

Essays in Macroeconomics

Jan Teresiński

Thesis submitted for assessment with a view to
obtaining the degree of Doctor of Economics
of the European University Institute

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Department of Economics

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Jan Teresiński

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Abstract

In the first two chapters of this thesis we analyze the impact of an increase in transfer payments - i.e., fiscal stimulus in a form of direct payments to individuals - on budget deficits. In the first chapter we study this issue theoretically and show that in a model with the cash-in-advance constraint on consumption and downward nominal wage rigidity the transfers multiplier is positive when the economy is below its full employment level. Increasing transfers in these circumstances relaxes the cash-in-advance constraint and effectively undoes the inefficiency caused by the wage rigidity. Since this results in higher income and consumption which are both taxed, the fiscal stimulus can possibly be self-financing - it pays for itself in a form of increased tax revenue. We also perform a quantitative analysis and show that under a plausible calibration of the model when the economy is far enough from the full employment, the transfer multipliers are large enough for the tax revenue to increase sufficiently so that the fiscal stimulus largely finances itself.

In the second chapter we analyze the self-financing nature of transfer payments empirically and estimate the impulse response functions of GDP, unemployment, consumption and debt to an increase in transfer payments on quarterly data from 1959Q2 to 1991Q4 using the local projection method and exogenous transfers shocks. We show that the stimulus in a form of higher transfers has more pronounced effects when unemployment is high than when it is low. Permanent transfers seem not to affect debt, while temporary transfers are estimated to reduce it after an initial increase, especially in the high unemployment regime - an increase in temporary transfers seems to be not only self-financing, but actually reducing debt when the economy recovers.

The third chapter is related to a different topic: we analyze how the terms of trade (TOT) - the ratio of export prices to import prices - affect total factor productivity (TFP). We provide empirical macroeconomic evidence based on the times series SVAR analysis and microeconomic evidence based on industry level data which shows that the terms of trade improvements are associated with a slowdown in the total factor productivity growth. Next, we build a theoretical model in which terms of trade improvement results in putting more resources into physical goods production at the expense of the research and development (R&D) sector, which in turn has a negative impact on knowledge development.

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Chapter 1

Self-financing transfers in a cash-in-advance economy with downward nominal wage rigidity

Abstract

In this chapter we study theoretically the impact of an increase in transfer payments - i.e., fiscal stimulus in a form of direct payments to individuals - on budget deficits. We show that in a standard competitive business cycle framework augmented with the cash-in-advance constraint on consumption and downward nominal wage rigidity the transfers payments multiplier is positive when the economy is below its full employment level. Increasing transfer payments in these circumstances relaxes the cash-in-advance constraint and effectively undoes the inefficiency caused by the wage rigidity. Since this results in higher income and consumption which are both taxed, the fiscal stimulus can possibly be self-financing - it pays for itself in a form of increased tax revenue. We also show that under a plausible calibration of the model and for the existing tax rates in the European Union when the economy is far enough from the full employment, the transfer multipliers are indeed big enough for the tax revenue to increase sufficiently so that the fiscal stimulus largely finances itself.

1.1 Introduction

The Great Recession and the coronavirus crisis renewed the interest in fiscal policy as an economic stabilization tool, especially in the context of monetary policy being constrained at the zero lower bound. The ratio of government expenditure to GDP increased significantly in the advanced economies, compared to the pre-crisis averages. Economic research has not remained deaf to these developments. There have been numerous papers analysing the effects of higher government pur-

chases on output and employment¹. Yet, surprisingly little work has been devoted to transfer payments defined as direct lump-sum payments to individuals despite the fact that transfer payments constitute an important share of GDP in advanced economies and were an important part of the actual fiscal stimulus packages²³.

In this chapter we analyze the impact of higher transfer payments on budget deficits. In particular, we are interested in conditions under which these transfers are self-financing, which means that they pay for themselves in a form of increased tax revenues. In a standard competitive Ricardian setting an increase in transfer payments translates into higher debt with no effect on output - representative agents just save the additional amount they receive in a form of government bonds. This setting is a useful benchmark but does not need to correspond to periods when output is depressed (otherwise a stimulus in a form of transfers would be useless). We show that in an alternative setting with spare capacity - unemployment above the natural rate introduced by downward nominal wage rigidity - when a part of household's consumption is subject to the cash-in advance constraint, an increase in transfers has positive multiplier effects by relaxing the cash-in-advance constraint and effectively undoing the inefficiency caused by the labor market frictions. As a result employment, output, income and consumption increase. This automatically generates higher tax revenue from taxing labor income, capital income and consumption which for a sufficiently large shock may be enough to cover the initial transfer expenses.

Our setting is a representative agent model - we show that even without heterogeneity the otherwise standard model with the two above-mentioned frictions (cash-in-advance constraint and downward nominal wage rigidity) is able to deliver a positive transfers multiplier. In reality transfer payments can be even more effective instrument, because they can be targeted to agents with higher marginal propensities to consume based on their holdings of liquid wealth. When transfers have multiplier effects, an increase in transfer payments is financed by extra tax revenue resulting from distortionary taxation. However, in order to study the self-financing nature of transfer payments we do not impose the balanced budget each period. We do not change the tax system - the tax rates on income and consumption are held constant and when the tax base expands, the tax revenues also increase. The amount of extra tax revenue collected tells us to what extent transfers are self-financing. The remaining amount (if transfer payments are not fully self-financed by additional tax revenue) is financed initially by bonds, but to prevent running the Ponzi schemes it is assumed that money supply is eventually adjusted to stabilize the debt. We also show that this policy improves

¹See Ramey (2011a) for a review of this literature.

²Notable exceptions in the literature include Oh and Reis (2012), Kaplan and Violante (2014), Romer and Romer (2016), Giambattista and Pennings (2017) and Mehrotra (2018) which we discuss below.

³Transfer payments of various kinds amount to around 20% of GDP in the OECD countries. Oh and Reis (2012) show that from the end of 2007 until the end of 2009 three quarters of the increase in the U.S. government expenditures were due to increases in transfers. Similarly, they show that in every OECD country where government spending increased over this period at least 30% of the increase was actually driven by transfers - with the median share of transfer payments of 64%.

the efficiency - welfare is higher with an increase in transfers than without. Moreover, the virtue of transfers is that they are lump-sum, hence they do not affect directly the decision rules of the agents and therefore the distortion of the policy restoring the labor market equilibrium is smaller than what would arise if other fiscal instruments were used. The investigation focuses on fiscal policy in a form of transfer payments and assumes that monetary policy is passive - the model features fiscal dominance.

Clearly the stimulus policy in a form of an increase in transfers payments is not a free lunch and the effects of higher transfer payments on budget deficits crucially depend on the setting considered. Our investigation shows that the lower the initial level of money supply and transfers, i.e. the lower the initial level of prices, the higher the multiplier is as long as the downward wage rigidity constraint binds. This means that the deeper the recession measured in terms of unemployment deviation from its natural level, the larger is the multiplier and the more likely are the transfer payments to pay for themselves. In particular, depending on the initial conditions (how far the economy is from the full employment) the transfers multiplier in the model could reach the level of 2.2 which corresponds to the self-financing result given the existing tax rates. In such a case an increase in transfers is large enough to generate sufficient additional tax revenue to make these extra payments self-financing. In our quantitative exercise under a plausible calibration the model delivers smaller multipliers which result in around three-quarters of an increase in transfers being self-financed with additional tax revenue generated thanks to the stimulus. The rest is assumed to be financed with seigniorage revenue so that the tax rates do not need to be raised in the future to stabilize the debt.

This chapter relates predominantly to the literature on the effects of fiscal spending stimulus on output and employment. Such a stimulus can take a form of both government purchases and transfers. However, as indicated above, most of the papers focus on the former - either in the neoclassical setting as in Baxter and King (1993) where government spending can have multiplier effects due to increased labor supply resulting from negative wealth effects (higher taxes in the future) and resulting higher investment due to complementarity between capital and labor in production - or in the New-Keynesian setting like in Cogan et al. (2010) where additional amplification is due to an expansion of production by those firms which cannot adjust prices. Christiano, Eichenbaum, and Rebelo (2011) and Woodford (2011) additionally show that the multiplier is likely to be larger when the zero lower bound on nominal interest rates binds⁴. As for the empirical studies, the multipliers are estimated either using restrictions on the contemporaneous effects on variables considered in the VAR system as in Blanchard and Perotti (2002), sign-restrictions as in Mountford and Uhlig (2009)

⁴Dupor, Li, and Li (2019) study a model in which government spending multiplier that is larger than one can arise as a consequence of wage stickiness. Similarly as in this chapter in case of transfers, higher government spending in their model increases prices, reduces real wages and increases employment, while it does not crowd out consumption, since higher labor income allows to afford higher consumption despite the lump-sum tax increase, so that negative wealth effects are muted.

or dummy variables indicating exogenous shifts in fiscal policy as in Ramey (2011b) and Barro and Redlick (2011). The first approach yields larger multipliers but close to one, other approaches result in smaller multipliers (below one).

The papers explicitly considering transfers effects on output usually focus on fiscal policy in non-representative-agent settings. These are either two agent models as in Giambattista and Pennings (2017) and Mehrotra (2018)⁵ or in the context of incomplete markets model with heterogeneous agents as in Oh and Reis (2012) and Kaplan and Violante (2014). Since they are very close to our analysis we analyze them in turn.

Giambattista and Pennings (2017) study a New Keynesian model featuring two types of households: Ricardian and hand-to-mouth agents. In their setting the lump-sum transfers to the credit-constrained households are financed by the lump-sum taxes levied on the unconstrained ones (so that the government runs a balanced budget). Transfers multiplier is zero under flexible prices but positive under sticky prices - monetary policy response to higher inflation does not affect consumption of the credit-constrained households. Additionally positive output gap translates into higher wages which are entirely spent by non-Ricardian agents, which boosts aggregate demand further. The transfers multiplier is larger than one especially if the share of credit-constrained is not too low and when the economy operates at the zero lower bound when higher inflation decreases the real interest rate beyond the possibilities of monetary policy.

Mehrotra (2018) analyzes non-targeted transfers (in a form of tax rebates) using a model with credit spreads which also includes two types of households: patient (savers) and impatient (borrowers). He finds that the transfer multiplier is close to zero when prices are flexible since then wealth effects lead to offsetting movements in hours worked by the households that provide and receive the transfer. Under sticky prices transfers have additional demand-side effects. However, if monetary policy is free to undo any effect of fiscal policy, the latter is irrelevant for determining aggregate output. Again, the transfer multiplier is substantial only at the zero lower bound and only if credit spreads are sufficiently responsive to changes in overall debt.

Oh and Reis (2012) study an incomplete markets New Keynesian model with heterogeneous agents facing health and income shocks. Transfers in their setting take the form of targeted lump-sum payments financed by lump-sum taxes. They find that transfers multipliers are positive due to agents heterogeneity (increased labor supply to finance higher transfers and higher consumption by shifting spending to those with higher marginal propensities to consume) and nominal rigidities (higher production and employment by firms not adjusting prices) but generally small.

The model of Kaplan and Violante (2014) features two assets: a low-return liquid asset and a high-return illiquid asset. In their model when wealthy households hold little or no liquid wealth

⁵Monacelli and Perotti (2011) also analyze two-agent New Keynesian model with borrowers and savers. On a side note of their analysis of government spending multiplier they show that fiscal expansion undertaken via a pure tax redistribution is neutral or quasi-neutral under flexible prices, while under sticky prices if it favors the constrained borrowers it generates an expansion in output, and a contraction when it favors the savers.

despite owning sizeable quantities of illiquid assets they behave as hand-to-mouth households, i.e., have large propensities to consume out of additional transitory income which can take a form of tax rebates paid in lump-sum fashion. These individuals are better off consuming transitory income rather than smoothing shocks because the latter involves paying transaction costs to tap into illiquid assets, holding large cash balances and foregoing high returns on illiquid assets or obtaining expensive credit. Kaplan and Violante document that one third of the US households can be attributed to the category of "wealthy hand-to-mouth". If they were not included in the model the fraction of credit-constrained households (usually around 10%) would be too small to generate a big enough response in the aggregate consumption spending.

Our analysis of transfers effects on the economy differs from those papers in several dimensions. First, we have a representative agent framework - in our setting heterogeneity is not needed for transfers to have multiplier effects, hence we provide a simpler setting in which such effects can arise. Second, our model features downward rigid nominal wages and unemployment - spare capacity in the economy is introduced via empirically plausible mechanism. However, nominal wages can freely adjust upwards and prices are fully flexible and there is a perfect competition on the goods and labor market. Again, the New Keynesian features (imperfect competition, sticky prices) are not necessary for our results⁶. Our model also includes distortionary taxation - a feature usually missing the above mentioned papers. Our final contribution is that in addition to the analysis of transfers multipliers we explicitly examine the effects of higher transfers on budget deficits, while most of the papers summarized above feature governments running balanced budgets, hence impose self-financing by adjusting taxes⁷.

This chapter also relates to the empirical work on the effects on transfers on consumption spending. Available microeconomic evidence suggests that the propensity to consume from transfers is sizeable. Johnson, Parker, and Souleles (2006) using the data from the Consumer Expenditure Survey estimate the causal effect of the disbursement of the 2001 federal income tax rebates on consumption expenditures exploiting the randomized timing of rebate receipt across households. They find that households spent cumulatively roughly two-thirds of their rebates during the period of receipt and subsequent three-month period. Responses were larger for households with low liquid wealth or low income. The same methodology is used in Parker et al. (2013) in their study on the impact of the 2008 tax rebates, called economic stimulus payments (ESPs), on consumption

⁶Broer, Krusell, and Öberg (2020) compare New Keynesian models with representative (RANK) and heterogeneous agents (HANK) and show that large fiscal multipliers in RANK result from implausible mechanism of falling profits which - together with higher taxes - bring about negative wealth effects and higher labor supply. These effects are much less pronounced in HANK where most of workers do not own stocks. The authors argue that nominal wage stickiness is a more plausible mechanism resulting in higher fiscal multipliers which are comparable between RANK and HANK models.

⁷DeLong and Summers (2012) in a reduced-form non-micro-founded model show that when interest rates are low enough, monetary policy is constrained at the zero lower bound and current output losses have hysteresis effects, expansionary fiscal policy is likely to be self-financing. In their setting government purchases might also be self-financing even without any hysteresis at all.

spending. They find slightly smaller responses for nondurable consumption than in case of the 2001 tax rebate, although the total increase in consumption was higher due to higher spending on durable goods. The randomized timing of the 2001 federal income tax rebates disbursement is also used by Agarwal, Liu, and Souleles (2007) who study the data on credit card accounts and estimate the monthly response of credit card payments. They find that, on average, consumers initially saved some of the rebate thereby paying down debt but later on over the subsequent year their spending increased. An increase in spending was particularly high for households likely to be liquidity constrained (with lower credit limits), while debt declined most for the unconstrained consumers. Broda and Parker (2008) also analyze the impact of the 2008 tax rebate payments to find that the typical family increased their spending on food, mass-merchandise and drug products by 3.5 percent when their rebate arrived, relative to a family yet to receive its rebate. This is estimated to directly boost demand for nondurable consumption by around 4.1 percent. More recently Karger and Rajan (2020) using anonymized transaction-level debit card data estimate that the marginal propensity to consume out of the Covid-19 Economic Impact Payments in the US was on average 48 percent.

Romer and Romer (2016) use their narrative approach to identify the macroeconomic effects of an increase in social security benefits. They find that permanent rise in those transfers has an immediate positive and significant impact on consumption, but not necessarily on industrial production or employment. Compared to tax changes, transfers effects are faster but less persistent and smaller. Párraga Rodríguez (2018) using their extended dataset finds an impact multiplier of permanent transfers of 0.2 and a cumulative multiplier as high as 2.8. In another paper - Párraga Rodríguez (2016) - she finds an output multiplier related to changes in old-age pensions in the EU equal to 0.5 on impact with a maximum cumulative response close to one. In the second chapter of this thesis we estimate the impact of an increase in transfers on debt using the local projection method and the shock series from Romer and Romer (2016). We find that increases in transfers actually seem to reduce debt over longer horizons in the high unemployment regime and do not increase it in the low unemployment regime.

More generally, this chapter also relates to the literature on how optimal fiscal and monetary policy can overcome the inefficiency caused by nominal rigidities. Adao, Correia, and Teles (2003) restrict the number of policy instruments to lump-sum taxes, money supply and nominal interest rates and show that in these circumstances optimal policy can undo the cash-in-advance constraint and nominal price-setting restriction, but not the monopolistic competition distortion. When lump-sum taxes are not available and distortionary taxes need to be used to collect revenue, the set of available tax instruments determines whether prices stickiness affects the optimal allocation. The models of Benigno and Woodford (2003), Schmitt-Grohé and Uribe (2004) and Siu (2004) feature only one tax and in this case nominal rigidity matters for optimal policies and allocations. However, as is shown by Correia, Nicolini, and Teles (2008) allowing for state-contingent labor and

consumption taxes, i.e., richer set of tax instruments, yields the optimal policy and equilibrium allocation independent of the degree of price stickiness. This would be also in our model which does not feature monopolistic competition - the only frictions are the cash-in-advance constraint and downward nominal wage rigidity, but we keep the tax rates constant and restrict the available policy instruments to transfer payments and money supply. We show that these two instruments are sufficient to undo the downward nominal wage rigidity constraint, while the efficiency in this case calls for inflating the economy more than it is required to undo the cash-in-advance constraint, so that the real wage is kept at the market clearing level. As from the government budget constraint point of view transfers are a liability and money supply provides the seigniorage revenue, the Ramsey planner needs to use both instruments in order to stabilize the government debt - once the full employment is attained, transfers do not have a self-financing nature and need to be financed with the seigniorage revenue in addition to the constant tax revenue. Equivalently, withdrawing money at the appropriate rate needs to be matched with decreasing non-negative transfers, otherwise it will result in the accumulation of debt in the hands of the public.

The rest of this chapter is organized as follows. In the next section we present our theoretical framework. We start with a standard competitive business cycle model with government levying taxes and paying lump-sum transfers to households which face the cash-in-advance constraint on a part of consumption. Next we augment the model with downward nominal wage rigidity. In section 1.3 we derive the transfers multiplier under the assumption that the wage rigidity constraint binds and hence the economy is below full employment. We also discuss how large the multiplier needs to be for an increase in transfer payments to be self-financing and analyze the optimal transfers policy in the Ramsey sense. In section 1.4 we quantitatively evaluate our model, showing that higher transfers under rigid wages improve welfare and make the recovery from the recession faster without imposing too much burden on the budget deficit. Section 1.5 concludes.

1.2 Models

In this section we present our framework used to illustrate the effects of an increase of transfer payments on budget deficits and in particular the conditions when such a stimulus pays for itself in a form of increased tax revenue. The first setting we consider is a competitive business cycle model with transfers and taxes in which households face the cash-in-advance constraint. Next we augment this model by adding downward nominal wage rigidity.

1.2.1 Benchmark model - competitive business cycle model with government

Our benchmark model is a standard competitive business cycle model with government which levies taxes and pays transfers to households. In the model prices and nominal wages are fully flexible. Time is discrete.

Households

The model features a large number of identical households. They choose the sequences of consumption $\{c_t\}_{t=0}^{\infty}$, labor supply $\{n_t\}_{t=0}^{\infty}$, bond holdings $\{b_{t+1}\}_{t=0}^{\infty}$ and money holdings $\{m_{t+1}\}_{t=0}^{\infty}$ to maximize the discounted lifetime utility

$$\max_{\{c_t, n_t, b_{t+1}, m_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(n_t))$$

where $\beta \in (0, 1)$ is the subjective discount factor $u(c)$ is twice continuously differentiable increasing and concave function and $v(n)$ is twice continuously differentiable increasing and convex function, subject to the sequence of the budget constraints

$$(1 + \tau^c)c_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = (1 - \tau^n)\frac{W_t}{p_t}n_t + b_t + \frac{m_t}{p_t} + \frac{TR_t}{p_t} + \pi_t \quad (1.1)$$

where τ^c is the tax rate on consumption, τ^n is the tax rate on labor income, TR_t are transfers paid to the household by government, R_t is the gross rate of return on one-period bonds held from t to $t+1$, p_t is the price level and π_t are firms' profits, and subject to the sequence of the cash-in-advance (CIA) constraints

$$(1 + \tau^c)\gamma_t c_t \leq \frac{m_t}{p_t} + \frac{TR_t}{p_t} \quad (1.2)$$

where γ_t reflects the part of gross consumption that needs to be paid in cash. In a heterogeneous agent setting this would correspond to the share of households whose consumption is cash-in-advance constrained. We allow γ_t to be time-varying - fluctuations in the share of the cash-in-advance constrained consumption would reflect the aggregate demand shocks caused by the liquidity shortages in the financial sector when less goods are available on credit. We also impose the following transversality condition

$$\lim_{t \rightarrow \infty} \left(\prod_{i=0}^{T-1} R_i^{-1} \right) \frac{b_{T+1}}{R_T} = 0$$

preventing from running the Ponzi schemes. Since transfers enter both budget constraint and can be used in making consumption purchases, an increase in transfer payments has positive wealth effects and relaxes the cash-in-advance constraint. In this model money demand is determined by the cash-in-advance constraint capturing explicitly the role of money as a medium of exchange in transactions. As transfers can also be used for consumption purchases they *reduce* demand for real money balances.

By the properties of the utility function budget constraint holds with equality. Since money does not pay interest in equilibrium, the return on bonds is higher than that on money, and hence we

restrict our attention to the equilibria in which the cash-in-advance constraint holds with equality⁸. Let λ_t denote the multiplier on the budget constraint and ν_t denote the multiplier on the CIA constraint. The Lagrangian of the household's problem is given by

$$L = \sum_{t=0}^{\infty} \left[\beta^t (u(c_t) - v(n_t)) + \lambda_t \left[(1 - \tau^n) \frac{W_t}{p_t} n_t + b_t + \frac{m_t}{p_t} + \pi_t + \frac{TR_t}{p_t} - (1 + \tau^c) c_t - \frac{b_{t+1}}{R_t} - \frac{m_{t+1}}{p_t} \right] + \nu_t \left(\frac{m_t}{p_t} + \frac{TR_t}{p_t} - (1 + \tau^c) \gamma_t c_t \right) \right]$$

First order conditions are given by

$$\beta^t u_c(c_t) = \lambda_t (1 + \tau^c) + \nu_t \gamma_t (1 + \tau^c) \quad (1.3)$$

$$\beta^t v_n(n_t) = \lambda_t (1 - \tau^n) \frac{W_t}{p_t} \quad (1.4)$$

$$\lambda_t = R_t \lambda_{t+1} \quad (1.5)$$

$$\frac{\lambda_t}{p_t} = \frac{\lambda_{t+1} + \nu_{t+1}}{p_{t+1}} \quad (1.6)$$

Combining (1.3) and (1.5) yields

$$\frac{u_c(c_t)}{1 + \tau^c} - \frac{\nu_t}{\beta^t} \gamma_t = \beta R_t \left(\frac{u_c(c_{t+1})}{1 + \tau^c} - \frac{\nu_{t+1}}{\beta^{t+1}} \gamma_{t+1} \right) \quad (1.7)$$

which is the Euler equation for bonds equalizing the marginal utility cost of forgoing one unit of consumption today with the discounted marginal utility gain in a form of higher consumption next period, adjusted for the cash-in-advance constraint holding with equality.

Combining (1.3) and (1.4) yields

$$v_n(n_t) = \frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{p_t} \left(u_c(c_t) - \frac{\nu_t}{\beta^t} \gamma_t (1 + \tau^c) \right) \quad (1.8)$$

which is the labor supply equation equalizing the marginal utility cost of higher labor supply with the marginal utility gain of higher after-tax wage income expressed in the utility units.

Finally combining (1.3) and (1.6) yields

$$\frac{\frac{u_c(c_t)}{1 + \tau^c} - \frac{\nu_t}{\beta^t} \gamma_t}{p_t} = \beta \frac{\frac{u_c(c_{t+1})}{1 + \tau^c} + \frac{\nu_{t+1}}{\beta^{t+1}} (1 - \gamma_{t+1})}{p_{t+1}} \quad (1.9)$$

which is the Euler equation for money and equalizes the marginal utility cost of holding an additional amount of money today with the marginal utility gain of higher money holdings next period.

⁸When the returns on bonds and money are equalized, the cash-in-advance constraint does not bind and this friction is disabled. See the Ramsey problem in section 1.3.5. below.

Firms

Firms are perfectly competitive and produce output using labor with production function

$$y_t = n_t^\alpha \quad (1.10)$$

with $\alpha \in (0, 1)$. Firm's optimization problem is given by

$$\max_{n_t} \{p_t n_t^\alpha - W_t n_t\}$$

First order condition is given by

$$\frac{W_t}{p_t} = \alpha n_t^{\alpha-1} \quad (1.11)$$

and implies that real wage is equal to the marginal product of labor. Profits in real terms are given by

$$\pi_t = \frac{p_t n_t^\alpha - W_t n_t}{p_t} = n_t^\alpha - \alpha n_t^{\alpha-1} n_t = (1 - \alpha) n_t^\alpha$$

Government

Government set the tax rates on consumption τ^c and labor income τ^n , collects the tax revenue, issues bonds b_{t+1} and money supply m_{t+1} and pays transfers to households TR_t . We assume that government consumption is zero. Government budget constraint is given by

$$\tau^c c_t + \tau^n \frac{W_t}{p_t} n_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = b_t + \frac{m_t}{p_t} + \frac{TR_t}{p_t}$$

We assume that tax rates are held constant. Unless employment and consumption change, any increase in transfers needs to be financed either by additional debt or seigniorage revenue. If it is financed by debt, under no change in tax revenue, seigniorage is assumed to eventually adjust so that the no Ponzi scheme condition is not violated⁹. Money supply is given by

$$m_t = M_t \quad (1.12)$$

where M_t is endogenously set by the government so that the time-zero government budget constraint

$$\sum_{t=0}^{\infty} q_t^0 \left(\tau^c c_t + \tau^n \frac{W_t}{p_t} n_t + \frac{m_{t+1}}{p_t} \right) = \sum_{t=0}^{\infty} q_t^0 \left(\frac{m_t}{p_t} + \frac{TR_t}{p_t} \right) + b_0$$

with $q_t^0 = \prod_{i=0}^{t-1} R_i^{-1}$ is satisfied. Hence, the model features fiscal dominance.

⁹Of course seigniorage is also a form of tax revenue where the tax base of real money balances is taxed with the inflation tax.

Feasibility and equilibrium

Feasibility constraint is given by

$$c_t = y_t \tag{1.13}$$

A competitive equilibrium is defined as follows.

Definition 1.2.1 (Competitive equilibrium). A perfect-foresight competitive equilibrium is an allocation $\{y_t, c_t, n_t, m_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, a price system $\{W_t, p_t, R_t\}_{t=0}^{\infty}$ and a sequence of multipliers $\{\nu_t\}_{t=0}^{\infty}$ satisfying equations (1.1 – 1.2, 1.7 – 1.13), that is households’ and firms’ optimality conditions are satisfied and markets clear, given the initial m_0, b_0 , the sequence of policy variables $\{TR_t, M_{t+1}\}_{t=0}^{\infty}$, the path for $\{\gamma_t\}_{t=0}^{\infty}$ and the tax rates τ^c, τ^n .

Since there is no labor market rigidity, this equilibrium is characterized by full employment: labor market is in equilibrium, labor demand and supply simultaneously determine employment and the real wage, and there is no unemployment. Employment determines output given production function and by goods market clearing consumption is equal to production. Changes in money supply, transfers and γ_t only affect prices via the cash-in-advance constraint, while nominal wages adjust immediately and the real economy remains unaffected.

1.2.2 Downward rigid nominal wages

In this subsection we introduce downward nominal wage rigidity into the model. Nominal wages adjustment is assumed to be constrained from below

$$W_t \geq \chi W_{t-1} \tag{1.14}$$

where parameter χ governs the degree of rigidity and therefore the speed of adjustment in the labor market¹⁰. In this case under the prevailing nominal wage it can happen that due to fluctuations in the share of the cash-in-advance constrained households γ_t acting as the aggregate demand shock, the (notional) labor supply denoted now by n_t^s and the actual labor demand n_t will not be equal to each other. With downward nominal wage rigidity there are two possible states of the world - households may be constrained or unconstrained in their labor supply provision. Equation (1.14) together with the (notional) labor supply and labor demand determines in which regime we are in. Firms’ problem is the same in the two cases and is analogous to the one described in 1.2.1. In case of households the decision problem can be thought of as taking place in two steps. First, households

¹⁰For the empirical evidence on nominal wage rigidity see for example Gottschalk (2005), Daly, Hobijn, and Lucking (2012), Barattieri, Basu, and Gottschalk (2014) and Schmitt-Grohé and Uribe (2016). The formulation (1.14) follows Schmitt-Grohé and Uribe (2016) who estimated χ for the periphery of the euro zone under the assumption that with rising unemployment (1.14) must be binding. They divided the nominal hourly labor cost growth by the productivity growth and the foreign (German) inflation to arrive at $\chi \in [0.990, 1.022]$.

solve the problem analogous to the one described in 1.2.1. and when choosing the sequence of control variables take into account the possibility of being constrained given the path for $\{\gamma_t\}_{t=0}^{\infty}$. This determines the optimal consumption, bond holdings and money holdings and the (notional) labor supply n_t^s determined by

$$\frac{v_n(n_t^s)}{u_c(c_t) - \frac{v_t}{\beta^t} \gamma_t (1 + \tau^c)} = \frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{p_t}$$

which is analogous to equation (1.8), but the amount of labor supplied it determines eventually may not be equal to labor demand. Labor demand resulting from firms' problem is still determined by

$$\frac{W_t}{p_t} = \alpha (n_t^d)^{\alpha-1}$$

which is equation (1.11). Given the notional labor supply, labor demand, price level and other variables, equations (1.8) and (1.11) determine the would-be market-clearing nominal wage. If such wage at t is below χW_{t-1} households are constrained and their actual labor supply is equal to labor demand, since we assume voluntary exchange. The effective labor supply and the actual employment are always equal to the labor demand, the short side of the market. Formally $n_t = \min\{n_t^d, n_t^s\} = n_t^d$.

