Solow (1956a) and recently modeled by Jones and Manuelli (1990). In a single-sector model where returns to scale are constant in the aggregate and returns to accumulation are bounded away from zero, however, non-accumulated factors of production earn a vanishing share of aggregate production if they are rewarded at marginal-productivity rates. If the aggregate production function is that given in (30), for example, then the competitive income share of labor is

$$\gamma = \frac{\partial F(K,L)}{\partial L}\frac{L}{F(K,L)} = \frac{1-\alpha}{1+(K/L)^{1-\alpha}B},$$

and tends to zero as K grows endogenously towards infinity.

Recall that, in Section 2.1's framework of analysis, the intercept $\bar{c} \neq 0$ of individual consumption functions has a crucial role in determining distributional dynamics: with $\bar{c} > 0$, an economy that grows towards a steady state would feature increasing inequality, as the higher average savings propensity of richer individuals reinforces wealth inequality. In many interesting models, however, the economy is capable of sustaining endless growth—because of exogenous technological progress, as in (26), or because capital accumulation does not endogenously deplete returns to investment, as in (30). If aggregate consumption does tend to grow at a proportional rate in the long run, relative-consumption dynamics must eventually become irrelevant, as a finite "required" consumption level constitutes an ever lower proportion of each individual's total consumption. Asymptotically, a growing economy behaves as if $\bar{c} = 0$ in

(18), or

$$U'(c) = c^{-\sigma}. \tag{31}$$

When preferences are in the form (31) and all savings earn the same rate of return R, the growth rate of consumption is constant across individuals and constant marginal utility ratios, as in (22), imply that ratios of consumption levels remain forever constant across individuals. Along a path of balanced long-run growth, savings behavior perpetuates whatever heterogeneity may exist across consumption and income levels.

In balanced growth, output and capital grow at the same rate as consumption,

$$\frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t} = \frac{C_{t+1}}{C_t} \equiv \theta :$$

Optimal savings choices associate a larger rate of return R with faster consumption growth.[18] In light of (23), balanced growth at a constant rate is associated with a constant income share $(1-\gamma)$ for accumulated factors, and a larger factor share for capital is associated with faster growth. This echoes the "classical" relationship between the two in (27), and in fact reflects a similar relationship between income sources and savings propensities. This is most clearly seen in the case where $\sigma = 1$ (the utility function is logarithmic) and the balanced growth rate is given by

$$\theta = \frac{1+R}{1+\rho} = \frac{1+(1-\gamma)\frac{Y}{K}}{1+\rho} \tag{32}$$

along the balanced growth path of standard optimizing models where agents have identical infinite planning horizons and utility functions. Consider the intertemporal budget constraint of an individual who owns k_t units of wealth and l units of the non-accumulated factor of production at time t, when the latter is compensated by a wage rate W_t. Since R is constant in balanced growth, we can write

$$\sum_{j=0}^{\infty} c_{t+j} \left(\frac{1}{1+R}\right)^j = k_t(1+R) + \sum_{j=0}^{\infty} l W_{t+j} \left(\frac{1}{1+R}\right)^j, \tag{33}$$

and, since a constant R implies a constant growth rate for both wages and individual consumption, inserting $c_{t+j} = c_t\theta^j$, and $W_{t+j} = W_t\theta^j$ in

[18] Even when income effects associate a higher R with lower savings, in fact, a higher rate of return unambiguously makes it possible to consume more in the future.

(33) yields

$$c_t = W_t l + (1+R)\left(1 - \frac{\theta}{1+R}\right) k_t = W_t l + (1+R-\theta)k_t. \qquad (34)$$

Across individuals, any difference in the amount of non-accumulated income $W_t l$ is reflected one-for-one in different consumption levels, and the propensity to consume non-accumulated-factor (or "labor") income is unitary just like the simple Classical or Post-Keynesian models outlined above. In fact, it is unnecessary (and would be suboptimal) to save any portion of the income flows accruing to non-accumulated factors of production when their wages grow at the same rate as each individual's optimal consumption, as is the case along a path of balanced growth. Any individual who happens to own only l and no accumulated factors of production enjoys a stream of income that grows like W and like desired consumption, never saves, and never accumulates any capital.

By contrast, and again consistently with Kaldorian behavioral assumptions, a portion of capital income *must* be saved for individual consumable resources to keep up with individual desired consumption paths. Savings by an individual who owns k_t and l units of the two factors are given by

$$Rk_t + W_t l - c_t = (R - (1+R-\theta))\, k_t = (\theta - 1)k_t, \qquad (35)$$

hence directly proportional to wealth and to accumulated-factor income. An individual who is a pure "capitalist," i.e., happens to own only an amount k of the accumulated factor of production, needs to save $(\theta-1)k$ for his wealth and income to increase at the same rate θ as consumption across periods.

Since savings behavior perpetuates any initial heterogeneity in the factor composition of income, the economy can feature a stable class structure, and the functional and personal distribution of income are strictly related to each other and to the economy's growth rate. In optimization-based models, causal relationships among factor shares, savings propensities, and growth are not as easy to identify as in Kaldor's equation (27), because in (32) the growth rate θ and/or the output/capital ratio Y/K are variables rather than given parameters as in (27). Further, savings propensities are attached to factor-income flows rather than to individuals belonging to different "classes," and the rate at which "profits" are saved is endogenous to preferences and distributional parameters.

Any balanced-growth path under optimal savings features relationships similar to (32) among factor income distribution, aggregate growth, and the capital/output ratio. If factor markets are cleared by price-taking economic interactions and lump-sum factor redistribution can address distributional issues, of course, then such relationships need not have the same distributional implications as in the class-based models of savings outlined above. Further, if decreasing returns to accumulation leave exogenous growth of the non-accumulated factor L as the only source of long-run growth then equations like (32) simply determine the endogenous steady-state capital/output ratio. Interactions between factor income distribution and macroeconomic phenomena are more complex and interesting in models which, following Romer (1986), specify the economy's technological and market structure so that returns to aggregate accumulation are constant. If the reduced form of the aggregate production function $F(K, L)$ is $A(L)K$, where

$$\partial F(.,.)/\partial K = A(L) \tag{36}$$

depends on L if non-accumulated factors have a productive role but is constant with respect to K, the capital/output ratio for given L is also independent of K, and the proportional growth rate of output is constant if the aggregate savings rate and $A(L)$ are constant:

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = \frac{Y - C}{K} = A(L)\frac{Y - C}{Y}. \tag{37}$$

If non-accumulated factors have a nonnegligible role in production, and accumulation does not encounter decreasing returns, aggregate returns to scale are not constant, but increasing. Since it is always conceptually possible to increase production by proportionately increasing all inputs or "replicating" identical microeconomic production units, decreasing returns to aggregate inputs can be ruled out on *a priori* grounds (Solow, 1956a). Replication arguments do not rule out increasing returns, however: as in Romer (1986, 1989, 1990), such *nonrival* factors as know-how, software, and other determinants of an economy's technological prowess can be simultaneously used in an arbitrary number of production units or processes, and need not increase in proportion to rival inputs to yield proportionately larger output at the aggregate or at the firm level. This makes it possible to rationalize increasing returns from first principles in many qualitatively realistic ways, and to model growth as endogenous to the economy's preferences, technology, and market structure. Intratemporal markets prices and factor payments for given K may be determined by competitive interactions if increasing returns are external to firms, as in Arrow's (1962) learning-by-doing model and Romer (1986). Inputs which are non-rival but excludable, such as patent-protected knowledge, imply increasing returns within each firm, and are naturally associated with market power in the models of Romer (1987), Grossman and Helpman (1991), and others. In general, isoelastic functional forms for technology and demand lead to "AK" reduced form functions which satisfy (36) when aggregate flows are measured as a price-weighted index of heterogeneous goods.[19]

For the present purpose of analyzing interactions between distribution and aggregate growth, the most relevant and general feature of this class of endogenous growth models is the simple fact that, under increasing returns, intertemporally efficient allocations cannot be decentralized in complete, competitive markets: since the sum total of marginal productivities exceeds aggregate production, the private remuneration of one or more factors of production must differ from its "social" counterpart. When the microeconomic structure of markets and production cannot be such as to guarantee that market equilibria are efficient then, as in class-based models of savings, the distribution of income flows across factors is obviously relevant to aggregate dynamics and, if factor ownership is heterogeneous, to resource distribution across individuals. Hence, aggregate growth and distribution hinge on policies, institutions, and politics (as in the models reviewed by Section 4 below) rather than on technological features only.

As pointed out by Rebelo (1991), efficient market interactions between accumulated and non-accumulated factors of production can support endogenous balanced growth in multi-sector growth models, as long as a "core" of accumulated factors reproduces itself without encountering decreasing returns.[20] Like in single-sector models of growth, savings

[19] Grossman and Helpman (1991) and Aghion and Howitt (1998) offer extensive and insightful reviews of these and other microeconomic foundations of endogenous growth.

[20] Externalities and other market imperfections do play an essential role in other multi-sector growth models, such as those of product and quality innovation proposed

propensities of individuals who happen to own accumulated and non accumulated factors in different proportions depend on income sources along such economies' balanced-growth paths because, again, an individual who owns no capital never needs or wants to accumulate any wealth. While aggregation of heterogeneous goods into homogeneous "capital" and "output" measures may be difficult from an accounting point of view (unless production functions satisfy separability conditions, as in Solow, 1956b), relative prices are unambiguously defined and easily interpreted as long as perfect, competitive markets support an efficient allocation (Dixit, 1977). Along the balanced growth equilibrium paths of multi-sector economies, taxes or other distortions which introduce wedges between factor incomes and marginal productivities affect the economy's growth rate, factor shares, and the relationship between the former and the latter in much the same way as in the simpler single-good models outlined above. Recent work on models of suboptimal endogenous growth under a variety of market imperfections has rekindled interest in distributional issues (see Section 4 below). If the distribution of income across factors owned by different individuals is allowed to play a substantive economic role, it unavoidably affects relative prices. From this survey's point of view, it may be interesting to note that factor-income distribution also affects the relative prices of capital and consumption in ways that are somewhat reminiscent of the Sraffa (1960) problem of how savings, investment, and "capital" might be measured in models where multiple capital goods are used in production and reproduction, and relative prices and the value of the aggregate stock of capital in terms of consumption generally depend on factor-income distribution.[21] .

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by Grossman and Helpman (1991) where growth is driven by production of K (which represents "knowledge" in these models) in a research and development sector which employs and compensates only labor, a non-accumulated private factor of production.

[21]Marglin (1984) and especially Kurz and Salvadori (1995) offer recent extensive treatments of these matters.

3 Investments, savings, and financial markets

The interactions between inequality and growth reviewed in Section 2 arise from factor-reward dynamics, and from heterogeneous sizes and compositions of individual factor bundles. None of the models encountered above explains what might generate such heterogeneity in the first place, however, and by assuming that all individuals' savings are allocated to similar investment opportunities the models surveyed so far strongly restrict the dynamic pattern of cross-sectional marginal utilities and consumption levels. Regardless of whether the homogeneous rate of return $r = R$ appropriately reflects "social" intertemporal trade-offs, or is distorted by imperfect intratemporal markets or policy instruments, inequality is eventually stable and may tend to disappear along balanced growth paths.

The literature reviewed in this Section allows for imperfections and/or incompleteness of the *intertemporal* markets where individual savings meet investment opportunities. In reality, financial market imperfections are presumably endogenous to microeconomic information and enforcement problems. What follows, however, focuses on the macroeconomic consequences rather than on the microeconomic sources of financial market imperfections, and aims at highlighting the basic mechanisms underlying many recent contributions also reviewed by Pagano (1993), King and Levine (1993), Bénabou (1996c), Greenwood and Smith (1997), and Levine (1997).

Realistic financial market imperfections introduce interesting interactions between distribution and macroeconomic phenomena, but also make it impossible to characterize the latter on a representative-individual basis. Under appropriate simplifying assumptions, however, macroeconomic models do feature meaningful linkages between resource distribution and aggregate dynamics when investment opportunities are heterogeneous. Most straightforwardly, the planning horizon of investment and savings differ across finitely-lived agents in the overlapping-generations models reviewed in Section 3.1 below. Further, *ex ante* investment opportunities may differ across individuals with different wealth if self-financing constraints are binding (Section 3.2), and *ex post* returns in consumption terms may differ across individuals if idiosyncratic risk cannot be pooled in the financial markets (Section 3.3). Since no financial-market counterpart could ever exist for a representative individual, neither self-financing nor borrowing constraints could be binding in a homogeneous economy and, more generally, not only the structure of financial markets but also the extent of inequality is relevant to macroeconomic outcomes and to the evolution of income inequality.

