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Empirical Evidence on Growth and Volatility

Michael Stastny and Martin Zagler

EUROPEAN UNIVERSITY INSTITUTE, FLORENCE
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Abstract

This paper empirically investigates the relationship between long-run economic growth and output volatility. There is an emerging theoretical literature on the topic which is inconclusive on the size and direction of the relationship. We analyze this relationship empirically for the time series experience of 21 OECD countries between the years 1961 and 2005. After applying a pooled OLS estimator and a series of robustness checks we conclude that there is strong empirical evidence for a positive relationship between output variability and economic growth.

Keywords
Growth, Volatility, Cycles, Innovation

JEL Classifications
E32; O33; O47

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1 Introduction

For a long time, the field of macroeconomics has between firmly divided between the analysis of the business cycle and the investigation of long-run determinants of economic growth. This distinction, however, is rather arbitrary and has been challenged by recent theoretical models and by empirical evidence that points to long-run performance being explained in part by business-cycle behavior and output variability. The aim of this paper is to empirically investigate the relationship between economic growth and output volatility.

The earliest theoretical argument for a relation between economic growth and the business cycle dates back to Schumpeter (1939), who argued that recessions provide a cleansing mechanism for the economy, where old technologies get replaced by newer technologies, and will be better adapt to economic growth thereafter. In a similar spirit Black (1981) argues that the average severity of a society's business cycle is largely a matter of choice. His idea was that economies face a positive risk-return trade-off in their choice of technology, as economic agents would choose to invest in riskier technologies only if the latter were expected to yield a higher return and hence, greater economic growth.

A series of papers have subsequently focused on the relationship between volatility and growth in exogenous growth models. On the one hand, the focus was on the impact of volatility on uncertainty, precautionary savings and hence accumulation of capital (cf. Boulding (1966), Leland (1968), Sandmo (1970)). On the other hand, Bernanke (1983) and Pindyck (1991) argue that if there are irreversibilities in investment, then increased volatility will lead to lower investment and hence lower capital accumulation. Both strands of literature have in common that they are based on exogenous growth models, hence whilst there may be transitional changes in growth rates due to changes in volatility, in the long-run economic growth will be exogenous.

More recently, in an endogenous growth model Aghion & Saint-Paul (1993) and Aghion et al. (2005) show that the sign of the relation depends on whether the activity that generates growth in productivity is a complement or a substitute to production. In the case where they are substitutes, since the opportunity cost of productivity-improving activities such as reorganizations or training falls in recessions, larger variability leads to higher long-term growth. This idea has recently been formalized in an endogenous growth framework by Jovanovich (2006).

A number of empirical studies on the relationship between growth and volatility has been conducted. Campbell & Mankiw (1987) were amongst the first to report permanent effects on the level of GDP from shocks to output growth, first for the US and later on for a selected sample of various countries (Campbell & Mankiw (1989)). Whilst it provides a confirmative test for models of exogenous growth and volatility, these studies fail to provide a test for models of endogenous growth and volatility.

The first empirical study that can be applied to endogenous growth models was done by Zarnowitz (1981). He identified periods of relatively high and relatively low economic stability by reviewing annual real GDP growth rates in the U.S. between 1882 to 1980 and accounts found in the literature on economic trends and fluctuations. He then calculated the yearly growth rate and the variance of the periods with high economic stability (group A) and low economic stability (group B). Though the mean growth rate of group A was higher, he could not reject the null hypothesis that the difference between the mean growth rates for groups A and B was due to chance.

The first econometric study investigating the link between growth, output variability—as measured by the standard deviation of the growth rate—and further macroeconomic variables was conducted by Kormendi & Mequire (1985). By averaging each country's time series experience into a single data point and estimating a cross-section of forty-seven observations, they found that higher output variability leads to higher economic growth. Grier & Tullock (1989), who used a pooled structure (five-year averaging) to account for both between- and within-country effects, confirmed Kormendi and Meguire's results.

The paper closest to ours is by Mills (2000). He applied various filters that are explicitly designed to capture movements in a time series that correspond to business-cycle fluctuations in twenty-two countries. Subsequently, he calculated the standard deviation of the output (filtered) series and visualized the bivariate relationship between growth and volatility by superimposing robust nonparametric curves on scatter plots. He found a positive relationship. In contrast to our paper, Mills (2000) suppresses all fluctuations of output at frequencies higher than his filter.

When analyzing the relationship between economic growth and output fluctuations, we are essentially investigating the first moment of the time series in first differences, and its corresponding second moment over the mean, i.e. the variance of the differentiated time series. There exists a standard econometric tool to analyze this relationship, the generalized auto-regressive conditional heteroscedacity (GARCH) class of models. And indeed, several authors have employed this methodology to analyze the relationship of output and volatility.

Ramey & Ramey (1995), using a panel structure, measured volatility as the standard deviation of the residuals in a growth regression consisting of the set of variables identified by Levine & Renelt (1992) as the important control variables for cross-country growth regressions. Ramey & Ramey (1995) use the estimated variance of the residuals in their regression, under the assumption that it differs across countries, but not time. In such, it can be considered an early predecessor of GARCH models[1]. They find a negative relation between long-run growth and volatility. By contrast, Caporale & McKiernan (1998) and Grier & Perry (2000) examined the issue from a pure time series perspective. Caporale & McKiernan (1998) ran an ARMA(1,2)-GARCH(0,1)-M model and Grier & Perry (2000) ran a complex bivariate GARCH(1,1)-M model for U.S. GDP growth. The former found a significant positive relationship while the latter found an insignificant positive relationship between growth and volatility.

The fact that these studies yield opposite results may come as a surprise. However, GARCH models were invented for financial time series, with a large number of observation. In Monte-Carlo simulations, presented in appendix A, we demonstrate that the widely-used and highly-sophisticated GARCH-in-mean models are inappropriate for this purpose as they require the estimation of too many parameters for the short time series that normally confront economists.

This leaves us with the more conventional approach of separating the time series into a trend and a cyclical component, and then investigate their relationship. There is a large number of filters available, most of them developed by the finance literature. We have decided to adopt the HP-filter. Our measure of

[1] With a single estimate per country, we cannot simulate their results as done in A

volatility is superior to any other measure of volatility we investigated due to its stability with respect to small changes in the data.

The empirical analysis presented here is based on the growth experience of twenty-one OECD countries between 1961 and 2005. After calculating the trend growth rate for each country using the HP-filter, we divided the data for each country into three, fifteen-year, non-overlapping sub-samples. For each sub-sample, the average growth rate and the volatility–based on the squared deviations of the actual growth rate from the trend growth rate–was computed. This not only mitigated the effect of assuming constant volatility and constant growth rates, the technique also accounted for the within-country variation of the volatility in our subsequent regression analysis. After running a series of robustness tests, we conclude that there is a significant positive relationship between output variability and growth. This relationship is robust against outliers and does not hinge on the sub-sample period chosen.

2 The Data

The data for this study came from the AMECO database.[2] It is the annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs (DG ECFIN). All 21 countries (Australia, Austria, Belgium, Canada, United Kingdom, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Mexico, Netherlands, Portugal, Spain, Sweden, Switzerland, Turkey, and USA) for which continuous annual series for gross domestic product at constant market prices per capita were recorded for the period of 1960-2005 were used for analysis.