This system requires the complementary slackness condition

$$(n_t^s - n_t)(W_t - \chi W_{t-1}) = 0 \tag{1.15}$$

which means that when the constraint (1.14) binds there is unemployment in the model and equations (1.8), (1.11), (1.15) determine the (notional) labor supply, labor demand and the level of nominal wages, while the effective labor supply and employment allocation is equal to labor demand. In such a case in the second step of solving their problem households take the labor income as given and maximize the present discounted utility with respect to c_t, b_{t+1} and m_{t+1} adjusting the optimal choice sequence accordingly, with the binding constraint feeding back to the prior decisions. Optimal choices are still described by (1.3), (1.5) and (1.6) or after combining them the Euler equation for bonds (1.7) and the Euler equation for money (1.9). If the constraint (1.14) does not bind, the labor market is in equilibrium, labor supply equals labor demand, while employment and nominal wages are determined by (1.8) and (1.11) simultaneously.

Unemployment u_t is defined as the difference between the notional labor supply and labor demand (the effective labor supply)¹¹

¹¹Alternative approach followed by Galí (2011) is to introduce imperfect competition on the labor market. Households (or labor unions) take labor demand as given and set wages in a staggered way, which results in nominal wage New Keynesian Phillips curve. Galí (2011) defines unemployment rate as the log difference between the labor force and actual employment, where labor force is actually analogous to our notional labor supply and is derived from the aggregated participation equation. The latter is based on the decision of the marginal supplier of labor who is willing to work if and only if the real wage for his labor type exceeds his disutility of labor.

$$u_t = n_t^s - n_t \quad (1.16)$$

Equilibrium with rationing

In the model with downward nominal wage rigidity the rationing on the labor market could occur, as we assume voluntary exchange and hence the short side of the market determines the allocation. The competitive equilibrium is defined as follows.

Definition 1.2.2 (Competitive equilibrium with rationing). A perfect-foresight competitive equilibrium with downward nominal wage rigidity, i.e., with the possibility of rationing is an allocation $\{y_t, c_t, n_t, m_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, a notional labor supply $\{n_t^s\}_{t=0}^{\infty}$, unemployment $\{u_t\}_{t=0}^{\infty}$, a price system $\{W_t, p_t, R_t, \}_{t=0}^{\infty}$ and a sequence of multipliers $\{\nu_t\}_{t=0}^{\infty}$ satisfying equations (1.1 – 1.2, 1.7 – 1.13, 1.15 – 1.16), that is households' and firms' optimality conditions are satisfied and goods, money and bond markets clear, while employment in the labor market is determined by firms (the short side of the market), given the initial m_0, b_0 , the sequence of policy variables $\{TR_t, M_{t+1}\}_{t=0}^{\infty}$, the path for $\{\gamma_t\}_{t=0}^{\infty}$ and the tax rates τ^c, τ^n .

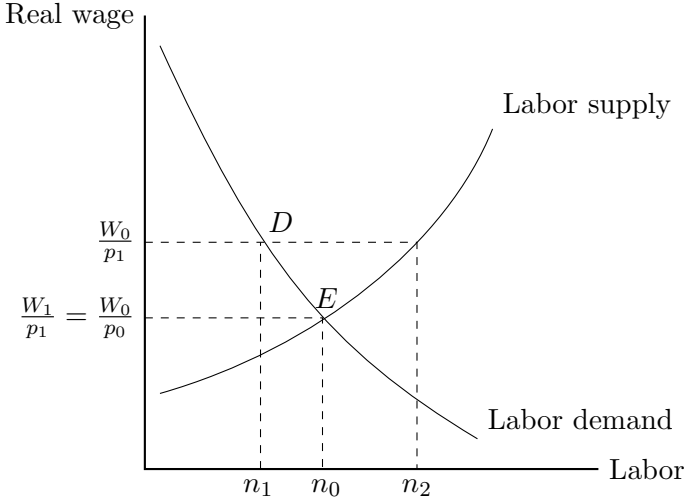
When downward nominal rigidity constraint binds, the notional labor supply is higher than labor demand and unemployment emerges. The speed of adjustment in the labor market depends on the degree of nominal wage rigidity. In this circumstances changes in transfers, money and γ_t may affect the real allocation. Contrary to the benchmark model, the equilibrium with downward nominal wage rigidity is not stationary in a sense that when rationing occurs nominal wages gradually adjust until the economy attains full employment. Hence, in the model with the nominal wage rigidity there are two possible types of equilibria - the usual stationary one with full employment and another one with rationing which gradually converges to the first one.

The dynamics of the labor market with rationing

Figure 1.1 illustrates the labor market of the model. Initially the labor market is in equilibrium at point E with the real wage equal to $\frac{W_0}{p_0}$ and employment equal to n_0 . When a negative aggregate demand shock (for instance an increase in a number of the cash-in-advance constrained households γ_t) hits, prices fall to p_1 , which increases the real wage to $\frac{W_0}{p_1}$. When nominal wages are flexible they fall to W_1 , thus restoring full employment and the previous level of the real wage. However, when they are downward rigid, they are stuck at W_0 and adjust only gradually with the speed of adjustment determined by χ . Then, involuntary unemployment emerges since the real wage $\frac{W_0}{p_1}$ is above the market clearing level - the notional labor supply (n_2) is higher than labor demand (n_1). As a result, the actual employment falls to n_1 which is determined by the short side of the market (point D) and only gradually converges to n_0 . Output is depressed as long as employment is below

n_0 . When the aggregate demand is depressed with prices at p_1 , there are two ways to restore the equilibrium in the labor market. One way is to wait for nominal wages to adjust - since they fall over time, real wages are also reduced and labor demand increases, while labor supply shrinks, so that the unemployment gap is closed. However, as long as the downward wage rigidity constraint is binding, the economy is facing depressed production, employment and consumption. Another solution is to exert an upward pressure on prices, so that they increase back to p_0 thus restoring the equilibrium in the labor market. This role is played by an increase in transfers in our setting, which results in a much faster recovery.

Figure 1.1: Labor market



1.3 Policy implications

In this section we derive the transfer multiplier for the model with downward nominal wage rigidity, discuss the Laffer curve implications of our setting and analyze how large the multipliers need to be for transfers to be self-financing. We also characterize the transfers level ensuring full employment in the labor market, discuss the optimal transfers policy under Ramsey optimal allocation and examine alternative fiscal policies which could achieve similar effects.

1.3.1 Multiplier

As long as the real wage is above market-clearing level, output is depressed and the economy moves along the labor demand curve (labor supply schedule is irrelevant as it determines only the notional labor supply). In such a case of binding downward nominal wage rigidity constraint with the binding cash-in-advance constraint we can derive the transfer multiplier in the following way. Since output is a function of employment and employment is a function of the real wage by (1.10)

and (1.11) we have

$$y_t = n_t^\alpha = \left(\frac{W_t}{\alpha p_t} \right)^{\frac{\alpha}{\alpha-1}}$$

By binding cash-in-advance constraint (1.2) and feasibility constraint (1.13) this becomes

$$y_t = \left(\frac{W_t}{\alpha \frac{m_t + TR_t}{(1+\tau^c)\gamma_t c_t}} \right)^{\frac{\alpha}{\alpha-1}} = \left(\frac{(1+\tau^c)\gamma_t y_t W_t}{\alpha(m_t + TR_t)} \right)^{\frac{\alpha}{\alpha-1}}$$

Now we can solve it for y_t which appears on both sides of the equation

$$\begin{aligned} y_t &= y_t^{\frac{\alpha}{\alpha-1}} \left(\frac{(1+\tau^c)\gamma_t W_t}{\alpha(m_t + TR_t)} \right)^{\frac{\alpha}{\alpha-1}} \iff y_t^{\frac{1}{1-\alpha}} = \left(\frac{(1+\tau^c)\gamma_t W_t}{\alpha(m_t + TR_t)} \right)^{\frac{\alpha}{\alpha-1}} \\ y_t &= \left[\left(\frac{(1+\tau^c)\gamma_t W_t}{\alpha(m_t + TR_t)} \right)^{\frac{\alpha}{\alpha-1}} \right]^{1-\alpha} = \left(\frac{(1+\tau^c)\gamma_t W_t}{\alpha(m_t + TR_t)} \right)^{-\alpha} = \left(\frac{(1+\tau^c)\gamma_t \chi W_{t-1}}{\alpha(m_t + TR_t)} \right)^{-\alpha} \end{aligned} \quad (1.17)$$

where the last equality follows from the binding downward wage rigidity constraint. Now we have y_t expressed as an increasing function of policy variables: transfer payments TR_t and current money supply m_t , and a decreasing function of the nominal wage $W_t = \chi W_{t-1}$. Higher degree of the wage rigidity χ reduces output, since it slows down the adjustment in the labor market. As the nominal wage is fixed at χW_{t-1} we take the derivative of (1.17) with respect to TR_t to obtain

$$\frac{dy_t}{dTR_t} = -\alpha \left(\frac{(1+\tau^c)\gamma_t \chi W_{t-1}}{\alpha(m_t + TR_t)} \right)^{-\alpha-1} (-1) \frac{(1+\tau^c)\gamma_t \chi W_{t-1}}{\alpha^2(m_t + TR_t)^2}$$

which simplifies to

$$\frac{dy_t}{dTR_t} = \frac{((1+\tau^c)\gamma_t \chi W_{t-1})^{-\alpha}}{\alpha^{-\alpha}(m_t + TR_t)^{1-\alpha}} = \frac{\alpha^\alpha}{((1+\tau^c)\gamma_t \chi W_{t-1})^\alpha (m_t + TR_t)^{1-\alpha}} \quad (1.18)$$

which is our impact transfer multiplier. It shows how output would change if at a given point in time the sequence for TR_t changed given the sequence for γ_t and M_t and the nominal wage $W_t = \chi W_{t-1}$. The value of multiplier is decreasing in m_t and TR_t . This is because if these variables are already high, prices are likely to be high and hence real wage to be low which makes the case of being far away from full employment less likely. High degree of the wage rigidity χ slows down the adjustment and therefore reduces the multiplier.

We define the total (long-run) multiplier as the total change in output since an increase in transfers at time $t = 0$ until infinity divided by the total change in transfers $\frac{\sum_{t=0}^{\infty} dy_t}{\sum_{t=0}^{\infty} dTR_t}$. While it is not possible to provide a closed-form formula of this multiplier, we calculate it in the numerical exercise in the subsection 1.4.7.

1.3.2 Laffer curve

In our setting one can also examine the Laffer curve implied by the fiscal stimulus in a form of higher transfers. We repeat the government budget constraint here for convenience:

$$\tau^c c_t + \tau^n \frac{W_t}{p_t} n_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = b_t + \frac{m_t}{p_t} + \frac{TR_t}{p_t}$$

Equivalently it can be written as

$$\tau^c c_t + \tau^n \frac{W_t}{p_t} n_t + \frac{m_{t+1}}{p_t} - \frac{m_t}{p_t} = \frac{TR_t}{p_t} + b_t - \frac{b_{t+1}}{R_t}$$

Now if we focus on self-financing case, transfers should increase output (income) and employment sufficiently so that tax revenues increase to be exactly equal to an increase in transfers. This implies that neither debt nor money supply need to be adjusted to balance the budget. Under no change in money supply and constant debt we have

$$\tau^c c_t + \tau^n \frac{W_t}{p_t} n_t = \frac{TR_t}{p_t} + b_t - \frac{b_t}{R_t}$$

which becomes

$$\tau^c c_t + \tau^n \frac{W_t}{p_t} n_t = \frac{TR_t}{p_t} + b_t \frac{R_t - 1}{R_t} \tag{1.19}$$

The left-hand side of (1.19) is given by

$$LHS(TR_t) = \tau^c c_t + \tau^n \frac{W_t}{p_t} n_t$$

The right hand side of (1.19) is given by

$$RHS(TR_t) = \frac{TR_t}{p_t} + b_t \frac{R_t - 1}{R_t}$$

Clearly for constant debt level, the right-hand side is increasing in transfers. To analyze the left-hand side we need to distinguish two regimes. Regime one would be the normal state of the economy either when wages are flexible or when they are downward rigid, but the constraint (1.14) does not bind. Then there is no unemployment and changes in transfers do not affect neither employment nor consumption. In that case the left hand side does not depend on transfers and is flat. This is illustrated in the left panel of Figure 1.2.

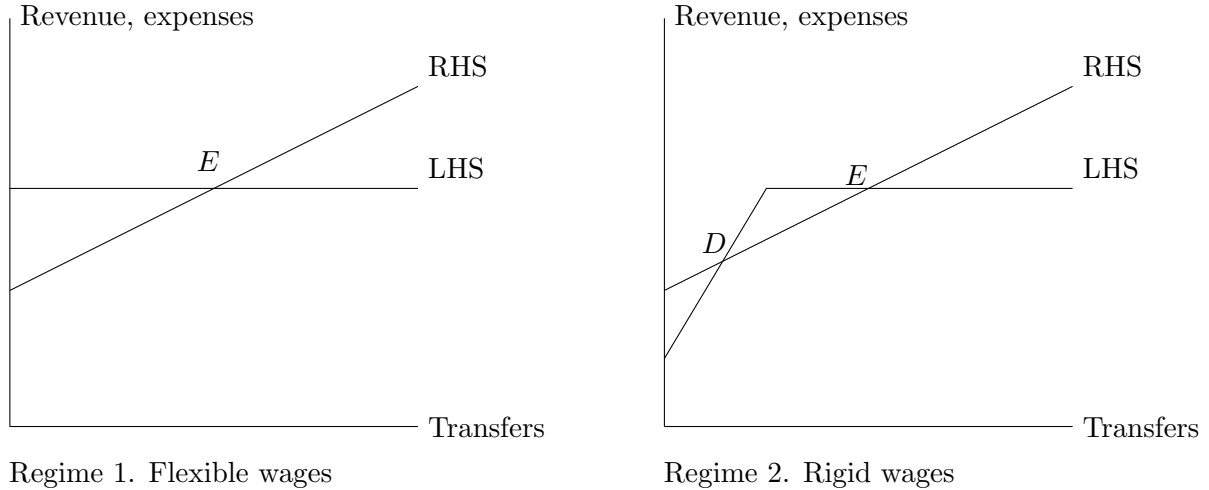


Figure 1.2: Balanced budget

Regime two would correspond to the situation when the constraint (1.14) binds. Then higher transfers increase prices, decrease real wages, increase employment, output and consumption. Hence, until we reach full employment, the left-hand side is increasing in transfers and then is flat. In this case there are two points at which the budget is balanced - one with lower transfers below full employment (being on the "wrong" side of the Laffer curve) and another one where higher transfers correspond to higher tax revenue and full employment. There are two values of transfers which are compatible with the balanced budget. This is illustrated in the right panel of Figure 1.2. Of course the exact shape of the left-hand side in case of binding wage rigidity constraint does not need to be linear and will depend on the specification of functional forms and parameters of the model.

1.3.3 Some pleasant fiscal algebra

In the previous subsections we have derived the theoretical transfer multiplier implied by the model, its primitives and parameters. We have also discussed how an increase in transfers can be self-financing by reaching higher-tax-revenue full-employment equilibrium. Now we wish to discuss a different question, namely how large the transfers multiplier needs to be for transfers to be self-financing. Previously, we have derived how large the multiplier is in the model depending on parameters and the realization of policy variables. Now we derive its required empirical counterpart - how much output needs to expand after an increase in transfers given the existing tax rates which as before are constant and treated as parameters. The question we are interested in is what multiplier the model has to deliver given the tax rates which the policy maker is facing.

An increase in transfers will be self-financing when the change in transfers is equal to the change in the tax revenue

$$dTransfers = dTaxRevenue$$

Substituting in for the tax revenue we get

$$dTR = \tau^c dc + \tau^n d\left(\frac{W}{p}n\right)$$

Now we can divide by the change in transfers

$$1 = \tau^c \frac{dc}{dTR} + \tau^n \frac{d\left(\frac{W}{p}n\right)}{dTR}$$

Since feasibility constraint implies $c_t = y_t$ we substitute output for consumption. We also multiply and divide the second component of tax revenue by dy

$$1 = \tau^c \frac{dy}{dTR} + \tau^n \frac{d\left(\frac{W}{p}n\right)}{dy} \frac{dy}{dTR}$$

Since $\frac{W}{p}n = \alpha \implies \frac{d\left(\frac{W}{p}n\right)y}{y^2} - \frac{W}{p}n \frac{dy}{y^2} = 0 \implies d\left(\frac{W}{p}n\right) = \alpha dy$ we obtain

$$1 = \tau^c \frac{dy}{dTR} + \alpha \tau^n \frac{dy}{dTR}$$

Now we factor out $\frac{dy}{dTR}$

$$\frac{dy}{dTR} (\tau^c + \alpha \tau^n) = 1$$

And solve for the multiplier

$$\frac{dy}{dTR} = \frac{1}{\tau^c + \alpha \tau^n}$$

To get the flavour of the size of this multiplier we plug in the tax rates and the labor share. We use the average tax rates on consumption and labor income in the European Union countries in 2016 (see the next section) and assume that $\alpha = 0.64$. Hence we obtain

$$\frac{dy}{dTR} = \frac{1}{0.225 + 0.64 \times 0.343} = \frac{1}{0.225 + 0.22} = 2.25$$

This means that if given the parameters the model-implied multiplier derived in the subsection 1.3.1 (see equation (1.18)) reaches the value of 2.25, an increase in transfers will be self-financing given the existing tax rates. In the appendix 1.6.1 we show that in a model with capital the multiplier necessary for the self-financing result is more or less the same (2.22).

1.3.4 Transfers ensuring full employment

The goal of this chapter is to show when an increase in transfers can be self-financing. As we discussed above, this happens when the economy is sufficiently far away from the full employment in which case higher transfers restore the equilibrium in the labor market and generate additional

tax revenue. Still, one may wonder what would be the level of transfer payments in order to close the gap between labor supply and demand, i.e., unemployment. Equation (1.8) implies that labor supply is given by

$$n_t^s = v_n^{-1} \left[\frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{p_t} \left(u_c(c_t) - \frac{\nu_t}{\beta^t} \gamma_t (1 + \tau^c) \right) \right]$$

while equation (1.11) implies that labor demand is given by

$$n_t = \left(\frac{W_t}{p_t \alpha} \right)^{\frac{1}{\alpha-1}}$$

Equalizing the two yields

$$v_n^{-1} \left[\frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{p_t} \left(u_c(c_t) - \frac{\nu_t}{\beta^t} \gamma_t (1 + \tau^c) \right) \right] = \left(\frac{W_t}{p_t \alpha} \right)^{\frac{1}{\alpha-1}}$$

By binding cash-in-advance constraint (1.2)

$$p_t = \frac{m_t + TR_t}{(1 + \tau^c) \gamma_t c_t}$$

this becomes

$$v_n^{-1} \left[\frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{\frac{m_t + TR_t}{(1 + \tau^c) \gamma_t c_t}} \left(u_c(c_t) - \frac{\nu_t}{\beta^t} \gamma_t (1 + \tau^c) \right) \right] = \left(\frac{W_t}{\frac{m_t + TR_t}{(1 + \tau^c) \gamma_t c_t} \alpha} \right)^{\frac{1}{\alpha-1}}$$

or equivalently

$$v_n^{-1} \left[\frac{(1 - \tau^n) W_t \gamma_t c_t}{m_t + TR_t} \left(u_c(c_t) - \frac{\nu_t}{\beta^t} \gamma_t (1 + \tau^c) \right) \right] = \left(\frac{W_t (1 + \tau^c) \gamma_t c_t}{(m_t + TR_t) \alpha} \right)^{\frac{1}{\alpha-1}}$$

This defines implicitly the level of transfers that is consistent with labor market equilibrium given the level of nominal wages. At the same time it also defines the level of money supply consistent with full employment. Then, the question is whether restoring labor market equilibrium can be achieved with either instrument. On one hand, the answer is affirmative, as both money and transfers enter the cash-in-advance constraint, hence affect the level of prices and provide a solution to downward nominal wage rigidity. On the other hand, transfers and money have different effects on the budget constraint - transfers appear only as a liability, while money provides seigniorage revenue. As we discuss below in the Ramsey problem, the optimal policy uses both transfers and money in order to stabilize the debt. When full employment is achieved, transfers do not have multiplier effects on output, do not generate extra tax revenue and therefore need to be financed with seigniorage.

1.3.5 Optimal Ramsey policy

In this subsection we analyze the optimal transfers policy in the Ramsey sense, i.e., maximizing household's welfare. We will show that while it is optimal to deflate the economy at the rate of time preference to undo the cash-in-advance constraint, in order to undo the downward nominal wage rigidity the economy needs to be inflated more to keep the real wage at the market clearing level. This can be achieved using the state-contingent labor and consumption taxes (as in Correia, Nicolini, and Teles, 2008), but here we show that, alternatively, lump-sum transfer payments and money supply can be used to attain the optimal allocation when taxes are constant. Both instruments need to be used in order to stabilize the government debt, since when full employment is attained, transfers do not generate extra tax revenue - in this case there is no self-financing and thus in addition to the constant tax revenue they need to be financed with the seigniorage revenue.

Our analysis is an unusual Ramsey problem. Often one aims to find the welfare-maximizing distortionary taxes to finance the exogenous stream of expenses when the lump-sum taxes are not available¹². We, however, treat the tax rates as parameters (we hold them fixed) and restrict the number of policy instruments to non-negative lump-sum transfers and money supply. These two instruments are used first to undo the cash-in-advance constraint and then to undo the nominal wage rigidity constraint. We do not have the exogenous stream of government spending to be financed - all distortionary tax revenue generated each period is spent on transfers and servicing of the government debt. The policy maker does not wish to change the tax rates and uses transfer payments (and money supply) to maximize welfare.

It is more convenient to set up the problem at time zero. Let $q_t^0 = \prod_{i=0}^{t-1} R_i^{-1}$ be the price of one period Arrow-Debreu securities where R_t is the gross rate of return on one-period bonds held from t to $t + 1$. We normalize $q_0^0 = 1$. Households time-zero budget constraint is given by

$$\sum_{t=0}^{\infty} q_t^0 \left((1 + \tau^c) c_t + \frac{m_{t+1}}{p_t} \right) = \sum_{t=0}^{\infty} q_t^0 \left((1 - \tau^n) \frac{W_t}{p_t} n_t + \frac{m_t}{p_t} + \frac{TR_t}{p_t} + \pi_t \right) + b_0 \quad (1.20)$$

Then the household's problem is given by

$$\max_{\{c_t, n_t, b_{t+1}, m_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(n_t))$$

subject to

$$\sum_{t=0}^{\infty} q_t^0 \left((1 + \tau^c) c_t + \frac{m_{t+1}}{p_t} \right) = \sum_{t=0}^{\infty} q_t^0 \left((1 - \tau^n) \frac{W_t}{p_t} n_t + \frac{m_t}{p_t} + \frac{TR_t}{p_t} + \pi_t \right) + b_0$$

¹²With the lump-sum taxes this problem is trivial.

and

$$(1 + \tau^c)\gamma_t c_t \leq \frac{m_t}{p_t} + \frac{TR_t}{p_t}$$

with the transversality condition

$$\lim_{t \rightarrow \infty} \left(\prod_{i=0}^{T-1} R_i^{-1} \right) \frac{b_{T+1}}{R_T} = 0$$

The Lagrangian is given by

$$\begin{aligned} L = & \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(n_t)) \\ & + \lambda \left[\sum_{t=0}^{\infty} q_t^0 \left((1 - \tau^n) \frac{W_t}{p_t} n_t + \frac{m_t}{p_t} + \frac{TR_t}{p_t} \right) + b_0 - \sum_{t=0}^{\infty} q_t^0 \left((1 + \tau^c) c_t + \frac{m_{t+1}}{p_t} + \pi_t \right) \right] \\ & + \sum_{t=0}^{\infty} \nu_t \left(\frac{m_t}{p_t} + \frac{TR_t}{p_t} - (1 + \tau^c) \gamma_t c_t \right) \end{aligned}$$

The first order conditions are given by

$$c_t : \beta^t u_c(c_t) = \lambda q_t^0 (1 + \tau^c) + \nu_t (1 + \tau^c) \gamma_t \quad (1.21)$$

$$n_t : \beta^t v_n(n_t) = \lambda q_t^0 (1 - \tau^n) \frac{W_t}{p_t} \quad (1.22)$$

$$m_{t+1} : \frac{\lambda q_t^0}{p_t} = \frac{\lambda q_{t+1}^0 + \nu_{t+1}}{p_{t+1}} \quad (1.23)$$

Firm's first order condition is still described by (1.11). It can be shown (see the appendix 1.6.2) that these first order conditions imply

$$q_t^0 = \frac{\beta^t v_n(n_t) n_0^{\alpha-1}}{v_n(n_0) n_t^{\alpha-1}}$$

$$\frac{p_{t+1}}{p_t} = \beta \frac{u_c(c_{t+1})}{u_c(c_t)}$$

These results together with the cash-in-advance constraint and the first order conditions allow us to express the present value budget constraint as (see the appendix 1.6.3 for the full derivation):

$$\frac{n_0^{\alpha-1}}{v_n(n_0)} \sum_{t=0}^{\infty} \beta^t \left[(1 - \gamma_t)(1 + \tau^c) c_t \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta(1 + \tau^c) \gamma_{t+1} c_{t+1} u_c(c_{t+1}) \frac{v_n(n_t)}{n_t^{\alpha-1} u_c(c_t)} + (\alpha \tau^n - 1) v_n(n_t) n_t - \frac{v_n(n_t) TR_{t+1}}{n_t^{\alpha-1} p_t} \right] = b_0$$

which is our implementability constraint. We can now define the Ramsey problem.