3.1 Finite lives

In the models reviewed above, the dynamics of each individual's income and wealth evolve over the same infinite horizon that is applicable to macroeconomic dynamics. One obvious reason why investment opportunities differ across individuals is the simple fact that not all individuals can participate in intertemporal markets, because some of them are not yet alive when the financial market is supposed to clear. Many of the models reviewed in this and the next Section indeed limit the time horizon of individual savings and investment problems, often for the sake of simplicity: since optimal consumption decisions across two periods are much more simply characterized than optimal forward-looking plans, an overlapping-generation structure recommends itself naturally to models which analyze explicitly other complex features of reality, such as uncertainty or politico-economic interactions.

Overlapping-generations models, however, are not just simpler versions of their infinite-horizon counterparts. When individuals have finite lifetimes within an infinite-horizon economy, aggregate income flows are distributed across generations as well as across factors and across individuals. While redistributing disposable income from accumulated to non-accumulated factors of production necessarily decreases the level and/or growth rate of income in infinite-horizon growth models, the same experiment is likely to increase the economy's savings propensity in a conventional Diamond (1965) overlapping-generations economy with two-period lifetimes. In that setting, all savings are performed by "young" agents who earn only (non-accumulated) labor income and solve a maximization problem in the form

$$\max_{c^y} \quad U(c^y) + \frac{1}{1+\rho} U(c^o)$$

$$\text{s.t.} \qquad c^o = (Wl - c^y)(1+R), \tag{38}$$

where c^y and c^o denote each agent's consumption when young and old respectively, and the rate of return R is the same for all individuals. The usual first-order condition

$$U'(c^y) = \frac{1+R}{1+\rho} U'(c^o) \tag{39}$$

and the budget constraint in (38) determine the two consumption levels. When utility is logarithmic ($U'(x) = 1/x$), income and substitution effects offset each other exactly and the individual savings rate

$$Wl - c^y = \frac{1}{2+\rho} Wl \tag{40}$$

is independent of the return rate R. Factor-income distribution, however, matters for aggregate accumulation, because in standard overlapping generations economies all (non accumulated) income is earned by young individuals, who also perform all of the aggregate economy's savings—while old individuals consume not only all of their (accumulated factor) income but also their stock of wealth. Accordingly, aggregate savings are given by

$$\Delta K_t = \frac{1}{2+\rho} W_t L - K_t = \left(\frac{1}{2+\rho} \gamma \frac{Y_t}{K_t} - 1 \right) K_t, \tag{41}$$

and are increasing in the share γ of the non-accumulated factor of production L in aggregate income.

To study the implications of this simple insight for macroeconomic dynamics, it is necessary to specify how the capital/output ratio and the factor share γ are determined by the economy's markets, policies, and technology. Uhlig and Yanagawa (1996) study a simple endogenous-growth economy where $Y_t/K_t = A(L)$ is constant, and discuss the effects of tax policies which shift disposable income from "capital" to "labor." Under the logarithmic specification of preferences above, aggregate savings are an increasing function of the share γ of non-accumulated factors in aggregate income, and so is aggregate capital growth, since

$$\frac{K_{t+1}}{K_t} = \left(\frac{\gamma}{2+\rho} \right) \frac{Y_t}{K_t}. \tag{42}$$

A lower rate of return on savings is likely to be associated with higher aggregate savings and to faster investment-driven growth also for more general specifications of preferences. For the opposite result to hold in a standard overlapping-generations model, in fact, the intertemporal elasticity of consumption substitution must be so high—i.e., σ must be so much lower than unity in the constant-elasticity case (31)—as to let substitution effects dominate not only the income effect, but also the effects of income redistribution across agents with different planning horizons.[22] The model's implications are not as sharp when young individuals can look forward to future wages. If agents still live for two periods, leave no bequests, and maximize loglinear objective functions, but are endowed with l^o units of labor in the second period of their life as well as with l^y units in the first, then consumption of the young is given by

$$c^y = \frac{1+\rho}{2+\rho}\left(W_t l^y + \frac{W_{t+1} l^o}{1+R}\right). \tag{43}$$

Shifting functional income distribution towards capital decreases W in all periods, but even when the utility function is logarithmic (and income and substitution effects offset each other) a higher R tends to decrease consumption and increase savings via wealth effects, i.e., because the present value of future wages is smaller. More complex and realistic but qualitatively similar effects are featured by continuous-time models where lifetimes are exponentially distributed (Bertola, 1996).

As in infinite-horizon models, growth can be sustained if some of the accumulated factors' contribution to aggregate production "spills over" to owners of non-accumulated factors, because of external effects and/or market imperfections; correspondingly, the private remuneration of savings will be lower than capital's contribution to future aggregate production. This, however, does not have the same normative implications as it would in the standard infinite-horizon model: a higher savings rate would increase the economy's growth rate and future income, but this

[22] A similar mechanism is at work when an infinite number of market participants lets asset prices deviate from their fundamental values (Grossman and Yanagawa, 1993). Like public debt or unfunded social security, asset bubbles transfer resources from the (saving) young to the (dissaving) old. As each generation finds it less necessary to rely on productive capital for consumption-smoothing purposes, investment-driven economic growth slows down.

cannot result in a Pareto improvement if the finitely-lived generations whose consumption is decreased is distinct from those which will enjoy the resulting income stream.[23] Jones and Manuelli (1992) and Boldrin (1992) show that standard discrete-time overlapping-generations models, where individuals own no capital in the first period of their lives, cannot sustain endogenous growth if factors are rewarded according to their marginal productivity. For intratemporal markets to support marginal-productivity-based income distribution, in fact, returns to scale to capital and labor together must be constant. As in the example (30) above, non-accumulated factors must then earn a vanishing share of aggregate production if returns to accumulation are asymptotically constant. Neoclassical markets assign an ever smaller share of aggregate production to labor at the same time as the economy accumulates an increasingly large stock of capital, and it must eventually become impossible for young capital-poor individuals to purchase with their savings the aggregate capital stock from older, about-to-die individuals.[24]

3.2 Self-financed investment

All the models reviewed so far allow agents to access an integrated financial market which, ruling out arbitrage, offers the same rate of return to all individuals. Other models rule out access to financial markets, so that investment must equal savings not only at the aggregate but also at the individual level, or otherwise link intertemporal investment opportunities to individual circumstances (such as the availability of col-

[23] Since an overlapping-generations economy features infinitely many market participants, its competitive equilibrium is not necessarily Pareto-efficient. As shown by Saint-Paul (1992a), however, dynamic efficiency is guaranteed to obtain if the marginal productivity of capital is independent of accumulation, as is the case along a path of endogenous balanced growth.

[24] Jones and Manuelli also show that an overlapping-generations economy can experience unbounded endogenous growth if, as is possible in multi-sector models, the price of capital in terms of consumption and wages declines steadily over time. The growth effects of policy interventions which redistribute income towards the early stages of individual lifetimes are similar to those outlined above, and can even make endogenous growth possible for an economy whose income would reach a stable plateau under *laissez-faire* markets: once again, however, such growth-enhancing policies affect intergenerational distribution.

lateral) rather than to equilibrium conditions in the financial market. Such capital-market imperfections are most realistic for educational investments and other forms of human capital accumulation, since labor income can hardly serve as collateral and investment returns generally accrue to heirs who are not legally bound to honor debts incurred by their parents.

The macroeconomic relevance of self-financing constraints clearly depends on the extent of inequality across individuals. Identical agents would not trade with each other even when allowed to do so, and the aggregate economy's accumulation path would simply resemble each (representative) individual's in that case. If resource levels and investment opportunities are both heterogeneous across individuals, conversely, the relationship between the two bears on the dynamics of distribution. Inequality tends to disappear if technology and markets offer less favorable rates of return to relatively rich individuals, and tends to persist and widen if the opposite is the case.

To isolate the role of self-financing constraints, it is useful to model investment opportunities as simply as possible. In the absence of financial market access, the accumulation constraint may be written in the form $k_{t+1} = f(k_t - c_t)$ at the level of each individual or family. Consumable resources are identified with wealth k_t (thus abstracting from any possible role of non-accumulated sources of income) and, unlike the budget constraint (1) above, the function $f(\cdot)$ which maps foregone consumption at time t into future resources is nonlinear. Simplicity is also a virtue with regard to savings decisions in this setting. Many studies of financial-market imperfections consider two-period planning problems with logarithmic objective functions, abstracting from the complex and ambiguous effects of current and expected future returns in more general models. The role of different investment opportunity sets in determining relative and aggregate accumulation dynamics may also be highlighted in the even simpler case where the savings rate is exogenously given, as in the models of Bencivenga and Smith (1991) and Piketty (1997), so that the dynamics of each individual's wealth,

$$k_{t+1} = f(sk_t), \tag{44}$$

depend essentially on the shape of the function $f(\cdot)$.

3.2.1 Convex investment opportunities

Since finitely-lived individuals face a smooth tradeoff between time spent in education and working time, educational investment opportunities may be modeled by a standard decreasing-returns function $f(\cdot)$ with $f'(\cdot) > 0$, $f''(\cdot) < 0$. As a simple example, consider the relative-wealth dynamics implied by $f(x) = x^{\alpha}$, $0 < \alpha < 1$. Denoting with $x_t \equiv k_t^i / k_t^j$ the wealth of individual i relative to that of individual j, we have

$$x_{t+1} = \frac{f(sk_t^i)}{f(sk_t^j)} = \left(\frac{k_t^i}{k_t^j}\right)^{\alpha} = (x_t)^{\alpha}.$$

Since $\alpha < 1$, relative wealth levels tend to converge to the stationary configuration $x_t = x_{t+1} = 1$ of this recursion, hence to complete equalization and, in the absence of exogenous non-accumulated income, complete stability of individual wealth levels. More general decreasing-returns functions such that

$$\lim_{x\to\infty} f'(x) = 0 \tag{45}$$

have similar implications. Relative wealth convergence is also implied by decreasing returns to individual accumulation in optimizing models, such as

$$\begin{aligned} \max \quad & U(c_t) + \frac{1}{1+\rho} V(k_{t+1}) \\ \text{s.t.} \quad & k_{t+1} = f(k_t - c_t), \end{aligned} \tag{46}$$

as long as the savings rate $s(k_t) = (k_t - c_t)/k_t$ does not depend too strongly on individual-specific investment returns (and certainly in the logarithmic case considered above). The two-period optimization problem (46) can be brought to bear on longer-horizon dynamics if the second-period utility $V(\cdot)$ is interpreted as a "warm glow" benefit of wealth bequeathed to one's descendants, along the lines of Andreoni (1989), or as the value function of an infinite-horizon optimization model. Of particular interest in the present context is the fact that if $V(\cdot)$ is the value function of an infinite-horizon model, then the dynamics converge to a unique steady state under the same conditions that would yield a well-defined competitive equilibrium for a neoclassical macroeconomy faced by the same problem as the individual considered (see, e.g., Chatterjee, 1994).[25]

Condition (45), in fact, is more than superficially related to (29). In neoclassical models where (29) is true, aggregate accumulation histories always converge to the same steady state level, while nondecreasing returns to accumulation would let an economy determine its own rate of long-run growth. Similarly, a cross-section of individual wealth levels may fail to converge if the return to investment and the savings rate eventually become constant as wealth increases. Under self-financing constraints, as noted by Chatterjee (1994), aggregate dynamics break down in a collection of side-by-side individual problems similar to that facing the representative individual or social planner of a neoclassical aggregate economy.

When savings are not allocated efficiently to investment opportunities by an integrated financial market, the level and dynamics of aggregate output are a function of all individual wealth levels $\{k^i\}$ rather than of the aggregate stock $K \equiv \int_{\mathcal{N}} k^i dP(i)$ only. As in Bénabou (1996c,d), interactions between distribution and aggregate dynamics can be analyzed in a parsimonious way if the form of individual production functions and of wealth distribution is appropriately restricted. If the accumulation technology at the individual level has a constant-elasticity form, as in $k^i_{t+1} = (k^i)^\alpha (k^i_t - c^i_t)^\beta$, then a constant investment rate s (as might be implied by optimization of similarly loglinear intertemporal objective functions) yields

$$k^i_{t+1} = (k^i_t)^{\alpha+\beta} (s)^\beta . \tag{47}$$

As long as $\alpha + \beta < 1$, the marginal return to individual i's investment is a decreasing function of individual resources k^i_t, and the next period's aggregate resources $K_{t+1} = \int_{\mathcal{N}} (k^i_t)^{\alpha+\beta} (s)^\beta \, dP(i)$ are smaller than they would be if the amount e^i invested in the production function $(k^i)^\alpha (e^i)^\beta$ did not need to coincide with that same individual's savings $k^i - c^i$.

