DEPARTMENT
OF ECONOMICS

Essays on Macroeconomic Policies and Household Heterogeneity

Gergő Motyovszki

Thesis submitted for assessment with a view to
obtaining the degree of Doctor of Economics
of the European University Institute

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European University Institute
Department of Economics

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I confirm that chapter 3 was jointly co-authored with Rana Sajedi and Lukasz Rachel and I contributed 85% of the work.
I confirm that chapter 1 has already been published in 2021 at the American Economic Journal: Macroeconomics, vol 13, issue 2, pp. 1-41. Copyright American Economic Association; reproduced with permission.

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Motyovszki Gergő

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Abstract

This thesis is composed of three independent chapters, but all centered around the broader topic of how macroeconomic policies interact with various aspects of household heterogeneity.

Monetary Policy and Inequality under Labor Market Frictions and Capital-Skill Complementarity We provide a new channel through which monetary policy has distributional consequences at business cycle frequencies. We show that an unexpected monetary easing increases labor income inequality between high and less-skilled workers. In particular, this effect is prominent in sectors intensive in less-skilled labor, that exhibit high degree of capital-skill complementarity (CSC) and are subject to matching inefficiencies. To rationalize these findings we build a New Keynesian DSGE model with asymmetric search and matching (SAM) frictions across the two types of workers and CSC in the production function. We show that CSC on its own introduces a dynamic demand amplification mechanism: the increase in high-skilled employment after a monetary expansion makes complementary capital more productive, encouraging a further rise in investment demand and creating a multiplier effect. SAM asymmetries magnify this channel.

Monetary-Fiscal Interactions and Redistribution in Small Open Economies Ballooning public debts in the wake of the covid-19 pandemic can present monetary-fiscal policies with a dilemma if and when *neutral* real interest rates rise, which might arrive sooner in emerging markets: policymakers can stabilize debts either by relying on fiscal adjustments (AM-PF) or by tolerating higher inflation (PM-AF). The choice between these policy mixes affects the efficacy of the fiscal expansion already today and can interact with the distributive properties of the stimulus across heterogeneous households. To study this, I build a two agent New Keynesian (TANK) small open economy model with monetary-fiscal interactions. Targeting fiscal transfers more towards high-MPC agents increases the output multiplier of a fiscal stimulus, while raising the degree of deficit-financing for these transfers also helps. However, precise targeting is much more important

under the AM-PF regime than the question of financing, while the opposite is the case with a PM-AF policy mix: then deficit-spending is crucial for the size of the multiplier, and targeting matters less. Under the PM-AF regime fiscal stimulus entails a real exchange rate depreciation which might offset "import leakage" by stimulating net exports, if the share of hand-to-mouth households is low and trade is price elastic enough. Therefore, a PM-AF policy mix might break the Mundell-Fleming prediction that open economies have smaller fiscal multipliers relative to closed economies.

Weak Wage Recovery and Precautionary Motives after a Credit Crunch During the economic recovery following the financial crisis many advanced economies saw subdued wage dynamics, in spite of falling unemployment and an increasingly tight labour market. We propose a mechanism which can account for this puzzle and work against usual aggregate demand channels. In a heterogeneous agent model with incomplete markets we endogenize uninsurable idiosyncratic risk through search-and-matching (SAM) frictions in the labour market. In this setting, apart from the usual precautionary saving behaviour, households can self-insure also by settling for lower wages in order to secure a job and thereby avoid becoming borrowing constrained. This channel is especially pronounced for asset-poor agents, already close to the constraint. We introduce a credit crunch into this framework modelled as a gradual tightening of the borrowing constraint (and utilizing a continuous time approach, known as HACT). The perfect foresight transition dynamics feature falling wages despite a tightening labour market and expanding employment. As households suddenly find themselves closer to the borrowing constraint, the increased precautionary motive drives them to accept lower wages in the bargaining process, while firms respond to this by posting more vacancies, leading to a tighter labour market and falling unemployment. If the household deleveraging pressure is persistent enough after the credit crunch, it can explain the weak wage recovery in spite of already stronger aggregate demand.

Acknowledgements

I am immensely thankful to my supervisor, *Evi Pappa*. Without her support and guidance throughout these years this PhD would not have been possible. She was always ready to deal with my issues, pointing me in the right direction and giving advice. I am very lucky to have had such a kind and easy-to-talk-to person as a colleague, and to have also gained a friend in the process. Co-authoring the first chapter of this thesis with her was an invaluable experience, which also lead to my first peer-reviewed publication. I am also grateful to our other co-author, *Juan J. Dolado* who was always ready to cheer us up and remind all of us that there is life beyond our work, too.