3 Methodology and Results

3.1 Modeling Trend and Volatility

We will investigate time series properties of a particular nature. In order to analyze the relationship between economic growth and volatility, we will ask whether a measure of volatility is correlated with changes in output growth. Several measures for both output growth and the volatility are feasible, and we will discuss them below. Whilst for economic growth, the change in the level of output–maybe averaged over several periods, which would be a trend–is a natural candidate, measures for the business cycle are volatility measures. Volatility refers to the spread or dispersion of all likely outcomes of a random variable. It is often measured as the sample standard deviation. Formally, we investigate a relationship such as,

$$g_t = \kappa + \gamma \sigma_t + u_t \tag{1}$$

where κ is a constant, γ is a parameter, and σ_t measures the standard deviation of the time series[3]. u_t is an error term. For a given time series, one could estimate the above equation (1), then use the estimator for the variance

[2] http://ec.europa.eu/economy_finance/indicators/annual_macro_economic_database/ameco_en.htm

[3] We refrain from including control variables in our estimation. Unless control variables would be correlated with the variance measure adopted, the estimator for γ remains unbiased.

σ^2 and reestimate the above equation until it converges.[4] This essentially what GARCH models do. Estimating a time-varying variance requires a long time series, a luxury we cannot afford for macroeconomic time series such as GDP. In appendix A, using Monte-Carlo simulations, we show that under reasonable parameter configurations, the variance of the estimator from its true variance is unacceptably large[5].

This leads us to the next best solution of estimating mean and variance separately.[6] The exercises is further complicated as both the mean and the standard deviation are not necessarily constant over time.[7] We will test for constancy over time using three types of unit root tests.

3.2 Unit Root Tests

One clear indication that the assumption of a constant mean and a constant variance of a time series cannot be maintained is when unit root tests point to the non-stationarity of the data. In this case, cross-country regressions based on sample mean and sample variance would lead to bogus results.

Testing for unit roots in the growth rate of GDP using the standard Augmented Dickey-Fuller[8] (ADF) test—with a constant and a trend in the regression equation—results in the failure to reject the null hypothesis of non-stationarity two-thirds of the time (5 % level of significance). Since the way in which classical hypothesis testing is carried out ensures that the null hypothesis is accepted unless there is overwhelming evidence against it and we want to point out that our series are non-stationary, the appropriate way to proceed is to use a test that has the null hypothesis of stationarity and the alternative of a unit root. A test with stationarity as null is the KPSS test. Kwiatkowski et al. (1992) start with the model

$$\begin{aligned} y_t &= \xi t + r_t + \epsilon_t \\ r_t &= r_{t-1} + u_t \end{aligned} \tag{2}$$

where $u_t \sim \text{iid}\left(0, \sigma_u^2\right)$, ϵ_t and u_t are independent, and the initial value r_0 is fixed. The ϵ_t satisfy the linear process conditions of Phillips & Solo (1989) (theorems 3.3,3.14) which allow for all ARMA processes, with either homogeneous or heterogeneous innovations.

The test for stationarity in this model is simply

$$H_0 : \sigma_u^2 = 0 \quad \text{vs.} \quad H_A : \sigma_u^2 > 0 \tag{3}$$

Most control variables that we can think of, such as policy variables, would work in favor, reducing the explanatory power of volatility on economic growth.

[4]It should also be noted that whenever one has an unbiased estimator for σ^2, the square root of $\hat{\sigma}^2$ is a biased–depending on the shape of the distribution and the sample size–estimator of σ due to Jensen's inequality, $\mathbb{E}\left[\hat{\sigma}\right] = \mathbb{E}\left[\sqrt{\hat{\sigma}^2}\right] < \sqrt{\mathbb{E}\left[\hat{\sigma}^2\right]} = \sqrt{\sigma^2} = \sigma$.

[5]This may be the reason why papers based on this methodology yield contrasting results.

[6]A measure for the spread of a distribution does not necessarily contain all information about its shape, so we can still miss some important features, unless the first two moments (mean and variance) are sufficient statistics to describe the entire distribution.

[7]The analysis by Kormendi & Mequire (1985) basically relies on this assumption.

[8]The number of lags used in the regression is $\text{trunc}\left((\text{length}\,(\text{series}) - 1)^{\frac{1}{3}}\right) = 3$. This corresponds to the suggested upper bound on the rate at which the number of lags should be made to grow with the sample size for the general ARMA(p,q) setup.

Country	$KPSS_\mu$	$KPSS_\tau$	ADF_τ
Australia	□	□	■
Austria	■	□	■
Belgium	■	□	■
Canada	□	□	■
England	□	□	□
Finland	□	□	■
France	■	■	■
Greece	■	■	■
Iceland	□	□	□
Ireland	□	□	■
Italy	■	□	□
Japan	■	■	■
Luxembourg	□	□	■
Mexico	□	□	■
Netherlands	■	□	■
Portugal	■	□	□
Spain	■	■	■
Sweden	□	■	■
Switzerland	□	□	□
Turkey	□	□	□
USA	□	□	□

Table 1: Unit Root Tests

We performed two tests[9], denoted by $KPSS_\mu$ and $KPSS_\tau$ based on a regression on a constant μ, and on a constant and a time trend τ, respectively. Even though both tests are very conservative, we reject the stationarity hypothesis in 45% and in 25% of the cases, respectively.

Table 1 shows the results for the ADF test and the two KPSS tests for each country. Black squares denote evidence for non-stationarity (ADF: non-rejection of the null hypothesis, KPSS: rejection of the null hypothesis) while white squares denote evidence for stationarity. Out of our sample of 21 countries, all three tests point to stationarity of the data for only five countries. To summarize, we obtain a dispersed picture, and have to reject the assumption that all series exhibit constancy over time in all countries. We will therefore resort to band-pass filters to identify the trend (growth) and cyclical component of the time series.

3.3 Separating Trend and Volatility

It is often assumed that the time series under investigation, Y_t, can be represented as a weighted sum of periodic functions of the form $\cos(\omega t)$ and $\sin(\omega t)$ where ω denotes a particular frequency:

$$Y_t = \mu + \int_0^\pi \alpha(\omega)\cos(\omega t)\,d\omega + \int_0^\pi \delta(\omega)\sin(\omega t)\,d\omega \tag{4}$$

An *ideal* band-pass filter is a linear transformation of Y_t that isolates the

[9]To estimate σ_u^2 the Newey-West estimator was used.

components that lie within a particular band of frequencies, i.e. the filter only passes frequencies in the range $\omega_L \leq \omega \leq \omega_H$. Applied to GDP growth rates, the filter eliminates very slow-moving ('trend') components and very high-frequency ('noise') components, while capturing intermediate components that correspond to business-cycle fluctuations. The variance of the filtered series, $\hat{g}_t$, could then serve as a measure of volatility.

However, since such an ideal band-pass filter is a moving average of infinite order and therefore requires infinite data, an approximation is necessary for practical applications. Mills (2000) employed the one suggested by Baxter & King (1995) and removed components with frequencies below two years and above eight years.

Building on the graduation method developed by Whittaker (1923) and Henderson (1924), Leser (1961) proposed a filter that is similar to the band-pass, one that has also been widely used in business-cycle research. In economics it is known as the Hodrick-Prescott (henceforth HP) filter. The HP filter is an approximate low-pass filter, i.e. it passes low frequencies but attenuates (or reduces) frequencies higher than the cutoff frequency.

The filtered series is obtained by solving:

$$\min_{\hat{g}_t} \left[\sum_{t=1}^{T} (y_t - \hat{g}_t)^2 + \lambda \sum_{t=2}^{T-1} \left((1-L)^2 \hat{g}_{t+1} \right)^2 \right] \tag{5}$$

where $L^n y_t = y_{t-n} \quad \forall n \in \mathbb{N}$. The first summation term in equation 5 concerns the fit (squared deviations), the second summation term the smoothness (squares of the second differences) of the filtered series. The parameter λ determines the importance of the smoothness relativ to the fit (trade-off). As $\lambda \to \infty$, $\hat{g}_t$ approaches a linear trend.

