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Price/Wage Staggering and Persistence*

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Abstract

Recent literature reaches contrasting conclusions on the ability of price/wage staggering models to generate output persistence. We derive fairly general results from a stylised log-linear model which encompasses most of the microfounded model of price/wage staggering. Our results highlight the features of the underlying economy which are crucial in generating output persistence: substitutability between goods and/or labour types and factor immobility. Moreover, we show that persistence do not generally depend on the particular value assigned to the intertemporal elasticity of labour supply, which has so far been the focus of this literature.

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1 Introduction

The overlapping contracts models of Calvo (1983) and Taylor (1980) have been widely used in the business cycle literature. Indeed, they not only introduce the nominal rigidity necessary for the impact effect of a monetary innovation, but also provide a nominal propagation mechanism which is in principle capable of generating persistence of the real effects of money shocks.

However, some recent papers (i.e., Chari et al. (1996) and Ascari (1997)) have seriously questioned the explanatory power of staggered price/wage setting in accounting for output persistence for reasonable parameter values. On the other hand, some others (i.e., Rotemberg and Woodford (1997) and Erceg (1997)) claim that their models can match the observed degree of persistence. Hence, it seems there is no consensus in the literature. Besides, comparison among existing models is far from straightforward since Chari et al. (1996) focus on price staggering, Ascari (1997) on wage staggering, Erceg (1997) on both and Rotemberg and Woodford (1997) employ the yeoman-farmer hypothesis.

In this paper, we provide a general framework to clarify the issue of persistence of real effects of money shocks in staggered wage/price models. Our aims are to: (i) highlight the differences between price and wage staggering; (ii) analyse which features of the underlying economy with superimposed price/wage staggering are crucial for generating output persistence; (iii) rank the different potential specifications according to their ability to generate output persistence. In order to do so, we build a stylised log-linear model that encompasses (most of) the existing microfounded models of staggered prices/wages.

Our results highlight that: (i) if the marginal cost curve is upward sloping then price staggering generally delivers a higher degree of persistence than wage staggering. (ii) Substitutability between goods and/or labour types plays the major role in generating persistence. In models with only substitutability between goods, price staggering naturally delivers higher persistence than wage staggering. The opposite is true in models with substitutability between labour types. (iii) To generate persistence together with some substitutability between goods and/or labour, we need some kind of factor immobility; in particular the distinction between free mobility and no mobility of labour is fundamental. No-mobility-of labour models (both "industrial" and "craft" union models) bring in new mechanisms that increase persistence, strengthening the effects of substitutability. Quantitatively, the difference is then going to rest upon the quantitative difference between the relevant elasticities of substitution (i.e., elasticities of labour demands). (iv) Although a substantial (in the sense of near random walk behaviour) degree of persistence is unlikely in price/wage staggering models, two models can deliver significant persistence: the yeoman farmer model (with price staggering) and the "craft" union models with wage staggering. (v) *Ceteris paribus* (i.e., for realistic values of all the other parameters), these conclusions do not depend on the particular value assigned to the intertemporal elasticity of labour supply, which has so far been the focus of this literature.

Note that the first result seems at odds with Andersen's (1998a) that wage staggering models deliver higher persistence than price staggering models. However, our analysis shows that Andersen's (1998a) finding is not due to an intrinsic difference between wage and price staggering models, but to the particular assumptions in the two cases there presented. The results are useful not only for interpreting the existing literature but also for those who might consider undertaking further research in this area. Specifically, the combination of nominal and real rigidities has recently received renewed attention (e.g., Jeanne (1998), Andersen (1998b), Kiley (1997), Bergin and Feenstra (1998), Ascari and Garcia (1998)), a line of research that is likely to continue in the near future. We therefore think that our analysis of the effects of the deep parameters of the underlying economy on the degree of persistence will prove enlightening.

2 A Perfectly Flexible Wage and Price Microfounded Model

In this section we sketch a perfectly flexible wage/price log-linear model which can be thought of as derived from a log-linearised version of a microfounded model. The model is very general in its formulation and could easily be derived as a log-linearised version of most models of monopolistic competition such as the one in Blanchard and Kiyotaki (1987) or the different versions presented by Dixon and Rankin (1994) in their survey. Even if all the equations are log-linear the model is not *ad hoc*; we will keep referring to the underlying microeconomic structure of the several versions of the model in the next section.

The size of the whole economy is normalised to 1 and thus the economy consists of a continuum of industries indexed by $i \in [0, 1]$.[1] Every industry produces a single differentiated perishable product. Households are indexed by $j \in [0, 1]$ and they live forever. All firms have the same technology and households have the same preferences. Preferences are CES over consumption goods which are gross substitutes. In order to ease the introduction of staggering in the next section, the supply side of the economy is divided into two sectors: A and B, of equal size (one half) for simplicity.

Firms are identical and labour is the only factor of production. From a short-run production function (in levels) of the form[2] $Y_t = \alpha L_t^{\sigma}$, the sectors' supply functions are (lower case variables denote log-

[1]The fact that we have a continuum (a 'large' number) of agents means that each firm and household takes the aggregate variables as given. Hence, there is no strategic interaction among them. We often just present the formulas for sectors, obtained simply by aggregating across typical firms or households belonging to the sector. However, it is not the same as having only one agent for each sector. If that was the case we would need to take strategic interactions into account.

[2]We interpret our production function as a short-term production function where capital is embedded in the fixed term and cannot be adjusted. Another way to look at it is to assume that capital is industry specific and hence cannot be moved across industries. See Erceg (1997) for an enlightening discussion of the implications of such an assumption.

deviations from steady state)

$$p_{A_t} = w_{A_t} + a y_{A_t} \tag{1}$$

$$p_{B_t} = w_{B_t} + a y_{B_t} \tag{2}$$

where $a = \frac{1-\sigma}{\sigma}$; p_{X_t} = price of sector X for $X = A, B$; w_{X_t} = nominal wage in sector X; y_{X_t} = sector X's real output. Note that (1) and (2) hold regardless of whether the goods market is competitive or monopolistic, as long as each firm faces a demand curve with constant elasticity.[3] The only difference between the two cases, in fact, is the presence of a constant mark-up (given that the elasticity of demand is constant) in the equation in levels for the monopolistic goods markets. Yet in the log-linearised version of the model the constants disappear. The two cases thus have the same log-linear formula for the supply functions of a typical firm.

Industries produce differentiated goods and θ is the elasticity of substitution among the goods.[4] Industries' demands are derived from a log-linearisation of the standard Dixit-Stiglitz formula

$$y_{A_t} = \theta(p_t - p_{A_t}) + y_t \tag{3}$$

$$y_{B_t} = \theta(p_t - p_{B_t}) + y_t \quad . \tag{4}$$

Households are also divided into two groups of equal size (one half), C and D. The standard first-order condition for labour supply (in levels) equates the real wage to the ratio of marginal disutility of labour and marginal utility of consumption $\left(\text{i.e., } \frac{W_t}{P_t} = \frac{-U_l}{U_c}\right)$. Assuming an additively separable utility function, we can write

$$w_{C_t} = \eta_{ll} l_{C_t} + \eta_{cc} c_{C_t} + p_t \tag{5}$$

$$w_{D_t} = \eta_{ll} l_{D_t} + \eta_{cc} c_{D_t} + p_t \tag{6}$$

where w_{S_t} = nominal wage in group S, for $S = C, D$; l_{S_t} = amount of labour supplied by a typical household in group S ; c_{S_t} = consumption

[3] This is the usual assumption in monopolistic competition macromodels.

[4] Despite the fact that industries produce differentiated goods, we can still regard a competitive goods market as encompassed by the formulas (1) and (2). This would be the case in a framework where there is a 'large' number of firms in each industry.

of a typical household in group S ; $\eta_{ll} = \frac{U_{ll}\, l}{U_l} > 0$ is the elasticity of the marginal disutility of labour with respect to labour; $\eta_{cc} = \frac{-U_{cc}\, c}{U_c} > 0$ is (minus) the elasticity of the marginal utility of consumption with respect to consumption. Throughout, we will assume that η_{ll} and η_{cc} are constant (as for most of the utility functions used in macromodels). Given that we have assumed an additively separable underlying utility function, η_{ll} and η_{cc} are the inverses of the intertemporal elasticity of substitution in labour supply and consumption respectively. Moreover, η_{cc} coincides with the income effect on labour supply.[5]

The presence of some heterogeneity across infinitely-lived households in macromodels creates an analytical problem. The usual way to deal with heterogeneity and avoid any distributional complications is to assume complete markets. Agents can then completely insure themselves against idiosyncratic shocks or unpredictable fluctuations. This implies that the marginal utility of consumption is equalised across households each period. Hence, given an additively separable utility function, households consume the same in each period. Since the goods market equilibrium implies $y_t = \frac{1}{2}(c_{C_t} + c_{D_t})$, each of the two groups consumes half of the real output each period, that is

$$\frac{1}{2}c_{C_t} = \frac{1}{2}y_t \tag{7}$$

$$\frac{1}{2}c_{D_t} = \frac{1}{2}y_t \quad . \tag{8}$$

Aggregate demand derives from a standard money demand equation[6]

$$c_t = y_t = m_t - p_t \tag{9}$$

[5] As was the case for the supply functions for the goods market, i.e., (1) and (2), the supply functions in the labour market, i.e., (5) and (6) hold both in a competitive labour market and in a labour market characterised by monopoly unions, provided labour demand functions have a constant elasticity.

