



# Fine cartels

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## Abstract

This paper studies a simple model of a repeated cartel that can punish using both voluntary fines and inefficient price wars. The idea is to use the fines in response to noisy signals of bad behavior and back it up with threats of price wars in response to the easily observed failure to pay the voluntary fines. The model is shown to deliver the insights of modern repeated game theory in an empirically accurate and tractable form.

**Keywords** Cartels · Monopoly · Repeated games · Fines · Price wars · Game theory · Industrial organization

**JEL Classification** A1 · D7 · D9

## 1 Introduction

This paper studies a simple model of a repeated cartel that can punish using both voluntary fines and inefficient price wars. The idea is to use the fines in response to noisy signals of bad behavior and back it up with threats of price wars in response to the easily observed failure to pay the voluntary fines. This model enables the analysis of less than full collusion due to informational imperfection, and makes possible comparative statics as in Levine et al. (2020).

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In the empirical literature on cartels the strongly symmetric equilibrium model of Green and Porter (1984), Rotemberg and Saloner (1986) or Abreu, Pearce and Stacchetti (1990) is often referenced as a theoretical explanation of cartel behavior despite the fact that there is strong evidence that cartels do not behave this way.<sup>1</sup> A fundamental insight from Fudenberg et al. (1994) work on repeated games with imperfect information is that collective punishments such as price wars are inefficient in comparison to punishments that involve transfer payments from guilty to innocent: transfer payments provide incentives without diminishing overall cartel profits. The underlying repeated game equilibria are complex and in some ways do not reflect how cartels actually operate. This paper shows that a simple model of a repeated cartel that can use both relatively efficient but voluntary fines and inefficient price wars delivers the insights of modern repeated game theory in an empirically accurate and tractable form. The insight is this: it is difficult to monitor individual firm output and/or price. Hence transfer payments should be used to provide incentives. As these transfer payments are voluntary there must be a punishment also for failing to pay a fine. This, however, can be done with a collective punishment such as a price war. The point is that while output and/or price are difficult to monitor whether or not firms pay their fines is easy to monitor. Hence there is little cost of using a collective punishment to enforce fines. The use of voluntary transfer payments are effective in the presence of noise and they convert a noisy signal of behavior into a sharp signal of adherence to the rules. From a theoretical point of view cartel equilibrium can be explicitly computed for every discount factor and are shown to be the solution of a mechanism design problem in which fines are chosen optimally subject to a simple constraint on size that depends upon the discount factor.

Before proceeding with the model, we examine the underlying facts. First: there is strong evidence that cartels do not use price wars to punish cheating. For example, much of the classical study of sugar cartels by Genesove and Mullin (2001) is devoted to debunking the idea that the behavior of the sugar cartel is consistent with strongly symmetric equilibrium and price wars as an enforcement device. Another example is cited in Levenstein and Suslow (2006) survey of the empirical literature: “after the adoption of an international price-fixing agreement in the bromine industry, the response to violations in the agreement was a negotiated punishment, usually a side-payment between firms, rather than the instigation of a price war... As repeatedly discovered by these cartel members, the threat of Cournot reversion is an inefficient way to sustain collusion.” Indeed one of the main conclusions of the survey is precisely the point that cartels do not generally use price wars or collective punishment to deter cheating. Harrington and Skrzypacz (2011) have similar evidence in the Lysine industry.

Second, the repeated game literature after the study of strongly symmetric equilibrium has moved on to show that from a theoretical point of view “the threat of Cournot reversion is an inefficient way to sustain collusion.” These models are difficult to solve, however, and there are only a scattering of special cases solved in the

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<sup>1</sup> See for example (Hyytinen, Steen and Toivanen 2018).

literature: Roberts (1985) provides little in the way of comparative static results, Athey and Bagwell (2001) study the possibility of attaining the first best when the discount factor is sufficiently high. These models and those of Cramton and Palfrey (1990) and Kihlstrom and Vives (1992) focus on the problem of private cost shocks. Here we focus on the more empirically relevant problem of observing output. More recently (Sannikov 2008) has shown how to compute equilibria in continuous time models for the principal agent model - but this computation is difficult with more than two players and has not yet been adapted to the cartel problem. Never-the-less fundamental to achieving good equilibria in these models is the idea that punishments should take the form of transfers as long as it is feasible to do so and that collective punishments such as a price war should be used only as a last resort.