3.4 Measuring Volatility

We are confronted with the situation whereby some GDP growth series appear to be stationary, while others appear to be trend-stationary, or even non-stationary. In the case of stationarity and trend-stationarity, the growth rate fluctuates around a constant and a linear trend, respectively. In the case of non-stationarity, the growth rate either fluctuates around a deterministic non-linear trend or a stochastic trend. Using different procedures to calculate the variance for each country could inadvertently result in data mining; therefore, we uniformly applied the same variance-extracting procedure to maintain consistency. We have chosen to use Hodrick-Prescott (HP) filtering to separate our data into a trend and a cyclical component after carefully researching a sequence of potential filtering methods.[10] The HP-filter not only exhibits the advantage of being well known in economics, it is also the only filter separating the series into only two components. All other decompositions split the sample into at least three components, and we would therefore have to ignore the higher frequencies from our analysis. The variance of the time series is obtained from

$$\hat{\sigma}^2_{\text{HP}} = \frac{1}{m-1} \sum_{t=1}^{m} (g_t - \hat{\mu}_t)^2, \tag{6}$$

[10] See the appendix for a full discussion.

where $\hat{\mu}_t$ is the Hodrick-Prescott filtered growth rate that is obtained by solving

$$\min_{\hat{\mu}_t}\left[\sum_{t=1}^{T}(g_t-\hat{\mu}_t)^2+\lambda\sum_{t=2}^{T-1}\left((1-L)^2\,\hat{\mu}_{t+1}\right)^2\right] \tag{7}$$

where $L^n y_t = y_{t-n} \quad \forall n \in \mathbb{N}$. The objective was to set the smoothing parameter such that for both types of stationarity, the filtered series would be a straight line. In case of non-stationarity, the filtered series should display the possible non-linear deterministic trend. Visual inspection (see figure A.3 to A.6 in the appendix suggested setting the smoothing parameter, λ, to 5000. The outcome is in line with our unit-root tests from the previous section.[11] England and the United States are stationary cases par excellence: the growth rate fluctuates around a constant value. Italy is a perfect case of trend-stationarity: the average growth rate has been declining since 1960 at a constant rate. Greece belongs in the nonstationary category: the trend growth rate was declining until the mid-1980s when it reached the bottom and started to increase again.

3.5 Results

Estimating the volatility and the average growth rate over the whole sample and running a cross-country regression afterwards would imply that we assume that both statistics are more or less stable. Visual inspection tells us that this is clearly not the case. Dividing the samples into sub-samples mitigated the effect of assuming constant volatility and constant trend growth rates. Furthermore, we end up with more data points. Of course, there is an upper-bound to the number of sub-samples since we still need enough data points to obtain a 'satisfactory' estimate of the variance (equation 6). Since the length of our time series is 45 (1961-2005) we decided to separate them into three (non-overlapping) sub-samples of length 15.[12] The resulting $3 * 21 = 63$ data points were pooled for our regression analysis.[13]

We are interested in the functional relationship between the growth rate of GDP, y, and our measure of its volatility, x. In a parametric approach, the obvious choice is linear,

$$y = \alpha + \beta x \tag{8}$$

We find a positive and significant relationship between the standard deviation and the growth rate of output,

$$y = \underset{(0.37)}{1.47} + \underset{(0.15)}{0.54}\,x \tag{9}$$

[11] Note that we have selected a λ very different from what can be found in the real business cycle literature. However, our objective, too, is very different. Whereas the real business cycle theory tries to eliminate very low frequencies (noise9), our ambition is very different: we try to split the GDP series into a trend and cyclical component.

[12] One robustness test we perform in the next chapter is splitting the sample into 2 or 4 groups. This does not alter our main findings.

[13] Pooled estimators impose the realistic assumption on our data set that the relationship between regressand and regressor is the same irrespective of whether we are looking across countries or over time within in a country, and that all the errors are drawn from the same distribution.

	$\hat{\alpha}$	**s.e.**	$\hat{\beta}$	**s.e.**
2 periods	1.49	0.46	0.53	0.18
3 periods	1.47	0.37	0.54	0.15
4 periods	1.96	0.33	0.33	0.13

Table 2: Regression estimates for different sample length

	$\hat{\alpha}$	**s.e.**	$\hat{\beta}$	**s.e.**
15-15-15	1.49	0.46	0.53	0.18
(6)-11-11-11-(6)	−1.48	0.61	0.24	0.04
(8)-15-15-(7)	−0.44	0.68	0.16	0.04

Table 3: Regression estimates for different sample length, omitting initial and final observations

where the number below the estimated coefficient indicate the standard error of the ordinary least square estimation. The regression can explain 17.7% of the variation, which is good, considering the fact that we did not include any other control variables and that we use cross country data. The result is certainly encouraging, as we find a significant relationship between economic growth and volatility. In order to confirm our results, we will conduct a series of robustness checks in the following chapter.

4 Robustness Analysis

4.1 Sample Variations

The first robustness check was to split the sample in different length. Whereas in the previous chapter, we used have split the sample in three, with a length of a single observation being 15 years, and a total of 63 observations, we have also split the sample period into 2 and 4 groups. This leads to the length of a single observation of 22 or 11 years respectively, with 42 or 84 observations. Our findings are summarized in table 2.

We obtain similar coefficient estimates for the 2-period split and the 3-period split, indicating robustness of our results. The coefficient remains statistically significant at the 5% level. The reason for the lower value may be due to the fact that 11 periods may be too short to compute the variance, and some variance is captured by the growth rates, which alter over the 4 observation periods.

The standard HP Filter is known to have problems detrending at the beginning and end of the sample period. For that reason, we created two additional series where we have eliminated the first 5 and 7 years respectively, and than split the remaining sample in three 11 year periods and two 15 year periods, respectively. The estimation results are presented in 3, and differ little from our previous results, continuing to show a positive and significant relation between economic growth and volatility.

	$\hat{\alpha}$	s.e.	$\hat{\beta}$	s.e.
Lin-Lin (8)	1.5	0.4	0.55	0.15
Log-Log (11)	1.7	1.1	0.46	0.13
Log-Lin (12)	0.5	0.1	0.17	0.05
Lin-Log (13)	1.6	0.3	1.40	0.37

Table 4: Regression estimates

4.2 Variants of Ordinary Least Squares

Regression analysis is concerned with the question of how y can be explained by x. This means a relation of the form

$$\begin{aligned} y_i &= m(x_i) + \epsilon_i \\ \mathbb{E}[Y|X=x] &= m(x). \end{aligned} \tag{10}$$

where m is a function in the mathematical sense. It determines how the *average* value of y changes as x changes. In a parametric approach, the obvious choice is linear, as discussed in the previous section, and functions whose parameters can be estimated by ordinary least squares after applying a linearizing transformation on the variables, like

$$m(x) = \alpha x^{\beta} \tag{11}$$

$$m(x) = e^{\alpha+\beta x} \tag{12}$$

$$m(x) = \alpha + \beta \ln x \tag{13}$$

In equation 11, β measures the elasticity[14] of $m(x)$ with respect to x. It can be written as $\ln m(x) = \ln \alpha + \beta \ln x$. In equation 12 β gives the proportionate change in $m(x)$ per unit change in x. Vice versa for equation 13.

