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Recurrent Neural Network-Based Robust Nonsingular Sliding Mode Control With Input Saturation for a Non-Holonomic Spherical Robot

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ABSTRACT We develop a new robust control scheme for a non-holonomic spherical robot. To this end, the mathematical model of a pendulum driven non-holonomic spherical robot is first presented. Then, a recurrent neural network-based robust nonsingular sliding mode control is proposed for stabilization and tracking control of the system. The designed recurrent neural network is applied to approximate compound disturbances, including external interferences and dynamic uncertainties. Moreover, the controller is designed in a way that avoids the singularity problem in the system. Another advantage of the proposed scheme is its ability for tracking control while there exists control input saturation, which is a serious concern in robotic systems. Based on the Lyapunov theorem, the stability of the closed-loop system has also been confirmed. Lastly, the performance of the proposed control technique for the uncertain system in the presence of an external disturbance, unknown input saturation, and dynamic uncertainties has been investigated. Also, the proposed controller has been compared with a Fuzzy-PID one. Simulation results show the effectiveness and superiority of the developed control technique.

INDEX TERMS Spherical robot, sliding mode control, recurrent neural network, external disturbance, unknown input saturation, control singularity.

I. INTRODUCTION

The pendulum-driven spherical robot can move by changing the position of its gravity centre. Indeed, the rotating of the pendulum generates the relocation of the mass centre position of the robot [1]–[3]. The spherical geometry is very suitable to use these robots for exploration in harsh environments, such as in the space, deserts, and earthquake ruins [4]–[7]. However, this kind of robot results in a nonlinear system with non-holonomic dynamics, which means that the dimensions of the state space model are more than the number of control inputs. This condition makes the tracking control of this robot difficult in real applications [8], [9].

Robotic systems are well-known samples of trajectory controllable mechanical ones. Nevertheless, their highly nonlinear dynamics, as well as uncertainties, cause a challenging control problem [10]–[12]. Hence, so far several studies have tried to solve the challenge of controlling a spherical robot.

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To this respect, a three-step technique has been developed by Li and Canny for the control of both position and orientation of a spherical robot [13]. A novel mechanism has also been proposed by Azizi and Naderi for controlling a spherical robot [14]. Precisely, they have investigated the dynamical model of the system and its control. Andani *et al.* [14] have proposed a sliding mode control (SMC) and a fuzzy SMC to control a spherical robot motion. They have demonstrated that the controlled system can track the desired path with minimum tracking error. However, they have not considered important issues, such as the control input limitation and the singularity problem. Kayacan *et al.* [15] have introduced another SMC with an online learning algorithm for spherical rolling robots. In one recent study, Roozegar *et al.* [8] have investigated the control and motion of a spherical robot on an inclined plane. They have proposed a terminal sliding mode control (TSMC) to maintain and control the robot on a variable slope.

One prevalent problem in TSMC is the singularity, which causes by some terms in the terminal sliding mode surface (this kind of singularity does not occur in SMC). To avoid this singularity problem, a saturation function has been introduced in [16] for dealing with the singularity problem in the case of chained nonlinear systems with matched perturbations. In this method, without changing the design of the controller, it was proposed to limit the control signal when singularity occurs. However, in this method, the stability of the closed-loop system where singularity occurs was not proven. A nonsingular TSMC method was presented in [17], which simply swaps the state variables in the conventional TSMC function while retaining the finite time convergence feature. Also, in [18] switching between TSMC and a linear hyper plane-based sliding mode was proposed. Another approach is to transfer the trajectory to a prescribed region in advance where no singularity occurs, which is the so-called two-phase control strategy [19]. It should be noted that these methods are adopting indirect approaches to avoid singularity. In [20], an adaptive non-singular integral TSMC has been presented for trajectory tracking of autonomous underwater vehicles with dynamic uncertainties and time-varying external disturbances, which can eliminate the singularity problem. In [21], a modified time-varying nonsingular TSMC manifold has been proposed to avoid the singularity problem.

In general, another type of singularity problem, which is the result of the dynamic of systems, may occur in control systems. In this kind of singularity, due to some terms that there exist in the functions of the system, the singularity will happen. This issue has been solved for TSMC in some studies [22]–[25]. However, there are a few studies that have solved this kind of singularity through SMC. Actually, most studies in this field have considered nonsingular approaches for TSMC, and solving this detrimental problem through SMC is neglected.

In most real-world applications, it is rare to found accurate information about the dynamics of the systems, and moreover, they are often in the presence of various disturbances [26]–[30]. Thus, in these cases it is very beneficial to apply a controller that is robust to unmodeled dynamics and external disturbances [31]–[33]. To this end, previous works have proposed several disturbance-observers with different control schemes for some systems [34], [35].

Neural networks have been presented as an appropriate tool for approximation of any unknown function [36], [37]. Thus, using this advantage, several research studies have applied neural networks for control purposes [38]–[40]. For instance, a neural network-based SMC has been designed by Guo *et al.* for an autonomous underwater vehicle [41]. However, few studies have shown that when there are unexpected changes in the system, recurrent neural networks (RNN) perform better than conventional feedforward neural networks [42]–[45]. To this respect, a RNN-based disturbance observer has been employed to fortify the robustness of the controller by Salgado and Chairez [46]. To approximate the uncertain dynamics of a MIMO system, Salgado *et al.* [47] have also developed a RNN-based observer for an adaptive SMC. The authors have proven that this control scheme can approximate the unknown states of a given nonlinear system and lessen the convergence time, as well as the oscillations in the steady-state responses. Fei and Lu [42] have presented an adaptive SMC by using a double loop RNN to approximate unknown dynamics. Zhang and Chu [48] have designed an adaptive SMC based on the local RNN to estimate the uncertainties for trajectory tracking control of an autonomous underwater robot. Xu *et al.* [49] have proposed a RNN-based robust tracking control to measure an online unknown nonlinear system function. The authors have shown that an RNN-based robust tracking control is able to significantly improve the performance of the controller. Also, there have been a lot of research works focusing on approximating time-variant functions using an RNN. For instance, Chow and Fang [45] have used RNN as an estimator to develop an algorithm that can approximate any trajectory tracking accurately. Feedback connections between layers of the recurrent neural network create sophisticated dynamics that can deal with time-varying outputs and estimate them. Li *et al.* [50] have shown the excellent ability of continuous-time recurrent neural networks in the estimation of dynamical time-variant systems.

On the other hand, since input saturation is a potential problem in many practical dynamic systems and has played an important role in many branches of control applications during the past decades, several valuable control schemes for uncertain nonlinear systems have been proposed up to now [51]–[53]. By employing the idea of auxiliary system design, Esfandiari *et al.* [53] have introduced nonsymmetric input saturation constraints for a class of uncertain nonaffine nonlinear systems with external disturbances. In [52], an adaptive backstepping approach has been introduced to control a single-input uncertain nonlinear system in the presence of external disturbances and input saturation. In that method, a Nussbaum function is employed to solve the problem of the saturation nonlinearity. By making use of the smooth nonlinear function of the control input signal, a non-affine pure-feedback stochastic nonlinear system has been investigated in [51]. More precisely, the proposed control guaranteed convergence of the tracking error to an arbitrarily small neighborhood around the origin in the sense of mean quartic value.