The model here takes a simplified yet empirically relevant approach based on the single-period model of Levine et al. (2020): we allow contemporaneous transfer payments such as fines. We show that in this setting collective punishment should be used only to ensure that fines are paid and never to punish signals of cheating. This idea is not a new one: it has been used in the relational contracting literature, for example, by Levin (2003) and a variety of oligopoly models use the same idea either to obtain near efficient equilibria or to give recursive characterizations of the set of equilibrium payoff. Papers that examine full or nearly full collusion include (Chan and Zhang 2009) and Harrington and Skrzypacz (2011). Papers that give recursive calculations of the equilibrium payoff set include (Goldlucke and Kranz 2012; Miller and Watson 2013), and Panov (2019). In contrast to these earlier results the model here enables simple and direct comparative static analysis when informational imperfections lead to less than fully efficient outcomes.

The results here are different than the anti-folk theorem literature such as Postlewaite and Roberts (1977), Radner (1980), and Fudenberg et al. (1998) where collective punishments fail because of large numbers. The results here are valid for any finite number of firms and any discount factor. The best cartel equilibrium itself is the solution to a simple mechanism design problem, and we give here a closed form solution for the special case in which demand and marginal cost are linear. By retaining the simplicity of strongly symmetric equilibrium while allowing contemporaneous transfer payments we are able to completely characterize cartel behavior as a function of the number of firms, market demand, cost, the discount factor, the difficulty of observing output, and the efficiency of transfer payments. This may be a useful foundation for future empirical studies. In addition anti-trust authorities use screening rules for industry or firm conduct to allocate scarce monitoring resources (see for example (Doan et al 2015)). In this context a clearer theoretical understanding of how collusion works in practice may lead to improved screening tools.

## 2 The model

We study a dynamic Cournot industry with  $N$  identical firms with common discount factor  $0 \leq \delta < 1$ . As is standard in the repeated game literature we use average present value throughout. In period  $t = 1, 2, \dots$  firm  $i$  produces output  $x_t^i \geq 0$ . Denote average firm output by  $x_t$ . The profit of firm  $i$  in period  $t$  is given by

$\bar{u}^i(x_t, x_t^i) = \bar{p}^i(x_t)x_t^i - c(x_t^i)$  where the individual firm price  $\bar{p}^i(x_t)$  is a random variable with support  $(0, \infty)$  and a firm independent mean  $E\bar{p}^i(x_t) = p(x_t)$ . With this formulation expected profits of a firm are

$$u(x_t, x_t^i) = p(x_t)x_t^i - c(x_t^i)$$

We assume that firm specific prices are not perfectly correlated so that  $\bar{p}^i(x_t)$  is private information and  $p(x_t)$  is not common knowledge.<sup>2</sup> We make the standard assumption that the expected price  $p(x_t)$  is smooth for  $p(x_t) > 0$ ,  $p'(x_t) < 0$ ,  $\lim_{x_t \rightarrow \infty} p(x_t) = 0$ , and  $c(x_t^i)$  is smooth with  $c'(x_t^i) > 0$ ,  $c''(x_t^i) \geq 0$ . We need also that there is a (symmetric pure strategy) Cournot equilibrium, and denote by  $x^n$  the worst such equilibrium and  $\underline{x}^n$  the equilibrium with the least level of output. For example, a standard argument implies that if we assume that for  $0 \leq ((N-1)/N)x_t + x_t^i/N \leq x^c$  we have  $p''(x_t)x_t^i + 2p'(x_t) - c''(x_t^i) < 0$  a Cournot equilibrium exists, and indeed, that there is a unique (symmetric pure strategy) Cournot equilibrium<sup>3</sup> in which case  $x^n = \underline{x}^n$ . However we require only the existence of a symmetric pure strategy Cournot equilibrium for our results.

The industry is governed by a cartel that sets a common quota  $\underline{x}^n \geq y_t \geq 0$  for all of the identical firms at the beginning of each period. After this production takes place. For each firm an independent public binary signal is observed of whether  $x_t^i \leq y_t$ , that is whether the quota was adhered to or not. The signal is either “good, adhered to the quota” or “bad, violated the quota.” If the quota is adhered to, that is,  $x_t^i \leq y_t$ , then a bad signal is generated with probability  $\pi > 0$ . If the quota is violated, that is  $x_t^i > y_t$ , the bad signal is generated with probability  $\pi_B > \pi > 0$ .<sup>4</sup> We denote by  $Z$  denote the total number of bad signals in the industry. Finally, after signals are commonly observed, firms may optionally choose to pay fines  $\phi_t^i$  where a fraction  $1 - \psi$  of the proceeds are distributed among the remaining firms and  $\psi$  of the fine is lost due to transaction costs with  $0 \leq \psi < 1$ .<sup>5</sup>

By restricting attention to perfect public strongly symmetric equilibrium we may use the results of Abreu, Pearce and Stacchetti (1990) to give a simple characterization of the best agreement achievable by the cartel. An *agreement* consists of a

<sup>2</sup> The function  $p(x_t)$  is common knowledge, but the realized value is not.