Table 4 summarizes the estimation results. All four models can account for about the same amount of variability in the growth rate (between 15 and 20 percent), with the lin-lin model (8, bold solid line) and the lin-log model (13, solid line) coming out leading (see figure 1). In both models the estimate for β is significantly different from zero (p-value < 0.001). The log-log model (11, dashed line) and the lin-log model (12, dot-dashed line) still exhibit coefficient that are significant at the 5% significance level.

The coefficients cannot be compared directly, so figure 1 draws the regression lines for all four models, showing that are all very similar in the relevant area, so that we can confirm the result of the previous chapter.

So far, we have based our regression on the standard deviation as a measure of volatility. Evidently, the variance, the square of the standard deviation, may also be an indicator of volatility. Although the coefficient in equation 11, which is far from 2, suggest otherwise, we run various polynomial regressions of the more general form

[14] The elasticity measures the percent change in $m(x)$ for a 1 percent change in x. $m(x)_\epsilon = \frac{m'(x)x}{m(x)} = \frac{d \ln m(x)}{d \ln x}$

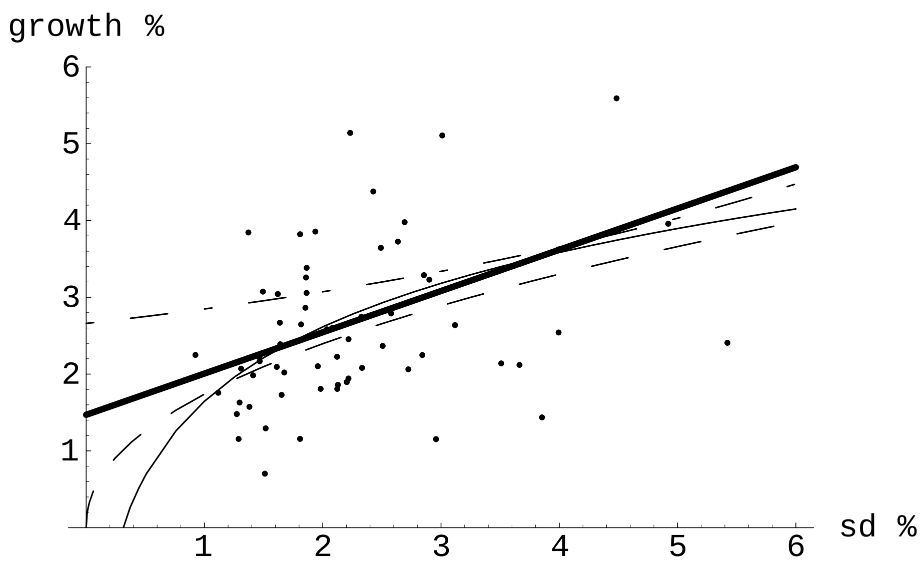

Figure 1: Scatterplot and Regression Lines

	$\hat{\alpha}$	**s.e.**	$\hat{\beta}$	**s.e.**	$\hat{\gamma}$	**s.e.**	R^2
Model 1	1.47	0.37	0.54	0.15			17.7
Model 2	2.19	0.21			8.26	2.52	15.0
Model 3	0.63	0.93	1.21	0.71	−11.57	11.77	19.0

Table 5: Regression estimates

$$m(x) = \alpha + \beta x + \gamma x^2 \tag{14}$$

We have tried higher order polynomials with no avail. The results for the estimation are presented in table 5. Whilst single variable models all yield statistically significant coefficient on the various measures of volatility, more complex models fail in obtaining these coefficients, probably due to correlation between independent variables. Among the first three models, we find that the version using the variance has a slightly higher explanatory power than the model which is based on the standard deviation, and hence preferable.

4.3 Robust Regression: M-Estimation

A statistical procedure is regarded as 'robust' if it performs reasonably well even when the assumption of the statistical model are not true. M-regression, the most common general method of robust regression introduced by Huber (1964), was specifically developed to be robust with respect to the assumption of normality (see Birkes & Dodge (1993)). Consider our linear model

$$y_i = \boldsymbol{x_i'}\boldsymbol{\beta} + \epsilon_i \tag{15}$$

for the ith of n observations. The fitted model is

$$y_i = \boldsymbol{x_i'b} + e_i \tag{16}$$

The general M-estimator minimizes the *objective function*

$$\sum_{i=1}^{n} \rho(e_i) = \sum_{i=1}^{n} \rho\left(y_i - \boldsymbol{x_i'b}\right) \tag{17}$$

where the function ρ gives the contribution of each residual to the objective function. Obviously, for least-squares estimation, $\rho(e_i) = e_i^2$. The Huber M-estimator uses a function ρ that is a compromise between e^2 and $|e|$:

$$\rho(e) = \begin{cases} e^2 & \text{for } |e| \leq \text{k} \\ 2k|e| - k^2 & \text{otherwise} \end{cases}$$

Tukey's biweight estimator is defined as:

$$\rho(e) = \begin{cases} \frac{k^2}{6}\left\{1 - \left[1 - \left(\frac{e}{k}\right)^2\right]^3\right\} & \text{for } |e| \leq \text{k} \\ \frac{k^2}{6} & \text{otherwise} \end{cases}$$

The value k for the Huber-M and Tukey's biweight estimator is called a tuning constant; smaller values of k produce more resistance to outliers, but at the expense of lower efficiency when the errors are normally distributed. We choose the pre-selected values of $k = 1.345\sigma$ for Huber's and $k = 4.685\sigma$ for Tukey's estimator (where σ is the standard deviation of the errors).

Figure 2 shows the regression lines for the OLS (red), Huber (blue), and Tukey (green) estimates. Both the Huber and the Tukey estimates of the slope are slightly lower than the OLS estimate, viz. 0.45 and 0.4, respectively, but still significantly different from zero. We can therefore still confirm the robustness of the OLS estimator presented in the previous chapter.

4.4 Detection of Influential Data Points

The purpose of any sample is to represent a certain population, actual or hypothetical. Influential data points or outliers[15] in a sample are likely to influence the sample-based estimates of the regression coefficients. There are many sources of outliers such as sampling a member not of that population, bad recording or measurement, errors in data entry, etc. For whatever reason they have come to exist, outliers will lessen the ability of the sample statistics to represent the population of interest. A common method of dealing with apparent outliers in a regression situation is to remove the outliers and then refit the regression line to the remaining points.

Since no data points that obviously qualify as an outlier could be found by visual inspection, we calculated Cook's distance for each observation. The 100(1-α)% joint confidence region for the parameter vector β is

$$\left(\hat{\beta} - \beta\right)' (X'X) \left(\hat{\beta} - \beta\right) \leq k\hat{\sigma}^2 F_{k,N-k,\alpha} \tag{18}$$

[15]Hawkins (1980) described an outlier as an observation that 'deviates so much from other observations as to arouse suspicions that it was generated by a different mechanism'. Outliers have also been labeled as contaminants (Wainer (1976))