To the best of the author's knowledge, very few attempts have been made to design a neural network-based controller for spherical robots [54], [55]. Moreover, none of these works have considered control input saturation in the system. However, as it is evident, because of the current limitations in real actuators, the bounds of control input should be considered in real-world systems [56]–[58]. Similarly, although the singularity problem can be induced to a large control input [59]–[62], this has not been taken to account in most previous studies on spherical robots. Moreover, the advantages presented by RNNs have still not been completely exploited for the control of this kind of robots. Hence, in the present work, a novel controller has been designed for an uncertain non-holonomic spherical robot in the presence of unknown disturbances, control singularity problem, and control input saturation. Precisely, an RNN has been combined with an SMC. Moreover, it has been demonstrated that the proposed RNN-based disturbance observer can identify time-varying disturbances and uncertainties when the robot is on a variable slope inclined plane. The control input saturation has also been taken into account for evaluating the performance of the robot in a practical, real-world scenario. Moreover, the proposed technique has been able to avoid the singularity problem in the spherical robot. The stability of the system has been proven by the Lyapunov stability theory and the Taylor expansions technique, even when control input limitations were considered.

Overall, the improvements reached by the proposed control method, regarding other previous works on control of spherical robots, can be summarized as follows:

(1) Ability to deal with the control input saturation and singularity problem through SMC, simultaneously.
(2) Access to the estimated disturbance and uncertainty by depicting the online assessed value of the overall disturbance.
(3) High-rate of accuracy in the disturbance estimation using the outputs of each step as the inputs of the next one in the RNN.
(4) Utilizing both activation function and biases in the neural network disturbance observer by taking to account the stability constraints.

The remainder of the work is organized as follows. Section 2 details non-holonomic spherical robot formulation. In Section 3, the RNN-based nonsingular SMC (RNN-based NSMC) is designed for the uncertain spherical robot. In Section 4, the proposed scheme is applied to control the motion of the system. Also, the performance of the proposed controller is compared with a Fuzzy-PID one. Lastly, conclusions are presented in Section 5.

II. MATHEMATICAL FORMULATION OF THE SYSTEM

A spherical robot is an active system which is led to a desired position and orientation by moving the pendulum. By adjusting the center of mass gravity, the motion of the robot could be controlled. A schematic representation of the system is depicted in Fig. 1, where points A, G, C, and P represent the center of the shell, the mass center of the robot, the contact point between the plane and robot, and the position of the pendulum, respectively. In what follows, $\hat{i}, \hat{j}$, and $\hat{k}$, respectively, denote the unit vectors in x, y, and z directions.

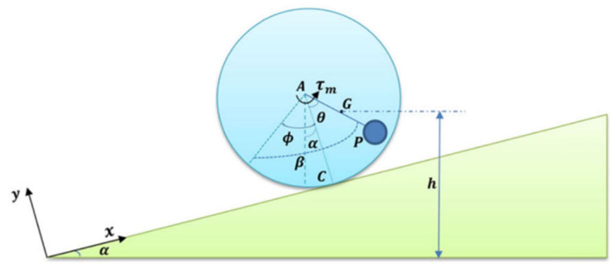

FIGURE 1. A spherical robot on an inclined surface.

The Euler–Lagrange equations for the system are given by

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \left(\frac{\partial \mathcal{L}}{\partial q}\right) = Q \tag{1}$$

where q illustrate the generalized coordinates, Q indicates the generalized external forces and $\mathcal{L}$ is the Lagrangian function of the system and is given by

$$\mathcal{L} = T - U, \tag{2}$$

where T indicates the total kinetic energy of the system and U denotes the potential energy. The total kinetic energy can be expanded as

$$T = T_{case} + T_{pendulum}, \tag{3}$$

where T_{case} and $T_{pendulum}$ are given by

$$T_{case} = \frac{1}{2} m_s \|V_A\|^2 + \frac{1}{2} I_A \|\omega_s\|^2, \quad \text{and} \tag{4}$$

$$T_{pendulum} = \frac{1}{2} m_p \|V_p\|^2, \tag{5}$$

where m_s is the mass of the spherical shell, V_A denotes the velocity of shell center, I_A is the spherical shell moment of inertia and $\omega_s = \left(\dot{\alpha} - \dot{\phi}\right)\hat{k}$ represents the angular velocity of the spherical shell, where ϕ is the rotation of the spherical shell relative to the inclined plane, and α is the angle of the inclined plane. Also, m_p and V_p indicate the mass of the pendulum and velocity of the pendulum, respectively.

According to the system that is shown in Fig. 1 the velocities are as

$$V_C = x\dot{\alpha}\hat{j}, \quad V_{A/C} = \rho\dot{\phi}\hat{i} \rightarrow \vec{V}_A = V_C + V_{A/C}$$
$$= \rho\dot{\phi}\hat{i} + x\dot{\alpha}\hat{j}, \text{ and} \tag{6}$$

$$V_P = V_A + V_{P/A}, \vec{V}_{P/A} = r\dot{\theta}(cos(\theta)\hat{i} + sin(\theta)\hat{j}), \tag{7}$$

where θ stands for the instantaneous angle of the pendulum, relative to the inclined plane.

ρ is radius of the spherical shell and r denotes the radius of the pendulum. The potential energy and external force can be then expressed as

$$U = Mg\left(x\,sin\left(\alpha\right) + \rho\,cos\left(\alpha\right) - r_G\,cos\left(\alpha+\theta\right)\right), \quad (8)$$

and

$$Q = \tau_m, \quad (9)$$

where r_G denotes the radius of the robot mass center and τ_m is the motor torque. If $\mathcal{L}$ is recomputed by Eq. (3) to Eq. (7), these equations will be obtained as

$$\left(I_c + m_p\rho^2\right)\ddot{\phi} + m_p\rho r\,cos\left(\theta\right)\ddot{\theta} - m_p\rho r sin\left(\theta\right)\left(\dot{\theta}^2 + \dot{\theta}\dot{\alpha}\right)$$
$$- I_A\ddot{\alpha} - M\rho\left(x_0 + \rho\phi\right)\dot{\alpha}^2 + Mg\rho sin\left(\alpha\right) = \tau_m, \quad (10)$$
$$m_p r^2\ddot{\theta} + m_p\rho r\,cos\left(\theta\right)\ddot{\phi} + m_p\rho r sin\left(\theta\right)\dot{\alpha}\dot{\phi}$$
$$+ m_p r\left(x_0 + \rho\phi\right) sin\left(\theta\right)\ddot{\alpha} + Mgr_G sin\left(\alpha+\theta\right) = \tau_m \quad (11)$$

