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Abstract

Since the writing of David Hume, in the eighteenth century, there has been a general agreement amongst economists that an increase in the stock of money leads, initially, to an increase in economic activity. Output and employment go up, the interest rate declines and prices respond weakly, if at all, to an increase in the quantity of money. Over time, these real effects die out and, in the long run, the only effect of higher money is higher prices. Most writers on the topic have attributed the real effects of money, in the short run, to mistaken expectations, non-market clearing or both. We argue instead, that neither of these channels is needed to explain the facts. We show that a competitive market-clearing model in which money enters the production function can reproduce the broad features of data. Our argument relies on an explanation of "price stickiness" that exploits a multiplicity of equilibria in a rational-expectations model.

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"…in every kingdom, into which money begins to flow in greater abundance than formerly, everything begins to take a new face: labour and industry gain life; the merchant becomes more enterprising, the manufacturer more diligent and skilful, and even the farmer follows his plough with greater alacrity and attention."

David Hume *Of Money*

1 Introduction

In simple equilibrium business cycle models that are amended to include money – it is difficult to set things up in a way that causes simulated time series to mimic real world data: in most equilibrium models there is too much price flexibility. Money shocks feed immediately into prices and these models display not only long run neutrality of money, but also short run neutrality. In the data, this is not what we observe. Instead, money shocks cause real output responses in the short run and only after a considerable period of time do prices adjust to insulate real quantities from nominal disturbances.[2]

There are two popular views of why equilibrium models fail. One-view holds that markets, expectations, or both are typically in disequilibrium. According to this view an amended version of the IS-LM model can accurately describe the world and the role of economic theory is to explain why prices do not clear markets in the short run. According to a second view, the correlations that we observe in the data

[2] The folk evidence for a transmission mechanism with these features extends as least as far as David Hume's essay "Of Money" from which our opening quote is taken. Formal analysis of macroeconomic time series using vector autoregressions points in the same direction. For a discussion of the monetary transmission mechanism based on the evidence from vector autoregressions see the recent article by Bernanke and Gertler (1995) in the *Journal of Economic Perspectives*.

are examples of reverse causation; output causes money rather than the other way around and hence there is no puzzle to be explained.

In this paper, we argue that there *is* a puzzle for equilibrium business cycle theory but this puzzle can be resolved within a market-clearing model in which agents have rational expectations. We argue that the world in which we live is one in which the assumption of rational expectations is insufficient to pin down a particular equilibrium. In fact, there are infinitely many beliefs that are consistent with rational expectations and market clearing. We argue that agents in the real world have resolved this multiplicity by coordinating on a particular equilibrium and that this equilibrium has the property that prices are predetermined one period in advance.

The foundation for our model can be found in Calvo (1979) and the argument that indeterminacy can be used to explain the observed behavior of prices has been made before.[3] There have however been few attempts to investigate the *empirical* plausibility of indeterminacy arising from the productive or utility producing role of money.[4] For this reason, most macroeconomists have tended to dismiss the idea that indeterminacy of equilibrium can explain the monetary transmission mechanism. In this paper we make the case for the multiple equilibrium approach to "price stickiness" by showing that a suitably calibrated model can fit the data well if one is prepared to accept a relatively flexible parameterization of preferences. Our model abstracts from capital accumulation and it assumes that labor and goods markets are competitive. We are nevertheless able to explain the stylized facts associated with the dynamics of interest rates, prices,

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[3] The fact that monetary models might display indeterminate equilibria has been known at least since the work of Brock (1974). Other authors who have studied this issue include Beaudry and Devereux (1993), Benhabib and Bull (1983), Benhabib and Farmer (1991), Farmer (1991:a), (1991:b), (1992), Farmer and Woodford (1997), Gray (1983), Lee (1993), Matsuyama (1991), Matheny (1992), (1998), Obstfelt and Rogoff (1983) and Woodford (1987), (1994).

[4] One such attempt is given by Benhabib and Farmer (1991), who rely on aggregate monetary externalities; another is by Beaudry and Devereux (1993), who rely on increasing returns to scale.

real balances, and output. Our theme is that, in simple monetary economies, equilibria can be represented as bounded solutions to a characteristic difference equation with a single state variable. Sometimes the characteristic difference equation has a unique bounded solution. Sometimes it does not. We argue that models that display multiple bounded solutions capture many of the features of the monetary transmission mechanism that are otherwise difficult to understand.

2 Setting up a Model Economy

Technology

We model production as a two-stage process. In the first stage, labor is combined with fixed factors of production in a neoclassical technology subject to decreasing returns-to-scale. We use the symbol L to refer to raw labor and S to refer to the state of technical progress. The first stage technology is described in equation (2-1):

(2-1) $$X_t = S_t L_t^a .$$

To describe technical progress we assume that S_t is a geometric random walk with drift g:

(2-2) $$log(S_t) = log(1+g) + log(S_{t-1}) + log(1+v_t), \quad E_{t-1}[v_t] = 0.$$

One could also assume that S_t is a trend stationary process in which the average growth rate is a deterministic function of time without changing the derivations of the equilibrium equations of motion.

To capture the idea that firms must engage in exchange with other agents we model a second stage of production by assuming that produced goods X are combined with real balances M/P according to the function $F(X, M/P)$. There is a large literature on non-Walrasian models of exchange that describes the role that money plays in facilitating transactions. We are unable to offer a coherent micro model of money in this paper and we choose instead to begin with the function $F(X, M/P)$ as a primitive and to study the implications of this approach for equilibrium. We use the notation Y_t to refer to commodities in the hands of the final user.

$$(2\text{-}3)\qquad Y_t = F\left(X_t, \left(\frac{M_t}{P_t}\right)\right) \equiv \left((1-a)X_t^{\,l} + a\left(\frac{M_t}{P_t}\right)^{l}\right)^{\frac{1}{l}}, \qquad 0 < a < 1, \quad l < 0.$$

We assume that $F(X, M/P)$ satisfies constant returns-to-scale and that it is increasing, concave and continuous. In our calibrated work, we use a constant elasticity-of-substitution production function with parameter l. The elasticity of substitution for this technology is equal to $1/(l-1)$ and a negative value for l reflects our prior belief that money and produced goods are complements, rather than substitutes.

Preferences

Our economy consists of a large number of representative families, each of which maximizes the utility function;

$$(2\text{-}4)\qquad Max\ \ U = E_0\left\{\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t U(C_t, L_t; S_t)\right\},$$

where S_t represents technological progress, C_t is consumption and L_t is time spent in market activities. We allow for the possibility that technical progress may influence the utility function for the reasons outlined by Benhabib, Rogerson and Wright (1991) in their work on home production. In order for this utility function to be consistent with balanced growth it must be homogenous (we assume homogeneity of degree 1-r) in C and S and in our calibrated examples, we use the function:

$$(2\text{-}5)\qquad U(C_t, L_t, S_t) = \frac{\left(C_t^{1-r} + AS_t^{1-r}\right)}{1-r}(L_t - B), \qquad r > 1, \quad A > 0, \quad B > 0.$$

The properties of utility that are important for behavior are marginal utilities (slope parameters) and their derivatives (curvature parameters). We have moved beyond a simple two-parameter family to describe utility because it will be important in our calibrated example that we can choose the curvature parameters of the utility function independently of the slope parameters. We have included the technology parameter S_t directly in the utility function to reflect the idea that the marginal utility of leisure is constant along the balanced growth path. As consumption increases, extra hours of work become more unpleasant because the household would like to spend more time enjoying its increased consumption goods. A second effect of productivity on utility occurs as time becomes more productive in all activities, including housework, thereby freeing more time that can be supplied both to the marketplace and to the enjoyment of leisure.

The Budget Constraint

Each family chooses how much time to spend in the activity of production, L_t, how much to consume of the commodities produced by other households, C_t, and how much to save in the form of money M_t and bonds B_t. The money supply in period t reflects the agent's choice M_{t-1} plus the transfer that they subsequently receive: $M_t = M_{t-1} + T_t$. Households choose sequences that maximize expected utility subject to the budget set defined by the constraints (2-6)– (2-8):

(2-6)
$$\frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t}\left(1+i_{t-1}\right) + F\left(S_t L_t^{\,a}, \left(\frac{M_{t-1}+T_t}{P_t}\right)\right) - C_t + \frac{T_t}{P_t}, \quad t = 1, \ldots,$$

(2-7)
$$B_0 = 0, \qquad M_0 = \overline{M}_0,$$

(2-8)
$$Lim_{s\to\infty} Q_t^s \left(\frac{M_s + B_s}{P_s}\right) \geq 0, \quad Q_t^s = \frac{P_s}{P_t} \prod_{v=1}^{s} \frac{1}{\left(1+i_v\right)}.$$

T_t represents a lump sum nominal transfer from the government that we include to allow for the disbursement of the seignorage revenues from money creation. We model fiscal policy by assuming that

(2-9) $B_t = 0, \quad \text{for all } t,$

and we define the rate of money creation from the identity:

(2-10) $M_t \equiv (1+m)(1+u_t)M_{t-1}, \qquad E_{t-1}[u_t] = 0,$

where m is the mean money growth rate and u_t is the unpredictable component of money growth.

We further assume that all output is consumed:

(2-11) $Y_t = C_t.$

The Equations that Describe Equilibrium

We have chosen a specification of our model that is consistent with the existence of a balanced growth path and in the remaining sections of the paper, we make use of the notation:

(2-12) $y_t = \frac{Y_t}{S_t}, \qquad c_t = \frac{C_t}{S_t}, \qquad m_t = \frac{M_t}{P_t S_t},$

to describe ratios of variables to the productivity trend S_t. In our simulations we will assume that S_t is a random walk with drift although the method we use will work equally well for trend stationary processes. We show in appendix A, that a competitive equilibrium will satisfy equations (2-13) – (2-15):

(2-13) $y_t = f(L_t, m_t), \quad \text{where} \quad f(L_t, m_t) = F\left(L_t^{\,a}, m_t\right),$ **Production function**

(2-14) $\frac{-u_L(c_t, L_t)}{u_c(c_t L_t)} = f_L(L_t, m_t), \quad \text{where} \quad \begin{matrix} u_L(c,L) \equiv U_L(c,L,1) \\ u_c(c,L) \equiv U_C(c,L,1) \end{matrix},$ **Labor demand and supply**

(2-15)
$$E_t\{x_{t+1} m_{t+1} u_c(c_{t+1}, L_{t+1})[1+i_t]\} = E_t\{x_{t+1} m_{t+1} u_c(c_{t+1}, L_{t+1})[1+f_m(L_{t+1}, m_{t+1})]\},$$
Demand for money

where $x_{t+1} = \frac{(1+g)^{1-r}(1+v_{t+1})^{1-r}}{(1+r)(1+m)(1+u_{t+1})}.$

Equation (2-13) is the production function expressed in terms of the variables y_t and m_t; recall that these are measured as ratios of Y_t and M_t / P_t to productivity-growth, S_t. Equation (2-14) is a first order condition for the choice of labor and equation (2-15) follows from the firm's optimal portfolio allocation between money and bonds.

In addition to these three static equations, the model delivers an Euler equation that represents the household's intertemporal tradeoff:

(2-16) $$m_t u_c(c_t, L_t) = E_t \left\{ \frac{(1+g)^{1-r}(1+v_{t+1})^{1-r}}{(1+r)(1+m)(1+u_{t+1})} \left[1 + f_m(L_t, m_{t+1})\right] m_{t+1} u_c(c_{t+1}, L_{t+1}) \right\}.$$ **Euler equation**

In a non-stochastic model equation (2-15) would reduce to:

(2-17) $$i_t = f_m(L_{t+1}, m_{t+1}),$$

which is relatively standard representation of a demand-for-money equation. For example, when the production function is CES, the right side of this expression is a power function of y_{t+1} / m_{t+1} and the equation can be log-linearized to give real balances as a function of income and the interest rate. Later in the paper we will make use of first order approximations in which we drop all second order terms. In the stochastic linearized model the demand-for-money expression takes the form:

(2-18) $$E_t\{log(i_t)\} = E_t\{\tilde{a}_1 \, log(m_{t+1}) + \tilde{a}_2 \, log(L_{t+1})\},$$

where $\tilde{a}_1$ and $\tilde{a}_2$ are coefficients of linearization. We include an expectation operator on the left side of this expression because we model the holding of government debt as a risky activity; in other words, we assume that i_t is not in the date t information set. To generate interest rate volatility in our calibrated model, we add a random variable to the interest rate to represent the influence of non-fundamental uncertainty on the asset markets. Since firms balance their portfolios daily, but the period of our model is annual, we argue that this is a good way of capturing the observed volatility of nominal interest rates in the data. If we were to drop this assumption and assume instead that i_t is known at date t, our simulated interest rate series would be much less volatile, but all other aspects of our simulations would be unaltered.