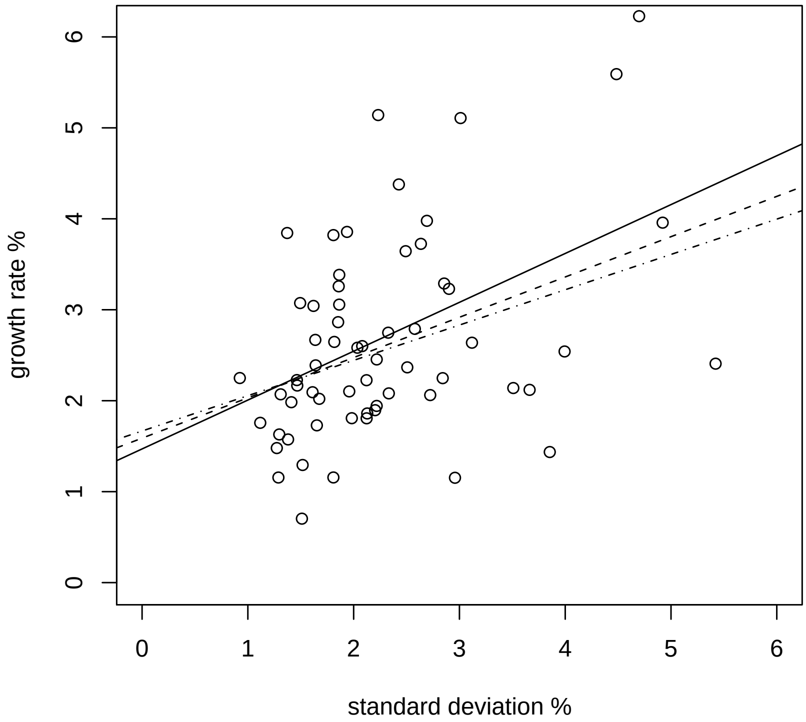

Figure 2: OLS, Huber-M, and Tukey's Biweight

Cook's Distance is defined as

$$C_i = \frac{\left(\hat{\beta} - \hat{\beta}_{-i}\right)' (X'X) \left(\hat{\beta} - \hat{\beta}_{-i}\right)}{k\hat{\sigma}^2} \tag{19}$$

The 100(1-α)% joint ellipsoidal confidence region for β given in 18 is centered at $\hat{\beta}$. The quantity C_i measures the change in the center of this ellipsoid when the ith observation is omitted, and thereby assesses its influence. C_i is the scaled distance between $\hat{\beta}$ and $\hat{\beta}_{-i}$. An alternate form of Cook's distance is

$$C_i = \frac{1}{k}\frac{h_{ii}}{(1-h_{ii})} r_i^2 \tag{20}$$

where h_{ii} is the leverage[16] and r_i the studentized residual[17] C_is that are above the threshold value of the $50th$ percentile of the F distribution with k and

[16]The leverage assesses how far away a value of the explanatory variable is from the mean value: the farther away the observation the more leverage it has. h_{ii} is the ith diagonal element of $X(X'X)^{-1}X'$. In the bivariate case $h_{ii} = \frac{1}{n} + \frac{(x_i - \bar{x})^2}{(n-1)s_x^2}$.

[17]The studentized residual is $r_i = \frac{e_i}{s_e\sqrt{1-h_{ii}}}$.

N-k degrees of freedom (in our case 0.7) are regarded as influential observations. According to this definition, as can be seen in 3, our sample does not contain any influential observations.

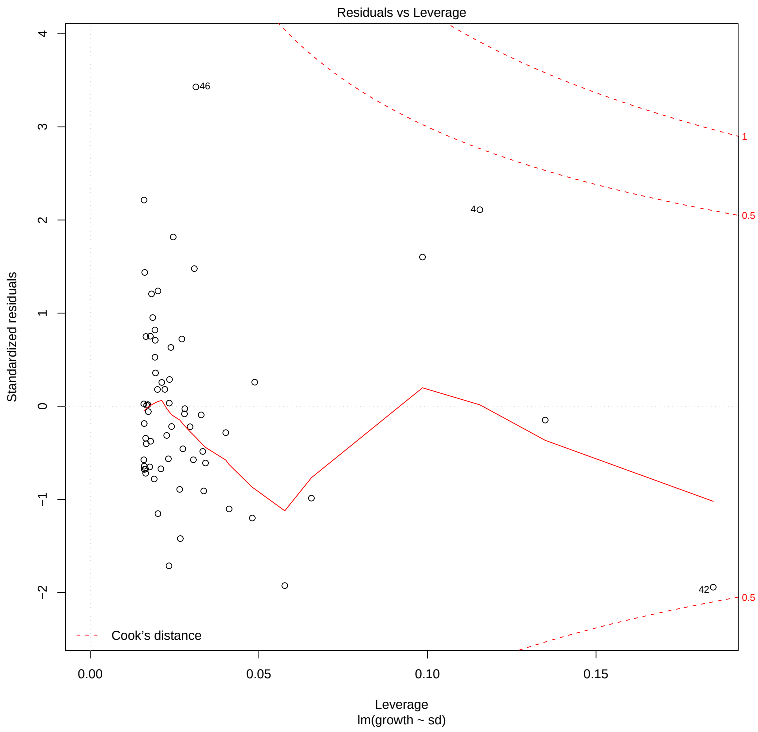

Figure 3: Influential Data Points

The most influential data points in our sample are $\text{Greece}_{1960-75}$ (#4) with a growth rate of 6.2% and a standard deviation of 4.7%, $\text{Turkey}_{1990-05}$ (# 42) with a growth rate of 2.4% and a standard deviation of 5.4%, and $\text{Japan}_{1960-75}$ (#46) with a growth rate of 7% and a sd of 3.2%. Running a OLS regression without those three data points yielded a slope of 0.46, wich is the same result as the one obtained by using the Huber-M-Estimator. Once again, this confirms our results of a positive and significant relationship between economic growth and volatility.

4.5 Nonparametric Estimation: Kernel Regression

Our final test of robustness is to use nonparametric estimation methods. The nonparametric approach does not assume any functional form for $m(x)$, but rather goes back to the statistical definition of conditional expectation:

$$m(x) = \mathbb{E}[Y|X = x] = \int_{-\infty}^{+\infty} y f_{Y|X}(y|x)\, dy = \frac{1}{f_X(x)} \int_{-\infty}^{+\infty} y f_{X,Y}(x, y)\, dy \tag{21}$$

Plugging in Kernel estimates for the marginal density, $f_X(x)$, and the joint density, $f_{Y,X}(y,x)$, delivers an estimate $m(x)$ of the conditional expectation at point x:

$$\frac{1}{\hat{f}_X(x)} \int_{-\infty}^{+\infty} y \hat{f}_{X,Y}(x,y)\, dy \tag{22}$$

This has become known as the Nadaraya-Watson estimator. Figure 4 shows two Nadaraya-Watson regression estimates, one with high bandwidth (dark blue line) and one with low bandwidth (light blue line). In the dense region, i.e. in the region where many data points are available, the estimates tell the same story as the OLS regression line, so it seems that there really is a linear relationship between volatility and growth. The Nadaraya-Watson estimates become very erratic in the region where the standard deviation is larger than 3.5%. This was to be expected, since only eight data points fall into this region.

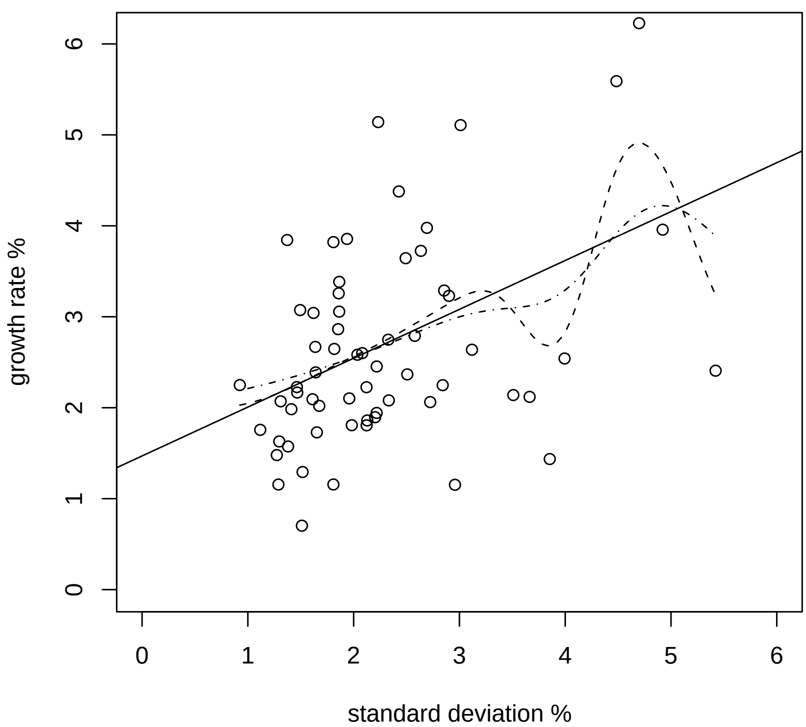

Figure 4: Nadaraya-Watson Estimates and OLS Regression Line

After running an entire series of robustness tests, from altering the sample, running non-linear versions of OLS regressions, M-estimations, checking against critical data points, and nonparametric methods, which all point toward a positive and significant relationship between economic growth and volatility, we are convinced about the robustness of our results indicated in the previous chapter.