I owe gratitude to *Alessia Campolmi*, who advised me during the beginning of my PhD and provided useful input to my thesis. She was also my MA thesis supervisor at the Central European University in Budapest where she introduced me to the world of New Keynesian macroeconomics, DSGE modelling and Matlab, which facilitated my current research path. She was the one who recommended to me the PhD program of the European University Institute, applying for which turned out to be a great decision.

I learned a lot from other macroeconomics faculty at the EUI, during various seminars, conferences and working groups. In particular, I thank *Árpád Ábrahám* who introduced me to heterogeneous agent macro, and *Ramón Marimon* who always made sure that the Department was buzzing with the current macro policy debates of the day. Having had numerous forums to discuss the actual issues of European monetary integration (especially during the ADEMU project!) helped direct our academic research towards policy relevant questions.

Part of my PhD was spent as a visiting researcher at the University College London. I thank my hosts, *Morten O. Ravn, Vincent Sterk* and *Ralph Lütticke* for their hospitality, and for taking the time to initiate me on solving HANK models. I am also indebted to *Florin Bilbiie* whose research on TANK models inspired me a lot, and who was always kindly encouraging me to pursue my research ideas whenever we met on various conferences.

One chapter of this thesis was written during my time as a trainee at the Bank of England in London. I am grateful for the opportunity to have worked on this project together with *Rana Sajedi* and *Łukasz Rachel*, and to have met other great economists at such a renowned policy institution.

I would like to express my gratitude to the European University Institute which provided the infrastructure and the best possible setting for academic research one can wish for. The scholarship given by EUI allowed me not only to pursue this PhD but also to spend 5 wonderful years in Tuscany, working in lovely villas among the green hills surrounding beautiful Florence. I thank all the staff of Villa La Fonte who made our daily life in the office run smoothly, Marco's alimentari in San Domenico who supplied us with delicious cappuccino, and trattoria Piatti e Fagotti whose superb dinners gave us something to look forward to during tense and stressful periods.

That my journey on the road of the PhD was not a lonely endeavour is due in most part to my amazing cohort. They were there when solving problem sets until late at night, there to provide emotional support when things were going bad, but there also outside work as friends, enjoying many nights out in Florence together and having the annual villa YOLO trips at the Tuscan countryside. And especially, they were there at my wedding in Hungary which meant a lot to me. I thank *Agnès, Alica, Ana, Carolina, Chiara, Chiara "piccola", Essi, Giorgos, Matteo, Rafa, Oliko* and *Simon*, for their friendship and for accompanying me on this journey.

The fact that I could even contemplate pursuing this PhD is undoubtedly due to the solid foundations I was privileged enough to receive beforehand. In this respect credit is due to the Central European University in Budapest whose master program equipped me with the the necessary toolkit, and in large part to the Rajk László College which broadened my professional perspective and instilled in me the ambition to become a socially responsible economist. I also thank my former colleagues from the central bank of Hungary for the lively economic policy discussions, for examplifying how to approach policy relevant questions with professional rigour, and for displaying integrity in the face of a suppressing regime. My 13-year-long subscription to *The Economist* was also invaluable in forming my economic world view.

Last but not least, I am forever indebted to my parents who turned me into the person I am today, and who provided me with the stable family background, loads of opportunities and unconditional love which is not to be taken for granted.

And to my wife and companion in life, *Pálma*, who followed me to Florence, stood by me, put up with me during hard times, always supported me and loved me throughout.

Contents

Introduction

This thesis is organized around the interaction between macroeconomic policies and household heterogeneity. In particular, it explores the distributional consequences of cyclical macro policies as well as how inequality influences the monetary-fiscal transmission mechanism and aggregate business cycle outcomes.

A recently growing literature has started to deviate from the previously dominant representative agent setup for macroeconomic modelling in order to investigate how monetary and fiscal policies might affect the distribution of income and wealth in the economy. The first chapter of this thesis falls into this category. This endeavour also required parting with the complete market assumption and introducing uninsurable idiosyncratic risk which gives rise to a non-degenerate wealth and income distribution as the full history of individual shocks starts to matter (as in the third chapter of this thesis). A reduced form way of capturing this phenomenon is having multiple, ex ante different types of households in models (like in the first two chapters of this thesis).