In the following two sections of the paper we will show how to use equations (2-13)–(2-16) to derive an approximate linear difference equation that characterizes an equilibrium. Following this discussion, we will calibrate the model and compare artificial time series generated by the model with time series from actual data.

3 Money and the Labor Market

In this section and the one that follows we discuss the operation of the labor market in our model and we explain a key assumption of our analysis; that leisure is an inferior good.[5] This assumption is essential to understanding the circumstances under which there can be multiple equilibria. In section 4 we compare our assumption that leisure is inferior, with a similar assumption, common in the New Keynesian literature, that there are important "real rigidities" in the labor market, in the sense of Ball and Romer (1990).

The Labor Market in a Standard Model

In figure 1 we depict a "standard view" of an equilibrium labor market. The downward sloping lines represent labor demand; these are the marginal product curves $f_L(L,m)$ that are shifted by changes in the quantity of real balances. The upward sloping lines are labor supply curves. They represent the household's willingness to supply labor as a function of the real wage holding constant consumption. If real balances increase from m_1 to m_2, the labor demand curve shifts to the right from $LD_1(m_1)$ to $LD_2(m_2)$. Since the economy now supplies more output, consumption increases and the labor supply curve shifts up

[5] For related work, see Matheny (1998).

from $LS_1(C_1)$ to $LS_2(C_2)$. The net effect is an increase in the real wage from w_1 to w_2 and an increase in employment from L_1 to L_2.

If one is interested in using a neoclassical model of the labor market to explain data one must ask how much do the labor demand and supply curves shift and what are their slopes? When one starts to answer this question it becomes clear that a "standard" view of an equilibrium labor market has little room for money to have big effects. The reason is that labor demand cannot be shifted very much by an increase in real balances since the amount that it moves is equal to the elasticity of output with resect to

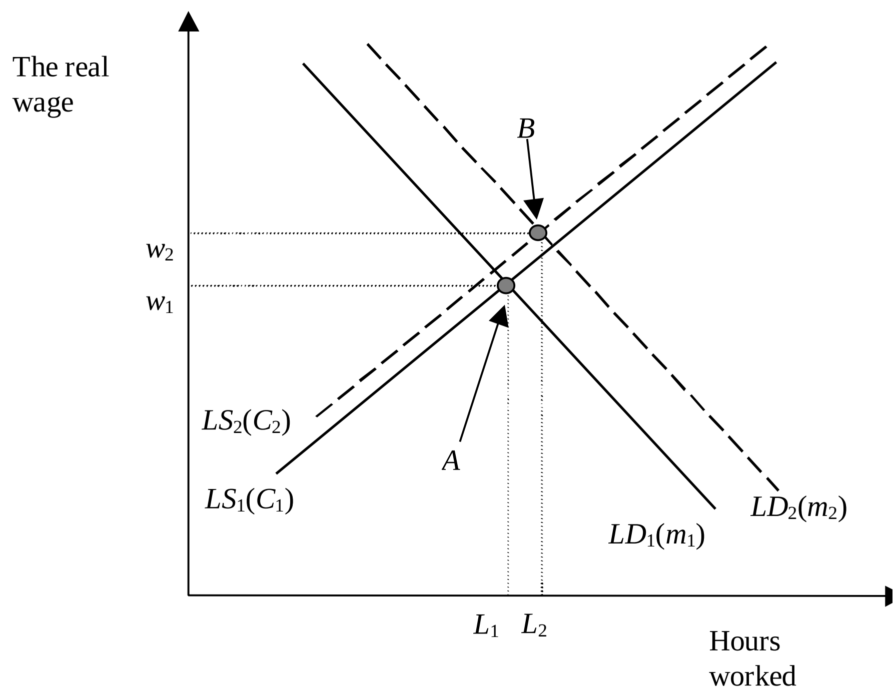

Figure 1: The Labor Market in a Standard Model

money: $\frac{mf_m}{f}$. This elasticity can be calibrated from data since in a competitive model one would expect to see firms equate the marginal product of money to the interest rate.

$$f_m(m) = i, \quad or \quad \frac{mf_m(m)}{f(m)} = \frac{im}{Y} = 0.01. \tag{3-1}$$

A straightforward calculation reveals that a reasonable number for the elasticity of output with respect to money is of the order of 1%; not a large enough number to be important if labor supply is parameterized as in most real business cycle models.

The Labor Market in Our Model

Attempts to estimate labor supply curves from first order conditions in aggregate data typically lead to estimates of a *negative* value for the slope of the labor supply curve.[6] An implication of these estimates, if one maintains a competitive view of the labor market, is that leisure is an inferior good. Equilibrium models force this interpretation on the data, in a model in which there is an important role for demand shocks, because consumption and hours worked are both pro-cyclical in the data. A representative household with standard preferences over leisure and consumption will choose to consume more leisure at the same time that it consumes more consumption implying that hours worked and consumption should move in opposite directions. To fit the fact that hours worked and consumption are both procyclical, equilibrium models must conclude that leisure is inferior.

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[6] See, for example, the paper by Farmer and Guo (1995).

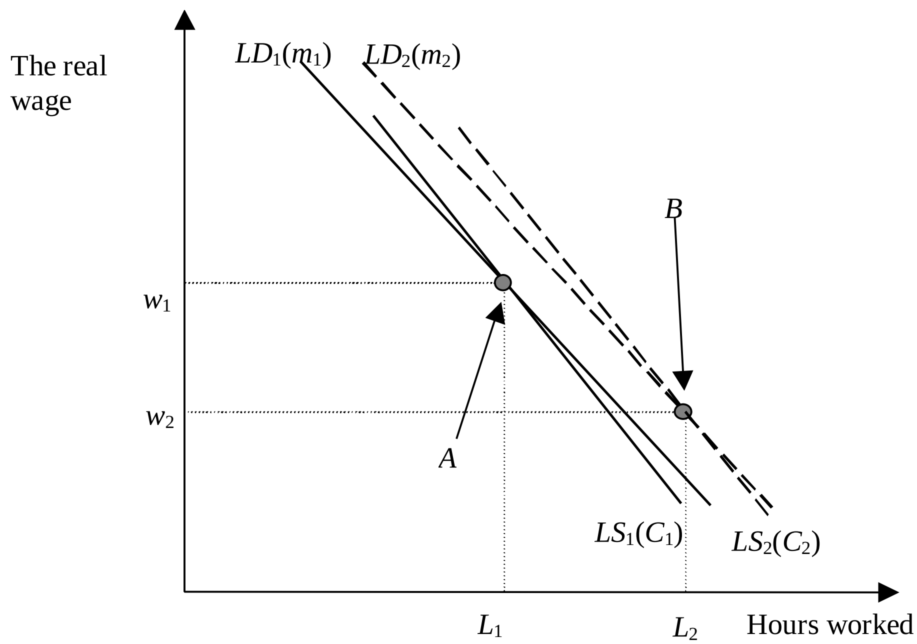

Figure 2: The Labor Market in Our Model

Figure 2 illustrates what happens in a model in which leisure is an inferior good. The qualitative features of this picture explain how our equilibrium model is able to generate a large effect of money even though money's share of income is small. An increase in money has two effects on employment, represented on the figure by shifts in the labor demand and supply curves. The first effect occurs as an increase in real balances causes firms to increase their demand for labor. On the diagram; the labor demand curve shifts to the right. The second effect occurs as increased production leads to increased consumption. The second effect causes the labor supply curve to shift up and it may cause firms to decrease or increase their labor supply at a given wage according to whether leisure is normal or inferior. The net effect of these shifts in the labor demand and supply curves is ambiguous since they cause employment to move in different directions.

The magnitude of the effects of a change in real balances on employment and output depends on how much the demand and supply curves shift and on their relative slopes. The fact that money's share of GDP is small implies that a 100% change in real balances causes at most a 1% shift of the labor demand curve. In a standard model, in which labor demand slopes down and labor supply slopes up, this could be translated, at most, into a 1% increase in employment. The maximum effect would occur when labor supply is horizontal. But if leisure is an inferior good, the labor supply curve slopes down and in this case employment can increase by more than the shift in the labor demand curve. It is this idea, illustrated in figure 2, which we exploit in our parameterization.[7]

Is the Inferiority of Leisure a Reasonable Assumption?

One is entitled to ask if our explanation of the effect of money on economic activity makes sense. Is it consistent with other facts about the labor market? We view our model as a useful abstraction. We believe that money does have big effects on employment and output, but the channels by which these effects operate are more complicated than we have described in this paper. An equilibrium model has no room for unemployment and yet most movements in hours worked at business cycle frequencies occur as workers move in and out of unemployment; not as a result of changes in labor force participation or of changes in hours worked by employed workers. We do not think that this invalidates an equilibrium approach to the labor market since a more sophisticated model that allows households to engage in an extra activity such as job search, is unlikely to change our main point. A more sophisticated model of the labor market would enable us to capture the idea that money may have big effects on employment, without assuming that leisure is an inferior good; an assumption that does not fit well with a priori reasoning about the way individuals make choices over alternatives.

A Formal Analysis of the Labor Market

In this subsection, we prove that money can have big effects on output, when leisure is inferior, by deriving two reduced form relationships; one between real balances and output and one between real balances and

[7] In our parameterization the labor supply curve slopes down more steeply than the labor demand curve and the initial increase in labor demand decreases employment and raises the real wage. This effect is small because the shift in the labor demand curve is small. The dominant effect occurs as consumption increases, labor supply shifts up, employment increases and the real wage falls.

employment. The elasticities of these two functions are related to the slopes of the labor demand and supply curves. When these slopes are close, the elasticities of the two functions are big.

The graphs depicted in figures 1 and 2 are expressions of the first order condition for labor market clearing. This can be expressed symbolically as follows:

(3-2)
$$\frac{-u_L(c_t, L_t)}{u_c(c_t, L_t)} \quad = \quad \frac{w_t}{S_t} \quad = \quad f_L(L_t, m_t),$$

where $u_L(c,L) \equiv U_L(c,L,1)$, $u_c \equiv U_c(c,L,1)$ and $f_L(L,m) \equiv aL^{a-1}F_X\left(\frac{L^a}{m},1\right)$.

The left side of this expression is the slope of an indifference curve and the right side is the marginal product of labor. Although we have not decentralized the labor market in our formal model, one could think of households supplying labor to firms. In this decentralized model the households would equate the slope of an indifference curve to the real wage; firms would equate the marginal product to the real wage.

Since, all output is consumed, one may replace c, in equation (3-2), by $f(L,m)$. The equation that results from this substitution describes employment, L, as an implicit function of real balances, m. Applying the implicit function theorem one can find a function $h(m)$ that describes how employment depends on money. This function is denoted $h(m)$ below:

(3-3) $$L \quad = \quad h(m).$$

If one substitutes the expression $h(m)$ back into the production function one can find a second expression that relates output to real balances:

(3-4) $$y \quad = \quad H(m) \quad \equiv \quad f[h(m),m].$$

The functions $h(m)$ and $H(m)$ play an important role in our analysis as they determine the way that labor supply and output respond to exogenous increases in real balances.

4 Indeterminacy Compared with a More Standard New Keynesian Approach

In this section we compare our model with a recent literature on the "New Keynesian Phillips Curve". A sample (by no means comprehensive) list of recent papers in this literature includes work by Ascari, (1997), Ascari and Garcia, (1999), Fuhrer and Moore (1995), Gali and Gertler (1998), Jeanne (1998), Kimball (1995), and Rotemburg and Woodford (1998:a), (1998:b).

The standard New Keynesian approach has the same underlying structure as the model in our paper. It differs in two important respects. First, money is included in the New Keynesian model in a way that makes the long run supply effects of real balances unimportant or non-existent. Second, the New Keynesian models are set up in such a way that there is a unique determinate equilibrium. In our paper we exploit indeterminacy to select an equilibrium in which nominal rigidities arise endogenously. In the more standard New-Keynesian approach, sticky prices are imposed by assuming that some fraction of firms face a cost of changing their price. The most common way to do this is to adapt the work of Calvo (1983) and to assume that a randomly selected fraction of firms is not permitted to change its price in each period.

A Common Problem

Before addressing the differences between the New Keynesian model and our indeterminacy approach, we will point out a challenge for both approaches. In the New Keynesian literature, following Ball and Romer (1990) it is common to distinguish between real and nominal rigidities. New Keynesians model nominal rigidities by assuming that some fraction of firms cannot adjust prices in any given period. In the context of a two period model, Ball and Romer (1990) showed that nominal rigidities are insufficient for monetary shocks to have large real effects. It must also be true that there are significant *real rigidities*.

