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Economic Geography with Spatial Econometrics: A'Third Way' to Analyse Economic Development and 'Equilibrium', with Application to the EU Regions

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Economic geography with spatial econometrics : a 'third way' to analyse economic development and 'equilibrium', with application to the EU regions

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Abstract

The main item of agreement between the 'new' and 'old' economic geography is the role of increasing returns in regional economic development. This provides a focal point for the model of this paper, which aims to highlight the existence of a 'third way' somewhere between the analysis provided by these two competing modes of explanation. Increasing returns are represented by the Verdoorn Law linking manufacturing output and productivity growth, which is augmented to include endogenous technical progress involving diffusion, spillover effects and putative human capital effects. The model is estimated using data for regions of the EU, thus emphasizing the need to confront theory with data. The approach of the paper thus avoids 'the lost scientific cause' of much of contemporary 'economic geography proper' and the constraints posed by the theory of 'new economic geography'. The implications of the model are explored and assumptions are imposed leading to a 'stochastic steady state' as an approximation to real world turbulence, and as an alternative to the Markov chain stochastic equilibrium suggested by Quah(1993). The paper shows that the implications of interregional spillovers are faster productivity growth and higher productivity levels, a trend that is accelerated with endogenously determined spillover. Without catch up, regional productivity levels diverge with no stable steady state and one region becomes increasingly dominant, but catch up ensures that cross-regional productivity growth rates tend to equality.

1. Introduction

Recently we have seen the advent of 'new economic geography' as a way of looking afresh at the causes of urban or regional concentration, problems that have traditionally been a concern of 'old' economic geography. The introduction to the book 'The Spatial Economy' by Fujita, Krugman and Venables, 1999, published on Krugman's website, the review by Ottaviano and Puga (1998), and the paper by Martin(1999), provide contrasting perspectives on these differing approaches. While the two approaches are evidently poles apart, it is interesting that both claim a particular branch of non-mainstream economics as more or less compatible with their contrasting viewpoints. Thus the literature on imperfect competition, increasing returns and cumulative causation processes, stemming back at least to Kaldor(1957) and Myrdal(1957) is, according to Martin(1999), consistent with some of the contemporary issues that are the concern of economic geographers. Similarly, Krugman(1991) acknowledges that Kaldor's vision of cumulative processes inspired subsequent work in new economic geography.

The aim of this paper is to take this point of contact between 'old' and 'new' economic geography as the basis of a modeling approach which avoids perceived limitations of both. Hence the paper develops and estimates a spatial econometric model which has at its core the Verdoorn Law (Verdoorn, 1949) linking manufacturing productivity growth to output growth which was also used by Kaldor (see also Fingleton and McCombie, 1998). This is relevant to contemporary modes of explanation, since the Verdoorn Law may be viewed as a forerunner of new growth theory, embodying increasing returns and endogenous technical progress. The model provides estimates of the significance of knowledge spillovers, first described by Marshall(1920) as one of a trio of reasons for spatial economic concentration. This has been 'downplayed' in new economic geography because 'it is hard to model'. Quite naturally much of the deductive mathematical theory underlying new economic geography has not been confronted by data since it was not designed principally with data analysis in mind. In contrast, combining economic geography with spatial econometrics emphasizes induction rather than deduction, the empirical testability of constructs at a fairly early stage prior to using them to infer trends. Such an approach is also at variance with 'the large-scale movement away from logical-positivism' (Martin 1999) that has influenced much of recent 'economic geography proper'.

While the present model seems to work quite well from an empirical point of view, the paper also highlights estimation problems that bring to the fore the question of whether or not a stable steady state exists. The paper emphasizes the unreality of a smooth progression to a deterministic steady state when compared to the observed turbulence of actual economies. This was also appreciated by Quah(1993) who suggested the 'stochastic equilibrium' of the Markov chain model as an alternative. While agreeing with the principle behind the choice of a Markov approach, the paper argues that it has limitations and uses instead the model developed in the paper to provide turbulent outcomes.

The final part of the paper explores further the dynamic implications of the model. In particular, the effects of interregional spillovers on productivity growth and levels are investigated in a 'laboratory' setting which facilitates endogenously determined interregional interaction. Thus the paper ends by moving a small way towards the abstraction which characterizes new economic geography, but from the opposite direction.

2. A model of regional economic growth

The dynamic Verdoorn Law propounds a linear relationship between the exponential growth rates of labour productivity (***p***) and output (***q***), so that

$$\boldsymbol{p} = \boldsymbol{m_0} + \boldsymbol{m}_1\boldsymbol{q} + x \qquad (1)$$
$$x \sim \mathbf{N(0,}\ s^2\mathbf{)}$$

In equation (1) the coefficient m_0 is the autonomous rate of productivity growth and m_1 is usually called the Verdoorn coefficient, the estimated value of which is quite consistently about 0.5 when the model is fitted to various data on manufacturing productivity growth and output growth. This indicates that a percentage point increase in output growth induces an increase in employment growth of about one-half of one percentage point and an equivalent increase in the growth of productivity. The error term x collects the other effects on ***p*** which in this initial specification are assumed to behave as random shocks. Hence, in its primitive form, the Verdoorn law is treated as a single equation and estimation is via OLS[1]. A number of issues are raised by this simple specification, since it excludes a number of ancillary variables suggested both by theory and by the applied literature. There is also the question of whether we need to account for endogeneity either in model structure or in estimation. The endogeneity of ***p*** can be justified by observing that since $\boldsymbol{p} = \boldsymbol{q} - \boldsymbol{e}$ (***e*** is employment growth), then $E(\boldsymbol{e}) = -\boldsymbol{m_0} + (1 - \boldsymbol{m_1})\boldsymbol{q}$. Employment growth is not an autonomous determinant of output growth, but is elastic due to commuting and labour migration. The presumed exogeneity of $\boldsymbol{q}$ has been challenged, as in the debate involving Rowthorn(1975a,b) and Kaldor(1975), and multi-equation systems of cumulative causation incorporating the dynamic Verdoorn Law commonly treat both $\boldsymbol{q}$ and $\boldsymbol{p}$ as mutually interdependent (Kaldor, 1970, Myrdal 1957, Dixon and Thirlwall 1975a,b, McCombie and Thirlwall 1994, Targetti and Foti 1997, Fingleton 1998a,b).

The Verdoorn Law may be seen as consistent with increasing returns, a feature emphasised in 'new economic geography' and long favoured by regional and urban economists working with internal and external (agglomeration) economies of scale. To see this, we commence with the conventional Cobb-Douglas[2] production function to demonstrate how a significant Verdoorn coefficient implies increasing returns as normally understood from this standpoint. We then develop this approach as a vehicle by which to introduce the additional features leading to the model of this paper. Assume therefore that an appropriate static underlying model is

$$\boldsymbol{Q} = \boldsymbol{A_0}\exp(l\boldsymbol{t})\boldsymbol{K}^{a}\boldsymbol{E}^{b} \qquad \textbf{\textit{(2)}}$$

in which l is the growth of total factor productivity or exogenous technical change, $\boldsymbol{Q}$, $\boldsymbol{K}$ and $\boldsymbol{E}$ are the levels of output, capital and employment. Note again that there is no constraint that there are constant returns to scale. In fact we know from Euler's theorem that the competitive equilibrium underpinning the neoclassical model requires that all factors are paid their marginal products and with increasing returns not all factors can be paid their marginal products, so we are admitting the possibility of a non-neoclassical world.
Taking natural logs and differentiating with respect to time, we obtain

$$\boldsymbol{q} = l + a\boldsymbol{k} + b\boldsymbol{e} \qquad \textbf{\textit{(3)}}$$

or equivalently, since $\boldsymbol{p} = \boldsymbol{q} - \boldsymbol{e}$, and allowing the presence of other effects (x),

$$\boldsymbol{p} = l/b + [(b - 1)/ b]\boldsymbol{q} + (a/b)\boldsymbol{k} + x \qquad \textbf{\textit{(4)}}$$

Equation (4) is seen to be a version of equation (1) but with the additional variable the growth of capital ($\boldsymbol{k}$). Hence ideally the Verdoorn Law should contain $\boldsymbol{k}$, but is omitted from this and many other cross-sectional analyses because, unfortunately, data on capital stock growth *per se* is for the most part unavailable at the level of regions. A standard approach in the literature is to use is the average share of real (gross) equipment investment in GDP as a proxy for $\boldsymbol{k}$, but even this may be unavailable.

If however we restrict the model by assuming that capital stock growth is equal to output growth (ie the capital – output ratio is constant), then $\boldsymbol{q}$ includes $\boldsymbol{k}$ which is omitted as an explicit term. The empirical basis for omitting $\boldsymbol{k}$ is the stylised fact that capital stock growth and output growth are both approximately the same in most developed economies[3]. Consequently equation (3) reduces to

$$\boldsymbol{p} = l/b + ((a + b - 1)/ b)\boldsymbol{q} + x \qquad \textbf{\textit{(5)}}$$

Observe from equation (5) that if $\boldsymbol{m_1} = ((a + b - 1)/ b) > 0$, then $(a + b) > 1$ and we have static returns to scale under the Cobb-Douglas production function.

Of course, as pointed out in the foregoing discussion, the Verdoorn Law *per se* is too simplistic to capture the nuances of regional growth variations and need to be somewhat enhanced by incorporating processes believed to be important at the regional level. In this paper, this development is achieved by endogenising technical progress (l) by relating it to productivity growth ($\boldsymbol{p}$) and some intrinsic regional characteristics, rather than treating it as an unexplained exogenous variable[4]. Let us attempt to justify first the dependence of l on $\boldsymbol{p}$. Given that we have already made the assumption that $\boldsymbol{k} = \boldsymbol{q}$, we now use the fact that this means that the growth of productivity ($\boldsymbol{q} - \boldsymbol{e}$) equates to the growth of capital per worker. Following some of the literature of endogenous growth theory (Lucas 1988, Barro & Sala-i-Martin 1995, p152), we choose to treat technical change as a function of capital accumulation (in the form of capital per worker). We assume that technical change is not fully internalized and so spills over to other firms and individuals within the region. Since at the level of EU (NUTS2) regions, regional boundaries are somewhat transparent and physically separated regions are often well connected, the spillover, it is hypothesized, will also involve other regions. The result is that firms and individuals capture externalities generated by productivity growth (*qua* capital accumulation) perhaps in neighbouring regions, or in important (high technology) regions elsewhere.

Since it is likely that a given region's productivity growth will have different effects on technical progress in different regions, we specify the following function

$$l = l^{\blacklozenge} + f\boldsymbol{p} + k\boldsymbol{p_o} \qquad \textbf{(6)}$$

In (6), $\boldsymbol{p}$ is intra-regional productivity growth and $\boldsymbol{p_o}$ denotes extra-regional productivity growth. It becomes clearer if we use matrix notation how $\boldsymbol{p_o}$ depends on the particular set of 'neighbours' for each region. The vector $\boldsymbol{p_o}$ is equal to the matrix product $\boldsymbol{Wp}$, with the cells of matrix $\boldsymbol{W}$ defining which regions influence technical progress. Hence $\boldsymbol{W}$ is a square matrix with n^2 cells defining the interaction between n regions. In the simplest case, $\boldsymbol{W}$ contains 1s and 0s linking pairs of regions, so the 1s in row i of $\boldsymbol{W}$ identify the regions interacting with region i. In practice, since we are dealing here with total manufacturing industry rather than a single chain of production, we assume that each region's influx of spillovers comes from all other regions (to varying extents depending on interregional distances and levels of technology). So it is by this matrix that remoteness impacts productivity growth. Regions that are remote have less spillover of knowledge, since transport costs reduce their interaction with neighbours. It is convenient to standardize $\boldsymbol{W}$ thus creating row totals equal to 1, in which case each element i of $\boldsymbol{Wp}$ is the weighted average of the other regions with weights proportional to the level of technology of their economies and their distance[5] from i. Thus high technology regions, even if they are physically remote, will have a large effect on region i since they will invariably contain industries driving technical progress in region i.

Thus far we have a region's technical progress depending on its capital accumulation represented by its productivity growth, and because technical progress is not contained by regional boundaries, we have included productivity growth in other regions. Of course, the spillover is two way and we assume it is simultaneous. Assume also that regions make technical progress at varying rates depending on internal conditions controlling the adoption and impact of technology diffusing from more advanced regions and countries. Note that this is not the same as the spillover effect already described. The key factor now is the level of technology of the recipient region, there is no suggestion that 'who your neighbours are' is a factor in this diffusion process. Assume that at any moment, there exists 'available for adoption' everywhere as it were, a given body of technical knowledge but whether or not a region adopts depends on the region's intrinsic characteristics. If it is an advanced region, then the available technology will make little or no impact, while a less developed region will benefit from adopting new technology. Realistically, Governments and the EU policy instruments will also be used to encourage the adoption of technology in less developed regions, and as regions develop, then the attraction of innovations diminishes and the strength of regional policy directed at innovation adoption weakens.

We attempt to capture this process by the variable ***G*** in equation (7),

$$l^{\blacklozenge} = p\boldsymbol{G} + d\boldsymbol{s} \qquad \textbf{\textit{(7)}}$$

in which $\boldsymbol{G_i} = (\boldsymbol{P}_{\mathrm{i}}^{*} - \boldsymbol{P}_{\mathrm{i}})/\boldsymbol{P}_{\mathrm{i}}^{*}$ is the start-of-period (and therefore exogenous) technology gap between region i and the leading technology region (*), proxied using the initial productivity levels $\boldsymbol{P}_{\mathrm{i}}$ and $\boldsymbol{P}_{\mathrm{i}}^{*}$ as the respective technology levels. The assumption is that $p > 0$, thus the larger the technology gap the faster the growth of technology. In summary, the mechanism is assumed to be the diffusion of innovations from high to low technology regions, enhanced by (a variety of) regional policy instruments becoming progressively weaker as catch-up occurs.

The second intrinsic regional characteristic is ***s***, the stock of human capital, which one would expect to influence innovation rates and innovation adoption.

$$\boldsymbol{s} = e + q\boldsymbol{l} + \mathrm{G}\boldsymbol{u} \qquad \textbf{\textit{(8)}}$$

Equation (8) assumes increasing human capital with decreasing peripherality (***l***), since peripheral regions are sparsely populated and culturally distinct from more central regions, and increasing human capital with increasing levels of urbanization (***u***).

Combining these, rearranging and simplifying, one obtains

$$\boldsymbol{p} = de/(b - f) + dq\boldsymbol{l}/(b - f) + d\mathrm{G}\boldsymbol{u}/(b - f) + p\boldsymbol{G}/(b - f) + k\boldsymbol{p_o}/(b - f)$$
$$+ (a + b - \boldsymbol{1})\boldsymbol{q}/(b - f) + x \qquad \textbf{\textit{(9)}}$$

or more simply

$$\boldsymbol{p} = r\boldsymbol{p_o} + \boldsymbol{b_0} + \boldsymbol{b_1 l} + \boldsymbol{b_2 u} + \boldsymbol{b_3 G} + \boldsymbol{b_4 q} + x \qquad \textbf{\textit{(10)}}$$

It is convenient to work with an equivalent matrix expression[6] for (10) which is

$$\boldsymbol{p} = r\boldsymbol{Wp} + \boldsymbol{Xb} + x \qquad \textbf{\textit{(11)}}$$
$$x \sim \mathrm{N}(\boldsymbol{0}, s^{2}\boldsymbol{I})$$

in which $\boldsymbol{p}$ is an n by 1 vector, r is a scalar representing the strength of autoregressive interaction, $\boldsymbol{W}$ is the n by n matrix, $\boldsymbol{X}$ is an n by $p^{''}$ matrix of regressors and $\boldsymbol{b}$ is a $p^{''}$ by 1 vector of coefficients. Fingleton(1998b) refers to equation (10) as an augmented spatial lag Verdoorn law.