Respectively. By defining the states of the system as $\phi = x_1, \dot{\phi} = x_2, \theta = z_1, \dot{\theta} = z_2$, the state space equations of the system are

$$\dot{x}_1 = x_2, \quad (12)$$
$$\dot{x}_2 = -\frac{1}{\left[I_c + m_p\rho^2 \sin^2 z_1\right]}\left(-m_p\rho r sin\left(z_1\right)\left(z_2^2 + z_2\dot{\alpha}\right)\right.$$
$$- I_A\ddot{\alpha} - M\rho\left(x_0 + \rho x_1\right)\dot{\alpha}^2 + Mg\rho sin(\alpha)$$
$$+ \rho\frac{cos\left(z_1\right)}{r}\left[-\left[m_p\rho r sin\left(z_1\right)\dot{\alpha}x_2 + m_p r\left(x_0 + \rho x_1\right)\right.\right.$$
$$\left.\left.\left. +\ sin\left(z_1\right)\ddot{\alpha} + Mgr_G sin\left(\alpha + z_1\right)\right]\right]\right)$$
$$+ \frac{\tau_m\left[1 - \rho\frac{cos(z_1)}{r}\right]}{\left[\left(I_c + m_p\rho^2 \sin^2 z_1\right)\right]} \quad (13)$$
$$\dot{z}_1 = z_2, \quad (14)$$
$$\dot{z}_2 = \frac{m_p\rho r cos\left(z_1\right)}{\left(I_c m_p r^2 + m_p^2\rho^2 r^2 sin^2 z_1\right)}$$
$$\times\left[Mg\rho sin\left(\alpha\right) - m_p\rho r sin\left(z_1\right)\left(z_1^2 + z_2\dot{\alpha}\right) - I_A\ddot{\alpha}\right.$$
$$\left. -\ M\rho\left(x_0 + \rho x_2\right)\dot{\alpha}^2\right] + \frac{\left(I_c + m_p\rho^2\right)}{\left(I_c m_p r^2 + m_p^2\rho^2 r^2 sin^2 z_1\right)}$$
$$\times\left[-m_p\rho r sin\left(z_1\right)\dot{\alpha}x_2 - m_p r\left(x_0 + \rho x_2\right) sin\left(z_1\right)\ddot{\alpha}\right.$$
$$\left. -\ Mgr_G sin\left(\alpha + z_1\right)\right] + \frac{\left[I_c + m_p\rho^2 - m_p\rho r cos\left(z_1\right)\right]}{\left(I_c m_p r^2 + m_p^2\rho^2 r^2 sin^2 z_1\right)}\tau_m \quad (15)$$

Eqs. (12)-(15) have been derived from the Lagrange equation and the total kinetic energy of the system. Combining Eqs. (13)-(15) and assuming that the $\dot{\alpha}$ and $\ddot{\alpha}$ are negligible, we have

$$\dot{x}_1 = x_2, \quad (16)$$
$$\dot{x}_2 = -\frac{1}{\left[I_c + m_p\rho^2 \sin^2 z_1\right]}\left(-m_p\rho r sin\left(z_1\right)\left(z_2^2\right)\right.$$
$$\left. +\ Mg\rho sin(\alpha) - \rho\frac{cos\left(z_1\right)}{r}\left[Mgr_G\,sin\left(\alpha + z_1\right)\right]\right)$$
$$+ \frac{\tau_m\left[1 - \rho\frac{cos(z_1)}{r}\right]}{\left[\left(I_c + m_p\rho^2 \sin^2 z_1\right)\right]} \quad (17)$$
$$\dot{z}_1 = z_2, \quad (18)$$
$$\dot{z}_2 = \frac{m_p\rho r cos\left(z_1\right)}{\left(I_c m_p r^2 + m_p^2\rho^2 r^2 \sin^2 z_1\right)}\left(\left[-m_p\rho r sin\left(z_1\right)\left(z_1^2\right)\right.\right.$$
$$\left.\left. +\ Mg\rho\sin\left(\alpha\right)\right]\right) + \frac{\left(I_c + m_p\rho^2\right)}{\left(I_c m_p r^2 + m_p^2\rho^2 r^2 \sin^2 z_1\right)}$$
$$\times[-Mgr_G sin(\alpha + z_1)] + \frac{\left[I_c + m_p\rho^2 - m_p\rho r\,cos(z_1)\right]}{\left(I_c m_p r^2 + m_p^2\rho^2 r^2 \sin^2 z_1\right)}\tau_m \quad (19)$$

This model contains enormous complexity. Thereby, reaching a favorable controller is the most challenging problem in this system, which motivates the rest of the present study.

III. CONTROLLER DESIGN

In the current section, an RNN-based NSMC is designed for a non-holonomic spherical robot, and the tracking convergence of the closed-loop system is proven. Suppose $\bar{u} = \tau_m$, we have the following nonlinear system:

$$\dot{x}_1 = x_2, \quad (20)$$
$$\dot{x}_2 = f_{xz} + \Delta f_{xz} + \left(g_{xz} + \Delta g_{xz}\right)\bar{u} + d(t), \quad (21\text{-a})$$
$$f_{xz} = -\frac{1}{\left[I_c + m_p\rho^2 sin^2 z_1\right]}\left(-m_p\rho r sin\left(z_1\right)\left(z_2^2\right)\right.$$
$$\left. +\ Mg\rho sin(\alpha) - \rho\frac{cos\left(z_1\right)}{r}\left[Mgr_G sin\left(\alpha + z_1\right)\right]\right) \quad (21\text{-b})$$
$$g_{xz} = \frac{\left[1 - \rho\frac{cos(z_1)}{r}\right]}{\left[\left(I_c + m_p\rho^2\ sin^2 z_1\right)\right]} \quad (21\text{-c})$$

where $d\left(t\right)$ denotates the external disturbances. These equations include uncertainties and disturbances against whom the system must be robust. According to the uncertain terms Δg_{xz} and Δf_{xz}, Eq. (21) can be rewritten as

$$\dot{x}_2 = f_{xz} + \left(g_{xz}\right)\bar{u} + D\left(t\right), \quad (22)$$

where $D(t)$ is defined as

$$D\left(t\right) = \Delta f_{xz} + \Delta g_{xz}\bar{u} + d\left(t\right). \quad (23)$$

A. SELECTING AN RNN-BASED CONTROLLER

Up to now, neural networks have been applied for enormous applications and have shown successful results in several problems. In [63], it is demonstrated that neural networks in conjunction with recursive least squares can be used effectively for model identification of nonlinear time-variant processes. Actually, if we combine a feedforward neural networks with a recursive method, then it can estimate time-varying functions as well.

In [64], it is proven that Multilayer feedforward networks are universal approximators and are capable of approximating any smooth function. However, feedforward networks need appropriate inputs to approximate any time-varying functions. Also, In [65], it is proven that RNNs are universal approximators even when they have only one layer.

An RNN is a powerful neural network that could be used for predicting complex uncertainties. In comparison with conventional feedforward neural networks, RNNs have better performance when changes in the system are unexpected [42], [66]. In addition, time-sequential can be stored through the recurrent weights of the network, and recurrent neurons can then reflect time sequences. Therefore, an RNN could estimate disturbances better than a conventional feedforward techniques [48], [67].

Especially, RNN, which possesses recursive features, is highly recommended for time-variant problems. Indeed, due to the recurrent information which has been indirectly stored in a neural cell, RNN provides short-term dependencies that create the capability of processing and learning time-varying smooth functions [50], [68], [69]. Hence, RNN is more applicable to the estimation of the time-evolving condition [70]–[72].

B. DESIGNING AN RNN-BASED NSMC

Consider ζ as an input vector, $\widehat{D}$ as the estimated disturbance, and Γ as a constant parameter which should be larger than the value of the compound disturbance, i.e., than all uncertainties and perturbations, we have

$$\widehat{D} = \Gamma f\left(\widehat{W_1}^T \zeta + \widehat{b}_1\right) + \varepsilon_n, \tag{24}$$

where $\widehat{W_1} = [\varpi_1, \varpi_2, \ldots, \varpi_n]^T$ is the vector of weights of the RNN, $\widehat{b}_1$ denotes bias, ε_n is the estimation error, and $f(X)$ is an activation function [56]–[58].

