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Monopolistic Competition, Multiproduct Firms and Optimum Product Diversity*

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Abstract

This paper tackles the issue of optimum product diversity in an imperfectly competitive market with small or large firms. First, it develops a quadratic utility model of monopolistic competition with horizontal product differentiation which avoids some of the main pitfalls of the S-D-S approach. Second, it extends the model to the case of multiproduct firms showing how product diversity is affected with respect to monopolistic competition. In particular, it is shown that monopolistic competition with single-product firms is the limiting case of oligopolistic competition with multiproduct firms when either varieties gets more and more differentiated or when the entry cost goes further and further down.

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1 Introduction

Product differentiation enlarges the scope of welfare analysis in the presence of imperfect competition. Pricing above marginal costs might well not be the main source of inefficiency in differentiated markets. The reason is that there are two other potential sources of inefficiency: the number of products supplied by the market (*product variety*) and the specification of the products made available to the consumers (*product selection*). As neatly posited by Spence (1976a, p.408):

> 'a significant fraction of the cost of imperfect competition may be due to the currently unmeasured cost of having too many, too few, or the wrong products.'

The two pillar papers that investigate these potential losses are Spence (1976b) and Dixit and Stiglitz (1977). Both are based on a similar interpretation of Chamberlin's (1933) ideas about monopolistic competition. Hence the common label of *S-D-S model*.

In the S-D-S model monopolistic competition emerges as a market structure determined by consumers' heterogeneous tastes and firms fixed requirements for limited productive resources.[1] On the demand side, the set of consumers with different tastes are aggregated into a representative consumer whose preferences exhibit *love of variety*: she demands varieties of a horizontally differentiated good and her utility is an increasing function not only of the amount of each variety consumed but also of the total number of varieties available. On the supply side, production exhibits economies of scale within varieties but no economies of scope across varieties. Consequently, each firm supplies one and only one variety (*monopolistic*). However, there are no entry or exit barriers so that profits are just enough to cover average cost (*competition*). Finally, there

[1] The assumption of a representative consumer displaying love of variety stands for a heterogenous population of consumers. And indeed, it can be shown that the CES utility may be used as an aggregate for such a population (see Anderson *et al.* (1992, Chs.3-4)).

are *many* firms so that firms do not interact directly through strategic interdependence but only indirectly through aggregate demand effects.

In this setting there is no obvious *a priori* on whether the market provides too many or too few varieties (Spence, 1976b). On the one hand, since revenues from producing a certain variety do not capture the corresponding consumer surplus, they may not cover costs even when the net social value of the variety is positive. This creates a potential bias towards too few varieties. On the other hand, when a new variety is introduced, it affects the profits of existing firms and gives rise to another external effect because the profit of the new entrant does not, in general, correspond to the net change in profits in the economy as a whole. If varieties are complements, this effect also favors too few of them. However, if varieties are substitutes, it fosters too many of them so that the net outcome is ambiguous (Stiglitz, 1986).

In addition, the presence of these countervailing distortions makes it difficult to establish *a priori* which kinds of varieties are likely to be supplied by the market. In the limit, even when the optimal number of varieties is provided, the available varieties may not be the optimal ones.

Therefore, we concur with Dixit and Stiglitz (1977, p.308) when they write:

> 'The general principle (...) is that a market solution considers profit at the appropriate margin, while a social optimum takes into account the consumer's surplus. However, applications of this principle come to depend on details of cost and demand functions'.

As a result, most of the investigations on the issue of optimum product diversification have been carried on through pedagogical examples with the aim of identifying those features of the market equilibrium which are important in determining whether there is likely to be an under-, over- or biased supply of variety (Stiglitz, 1986).

Spence (1976b) proposes two models. Both models feature two goods, a homogeneous good (the numéraire) and a composite good of horizontally differentiated varieties. Utility is additive in the two goods giving his model a strong partial equilibrium flavor. His aim is to relate product diversity to the *own- and cross-elasticities between varieties.* The first model captures love for variety via a generalized quadratic sub-utility function. The number of varieties is kept constant in order to analyze product selection: supply is biased against products whose revenues capture small fractions of the contributed surplus (i.e., have steep demand functions maybe due to a naturally limited set of buyers with extreme variegation in the valuation of the product). In the second model love for variety is translated into a generalized CES sub-utility function. Unlike the previous model, in order to gain insight on product diversification, varieties are assumed to enter utility symmetrically and their number is endogenized. The market outcome is compared with the social optimum in two cases. The first best optimum, obtained when lump sum transfers are available, and the second best optimum, arising when they are not (in which case profits cannot be negative). In the CES case, at the first best optimum firms make negative profits and there are more varieties than at the market equilibrium, while existing varieties are supplied in the same quantity. By contrast, the second best optimum and the market equilibrium are shown to coincide.

Dixit and Stiglitz (1977) complement Spence (1976b) by tackling the issue of *intersectoral substitution* in a general equilibrium framework. They concentrate on the case where the varieties in the differentiated commodity sector are good substitutes among themselves, but poor substitutes for the numéraire commodity. While their results stemming from intrasectoral elasticities mirror Spence's, additional insights are provided on the role of intersectoral substitutability. In particular, they focus on a 'central case' (pp.298-302), which differs from Spence's second model by assuming finite rather than infinite elasticity of intersectoral substitution. In the central case, as in Spence (1976b), the market equilibrium coincides with the second best optimum while the first best optimum has a greater number of firms. Moreover, the elasticity of intersectoral substitution governs the resource allocation between the sectors. From the technical standpoint, it also plays a central role in the proof of uniqueness of equilibrium and in the second-order conditions for an optimum.