3. Estimation : methods

In this section, four alternative estimation methods are discussed, namely OLS, Maximum Likelihood (ML), Instrumental Variables or Two Stage Least Squares (IV or 2SLS), and Bootstrap estimation. In fact, estimation of spatial autoregressive models such as equation (10) has most frequently been via ML since, in the spatial case, the spatially lagged variable correlates with the error term (Ord 1975, Anselin 1988). The consequence is that OLS is biased and inconsistent because a necessary asymptotic condition is violated. With the spatial lag represented by the matrix product $\boldsymbol{Wp}$ and the independent identically distributed error term by x, $\text{plim}\ n^{-1}((\boldsymbol{Wp})'\ x) = \text{plim}\ n^{-1}\ x'\boldsymbol{W}(\boldsymbol{I} - r\boldsymbol{W})^{-1}x \neq 0$ when $r \neq 0$, hence inconsistency results from the presence of the quadratic form. In contrast, in the time series case, with $\boldsymbol{W}$ structured as for time series (see note 6) and serially uncorrelated errors, $\text{plim}\ n^{-1}((\boldsymbol{Wp})'\ x) = 0$. So, while the small sample properties of the estimator are influenced by the presence of the lagged variable (the estimator is biased), it is consistent and valid for asymptotic inference (Anselin, 1988).

The likelihood for the spatial autoregressive model is

$$\boldsymbol{L} = |Q|(s^{n}(2p)^{n/2})^{-1}\exp(-x'x/2s^{2}) \qquad \textbf{\textit{(12)}}$$

in which the term $Q = (\boldsymbol{I} - r\boldsymbol{W})$ comes from the transformation from the vector of standard normal independent error terms to the vector $\boldsymbol{p}$ given by the Jacobian $\boldsymbol{J}$, where

$$\boldsymbol{J} = |\,\partial x / \partial \boldsymbol{p}\,| = |Q| \qquad \textbf{\textit{(13)}}$$

$$x = (Q\boldsymbol{p} - \boldsymbol{Xb}) \qquad \textbf{\textit{(14)}}$$

It follows that

$$\text{Ln}\ \boldsymbol{L} = \text{constant} - n/2\ \ln s^{2} - x'x/2s^{2} + \ln|Q| \qquad \textbf{\textit{(15)}}$$

in which $x'x$ is the sum of squares of errors.

Since $s^2 = \{\boldsymbol{p}'(\boldsymbol{I} - r\boldsymbol{W})'\boldsymbol{R}(\boldsymbol{I} - r\boldsymbol{W})\boldsymbol{p}\}/n$ and $\boldsymbol{R} = \boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$, by a process of substitution (see Upton and Fingleton 1985), we obtain the following expression in terms of r

$$\boldsymbol{M} = \ln(n s^2) - (2/n)\ln|Q| \qquad \textbf{\textit{(16)}}$$

Therefore the ML estimate of r is the value that minimizes the negative log profile likelihood $\boldsymbol{M}$.

There is however an important disadvantage associated with ML, namely the restricted parameter framework. Since it involves the error sum of squares, the concentrated likelihood has similarities to OLS, but the additional ('penalty function') term equal to the log of the determinant of the Jacobian separates the two estimators. Since the log determinant tends to infinity as r approaches the singularities at 1/i where i denotes eigenvalues of the matrix $\boldsymbol{W}$[7], it only makes sense to omit the singular points of $(\boldsymbol{I} - r\boldsymbol{W})$ for certain real values of r, such as at $r = 1/i_{max}$ and $r = 1/i_{min}$ where the log determinant is infinite. In practice, the model parameter estimates are obtained by searching[8] within the stable range defined by $1/i_{max}$ and $1/i_{min}$, where i_{max} is the largest eigenvalue or characteristic root and i_{min} is the smallest (ie the largest negative eigenvalue). In the stable range the parameter space is compact, but outside singular points interrupt the continuum of feasible parameter values. Standardizing $\boldsymbol{W}$ so that the rows sum to 1 is not essential, although doing this means that $i_{max} = 1$ and the upper bound of the stable range is equal to one (see Upton and Fingleton 1985, Haining 1990, Anselin 1988, Kelejian and Robinson,1995). Figure 1, the negative log profile likelihood, illustrates this. The corresponding parameter estimates are given in Table 2.

The outcome is that convergence is automatically imposed by ML estimation, rather than being an open question, because under ML there will always be a stable solution. However, it is precisely the possibility of non-convergence which is of interest here, and attention focuses on $r = 1$, a singularity at which the convergence process (ie the model (10)) is not defined.

IV estimation avoids the limitation of a restricted parameter framework, but introduces other problems. In fact, before the advent of modern software (ie SPACESTAT), it was easier to implement since it does not involve nonlinear optimization. None the less, its application to spatial models has been limited (Haining 1978, Bivand 1984, Anselin 1984).

As explained by Anselin(1988), a major problem for IV estimation is finding the proper set of instruments[9]. Let us commence by separating out the endogenous spatial lag $\boldsymbol{Wp}$ and denote the endogenous and exogenous variables of $\boldsymbol{X}$ as $\boldsymbol{X_I}$ and $\boldsymbol{X_x}$ respectively, so that the set of regressors is the n by $p^{\blacklozenge}+1$ matrix $\boldsymbol{Y} = (\boldsymbol{X_I}, \boldsymbol{X_x}, \boldsymbol{Wp})$. Clearly, we require for IV estimation the n by $q^{\blacklozenge}$ matrix of instruments $\boldsymbol{Z}$ comprising $\boldsymbol{X_x}$ plus some additional instruments to be described below. Define $\boldsymbol{P_z} = \boldsymbol{Z}(\boldsymbol{Z'Z})^{-1}\boldsymbol{Z'}$ as the symmetric and idempotent projection matrix. Hence $\boldsymbol{Y_p} = \boldsymbol{P_zY}$ and the IV estimate of $\boldsymbol{b_{IV}} = (r, \boldsymbol{b})$ is

$$\text{Est } \boldsymbol{b_{IV}} = (\boldsymbol{Y'}\, \boldsymbol{P_zY})^{-1}(\boldsymbol{Y'P_zp}) = (\boldsymbol{Y'_pY_p})^{-1}(\boldsymbol{Y'_pp}) \qquad \textbf{\textit{(17)}}$$

The matrix $\boldsymbol{Y'}\,\boldsymbol{P_zY}$ is nonsingular and can be inverted, despite the fact that $\boldsymbol{P_z}$ is singular, assuming that $\boldsymbol{Y}$ and $\boldsymbol{Z}$ are full column rank with $q^{\blacklozenge} \geq p^{\blacklozenge}+1$, and we assume also that the matrices (1/n)$\boldsymbol{Z'Z}$ and (1/n) $\boldsymbol{Z'Y}$ tend in probability to matrices of finite constants with full column rank (see Bowden and Turkington 1984, Kelejian and Prucha,1998). Since the vector (1/n) $\boldsymbol{Z'u}$ converges in probability to zero, then

$$\text{Est } \boldsymbol{b_{IV}} - \boldsymbol{b} = [\boldsymbol{Y'Z}/\text{n}\,(\boldsymbol{Z'Z}/\text{n})^{-1}\boldsymbol{Z'Y}/\text{n}]^{-1}\, \boldsymbol{Y'Z}/\text{n}(\boldsymbol{Z'Z}/\text{n})^{-1}\, \boldsymbol{Z'u}/\text{n} \qquad \textbf{\textit{(18)}}$$

tends to zero in probability. In summary, standard theory shows that, given that the instruments are assumed to be asymptotically correlated with the regressors and asymptotically uncorrelated with the errors, $\boldsymbol{b_{IV}}$ is a consistent estimator of $\boldsymbol{b}$.

We obtain the instruments for the data matrix $\boldsymbol{Y}$ using an estimate of the matrix of expected values $\text{E}(\boldsymbol{Y}) = (\boldsymbol{X_x}, \text{E}(\boldsymbol{X_I}), \boldsymbol{W}\text{E}(\boldsymbol{p}))$ given by $\boldsymbol{Y_p} = (\boldsymbol{X_x}, \boldsymbol{X_{Ip}}, \boldsymbol{Wp_p})$ in which $\boldsymbol{Wp_p} = \boldsymbol{P_zWp}$ and $\boldsymbol{X_{Ip}} = \boldsymbol{P_zX_I}$. However, it turns out that the more closely we try to approximate to $\boldsymbol{W}\text{E}(\boldsymbol{p})$, the more difficult it becomes to retain the full column rank of $\boldsymbol{Z}$ as required by the foregoing theory. This problem was first highlighted by Kelejian and Robinson (1993) and Kelejian and Prucha(1998) who observe that since
$\text{E}(\boldsymbol{p}) = (\boldsymbol{I} - r\boldsymbol{W})^{-1}\boldsymbol{Xb}$, and assuming $|r| < 1$,

$$\text{E}(\boldsymbol{p}) = [\textstyle\sum r^{i}\boldsymbol{W}^{i}]\boldsymbol{Xb} \qquad \textbf{\textit{(19)}}$$

where the summation is from i = 0 to ∞ and $\boldsymbol{W}^0 = \boldsymbol{I}$. This implies that $\boldsymbol{W}\text{E}(\boldsymbol{p})$ is a linear combination of the columns of the matrices $(\boldsymbol{X}, \boldsymbol{WX}, \boldsymbol{W}^2\boldsymbol{X}, \boldsymbol{W}^3\boldsymbol{X}\ \ldots)$. If an attempt is made to approximate $\boldsymbol{W}\text{E}(\boldsymbol{p})$ closely by including high order spatial lags in $\boldsymbol{Z}$ then the danger is the existence of linear dependence among the columns of $\boldsymbol{Z}$. This can be easily shown empirically by generating independent random vectors for $\boldsymbol{X}$ and progressively adding $\boldsymbol{WX},\boldsymbol{WX},\boldsymbol{W}^2\boldsymbol{X},\boldsymbol{W}^3\boldsymbol{X}$ etc to $\boldsymbol{Z}$. The inevitable result is that ultimately the columns become linearly dependent.

It follows that $\boldsymbol{Z}$ should be a subset of linearly independent columns of, for example, $(\boldsymbol{X},\boldsymbol{WX},\boldsymbol{W}^2\boldsymbol{X})$, or with a large number of regressors, of $(\boldsymbol{X},\boldsymbol{WX})$, in order to ensure full column rank and avoid problems associated with overidentification. We therefore endeavour to make $\boldsymbol{Wp_p}$ an approximation to $\boldsymbol{W}\mathrm{E}(\boldsymbol{p})$ and $\boldsymbol{X_{Ip}}$ an approximation to $\mathrm{E}(\boldsymbol{X_I})$ by forming $\boldsymbol{Z}$ as the set of exogenous variables and their low order spatial lags, plus any additional instruments correlating with $\boldsymbol{X_I}$. Kelejian and Prucha(1998) developed this approach in the wider context of a spatial autoregressive model which also has an autoregressive error process, and provide mathematical detail and proofs relating to an estimation procedure for this more complex model.

The fourth estimation method, Bootstrap estimation, provides a more robust approach avoiding strong error assumptions. Using IV to obtain the initial residual vector, k=1,…,999 vectors of pseudo errors are obtained by random resampling (with replacement) from the residual vector and for vector k a vector of pseudo observations are obtained from $\boldsymbol{p}_k = (\boldsymbol{I}-r\boldsymbol{W})^{-1}(\boldsymbol{Xb} + \boldsymbol{e}_k)$ using the initial IV estimates of $\boldsymbol{b}$ and r. The k'th (IV) estimates of $\boldsymbol{b}$ and r are based on the spatial lag $\boldsymbol{Wp}_k$. This approach is similar to that Bootstrap method for simultaneous equation systems, and is preferable to sampling from the joint density $(\boldsymbol{p},\boldsymbol{Wp}, \boldsymbol{X})$ since in the latter the data are spatially dependent rather than equiprobable cases as desired, and resampling would not preserve the spatial structure of the data, as explained by Anselin(1988).

4. Estimation : results

The data, covering 178 NUTS 2 regions (13 countries) of the EU, are taken from Cambridge Econometrics' European Regional Databank which is itself based on the (nominal prices, local currency) EUROSTAT series. Cambridge Econometrics fill gaps by interpolation, establish consistency with national series, and deflate using, in the absence of regional deflators, national deflators. The outcome is Gross Value Added measured presently in constant (1985) ecus[10]. Productivity growth ($\boldsymbol{p}$) is represented by the average annual (exponential) growth of manufacturing (and energy) gross value added per worker over the period 1975-1995. Similarly, output growth ($\boldsymbol{q}$) is the average annual growth of manufacturing gross value added over the period. While the data processing described above may create some measurement errors, this database is probably the most consistent and accurate available for the EU as a whole.

Table 1 OLS estimates of Spatial lag model

variable	parameter	estimate	t-value	Standard error
Wp	*r*	0.7420	5.5789	0.1330
constant	$\boldsymbol{b_0}$	-0.0219	-4.4242	0.0050
l	$\boldsymbol{b_1}$	-0.0148	-5.4212	0.0027
u	$\boldsymbol{b_2}$	0.0084	2.7087	0.0031
G	$\boldsymbol{b_3}$	0.0642	7.4848	0.0086
q	$\boldsymbol{b_4}$	0.4867	7.8374	0.0621
	R^2 (corrected)	0.431		
	R^2(squared correlation)	0.532		
	σ^2	0.0001679		

Table 2 ML estimates of Spatial lag model

variable	parameter	estimate	t-value	Standard error
Wp	*r*	0.6422	7.1909	0.0893
constant	$\boldsymbol{b_0}$	-0.0193	-4.8496	0.0040
l	$\boldsymbol{b_1}$	-0.0149	-5.4659	0.0026
u	$\boldsymbol{b_2}$	0.0083	2.7470	0.0030
G	$\boldsymbol{b_3}$	0.0642	7.5990	0.0084
q	$\boldsymbol{b_4}$	0.4960	8.2388	0.0602
	R^2	0.5266		
	R^2(squared correlation)	0.5460		
	σ^2	0.000163		

The inconsistent OLS estimates of equation (10) are given in Table 1. Table 2 contains a summary of the ML estimation of the model, assuming ***q*** is exogenous. Note that the Verdoorn coefficient is very close to the value of 0.5 that is commonly associated with the basic Verdoorn Law (equation (1)) even though in this instance we are estimating the augmented spatial lag version. The inference (see below) is that we have increasing returns to scale, with faster output growth inducing faster productivity growth. The parameter estimate of each of the variables is significantly different from zero and correctly signed, hence productivity growth increases with urbanization and with the start-of-period technology gap, and diminishes with increasing peripherality. In addition there is a highly significant spatial externality with each region's productivity growth interacting simultaneously and positively with productivity growth in the connected regions.

