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**EUROPEAN UNIVERSITY INSTITUTE  
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# Endogenous Growth through Selection and Imitation\*

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## Abstract

A simple dynamic general equilibrium model is set up in which firms face idiosyncratic productivity shocks. Firms whose productivity has fallen too low exit, and entrants try to imitate the best practice of existing firms, so that the expected productivity of entering firms is a function of current average productivity. Because of the resulting selection and imitation process, aggregate productivity grows endogenously. When calibrated to U.S. data, the model suggests that around one-fifth of productivity growth is due to such a selection and imitation effect.

*JEL codes:* B52, O3, O41.

*Keywords:* endogenous growth; selection; imitation; firm entry and exit.

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# 1 Introduction

The competitive struggle among heterogeneous firms is among the defining features of a market economy. Not only does this struggle drive the price of goods down to their marginal cost of production; it also ensures that those goods are produced efficiently. Firms which are unable to do that must eventually exit the market, and are replaced by new, more efficient firms. One way to interpret this mechanism is that competition allows for the selection of good ideas. Productivity growth is driven by the trial of successive ideas and the weeding out of bad ones. Successful ideas will be copied by entering firms, causing productivity to grow endogenously through a continuous process of selection and imitation.

The idea that there can be economic growth through selection dates back at least to the seminal work of Nelson and Winter (1982) on evolutionary economics. The strand of literature that has followed it considers the process of growth in analogy to the process of natural selection, in which only the fittest survive, and where ‘efficient’ behaviour is transmitted to future generations in the form of genes. However, this literature generally focuses on how behavioural rules evolve in a world of bounded rationality. Nevertheless, as this paper shows, there is no inherent contradiction between a mechanism of growth through selection and rational expectations.

One of the few to explicitly model the outcome of selection in terms of growth is Conlisk (1989). He sets up a simple model in which the productivity of new plants is a random draw whose mean depends on current average productivity; labour is then moved from the least productive old plants towards entering plants, causing the former to shut down. As a result, the economy grows at an endogenous rate, which crucially depends on the variance of the random draw of new plants. One of the drawbacks of the model is that only the entry process is stochastic, which is strongly rejected by the data. Furthermore, as is common in the evolutionary economics literature, firms operate in a setting of bounded rationality; in practice, this generally means that the number of entering and exiting firms is set exogenously, which precludes any meaningful statements about the quantitative implications of such models.

The present paper proposes to fill that gap by trying to quantitatively link the selection process going on at the firm level to the rate of growth of the aggregate economy. This is done by setting up a dynamic general equilibrium, rational expectations model with mean-preserving idiosyncratic productivity shocks to firms; in other terms, the expected growth rate of firm-specific productivity is zero. One way to interpret those shocks is to imagine that each firm represents an idea, or variation of an idea, and that firms try to improve the execution of this idea by progressively making small changes to the production process. The outcome of

those changes might be uncertain, although their expected impact on productivity will probably be positive. Also, other existing firms might find it difficult to emulate at least some of these changes, leading to heterogeneity in productivity levels across firms. Entering firms will then try to implement as a whole the production processes of those firms which they think perform best, and after that will focus on making small changes to these processes; some of these changes will be inspired by what other firms in the economy do.

This means that there are two channels through which more efficient production processes spread across the economy. The first is through spillovers between existing firms, and concerns ideas which are relatively easily transferred from one firm to another; the extreme case in which all firms can implement such ideas costlessly is sometimes referred to as neutral technological progress. The second is through spillovers from existing towards entering firms, and concerns ideas whose implementation require for example very different organisational structures, and which entering firms might find much easier to implement; this is sometimes called disruptive (or non-neutral) technological progress, and should be seen in analogy to the concept of capital-embodied technical change, which stresses that certain technologies can only be implemented by setting up new plants. We focus on this second channel of growth through selection and imitation.

However, a crucial difference between models of embodied technological change in the line of Greenwood, Hercowitz, and Krusell (1997) and our model is that in the former, the economy grows through an expansion in the number of capital goods any given firm can produce. In models of growth through selection such as ours, productivity growth does not happen through an expansion in the frontier of production possibilities; in principle, firms of any productivity may coexist at any point in time. However, the ‘best practice’ of highly productive firms does not spread instantly to other firms in the economy, but is passed over gradually to new firms. The relevant knowledge frontier is then the technology which is used to make new firms.