Despite some other important contributions in the field (see, e.g. Hart (1985a, b), Wolinsky (1986), Deneckere and Rothschild (1992)), it is fair to say that most applications of monopolistic, not to say imperfect, competition theory to trade (Helpman and Krugman (1985)), growth (see, e.g. Romer (1990)), geography (see, e.g. Krugman (1991)), or macroeconomics (see, e.g. Blanchard and Kiyotaki (1987)), have been carried out under the S-D-S model (see Matsuyama (1995) for a recent survey). It is therefore important to know the general validity of the conclusions derived under the S-D-S model.

The aim of this paper is twofold. First, we want to present an alternative model to the S-D-S one and check the robustness of its conclusions. In particular, it is our contention that our model can serve as an alternative to the S-D-S model, especially because of its better tractability when dealing with strategic considerations. Second, we wish to supplement the foregoing classical analyses by investigating the issue of product diversification in the presence of multiproduct firms. This issue is not dealt with by Dixit and Stiglitz (1977) while Spence's (1976b) '[f]or reasons of space, (...) simply sketch[es] the argument and its implications' (p.225). In a world where flexible manufacturing and multiproduct strategies become more and more relevant in business, this is certainly a useful extension.

To this end, we use *a quadratic utility model of horizontal product differentiation* which has often been employed in industrial organization (see, e.g. Dixit (1979) and Vives (1990)). There are several good reasons for choosing this model First, it leads to very simple closed form for the equilibrium outcomes that can serve as a building block in much broader contexts. Second, it provides a first-order approximation of any demand system which is satisfactory once these demands are used to describe the market in the vicinity of the equilibrium. Hence it allows for a reconciliation of the two main approaches to monopolistic competition used in general equilibrium, that is, the 'objective' demand approach taken by Gabszewicz and Vial (1972) and the 'subjective' demand one suggested by Negishi (1961). Our model is therefore a natural starting point to explore more general models for which the two approaches are formally equivalent. Third, it can be interpreted as a setting in which firms are boundedly rational in that they extrapolate linearly their demand around the solution to the first order conditions as in Bonanno (1988) and Gary-Bobo (1989). Last, unlike the CES demands, the linear demands do not satisfy the restrictive condition of independence from irrelevant alternatives according to which the ratio of demands for any two varieties is independent of the other varieties' prices.

However, since we assume that preferences are linear in the homogeneous good, the model can be used to study allocative issues but not redistributive ones. Furthermore, we consider *a continuum of potential varieties* as an alternative approach to the modelling of the supply side. Such a formulation is based on Aumann's assumption of an atomless economy in which firms are represented by points inside a (distance-free) continuum of corresponding varieties (see Spence (1976b, section 5) for a similar idea as well as the original formulation developed by Dixit and Stiglitz in their working paper). This approach has become the standard choice adopted in the various applications of the S-D-S framework (see, e.g., Matsuyama (1995)). As we will argue below, the atomless assumption is probably the most natural way to capture Chamberlin's (1933) intuition regarding the working of a 'large group' industry. This also allows us to get rid of the integer problem which often leads to inelegant results and cumbersome developments.

The atomless approach does not necessarily implies that firms behave in a nonstrategic manner, as applications of the S-D-S models seem to suggest. Instead, in our setting each firm must figure out what will be the total output in equilibrium when choosing its own quantity. This is not what we typically encounter in a differentiated oligopolistic market when (almost all) individual decisions made by the other firms are needed by each firm. Here, we have a setting in which *each firm must know only a global statistics about the market but not its details.* We agree with Chamberlin's (1933, p.74) when he writes:

> 'Theory may well disregard the interdependence between markets whenever business men do, in fact, ignore it.'

However one should not go too far into this direction. We believe that using a statistics of the market is a particularly appealing way to capture the idea of monopolistic competition because it saves the essence of competition by forcing each firm to account for the aggregate behavior of its competitors through the total industry output (or, alternatively, the average price index). In our opinion, this is a sort of minimum requirement for the word 'competition' to be meaningfully associated with the word 'monopoly'. On the contrary, because of the particular functional form chosen in the central S-D-S model, firms do not need such an information and behave like pure monopolists. As a result, while in the S-D-S model the market equilibrium reflects only cost and preference considerations, in our framework the equilibrium also accounts for the mass of competing firms and, therefore, for various intensities of competition.

The atomless economy seems to be the ideal point of departure for our investigations of multiproduct firms. Indeed, a multiproduct firm can be most conveniently modelled as an *atom*, viz. a strictly positive measure subset of varieties which includes no proper subset of strictly positive measure. An atom is therefore a non-negligible participant to the market (Gabszewicz and Shitovitz, 1992). We investigate the properties of the market equilibrium in relation to the number of active firms. In particular, we identify conditions under which monopolistic competition may be a good approximation of the more realistic model of oligopolistic competition with multiproduct firms.

Among the results obtained below, we wish to single out the following ones. First, under monopolistic competition with single-product firms the market tends to overprovide diversity when the representative consumer has a weak preference for variety and when fixed costs are low relative to the market size, a result which illustrates Chamberlin's excess capacity 'theorem'. Second, we also show that the second best optimum leads to a smaller number of varieties sold at a lower price in a larger quantity than the market solution. In other words, in our setting the market solution differs from the second best solution unlike Spence (1976b) and Dixit and Stiglitz (1977) who showed equivalence under the CES. Third, in oligopolistic competition with multiproduct firms, the product range provided by each firm narrows with the number of firms but the entire range of available varieties expands. Last, we are able to connect the two models through the following result: when the degree of product differentiation is high or when the economies of scope are weak, firms have few incentives to provide several varieties and the market outcome looks like the monopolistically competitive one.