Thus far $\boldsymbol{q}$ has been exogenous, despite the earlier suggestions to the contrary. Previous work admitting endogenous $\boldsymbol{q}$ (see Fingleton and McCombie 1998) found that in practice instrumenting $\boldsymbol{q}$ (using lagged $\boldsymbol{q}$ or the rank of $\boldsymbol{q}$) made little difference to the interpretation. There is some evidence from Hausman's test of the joint endogeneity of $\boldsymbol{Wp}$ and $\boldsymbol{q}$, though the level of significance is marginal (the F ratio of has a p-value of 0.04 in the $F_{4,168}$ distribution) and may be attributable to $\boldsymbol{Wp}$.

Assume $\boldsymbol{q}$ is endogenous and use instrumental variables for both $\boldsymbol{q}$ and $\boldsymbol{Wp}$. Given the earlier problems of linear dependence, we follow standard practice and use only first spatial lags as $\boldsymbol{Wp}$ instruments. Since in equation (9) $\boldsymbol{G}$, $\boldsymbol{l}$ and $\boldsymbol{u}$ are taken as exogenous, the matrix $\boldsymbol{Z}$ comprises ($\boldsymbol{G},\boldsymbol{l},\boldsymbol{u},\boldsymbol{WG}, \boldsymbol{Wl}, \boldsymbol{Wu}, \boldsymbol{q_R}$). A check (using the LINDEPENDENCE macro of GENSTAT) confirms the linear independence of the columns of $\boldsymbol{Z}$ and the solution to equation (17) provides the estimates. Of course an identical set of IV estimates of $\boldsymbol{b_{IV}}$ is provided by two stage least squares (2SLS) in which the endogenous variables ($\boldsymbol{Wp},\boldsymbol{q}$) are first regressed on all the instruments ($\boldsymbol{q_R}, \boldsymbol{G},\boldsymbol{l},\boldsymbol{u},\boldsymbol{WG}, \boldsymbol{Wl}$ and $\boldsymbol{Wu}$) and the fitted values from these plus the exogenous variables $\boldsymbol{G},\boldsymbol{l},\boldsymbol{u}$ are the regressors and $\boldsymbol{p}$ is the regressand in the second stage.

The additional instrument $\boldsymbol{q_R}$ introduced for endogenous $\boldsymbol{q}$ is an adaptation of a method suggested in the context of the errors in variables problem by Durbin (1954), which uses as an instrumental variable rank orders (1,2,3 etc....denoting the highest, second, third etc value) in place of an endogenous variable. Evidently this approach produces consistent estimates under fairly general conditions (see Johnston 1984) although Bowden and Turkington (1984) and Maddala(1988) warn that if the errors are large, the ranks will be correlated with the errors and the estimators inconsistent.

Table 3 IV estimates of Spatial lag model (endogenous ***q***)

variable	parameter	estimate	t-value	Standard error
Wp	*r*	0.9707	5.0226	0.1933
constant	b_0	-0.0270	-4.5864	0.0059
l	b_1	-0.0164	-5.7116	0.0029
u	b_2	0.0071	2.2441	0.0032
G	b_3	0.0656	7.5792	0.0087
q	b_4	0.4027	5.7198	0.0704
	R^2	0.5567		
	R^2(squared correlation)	0.5368		
	σ^2	0.000166		

Table 4 Bootstrap estimates of Spatial lag model(999 replications)

variable	parameter	estimate	t-value	Standard error
Wp	*r*	0.8716	5.5371	0.1574
constant	b_0	-0.0303	-1.8610	0.0163
l	b_1	-0.0158	-5.8293	0.0027
u	b_2	0.0076	2.3802	0.0032
G	b_3	0.0661	7.6519	0.0086
q	b_4	0.4100	6.1189	0.0670
	R^2	0.7443		
	R^2(squared correlation)	0.4630		
	σ^2	0.000113		

The results of the IV estimation are summarized in Table 3. The overall fit of the model is reasonably good (R^2=0.56, squared correlation between observed and fitted values of the dependent variable = 0.54) with significant and correctly signed parameter estimates. However, the most striking feature of the estimates is the proximity of the endogenous spatial lag coefficient to the singularity at *r* =1 (using the region from plus to minus two standard errors from the estimated *r* to define proximate). Note that we cannot be entirely sure that the estimated *r* is actually consistent with a true *r* =1 since, in order to test H_0 : *r* =1, we require the sampling distribution of estimated *r* and t when H_0 is true, which for a spatial unit root is currently unknown. The development of a methodology comparable to that for time series has only recently begun (Fingleton, 1999a) and the multilateral dependence inherent in spatial processes can introduce complications. The hypothesis that r =1 is therefore without rigorous foundation, but seems a distinct possibility. Thus, with an unrestrained parameter space, avoiding linear dependencies and allowing for the endogeneity of $\boldsymbol{q}$, the indication is that the model (9) is indeterminate.

The Bootstrap parameter estimates, given in Table 4, are the means of the empirical parameter distributions, and the Bootstrap variances are the dispersions in these empirical distributions. The estimates obtained tend to reaffirm the results obtained by IV, although estimated r is now roughly one standard deviation below the singularity.

Since the model with endogenous $\boldsymbol{q}$ is (possibly) indeterminate, we assume exogeneity. With only $\boldsymbol{Wp}$ endogenous, $\boldsymbol{q}$ and $\boldsymbol{Wq}$ now are added to the instruments for $\boldsymbol{Wp}$ and the resulting estimate of coefficient r is now about two standard errors below the singularity, and well within the stable range.

Table 5 IV estimates of Spatial lag model ($\boldsymbol{q}$ exogenous)

variable	Parameter	Estimate	t-value	Standard error
$\boldsymbol{Wp}$	r	0.7336	4.7950	0.1530
constant	$\boldsymbol{b_0}$	-0.0209	-4.0043	0.0052
$\boldsymbol{l}$	$\boldsymbol{b_1}$	-0.0151	-5.3890	0.0028
$\boldsymbol{u}$	$\boldsymbol{b_2}$	0.0089	2.8876	0.0031
$\boldsymbol{G}$	$\boldsymbol{b_3}$	0.0626	7.2824	0.0086
$\boldsymbol{q}$	$\boldsymbol{b_4}$	0.4992	8.0449	0.0621
	R^2	0.5419		
	R^2(squared correlation)	0.5468		
	σ^2	0.0001624		

This is shown by Table 5 which reaffirms the earlier finding of a very significant simultaneous interaction across regions. According to the underlying model, this is due to non-internalised technical change arising from capital accumulation being captured in other regions. Also increasing returns are inferable from the Verdoorn coefficient, assuming, as seems reasonable, that all coefficients in equation (9) apart from e

(unknown *a priori*) and q are positive. The parameter estimates in Table 5 indicate that $b > f$ and since $\boldsymbol{b_4} = (a + b - 1)/(b - f)$ then the fact that estimated $\boldsymbol{b_4}$ is significantly greater than 0 means that $(a + b) > 1$. The Table 5 estimates also indicate a significant catch-up term and significant urbanization and peripherality effects attributed to human capital.

5. Convergence with spatial effects – simulation methodology

In the previous Sections it was pointed out that the model is indeterminate, and therefore cannot converge to a steady state, at the singular points of $(\boldsymbol{I} - r\boldsymbol{W})$. In this Section we therefore confine attention to the feasible parameter space, primarily the stable range of the compact region $1/i_{max} > r > 1/i_{min}$. We also briefly explore the nature of ‘convergence’ outside this compact region. Even within the stable region, there are other conditions required for smooth convergence to a steady state. One is the existence of the catch up mechanism, without which regions diverge. An additional condition is an absence of stochastic disturbances. In Part 6 we then introduce disturbances and focus on stochastic outcomes. In this set up, we cannot conceive of a role for maximising decisions by rational individuals determining the dynamics and a single equilibrium. It is interesting that micro-foundations also have little to offer Fujita, Krugman and Venables (1999). It is pertinent to quote at length from their rationalization. Thus

‘to insist that models of economic geography explicitly model firms and households as making intertemporal decisions based on rational expectations would greatly complicate an already difficult subject. It is very tempting to take a shortcut: to write down static models, then impose ad hoc dynamics on those models’

‘Ad hoc dynamics have been very much out of fashion in economics for the past 25 years; dynamics are supposed to emerge from rational, maximising decisions by individual agents. Yet what is one to do when a model predicts the existence of multiple equilibria, as geography models usually do?’

‘In short, we believe that we are right to give in to the temptation to sort out equilibria using simple, evolutionary dynamic stories, even though the models do not ground these dynamics in any explicit decision-making over time.’

With these limitations in mind, we show the implications in terms of dynamics of the spatial autoregressive model by using appropriate parameter estimates to drive the model forward to a (deterministic) steady state. As mentioned above, under certain assumptions the deterministic steady state exists and is very easy to obtain analytically. In practice an equivalent iterative solution is preferred and this leads to the method for 'stochastic equilibrium'. We commence with a re-expression of model (10), which is

$$\boldsymbol{p} = r\boldsymbol{W}\boldsymbol{p} + \boldsymbol{X}\boldsymbol{b} + x \qquad \textbf{\textit{(20)}}$$

$$x \sim \mathrm{N}(\boldsymbol{0},\ s^2\boldsymbol{I})$$

$$(\boldsymbol{I} - r\boldsymbol{W})\boldsymbol{p} = \boldsymbol{X}\boldsymbol{b} + x$$

$$\mathrm{E}(\boldsymbol{p}) = (\boldsymbol{I} - r\boldsymbol{W})^{-1}\boldsymbol{X}\boldsymbol{b} \qquad \textbf{\textit{(21)}}$$

Assume a steady state exists, then at steady state the proportional rate of growth of $\boldsymbol{R}$, $\boldsymbol{R}^{\bullet}/\boldsymbol{R}$, equals zero. Since $\boldsymbol{R}^{\bullet}/\boldsymbol{R} = \mathrm{E}(\boldsymbol{p} - \boldsymbol{p}^*)$, then

$$\mathrm{E}(\boldsymbol{p} - \boldsymbol{p}^*) = (\boldsymbol{I} - r\boldsymbol{W})^{-1}\boldsymbol{X}\boldsymbol{b} - (\boldsymbol{I} - r\boldsymbol{W})^{-1}\boldsymbol{X}^*\boldsymbol{b} = \boldsymbol{0} \qquad \textbf{\textit{(22)}}$$

Which can be re-expressed as

$$\mathrm{E}(\boldsymbol{p} - \boldsymbol{p}^*) = (\boldsymbol{I} - r\boldsymbol{W})^{-1}(\boldsymbol{X} - \boldsymbol{X}^*)\boldsymbol{b} = \boldsymbol{0} \qquad \textbf{\textit{(23)}}$$

In equation (22) and equation (23), $\boldsymbol{X}^*$ has the same dimensions as $\boldsymbol{X}$, but each row of $\boldsymbol{X}^*$ is equal to the productivity leader's row of $\boldsymbol{X}$. In order to obtain an expression for $\boldsymbol{G}^e$ and hence the steady state vector $\boldsymbol{R}^e$, we remove $\boldsymbol{G}$ from $\boldsymbol{X}$, thus creating matrix $\boldsymbol{X}^{\blacklozenge}$ with the corresponding vector of (reduced) coefficients denoted by $\boldsymbol{b}^{\blacklozenge}$. The (n x 1) matrix $\boldsymbol{X}$ ¢contains $\boldsymbol{G}$ while the coefficient corresponding to $\boldsymbol{G}$ is denoted by $\boldsymbol{b}$ ¢, hence at steady state

$$(\boldsymbol{I} - r\boldsymbol{W})^{-1}\boldsymbol{X}^{\blacklozenge}\boldsymbol{b}^{\blacklozenge} + (\boldsymbol{I} - r\boldsymbol{W})^{-1}\boldsymbol{X}^2\boldsymbol{b}^2 - (\boldsymbol{I} - r\boldsymbol{W})^{-1}\boldsymbol{X}^*\boldsymbol{b} = \boldsymbol{0} \qquad \textbf{\textit{(24)}}$$

And thus

$$\boldsymbol{X}^2 = \boldsymbol{G}^e = (\boldsymbol{X}^*\boldsymbol{b} - \boldsymbol{X}^\blacklozenge\boldsymbol{b}^\blacklozenge)(\boldsymbol{b}^2)^{-1} \qquad (25)$$

$$\boldsymbol{R}^e = \boldsymbol{U} - (\boldsymbol{X}^*\boldsymbol{b} - \boldsymbol{X}^\blacklozenge\boldsymbol{b}^\blacklozenge)(\boldsymbol{b}^2)^{-1} \qquad (26)$$

In which $\boldsymbol{R}^e$ denotes the steady state vector of productivity level ratios and $\boldsymbol{U}$ is a vector of 1s.

Critically, as mentioned above, if $\boldsymbol{b}^2 = 0$, there is no steady state, so the presence of the term $\boldsymbol{G}$ is necessary for a stable solution. Also, observe that $r\boldsymbol{Wp}$ is absent from (16) because $\boldsymbol{p}$ is constant across regions in the steady state, so the weighted average $\boldsymbol{Wp}$ will be a constant. This means that whether cross-region spillovers are weak or strong makes no difference to steady state productivity gaps. However the existence of spillovers causes faster steady state productivity growth and thus higher productivity levels than would otherwise occur, *ceteris paribus*. Also, if an autoregressive process exists but is omitted from the model, the estimates of $\boldsymbol{b}$ are biased (Anselin 1988), hence the steady state vector $\boldsymbol{R}^e$ will be biased.

As also mentioned above, we obtain precisely the same vector $\boldsymbol{R}^e$ by iteration, as defined by equations (27a) to (27e). In these, $\boldsymbol{R}$ for each iteration obtained from E($\boldsymbol{p}$) based on a revised matrix $\boldsymbol{X}$ since $\boldsymbol{G} = 1- \boldsymbol{P}/\boldsymbol{P}^* = 1- \boldsymbol{R}$ changes as $\boldsymbol{P}$ and $\boldsymbol{P}^*$ change with E($\boldsymbol{p}$) and E($\boldsymbol{p}^*$). Hence, with v denoting column v of $\boldsymbol{X}$,

$$\mathrm{E}(\boldsymbol{p}_t) = (\boldsymbol{I} - r\boldsymbol{W})^{-1}\boldsymbol{X}_t\boldsymbol{b} \qquad (27a)$$

$$\boldsymbol{P}_{t+1} = \boldsymbol{P}_t\exp(\mathrm{E}(\boldsymbol{p}_t)) \qquad (27b)$$

$$\boldsymbol{P}^*_{t+1} = \boldsymbol{P}^*_t\exp(\mathrm{E}(\boldsymbol{p}^*_t)) \qquad (27c)$$

$$\boldsymbol{G}_{t+1} = 1 - (\boldsymbol{P}_{t+1}/\ \boldsymbol{P}^*_{t+1}) \qquad (27d)$$

$$\boldsymbol{X}_{t+1,v} = \boldsymbol{G}_{t+1} \qquad (27e)$$

We use this iteration to illustrate the influence of $r\boldsymbol{Wp}$ on the transitional dynamics, and the independence of the steady state under stable conditions. The outcome, for an artificial set of 10 regions with arbitrary $\boldsymbol{X}$, $\boldsymbol{b}$ and $\boldsymbol{W}$ matrices, the so-called regional laboratory, is summarized by Figures 2, 3 and 4. To produce these Figures, everything apart from r is held constant. Figure 2 shows the dynamics leading to steady state with r

= 0, Figure 3 is for r = 0.6 and Figure 4 is for r = 0.95. The graphs plot $\boldsymbol{R}$ against iteration number.