Ball and Romer (1990) defined real rigidity as a property of a static model. The New Keynesian literature has extended the Ball-Romer idea and shown it also to be relevant to dynamic models. In the

New Keynesian model, real rigidity is expressed as a property of an equation relating the real wage to output. In some versions of the model this equation is derived by combining a labor supply equation with the production function. In others, (Jeanne (1998) for example) it is assumed to derive from union bargaining. A log linear approximation to this equation would take the form:

$$w_t = l_0 + l_1 y_t . \tag{4-1}$$

In a model with an equilibrium labor market, equation (4-1) would come from the labor market equilibrium condition:

$$w_t = \frac{-U_L\left(Y, F^{-1}(Y)\right)}{U_C\left(Y, F^{-1}(Y)\right)}, \tag{4-2}$$

where Y is consumption (equal to output) and $F^{-1}(Y)$ is the labor input required to produce Y. Ball and Romer's definition of a *real rigidity* is equivalent to the assumption that l_1 is small and in this case real wages will be relatively insensitive to changing labor market conditions.

Ball and Romer's definition is important because it can be shown (see Kimball (1995) or Jeanne (1998)) that in dynamic New Keynesian models real rigidities are also necessary for *persistence* of monetary policy shocks. Indeed, in a recent paper, Chari, Kehoe and McGrattan (1996) claim that New Keynesian models are unable to generate persistence because any reasonably calibrated model of the labor market must display a value of l_1 larger than one. A value of l_1 greater than one can be shown to be much too big to generate persistent effects of monetary policy shocks.

The Ball-Romer definition of real rigidities is exactly the condition that is needed in our model for monetary policy to have large real effects, and therefore for the model we describe to have an indeterminate equilibrium. In our model the firm equates the marginal product of labor to the real wage:

(4-3) $$f_L\left(F^{-1}(Y),m_t\right) = w_t = \frac{-U_L\left(Y,F^{-1}(Y)\right)}{U_C\left(Y,F^{-1}(Y)\right)},$$

and it sufficient for money to have a large effect on output, that the real wage be relatively rigid. For example, when the exchange technology is CES, the left side of (4-3) takes the form:

(4-4) $$f_L\left(F^{-1}(Y),m_t\right) = \left(\frac{Y}{m}\right)^l,$$

and in this case the elasticity of aggregate supply with respect to real balances is equal to one when the real wage is rigid using the definition of Ball and Romer. One assumption, which guarantees that the real wage is rigid in this sense, is that leisure is inferior. For example, if $U(C,L) = -\frac{L}{C}$ and $Y = C = L$ then $\frac{-U_L}{U_C} = 1$. This discussion implies that real rigidities are necessary for Neo-Keynesian theories of price stickiness *and* for the indeterminacy model to explain the persistence of monetary policy shocks.[8] Our assumption that leisure is inferior and that the labor market is competitive is one route to real rigidities. Relative wage concern (as in Ascari and Garcia (1999)), segmented labor markets (as in Rotemburg and Woodford (1998:a) or union bargaining (as in Jeanne (1998)) are some of the alternatives that have been exploited in the New Keynesian literature.

Two Differences between the Two Approaches

In this section we point out two important differences between the New Keynesian and indeterminacy models. First, the indeterminacy approach requires the existence of an important long-run effect of real balances on output; higher real balances imply higher output. Since real balances are inversely related to inflation in a long run balanced growth path this property implies the existence of a

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[8] Michael Kiley (1997) makes a similar point. He shows, in models with increasing returns to scale, that increasing returns leads to real rigidities *and* to indeterminacy.

positively sloped “long-run Phillips curve”. The New-Keynesian models may also exhibit a relationship of this kind, but it is not central to the theory and one could seek evidence against indeterminacy by establishing that no such relationship exists.

A second difference between the New Keynesian theories, and the indeterminacy approach relates to whether purely anticipated changes in monetary policy will have real effects on output. The short run New Keynesian aggregate supply curve includes both backward and forward looking elements. Backward looking elements enter the equation because some fraction of agents is unable to alter its price; forward looking elements enter because when agents do alter their price they must forecast the entire future path of all endogenous variables in order to make a rational price setting decision. The indeterminacy approach, in contrast, does not require forward-looking behavior by price setters because we exploit indeterminacy to select a backwards looking equilibrium.

This difference suggests a second possible test to distinguish the two approaches that would exploit different predictions in the face of a change in the money supply rule. The new Keynesian approach is partly forward looking and would predict a jump in the inflation rate in the face of a change in the rule generating money growth. The indeterminacy approach is purely backward looking and predicts no such jump.[9]

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[9] Farmer (1991:a) makes the point that there are circumstances in which the Lucas Critique does not apply in rational expectations models with indeterminacy. The model in this paper is one such case. There is some evidence that forward looking behavior is not important, contained in work by David Hendry and Carlo Favero (1992). These authors find periods in U.K. data when there are breaks in the money supply process, without simultaneous accompanying breaks in the equation describing the behavior of prices. This is exactly the kind of evidence that is needed to discriminate between backward and forward looking models of aggregate supply. The Hendry-Favero results suggest that the indeterminacy explanation of persistence holds some promise.

5 Equilibrium in the Model Economy

In this section we begin a formal analysis of the properties of equilibria in our model. As is common in recent RBC literature, we study equilibria in a linear approximation around a balanced growth path. The existence of such a path is established in Appendix B.[10]

Employment, Output and Real Balances

We begin by finding linearized versions of the functions $h(m)$ and $H(m)$ that we described in section 3. To derive these expressions, we first linearize the production function and the labor market equation: (details are given in Appendix C). In these expressions, and in our subsequent discussion, we use the symbol $d\,log(x)$ to mean the log-difference, $log(x)-log(x^*)$, of a variable x, $x \in \{y, L, m\}$ from x^*. The equations are represented as follows:

(5-1) $$d\,log(L_t) = e_h\ d\,log(m_t) \qquad e_h \equiv \frac{a_2}{b-a_1}.$$

(5-2) $$d\,log(y_t) = e_H\ d\,log(m_t), \qquad e_H \equiv \frac{ba_2}{b-a_1}.$$

In Appendix C we show that the parameter b is given by the expression $b \equiv \dfrac{a\ l - (1-s_L)}{l\ +\ (r-1)(1-s_c)}$ where s_c and s_L are functions of the parameters of utility (see Appendix B for details). The parameter a_2 is small; this is the elasticity of output with respect to real balances and we have argued that it is of the order of 1%. It follows from the definition of e_h and e_H that for real balances to have a big effect on

[10] To derive this path we set $v_t = 0$ and $u_t = 0$ for all t and we let y*, L*, c* and m* represent the stationary values of y_t, c_t, and m_t that satisfy equations (2-13) – (2-16).

employment and output, b must be close to $a_{1.}$ This condition is equivalent to the assumption that the labor demand and supply curves have similar slopes.

The Characteristic Equation: A Dynamic Equation to Characterize Equilibrium

The solution to a rational expectations model is characterized by a joint probability distribution over sequences of real balances $\{m_t\}_{t=1}^{\infty}$. In this section we derive a functional equation (we call this the characteristic equation) that must be obeyed by the equilibrium probability distribution. We derive the characteristic equation from the Euler equation, (2-16), by substituting into it expressions for employment and output, as functions of real balances. When the elasticities of these functions are large we say that money is important in production. We will show that when money is important in production, the economy has a continuum of indeterminate equilibria.

Using the production function and the labor market equation we have shown that employment and output can be written as functions of real balances; we referred to these as $h(m)$ and $H(m)$. By substituting the expressions $h(m)$ and $H(m)$ into the Euler equation (2-16) we arrive at the *characteristic equation* for our economy.

(5-3)

$$G(m_t) \;=\; E_t\left\{\frac{(1+g)^{1-r}(1+v_{t+1})^{1-r}}{(1+r)(1+m)(1+u_{t+1})}G(m_{t+1})X(m_{t+1})\right\},$$

$$G(m_t) \equiv m_t u_c\big(H(m_t),h(m_t)\big), \quad X(m_t) \equiv 1 + f_m\big(h(m_t),m_t\big).$$

Linearizing (5-3) around the balanced growth path gives the linearized characteristic equation:

(5-4)
$$e_G d\,log(m_t) \;=\; E_t\{(e_G + e_X)d\,log\,m_{t+1} \;+\; (1-r)v_{t+1} \;-\; u_{t+1}\},$$

where e_G and e_X are the elasticities of the functions $G(m)$ and $X(m)$. For our choice of utility and production functions, these are given by the expressions:[11]

(5-5) $$e_G = 1 - re_H + s_L e_h, \qquad e_X = \frac{f_m}{1+f_m}(1-l)(e_H - 1).$$

More compactly, we can write the linearized characteristic equation as follows:[12]

(5-6)
$$d\,log(m_t) = E_t\left\{\frac{1}{b_1}d\,log\,m_{t+1} - \frac{b_2}{b_1}v_{t+1} - \frac{b_3}{b_1}u_{t+1}\right\},$$
$$b_1 \equiv \frac{e_G}{e_G + e_X}, \quad b_2 \equiv \frac{(r-1)}{(e_G + e_X)}, \quad b_3 \equiv \frac{1}{(e_G + e_X)}.$$

In the following subsection we will discuss the properties of this equation and compare our model, in which there may be multiple equilibria, with other more familiar rational expectations models in which equilibrium is unique.

When Does the Characteristic Equation have a Unique Solution?

An equation like (5-6) is typical in monetary rational expectations models. Often these models are derived in the context of a two-parameter family of utility functions in which utility is separable. By

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[11] The parameter e_G is given by the expression $e_G \equiv 1 + \frac{u_{cc}c}{u_c}e_H + \frac{u_{cL}L}{u_c}e_h$. For our choice of utility function, $\frac{u_{cc}c}{u_c} \equiv r$ and $\frac{u_{cL}L}{u_c} \equiv s_L$, where the parameter $s_L \equiv \frac{L^*}{L^* - B}$. The parameter e_X is the elasticity of $1+f_m$. For our technology this can be written as $1+f_m = 1 + a\left(\frac{H(m)}{m}\right)^{1-l}$. Taking the log derivative of this function leads to the expression: $e_X = \frac{f_m}{1+f_m}(1-l)(e_H - 1)$. The linearization of the utility function is discussed further in section 8.

[12] We have chosen to express the parameters in this way to simplify expressions for the backwards dynamics.

iterating the characteristic equation into the future, one obtains the following expression for deviations of real balances from their balanced growth path:

(5-7) $$d\,log(m_t) = E_t\left\{\sum_{t=1}^{\infty}\left(\frac{1}{b_1}\right)^{t-1}\left(\frac{b_2}{b_1}v_{t+i} - \frac{b_3}{b_1}u_{t+1}\right) + \lim_{T\to\infty}\left(\frac{1}{b_1}\right)^{T} d\,log(m_{t+T})\right\}.$$

It is typical, in simple two parameter models, for one to obtain a restriction of the form

(5-8) $$|b_1| > 1.$$

If inequality (5-8) holds, then the term, $\lim_{T\to\infty}\left(\frac{1}{b_1}\right)^{T} d\,log(m_{t+T}) = 0$. Since $E_t(v_{t+i}) = E_t(u_{t+i}) = 0$ for all i>0, it follows that:

(5-9) $$d\,log(m_t) = 0,$$

which implies that when $|b_1| > 1$, $m_t = m^*$ is the unique rational expectations equilibrium.

An implication of the monetary model with a unique determinate equilibrium is that a monetary shock will be absorbed 100% in prices at the moment it occurs since real balances will adjust immediately to keep m_t on its balanced growth path m^*.[13] In other words, determinacy (in the absence of some kind of sticky price mechanism, such as menu costs or other nominal rigidities) implies that money cannot have real effects either in long run or in the short run.

When Does the Characteristic Equation have a Multiple Solutions?

What happens if one chooses a more flexible parameterization of the utility function as we have done in this paper? The answer is that if there are important real rigidities (we capture real rigidities by

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[13] If the monetary shock or the real shock is auto-correlated, this expression will be a little more complicated since the persistence of shocks will introduce an endogenous dynamic to real balances. But the basic point that prices will be highly flexible in this model will survive.

making leisure an inferior good) then the parameter b_1 may be less than one in absolute value and in this case the $\lim_{T\to\infty}\left(\frac{1}{b_1}\right)^T d\,log\left(m_{t+T}\right)$ will not exist. There will still however, be solutions to (5-6). Indeed, in this case there will exist a continuum of rational expectations equilibria of the kind discussed in Farmer and Woodford (1997).