The remaining part of the paper is organized as follows. Section 2 constructs a tractable model of monopolistic competition with horizontal product differentiation. Section 3 solves for the market equilibrium as well as the first and second best optima. It is shown that this model, while as easy to handle as the model of Dixit and Stiglitz (1977), yields richer results in terms of optimum product diversity avoiding some of the main pitfalls of the S-D-S approach pointed out by Hart (1985a, b) and Bénassy (1996). Section 4 deals with the case of multiproduct firms. The fact that each firm is now an atom in the variety space shows *how the nature of competition is affected by the switch from single-product to multiproduct firms.* In particular, diversity is affected with respect to the single-product monopolistically competitive model because of the trade-off faced by a multiproduct firm between the following effects. The first one is the market expansion effect generated by proliferation; the second one is the cannibalization effect due to competition within each firm's product line while the third one comes from the additional cost a firm bears each time it launches a new variety. The first effect fosters product diversity while the other two push toward less diversity. Section 5 concludes.

2 The Model

There is only one factor, say labor, which is in fixed supply, say 1. There are two goods. The first good is homogeneous and is produced under constant returns to scale and perfect competition. This good is chosen as the numéraire. The other good is a horizontally differentiated product which is supplied under increasing returns to scale and imperfect competition. To capture the essence of monopolistic competition, we want each firm to have a negligible impact on the market outcome. To this end, we assume that there is a continuum N of firms so that all the unknowns of our model are described by *density* functions. Because of competition, any two firms do not want to sell the same variety, thus implying a one-to-one relationship between firms and varieties.

Consider a representative consumer whose preferences are described by the following quasi-linear utility function which is supposed to be symmetric in all varieties:

$$U(q_0; q(i), i \in [0,N]) = K + \alpha \int_0^N q(i)di - \frac{1}{2}\beta \int_0^N q(i)^2 di - \gamma \int_0^N \int_0^N q(i)q(j)didj + q_0 \quad (1)$$

where $q(i)$ is the quantity of variety $i \in [0,N]$ and q_0 the quantity of the numéraire. The parameters in (1) are such that $\alpha > 0$ and $\beta \geq \gamma > 0$, while K is a constant. In this expression, α is a measure of the size of the market (since it expresses the intensity of preferences for the differentiated product), whereas a large value for β means that the representative consumer is biased toward a dispersed consumption of varieties, thus reflecting a love for variety.[2] For a given value of β, the parameter γ expresses the substitutability between varieties: the higher γ, the closer substitutes the varieties.[3] Finally, we assume that the initial endowment $\overline{q}$ in the numéraire is large enough for the consumption of the numéraire to be strictly positive at the market and optimal solutions.

We may identify a simple condition on the parameters β and γ such that the representative consumer shows a taste for variety. In the spirit of Bénassy (1996), we say that the consumer has a *taste for variety* if she prefers n varieties (when n is now an integer) in quantity q each to

[2] The intuition behind this interpretation is very similar to the the one that stands behind the Herfindahl index used to measure industrial concentration. Controlling for the total amount of the differentiated good consumption, the absolute value of the quadratic term in (1) increases with concentration of consumption on few varieties, thus decreasing utility.

[3] The quadratic utility model may be given a disaggregate foundation in terms of the spatial model of product differentiation. In such a context, γ is an inverse measure of the distance between varieties (see Anderson *et al.* (1992, Ch.5)).

a single variety supplied in quantity nq, for all $q > 0$ and all $n \geq 2$. Rewriting (1) when n is an integer yields

$$U(q_0; q(i), i \in \{1, ..., n\}) = K + \alpha \sum_{i=1}^{n} q(i) - \frac{1}{2}\beta \sum_{i=1}^{n} q(i)^2 - \gamma \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} q(i)q(j) + q_0$$

Evaluating this expression at $q(i) = q$ for all $i = 1, ..., n$ and using the utility $U = K + \alpha Q - \frac{1}{2}\beta Q^2$ when a single variety is supplied in quantity $Q = nq$, we see that Bénassy's condition becomes

$$K + \alpha nq - \frac{1}{2}\beta nq^2 - \gamma n(n-1)q^2 > K + \alpha nq - \frac{1}{2}\beta(nq)^2$$

which holds if and only if

$$\beta > 2\gamma \tag{2}$$

In words, *the quadratic utility function exhibits a preference for variety when the product is differentiated enough.* Clearly, the smaller is γ and/or the larger is β, the stronger is the preference for variety.

In many applications of the S-D-S model, a Cobb-Douglas preference on the homogeneous and differentiated goods with CES subutility is considered, while we assume instead a *quasi-linear preference with a quadratic subutility.* Observe that both utilities correspond to two rather extreme cases: the former assumes a unit elasticity of substitution, the latter an infinite elasticity between the differentiated product and the numéraire. Finally, since the marginal utility of the numéraire is constant, our model can be used in welfare comparisons while this is not necessarily true with the CES (see Anderson *et al.* (1992, Ch.3)).

The representative consumer is endowed with one unit of labor. Her budget constraint can then be written as follows:

$$\int_0^N p(i)q(i)di + q_0 = w + \overline{q} \tag{3}$$

where w is the wage and $p(i)$ the price of variety i.

Solving (3) for the numéraire consumption, substituting the corresponding expression into (1) and solving the first order conditions with respect to $q(i)$ yields inverse demands:

$$\alpha - \beta q(i) - \gamma Q = p(i), \qquad i \in [0, N] \tag{4}$$

where

$$Q \equiv \int_0^N q(j)dj \tag{5}$$

is the total market output of the differentiated product.

