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The bargaining family revisited: a Comment *

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Abstract

This paper reexamines results of Konrad and Lommerud (2000). They construct a two-stage game model of the family. We prove that their result crucially depends on their linear payoff function and obtain an opposite result if the interaction within the family is represented by a non-linear function; that is, the interaction exhibits strategic complementarity.

Classification Numbers: D13, J24

Key words: public goods, cooperative game, non-cooperative game

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1 Introduction

Since the seminal paper of Becker (1981), many papers have examined issues on resource allocation within the family (See for example, Chiappori (1988) and Chiappori (1992)). In these works, it is postulated that the family maximizes a joint "welfare function". On the contrary, Konrad and Lommerud (2000) construct a two-stage game model of the family. In their model, educational investments are non-cooperatively undertaken in the first stage of the two-stage game prior to marriage, which means human capital investment by family members; that is, husband and wife. After marriage (in stage 2), their supply decisions of public goods are determined.[1] Using this model, they derive important implications concerning the private provision of family public goods.[2] First, in their proposition 1, they show that at the non-cooperative Nash equilibrium, a joint increase in contributions to the public good is a Pareto improvement. Second, in their proposition 2, they show that in the non-cooperative equilibrium, a joint decrease in education investment is a Pareto improvement.

Although Konrad and Lommerud's results have important implications for understanding the nature of private provision of public goods within the family, their result crucially depend on their assumption on the functional form of the family members' payoff function. They adopt a linear payoff function, which eliminates any strategic interaction within the family. Although the assumption of the linear functional form is admissible for analyses of a benchmark case, linearity fails to capture strategic interaction in models of household production. For example, in the case of child rearing, husbands and wives can play different roles. An increase in provision of child rearing by husband raises efficiency or productivity of child rearing by his wife. This interaction is known as strategic complementarity.[3] Their linear functional form cannot

[1] For another example of the model of the family, it is useful to see the overview of Bergstrom (1997).

[2] Example of the family public good are child rearing, cleaning house, and well-being parents.

[3] See Bulow, Geanakoplos, and Klemperer (1985) as for the strategic complementarity.

take account of this kind of relationship in the family. By considering this strategic complementarity in the family, we prove an opposite result that in Proposition 2 of Konrad and Lommerud (2000).

The remainder of this paper is organized as follows. Section 2 is divided into two subsections. Section 2.1 analyzes the stage 2 outcomes. Section 2.2 derives the education choices in stage 1. Section 3 concludes this paper.

2 The Model

In this section, we construct a two–stage game á la Konrad and Lommerud (2000) by taking account of the strategic interaction within the family. Following Konrad and Lommerud (2000), we consider the family, consisting of two agents (a husband and a wife) denoted by $i = f, m$. Each agent has the following payoff function:

$$u_i = x_i + G(g_f, g_m) - \frac{\alpha}{2} g_i^2 - \frac{\beta}{2} w_i^2, \quad i \in \{f, m\} \text{ and } i \neq j. \tag{1}$$

Here $G(g_f, g_m)$ is the payoff from providing public good. It is given by

$$G(g_f, g_m) = g_f - \delta g_f^2 + g_m - \delta g_m^2 + \gamma g_f g_m, \tag{2}$$

where g_f and g_m show the provisions of public goods of agents $i = f, m$. We assume that the parameter γ takes a positive value. This indicates that the relationship between agents within the family exhibits strategic complementarity. If $\delta = \gamma = 0$, the payoff function is the same as Konrad and Lommerud (2000). Further, x_i is the individual's consumption of private goods and is determined as follows:

$$x_i = w_i(y - g_i), \quad i \in \{f, m\} \tag{3}$$

where w_i is the labor market wage to be determined below, and y is i's maximum time available for working in the labor market and for contributions to the public good. Thus, $(y - g_i)$ measures the labor supply of agent i. Apart from reducing the time available for labor market activities, contributing to the public good has some "psychic" cost that is measured by a convex cost function, $a(g_i) = \frac{\alpha}{2}g_i^2, i \in \{f, m\}$ and $(\alpha > 0)$. If an agent makes an effort to have the education, it leads to higher education level of the agent and also brings the higher wage rate. Therefore, the wage rate also indicates the education level. We assume that the marginal cost of education is positive and increasing; that is, the education cost is given by $b(w_i) = \frac{\beta}{2}w_i^2, i \in \{f, m\}$ and $(\beta > 0)$.