[25] If future wealth's contribution to utility has a "warm glow" interpretation, however, then multiple equilibria are possible. For example, Galor and Tsiddon (1997) let k_{t+1} depend on k_t as well as on $k_t - c_t$, and show that the recursion implied by optimal savings behavior can have multiple fixed points, depending on third-order and mixed derivatives. The resulting wealth dynamics are similar to those (reviewed below) generated by non-convex investment opportunity sets, which are perhaps easier to interpret.

In general, the output loss due to inefficient allocation across decreasing-returns investment opportunities is an increasing function of the degree of heterogeneity across individuals. For the constant-elasticity specification (47), a closed-form expression is available if, following Bénabou, $\log k^i \sim N(m, \Sigma^2)$ (i.e., initial wealth is lognormally distributed). As a function of the average level of wealth and of its dispersion, aggregate production in the second period is given by[26]

$$(s)^\beta \int_{\mathcal{N}} (k_t^i)^{\alpha+\beta} dP(i) = (s)^\beta\, E[(k_t^i)^{\alpha+\beta}] = s^\beta K_t^{\alpha+\beta} e^{(\alpha+\beta)(\alpha+\beta-1)\frac{\Sigma^2}{2}}, \quad (48)$$

and with $\alpha+\beta < 1$ less inequality (a lower value of Σ) reduces the extent to which self-financing constraints are binding, and increases output. If second-period resources could be redistributed, there would be no reason for inequality to reduce the economy's efficiency. The social planning problem

$$\begin{aligned} \max \quad & \int_{\mathcal{N}} (k_t^i)^\alpha \left(e^i\right)^\beta dP(i) \\ \text{s.t.} \quad & \int_{\mathcal{N}} e^i dP(i) = K - C = sK \end{aligned} \quad (49)$$

is solved if the marginal efficiency of investment is the same across all investment opportunities, i.e., if

$$e^i = \frac{(k^i)^{\frac{\alpha}{1-\beta}}}{\int (k^{i'})^{\frac{\alpha}{1-\beta}} dP(i')} sK$$

is invested in individual i's production function. The exponent of k^i is less than unity if $\alpha+\beta < 1$: to equalize marginal productivity across individuals, the amount invested increases less than proportionately to the initial endowment of individual i. When Pareto optimal redistribution of resources cannot be implemented at time $t+1$ by financial markets, then taxation and subsidies come into play along the lines of models reviewed in Section 4 below.

[26] These derivations make use of the fact that then $\log\left((k^i)^{\alpha+\beta}\right) \sim N((\alpha+\beta)m, (\alpha+\beta)^2\Sigma^2)$, and that, since $K_t = E[k_t^i] = e^{m+\Sigma^2/2}$, $e^{(\alpha+\beta)m+(\alpha+\beta)\Sigma^2/2} = K_t^{\alpha+\beta}$.

The accumulation equation (47) implies that the relative wealth of any two individuals evolves according to

$$\frac{k^i_{t+1}}{k^j_{t+1}} = \left(\frac{k^i_t}{k^j_t}\right)^{\alpha+\beta} \tag{50}$$

and converges to unity if $\alpha + \beta < 1$ (hence, Σ converges to zero unless idiosyncratic shocks are added to each individual's accumulation equation). The aggregate expression in (48) makes it straightforward to study the dynamics of aggregate consumption and/or of its distribution. The growth rate of capital is given by

$$\frac{K_{t+1}}{K_t} = s^\beta K_t^{\alpha+\beta-1} e^{(\alpha+\beta)(\alpha+\beta-1)\frac{\Sigma^2}{2}}, \tag{51}$$

and is a decreasing function of K_t for given Σ if $\alpha + \beta < 1$. Hence, long-run growth of aggregate consumption and capital cannot be endogenously determined by savings decisions under the same conditions that imply convergence of cross-sectional wealth levels. This coincidence of individual and aggregate convergence implications is not surprising, since this simple model of self-financed investment represents the evolution of a cross-section of individual wealth levels as a collection of atomistic accumulation problems similar to each other, and to their own aggregate counterpart.

More general models, however, need not feature convergence or lack thereof at both the individual and aggregate levels. In the present setting, if the market allocation were the one that solves the social planning problem (49) investment and production would be immediately and completely equalized in cross-section but, depending on the specification of utility functions, consumption may or may not converge across individuals under complete financial markets. Conversely, an economy can be capable of endless endogenous growth even as individuals within it converge towards each other. This is the case if, despite their inability to interact in financial markets, individuals engaged in self-financed accumulation programs interact with each other and influence aggregate growth through *non*-market channels: externalities and spillovers across saving programs can sustain aggregate growth even when each individual's savings would eventually cease to fuel his own income's growth if performed in isolation. As in Tamura (1991), models can be specified where (29) is violated at the aggregate level, so that economy-wide growth proceeds at an endogenous rate in the long run, but (45) holds at the individual level, to imply convergence in the cross-sectional distribution of wealth.[27] To illustrate the point in the present context, let the specification of individual accumulation constraints feature an aggregate knowledge spillover, as in

$$k^i_{t+1} = (k^i)^\alpha (k^i_t - c^i_t)^\beta K_t^{1-\alpha-\beta}. \tag{52}$$

This leaves (50) unchanged, but alters (51) to read

$$\frac{K_{t+1}}{K_t} = s^\beta e^{(\alpha+\beta)(\alpha+\beta-1)\frac{\Sigma^2}{2}}. \tag{53}$$

The aggregate growth rate is independent of the aggregate capital stock K_t, hence does not tend to decline as production grows (its dynamics, if any, will instead reflect changes in the degree of wealth and income inequality, here denoted Σ and kept constant for simplicity).[28]

3.2.2 Increasing returns to individual investments.

In the neoclassical model of Section 2.1, factor remuneration on the basis of marginal productivity was a logical possibility only if production functions had non-increasing returns. It was then natural to let the marginal productivity of capital be decreasing in the capital intensity of production, and possibly so strongly decreasing as to satisfy (29). When self-financing constraints make it impossible to trade accumulated factors of production on a competitive market, however, returns can be increasing at the level of individual production units.

To see how investment nonconvexities may lead to divergent wealth dynamics and segmentation, consider an economy where the individual

[27] Barro and Sala i Martin (1997) and others study models where growth is sustained by realistic knowledge spillovers, such as those generated by imitation of new inventions.

[28] Convergence can occur within sub-economies, or neighborhoods, if the non-market interactions that allow aggregate growth to proceed forever occur within such units. Bénabou (1996a,b), Durlauf (1996), and others propose and study models of endogenous neighborhood choice and discuss their implications for the dynamics of distribution and of aggregate variables.

savings rate s is again a given constant, but let investment opportunities feature an indivisibility similar to that studied by Galor and Zeira (1993) in an optimizing context. At time t, individual or family i consumes a fraction $1-s$ of available resources, which consist of labor income w_t^i as well as of accumulated wealth k_t^i. The amount $s(k_t^i+w_t^i)$ that is saved at time t can earn a net rate of return R in the financial market, but part of it may be invested in an indivisible educational opportunity instead. Against payment of a given cost $\bar{x}_t$, purchase of education ensures that in the next period $w_{t+1}^i = w^S$, the labor income flow of a skilled worker, rather than the unskilled wage rate w^N. Hence, individual i's resources evolve according to

$$k_{t+1}^i + w_{t+1}^i = (1+R)s(k_t^i + w_t^i) + w^N \qquad (54)$$

if the investment in education is not undertaken, and to

$$k_{t+1}^i + w_{t+1}^i = (1+R)\left[s(k_t^i + w_t^i) - \bar{x}_t\right] + w^S \qquad (55)$$

if education is purchased.

Clearly, the indivisible educational opportunity is relevant only if it offers higher returns than financial investment,

$$w^S - w^N > (1+R)\bar{x}_t, \qquad (56)$$

in which case it would be efficient to educate all individuals if aggregate resources suffice to do so, i.e., if $s(K_t + W_t L) \geq \bar{x}_t$. If $s(k_t^i + w_t^i) < \bar{x}_t$ for some i and education must be self-financed, however, then some resources will inefficiently earn only the financial return R even as some opportunities for educational investment remain unexploited, because financial market imperfections make it impossible to reap the fruits of investment in others' education.

If $\bar{x}_t = \bar{x}$ is constant and $(1+R)s < 1$, then the wealth paths of individuals who always earn different wages converge to heterogeneous steady states. In the case of poor individuals who cannot afford education and earn only w^N, wealth follows the dynamics in (54) and may always remain too low to afford education. Symmetrically, the wealth of individuals who are initially rich enough to afford education may always suffice to make education affordable for them. Thus, there exist configurations of parameters such that all individuals with initial resources below the critical level $\bar{x}/s$ never purchase education and, if their wealth is initially above the steady state level, become increasingly poor over time, while individuals whose resources are even only marginally higher than $\bar{x}/s$ follow a path of increasing wealth and consumption.

Such distributional dynamics can be embedded in more or less complex and realistic models of macroeoconomic dynamics. Galor and Zeira (1993) interpret their similar, more sophisticate model as a small open economy, where the rate of return on financial investment is given at the world level, and discuss possible interactions across individual problems in the case where the wage paid to unskilled workers depends on the amount of labor supplied to a sector which uses no internationally mobile capital. Other models feature dynamic interactions among individual-level savings and investment problems. The model of Aghion and Bolton (1997) determines interest rates endogenously, and features a "trickle down" mechanism by which aggregate growth eventually brings all individuals to take advantage of the more favorable opportunities afforded by their non-convex investment sets. In the context of the simple model above, any fixed $\bar{x}$ would similarly become irrelevant if aggregate wages grew along with aggregate capital. The poverty traps would not disappear, however, if the cost $\bar{x}_t$ of education grows in step with aggregate income and wages, as might be realistic if it is specified in terms of labor.

The prediction that equality is associated with better efficiency and faster growth under self-financing constraints can be overturned if investment projects are indivisible: in the context of the simple example above, if $s(K_t + W_t L) < \bar{x}_t$ (i.e., the aggregate economy is so poor as to be unable to educate all its members) then an egalitarian allocation of resources would make self-financing constraints binding for all individuals, and prevent all savings from earning the higher of the two rates of return in (56). Since the speed of further aggregate development depends on the initial distribution of resources under these circumstances, macroeconomic dynamics are generally path-dependent and may feature multiple equilibria. The point is relevant in the context of the model analyzed by Acemoglu and Zilibotti (1997), who abstract from distributional issues by assuming that all individuals are identical within each generation, and in many other models where individual returns are increasing in the size of investment, such as those proposed by Banerjee and Newman (1993) and Perotti (1993) where individual-level increasing returns interact with complex and realistic financial-market imperfections and endogenously determined redistributive policies.

3.3 Idiosyncratic uncertainty

The dynamics of individual and aggregate income, consumption, and wealth levels were deterministic in all the models above. In reality, random shocks are certainly relevant to the evolution of aggregate resources and of their distribution. Individual-level or "idiosyncratic" uncertainty, however, would be completely irrelevant if exchanging contingent securities in perfect and complete financial markets made it possible for individuals to smooth consumption not only over time, as in the models of Section 2, but also across different realizations of exogenous random events. As in Section 2.1.2, the equilibrium allocation under complete markets can be interpreted in terms of a social planning problem. Exchange of state- and time-contingent claims in financial markets ensures that different individuals' marginal utilities always remain proportional, as in (22), and the constants of proportionality ω_i depend on the individual i's endowment of factors of production. As shown by Cochrane (1991) in more detail, the specification that is most relevant for macroeconomic purposes—where utility functions are the same across individuals and have the isoelastic form (31)—implies that consumption levels should remain proportional to each other at all times. Thus, under complete markets, idiosyncratic events have no implications for aggregate dynamics, which can be analyzed on a representative-individual basis, or for distribution, which is determined once and for all by initial conditions.