Since firm i faces the demand function (4), it can be seen that firms do not interact directly. However they do interact indirectly through an aggregate demand effect as shown by the presence of Q in (4). Thus, though each firm is negligible to the market, when choosing its output level it must figure out what the total output (5) will be. Stated differently, we have a model in which *a firm correctly neglects its impact on the market but must explicitly account for the impact of the market on its profit.*[4] It then follows from our discussion above that assumption (i) discussed in 2.1 is not inherent to monopolistic competition models. Furthermore, unlike the central model of Dixit and Stiglitz, the elasticity of (4) is variable.

Technology in the homogeneous production requires one unit of labor in order to produce one unit of output. The choice of this good as the numéraire implies that in equilibrium $w = 1$. Technology in manufacturing requires $F < 1$ units of labor in order to produce any amount of a variety and the marginal cost of production of a variety is equal to zero. This simplifying assumption, which is standard in many models of industrial organization, makes sense here because our preferences imply that firms use an absolute markup instead of a relative one when choosing prices. In Dixit and Stiglitz (1977), there would be no equilibrium under such an assumption.

The assumptions above imply a perfectly symmetric setting. Hence we may focus on an equilibrium in which all firms produce the same output q sold at the same price p, and concentrate on the issue of product diversity without being concerned about product selection. This implies that the budget constraint (3) can be rewritten as follows:

$$Npq + q_0 = 1 + \overline{q}$$

[4]The same holds in the Dixit-Stiglitz model with a continuum of firms but there each firm must account for the price index only when they make their entry decision. In our model, this statistics also influences the output choice.

Finally, entry and exit are free so that profits are zero in equilibrium. Before proceeding, it is worth stressing here that our model builds on the recent debate on the *approximation approach* to closed form solutions in the S-D-S model. This debate arises from the fact that the analytical results by Dixit and Stiglitz (1977) and those in the first model by Spence (1976b) are obtained using an approximate solution method based on the assumption that, if there are *many firms*, (i) their influence on the aggregate price index can be ignored as well as (ii) their direct interactions in the product and labor markets (i.e. each producer ignores the cross-price elasticities). Subsequent research has mainly focused on the question of *how many is 'many'*. Yang and Heijdra (1993) propose to relax assumption (i). Limiting their model to the special case of unit elasticity of intersectoral substitution, they show that, at the market outcome, Dixit and Stiglitz's central model underestimates the number of firms, underestimates prices and overestimates quantities. In their reply, Dixit and Stiglitz (1993) observe that Yang and Heijdra's approach is still an (possibly inconsistent) approximation because, if it disregards assumption (i) it still keeps assumption (ii). d'Aspremont *et al.* (1996) move one step further by allowing firms to take into account the general equilibrium income effects of their decisions via the labor market. They show that, in the central model, Yang and Heijdra's approach also underestimates the number of active firms even though more slightly than Dixit and Stiglitz's. Moreover, they show that in an enlarged model with labor as an additional good, Yang and Heijdra's approach does not necessarily lead to an outcome closer to the true solution than Dixit and Stiglitz's.

3 Single-product Firms and Monopolistic Competition

3.1 The market equilibrium

The representative firm maximizes profit:

$$\Pi(i) = p(i)q(i) - F \tag{6}$$

subject to (4) with respect to its output $q(i)$. Using symmetry among firms in the first order conditions gives equilibrium quantity:

$$q = \frac{\alpha}{2\beta + \gamma N} \tag{7}$$

and, using (4), equilibrium price:

$$p = \frac{\alpha\beta}{2\beta + \gamma N} \tag{8}$$

as functions of the number of active firms N. This is a significant departure from Spence (1976b) and Dixit and Stiglitz (1977) in which the equilibrium price is a constant relative markup times marginal cost. By contrast, the equilibrium price is here given by an absolute markup (recall that marginal cost is zero) which decreases with the number of firms. The markup also rises when there is more product differentiation (γ falls). It is worth noting that these effects concur with what is observed in industrial organization models of product differentiation (Archibald *et al.* (1986); Anderson *et al.* (1992)), while they are not part of the S-D-S model.

In equilibrium, we also have:

$$\alpha - \beta q - \gamma N q = \alpha - (\beta + \gamma N) q = p \tag{9}$$

which shows how increasing product diversity N within the industry shifts downward a firm's demand by reducing the value of its intercept. Accordingly, (4) might be interpreted as Chamberlin's dd-demand curve when the size of the industry is given, whereas (9) would correspond to his DD-curve. As usual, the DD-curve is steeper than the dd-curve but has a larger intercept.

The equilibrium value of N can be found by setting (6) equal to zero after substituting for (7) and (8):

$$N^* = \frac{\alpha\sqrt{\beta/F} - 2\beta}{\gamma} \tag{10}$$

As expected, for $\gamma > 0$, the equilibrium number of firms increases when varieties are more differentiated but decreases when fixed cost rises. Also,

$N^* > 0$ if and only if

$$\alpha > 2\sqrt{\beta F} \tag{11}$$

that is, if and only if the fixed cost of a firm is small relative to market size.

Substituting (10) in (7) gives the equilibrium quantity:

$$q^* = \sqrt{F/\beta} \tag{12}$$

while introducing (10) in (8) gives the equilibrium price:

$$p^* = \sqrt{\beta F} \tag{13}$$

so that revenues $(q^* p^*)$ are just sufficient to cover the fixed cost.