We consider the following two–stage game. In stage 1, each agent simultaneously chooses his or her education level. In stage 2, taking the choice of education levels in stage 1 into account, each agent simultaneously determines how much time he or she devotes to activities that contribute to the family public good. We seek the Nash equilibrium of the stage 2 game given an education choice (w_f, w_m) in stage 1. Having solved this problem contingent on given educational choices, we derive the stage 1 subgame perfect Nash equilibrium choices of labor market productivities.

2.1 Stage 2 outcomes

In this subsection, we calculate the non-corporative Nash equilibrium with taking the education levels w_f and w_m as given. The wife maximizes her own utility given the provision of public good by her husband, and vice versa. The first–order conditions of these problems are given by

$$\frac{\partial u_i}{\partial g_i} = -w_i + 1 - 2\delta g_i + \gamma g_j - \alpha g_i = 0, \quad i, j \in \{f, m\} \text{ and } i \neq j. \tag{4}$$

The first and last terms in the second equation of (4) show the marginal cost from the provision of family goods and the second to fourth terms show the marginal benefit. Equations (4) define the reaction functions of the agents as follows:

$$g_i = \frac{1}{\alpha + 2\delta}(1 + \gamma g_j - w_i), \quad i, j \in \{f, m\} \text{ and } i \neq j.$$

Following Konrad and Lommerud (2000), we assume that $1 - w_i > 0$. The slopes of these reaction functions are given by

$$\frac{\partial g_i}{\partial g_j} = \frac{\gamma}{\alpha + 2\delta}, \quad i \in \{f, m\} \text{ and } i \neq j.$$

This implies that the reaction curves of the agents are upward sloping. It should be noted that there is no reaction function in Konrad and Lommerud (2000). Their reaction curve can be depicted as a horizontal line.

When the education levels of the agents change, the reaction curves shift. We can calculate these shifts as follows:

$$\frac{\partial g_f}{\partial w_f} = -\frac{1}{\alpha + 2\delta}, \quad \frac{\partial g_m}{\partial w_m} = -\frac{1}{\alpha + 2\delta}.$$

Consequently, an increase in education level raises the opportunity cost of the provision of family goods, and thus decreases the contribution to family public good.

To guarantee an interior solution, we assume the following:

Assumption 1 $\gamma < 2\delta$

Solving (4), we obtain the second-stage Nash equilibrium $g_i^* \in (0, y)$ depending on the levels of education that the agents choose in the first-stage:

$$g_f^*(w_f, w_m) = \frac{(1 - w_f)(\alpha + 2\delta) + (1 - w_m)\gamma}{(\alpha + 2\delta)^2 - \gamma^2}, \tag{5}$$

$$g_m^*(w_f, w_m) = \frac{(1 - w_m)(\alpha + 2\delta) + (1 - w_f)\gamma}{(\alpha + 2\delta)^2 - \gamma^2}. \tag{6}$$

We examine how the second-stage Nash equilibrium depends on the levels of education. From equations (5) and (6), it is clear that $\partial g_f^*/\partial w_f < 0, \partial g_f^*/\partial w_m < 0, \partial g_m^*/\partial w_m < 0$ and $\partial g_m^*/\partial w_f < 0$.