To see why real rigidities are important in generating indeterminacy, notice that the critical parameter b_1 is related to the elasticities of the functions $G(m)$ and $X(m)$ in the following way: $b_1 = e_G / \left(e_G + e_X\right)$. For standard parameterizations of technology, e_X is small and negative. It follows that a sufficient condition for b_1 to be positive and less than one (and therefore for the existence of indeterminate equilibria), is for e_G to be negative. For our choice of functional forms, e_G can be expressed as follows, (see footnote 11) $e_G = 1 - re_H + s_L e_h$, where $r = u_{cc}c / u_c$ and $s_L = u_{cL} L / u_c$ are curvature parameters of the utility function evaluated along the balanced growth path. Inspection of the definition of e_G reveals that indeterminacy is more likely (e_G is more likely to be negative) when e_H and r are large and s_L and e_h are small. It is the introduction of inferiority of leisure that allows us simultaneously to make e_G large and s_L relatively small thereby causing e_G to be negative.

When b_1 is between zero and one, the backward equation ($d\,log\left[m_{t+1}\right]$ expressed as a function of $d\,log\left[m_t\right]$) will be stable. In this case, the following stochastic difference equation may be used to construct probability distributions over real balances that satisfy the characteristic equation (5-6) and which are, therefore, valid rational expectations equilibria.

(5-10) $$d\,log\left(m_{t+1}\right) = b_1 d\,log\left(m_t\right) + b_2 v_{t+1} + b_3 u_{t+1} + e_{t+1}.$$

The variable e_{t+1} represents non-fundamental uncertainty, referred to in the literature as sunspots or animal spirits. Since e_{t+1} can be chosen arbitrarily, the model with $|b_1| < 1$ will possess multiple rational expectations equilibria. In the following section of the paper, we explore this issue.

6 Selecting an Equilibrium: Beliefs as Fundamentals

In models in which the equilibrium is indeterminate, the fundamentals of the economy are insufficient to pin down behavior. But agents must still make forecasts of the future and decide how to act. Some authors have argued that models with indeterminacy are bad models because they do not make predictions about what will occur. In this section we propose a resolution of this problem by supplementing our model with a *belief function* to which we attribute the same methodological status as preferences, endowments and technology. We parameterize the belief function and we argue that the parameters of the belief function can be estimated in the same way as the parameters of utility and technology.[14]

To make our argument precise we will define three related objects: the belief function, characteristic equation, and the equilibrium price function. The characteristic equation is a functional equation that must be satisfied in equilibrium. The equilibrium price function is a stochastic difference equation that describes how prices evolve in a rational expectations equilibrium. The characteristic equation and the equilibrium price function are related by the fact that the equilibrium price function must generate sequences of prices that satisfy the characteristic equation.

In a rational expectations model with a unique equilibrium the belief function plays no role since the probability distribution of beliefs is endogenously determined by the condition that it must coincide with the probability distribution of prices. But in models with multiple rational expectations equilibria there are many equilibrium price functions. Our resolution of this multiplicity is to select an equilibrium by choosing the belief function and allowing it to select an equilibrium.

The Belief Function

Before the advent of rational expectations it was typical to model expectations with a rule of the form:

(6-1) $$P_{t+1}^{E} = \Psi(X_t),$$

where P_{t+1}^{E} is the agents' subjective expectation of the price level at date t+1 and $\Psi(X)$ is a belief function that explains how agents' forecasts of the future depend on the present. The term X_t represents all information available at date *t*. In a model with a unique rational expectations equilibrium, the exogenous specification of a belief function is unnecessary since the function $\Psi(X)$ must implement the unique rational expectations equilibrium. But in a model with multiple rational expectations equilibria, it becomes necessary once more to specify how individuals predict.

The Characteristic Equation

To keep our argument concise we will shut down all real shocks and we will study the special case in which the money supply is a random walk. The basic points that we want to make do not depend on these assumptions although they do simplify some of the algebra. An equilibrium must satisfy the characteristic equation (5-6). Since all the shocks in our model have zero conditional means, this equation can be written as follows:

(6-2) $$log(M_t) - log(P_t) = E_t\left\{\frac{1}{b_1}\left[log(M_{t+1}) - log(P_{t+1})\right]\right\}.$$

In models with indeterminacy the characteristic equation has multiple bounded solutions, a fact that is widely perceived to be a problem for rational expectations because it is not clear how a particular equilibrium would be established. Our view is that the problem lies not with the equations of the general equilibrium model, but from the fact that these equations are incomplete. In the multiple equilibrium world one must supplement the equilibrium equations with a separate rule in the class (6-1) that models the

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[14] The concept of a belief function is discussed in Farmer (1993) and has been generalized by Matheny (1999) to a large set of linear rational expectations models in which equilibria are indeterminate.

process by which agents forecast. Within the class of all such rules, some will implement a rational expectations equilibria and it is on these that we will focus.

The Equilibrium Price Function

Since the set of rational expectations equilibria is extremely large, we will restrict our attention to a subset of equilibria that can be represented as solutions to the following stochastic difference equation:

$$\text{(6-3)} \qquad \left[log\left(M_{t+1}\right) - log\left(P_{t+1}\right)\right] = b_1\left[log\left(M_t\right) - log\left(P_t\right)\right] + e_{t+1}.$$

It is important to keep equation (6-3) distinct from the characteristic equation (6-2). Equation (6-3) describes the actual evolution of real balances in a particular rational expectations equilibrium. We refer to it as an *equilibrium price function* since for given M_{t+1} the equation determines the price level in period $t+1$. To verify that a proposed price function is indeed an equilibrium, one must ensure that a sequence of real balances generated by equation (6-3) satisfies equation (6-2).

Equation (6-3) represents not one, but many, equilibrium price functions. Different members of the class are determined by specifying a rule for generating the sequence of sunspot variables $\{e_t\}$. In the remaining part of this section we will focus our attention on equilibria for which the sunspot process $\{e_t\}$ is a linear function of the money shock $\{u_t\}$; that is, on equilibria in the restricted class:

$$\text{(6-4)} \qquad \left[log\left(M_{t+1}\right) - log\left(P_{t+1}\right)\right] = b_1\left[log\left(M_t\right) - log\left(P_t\right)\right] + y\ u_{t+1}, \quad u_{t+1} \equiv \left[log\left(M_{t+1}\right) - log\left(M_t\right)\right].$$

These equilibria are interesting since they are able to explain why nominal shocks have real effects.

How the Belief Function Implements an Equilibrium

Writing down an equilibrium for our model is an important first step, but we must also illustrate how any particular equilibrium comes about. We will develop the idea in the following sections that an equilibrium is supported by a belief function. Specifically, we will show that rational expectations equilibria in the class (6-4) are supported by the belief function:

$$(6\text{-}5)\qquad \begin{aligned} &log\left(P_{t+1}^{E}\right)=q_1\,log\left(M_t\right)+q_2\,log\left(M_{t-1}\right)+q_3\,log\left(P_{t-1}\right)\\ &q_1\equiv\left(1-b_1 y\right),\quad q_2\equiv b_1\left(y-b_1\right),\quad q_3=b_1^{\,2}, \end{aligned}$$

where the parameter b_1 depends on the fundamentals of the economy and y parameterizes beliefs. We will show that when agents forecast with equation (6-5) in every period, actual prices will follow the same process. In other words, the belief function (6-5) is self-fulfilling. We propose to treat the parameter y as a "deep parameter" that has the same methodological status as preferences and technology. Taking this approach implies that equilibrium is unique since for any given belief function there is only one possible rational expectations equilibrium. For almost all values of y, the real economy will respond to nominal shocks in the short run.

Why Lagged Prices Must Appear in the Belief Function

In this section we provide a method that can be use to construct a family of belief functions, each of which implements a different rational expectations equilibrium. Our method starts from a proposed equilibrium price function and lags it one period to remove the influence of the current price. This step is important, for the reasons that we explain below.

Think of the characteristic equation as an equilibrium condition between a demand and supply function in which expectations appear because demand depends on the beliefs of agents about the future. Using the symbols M^S and M^D to mean money supply and demand, in equilibrium it must be true that:

$$(6\text{-}6)\qquad log\left(M_t^{S}\right)-log\left(P_t\right)\;=\;log\left(M_t^{D}\right)-log\left(P_t\right)\;=E_t\,\frac{1}{b_1}\left\{log\left(M_{t+1}^{S}\right)-log\left(P_{t+1}\right)\right\}$$

The left-side of this equation is the real supply of money. The right side is the demand for real money which depends on the expected value of the future money supply and on the future price level. To complete an explanation of how the economy achieves asset market equilibrium we need to explain how agents forecast the future values of these variables.

Suppose that agents forecast $\left\{log\left(M_{t+1}^{S}\right)-log\left(P_{t+1}\right)\right\}$ using the equilibrium price function (6-4). Substituting equation (6-4) in to (6-6) leads to the identity:

$$\text{(6-7)} \qquad log\left(M_t^S\right)-log\left(P_t\right) = log\left(M_t^D\right)-log\left(P_t\right) = log\left(M_t^S\right)-log\left(P_t\right),$$

which will be satisfied by *any* price level. We have shown that if agents use the equilibrium price function to forecast the future, then the equality of demand and supply cannot be simultaneously used to determine the price level. There is circularity here since the price level carries a signal both about equilibrium and about future prices. This circularity can be avoided if agents enter the period with beliefs about future prices that are insensitive to current prices. We will show below, that it is possible to find a belief function that is independent of the current price that can implement a *particular* rational expectations equilibrium.

How to Construct a Belief Function

In this section we show how to construct a belief function. Suppose that instead of using the equilibrium price function itself, we iterate the right-hand-side of equation (6-4) one period so that forecasts of the period $t+1$ price do not depend on the price at date t. This construction leads to the proposed belief function:

$$\text{(6-8)} \qquad \left[log\left(M_{t+1}\right)-log\left(P_{t+1}\right)\right]=b_1{}^2\left[log\left(M_{t-1}\right)-log\left(P_{t-1}\right)\right]+y\ \ u_{t+1,}+b_1 y\ \ u_t,$$

which, by rearranging terms and taking expectations conditional on information at date *t*, can be shown to be equivalent to equation (6-5).[15]

We must now verify that this function is consistent with rational expectations. To do this we will plug it into the right-hand-side of the characteristic equation, (6-2) and apply the expectations operator to the right-hand-side. This operation leads to the expression

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[15] Recall that $u_{t+1}\equiv log\left(M_{t+1}\right)-log\left(M_t\right)$ and that $E_t\ log\left(M_{t+1}\right)=log\left(M_t\right)$.

$$\log\left(M_t^S\right)-\log(P_t)=\log\left(M_t^D\right)-\log(P_t)=\frac{1}{b_1}\left\{b_1{}^2\left[\log(M_{t-1})-\log(P_{t-1})\right]+b_1 y\ \ u_t\right\},$$

(6-9) $$\Rightarrow$$

$$\log\left(M_t^S\right)-\log(P_t)=b_1\left[\log(M_{t-1})-\log(P_{t-1})\right]+y\ \ u_t\,.$$

If agents use the belief function (6-8) to forecast real balances in period $t+1$, then the equilibrium price function in period t will be given by equation (6-9). By iterating the right-hand-side one period to eliminate $\log(M_{t-1})-\log(P_{t-1})$ it follows that real balances in period t will follow the process:

(6-10) $$\left[\log(M_t)-\log(P_t)\right]=b_1{}^2\left[\log(M_{t-2})-\log(P_{t-2})\right]+y\ \ u_t+b_1 y\ \ u_{t-1}\,.$$

Equation (6-10) is an alternative representation of the equilibrium price function and it is one that coincides with the belief function, equation (6-8). We have shown that when agents forecast future prices in this way, the economy will be in a rational expectations equilibrium. Furthermore, if we are prepared to treat the parameter y as a primitive, in the same way as we treat the parameters of preferences and technology, this rational expectations equilibrium is unique. For a give value of y, the belief function (6-8) can sustain *one and only one rational expectations equilibrium*.

7 Two Special Cases

Two of the equilibria supported by the belief function (6-5) are special since they correspond to polar views about price flexibility in the economy. In our first example the parameter y is equal to 0. We call this case the quantity-theoretic economy (after the Quantity Theory of Money) because nominal shocks feed immediately into prices and money is neutral in both the long run and the short run. In our second example the parameter y is equal to 1. We call this case the fixed price economy because the price level is predetermined one period in advance, nominal shocks feed immediately into quantities and prices respond only asymptotically.