As in learning-by-doing models (see for example Romer, 1996), in which the amount of innovation depends on the economy-wide output, technological progress is a costless externality. However, in this model, the spread of ‘best practices’ happens through technology spillovers from existing to new firms; this is modeled by assuming that entering firms start with a productivity level which depends on the current average level in the economy, and that the evolution of productivity at a given firm then follows an autoregressive process.

The concept of growth through selection has much in common with the idea of Schumpeterian creative destruction. In the interpretation of creative destruction by Aghion and Howitt (1992), growth is generated by a random sequence of quality-improving, sector-specific innovations; better products or technologies

render previous ones obsolete, and this occurs through the replacement of the incumbent sectoral monopolist by a new firm. An analogous mechanism is at work in models of growth through selection, except that it is not the firm of a given sector, but the marginal firm (i.e., the least profitable of all firms) that is rendered obsolete.

We assume that the capital stock of a firm is fixed at entry, and exiting firms recover part of their invested capital, which can then be re-used by new firms. Hence, and unlike most vintage capital models, the technology for producing capital goods in our model is not putty-clay, since existing capital can be scrapped and re-used (at a cost).

Although the idea of selection has its origin in the evolutionary economics literature, this paper, at least from a modelling standpoint, has more in common with models of industrial evolution, which notably includes papers by Jovanovic (1982) and Hopenhayn (1992). While both papers model idiosyncratic shocks hitting firms each period, the former considers a setup of imperfect information: firms do not directly observe their own productivity level, which leads inefficient plants to delay exit until they have sufficient information. The latter sets up a model with endogenous firm size in order to replicate cross-sectional properties - across size and age cohorts - in the data. However, since the technology of entering firms improves at an exogenous rate, neither of the two models is able to estimate the effect of selection on growth.

Campbell (1998) looks at the business cycle implications of entry and exit. His model is similar to ours except for the fact that he abstracts from imitation, assuming instead that the productivity of entering firms grows at an exogenous rate. Melitz (2003) sets up a model with heterogeneous firms in which the exposure to trade results in an endogenous one-off productivity increase through the entry into the export market of the most productive firms, and the closure of the least productive ones. A number of recent papers dealing with firm entry and exit, among them Comin and Mulani (2005), model firm-level heterogeneity by assuming monopolistic competition. From a quantitative point of view, this approach has the disadvantage of greatly increasing the number of required parameters. In order to keep the complexity of the model to a minimum, we choose to limit ourselves to the case of perfect competition, which greatly facilitates the task of taking the model to the data.

The present paper is closely related to Luttmer (forthcoming), who looks at the effects of selection and imitation on long-run growth, as we do; his model differs from ours *inter alia* by the absence of capital and his assumption of monopolistic competition. His work also uses a different calibration strategy, which focuses on matching empirical firm size distributions. He estimates that around half of U.S. output growth can be attributed to selection and imitation, which he interprets



as an upper bound; we find a contribution of selection and imitation of one-fifth, which we interpret as a lower bound.

The remainder of the paper is organised as follows: section 2 describes the model; section 3 deals with its calibration; section 4 looks at the results, and section 5 concludes.

## 2 The Model

### 2.1 Technology

The economy consists of a continuum of firms producing all the same homogeneous good but differing in their productivity levels. The output  $Y_i$  of any given firm  $i$  depends on its capital and labour inputs  $K_i$  and  $L_i$ , its firm-specific productivity  $Z_i$ , and on firm-neutral productivity  $A$ :

$$Y_{i,t} = (A_t Z_{i,t} K_{i,t})^\alpha L_{i,t}^{1-\alpha}. \quad (1)$$

While labour can be adjusted at any point in time, a firm's capital stock is chosen at entry, and cannot be changed anymore afterwards. Since the production function displays constant returns to scale in  $K_i$  and  $L_i$ , the size of firms is a priori undetermined. Nevertheless, for notational convenience, we will define a firm as consisting of one unit of capital, as in Campbell (1998).