Now we turn to characterizing the efficient outcome. We define the social welfare as $SW \equiv \sum_{i=j,m} u_i$ for simplicity. The efficient allocation must satisfy the following first–order conditions:

$$\frac{\partial SW}{\partial g_i} = -w_i + 2(1 - 2\delta g_i + \gamma g_j) - \alpha g_i = 0, \quad i, j \in \{f, m\} \text{ and } i \neq j. \tag{7}$$

The first and last terms in the second equation of (7) show the marginal cost from the provision of family goods and the second and third terms also show the marginal benefit. Solving (7), we obtain the efficient outcomes $g_i^e \in (0, y)$ in the stage 2 as follows:

$$g_f^e(w_f, w_m) = \frac{(2 - w_f)(\alpha + 4\delta) + (2 - w_m)2\gamma}{(\alpha + 4\delta)^2 - (2\gamma)^2}, \tag{8}$$

$$g_m^e(w_f, w_m) = \frac{(2 - w_m)(\alpha + 4\delta) + (2 - w_f)2\gamma}{(\alpha + 4\delta)^2 - (2\gamma)^2}. \tag{9}$$

We can compare the Nash equilibrium with the efficient outcome as follows:

Proposition 1 *For a given level of education, in the non-cooperative Nash equilibrium, a joint increase in contributions to the public good is a Pareto improvement.*

Proof.

$$\begin{aligned} g_i^e - g_i^* &= \frac{1}{[(\alpha + 4\delta)^2 - (2\gamma)^2][(\alpha + 2\delta)^2 - (\gamma)^2]} \Big(\alpha(\alpha + 2\delta + \gamma)(\alpha + 4\delta) + 2\alpha\gamma(2\delta + \gamma) \\ &+ \; w_i[2\alpha^2\delta + (3\alpha + 4)(4\delta^2 - \gamma^2)] + w_j[2\gamma(4\delta^2 - \gamma^2)] + \alpha^2\gamma(2 - w_j) \Big) > 0, \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i, j \in \{f, m\} \text{ and } i \neq j. \end{aligned}$$

Thus we have $g_i^e > g_i^*$. ■

The intuition of this result is as follows. In the efficient allocation, the agents maximize the joint utility function, the opportunity cost of the provision of family goods at the efficient outcome is smaller than that of Nash equilibrium regime, thus the agents have incentives to increase the provision of family goods at the efficient outcome. This results is similar that in Proposition 1 of Konrad and Lommerud (2000).

2.2 The educational choice

In this subsection, we consider the choice of education levels in stage 1. The equilibrium choice depends on whether behaviors in stage 2 are characterized by non-cooperative mode or by cooperative mode. We consider these cases respectively.

First, we consider the case where agents behave non–cooperatively in stage 2. Taking the second stage equilibrium allocation into account, the wife maximizes the following her own utility given the education level by her husband, and vice versa.

$$\begin{aligned} u_i &= w_i[y - g_i^*(w_f, w_m)] + g_i^*(w_f, w_m) - \delta g_i^*(w_f, w_m)^2 + g_j^*(w_f, w_m) - \delta g_j^*(w_f, w_m)^2 \\ &+ \gamma g_f^*(w_f, w_m) g_m^*(w_f, w_m) - \frac{\alpha}{2} g_i^*(w_f, w_m)^2 - \frac{\beta}{2} w_i^2, \ \ i, j \in \{f, m\} \text{ and } i \neq j. \quad (10) \end{aligned}$$

where $g_i^*(w_f, w_m)$ is given by (5) and (6). The educational choice, w_i that maximizes (10) is depends on w_j and is determined by the following first-order condition:

$$\begin{aligned} &\Gamma^i(w_f, w_m) \equiv \frac{\partial u_i}{\partial w_i} \\ &= y - g_i^*(w_f, w_m) + [1 - 2\delta g_j^*(w_f, w_m) + \gamma g_i^*(w_f, w_m)] \frac{\partial g_j^*(w_f, w_m)}{\partial w_i} - \beta w_i \\ &= y - \frac{(1 - w_i)(\alpha + 2\delta) + (1 - w_j)\gamma}{(\alpha + 2\delta)^2 - \gamma^2} \\ &+ \Big(1 - 2\delta \frac{(1 - w_j)(\alpha + 2\delta) + (1 - w_i)\gamma}{(\alpha + 2\delta)^2 - \gamma^2} + \gamma \frac{(1 - w_i)(\alpha + 2\delta) + (1 - w_j)\gamma}{(\alpha + 2\delta)^2 - \gamma^2}\Big) \\ &\Big(- \frac{\gamma}{(2 + \alpha)^2 - \gamma^2}\Big) - \beta w_i = 0, \qquad i, j \in \{f, m\}, \text{ and } i \neq j. \quad (11) \end{aligned}$$

The necessary condition is classified into three parts. The first and second terms in the second equation of (11) show the marginal benefit from the education, the third term shows the strategic effects. As can be seen in the previous subsection, please note that the payoff function (2) exhibits the strategic complementarity in a family.