The Quantity Theoretic Economy

Equilibrium is determined by the difference equation:

(7-1) $$log(M_t) - log(P_t) = b_1[log(M_{t-1}) - log(P_{t-1})],$$

which is the special case of (6-9) when $y = 0$. For this case, the log of real balances converges to zero and asymptotically:

(7-2) $$\frac{M_t}{P_t} = 1, \quad \text{for all } t,$$

which implies that the price level in equilibrium is equal to the money stock. Since the money supply is a random walk, the expected price level one period ahead must also equal the current period's money stock. This intuition is borne out by the belief function:

(7-3) $$log(P_{t+1}^E) = log(M_t) - b_1^2[log(M_{t-1}) - log(P_{t-1})].$$

As $log(M_{t-1}) - log(P_{t-1})$ converges to zero, the economy converges to a steady state equilibrium. In the steady state, next period's expected price is equal to the current period money stock.

The Predetermined Price Equilibrium

A second interesting case occurs when $y = 1$. In this case equilibrium is determined by the equation:

(7-4) $$log(M_t) - log(P_t) = b_1[log(M_{t-1}) - log(P_{t-1})] + u_t.$$

Using the fact that the money shock u_t is equal to $log(M_t) - log(M_{t-1})$ we can derive an equation that describes how the price level will be determined in equilibrium:

(7-5) $$log(P_t) = log(M_{t-1}) - b_1[log(M_{t-1}) - log(P_{t-1})].$$

Since only lagged variables appear on the right-hand-side of this equation, the price level must be predetermined at date t. It is this sense in which our model leads to a description of an economy in which prices may be "sticky". There are no barriers to prevent prices from responding to new information. Instead, it is the way that individuals use that information to adapt their beliefs about *future* inflation that causes prices to respond slowly to nominal shocks.

8 Calibrating the Model

In this section we calibrate our model and investigate its implications for the moments of simulated data.

The Production Function

Our production function has three parameters; the elasticities of output with respect to labor and money and the elasticity of substitution between money and real balances. The assumption of competitive markets implies that a_1 and a_2, the elasticities of the production function with respect to labor and money, are equal to average factor shares, since competitive firms equate marginal products to prices:

$$(8\text{-}1) \qquad a_1 = \frac{wL}{Y} = 0.66, \qquad a_2 = \frac{iM}{PY} = 0.01.$$

The numbers 0.66 and 0.01 are averages in U.S. data for the period from 1900 to 1996.

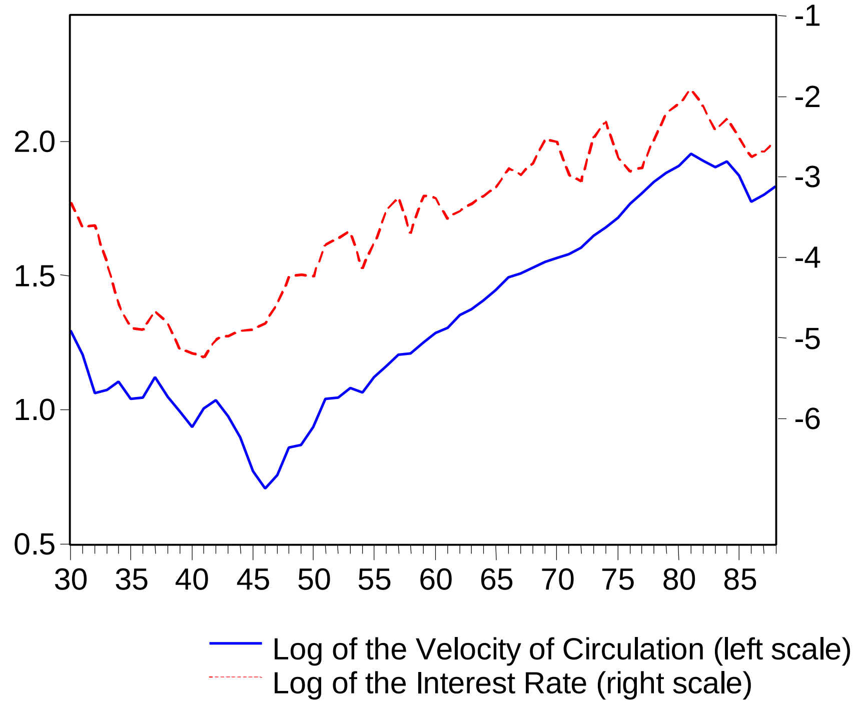

Figure 3: The Interest Rate and the Velocity of Circulation in the Data

We calibrate the elasticity of substitution parameter from demand-for-money studies. Equation (2-15) evaluated along the balanced growth path implies that:

(8-2) $$f_m\left(L^*, m^*\right) \equiv a\left(\frac{m^*}{y^*}\right)^{l-1} = \bar{i}\,.$$

In U.S. data, the velocity of circulation m/y and the interest rate i are non-stationary variables in the period from 1929 to 1988. The annual data is graphed in Figure 3 where the low frequency relationship between velocity and the interest rate is apparent to the eye. Our model implies that as the interest rate trends upwards, the velocity of circulation should grow at the rate $1/(1-l)$. Using evidence from the co-integrating relationship between velocity and the interest rate we choose $l=-1$. This choice implies that the interest elasticity of money demand should be –0.5, a number that is consistent with an estimate of the co-integrating relationship between velocity and the interest rate in the data depicted in Figure 3.[16]

The Utility Function

The utility function parameters that influence behavior are the elasticity of utility with respect to labor supply and consumption and the elasticities of marginal utilities evaluated along the balanced growth path. For our parameterized example these are given by the expressions:

(8-3) $$d_c \equiv \frac{c^* u_c(c^*,L^*)}{u(c^*,L^*)} = s_c(1-r), \qquad d_L \equiv -\frac{L^* u_L(c^*,L^*)}{u(c^*,L^*)} = s_L,$$

(8-4) $$\begin{aligned} d_{cc} &\equiv \frac{c^* u_{cc}(c^*,L^*)}{u_c(c^*,L^*)} = -r, & d_{cL} &\equiv -\frac{c^* u_{cL}(c^*,L^*)}{u_c(c^*,L^*)} = s_L, \\ d_{Lc} &\equiv -\frac{c^* u_{Lc}(c^*,L^*)}{u_L(c^*,L^*)} = s_c(1-r), & d_{LL} &\equiv \frac{L^* u_{LL}(c^*,L^*)}{u_L(c^*,L^*)} = 0. \end{aligned}$$

The parameters s_L and s_c are the functions of the steady values c^* and L^* described below:

(8-5) $$s_c \equiv \frac{c^{*1-r}}{c^{*1-r}+A}, \qquad s_L \equiv \frac{L^*}{L^*-B}.$$

Evidence concerning the values of the utility function parameters comes from several sources. One restriction comes from the first order condition for labor supply which implies that:

(8-6) $$-\frac{Lu_L(c,L)}{cU_c(c,L)} = \frac{s_L}{s_c(r-1)} = \frac{wL}{C} = 1.$$

[16] There are a number of recent studies of the demand-for-money that exploit the low frequency movements in the data to estimate money demand functions. See for example, Hoffman, D. L., R. H. Rasche, and M. A. Tieslau (1995) who obtain similar estimates to our U.S. estimate using data from the U.S., Japan, Canada, the U.K. and West Germany.

Since the wage bill has historically been equal to consumption in US data we choose a parameterization that sets the ratio of the parameters s_L and $s_c(r-1)$ to unity. Since our model sets $C_t = Y_t$, ignoring investment, we were forced to choose between setting this ratio to unity, conforming to the model assumption that consumption equals GDP, or 0.8, conforming to the empirical observation that total consumption is 80% of GDP in the data. In our calibrations we experimented with both assumptions and found very little difference in the reported simulations providing we chose our one free parameter s_L, to keep the slopes of demand and supply of labor close to each other. We discuss this issue further in the final paragraph of this section.

The parameter r is often referred to as the coefficient of relative risk aversion although it could equally well be described as the elasticity of intertemporal substitution. This parameter governs the willingness of the household to accept consumption plans that fluctuate through time as well as across states of nature.[17] The literature on "reasonable values" for r varies from the a priori argument by Ken Arrow that utility is logarithmic, which implies that $r = 1$, to attempts to explain the equity premium puzzle using risk aversion which require values of r of the order of 50 or more. Our reading of the literature suggests that a value in the range of 2 to 5 would be accepted as reasonable. Our benchmark model chooses $r = 4.09$ which is in the upper end of this range. Low values are not consistent, in our model, with multiple equilibria.

The fact that d_{LL} is zero follows from the fact that we chose to model utility as linear in labor supply. This assumption helps us to capture the fact that hours-worked are highly volatile in the data and it

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[17] In models in which one assumes that preferences are additively separable Von-Neumann Morgenstern functions defined over consumption sequences, the parameter that governs risk aversion is the same parameter that governs intertemporal substitution. Hence there is some confusion over nomenclature. For an excellent discussion of these issues see the paper by Philippe Weil (1990).

has become common in equilibrium business cycle models following the work by Hansen (1984) and Rogerson (1988).

Given the restrictions described above we are free to choose one parameter. We chose this parameter, s_L, to maximize the chance of our model to describe an indeterminate equilibrium by picking a value for which the elasticity of the function e_H is large.

Summarizing Our Parameterization

In Table 1 we summarize the information on our calibrated parameters and the evidence that we used to

Table 1: Parameter	*Magnitude*	*Evidence*
a_1	0.66	Labor's share of Income
a_2	0.01	Money's Share of Income
l	-1	Cointegrating relationship of velocity and the interest rate
r	4.09	Asset market studies (1 is log preferences)
s_L	1.84	Makes e_H equal to 0.75

Table 2: Parameter restrictions	*Reason*
$s_L = (r-1)s_c$	First order conditions in the labor market
$a(1-a_2) = a_1$	Constant returns-to-scale
$e_G = 1 - re_H + s_L e_h$	Definition of $G(m)$
$e_X = \frac{f_m}{1+f_m}(1-l)(e_H - 1)$	Definition of $X(m)$ (this also exploits the functional form of f)

choose these values. Table 2 summarizes the restrictions that we used to pick two remaining parameters, s_c and the elasticity of labor in the first stage production function a. Finally, in Table 3 we note the values of the derived parameters of the model. The parameter s_c measures departures of the utility function from the standard case in which the parameter A would be set to 0 implying $s_c = 1$. We are able to set s_c different from 1, and still maintain balanced growth, by allowing the utility function to depend directly on the productivity shock.

Table 3 *Derived parameters*	*Magnitude*	*Interpretation*
s_c	0.59	Preference parameter
b	0.67	Reduced form elasticity of y w.r.t. L
e_h	1.12	Reduced form elasticity of L w.r.t. m
e_H	0.75	Reduced form elasticity of y w.r.t. m
e_X	-0.024	Reduced form elasticity of $(1+i)$ w.r.t. m
e_G	-0.011	Elasticity of $mu_c(H(m),h(m))$ w.r.t. m
b_1	0.3	Slope coefficient of characteristic equation

The parameter b is related to the slope of the labor demand and supply curves. Our explanation of labor market dynamics is very sensitive to b and to make our explanation work we must choose s_L and r in such a way that b is very close to a. This is another way of saying that when the direct effect of money is small, the slopes of the labor demand and supply curves must be very close.

The parameters e_h, and e_H are the elasticities of the functions $h(m)$ and $H(m)$ and they determine the responsiveness of employment and output to money shocks. The parameter e_X measures the sensitivity of one plus the marginal product of money to changes in real balances. Since the marginal product of money is equated in equilibrium to the interest rate, this parameter also determines the elasticity of interest rate fluctuations with respect to money shocks. Finally, b_1 is the slope of the characteristic equation. It is this parameter that determines whether the equilibrium is indeterminate. Indeterminacy requires $|b_1|<1$.

9 Evidence From Simulated Data

In this section we illustrate the idea that prices may be "sticky" in equilibrium by simulating data from our model economy. In our simulations we choose the parameter y to equal 1; in other words, we simulated a predetermined price equilibrium.[18]

How We Simulated Our Data

To facilitate comparison with actual data, we fed shocks into our model, recovered from actual U.S. data. For the sequence $\{u_t\}$ we used the log growth rate of U.S. M1 and for the productivity shock $\{S_t\}$ we used the Solow residual, computed as $log(S_t) = log(Y_t) - 0.67\,log(L_t) - 0.33\,log(K_t)$.[19] Figure 4 illustrates the behavior of the log difference of the Solow Residual and the log difference of M1 over the period 1930 through 1988.

Our simulated data was constructed by first generating a sequence of 59 values for $log(m_t)$ by iterating the equation

(9-1) $$log(m_t) = b_1\, log(m_{t-1}) + u_t, \qquad m_0 = 0,$$

where $b_1 = 0.3$ and $\{u_t\}$ was the sequence of actual log money supply growth rates. Next, we generated the stationary series $\{d\overline{m}_t\}_{t=1930}^{1988}$ from the equation:

(9-2) $$d\,log(\overline{m}_t) = log(m_t) - log(m_{t-1}) + v_t.$$

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[18] A copy of the Gauss code that we used to simulate our data is available at http://www.iue.it/Personal/Farmer/Pdf%20Files/DataAppendixfor%20MonTran.pdf .