These new model laboratories then uncovered interesting channels through which inequality also affects the aggregate economy and the transmission of macroeconomic policies – and not just the other way around. The second and third chapter of this thesis fall into this category. The two most notable channels are the marginal propensity to consume (MPC) heterogeneity channel on the one hand (featured in the second chapter), and the precautionary saving channel on the other hand (featured in the third chapter). An aggregate demand stimulus is much more powerful when it also affects high-MPC agents who consume most of the temporary increase in their income – as opposed to the consumption-smoothing optimizing representative agent. The rise their consumption stimulates aggregate demand further, giving rise to a Keynesian Cross type of amplification. Countercyclical variations in uninsured risk can also provide amplification of aggregate demand: in response to higher income risk during recessions, households increase precautionary savings and self-insurance, further depressing aggregate demand.

The first chapter focuses on the effects of monetary policy on wage inequality between skilled and unskilled workers in the presence of capital-skill complementarity and asymmetric frictions in the labour market. Utilizing a New Keynesian framework with incomplete financial markets we find that when high skilled workers are complementary to capital and they also face a lower difficulty of finding new jobs, then these two sources of heterogeneity interact and amplify each other such that an unexpected aggregate demand stimulus leads to rising wage inequality.

The second chapter explores how the presence of borrowing constrained "hand-to-mouth" households and redistribution towards them affects the transmission of monetary-fiscal policies in a small open economy. Targeting fiscal transfers more towards high-MPC agents increases the output multiplier of a fiscal stimulus, while raising the degree of deficit-financing for these transfers also helps. However, precise targeting is much more important than the question of financing when monetary policy actively stabilizes inflation, while the opposite is the case with a policy mix where monetary policy accommodates fiscal expansion by keeping interest rates low: then deficit-spending is crucial for the size of the multiplier, and targeting matters less. Under the latter regime fiscal stimulus entails a real exchange rate depreciation which might offset "import leakage" by stimulating net exports, if the share of hand-to-mouth households is low and trade is price elastic enough. Therefore, such a policy mix might break the Mundell-Fleming prediction that open economies have smaller fiscal multipliers relative to closed economies.

The third chapter attempts to explain the weak real wage recovery following the financial crisis by proposing a mechanism based on household heterogeneity in the presence of uninsured idiosyncratic unemployment risk. In this setting, apart from the usual precautionary saving behaviour, households can self-insure also by settling for lower wages in order to secure a job and thereby avoid becoming borrowing constrained. This channel is especially pronounced for asset-poor agents, already close to the constraint. We introduce a credit crunch into this framework and can replicate falling wages despite a tightening labour market and expanding employment. As households suddenly find themselves closer to the borrowing constraint, the increased precautionary motive drives them to accept lower wages in the bargaining process, while firms respond to this by posting more vacancies, leading to a tighter labour market and falling unemployment. If the household deleveraging pressure is persistent enough after the credit crunch, it can explain the weak wage recovery in spite of already stronger aggregate demand.

Chapter 1

Monetary Policy and Inequality under Labor Market Frictions and Capital-Skill Complementarity

joint with Juan J. Dolado and Evi Pappa [1] [2]

1.1 Introduction

During the last two decades growing inequality has become a key topic in the public debate, mainly pointing to long-term trends driven by technological change and globalization. However, following the financial crisis and the extreme measures central banks took to fight it, many questions have arisen about how monetary policy might affect inequality at business cycle frequencies. There are contrasting views on this issue. On the one hand, concerns have been expressed that the highly accommodative monetary policy stance in advanced economies, as with unconventional quantitative easing, favors richer households disproportionately, thereby contributing to more unequal income and wealth distributions. On the other hand, there are opinions supporting the opposite view, namely, that expansionary monetary policy reduces inequality because borrowers become

[1]We are indebted to Benjamin Moll and three anonymous referees for many insightful comments which helped improve the paper substantially; to Arpad Abraham, Alessia Campolmi, Edouard Challe, Andresa Lagerborg, and Luis Rojas for useful inputs on earlier versions of this paper; and to Christian J.Meyer and Egon Tripodi for excellent research assistance. We also thank Jerome Adda, Juan Jimeno, Nicola Pavoni, Carlos Thomas, and participants in seminars at Bank of Spain, Bocconi University, CBS, Cemfi, Cunef, De Nederlandsche Bank, ADEMU-Banque de France Conference (EUI), EABCN Conference on The Effects of Unconventional Monetary Policy (UPF), and Bank of England for helpful remarks.

[2]This paper has been accepted for publication in the *American Economic Journal: Macroeconomics*.

better off than savers.