5 Conclusion

The contribution of this paper is twofold. First the empirical result of a robust and positive relationship between economic growth and volatility should stimulate and support further theoretical research in the field, which is growing in magnitude and importance. Second, the paper suggests an empirical method to analyze the relationship between economic growth and volatility. We use the well-known Hodrick-Prescott filter to separate GDP time series into a trend component and a cyclical component, and then use period averages to obtain statistics for growth and volatility. This method is preferential to other band-pass filtering techniques, but also with respect to GARCH methods, which are wholly unfit for short time series such as national accounting data.

Using the time series experience of twenty-one OECD countries between 1961 and 2005, we have presented strong empirical evidence for a positive relationship between output variability and economic growth. This relationship is robust against outliers and also shows up in a non-parametric setting. A case can be made that our measure of output variability is more suitable than the ones used in previous work for time series of economic growth.

These results have to be treated with care, particularly when making policy implications. Whilst we find that there is a positive and significant relation between economic growth and volatility, we refrain from making any comment on causality. Factors that increase volatility, such a pro-cyclical fiscal or monetary policy probably will not alter the growth pattern of the economy. We do believe in "innovative risk", or the concept that an innovation, which will induce economic growth, is intrinsically risky, and therefore we should observe a positive relation between growth and volatility in the data, as we indeed do. Whilst it is true, at least at the margin, that an increase in innovation would lead to faster economic growth, this will come at the cost of higher volatility. We think that economic stability is welfare enhancing, and therefore policymakers face a trade-off between economic growth and volatility.

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APPENDIX

A Garch-in-Mean Regression Models

In the GARCH-in-Mean (GARCH-M) model the conditional variance of the error term is used as an explanatory variable in the equation (1) for the conditional mean of the variable to be explained. The error term follows a GARCH(p,q) model

$$u_t = \sigma_t \epsilon_t \tag{A.1}$$

where $\epsilon_t \sim IID(0,1)$ and σ_t^2, the conditional variance of u_t conditional on all the information up to time $t-1$, $\mathcal{F}_{t-1}$, is given as:

$$\mathbb{E}\left[u_t^2|\mathcal{F}_{t-1}\right] = \sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j u_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{A.2}$$

All coefficients in equation A.2 are necessarily non-negative. Nelson (1990) showed that a GARCH(1,1) process is strictly stationary when $\mathbb{E}[\log(\alpha\epsilon_t^2 + \beta)] < 0$. When $\epsilon_t \sim N(0,1)$, the condition for strict stationarity is weaker then the condition for covariance stationarity $\alpha + \beta < 1$.

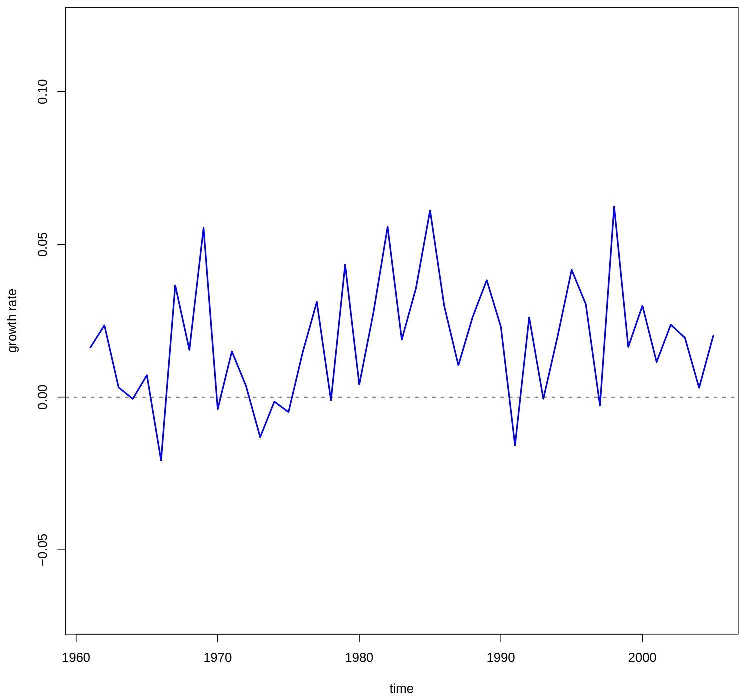

Figure A.1: Trajectory of a GARCH(1,1)-M process

Figure A.1 shows a trajectory of a GARCH(1,1)-M process. The risk premium parameter, γ, was set to 2, a value in between those obtained by the

GARCH(0,1)-M model of Caporale et al. (0.7) and the bivariate GARCH(1,1)-M model Grier et al. (3.5). The parameters for the variance equation, α and β, were set to 0.1 and 0.8, respectively. These values are common in finance (see for instance Tsay (2005)) and close to the ones obtained by Grier & Perry (2000) (0.2 and 0.7).[18] Though it seems that such processes are capable of producing series that resemble actual GDP growth rates, unfortunately, very long time series (n » 2500) are required for estimating such processes efficiently.

In a small Monte-Carlo simulation running 100 realizations of a GARCH(1,1)-M process with t = 1,...,200 and with the parameters as given above and re-estimating the process yielded the distribution of the GARCH-in-Mean effect, $\hat{\gamma}$ as shown in figure A.2.

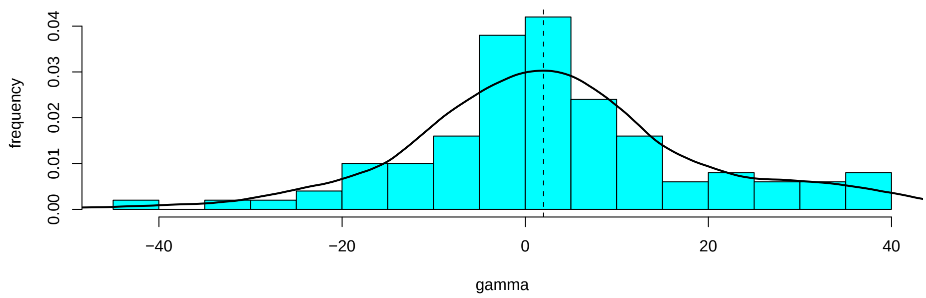

Figure A.2: Histogram and Empirical Density Function

The average is close to the true mean of our simulation (3 instead of 2) but the standard deviation of 15 is unacceptably large. In 25 percent of our simulation we obtained an estimate for γ that was at least twice as large but had the opposite sign (-4 instead of 2). Apart from this technical obstacle, the implication of the fact that the measure for volatility is based solely on forecast uncertainty seems to be not fully understood when the mean equation 1 contains additional regressors.