[6] Note that we impose a constant-velocity-of-circulation aggregate demand function because we want to focus on the supply side of the model. Even if this aggregate demand looks static, (9) can be derived from intertemporal optimisation in two cases. First, as Bénassy (1995) shows, the velocity of circulation of money is constant in an intertemporal optimising model whenever money is injected only through interest

where consumption is equal to output, since capital is fixed. The aggregate price level is just the average of the two sectors' prices (since firms belonging to the same sectors will set the same price independently)

$$p_t = \frac{1}{2}(p_{A_t} + p_{B_t}) \quad . \tag{10}$$

Moreover, (3), (4) and (10) yield

$$y_t = \frac{1}{2}(y_{A_t} + y_{B_t}) \tag{11}$$

which simply states that aggregate output is given by the weighted average of outputs in a typical industry of each sector.

So far we have presented a rather general perfectly flexible wage/price log-linear model whose equations are consistent with virtually any monopolistic competition macromodel. However, we have a perfectly flexible wage/price model with 14 variables (i.e., y_t, p_t, p_{A_t}, p_{B_t}, w_{A_t}, w_{B_t}, y_{A_t}, y_{B_t}, l_{C_t}, l_{D_t}, w_{C_t}, w_{D_t}, c_{C_t}, c_{D_t}), one exogenous variable, i.e. m_t, and 10 equations ((9), (10), (1), (2), (3), (4), (5), (6), (7), (8)). The four missing equations are the sectors' labour demands, l_{C_t} and l_{D_t} and two equations describing the links between w_{A_t}, w_{B_t}, and w_{C_t}, w_{D_t}. These equations will be different according to the type of labour market structure we assume. As we will see, different labour market structures turn out to be crucial for the persistence properties of the staggered version of the model.

To introduce staggering in the model we proceed in the following way. The first step is to close the perfectly flexible wage/price model, that is, to write down the missing equations according to the different labour market structures we will analyse in the next section. The second step is to find the optimal rule in the perfectly flexible wage/price model

payments on bonds. Second, given an additively separable utility function as in Ascari (1997), the log-linearised aggregate demand is equal to $y_t = m_t - p_t + z_t$, where the last term evolves according to a pure forward-looking equation. Following a temporary i.i.d. money shock, z_t jumps immediately to the (unchanged) steady state, thus it stays constant. If money shocks are temporary and not autocorrelated then z_t has no effect on persistence and we can just impose (9) on the model.

for the nominal variable we want to stagger (price or wage), given the labour market structure. The third step transforms the perfectly flexible wage/price model into a staggered model by assuming that the nominal variable of interest: (i) is set for two periods in a staggered fashion; (ii) is simply given by the average of today's and tomorrow's optimal rules (which have just been found for the perfectly flexible wage/price model in step two). Thus, our approach is very similar to Blanchard and Fischer (1989), Chapter 8. The final step is to express the model in the following reduced form[7]

$$x_t = \frac{1}{2}(p_t + p_{t+1}) + \frac{1}{2}\,\gamma\,(y_t + y_{t+1}) \tag{12}$$

$$p_t = \frac{1}{2}(x_{t-1} + x_t) + hy_t \tag{13}$$

$$y_t = m_t - p_t \quad . \tag{14}$$

where x denotes the staggered variable of interest (sectors' price or wage). The solution for the stable root of this system is given by

$$\lambda = \frac{1 - \sqrt{\frac{h+\gamma}{h+1}}}{1 + \sqrt{\frac{h+\gamma}{h+1}}} \equiv \frac{1-\sqrt{R}}{1+\sqrt{R}} \quad . \tag{15}$$

λ also represents the persistence root for output and will be the focus of the analysis. For convenience in the next sections we will actually focus on R because of the direct link between λ and R: the higher R, the lower λ and persistence.

We investigate the persistence properties (i.e., R) of several different models according to the type of labour market we consider and the nominal variable (price or wage) we stagger. We show how each of these different models corresponds to a model in the literature and can be expressed in the same reduced form as above. The implications of the

[7] Quite obviously, the same result can be obtained by directly staggering the equation for the nominal variable of interest in the perfectly flexible model and then substituting out to get the reduced form. It does not matter the order in which step two and three are performed. Moreover, future variables should be expected. Given that we will analyse log-linear models, though, the expectation operator has no effect on persistence properties. Thus, we will omit it for simplicity.

alternative economic structures for persistence will then be immediately evident.

3 Staggered Models

In this section we present different wage/price staggering models, derived from the perfectly flexible wage/price model of the previous section. The discussion and comparison among all these different models is postponed to section 4.

3.1 Staggered Prices and Perfect Labour Mobility: Chari et al. (1996)

First, we need to close the perfectly flexible wage/price model. If labour is completely mobile across sectors then households supply labour to both sectors in the economy and the wage is equalised across households and firms, i.e. $w_{A_t} = w_{B_t} = w_{C_t} = w_{D_t}$. Given (5), (6), (7) and (8), the two groups of households supply the same amount of labour. Thus

$$l_{C_t} = l_{D_t} = l_t = \frac{1}{\sigma}\, y_t \tag{16}$$

and this gives the sectors' labour demands in the perfectly flexible wage/price model.

Second, we find the optimal rule in the perfectly flexible wage/price model for the variable we want to stagger: sectors' prices. Substituting (16) in (5) and (6), and using (7) and (8), yields[8]

$$w_{C_t} = w_{D_t} = w_t = p_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t \equiv p_t + \widetilde{\gamma}\; y_t \quad . \tag{17}$$

[8] To avoid confusion, it is better to stress that we denote $\widetilde{\gamma}$ as the elasticity of real wage with respect to aggregate output ($w_{X_t} - p_t = \widetilde{\gamma} y_t$) and $\overline{\gamma}$ as the elasticity of sectors' price with respect to aggregate output ($p_{X_t} - p_t = \overline{\gamma} y_t$). γ is instead the elasticity of the staggered variable with respect to aggregate output in the reduced form (12). Thus γ will coincide with $\widetilde{\gamma}$ in staggered wage models and with $\overline{\gamma}$ in staggered price models.

Use this expression in (1) and (2) and substitute out for sectors' output by making use of (3) and (4) to obtain

$$p_{A_t} = p_{B_t} = p_t + \left(\frac{\widetilde{\gamma} + a}{1 + a\theta}\right) y_t \equiv p_t + \overline{\gamma} y_t \qquad (18)$$

which is the optimal pricing rule of sector-A firms as a function of p_t and y_t.

Third, we introduce staggering. We then suppose that each firm in a given sector, acting independently, sets its price for two periods in a staggered fashion. That is, firms in sector A fix the price in even periods, firms in sector B fix it in odd periods. Thus

$$p_{i_{t+s}} = p_{i_{t+s+1}} = \frac{1}{2}(p_{t+s} + p_{t+s+1}) + \frac{1}{2}\overline{\gamma}(y_{t+s} + y_{t+s+1}) \qquad (19)$$

$$for\ i \in A \text{ and } for\ s = 0, 2, 4...$$

$$p_{i_{t+s-1}} = p_{i_{t+s}} = \frac{1}{2}(p_{t+s-1} + p_{t+s}) + \frac{1}{2}\overline{\gamma}(y_{t+s-1} + y_{t+s}) \qquad (20)$$

$$for\ i \in B \text{ and } for\ s = 0, 2, 4... \quad .$$

Aggregating across sectors and denoting the staggered variable by x_t, the reduced form of the model can be written as[9]

$$x_t = \frac{1}{2}(p_t + p_{t+1}) + \frac{1}{2}\ \overline{\gamma}\ (y_t + y_{t+1})$$