The natural logarithm of the firm-specific productivity level  $Z_{i,t}$  for any given firm  $i$  follows a random walk:

$$\begin{aligned} \ln(Z_{i,t+1}) &= \ln(Z_{i,t}) + \varepsilon_{i,t}, \\ \varepsilon_{i,t} &\sim N(Z_0, \sigma^2), \end{aligned} \quad (2)$$

where  $Z_0$  is set such that the expected growth rate of firm-specific productivity within any given firm is zero.  $\varepsilon_{i,t}$  is independently and identically distributed across firms. The cumulative distribution of firms across productivity levels and the corresponding density function are denoted by  $\Phi_t(Z)$  and  $\phi_t(Z)$ , respectively.

Average firm-specific productivity is defined as

$$\bar{Z}_t = \int_0^\infty Z d\Phi_t(Z).$$

The (gross) growth rate of  $\bar{Z}$  is endogenous and we denote it by  $\gamma_z$ . We assume that firms observe their productivity after having made decisions about whether to enter or exit, but before deciding on their labour input. Firm-neutral technology  $A_t$  grows at a constant (gross) exogenous rate  $\gamma_a$ , where  $\gamma_a > 1$ :

$$A_{t+1} = \gamma_a A_t. \quad (3)$$

Given that capital is a fixed input and since their relative productivity may decline over time, firms have the option of costlessly and definitively ceasing production at any point in time. In this case the firm is scrapped, and its capital can be transformed into new capital at a rate  $\theta < 1$ .

The time-to-build for new firms is one period. Each entering firm  $i$  draws its productivity  $Z_{i,t}$  from a log-normal distribution with mean  $\mu_{e,t}$  and standard error  $\sigma_e$ . The probability density function for the productivity of an entering firm is denoted by  $\varphi_{e,t}(Z)$ .  $\mu_{e,t}$  is set such that the average (firm-specific) productivity of entering firms,  $\bar{Z}_{e,t}$ , is a constant fraction  $\psi_e$  of the average productivity of existing firms,  $\bar{Z}_t$ :

$$\bar{Z}_{e,t} = \psi_e \bar{Z}_t. \quad (4)$$

Equation (4) is a simple way of formalising imitation. It states that entering firms' expected productivity depends linearly on the average productivity of existing firms.

Total output is given by

$$Y_t = \int_i Y_{i,t} di = \int_0^\infty Y_t(Z) d\Phi_t(Z), \quad (5)$$

where  $Y_t(Z)$  is the output of a firm with productivity  $Z$ . Labour inputs satisfy

$$\int_0^\infty L_t(Z) d\Phi_t(Z) = 1,$$

where  $L_t(Z)$  is the amount of labour employed at a firm of type  $Z$ . Output has to be allocated between consumption  $C_t$  and investment  $I_t$ :

$$Y_t = C_t + I_t. \quad (6)$$

The total amount of scrap recovered from exiting firms is equal to  $\theta X_t$ , where  $X_t$  is the number of firms choosing to exit after having produced in period  $t$ . Capital depreciates at a rate  $\delta$ ; we assume that firms always replace depreciated capital. New firms in  $t + 1$  are created using net investment and recovered scrap from period  $t$ , so that the number of firms one period ahead is:

$$K_{t+1} = K_t - X_t + \underbrace{I_t - \delta K_t + \theta X_t}_{\text{entering firms}}, \quad (7)$$

where  $K_t = \int_0^\infty d\Phi_t(Z)$  is the total number of firms.

To map one period's productivity distribution  $\phi_t(Z)$  into next period's, one has to take into account (i) idiosyncratic shocks hitting firms, (ii) the disappearance of those firms which choose to shut down, and (iii) entrance of new firms. Since

there is a continuum of firms in the economy, the evolution of the distribution of firms across productivity levels is deterministic even though each particular firm experiences random shocks. The transition function for the distribution of firms across productivity levels is

$$K_{t+1}\phi_{t+1}(Z) = \underbrace{K_t \int_0^\infty \varphi_t(Z/Z') d\Phi_t(Z')}_{\text{existing firms}} + \underbrace{(I_t - \delta K_t + \theta X_t) \varphi_{e,t}(Z)}_{\text{entering firms}} - \underbrace{X_t(Z)}_{\text{exiting firms}}, \quad (8)$$

where  $X_t(Z)$  is the number of firms at productivity level  $Z$  which choose to exit after having produced in period  $t$ , and  $\varphi_t(Z)$  is the probability density function for the exponential of the idiosyncratic productivity shocks to firms  $e^{\varepsilon_{i,t}}$ .