In the current study, we have used SoftSign as an activation function due to the fact that SoftSign is smoother than tanh and sigmoid, and approaches its saturation regime much slower. Consequently, compared with sigmod and tanh, SoftSign prevents the chattering in the estimation process and is less likely to oscillate between minimum and maximum bounds [73]. Even though tanh and softsign functions are closely related, tanh converges exponentially, whereas softsign converges polynomially. Softsign functions produce outputs in scale of $[-1, +1]$; hence, $\Gamma f\left(\widehat{W_1}^T \zeta + \widehat{b}_1\right)$ will be in $[-\Gamma, \Gamma]$.

Adaptive laws for updating the bias and weights of the RNN are designed as

$$\dot{\widehat{b}}_1 = -\Gamma S_2 \dot{f}\left(\widehat{W_1}^T \zeta + \widehat{b}_1\right), \quad \text{and} \tag{25}$$

$$\dot{\widehat{W_1}} = -S_2 \dot{f}\left(\widehat{W_1}^T \zeta + \widehat{b}_1\right)\zeta, \tag{26}$$

respectively, where S_2 is the sliding surface that will be determined as described below. These updating rules with utilizing the backpropagation method update the weights and bias of the recurrent neural network. The optimal value of compound disturbance which observer can estimate is modeled as

$$D_t = \Gamma f\left(W_1^T \zeta + b_1\right), \tag{27}$$

where the input vector ζ includes $S_1, t \cdot S_2, X_1, X_2$. The RNN operation is shown in Fig. 2(a), whereas Fig.2 (b) shows the procedure of the proposed controller. The RNN disturbance observer has been combined with a SMC to compensate the effects of control singularity, input saturation, and external disturbance.

Assumption 4: There exist ideal vector of weights and bias of the RNN such that $|\varepsilon_n| < \varepsilon_m$ with constant $\varepsilon_m > 0$ for all ζ.

Assumption 5: The activation function $f(*)$ is bounded. By virtue of Assumption 4, it can be concluded that $\widehat{D}$ is also bounded.

The manifolds of the sliding surface are designed as

$$S_1 = e = x_1 - x_{1d}, \quad \text{and} \tag{28}$$

$$S_2 = \dot{e} + \alpha_c e = \dot{S}_1 + \alpha_c S_1, \tag{29}$$

where e indicates the error of the system, which can be measured by a sensor. Moreover, α_c is a positive parameter that should be designed. The first-time derivative S_2 is then

$$\dot{S}_2 = \dot{x}_2 - \dot{x}_{2d} + \alpha_c (x_2 - x_{2d}). \tag{30}$$

Due to the limitation of the control force, the input saturation function should be considered in practical, real-world applications. In the present study, the amount of the controller input has been restricted by u_{max} and u_{min}. Hence, the restricted control input ($\bar{u}$) is considered as [74]:

$$\bar{u} = \begin{cases} u_{max} & if\ u > u_{max} \\ u & if\ u_{max} \geq u \geq u_{min} \\ u_{min} & if\ u < u_{min} \end{cases} \tag{31}$$

where u_{max} and u_{min} are the bounds for the input signal and u is the control signal, which is designed as

$$u = \frac{g_{xz}}{(g^2{}_{xz} + \varepsilon)} u_0, \tag{32}$$

where u_0 will be described below, and ε is a positive parameter. Based on Eq. (21-c) when $\cos(z_1) = \frac{r}{\rho}$, the singularity problem occurs, and in this study, thanks to the denominator of Eq. (32), we will rid of the singularity problem. A simple calculation yields

$$\frac{g_{xz}^2}{(g^2{}_{xz} + \varepsilon)} = 1 - \frac{\varepsilon}{(g^2{}_{xz} + \varepsilon)}. \tag{33}$$

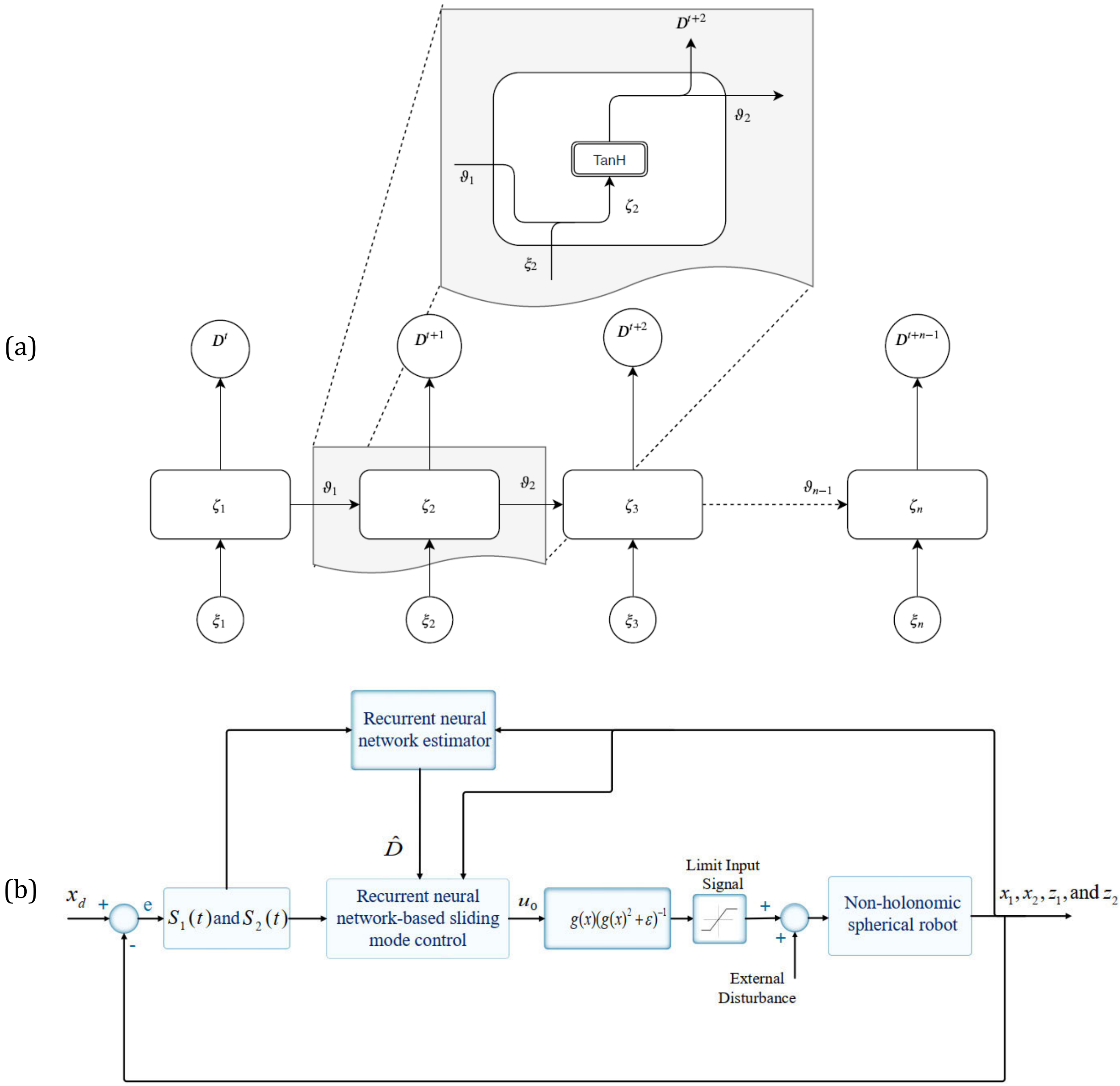

FIGURE 2. **(a) Internal structure of the proposed RNN-based NSMC disturbance observer. (b) Global block diagram of the proposed RNN-based NSMC for a non-holonomic spherical robot.**