Assume that necessary conditions for smooth dynamics to a stable steady state exist, thus $\boldsymbol{b}^2 \neq 0$ and there are no stochastic disturbances. Assume also r takes values outside the stable range defined by $1/i_{max}$ and $1/\ i_{min}$, acknowledging that outside the stable range the model is 'well defined' (Kelejian and Robinson 1995) so long as the singularities are avoided. This leads us to briefly consider what kind of equilibrium, if any, exists in this region. To take one example, with r = 1.05, which lies between the singular point at r = 1 and the next one at r = 1.085069, we have a characteristically explosive or non-stable process, as illustrated by the simulation in Figure 5.

6. Convergence with spatial effects – simulation empirics

The parameter estimates indicate that, ignoring stochastic disturbances, conditions exist for smooth convergence to a stable steady state. This is illustrated by applying the iteration (27) to the fitted model for the EU regional data set. We hold the variables $\boldsymbol{u}$ and $\boldsymbol{l}$ constant but allow $\boldsymbol{G}$ to change because of the link between E($\boldsymbol{p}$) and $\boldsymbol{G}$. We also hold $\boldsymbol{q}$ (hence $\boldsymbol{k}$) constant across regions at the EU annual average calculated over the period 1975-1995 (0.01731). This might not be so bad an assumption with open markets in a single European economy, if the same policy instruments and market conditions are assumed to hold for each region, and is preferred to retaining the disequilibrium growth rates. This seems acceptable under advanced European economic integration which lowers interregional barriers and enhances market penetration. High demand in faster growing regions will be satisfied by output in other regions, so there will be a tendency for output growth to be equilibrated across regions.

Figure 6 gives the dynamics leading to the steady state vector[11] $\boldsymbol{R^e}$ based on the Table 5 IV estimates. The inference from Figure 6 is a lower productivity gap, with the average ratio rising from about 0.6 to 0.8 of the leaders' productivity level at steady state. While there is considerable catch up, the steady states remain dispersed.

7. 'Stochastic equilibrium'

Thus far the model copies neoclassical models in so far as, under certain assumptions, it too predicts stable steady states. The notion of a smooth progression to a stable equilibrium was a target of criticism by Quah(1993), who made the point that growth trends in actual economies do not appear to be stable and smooth. This observation is certainly true of the EU regions. As Figure 7 shows, there is considerable ***R*** turbulence over 1975-95 compared to the idealised paths to equilibrium (Figure 6) under the model.

In order to eliminate any unrealistic certainty from our model predictions and to inject realism, the assumption is that at any instant productivity growth is disturbed by random shocks. This means that the model is open to a stream of unknown external factors, for example policy and institutional changes, historical events and abrupt social and environmental changes that are exogenous to the production system and parodied by stochastic disturbances. Assume, for simplicity, independence over time and space. Otherwise we might conjecture fitting a model like that of Kelejian and Prucha (1998) entailing both autoregressive spatial lag and autoregressive disturbances. Of course this assumption is already part of our model structure in the form of x in equation (11). The impact of a single shock is illustrated by Figures 8 and 9 (which one might compare with Figure 3, which is an otherwise identical process). At t = 10, a shock to productivity in a single region simultaneously impact productivity growth in other regions, the extent of impact depending on the structure of the ***W*** matrix. This affects subsequent productivity growth but is impermanent. Assume that the shocks are recurrent, rather than 'one-off', injecting turbulence along the path to 'equilibrium'. Iteration (28a to 28g) introduces recurrent disturbances so that there is no possibility of the impacts dying out giving a smooth path to steady state.

$$x_t = \mathrm{N}(0, s^2\boldsymbol{I}) \qquad \textbf{\textit{(28a)}}$$

$$\boldsymbol{p_t} = (\boldsymbol{I} - r\boldsymbol{W})^{-1}(\boldsymbol{X_t b} + x_t) \qquad \textbf{\textit{(28b)}}$$

$$\boldsymbol{P_{t+1}} = \boldsymbol{P_t}\exp(\boldsymbol{p_t}) \qquad \textbf{\textit{(28c)}}$$

$$\boldsymbol{P^*_{t+1}} = \boldsymbol{P^*_t}\exp(\boldsymbol{p^*_t}) \qquad \textbf{\textit{(28d)}}$$

$$\boldsymbol{G_{t+1}} = 1 - (\boldsymbol{P_{t+1}} / \boldsymbol{P^*_{t+1}}) \qquad \textbf{\textit{(28f)}}$$

$$X_{t+1,v} = G_{t+1} \qquad (28g)$$

The process is realised for the 178 regional economies using the estimates in Table 5, including s^2 = 0.0001624, as summarised by Figure 10. As is apparent, a sequence of one hundred iterations per realisation generates more or less 'stable' turbulence.

In Figure 10 the paths traced by the individual economies are dependent, a form of 'sticky mobility'. Productivity growth never becomes equalised as in the deterministic model, with the effect that regions perpetually interact producing cycles of fast and slow growth as the net outcome of shocks simultaneously transmitted across regions. In addition, when the technology leadership changes, as a result of a faster growing region replacing a slower one, the technology gap widens. A large downward shock to the leader's growth reduces the technology gap. These interactions are apparent in the topography of Figure 10.

Rerunning using different random number streams produces peaks and valleys are in different positions. Therefore, since Figure 10 is just one of many realisations, we need to generate a large number to have a more accurate picture of 'equilibrium'. As an illustration, Table 6 summarises 100 different realisations for a few NUTS 2 regions of the EU ranging from a very high ranking region (Antwerp) to a very low ranking region (Crete), plus a few other selected intermediate regions of interest.

This approach can be compared with the Markov chain approach suggested by Quah(1993) to model turbulent dynamics. An attractive feature of Markov chains is the presence of stochastic equilibrium, which is the stable vector of probabilities of different levels of productivity. At equilibrium, the state probabilities that are fixed, but regions can migrate from one state to another thus reflecting some of the turbulence of the real world.

There are a number of limitations of the Markov chain approach which have been highlighted by Fingleton(1997, 1998b, 1999b,c). One is that it does not take explicit account of inherent differences between regions affecting productivity growth and steady states. Secondly, it is not clearly related to any one specific underlying economic theory. Thirdly, it ignores the role of spatial interaction and consequent 'sticky mobility', which has been shown to be significant in the empirical results in this paper. Thus, while the concept of stochastic equilibrium is an attractive one, Markov chains are not the ideal mode of analysis. For instance since the economic theory is obscure, it would be hard to create alternative scenarios by manipulating driving variables.

Table 6 The distribution of stochastic equilibrium outcomes for selected regions

		Region			
	1	2	3	4	5
.95	71	0	0	0	0
.90	27	0	0	0	0
.85	2	1	0	0	0
.80	0	18	0	0	0
.75	0	49	7	0	0
.70	0	30	39	6	0
.65	0	2	45	41	0
.60	0	0	9	43	0
.55	0	0	0	10	0
.50	0	0	0	0	0
.45	0	0	0	0	1
.40	0	0	0	0	23
.35	0	0	0	0	68
.30	0	0	0	0	8
.25	0	0	0	0	0

Key :
1. Antwerpen, Belgium
2. E. Anglia, UK
3. Toscana, Italy
4. Ireland
5. Kritti, Greece

8. Endogenous spatial interaction

The focus on the conditions leading to some kind of steady state has up to now ignored endogeneity involving the matrix ***W***. Endogeneity is a consequence of ***W*** being a function of the level of technology, thus

$$\ddot{\boldsymbol{W}}_{ij,t+1} = \boldsymbol{P}_{j,t+1}^{\alpha} / \boldsymbol{d}_{ij}^{\gamma} \qquad (29a)$$

$$\boldsymbol{W}_{ij,t+1} = \ddot{\boldsymbol{W}}_{ij,t+1} / S_j \, \ddot{\boldsymbol{W}}_{ij,t+1} \qquad (29b)$$

and by the fact that $\boldsymbol{P}_{t+1} = \boldsymbol{P}_t \exp(\boldsymbol{p}_t)$. Thus far we have simplified the construction of $\boldsymbol{W}$ by assuming the steady state $\boldsymbol{W}$, with $\boldsymbol{p}$ constant so that the link between $\boldsymbol{P}$ and $\boldsymbol{p}$ is of no consequence for $\boldsymbol{W}$ since $\boldsymbol{W}_{ij,t+1} = \boldsymbol{W}_{ij,t}$. Once we admit spatially varying $\boldsymbol{p}$ then this is not the case. Geography becomes mutable. The intuitive outcome is that productivity growth differences will strengthen interaction between fast growing regions. However, it turns out that while out-of-equilibrium dynamics are altered, the catch up term in the model dictates that regions still converge to the same productivity growth rates whether or not $\boldsymbol{W}$ is endogenous. This is illustrated for the laboratory set up by Figures 11 and 12. Figure 11 shows the productivity growth dynamics for exogenous $\boldsymbol{W}$ and Figure 11 is the same set up except that $\boldsymbol{W}$ is endogenous. The growth dynamics are different and therefore the steady state levels differ with endogenous $\boldsymbol{W}$ producing higher productivity levels at any one time. However the inexorable tendency for productivity growth rates to become equalised in the steady state causes the productivity level ratios (R) to be equal under both exogenous and endogenous $\boldsymbol{W}$.

Although catch up is an empirical reality, it is instructive to look at what happens in its absence. Simply assuming exogenous $\boldsymbol{W}$ results in productivity growth rates failing to converge, but remaining constant at the levels determined by output growth rates, interregional spillovers and intrinsic differences between regions. This is illustrated by Figure 13, which is deceptively simple because it implies that regional productivity levels diverge. Note that interregional spillover is not the sole reason for regional divergence, but it adds to the rate of divergence by increasing the productivity growth rates. However, it could also cause regions to diverge in a similar fashion by causing the (higher) productivity growth rates to be more similar. We can show experimentally that if two regions are well connected then their (higher) productivity growth rates converge. Assume 10 regions with (implicit) productivity growth rates 0.01 to 0.1, ordered in sequence so that region 1 has growth rate 0.01, region 2 has 0.02 and so on. Call this the vector $\boldsymbol{p}^{\circ}$. These are not the actual growth rates $\boldsymbol{p}$ since we assume $\boldsymbol{p} = \boldsymbol{p}^{\circ} + \boldsymbol{W}\boldsymbol{p}^{\circ}$ or $\boldsymbol{p}^{\circ} = (\boldsymbol{I} + r\boldsymbol{W})^{-1}\boldsymbol{p}$ in other words we assume growth is enhanced by externalities with spillover from other regions. Assume also a very simple $\boldsymbol{W}$ matrix with just two regions interacting, hence $\boldsymbol{W}$ is a 10 by 10 matrix of zeros except $\boldsymbol{W}_{12} = 1$ and $\boldsymbol{W}_{21} = 1$. Assuming $r = 0.5$, then $\boldsymbol{p}_1 = 0.02$ and $\boldsymbol{p}_2 = 0.025$ but the rest remain as before, so spatial interaction reduces the difference from $D\boldsymbol{p}^{\circ} = 0.01$ to $D\boldsymbol{p} = 0.005$ and raises the growth rates, compared with what they would have otherwise been with $\boldsymbol{W} = 0$ throughout. In general the presence of the externality causes growth rates to be higher and growth rate differences to be lower than otherwise. Assume we have complete connectivity between regions so that ***W*** contains values equal to 1/9 apart from zeros on the main diagonal. A consequence is that the implicit growth rate differences of $D\boldsymbol{p}^{\circ}$ = 0.01 between successive regions reduce to actual differences equal to $D\boldsymbol{p}^{\circ} - rD\boldsymbol{p}^{\circ}/9 = 0.009444$. Increasing spatial interaction by allowing r to approach 1 reduces differences further and increases growth rates. The rule $D\boldsymbol{p}^{\circ} - rD\boldsymbol{p}^{\circ}/9 = D\boldsymbol{p}$ also applies to a random vector $\boldsymbol{p}^{\circ}$ so positive and negative differences are closer to zero. However this is only a special case when complete connectivity exists, with arbitrary ***W*** it is the more connected regions growth rates that converge.

We have seen that different ***W*** produce different productivity growth rates in the long run when catch up is eliminated. We now look at what happens when ***W*** is endogenously determined and as explained above depends on the growth of productivity. Allowing this to happen in our laboratory produces Figure 14, which can be compared to Figure 13 which is the same but for fixed ***W***. Initially there is a rising trend with some regions' productivity growth rates move together, some move apart. Interaction between regions that are growing fast is strengthened, causing them to grow faster and converge in growth rate. Endogenously determined interaction produces faster growth than otherwise, but why does it automatically tend to level growth rates? The reason is that with one region growing faster it eventually becomes very dominant with a much higher productivity level than the other regions. Figure 15 illustrates the long term consequences of the growth rates of Figure 14, the productivity levels ratios are tending to zero. Since ***W*** depends on productivity levels, this means that the ***W*** matrix tends towards domination by a single fastest growing region. In the limit the ***W*** cells tend to zero except for the column for the fastest growing region, which tend to one. This means that the productivity growth of the fastest growing region is tending to become the only factor involved in the slower growing regions interregional interaction. The productivity growth of the fastest growing region in turn depends solely on the productivity growth in the second fastest region. Thus there is also one cell in the row for the fastest growing region tending to one. The ***W*** matrix always tends to a constant with these characteristics when there is one region with faster productivity growth than the others. This tendency for endogenously determined ***W*** matrices to tend to a constant as the dominant region becomes increasingly dominant explains why the growth rates level off, since there is no longer the reinforcing effect which occurred before ***W*** stabilised.

Finally, let us explore the results of introducing endogenously determined interaction into the full scale simulation portrayed by Figure 10, which included all the variables including catch up and stochastic turbulence.

Because of the complex of factors operating simultaneously, the theoretical tendency to polarise is contained. In fact, since the manufacturing productivity growth rates are small in relation to the productivity levels, allowing them to feed back to the levels makes little difference to the ***W*** matrix at least for up to 100 iterations, although there are perceptible effects. The effects are illustrated by comparing two contrasting groups, each containing six regions. The first group, comprising Greater London, Ile de France, Bruxelles, Stuttgart, Lombardia and Dusseldorf, consists of core regions with on the whole high levels of manufacturing technology. One should expect these to mutually interact and for this to strengthen as a result of this interaction. The second group consisting of Corse, Sicilia, Highlands and Islands of Scotland, Ireland, Kritti and Extremadurra, is scattered and of variable technology. We would not expect these to interact much or for interaction to be reinforced over time. Figure 16 plots the productivity levels ratios of the core regions, and Figure 17 is for the scattered peripheral regions. It appears that the core regions are a more co-ordinated group that is tending to move in unison compared with the peripheral regions, as one might expect with strong and increasing regional interaction. Of course we have built these features into our outcomes by endogenising ***W***, so it is no surprise that they are apparent. These are simply illustrations of how endogenous interaction might work in practice, and much more econometric work is needed to evaluate the role and significance of endogenous interaction in a multivariate situation.