We have the sufficient conditions of the form

$$\Gamma_1^i(w_i, w_j) \equiv -\beta + \frac{\alpha + 2\delta}{(\alpha + 2\delta)^2 - \gamma^2} + \frac{\alpha\gamma^2}{[(\alpha + 2\delta)^2 - \gamma^2]^2} < 0, , \ \ i, j \in \{f, m\}, \text{ and } i \neq j. \quad (12)$$

where $\Gamma_1^i(w_i, w_j) \equiv \partial\Gamma^i / \partial w_i$.

To guarantee the sufficient condition and that w_i has an interior solution, we assume the following:

Assumption 2

$$\beta > \max\left\{ \frac{(\alpha + 2\delta)^3 - 2\delta\gamma^2}{[(\alpha + 2\delta)^2 - \gamma^2]^2}, \frac{\alpha + 4\delta}{(\alpha + 4\delta)^2 - (2\gamma)^2} \right\}$$

The sufficient condition is satisfied due to Assumption 2. Thus, we can obtain the following reaction functions defined by $\Gamma^i(w_i, w_j) = 0, i \in \{f, m\}$.

$$w_i = \frac{[(\alpha+2\delta)^2 - \gamma^2]^2 y + \alpha\gamma(\alpha+2\delta)w_j - (\alpha+2\delta+\gamma)[(\alpha+2\delta)^2 + \gamma(\alpha-\gamma)]}{2\gamma^2\delta - (\alpha+2\delta)^3 + \beta[(\alpha+2\delta)^2 - \gamma^2]^2}, \quad i, j \in \{f, m\}, \text{ and } i \neq j. \tag{13}$$

Because of Assumption 2, the reaction curves of husband and wife become upward sloping.

Solving (13), we get the Nash equilibrium of the two stage game as follows:

$$w_i^* = \frac{[(\alpha+2\delta)^2 - \gamma^2]^2 y - (\alpha+2\delta+\gamma)[(\alpha+2\delta)^2 + \gamma(\alpha-\gamma)]}{\beta[(\alpha+2\delta)^2 - \gamma^2]^2 + 2\gamma^2\delta - (\alpha+2\delta)^3 - \alpha\gamma(\alpha+2\delta)}, \quad i \in \{f, m\}. \tag{14}$$

Next, let us consider the efficient outcome. The efficient education levels are derived by choosing the wage rate that maximizes the following:

$$\begin{aligned} SW &= w_i[y - g_i^e(w_f, w_m)] + w_j[y - g_j^e(w_f, w_m)] \\ &+ 2[g_f^e(w_f, w_m) - \delta g_f^e(w_f, w_m)^2 + g_m^e(w_f, w_m) - \delta g_m^e(w_f, w_m)^2 \\ &+ \gamma g_f^e(w_f, w_m) g_m^e(w_f, w_m)] - \frac{\alpha}{2} g_i^e(w_f, w_m)^2 - \frac{\beta}{2} w_i^2 - \frac{\alpha}{2} g_j^e(w_f, w_m)^2 - \frac{\beta}{2} w_j^2, \\ & \qquad i, j \in \{f, m\} \text{ and } i \neq j \end{aligned} \tag{15}$$

where $g_i^e(w_f, w_m), i \in \{f, m\}$ is defined by (8) and (9). The necessary conditions are given by

$$\begin{aligned} \Gamma^{i,e}(w_i, w_j) &\equiv \frac{\partial SW}{\partial w_i} = y - g_i^e - \beta w_i \\ &= y - \frac{(2 - w_i)(\alpha + 4\delta) + (2 - w_j)2\gamma}{(\alpha+4\delta)^2 - (2\gamma)^2} - \beta w_i = 0, \ i, j \in \{f, m\} \text{ and } i \neq j. \end{aligned} \tag{16}$$