Observe that we have chosen to conduct our analysis in terms of quantity-setting firms, as in Spence (1976b) and Dixit and Stiglitz (1977). It is worth noting that the same results could have been obtained with price-setting firms as shown in the appendix. While *the choice of quantity or price as a decision variable* is a crucial one in oligopolistic competition, it *does not seem to be essential for monopolistic competition.*

3.2 The first best optimum

A planner seeking the first best maximizes the utility of the representative consumer evaluated at a symmetric outcome

$$U = K + \alpha N q - \frac{\beta}{2} N q^2 - \gamma N^2 q^2 + q_0$$

with respect to q and N subject to the resource constraint:

$$FN + q_0 = 1 + \overline{q}$$

The first order condition on q requires:

$$\alpha - \beta q - 2\gamma N q = 0 \tag{14}$$

while the first order condition on N means that:

$$\alpha q - \frac{\beta}{2}q^2 - 2\gamma N q^2 - F = 0$$

Together, these conditions imply that the optimum output of each variety is:

$$q^o = \sqrt{2F/\beta} \tag{15}$$

while the optimal number of varieties of the differentiated product is given by:

$$N^o = \frac{\alpha\sqrt{\beta/2F} - \beta}{2\gamma} \tag{16}$$

Given any $\gamma > 0$, N^o is positive if and only if

$$\alpha > \sqrt{2\beta F} \tag{17}$$

We now check that the shadow price of a variety equals marginal cost. At the optimum, the shadow price p^o is the same across varieties and, following Dixit and Stiglitz, it must satisfy:

$$Np^o = \frac{\partial U/\partial q}{\partial U/\partial q_0}$$

which says that the marginal rate of substitution between the differentiated product and the numéraire is equal to the price ratio. Using (14), we obtain:

$$\frac{\partial U/\partial q}{\partial U/\partial q_0} = N(\alpha - \beta q - 2\gamma N q) = 0$$

implying that the shadow price p^o is zero, that is, marginal cost.

Using (12) and (15), it is readily verified that we always have $q^o > q^*$ and $p^o < p^*$. In order to compare the equilibrium and optimum numbers of firms, α must satisfy both (11) and (17), that is, the former which is the more stringent.

Clearly, we have $N^* > N^o$ if and only if

$$\alpha > \overline{\alpha} \equiv \frac{3\sqrt{2}}{2\sqrt{2}-1}\sqrt{\beta F} \tag{18}$$

The results above mean that, in monopolistic competition, *the equilibrium number of varieties may be larger than the optimum number,* a result in sharp contrast to that obtained by Spence (1976b) and Dixit and Stiglitz (1977) in the CES case.[5] In particular, we see that *the market tends to overprovide variety* (an example of Chamberlin's excess capacity) *when the representative consumer has a weak preference for variety* (see (2)) *and when fixed cost F are low compared to the market size*, a very intuitive result. Consequently, the quadratic utility model allows us to qualify the assertion made by Dixit and Stiglitz (1977) about the generality of their model: "With variable elasticities (...) there [is] some presumption that the market solution would be characterized by too few firms in the monopolistically competitive sector" (p.308). This difference in results is due to the fact that, in the S-D-S model, the equilibrium price is so low[6] that the incumbents' profits are low enough to deter entry, thus leading to insufficient product diversity. On the contrary, in our model, the equilibrium price varies with the number of firms, thus allowing the incumbents to charge a higher price and to earn higher profits (letting N go to infinity in our setting would lead to zero price, negative profits and no entry). In general, *the comparison between the market equilibrium and the optimum outcomes is therefore ambiguous*, as already noticed by Meade (1974) in a different, less formal setting.

By contrast, when the product is very differentiated in that (18) holds, the market always overprovides product diversity, thus confirming other existing analyses (see, e.g. Archibald *et al.* (1986); Anderson *et al.* (1995)).

3.3 The second best optimum

Since profits are always negative and equal to $-F$ at the first best optimum, a planner constrained by the absence of lump-sum transfers may

[5] Recall that Salop (1979) shows that excess variety characterizes spatial models of monopolistic competition.

[6] If one considers the sequence of Nash price equilibria of games involving an integer number N of firms, the S-D-S equilibrium price is the limit of this sequence when $N \to \infty$ (see Anderson *et al.* (1992, Ch.7)).

choose to maximize:

$$U = K + \alpha N q - \frac{\beta}{2} N q^2 - \gamma N^2 q^2 + q_0 \tag{19}$$

(where we have used again symmetry among varieties) with respect to q and N subject to the consumer budget constraint:

$$Npq + q_0 = 1 + \overline{q} \tag{20}$$

the zero profit condition:

$$pq - F = 0 \tag{21}$$

and the demand function:

$$p = \alpha - (\beta + \gamma N)q \tag{22}$$

Plugging (22) into (20) as well as into (21) and the corresponding results into (19), the constrained planner maximizes:

$$U = K + \frac{\beta}{2\gamma}(\alpha q - \beta q^2 - F)$$

Thus, the second best quantity is:

$$q^S = \frac{\alpha}{2\beta}$$

which is the quantity that a firm would choose if disregarding the impact of competition on its profit, viz. acting as a monopolist. The corresponding price is:

$$p^S = \frac{2\beta F}{\alpha}$$

By (21) and (22), this implies:

$$N^S = \frac{\beta(\alpha^2 - 4\beta F)}{\alpha^2 \gamma}$$

which is positive if and only if (11) holds.