Substituting Eq. (33) into Eq. (22), Eq. (34) can be obtained as

$$\begin{aligned}\dot{x}_2 &= f_{xz} + (g_{xz})(u + \Delta\bar{u}) + D(t)\\ &= f_{xz} + u_0 + g_{xz}\Delta\bar{u} + D(t) - \frac{\varepsilon}{(g^2{}_{xz} + \varepsilon)}u_0,\end{aligned} \tag{34}$$

where $\Delta\bar{u} = \bar{u} - u$ and its value is unknown. By considering impacts of the nonsymmetric input limitation, the compound disturbance can be expressed as

$$D_t = g_{xz}\Delta\bar{u} + D(t) - \frac{\varepsilon}{(g^2{}_{xz} + \varepsilon)}u_0, \tag{35}$$

in which Actually, the uncertainties and the disturbances are assumed to satisfy $|D_t| < \Gamma$. Considering Eq. (35), then, Eq. (34) can be written as

$$\dot{x}_2 = f_{xz} + u_0 + D_t. \tag{36}$$

Finally, by considering the singularity problem and control input saturation, the RNN-based NSMC is designed as

$$\begin{aligned}u_0 = -\alpha_c(x_1 - x_{1d}) - \delta S_2 - \psi\, sign(S_2) - f_{xz} - \widehat{D} + \dot{x}_{2d}\\ - \alpha_c(x_2 - x_{2d}),\end{aligned} \tag{37}$$

where δ and ψ are positive parameters and ψ should be a large constant to fulfill $\psi > |\varepsilon_n|$.

The stability of the closed-loop system based on the control laws described in Eqs. (37)-(38) is demonstrated with the following theorem.

Theorem 1 Under the proposed RNN-based NSMC, Eqs. (24) an (38), the uncertain spherical robot converges to the desired trajectory in the presence of unexpected disturbances, singularity, and input saturation.

Proof: Supposing a Lyapunov function candidate as

$$V = \frac{1}{2}\alpha_c S_1^2 + \frac{1}{2}S_2^2 + \frac{1}{2}\tilde{W}_1^T\Gamma\tilde{W}_1 + \frac{1}{2}\tilde{b_1}^2, \quad (38)$$

where $\tilde{W}_1 = \widehat{W_1} - W_1$ and $\tilde{b}_1 = \widehat{b}_1 - b_1$, considering $\dot{\tilde{b}}_1 = \dot{\widehat{b}}_1$, $\dot{\tilde{W}}_1 = \dot{\widehat{W}}_1$, and the first-time derivative of V is

$$\begin{aligned}\dot{V} &= \alpha_c S_1\dot{S}_1 + S_2\dot{S}_2 + \tilde{W}_1^T\Gamma\dot{\widehat{W}}_1 + \tilde{b_1}\dot{\widehat{b}}_1\\ &= \alpha_c S_1(S_2 - \alpha_c S_1) + S_2\dot{S}_2 + \tilde{W}_1^T\Gamma\dot{\widehat{W}}_1 + \tilde{b_1}\dot{\widehat{b}}_1, \quad (39)\end{aligned}$$

substituting Eqs. (29)-(30) and the proposed control law described in Eq. (37) into Eq. (39), yields

$$\begin{aligned}\dot{V} &= S_2\left(\dot{S}_2 + \alpha_c S_1\right) - \alpha_c S_1^2 + \tilde{W}_1^T\Gamma\dot{\widehat{W}}_1 + \tilde{b_1}\dot{\widehat{b}}_1\\ &= S_2\left(\alpha_c S_1 + \dot{x}_2 - \dot{x}_{2d} + \alpha_c\left(x_2 - x_{2d}\right)\right)\\ &\quad - \alpha_c S_1^2 + \tilde{W}_1^T\Gamma\dot{\widehat{W}}_1 + \tilde{b_1}\dot{\widehat{b}}_1. \quad (40)\end{aligned}$$

Then, using Eq. (36), Eq. (40) can be written as follows

$$\begin{aligned}\dot{V} &= S_2\left(\alpha_c S_1 + f_{xz} + u_0 + D_t - \dot{x}_{2d} + \alpha_c\left(x_2 - x_{2d}\right)\right)\\ &\quad - \alpha_c S_1^2 + \tilde{W}_1^T\Gamma\dot{\widehat{W}}_1 + \tilde{b_1}\dot{\widehat{b}}_1. \quad (41)\end{aligned}$$

Considering Eqs. (41) and (37), Eq. (41) can be expressed as

$$\begin{aligned}\dot{V} &= S_2\left(-\delta S_2 - \psi sign(S_2) - \widehat{D} + D_t\right) - \alpha_c S_1^2 + \tilde{W}_1^T\Gamma\dot{\widehat{W}}_1\\ &\quad + \tilde{b_1}\dot{\widehat{b}}_1. \quad (42)\end{aligned}$$

According Eqs. (24) and (27), we have:

$$\begin{aligned}\dot{V} &= S_2\Big(-\delta S_2 - \psi sign(S_2) - \Gamma f\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right) - \varepsilon_n\\ &\quad + \Gamma f(W_1^T\zeta + b_1)\Big) - \alpha_c S_1^2 + \tilde{W}_1^T\Gamma\dot{\widehat{W}}_1 + \tilde{b_1}\dot{\widehat{b}}_1. \quad (43)\end{aligned}$$

Using Taylor expansion of $f\left(W_1^T\zeta + b_1\right)$ about $\widehat{W_1}^T\zeta + \widehat{b}_1$, we have

$$\begin{aligned}f\left(W_1^T\zeta + b_1\right) &= f\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right) + \dot{f}\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right)\\ &\quad \times(\tilde{W}_1^T\zeta + \tilde{b_1}), \quad (44)\end{aligned}$$

in which, $f\left(W_1^T\zeta + b_1\right)$ is linearized by employing the mathematical Taylor polynomial. By substituting Eq. (44) into Eq. (43) yields

$$\begin{aligned}\dot{V} &= S_2\Big(-\delta S_2 - \psi sign(S_2) - \Gamma f\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right) - \varepsilon_n\\ &\quad + \Gamma f\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right) + \Gamma\dot{f}\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right)(\tilde{W}_1^T\zeta + \tilde{b_1})\Big)\\ &\quad - \alpha_c S_1^2 + \tilde{W}_1^T\Gamma\dot{\widehat{W}}_1 + \tilde{b_1}\dot{\widehat{b}}_1. \quad (45)\end{aligned}$$

Finally, considering the updating rules described in Eqs. (25) and (26), we have

$$\begin{aligned}\dot{V} &= S_2\Big(-\delta S_2 - \psi sign(S_2) - \Gamma f\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right) - \varepsilon_n\\ &\quad + \Gamma f\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right) + \Gamma\dot{f}\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right)(\tilde{W}_1^T\zeta + \tilde{b_1})\Big)\\ &\quad - \alpha_c S_1^2 - \tilde{W}_1^T\Gamma S_2\dot{f}\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right)\zeta\\ &\quad - \tilde{b_1}\Gamma S_2\dot{f}\left(\widehat{W_1}^T\zeta + \widehat{b}_1\right) = S_2\left(-\delta S_2 - \psi sign(S_2) - \varepsilon_n\right)\\ &\quad - \alpha_c S_1^2, \quad (46)\end{aligned}$$

and having in mind that $\boldsymbol{\psi} > |\boldsymbol{\varepsilon_n}|$, the following inequality is obtained

$$\dot{V} \leq -\delta S_2^2 - \alpha_c S_1^2. \quad (47)$$

At the present stage, the proof is completed, and the states of the system converge to the commanded values, even when there exist external disturbances, unknown input saturation, and the singularity problem.