[19] Y_t is GDP, L_t is full and part time equivalent employees and K_t was constructed from the U.S. investment data using a perpetual inventory method. Details can be found in Farmer and Ohanian (1998).

The notation $d\,log(\overline{m}_t)$ stands for the first difference of the log of real balances and v_t is the first difference of the log of the productivity shock. Equation (9-2) comes from differencing the identity,

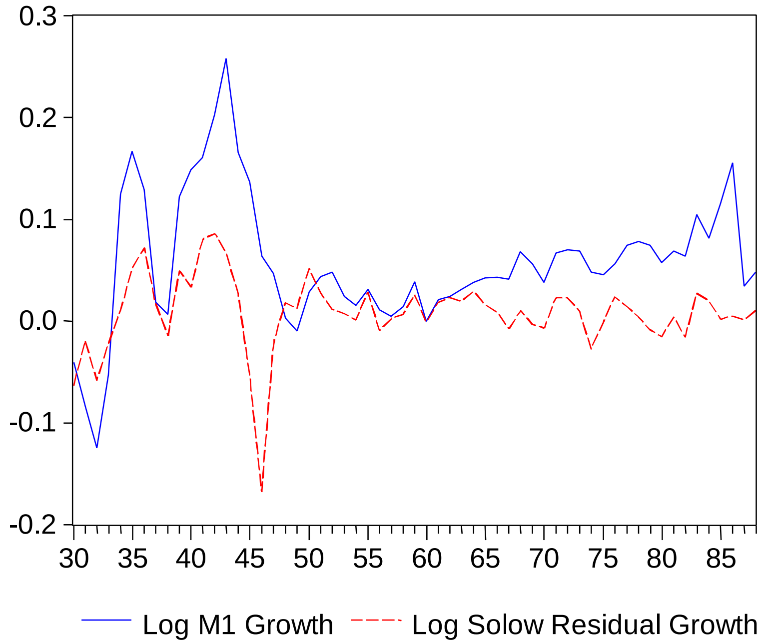

Figure 4: The Solow Residual and M1 Growth

$log\left(\frac{M_t}{P_t}\right) \equiv log(m_t) + log(S_t)$. For the series $\{v_t\}_{t=1930}^{1988}$ we fed in the actual values of the log difference of the Solow residual taken from the U.S. data. Then we constructed the series $\{d\,log(y_t)\}_{t=1930}^{1988}$ by taking the difference of equation (5-2);

(9-3) $$d\,log(y_t) = e_H\left[log(m_t) - log(m_{t-1})\right] + v_t.$$

Interest Rate Volatility

The interest rate in our model is found from linearizing equation (2-15).

$$E_t\left\{x_{t+1} m_{t+1} u_c\left(c_{t+1}, L_{t+1}\right)\left[1+i_t\right]\right\} =$$

$$E_t\left\{x_{t+1} m_{t+1} u_c\left(c_{t+1}, L_{t+1}\right)\left[1+f_m\left(L_{t+1}, m_{t+1}\right)\right]\right\},$$

(9-4) where $x_{t+1} = \dfrac{(1+g)^{1-r}\left(1+v_{t+1}\right)^{1-r}}{(1+r)(1+m)\left(1+u_{t+1}\right)}.$

$$d\,log\left(i_t\right) = (l-1)\left(1-e_H\right)\left[log\left(m_{t+1}\right) - log\left(m_t\right)\right] + w_t.$$

The first line of this expression is the asset market equilibrium equation. By including the interest rate inside the expectation operator on the left side of (9-4) we are implicitly assuming that bonds are not perfectly safe assets. Since the period of our model is a year, and since portfolios are rebalanced daily, this does not seem an unreasonable assumption. It has the advantage of allowing us to capture observed interest rate volatility. The final line of equation (9-4) is the linear equation we used to simulate the series $d\,log\left(i_t\right)$.

To capture the fact that the interest rate in real data is relatively volatile, we added the sequence of random variables $\left\{w_t\right\}_{t=1930}^{1988}$ to our simulated interest rate series. To generate $\left\{w_t\right\}$ we took a sequence of mean zero normal random variables with a variance of .065, a number chosen to replicate the observed standard deviation of interest rate fluctuations in the data.

Characteristics of the Simulated Data

Figures 5 and 6 graph the actual series for the log differences of GDP, real balances and the interest rate against simulated data for a single simulation and Table 4 compares the volatility of the data with the volatility of the simulated series.

Table 4: Standard Deviations	*GDP*	*Real Balances*	*Interest Rate*
Std. Dev. (Simulation)	0.068	0.080	0.23
Std. Dev. (Actual Data)	0.066	0.065	0.26

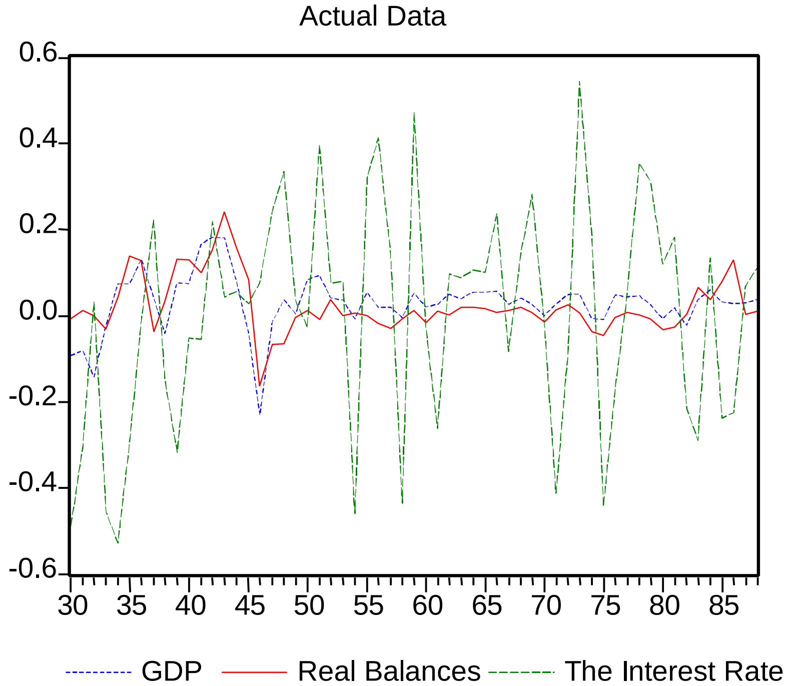

Figure 5: GDP, Real Balances and the Interest Rate in US Time Series from 1930 through 1988

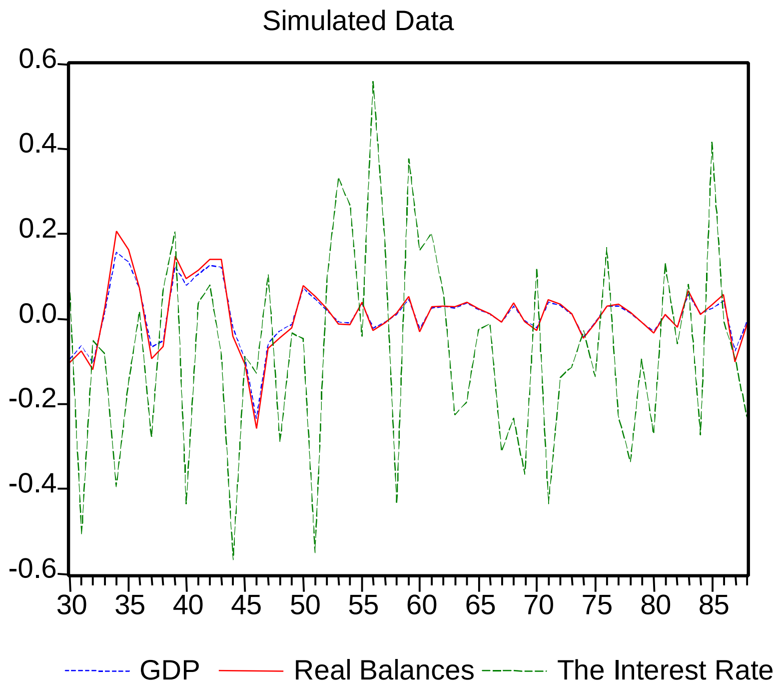

Figure 6: GDP, Real Balances and the Interest Rate in Simulated Data Using Actual Money growth and the Actual Solow Residual as Shocks

Table 5: Correlation Simulated of Actual Data	*GDP*	*Real Balances*	*Interest Rate*
GDP	1.000000	0.997161	-0.021490
Real Balances	0.997161	1.000000	-0.010626
Interest Rate	-0.021490	-0.010626	1.000000

Table 6: Correlation Matrix of Actual Data	*GDP*	*Real Balances*	*Interest Rate*
GDP	1.000000	0.656839	0.192373
Real Balances	0.656839	1.000000	-0.180435
Interest Rate	0.192373	-0.180435	1.000000

Tables 5 and 6 present the correlation matrix of the simulated and actual series. It is apparent from these tables that, in the simulations, real balances move a little too closely with GDP. The interest rate also has the wrong correlation with GDP. However, the broad features of actual and simulated series are similar.

To get a better feel for the dynamics of the model, compared with data, we estimated a three variable vector autoregression on actual and simulated data series. In each case we included two lags of the log difference of money growth , the log difference of GDP growth and the log difference of the interest rate. We used actual data on GDP and the interest rate in one case and data simulated from a single run of the model in the other. Since the model was driven by the actual log difference of nominal money growth, we used the actual money growth series in both cases.

In actual data we also looked at a four variable autoregression, including real balances in the system, with similar results. We could not run a four variable system on the simulated data as the four variable simulated system is singular: there are only three independednt shocks. To check that this did not affect the qualitative features of the results we experimented with two different three variable VAR's; one with GDP, the interest rate and nominal money and one with real balances, the interest rate and