[9] As explained in footnote 7 above, we can reverse the order of steps two and three. That is, the same expressions can be obtained by staggering at the outset (1) and (2) for a typical firm in each sector. For a typical firm i in sector A

$$p_{i_t} = p_{i_{t+s+1}} = \frac{1}{2}(w_{t+s} + w_{t+s+1}) + \frac{a}{2}(y_{i_{t+s}} + y_{i_{t+s+1}}) \qquad for\ i \in A \text{ and } for\ s = 0, 2, 4...$$

and then aggregate across sector A and substitute out for w_t and y_{A_t} using (17) and (3).

$$p_t = \frac{1}{2}(x_{t-1} + x_t)$$
$$y_t = m_t - p_t$$

where $\overline{\gamma} = \left(\frac{\widetilde{\gamma}+a}{1+a\theta}\right)$. This reduced form corresponds to the one of the previous section (i.e., (12), (13) and (14)) with $\gamma = \overline{\gamma}$ and $h = 0$. The solution for the root governing output is

$$\lambda_1 = \frac{1-\sqrt{R_1}}{1+\sqrt{R_1}} \quad ; \quad \text{where} \quad R_1 \equiv \overline{\gamma} = \frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{\sigma + \theta\ (1-\sigma)} \quad . \tag{21}$$

We can also write the model as follows. Consider the staggered version of (1)

$$x_t = \frac{1}{2}(w_t + w_{t+1}) + \frac{a}{2}(y_{A_t} + y_{A_{t+1}}) \tag{22}$$

which can be expressed as

$$\begin{aligned} x_t &= \frac{1}{2}(x_{t-1} + x_{t+1}) + (\omega_t + \omega_{t+1}) + a(y_{A_t} + y_{A_{t+1}}) = \\ &= \frac{1}{2}(x_{t-1} + x_{t+1}) + \left(\frac{1}{1+a\theta}\right)(\omega_t + \omega_{t+1}) + \left(\frac{a}{1+a\theta}\right)(y_t + y_{t+1}) \end{aligned} \tag{23}$$

where $\omega_t = (w_t - p_t)$ represents the real wage. If $\sigma = 1$ (constant returns to labour), then $a = 0$ and equation (23) exactly matches equation (46) in Chari et al. (1996), p. 13. Moreover, in this case $\overline{\gamma} = \widetilde{\gamma} = \eta_{ll} + \eta_{cc}$, exactly as their γ on page 15.

The model is also a generalisation of the *ad hoc* price staggering model of Andersen (1998a), which is obtained by setting $\sigma = 1$.

3.2 Staggered Prices and No Mobility of Labour: the Yeoman-Farmer Model: Blanchard and Fischer (1989) and Rotemberg and Woodford (1997)

In this case, households of group C work for firms in sector A while households of group D work for firms in sector B. The model is equivalent to the yeoman-farmer model where each household produces a differentiated good and there is no labour market. Thus, the missing equations in the perfectly flexible wage/price model are: $l_{C_t} = l_{A_t} = \frac{y_{A_t}}{\sigma}$, $w_{A_t} = w_{C_t}$ and $l_{D_t} = l_{B_t} = \frac{y_{B_t}}{\sigma}$, $w_{B_t} = w_{D_t}$. The wage equation in the perfectly flexible wage/price model is given by

$$\begin{aligned} w_{A_t} &= \eta_{ll} \frac{y_{A_t}}{\sigma} + \eta_{cc} y_t + p_t = \\ &= p_t + \left[\frac{\eta_{ll} + \eta_{cc}(1+\theta a)\sigma}{\sigma(1+\theta a) + \theta \eta_{ll}} \right] y_t = p_t + \widetilde{\gamma}\, y_t \quad . \end{aligned} \tag{24}$$

(24) corresponds to (17) in the previous model, and we can proceed following the same steps as before to derive the optimal pricing rule for the farmer. However, apart from the different expression for $\widetilde{\gamma}$, the model is analytically equivalent to the previous one. Thus, we know the solution is $\lambda_2 \equiv \frac{1-\sqrt{R_2}}{1+\sqrt{R_2}} = \frac{1-\sqrt{\overline{\gamma}}}{1+\sqrt{\overline{\gamma}}}$ where $\overline{\gamma} = \frac{\widetilde{\gamma}+a}{1+a\theta}$. Again, the persistence properties of this model depend only on

$$R_2 = \overline{\gamma} = \frac{\eta_{ll} + \sigma \eta_{cc} + 1 - \sigma}{\sigma + \theta\,(1-\sigma) + \theta \eta_{ll}} \quad . \tag{25}$$

The model encompasses two recent yeoman-farmer models: Blanchard and Fischer (1989) and Rotemberg and Woodford (1997).[10] Normally, a standard yeoman-farmer model includes a term $V(y_i)$ in the utility function to represent the disutility from producing for the farmer. The standard first order condition that gives the optimal price in a farmer model is (in levels)

$$\frac{P_i}{P} = \left(\frac{\theta}{\theta - 1} \right) \left(\frac{V_{y_i}(y_i)}{U_{c_i}(c_i)} \right) \quad . \tag{26}$$

It states that the optimal ratio of the price of the good produced by the farmer to the aggregate price level is given by a constant mark-up over the

[10] This is quite a strong claim but should be thought as limited to the issue we are concerned with here (i.e., the elasticity of the real wage with respect to output and its implication for persistence). In reality, apart from this point the two models are very much different. Rotemberg and Woodford (1997): (i) have a Calvo-type structure for price staggering; (ii) have other rigidities to help the model match the impulse response function of an unrestricted VAR; (iii) are mainly concerned with another issue: computing optimal monetary policy in an optimising framework.

ratio of marginal disutility from effort in production to marginal utility of consumption. Log-linearising (26) for sector A households yields

$$p_{A_t} - p_t = \eta_{yy} y_{A_t} + \eta_{cc} y_t \tag{27}$$

where $\eta_{yy} = (V_{y_i y_i} y_i / V_{y_i}) > 0$ is the elasticity of the marginal disutility of production. Given our production function, i.e., $Y_t = \alpha L_t^{\sigma}$, simple algebra shows that $\eta_{yy} = (\eta_{ll}/\sigma + a)$. Substituting (3) in (27) gives

$$p_{A_t} - p_t = \left(\frac{\eta_{yy} + \eta_{cc}}{1 + \theta\eta_{yy}}\right) y_t = \left(\frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{\sigma + \theta\,(1 - \sigma) + \theta\eta_{ll}}\right) y_t = \overline{\gamma} y_t \tag{28}$$

which is the optimal pricing rule for the perfectly flexible wage/price model. Performing then step three, that is considering (28) under staggering, shows our model to be equivalent to Rotemberg and Woodford (1997) (see their formula for κ at p.316).[11]

Our model is also a generalisation of the Blanchard and Fischer (1989), Chapter 8, model, whose specification of preferences implies a zero income effect on labour supply (i.e., $\eta_{cc} = 0$). Then

$$p_{A_t} = p_t + \left[\frac{\eta_{yy}}{1 + \eta_{yy}\theta}\right] y_t = \left[\frac{1 + (\theta - 1)\eta_{yy}}{1 + \eta_{yy}\theta}\right] p_t + \left[\frac{\eta_{yy}}{1 + \eta_{yy}\theta}\right] m_t \tag{29}$$

which is equivalent to equation (9) of Blanchard and Fischer (1989), p. 385.[12]

[11] Again we stress that this claim is subject to the caveats of footnote 10. However, in order to match the data, the Rotemberg and Woodford (1997) model should generate some endogenous persistence. As far as persistence is concerned, the key parameter in their model is κ (see p. 316), which exactly corresponds to our formula for $\overline{\gamma}$ if one abstracts from the terms due to the particular definition of the staggered variable and to the Calvo-type structure (respectively (1-α)/α and (1-$\alpha\beta$) in their notation on p. 316).

[12] They use the following utility function for the yeoman farmer: $U_i = \left(\frac{C_i}{d}\right)^d \left(\frac{M_i/P}{1-d}\right)^{1-d} - \left(\frac{f}{\beta}\right) Y_i^{\beta}$. Hence, the elasticity of the marginal disutility of production is given by $(\beta - 1)$. Replacing η_{yy} by $(\beta - 1)$, equation (29) exactly matches theirs.