Definition 1.3.1 (Ramsey equilibrium). A full commitment Ramsey equilibrium is a competitive equilibrium such that $\{c_t, n_t\}_{t=0}^{\infty}$ solve

$$\max_{\{c_t, n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(n_t))$$

subject to

(i) the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \frac{n_0^{\alpha-1}}{v_n(n_0)} \left[(1 - \gamma_t)(1 + \tau^c) c_t \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta(1 + \tau^c) \gamma_{t+1} c_{t+1} u_c(c_{t+1}) \frac{v_n(n_t)}{n_t^{\alpha-1} u_c(c_t)} + (\alpha \tau^n - 1) v_n(n_t) n_t - \frac{v_n(n_t)}{n_t^{\alpha-1}} \frac{TR_{t+1}}{p_t} \right] = b_0$$

(ii) the feasibility constraint

$$c_t = n_t^{\alpha}$$

given the initial b_0 , the path for $\{\gamma_t\}_{t=0}^{\infty}$ and the tax rates τ^c, τ^n .¹³

Having derived the implementability constraint we can solve for the Ramsey plan by simply maximizing household's utility subject to the feasibility constraint and the implementability constraint. Government budget constraint as a mirror image of household constraint is redundant. The Lagrangian is given by

$$\begin{aligned} L &= \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(n_t)) \\ &+ \sum_{t=0}^{\infty} \beta^t \theta_t (n_t^{\alpha} - c_t) \\ &+ \Phi \left[b_0 - \frac{n_0^{\alpha-1}}{v_n(n_0)} \sum_{t=0}^{\infty} \beta^t \left[(1 - \gamma_t)(1 + \tau^c) c_t \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta(1 + \tau^c) \gamma_{t+1} c_{t+1} u_c(c_{t+1}) \frac{v_n(n_t)}{n_t^{\alpha-1} u_c(c_t)} + (\alpha \tau^n - 1) v_n(n_t) n_t - \frac{v_n(n_t)}{n_t^{\alpha-1}} \frac{TR_{t+1}}{p_t} \right] \right] \end{aligned}$$

or equivalently

$$\begin{aligned} L &= u(c_0) - v(n_0) + \beta \sum_{t=0}^{\infty} \beta^t (u(c_{t+1}) - v(n_{t+1})) \\ &+ \theta_0 (n_0^{\alpha} - c_0) + \beta \sum_{t=0}^{\infty} \beta^t \theta_{t+1} (n_{t+1}^{\alpha} - c_{t+1}) \\ &+ \Phi \left[b_0 - \frac{n_0^{\alpha-1}}{v_n(n_0)} \sum_{t=0}^{\infty} \beta^t \left[(1 - \gamma_t)(1 + \tau^c) c_t \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta(1 + \tau^c) \gamma_{t+1} c_{t+1} u_c(c_{t+1}) \frac{v_n(n_t)}{n_t^{\alpha-1} u_c(c_t)} + (\alpha \tau^n - 1) v_n(n_t) n_t - \frac{v_n(n_t)}{n_t^{\alpha-1}} \frac{TR_{t+1}}{p_t} \right] \right] \end{aligned}$$

The first order conditions for $t = 0$ are given by:

$$c_0 : u_c(c_0) = \theta_0 + \Phi(1 - \gamma_0)(1 - \tau^c) - \Phi\beta(1 + \tau^c)\gamma_1 c_1 u_c(c_1) \frac{1}{u_c(c_0)^2} u_{cc}(c_0) \quad (1.24)$$

¹³This is a perfect-foresight equilibrium. The introduction of uncertainty would result in the future realizations being weighted with the respective probabilities in the Ramsey problem.

$$n_0 : v_n(n_0) = \theta_0 \alpha n_0^{\alpha-1} \quad (1.25)$$

$$-\Phi \left[\frac{(\alpha-1)n_0^{\alpha-2}v_n(n_0) - n_0^{\alpha-1}v_{nn}(n_0)}{v_n(n_0)^2} \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma_t)(1+\tau^c)c_t \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta(1+\tau^c)\gamma_{t+1}c_{t+1}u_c(c_{t+1}) \frac{v_n(n_t)}{n_t^{\alpha-1}u_c(c_t)} + (\alpha\tau^n - 1)v_n(n_t)n_t - \frac{v_n(n_t)}{n_t^{\alpha-1}} \frac{TR_{t+1}}{p_t} \right] \right]$$

$$-\Phi \frac{n_0^{\alpha-1}}{v_n(n_0)} \left[-\frac{v_{nn}(n_0)n_0^{\alpha-1} - v_n(n_0)(\alpha-1)n_0^{\alpha-2}}{n_0^{2(\alpha-1)}} \left[(1-\gamma_0)(1+\tau^c)c_0 + \beta(1+\tau^c)\gamma_1c_1 \frac{u_c(c_1)}{u_c(c_0)} - \frac{TR_1}{p_0} \right] + (\alpha\tau^n - 1)[v_{nn}(n_0)n_0 + v_n(n_0)] \right]$$

The first order conditions for $t \geq 1$ are given by

$$c_{t \geq 1} : \beta^t u_c(c_t) = \beta^t \theta_t + \Phi \frac{n_0^{\alpha-1}}{v_n(n_0)} \left[\beta^t (1-\gamma_t)(1+\tau^c) \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta^t (1+\tau^c) \gamma_t (u_c(c_t) + c_t u_{cc}(c_t)) \frac{v_n(n_{t-1})}{n_{t-1}^{\alpha-1} u_c(c_{t-1})} \right] \quad (1.26)$$

$$n_{t \geq 1} : \beta^t v_n(n_t) = \theta_t \beta^t \alpha n_t^{\alpha-1} \quad (1.27)$$

$$-\Phi \frac{n_0^{\alpha-1}}{v_n(n_0)} \beta^t \left[-\frac{v_{nn}(n_t)n_t^{\alpha-1} - v_n(n_t)(\alpha-1)n_t^{\alpha-2}}{(n_t^{\alpha-1})^2} \left[(1-\gamma_t)(1+\tau^c)c_t + \beta(1+\tau^c)\gamma_{t+1}c_{t+1} \frac{u_c(c_{t+1})}{u_c(c_t)} - \frac{TR_{t+1}}{p_t} \right] \right]$$

$$-\Phi \frac{n_0^{\alpha-1}}{v_n(n_0)} \beta^t (\alpha\tau^n - 1)[v_{nn}(n_t)n_t + v_n(n_t)]$$

These four first order conditions together with the feasibility and implementability constraints characterize the optimal Ramsey allocation given the path of γ_t and the constant tax rates τ^c, τ^n . Using this optimal allocation one can recover prices. As there is no friction in the labor market, this is a full employment allocation. Since the only friction in this setting is the cash-in-advance constraint, the optimality requires deflating the economy at the rate of time preference to undo this friction. To see this divide (1.22) by (1.21) to obtain

$$\frac{v_n(n_t)}{u_c(c_t)} = \frac{\lambda q_t^0 (1 - \tau^n) \frac{W_t}{p_t}}{\lambda q_t^0 (1 + \tau^c) + \nu_t (1 + \tau^c) \gamma_t}$$

When the economy is deflated at the rate of time preference, the return on money is equal to that of bonds and therefore the cash-in-advance constraint does not bind. In this case the multiplier ν_t is zero and the efficiency is restored - the marginal rate of substitution between leisure and consumption is equal to the (after tax) marginal rate of transformation which by firm's first order condition is equal to the real wage.

When it comes to recovering policy variables, since both money supply and transfers enter the cash-in-advance constraint, they could be seen as perfect substitutes in delivering the optimal allocation. However, as they have different effects on the government budget constraint - transfers being a liability and money providing the seigniorage revenue, the Ramsey planner uses the combination of the two which ensures that debt is stabilized. From the time $t = 1$ on, once the stationary allocation (denoted with variables without the time subscript) is attained, in each period the choice

of transfers and money supply delivering the optimal allocation and the appropriate deflation must be such that

$$\frac{TR_t}{p_t} = \tau^c c + \tau^n \frac{W_t}{p_t} n + \frac{1-R}{R} b + \frac{M_{t+1}}{p_t} - \frac{M_t}{p_t}$$

so that the budget is balanced. Under full employment transfers do not have multiplier effects on output, do not generate additional tax revenue and therefore in addition to the constant tax revenue need to be financed with the seigniorage revenue.

Downward rigid nominal wages

Now we proceed to show that when nominal wages are downward rigid the Ramsey planner finds it optimal to inflate the economy more than undoing the cash-in-advance constraint calls for, in order to avoid hitting the nominal wage rigidity constraint and to keep the real wage at the market clearing level. As before, she uses both transfers and money supply to achieve this goal so that the government debt is stabilized.

When nominal wages are downward rigid as in the subsection 1.2.2 the Ramsey planner is facing an additional constraint

$$W_{t+1} \geq \chi W_t$$

for $t \geq 0$. Equivalently

$$\frac{W_{t+1} p_{t+1}}{p_{t+1} p_t} \geq \chi \frac{W_t}{p_t}$$

Plug in for $\frac{p_{t+1}}{p_t}$ and use firm's first order condition to obtain

$$\alpha n_{t+1}^{\alpha-1} \beta \frac{u_c(c_{t+1})}{u_c(c_t)} \geq \chi \alpha n_t^{\alpha-1}$$

Rearrange to obtain

$$\beta n_{t+1}^{\alpha-1} u_c(c_{t+1}) \geq \chi n_t^{\alpha-1} u_c(c_t)$$

Denote the multiplier of this constraint as ξ_t . Now the Ramsey problem is defined as follows.

Definition 1.3.2 (Ramsey equilibrium with downward nominal wage rigidity). A full commitment Ramsey equilibrium with downward nominal wage rigidity is a competitive equilibrium such that $\{c_t, n_t\}_{t=0}^{\infty}$ solve

$$\max_{\{c_t, n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (u(c_t) - v(n_t))$$

subject to

(i) the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \frac{n_0^{\alpha-1}}{v_n(n_0)} \left[(1-\gamma_t)(1+\tau^c) c_t \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta(1+\tau^c) \gamma_{t+1} c_{t+1} u_c(c_{t+1}) \frac{v_n(n_t)}{n_t^{\alpha-1} u_c(c_t)} + (\alpha \tau^n - 1) v_n(n_t) n_t - \frac{v_n(n_t)}{n_t^{\alpha-1}} \frac{TR_{t+1}}{p_t} \right] = b_0$$

(ii) the feasibility constraint

$$c_t = n_t^\alpha$$

(iii) the downward nominal wage rigidity

$$\chi n_t^{\alpha-1} u_c(c_t) \leq \beta n_{t+1}^{\alpha-1} u_c(c_{t+1})$$

given the initial b_0 , the path for $\{\gamma_t\}_{t=0}^\infty$ and the tax rates τ^c, τ^n .

The optimal allocation is described by the following first order conditions. The first order conditions for $t = 0$ are given by:

$$c_0 : u_c(c_0) = \theta_0 + \Phi(1 - \gamma_0)(1 - \tau^c) - \Phi\beta(1 + \tau^c)\gamma_1 c_1 u_c(c_1) \frac{1}{u_c(c_0)^2} u_{cc}(c_0) + \xi_0 \chi n_0^{\alpha-1} u_{cc}(c_0) \quad (1.28)$$

$$n_0 : v_n(n_0) = \theta_0 \alpha n_0^{\alpha-1}$$

$$\begin{aligned} & -\Phi \left[\frac{(\alpha - 1)n_0^{\alpha-2} v_n(n_0) - n_0^{\alpha-1} v_{nn}(n_0)}{v_n(n_0)^2} \sum_{t=0}^{\infty} \beta^t \left[(1 - \gamma_t)(1 + \tau^c) c_t \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta(1 + \tau^c) \gamma_{t+1} c_{t+1} u_c(c_{t+1}) \frac{v_n(n_t)}{n_t^{\alpha-1} u_c(c_t)} + (\alpha \tau^n - 1) v_n(n_t) n_t - \frac{v_n(n_t)}{n_t^{\alpha-1}} \frac{TR_{t+1}}{p_t} \right] \right] \\ & -\Phi \frac{n_0^{\alpha-1}}{v_n(n_0)} \left[-\frac{v_{nn}(n_0) n_0^{\alpha-1} - v_n(n_0)(\alpha - 1) n_0^{\alpha-2}}{n_0^{2(\alpha-1)}} \left[(1 + \gamma_0)(1 + \tau^c) c_0 + \beta(1 + \tau^c) \gamma_1 c_1 \frac{u_c(c_1)}{u_c(c_0)} - \frac{TR_1}{p_0} \right] + (\alpha \tau^n - 1) [v_{nn}(n_0) n_0 + v_n(n_0)] \right] \\ & + \xi_0 \chi (\alpha - 1) n_0^{\alpha-2} u_c(c_0) \end{aligned} \quad (1.29)$$

The first order conditions for $t \geq 1$ are given by

$$\begin{aligned} c_{t \geq 1} : \beta^t u_c(c_t) &= \beta^t \theta_t + \Phi \frac{n_0^{\alpha-1}}{v_n(n_0)} \left[\beta^t (1 - \gamma_t)(1 + \tau^c) \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta^t (1 + \tau^c) \gamma_t (u_c(c_t) + c_t u_{cc}(c_t)) \frac{v_n(n_{t-1})}{n_{t-1}^{\alpha-1} u_c(c_{t-1})} \right] \\ & + \beta^t n_t^{\alpha-1} u_{cc}(c_t) [\xi_t \chi + \xi_{t-1}] \end{aligned} \quad (1.30)$$

$$n_{t \geq 1} : \beta^t v_n(n_t) = \theta_t \beta^t \alpha n_t^{\alpha-1}$$

$$\begin{aligned} & -\Phi \frac{n_0^{\alpha-1}}{v_n(n_0)} \beta^t \left[-\frac{v_{nn}(n_t) n_t^{\alpha-1} - v_n(n_t)(\alpha - 1) n_t^{\alpha-2}}{(n_t^{\alpha-1})^2} \left[(1 - \gamma_t)(1 + \tau^c) c_t + \beta(1 + \tau^c) \gamma_{t+1} c_{t+1} \frac{u_c(c_{t+1})}{u_c(c_t)} - \frac{TR_{t+1}}{p_t} \right] \right] \\ & -\Phi \frac{n_0^{\alpha-1}}{v_n(n_0)} \beta^t (\alpha \tau^n - 1) [v_{nn}(n_t) n_t + v_n(n_t)] \\ & + \beta^t (\alpha - 1) n_t^{\alpha-2} u_c(c_t) [\xi_t \chi + \xi_{t-1}] \end{aligned} \quad (1.31)$$

These four first order conditions together with the feasibility, the implementability and the wage rigidity constraints characterize the optimal Ramsey allocation. Using this allocation one can recover prices. As before to undo the cash-in-advance constraint friction it is necessary to deflate the economy at the rate of time preference. However, in this case because of the wage rigidity, in order to ensure full employment, the Ramsey planner needs to inflate the economy more (deflate less) to keep the real wage at the full employment level. It is optimal to do so, because when nominal wage rigidity constraint binds, employment falls, which has a negative impact on utility. To see this consider a change in utility $\sum_{t=0}^{\infty} \beta^t [u(n_t^\alpha) - v(n_t)]$ (where we have used the feasibility condition $c_t = y_t = n_t^\alpha$) given by a reduction in employment $dn_t < 0$

$$dU_t = \beta^t [u_c(n_t^\alpha) \alpha n_t^{\alpha-1} - v_n(n_t)] dn_t = \beta^t \left[u_c(n_t^\alpha) \frac{W_t}{p_t} - v_n(n_t) \right] dn_t < 0$$

where the second equality follows from firm's first order condition. The change in utility dU_t is negative, since the expression $\left[u_c(n_t^\alpha) \frac{W_t}{p_t} - v_n(n_t) \right]$ is positive. It can be seen by dividing (1.21) by (1.22)

$$\frac{u_c(c_t)}{v_n(n_t)} = \frac{\lambda q_t^0 (1 + \tau^c) + \nu_t (1 + \tau^c) \gamma_t}{\lambda q_t^0 (1 - \tau^n) \frac{W_t}{p_t}}$$

and rearranging

$$u_c(c_t) \frac{W_t}{p_t} = \frac{\lambda q_t^0 (1 + \tau^c) + \nu_t (1 + \tau^c) \gamma_t}{\lambda q_t^0 (1 - \tau^n)} v_n(n_t) > v_n(n_t)$$

Even when the cash-in-advance constraint does not bind, i.e., $\nu_t = 0$, we have that $u_c(c_t) \frac{W_t}{p_t} > v_n(n_t)$, because due to taxes, the employment is inefficiently low also without the CIA friction.

Since nominal wages cannot fall by more than χ as long as $\chi > \beta$, deflating the economy at β will result in hitting the constraint $W_t \geq \chi W_{t-1}$ and therefore will drive up the real wage above the full employment level. In order to avoid this effect, the economy needs to be deflated less, that is at χ (hence inflated more than β), so that the real wage remains constant.

As in the case of flexible wages money supply and transfers could be seen as perfect substitutes in delivering the optimal allocation, since both of them enter the cash-in-advance constraint. However, as they have different effects on the government budget constraint - transfers are a liability and money supply provides seigniorage revenue, the Ramsey planner uses the combination of the two which ensures that debt is stabilized. Again, from the time $t = 1$ on, once the stationary allocation (denoted with variables without the time subscript) is attained, in each period the choice of transfers and money supply delivering the optimal allocation and the appropriate deflation must be such that

$$\frac{TR_t}{p_t} = \tau^c c + \tau^n \frac{W_t}{p_t} n + \frac{1 - R}{R} b + \frac{M_{t+1}}{p_t} - \frac{M_t}{p_t}$$

so that the budget is balanced. This is because when full employment is attained, transfers do not have multiplier effects on output and do not generate extra tax revenue - there is no self-

financing in this case and thus in addition to the constant tax revenue transfers need to be financed with the seigniorage revenue. Withdrawing money at the appropriate rate needs to be matched with decreasing path of transfers. Hence, the government budget constraint or equivalently the implementability constraint helps to pin down the paths of the policy variables.

1.3.6 Alternative fiscal policies

This chapter focuses on transfers as a fiscal policy stabilization tool. Nevertheless, one may wonder whether different fiscal policy instruments can act in a similar way and overcome the inefficiency caused by downward wage rigidity. The inspection of the model with rigid wages suggests that changes in the tax rates τ^n and τ^c can possibly achieve a similar goal.

As is suggested by equation (1.8) labor supply depends on the effective tax rate on labor income $\frac{1-\tau^n}{1+\tau^c}$. Unemployment in the model arises when labor supply is higher than labor demand under the prevailing real wage. Hence, to restore the equilibrium without changing transfers, one could *decrease* labor supply, for instance by *increasing* τ^n . Surprisingly then, closing the unemployment gap calls for increasing labor income taxes¹⁴. However, a decrease in labor supply to the level consistent with labor demand, while restoring the labor market equilibrium is associated with lower employment and output than in the initial full-employment pre-recession case.

Another possibility is to increase prices and therefore reduce real wages via decreasing τ^c . Similarly to higher transfers this affects the cash-in-advance constraint (1.2) and translates into higher inflationary pressure which again restores the labor market equilibrium. However, a decrease of the tax rate on consumption also decreases the effective tax rate on labor income and as a result expands labor supply, so the opposite forces are at play in this case. For a sufficient reduction of the consumption tax, prices should increase sufficiently so that real wages are reduced to the level equating labor demand with an expanded labor supply¹⁵.

The above solutions are based on changing the distortionary tax rates and therefore directly distort the economic decisions of the agents - even when taxes are temporarily reduced, they need to be subsequently increased to balance the budget, which results in costly tax rate fluctuations. The virtue of our transfers is that they are lump-sum, so that the distortion of the policy leading to full employment is of second order and thus minimized¹⁶.

1.4 Quantitative analysis

In this section we perform the quantitative evaluation of the model presented in section 1.2.

¹⁴Increasing τ^c also reduces labor supply, but at the same time tightens the CIA constraint, hence reduces aggregate demand and prices further.

¹⁵This could also be accompanied with an increase in τ^n so that labor supply does not expand too much.

¹⁶Still the tax revenue must be collected via distortionary taxation with the constant tax rates.

1.4.1 Adding capital and capital taxes

To make the model more realistic we add capital and capital taxes. Therefore, budget constraint (1.1) becomes

$$(1 + \tau^c)c_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} + k_{t+1} = (1 - \tau^n)\frac{W_t}{p_t}n_t + (1 - \tau^k)r_t k_t + (1 - \delta)k_t + b_t + \frac{m_t}{p_t} + \frac{TR_t}{p_t} + \pi_t \quad (1.32)$$

where k_t is capital, r_t is the rental rate of capital, τ^k is the tax rate on capital income and δ is the depreciation rate. At t household choose k_{t+1} in addition to c_t, n_t, b_{t+1} and m_{t+1} .

The optimality condition for capital choice is

$$\lambda_t = \lambda_{t+1}((1 - \tau^k)r_{t+1} + (1 - \delta)) \quad (1.33)$$

with λ_t being the Lagrange multiplier on the budget constraint as before. Using (1.3) and (1.33) we obtain the Euler equation for capital

$$\frac{u_c(c_t)}{1 + \tau^c} - \frac{\nu_t}{\beta^t}\gamma_t = \beta((1 - \tau^k)r_{t+1} + (1 - \delta)) \left(\frac{u_c(c_{t+1})}{1 + \tau^c} - \frac{\nu_{t+1}}{\beta^{t+1}}\gamma_{t+1} \right) \quad (1.34)$$

equalizing the marginal utility cost of forgoing one unit of consumption today with the discounted marginal utility gain of higher returns on capital next period expressed in utility units, adjusted for the binding cash-in-advance constraint.

Firms are now facing the Cobb-Douglas production function

$$y_t = k_t^{1-\alpha} n_t^\alpha \quad (1.35)$$

with $\alpha \in (0, 1)$ and positive and decreasing marginal products of labor and capital. Firms problem is now

$$\max_{n_t, k_t} p_t k_t^{1-\alpha} n_t^\alpha - p_t r_t k_t - W_t n_t$$

We obtain the following first order conditions

$$(1 - \alpha)k_t^{-\alpha} n_t^\alpha = r_t \quad (1.36)$$

$$\alpha k_t^{1-\alpha} n_t^{\alpha-1} = W_t/p_t \quad (1.37)$$

which equalize the marginal product of capital with its rental rate and the marginal product of labor with the real wage. Profits in real terms are given by

$$\pi_t = \frac{p_t k_t^{1-\alpha} n_t^\alpha - p_t r_t k_t - W_t n_t}{p_t} = k_t^{1-\alpha} n_t^\alpha - (1 - \alpha)k_t^{-\alpha} n_t^\alpha k_t - \alpha k_t^{1-\alpha} n_t^{\alpha-1} n_t = 0$$

Government budget constraint becomes

$$\tau^c c_t + \tau^n \frac{W_t}{p_t} n_t + \tau^k r_t k_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = b_t + \frac{m_t}{p_t} + \frac{TR_t}{p_t}$$

and, finally, the feasibility constraint is now

$$c_t + k_{t+1} = y_t + (1 - \delta)k_t$$

1.4.2 Calibration

We assume that the utility function takes the following constant relative risk aversion form

$$u(c) - v(n) = \frac{c^{(1-\sigma)}}{1-\sigma} - \frac{n^{(1+\psi)}}{1+\psi}$$

The calibration of the model is summarized in Table 1.1. We set the labor share α at 0.64. The subjective discount factor β takes the value of 0.95. The coefficient of relative risk aversion σ is set at 2, while the Frisch elasticity of labor supply ψ is set at 1. The depreciation rate of capital δ takes the value of 0.025. These are the standard value used in the business cycle literature.

We assume that the parameter governing the downward wage rigidity χ takes the value of 0.99 which is a conservative estimate of this parameter for the euro zone periphery countries in the Great Recession by Schmitt-Grohé and Uribe (2016)¹⁷. The steady-state share of cash-in-advance constrained households proxied by the share of credit-constrained households¹⁸. The average estimate of this parameter for the euro zone by Martin and Philippon (2017) is 0.48. Following this γ is set at 0.48.

The tax rates of consumption τ^c and labor income τ^n are set at 22.5% and 34.3%, respectively. The tax rate on capital τ^k is proxied by the corporate income tax rate and is set at 22.5%. These were the average tax rates for these categories in the EU countries in 2016¹⁹.

¹⁷This parameter governs the speed of adjustment in the labor market. Lower values are associated with a faster recovery and less room for a policy intervention. Higher values correspond with a prolonged slowdown and a lower tax base.

¹⁸This is a conservative approach - the share of cash-in-advance households in consumption is likely to be larger.

¹⁹See https://ec.europa.eu/taxation_customs/business/economic-analysis-taxation/data-taxation_en We refer to the implicit tax rate on consumption defined as the ratio of the revenue from all consumption taxes to the final consumption expenditure of households and the implicit tax rate on labor income defined as the ratio of taxes and social contributions on employed labor income to total compensation of employees and payroll taxes. These categories are more in line with our model setting than the statutory tax rates. The implicit tax rate on capital is not available in this dataset.

Table 1.1: Calibration

Parameter	Description	Value
α	Labor share	0.64
β	Discount factor	0.95
σ	Coefficient of relative risk aversion	2
ψ	Frisch elasticity of labor supply	1
δ	Depreciation rate for capital	0.025
χ	Downward wage rigidity	0.99
γ	Steady-state share of CIA constrained households	0.48
τ^c	Tax rate on consumption	22.5%
τ^n	Tax rate on labor income	34.3%
τ^k	Tax rate on capital income	22.5%

1.4.3 Experiments

In our experiments we assume that the economy starts at the flexible wage steady state and then enters a period when output is below the full employment level. The recession is introduced by an increase in the share of the cash-in-advance constrained households γ_t from the steady-state value of 0.48 to 0.517. The latter under downward rigid wages corresponds to the unemployment rate increasing to 20%. It is assumed that such an increase in the number of constrained households lasts precisely for two years (eight quarters). It corresponds to the negative shocks to aggregate demand and can be thought of as resulting from a liquidity crisis in the financial sector making credit less available and credit conditions more stringent. Since this is a perfect foresight model agents know the paths of exogenous variables, in particular that of γ_t , and understand in which type of the economy they live (i.e., with flexible or downward rigid nominal wages). We analyze three cases:

- Flexible-wages benchmark
- Downward rigid nominal wages without a change in transfers
- Downward rigid nominal wages with an increase in transfers

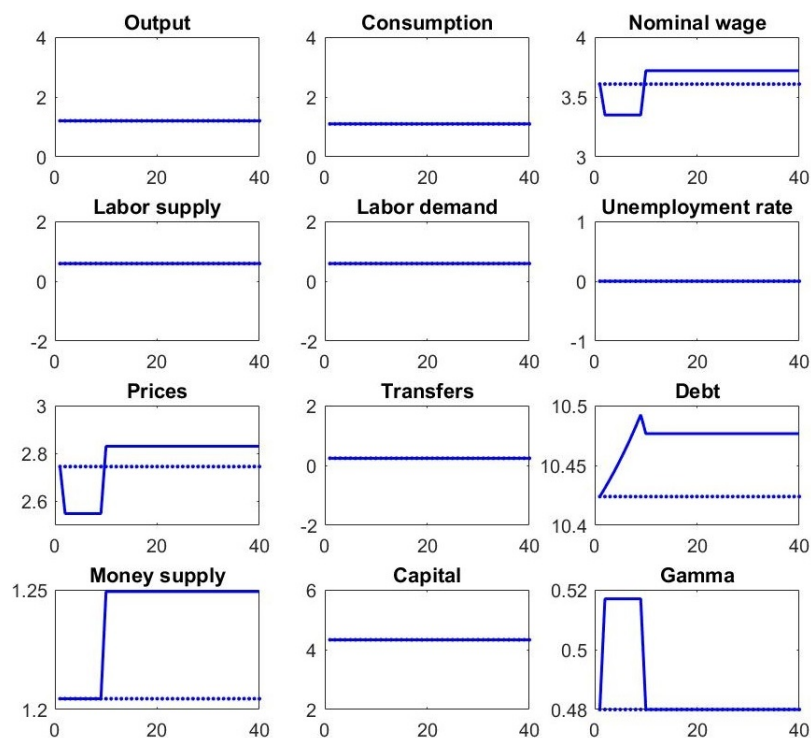
The second and the third case differ in the sequence of transfers facing the agents in the economy. In the third case government increases transfers by 20% (corresponding to the level of unemployment) one period after unemployment emerges for seven quarters, i.e. for the remaining time of the economic slowdown when the credit conditions are worsened (γ_t is increased), in order to mitigate it, while in the second case it does nothing. After the experiment money supply is adjusted so that debt is stabilized. The end values of the real variables in the simulation in each case are set at

the flexible-wages steady state. In the following subsections we analyze the particular cases. The model is solved simultaneously for all equations for every period using Levenberg Marquardt mixed complementarity problem which allows for the inequality constraints on endogenous variables.

1.4.4 Benchmark model

The time paths of the selected variables of the benchmark model are presented on Figure 1.3.

Figure 1.3: Time paths of the selected variables of the benchmark model.



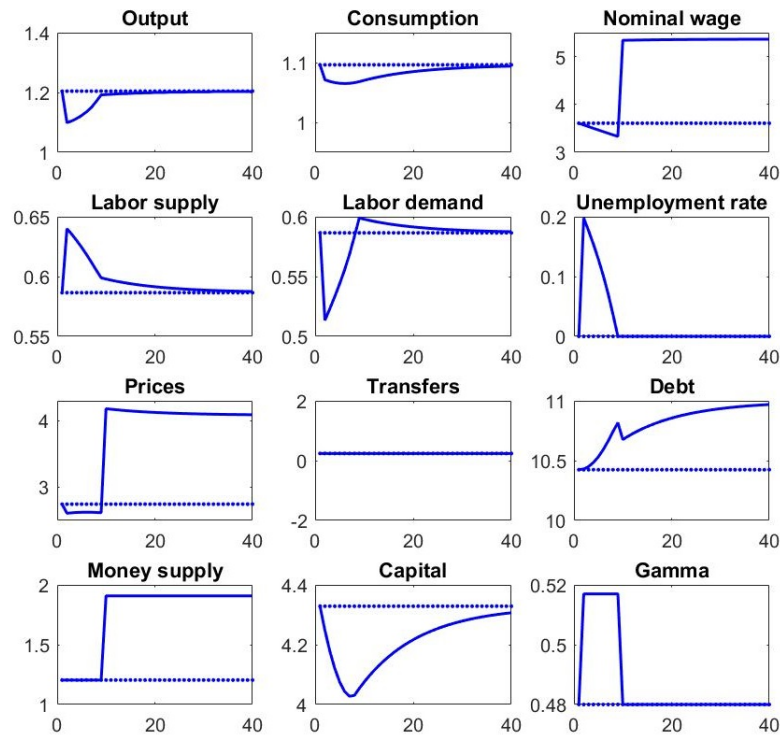
Note: Solid line - time path. Dotted line - steady state. Horizontal axis - time periods (quarters).

We can see that after tightening of the cash-in-advance constraint aggregate demand falls and prices drop and since the constraint is tighter for eight quarters prices stay below their initial level. As in the current case nominal wages are flexible, they follow the pattern of prices and fall. There is no unemployment, labor supply and demand stay at their steady state levels. The same is true for output, consumption and capital. There is no need to increase transfers. Since lower prices correspond to higher real value of transfers, real value of debt initially increases. After the designed recession money supply needs to be adjusted upwards in order to stabilize the debt. Hence, after the recession prices jump upwards and stay at the higher level. Again, only nominal wages adjust and there is no change in real variables.

1.4.5 Downward rigid nominal wages

The time paths of the selected variables of the model with downward rigid nominal wages are presented on Figure 1.4.

Figure 1.4: Time paths of the selected variables of the model with downward rigid nominal wages.



Note: Solid line - time path. Dotted line - steady state. Horizontal axis - time periods (quarters).