In reality, of course, not all idiosyncratic (hence potentially insurable) risk is traded in perfect and complete markets, and any discussion of dynamic income inequality must explicitly allow for imperfect insurance.[29] For the purpose of characterizing the macroeconomic relevance of distribution dynamics, it will be helpful to assume that the economy of interest is populated by so many atomistic individuals that, by a law of large numbers, aggregate dynamics are deterministic if all uncertainty is idiosyncratic, yet uninsurable because financial markets

[29] In a complete-markets setting, in fact, income flows could hardly be defined and measured, since Arrow-Debreu market participants should own portfolios of contingent claims rather than bundles of production factors.

fall short of completeness in one or more respects. The distributional implications of uninsurable idiosyncratic uncertainty are qualitatively straightforward, and perhaps most immediately illustrated in the context of the models with self-financed investment opportunities reviewed above, where idiosyncratic events may generate or regenerate inequality across individuals. In stochastic versions of models where, like in Section 3.2.1, self-financing constraints imply higher investment returns for poor individuals, wealth levels converge asymptotically to a distribution rather than to complete equalization; in steady state, ongoing random shocks offset the mean reversion induced by savings and investment returns (Loury, 1981; Benabou, 1996b). In models where, as in Section 3.2.2, indivisibilities and fixed costs imply locally divergent wealth dynamics, idiosyncratic shocks may or may not ensure that stochastic paths of wealth accumulation converge to a single ergodic distribution; if the tendency towards segmentation is strong enough, initial conditions and one-time events can have long-run implications (see, e.g., Piketty, 1997).

The macroeconomic implications of idiosyncratic uninsurable uncertainty are more subtle. By definition, idiosyncratic events cancel out in the aggregate. As illustrated by the simple models outlined below, however, random investment opportunities affect individual behavior in ways that do bear on aggregate dynamics, and this is the case even when utility and savings functions obey the restrictions introduced and discussed in Section 2—so that a one-time redistribution of resources would leave unaltered the aggregate propensity to save in a certainty framework.

Assets available to each individual may yield idiosyncratic random returns, and the risk associated with investment in individual-specific assets may be uninsurable. Further, uncertainty about future non-accumulated income is relevant to savings decisions whenever available assets' payoffs cannot isolate individual consumption from idiosyncratic events. In general, the microeconomic consumption/savings problem of an individual may feature uncertain returns to endogenously accumulated wealth, borrowing constraints, and/or random flows of non-accumulated factor income in the dynamic accumulation constraint (1). Many of the relevant insights can again be obtained from a simple two-period specification. Consider the problem

$$\max_{c_t} \qquad U(c_t) + \frac{1}{1+\rho} E_t[V(k_{t+1}; w_{t+1}, l_{t+1}, ...)]$$

$$\text{s.t.} \qquad k_{t+1} = f(k_t - c_t; z_{t+1}) + w_{t+1}l_{t+1}, \qquad c_t \leq \bar{c},$$

where the realization of the z_{t+1} determinant of investment returns and/or the amount of non-accumulated income $w_{t+1}l_{t+1}$ are random as of time t, and borrowing limits may impose an upper bound on current consumption. Under the usual regularity conditions, the necessary and sufficient condition for choice of c_t reads

$$U'(c_t) = \frac{1}{1+\rho} E_t[f'(k_t - c_t; z_{t+1})V'(k_{t+1}; ...)] + \mu_t, \qquad (57)$$

where the Kuhn-Tucker shadow price μ_t is positive if the borrowing constraint is binding, and zero otherwise. As in simpler settings such as (46) above, the implications of more complex multi-period problems are qualitatively similar, since the second term in the two-period problem's objective function can be interpreted as the value function of utility-maximization problems over longer planning horizons. What follows uses (57) to characterize individual savings behavior and its implications for aggregate accumulation and inequality in two complementary special cases: that where non-accumulated income is certain (and, for simplicity, equal to zero), but returns to accumulation are partly or wholly individual-specific; and that where returns to accumulation are constant, but non-accumulated income is subject to idiosyncratic shocks.

3.3.1 Uncertain returns to accumulation

Realized returns to accumulation may be heterogeneous across individuals not only because capital-market imperfections require partial or complete self-financing of investments, but also because they make it difficult or impossible to avoid exogenous rate-of-return risk. To focus on the latter phenomenon, consider the case where an individual's investment opportunity set offers stochastic constant returns, i.e., let

$$f(k - c; z_{t+1}) = (k - c)(1 + r(z^i_{t+1})) :$$

unit investment returns depend on an exogenous "state of nature" realization z^i_t, but are independent of wealth and investment levels, implying that self-financing constraints (if any) would not affect distributional and aggregate dynamics through the mechanisms reviewed above under certainty.

In (57), the extent to which investment risk influences individual-specific returns and consumption growth depends on the degree of financial market completeness on the one hand, and on the proportion of individual savings channeled through risky assets on the other. When a "stock market" is open, access to less risky (hence more favorable) investment opportunities may or may not increase the savings rate, depending on the balance of income and substitution effects. The point can be illustrated simply in the case where non-accumulated or "labor" income is absent and all period utility functions have the isoelastic form (31). Under these circumstances, (57) can be rearranged to read

$$\left(\frac{c_t^i}{k_t^i - c_t^i}\right)^{-\sigma} = \frac{1}{1+\rho} E_t\left[(1 + r(z_t^i))^{1-\sigma}\right]. \qquad (58)$$

The left-hand side of (58) is a decreasing function of c_t^i for all $\sigma > 0$. Its right-hand side is constant if $\sigma = 1$: as usual, savings are independent of the rate of return on investment under logarithmic utility and in the absence of non-accumulated income, since income and substitution effects cancel each other and there are no wealth effects. When $\sigma \neq 1$, the effects of rate-of-return uncertainty depend on whether the function whose expectation is taken on the right-hand side of (58) is convex or concave. When the elasticity of intertemporal substitution is small ($\sigma > 1$), then $(1+r)^{1-\sigma}$ is a convex function of r and, by Jensen's inequality, wider dispersion of investment returns around a given mean increases the right-hand side of (58). Hence, first-period consumption is decreased—and savings are increased—by higher rate-of-return uncertainty. If $\sigma < 1$ instead, lower rate of return uncertainty at the individual level increases each individual's savings rate: roughly speaking, when income effects dominate substitution effects then a more favorable investment opportunity set leads to lower savings.

The basic insight illustrated above is relevant in different ways to various models proposed in the literature. An overlapping-generations structure is convenient when the risk structure of returns is less than trivially endogenous to individual choices, and many models adopt it to study the implications for growth and distribution of (idiosyncratic) risk in investment returns and liquidity constraints. The simple results obtained in a two-period framework are qualitatively similar to those obtained in infinite-time horizon models. In the absence of non-accumulated income dynamics, an infinite horizon program can be analyzed as a sequence of two-period problems in the form (46) if $V(\cdot)$ is viewed as a Bellman value function, whose functional form is the same as that of the period utility function $U(\cdot)$ if the latter belongs to the quasi-homothetic class of which (18) is an example (see e.g. Merton, 1971). Continuous-time specifications of rate-of-return uncertainty yield closed-form solutions, which Obstfeld (1994) applies to the financial market integration issues of interest here.[30]

As only the mean rate of return matters for aggregate saving's contribution to future output and growth, when $\sigma > 1$ then forms of financial market development that simply allow individuals to pool idiosyncratic rate-of-return risk are associated with slower growth. In fact, since the savings rate satisfying (58) is the same for all individuals faced by the same *ex ante* investment opportunity set, aggregating across individuals yields

$$\begin{aligned} K_{t+1} &= \int_{\mathcal{N}} (k_t^i - c_t^i)(1 + r(z_t^i)) dP(i) = \\ & \left(\frac{K - C}{K} \right) \int_{\mathcal{N}} k_t^i (1 + r(z_t^i)) dP(i); \end{aligned}$$

and if realized returns are uncorrelated to individual wealth levels, then

$$\int_{\mathcal{N}} k_t^i (1 + r(z_t^i)) dP(i) = \int_{\mathcal{N}} k_t^i dP(i) \, E_t[(1 + r(z_t^i))] \equiv K_t(1 + R),$$

hence

$$\frac{K_{t+1}}{K_t} = \left(\frac{K - C}{K} \right) (1 + R). \tag{59}$$

The realized mean return R is a given parameter if all uncertainty is idiosyncratic, and idiosyncratic uncertainty has aggregate effects only through the savings propensity. If $\sigma > 1$, an economy without financial

[30] Obstfeld, like Devereux and Smith (1994) and Devereux and Saito (1997), emphasizes the implications of financial market integration in an international context, along the lines of the discussion in Section 5.1 below. Obstfeld also studies a more general case where utility is not additively separable over time and, as in Weil (1990), makes it clear that the effects of investment opportunities on consumption growth are mediated by σ in its role as the inverse of the intertemporal elasticity of substitution, rather than as the coefficient of relative risk aversion.

markets produces a larger amount of aggregate resources even as it distributes it more unevenly across its consumers/investors—whose welfare is, however, lowered *ex ante* by consumption volatility, and quite imperfectly approximated by conventional output measures (Devereux and Smith, 1994, compute and discuss welfare measures).

If individuals can control the riskiness of their investment portfolios, risk pooling is generally relevant to the investment efficiency of any given volume of savings, or to the size of the aggregate return R in (59). Risky investments must be more productive (on average) than safe ones if they are ever undertaken. Hence, aggregate productivity is higher and growth is *ceteris paribus* faster when risk pooling makes it individually optimal to reduce the portfolio share of safe, low-expected-return assets, and increase that of (well diversified) high-return risky assets. Saint-Paul (1992b), Obstfeld (1994), Devereux and Saito (1997) formulate and solve models where this effect has a role.[31]

Distributional implications. Models where returns to accumulation are idiosyncratically uncertain obviously rationalize *ex post* inequality over any finite horizon. In infinite-horizon models, inequality would simply increase without bounds if returns to investment were continuously perturbed by idiosyncratic shocks, and were unrelated to wealth levels.[32] In general, financial markets offer better insurance against idiosyncratic income and consumption uncertainty, and a more efficient allocation of aggregate savings across investment opportunities. Across economies at different levels of financial development, accordingly, higher production and faster growth should be associated with more stable inequality. Recent work brings this insight to bear on time-series developments, allowing the evolution of financial markets to be endogenously re-

[31] Similarly, a well-developed financial market lets savings be allocated more efficiently when new capital takes time to become productive and, as in the model proposed by Bencivenga and Smith (1991), individual portfolios are biased to more liquid but less productive assets when financial institutions ("banks") are not available to smooth liquidity risk across heterogeneous individuals; see also Greenwood and Smith (1997).

[32] Models like Bénabou's (1996c,d) feature uncertain returns to investment, but also self-financing constraints, which generate mean-reverting wealth dynamics as in Tamura (1991).

lated to growth and wealth dynamics. Greenwood and Jovanovic (1990), Saint-Paul (1992b), and other models surveyed by Greenwood and Smith (1997) let it be costly for individuals to access an intermediated financial market. The implications of costly access to the favorable investment opportunities offered by organized financial markets depend on distribution as well as on the level and expected growth rate of income, and are similar to those of the indivisibilities and fixed costs in individual investment opportunity sets reviewed in Section 3.2.2 above.[33] Depending on the distribution of resources, a more or less large fraction of the population may be able to afford participation when its costs are partly fixed at the individual's level. Since relative welfare levels are completely stabilized across those individuals who do participate in the financial market the dynamic paths of aggregate output and cross-sectional inequality are jointly determined and, as in the simpler setting discussed in Section 3.2.2, fixed participation costs may become irrelevant if growth "trickles down" so as to eventually lead all individuals to enter the financial market.[34]

3.3.2 Liquidity constraints and uninsurable endowment risk

Consider a two-period problem where the rate of return is certain, as in (38), but non-accumulated income accrues in the second as well as in the first period:

$$\max \qquad U(c_t^y) + \frac{1}{1+\rho} U(c_{t+1}^o) \tag{60}$$

$$\text{s.t.} \qquad c_{t+1}^o = (W_t l_t^y - c_t^y)(1+R) + W_{t+1} l_{t+1}^o, \tag{61}$$

where l_t^y denotes the labor endowment of individuals who are young at time t, and l_{t+1}^o that of the same individuals when old at time $t+1$. Let

[33] The model of Acemoglu and Zilibotti (1997), in fact, features better diversification in a more developed economy because investment projects are indivisible, rather than because of assumptions regarding financial market set-up costs.

[34] The Greenwood and Jovanovic (1990) model predicts convergence to a stable distribution of welfare (and, since utility is logarithmic, of consumption and wealth). Some individuals' wealth levels may never become high enough to induce them to enter the financial market. Even in that case, however, all of the economy's wealth is asymptotically invested in the financial market, for individuals may remain out of it only if their wealth becomes negligible in relative terms.