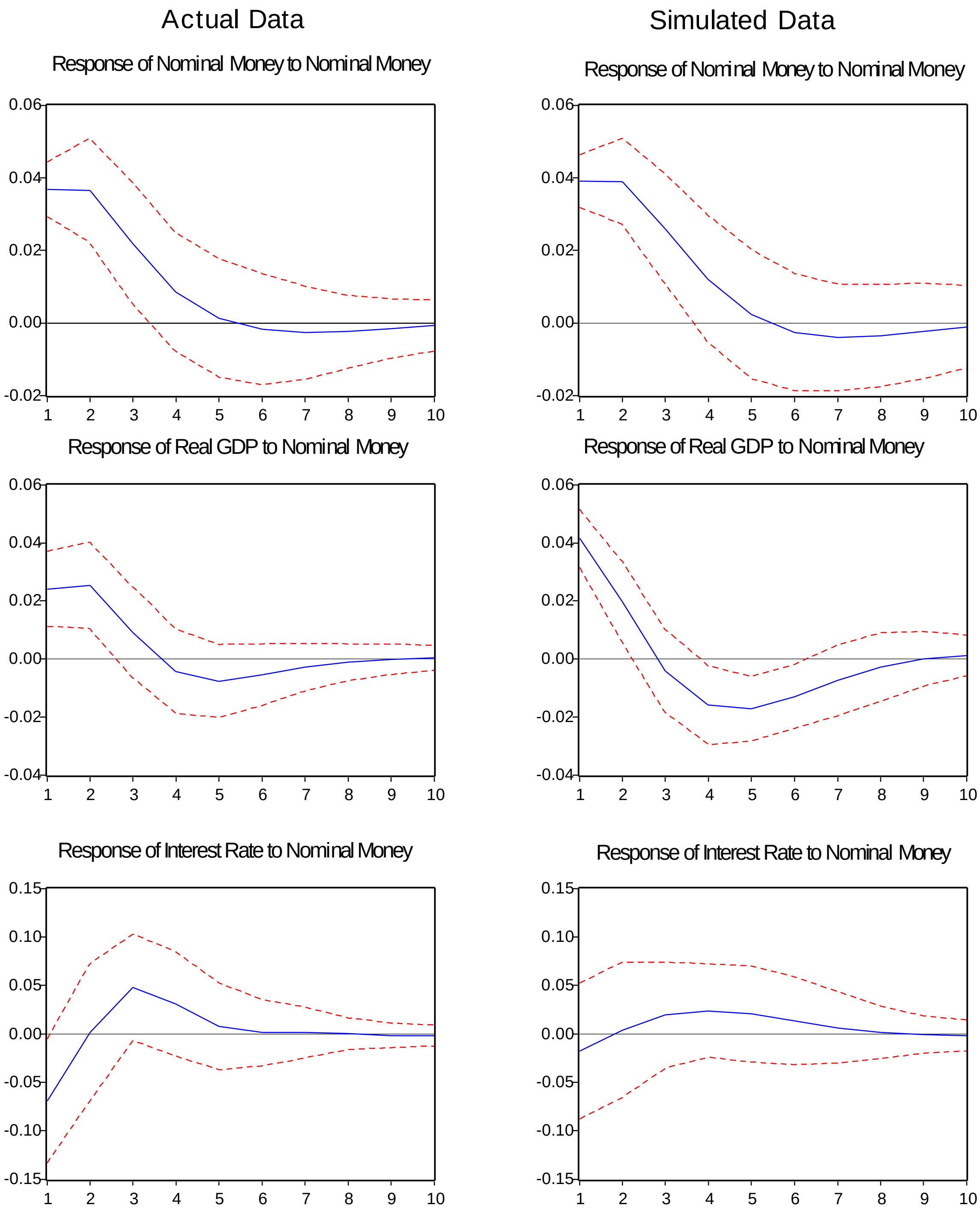

Figure 7: Impulse Responses to a Money Growth Shock

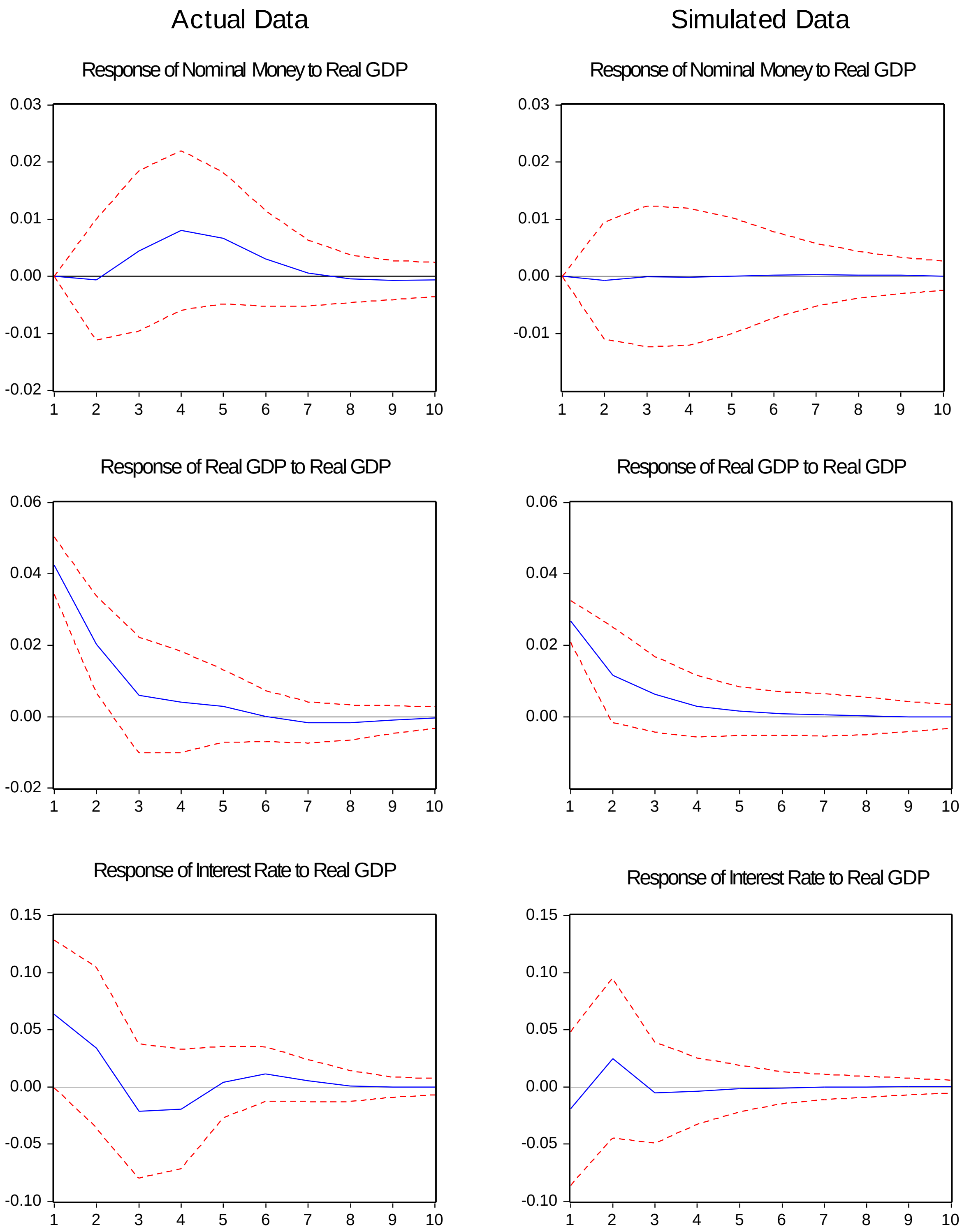

Figure 8: Impulse Responses to a real GDP Shock

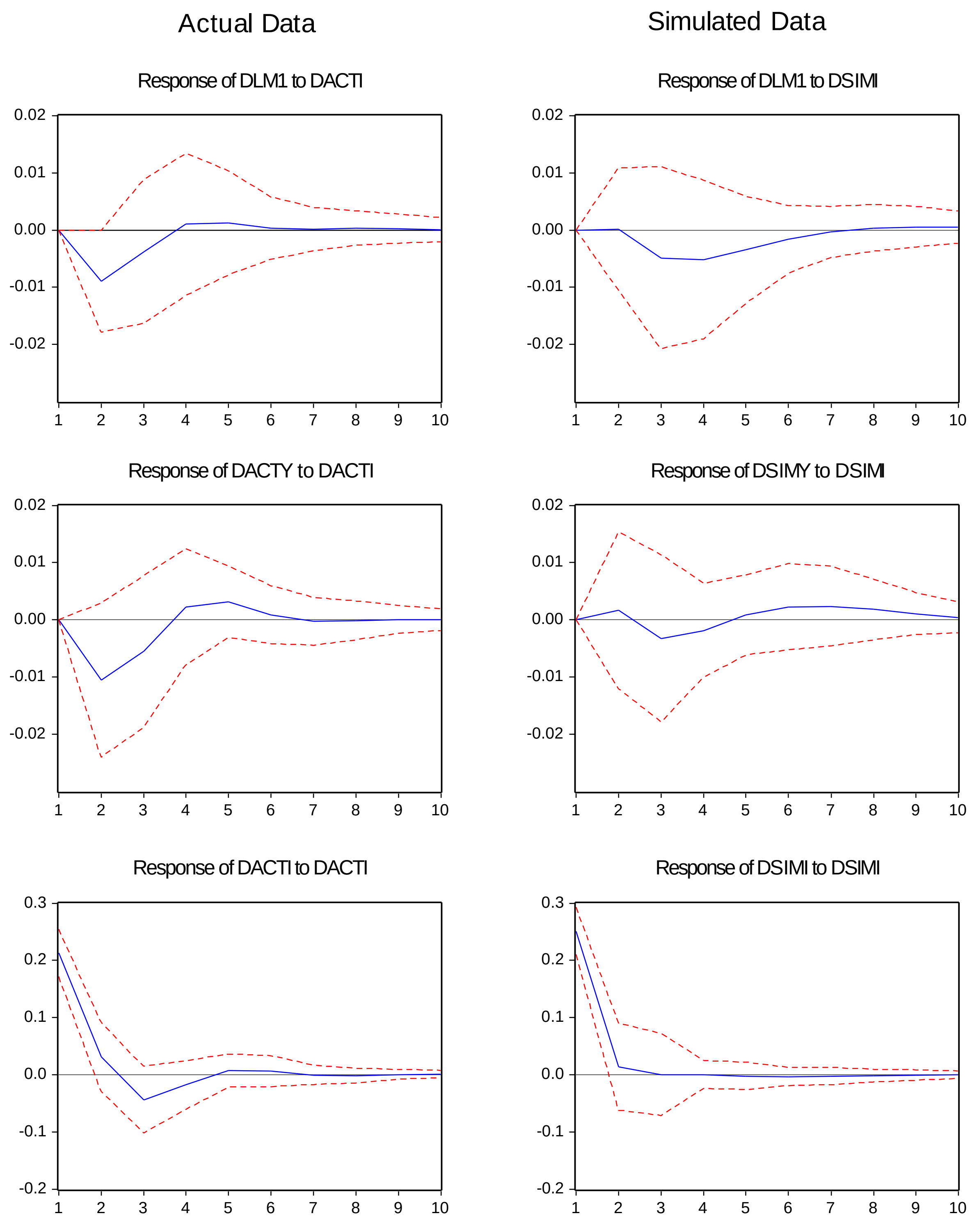

Figure 9: Impulse Responses to an Interest Rate Shock

nominal money. We also ran the four variable VAR on the actual data and compared the impulse responses with each of the three variables systems to make sure that the qualitative features of the systems did not change.

The main findings from the comparison of these two sets of figures is the broad similarity in the qualitative and quantitative nature of the responses of the model economy with that of the data. Notice in particular, the response of the real economy to a monetary shock depicted in the second panel of figure 7.

The Labor Market

It is perhaps worth drawing attention to one aspect of our model that is a common failing of equilibrium models in which demand shocks play an important role. Since output movements are generated by

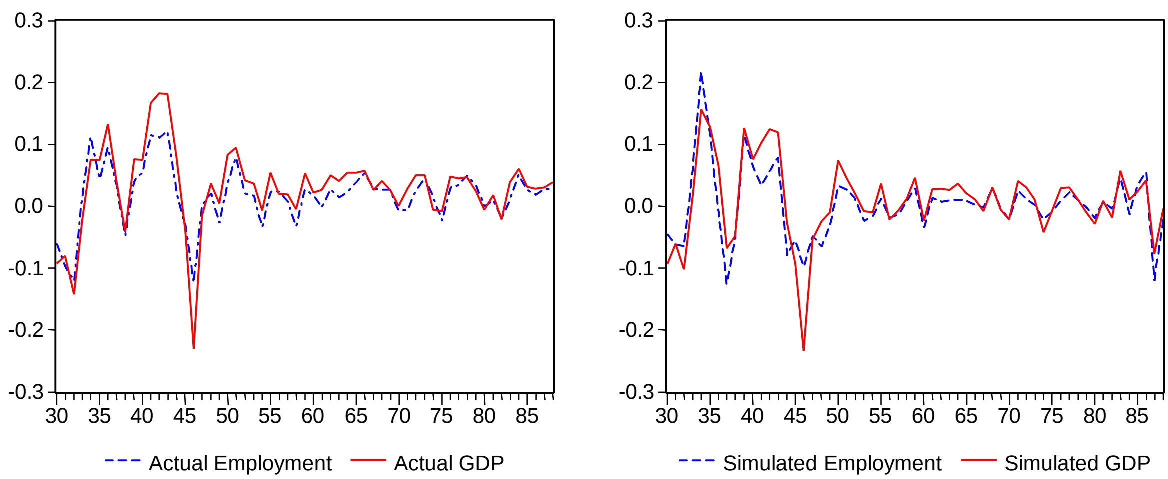

Figure 10: Employment and GDP in Actual and Simulated Data

movements along a concave neoclassical production function, productivity is predicted to be counter-cyclical. In Figure 10 we compare output and employment series from actual data with output and employment series from our simulations. Notice that in the data and in the model, GDP is more volatile than employment. Table 7 compares the standard deviations of these series, .048 for employment and .065 for GDP in the actual data: .055 for employment and .064 for GDP in simulated data. This feature of the data often presents a problem for models that are driven by demand side shocks because when most movements in GDP and employment are along a neo-classical production function output should be less volatile than employment. Our model does well in this dimension because, although demand side shocks can be important, a significant part of business fluctuations in the model are driven by productivity disturbances.

Table 7: Volatility Data	*Actual Employment*	*Actual GDP*	*Simulated Employment*	*Simulated GDP*
Std. Dev.	0.048231	0.065772	0.055441	0.064056

A second aspect of the labor market behavior of the model that is worth pointing out is its ability to capture the covariation of real wages with employment. It has been pointed out by a number of authors that the actual covariance of real wages and employment is low. In a one shock model, the covariance should be high. The resolution is to add demand side shocks as in the work by Christiano and Eichenbaum (1992:2).

Figure 11: Scatter Plots of the Real Wage against Employment in Actual and Simulated Data. (Variables are Log. Differences).

Since our model is driven by both demand and supply shocks it is perhaps unsurpising that we are able to replicate this feature of the actual data. Figure 11 presents scatter plots of the log difference of the real wage and employment in actual and simulated data.