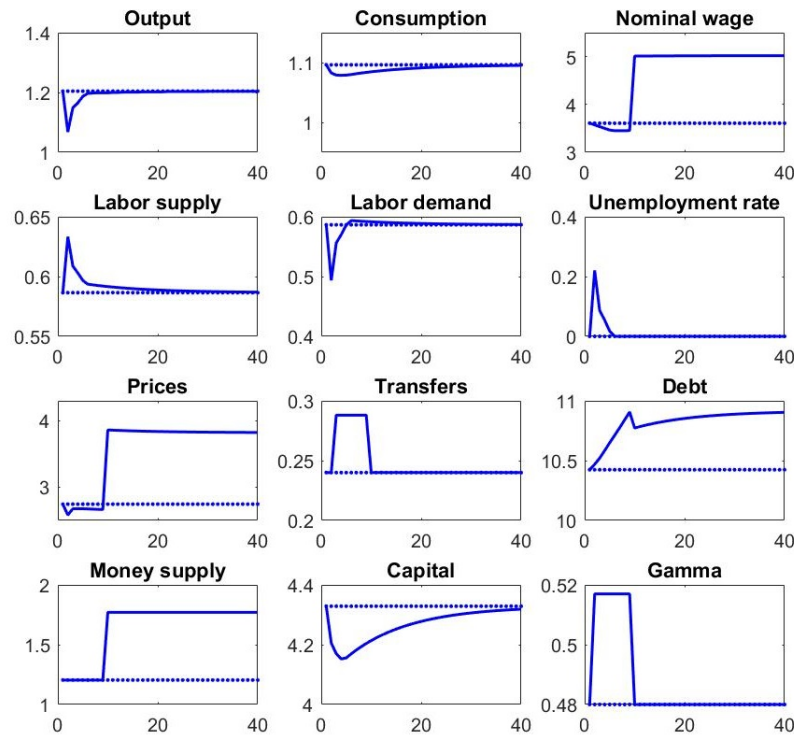
Again after the tightening of the cash-in-advance constraint and resulting drop in aggregate demand prices fall and since the constraint is assumed to stay tighter than the steady state level for eight quarters, prices also stay below their initial level. As nominal wages are now downward rigid, they do not adjust immediately and fall only slowly at the rate χ . Since the real wage is now above the market clearing level, labor supply is higher than labor demand and this results in involuntary unemployment. As now the labor demand determines employment and it is below the steady state level, output falls and so does consumption. Capital falls as it is less productive due to a drop in employment and the reduction in investments also reflects the consumption smoothing. We assume that nominal transfers are fixed and do not change in this case. Hence, the equilibrium in the labor market is restored only via a gradual fall in the nominal wage. Debt increases both because of higher real value of transfers and lower tax revenues. After the designed recession money supply is adjusted upwards in order to stabilize the debt. Hence, after the recession prices again

jump upwards and stay at the higher level. The nominal wages freely adjust upwards and also stay at the higher level.

1.4.6 Downward rigid nominal wages with an increase in transfers

The time paths of the selected variables of the model with downward rigid nominal wages and an increase in transfers are presented on Figure 1.5.

Figure 1.5: Time paths of the selected variables of the model with downward rigid nominal wages and an increase in transfers.



Note: Solid line - time path. Dotted line - steady state. Horizontal axis - time periods (quarters).

As before after the cash-in-advance constraint gets tighter prices drop and since the constraint is assumed to stay tighter than the steady state level for eight quarters, prices also stay below their initial level. Since nominal wages are again downward rigid, they do not adjust immediately and fall only gradually at the rate χ . The real wage is again above the market clearing level, and therefore labor supply is higher than labor demand. This results in involuntary unemployment. Since under rigid wages the labor demand determines employment which is now below the steady state level, output falls and so does consumption. Capital falls again since it is less productive and consumption smoothing induces a reduction in investment. Now, however, we assume that nominal

transfers are increased one-period after the recession starts for seven quarters. What we can see is that unemployment falls much quicker than in the case with no fiscal stimulus. This is because higher transfers increase prices, which reduces real wages, effectively undoing the inefficiency caused by the wage rigidity. Real wages are reduced much faster to the full employment level. For the current path of the real wage labor supply is smaller and labor demand is higher and hence the unemployment gap closes faster than before. In this scenario debt increases because of higher real value of initial transfers, lower tax revenues and subsequent higher transfers. However, when employment, output and consumption increase, this increases the tax revenue from labor and capital income and consumption taxes. As before after the designed recession money supply is adjusted upwards in order to stabilize the debt. However, the necessary adjustment of the money supply is smaller than in the previous case thanks to the extra tax revenue generated due to a partial self-financing of transfers. Because of the monetary easing after the recession prices again jump upwards and stay at the higher level. The nominal wages freely adjust upwards and also stay at the higher level.

1.4.7 Quantitative results

In this subsection we compare the results of the two last cases in order to analyze the effects of higher transfer payments. Comparing the effects on output one can calculate the impact multiplier defined here as the difference between output under rigid wages without and with the change in transfers divided by this change in transfers within the first period of a change in transfers. This multiplier takes the value of 0.90 in our calibration. The total (long-run) multiplier defined here as the total difference in output under rigid wages without and with the change in transfers divided by the difference between transfers path in those both cases is 1.03. These multipliers are too small for higher transfers to be fully self-financing (see subsection 1.3.3).

Table 1.2 summarizes some other quantitative results. Welfare is calculated as the consumption equivalent variation of the utility versus the benchmark model. In both cases the model with rigid wages has smaller welfare than the model with no rigidities. However, we can see that the welfare is higher under the case with fiscal stimulus - with higher transfers welfare is lower only by 7.90% than in the benchmark model, while without an increase in transfers the utility is lower by 15.39% in consumption equivalent terms. Budget deficit in period 3 (when we provide fiscal stimulus) is higher under higher transfers. However, it does not increase as much as the increase in transfers themselves. This is because this policy generates additional tax revenue and higher transfers are partially (around 34%) self-financing. We can also see how much debt in period $t = 9$ when the recession ends differs between the two cases. It turns out that the difference is 0.09, while the transfers were higher by $7 \times 0.048 = 0.336$, which means that around 73% of this increase financed itself. The rest needs to be financed by seigniorage revenue in order to stabilize the debt.

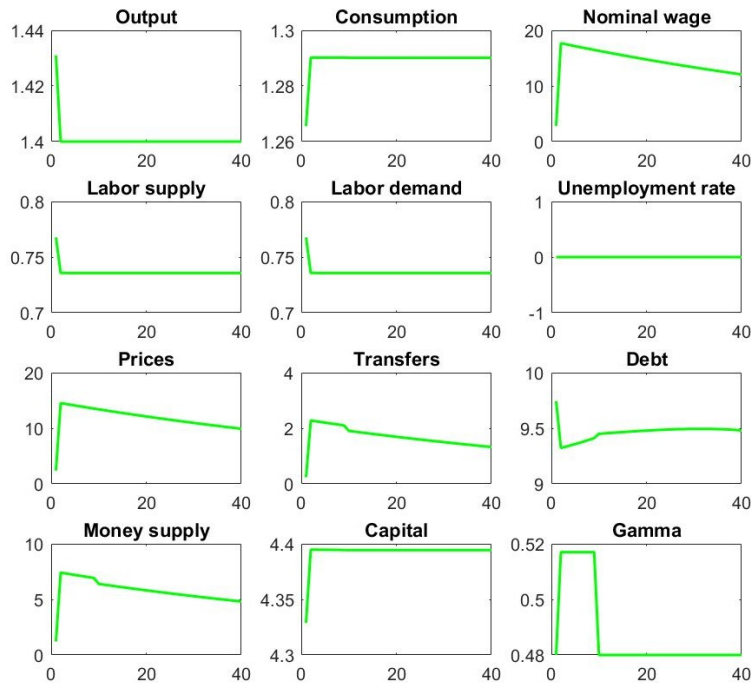
Table 1.2: Quantitative results

	Regime		
	Rigid wages	Rigid wages with $\Delta TR = 0.0480$	Self-financing
Welfare - consumption equivalent variation vs. benchmark model	-15.39 %	-7.90%	
Deficit at $t = 3$	0.0204	0.0520	$1 - \frac{0.0520 - 0.0204}{0.0480} = 34.2\%$
Debt at $t = 9$	10.8169	10.9069	$1 - \frac{10.9069 - 10.8169}{7 \times 0.0480} = 73.2\%$

1.4.8 Ramsey optimal policy

In this subsection we simulate the optimal policy according to the Ramsey allocation described in subsection 1.3.5. The simulation is presented on Figure 1.6.

Figure 1.6: Time paths of the selected variables of the model with rigid wages for the Ramsey economy.



Note: Solid line - time path. Horizontal axis - time periods (quarters).

The allocation from period one onwards differs from the allocation for period zero - there is a time inconsistency in the optimal allocation. Ramsey planner ensures full employment so that output is at the full-employment level and consumption is constant from period one onwards. Optimal policy absent wage rigidity would call for deflating the economy at the rate of time preference to disable the cash-in-advance friction. However, with rigid wages the economy needs to be deflated at the lower rate (hence more inflated) which is equal to the nominal wage rigidity constraint lower bound so that real wage stays at the market clearing level. Moreover, when the cash in advance constraint is more binding (higher γ) in order to ensure appropriate deflation money supply and transfers need to grow at higher rate (actually fall at the lower rate), hence the kink on the graphs. As it was indicated in the theoretical section, Ramsey planner could use either money supply or transfers to generate the appropriate deflation, however, she needs to use both instruments to deliver the optimal allocation and satisfy the government budget constraint. This is because transfers are not self-financing in this case (once we are at full employment they do not generate extra tax revenue) and money supply needs to be adjusted so that the time zero budget constraint (or equivalently the implementability constraint) is satisfied. The presented simulation involves a combination of money supply and transfers delivering optimal allocation with money supply adjusted so that the government debt is stabilized.

1.5 Conclusions

In this paper we have analyzed the impact of an increase in transfer payments on budget deficits. We have shown that this impact depends on whether the economy operates at or below full capacity. In particular, we have examined the case of the cash-in-advance economy with downward nominal wage rigidity in which the negative demand shock results in unemployment. In such a situation an increase in transfer payments relaxes the cash-in-advance constraint and effectively undoes the inefficiency caused by the wage rigidity. As a result, higher transfers have multiplier effects on output and via higher income and consumption generate additional tax revenue thanks to the expanded tax base. The multiplier depends on the initial level of money supply, transfers and nominal wages.

We have also shown that for large enough multiplier an increase in transfer payments can be self-financing. Our quantitative analysis suggests that under a plausible calibration of the model and for the existing tax rates in the European Union the transfer multipliers are indeed sufficient for the tax revenue to increase enough so that the stimulus largely pays for itself - around three-quarters of an increase in transfers are self-financing. Further research could focus on whether similar results can be obtained in an alternative setting where spare capacity of the economy, i.e., unemployment is introduced via search and matching frictions in the labor market.

1.6 Appendix

1.6.1 Some pleasant fiscal algebra with capital

In this section we show how large the multiplier needs to be in order for transfers to be self-financing in a model with capital. The production function is given by

$$y = k^{1-\alpha}n^\alpha$$

As before, the change in transfers needs to be equal to the change in the tax revenue

$$dTransfers = dTaxRevenue$$

Substituting in for the tax revenue we get

$$dTR = \tau^c dc + \tau^n d\left(\frac{W}{p}n\right) + \tau^k d(rk)$$

where τ^k is the tax rate on capital income. Now we again divide by the change in transfers

$$1 = \tau^c \frac{dc}{dTR} + \tau^n \frac{d\left(\frac{W}{p}n\right)}{dTR} + \tau^k \frac{d(rk)}{dTR}$$

and multiply and divide by dy

$$1 = \tau^c \frac{dc}{dy} \frac{dy}{dTR} + \tau^n \frac{d\left(\frac{W}{p}n\right)}{dy} \frac{dy}{dTR} + \tau^k \frac{d(rk)}{dy} \frac{dy}{dTR}$$

The expression $\frac{dc}{dy}$ is the marginal propensity to consume (MPC)

$$1 = \tau^c MPC \frac{dy}{dTR} + \tau^n \frac{d\left(\frac{W}{p}n\right)}{dy} \frac{dy}{dTR} + \tau^k \frac{d(rk)}{dy} \frac{dy}{dTR}$$

Since $\frac{W}{p}n = \alpha y \implies \frac{d\left(\frac{W}{p}n\right)y}{y^2} - \frac{W}{p}n \frac{dy}{y^2} = 0 \implies d\left(\frac{W}{p}n\right) = \alpha dy$ and $\frac{rk}{y} = (1-\alpha) \implies d(rk) = (1-\alpha)dy$ we obtain

$$1 = \tau^c MPC \frac{dy}{dTR} + \alpha \tau^n \frac{dy}{dTR} + (1-\alpha) \tau^k \frac{dy}{dTR}$$

Again we factor out $\frac{dy}{dTR}$

$$\frac{dy}{dTR} (\tau^c MPC + \alpha \tau^n + (1-\alpha) \tau^k) = 1$$

And solve for the multiplier

$$\frac{dy}{dTR} = \frac{1}{\tau^c MPC + \alpha\tau^n + (1-\alpha)\tau^k}$$

Assuming that the marginal propensity to consume out of transfers is 0.67 as found by Johnson, Parker, and Souleles (2006) and labor share is 0.64, using the tax rates on consumption and labor income as above and adding average corporate income tax rate in the European Union we obtain

$$\frac{dy}{dTR} = \frac{1}{0.225 \times 0.67 + 0.64 \times 0.342 + 0.36 \times 0.225} = \frac{1}{0.15 + 0.22 + 0.08} = 2.22$$

which is roughly the same as the result in the main text.

1.6.2 Results used in subsection 1.3.5

In this section we derive some results used in deriving the Ramsey allocation in subsection 1.3.5. As we normalize $q_0^0 = 1$ from (1.21) at time $t = 0$ we obtain λ

$$u_c(c_0) = \lambda(1 + \tau^c) + \nu_0(1 + \tau^c)\gamma_0 \iff \lambda = \frac{u_c(c_0)}{1 + \tau^c} - \nu_0\gamma_0$$

Using (1.21) and (1.22) at $t = 0$ we obtain ν_0

$$\frac{u_c(c_0)}{v_n(n_0)} = \frac{\lambda(1 + \tau^c) + \nu_0(1 + \tau^c)\gamma_0}{\lambda(1 - \tau^n)\frac{W_0}{p_0}} = \frac{(\frac{u_c(c_0)}{1 + \tau^c} - \nu_0\gamma_0)(1 + \tau^c) + \nu_0(1 + \tau^c)\gamma_0}{(\frac{u_c(c_0)}{1 + \tau^c} - \nu_0\gamma_0)(1 - \tau^n)\frac{W_0}{p_0}} = \frac{u_c(c_0)}{(\frac{u_c(c_0)}{1 + \tau^c} - \nu_0\gamma_0)(1 - \tau^n)\frac{W_0}{p_0}}$$

Hence:

$$\nu_0 = \frac{u_c(c_0)}{\gamma_0(1 + \tau^c)} - \frac{v_n(n_0)}{\gamma_0(1 - \tau^n)\frac{W_0}{p_0}} \implies \lambda = \frac{v_n(n_0)}{(1 - \tau^n)\frac{W_0}{p_0}}$$

Then from (1.22) we can obtain q_t^0

$$\beta^t v_n(n_t) = \lambda q_t^0 (1 - \tau^n) \frac{W_t}{p_t}$$

Plug in for λ

$$\beta^t v_n(n_t) = \frac{v_n(n_0)}{(1 - \tau^n)\frac{W_0}{p_0}} q_t^0 (1 - \tau^n) \frac{W_t}{p_t} = \frac{v_n(n_0)}{\frac{W_0}{p_0}} q_t^0 \frac{W_t}{p_t}$$

Using firm's FOC we obtain

$$q_t^0 = \frac{\beta^t v_n(n_t) \frac{W_0}{p_0}}{v_n(n_0) \frac{W_t}{p_t}} = \frac{\beta^t v_n(n_t) \alpha n_0^{\alpha-1}}{v_n(n_0) \alpha n_t^{\alpha-1}} = \frac{\beta^t v_n(n_t) n_0^{\alpha-1}}{v_n(n_0) n_t^{\alpha-1}}$$

Also

$$\lambda q_t^0 = \frac{\beta^t v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}$$

From (1.21) and (1.22)

$$\frac{u_c(c_t)}{v_n(n_t)} = \frac{\lambda q_t^0 (1 + \tau^c) + \nu_t (1 + \tau^c) \gamma_t}{\lambda q_t^0 (1 - \tau^n) \frac{W_t}{p_t}}$$

we obtain $\nu_t \gamma_t$

$$\nu_t \gamma_t = \frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{p_t} \frac{u_c(c_t)}{v_n(n_t)} \lambda q_t^0 - \lambda q_t^0$$

Plug in for λq_t^0

$$\nu_t \gamma_t = \frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{p_t} \frac{u_c(c_t)}{v_n(n_t)} \frac{\beta^t v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}} - \frac{\beta^t v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}$$

From (1.23) and (1.21)

$$\frac{\lambda q_t^0}{p_t} = \frac{\lambda q_{t+1}^0 + \nu_{t+1}}{p_{t+1}} \iff \frac{\frac{\beta^t u_c(c_t)}{1 + \tau^c} - \nu_t \gamma_t}{p_t} = \frac{\frac{\beta^t u_c(c_{t+1})}{1 + \tau^c} - \nu_{t+1} \gamma_{t+1} + \nu_{t+1}}{p_{t+1}}$$

we obtain

$$\frac{p_{t+1}}{p_t} = \frac{\frac{\beta^t u_c(c_{t+1})}{1 + \tau^c} - \nu_{t+1} \gamma_{t+1} + \nu_{t+1}}{\frac{\beta^t u_c(c_t)}{1 + \tau^c} - \nu_t \gamma_t}$$

which can be solved for ν_{t+1}

$$\nu_{t+1} = \frac{p_{t+1}}{p_t} \left(\beta^t \frac{u_c(c_t)}{1 + \tau^c} - \nu_t \gamma_t \right) - \beta^{t+1} \frac{u_c(c_{t+1})}{1 + \tau^c} + \nu_{t+1} \gamma_{t+1}$$

From (1.21) evaluated at $t + 1$ and (1.22) evaluated at t

$$\frac{u_c(c_{t+1})}{v_n(n_t)} = \beta^{-1} \frac{\lambda q_{t+1}^0 (1 + \tau^c) + \nu_{t+1} (1 + \tau^c) \gamma_{t+1}}{\lambda q_t^0 (1 - \tau^n) \frac{W_t}{p_t}}$$

Rearrange

$$\frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{p_t} \frac{u_c(c_{t+1})}{v_n(n_t)} = \beta^{-1} \frac{\lambda q_{t+1}^0 + \nu_{t+1} \gamma_{t+1}}{\lambda q_t^0}$$

Add $\beta^{-1} \frac{\nu_{t+1} (1 - \gamma_{t+1})}{\lambda q_t^0}$ on both sides

$$\frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{p_t} \frac{u_c(c_{t+1})}{v_n(n_t)} + \beta^{-1} \frac{\nu_{t+1} (1 - \gamma_{t+1})}{\lambda q_t^0} = \beta^{-1} \frac{\lambda q_{t+1}^0 + \nu_{t+1} \gamma_{t+1}}{\lambda q_t^0} + \beta^{-1} \frac{\nu_{t+1} (1 - \gamma_{t+1})}{\lambda q_t^0}$$

Equivalently

$$\frac{1 - \tau^n}{1 + \tau^c} \frac{W_t}{p_t} \frac{u_c(c_{t+1})}{v_n(n_t)} + \beta^{-1} \frac{\nu_{t+1} (1 - \gamma_{t+1})}{\lambda q_t^0} = \beta^{-1} \frac{\lambda q_{t+1}^0 + \nu_{t+1}}{\lambda q_t^0} = \beta^{-1} \frac{p_{t+1}}{p_t}$$

where the second equality follows from (1.23). Then

$$\frac{p_{t+1}}{p_t} = \beta \frac{1 - \tau^n}{1 + \tau^c} \frac{W_t u_c(c_{t+1})}{p_t v_n(n_t)} + \frac{\nu_{t+1}(1 - \gamma_{t+1})}{\lambda q_t^0} = \beta \frac{1 - \tau^n}{1 + \tau^c} \alpha n_t^{\alpha-1} \frac{u_c(c_{t+1})}{v_n(n_t)} + \frac{\nu_{t+1}(1 - \gamma_{t+1})}{\lambda q_t^0}$$

where the second equality follows from (1.11). Plug in for λq_t^0

$$\frac{p_{t+1}}{p_t} = \beta \frac{1 - \tau^n}{1 + \tau^c} \alpha n_t^{\alpha-1} \frac{u_c(c_{t+1})}{v_n(n_t)} + \frac{\nu_{t+1} - \nu_{t+1} \gamma_{t+1}}{\frac{\beta^t v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}}$$

Plug in for ν_{t+1} obtained above

$$\frac{p_{t+1}}{p_t} = \beta \frac{1 - \tau^n}{1 + \tau^c} \alpha n_t^{\alpha-1} \frac{u_c(c_{t+1})}{v_n(n_t)} + \frac{\frac{p_{t+1}}{p_t} (\beta^t \frac{u_c(c_t)}{1 + \tau^c} - \nu_t \gamma_t) - \beta^{t+1} \frac{u_c(c_{t+1})}{1 + \tau^c} + \nu_{t+1} \gamma_{t+1} - \nu_{t+1} \gamma_{t+1}}{\frac{\beta^t v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}}$$

Cancel out

$$\frac{p_{t+1}}{p_t} = \beta \frac{1 - \tau^n}{1 + \tau^c} \alpha n_t^{\alpha-1} \frac{u_c(c_{t+1})}{v_n(n_t)} + \frac{\frac{p_{t+1}}{p_t} (\beta^t \frac{u_c(c_t)}{1 + \tau^c} - \nu_t \gamma_t) - \beta^{t+1} \frac{u_c(c_{t+1})}{1 + \tau^c}}{\frac{\beta^t v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}}$$

Plug in for $\nu_t \gamma_t$ obtained above

$$\frac{p_{t+1}}{p_t} = \beta \frac{1 - \tau^n}{1 + \tau^c} \alpha n_t^{\alpha-1} \frac{u_c(c_{t+1})}{v_n(n_t)} + \frac{\frac{p_{t+1}}{p_t} (\beta^t \frac{u_c(c_t)}{1 + \tau^c} - \frac{1 - \tau^n}{1 + \tau^c} \frac{W_t u_c(c_t)}{p_t v_n(n_t)} \frac{\beta^t v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}} + \frac{\beta^t v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}) - \beta^{t+1} \frac{u_c(c_{t+1})}{1 + \tau^c}}{\frac{\beta^t v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}}$$

Simplify

$$\frac{p_{t+1}}{p_t} = \beta \frac{1 - \tau^n}{1 + \tau^c} \alpha n_t^{\alpha-1} \frac{u_c(c_{t+1})}{v_n(n_t)} + \frac{\frac{p_{t+1}}{p_t} (\frac{u_c(c_t)}{1 + \tau^c} - \frac{1 - \tau^n}{1 + \tau^c} \frac{W_t u_c(c_t)}{p_t v_n(n_t)} \frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}} + \frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}) - \beta \frac{u_c(c_{t+1})}{1 + \tau^c}}{\frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}}$$

Rearrange

$$\frac{p_{t+1}}{p_t} \frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}} = \beta \frac{1 - \tau^n}{1 + \tau^c} \alpha n_t^{\alpha-1} \frac{u_c(c_{t+1})}{v_n(n_t)} \frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}} + \frac{p_{t+1}}{p_t} (\frac{u_c(c_t)}{1 + \tau^c} - \frac{1 - \tau^n}{1 + \tau^c} \frac{W_t u_c(c_t)}{p_t v_n(n_t)} \frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}} + \frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}) - \beta \frac{u_c(c_{t+1})}{1 + \tau^c}$$

Simplify

$$\frac{p_{t+1}}{p_t} \frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}} = \beta \frac{u_c(c_{t+1})}{1 + \tau^c} \alpha + \frac{p_{t+1}}{p_t} (\frac{u_c(c_t)}{1 + \tau^c} - \frac{u_c(c_t)}{1 + \tau^c} \alpha + \frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}) - \beta \frac{u_c(c_{t+1})}{1 + \tau^c}$$

Factor out

$$\frac{p_{t+1}}{p_t} (\frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}} - (1 - \alpha) \frac{u_c(c_t)}{1 + \tau^c} - \frac{v_n(n_t)}{(1 - \tau^n) n_t^{\alpha-1}}) = \beta \frac{u_c(c_{t+1})}{1 + \tau^c} (\alpha - 1)$$

Cancel out

$$\frac{p_{t+1}}{p_t} \left(-(1-\alpha) \frac{u_c(c_t)}{1+\tau^c} \right) = \beta \frac{u_c(c_{t+1})}{1+\tau^c} (\alpha-1)$$

Simplify to obtain

$$\frac{p_{t+1}}{p_t} = \beta \frac{u_c(c_{t+1})}{u_c(c_t)}$$

1.6.3 Deriving the implementability constraint

In this section we derive the implementability constraint. Since the cash-in-advance constraint holds with equality from (1.20) we obtain

$$\sum_{t=0}^{\infty} q_t^0 \left((1-\gamma_t)(1+\tau^c)c_t + \frac{m_{t+1}}{p_{t+1}} \frac{p_{t+1}}{p_t} \right) = \sum_{t=0}^{\infty} q_t^0 \left((1-\tau^n) \frac{W_t}{p_t} n_t + \pi_t \right) + b_0$$

Substitute for $\frac{m_{t+1}}{p_{t+1}}$

$$\sum_{t=0}^{\infty} q_t^0 \left((1-\gamma_t)(1+\tau^c)c_t + \left((1+\tau^c)\gamma_{t+1}c_{t+1} - \frac{TR_{t+1}}{p_{t+1}} \right) \frac{p_{t+1}}{p_t} \right) = \sum_{t=0}^{\infty} q_t^0 \left((1-\tau^n) \frac{W_t}{p_t} n_t + \pi_t \right) + b_0$$

Rearrange to obtain

$$\sum_{t=0}^{\infty} q_t^0 \left[(1-\gamma_t)(1+\tau^c)c_t + (1+\tau^c)\gamma_{t+1}c_{t+1} \frac{p_{t+1}}{p_t} - \left((1-\tau^n) \frac{W_t}{p_t} n_t + \pi_t \right) - \frac{TR_{t+1}}{p_t} \right] = b_0$$

Plug in wages and profits from firm's problem

$$\sum_{t=0}^{\infty} q_t^0 \left[(1-\gamma_t)(1+\tau^c)c_t + (1+\tau^c)\gamma_{t+1}c_{t+1} \frac{p_{t+1}}{p_t} - \left((1-\tau^n)\alpha n_t^{\alpha-1}n_t + (1-\alpha)n_t^\alpha \right) - \frac{TR_{t+1}}{p_t} \right] = b_0$$

Rearrange to obtain

$$\sum_{t=0}^{\infty} q_t^0 \left[(1-\gamma_t)(1+\tau^c)c_t + (1+\tau^c)\gamma_{t+1}c_{t+1} \frac{p_{t+1}}{p_t} + (\alpha\tau^n - 1)n_t^\alpha - \frac{TR_{t+1}}{p_t} \right] = b_0$$

Now we plug in for the prices $q_t^0, \frac{p_{t+1}}{p_t}$ from the appendix 1.6.2.

$$\sum_{t=0}^{\infty} \frac{\beta^t v_n(n_t)}{v_n(n_0)} \frac{n_0^{\alpha-1}}{n_t^{\alpha-1}} \left[(1-\gamma_t)(1+\tau^c)c_t + (1+\tau^c)\gamma_{t+1}c_{t+1} \frac{\beta u_c(c_{t+1})}{u_c(c_t)} + (\alpha\tau^n - 1)n_t^\alpha - \frac{TR_{t+1}}{p_t} \right] = b_0$$

Rearrange to obtain our implementability constraint

$$\frac{n_0^{\alpha-1}}{v_n(n_0)} \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma_t)(1+\tau^c)c_t \frac{v_n(n_t)}{n_t^{\alpha-1}} + \beta(1+\tau^c)\gamma_{t+1}c_{t+1}u_c(c_{t+1}) \frac{v_n(n_t)}{n_t^{\alpha-1}u_c(c_t)} + (\alpha\tau^n - 1)v_n(n_t)n_t - \frac{v_n(n_t)TR_{t+1}}{n_t^{\alpha-1}p_t} \right] = b_0$$

Chapter 2

What are the effects of higher transfer payments on debt? Are transfers self-financing?

Abstract

In this chapter we empirically analyze the self-financing nature of transfer payments by estimating the impulse response functions of GDP, unemployment, consumption and debt to an increase in transfer payments on quarterly data from 1959Q2 to 1991Q4 using the local projection method and exogenous transfers shocks identified by Romer and Romer (2016). We show that the stimulus in a form of higher transfers has more pronounced effects when unemployment is high than when it is low, hence when there is more spare capacity in the economy. Permanent transfers shocks initially cause GDP to fall and unemployment to rise before stimulative effects arrive, a phenomenon practically not observed in case of temporary transfers, which makes the latter more suitable for stimulative purposes. Permanent transfers seem not to affect debt, while temporary transfers are estimated to reduce it after an initial increase, especially in the high unemployment regime - an increase in temporary transfers seems to be not only self-financing, but actually reducing debt when the economy recovers. In any case, the positive stimulative effects arrive only after two years.

2.1 Introduction

The two recent global economic crises - the Great Recession and the coronavirus crisis - renewed the interest in fiscal policy as a tool of macroeconomic stabilization. We have observed a significant increase in the ratio of government expenditure to GDP in the advanced economies in comparison to the pre-2009 averages. These developments were also reflected in economic research. However, while there has been a large number of papers examining the effects of higher government purchases on output and employment, surprisingly little work has been dedicated to transfer payments defined

as direct payments to individuals¹². This is astonishing, given the fact that transfer payments represent an important share of GDP in the advanced economies and furthermore they were a large part of the actual fiscal stimulus programs³.

In this chapter in order to analyze empirically the self-financing nature of transfer payments we estimate the impact of an increase in transfer payments on government debt in addition to GDP, unemployment and consumption. The research question of the chapter is what the effects of higher transfer payments on debt are. There are always concerns about the public finance sustainability and in this analysis we ask how much burden extra transfer payments impose on debt. In particular, we are interested if and when these additional transfers are self-financing, i.e., they pay for themselves in a form of an increased tax revenue. As this clearly depends on the multiplier effects which this kind of stimulus generates, we also estimate cumulative multipliers defined as total change in GDP divided by the total change in transfer payments. The model is estimated using the local projection method on the quarterly data from the U.S. for the period 1959Q2-1991Q4. As the method requires the series of previously identified shocks, we use the narrative series identified by Romer and Romer (2016). We also estimate the state-dependent model depending on whether unemployment is high (above the average of the sample) or low (otherwise).