B Growth Rates and the HP filter

[18]The intercepts were set to $\omega = 0.0001$ and $\kappa = 0.005$, respectively and $\epsilon \sim \mathcal{N}(0,1)$

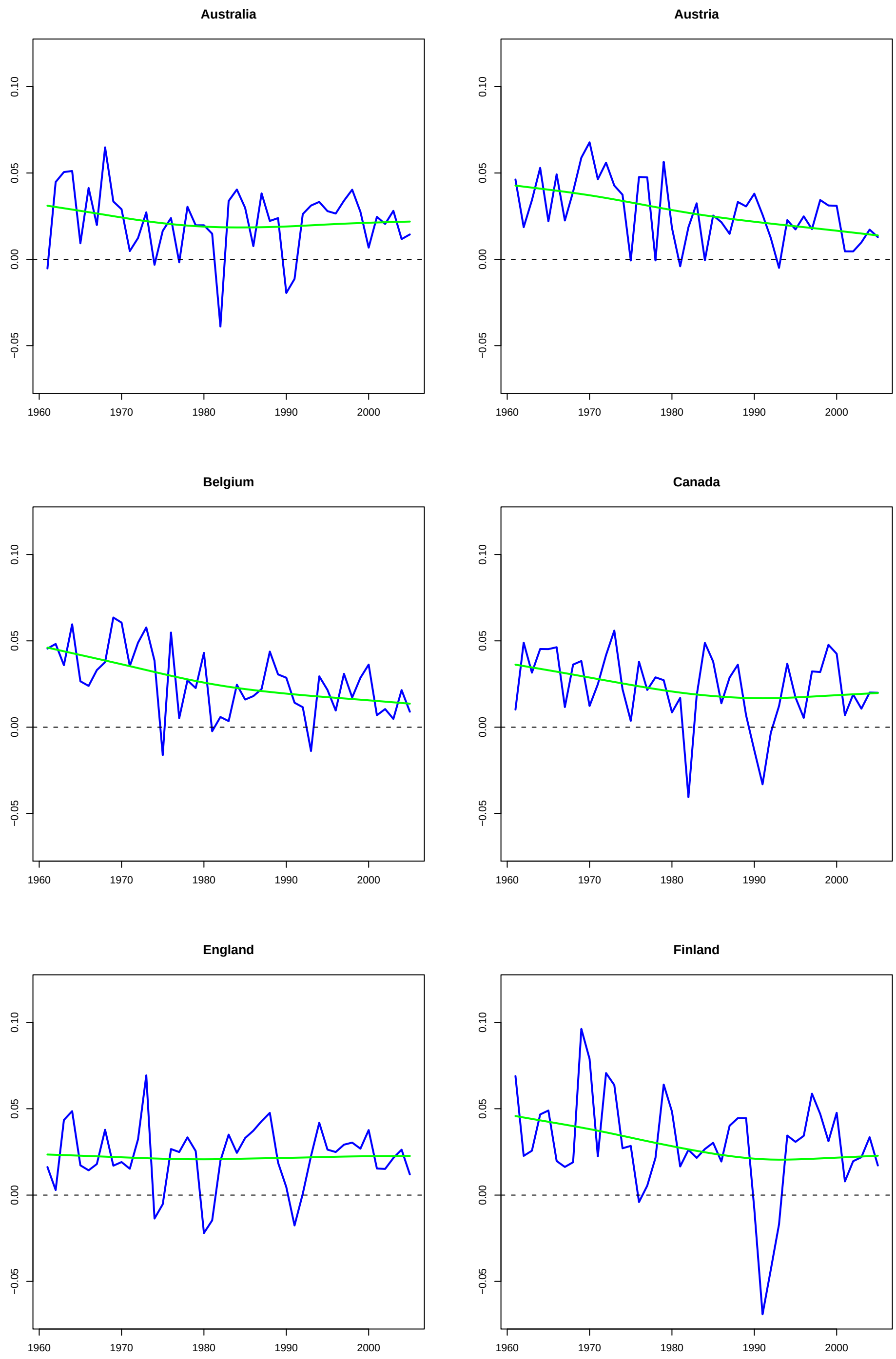

Figure A.3: Growth Rates of Selected Countries 1

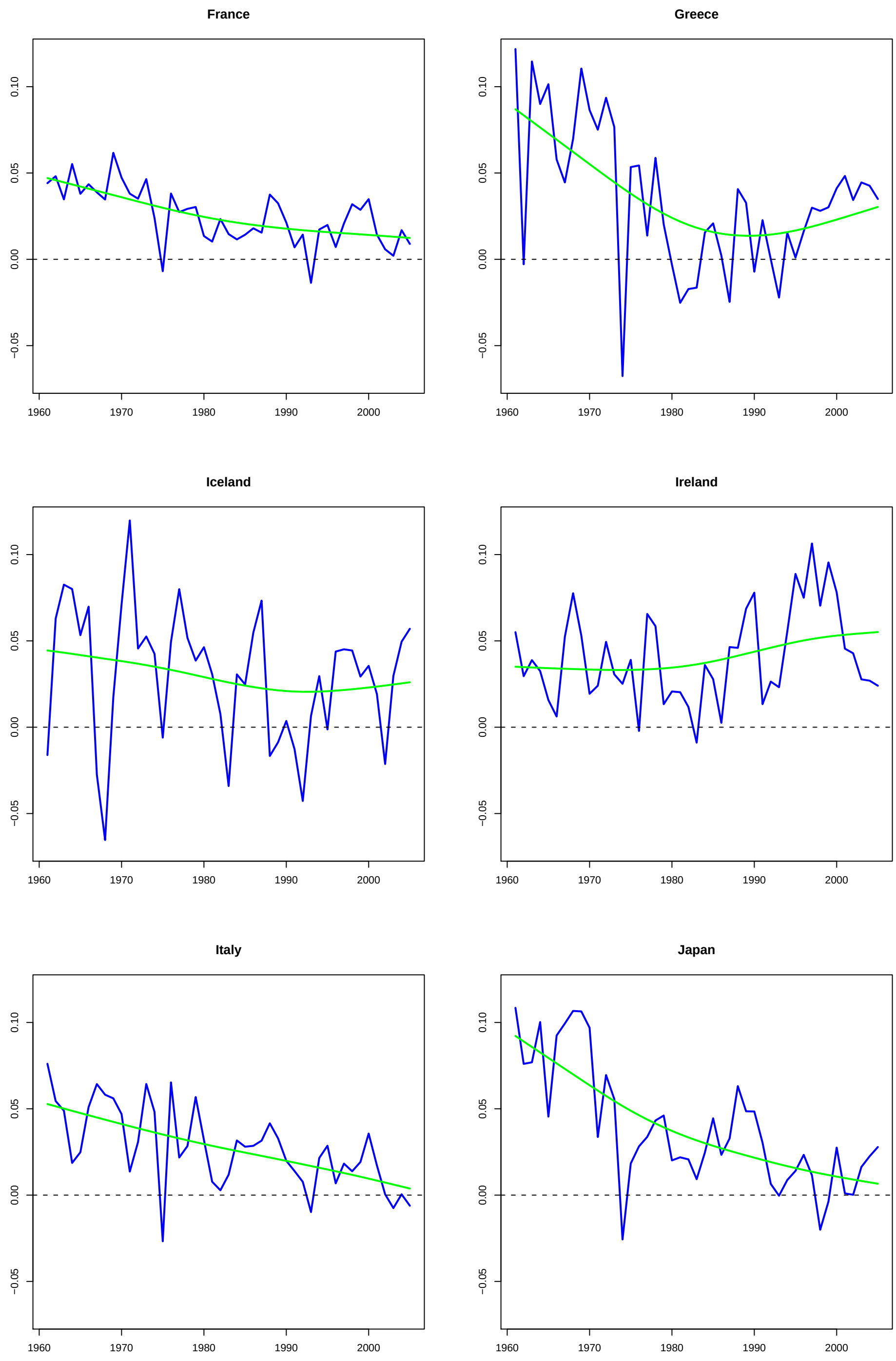

Figure A.4: Growth Rates of Selected Countries 2