9. Conclusions

The paper has proposed a model of regional economic productivity growth for the EU regions as a 'third' way to analyze regional development somewhere between 'new' and 'old' economic geography. The approach adopted, labeled 'economic geography with spatial econometrics', places emphasis on inductive analysis at an early stage rather than deduction, so in a sense tackles the problem of geographical concentration from the opposite direction to new economic geography. The main empirical findings of the paper are that there are significant increasing returns to scale, at least in the EU manufacturing context, and significant externalities with technical progress assumed to depend on spillovers from capital accumulation which cross regional boundaries. Also, low productivity regions have, allowing for other factors, seen faster productivity growth and technological catch up. However, intrinsic regional differences presumed to relate to human capital stocks varying with peripherality and urbanization also account for productivity growth differences.

The preferred fitted model is used to illustrate long run dynamic implications. The model converges to a stable steady state despite the existence of increasing returns, because the catch up element in the model causes productivity growth rates to become equalized. The stable steady state is a dispersed equilibrium because of the intrinsic differences between regions. Admitting exogenous shocks implies some form of 'stochastic equilibrium'. This is preferred to Markov chain stochastic equilibrium because it captures the 'sticky mobility' of regions, 'permanent' interregional differences and supposes an explicit underlying model.

The paper also examines dynamic implications of the model in the presence of dynamic endogenously determined interregional interaction. Eliminating catch up, static interaction plus increasing returns result in stable productivity growth rates but the differences between regions ensure increasing disparities between productivity levels. This tendency is exacerbated by endogenous interregional interaction that produces rising rather than stable productivity growth rates. At an extreme level of geographical concentration growth rates stabilize so we have non-accelerating regional divergence, but only after productivity levels have become higher and growth rates faster than under exogenous interregional interaction.

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11. Notes

1. One minor problem with the specification is the spurious correlation resulting from the fact that **p** = **q** – **e**, where **e** is the growth of employment. This is best seen in the simple regression context E($\boldsymbol{p}$) = $\boldsymbol{b_0} + \boldsymbol{b_1} \boldsymbol{q}$. The main consequence of this is the inflated value for R^2 which results, and which is avoided by regressing **e** on **q** in order to obtain the correct R^2. The resulting regression coefficients are simple functions of the original ones, since E(**e**) = $-\boldsymbol{b_0} + (1-\boldsymbol{b_1})\mathbf{q}$.

2. The appropriateness of a production function as an underlying static model has been questioned. Kaldor(1957) viewed the Verdoorn Law as a linear technical progress function, considering as arbitrary and artificial 'any sharp or clear cut distinction between the movement along a "production function" with a given state of knowledge and a shift in the "production function" caused by a change in the state of knowledge' (see also Harris and Lau 1998, McCombie and Thirlwall 1994, Aghion and Hewitt 1998).

3. In fact this assumption is not a bad one, corresponding to one of Kaldor's stylized facts discussed by Barro & Sala-i-Martin (1995).

4. An earlier related approach (eg Fingleton & McCombie 1998) assumed that technical progress is partly induced by output growth, as in

$$l = l^{\blacklozenge} + h\boldsymbol{q}$$

5. In fact the cell (i,j) of the $\boldsymbol{W}^{\ddot{}}$ matrix (ie prior to standardization) are given by

$$\boldsymbol{W}^{\ddot{}}_{ij} = \boldsymbol{Q}_{j0}^{\,a}\boldsymbol{d}_{ij}^{\,-g}$$

in which $\boldsymbol{Q}_{j0}$ denotes the level of output of economy j at time 0 and $\boldsymbol{d}_{ij}$ denotes great circle distance between the centres of regions i and j. The coefficients α and γ are set to the value 2. This is thus a broader (steady-state) measure of interaction than level of technology *per se*, but encompasses technology since $\boldsymbol{Q} = \boldsymbol{PE}$. In later simulations, $\boldsymbol{W}$ is endogenised with respect to $\boldsymbol{P}$.

6. There are clear analogies here with autoregressive time series processes. Assume for example

$$\boldsymbol{Y_t} = y\, \boldsymbol{Y_{t-1}} + w_{\boldsymbol{t}}, \quad \boldsymbol{t} = 2,\ldots,\boldsymbol{T}$$
$$w_{\boldsymbol{t}} \sim \mathrm{N}(0,\ s^{\boldsymbol{2}})$$
$$\boldsymbol{Y_1} = 0$$

This can also be written in matrix terms as

$$\boldsymbol{Y} = y\, \boldsymbol{WY} + w$$
$$w \sim \mathrm{N}(0,\ s^{\boldsymbol{2}}\boldsymbol{I})$$

In which $\boldsymbol{W}$ here is defined as a $\boldsymbol{T}$ by $\boldsymbol{T}$ matrix of 1s and 0s with
1s located on the minor diagonal in cells (2,1), (3,2), …($\boldsymbol{T}$,$\boldsymbol{T}$-1). Given this $\boldsymbol{Y_t}$ values exactly equal to those produced by the straightforward repeated sequential calculation of the first equation are given by

$$\boldsymbol{Y} = (\boldsymbol{I} - y\ \boldsymbol{W})^{-1} w$$
$$w_1 = 0$$
$$w \sim N(0,\ s^2 \boldsymbol{I})$$

Provided the 'stream' of random numbers is the same in each case.

7. Similarly the singular points are the roots of the polynomial equation $|\boldsymbol{I} -\ r\boldsymbol{W}\ | = 0$ and if $\boldsymbol{W}$ is an n x n matrix, there will be there will be up to n real distinct roots for the polynomial equation.

8. The usual method is a bisection search as in SPACESTAT.

9. IV or 2SLS estimation has frequently been carried out in cross-sectional growth analysis by treating lagged variables as predetermined, for example Barro and Sala-i-Martin(1995) use the average investment ratio for 1960-64 as an instrument for the average for 1965-75, although it is not always certain that using a lagged variable as an instrumental variable will solve the problem by being independent of the error term (Barro and Sala-i-Martin, 1995, argue that lag values are reasonable candidates as instruments because the correlation of the residuals in the growth regressions between their two periods is insubstantial).

10. Since Groningen and Flevoland have anomalous manufacturing GVA and GVA per worker values (largely due to fluctuations in gas production in Groningen and possibly also due to commuting), in these cases the Dutch national averages were used. In the case of Hamburg, it is apparent that commuting may also be a distorting influence because of the (NUTS 1) region's small spatial extent. Hence a more appropriate definition of the city was used, the travel to work area (ROR05) which comprises the NUTS 1 region of Hamburg and the surrounding Kreise that qualify as part of the functional urban area. For example, in 1990, the Hamburg TTWA had a population of 2.9m people, compared with 1.6m people for the NUTS 1 region. This provides a more realistic per worker GVA.

11. Note that this vector is obtained by iteration rather than by calculating equation (26). With unchanging leadership, both methods give the same results. If productivity leadership changes, the consequence of applying equation (26) is $\boldsymbol{R^e} > 1$ for some regions that catch-up and surpass the original leader. On the other hand calculating $\boldsymbol{P^*}$ in equation (17c) at each iteration, as has been done in practice, makes $\boldsymbol{R^e} > 1$ impossible and this seems more realistic since regions will be tending to catch up the current productivity leader rather than the initial one.

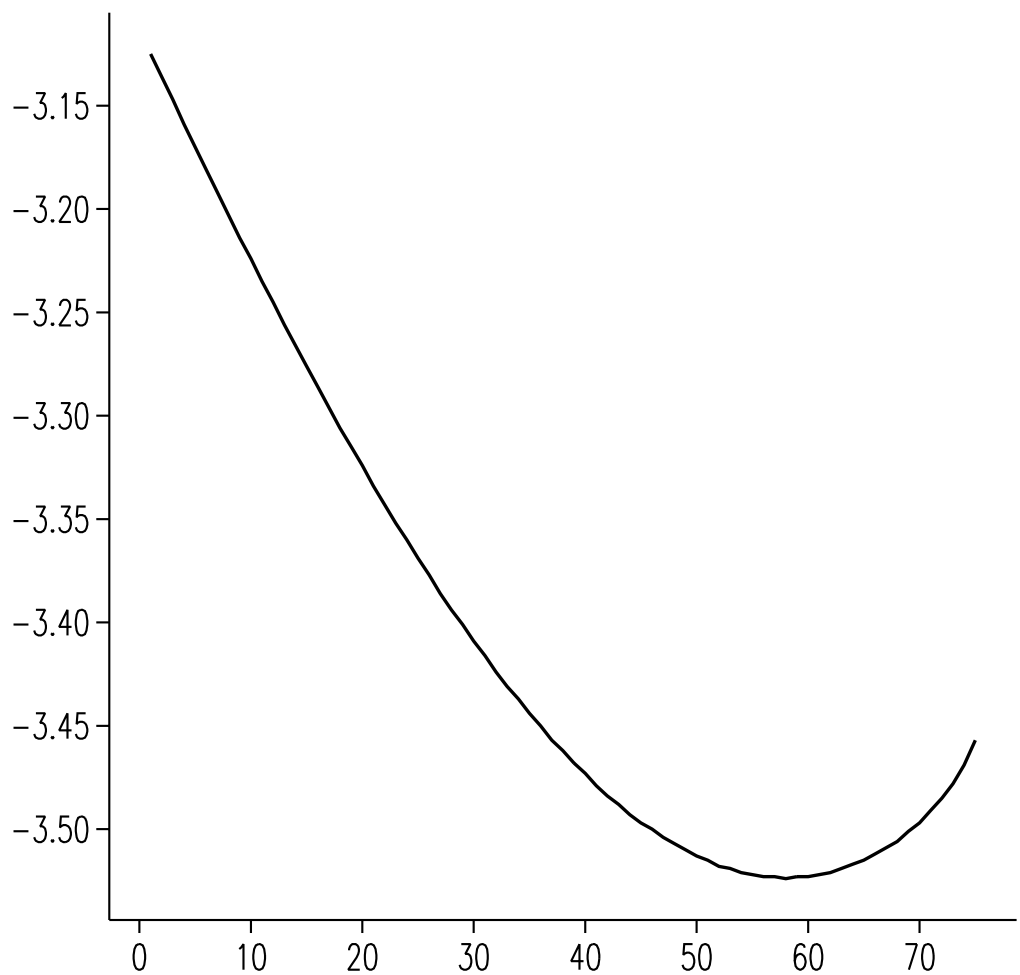


Figure 1

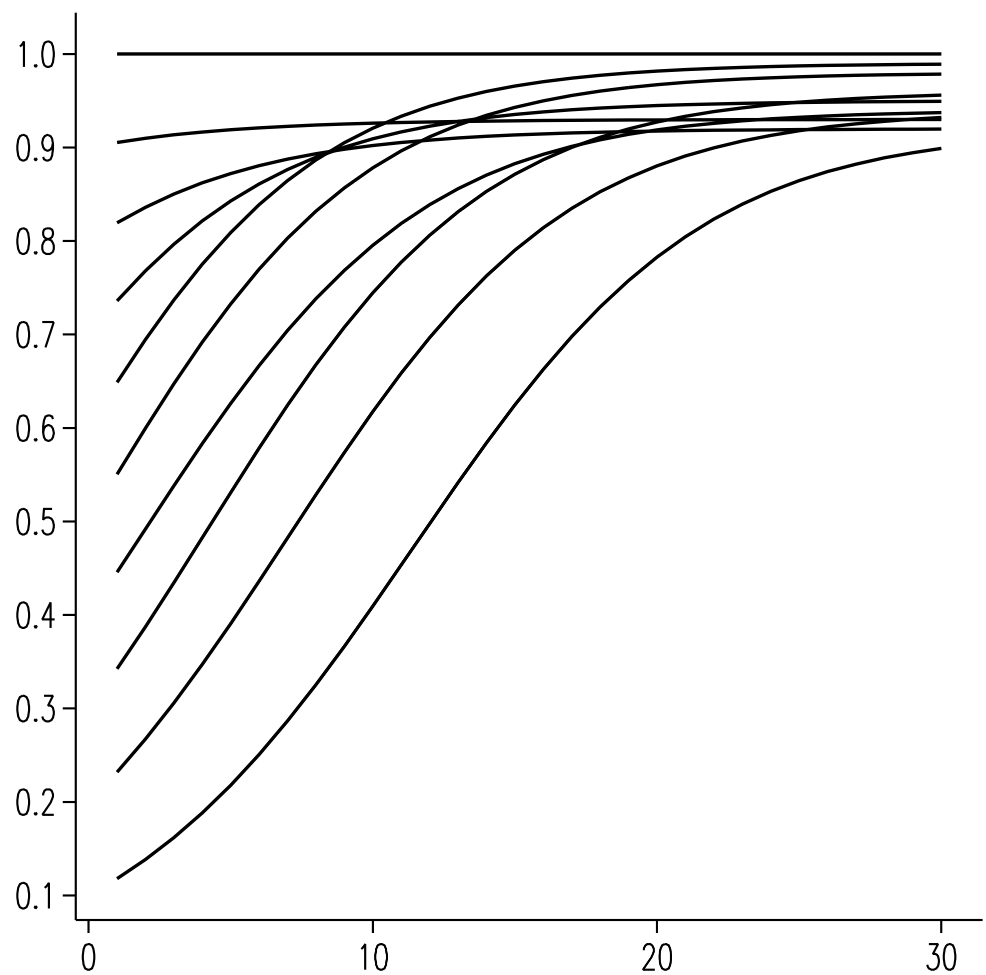
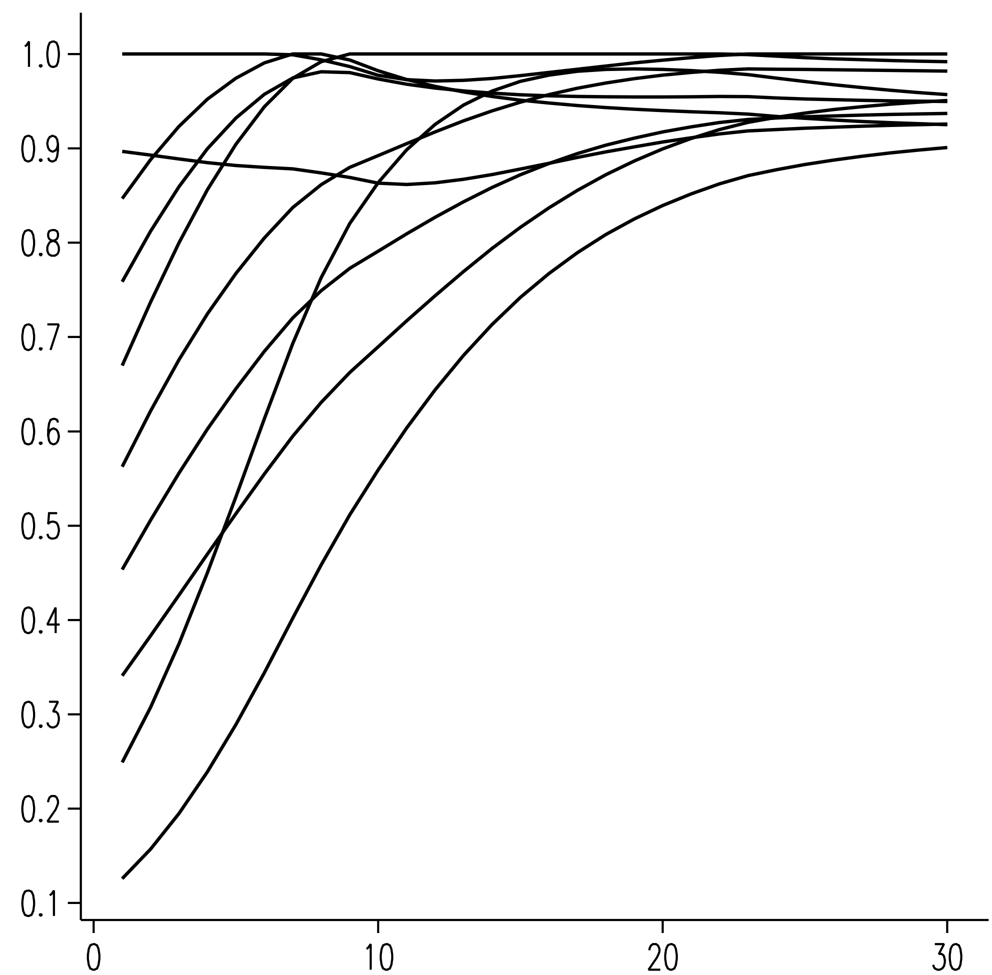