$l_t^o = l_{t+1}^o = \ldots = l^o$ be constant over time, and similarly let $l_t^y = l_{t+1}^y = \ldots = l^y$ for all t; in a growing economy, of course, different market wage rates may reward the individual's given labor supply at times t and $t+1$. Consumption choices satisfy (57) with $V(\cdot) = \frac{1}{1+\rho}U(\cdot)$:

$$U'(c_t^y) = \frac{1+R}{1+\rho}E_t[U'\left((W_t l^y - c_t^y)(1+R) + W_{t+1}l^o\right)] + \mu_t. \qquad (62)$$

If $U'(c) = c^{-\sigma}$, it is easy to verify that $W_t l_t^y - c^y > 0$ (savings are positive) if

$$\frac{W_{t+1}}{W_t}\frac{l^o}{l^y} < \left(\frac{1+R}{1+\rho}\right)^{\frac{1}{\sigma}} \equiv \xi. \qquad (63)$$

This condition is trivially satisfied if $l^o = 0$, as in the standard overlapping-generations model (38). In more general models, however, the growth rate of wages may exceed the desired growth rate of consumption, at least over part of an individual's life: if the inequality in (63) is reversed, then young individuals would wish to borrow, and if they are not allowed to do so then $\mu_t > 0$ in (62). Under certainty, if exogenous earnings increase faster than desired consumption then binding liquidity constraints imply larger savings in the aggregate.

A further "precautionary" increase in savings occurs if future non-accumulated income is random, future consumption is uninsurably uncertain, and the utility function has a positive third derivative (see Deaton, 1991, and Carroll, 1992, for recent discussion of such phenomena). Ljungqvist (1993, 1995) and Jappelli and Pagano (1994) explore the growth implications of precautionary savings in overlapping-generations settings. When endogenous growth is driven by productivity spillovers, then liquidity constraints may improve every individual's welfare if the distortion of consumption patterns over each generation's lifetime is more than offset by the faster consumption growth induced by external effects. Besides distorting intertemporal consumption patterns relative to what would be optimal for the given private rate of return on savings, in fact, liquidity constraints also reduce individual borrowing, hence increase aggregate savings. To the extent that each generation's savings affect its own wages through external effects (and the social return on savings is higher than the private one, as is plausible in an endogenous-growth model), higher savings may bring each generation closer to the truly optimal lifecycle pattern of consumption.[35]

The direction of the inequality in (63) depends not only on the lifetime pattern of labor endowments l_t^y, l_{t+1}^o and on the taste parameters ρ, σ, but also on the rate of return on savings R and on the growth rate of wages W_{t+1}/W_t, either or both of which are generally endogenous in macroeconomic equilibrium. The models of Laitner (1979a,b, 1992), Aiyagari (1994), Aiyagari and McGrattan (1995), and others study wealth accumulation in general-equilibrium settings with exogenous or endogenous growth, and focus on the macroeconomic implications of savings intended to provide a "precautionary" cushion against idiosyncratic bad luck. In the notation adopted here, the rate of return on wealth R is both constant over time and common across individuals, while the income stream $\{l_t^i W_t\}$ is random and exogenous to the individual's consumption and savings choices. In analogy to R, one may let $w = W$ be homogeneous across individuals, and ascribe all uncertainty to the individual- and period-specific endowment l_t^i of the non-accumulated factor L. This also ensures that uncertainty does not become irrelevant in proportional terms if W_t grows over time.

Distributional implications. Across individuals who may lend or borrow (subject to solvency constraints) but bear uninsurable risk, inequality tends to increase over time. To see this, consider that (57) with $f'(\cdot) = 1 + R$ and—in the absence of liquidity constraints— $\mu_t = 0$ implies:

$$U'(c_t^i) = \frac{1+R}{1+\rho}\left(U'(c_{t+1}^i) - \epsilon_{t+1}^i\right), \tag{64}$$

where ϵ_t^i denotes the unpredictable difference between the expected and realized marginal utility of individual i's consumption. If $R = \rho$, each individual's marginal utility follows the driftless process

$$U'(c_{t+1}^i) = U'(c_t^i) + \epsilon_{t+1}^i, \tag{65}$$

and marginal utility differentials across individuals also have unpredictable increments over time if, as we assume, all uncertainty is idiosyncratic.

[35] De Gregorio (1996) studies the interaction of such welfare-enhancing effects of financial market imperfections with the investment distortions implied by self-financing constraints.

Like the probability distribution of a random-walk process, the cross-sectional distribution of consumption and welfare levels tends to widen. As noted by Atkeson and Lucas (1992), such lack of mean reversion in relative welfare levels is a general feature of efficient allocations under private information, which prevents full insurance but does not reduce the desirability of consumption smoothing over time. The same efficiency considerations that imply stability of relative marginal utilities in the first-best setting of Section 2.1.2 imply unpredictability of marginal-utility shocks when the planner's welfare weights need to be revised so as to maintain incentive compatibility under asymmetric information.[36]

If welfare is bounded below, however, marginal utility processes follow a renewal process with a well-defined ergodic distribution. Heuristically, an upper bound on marginal utility imposes a reflecting barrier on the nonstationary process (65), and past experiences become irrelevant whenever the barrier is reached. Liquidity constraints do impose such lower bounds on consumption and welfare, and effectively truncate individual planning horizons at the (random) times when binding constraints make past accumulation irrelevant to future consumption and welfare.[37] If liquidity constraints are binding with positive probability (and bind at any given time for a finitely positive fraction of a large economy's population), then individual marginal utilities are increasing on average (and utility levels drift downwards) over time, or $R < \rho$ in

$$U'(c_t^i) = \frac{1+R}{1+\rho}\left(U'(c_{t+1}^i) - \epsilon_{t+1}^i\right).$$

Since financial markets offer less favorable lending opportunities to richer individuals when poor individuals find it impossible to borrow, consumption-smoothing individuals decumulate wealth on average when they are rich

[36]Finite individual lifetimes or planning horizons, of course, limit the extent to which wealth and welfare levels can drift randomly away from each other. Deaton and Paxson's (1994) empirical work supports the implication that consumption inequality should be increasing with age within consumer cohorts.

[37]The implications of binding liquidity constraints, in fact, are in many ways similar to those of finite lifetimes in overlapping-generation models (Laitner, 1979a). Tighter bounds on consumption and wealth dynamics than those required by simple solvency may reflect nonnegativity constraints on bequests, limited possibilities to use future labor income as collateral, and/or welfare lower bounds implied by redistribution policies (Atkeson and Lucas, 1995).

enough to self-insure, i.e., draw on assets so as to buffer the effects of exogenous income shocks on their consumption. This behavior implies that accumulated wealth, driven by a cumulation of stationary labor-income realizations, follows a nonstationary process with negative drift and a reflecting barrier at the lowest level consistent with borrowing constraints, and has a well-defined distribution in the long run. Since consumption cannot be sheltered forever from labor-income uncertainty, consumers with decreasing absolute risk aversion find it optimal to transfer resources from present (and certain) to future (and uncertain) consumption by "precautionary" or "buffer-stock" savings. To ensure homotheticity of the objective function and avoid a trending savings rate in a growing environment, the period utility function must take the isoelastic form (31).[38] While a perfectly insured consumer would have a constant propensity to save out of current resources if the period utility function $U(\cdot)$ has the form (31), in the solution of (16) savings are luxuries—i.e., a higher proportion of available resources is saved by richer individuals—if labor income is uncertain (Laitner, 1979a). This provides a better rationale for wealth-dependent savings rates than the positive $\bar{c}$ required consumption levels studied above in a certainty setting, since any finite $\bar{c}$ would become asymptotically irrelevant in a growing economy unless required consumption is specified in relative terms.[39]

For the canonical isoelastic specification (31) of preferences, savings propensities depend in intuitive and realistic ways on both the level and the factor composition of individual and aggregate income flows. The propensity to consume out of wealth is higher for richer individuals, who are less concerned with (heavily discounted) future consumption volatility; the propensity to consume out of non-accumulated income depends,

[38] Closed-form solutions for precautionary-savings problems are available if absolute, rather than relative, risk aversion is constant (see Caballero, 1991). Under a constant absolute risk aversion specification, however, assets do not behave as a buffer stock; consumption responds fully to income innovations, and this has inconvenient and unrealistic implications for aggregate analysis (Irvine and Wang, 1994).

[39] Rebelo (1992), Atkeson and Ogaki (1996), and their references formulate and solve models of this type and assess their empirical relevance. The Uzawa (1968) assumption that the discount rate ρ is an increasing function of current utility (and wealth) has the unintuitive implication of a decreasing wealth elasticity of savings, yet it is often adopted in macroeconomic applications where asymptotic stability of wealth accumulation is needed.

in accordance with permanent-income theory, on whether the current flow is above or below its long-run expected level. This class of models can rationalize an increasing income elasticity of savings without resorting to ad-hoc assumptions on the form of utility, which would imply increasing rates of accumulation and growth in a growing economy where all agents become richer over time. The empirical realism of these models can be enhanced in a variety of ways, most notably allowing for realistic lifecycle patterns of labor earnings and wealth as in Laitner (1992).

The macroeconomic implications of such microeconomic behavior are qualitatively straightforward, but somewhat difficult to study because closed-form solutions are not available. As each individual attempts to self-insure against idiosyncratic risk, aggregate accumulation is more intense for any given rate of return and expected accumulation rate. If this results in a higher aggregate wealth-to-output ratio, steady-state equilibrium is restored by a decline of the rate of return on savings along a neoclassical factor price frontier (as in the models of Laitner, 1979a,b, and Aiyagari, 1994); if the marginal and average return of wealth accumulation is constant instead, as in endogenous growth models, the higher propensity to accumulate capital increases the average growth rate of consumption and non-accumulated factor incomes, and the latter restores equilibrium as a larger expected flow of future income makes it less necessary for individuals to rely on accumulation to boost future consumption levels. One of the models that Devereux and Smith (1994) specify and solve in an international framework of analysis is isomorphic to a macroeconomic model where infinitely lived individuals can neither borrow nor lend, and can only use self-financed investment for consumption-smoothing purposes. Like in the overlapping-generations model of Jappelli and Pagano (1994), precautionary savings induced by additive ("labor income") uninsurable shocks can accelerate endogenous growth to the point that welfare is higher under financial autarchy than under perfect insurance.

4 Politics and institutions

In a neoclassical economy with complete competitive markets, one-time redistribution could and should resolve any distributional issues without compromising the efficiency of macroeconomic outcomes. The appropriate lump-sum redistribution instruments, however, are simply not available in the absence of complete intertemporal markets. At the same time as distortions (such as taxes, subsidies, and market imperfections) decrease the size of the economic "pie" available to a hypothetical representative individual or to a social planner with access to lump-sum redistribution, they also alter the way economic welfare is shared among individuals. Hence, distribution and macroeconomics interact not only through the channels surveyed in the previous sections, but also by influencing the extent to which distortionary policies are implemented in politico-economic equilibria.[40]

The point is relevant to any model where policy is allowed to play a role, but perhaps most relevant in this survey's context when taxes and other relative price distortions can affect an economy's endogenous rate of growth, i.e., when they alter private incentives to allocate resources to the sector or sectors where a "core" of accumulated factors can reproduce itself without encountering decreasing returns (Rebelo, 1991). Since many such models feature increasing returns, missing markets, or imperfectly competitive market interactions, policy interventions meant to offset *laissez-faire* inefficiencies and distortions play a prominent role in this context. Accordingly, recent work (also surveyed by Bénabou, 1996c, and Persson and Tabellini, 1998) has focused on the growth implications of distributional tensions.

To illustrate the macroeconomic impact of distortionary policies and the political mechanisms linking distributional tensions to equilibrium distortions, consider the simplest model encountered above, where individual savings decisions aim at maximizing

$$\log(c_t^i) + \frac{1}{1+\rho} \log(c_{t+1}^i), \tag{66}$$

[40] Related incentive mechanisms may also be relevant in some contexts. Even selfish individuals may be concerned with inequality when it is so wide as to make predatory activities preferable to market participation for poor individuals, and costly defensive activities necessary for richer individuals; Grossman and Kim (1996) and their references analyze in detail the microeconomic determinants and macroeconomic implications of predatory activity. Distributional issues are also directly relevant when individuals' relative standing bears on their economic welfare and their savings decisions, as in the model of Cole, Mailath, and Postlewaite (1992).

and specify the budget constraint in such a way as to account for individual resource heterogeneity and for taxation. Let individual i's first- and second-period consumption levels be given by

$$c_t^i = w_t^i - k_t^i, \quad c_{t+1}^i = (1+(1-\tau)R)k_t^i + S,$$

where the exogenous endowment w_t^i and the portion k_t^i saved out of it are individual-specific, while the gross return on savings R, the tax rate τ applied to it, and the subsidy S are the same for all individuals. Taking both τ and S as given, the individually optimal consumption choice is

$$c_t^i = \frac{1+\rho}{2+\rho}\left(w^i + \frac{S}{1+(1-\tau)R}\right). \tag{67}$$

With a logarithmic utility function, the lower net rate of return implied by a higher tax rate τ has offsetting income and substitution effects. The subsidy, however, unambiguously increased first-period consumption, to an extent that depends on the wealth effect of the tax-determined rate of return.