IV. NUMERICAL SIMULATIONS

Herein, the performance and effectiveness of the proposed RNN-based NSMC are demonstrated. The design parameters of the controller were chosen as $\alpha_c = \psi = 20$, $\delta = 10$, and the initial weights in the RNN were considered as uniform random functions in the range (0,10). The system's parameters were supposed as $m_p = 0.639\ kg$, $M = 1.139\ kg$, $\rho = 0.2\ m$, $r_G = 0.101\ m$, and $I_c = 0.05\ m^4$. Also, to investigate the robust performance of the suggested control, the external disturbance was assumed as $d(t) = 5\cos\left(t^2\right)\frac{rad^2}{s}$ (By considering $d_{max} = 5\frac{rad^2}{s}$) and initial conditions for the system were $[X_1(0), X_2(0), Z_1(0), Z_2(0)] = [0.1, 0, 0, 0]$. The sampling time for simulations were chosen to be 0.01 second.

A. TRACKING CONTROL

In this section, the performance of the RNN-based NSMC on position tracking is illustrated. For this purpose, the slope of the plane was varying by $\boldsymbol{\alpha(t) = \frac{\pi}{8} + \frac{\pi}{12}sin\left(\frac{\pi}{40}t\right)rad}$ and the control input was limited to the values $\boldsymbol{u_{max} = 6\ N.m,\ and\ u_{min} = -5\ N.m}$. Then, Fig. 3 presents the simulation results. It can be easily observed that the proposed RNN-based NSMC can track the desired reference signal in the presence of disturbances, input saturation, and dynamic uncertainties. The maximum absolute angle of the pendulum is less than 0.4 rad. Therefore, it can be verified that the angle of the pendulum does not exceed $\frac{\pi}{2}$ rad, which shows that these results are appropriate for a practical system.

On the other hand, Fig. 4 demonstrates that the RNN-based disturbance observer can identify disturbances and uncertainties precisely. In fact, Fig. 4(a) depicts the torque input, which implies that the proposed RNN-based NSMC has been saturated, and Fig 4. (b) shows the estimated disturbance.

To show more extensively the capacity of the RNN-based approximation and the proposed control scheme in tracking

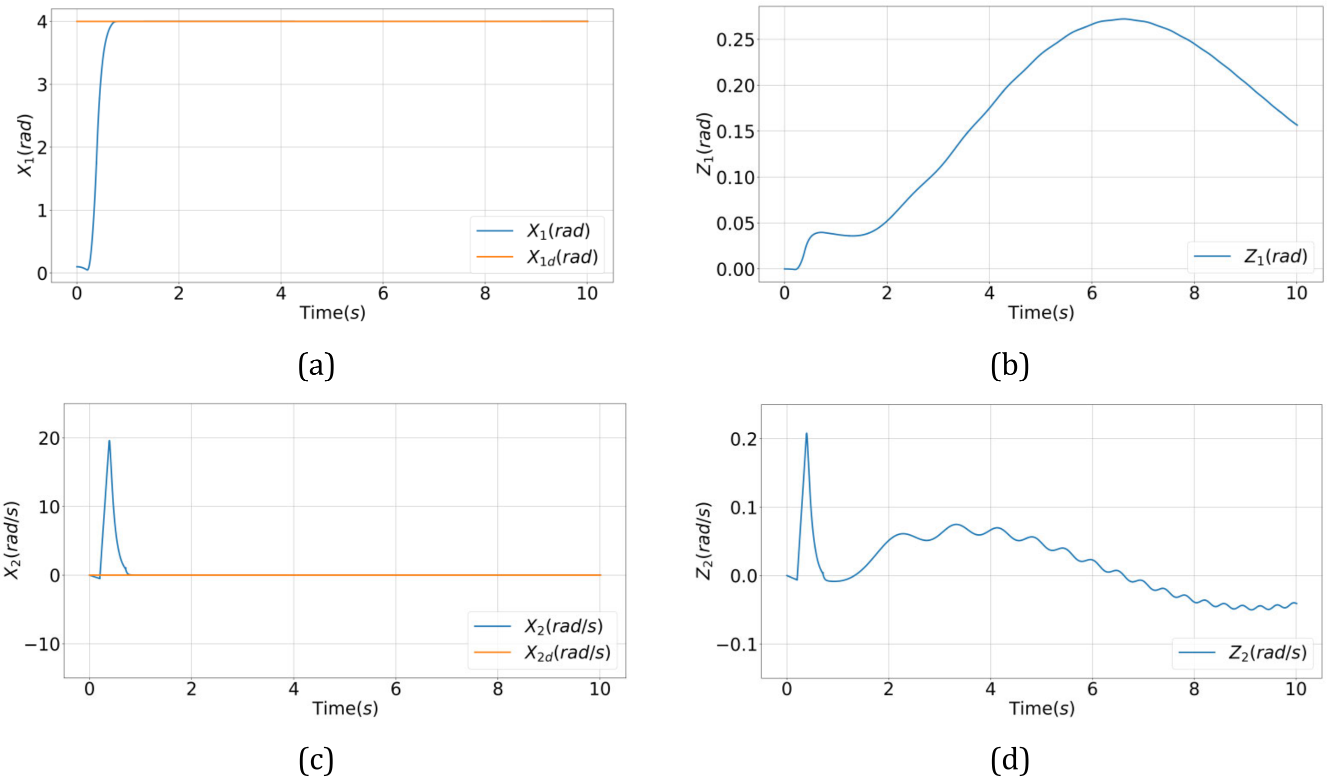

FIGURE 3. **Time history of the closed-loop system for a step input signal.**

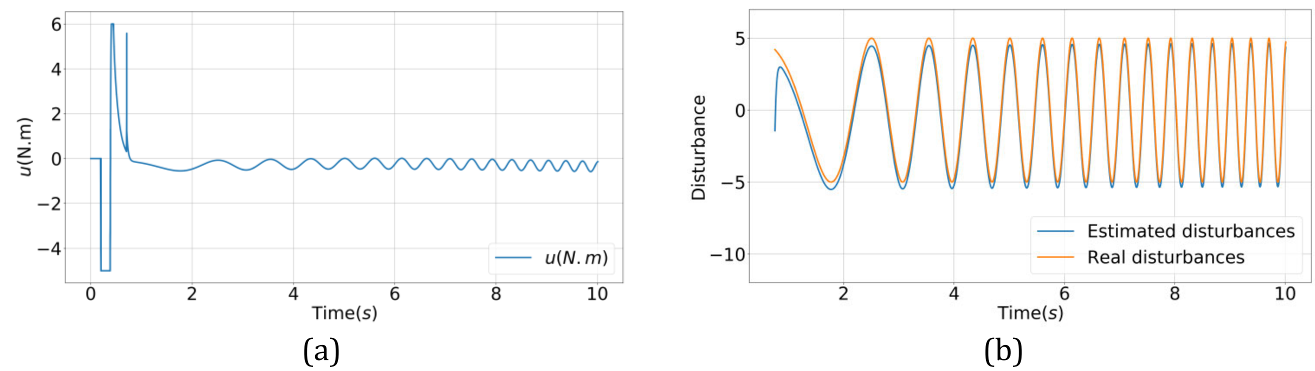

FIGURE 4. **(a) Time history of the control input when there are input saturation (b) Actual and estimated values of the disturbance d(t).**