The first and second terms in the second equation of (16) show the marginal benefit and the third term shows the marginal cost. The sufficient conditions are satisfied due to Assumption 2; that is,

$$\Gamma_1^{i,e}(w_i, w_j) \equiv \frac{\partial \Gamma^{i,e}(w_i, w_j)}{\partial w_i} = \frac{\alpha + 4\delta}{(\alpha+4\delta)^2 - (2\gamma)^2} - \beta < 0, \quad i, j \in \{f, m\} \text{ and } i \neq j$$

From (16), we have the following condition:

$$w_i = \frac{2\gamma w_j + [(\alpha+4\delta)^2 - (2\gamma)^2]y - 2(\alpha+4\delta+2\gamma)}{\beta[(\alpha+4\delta)^2-(2\gamma)^2] - (\alpha+4\delta)}, \quad i,j \in \{f,m\}, \quad i \neq j. \tag{17}$$

Solving (17), we obtain the efficient wage levels as follows:

$$w_i^e = \frac{[(\alpha+4\delta)^2 - (2\gamma)^2]y - 2(\alpha+4\delta+2\gamma)}{\beta[(\alpha+4\delta)^2-(2\gamma)^2] - (\alpha+4\delta+2\gamma)}, \quad i \in \{f,m\}. \tag{18}$$

Proposition 2 *Suppose that the following inequality holds:*

$$1 - \alpha\beta - (2\delta-\gamma)y > 0,$$

Then, a joint increase in education investment is a Pareto improvement.

Proof.

$$w_i^e - w_i^* = \frac{(\alpha+2\delta+\gamma)(\alpha+4\delta+2\gamma)[(\alpha^2+\alpha(4\delta-\gamma)+(\gamma-2\delta)^2]}{X}\Big[1-\alpha\beta-(2\delta-\gamma)y\Big] > 0, \quad i,j \in \{f,m\},$$

where $X \equiv \{\beta[(\alpha+4\delta)^2-(2\gamma)^2]-(\alpha+4\delta+2\gamma)\}\{\beta[(\alpha+2\delta)^2-\gamma^2]^2+2\delta\gamma^2-(\alpha+2\gamma)^3-\alpha\gamma(\alpha+2\delta)\} > 0$. Thus, if $1-\alpha\beta+(2\delta-\gamma)y>0$. ■

Proposition 2 holds when α is sufficiently small. When $\gamma=\delta=0$, the subgame perfect Nash equilibrium (13) and the efficient allocation (18) are respectively rewritten as $w_i = \frac{\alpha y-1}{\alpha\beta-1}$, $i \in \{f,m\}$ and $w_f^e = \frac{\alpha y-2}{\alpha\beta-1}$, $i \in \{f,m\}$. By comparing these values, we have $w_i > w_i^e$, $i \in \{f,m\}$. This result is that of Konrad and Lommerud (2000). In contrast, when $\gamma>0$ and $\delta>0$, we have the opposite result that in Proposition 2 of Konrad and Lommerud (2000). Their proposition states that agents acquire higher education levels at the non–cooperative equilibrium than those at the efficient outcome. When the relationship between husband and wife exhibits strategic complementarity, the agents acquire lower education levels at the non–cooperative equilibrium than those at the efficient outcome.

We can consider the policy of educational subsidy. Consider a subsidy z on educational expenditure, this subsidy changes the cost function $\beta w_i^2/2$ to $(1-z)\beta w_i^2/2, i \in \{f, m\}$. This subsidy decreases marginal cost of education and increases education level. Thus a subsidy on education expenditure is a Pareto improving. This policy is also contrary to Konrad and Lommerud's proportional tax policy on education expenditure.

3 Conclusion

This paper extends the model of Konrad and Lommerud (2000). We prove that their result crucially depends on the functional form of the family members' linear payoff function. The linearity fails to capture strategic interaction in models of household production. By considering the strategic interaction in the family, we prove an opposite result that in Proposition 2 of Konrad and Lommerud (2000).

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