Note also that (11) implies that $p^S < p^*$, that is, *the second best price is always below the market price*. Using again (11), we see that $q^S > q^*$. It remains to compare N^* and N^S. We have

$$\frac{\alpha\sqrt{\beta/F} - 2\beta}{\gamma} > \frac{\beta(\alpha^2 - 4\beta F)}{\alpha^2\gamma}$$

which is equivalent to

$$1 > \frac{\sqrt{\beta F}\left(\alpha + 2\sqrt{\beta F}\right)}{\alpha^2}$$

which holds because of (11) so that $N^* > N^S$.

To sum-up: *the second best optimum leads to a smaller number of varieties sold at a lower price in a larger quantity than the market solution*. Hence, in our setting, the market solution does differ from the second best solution unlike Spence (1976b) and Dixit and Stiglitz (1977) who showed equivalence under the CES.[7]

4 Competition with Multiproduct Firms

So far we have considered a one-to-one relationship between firms and varieties. We now investigate the implications of having *a finite number of multiproduct firms, each supplying a non-negligible set of varieties.* For that, it is assumed that, in addition to the cost associated with each supplied variety, each firm has to pay an entry cost regardless of the size of its product range, thus implying that economies of scope are present. Because multiproduct firms are 'large actors', they now interact in a more strategic way than in the section above. In other words, firms now behave like oligopolists. In addition, the possibility of choosing its product range gives each firm an additional tool to challenge its competitors.

[7] This coincidence result depends crucially on the CES assumption (Spence, 1976b; Dixit and Stiglitz, 1977) and on the related "accidental implicit choice of the 'taste of variety' parameter" (Benassy, 1996).

We want to know whether this increased interaction improves upon the market equilibrium described in the foregoing. To this end, we consider a very simple setting in which each active firm, first, chooses its product range and, then, the quantity of each variety it supplies. In the same spirit as in the section above in which a firm can be identified with the variety it supplies, we assume here that each firm has access to *an exclusive continuum of potential symmetric varieties.* However, since launching a variety involves some positive cost, a firm does not necessarily produce all these varieties and must choose how many of them to supply. Formally, this implies that the strategy spaces are such that the overlap of the product lines of any two firms is a zero measure set. The market solution is given by a subgame perfect Nash equilibrium; we look for a symmetric equilibrium and will describe the equilibrium path only. Since our purpose is to keep the analysis as intuitive as possible, we will refrain from providing a full description of the game.

Let M be a given (integer) number of multiproduct firms facing an identical entry cost G and an identical fixed cost F per variety. We denote by $\Omega_i \subseteq R_+$ the set of varieties ω produced by firm i $(= 1, ..., M)$. Because of our symmetry assumption, it will be convenient to describe its size by Ω_i too. The total production cost of firm i is then given by $G + F\Omega_i$ .Firm i maximizes its profit given by:

$$\Pi_i = \int_{\omega \in \Omega_i} [p_i(\omega) q_i(\omega) - F] d\omega - G \tag{23}$$

where $p_i(\omega)$ is the price of variety ω supplied by firm i and $q_i(\omega)$ the quantity of this variety. Demand for variety ω is still defined by (4). However, because a firm now controls a non-negligible set of varieties, the total output Q is no longer given to the firm since it varies with its own output. Since we focus on a symmetric outcome, we may assume that all the other firms choose the same product range $\Omega \subseteq R_+$. Consequently, we have:

$$Q_i = \int_{\omega \in \Omega_i} q_i(\omega) d\omega + (M-1) \int_{\omega \in \Omega} q(\omega) d\omega \tag{24}$$

where the first integral corresponds to the demand for the varieties produced by firm i while the second integral refers to those produced by its competitors. Note the difference with the monopolistic competition model where the total output of the industry (5) is unaffected by a single firm's decision, while here it varies with each firm's decision as shown by (24).

The solution of the second stage subgame is obtained as follows. Because we have symmetry among varieties within each firm's product line, the quantity supplied by a firm is the same across its varieties. In other words, we have $q_i(\omega) = q_i$ for all $\omega \in \Omega_i$ and $q(\omega) = \widehat{q}$ for all $\omega \in \Omega$. Consequently, (23) may rewritten as follows:

$$\Pi_i = (p_i q_i - F)\Omega_i - G$$

whereas (24) becomes

$$Q_i = q_i \Omega_i + (M-1)\widehat{q}\Omega$$

Hence we have

$$\Pi_i = \left\{ [\alpha - \beta q_i - \gamma[q_i \Omega_i + (M-1)\widehat{q}\Omega] \, q_i - F \right\} \Omega_i - G$$

Applying the first order condition with respect to q_i, the equilibrium output q_i^* must satisfy:

$$\alpha - 2\beta q_i^* - 2\gamma \Omega_i q_i^* - \gamma(M-1)\widehat{q}\Omega = 0$$

so that the equilibrium quantity of each variety provided by firm i is:

$$q_i^*(\Omega_i, \Omega) = \frac{\alpha - \gamma(M-1)\widehat{q}\Omega}{2(\beta + \gamma\Omega_i)} \tag{25}$$

The equilibrium quantity of each supplied variety thus decreases with the size of the firm i's product range. Cannibalization arises despite the fact that the representative consumer (who loves variety) spends more on the differentiated product when the number of varieties rises. Indeed, she chooses to distribute her higher expenditure in a way that reduces the consumption of each variety. Consequently, brand proliferation is not just limited by the fixed cost associated with the launching of a new variety. It is also limited by competition within the product line.

The equilibrium price charged by firm i for all its varieties is:

$$p_i^*(\Omega_i, \Omega) = \frac{\alpha - \gamma(M-1)\widehat{q}\Omega}{2} \tag{26}$$

which is decreasing in Ω because a wider array of products supplied by rivals makes firm i's market more competitive.