Figure 2



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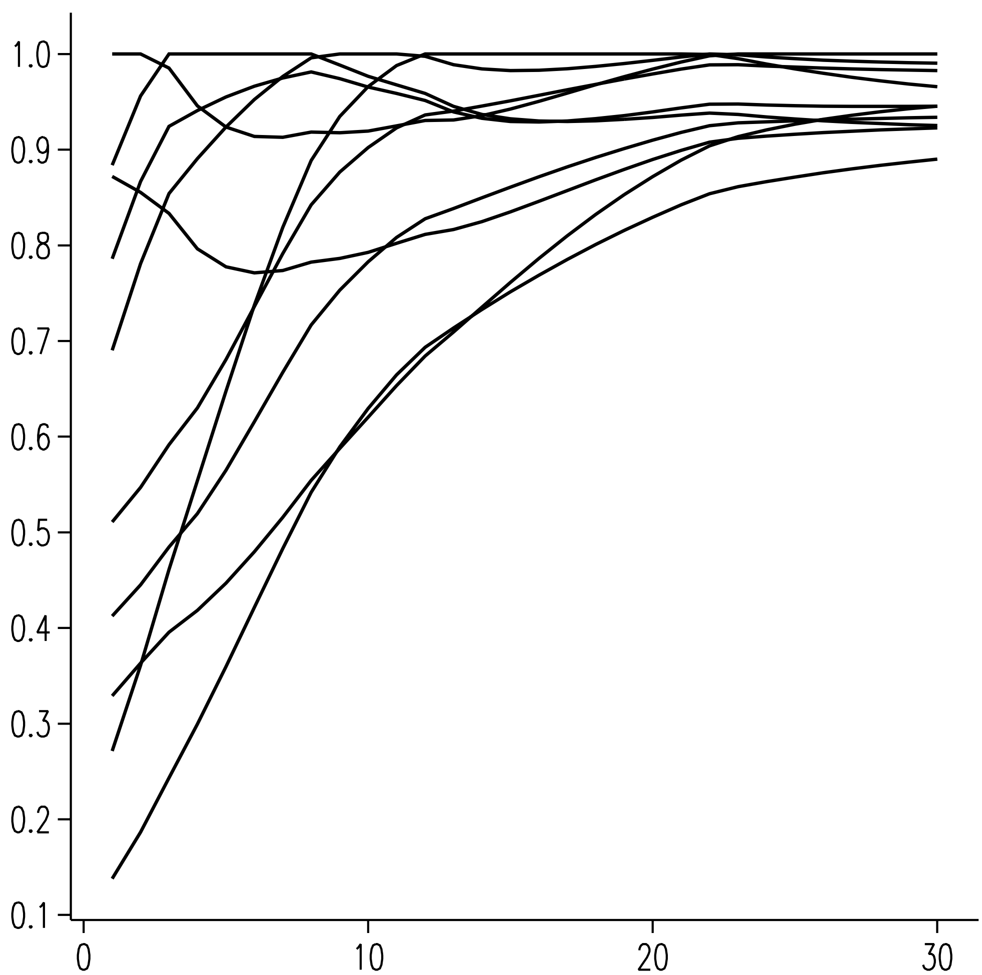

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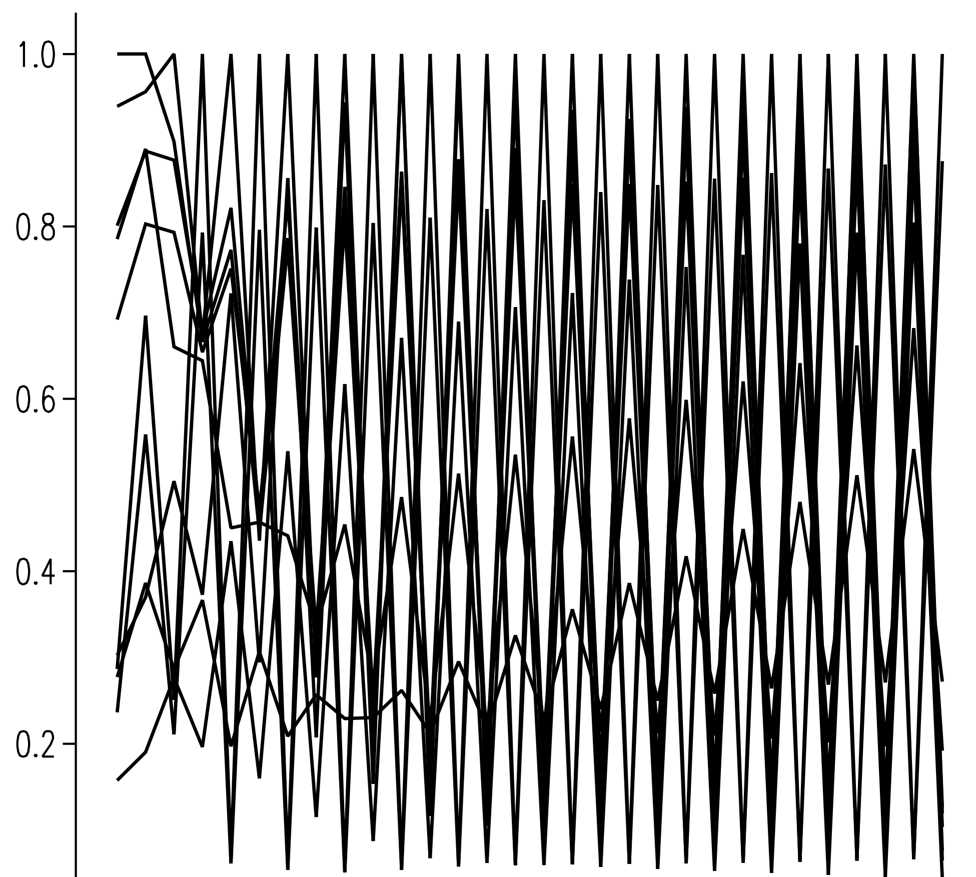

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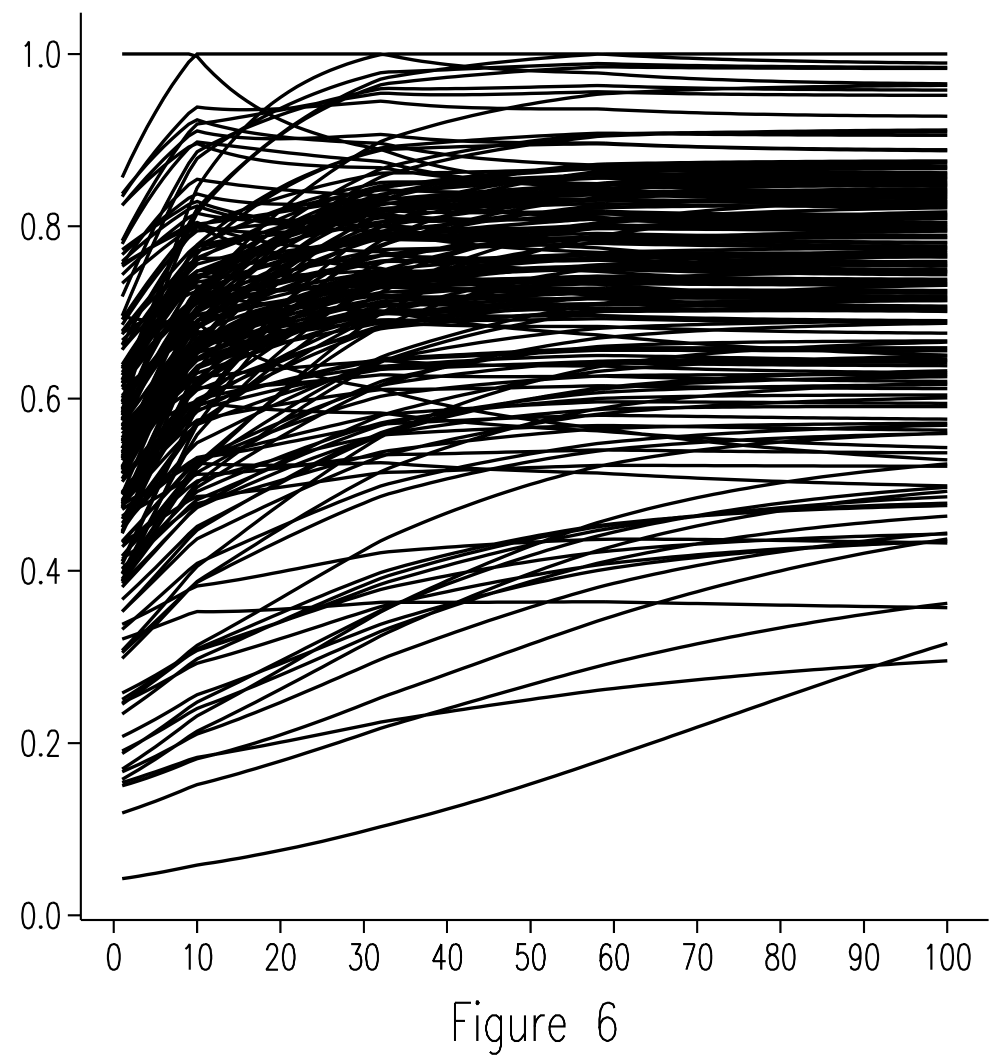

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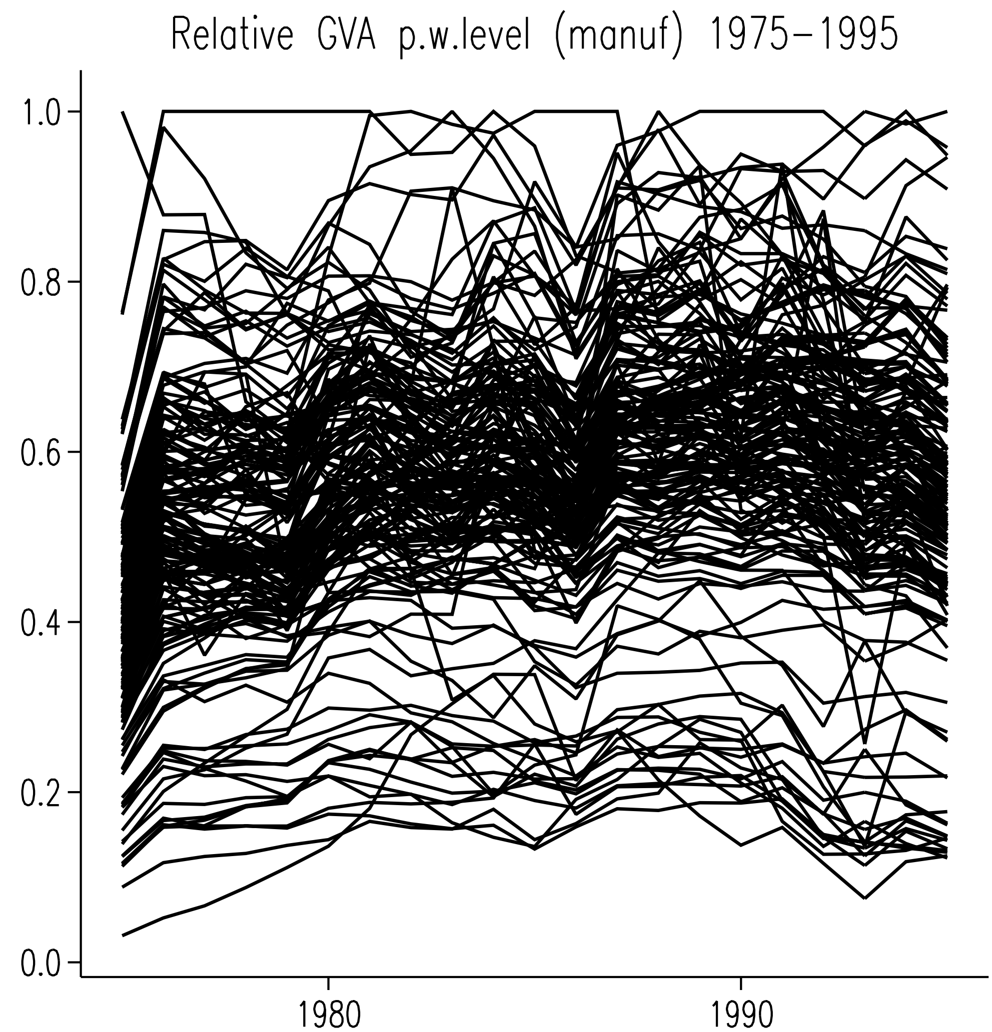
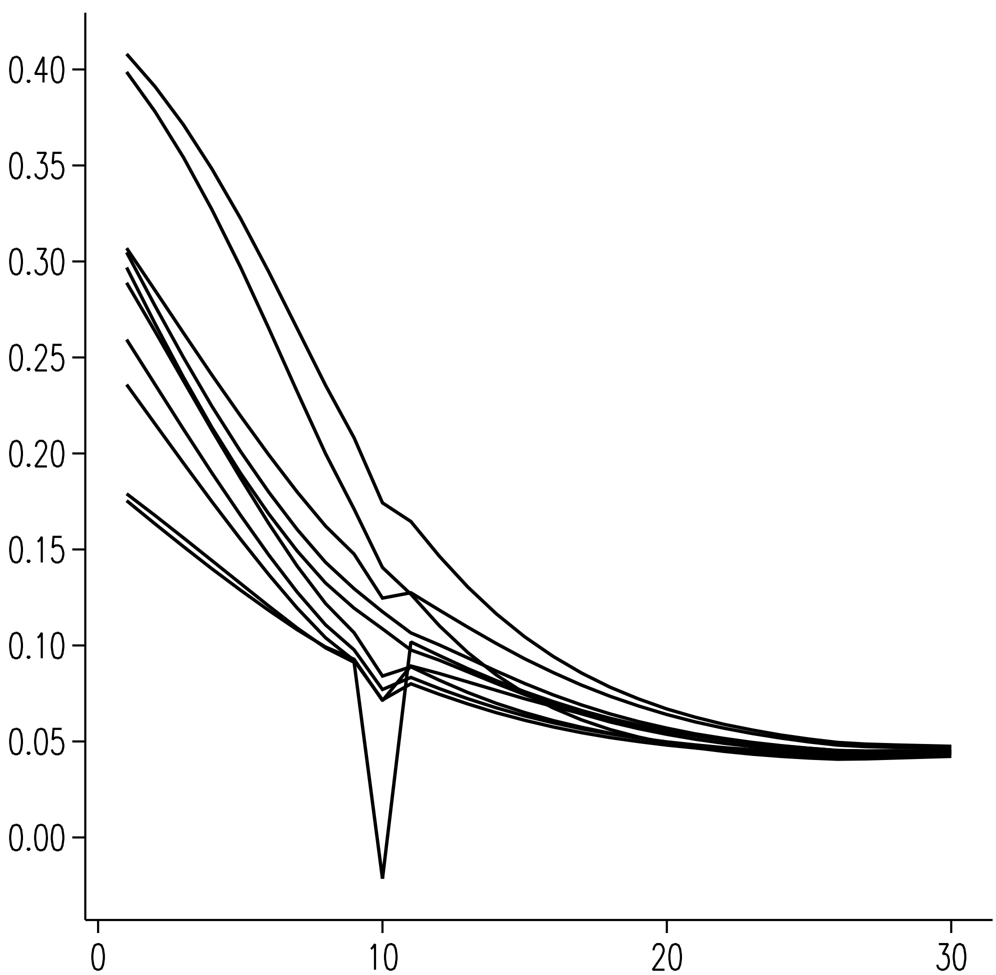