What we find is that the stimulus in a form of higher transfers has more pronounced effects when unemployment is high than when it is low, hence when there is more spare capacity in the economy. Interestingly, permanent transfers initially cause GDP to fall and unemployment to rise before stimulative effects arrive, which is practically not observed in case of temporary transfers. Permanent transfers are estimated not to affect debt, while temporary transfers seem to reduce it after an initial increase, especially in the high unemployment regime, possibly thanks to extra tax revenue generated due to the tax base expansion. Our results suggest that temporary transfers are more suitable for stimulative purposes, as they do not bring about negative effects observed in the case of permanent transfers and mitigate better the effects of the economic slowdown. Clearly, the stimulus should be only provided in the periods when unemployment is high. It seems that an increase in temporary transfers is not only self-financing, but actually reduces debt when the economy recovers. Nevertheless, the positive stimulative effects arrive only after two years which reduces the usefulness of this policy for fast recovery. We also estimate the cumulative transfers multipliers to find positive ones in case of temporary transfers (as high as 3.6 in the high unemployment regime) and negative ones in case of permanent transfers.

¹The literature on government purchases multipliers is reviewed in Ramey (2011a).

²There are some notable exceptions in the literature, dealing with transfers payments: Oh and Reis (2012), Kaplan and Violante (2014), Romer and Romer (2016), Párraga Rodríguez (2016, 2018), Giambattista and Pennings (2017) and Mehrotra (2018).

³The share of transfer payments in GDP in the OECD countries is around 20%. Oh and Reis (2012) estimate that from the end of 2007 until the end of 2009 around 75% of the increase in the U.S. government expenditures was due to increases in transfers. They also show that in every OECD country where government spending increased over this period at least 30% of this increase was driven by transfers - with the median share of transfer payments of 64%.

This chapter contributes to the empirical literature on the stimulative effects of fiscal policy. As indicated above, this literature focused mostly on the effects of government purchases. The multipliers are estimated either using restrictions on the contemporaneous effects on variables considered in the VAR system as in Blanchard and Perotti (2002), sign-restrictions as in Mountford and Uhlig (2009) or dummy variables indicating exogenous shifts in fiscal policy as in Ramey (2011b) and Barro and Redlick (2011). The first approach yields larger multipliers but close to one, other approaches result in smaller multipliers (below one). Ramey and Zubairy (2018) use local projections state-dependent model to investigate whether the size of multipliers differs in high and low unemployment periods. They find little evidence of state dependence since both output and government spending respond more strongly in slack times. On the contrary, Auerbach and Gorodnichenko (2012) using the a smooth transition VAR model find that increased government purchases have a much larger impact in recessions.

The most prominent paper estimating the impact of a stimulus in a form of transfers is Romer and Romer (2016). These authors use their narrative approach to identify the macroeconomic effects of an increase in social security benefits. They find that permanent rise in those transfers has an immediate positive and significant impact on consumption, but not necessarily on industrial production or employment. Temporary transfers have much smaller impact on consumption. The shocks series identified by Romer and Romer (2016) end in 1991, because since then the cost-of-living adjustments in transfers were automatic. Despite this fact, Parraga-Rodriguez (2018) extended this sample to 2007. To remove the predictable component of the series she regresses the extension of the narrative series on inflation and uses the residuals of this regression as shocks in proxy SVAR. She finds an impact multiplier of permanent transfers of 0.2 and a cumulative multiplier as high as 2.8. Parraga-Rodriguez (2016) estimates a dynamic panel model for the EU countries using the data from a confidential dataset compiled by public finance experts from the European System of Central Banks on discretionary fiscal actions. She finds that the output multiplier related to changes in old-age pensions is 0.5 on impact with a maximum cumulative response close to one.

This chapter extends the existing work in several dimensions. First, the model is estimated using local projections state-dependent model which recently became quite popular in evaluating the impact of fiscal policy on macroeconomic developments. The method allows to easily incorporate different regimes under which the policy operates. Second, we estimate the impulse responses of variables that were missing in the original Romer and Romer (2016) study and in particular that of debt, which allows us to explicitly analyze the self-financing nature of transfers. Third, we also estimate the transfers multipliers under different unemployment regimes.

This chapter is also a companion empirical investigation to our theoretical work on self-financing transfers presented in the first chapter of this thesis. We have shown there that in a model with the cash-in-advance constrained households and downward nominal wage rigidity, an increase in transfer payments is likely to be partially self-financing and the degree of this phenomenon depends

on the depth of the recession. Social security payments considered here are transfers to a specific group of people who cannot adjust their labor supply. However, an increase in transfers relaxes their financial constraints, increases consumption and under the assumption of demand determined output also employment and payroll tax revenue to social security system. In general, if the US social security system has a surplus, it invests in intra-governmental bonds, if it has a deficit, it redeems these bonds. Now, if an increase in transfers is fully self-financing, it generates extra payroll tax revenue to pay for it. If not, any deficit is covered from redeemed bonds which in turn must be financed by standard borrowing from the public. Hence, if higher transfers are not self-financing, they contribute to extra debt in a consolidated government budget constraint.

The rest of this chapter is organized as follows. In the next section we present our data and Romer and Romer (2016) shocks. In section 2.3 we discuss our estimation methodology. In section 2.4 we present and interpret our results. Section 2.5 concludes.

2.2 Data

The impulse response functions and resulting multipliers are estimated on quarterly data for the U.S. from 1959Q2 to 1991Q4. The time frame is constrained by data availability - the quarterly data for consumption start only 1959 and Romer and Romer's (2016) shocks end in 1991. The dependent variables include GDP, unemployment, consumption, debt and transfers, with their lagged values used as controls in the regression (see section 2.3). GDP is the Real Gross Domestic Product divided by Total Population, which results in real GDP per capita. Unemployment is Civilian Unemployment Rate in percent. Consumption is Personal Consumption Expenditures divided by GDP Implicit Price Deflator and Total Population, which results in real consumption per capita. Debt is Market Value of Gross Federal Debt divided by GDP Implicit Price Deflator, which results in real debt. Transfers are Personal current transfer receipts: Government social benefits to persons: Social security divided by Total Population, which results in transfers per capita. As additional controls we also use government spending which is Government Consumption Expenditures and Gross Investment divided by GDP Implicit Price Deflator and Total Population, which results in real government spending per capita and nominal wages which is Compensation of employees, paid: Wages and salaries divided by Total population, which results in nominal wages per capita. GDP, consumption, debt, transfers, government spending and nominal wages are logged. All variables come from the FRED database.

The shocks used to compute the impulse response functions are the shocks series identified by Romer and Romer (2016) based on their analysis of fiscal policy documents relevant for social security, in particular the *Social Security Bulletin*, providing the information on the size, timing, permanence and motivation of the measure. These shocks series include exogenous changes in aggregate social security payments and exclude those increases in transfers made for explicitly

countercyclical reasons. One-time payments were classified as temporary, while permanent cost-of-living adjustments and the extensions of benefits were classified as permanent. The primary motivation for permanent changes was adjusting for inflation and expanding the coverage, as well as the equity and fairness considerations. Temporary changes in transfers were related to catching up with inflation, the recalculation of benefits and adjustments for miscalculations. The shocks expressed as a percent of personal income are depicted in Figure 2.1. As we can see, the increases in transfer payments were infrequent, irregular and somewhat erratic, so there is a substantial variation to exploit.

Figure 2.1: Romer and Romer (2016) shocks

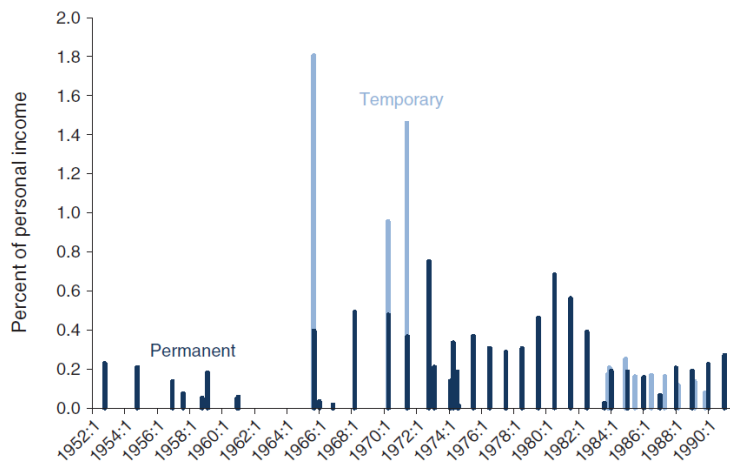
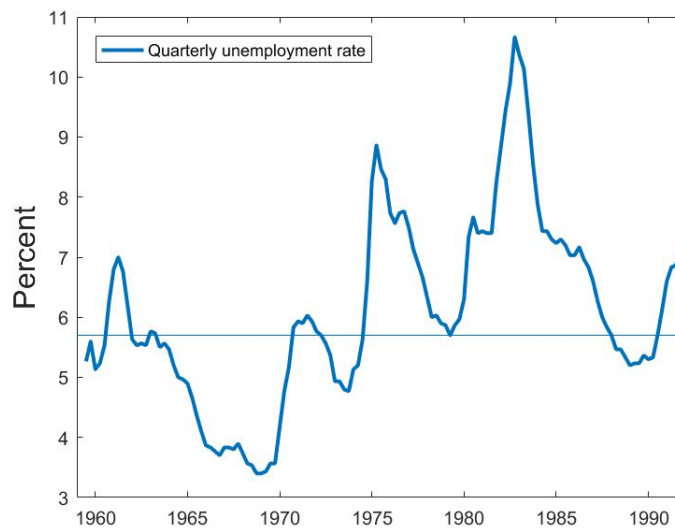


Figure 2.2: Quarterly unemployment rate in the US



Since our theory presented in the first chapter of this thesis predicts that transfer payments should have more pronounced effects when there is more slack in the economy, we also estimate a state-dependent model. The two regimes considered are high and low unemployment, depending on whether unemployment is above or below the sample average (5.7 percent), respectively. Figure 2.2 presents unemployment development over the sample period considered. We can see that unemployment was having a downward trend in the 1960s, an upward trend in the 1970s and a downward trend again in the 1980s. The economy was in the low unemployment regime mostly during the first half and in the high unemployment regime during the second half of the period considered.

2.3 Estimation methodology

The impulse response functions of the variables of interest are estimated using the local projections method proposed by Jordà (2005) which has become quite popular recently due to its undeniable advantages. The method does not require to impose dynamic restrictions to estimate the impulse response functions as in vector autoregressions (VARs) and is very flexible in terms of estimating state-dependent and non-linear effects of shocks. The impulse responses can be estimated using simple regression techniques such as ordinary least squares.

Linear model specification takes the following form:

$$x_{t+h} = \alpha_h + \beta_h \varepsilon_t + \psi_h(L)z_{t-1} + trend_t + u_{t+h}$$

where x_{t+h} is the variable of interest, α_h is a constant term, β_h is the coefficient of the impulse response, ε_t is the shock, $\psi(L)$ is the polynomial of the lag operator, z_{t-1} is the vector of controls, $trend_t$ is the time trend and u_{t+h} is the error term. The estimated coefficient β_h captures the response of x at $t+h$ to an unanticipated shock at t . In order to estimate the impulse responses at horizon h one needs to estimate $h+1$ regressions.

The state-dependent model features two different states depending on the situation on the labor market distinguished using the indicator function with $\mathbb{1}_t^{High} = 1$ if unemployment is high and $\mathbb{1}_t^{High} = 0$ if unemployment is low. Transfers should have more pronounced effects when there is more spare capacity in the economy. We define high unemployment as the periods when the unemployment rate is above the average of the sample (5.7 percent). The state-dependent model specification takes the following form:

$$x_{t+h} = \mathbb{1}_{t-1}^{High} [\alpha_{h,High} + \beta_{h,High} \varepsilon_t + \psi_{h,High}(L)z_{t-1}] + (1 - \mathbb{1}_{t-1}^{High}) [\alpha_{h,Low} + \beta_{h,Low} \varepsilon_t + \psi_{h,Low}(L)z_{t-1}] + trend_t + u_{t+h}$$

where $\mathbb{1}_t^{High}$ is the indicator function, x_{t+h} is the variable of interest, $\alpha_{h,High}$ is a constant term when unemployment is high, $\alpha_{h,Low}$ is a constant term when unemployment is low, $\beta_{h,High}$ is the coefficient of the impulse response when unemployment is high, $\beta_{h,Low}$ is the coefficient of the impulse response when unemployment is low, ε_t is the shock, $\psi_{h,High}$ is the polynomial of the lag operator when unemployment is high, $\psi_{h,Low}$ is the polynomial of the lag operator when unemployment is low, z_{t-1} is the vector of controls, $trend_t$ is the time trend and u_{t+h} is the error term. The indicator function is indexed at $t-1$ so that the state of the economy under consideration does not depend on contemporaneous impact of the shock.

The variables for which the impulse response functions are estimated include log real GDP per capita, log real consumption per capita, unemployment rate and log real debt. The controls include lagged values of log real GDP per capita, log real consumption per capita, unemployment rate and log real debt, as well as log real government spending per capita and log nominal wages per capita. Four lags are considered. The time trend is cubic. The model is estimated on quarterly data using the ordinary least squares. Since this estimation results in serial correlation of the standard errors, Newey-West robust standard errors for estimated coefficients are used. The cumulative multipliers are estimated by dividing the sum of estimated impulse responses β_h of GDP by the sum of estimated impulse responses of transfers multiplied by the average share of transfers in GDP in the sample.

2.4 Results

The estimation results are presented on Figures 2.3-2.6.

Figure 2.3: Temporary transfers shock - linear model

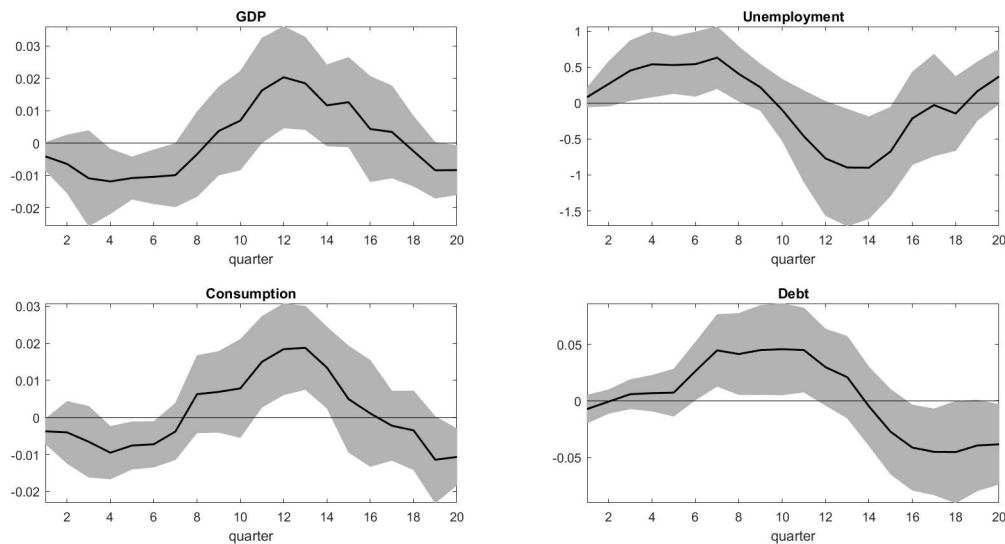


Figure 2.3 presents the estimated impulse response function to temporary transfers shock in the linear model. We can see that a one-percent increase in temporary transfers initially *reduces* GDP by around one percent and increases unemployment by 0.5 percentage point. This could be associated with the anticipation of higher taxes needed to finance an increase in the benefits. It may be also related to the fact that the increases in transfer payments mainly to compensate for inflation were paid after the periods of the economic boom when the economy was already slowing down. Even though not used for countercyclical reasons, transfer payments still relaxed the financial constraints and allowed to mitigate an increase in unemployment as in our first thesis chapter, which can be also seen in the state-dependent model. If it had not been for the transfers, a rise in unemployment would have probably been much higher. The stimulus has positive effects only after around two and a half years. At its peak, GDP increases by 2 percent, while unemployment falls by 1 percentage point. The consumption response follows that of GDP, decreasing by 0.5 percent and later increasing by 1.5 percent until the effects die out. Contrary to the permanent income hypothesis, transitory changes in income are not spread over the life cycle, but have a substantial positive impact on consumption which arrives after the initial fall, though. Debt increases by 5 percent and then gradually falls possibly as a result of extra tax revenue generated thanks to the stimulus, although the fall is barely statistically significant.

Figure 2.4: Temporary transfers shock - state-dependent model

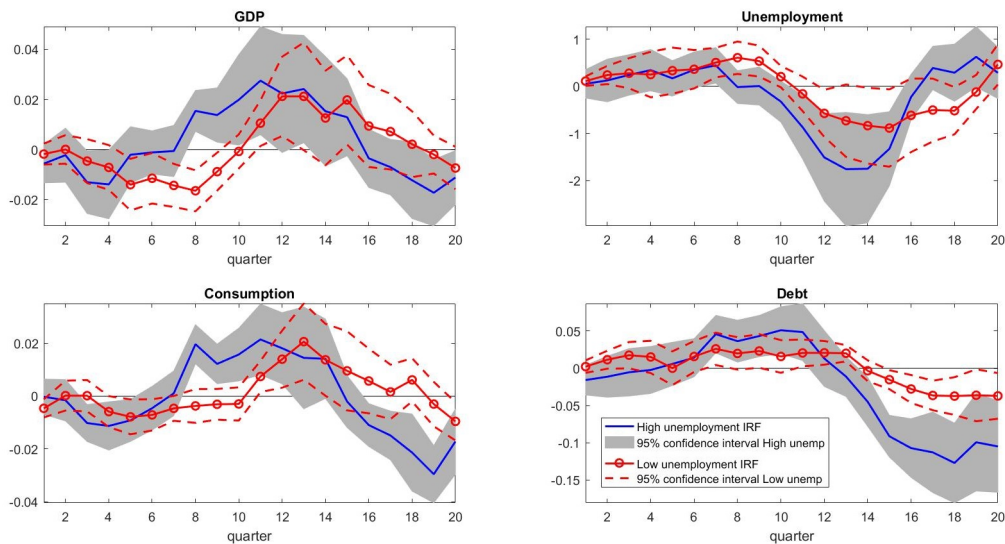


Figure 2.4 presents the estimated impulse response function to temporary transfers shock in the state-dependent model. In the high unemployment regime initially GDP also falls as in the linear model, but this response is only slightly significant, while unemployment is unaffected. As before we could attribute a slight fall in GDP due to the fact that transfers were increased after the economic boom when the economy was slowing down. Transfers relaxed the financial constraints

and mitigated negative developments in the labor market. If it had not been for the transfers, unemployment would have probably increased. These initial periods in the high-unemployment regime resemble figure 1.5 from the first chapter, where higher transfers alleviate the negative consequences of the economic slowdown. After less than two years GDP starts increasing and reaches its peak at a 3 percent increase. At the same time unemployment falls as much as 2 percentage points. Consumption follows the GDP pattern and increases at best by 2 percent. We also observe a decline in consumption at the end of the analysis horizon. Debt increases by 5 percent, but then after 4 years once the economy recovers, it falls by remarkable 10 percent, potentially due to the additional tax revenue generated by the stimulus. This suggests that a temporary increase in transfer payments when unemployment is high is not only self-financing but actually reduces debt in the medium-term horizon.

In the low unemployment regime GDP falls significantly by around 2 percent and recovers only after 3 years. An increase of 2 percent is, however, only marginally significant. Transfers are less powerful here as there is less slack in the economy. For the first three years unemployment is not affected and later slightly falls, but again this is marginally significant. Consumption and debt are also practically not affected for three years and later consumption increases by two percent, while debt falls by 2.5 percent. This suggests that under low unemployment temporary transfers increases have less significant effects, are associated with a fall in GDP initially and do not reduce debt as much as possible when the labor market is slack.

Figure 2.5: Permanent transfers shock - linear model

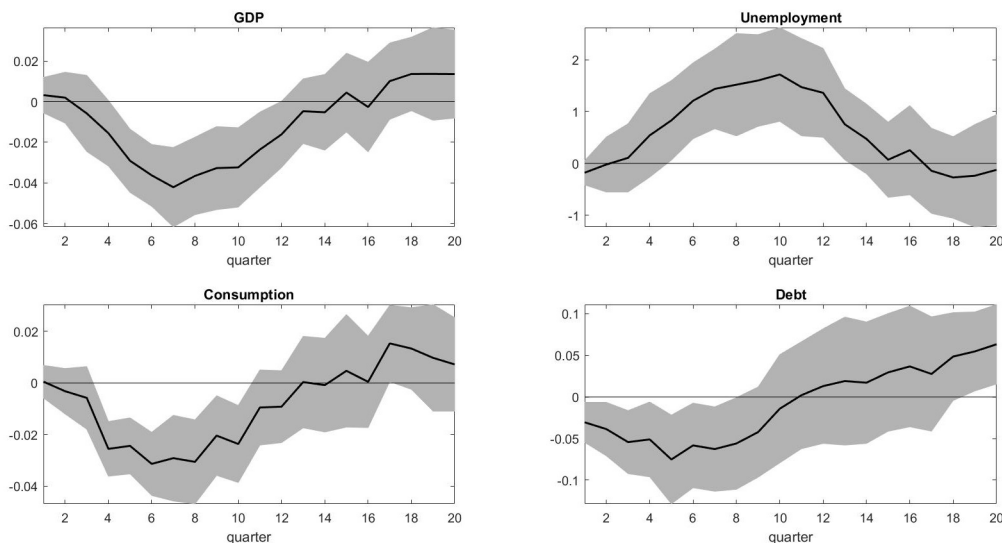


Figure 2.5 presents the estimated impulse response function to permanent transfers shock in the linear model. We can see that a one-percent increase in permanent transfers reduces GDP by around 3 percent and increases unemployment by 1.5 percentage point. Subsequent increase in GDP

is only marginally significant, while a reduction in unemployment is not statistically significant. Consumption follows GDP and falls by 2.5 percent at the trough. Contrary to the permanent income hypothesis, permanent increases in income not only do not translate one to one into a permanent rise in consumption, but actually reduce it, possibly because of the anticipation of higher taxes needed to finance the benefits in the future. As before, since these transfers were distributed mainly to compensate for inflation after the economic booms, if they had been lower, we would observe even more pronounced negative effects. As the theoretical model from the first chapter predicts, transfers mitigate these effects by relaxing the financial constraints and making the labor market recovery faster. We see that a temporary fall in the debt is followed by a gradual increase, as the permanent fiscal burden builds up. However, this pattern is only marginally significant.

One might be concerned about a negative response of consumption to the positive transfers shock. However, this result is actually consistent with what Romer and Romer (2016) report in the on-line appendix to their paper when they use the same sample period as we do (their Figure B1). Unfortunately they only report first four quarters of the response. Moreover, Párraga Rodríguez (2018) in the appendix of her paper also finds a negative response of output after a slight increase for the sample period we are interested in (her Figure A9). In the appendix 2.6.2 of this chapter we estimate the impulse response functions using an alternative approach - a structural vector autoregression model where the shock to transfers is identified using the short-run restrictions and we find a similar pattern of impulse responses.

Figure 2.6: Permanent transfers shock - state-dependent model

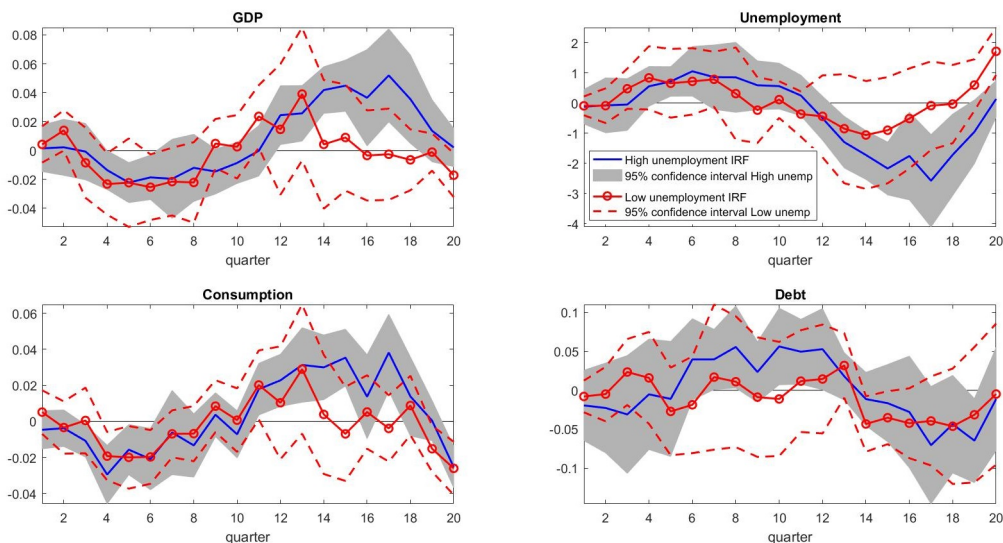


Figure 2.6 presents the estimated impulse response function to permanent transfers shock in the state-dependent model. In the high unemployment regime GDP clearly falls by around 2 percent and recovers only after 3 years increasing by 4 percent at its peak. Unemployment has a reverse

impulse response - it increases initially by 1 percentage point and later falls by 2 percentage points. Consumption follows GDP, but seems to be a bit more volatile. After a fall of 2 percent we observe an increase of a similar magnitude. Again, one could argue through the lens of the theoretical model from the first chapter that had the transfers not been increased, the initial negative effects would have been much larger. The response of debt is largely insignificant with a marginal fall at the end of the analysis horizon, possibly again due to extra tax revenue generated. In the low unemployment regime GDP, unemployment and debt are practically unaffected by an increase in transfer. Consumption drops by 2 percent one year after the stimulus and its subsequent increase is not significant.

From the estimated impulse responses one can draw the following lessons. As suggested by the theory from the first chapter, transfers payments have more pronounced effects when the labor market is slack. When it is tight permanent changes in transfers have negligible effects. Permanent transfers also cause a significant fall in GDP even when unemployment is high until their bring about stimulative effects, while temporary transfers practically do not cause the initial negative effects. This suggests that temporary transfers are more suitable for stimulative purposes. Still, the positive effects of the stimulus are only observed after at least two years. One could argue that if it had not been for transfers, the negative effects of the economic slowdown would have been much more prominent.

When it comes to debt, temporary transfers seem to reduce it after an initial increase thanks to the additional tax revenue generated, while permanent transfers do not have significant effects on debt. The reduction in debt is quite substantial in periods of high unemployment, which suggests that temporary transfers are not only self-financing, but can actually reduce debt. A significant increase in debt after an increase in temporary transfers in the low unemployment regime is not observed.

The estimated cumulative multipliers are presented in Table 2.1.

Table 2.1: Estimated cumulative multipliers

	Linear model	High unemployment	Low unemployment
Temporary transfers	0.1010	3.6299	0.1984
Permanent transfers	-0.3518	-1.7019	-0.3020

We can see that in case of temporary transfers the estimated multipliers are positive. In the linear model the multiplier is quite small, it takes a value of 0.10. However, in the high unemployment regime this multiplier is much higher and reaches 3.63, while in the low unemployment regime it equals 0.20. In case of permanent transfers the estimated cumulative multipliers are negative.

For the linear model and in the low unemployment regime this is because the cumulative response of GDP was negative, which results in the estimated multipliers of -0.35 and -0.30, respectively. In case of high-unemployment regime the cumulative response of GDP is positive, but the cumulative response of transfers themselves to the shock is negative, which results in negative multiplier of -1.70⁴.

One could be concerned that the estimated impact of transfers shocks on debt is not only the result of stimulative effects brought about by an increase in transfer payments and resulting expansion of the tax base, but also legislative changes in taxes. In order to address this concern we reestimate the model using tax changes identified by Romer and Romer (2010) as additional control variables. We use both all tax changes and those explicitly introduced to finance the social security needs. We find that the results of the estimation are practically not affected.

2.5 Conclusions

In this chapter we have estimated the impulse response functions of GDP, unemployment, consumption and debt to an increase in transfer payments on quarterly data from 1959Q2 to 1991Q4 using the local projection method and exogenous transfers shocks identified by Romer and Romer (2016). We have found that the stimulus in a form of higher transfers has more pronounced effects when unemployment is high than when it is low, hence when there is more spare capacity in the economy, confirming our theory from the first chapter. Permanent transfers initially cause GDP to fall and unemployment to rise before stimulative effects arrive, which is practically not observed in case of temporary transfers. Permanent transfers seem not to affect debt, while temporary transfers seem to reduce it after an initial increase, especially in the high unemployment regime, potentially thanks to the additional tax revenue generated by the tax base expansion. The estimated cumulative multipliers are positive in case of temporary transfers - with the multiplier as high as 3.62 in the high unemployment regime - and negative in case of permanent transfers.

Our analysis suggests that temporary transfers are more suitable for stimulative purposes, as they do not cause negative effects observed in the case of permanent transfers and mitigate better the effects of the economic slowdown. The stimulus should be only provided in the periods when unemployment is high. It seems that an increase in temporary transfers is not only self-financing, but actually reduces debt when the economy recovers. It is important to note that positive stimulative effects arrive only after two years, which reduces the usefulness of this policy for fast recovery.