The second-staged equilibrium profit of firm i is then given by (23) after having substituted for (25) and (26):

$$\Pi_i(\Omega_i, \Omega) = \frac{[\alpha - \gamma(M-1)\widehat{q}\Omega]^2}{4(\beta + \gamma\Omega_i)}\Omega_i - F\Omega_i - G \tag{27}$$

which describes the payoff of firm i in the first stage game, conditional upon the fact that the others choose the product range Ω.

The expression (27) has a unique and finite maximum since $\Pi_i(\Omega_i, \Omega)$ is strictly concave in Ω_i and becomes arbitrarily small for large enough Ω_i. In order to determine the equilibrium product range at the symmetric equilibrium of the first stage game, we differentiate (27) with respect to Ω_i, set $\Omega_i = \Omega$ and obtain after some simple manipulations:

$$\frac{\alpha - \gamma(M-1)\widehat{q}\Omega}{\beta + \gamma\Omega} = 2\sqrt{F/\beta} \tag{28}$$

In a symmetric equilibrium, it must be that $q_i^*(\Omega_i, \Omega) = \widehat{q}$ and $\Omega_i = \Omega$. By substitution in (25), we obtain:

$$\widehat{q} = \frac{\alpha - \gamma(M-1)\widehat{q}\Omega}{2(\beta + \gamma\Omega)}$$

which, together with (28), yields the equilibrium output which is still equal to (12):

$$q^* = \sqrt{F/\beta}$$

Observe that the quantity of each variety supplied by the multiproduct firm in oligopolistic competition is the same as that produced by the single-product firm under monopolistic competition. Moreover, the equilibrium product range is given by:

$$\Omega^*(M) = \frac{\alpha\sqrt{\beta/F} - 2\beta}{\gamma(M+1)}$$

For Ω^* to be positive, we need condition (11) to hold. Clearly, the lower the fixed cost associated with the launching of a new variety or the higher the degree of product differentiation, the wider is the firm's product range, as in Anderson and de Palma (1992).

It thus appears that, though the quantity supplied of each variety is independent of the number of firms, *the number of varieties supplied by a firm decreases as the number of firms rises.* In the limit, $\Omega^* \to 0$ when $M \to \infty$, namely each firm wants to sell a single product (formally, a zero measure set of varieties) when the number of firms is arbitrarily large.

Furthermore, the total number of varieties is given by:

$$N^*(M) \equiv M\Omega^* = \frac{\alpha\sqrt{\beta/F} - 2\beta}{\gamma}\frac{M}{M+1} \tag{29}$$

which converges from below toward (10) when $M \to \infty$.

Using (26), the equilibrium price common to all firms and varieties is:

$$p^*(M) = \frac{(M-1)\sqrt{\beta F} + \alpha}{(M+1)} \tag{30}$$

When $M \to \infty$, this price converges from above toward (13), that is, the equilibrium market price in the single-product monopolistic competition case.

We can now determine the number of firms that are active in equilibrium as a function of the entry cost G. This requires finding M (which is now treated as a real number) such that:

$$p^*(M)q^*\Omega^*(M) - F\Omega^*(M) - G = 0$$

the solution of which is easily calculated as:

$$M^* = \frac{\alpha - 2\sqrt{\beta F}}{\sqrt{\gamma G}} - 1 \tag{31}$$

For M^* to be positive, we need

$$\alpha > 2\sqrt{\beta F} + \sqrt{\gamma G} \tag{32}$$

which is more stringent than (11). In other words, (32) is the condition to be imposed to the parameters of the economy for a positive number of firms to offer more than one variety.

The corresponding equilibrium price may be obtained by substituting (31) into (30) as follows:

$$p^* = \sqrt{\beta F} + \sqrt{\gamma G}$$

which converges from above towards the monopolistic competitive price (13) when $G \to 0$ (no scope economies) and/or $\gamma \to 0$ (varieties are independent). In addition, if $F \to 0$, $p^* = 0$, that is, the competitive level. As for the long run equilibrium product range, it is obtained by plugging (31) into (29):

$$\Omega^* = \sqrt{\frac{\beta}{\gamma}\frac{G}{F}}$$

Clearly, the equilibrium product range expands with love for variety, scope economies and the degree of product differentiation while it narrows with the cost of launching an additional variety. Hence, in choosing their product line, firms trade the potential gains of economies of scope (G) and of the potential increase in market power generated by an expansion of their product line (the interplay between β and γ) against the corresponding launching costs (F).

The total number of varieties supplied by the multiproduct firms given by

$$N^* \equiv M^*\Omega^* = \frac{\sqrt{\beta/F}(\alpha - 2\sqrt{\beta F} - \sqrt{\gamma G})}{\gamma} \tag{33}$$

is smaller than the number of varieties under monopolistic competition as shown by comparing (10) and (33). When $G \to 0$, the number M^* of firms becomes arbitrarily large and the range of varieties N^* converges toward (10). Since the ratio between (10) and (33) goes to one when $\gamma \to 0$, we can say that the two market product ranges tend to coincide when the degree of product differentiation is large.

As expected, the total number of firms M^* rises with the market size (α) and the degree of product differentiation ($1/\gamma$) whereas it decreases with the launching cost of a variety (F), the economies of scope (G) and the love for variety (β). In particular, when $G \to 0$ and/or $\gamma \to 0$, the number M^* of firms becomes arbitrarily large, and *the market with multiproduct firms supplies almost the same range of varieties at almost the same price as under monopolistic competition.*

In view of these results, it is fair to conclude that *monopolistic competition is the limiting case of oligopolistic competition with multiproduct firms as either varieties get more and more differentiated or economies of scope get weaker and weaker.*[8] In a metaphorical sense, as scope economies vanish each atomic firm dissolves in an ocean of negligible firms offering each a single variety.