Relative GVA p.w.level (manuf) 1975-1995
1.0
0.8
0.6
0.4
0.2
0.0
1980
1990

Figure 7



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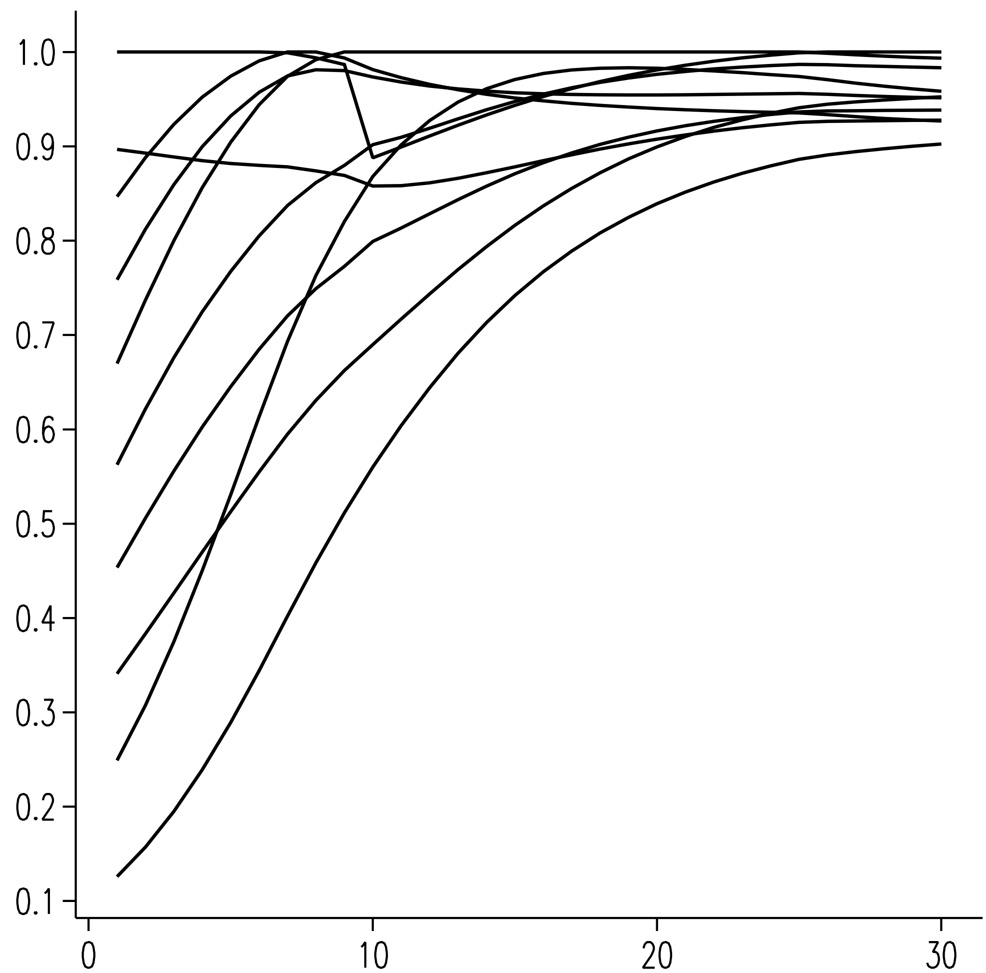

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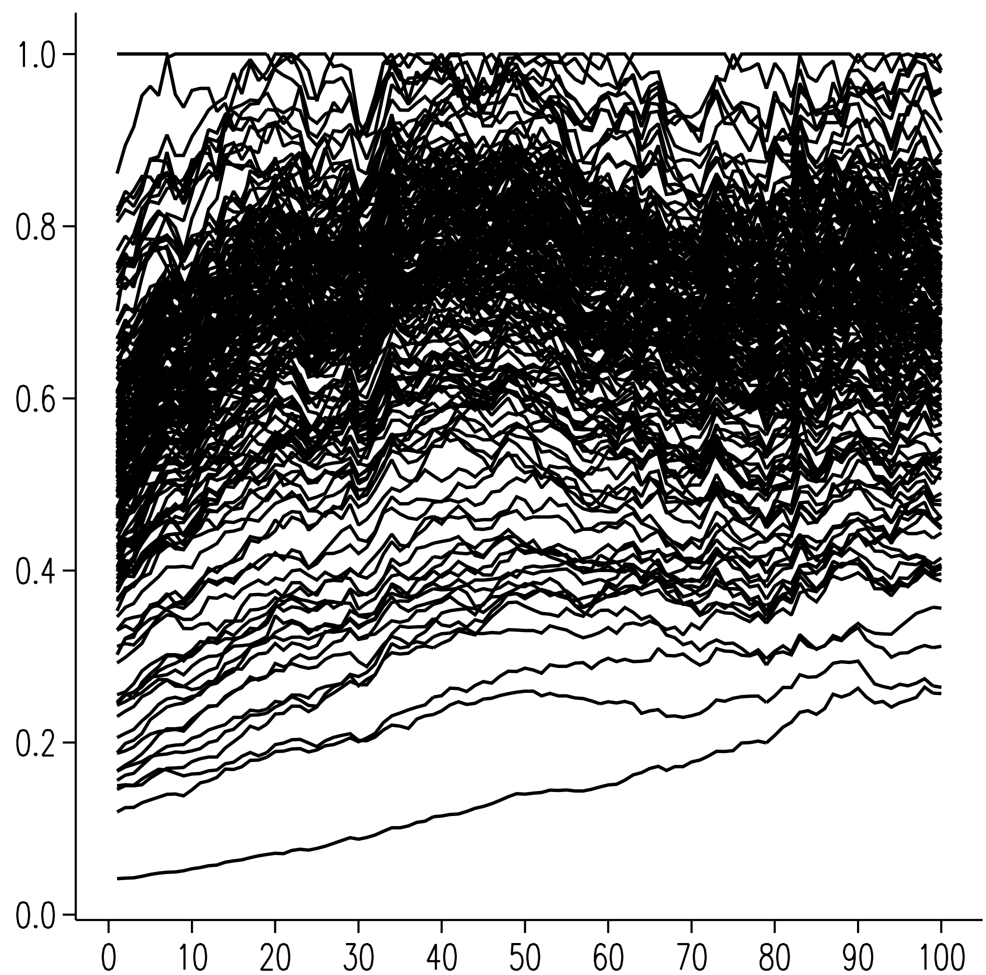

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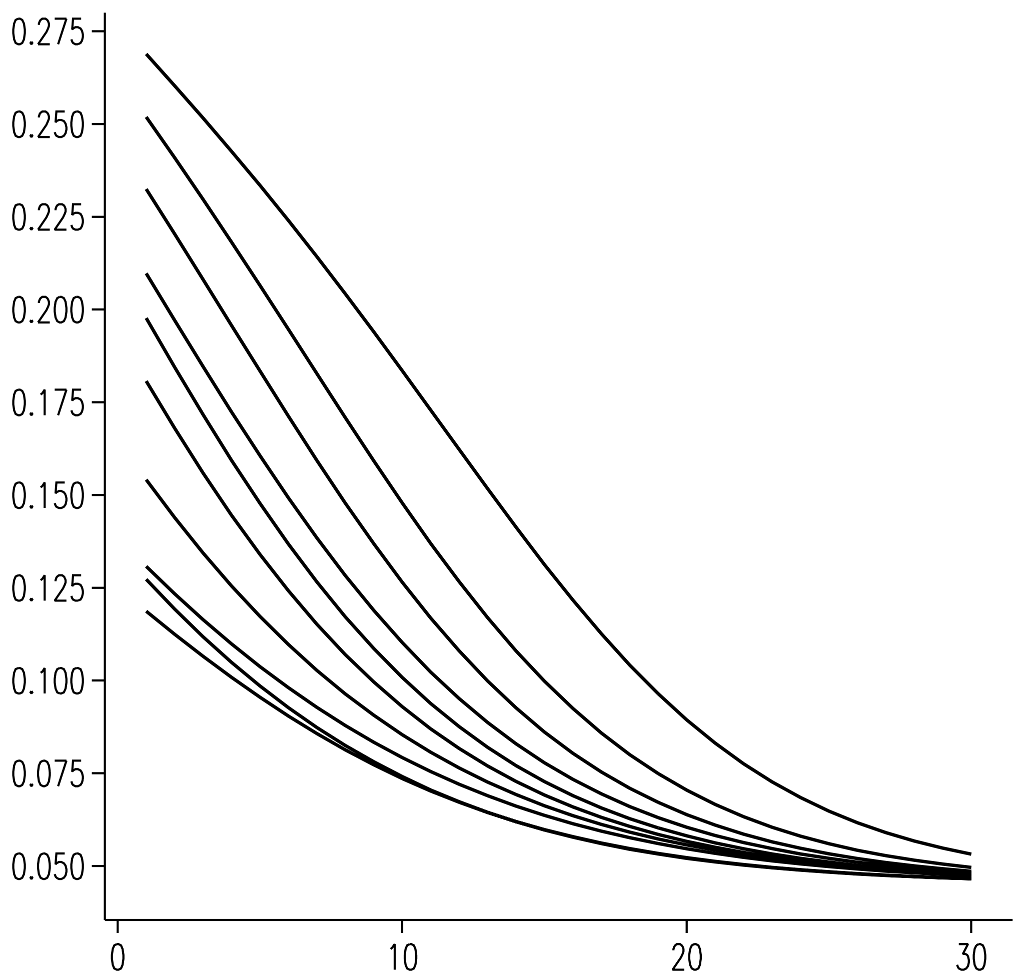

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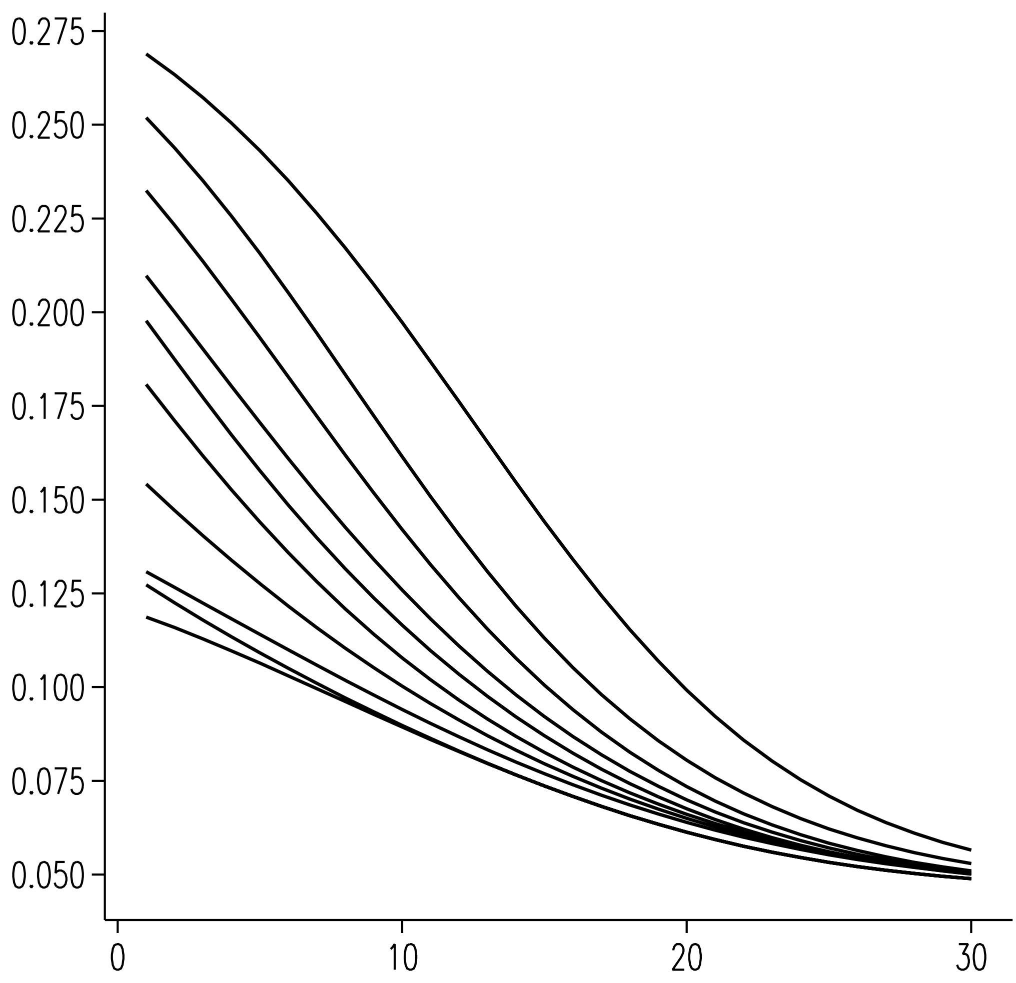