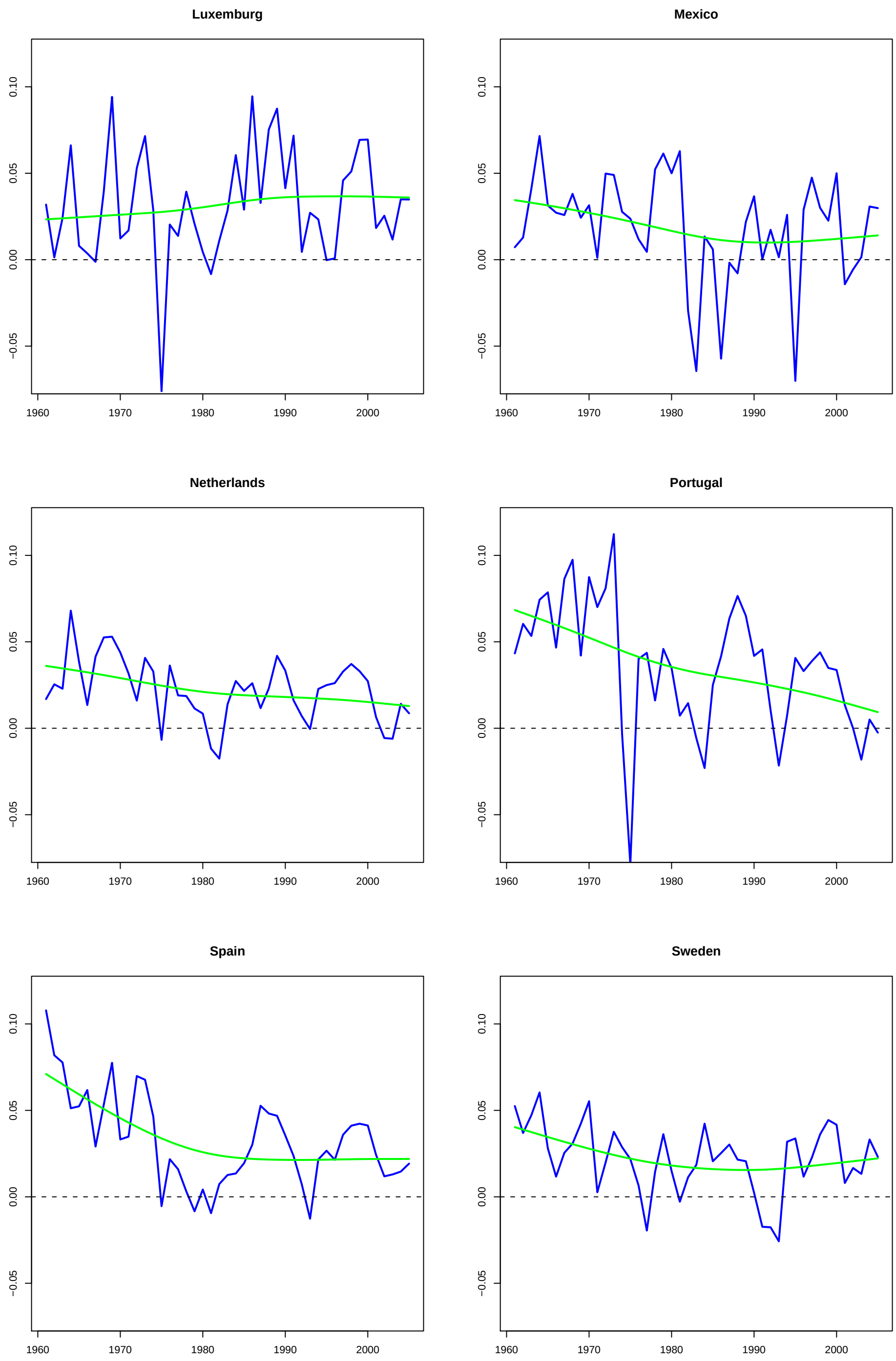

Figure A.5: Growth Rates of Selected Countries 3

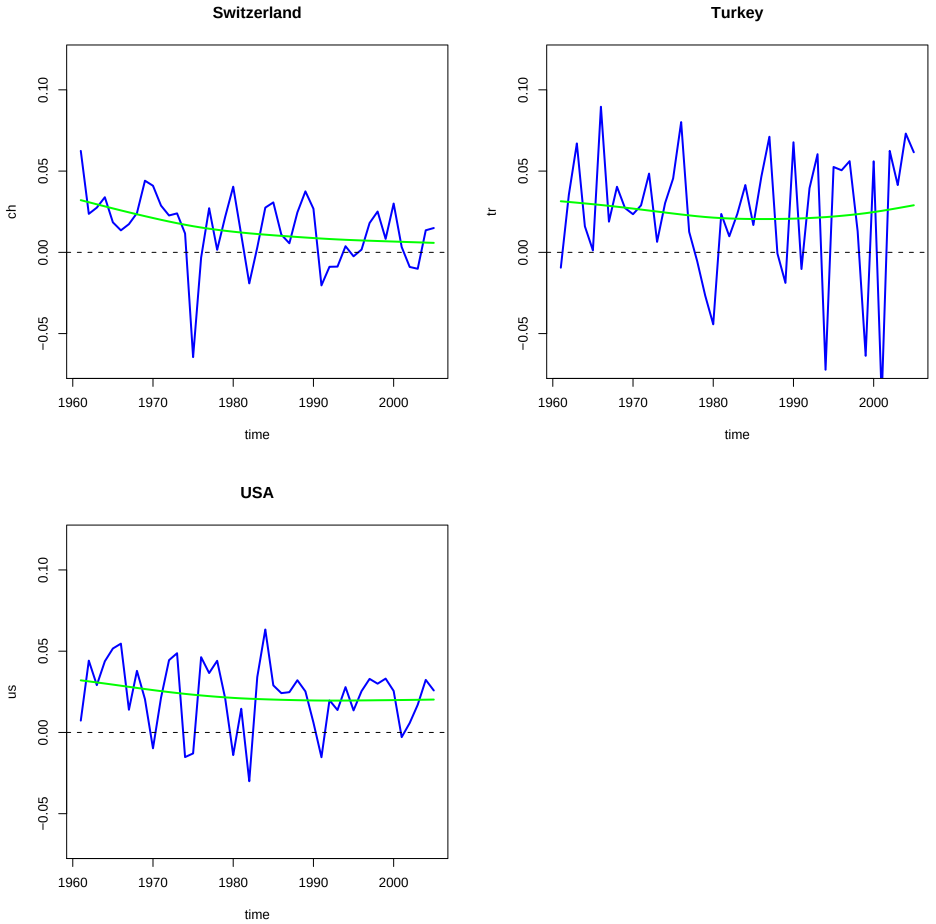

Figure A.6: Growth Rates of Selected Countries 4

Corresponding Author:

Martin Zagler
Vienna University of Economics
Augasse 2-6
A1090 Vienna, Austria
http://www.wu-wien.ac.at/inst/vw1/zagler/
Email: zagler@wu-wien.ac.at