control, the designed controller has been used for another trajectory target. Thus, Figs. 5 and 6 depict the state of the system, control signal, and the estimated disturbances for a ramp input signal. The numerical results conspicuously demonstrate that, using the proposed RNN-based NSMC, the spherical robot can track the desired trajectories, even when there exist uncertainties and external disturbances.

B. COMPARISON BETWEEN THE PROPOSED METHOD AND FUZZY-PID CONTROLLER

To illustrate the benefits of the proposed control scheme, its performance has been compared with a Fuzzy-PID controller. This algorithm was mainly selected by two reasons. On the one hand, many previous works have proven that neural network-based disturbance estimators considerably improves the performance of many controllers [75]. On the other hand, Fuzzy control has been suggested as suitable for complex robots, whose models cannot be easily established from a mathematical point of view [76]. Moreover, a Fuzzy-PID controller has been recently proposed to control a spherical robot with excellent performance [47].

Consequently, the method developed by Roozegar *et al.* [77] has been implemented and analyzed in the present work. Although all details for this controller can be found in [47], it should be noted that the gains for

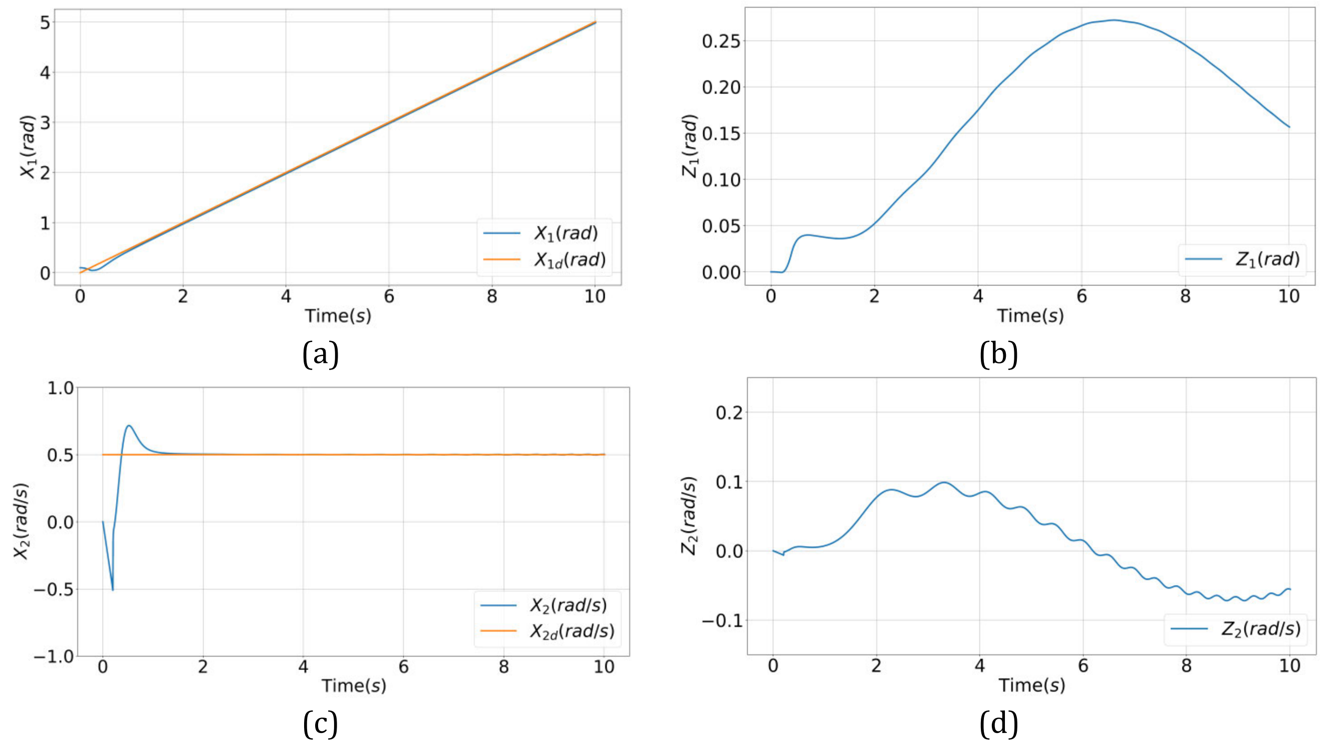

FIGURE 5. Time history of the closed-loop system's response for a ramp input signal.

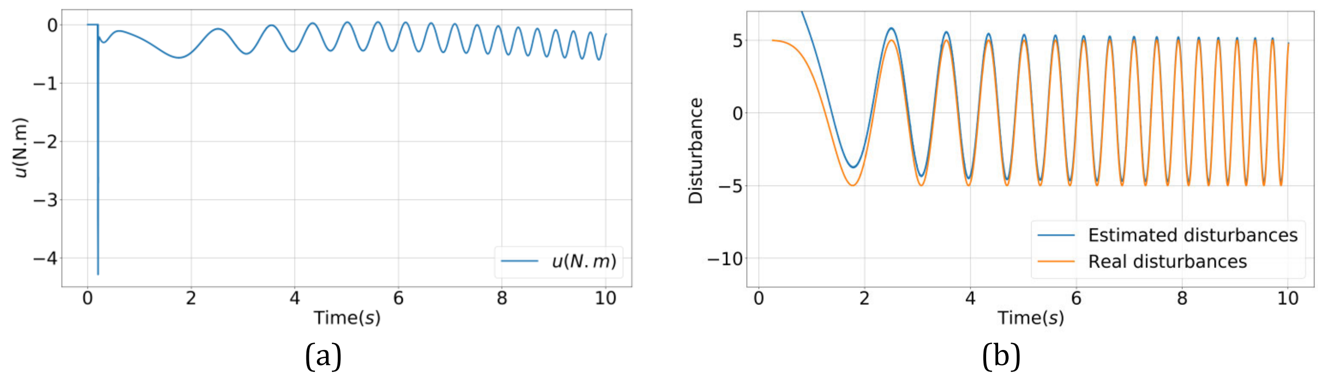

FIGURE 6. (a) Control input with input saturation for a ramp input signal (b) Actual and estimated values of the disturbance d(t).

TABLE 1. Parameters of the Fuzzy-PID control scheme.