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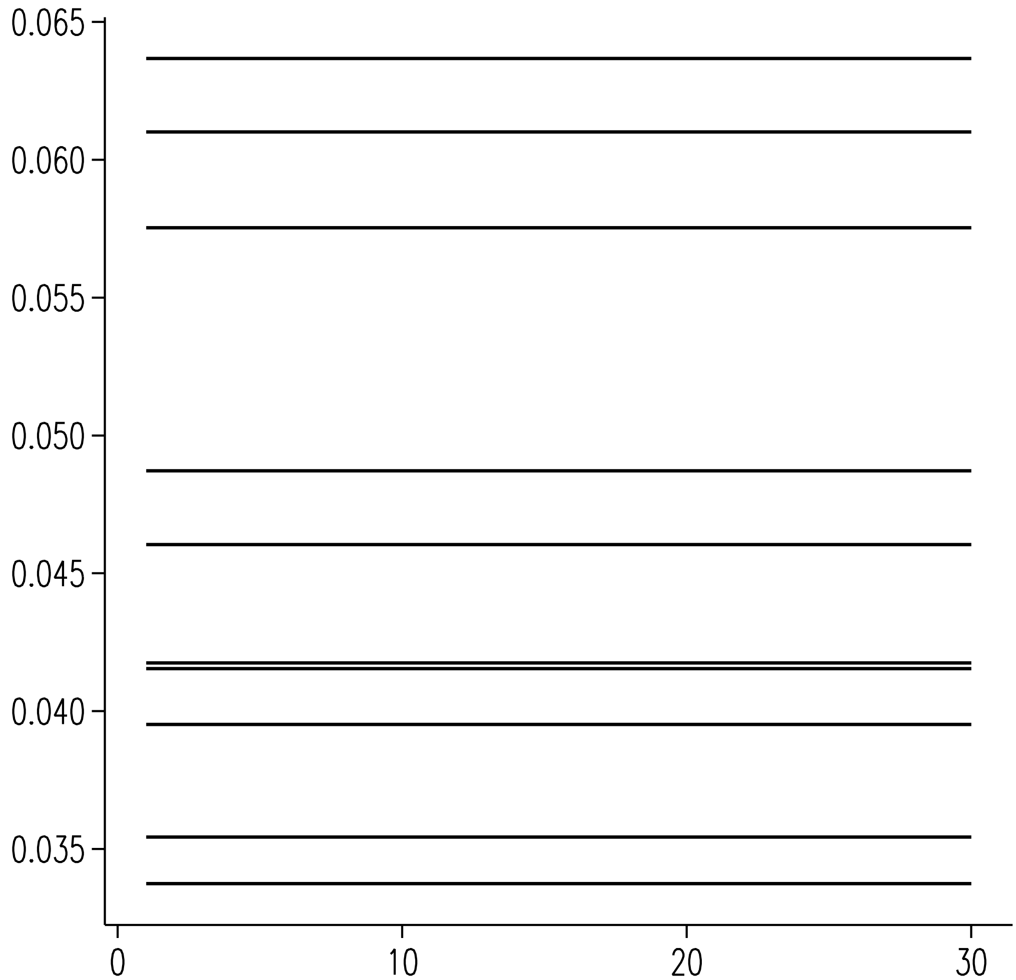

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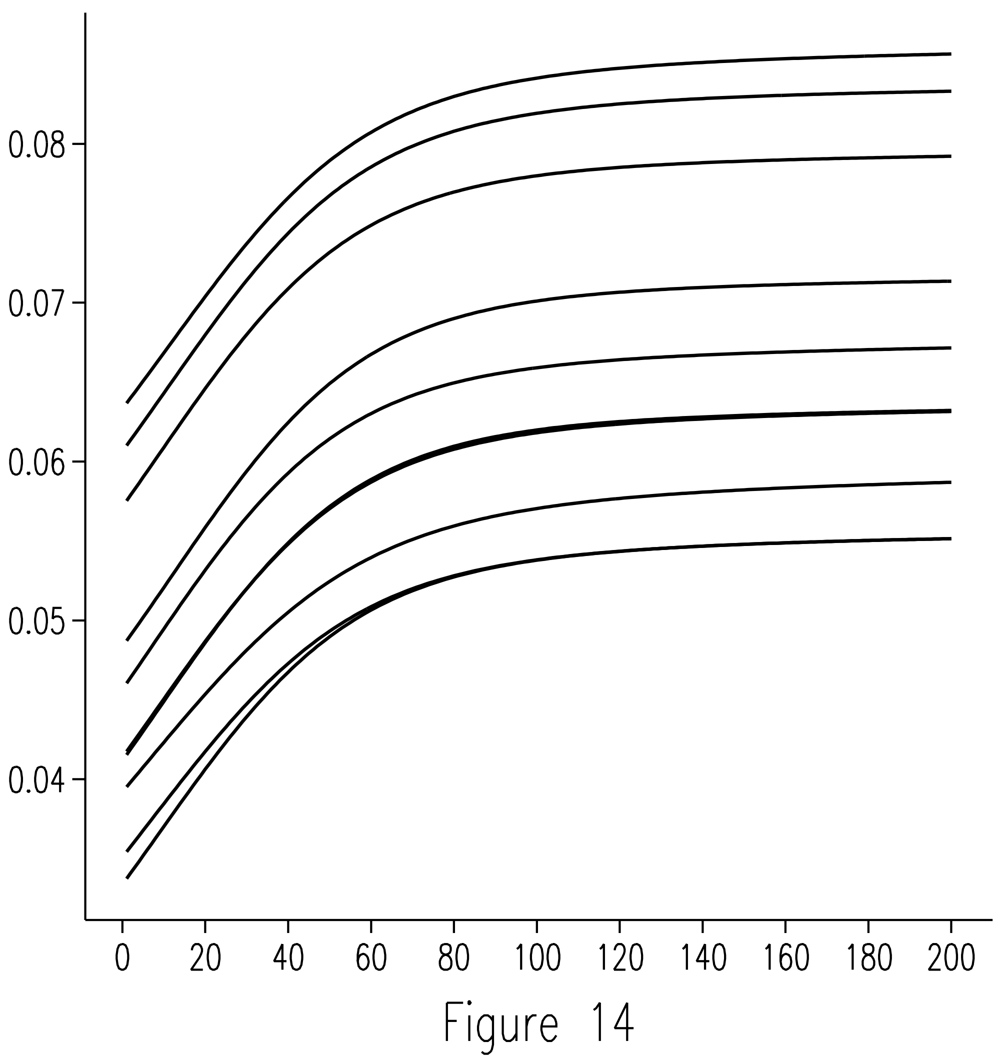

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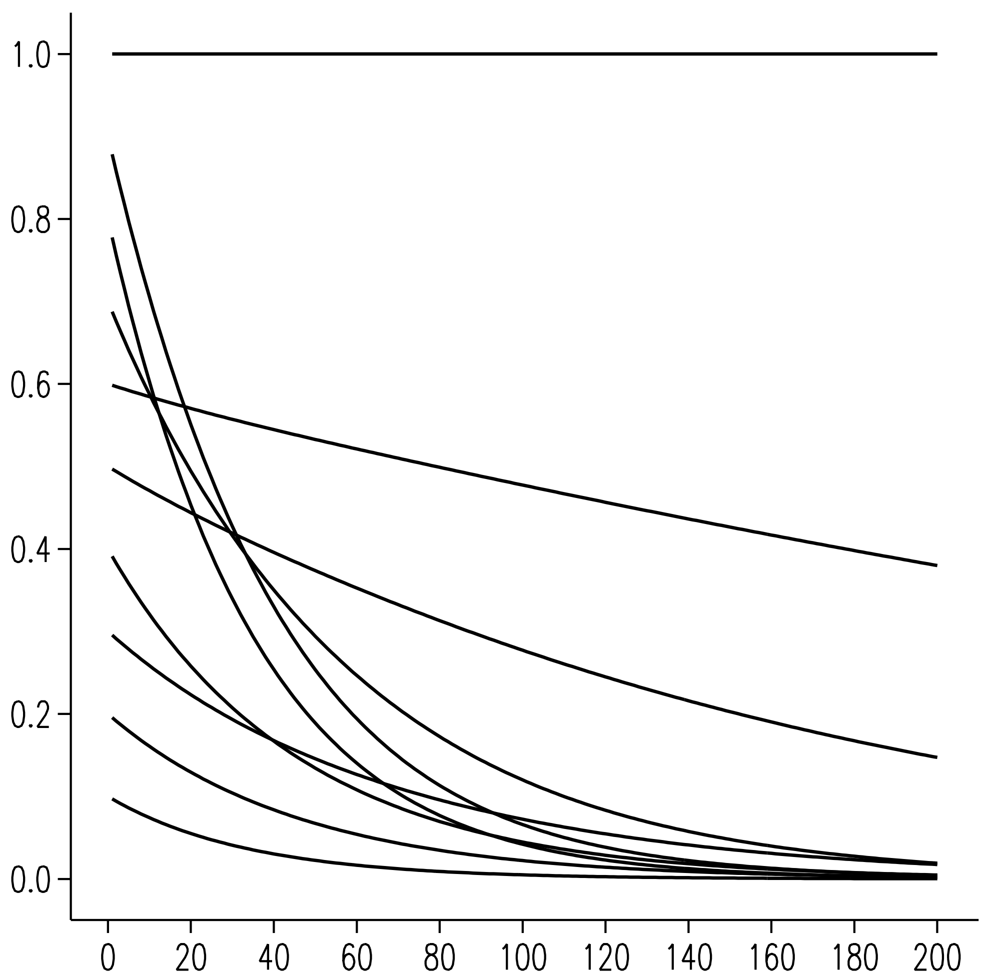

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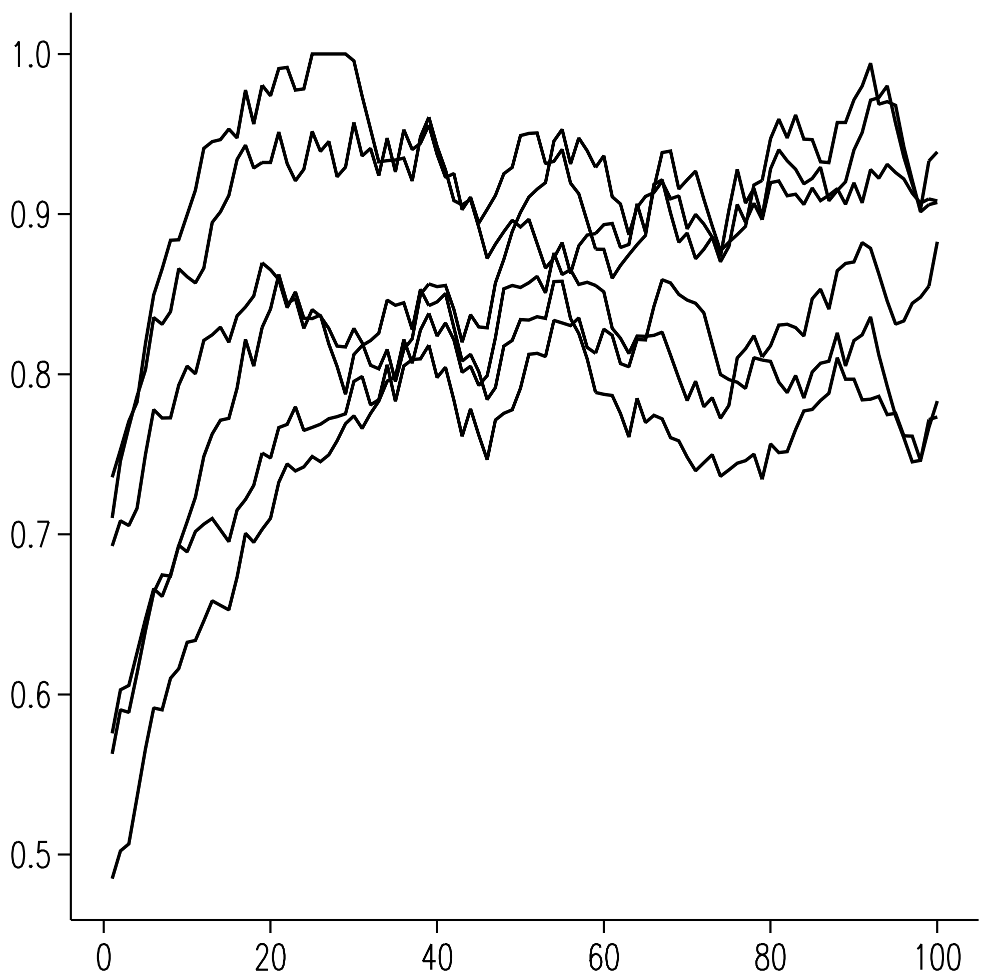

Figure 16

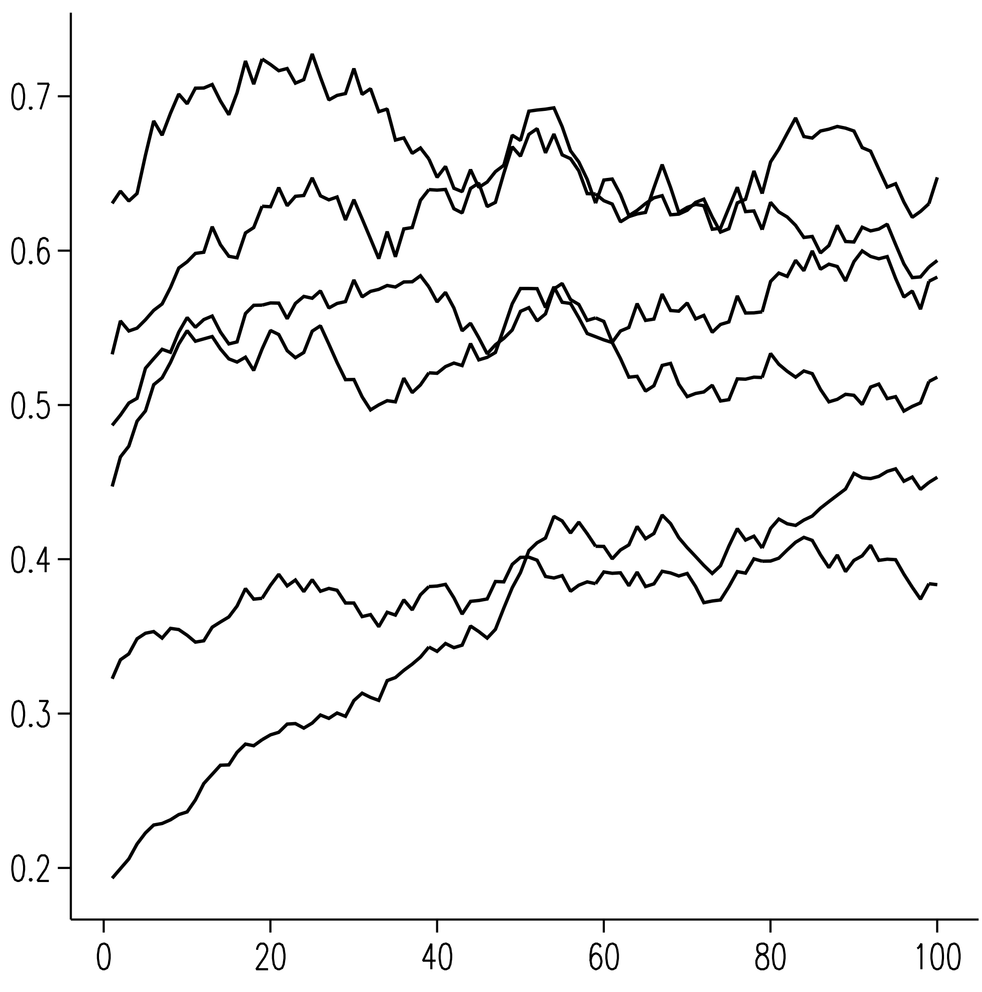

Figure 17