It remains to compare the market outcome to the optimum. Since there are economies of scope, it is always optimal to have a single firm producing the entire range of varieties. In others words, we have $M^o = 1$ so it must be that $N^o = \Omega^o$.[9] It is then obvious that the optimal number of varieties is still given by (16). Hence, though the market with multiproduct firms always involves too many firms, the number of varieties may be too large or too small depending on the difference between (33) and (16).

Since the number of varieties is always smaller under oligopolistic competition with multiproduct firms than under monopolistic competition with single-product firms (see (33)), the market outcome with multiproduct firms is even further away from the optimum whenever the monopolistic competitive market underprovides variety, namely when (18) does not hold. When by contrast this condition holds, we know that the monopolistic competitive market overprovides variety. This is not necessarily the case under oligopolistic competition where it is readily verified

[8] When G and γ converge together to zero, we assume that the former decreases faster than the latter in order to avoid perverse limiting behavior of Ω^*.

[9] Note that Anderson and de Palma (1992) obtain the same result when intrafirm heterogeneity and interfirm heterogeneity are the same.

that the market overprovides variety if and only if

$$\alpha > \frac{\sqrt{2}}{2\sqrt{2}-1}(3\sqrt{\beta F} + 2\sqrt{\gamma G}) \tag{34}$$

This condition is clearly more stringent than (18). Therefore, when (18) holds but (34) does not, monopolistic competition provides too much diversity whereas oligopolistic competition does not supply enough at the market outcome. In other words, *market structure matters for the comparison between the equilibrium and the optimum*.

The foregoing analysis suggests a new testable property of market structure with respect to product differentiation and scope economies. Product differentiation is low and scope economies are weak in industries where firms offer each a narrow product line, whereas firms with wide product lines are more likely to be observed in markets in which products are very differentiated and economies of scope are strong. Furthermore, with high product differentiation and weak scope economies the market structure is likely to involve many firms and a great deal of product diversity, whereas few firms and little product diversity should be observed with low product differentiation and strong scope economies.

5 Concluding Remarks

Our purpose was twofold. First, we have designed an alternative model of monopolistic competition which is both tractable and intuitively plausible. It turns out that this model allows us to formalize the idea of monopolistic competition in a way that places it between oligopolistic competition and perfect competition. We have shown that this model, while easier to handle than the model by Dixit and Stiglitz (1977), yields richer results in terms of optimum product diversity while avoiding some of the main pitfalls of the S-D-S approach. Given the high flexibility of the model proposed, we may reasonably expect it to be useful in applications where the S-D-S model have led to new and relevant economic results. The advantages of doing so are manifold. In particular, even if our model does not seem appealing to study long run growth of a balanced kind, it should permit to test the robustness of existing results often criticized for their lack of generality. If so, this would suggest the existence of a whole class of monopolistically competitive economies for which some of the main results derived in trade, growth, geography, or macroeconomics, hold true. A first attempt undertaken by the authors in economic geography shows both the robustness of the main results as well as the limits of the approach followed in the literature (Ottaviano and Thisse, 1998). In addition, we conjecture that the model could be suitable to study a broader set of issues once we allow the parameters of the utility function (1) to be dependent on the income level and/or the size of the product range, an assumption that would reflect changes in preferences as the economy expands.

Second, we have explored a topic so far untouched, namely the relationships between monopolistic competition with single-product firms and oligopolistic competition with firms supplying wide arrays of varieties. The real world is repleaded with such big oligopolists and it is important to assess the quality of the approximation made by using monopolistic competition in applications. It seems to us that the conclusions derived in the foregoing are positive enough to justify further explorations of the theoretical foundations of monopolistic competition and to test the validity of propositions obtained in applications against alternative formulations of monopolistic competition. In particular, we have shown the existence of a trade-off faced by multiproduct firms between economies of scope, cannibalization, and the cost of expanding the product line.

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Appendix

We first derive the demand system and then show that the market outcome with price-making firms is identical to that with quantity-setting firms under monopolistic competition.

We know from (4) that the inverse demands are given by:

$$p(i) = \alpha - \beta q(i) - \gamma \int_0^N q(j)dj \tag{35}$$

Integrating over varieties yields

$$\int_0^N p(i)di = \alpha N - \beta \int_0^N q(i)di - \gamma N \int_0^N q(j)dj$$

from which we obtain

$$\int_0^N q(i)di = \frac{\alpha N - \int_0^N p(i)di}{\beta + \gamma N}$$

Introducing into (35) leads to

$$p(i) = \alpha - \beta q(i) - \gamma \frac{\alpha N - \int_0^N p(i)di}{\beta + \gamma N}$$

from which it follows that

$$\beta q(i) = \frac{\alpha\beta}{\beta + \gamma N} - p(i) + \frac{\gamma}{\beta + \gamma N} \int_0^N p(i)di$$

Adding and substracting $N\gamma p(i)/(\beta + \gamma N)$, we get after simplifications

$$q(i) = \frac{\alpha}{\beta + \gamma N} - \frac{1}{\beta + \gamma N} p(i) + \frac{\gamma}{\beta(\beta + \gamma N)} \int_0^N [p(j) - p(i)]dj \qquad (36)$$

Profits are still given by (6) where $q(i)$ is now given by (36). Differentiating the corresponding expression with respect to price and focussing on a symmetric equilibrium yields (8) and (7).