<sup>3</sup> The equilibrium condition is that  $p'(x_t)x_t + p(x_t) - c'(x_t)$  be zero and the given condition assures that the LHS is downward sloping so that there is a unique zero.

<sup>4</sup> The bad signal should be thought of as evidence of a quota violation. Provided that this evidence is verifiable it need not be public in the sense of being commonly observed by all firms in the cartel. Even if the evidence is private to one or a few firms they can credibly communicate the signal to the rest of the cartel making it *de facto* public. Generally speaking the literature on communications in repeated games with private signals (see, for example, Fudenberg and Levine 2007) is complicated by the need to provide incentives for information revelation. That is not an issue here as firms wish to see their rivals fined so that they can receive their share of the proceeds. Firms may also have private non-verifiable evidence of quota violations by rivals: following the literature on perfect public equilibrium (see, for example, Fudenberg, Levine and Maskin 1994) we assume that the cartel does not try to use this additional private information. Similarly firms do not use their individual private information about price.

<sup>5</sup> So for, for example, if there are two firms and both are required to pay a fine  $\phi$  then each pays the fine to the other, paying  $\phi$  and receiving  $(1 - \psi)\phi$  for a net per firm loss of  $\psi\phi$ .

quota  $y$ , the rule that firms produce to quota<sup>6</sup>  $x_t^i = y$ , a system of required fines  $\phi(Z)$  paid by firms with bad signals, termination (of the cartel) probabilities  $Q(Z)$  and the rule that if any firm fails to pay a required fine termination (of the cartel) takes place with probability one.<sup>7</sup> A *strongly symmetric profile* is an agreement along with the rule that if termination (of the cartel) takes place each firm will produce at the worst Cournot level  $x^n$  forever and otherwise the agreement will continue for another period. Our notion of equilibrium is strongly symmetric subgame perfect equilibrium: we say that an agreement is *incentive compatible* if in the strongly symmetric profile every firm is willing to pay the fine and no firm wishes to deviate from the quota. Our goal is to characterize the *best agreement*: the incentive compatible agreement that yields the highest per firm profit.

Note that the assumption that firms face idiosyncratic prices together with perfect public equilibrium rules out the use of firm’s own price information in cartel enforcement. While this a useful theoretical simplification it is also empirically relevant. Levenstein and Suslow (2006) survey of the empirical literature discusses “Monitoring Output and Prices” but in fact only discusses the monitoring of output. In their discussion of “Cartel Breakdown” there is no indication that fluctuations in individual firm prices or market prices play a role in enforcement: in general it appears that the information used by cartels lies in evidence of adherence to cartel rules as in this model, not on data about market prices.

### 3 The theorem

Denote the worst Cournot utility by  $u^n = u(x^n, x^n)$ . Define the greatest utility from deviating from a quota as  $u^B(y) = \max_{x^i} u((N - 1)/N)y + x^i/N, x^i)$ . A crucial aspect of the model is that the transactional loss from a fine  $\psi < 1$  so at least some small part of the fine is received by cartel members. In other words, the social cost of using a fine as punishment is strictly less than the size of the punishment. By contrast, if the cartel is terminated, the social cost of the punishment is at least equal to the size of the punishment. This suggests that it is always better to use fines, and our first result shows that this is true: termination of the cartel should be used only to enforce the payment of fines.

**Theorem 1** *In any best agreement  $Q(Z) = 0$ .*

**Proof** Let  $\bar{u}$  be the utility from the best agreement. Then  $\bar{u} \geq u^n$  and if  $\bar{u} = u^n$  then there is no loss of generality in taking  $Q(Z) \equiv 0$ . Hence to prove  $Q(Z) = 0$  we may assume  $\bar{u} > u^n$ .

Define the collective punishment as  $q(Z) = (\delta/(1 - \delta))Q(Z)[\bar{u} - u^n]$ . Suppose that  $N - 1$  firms adhere to the quota and let  $\Pi(Z)$  be the probability that they generate

<sup>6</sup> Since we assume the quota is no greater than the least Cournot equilibrium output  $y \leq x^n$  no firm will produce less than the quota.