## 2.2 Firm Entry and Exit

Preferences are represented by

$$U = \sum_{t=0}^{\infty} \beta^t \ln(C_t), \quad (9)$$

where  $C_t$  denotes consumption, and the discount factor is  $\beta$ , with  $\beta \in (0, 1)$ . The representative household maximises its life-time utility (9) subject to a standard budget constraint. Given that firms are atomistic, and all shocks are independently and identically distributed across firms, households may diversify any individual risk by owning a positive measure of firms. The first order condition for consumption yields the usual Euler equation:

$$\frac{C_{t+1}}{C_t} = \beta(1 + r_t). \quad (10)$$

Let  $V_t(Z)$  be the time  $t$  value of a firm with productivity  $Z$ . If the firm chooses to exit, its value is equal to the scrap value of its capital, which is  $\theta$ . If it chooses to stay, its value is equal to its current profits plus its expected discounted value in the next period. The optimal policy then involves choosing a ‘reservation’ productivity level  $Z_t^*$  at which firms are indifferent between staying and exiting:

$$\int_0^\infty \left[ \Pi_t(Z') + \frac{1}{1+r_t} V_{t+1}(Z') \right] \varphi_t(Z'/Z_t^*) dZ' = \theta. \quad (11)$$

The value function is given by

$$V_t(Z) = \begin{cases} \int_0^\infty \left[ \Pi_t(Z') + \frac{1}{1+r_t} V_{t+1}(Z') \right] \varphi_t(Z'/Z) dZ' & \text{if } Z \geq Z_t^*, \\ \theta & Z \leq Z_t^*, \end{cases} \quad (12)$$

where  $\Pi_t(Z)$  is current profits:

$$\Pi_t(Z) = (A_t Z)^\alpha L_t(Z)^{1-\alpha} - W_t L_t(Z) - \delta. \quad (13)$$

Optimal employment at any given firm is determined by the first order condition for  $L_t(Z)$  on equation (13):

$$W_t L_t(Z) = (1 - \alpha) Y_t(Z). \quad (14)$$

Integrating both sides of equation (14) with respect to  $\Phi_t(Z)$  yields the following expression for the wage rate, which implies that the labour income share is  $1 - \alpha$ :

$$W_t = (1 - \alpha) Y_t. \quad (15)$$

Under free entry, expected profits from entering are driven to zero:

$$\int_0^\infty \varphi_{e,t}(Z) V_{t+1}(Z) dZ = 1 + r_t. \quad (16)$$

### 2.3 Embodied Technological Progress

In this section, we show that the above setup can be written as a neoclassical growth model with capital-embodied technological change in which the relative price of (productivity-adjusted) capital and the depreciation rate is endogenous.

To compute the aggregate technology, we follow Solow (1957) in defining the ‘effective’ (i.e., productivity-adjusted) aggregate capital stock as

$$\hat{K}_t = K_t \int_0^\infty Z d\Phi_t(Z).$$

Notice that this implies that average productivity  $\bar{Z}_t$  is equal to  $\hat{K}_t/K_t$ . From (1) and (14), after substituting  $W$  for (15), firms of type  $Z$  produce

$$Y_t(Z) = A_t Z Y_t^{\frac{\alpha-1}{\alpha}}. \quad (17)$$

Then, from (5),

$$Y_t = \left( A_t \hat{K}_t \right)^\alpha. \quad (18)$$

Combining the equations above with (13), it can be shown that the share of total profits accruing to firms with productivity level  $Z$  corresponds to their share of total effective capital:

$$\Pi_t(Z) = \alpha \frac{Z}{\hat{K}_t} Y_t - \delta. \quad (19)$$

Notice that, abstracting from entry and exit, firm-specific productivity shocks as defined in equation (2) do not affect the stock of productivity-adjusted capital  $\hat{K}_t$ , given that these shocks are mean-preserving in  $Z$ . The change in productivity-adjusted capital from one period to the next is then given by difference between (productivity-adjusted) capital added by entering firms, and that destroyed by exiting firms:

$$\hat{K}_{t+1} = \hat{K}_t + \underbrace{\psi_e \bar{Z}_t (I_t + \theta X_t - \delta K_t)}_{\text{entry}} - \underbrace{\psi_{x,t} \bar{Z}_t X_t}_{\text{exit}}, \quad (20)$$

where  $\psi_{x,t}$  is the average productivity of exiting firms relative to existing ones. Re-arranging terms, we get

$$\hat{K}_{t+1} = (1 - \hat{\delta}_t) \hat{K}_t + \hat{I}_t, \quad (21)$$

where  $\hat{\delta}_t = \delta + (\psi_{x,t} - \psi_e \theta) \frac{X_t}{K_t}$  is the endogenously-determined depreciation rate for productivity-adjusted capital, and  $\hat{I}_t = \psi_e \bar{Z}_t I_t$  is productivity-adjusted investment.

If the average firm-specific productivity  $\bar{Z}$  grows over time, then this implies that each cohort of entering firms has a higher productivity than the previous one. As a result, the productivity-adjusted capital stock grows faster than output.  $\psi_e \bar{Z}_t$  can also be interpreted as the inverse of the price of (productivity-adjusted) capital in terms of consumption goods, so that part of productivity growth is specific to the investment sector, as in Greenwood, Hercowitz, and Krusell (1997).

In order for  $\bar{Z}$  to grow at a positive rate, it is merely necessary that entering firms be more productive on average than exiting firms. Note that increases in average productivity  $\bar{Z}$  are exclusively due to the exit of unproductive firms and their replacement by more productive ones, given that idiosyncratic productivity shocks experienced by individual firms are mean-preserving; we therefore refer to the productivity growth which is due to increases in  $\bar{Z}$  as firm-embodied technological progress, given that it is embodied in new firms.

The law of motion for non-adjusted capital is

$$K_{t+1} = (1 - \tilde{\delta}) K_t + I_t, \quad (22)$$

where  $\tilde{\delta} = \delta + (1 - \theta) \frac{X_t}{K_t}$  is the endogenously determined depreciation rate of (non-adjusted) capital, which includes exogenous physical depreciation  $\delta$  as well as capital lost because of exiting firms.

## 2.4 Balanced Growth

The aim of this section is to transform the model in a way which makes all variables constant along a balanced growth path, which is defined as a situation in which both the aggregate variables and the distribution of firm-specific variables across *relative* productivity levels grow at constant rates. Relative productivity  $z = Z/\bar{Z}$  is defined as the productivity level of a given variable relative to average productivity  $\bar{Z}$ .

To find the appropriate transformation, notice that the resource constraint (6), the budget constraint for households and the transition function for aggregate capital (22) imply that  $Y$ ,  $C$  and  $I$  must all grow at the same (gross) rate, say  $g$ , along such a balanced growth path. Furthermore, equation (20), divided on both sides by  $\hat{K}_t$ , implies that  $\hat{K}_t$  and  $\hat{X}_t$  grow at a rate  $g \cdot \gamma_z$ . From the equation for aggregate output (18), we then obtain the following relationship between firm-neutral technological progress  $\gamma_a$ , firm-specific technological progress  $\gamma_z$ , and the growth rate of aggregate output  $g$ :

$$g = (\gamma_a \gamma_z)^{\frac{\alpha}{1-\alpha}}, \quad (23)$$

so that the rate of growth which is due to firm-embodied technological progress is  $\gamma_z^{\frac{\alpha}{1-\alpha}}$ .