Objective	K_p	K_d	K_i
Minimum	35	0.005	0.0008
Maximum	45	0.02	0.002

the control scheme are given in Table 1. Moreover, input is divided into 7 Fuzzy logic values, including zero (ZO), positive small (PS), positive medium (PM), positive big (PB), negative big (NB), negative medium (NM), and negative small (NS). The degree of error and its time derivative are expressed by these linguistic variables. Then, membership functions are depicted in Fig. 7 and the fuzzy rule table is listed in Table 2.

Figure 8 depicts the time history of the system with the proposed RNN-based control scheme as well as the implemented Fuzzy-PID controller used for comparison. In this simulation, both controllers have been applied to the robot after 0.2 seconds. As can be seen, both algorithms converge to the desired position. Nonetheless, it is noteworthy that when we have tried to consider the same input saturation for the Fuzzy PID controller, we could not obtain a proper result. Hence, we inevitably applied the fuzzy PID controller without control saturation, which means a great drawback for practical, real-world applications. As can also be observed in Figs. 8(a) and (b), the proposed RNN-based NSMC was

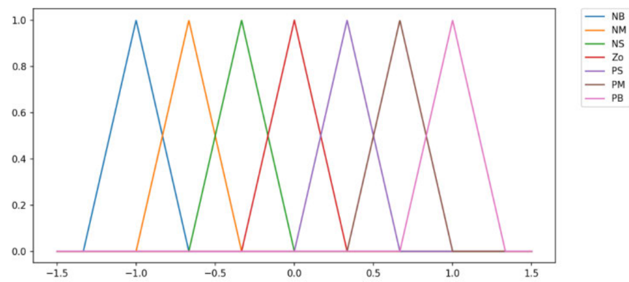

FIGURE 7. Membership functions for the Fuzzy-PID controller used for comparison.

TABLE 2. Fuzzy rule table for the fuzz-PID controller used for comparison.

	ė					
e	NB	NM	NS	ZO	PS	PM
NB	NB	NB	NB	NB	NM	NS
NM	NB	NB	NB	NM	NS	ZO
NS	NB	NB	NM	NS	ZO	PS
ZO	NB	NM	NS	ZO	PS	PM
PS	NM	NS	ZO	PS	PM	PB
PM	NS	ZO	PS	PM	PB	PB
PB	ZO	PS	PM	PB	PB	PB

faster than the Fuzzy-PID controller. Moreover, the simulation also confirmed that the proposed RNN-based controller was able to overcome the external disturbance and uncertainties better than the Fuzzy-PID. Furthermore, the Fuzzy-PID produced a high-frequency oscillation in the response of the system.

On the other hand, it should also be noted that conventional SMC generates chattering for uncertain systems due to the existence of the sign function in the control input. This phenomenon then causes vibration in the system, because it takes time to converge the sliding surface to zero. However, Fig. 8 shows that the proposed RNN-based NSMC is able to substantially minimize this problem. Actually, one of the major issues that causes chattering in a conventional SMC is the existence of the uncertainties and disturbances. However,, as it is shown in Fig. 4 (b) and 6 (b), the controller proposed in the present study is able to estimate quickly and accurately the uncertainties and disturbances. Actually, our numerical results confirm that after 3 second the estimation errors for ramp and step input respectively are less than 9% and 2% in which the estimation errors ($\boldsymbol{d_e}$) is calculated in (48), as shown at the bottom of the page.

Therefore, the vibrations in the response of the system will be significantly decreased in comparison with a conventional SMC.

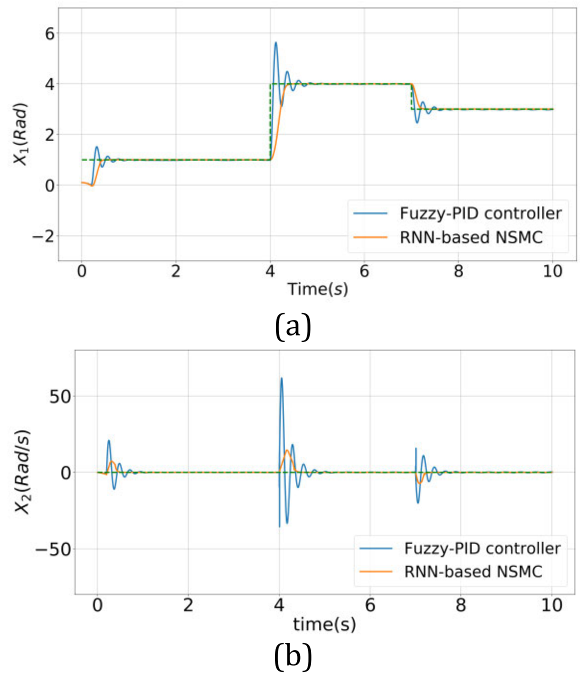

FIGURE 8. Comparison of the (a) angular position and (b) angular velocity reached by a spherical robot with the proposed controller and the Fuzzy-PID one sued for comparison.

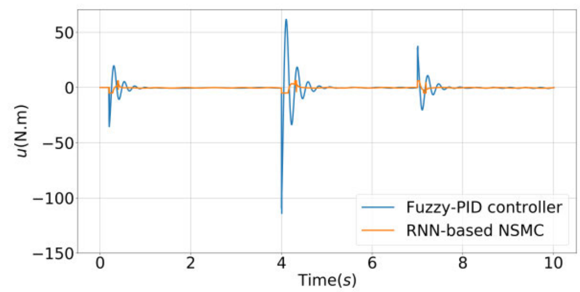

FIGURE 9. Comparison of the control inputs obtained by the proposed RNN-based controller and the Fuzzy-PID one.

The control input signals for both control schemes are depicted in Fig. 9. As can be seen, in both cases the input signals dropped to zero with some oscillation. Nonetheless, it can be noticed that, when the target changed, the control input for the Fuzzy-PID controller had a big overshoot. Otherwise, the control input for the proposed RNN-based controller presented a smaller overshoot and quicker convergence time. Thus, this figure corroborates that the proposed control approach needed less control effort compared to the Fuzzy-PID controller. To summarize, the proposed control method had smaller oscillations, faster response, and less control effort. Hence, the numerical simulations have confirmed the superiority of the proposed RNN-based NSMC over the Fuzzy-PID controller.

$$d_e = \frac{|\text{ACTUAL DISTURBANCE} - \text{ESTIMATED DISTURBANCE}|}{\text{MAX (ACTUAL DISTURBANCE)}} \tag{48}$$

V. CONCLUSION

In the present study, an RNN-based NSMC has been designed to control the motion of a spherical robot. Unknown uncertainties, including external disturbances and the control input saturation, have been quickly and accurately approximated by the approach, thus leading to successful control. Also, the prevention of the singularity problem has been taken into consideration for the control scheme. In fact, using the Lyapunov stability theorem and employing Taylor series method, stability, and robustness of the controller against uncertainties, disturbances, control input saturation, and singularity have been guaranteed. Finally, the proposed controller has been compared with a Fuzzy-PID control scheme. The obtained simulation results have shown that the response of the system is smoother when the proposed RNN-based controller was used. Nonetheless, as a future suggestion, updating control gains of the proposed control scheme in a Fuzzy environment can improve the output response of the system. Moreover, an extension of the proposed controller could also be used for fractional-order systems.

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