10 Conclusions

The idea that general equilibrium models can generate indeterminate equilibria has been understood for some time although it is only recently that such models have been calibrated to fit existing data. Two criticisms are frequently leveled at economic models with indeterminacy. The first is that the degree of increasing returns required to generate indeterminacy is implausible. The second is that models with a multiplicity of equilibria cannot be used to make concrete predictions. In this paper we have addressed both of these criticisms by supplementing a monetary model with a model of how agents form beliefs. In our model, indeterminacy arises for parameter values that we argue are plausible, even when the technology satisfies constant returns-to-scale.

Perhaps the most unsatisfactory element of our explanation of the monetary transmission mechanism is our reliance on voluntary fluctuations in labor supply to explain employment variation. In this regard, we are following in the tradition of real business cycle models. We believe that the equilibrium approach to the labor market is the right one, although we would prefer a more sophisticated model, perhaps based on search theory, with a role for unemployment. Developments of this kind may add realism to the model, an important consideration if we wish our explanation to have an impact on the monetary policy debate. But it is unlikely to alter our main conclusions.

We have argued that models of multiple equilibria are not devoid of predictive content. In fact, each of the equilibria that might arise has a very different concrete prediction for the behavior of data.[20]

[20] For an elaboration this point see the paper by Farmer and Guo (1995) and the discussion by Aiyagari (1995).

Provided one imposes the discipline that agents form expectations in a stable way, the existence of indeterminacy should provide no more of a problem for econometricians than the assumption that utility functions are stable over time. In recent literature, a number of authors have exploited the idea that equilibria may be indeterminate to generate explanations of business cycles that are driven and propagated by "animal spirits".[21] In this paper we have argued that equilibrium models in which there may be an indeterminate set of equilibria may also be used to explain why monetary policy has real efffects.

[21] See the collection of papers on this issue in the *Journal of Economic Theory*, Vol. 63 no. 1, 1994.

11 Appendix A

To derive equations (2-13)–(2-16) we combine the production function with the first-order conditions for the household's decision problem and the market clearing equation. Equation (2-13) follows directly from the fact that the function $F(X, M/P)$ is homogenous of degree 1. To derive the first order conditions for the household's maximization problem we substitute the budget constraint, (2-6) into the objective function. Maximizing utility with respect to L_t leads to the first order condition:

$$(11\text{-}1)\quad U_C(C_t, L_t, S_t) S_t a L_t^{a-1} F_X\left(S_t L_t^{a}, \left(\frac{M_t}{P_t}\right)\right) = -U_L(C_t, L_t, S_t).$$

Since $F(X, M/P)$ is homogenous of degree 1, it follows that F_X (the derivative of F with respect to $X = SL^a$) is homogenous of degree zero in X and M/P. Since U is homogenous of degree 1-*r* in *C* and *S*, U_C and U_S are homogenous of degree –*r* and U_L is homogenous of degree 1-*r* in *C* and *S*. Using these results and the fact that $f_L(L, m) \equiv aL^{a-1} F_X(L^a, m)$ we can divide (11-1) by *S* to generate equation (2-14) in the text.

The first order conditions for money and bonds imply:

$$(11\text{-}2)\quad \frac{1}{P_t} U_C(C_t, L_t, S_t) = E_t\left\{\frac{1}{1+r}\frac{1}{P_{t+1}} U_C(C_{t+1}, L_{t+1}, S_{t+1})\left[1 + F_m\left(S_{t+1} L_{t+1}^{a}, \left(\frac{M_t + T_{t+1}}{P_{t+1}}\right)\right)\right]\right\},$$

$$(11\text{-}3)\quad \frac{1}{P_t} U_C(C_t, L_t, S_t) = E_t\left\{\frac{1}{1+r}\frac{1}{P_{t+1}} U_C(C_{t+1}, L_{t+1}, S_{t+1})[1 + i_t]\right\}.$$

Multiplying each equation by M_t and exploiting homogeneity gives:

(11-4)
$$\frac{M_t}{P_t} S_t^{-r} u_c(c_t, L_t) = E_t\left\{\frac{1}{1+r}\frac{M_{t+1}}{P_{t+1}}\frac{M_t}{M_{t+1}} S_{t+1}^{-r} u_c(c_{t+1}, L_{t+1})\left[1 + F_m\left(L_{t+1}^{a}, \left(\frac{M_t + T_{t+1}}{S_{t+1}P_{t+1}}\right)\right)\right]\right\},$$

(11-5)
$$\frac{M_t}{P_t} S_t^{-r} u_c(c_t, L_t) = E_t\left\{\frac{1}{1+r}\frac{M_{t+1}}{P_{t+1}}\frac{M_t}{M_{t+1}} S_{t+1}^{-r} u_c(c_{t+1}, L_{t+1})[1 + i_t]\right\},$$

which can be written as follows:

(11-6)
$$m_t u_c(c_t, L_t) = E_t\left\{\frac{1}{1+r} m_{t+1}\frac{M_t}{M_{t+1}}\left(\frac{S_{t+1}}{S_t}\right)^{1-r} u_c(c_{t+1}, L_{t+1})[1 + f_m(L_{t+1}, m_{t+1})]\right\},$$

(11-7)
$$m_t u_c(c_t, L_t) = E_t\left\{\frac{1}{1+r} m_{t+1}\frac{M_t}{M_{t+1}}\left(\frac{S_{t+1}}{S_t}\right)^{1-r} u_c(c_{t+1}, L_{t+1})[1 + i_t]\right\}.$$

Equation (2-16) follows from equation (11-7) using also equations (2-2) and (2-10), (the assumptions that the real productivity shock and the money supply process are geometric random walks with drift). Equation (2-15) follows from equating the right-hand-sides of equations (11-6) and (11-7).

12 Appendix B

In this appendix we establish the existence of a solution to equations (2-13)–(2-16), evaluated, along a balanced growth path for specific functional forms. We seek values $\{y^*, c^*, L^*, m^*, i^*\}$ that solve the steady state equations:

(12-1) $c^* = y^* = f(L^*, m^*) \qquad \Rightarrow \quad y^* = \left((1-a)L^{*al} + am^{*l}\right)^{\frac{1}{l}},$

$$\text{(12-2)}\quad \frac{-u_L(y^*,L^*)}{u_c(y^*,L^*)} = f_L(L^*,m^*), \Rightarrow \quad \frac{\left(y^{*1-r}+A\right)}{(r-1)y^{*-r}\,(L^*-B)} = (1-a)aL^{*a\,l-1}\,y^{*1-l} \quad ,$$

$$\text{(12-3)}\quad i^* = f_m(L^*,m^*) \qquad \Rightarrow \qquad i^* = a\left(\frac{y^*}{m^*}\right)^{1-l},$$

$$\text{(12-4)}\quad \frac{(1+r)(1+m)}{(1+g)^{1-r}} = \left[1+f_m(L^*,m^*)\right] \quad \Rightarrow \quad \frac{(1+r)(1+m)}{(1+g)^{1-r}} = 1+a\left(\frac{y^*}{m^*}\right)^{1-l}.$$

For our choice of functional form, for some choices of monetary policy there may be no balanced growth path. For other policies and parameter values there may be multiple balanced growth paths. We are interested in choosing the parameters of our model that are consistent with observed moments of the data. To this end, rather than start with a utility function and derive the balanced growth path, we begin with a given balanced growth path and show that there is a utility function that generates it.

Let the balanced growth path of the model be given by $\{i^*, L^*, m^*, y^*, w^*\}$ where w^* represents the ratio of the real wage to the technology parameter S evaluated along a balanced growth path. Let a, g, l and m represent fundamental parameters of preferences, technology and monetary policy. Define the parameters s_L and s_C by the equations:

$$\text{(12-5)}\quad 0 < s_C = \frac{y^{*1-r}}{y^{*1-r}+A} < 1,$$

$$\text{(12-6)}\quad s_L = \frac{L^*}{L^*-B} > 1\ .$$

We now establish that for a given growth path $\{i^*, L^*, m^*, y^*, w^*\}$, there exist real numbers $r > 1$, $r > 0$, $a > 0$, $A > 0$ and $0 < B < L^*$ such that $\{i^*, L^*, m^*, y^*, w^*\}$ is a balanced growth path of the model generated by a representative household with preferences $U=\frac{C^{1-r}+AS^{1-r}}{1-r}\left(L-B\right)$ and rate of time preference r,

using the technology $Y=\left((1-a)S^{l}L^{a\,l}+a\left(\frac{M}{P}\right)^{l}\right)^{\frac{1}{l}}$.

From equations (12-3) and (12-4) it follows that the interest rate along the balanced growth path is given by:

(12-7) $$i^{*}=\frac{(1+r)(1+m)}{(1+g)^{1-r}}-1.$$

Since i^* is observable, for given g, m and r, equation (12-7) determines r. From (12-3) it follows that

(12-8) $$a=i^{*}\left(\frac{m^{*}}{y^{*}}\right)^{1-l}.$$

We now derive a parameter restriction that follows from the first order conditions of the model. From equation (12-2):

(12-9) $$-\frac{u_L}{u_c}\frac{L^*}{y^*}=\frac{y^{*1-r}+A}{(r-1)y^{*-r}}\frac{1}{L^*-B}\frac{L^*}{y^*}=\frac{L^*}{y^*}f_L=\frac{w^*L^*}{y^*}.$$

where we use w^* to mean the real wage. The first equality follows from exploiting the functional form of utility and the last equality follows from matching the ratio of the wage bill to GDP predicted by the model to US data. Since $y^*=c^*$ in our model, and since the wage bill is approximately equal to consumption, we set $\frac{w^*L^*}{y^*}$ equal to one.[22] Using the definition of s_C and s_L it follows that the parameters r, s_L and s_c must be related by the restriction:

[22] In calibrations we also experimented by setting this ratio to 0.8 with little difference to our simulations.

(12-10) $$r \quad = \quad 1+\frac{s_L}{s_C}.$$

Equation (12-10) is imposed in our calibration.

Equations (12-1)–(12-4) define a balanced growth. We have established that there exist values of r, r and a such that these equations are satisfied for any observed path $\{i^*, L^*, m^*, y^*, w^*\}$. The existence of parameters A and B for any y^* and L^* follows from the definitions of s_C and s_L.

13 Appendix C

We begin by log linearizing the production function. The parameters a_1 and a_2 are the elasticities of output with respect to labor and real balances evaluated along a balanced growth path:

$$d\,log(y_t) = a_1 d\,log(L_t) \quad + \quad a_2 d\,log(m_t),$$

(13-1) $$\text{where} \quad a_1 \equiv \frac{(1-a)L^{*al}}{(1-a)L^{*al} + m^{*l}}, \qquad . \qquad \textbf{Production function}$$

$$\text{and} \quad a_2 \equiv \frac{am^{*l}}{(1-a)L^{*al} + m^{*l}}$$

The labor market equation takes the form:

(13-2) $$\frac{-u_L(c_t, L_t)}{u_c(c_t, L_t)} \quad = \quad f_L(L_t, m_t),$$

which, in the case of our model has the specific representation:

(13-3) $$\frac{\left(y_t^{1-r} + A\right)}{y_t^{-r}(r-1)(L_t - B)} \quad = \quad (1-a)\frac{L_t^{al-1}}{y_t^{l-1}}.$$

Log-linearizing (13-3) leads to the expression:

$$\left[r + (1-r)s_c\right] d\,log(y_t) \;-\; s_L d\,log(L_t) \quad = \quad (a\;l - 1) d\,log(L_t) \;-\; (\,l - 1) d\,log(y_t),$$

where $$s_c \;\equiv\; \frac{y^{*1-r}}{y^{*1-r} \;+\; A}, \;\; s_L \;\equiv\; \frac{L^*}{L^* - \;B}.$$

The linear form of this equation for our choice of utility and production functions can be expressed more compactly as:

(13-4) $$d\,log(y_t) \;=\; bd\,log(L_t) \qquad b \;\equiv\; \frac{a\;l \;-\; (1 - s_L)}{l \;+\; (r-1)(1-s_c)}.$$ **Labor market equation**

Putting together equations (13-1) and (13-4) one can solve for $d\,log(y)$ and $d\,log(L)$ as functions of $d\,log(m)$:

(13-5) $$d\,log(L_t) \;=\; e_h\; d\,log(m_t) \qquad e_h \;\equiv\; \frac{a_2}{b - a_1}\;.$$

(13-6) $$d\,log(y_t) \;=\; e_H\; d\,log(m_t), \qquad e_H \;\equiv\; \frac{ba_2}{b - a_1}.$$

These are linearized versions of the functions $h(m)$ and $H(m)$ that we described in the text.

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