Both τ and S can be negative (to represent an investment subsidy financed by lump-sum taxes), and the two policy instruments are related to each other through the government's budget constraint if the per-capita subsidy is financed by taxing the income RK_t, of capital in the second period, so that

$$S = \tau R K_t \tag{68}$$

for $K_t = \int_{\mathcal{N}} k_t^i dP(i)$ the aggregate capital stock at the end of the first period. Since

$$k_t^i = w^i - c^i = \frac{w^i}{2+\rho} - \frac{1+\rho}{2+\rho}\frac{S}{1+(1-\tau)R},$$

aggregating, denoting $\int w_t^i dP(i) = W_t$, and using (68) yields

$$K_t = \frac{W_t}{2+\rho} - \frac{1+\rho}{2+\rho}\frac{\tau R K_t}{1+(1-\tau)R}.$$

Solving for the equilibrium level of K_t, we find that

$$K_t = \left(2+\rho+\frac{(1+\rho)\tau R}{1+(1-\tau)R}\right)^{-1} W_t.$$

Hence, a higher tax rate τ unambiguously reduces the aggregate capital stock in the second period. The insight is more general than the simple model considered here. Rate of return taxes only have substitution effects when their revenues are rebated in lump-sum fashion, and under any homothetic objective function individual savings choices can be aggregated to yield the same qualitative results as in the logarithmic case considered here (Persson and Tabellini, 1994). Quite intuitively, a positive tax rate on investment returns and a lump-sum consumption subsidy make savings less attractive, for each individual can to some extent rely on taxation of others' savings to finance future consumption. In equilibrium, individuals free ride on each other's choices to postpone consumption, and less capital is accumulated.

4.1 Political sources of distortionary taxation

It is of course far from surprising to find that taxing the income of an endogenously supplied factor, like k^i_{t+1} in this simple model, decreases private supply incentives and has negative effects on macroeconomic efficiency. Such effects would be present even in a representative-individual macroeconomy where $w^i = W$ for all i. Recent research aims at highlighting how such outcomes, while clearly undesirable from the representative individual's point of view, may be rationalized by explicit consideration of redistributive motives in the politico-economic process that presumably underlies policy choices in reality.

To illustrate the insight in the context of the simple model introduced above, note that the Euler equation implies

$$c^i_{t+1} = \frac{1 + (1-\tau)R}{1+\rho} c^i_t$$

for the simple example's logarithmic objective function. Hence, maximized individual welfare may be written

$$\frac{2+\rho}{1+\rho} \log(c^i_t) + \frac{1}{1+\rho} \log\left(\frac{1+(1-\tau)R}{1+\rho}\right).$$

Using (67) and (68), and neglecting irrelevant constants, individual welfare depends on the tax rate according to

$$V(\tau) = (2+\rho)\log\left(w^i + \frac{\tau W R}{2+\rho+R(2+\rho-\tau)}\right) + \log\left(1+(1-\tau)R\right). \tag{69}$$

Each individual's welfare is increased by the tax and subsidy package's impact on the two consumption levels, represented by the first term on the right-hand side of (69). Differentiating this term, it is easy to show that the welfare effect of a higher τ is more positive for small values of w^i: intuitively, relatively poor individuals' consumption levels are subsidized by taxing the higher savings of richer individuals.

All individuals' welfare is also decreased by the distorted intertemporal pattern of consumption, represented by the last term in the expression above: differentiating, it is easy to show that the two marginal effects offset each other at $\tau = 0$ if $w^i = W$, i.e., if the welfare expression refers to a representative individual's welfare whose welfare is maximized by the savings choices implied by an undistorted intertemporal rate of transformation. For individuals with $w^i < W$, however, the level effect is larger than the slope effect at $\tau = 0$, and welfare is maximized at a positive level of τ. Hence, relatively poor individuals prefer strictly positive tax rates, because from their point of view the benefits of redistribution more than offset the welfare loss from a distorted intertemporal consumption pattern. Conversely, for those endowed with more resources than the representative individual ($w^i > W$) a policy of investment subsidization and lump-sum taxes would be preferable to the laissez-faire outcome.

As noted by Persson and Tabellini (1994), realistic skewness of income distribution associates higher inequality with a higher percentage of relatively poor individuals. For example, a democratic one-person-one-vote political process should generally result in redistribution-motivated distortions, because the median voter is poorer—to an extent that depends on the degree of inequality—than the average (representative) individual. Other political decision processes will also yield interior solutions for tax rate τ, as individuals (or coalitions of individuals) weigh the costs and benefits of redistribution and distortions from their own point of view.

The insight can be brought to bear on macroeconomic growth if the simple two-period model above is embedded within a longer-horizon aggregate economy. Persson and Tabellini (1994) let each generation's initial resources, denoted W in the derivations above, depend on the previous generation's savings decisions through external effects. Then, the simple insights afforded by the two-period savings decision carry over directly to aggregate dynamics, since all economic and political interactions occur within a closed set of individuals alive at the same time: a higher level of exogenous inequality is associated with more intense redistributive tensions and, in situations where distortionary taxation is used for redistributive purposes, with slower growth. Persson and Tabellini (1994) offer evidence in support of this simple and realistic insight. Further and more detailed empirical work (briefly reviewed in Section 5 below) is less supportive, and other theoretical models also suggest more complex linkages between inequality, redistribution, and economic performance.

4.2 Dimensions of heterogeneity and distribution

As in the simple model above, distortionary redistribution can be a political equilibrium outcome only if individual agents' endowments are cross-sectionally heterogeneous ($w^i \neq W$ for at least some i). In fact, identical individuals—like a hypothetical social planner—would never want to decrease economic efficiency. The extent and character of heterogeneity, however, need not be as immediately associated with the size distribution of income as in the model outlined above.

Bertola (1993) and Alesina and Rodrik (1994) study policy determination in models of endogenous growth which, like those outlined in Section 2.2.2, feature balanced paths of endogenous growth with no transitional dynamics. In these models, the speed of growth is directly related to the private rate of return on savings and investment decisions, hence to the portion of aggregate production accruing to accumulated factors of production. Explicit discussion of policy choices is particularly important in this context, because the underlying economic models allow for market imperfections and/or for an explicit role of government expenditure (and for increasing returns to scale at the aggregate level) in order to obtain constant returns to accumulation. Thus, policy intervention would generally be desirable even from a representative individual's point of view.

If ownership of accumulated and/or of nonaccumulated factors of production is not evenly spread across all individuals, however, then factor-income distribution affects not only the aggregate growth rate, but also the distribution of income and welfare across individuals. The extent of such heterogeneity and the character of political interactions are crucial determinants of policy choices and, through them, of macroeconomic growth outcomes.

To illustrate this point, consider a simple discrete-time version of the relevant models. If all individuals aim at maximizing logarithmic utility flows discounted at rate ρ over an infinite horizon, a constant rate of return R on savings implies that all consumption flows grow according to

$$\frac{c_{t+1}}{c_t} = \frac{C_{t+1}}{C_t} = \frac{1+R}{1+\rho} \equiv \theta,$$

and the welfare level of individual i can be written

$$\sum_{j=0}^{\infty} \left(\frac{1}{1+\rho}\right)^j \log(c_t^i \theta^j) = \log(c_t^i)\frac{1+\rho}{\rho} + \log(\theta)\frac{1+\rho}{\rho^2}$$

as of time t. The budget constraint, as in (34), implies that

$$c_t^i = W_t l^i + (1+R-\theta)k_t^i \tag{70}$$

for an individual who owns a constant number l^i of units of the non-accumulated factor (each earning W_t at time t) and k_t^i units of the accumulated one at time t (earning a constant gross rate of return $1+R$). If the output/capital ratio is a constant A, then using $W = \gamma A K_t / L$ and $R = (1-\gamma)A$ in (70) yields

$$c_t^i = \left(\gamma A \frac{l^i}{L} + \frac{\rho}{1+\rho}(1+(1-\gamma)A)\frac{k_t^i}{K_t}\right) K_t, \tag{71}$$

and makes it possible to write individual welfare as a function of the factor share γ of non-accumulated factors of production. Disregarding irrelevant constants and the level of the aggregate capital stock K_t, which affects all welfare levels equally, the relevant expression reads

$$V(\gamma) = \log\left(\gamma A \frac{l^i}{L} + \frac{\rho}{1+\rho}(1+(1-\gamma)A)\frac{k_t^i}{K_t}\right) + \log(1+(1-\gamma)A)\frac{1}{\rho}. \tag{72}$$

Like the savings tax rate in the two-period model above, different values of γ affect consumption levels (in a way that depends on initial endowments) on the one hand, and the slope of (all individuals') consumption paths on the other.

The slope effect of a smaller γ is unambiguously positive: faster growth benefits all individuals' welfare, and is the equilibrium outcome in this model if a larger share of aggregate output is paid to accumulated factors of production. Faster investment-driven growth must be financed by lower consumption levels, however, and the impact of a smaller γ on initial consumption depends on the factor composition of individual income sources. For the representative individual, $l^i/L = k^i/K \equiv 1$ by definition, and welfare is maximized when $\gamma = 0$ and the private gross return $1 + R$ coincides with the aggregate transformation factor $1 + A$. Equally unsurprisingly, the welfare of an individual i who happens to own only non-accumulated factors of production (so that $k^i = 0$) is far from being maximized by $\gamma = 0$, which implies zero consumption and an infinitely negative welfare level.

More generally, the preferred value of γ depends on the *relative* size of k^i and l^i. Heterogeneity of factor income sources may (but need not) be related to the size distribution of income that was relevant in the above model, for example because accumulated wealth is more unequal than other sources of income; when a political process leads to implementation of policies (such as taxes and subsidies) which bear on the after-tax income shares of the two factors, its outcome generally depends on the distribution of political power across constituencies (or "classes") with different income sources (Bertola, 1993).

4.3 The menu and timing of policies

The simple models outlined above illustrate the general insights that distributional tensions can have macroeconomic effects when they result in distortionary policies. Their results, of course, hinge on the details of politico-economic interactions on the one hand, and on the specific distortionary instrument used for redistributional purposes on the other. In models where distribution-motivated policy interventions unavoidably distort incentives, individuals trade their preference for a large share of the social pie against the size of the latter, and it may be possible to obtain and characterize interior politico-economic equilibria.[41] In prac-

[41]For well-defined voting equilibria to exist, it is generally necessary to limit the extent and character of heterogeneity across agents in such a way as to ensure that preferences over packages of different policy instruments are well-behaved (single-

tice, of course, more than one instrument is generally available to pursue distributional objectives and, like imperfect and incomplete financial markets, political interactions can be specified in many different ways. While the simple illustrative models above can characterize sharply the distortionary effects of political interactions by focusing on simple policy instruments, more complex models recognize that many different policies may be separately or simultaneously implemented in reality.

While in the simple models outlined above distributional tensions clearly reduce aggregate efficiency, redistribution can have beneficial effects on representative-agent welfare when it substitutes missing markets. Human capital accumulation is most likely to be distorted by self-financing constraints and uninsurability, and is often targeted by policy interventions (see Glomm and Ravikumar, 1992, for a simple model of the implications of private or public education schemes for growth and distribution). In the models reviewed in Section 3.2.1 and studied in more detail by Bénabou (1996c,d), self-financing constraints prevent relatively poor agents (and the aggregate economy) from taking advantage of high returns from investment in their own education. When the *status quo* cross-sectional allocation of savings is distorted by self-financing constraints, a more equal distribution improves the efficiency of investment allocation, and is associated with higher output levels (or faster growth). Since inefficient investment patterns (whether caused by self-financing constraints, or by the incentive effects of redistribution-motivated policies) are unanimously disliked, politico-economic interactions will tend towards efficiency whenever it can be achieved independently of distribution.

As the efficiency benefits of redistribution depend on the extent of inequality, but only the relatively poor ones gain from the redistributive aspects of investment subsidies, political support for such redistributive policies as education subsidies is generally not a monotonic function of *status quo* inequality. In the models proposed by Bénabou (1996c,d), which introduce tractable specifications of tax and subsidy schemes in loglinear budget constraints in the form (47), the relative importance of efficiency-enhancing and redistributive effects in political interactions depends on the dispersion and skewness of income distribution, and on the

peaked).

distribution of political power across income levels. Further, since policies that affect *ex post* inequality feed back into their own political sustainability in a dynamic environment, multiple equilibria are possible: at relatively low levels of inequality, political equilibrium entails efficiency-enhancing redistribution and smaller income dispersion increases future political support for more redistribution, while symmetric reinforcing effects can be featured by high-inequality, low-redistribution dynamic trajectories.