One can then define transformations which make all the variables in the model stationary; transformed variables are denoted by lower-case letters. Specifically, first set  $j_t = J_t/g^t$  for  $J = Y, C, I, K, X$  and  $W$ ; second, set  $j_t = J_t/(g\gamma_z)^t$  for  $J = \hat{K}$  and  $\hat{X}$ ; third, set  $j_t(z) = J_t(z\bar{Z}_t)$  for  $J = Y$  and  $L$ ; fourth, set  $v_t(z) = V_t(z\bar{Z}_t)/g^t$ ,  $\hat{\Phi}_t(z) = \Phi_t(z\bar{Z}_t)$ ,  $\hat{\phi}_t(z) = \phi_t(z\bar{Z}_t)$ , and  $\hat{\varphi}_{e,t}(z) = \varphi_{e,t}(z\bar{Z}_t)$ . Furthermore, aggregate firm-neutral productivity  $a_t = A_t/\gamma_a^t$  is normalised to one, while the average of firm-specific relative productivity  $\bar{z}_t = \int_0^\infty z d\hat{\Phi}_t(z)$  is equal to one by definition. The equilibrium equations of the model can then be rewritten in terms of these transformed variables.

A stationary equilibrium then consists of values for  $v(z)$ ,  $\hat{\phi}(z)$ ,  $i$  and  $z^*$  which satisfy the equilibrium conditions in equations (24) through (27) as described below. Dropping time subscripts, the value function for a firm with a relative productivity of  $z$  is

$$v(z) = \begin{cases} \int_0^\infty \left[ \alpha z' k^{\alpha-1} - \delta + \frac{\beta}{g} v(z') \right] \hat{\varphi}(\gamma_z z'/z) dz' & \text{if } z \geq z^*, \\ \theta & z < z^*, \end{cases} \quad (24)$$

where  $k = \int_0^\infty d\hat{\Phi}(z)$ .  $\pi(z) = \alpha z k^{\alpha-1} - \delta$  is obtained from equation (19) and from the fact that average relative productivity  $\bar{z} = \hat{k}/k$  is equal to one, and  $r = g/\beta - 1$  is determined by the Euler equation (10). Equation (24) takes into account the

fact that, since firm-specific productivity  $z$  follows a (stationary) random walk while average productivity  $\bar{z}$  grows at an expected rate of  $\gamma_z$ , any given firm will in expected terms see its relative productivity  $z$  decline over time at that same rate  $\gamma_z$ . The transition function for the distribution of capital across relative productivity levels is

$$\hat{\phi}(z) g \gamma_z = \int_{z^*}^{\infty} \varphi [z / (z' \gamma_z)] d\hat{\Phi}(z') + [i + \theta x - \delta k] \hat{\varphi}_e(z), \quad (25)$$

where  $x = \int_{-\infty}^{z^*} \kappa(z) dz$ .

The expected value of an entering firm is equal to the cost of buying one unit of capital, while the value of a firm with relative productivity  $z^*$  is equal to the scrap value of that capital:

$$g/\beta = \int_0^{\infty} \hat{\varphi}_e(z) v(z) dz, \quad (26)$$

$$\theta = \int_0^{\infty} \left[ \alpha z' k^{\alpha-1} - \delta + \frac{\beta}{g} v(z') \right] \varphi [z' / (z^* \gamma_z)] dz'. \quad (27)$$

The model distinguishes itself from the evolutionary economics literature in the line of Nelson and Winter (1982) through equations (26) and (27), which state that entry and exit follow rational, instead of adaptive, expectations. It distinguishes itself from the industrial evolution literature, which notably includes papers by Jovanovic (1982) and Hopenhayn (1992), through equation (25), which states that entering firms' productivity is not exogenous but instead depends on the productivity of existing firms.

The model is solved numerically, following a method which is described in appendix A.

### 3 Calibration

The aim of this section is to study the behaviour of a parameterised version of the model economy, in order to assess the quantitative impact of selection and imitation (that is, firm-embodied technological progress) on productivity growth in the U.S., in analogy to Greenwood, Hercowitz, and Krusell (1997), who estimate the contribution of investment-embodied technological change on productivity growth.

The length of a period is set to one quarter. The parameters which need to be calibrated are the technology parameter  $\alpha$ , the discount rate  $\beta$ , the exogenous exit probability  $\delta$ , the growth rate of output  $g$ , the variance of idiosyncratic shocks to existing firms  $\sigma$  and to entering firms  $\sigma_e$ , the average relative productivity of entering firms  $\psi_e$ , and the scrap value of capital  $\theta$ .