<sup>7</sup> Because the payment of fines is perfectly observed this is without loss of generality.

exactly  $Z$  bad signals. Note that since the quota is no greater than the least Cournot equilibrium output<sup>8</sup>  $y \leq \underline{x}^n$  no firm will wish to produce less than the quota. Hence the only incentive constraint is that firms weakly prefer producing to quota to deviating to a higher output and receiving  $u^B(y)$  with a higher probability of punishment:

$$\begin{aligned} u(y, y) - \sum_{Z=0}^{N-1} \Pi(Z) [(1 - \pi)q(Z) + \pi(\phi(Z + 1) + q(Z + 1))] \\ \geq u^B(y) - \sum_{Z=0}^{N-1} \Pi(Z) [(1 - \pi_B)q(Z) + \pi_B(\phi(Z + 1) + q(Z + 1))] \end{aligned}$$

which may be written as

$$u^B(y) - u(y, y) \leq (\pi_B - \pi) \sum_{Z=0}^{N-1} \Pi(Z) [\phi(Z + 1) + (q(Z + 1) - q(Z))]. \quad (3.1)$$

The incentive constraint for paying fines is

$$\phi(Z) \leq (\delta/(1 - \delta))(1 - Q(Z))[\bar{u} - u^n] = (\delta/(1 - \delta))[\bar{u} - u^n] - q(z). \quad (3.2)$$

Per firm profits when all firms adhere to the quota are

$$u(y, y) - \sum_{Z=0}^{N-1} \Pi(Z) [(1 - \pi)q(Z) + \pi(\psi\phi(Z + 1) + q(Z + 1))]. \quad (3.3)$$

Start with an incentive compatible plan in which  $Q(Z_0) > 0$  and consider increasing  $\phi(Z_0)$  by  $r$  and decreasing  $q(Z_0)$  by  $r$ . The RHS of 3.1

$$\begin{aligned} (\pi_B - \pi) \sum_{Z=0}^{N-1} \Pi(Z) [\phi(Z + 1) + (q(Z + 1) - q(Z))] \\ + \mathbf{1}(Z_0 \leq N - 1)(\pi_B - \pi)\Pi(Z_0)r, \end{aligned}$$

where  $\mathbf{1}$  is the indicator function, so the incentive constraint for adhering to the quota is satisfied. It is similarly clear that 3.2 remains satisfied.

Per firm profits are

$$\begin{aligned} u(y, y) - \sum_{Z=0}^{N-1} \Pi(Z) [(1 - \pi)q(Z) + \pi(\psi\phi(Z + 1) + q(Z + 1))] \\ + \mathbf{1}(Z_0 \geq 1)\Pi(Z_0 - 1)\pi(1 - \psi)r + \mathbf{1}(Z_0 \leq N - 1)\Pi(Z_0)(1 - \pi)r \end{aligned}$$

which is strictly increasing in  $r$ . We conclude that  $Q(Z) \equiv 0$ .  $\square$

<sup>8</sup> Note that we did not assume that there is a unique Cournot equilibrium.

Define  $\hat{\phi}(y) \equiv [1/(\pi^B - \pi)](u^B(y) - u(y, y))$ . Because termination of the cartel is used only to enforce the payment of fines the punishment from termination imposes a simple constraint on the size of the fines used to enforce quotas. This enables us to reduce the dynamic problem of finding the best agreement to the simple static mechanism design problem of maximizing one-period utility net of a cost of enforcing the agreement. This is our second result.

**Theorem 2** *In any best agreement the quota  $y^a$  is a solution of the static mechanism design problem  $\max_y u(y, y) - \psi \pi \hat{\phi}(y)$  subject to  $\hat{\phi}(y) \leq (\delta/(1 - \delta))[u(y, y) - \psi \pi \hat{\phi}(y) - u(x^n, x^n)]$  and any such solution is part of a best agreement in which  $\phi(Z) = \hat{\phi}(y)$ .*