In general, when the menu of available policy instruments is so wide as to make it possible to target both efficiency and distribution, then aggregate outcomes are much less likely to be affected by inequality, and distributional issues can be separated from macroeconomic performance in much the same way as in the complete markets case. Most relevantly, investment efficiency can be preserved by appropriately targeted subsidies even as politico-economic determination of tax rates pursues distributional objectives. In the models studied by Bénabou (1996c,d), where the accumulated factor is human capital, efficiency can be pursued by education subsidies (or by state-financed education) rather than by progressive taxation schemes. Individual agents, regardless of their income level, unanimously agree that efficiency should be achieved. This objective does not interfere with heterogeneous incentives to redistribute income when the latter can be pursued by a separate instrument. Similarly, Bertola (1993) finds that capital-poor individuals would obviously vote against policies that increase the growth rate of the economy by reducing their share of aggregate income, but would favor policy packages that restore growth-rate efficiency by subsidizing investment. In general, a wider menu of potentially distortionary policy instruments makes it easier for redistribution-motivated policy interventions to preserve efficiency, and brings macroeconomic outcomes closer to those that would be realized if distributional issues could be resolved by lump-sum instruments.

While once-and-for-all choices from a wide menu of policies can in principle minimize the distortionary consequences of politically desirable redistribution, the kind of one-time redistribution that would support the textbook separation of efficiency and distribution is hardly feasible in realistic dynamic settings. In the extreme case where the menu of available policies indeed includes a lump-sum redistribution instrument—e.g., taxation of the first-period endowment w^i in the two-period model of Section 4.1—nothing should in principle prevent macroeconomic efficiency but, since each individual would simply want to appropriate as large a share of aggregate resources as possible, it would be impossible to characterize interior political equilibria. In more complex dynamic models, and in reality, only distortionary instruments are available: in any situation where binding, complete intertemporal contracts are not available, in fact, "lump sum" redistribution is generally feasible only at the beginning of time, and can hardly be discussed or implemented in real time. Like capital income taxation in the simple model of Section 4.1, threats of "one-time" expropriation in an ongoing dynamic environment and lax enforcement of property rights loosens the link between individual supply decisions and individual consumption levels and, in the models proposed by Tornell and Velasco (1992), Benhabib and Rustichini (1996), and others, slows down capital accumulation.

Dynamic interactions between policy choices and political sustainability are potentially much more complex than simple models make it, and so is the relationship between *ex ante* or *ex post* inequality and macroeconomic outcomes. When taxation is decided *ex post*, or when predatory activity is made possible by imperfect protection of property rights, then the rational expectation of redistributive pressure affects incentives to save and invest even when all agents face identical problems and no redistribution takes place *ex post*. Distributional tensions are present and distortionary even when agents are and remain homogeneous, for the simple *fear* of ex-post expropriation tends to remove incentives to save and invest (Bénabou, 1996c). Recent work by Krusell and Rios Rull (1992), Krusell, Quadrini, and Rios-Rull (1997), Huffman (1996) applies such intertemporal insights to the analysis of political decision processes focused on simple policy instruments, drawing a useful parallel to time inconsistency issues in optimal taxation problems from a representative-agent's perspective. Numerical results are qualitatively consistent with the outcome of simpler equilibrium notions: as in the simple models of Sections 4.1 and 4.2, capital-poor agents are less inclined to reward investment (and speed up growth) than the representative agent. The resulting equilibrium tax rate is different, however, when policy choices are made every period within a dynamic framework of analysis rather than once and for all, as in the simpler models outlined above and in Bertola

(1993), Persson and Tabellini (1994), Alesina and Rodrik (1994); even identical individuals, in fact, should generally refrain from supporting first-best investment rewards if they think they can free-ride on their own future time-consistent choices.

Symmetrically, when taxation and redistribution policies are decided before the realization of exogenous income inequality is known, the observed intensity of *ex post* fiscal redistribution may mimic that which would be implied by intertemporal contingent contracts. In reality, of course, imperfect insurance reflects incomplete or asymmetric information and, unless tax-based redistribution can exploit superior sources of information, *ex post* redistribution meant to shelter individual consumption from undesirable fluctuations should generally worsen the economy's allocative efficiency at the same time as it reduces *ex post* cross sectional inequality.

As a perhaps trivial example of how policy-based redistribution may improve *laissez-faire* efficiency, however, consider how different the role of taxes and subsidies would be if the expected value of an objective function similar to (66) were maximized under the constraints

$$c_t^i = w_t^i - k_t^i, \quad c_{t+1}^i = (1-\tau)w_{t+1}^i + (1+R)k_t^i + S, \tag{73}$$

where w_{t+1}^i is idiosyncratically random as of time t. If for some reason individuals find it impossible to stipulate insurance contracts, or if such contracts are even slightly costly to write and enforce, all would unanimously agree that $\tau = 1$, $S = \mathrm{E}_t\left[w_{t+1}^i\right]$ is a welfare-increasing set of taxes and subsidies (and, since a non-random second-period consumption would eliminate precautionary savings, higher welfare would be associated with slower aggregate consumption growth). Less benign, but qualitatively similar implications for the role of redistribution can be drawn from models where individuals are not *ex ante* identical. If $\mathrm{E}_t\left[w_{t+1}^i\right]$ is heterogeneous across individuals in (73), then those individuals who expect relatively large exogenous income flows will be opposed to complete equalization of second-period incomes. As long as their second period income is uninsurably uncertain, however, even the richest individuals will favor at least partial redistribution. In politico-economic equilibrium, the extent and character of redistribution will then depend not only on the dynamics of *status quo* inequality, but also on the aggregate economy's dynamics. As in the model of Wright (1996), in fact, the insurance properties of *ex post* redistributive taxation may be made more or less desirable by faster growth of average labor-income endowments. Since future taxes and subsidies play the role of otherwise non-existent financial investment opportunities in this type of model, the sign of growth effects—like that of many others discussed above—depends on whether the intertemporal elasticity of substitution is larger or smaller than unity. If $\sigma > 1$ in the canonical specification (31) of preferences, then faster growth is associated with less ex-post redistribution in a politico-economic equilibrium where the character of the economy's safety net is decided once and for all, at a constitutional stage, by individuals whose income is currently high but who fear future bad luck.

5 Empirical evidence

Like all empirical work, tests of the theoretical mechanisms reviewed above and measures of their relevance are constrained by data availability. Studies of relationships between growth rates and income distribution across countries can rely on data collected for national income accounting purposes, and similar data are often available for smaller regional units. The relevant literature is reviewed in subsection 5.1 below, while subsection 5.2 summarizes the strategies and findings of recent research on relationships between economic inequality at the level of individuals or households within countries and country-level macroeconomic performance.

5.1 Convergence across countries

Each of the models reviewed in the previous Sections has specific predictions as to divergence or convergence of incomes over time within a macroeconomic entity. Two basic mechanisms lead to convergence (divergence) across individuals: relatively rich individuals may save less (more) than poor ones and/or obtain lower (higher) returns on their investment in physical or human capital. As noted when discussing equation (45) above, many of the relevant insights are similar to those applicable to macroeconomic growth dynamics. Accordingly, the theories reviewed above can be brought to bear on cross-country evidence if each country's aggregate income dynamics are interpreted on a representative individual basis—so that, in terms of the notation used above, the index i refers to any or all of the indistinguishable individuals inhabiting each country. While such an interpretation is clearly less than fully satisfactory, ready availability of of the relevant aggregate—notably the Summers and Heston (1991) harmonized data set and its updates—has generated an extensive body of literature, reviewed in this section.

The data indicate that growth rates of *per capita* income are hardly any faster on average in relatively poor countries than in richer ones (see, e.g., Canova and Marcet, 1995). Since country-level growth rates vary widely over time, measures of income inequality display substantial divergence in the post-war period.[42] A standard approach to the interpretation of cross-country growth dynamics views each country as a macroeconomy of the type encountered in Section 2.1, within which all savings earn the same rate of return (and measured inequality may or may not be evolving over time). Rates of return, however, are allowed to differ across countries, reflecting an absence or imperfection of financial market interactions across the borders of different jurisdictions. In the limit case where economies are completely closed to international capital flows, then each country's national income dynamics should be similar to those of individual incomes under self-financing constraints in Section 3.2 above, and a given technology should offer higher returns to accumulation in relatively capital-poor locations. The empirical fact that poor countries do not grow noticeably faster than rich ones is hard to interpret from the standpoint of models where investment must be self-financed. At the country level, in fact, the degree of concavity of self-financed investment functions like (44) is to some extent measurable if marginal productivities are well approximated by market rates of return. It is standard to view the income share of labor as an empirical counterpart to the share of non-accumulated factors γ in (23), and investment in physical capital as the empirical counterpart of accumulation as in (5). While these variables are less easy to measure in practice than to define in the simple theoretical framework above,[43] a share of capital of about

[42]As argued by Pritchett (1997), cross-country inequality was likely much narrower in pre-industrial times, since recent growth rates cannot be extrapolated backward without violating reasonable lower bounds on subsistence income levels.

[43]E.g., because labor income has to be disentangled from profits and rents in the

one third in aggregate income implies that transitional dynamics towards steady state output levels (or towards balanced growth) should be very fast indeed when only the capital intensity of production determines income inequality across countries.[44]

Under the maintained hypothesis that capital does not flow across countries and that individual countries' data provide independent observations of similar economic processes, lack of convergence may be viewed as supporting evidence for models of country-specific endogenous growth. As in Romer (1986) and the subsequent literature, macroeconomic models of growth predict that growth rates need not decrease over time if returns to capital accumulation are allowed to be asymptotically constant or increasing, and the measured share of capital—which, as in Section 2.2.2 above, is an important determinant of long-run growth rates—reflects less than perfectly competitive market interactions as well as the capital's aggregate productivity. When applied to cross-country observations, these models would indeed imply that differences in *per capita* levels of capital and production should persist indefinitely, in the sense that no mean reversion is expected while the distribution of per capita incomes widens over time if countries experience idiosyncratic shocks.

This oversimplified contrast between different interpretations of cross-country growth experiences leads naturally to considering more flexible models. Two interrelated strands of recent empirical work on convergence issues are particularly easy to relate to the models outlined in the previous Sections.

Even when technology is the same across countries which self-finance their accumulation, neoclassical growth models can be consistent with evidence of persistent inequality if countries converge to different steady state capital stocks. An extensive body of empirical work—surveyed by Barro and Sala-i-Martin (1994), Barro (1997), and De La

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case of the self-employed, and because capital stocks have to be constructed from investment data and depreciation assumptions by perpetual-inventory methods.

[44] The calibration exercises of King and Rebelo (1993) indicate that observed time-series patterns of country-specific growth rates are much too smooth to be consistent with the observed income share of capital and with realistic elasticities of intertemporal substitution in consumption. In a cross-country context, rates of return to accumulation should not only differ dramatically across countries with different initial conditions, but also vanish very quickly.

Fuente (1997)—detects a mild but nonnegligible negative effect of initial income on subsequent growth after controlling for savings rates, population growth rates, and other determinants of a neoclassical economy's steady state output.[45] The role of such controlling factors is of independent interest. In fact, empirical work which applies representative-individual specifications to aggregate data is most convincingly motivated when it is focused on phenomena which feature interesting variation and insightful theoretical implications across the borders of countries. The size of government budgets, the character of property rights protection, and other policy variables indeed play significant roles in those regressions.[46] The rate of "conditional" convergence towards country-specific steady states, while statistically significant, is slow, and again hard to interpret if accumulated factors are identified with physical capital.[47] However, the distinction between accumulated (or reproducible) and non-accumulated factors which has played a key role throughout this Chapter need not coincide with the standard definition of reproducible factors as physical capital. If a portion of measured labor income accrues to (accumulated) human capital rather than to raw labor, for example, then a larger share of aggregate income is paid to accumulated factors in the absence of external effects.[48] Mankiw, Romer, and Weil (1992) specify such an "augmented" model of growth convergence, estimate it using school enrollment data as a proxy for human capital accumulation, and

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[45]As pointed out by Carroll and Weil (1994), however, saving and growth rates are jointly endogenous in general, and the direction of causality may differ from that implicit in standard growth regressions.

[46]As noted by Barro (1997), the dynamics of the variables determining country-specific steady states are hard to characterize with available data; they may, however, be as important as the convergent dynamics emphasized by cross-sectional empirical work.

[47]Sala-i-Martin (1996a,b) finds similar rates of unconditional convergence across regions within the same countries, and argues that —since (omitted) conditioning variables should be less heterogeneous across regions than across countries—this offers additional support for conditional convergence specifications.