In order to impose some rigour on the quantitative analysis, the procedure advanced by Kydland and Prescott (1982) is followed. The parameters in the model are set such that along the balanced growth path a number of economic variables assume their average values observed for U.S. data. These average values are taken from two distinct sources: from the U.S. National Income and Product Accounts and from studies on establishment-level evidence from the Longitudinal Research Database (LRD), which tracks between 55'000 and 300'000 establishments in the US manufacturing sector.<sup>1</sup> The government sector is netted out of GDP, given that the selection mechanism which is at work in the model is specific to a competitive private sector. The sample period for NIPA data is chosen to coincide with that of the available evidence for LRD data. This is done so that average productivity growth within the sample period is comparable across the two data sets, in order to obtain a consistent estimate of the contribution of selection and imitation to aggregate productivity growth.

Average values for quarterly NIPA data for the years 1972 to 1988 yield a capital income share of .32; a depreciation rate of 1.67% which we set to match the endogenous depreciation rate  $\tilde{\delta}$  in equation (21); a quarterly growth rate of per capita output of .46%; and a capital-output ratio of 11.65.

Using LRD data, Foster, Haltiwanger, and Krizan (2001) estimate that the (output-weighted) average productivity of establishments which have entered between 1972 and 1988 relative to continuing plants in 1988 is .99, while the corresponding number for establishments which have exited within this time period is .96; Campbell (1998) reports that between 1972 and 1978 the average quarterly employment-weighted exit rate of establishments, which in our model corresponds to  $\psi_x \frac{X}{K}$ , and that of establishments which are less than one year old, are .83% and 1.64% respectively. Table 1 contains a summary of the calibrated parameters.

## 4 Results

Figure 1 shows the steady-state distribution of firms along productivity levels. The exit threshold of establishments,  $z^*$ , is at 81.78% of average productivity. The calibrated value for the scrap value of capital,  $\theta$ , is close to that of Campbell (1998), even though his calibration strategy is quite different. Also, the fact that the estimated variance of the productivity shock to entering establishments,  $\sigma_e^2$ , is several orders of magnitude larger than the variance of the shock to existing establishments,  $\sigma^2$ , is consistent with the finding by Bartelsman and Dhrymes

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<sup>1</sup>For a review of productivity studies on the LRD see Bartelsman and Dhrymes (1998) and Caves (1998).



Table 1: Parameterisation

| Parameter   | Chosen Value |
|---|--------------|
| Capital share of income, $\alpha$                 | .32          |
| Discount rate, $\beta$                            | .994         |
| Depreciation rate, $\delta$                       | .0159        |
| Growth rate of output, $g - 1$                    | .0046        |
| Variance of shock to new firms, $\sigma_e^2$      | .1142        |
| Variance of shock to all firms, $\sigma^2$        | .0177        |
| Relative productivity of entering firms, $\psi_e$ | 0.99         |
| Scrap value of firms, $\theta$                    | .832         |

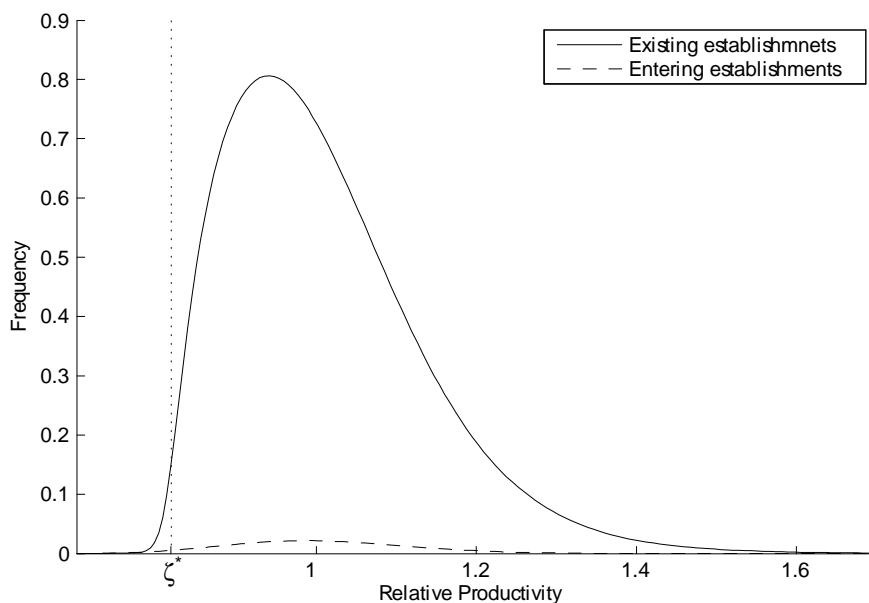
(1998) that young plants face substantially more productivity uncertainty than their older counterparts.