⁴The impulse responses of transfers to temporary and permanent shocks are presented in the appendix 2.6.1 to this chapter.

2.6 Appendix

2.6.1 Impulse response functions of transfers

Figure 2.7: Impulse response of transfers to the temporary transfers shock - linear model

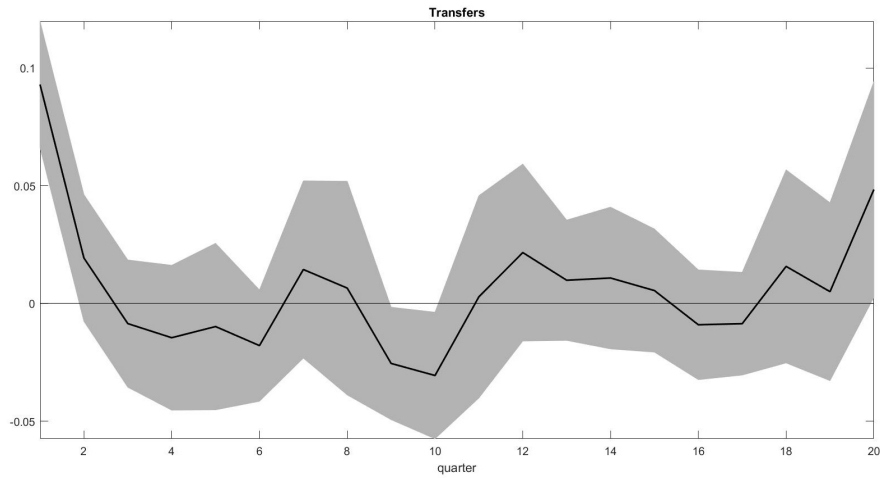


Figure 2.8: Impulse response of transfers to the temporary transfers shock - state-dependent model

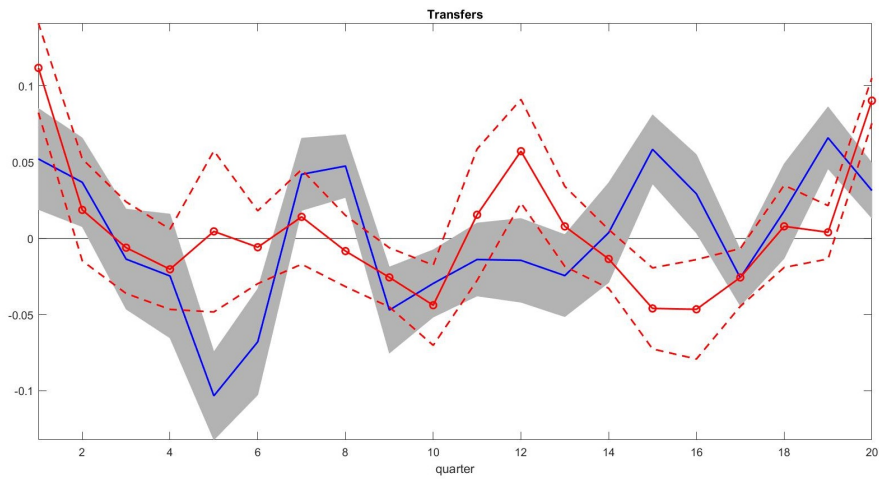


Figure 2.9: Impulse response of transfers to the permanent transfers shock - linear model

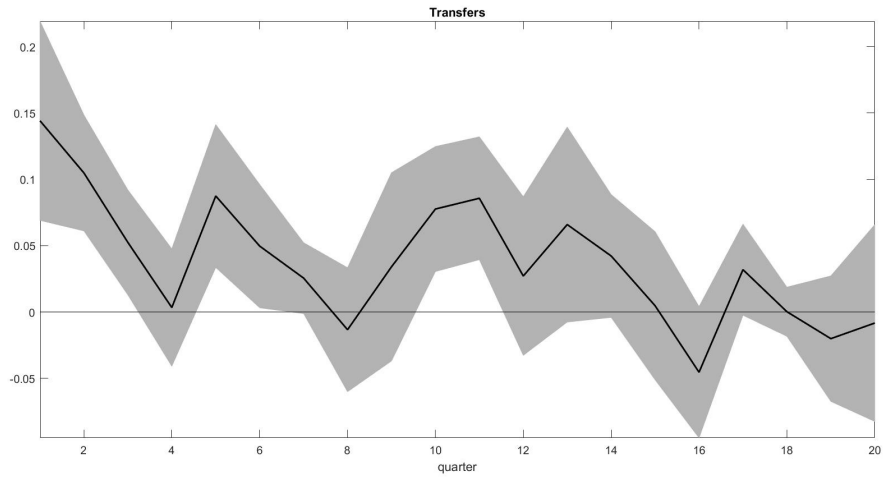
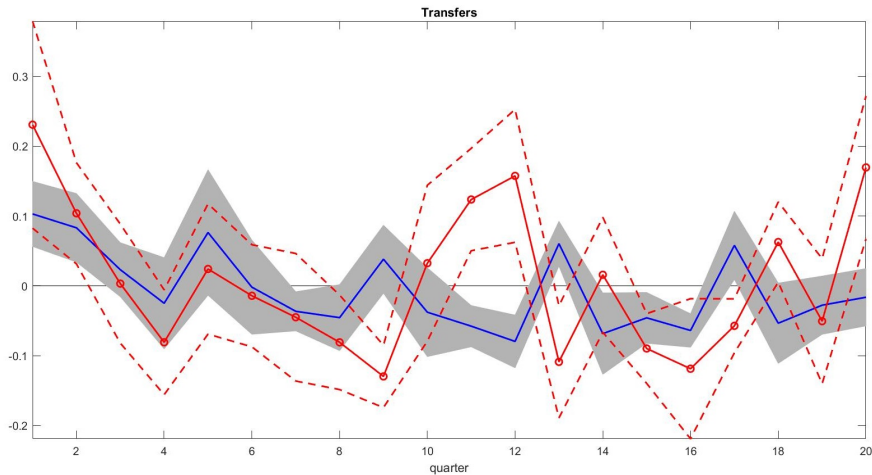


Figure 2.10: Impulse response of transfers to the permanent transfers shock - state-dependent model



2.6.2 Structural VAR analysis

As a robustness check in this subsection we estimate a structural vector autoregression (VAR) model with four lags using the same dataset as in the main text. Hence, the time series used are GDP, unemployment, consumption, debt and transfer payments. We also control for government spending and taxes. Structural model is given by

$$AY_t = B(L)Y_t + u_t$$

where Y_t includes transfers, government spending, taxes, GDP, unemployment, consumption and debt, u_t is a vector of structural shocks and $A, B(L)$ are matrices of coefficients. The reduced form

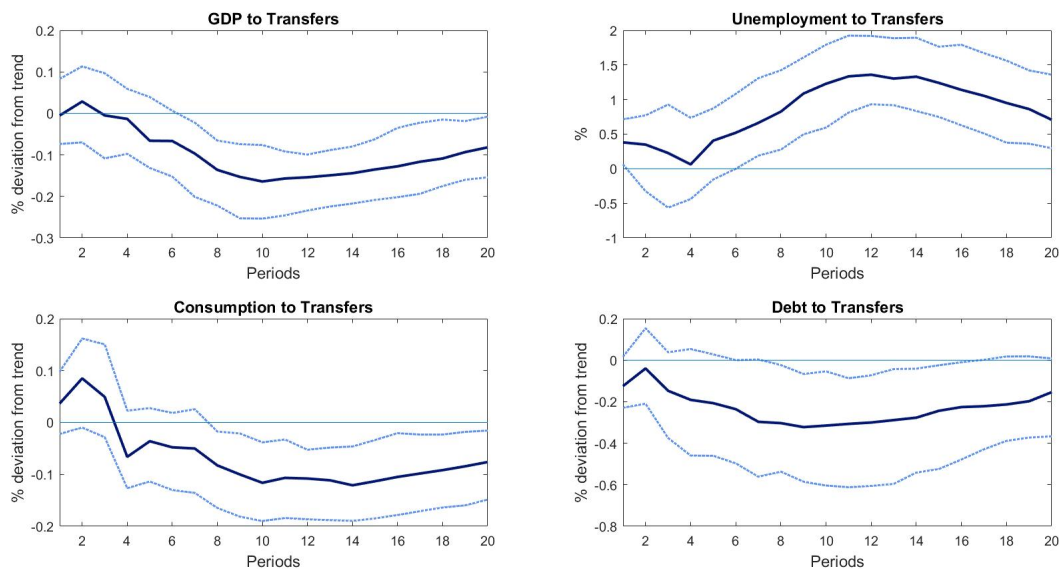
of the model is given by

$$Y_t = C(L)Y_t + \varepsilon_t$$

where $\varepsilon_t = A^{-1}u_t$ is a vector of reduced form residuals and $C(L) = A^{-1}B(L)$ is a matrix of coefficients. The reduced form model can be estimated with ordinary least squares.

We identify the transfers shock using short-run restrictions as in Blanchard and Perotti (2002), that is we recover structural shock from reduced form residuals by Cholesky ordering of variables where the policy variable is ordered first. The assumption here is that it takes time for policy makers to implement the policy and therefore transfers do not respond to shocks to other variables contemporaneously. We have also experimented with different ordering of policy variables - with government spending and taxes ordered first and transfers second - but the results are similar under different specifications. The impulse response functions of the variables of interest are presented on Figure 2.11.

Figure 2.11: Impulse responses to transfers shocks identified in a VAR model using short-run restrictions



We can see that the impulse responses obtained in this specification resemble those obtained from local projection of permanent transfers shocks identified by Romer and Romer (2016). In particular, GDP and consumption fall and unemployment rises as a response to transfers shock. In case of the first two variables the order of magnitude of the reaction is smaller than in the case of local projection. There is a slight increase in consumption and GDP in the first quarters, but it is not significant. Finally, transfers shocks seem to reduce debt, but to a lesser extent than in the case of local projection estimates.

Chapter 3

Total factor productivity and the terms of trade

Abstract

In this chapter we analyze how the terms of trade (TOT) - the ratio of export prices to import prices - affect total factor productivity (TFP). We provide empirical macroeconomic evidence based on the times series SVAR analysis and microeconomic evidence based on industry level data from the European Union countries which shows that the terms of trade improvements are associated with a slowdown in the total factor productivity growth. Next, we build a theoretical model which combines open economy framework with the endogenous growth theory. In the model the terms of trade improvements increase demand for labor employed in exportable goods production at the expense of technology production (research and development - R&D) which leads to a shift of resources from knowledge development towards physical exportable goods. This reallocation has a negative impact on the TFP growth. Under a plausible calibration the model is able to replicate the observed empirical pattern.

3.1 Introduction

Terms of trade (TOT) - the ratio of export prices to import prices - is one of the most important variables in open economies. Studies (Mendoza, 1995, Kose, 2002) show that the terms of trade shocks are important drivers of business cycles and explain significant fraction of output variability. Total factor productivity (TFP), often treated as an exogenous process, is a key driving force of growth models, while TFP shocks play a crucial role in business cycles models. In this chapter we analyze whether and how total factor productivity in an open economy responds to changes in the terms of trade. This inquiry allows to improve the understanding of TFP determinants in open economies.

Clearly, the relationship between the terms of trade and TFP can work both ways. When TFP is treated as exogenous it cannot be affected by the terms of trade, while improvements in TFP decrease marginal costs and therefore reduce domestic prices. In this chapter, however, we focus on the reverse relationship and isolate the impact of the terms of trade on endogenously determined TFP. One may think of two ways how TFP may be affected by the terms of trade. On the one hand, given limited resources improvements of the terms of trade might increase the incentives to put more resources in physical goods production (as it is more profitable to produce goods for exports and imported inputs are cheaper) at the expense of spending on research and development (R&D), which slows down the TFP growth (substitutability channel). On the other hand, it might be the case that since an improvement in the terms of trade makes the whole economy richer it allows to expand both physical goods production and R&D activities (complementarity channel).

We show that the first channel is more empirically relevant (the terms of trade improvements slow down the TFP growth) and explore a substitution between physical goods production and investment in research and development. In our setting once the terms of trade improve, a country exports more and shifts resources away from knowledge production sector, which decreases the TFP growth. On the contrary, when for instance foreign competition drives down the prices of goods which a country sells (its exports) more investment in productivity is needed and desired in order to break even.

The research question of this chapter - whether changes in the terms of trade explain total factor productivity development - is addressed both empirically and theoretically. First, we test the relationship between the terms of trade and TFP in twelve European Union (EU) open economies using a structural vector autoregressive (SVAR) model applied to these macroeconomic time series from the OECD database and show that on impact detrended TFP responds negatively to the positive structural terms of trade shocks. We also provide microeconomic evidence based on industry-level data from the Competitiveness Research Network (CompNet) database and show that improvements in the terms of trade are associated with a slowdown in TFP growth at the level of particular sectors in the EU countries considered.

Next, we show how this empirical pattern can be explained in a theoretical framework. We build a model which combines open economy framework including importable, exportable and nontradable goods with the endogenous growth theory. Open economy setting allows us to gain additional insights of how and when technology determining TFP is developed. In the model there is a separate knowledge-producing sector. Terms of trade improvements increase demand for labor in physical exportable goods production at the expense of labor employed in R&D sector. In the latter employment decreases, which has a negative impact on TFP.

Finally, we ask how well this theoretical model matches with the empirical evidence. We show that under a plausible calibration the model produces the desired responses and is able to replicate the above-mentioned empirical relationship. At the same time since the terms of trade shocks

increase output and decrease endogenously-determined component of TFP (via lower R&D employment) the latter is countercyclical in the model. This is at odds with the data in which both TFP and R&D are procyclical. The negative correlation between endogenous component of TFP and output is a result of the terms of trade shocks studied in isolation. Once the exogenous TFP shocks are included in the model, the positive correlation between output and overall TFP is restored, while the terms of trade shocks only weaken it.

This chapter relates to several strands of literature. Our theoretical model builds upon the endogenous growth literature which endogenizes technological change process. In the spirit of the seminal contributions by Romer (1986, 1990) the technological progress in our model is a result of profit maximizing behavior. The structure of the economy which features a physical goods production sector and a separate knowledge producing sector relates our work to Uzawa (1965), Lucas (1988) and Rebelo (1991). In our setting this framework is embedded into an open economy model with importable, exportable and non-tradable goods as in Schmitt-Grohé and Uribe (2018) which builds on the classic work of Mendoza (1995). The main theoretical contribution of our chapter comes from combining these two strands of literature to explain how TFP endogenously responds to changes in the terms of trade.

This chapter provides also empirical evidence on the impact of the terms of trade on total factor productivity. Empirical literature on endogenous determinants of total factor productivity is extensive. Closely related to our work Miller and Upadhyay (2000) using macroeconomic evidence show that higher openness, more outward orientation and higher human capital have significant positive effects on total factor productivity. Similarly Alcalá and Ciccone (2004) find that international trade has an economically significant and statistically robust positive effect on productivity. More recently Mayer, Rüth, and Scharler (2016) using sign restrictions in SVAR framework show that total factor productivity responds endogenously to exogenous spending demand shocks.

The impact of the terms of trade on TFP did not raise too much attention in the literature so far. Notable exception is the paper by Kehoe and Ruhl (2008). They start with an observation that theoretically the terms of trade shocks seem to have equivalent effects to that of productivity, while from purely accounting point of view changes in the terms of trade do not affect real GDP nor TFP calculated as the residual from real output after subtracting the contribution of properly deflated inputs¹. However, this observation is inconsistent with the empirical correlations between the terms of trade and TFP they document which calls for a mechanism capturing any possible causal relationship between the two. Kehoe and Ruhl analyze the data for the United States and Mexico and find that sharp deteriorations in the terms of trade were accompanied by drops in real GDP most of which were driven by drops in TFP. However, they also show that in case of

¹The impact of terms of trade measurement on TFP growth is also a theme of Feenstra et al. (2013) who claim that once the terms of trade are overestimated, this results in higher than actual TFP growth. Since we find a negative relationship between the two, this actually strengthens our findings - when the terms of trade are overstated, if they were properly measured, TFP growth will be even lower after their improvement than what we find.

Switzerland, for instance, the terms of trade improvements were associated with declines in GDP and TFP, so their evidence is inconclusive. Additional evidence can be found in Gopinath and Neiman (2014) who using a calibrated model show how during the Argentine crisis an increase in import prices (i.e. worsening of the terms of trade) of intermediate inputs led to a significant decline in productivity. Our investigation differs in terms of method and findings - we apply the structural VAR framework and find the opposite effect: the terms of trade improvements are associated with declines in detrended TFP in the European Union countries. The closest to our work is the paper by Bardález and Zea (2014) who also estimate the impact of the terms of trade on TFP using the long-run restrictions in a structural VAR model for Chile, Mexico and Peru to find that the terms of trade improvements have a positive impact on TFP. These authors, though, use the TFP series obtained from the Kalman filter and do not have endogenous technological change in their model.

Microeconomic evidence on the possible effects of the terms of trade on TFP is only indirect. Galdon-Sanchez and Schmitz (2002), Schmitz (2005) and Dunne, Klimek, and Schmitz (2010) using industry-level data show how competitive pressure driving down the prices increased total factor productivity in iron-ore and cement industries. More recently Alfaro et al. (2017) analyze the effects of the real exchange rate (the relative price of foreign basket in terms of domestic baskets) and show that these effects are not uniform. In Asian emerging countries real depreciations improve TFP at the firm level, while the opposite is true for European emerging economies. They do not find significant effects in case of developed countries. Our study is the first one to analyze the effects of aggregate TOT shocks on industry level TFP.

The main finding of this chapter - terms of trade improvements slowing down the TFP growth - resembles the resource curse (the Dutch disease): discovery of natural resources may have a negative impact on economic performance. This literature is reviewed in Frankel (2010) and Ploeg (2011). Recently, this issue in a different variant was studied by Benigno and Fornaro (2014) who show that similar effects (weak productivity growth) might be related to abundant access to foreign capital. Easy credit expands consumption and while additional tradable goods can be imported, the production of nontradable goods needs to increase. In the model of Benigno and Fornaro productivity growth is increasing in labor employed in tradable sector, so shifting productive resources away from this sector deteriorates productivity. In our setting similar effects are associated with the terms of trade improvement - resources are shifted away from R&D into physical goods production which slows down the productivity growth.

The rest of this chapter is organized as follows. In the next section we present the empirical evidence showing how TFP reacts to changes in the terms of trade based on the data from the EU countries. Section 3.3 presents our theoretical model which combines open economy framework with endogenous growth models. In section 3.4 we calibrate and simulate the model to show its ability of replicating the empirical relationships. Section 3.5 concludes.

3.2 Empirical evidence

In this section we present our empirical evidence showing how total factor productivity responds to changes in the terms of trade. The first subsection below deals with macroeconomic evidence, the next one - with microeconomic evidence. In the third subsection we discuss some evidence on the relationship between R&D spending and the terms of trade.

3.2.1 Macroeconomic evidence

In this subsection we analyze how the overall country TFP responds to changes in the terms of trade. We test it using a structural bivariate VAR model with two lags. We estimate the VAR system using quadratically-detrended time series of the total factor productivity index (a residual of the change in aggregate output that cannot be accounted for by the change in combined inputs) and the terms of trade index (the ratio of the price index for exports of goods and services to the price index for imports of goods and services). The annual data used include the period 1985-2016 and come from the OECD database. We estimate the model country by country for Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal, Spain, Sweden and the United Kingdom. The choice of the countries was determined by the data availability. All analyzed countries are relatively open economies and the European Union member states, hence to some extent homogeneous. The share of their trade in GDP is presented in Table 3.7. in the appendix 3.6.1. of this chapter.

We identify the structural shock using the long-run restrictions (Blanchard and Quah, 1989). We impose the restriction that TFP in the long-run can only be affected by its own shocks while the terms of trade shock is assumed to have no long run effects on TFP². Hence the VAR model takes the following form:

$$\begin{bmatrix} TFP_t \\ TOT_t \end{bmatrix} = \begin{bmatrix} \psi_{11}(L) & \psi_{12}(L) \\ \psi_{21}(L) & \psi_{22}(L) \end{bmatrix} \begin{bmatrix} \epsilon_t^{TFP} \\ \epsilon_t^{TOT} \end{bmatrix}$$

where TFP_t is total factor productivity, TOT_t are the terms of trade, $\psi_{ii}(L)$ are polynomials of the lag operator, ϵ_t^{TFP} is a structural TFP shock and ϵ_t^{TOT} is a structural terms-of-trade shock. We make the usual assumption that these shocks are orthogonal and serially uncorrelated. Our restriction corresponds to $\psi_{12}(1) = 0$ ³.

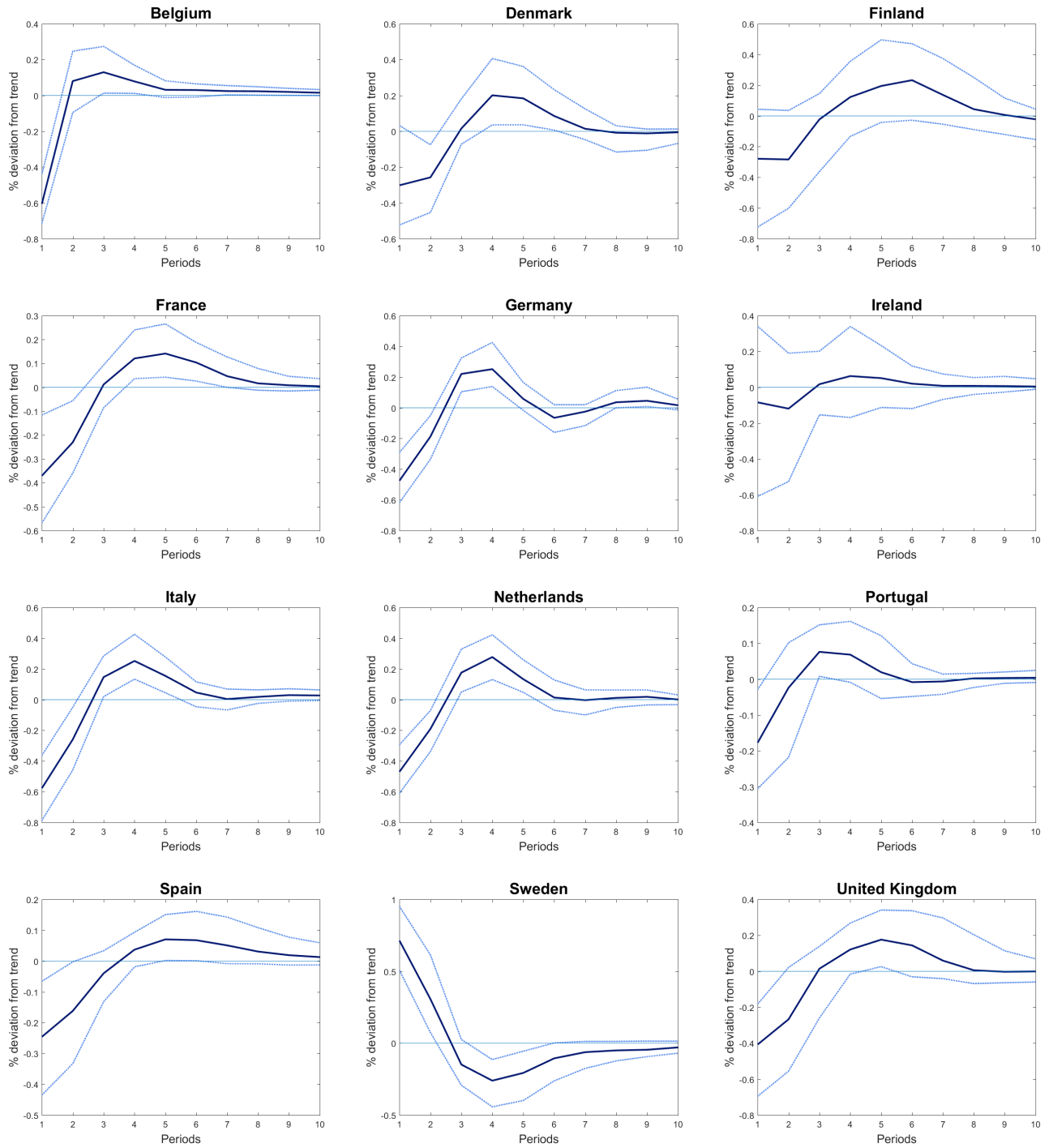
The impulse response functions of TFP to structural terms of trade shocks are presented in

²Alternative approach would be to use the short run restrictions. However, this is not suitable in our case given the annual frequency of our data and the fact that in order to identify the structural shocks this requires to assume that depending on the ordering one of the variables (either TFP or the terms of trade) is not affected by the other one (the terms of trade or TFP, respectively) contemporaneously – the assumption we are not willing to make.

³We have also tried a trivariate VAR by including GDP in the system. This did not affect significantly the impulse responses of TFP. Given the limited number of observations we have (31) including more variables in the system quickly results in running out of the degrees of freedom.

Figure 3.1.

Figure 3.1: Impulse responses of TFP to TOT shocks



Note: Solid line - impulse response. Dotted line - 68% confidence intervals.

Source: Author's calculations.

We can see from the graphs that for 9 out of 12 countries considered we find a negative and significant response of TFP to a positive structural shock to the terms of trade on impact⁴. Hence, improvements in the terms of trade have a negative impact on detrended TFP. In the next subsection we present microeconomic evidence capturing the same pattern.

3.2.2 Microeconomic evidence

In this subsection we analyze how the industry level TFP responds to changes in the terms of trade. In our empirical investigation we use the data from the Competitiveness Research Network (CompNet) which is a European Union firm-level based dataset and provides some moments of the distribution of available variables⁵. As a dependent variable we use a change in mean TFP in particular industries. TFP in the dataset is computed as Solow residual in production function of the real value added after subtracting the inputs of labor, materials and capital in real terms. Our dataset contains 22 manufacturing industries⁶ for 10 countries: Austria, Belgium, Estonia, Finland, Germany, Italy, Lithuania, Portugal, Slovenia and Spain in the period 1996-2012. Again, the choice of the countries was constrained by the data availability. All analyzed countries are open economies and the European Union member states, hence to some extent homogeneous. The share of their trade in GDP are presented in Table 3.7. in the appendix 3.6.1. of this chapter.

The estimated regression takes the following form

$$\Delta TFP_{sct} = \alpha + \beta \Delta TOT_{ct} + \eta_s + \nu_c + \gamma_t + \varepsilon_{sct}$$

where TFP_{sct} is the total factor productivity in time t , sector s and country c , TOT_{ct} are the terms of trade in time t and country c , η_s captures the sector fixed effect, ν_c captures the country fixed effect, γ_t captures the time fixed effect and ε_{sct} is the error term.

The data on the terms of trade - the ratio of the price index for exports of goods and services to the price index for imports of goods and services - are taken from the OECD database as before. As we can see, for all sectors at time t and country c face the same level of the terms of trade - there is more variation in the total factor productivity than in the terms of trade series. The regression is performed under the assumption that particular industries are unlikely to affect the overall country terms of trade index. Under this assumption there is no endogeneity problem and the terms of trade shocks can be treated as exogenous shocks to particular industries⁷.

⁴The confidence level is 68%, which is common in the literature, following Sims and Zha (1999).

⁵The data provider indicates that data collection rules and procedures across countries are different, and out of CompNet's control. Hence, despite all efforts made to improve sample comparability across countries some country samples might still suffer from biases. For a more detailed account of raw data characteristics and sample biases, please refer to the ECB Working Paper 1764 (Lopez-Garcia and Di Mauro, 2015)

⁶The list of industries is available in Table 3.8. in the appendix of this chapter.

⁷Unfortunately the data on the industry-specific terms of trade indexes are not available and calculating them is out of the scope of this chapter. Still, such a measure would clearly suffer from the endogeneity problem.

The regression results are presented in Table 3.1.

Table 3.1: Microeconomic evidence - regression results

Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Δ TFP	Δ TFP	Δ TFP	Δ TFP	Δ TFP	Δ TFP	Δ TFP	Δ TFP
Δ TOT	-.4950*** (.0544)	-.4950*** (.0543)	-.4959*** (.0555)	-.3068*** (.0721)	-.4958*** (.0555)	-.3109*** (.0714)	-.2861*** (.0761)	-.2863*** (.0762)
Sector dummies	NO	YES	NO	NO	YES	YES	NO	YES
Country dummies	NO	NO	YES	NO	YES	NO	YES	YES
Year dummies	NO	NO	NO	YES	NO	YES	YES	YES
Mean TFP	61.1572	61.1572	61.1572	61.1572	61.1572	61.1572	61.1572	61.1572
Number of obs.	2591	2591	2591	2591	2591	2591	2591	2591
R^2	0.0296	0.0613	0.0481	0.0804	0.0800	0.1126	0.0989	0.1311

Standard deviation in parenthesis. Legend: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

Source: Author's calculations.

As we can see the regression results suggest that improvements in the terms of trade are associated with a reduction in changes of TFP. This result is robust under different specifications, including various control variables - sector, countries and year dummies as well as their combinations. We can also interpret the results in the following way. Sectoral TFP improves when relative prices of goods a given country sells (exports) go down and when relative price of goods a given country buys (imports) go up. This is consistent with previous economic evidence suggesting that foreign competition driving down domestic tradable goods prices induces improvements in productivity.

Table 3.2. presents the results of robustness checks of the empirical analysis of the microeconomic data. The second column repeats the result from the last column of Table 3.1. for convenience. In the third column of Table 3.2. we can see that improvements in TOT reduce changes in TFP even after including non-manufacturing sectors in the sample⁸. However, this result does not hold when non-manufacturing sectors are considered separately.