3.3 Staggered Wages and Perfect Labour Mobility: the *ad hoc* models

The combination of wage staggering and perfect labour mobility is unusual in microfounded models. The reason is very simple: there are no convincing microfoundations for this case. Indeed, if wages are staggered it means that someone must have the power to set them. That is, staggered wages should go together with monopoly unions which set the wages. The unions must therefore enjoy market power and there cannot be perfect labour mobility or a competitive labour market. However, this case is of interest to illustrate the difference between price and wage staggering. Furthermore, this is the so-called "expected-market–clearing-case" employed by the *ad hoc* literature of the 70's and 80's on staggered wage models (e.g., Gray (1976), Fischer (1977) and Taylor (1979)). Gray's (1976), Fischer's (1977) or Taylor's (1979) types of nominally rigid labour contract are set in order to achieve an *ad hoc* target wage level, which is the one that clears the labour market 'in expectation'.[13] In terms of our microfounded perfectly flexible wage/price model, this assumption implies that the workforce in each firm is equally divided between the two groups of workers. Then the reference wage for all the firms in each period is equal to $w_{A_t} = w_{B_t} = (1/2)\left(w_{C_t} + w_{D_t}\right)$. Moreover, all the firms would charge the same price and produce the same level of output. Hence, the one-period nominal wage contract will simply be

$$w_t = p_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t = p_t + \widetilde{\gamma}\ y_t \tag{30}$$

and under staggering

$$x_t = \frac{1}{2}(p_t + p_{t+1}) + \frac{1}{2}\widetilde{\gamma}(y_t + y_{t+1}) \quad . \tag{31}$$

The aggregate price level is given by

$$p_t = p_{A_t} = p_{B_t} = \frac{1}{2}(x_{t-1} + x_t) + ay_t \quad . \tag{32}$$

[13] Therefore, the assumption that employment is always on the labour demand curve is inconsistent with optimisation. This is not true in monopoly union models. There, the wage is above the competitive wage, and ex post it is optimal for the household-union to satisfy an unexpected increase in labour demand.

The reduced form of the model corresponds to the one in section 2 with $\gamma = \widetilde{\gamma}$ and $h = a = \frac{1-\sigma}{\sigma}$. The solution is given by $\lambda_3 = \frac{1-\sqrt{\frac{h+\gamma}{h+1}}}{1+\sqrt{\frac{h+\gamma}{h+1}}} \equiv \frac{1-\sqrt{R_3}}{1+\sqrt{R_3}}$, where

$$R_3 = \sigma\widetilde{\gamma} + 1 - \sigma = \eta_{ll} + \sigma\eta_{cc} + 1 - \sigma \quad . \tag{33}$$

This model also corresponds to the *ad hoc* staggered wage model in Andersen (1998a) when $\widetilde{\gamma} = 0$ is imposed. This restriction is at the root of Andersen's (1998a) result that wage staggering models are potentially able to deliver persistence while price staggering models are not. The intuition provided by Andersen (1998a) states that the adjustment burden in wage staggering models is borne by prices, while wages do not react to excess demand conditions. While this is correct as a description of the impact effect of a money shock when wages are preset (i.e., flat labour supply curve), the argument cannot be automatically transferred to the dynamic model. In other words, when workers renegotiate a new wage they will take into account the expected (labour) demand conditions (hence $\widetilde{\gamma}$ should be different from 0).

3.4 Staggered Wages and No Mobility of Labour: Ascari (1997)

Here, households of group C only work for firms in sector A while households of group D work for firms in sector B. Then, as in the yeoman-farmer model, equation (24) holds and staggering yields

$$x_t = \frac{1}{2}(p_t + p_{t+1}) + \frac{1}{2}\widetilde{\gamma}(y_t + y_{t+1}) \tag{34}$$

where now $\widetilde{\gamma} = \left[\frac{\eta_{ll}+\eta_{cc}(1+\theta a)\sigma}{\sigma(1+\theta a)+\theta\eta_{ll}}\right]$. The model is then analytically equivalent to the previous one, and the solution just depends on

$$R_4 = 1 + \sigma(\widetilde{\gamma} - 1) = \frac{[\sigma + \theta(1-\sigma)]\,[\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma]}{\sigma + \theta(1-\sigma) + \theta\eta_{ll}} \quad . \tag{35}$$

$\widetilde{\gamma}$ can be written as

$$\widetilde{\gamma} = \left[\frac{\eta_{ll} + \eta_{cc}(\sigma + \theta(1-\sigma))}{(\sigma + \theta(1-\sigma)) + \theta\eta_{ll}}\right] = \left[\frac{\frac{\varepsilon}{\theta}\eta_{ll} + \eta_{cc}}{1 + \varepsilon\eta_{ll}}\right] \tag{36}$$

which corresponds to the expression for g in Ascari (1997).

3.5 Staggered Wages and Craft Unions: Blanchard and Kiyotaki (1987) and Erceg (1997)

Another hypothesis used very widely to depict the labour market in microfounded models is that of Blanchard and Kiyotaki (1987). This hypothesis has been employed recently in works focusing on the persistence issue (e.g., Kim (1996), Erceg (1997)). The labour market is assumed to be composed of a large number of households that supply differentiated labour inputs. Firms regard each household's labour services as an imperfect substitute for the labour services of other households. Then, households who provide a particular labour service group together as a union, and act as wage-setters in the labour market. This labour market structure is sometimes called a "craft" union structure, while the one presented in the previous sections is an "industrial" union structure (see e.g., Dixon and Rankin (1994)). Indeed, in the first case unions are organised by labour skills, while in the second unions are characterised as specific to the industry to which its members supply labour. Note further that both cases imply a different kind of labour immobility. In a "craft" union labour market structure labour cannot move across skills, while in a "industrial" union labour market structure workers cannot move across industries.

With respect to the perfectly flexible wage/price model of section 2, the production function for firm i is now CES, that is (in levels)

$$Y_{i_t} = \left[\int_j L_{ij_t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\sigma\phi}{(\phi-1)}} \tag{37}$$

where ϕ is the elasticity of technical substitution between different types of labour inputs. This production function yields the following constant elasticity demand for labour type j of firm i

$$L_{ij_t} = \left[\frac{W_{j_t}}{W_t}\right]^{-\phi} Y_{i_t}^{\frac{1}{\sigma}} \tag{38}$$

where $W_t = \left[\int_j W_{j_t}^{1-\phi}\right]^{\frac{1}{1-\phi}}$ is the wage index (which exactly parallels the standard Dixit-Stiglitz price index for differentiated goods). Since all the firms face the same wage index, they will produce the same level of output. We can aggregate across firms and then log-linearise (38) to obtain the labour demand for labour type j in the whole economy (in log-deviations)

$$l_{j_t} = \phi(w_t - w_{j_t}) + \frac{1}{\sigma} y_t \quad . \tag{39}$$

Within each cohort of households, a symmetric equilibrium holds, that is they will fix the same wage. Hence, the missing equations to close the perfectly flexible wage/price model are

$$l_{C_t} = \phi(w_t - w_{C_t}) + \frac{1}{\sigma} y_t \tag{40}$$

$$l_{D_t} = \phi(w_t - w_{D_t}) + \frac{1}{\sigma} y_t \tag{41}$$

where

$$w_t = \frac{1}{2}(w_{C_t} + w_{D_t}) \quad . \tag{42}$$

Equation (40) matches equation (30) in Erceg (1997).

Substituting (40) and (41) respectively into (5) and (6), we get the optimal wage setting rule for households of group C and D, that is[14]

$$w_{C_t} = \left(\frac{1}{1+\phi\eta_{ll}}\right) \left[\phi\eta_{ll} w_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t + p_t\right] \tag{43}$$

$$w_{D_t} = \left(\frac{1}{1+\phi\eta_{ll}}\right) \left[\phi\eta_{ll} w_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t + p_t\right] \quad . \tag{44}$$

[14] Note that, unsurprisingly, (43) and (44) immediately imply that in the static model the two groups of households set the same wage, that is, a symmetric equilibrium holds. This is obviously not the case in the wage staggering model because the two cohorts of households set the wage in different periods.