[48]Mulligan and Sala-i-Martin (1997) discuss how a measure of the relevant human capital may be constructed from wage and schooling data under the hypothesis that factor-income distribution reflects marginal productivities. Benhabib and Spiegel (1994) discuss various possible specifications for empirical counterparts of the theory's human capital stock.

find that once savings and population growth rates (but not technology) are allowed to be heterogeneous across countries the model is capable of interpreting the evidence if the output share of human capital is about one-third.

Further, country observations certainly do not carry information about completely separate experiences. As originally pointed out by Feldstein and Horioka (1980), savings and investment rates covary strongly across countries, but factors and goods do flow across countries' borders (see Obstfeld, 1995, for references to recent contributions). In the extreme case where countries belong to an integrated world economy, country i's per capita national income can be written in the form

$$y^i = Rk^i + Wl^i \tag{74}$$

if k^i and l^i denote the per capita amounts of accumulated and non-accumulated factors of production owned by its residents, and R and W denote factor prices in the world market. Since the expression in (74) coincides with the definition of an individual's income in the models of Section 2, those models yield the same implications for the dynamics of income across countries as they did above for the evolution of income inequality within a single macroeconomy. As in the model of Section 2.1.1, a common rate of return on heterogeneous wealth levels implies convergence if the propensity s to save out of total income is a given constant, since the proportional growth rate of national income is larger for countries who earn a lower portion of their income from accumulated factors. As long as complete specialization does not occur—hence in the single-good framework of the simplest macroeconomic models—unrestrained mobility of just one of the factors generally suffices to equalize the price of both. Further, trade can effectively substitute for factor mobility. Ventura (1997) models the world economy as a collection of fully integrated small open economies which, by trading intermediate inputs, can essentially rent each other's baskets of accumulated and non-accumulated factors even in the absence of financial capital flows. In this setting, conditioning upon differences in productive efficiency (or, in the notation used here, on the available amount of non-accumulated factors) yields empirically plausible convergence rates if the rate of return to aggregate accumulation is (mildly) decreasing along the transition to a steady state of endogenous or exogenous growth. Caselli and Ventura (1996) suggest that findings of "conditional" convergence may also be interpreted by a model similar to those reviewed in Section 2.1.2, allowing for heterogeneity in $\bar{c}$, and emphasize that incomes do converge towards each other when such "required consumption" parameters are negative. Smaller degrees of international integration are featured in other models, such as those proposed by Barro, Mankiw, and Sala-i-Martin (1995) where physical, but not human capital accumulation may be financed across country borders. Then, *per capita* domestic production differentials reflect differences in *per-capita* human capital levels, even if technologies are identical, and imply slow convergence of domestic production levels if the elasticity of production differentials with respect to (slowly) evolving human capital stocks is large.

The models underlying such interpretations of the empirical evidence rule out at least some economic interactions across countries, but maintain the assumption of identical technologies and homogeneous long-run growth rates. In reality, different technologies (or different *per capita* endowments of non-accumulated factors) presumably do play an important role in determining cross-sectional inequality and income dynamics, and less than instantaneous technological spillovers across countries may explain much of the observed dynamics of per capital incomes.[49]

More generally, the extent to which optimal savings behavior and factor accumulation lead to convergence depends on the character of investment opportunity sets, hence on the extent of cross-country financial market integration. The various insights discussed above are in general as important at the cross-country level as in the income distribution models of Section 3 above. The empirical results of Durlauf and Johnson (1994), Quah (1996a,b) and others indicate that convergence is stronger within subsets of countries with similar income levels, suggesting that lack of overall convergence is the result of increasing polarization of income levels across groups of countries which *do* converge towards each other. These findings are consistent with a country-level interpretation of models where, like in Section 3.2.2 above, non-convex investment opportunity sets generate poverty traps.

Obstfeld (1994), Devereux and Saito (1997), and others propose

[49] Ciccone (1996) explicitly allows countries' production to exert external effects on each other, and can estimate the strength of these effects under the identifying assumption that external effects should be more important across neighboring countries.

cross-country interpretations of models (reviewed in Section 3.3.1 above) of the growth and distributional implications of financial market integration. Again in an international framework of analysis, Devereux and Smith (1994) propose a model featuring both rate-of-return (as in Section 3.3.1) and endowment (as in Section 3.3.2) cross-country risk. Marcet and Marimon (1992) model international capital flows taking realistic information asymmetries and default risk into account, along the lines of closed-economy models of distribution discussed above. Ghosh and Ostry (1994) study the current account implications of precautionary-savings behavior; Ogaki, Ostry, and Reinhart (1996) and their references study macroeconomic savings data in an optimizing framework of analysis, treating all individuals within each country on a representative-agent basis. Daniel (1997) points out that the role of precautionary savings in ensuring stability of income, wealth, and consumption distributions can be as important at the cross-country level as within closed economies: lower wealth levels make precautionary behavior a more important determinant of saving behavior, and poor countries tend to accumulate wealth faster than rich ones for the reasons outlined above.

5.2 Growth and inequality within countries

The closed-economy models of Sections 2 and 3 above featured many different channels of interaction between distribution across individuals and macroeconomic growth. In the context of the models reviewed in Section 5.1, the distribution of income across countries might in principle bear similarly on issues of worldwide growth. The extent of *per capita* inequality across countries, however, is only indirectly relevant to the arguably more important issue of inequality across individuals, and empirical work on growth and distribution prefers to relate country-specific growth performances to the relatively limited information available on income distribution within countries.

The relevant literature dates back to at least Kuznets (1955). Finding statistical evidence of decreasing inequality along the growth path of developed countries, Kuznets discussed how it might be interpreted along much the same lines as those of subsequent theoretical literature (and of this Chapter)—arguing in particular how redistributive policies and finite lifetimes may offset theoretical mechanisms of wealth concentration—but privileged structural change as a source of U-shaped inequality dynamics along economic development paths,[50] while offering a lucid discussion of how data limitation may limit the progress of any such empirical work. While Kuznets's original U-curve intuition has remained somewhat elusive in available data, the subsequent literature, recently reviewed and extended by Bénabou (1996c), confirms that income distribution is far from unrelated to macroeconomic phenomena. The empirical evidence is far from settled (see Benabou 1996c, Aghion and Howitt 1998, Ch.9, and Deininger and Squire 1996a,b for recent surveys and updates). It does indicate, however, that relatively high degrees of income inequality are tenuously associated, in cross-country growth regressions, with relatively low income levels and slow growth rates. This is perhaps suprisingly less ambiguous than the predictions of theoretical models, where higher inequality may be associated with faster or slower growth; and it is certainly interesting to find that measures of within-country inequality are not unrelated to macroeconomic growth performances, as would be implied by the neoclassical models of Section 2, where savings and investment rates are by construction independent of income and consumption inequality or, indeed, by the representative-agent view of country-level data implicit in Section 5.1's perspective on international income convergence.

Though the direction of causality is perhaps unavoidably difficult to ascertain in practice, this empirical evidence can in principle be brought to bear on those among the models reviewed above which identify specific causal links running from inequality to income growth—namely, on models of financial market imperfections from Section 3, and models of factor share determination and politico-economic interactions from Sections 2.2.2 and 4.

Extensive theoretical and empirical work on the role of financial markets in economic development has recently been reviewed by Levine (1997), and somewhat inconclusive evidence on the interaction of simple indicators of financial development with inequality is discussed by Bénabou (1996c). Some evidence is also available on the theoretical role of relative factor prices and factor shares. Lindert and Williamson (1985) argue that most of the variability in personal income distribution (across

[50] The class of single-sector macroeconomic models reviewed here has no room for structural change, of course. Adelman and Robinson (1989) survey subsequent related work in the field of development economics.

time and space) is due to variations in factor rewards rather than by variations in individuals' factor bundles, and Bourguignon and Morrisson (1990) find that land concentration and mineral resource endowments are closely associated with measures of inequality in cross-country data. While the dynamics of aggregate income and of inequality do not appear to be causally related in any obvious way, the evidence in Deininger and Squire (1996b) and Persson and Tabellini (1992) suggests that land ownership concentration appears unambiguously relevant to subsequent economic growth. If land ownership proxies for the distribution of wealth, such findings lend support to the idea that an inegalitarian wealth distribution exacerbates financial market imperfections, with particularly strong implications for accumulation of human capital and of other factors which are likely to face binding self-financing constraints. The empirical relevance of land as a source of income may also indicate that, as in the Classical framework of analysis and in the optimization-based models of Section 2.2.2, the role of factor income distribution in the determination of macroeconomic growth is to some extent independent of that of the size distribution of income.

As to politico-economic channels of causation from inequality to growth, it is difficult to disentangle *ceteris paribus* effects of inequality (even when pre-tax inequality is treated as an exogenous variable) from those of the distortionary policies that may, depending on the structure of political interactions, be equilibrium outcomes for different levels of inequality. The careful work of Perotti (1996) indicates that, across countries, before-tax inequality tends to be compounded rather than reduced by fiscal policies, contrary to what would be implied by models where (high) inequality causes slow growth via redistribution. These results are perhaps more indicative of theoretical models' simplicity than of the practical relevance of theoretical models. Distributional tensions do matter, but they presumably do so in many subtle ways, by creating institutional and market conditions more or less conducive to efficiency and to adequate incentives to private investment. As Bénabou (1996c) points out, when *status quo* inequality slows down growth by decreasing the efficiency of investment allocation, transfers and subsidies should indeed be conducive to more equal investment opportunities and faster growth. Moreover, the threat of expropriation is sufficient to reduce investment incentives and, if the status quo degree of inequality is preserved by the resulting low investment level, no actual transfers need be observed. The empirical literature in this field addresses this potential mechanism by introducing measures of politicol-economic instability and of property right enforcement in cross-sectional growth regressions (Alesina and Perotti, 1996).

6 Other directions of research

This Chapter has focused on interactions of income and wealth distribution with growth and accumulation and, even within this already narrow scope, has necessarily neglected many relevant aspects of the literature. Of course, many other aspects of macroeconomic performance—such as inflation, which plays a prominent role in Barro's (1997) growth regressions—are theoretically and empirically related to growth, and to inequality (see e.g. Beetsma and van der Ploeg, 1996, for an exploration of linkages between inequality and inflation). This brief concluding section outlines how recent and less recent research has addressed three sets of issues which, though eminently relevant to the subject matter, have found little or no room in this Chapter.

First, almost all of the simple models and insights of the Chapter have been framed in terms of a single-good macroeconomy, with only a passing reference at the end of Section 2.2.2 to ways in which relative prices, income distribution, and aggregate dynamics may be jointly determined in the context of multi-sector models with many output goods. Such issues are central to many classical models of long-run growth and value determination, of course, and may to some extent be analyzed abstracting from capital accumulation (as in Pasinetti, 1993). Recent contributions study the macroeconomic role of sectoral output composition in a representative-agent setting (Kongsamut, Rebelo, and Xie, 1997), or in models where heterogeneous (Glass, 1996) or non-homothetic (Zweimüller, 1995) preferences can make income distribution relevant to the speed of innovation-driven growth.

Second, the Chapter has analyzed distribution and growth in a long-run setting, with only limited attention to transitional dynamics in Sections 2 and 5.1. It is of course difficult to isolate long-run, accumulation-driven dynamics from cyclical phenomena in empirical data, and all variables, parameters, and functions featured by the theoretical models above could in principle be allowed to depend on aggregate shocks. Realistic macroeconomic models would need to account for cyclical unemployment, for discrepancies between intended savings and investment, for monetary exchange technologies, and for price stickiness. Of course, it is extremely complex to model all or most of such features without relying on the representative agent paradigm. Cyclical dynamics have been studied in the context of models that, like those outlined in Section 2.2, took for granted a link between factor-income sources and savings propensities, without rationalizing consumption choices on a forward-looking basis; see, for example, Goodwin (1969) for a model of unemployment-based distributive dynamics and endogenous cycles. Many recent contributions exploring the role of incomplete financial markets in the "real business cycle" extension of neoclassical growth models could have been reviewed here but for space constraints. Interested readers should consult the methodological survey by Rios-Rull (1995), the empirically motivated analysis of Krusell and Smith (1997), and their references.

Finally, while wealth-driven inequality has played the most important role in this survey, earned-income inequality is far from unrelated to macroeconomic dynamics—and may be analyzed along much the same lines as those of this Chapter if not only standard savings and investment choices, but also the accumulation of human capital and the introduction of new technologies respond to economic incentives and politically determined policies (see Aghion and Howitt 1998, Chapter 9 for an introduction to these and other issues, and references to the literature).

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