In column (11) and (12) we test the importance of trade openness by using the exporters share in the industry as an explanatory variable in the regression. The share of exporting firms in the industry is taken from the Eurostat database. As we can see in specification (11), the higher the share of exporters in the industry, the greater are changes in the productivity. Once we interact changes in the terms of trade index with the share of exporters in (12), we can see that the greater

⁸The list of non-manufacturing industries is available in Table 3.9. in the appendix 3.6.1. of this chapter.

is the share, the more TFP worsens once the terms of trade improve, while the non-interacted coefficient is no longer significant. As one would expect, it is the engagement in international trade that drives TFP growth slowdown after the terms of trade improvements. In columns (13) and (14) we also include the lagged changes in the terms of trade in the regression, but only the second lag is significant. Finally, in column (15) in order to address potential endogeneity concerns we estimate the instrumental variables regression, where the change in the terms of trade is instrumented with its lagged value. We find that the negative relationship between TFP and the terms of trade is preserved under this specification.

Table 3.2: Microeconomic evidence - robustness checks

Sample	Manufact	All	Non-manufact	Manufact	Manufact	Manufact	Manufact	Manufact
Model	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	Δ TFP	Δ TFP	Δ TFP	Δ TFP	Δ TFP	Δ TFP	Δ TFP	Δ TFP
Δ TOT	-.2863*** (.0762)	-.1019* (.0509)	.0169 (.0653)	-.2927*** (.0765)	.1511 (.1300)	-.3058*** (.0811)	-.3712*** (.0852)	-.5467* (.2148)
Share of exporters				7.0495*** (1.7632)	6.8303*** (1.7556)			
Share of exporters x Δ TOT					-1.2060*** (.2863)			
Lagged Δ TOT (t-1)						.0470 (.0786)	.0560 (.0829)	
Lagged Δ TOT (t-2)							-.1768* (.0832)	
Sector dummies	YES	YES	YES	YES	YES	YES	YES	YES
Country dummies	YES	YES	YES	YES	YES	YES	YES	YES
Year dummies	YES	YES	YES	YES	YES	YES	YES	YES
Mean TFP	61.1572	55.4635	51.7071	62.3931	62.3931	61.1572	61.1572	61.1572
Number of obs.	2591	6295	3704	2563	2563	2591	2591	2591
R^2	0.1311	0.0678	0.0366	0.1390	0.1452	0.1387	0.1496	0.1277

Standard deviation in parenthesis. * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

Source: Author's calculations.

It might be also the case that when the terms of trade improve more less-productive firms with higher prices are able to sell their goods abroad. Since they are less productive overall TFP in the industry might fall because of these new entrants. Similarly, when the terms of trade worsen less productive firms exit and overall TFP increases. In Table 3.3 we test whether the negative relationship between changes in TFP and the terms of trade improvement is preserved conditional on industry dynamics using the World Bank Exporter Dynamics database. Unfortunately the data corresponding with our sample are available only for Germany (2011-2012), Portugal (2008-2012) and Spain (2007-2012) which decreases the number of observations. The results suggest that the number of new entrants/exiters and the change of new entrants/exiters do not affect our main

result.

Table 3.3: Microeconomic evidence - new entrants and exiters

Model	(16)	(17)	(18)	(19)	(20)
	Δ TFP	Δ TFP	Δ TFP	Δ TFP	Δ TFP
Δ TOT	-1.2218*	-1.2073*	-1.3103*	-1.2612*	-1.2381*
	(.5377)	(.5395)	(.5469)	(.5406)	(.5453)
New entrants		.0007			
		(.00019)			
Δ New entrants			-.00054		
			(.00062)		
Exiters				.00064	
				(.00065)	
Δ Exiters					.00105
					(.00126)
Sector dummies	YES	YES	YES	YES	YES
Country dummies	YES	YES	YES	YES	YES
Year dummies	YES	YES	YES	YES	YES
Mean TFP	62.2994	62.2994	62.2994	62.2994	62.2994
Number of obs.	260	260	260	260	260
R^2	0.1603	0.1616	0.1628	260	260

Standard deviation in parenthesis. * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

Source: Author's calculations.

To summarize, we can see that both macroeconomic evidence based on SVAR analysis and microeconomic evidence based on industry level data suggests that the terms of trade improvements are associated with a slowdown in the total factor productivity growth⁹. In the next section we describe the model which aims at explaining this phenomenon. First, however, we discuss the relationship between R&D spending and the terms of trade in the next subsection below.

3.2.3 Evidence on the relationship between R&D and the terms of trade

Since our channel of the terms of trade shock propagation to TFP is research and development activity, in this subsection we provide some evidence on how R&D correlates with the terms of

⁹Unfortunately our microeconomic sample is too short to repeat the SVAR analysis separately for manufacturing and non-manufacturing industries based on microeconomic data and obtain meaningful results.

trade. We use quadratically detrended data on R&D spending, i.e., the total expenditure on R&D carried out by all resident companies, research institutes, university and government laboratories and the terms of trade series from the OECD database. The results of the regression of R&D spending on the terms of trade are presented in Table 3.4.

Table 3.4: R&D on TOT regression

Country	Regression coefficient	Standard error	R^2	Sample period
Belgium	0.1729	1.2130	0.0009	1993-2016
Denmark	-0.8980	1.0650	0.0483	2001-2016
Finland	-1.6031**	0.5321	0.2323	1985-2016
France	0.5047	0.2476	0.1217	1985-2016
Germany	-0.2844	0.2231	0.0514	1985-2016
Ireland	-0.8496	0.5512	0.0734	1985-2016
Italy	0.3336	0.2688	0.0488	1985-2016
Netherlands	-0.1765	0.4769	0.0045	1985-2016
Portugal	-1.7011	0.8601	0.1154	1985-2016
Spain	0.9430*	0.4534	0.1260	1985-2016
Sweden	-0.9075	1.3203	0.0379	2003-2016
United Kingdom	0.2211	0.4163	0.0093	1985-2016

Source: Author's calculations.

As we can see, the relationship between the detrended R&D spending and the terms of trade is estimated to be negative in the majority of cases. However, in general it is not statistically significant. Clearly, there are many determinants of the R&D spending other than the terms of trade developments, which is also suggested by very low R^2 in some cases. Another caveat is the fact that the sample size for some countries (Belgium, Denmark and Sweden) is quite small. Unfortunately our microeconomic dataset (CompNet) does not include the data on R&D spending for individual firms.

3.3 Model

In this section we describe our model which combines open economy framework featuring importable (M), exportable (X) and non-tradable (N) goods with the endogenous growth theory. Importable goods are defined as goods that are domestically consumed, produced and imported, but not exported. Exportable goods are defined as goods that are domestically consumed, produced

and exported, but not imported. Nontradable goods are defined as goods that are domestically consumed and produced, but neither imported nor exported¹⁰.

Our model is a small open economy (SOE) model - a country takes export and import prices as well as world interest rate as given and faces perfectly elastic demand for goods it exports¹¹. We start with a description of households problem, exportable goods producer profit maximization and technology producer (R&D sector) profit maximization. The latter endogenously determines the technology level¹². Optimality conditions of these three agents allow us to describe our main mechanism in which the terms of trade improvements slow down the TFP growth.

In the subsequent subsections we describe the remaining elements of the model. We start with the maximization problems of the importable and non-tradable goods producers which are standard perfectly competitive firms. The introduction of the former is necessary to have relative prices of export goods to import goods, i.e., the terms of trade in the model. The introduction of the latter is needed to soften the effects of the terms of trade which otherwise would be implausibly large. In next subsections we describe the evolution of the debt interest rate and the terms of trade process, market clearing and the definition of competitive equilibrium. In the model importable good is treated as numeraire with its price $P_t^m = 1$.

3.3.1 Households

The model features a large number of identical households. At time t households choose consumption c_t labor supply to importables l_t^m , exportables l_t^x and nontradables l_t^n production sector, labor supply to technology production sector h_t , capital supply to importables k_{t+1}^m , exportables k_{t+1}^x and nontradables k_{t+1}^n production sector, and the level of future debt d_{t+1} to maximize the expected discounted lifetime utility

$$\max_{\{c_t, l_t^m, l_t^x, l_t^n, h_t, k_{t+1}^m, k_{t+1}^x, k_{t+1}^n, d_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t^m, l_t^x, l_t^n, h_t)$$

where E_0 is the expectations operator, $\beta \in (0, 1)$ is the subjective discount factor and $U(\cdot)$ is twice continuously differentiable utility function which is increasing and concave in consumption ($U_1 > 0, U_{11} < 0$) and decreasing and concave in labor supply ($U_2 < 0, U_{22} < 0, U_3 < 0, U_{33} < 0, U_4 < 0, U_{44} < 0, U_5 < 0, U_{55} < 0$),

subject to the sequential budget constraint (expressed in terms of importable goods)

$$p_t^f c_t + p_t^r d_t + p_t^f [k_{t+1}^m + k_{t+1}^x + k_{t+1}^n]$$

¹⁰Clearly those sectors cannot be easily mapped to the actual industries. However, this classification is a useful modelling device and is common in the literature.

¹¹Hence, when the terms of trade improve foreign demand for domestic goods does not fall.

¹²In the model setting we use the term technology and knowledge interchangeably.

$$= p_t^\tau \frac{d_{t+1}}{1+r_t} + w_t^m l_t^m + w_t^x l_t^x + w_t^n l_t^n + w_t^h h_t + r_t^{km} k_t^m + r_t^{kx} k_t^x + r_t^{kn} k_t^n + p_t^f (1-\delta)(k_t^m + k_t^x + k_t^n) - \tau_t$$

where p_t^f is the relative price of the final composite good, p_t^τ is the relative price of the composite tradable good, w_t^m, w_t^x, w_t^n are wages earned for work in importables, exportables and nontradables production sector, respectively, w_t^h are wages earned in technology production sector, $r_t^{km}, r_t^{kx}, r_t^{kn}$ are the rental income for capital services to importables, exportables and nontradables production sector, respectively, r_t is the interest rate on external debt and τ_t is the lump-sum tax, and the non-Ponzi scheme condition

$$\lim_{T \rightarrow \infty} \left(\prod_{i=0}^{T-1} (1+r_i)^{-1} \right) \frac{d_{T+1}}{1+r_T} = 0$$

The first order conditions of the households problem are given by

$$[c_t :] \quad U_1(c_t, l_t^m, l_t^x, l_t^n, h_t) = \lambda_t p_t^f \quad (3.1)$$

$$[l_t^m :] \quad -U_2(c_t, l_t^m, l_t^x, l_t^n, h_t) = \lambda_t w_t^m \quad (3.2)$$

$$[l_t^x :] \quad -U_3(c_t, l_t^m, l_t^x, l_t^n, h_t) = \lambda_t w_t^x \quad (3.3)$$

$$[l_t^n :] \quad -U_4(c_t, l_t^m, l_t^x, l_t^n, h_t) = \lambda_t w_t^n \quad (3.4)$$

$$[h_t :] \quad -U_5(c_t, l_t^m, l_t^x, l_t^n, h_t) = \lambda_t w_t^h \quad (3.5)$$

$$[k_{t+1}^m :] \quad \lambda_t p_t^f = \beta E_t \lambda_{t+1} [r_{t+1}^{km} + (1-\delta)p_{t+1}^f] \quad (3.6)$$

$$[k_{t+1}^x :] \quad \lambda_t p_t^f = \beta E_t \lambda_{t+1} [r_{t+1}^{kx} + (1-\delta)p_{t+1}^f] \quad (3.7)$$

$$[k_{t+1}^n :] \quad \lambda_t p_t^f = \beta E_t \lambda_{t+1} [r_{t+1}^{kn} + (1-\delta)p_{t+1}^f] \quad (3.8)$$

$$[d_{t+1} :] \quad \lambda_t p_t^\tau = \beta(1+r_t)E_t \lambda_{t+1} p_{t+1}^\tau \quad (3.9)$$

where λ_t is the Lagrange multiplier on the budget constraint. Condition (3.1) equates the marginal utility of consumption to its price multiplied by the Lagrange multiplier which reflects marginal utility of income. Conditions (3.2) to (3.5) equate the marginal disutility of labor to the marginal utility gain due to higher consumption. Conditions (3.6) to (3.8) reflect the Euler equations for different types of capital equating the marginal utility of forgoing one unit of consumption today with the marginal benefit - the expected discounted return on capital expressed in tomorrow's utility units. Finally, condition (3.9) is the Euler equation for external debt and equates the marginal utility of obtaining one more unit of consumption today with the marginal cost - the expected discounted payment of debt expressed in tomorrow's utility units.

3.3.2 Exportable goods producer

Firms producing exportable goods are perfectly competitive and maximize profits:

$$\max_{\{l_t^x, k_t^x\}} \text{tot}_t y_t^x - w_t^x l_t^x - r_t^{kx} k_t^x$$

subject to the production function

$$y_t^x = A_t z_t F^x(k_t^x, l_t^x) \quad (3.10)$$

where tot_t are the terms of trade - the relative price of exportable goods in terms of importable goods, y_t^x is exportable goods production, l_t^x is labor employed in exportable goods production, k_t^x is capital employed in exportable goods production, w_t^x is wage rate in exportable goods production and r_t^{kx} is capital rental rate in exportable goods production, z_t is a technology shock and A_t is endogenously determined technology level. Function $F^x(k^x, l^x)$ is assumed to be twice continuously differentiable, constant returns to scale with positive and decreasing marginal products of the inputs.

First order conditions of exportable goods producer are given by

$$[l_t^x :] \quad \text{tot}_t A_t z_t F_2^x(k_t^x, l_t^x) = w_t^x \quad (3.11)$$

$$[k_t^x :] \quad \text{tot}_t A_t z_t F_1^x(k_t^x, l_t^x) = r_t^{kx} \quad (3.12)$$

At the optimum the prices of factors of production are equal to the market value of their marginal products.

3.3.3 Technology producer

The producer of technology is assumed to maximize the expected discounted stream of profits by solving the following intertemporal maximization problem:

$$\max_{\{A_{t+1}, h_t\}} \left\{ E_0 \sum_{t=0}^{\infty} \prod_{i=0}^{t-1} \frac{1}{1+r_i} (A_{t+1} - w_t^h h_t) \right\}$$

subject to the law of motion of the technology

$$A_{t+1} - A_t = \Theta A_t z_t h_t^\gamma \quad (3.13)$$

where A_t is the current level of technology (endogenously determined in $t-1$), z_t is a technology shock, w_t^h is a wage in knowledge production, h_t is labor employed in knowledge production, while Θ and γ are parameters of the knowledge production function¹³.

¹³In order for the technology growth rate to be stable, we need to assume that along the balance growth path labor employed in the knowledge production h_t (hours worked) does not grow over time - see section 3.6.2. in the

The first order condition of the problem is given by:

$$[h_t :] \quad \Theta A_t z_t \gamma h_t^{\gamma-1} = w_t^h \quad (3.14)$$

The marginal product of labor employed in knowledge production is equated to the wage in the sector.

3.3.4 The main mechanism

Having derived the optimality conditions for households, exportable goods producer and technology producer we are ready to discuss our main mechanism. By (3.3) and (3.11) we have that

$$\lambda_t = -\frac{U_3(c_t, l_t^m, l_t^x, l_t^n, h_t)}{w_t^x} = -\frac{U_3(c_t, l_t^m, l_t^x, l_t^n, h_t)}{tot_t A_t z_t F_2^x(k_t^x, l_t^x)}$$

By (3.5) and (3.14) we have that

$$\lambda_t = -\frac{U_5(c_t, l_t^m, l_t^x, l_t^n, h_t)}{w_t^h} = -\frac{U_5(c_t, l_t^m, l_t^x, l_t^n, h_t)}{\Theta A_t z_t \gamma h_t^{\gamma-1}}$$

This implies that

$$-\frac{U_3(c_t, l_t^m, l_t^x, l_t^n, h_t)}{tot_t A_t z_t F_2^x(k_t^x, l_t^x)} = -\frac{U_5(c_t, l_t^m, l_t^x, l_t^n, h_t)}{\Theta A_t z_t \gamma h_t^{\gamma-1}}$$

When the terms of trade improve the left-hand side of the above expression goes down. In order for the equality to be satisfied, the right-hand side needs to go down as well. Since it is increasing in h_t (denominator corresponds to labor supply and hence is increasing in h_t ¹⁴, while numerator is marginal product of labor and thus is decreasing in h_t), h_t needs to fall after the terms of trade improve. A fall in h_t has a negative impact on TFP growth. This can be also seen in the following way. Equations (3.3), (3.5) and (3.11) imply that:

$$w_t^h = \frac{U_5(c_t, l_t^m, l_t^x, l_t^n, h_t)}{U_3(c_t, l_t^m, l_t^x, l_t^n, h_t)} tot_t A_t z_t F_2^x(k_t^x, l_t^x)$$

This and (3.14) yields:

$$h_t = \left(\frac{w_t^h}{\Theta A_t z_t \gamma} \right)^{\frac{1}{\gamma-1}} = \left(\frac{\frac{U_5(c_t, l_t^m, l_t^x, l_t^n, h_t)}{U_3(c_t, l_t^m, l_t^x, l_t^n, h_t)} tot_t A_t z_t F_2^x(k_t^x, l_t^x)}{\Theta A_t z_t \gamma} \right)^{\frac{1}{\gamma-1}}$$

appendix.

¹⁴We assume that the substitution effect dominates over the income effect.

We can apply the implicit function theorem to the above expression (we skip the arguments of the utility function) which yields:

$$\frac{dh_t}{dtot_t} = \frac{-\frac{1}{\gamma-1} \left(\frac{\frac{U_5(\cdot)}{U_3(\cdot)} tot_t A_t z_t F_2^x(k_t^x, l_t^x)}{\Theta A_t z_t \gamma} \right)^{\frac{1}{\gamma-1}-1} \frac{U_5(\cdot)}{U_3(\cdot)} A_t z_t F_2^x(k_t^x, l_t^x)}{\frac{1}{\gamma-1} \left(\frac{\frac{U_5(\cdot)}{U_3(\cdot)} tot_t A_t z_t F_2^x(k_t^x, l_t^x)}{\Theta A_t z_t \gamma} \right)^{\frac{1}{\gamma-1}-1} \frac{A_t z_t F_2^x(k_t^x, l_t^x)}{\Theta A_t z_t \gamma} \frac{U_{55}(\cdot) U_3(\cdot) - U_5(\cdot) U_{35}(\cdot)}{(U_3(\cdot))^2} - 1} < 0$$

Since $\gamma < 1$ the numerator is positive, while the denominator is negative¹⁵, the whole expression is negative. Then, by (3.13) implying $\frac{dA_{t+1}}{dh_t} > 0$ we have that

$$\frac{dA_{t+1}}{dtot_t} = \frac{dA_{t+1}}{dh_t} \frac{dh_t}{dtot_t} < 0$$

so that the terms of trade improvements are associated with a slowdown in the TFP growth. Clearly this analysis is keeping other variables unchanged, while in general equilibrium they would also be affected. However, the negative impact of the terms of trade improvements on technology is also illustrated in our numerical simulation in section 3.4.

The result that increasing terms of trade have a negative impact on future productivity is in line with empirical facts described in section 3.2. What is the intuition behind it? The terms of trade improvements encourage to put more resources into physical exportable good production at the expense of knowledge production. Demand for labor employed in physical goods production increases, which increases wages and employment in the sector. Wages in all sectors are connected by the household's optimality conditions. When the terms of trade drive up wages in exportable production sector they also increase salaries in R&D production. Since the marginal product of labor in this sector is unchanged, the labor demand in this sector is lower under these higher wages. The employment in technology production sector decreases which leads to a deterioration in an endogenously determined component of the TFP level.

Alternative technology production function

In the current version of the technology production function described by (3.13) growth of the technology increases in the employment in the R&D sector. Since the terms of trade improvements increase employment in exportable sector and drive up wages in this sector, by (imperfect) wage equalization employment in R&D sector decreases and TFP growth slows down. One could, however, imagine a different technology production function where technology would be increasing in

¹⁵The denominator is negative as long as $U_{55}(\cdot)U_3(\cdot) > U_5(\cdot)U_{35}(\cdot)$ which we assume holds. This condition can be equivalently written as $\frac{U_{55}(\cdot)}{U_5(\cdot)} > \frac{U_{35}(\cdot)}{U_3(\cdot)}$ or $\frac{U_{55}(\cdot)}{U_5(\cdot)} h_t > \frac{U_{35}(\cdot)}{U_3(\cdot)} h_t$ which means that the elasticity of the marginal disutility of working in R&D sector with respect to labor supply to R&D sector is higher than the elasticity of the marginal disutility of working in exportable sector with respect to labor supply to R&D sector.

the amount of labor employed in exportable sector capturing learning-by-doing effects (for example as in Benigno and Fornaro, 2014). In our setting it would take the following form:

$$A_{t+1} - A_t = \Theta A_t z_t (l_t^x)^\gamma$$

In such a case terms of trade improvement would be associated with an acceleration of the TFP growth.

3.3.5 Remaining elements of the model

Importable goods producer

Firms producing importable goods are perfectly competitive and maximize profits:

$$\max_{\{l_t^m, k_t^m\}} y_t^m - w_t^m l_t^m - r_t^{km} k_t^m$$

subject to the production function

$$y_t^m = A_t z_t F^m(k_t^m, l_t^m) \quad (3.15)$$

where y_t^m is importable goods production, l_t^m is labor employed in importable goods production, k_t^m is capital employed in importable goods production, w_t^m is wage rate in importable goods production, r_t^{km} is capital rental rate in importable goods production, z_t is a technology shock and A_t is endogenously determined technology level. Function $F^m(k^m, l^m)$ is assumed to be twice continuously differentiable, constant returns to scale with positive and decreasing marginal products of the inputs.

First order conditions of importable goods producer are given by

$$[l_t^m :] \quad A_t z_t F_2^m(k_t^m, l_t^m) = w_t^m \quad (3.16)$$

$$[k_t^m :] \quad A_t z_t F_1^m(k_t^m, l_t^m) = r_t^{km} \quad (3.17)$$

At the optimum the prices of factors of production are equal to the market value of their marginal products.

Non-tradable goods producer

Firms producing non-tradable goods are perfectly competitive and maximize profits:

$$\max_{\{l_t^n, k_t^n\}} p_t^n y_t^n - w_t^n l_t^n - r_t^{kn} k_t^n$$

subject to the production function

$$y_t^n = A_t z_t F^n(k_t^n, l_t^n) \quad (3.18)$$

where p_t^n is the relative price of non-tradable goods in terms of importable goods, y_t^n is non-tradable goods production, l_t^n is labor employed in non-tradable goods production, k_t^n is capital employed in non-tradable goods production, w_t^n is wage rate in non-tradable goods production r_t^{kn} is capital rental rate in non-tradable goods production, z_t is a technology shock and A_t is endogenously determined technology level. Function $F^n(k^n, l^n)$ is assumed to be twice continuously differentiable, constant returns to scale with positive and decreasing marginal products of the inputs.

First order conditions of non-tradable goods producer are given by

$$[l_t^n :] \quad p_t^n A_t z_t F_2^n(k_t^n, l_t^n) = w_t^n \quad (3.19)$$

$$[k_t^n :] \quad p_t^n A_t z_t F_1^n(k_t^n, l_t^n) = r_t^{kn} \quad (3.20)$$

At the optimum the prices of factors of production are equal to the market value of their marginal products.

Composite tradable goods

The composite tradable good is produced using an increasing, concave and linearly homogeneous aggregator function:

$$a_t^\tau = G(a_t^m, a_t^x) \quad (3.21)$$

where a_t^m is the domestic absorption of importable goods and a_t^x is the domestic absorption of exportable goods. The maximization problem takes the following form

$$\max_{a_t^m, a_t^x} p_t^\tau G(a_t^m, a_t^x) - a_t^m - tot_t a_t^x$$

where p_t^τ is the relative price of the composite tradable goods in terms of importables. Assuming perfect competition in the composite tradable goods production process, the first order conditions are given by

$$p_t^\tau G_1(a_t^m, a_t^x) = 1 \quad (3.22)$$

$$p_t^\tau G_2(a_t^m, a_t^x) = tot_t \quad (3.23)$$

Composite final goods

The composite final good is produced using an increasing, concave and homogeneous of degree one aggregator function:

$$H(a_t^\tau, a_t^n)$$

where a_t^τ is the tradable composite good and a_t^n is the domestic absorption of nontradable goods. The maximization problem takes the following form

$$\max_{a_t^\tau, a_t^n} p_t^f H(a_t^\tau, a_t^n) - p_t^\tau a_t^\tau - p_t^n a_t^n$$

where p_t^f is the relative price of the final goods in terms of importables. Assuming perfect competition in the composite final goods production process, the first order conditions are given by

$$p_t^f H_1(a_t^\tau, a_t^n) = p_t^\tau \quad (3.24)$$

$$p_t^f H_2(a_t^\tau, a_t^n) = p_t^n \quad (3.25)$$

Financing the externality

The technology developed by the knowledge production sector is freely used by the physical goods production sectors. Since they do not pay for it, there is an externality in the model. This externality whose cost is equal to the wage bill in the technological sector $w_t^h h_t$ is financed by the lump sum tax levied on household

$$w_t^h h_t = \tau_t \quad (3.26)$$

The tax τ_t adjusts so that the feasibility constraint of the economy is not violated.

Debt-elastic interest rate premium

The interest rate on debt is assumed to evolve according to

$$r_t = r^* + p(d_{t+1}) \quad (3.27)$$

where r^* is the world interest rate and the function $p(\cdot)$ is assumed to be increasing and takes the form

$$p(d) = \psi(e^{d-\bar{d}})$$

where \bar{d} is the steady state level of debt. This debt-elastic interest rate premium is necessary to ensure a stationary equilibrium process for external debt.

Market clearing, import and export

In equilibrium the demand for final goods must equal their supply:

$$c_t + k_{t+1}^m + k_{t+1}^x + k_{t+1}^n - (1 - \delta)(k_t^m + k_t^x + k_t^n) = H(a_t^\tau, a_t^n) \quad (3.28)$$

Since non-tradable goods by definition are consumed only domestically, their market has to clear so that demand for nontradables is equal to their production:

$$a_t^n = y_t^n \quad (3.29)$$

In our setting import is given by

$$m_t = a_t^m - y_t^m \quad (3.30)$$

and export is given by:

$$x_t = tot_t(y_t^x - a_t^x) \quad (3.31)$$

Then from households' budget constraint and by producers making zero profits and by (3.26):

$$m_t - x_t + p_t^r d_t = p_t^r \frac{d_{t+1}}{1 + r_t} \quad (3.32)$$

which is the economy-wide resource constraint.

Exogenous processes

We assume that the terms of trade follow a univariate first-order autoregressive (AR(1)) process of the form

$$\ln \frac{tot_t}{\overline{tot}} = \rho_{tot} \ln \frac{tot_{t-1}}{\overline{tot}} + \sigma^{tot} \varepsilon_t^{tot} \quad (3.33)$$

where $\overline{tot} > 0$ is the deterministic level of the terms of trade, $\rho_{tot} \in (-1, 1)$ is the serial correlation of the process and $\sigma^{tot} > 0$ is the standard deviation of the innovation to the terms of trade.

We also assume that the technology shock follows a univariate first-order autoregressive (AR(1)) process of the form

$$\ln \frac{z_t}{\bar{z}} = \rho_z \ln \frac{z_{t-1}}{\bar{z}} + \sigma^z \varepsilon_t^z \quad (3.34)$$

where $\bar{z} > 0$ is the deterministic level of the technology shock normalized to one, $\rho_z \in (-1, 1)$ is the serial correlation of the process and $\sigma^z > 0$ is the standard deviation of the innovation to the technology shock.

Competitive equilibrium

Definition 3.3.1 (Competitive equilibrium). A competitive equilibrium is:

a set of prices $\{r_t^{km}, r_t^{kx}, r_t^{kn}, w_t^m, w_t^x, w_t^n, w_t^h, p_t^f, p_t^r, p_t^n, r_t\}_{t=0}^\infty$,

an allocation $\{k_{t+1}^m, k_{t+1}^x, k_{t+1}^n, l_t^m, l_t^x, l_t^n, h_t, A_{t+1}, y_t^m, y_t^x, y_t^n, c_t, a_t^m, a_t^x, a_t^n, a_t^r, m_t, x_t, d_{t+1}\}_{t=0}^\infty$,

a sequence of multipliers $\{\lambda_t\}_{t=0}^\infty$ and a tax system $\{\tau_t\}_{t=0}^\infty$

which satisfy equations (3.1) to (3.32) such that households' and firms' optimality conditions are satisfied and markets clear given the stochastic processes $\{tot_t, z_t\}_{t=0}^\infty$ described by (3.33) and (3.34)

and the initial conditions $k_0^m, k_0^x, k_0^n, d_0, A_0, tot_{-1}, z_{-1}$.

3.4 Quantitative model evaluation

In this section we perform a quantitative evaluation of the theoretical model presented in the previous section. As TFP is non-stationary the trending variables in the model are normalized by the one-period lagged TFP level for the purpose of this quantitative evaluation. The model has a stationary equilibrium in terms of the normalized variables. The model is solved using a second-order perturbation method.

3.4.1 Functional forms

We assume that the utility function is of constant relative risk aversion (CRRA) in a quasilinear composite of consumption and labor:

$$U(c, l^m, l^x, l^n, h) = \frac{[c - L(l^m, l^x, l^n, h)]^{1-\sigma} - 1}{1 - \sigma}$$

where

$$L(l^m, l^x, l^n, h) = \frac{(l^m)^{\omega_m}}{\omega_m} + \frac{(l^x)^{\omega_x}}{\omega_x} + \frac{(l^n)^{\omega_n}}{\omega_n} + \frac{(h)^{\omega_h}}{\omega_h}$$

with parameters $\sigma, \omega_m, \omega_x, \omega_n, \omega_h > 0$. This specification ensures that sectoral labor supplies are wealth inelastic¹⁶. The wage elasticities of labor supply are given by $\frac{1}{1-\omega}$.