Before staggering these two equations, it is convenient to substitute for the wage index using (42) into (43) and (44) to get

$$w_{C_t} = \left(\frac{1}{1+\frac{\phi\eta_{ll}}{2}}\right)\left[\frac{\phi\eta_{ll}}{2}w_{D_t} + \left(\frac{\eta_{ll}}{\sigma}+\eta_{cc}\right)y_t + p_t\right] \tag{45}$$

$$w_{D_t} = \left(\frac{1}{1+\frac{\phi\eta_{ll}}{2}}\right)\left[\frac{\phi\eta_{ll}}{2}w_{C_t} + \left(\frac{\eta_{ll}}{\sigma}+\eta_{cc}\right)y_t + p_t\right] \quad . \tag{46}$$

Now, using these two formulas to derive the staggered wage model, we can write the supply side as

$$\begin{aligned} x_t \quad = \quad & \left(\frac{1}{1+\frac{\phi\eta_{ll}}{2}}\right)\frac{1}{2}\left\{\left[\frac{\phi\eta_{ll}}{2}x_{t-1} + \left(\frac{\eta_{ll}}{\sigma}+\eta_{cc}\right)y_t + p_t\right]\right. \\ & \left. + \left[\frac{\phi\eta_{ll}}{2}x_{t+1} + \left(\frac{\eta_{ll}}{\sigma}+\eta_{cc}\right)y_{t+1} + p_{t+1}\right]\right\} \end{aligned} \tag{47}$$

$$p_t = p_{A_t} = p_{B_t} = w_t + ay_t = \frac{1}{2}(x_t + x_{t-1}) + ay_t \tag{48}$$

which, substituting out for x_{t-1} and x_{t+1} in (47), delivers the following reduced form

$$x_t = \frac{1}{2}\left(p_t + p_{t+1}\right) + \frac{1}{2}\left[\frac{\frac{\eta_{ll}}{\sigma}+\eta_{cc}-a\phi\eta_{ll}}{1+\phi\eta_{ll}}\right](y_t + y_{t+1}) \tag{49}$$

$$p_t = w_t + ay_t = \frac{1}{2}(x_t + x_{t-1}) + ay_t$$

$$y_t = m_t - p_t$$

where now $\widetilde{\gamma} = \left[\frac{\frac{\eta_{ll}}{\sigma}+\eta_{cc}-a\phi\eta_{ll}}{1+\phi\eta_{ll}}\right]$. The model again corresponds to the one in section 2 with $\gamma = \widetilde{\gamma}$ and $h = a$ and the solution hence is

$$R_5 = 1 + \sigma(\widetilde{\gamma} - 1) = \frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{1+\phi\eta_{ll}} \quad . \tag{50}$$

First, as Erceg (1997) noted, the group of households adjusting its wage upward following a positive money shock realises that it will experience some reduction in relative demand for its labour skill, according to the elasticity ϕ. This will make it choose a lower nominal wage increase. The intuition is confirmed by the following equation, obtained by substituting for the price level in (49)

$$x_t = \frac{1}{2}\left(x_{t-1} + x_{t+1}\right) + \left[\frac{\frac{\eta_{ll}}{\sigma} + \eta_{cc} + a}{1 + \phi\eta_{ll}}\right](y_t + y_{t+1}) \quad . \tag{51}$$

This is the equivalent of equation (31) in Erceg (1997). As $\phi \to \infty$ the coefficient on the aggregate demand term tends to zero and persistence tends to a unit root. In this case, the elasticity of sustitution between labour types is so high that wages would not change at all (note that $\gamma \to -a$ in (49) and $R_5 \to 0$).

Second, if $\phi = 0$ we are back to the "expected-market–clearing-case" of section 3.3. This explicitly demonstrates why the "expected-market–clearing-case" cannot be supported by any sensible microfoundations. In fact, a necessary condition for an interior solution of the "craft" union model is $\phi > 1$. If $\phi = 0$ then the monopoly union would face a demand curve with zero elasticity and would fix an infinite wage.

If we use the Blanchard and Kiyotaki (1987) labour market framework to derive implications for persistence in the price staggering model, we are obviously back to the case in section 3.1. In fact, if all households reset their wages in each period then a symmetric equilibrium holds and all will choose the same wage. Wages will be equalised across different "crafts". Analytically, it is like having just one type of labour and the parameter ϕ becomes irrelevant for the dynamics of the model. The optimal wage rule in the perfectly flexible wage/price model will simply be $w_{C_t} = w_{D_t} = w_t = p_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t$ as in (17). We get the same reduced form model as in section 3.1.

3.6 Liquidity Constraints

So far we have assumed the existence of complete markets, which implies that households always consume half of aggregate real output. In this section we make the other extreme assumption instead: workers are completely liquidity constrained. This assumption actually only matters in the "industrial" union - no labour mobility cases. Then, the liquidity constraint hypothesis would imply that households consume an amount equal to the output of the sector to which they supply labour (i.e., $c_{C_t} = y_{A_t}$, $c_{D_t} = y_{B_t}$).[15]

In the yeoman farmer model the marginal effort cost in production changes and equation (24) becomes

$$w_{A_t} - p_t = \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_{A_t} \tag{52}$$

while the rest of the model stays the same. Thus, repeating steps one to three, it yields

$$R_6 = \gamma = \left[\frac{a+\psi}{1+(a+\psi)\theta}\right] = \frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{\sigma + \theta\ (1-\sigma) + \theta\eta_{ll} + \theta\sigma\eta_{cc}} \quad . \tag{53}$$

Similarly, in the wage staggering case we have

$$R_7 = \frac{[\sigma + \theta(1-\sigma)]\,[\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma]}{\sigma + \theta\ (1-\sigma) + \theta\eta_{ll} + \theta\sigma\eta_{cc}} \quad . \tag{54}$$

4 Wage/Price Staggering and Persistence

We divide this section into three parts. In 4.1 we list some analytical results for the effects of the different parameters on the degree of persistence in the various models. In 4.2 we quantitatively assess the capability of the different models to generate persistence. In 4.3 we discuss the interpretation of the results in 4.1. and 4.2.

4.1 Analytical Results

As said above, according to $\lambda_i = \frac{1-\sqrt{R_i}}{1+\sqrt{R_i}}$, R_i is sufficient to describe the persistence properties of the models (the higher R_i, the lower persistence). Table 1 reports the different solutions delivered by the models.

[15] A further assumption is actually needed: households only receive profits from firms in their own sector.

	Price Staggering	Wage Staggering
Perfect labour mobility	$R_1 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{\sigma+\theta\ (1-\sigma)}$	$R_3 = \eta_{ll} + \sigma\eta_{cc} + 1 - \sigma$
No mobility of labour ("Industrial" unions)	$R_2 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{\sigma+\theta\ (1-\sigma)+\theta\eta_{ll}}$	$R_4 = \frac{[\sigma+\theta(1-\sigma)][\eta_{ll}+\sigma\eta_{cc}+1-\sigma]}{\sigma+\theta\ (1-\sigma)+\theta\eta_{ll}}$
Liquidity constraint	$R_6 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{\sigma+\theta\ (1-\sigma)+\theta\eta_{ll}+\theta\sigma\eta_{cc}}$	$R_7 = \frac{[\sigma+\theta(1-\sigma)][\eta_{ll}+\sigma\eta_{cc}+1-\sigma]}{\sigma+\theta\ (1-\sigma)+\theta\eta_{ll}+\theta\sigma\eta_{cc}}$
"Craft" unions	$R_1 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{\sigma+\theta\ (1-\sigma)}$	$R_5 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{1+\phi\eta_{ll}}$

Table 1: Persistence Properties of the Different Models

Proposition 1. *Look at Table 1 by rows. In all cases but the last one, for a particular model of price staggering* $R_{ws} = [\sigma + \theta(1-\sigma)]R_{ps}$*, that is, the corresponding model of wage staggering exhibits a value of* R *which is* $[\sigma+\theta(1-\sigma)]$ *times higher than the corresponding price staggering model. Given that* $0 < \sigma \leq 1$ *and* $\theta > 1$*, wage staggering models always deliver lower (or at most equal) output persistence than price staggering models. With regard to the "craft" union case, the condition for* $R_1 < R_5$ *is:* $\phi < \frac{(\theta-1)(1-\sigma)}{\eta_{ll}}$.

Proposition 2. *Look at Table 1 by columns.*

(i) Whatever the staggered nominal variable, prices or wages, persistence is lowest if there is perfect labour mobility. That is, constraints on labour mobility (both "industrial" and "craft" union cases) tend to increase persistence.