**Proof** We continue to let  $\bar{u}$  be the utility from the best agreement. Define  $\Phi = \sum_{Z=0}^{N-1} \Pi(Z)\phi(Z + 1)$ . Since  $\Pi(Z)$  is the probability that  $N - 1$  firms generate exactly  $Z$  bad signals, we have  $\sum_{Z=0}^{N-1} \Pi(Z) = 1$ . Since  $Q(Z) = q(z) = 0$  the objective function is to maximize  $u(y, y) - \psi \pi \Phi$ , the incentive constraint for adhering to the quota is  $u^B(y) - u(y, y) \leq (\pi_B - \pi)\Phi$ , and the incentive constraint for paying fines is  $\phi(Z) \leq (\delta/(1 - \delta))[\bar{u} - u^n]$ . The fine paying constraint may also be written as  $\max_Z \phi(Z) \leq (\delta/(1 - \delta))[\bar{u} - u^n]$  from which it is clear that  $\Phi \leq (\delta/(1 - \delta))[\bar{u} - u^n]$ , while conversely if that is the case then  $\phi(Z) = \Phi$  satisfies the constraint. Hence the constraint  $\Phi \leq (\delta/(1 - \delta))[\bar{u} - u^n]$  suffices. Since the objective function is decreasing in  $\Phi$  the quota adherence constraint must hold with exact equality  $u^B(y) - u(y, y) = (\pi_B - \pi)\Phi$ , which is to say  $\Phi = \hat{\phi}(y)$ . Plugging in  $\bar{u} = u(y, y) - \psi \pi \hat{\phi}(y)$  then gives the result.  $\square$

This mechanism design problem has an important feature. If the discount factor is large enough the constraint on the size of the fine does not bind, and for discount factors greater than this critical level we need only solve the unconstrained mechanism design problem. This problem is independent of the discount factor.

**Corollary 1** *If we denote by  $\bar{\phi}$  the minimum of  $\hat{\phi}(y^a)$  over unconstrained solutions to the problem  $\max_y u(y, y) - \psi \pi \hat{\phi}(y)$  and  $\bar{U}$  the utility then for  $\bar{\delta} \equiv \bar{\phi}/(\bar{\phi} + \bar{U} - u^n)$  we have  $\bar{\delta} < 1$  and for  $\delta \geq \bar{\delta}$  utility from the optimal agreement is  $\bar{U}$ , that is, the constraint does not bind.*

**Proof** The only thing to be proven here is  $\bar{U} > u^n$ . If we set  $\theta = \psi \pi / (\pi^B - \pi)$  we can write the objective as  $u(y, y) - \theta(u^B(y) - u(y, y))$ . Consider the derivative with respect to  $y$  at  $y = x^n$ . From the envelope theorem the derivative of the second part  $(u^B(y) - u(y, y))$  is zero, so that the derivative is just that of  $u(y, y)$ , that is, the monopoly profit. But under our standard regularity conditions the derivative of monopoly profit with respect to output is strictly negative at the Cournot equilibrium so we are done.  $\square$

## 4 The square root rule

Given a particular profit function  $u(x_t, x_t^j)$  the static mechanism design problem can be readily solved. Here we give the solution in the quadratic case. We normalize slope of demand to 1 and take marginal cost to be constant and equal to zero so the competitive equilibrium is  $x_t = x_t^j = 1$ . Profits are then given by

$$u(x_t, x_t^j) = (1 - x_t)x_t^j.$$

It will be convenient to work with the *patience*  $\Delta = \delta/(1 - \delta)$  rather than the discount factor. Define also the constants  $\lambda = 1/(\pi_B - \pi)$ ,

$$A = \frac{2N}{N+1} - 1$$

$$B = \left[ \frac{4N}{\lambda(N+1)^2} + \psi\pi \right]$$

and

$$D = \frac{2\sqrt{N}}{\sqrt{\lambda(N+1)}}.$$

Note that the (unique) Cournot equilibrium is

$$x^n = \frac{N}{N+1}.$$

**Theorem 3** *The critical cutoff is  $\bar{\Delta} = 1/B$ . For  $\Delta \leq \bar{\Delta}$*

$$\sqrt{\bar{\phi}} = \frac{\Delta A}{1 + \Delta B}$$

*with the corresponding optimal quota equal to*

$$y = x^n - D \frac{\Delta A}{1 + \Delta B}.$$

*and utility*

$$\bar{u}(\phi) - u^n = A^2 \frac{\Delta}{(1 + \Delta B)^2}.$$

*For  $\Delta \geq \bar{\Delta}$  we have  $\phi = \bar{\phi} = (A/(2B))^2$  with the corresponding optimal quota equal to*