The production technologies in importable, exportable and nontradable sectors are assumed to be Cobb-Douglas

$$F^m(k^m, l^m) = (k^m)^{\alpha_m} (l^m)^{1-\alpha_m}$$

$$F^x(k^x, l^x) = (k^x)^{\alpha_x} (l^x)^{1-\alpha_x}$$

$$F^n(k^n, l^n) = (k^n)^{\alpha_n} (l^n)^{1-\alpha_n}$$

with parameters $\alpha_m, \alpha_x, \alpha_n \in (0, 1)$. The aggregators used in the production of composite tradable and final goods take the constant elasticity of substitution form

$$G(a_t^m, a_t^x) = \left[\chi_m (a_t^m)^{1-\frac{1}{\nu_{mx}}} + (1 - \chi_m) (a_t^x)^{1-\frac{1}{\nu_{mx}}} \right]^{\frac{1}{1-\frac{1}{\nu_{mx}}}}$$

$$H(a_t^\tau, a_t^n) = \left[\chi_\tau (a_t^\tau)^{1-\frac{1}{\nu_{\tau n}}} + (1 - \chi_\tau) (a_t^n)^{1-\frac{1}{\nu_{\tau n}}} \right]^{\frac{1}{1-\frac{1}{\nu_{\tau n}}}}$$

¹⁶Imperfect substitutability of labor in different sectors was chosen for computational reasons (this gives us separate labor supply schedules for each sector) and is not necessary for our results - the mechanism is preserved also under perfect substitutes. Similarly allowing for income effects does not affect the results as long as the substitution effect dominates over the wealth effect.

with parameters $\chi_m, \chi_\tau \in (0, 1)$ and $\nu_{mx}, \nu_{\tau n} > 0$.

3.4.2 Calibration

The calibration of the model is summarized in Table 3.5. The time unit is a year. We follow a standard calibration of the MXN model (see Schmitt-Grohé and Uribe, 2018). The coefficient of the relative risk aversion σ is set at 2 which is a usual value used in business cycle literature. The subjective discount factor β takes value of 0.95. The parameter ω is set at 1.455 so that labor supply elasticity $\frac{1}{\omega-1}$ equals 2.2 as in Mendoza (1991) and is the same in all sectors. Parameters of the production function (capital share in production) are given by $\alpha_m = 0.33$, $\alpha_x = 0.33$, $\alpha_n = 0.25$. The latter reflects higher labor share in production of nontradable goods sector comparing to importable and exportable sectors.

The intratemporal elasticity of substitution between exportable and importable absorption ν_{mx} is set at 1. Available quarterly estimates are usually below one (see e.g. Corsetti, Dedola, and Leduc (2008)), while those based on 5-10 years data averages find it to be above one. Setting it to unity is reasonable for annual frequency model. The parameter χ_m reflecting the share of importables in tradable goods aggregator is set at 0.9 to match the average share of import in total trade for the analyzed countries over the sample period (49.01%). The intratemporal elasticity of substitution between tradable and nontradable absorption $\nu_{\tau n}$ is set at 0.5 which is based on Akinci (2011). The parameter χ_τ reflecting the share of tradables in composite goods aggregator is set at 0.36 to match the average trade share of nontradables (proxied by services) in GDP for the analyzed countries over the sample period (62.71%).

The capital depreciation rate δ is set at 0.1 which is standard. The world interest rate r^* is set at 0.04. The parameter governing the debt elasticity of the country premium ψ takes value of 0.08. The steady state debt \bar{d} is set at 4.9. These two parameters are set to match the average trade balance share in GDP for the analyzed countries over the sample period (2.38%).

The steady state level of the terms of trade \bar{tot} takes value of 1. The autocorrelation of the terms of trade process is set at 0.46 which is the median of the estimates for the countries in our macroeconomic sample. The standard deviation of the terms of trade innovation σ_{tot} is set at 0.0166 which is the median of the estimates of the countries in our macroeconomic sample. The autocorrelation of the technology shock process is set at 0.72 which is the median of the estimates for the countries in our macroeconomic sample. The standard deviation of the terms of trade innovation σ_{tot} is set at 0.0114 which is the median of the estimates of the countries in our macroeconomic sample.

Besides the MXN model parameters additionally we need to set the parameters of the knowledge production function. We set $\Theta = 1$. Porter and Stern (2000) estimate equation (3.13) and find that $\gamma \in (0.2, 0.48)$ for different sets of controls. Here following these estimates we set γ at 0.4.

Table 3.5: Calibration

Parameter	Description	Value
σ	Coefficient of the relative risk aversion	2
β	Subjective discount factor	0.95
ω^m	$\frac{1}{\omega^m-1}$ = Importable goods labor supply elasticity	1.455
ω^x	$\frac{1}{\omega^x-1}$ = Exportable goods labor supply elasticity	1.455
ω^n	$\frac{1}{\omega^n-1}$ = Nontradables goods labor supply elasticity	1.455
ω^h	$\frac{1}{\omega^h-1}$ = Technology sector labor supply elasticity	1.455
α_m	Capital share in importable goods sector	0.33
α_x	Capital share in exportable goods sector	0.33
α_n	Capital share in nontradable goods sector	0.25
ν_{mx}	The elasticity of substitution between exportable and importable absorption	1
χ_m	The importables share parameter	0.9
$\nu_{\tau n}$	The elasticity of substitution between tradable and nontradable absorption	0.5
χ_τ	The tradables share parameter	0.36
δ	Capital depreciation rate	0.1
ψ	Parameter governing the debt elasticity of the country premium	0.08
r^*	World interest rate	0.04
\bar{d}	Steady state debt	20.47
\bar{tot}	Steady state TOT	1
ρ_{tot}	TOT autocorrelation coefficient	0.46
σ_{tot}	Standard deviation of TOT process innovation	0.0166
ρ_z	Autocorrelation coefficient of technology shock	0.72
σ_z	Standard deviation of technology shock innovation	0.0114
Θ	Shift parameter of the knowledge production function	1
γ	Parameter of the knowledge production function	0.4

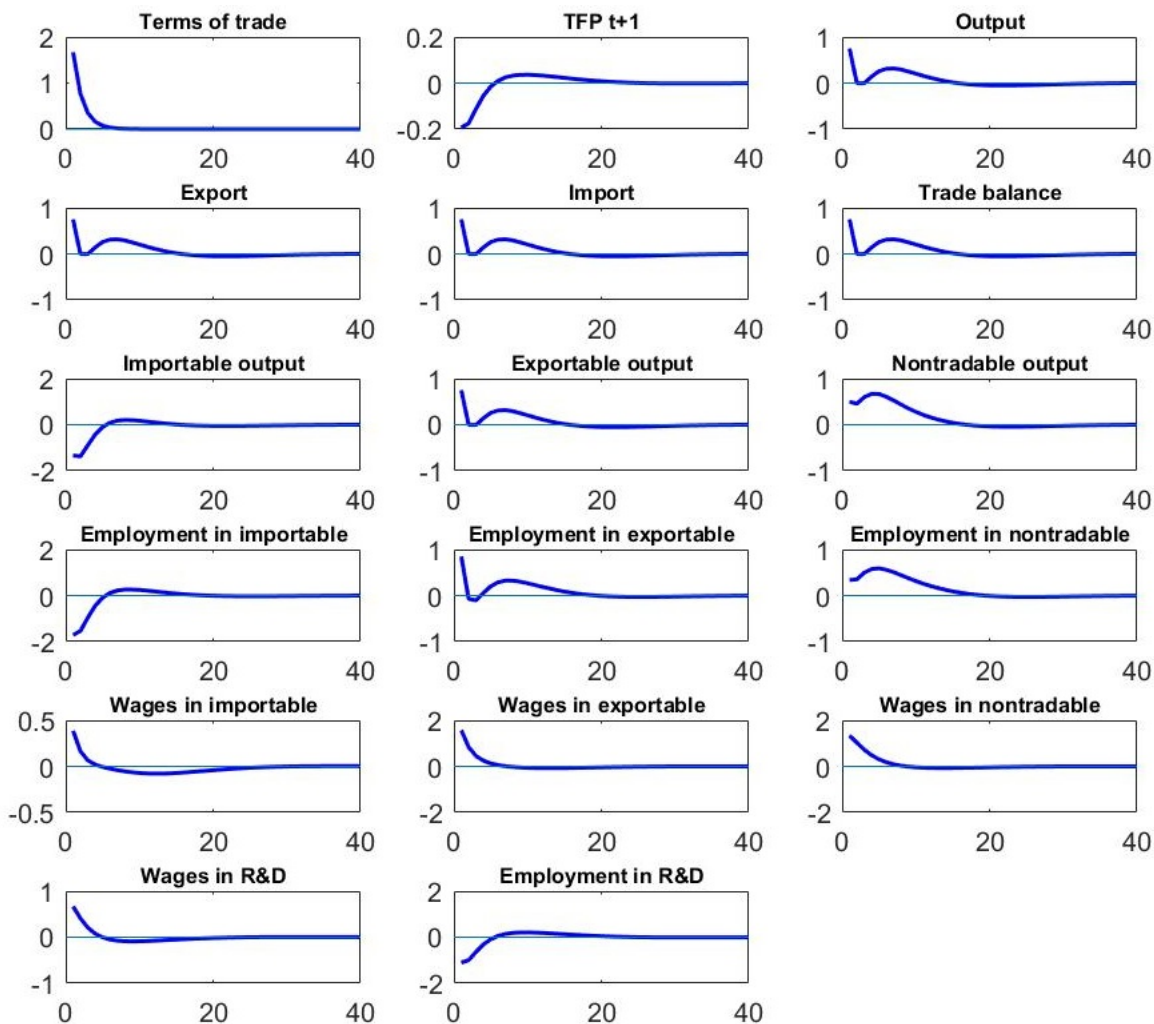
3.4.3 Model responses

Figure 3.2 shows the impulse response functions of selected model variables in terms of the percentage deviations from the steady state to the positive terms of trade shock conditional on the technology shock being switched off. As we can see, as a result of the shock (future) TFP drops, while output increases. There is an increase in exports and exportable output. Imports increase both because they are relatively cheaper and due to positive economy-wide income effect of the terms of trade improvement. This increase is smaller than an increase in exports (in absolute terms) so that the trade balance improves. This is in line with the Harberger-Laursen-Metzler effect which predicts trade balance improvements after positive terms of trade shocks. It deteriorates slightly

afterwards due to positive income shock related to the terms of trade improvement.

Domestic production of importable goods decreases as it is relatively less profitable. Non-tradable output increases due to the positive income effect of the terms of trade improvement. Consistently with output in each sector employment in exportables and non-tradable production increases and employment in importables production decreases. Relative (to the price of importables) wages increase in all sectors.

Figure 3.2: Impulse responses of selected model variables to the terms of trade shock



Note: Vertical axis - percentage deviations from the steady state. Horizontal axis - time periods (years).

Finally, we can see that wages in R&D sector increase, while employment in R&D decreases. The terms of trade improvement increases demand for labor employed in physical goods production

and drives up wages in this sector. However, since all wages are connected by households' optimality conditions wages in R&D also increase. Since the marginal product of labor in R&D is unchanged, the labor demand in this sector corresponding with these higher wages is lower. Hence, we observe a drop in demand for this kind of labor and shifting resources from R&D towards physical exportable goods production. As a result, next period TFP decreases. Thus, consistently with empirical evidence, the model predicts a decrease in TFP after the terms of trade improvement¹⁷.

The terms of trade shock increases output and at the same time decreases TFP in the model. Because of that TFP and R&D spending - which is our channel of shock propagation - are counter-cyclical. It is at odds with the data where TFP and R&D spending are procyclical¹⁸. This result capturing (data-consistent) negative relationship between the terms of trade and TFP holds only for a model which does not feature other shocks. With technology shocks z_t in force, output increases after a positive productivity shock which results in positive correlation between the two, even though positive terms of trade shocks have a negative impact on TFP.

Below in Table 3.6 we present the moments generated by the model with both technology and terms of trade shocks operating. As mentioned above, the average share of import in total trade, the average trade share of nontradables in GDP and the average trade balance share in GDP were targeted in setting parameters χ^M , χ^τ and \bar{d} . We can see that the model is doing quite well in capturing the autocorrelation of TFP, R&D spending (proxied in the model by the wage bill in the technology production sector) and output, as well as the standard deviation of TFP. The standard deviation of output is slightly higher in the model comparing to the data, while standard deviation of TFP is slightly lower. Finally, the positive correlation between TFP and output, as well as R&D and output is achieved in the model once technology shocks are operating.

¹⁷Since all sectors share the same TFP the effects of the terms of trade shock on TFP are the same across sectors. However, the microeconomic evidence we presented suggests that these effects are heterogeneous depending on the level of tradability of goods produced. In the model this could be achieved by fixing the endogenous part of the TFP in nontradable sector at the steady state level so that TFP fluctuations only in tradable industries (and as a result the overall country-wide TFP fluctuations) would be affected by the terms of trade shocks.

¹⁸Indeed the average correlation between TFP and output per capita in analyzed countries for the analyzed period is 0.71 while the average correlation between R&D spending and output per capita in analyzed countries for the analyzed period is 0.31 for quadratically detrended series.

Table 3.6: Targeted and non-targeted moments

Statistic	Data	Model
Targeted moments		
Average share of import in total trade	49.01%	48.50%
Average trade share of nontradables in GDP	62.71%	62.26%
Average trade balance share in GDP	2.38%	2.38%
Non-targeted moments		
Standard deviation output	2.71	3.66
Autocorrelation output	0.76	0.79
Standard deviation TFP	1.57	0.97
Autocorrelation TFP	0.72	0.72
Standard deviation R&D spending	3.70	3.00
Autocorrelation R&D spending	0.70	0.82
Correlation output vs. TFP	0.71	0.81
Correlation output vs. R&D	0.31	0.84

Source: Author's calculations.

3.5 Conclusions

In this chapter we have analyzed how the terms of trade affect total factor productivity. Using the data for the European Union countries we have shown that macroeconomic evidence based on times series SVAR analysis suggests that the structural terms of trade shocks have a negative impact on total factor productivity. Consistently, empirical microeconomic evidence based on industry level data suggests that improvements in terms of trade are associated with a slowdown of the total factor productivity growth at the sectoral level.

Next, we have built a theoretical model which combines open economy framework with the endogenous growth theory. In the model the terms of trade improvements lead to a shift of resources from R&D production towards physical exportable goods. Employment in exportables sector increases, while the opposite happens in knowledge production sector due to a drop in labor demand. As a result, total factor productivity decreases. We have also shown that under a plausible calibration the model is able to produce this mechanism and thus replicate the observed empirical pattern.

3.6 Appendix

3.6.1 Tables

Table 3.7: Trade shares of the countries used in empirical investigation

Country	Average share of exports+imports in GDP over 1985-2016
Austria	83.46
Belgium	134.73
Denmark	81.97
Estonia	143.01*
Finland	66.54
France	49.93
Germany	61.41
Ireland	150.78
Italy	46.73
Lithuania	117.08*
Netherlands	121.25
Portugal	65.32
Slovenia	118.99*
Spain	49.81
Sweden	74.66
United Kingdom	52.18

* over the period 1995-2016

Table 3.8: Manufacturing industries in the microeconomic dataset

Manufacture of food products
Manufacture of beverages
Manufacture of tobacco products
Manufacture of textiles
Manufacture of wearing apparel
Manufacture of leather and related products
Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
Manufacture of paper and paper products
Printing and reproduction of recorded media
Manufacture of chemicals and chemical products
Manufacture of basic pharmaceutical products and pharmaceutical preparations
Manufacture of rubber and plastic products
Manufacture of other nonmetallic mineral products
Manufacture of basic metals
Manufacture of fabricated metal products, except machinery and equipment
Manufacture of computer, electronic and optical products
Manufacture of electrical equipment
Manufacture of machinery and equipment
Manufacture of motor vehicles, trailers and semitrailers
Manufacture of other transport equipment
Manufacture of furniture
Other manufacturing

Table 3.9: Non-manufacturing industries in the microeconomic dataset

Repair and installation of machinery and equipment
Construction of buildings
Civil engineering
Specialised construction activities
Wholesale and retail trade and repair of motor vehicles and motorcycles
Wholesale trade, except of motor vehicles and motorcycles
Retail trade, except of motor vehicles and motorcycles
Land transport and transport via pipelines
Water transport
Air transport
Warehousing and support activities for transportation
Postal and courier activities
Accommodation
Food and beverage service activities
Publishing activities
Motion picture, video and television programme production, sound recording and music publishing activities
Programming and broadcasting activities
Telecommunications
Computer programming, consultancy and related activities
Information service activities
Real estate activities
Legal and accounting activities
Activities of head offices; management consultancy activities
Architectural and engineering activities; technical testing and analysis
Scientific research and development
Advertising and market research
Other professional, scientific and technical activities
Veterinary activities
Rental and leasing activities
Employment activities
Travel agency, tour operator and other reservation service and related activities
Security and investigation activities
Services to buildings and landscape activities
Office administrative, office support and other business support activities

3.6.2 Growth of the technology

In a general case (3.13) would be given by

$$A_{t+1} - A_t = \Theta A_t^\theta z_t h_t^\gamma$$

Then the growth rate of the technology is given by

$$g_t^A = \frac{A_{t+1} - A_t}{A_t} = \Theta A_t^{\theta-1} z_t h_t^\gamma$$

Itself grows at

$$\frac{g_{t+1}^A - g_t^A}{g_t^A} = \gamma n + (\theta - 1)g_t^A$$

where $n = \frac{h_{t+1} - h_t}{h_t}$. To have a stable growth path, i.e., $\frac{g_{t+1}^A - g_t^A}{g_t^A} = 0$ which is positive we need either $n = 0$ and $\theta = 1$ or $\theta < 1$ for $n > 0$. In the latter case

$$g_t^A = \frac{\gamma n}{1 - \theta}$$

For simplicity we assume the former.

Bibliography

- Adao, Bernardino, Isabel Correia, and Pedro Teles (2003). “Gaps and triangles”. In: *The Review of Economic Studies* 70.4, pp. 699–713.
- Agarwal, Sumit, Chunlin Liu, and Nicholas S. Souleles (2007). “The reaction of consumer spending and debt to tax rebates - evidence from consumer credit data”. In: *Journal of Political Economy* 115.6, pp. 986–1019.
- Akinci, Ozge (2011). “A note on the estimation of the atemporal elasticity of substitution between tradable and nontradable goods”. In: *Manuscript, Columbia University*.
- Alcalá, Francisco and Antonio Ciccone (2004). “Trade and productivity”. In: *The Quarterly Journal of Economics* 119.2, pp. 613–646.
- Alfaro, Laura et al. (2017). *The Real Exchange Rate, Innovation and Productivity*. Tech. rep. Harvard Business School.
- Auerbach, Alan J. and Yuriy Gorodnichenko (2012). “Fiscal multipliers in recession and expansion”. In: *Fiscal policy after the financial crisis*. University of Chicago Press, pp. 63–98.
- Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk (2014). “Some evidence on the importance of sticky wages”. In: *American Economic Journal: Macroeconomics* 6.1, pp. 70–101.
- Bardález, Paul Castillo and Youel Rojas Zea (2014). “Terms of Trade and Total Factor Productivity: Empirical evidence from Latin American emerging markets”. In: *Central Bank of Peru Working Paper* 12.
- Barro, Robert J. and Charles J. Redlick (2011). “Macroeconomic effects from government purchases and taxes”. In: *The Quarterly Journal of Economics* 126.1, pp. 51–102.
- Baxter, Marianne and Robert G. King (1993). “Fiscal policy in general equilibrium”. In: *The American Economic Review*, pp. 315–334.
- Benigno, Gianluca and Luca Fornaro (2014). “The financial resource curse”. In: *The Scandinavian Journal of Economics* 116.1, pp. 58–86.
- Benigno, Pierpaolo and Michael Woodford (2003). “Optimal monetary and fiscal policy: A linear-quadratic approach”. In: *NBER Macroeconomics Annual* 18, pp. 271–333.

- Blanchard, Olivier and Roberto Perotti (2002). “An empirical characterization of the dynamic effects of changes in government spending and taxes on output”. In: *the Quarterly Journal of Economics* 117.4, pp. 1329–1368.
- Blanchard, Olivier J. and Danny Quah (1989). “The dynamic effects of aggregate demand and supply disturbances”. In: *American Economic Review* 79.4, pp. 655–673.
- Broda, Christian and Jonathan Parker (2008). *The impact of the 2008 tax rebates on consumer spending: Preliminary evidence*. Mimeo. Northwestern University.
- Broer, Tobias, Per Krusell, and Erik Öberg (2020). *Fiscal Multipliers: A Heterogeneous-Agent Perspective*. Mimeo. IIES.
- Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo (2011). “When is the government spending multiplier large?” In: *Journal of Political Economy* 119.1, pp. 78–121.
- Cogan, John F. et al. (2010). “New Keynesian versus old Keynesian government spending multipliers”. In: *Journal of Economic Dynamics and Control* 34.3, pp. 281–295.
- Correia, Isabel, Juan Pablo Nicolini, and Pedro Teles (2008). “Optimal fiscal and monetary policy: Equivalence results”. In: *Journal of Political Economy* 116.1, pp. 141–170.
- Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc (2008). “International risk sharing and the transmission of productivity shocks”. In: *The Review of Economic Studies* 75.2, pp. 443–473.
- Daly, Mary, Bart Hobijn, Brian Lucking, et al. (2012). “Why has wage growth stayed strong?” In: *FRBSF Economic Letter* 10.2.
- DeLong, J. Bradford and Lawrence H. Summers (2012). “Fiscal policy in a depressed economy”. In: *Brookings Papers on Economic Activity*, pp. 233–274.
- Dunne, Timothy, Shawn Klimek, and James A. Schmitz (2010). *Competition and Productivity: Evidence from the Post WWII US Cement Industry*. Tech. rep. US Census Bureau, Center for Economic Studies.
- Dupor, Bill, Jingchao Li, and Rong Li (2019). “Sticky wages, private consumption, and Fiscal multipliers”. In: *Journal of Macroeconomics* 62, p. 103157.
- Feenstra, Robert C. et al. (2013). “Effects of terms of trade gains and tariff changes on the measurement of US productivity growth”. In: *American Economic Journal: Economic Policy* 5.1, pp. 59–93.
- Frankel, Jeffrey A. (2010). *The natural resource curse: a survey*. Tech. rep. National Bureau of Economic Research.
- Galdon-Sanchez, Jose E. and James A. Schmitz (2002). “Competitive pressure and labor productivity: World iron-ore markets in the 1980’s”. In: *American Economic Review* 92.4, pp. 1222–1235.
- Galí, Jordi (2011). *Unemployment fluctuations and stabilization policies: a new Keynesian perspective*. MIT press.

- Giambattista, Eric and Steven Pennings (2017). “When is the government transfer multiplier large?” In: *European Economic Review* 100, pp. 525–543.
- Gopinath, Gita and Brent Neiman (2014). “Trade adjustment and productivity in large crises”. In: *American Economic Review* 104.3, pp. 793–831.
- Gottschalk, Peter (2005). “Downward nominal-wage flexibility: real or measurement error?” In: *Review of Economics and Statistics* 87.3, pp. 556–568.
- Johnson, David S., Jonathan A. Parker, and Nicholas S. Souleles (2006). “Household expenditure and the income tax rebates of 2001”. In: *American Economic Review* 96.5, pp. 1589–1610.
- Jordà, Òscar (2005). “Estimation and inference of impulse responses by local projections”. In: *American Economic Review* 95.1, pp. 161–182.
- Kaplan, Greg and Giovanni L. Violante (2014). “A model of the consumption response to fiscal stimulus payments”. In: *Econometrica* 82.4, pp. 1199–1239.
- Karger, Ezra and Aastha Rajan (2020). *Heterogeneity in the Marginal Propensity to Consume: Evidence from Covid-19 Stimulus Payments*. Working Paper 2020-15. FRB of Chicago.
- Kehoe, Timothy J. and Kim J. Ruhl (2008). “Are shocks to the terms of trade shocks to productivity?” In: *Review of Economic Dynamics* 11.4, pp. 804–819.
- Kose, M. Ayhan (2002). “Explaining business cycles in small open economies: How much do world prices matter?” In: *Journal of International Economics* 56.2, pp. 299–327.
- Lopez-Garcia, Paloma and Filippo Di Mauro (2015). *Assessing European competitiveness: the new CompNet microbased database*. Tech. rep. European Central Bank Working Paper no. 1764.
- Lucas, Robert E. Jr. (1988). “On the mechanics of economic development”. In: *Journal of Monetary Economics* 22.1, pp. 3–42.
- Martin, Philippe and Thomas Philippon (2017). “Inspecting the mechanism: Leverage and the great recession in the eurozone”. In: *American Economic Review* 107.7, pp. 1904–37.
- Mayer, Eric, Sebastian Rùth, and Johann Scharler (2016). “Total factor productivity and the propagation of shocks: Empirical evidence and implications for the business cycle”. In: *Journal of Macroeconomics* 50, pp. 335–346.
- Mehrotra, Neil R. (2018). “Fiscal Policy Stabilization: Purchases or Transfers?” In: *International Journal of Central Banking* 14.2, pp. 1–50.
- Mendoza, Enrique G. (1991). “Real business cycles in a small open economy”. In: *The American Economic Review*, pp. 797–818.
- (1995). “The terms of trade, the real exchange rate, and economic fluctuations”. In: *International Economic Review*, pp. 101–137.
- Miller, Stephen M. and Mukti P. Upadhyay (2000). “The effects of openness, trade orientation, and human capital on total factor productivity”. In: *Journal of Development Economics* 63.2, pp. 399–423.

- Monacelli, Tommaso and Roberto Perotti (2011). “Redistribution and the Multiplier”. In: *IMF Economic Review* 59.4, pp. 630–651.
- Mountford, Andrew and Harald Uhlig (2009). “What are the effects of fiscal policy shocks?” In: *Journal of Applied Econometrics* 24.6, pp. 960–992.
- Oh, Hyunseung and Ricardo Reis (2012). “Targeted transfers and the fiscal response to the great recession”. In: *Journal of Monetary Economics* 59, S50–S64.
- Parker, Jonathan A. et al. (2013). “Consumer spending and the economic stimulus payments of 2008”. In: *American Economic Review* 103.6, pp. 2530–53.
- Párraga Rodríguez, Susana (2016). “The aggregate effects of government income transfer shocks: EU evidence”. In: *Documentos de trabajo/Banco de España*, 1629.
- (2018). “The dynamic effects of public expenditure shocks in the United States”. In: *Journal of Macroeconomics* 56, pp. 340–360.
- Ploeg, Frederick Van der (2011). “Natural resources: curse or blessing?” In: *Journal of Economic Literature* 49.2, pp. 366–420.
- Porter, Michael E. and Scott Stern (2000). *Measuring the “ideas” production function: Evidence from international patent output*. Tech. rep. National Bureau of Economic Research.
- Ramey, Valerie A. (2011a). “Can government purchases stimulate the economy?” In: *Journal of Economic Literature* 49.3, pp. 673–85.
- (2011b). “Identifying government spending shocks: It’s all in the timing”. In: *The Quarterly Journal of Economics* 126.1, pp. 1–50.
- Ramey, Valerie A. and Sarah Zubairy (2018). “Government spending multipliers in good times and in bad: evidence from US historical data”. In: *Journal of Political Economy* 126.2, pp. 850–901.
- Rebelo, Sergio (1991). “Long-run policy analysis and long-run growth”. In: *Journal of Political Economy* 99.3, pp. 500–521.
- Romer, Christina D. and David H. Romer (2016). “Transfer payments and the macroeconomy: The effects of social security benefit increases, 1952-1991”. In: *American Economic Journal: Macroeconomics* 8.4, pp. 1–42.
- Romer, Paul M. (1986). “Increasing returns and long-run growth”. In: *Journal of Political Economy* 94.5, pp. 1002–1037.
- (1990). “Endogenous technological change”. In: *Journal of Political Economy* 98.5, Part 2, S71–S102.
- Schmitt-Grohé, Stephanie and Martín Uribe (2004). “Optimal fiscal and monetary policy under sticky prices”. In: *Journal of Economic Theory* 114.2, pp. 198–230.
- Schmitt-Grohé, Stephanie and Martín Uribe (2016). “Downward nominal wage rigidity, currency pegs, and involuntary unemployment”. In: *Journal of Political Economy* 124.5, pp. 1466–1514.
- Schmitt-Grohé, Stephanie and Martín Uribe (2018). “How Important are Terms-Of-Trade Shocks?” In: *International Economic Review* 59.1, pp. 85–111.

- Schmitz, James A. (2005). “What determines productivity? Lessons from the dramatic recovery of the US and Canadian iron ore industries following their early 1980s crisis”. In: *Journal of Political Economy* 113.3, pp. 582–625.
- Sims, Christopher A. and Tao Zha (1999). “Error bands for impulse responses”. In: *Econometrica* 67.5, pp. 1113–1155.
- Siu, Henry E. (2004). “Optimal fiscal and monetary policy with sticky prices”. In: *Journal of Monetary Economics* 51.3, pp. 575–607.
- Uzawa, Hirofumi (1965). “Optimum technical change in an aggregative model of economic growth”. In: *International Economic Review* 6.1, pp. 18–31.
- Woodford, Michael (2011). “Simple analytics of the government expenditure multiplier”. In: *American Economic Journal: Macroeconomics* 3.1, pp. 1–35.