(ii) Liquidity constraints tend to increase persistence. Indeed, models with liquidity constraints exhibit a lower value of R *compared to the same models without such constraints.*

Additional results

(1) Effect of η_{ll}.

$$\begin{aligned}
\frac{\partial R_i}{\partial \eta_{ll}} &> 0 && \mathit{for}\ i = 1,3,6,7\ ; \\
\frac{\partial R_i}{\partial \eta_{ll}} &\gtrless 0 \iff && \frac{1}{\theta} \gtrless \eta_{cc}\ ,\ \mathit{for}\ i = 2,4; \\
\frac{\partial R_5}{\partial \eta_{ll}} &\gtrless 0 \iff && \frac{1}{\phi} \gtrless 1 + \sigma(\eta_{cc} - 1)\ .
\end{aligned}$$

The effect of η_{ll} is an interesting and delicate issue, as we know from the existing literature. Simple intuition suggests that $\uparrow \eta_{ll} \Longrightarrow \uparrow \gamma \Longrightarrow \uparrow R \Longrightarrow \downarrow \mathit{persistence}$ as found by Blanchard and Fischer (1989) and Chari et al. (1996). Thus they conclude that a low value of η_{ll} (i.e., a high intertemporal elasticity of substitution in labour supply) is necessary to generate persistence. Here we show that, depending on the particular set up of the model, the intuition may or may not hold. In particular, (i) it holds for models with perfect labour mobility; (ii) it is not likely to hold for standard calibration values in the no-labour mobility cases (both in "industrial" and in "craft" union models); (iii) it holds again when liquidity constraints are added to these models. Note that it is the underlying economic structure chosen and *not* the difference between price and wage staggering that matters.[16]

(2) Effect of η_{cc}.

$$\frac{\partial R_i}{\partial \eta_{cc}} > 0 \qquad \mathit{for\ all\ the\ models.}$$

[16]Note that Blanchard and Fischer (1989) in a yeoman farmer model have the same conclusion with respect to the effect of η_{ll} as Chari et al. (1996) in their price staggering model with free mobility of labour. This is because Blanchard and Fischer (1989) use a particular utility function with zero income effects on labour supply (i.e., $1/\theta > \eta_{cc} = 0$).

This suggests that a specification of preferences with a high intertemporal elasticity of substitution in consumption (low income effect on labour supply) is a promising route to generating output persistence, as already suggested by Chari et al. (1996) or in Ascari (1997). However, the likely magnitude of these derivatives changes from model to mode,l being particularly low for liquidity constraint models.

(3) Effect of σ.[17]

$$
\begin{aligned}
\frac{\partial R_1}{\partial \sigma} &\gtrless 0 &\iff& \quad \eta_{cc} \gtrless \frac{1}{\theta} - \left[\frac{\theta-1}{\theta}\right]\eta_{ll};\\
\frac{\partial R_2}{\partial \sigma} &\gtrless 0 &\iff& \quad \eta_{cc} \gtrless \frac{1}{\theta}\ ;\\
\frac{\partial R_i}{\partial \sigma} &\gtrless 0 &\iff& \quad \eta_{cc} \gtrless 1 \quad for\ i = 3,5\\
\frac{\partial R_4}{\partial \sigma} &< 0 &\Longleftarrow& \quad \eta_{cc} \leq 1 \quad ;
\end{aligned}
$$

Simple intuition would suggest that $\sigma = 1$ is the maximum degree of nominal rigidity (i.e., flat marginal cost curve) and so this case would deliver the maximum degree of persistence. On the contrary, in staggered price models, persistence is decreasing in σ for realistic parameter values. It is more difficult to reach any definite conclusion for staggered wage models.[18]

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[17] The cases for R_6 and R_7 are not presented because the conditions are very complicated and meaningless expressions.

[18] However, $\eta_{cc} \leqslant 1$ is a sufficient, but not necessary condition for persistence to be increasing in σ for case 4. The overall condition for $\partial R_4/\partial\sigma$ to be negative is

$$\frac{\partial R_4}{\partial \sigma} < 0 \Leftrightarrow \eta_{cc} < 1 + \frac{\theta\eta_{ll}(\theta-1)(\eta_{ll}+\sigma\eta_{cc}+1-\sigma)}{[\sigma+\theta(1-\sigma)][\sigma+\theta(1-\sigma)+\theta\eta_{ll}]}$$

which, substituting standard calibration values (see below), is very likely to be satisfied: $1 < 11.4$.

(4) Effect of θ.

$$\frac{\partial R_i}{\partial \theta} < 0 \qquad for\ i = 1,2,4,6,7$$
$$\frac{\partial R_i}{\partial \theta} = 0 \qquad for\ i = 3,5$$

An increase in θ therefore tends to increase persistence. Similarly, in the wage staggering - "craft" union model $(\partial R_5/\partial \phi) < 0$.

4.2 Quantitative Results

In this section we want to address the following question: *are any of these models likely to deliver high persistence?*

First, note that persistence is rapidly decreasing in R, for low values of R. Thus, only values of R very close to zero can deliver some notable persistence. For example, if we quite arbitrarily define a significant degree of persistence to be a value of λ of at least 0.5, then R should not be higher than 0.11.

Given the calibration literature, we take as indicative benchmark values: $\sigma = 0.67$, $\theta = 6$, $\eta_{cc} = 1$, $\eta_{ll} = 5$ and $\phi = 10$.[19] For our benchmark case, the values of R and the implied values for persistence in the different models are the following:

$$\begin{aligned} R_1 &= 2.264 & \lambda_1 &= -0.2 \\ R_2 &= 0.184 & \lambda_2 &= 0.4 \\ R_3 &= 6 & \lambda_3 &= -0.42 \\ R_4 &= 0.487 & \lambda_4 &= 0.18 \\ R_5 &= 0.118 & \lambda_5 &= 0.49 \end{aligned}$$

[19] This latter value is the one used by Erceg (1997). Moreover, η_{ll} is actually quite difficult to tie down and, given Pencavel's (1986) results, could range from 1 to infinity.

$$R_6 = 0.164 \qquad \lambda_6 = 0.42$$
$$R_7 = 0.43 \qquad \lambda_7 = 0.21$$

While there are some slight differences between price staggering and wage staggering models, the critical difference arises from the labour mobility assumption. In fact, models with free mobility of labour are likely to deliver a *negative* root for output persistence. However, in the "industrial" and "craft" union cases there seems to be a quantitative difference between price and wage staggering models (i.e., R_2 vs. R_4 and R_1 vs. R_5). In "industrial" union models price staggering delivers more persistence, while in "craft" union models wage staggering delivers more persistence. As a conclusion, *only two classes of models can deliver a substantial degree of persistence: the yeoman farmer model (i.e., price staggering and no labour mobility across industries and the model with wage staggering and no labour mobility across skills ("craft" unions).* Moreover, even if liquidity constraints do increase persistence, their quantitative importance seems negligible.

4.3 The sources of persistence

Given our general framework, we can detect how the different hypothesis on the structure of the model, preferences and technology combine in inducing persistence. As clarifying examples, we may also refer to the models in the recent literature. The results highlight the driving forces that can make a staggering model generate persistence. In particular, three main points will be the focus of three following sections.

4.3.1 Does the intertemporal elasticity of labour supply really matters?

While the literature has so far emphasised the intertemporal elasticity of labour supply as the key parameter, we will show that focusing only on this parameter may be misleading. Indeed, with respect to the Chari et al. (1996) model of staggered prices and perfect labour mobility, our results can explain why they conclude that a staggered price model could never deliver any notable persistence. In their Section 4, Chari et al. (1996) show that setting $\sigma = 1$ in their staggered price model, the sensitivity of the real wage to output is: $R_1 = \gamma = \overline{\gamma} = \widetilde{\gamma} = \eta_{ll} + \eta_{cc}$. Hence, they conclude that since η_{ll} should be at least 1, $\gamma \geqslant 1$. That is, even assuming zero income effects, γ is too high to generate any persistence at all. Central to the argument is the fact that persistence is increasing (γ is decreasing) in the intertemporal elasticity of labour supply, which is believed to be very low.[20]

However, as stated by *(1)*, the argument is likely to be reversed for models with no mobility of labour. Indeed, the no-labour-mobility models reach a minimum for R when η_{ll} tends to infinity, which goes exactly against the critics of nominal rigidity propagation mechanism models. Nevertheless, in contrast with what has been suggested so far by the literature, in the no-labour-mobility models *the degree of persistence seems extremely insensitive to the value of the intertemporal elasticity of substitution of labour supply.* Consider just plausible values for η_{ll}: $\eta_{ll} \in [1, \infty)$. Then, *ceteris paribus*, in the yeoman farmer model R_2 varies from

[20] It is worth noting that, since $\partial R_1/\partial \sigma$ is likely to be positive and by assuming that $\sigma = 1$, Chari et al. (1996) section on 'intuition' actually presents a case biased against persistence (obviously, only with regard to σ). In other words, their γ is biased upwards. Nevertheless, their main argument is valid since their model exhibits staggered prices and perfect labour mobility.

0.23 to 0.17, that is, $\lambda_2 \in [0.35, 0.42]$, in the staggered wage - "industrial" union model R_4 varies from 0.61 to 0.44, that is, $\lambda_4 \in [0.12, 0.2]$ and in the staggered wage - "craft" union model R_5 varies from 0.18 to 0.1, that is, $\lambda_4 \in [0.4, 0.52]$. Hence, *ceteris paribus* (i.e., for realistic values of the other parameters of the model), the intertemporal elasticity of labour supply is *not* a key parameter of the model with respect to its ability to generate persistence. In other words, η_{ll} alone cannot substantially change the persistence properties of these models.