$$y = x^n - \frac{1}{2B} DA$$

*and utility gain  $\bar{U} - u^n = A^2/(4B)$ .*



The global picture here is that indicated by Corollary 1: finer go up, utility goes up, and quotas go down as  $\delta$  or  $\Delta$  increase up to a point  $\bar{\delta}, \bar{\Delta}$  at which the constraint no longer binds. What is new and interesting is what happens for low discount factors. Here there is a square-root rule: the square root of the fine is locally linear in  $\Delta$  with derivative equal to  $A$ . By contrast the quota declines approximately linearly with  $\Delta$  with derivative equal to  $-DA$  and utility increases with derivative  $A^2$ . This result is general because locally near the competitive equilibrium supply and demand are approximately linear. What this means is that when the discount factor is very low quotas decrease and utility increases much faster than the fine. As the markup is locally a linear function of the quota, it also means that markups also increase approximately linearly, which is to say much faster than the fines.

In short, when collective punishment is weak due to a low discount factor there can be a significant markup with a very low fine. This can be important for anti-trust authorities in an investigation triggered by a high markup. A low “fine” might not be a monetary penalty at all. As an example an industry might have an annual awards banquet with a “CEO of the year award.” If that award was given or denied on the basis of signals of adhering to a quota this might be enough of a “fine” to induce substantial markups. Hence, rather than looking for evidence of fines, investigators might also look for evidence that the award was based on adherence to collusive practices.

**Proof** For any given quota  $y$  the profit of firm  $i$  is

$$\left[ 1 - \left( \frac{N-1}{N} \right) y - \frac{x^i}{N} \right] x^i$$

and the corresponding best response is

$$x^B = \frac{1 - \left( \frac{N-1}{N} \right) y}{2/N}.$$

This enables us to explicitly compute the utility gain

$$u^B(y) - u(y, y) = (1/N)(x^B - y)^2 = (1/N) \left( \frac{1 - \left( \frac{N-1}{N} \right) y}{2/N} \right)^2.$$

When the incentive constraint holds with equality  $\phi = \lambda(u^B(y) - u(y, y))$ . Substituting in

$$\phi = \lambda(1/N) \left( \frac{1 - \left( \left( \frac{N-1}{N} \right) y \right)}{2/N} \right)^2.$$

As this is strictly increasing we can invert it to find the optimal quota as a function of the fine

$$y = \frac{1 - \left(\frac{(2/N)^2}{\lambda(1/N)}\right)^{1/2} \sqrt{\phi}}{\left(\frac{N+1}{N}\right)}.$$

Since  $u(y, y) = (1 - y)y$

$$\begin{aligned} \bar{u}(\phi) &= y - y^2 - \psi \pi \phi \\ &= \frac{1 - \left(\frac{(2/N)^2}{\lambda(1/N)}\right)^{1/2} \sqrt{\phi}}{\left(\frac{N+1}{N}\right)} - \left(\frac{1 - \left(\frac{(2/N)^2}{\lambda(1/N)}\right)^{1/2} \sqrt{\phi}}{\left(\frac{N+1}{N}\right)}\right)^2 - \psi \pi \phi. \end{aligned}$$

Notice that when  $\phi = 0$  this is the utility from the Cournot equilibrium so  $\bar{u}(\phi) - u^n$  is equal to this expression net of the constant term, that is has the form

$$\bar{u}(\phi) - u^n = A\sqrt{\phi} - B\left(\sqrt{\phi}\right)^2.$$

Multiplying out the terms gives the constants  $A, B$ .

The unconstrained optimal fine is then  $\bar{\phi} = (A/(2B))^2$  with  $\bar{U} - u^n = A^2/(4B)$ . The constraint is  $\phi \leq \Delta(\bar{u}(\phi) - u^n)$ . This holds with equality at  $\bar{\phi}$  when  $\Delta = 1/B$ . The corresponding optimal quota are then as stated in the Theorem.

For  $\Delta \leq 1/(4B)$  the constraint is  $\left(\sqrt{\phi}\right)^2 = \Delta A\sqrt{\phi} - \Delta B\left(\sqrt{\phi}\right)^2$  and solving for  $\sqrt{\phi}$  and back substituting gives also  $y$  and the utility gain result.  $\square$

## 5 Conclusion

We study a repeated cartel problem in which cartels can use fines as well as price wars. We show that best agreement is the solution to a simple mechanism design problem, and we give here a closed form solution with linear demand and marginal cost. By retaining the basic simplicity of strongly symmetric equilibrium we are able to completely characterize cartel success and behavior as a function of the number of firms, market demand, cost, the discount factor, the difficulty of observing output, and the efficiency of transfer payments. This may be a useful foundation for future empirical studies.

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