While the model is calibrated on the observed average employment-weighted exit rate, which is .83%, it also broadly matches the non-weighted exit rate, which is .98 in the model and 1.04 in the data. This means that the model replicates the size of exiting establishments relative to continuing ones.

One of the key results of the paper concerns the proportion of aggregate productivity growth which is due to establishment-embodied technological progress. The quarterly growth rate of  $\bar{Z}$  implied by the model is .20%. From the discussion in section 2.4, this implies that the yearly growth rate due to entry, exit and imitation is .36%, which corresponds to 20% of total productivity growth. This number can be compared to microeconomic studies of establishment-level productivity decomposition. Foster, Haltiwanger, and Krizan (2001) estimate that in the U.S. manufacturing sector, between 48% and 65% of productivity growth takes place within establishments, with the remainder coming from either the reallocation of inputs from unproductive to more productive establishments, or from entry and exit. We argue that our estimate of the contribution of selection and imitation to productivity growth should be considered a lower bound, because we match the exit rate for the manufacturing sector, which is around 4% per year; Luttmer (forthcoming), with a model similar to ours, matches an annual firm entry rate for the overall economy of 11.6%, and finds that the contribution of selection and imitation is around one-half.

Our results are also closely related to Greenwood, Hercowitz, and Krusell (1997), who find that 60% of post-war U.S. productivity growth is due to technical change which is embodied in capital, and to Atkeson and Kehoe (2005), who estimate that over one-third of the payments received by plant owners are due to

Figure 1: Distribution of Establishments along Productivity Levels



plant-specific knowledge (i.e., to organisational capital).

## 5 Conclusion

A model was set up in which firms face idiosyncratic productivity shocks; entry and exit are endogenous, and entering firms start with a productivity level which depends on the average productivity in the economy. This is shown to result in aggregate growth even in the absence of an exogenous positive trend in productivity growth at individual firms, through a process of selection and imitation. The parametrised version of the model economy suggests that at least one-fifth of U.S. productivity growth is due to such a selection effect.

The idea of growth through selection does also have some policy implications, although they are not formally investigated here. Chief among them is the fact that since the growth effect of selection turns out to be quite substantial, protecting firms by setting up entry barriers or by not allowing them to fail can have a sizeable effect not only on real income levels through higher prices, but also on long-run growth rates. As an illustration, Levinsohn and Petrin (1999) cite an article by *The Economist*<sup>2</sup> suggesting that Japan's poor economic performance during the

<sup>2</sup>See the June 20, 1998 issue containing the article "Japan's Economic Plight."

1990s has been due at least in part to a Japanese aversion to ‘outright failure’ of firms.

## A Algorithm

The numerical algorithm that is used to solve for the stationary equilibrium of the model is the following:

1. Guess the equilibrium average growth rate of firm-specific productivity  $\gamma_z$ .
2. Guess the aggregate stock of capital,  $k$ . Iterate on the firm’s value function  $v(z)$  given by equation (24) until convergence is reached. In practice,  $z$  is discretised into a matrix of dimension  $[500 \times 1]$ . Given that  $v(z)$  is decreasing in  $k$ , use the free entry condition in (26) to update  $k$  through a bisection method, and iterate until convergence is reached.
3. Use (27) to determine  $z^*$ .
4. Guess a distribution for  $\kappa(z)$ . Iterate over the capital transition function (25) until  $\kappa(z)$  converges; at each iteration, set  $i$  such that the capital stock implied by (25) corresponds to the value guessed at step 2.
5. Given that average productivity, which is equal to one at equilibrium, is a decreasing function of  $\gamma_z$ , use  $\bar{z} = \int_0^\infty z\kappa(z) dz$  to update  $\gamma_z$  through a bisection method.

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