Rather than taking arbitrary numerical examples, our framework allow us to illustrate this point with an useful extreme example from the literature: the calibration of Rotemberg and Woodford (1997). The benchmark calibration of their yeoman farmer model is the following: $\eta_{cc} = 0.16$, $\eta_{yy} = 0.47$, $\sigma = 0.75$ and $\theta = 7.88$, which delivers a low value of $\overline{\gamma} = 0.134$ in (28).[21] However, they stress that their results do not rely on high labour supply elasticity (i.e., a low value of η_{yy}). Indeed, look at the formula for $\overline{\gamma}$, i.e., (28). With such values of η_{cc} and θ, if $\eta_{yy} \in [0, \infty)$ then $\overline{\gamma} \in [\eta_{cc} = 0.16, 1/\theta = 0.13]$. That is, the value of η_{yy} (and hence of η_{ll}) is basically unimportant as far as persistence is concerned, since $\overline{\gamma}$ does not change very much (alternatively, very marginal changes in the value of θ can keep $\overline{\gamma} = 0.134$).

This example shows not only that the value of η_{ll} can be unimportant, but also that instead the interrelation between η_{ll} and η_{cc} is impor-

[21] They acknowledge the fact that such a low value of η_{yy} is difficult to believe since they also suggest it implies an intertemporal substitution in labour supply of 9.5 (i.e., $\eta_{ll} = 0.105$). However, they calculate $\eta_{ll} = \sigma(\widetilde{\gamma} - \eta_{cc})$, upon which they calibrate $\eta_{ll} = 0.75(0.3 - 0.16) = 0.105$. The formula for η_{ll} as a function of $\widetilde{\gamma}$, however, is consistent with a competitive labour market, as (17) shows. Simple algebra, instead, shows that in a yeoman farmer model the consistent formula for η_{ll} as a function of $\widetilde{\gamma}$ is: $\eta_{ll} = \sigma(1 + \theta a)(\eta_{cc} - \widetilde{\gamma})/(\theta\widetilde{\gamma} - 1)$, which given their parameter values yields a negative (?) η_{ll}.

tant. For example, in the constant returns to labour case, as shown in Chari et al. (1996) and in Ascari (1997), particular (and peculiar) assumptions on the form of the utility function could make all the models deliver very high persistence. Specifically, a high intertemporal elasticity of labour supply (i.e., $\eta_{ll} \to 0$) and of consumption (i.e., $\eta_{cc} \to 0$) make $R_i \to 0$ and $\lambda_i \to 1$ for all the models.

In conclusion, though the literature suggests that everything rests on the value of the intertemporal substitution in labour supply, the above results show that to focus only on this parameter can be misleading. In models with staggering and labour mobility, persistence is decreasing with η_{ll}, while the contrary is true for models with staggering and no-labour-mobility. However, this is not the reason why the first class of models cannot deliver any persistence, while the second class can. Changes in η_{ll} alone do not substantially change the capability of the models to generate persistence.[22]

4.3.2 Are the dynamic implications of price vs. wage staggering any different?

To illustrate the difference between price and wage staggering, let us consider the first three model structures in Table 1. Looking at them by rows, we saw that, given a particular model structure, $R_{ws} = [\sigma + \theta(1 - \sigma)]R_{ps}$, and this implies $R_{ws} \geq R_{ps}$. To understand why, we need to go back to the reduced form model

$$x_t = \frac{1}{2}(p_t + p_{t+1}) + \frac{1}{2}\,\gamma\,(y_t + y_{t+1}) \tag{55}$$

[22]Indeed, in the Chari et al. (1996) case we could have also written: since $R_1 = \gamma = \widetilde{\gamma} = \eta_{ll} + \eta_{cc}$ and η_{cc} is around 1, then $\gamma \geq 1$; that is, even assuming infinite elasticity of labour supply, γ is too high to generate persistence.

$$p_t = \frac{1}{2}(x_{t-1} + x_t) + hy_t \tag{56}$$

$$y_t = m_t - p_t \tag{57}$$

where the solution is given by $\lambda = \frac{1-\sqrt{\frac{h+\gamma}{h+1}}}{1+\sqrt{\frac{h+\gamma}{h+1}}} \equiv \frac{1-\sqrt{R}}{1+\sqrt{R}}$. γ is the elasticity of the staggered variable with respect to aggregate output. In wage staggering models $\gamma = \widetilde{\gamma}$ and $h = a = \frac{1-\sigma}{\sigma}$; in price staggering models $\gamma = \overline{\gamma} = \frac{\widetilde{\gamma}+a}{1+a\theta}$ and $h = 0$. It is easily checked that these two differences determine $R_{ws} = [\sigma + \theta(1-\sigma)]R_{ps}$. Furthermore, if $\sigma = 1$ then trivially $\gamma = \widetilde{\gamma} = \overline{\gamma}$, $h = a = 0$ and $R_{ws} = R_{ps}$. When $\sigma < 1$, returns to labour are decreasing and, given our production function, the marginal cost curve slopes upwards. Thus, for a given wage level, the price has to increase with production, as shown by (1). Note that

$$p_{A_t} = w_{A_t} + ay_{A_t} = w_{A_t} + a[\theta(p_t - p_{A_t}) + y_t]$$

The crucial term is ay_{A_t}. For a given wage, the marginal cost increases with production and so does the price. However, this causes an increase in the relative price for the price resetting firm and thus a loss in demand which counteracts the rise in marginal costs. In price staggering models this effect is directly embedded in the dynamic rule for the staggered variable (i.e., $\gamma = \overline{\gamma} = \frac{\widetilde{\gamma}+a}{1+a\theta}$ and $h = 0$). In wage staggering models this effect works only through the aggregate relation (56) and is not in the dynamic rule for the staggered variable (i.e., $\gamma = \widetilde{\gamma}$ and $h = a = \frac{1-\sigma}{\sigma}$). For example, for the perfect labour mobility case, in both price and wage staggering models all the firms face the same wage. Hence, in the wage staggering model the wage resetting households are only interested in the aggregate effect (thus θ does not play any role). The price staggering model would deliver the same result if the marginal cost curve is flat (i.e., constant returns to labour), since then the optimal price would just depend on the wage. In contrast, whenever there are decreasing returns, price resetting firms should also look at the quantity to be produced, given the wage. Hence, the optimal price depends on demand and the relative price effect increases stickiness.[23] This explain why, as shown by *(3)*, persistence is decreasing in σ for realistic values of the parameters in staggered price models (i.e., the lower is σ the steeper is marginal cost).

To sum up, if there are constant returns then price and wage staggering models deliver the same amount of persistence.[24] When the marginal cost curve is upward sloping instead, $R_{ws} = [\sigma + \theta(1 - \sigma)]R_{ps}$ and thus price staggering delivers a higher degree of persistence than wage staggering.[25]

4.3.3 Elasticity of demand and factor mobility

More generally, the difference in the capability of the models to generate persistence is given by the structure of the model. In particular, there are two very related issues: (i) the elasticity of demand; (ii) the constraint on factor mobility.

First, the elasticities of demand (of consumption goods and labour) are the key parameters in all the staggering models. The intuition is straightforward. Following a money shock, endogenous nominal stickiness arises if price (wage)-setting agents choose not to change their prices

[23] This does not mean that the relative price effect does not play any role in all the wage staggering models. For example, in the "industrial" union case the relative price effect determines l_{A_t} through y_{A_t}, and hence w_{A_t}, as (24) shows.

[24] A case apart is "craft" unions, where the effect of substitutability between skills does not play any role in the dynamics of the price staggering model.

[25] This would suggest that if freely mobile capital is added to the model then the effect would cancel out. However, this would not be the case wherever, as plausible, capital cannot be fully adjusted along the business cycle (e.g., capital cannot be installed without some adjustment costs and/or has some firm-specific properties), see Erceg (1997).

(wages) by a large amount when they can reset them. In the above models, they would be willing to do so only for one reason: to preserve demand. They recognise that, following a positive money shock, if they reset a higher price (wage) then they will lose demand for goods (labour) with respect to the other firms (workers) locked into the contract already signed one period before. The higher is the elasticity of substitution, the higher the elasticity of demand, the higher the loss in demand, the closer the new reset price (wage) to the existing ones, and the higher is stickiness. Then, depending on the structure of the model, the goods market (and hence the elasticity of substitution between goods, i.e., θ) or the labour market (and hence the elasticity of substitution between skills, i.e., ϕ) plays the pivotal role. In the first case, price staggering would naturally deliver more persistence than wage staggering (i.e., $R_2 < R_4$) and vice versa in the second case (i.e., $R_5 < R_1$).

The other very related issue is factor mobility: the lower is factor mobility the stronger is the effect just described in the previous paragraph, that is, the more substitutability matters. This point was actually already suggested in the previous section. If capital is not freely mobile across industries then firms face an upward sloping marginal cost curve. Thus the price depends on demand and hence, through the relative price effect, price stickiness increases.

More importantly, we saw that restrictions on labour mobility can increase persistence considerably. Let us consider the price staggering model in the perfect mobility and "industrial" union cases (i.e., R_1 and R_2). Analytically, the only difference between the two models comes from the marginal cost of supplying labour, that is, equation (5). In the perfect labour mobility case there is a common labour market and thus $l_{C_t} = l_{D_t} = l_t$. All the sectors have to pay the same wage according to the marginal cost of working of the average household. Hence, following a money supply shock, aggregate demand increases. The non-resetting sectors will satisfy this extra demand at given prices. To do so, they have to bid for new workers. Given the common labour market, the increase in the nominal wage is transmitted to the price-resetting sector that faces a rise in marginal cost. When labour is immobile across industries, instead, the segmented labour market cuts off this cost transmission mechanism. Furthermore, in this case the relevant labour demand is the "industrial" labour demand, since workers are sector specific (i.e., $l_{C_t} = l_{A_t} \neq l_t$). Hence, the marginal cost of working of households employed in the price resetting industries depends only on the industrial labour demand (l_{A_t}) and not on aggregate labour demand (l_t). This makes the condition in the price resetting sector labour market depend heavily on the elasticity of the "industrial" labour demand. Higher wages affect the price of the good produced by the industry. This in turn affects the demand (through θ) for the good, reducing the demand for labour. Algebraically, this difference is highlighted by the term $\theta\eta_{ll}$ in the denominator of R_2. The fact that the workers face a different labour demand is what distinguishes the two classes of models, making marginal cost rise much slower for the resetting sector in the no labour mobility case. Indeed, given the likely magnitudes of θ and η_{ll}, this effect quantitatively makes a high difference, as shown above.

Similarly, in the "craft" union case, workers cannot move across skills. Then, in the wage staggering model, the marginal cost of working for households belonging to a particular "craft" union depends only on the "craft" labour demand. The elasticity of demand is just the elasticity of substitution between skills (ϕ). Thus, the term $\phi\eta_{ll}$ appears in the denominator of R_5.[26] To see how this case is symmetric to the

[26] In the "craft" union, price staggering case, as we know, ϕ does not play a role because a symmetric equilibrium holds in the labour market.

"industrial" union wage staggering case, let us calculate the elasticity of labour demand for an "industrial" union. Substitute (1) into (3) and use $y_{A_t} = \sigma l_{A_t}$, to find

$$l_{A_t} = \left(\frac{\theta}{\sigma + \theta(1-\sigma)} \right) (p_t - w_{A_t}) + \left(\frac{1}{\sigma + \theta(1-\sigma)} \right) y_t \equiv \varepsilon (p_t - w_{A_t}) + \frac{\varepsilon}{\theta} y_t$$

Hence, the elasticity of the demand for labour with respect to the money wage for an "industrial" union is equal to ε. We can simply rewrite R_4 as

$$R_4 = \frac{[\sigma + \theta(1-\sigma)]\,[\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma]}{\sigma + \theta\ (1-\sigma) + \theta\eta_{ll}} \equiv \frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{1 + \varepsilon\eta_{ll}}$$

which exactly corresponds to R_5, the only difference being the different elasticity of labour demand for a "craft" union (ϕ) and an "industrial" union (ε).

Finally, by considering the yeoman farmer model (i.e., price staggering and no labour mobility across industries) with liquidity constraints we can identify the following effects

$$R_6 = \frac{1}{\underbrace{\sigma + \theta\ (1-\sigma)}_{\text{2) no capital mobility + price staggering}}} \frac{\overbrace{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}^{\text{1) common effect}}}{1 + \underbrace{\varepsilon\eta_{ll}}_{\text{3) no labour mobility}} + \underbrace{\varepsilon\sigma\eta_{cc}}_{\text{4) liquidity constraints}}}.$$

1) is the "pure" staggering effect, in the sense that it is common to all models; 2) has been described in the previous section and 3) in the present section. 4) is due to liquidity constraints, which change the income effect on labour supply. The effect parallels the no labour mobility effect. As the constraint on labour mobility affects the marginal disutility from working, in a similar way liquidity constraints affect the marginal utility of consumption. In this case, it is the elasticity of "industrial" income with respect to the money wage ($\varepsilon\sigma$) which is relevant. However, we have

already noticed that while liquidity constraints enhance persistence, their quantitative importance is likely to be small.

To generate persistence we need factor immobility and some substitutability between goods and/or labour. Quantitatively, the difference then rests upon the quantitative difference between the relevant elasticities. The fact that ε $(= 2.26)$ is much lower than ϕ creates the difference between R_4 and R_5. Moreover, the quantitative difference between the two models able to deliver significant persistence (namely, the yeoman farmer model and the wage staggering model with skills substitutability) also depends upon the quantitative difference between the relevant elasticities (i.e., θ and ϕ). If we calibrate $\theta = 10$[27] then in the yeoman farmer case we get $R_2 = 0.11$ and $\lambda_2 = 0.5$, which are basically equivalent to the benchmark case for the R_5 and λ_5.[28]

A final remark is needed. The perfectly flexible wage/price model presented here is quite stylised, but also quite general as we tried to show above. Indeed, since it encompasses most of the microfounded models of staggering in the literature, it can be thought of as derived from the log-linearisation of a more general microfounded model. However, implicitly the log-linearised model presumes another *not innocuous assumption*. The model is log-linearised around a particular steady-state with constant money supply (i.e., zero inflation steady state). In fact, the policy parameters (or the inflation trend) do not appear in the model. However,

[27] Often in the literature (e.g., Chari et al. (1996)) a CES function is used to describe the technology for producing final goods from intermediate goods. It follows that θ is a technology parameter representing the elasticity of substitution between inputs and thus the elasticity of demand for intermediate goods. Chari et al. (1996) calibrate it equal to 10.

[28] Moreover, this would suggest that combining price *and* wage staggering in a "craft" union model could deliver substantial persistence. This intuition is developed by Erceg (1997).

Ascari (1997) showed the degree of persistence to be decreasing considerably in the steady state inflation trend. This point has to be take into account and combined with the results above.

5 Conclusions

We have derived a general, stylised log-linear model which encompasses most of the microfounded models of price/wage staggering. Our framework shows how the different hypothesis on the structure of the model, preferences and technology combine in inducing persistence. We were then able to show why different staggering models in the literature came to very different conclusions. Our general framework provides new insights on the importance of the underlying economic structure for the ability of staggered price/wage models to explain the persistence of real effects of money shocks. The main conclusions are:

- In monopolistic competition, and hence game-theory-free frameworks, *substitutability between goods and/or labour types* plays the major role in generating persistence. In models with only substitutability between goods, price staggering naturally delivers higher persistence than wage staggering (if the marginal cost curve is upward sloping). The opposite is true in models with substitutability between labour types (the "craft" union model).

- To generate persistence together with some substitutability between goods and/or labour we need some kind of *factor immobility*; in particular, the distinction between free mobility and no mobility of labour is fundamental. No-mobility-of labour models (both "industrial" and "craft" union models) bring in new mechanisms that increase persistence, strengthening the effects of substitutability. Quantitatively, the difference rests upon the quantitative difference between the relevant elasticities of substitution (i.e., elasticities of factor demands)

- *Liquidity constraints tend to (marginally) increase persistence.*

- While in these models a substantial (in the sense of near random walk behaviour) degree of persistence is unlikely, two models can deliver *significant persistence: the yeoman farmer model (with price staggering) and the "craft" union model with wage staggering.* It is not then by chance that the two models in the literature which claim to be able to generate a contract multiplier are Rotemberg and Woodford (1997) and Erceg (1997). The first is a yeoman-farmer model, the second is a "craft" union model.[29]

- *Ceteris paribus (i.e., for realistic values of all the other parameters), these conclusions do not depend on the particular value assigned to the intertemporal elasticity of labour supply,* which so far has been the focus of this literature. This suggests that the importance of the intertemporal elasticity of labour supply in generating persistence in staggered wage/price models may have been somewhat overstated.

[29] Some others actually share the same claim, but in a somewhat non-standard framework and hence their models cannot be encompassed by our model. For example, Andersen (1998b) considers a particular utility function of a monopoly union, Ascari and Garcia (1998) consider the existence of relative wage concern, Bergin and Feenstra (1998) consider a translog form for preferences.

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