Of course central banks consider the economy *as a whole* when setting monetary policy. As pointed out by Bernanke (2015), distributional issues should not be the concern of monetary authorities but rather be addressed by democratically elected officials (e.g. through fiscal policy). A corollary of this view is that monetary policy can best contribute to social welfare by promoting aggregate economic stability, which can be beneficial from an inequality perspective. Notwithstanding this, it is increasingly acknowledged that the short-run effects of monetary policy on inequality could matter for its optimal design. Taking these effects into account might have welfare implications for various systematic monetary strategies, while inequality might also interact with the different channels of the monetary transmission mechanism. As a result, a recent strand of the literature has started to analyze how these issues are related.

The channels through which monetary policy affects inequality are complex. Interest rate changes can have different effects on savers and borrowers across the wealth distribution (the savings-redistribution channel). Asset prices (with various maturities) can react in different ways to changes in interest rates and/or inflation which in turn can influence inequality between the holders of these assets (the interest-sensitivity channel). Different household preferences and differing financial market access can also introduce heterogeneity in the effects of monetary policy (the household heterogeneity channel). More indirectly, the aggregate demand expansion engineered by monetary loosening can affect the outcomes of workers and capital owners differently, insofar as wages and profits change by different amounts (the income composition channel). Finally, the wages and employment of different types of workers can also exhibit heterogeneous responses, depending on unemployment risk, asymmetric wage rigidity, and labour market institutions (the earnings heterogeneity channel).[3]

The balance of all the above-mentioned forces is ambiguous and thus can only be determined using quantitative methods. Our main goal in the present paper is to focus only on one of the channels, namely labor earnings heterogeneity, leaving aside other sources of heterogeneity (like, e.g. the wealth distribution). In particular, we uncover a new mechanism through which monetary policy affects labor-income inequality by investigating the interaction between capital-skill complementarity (CSC hereafter) in production, and labor market heterogeneity in search-and-matching (SAM) frictions of high vs. low-skilled workers. Skill-biased technological change has been traditionally considered as one of the main determinants of the growing trend in labor income inequality, as reflected by

[3]For more details on the different channels between monetary policy and inequality see Bell et al. (2012), Amaral (2017), Coibion et al. (2017) and Heathcote, Perri and Violante (2010).

increasing gaps between the wages of high-skilled and low-skilled workers (skill premium) and their employment rates (relative employment). However, to the best of our knowledge, there has not been any analysis on the role played by the interaction of CSC and SAM frictions in explaining the effects of monetary policy on these gaps over the business cycle.

We start motivating our analysis by reporting the effects of monetary policy shocks on the skill premium and relative employment both for the aggregate US economy and six different sectors, using data from the Current Population Survey (CPS). Our main finding is that a monetary expansion (i.e., an unexpected reduction of 100 bp. in the annualized Federal Funds rate) increases on impact the skill premium and the relative employment of skilled vs. unskilled workers by 0.4 pp. and 0.35 pp., respectively, and that these effects are fairly long lasting. When the labor force is broken down into six sectors we report similar responses to an expansionary monetary shock in Manufacturing and in Wholesale and Retail Trade. Notably, these two sectors exhibit a large share of unskilled workers, underwent intense technological changes leading to massive restructuring and reallocation of activity during the 1990s (see, Foster, Haltiwanger and Krizan (2006)), and are characterized by similar degrees of CSC (see Blankenau and Cassou (2011)) and matching inefficiencies (see Sahin et al. (2014)). Hence, the characteristics exhibited by these two sectors seem to mimic the main ingredients of our model.

To rationalize these empirical findings, we build a model within the family of New Keynesian models with CSC embedded in the production function and SAM frictions á la Diamond-Mortensen-Pissarides (Blanchard and Galí, 2010) affecting high and low-skilled workers in an asymmetric fashion. CSC is captured through the elasticity of substitution between high-skilled labor and capital being below unity (making them complements), while it is above unity between low-skilled labor and capital (making them substitutes). High-skilled workers also face lower SAM-frictions in the form of lower separation rates, higher bargaining power and better matching efficiency. Krusell et al. (2000) introduced a CSC technology to study the effects of skill-biased technological change on the U.S. skill premium in the medium and long run, while Lindquist (2004) has shown that CSC is crucial to explain the behavior of the skill premium and labor income inequality at business cycle frequencies. As regards the asymmetric nature of SAM-frictions, Barnichon and Figura (2015) report that more educated workers have higher search efficiency despite the presumption that their labour market in thinner. Dolado, Jansen and Jimeno (2009) argue that a potential explanation of this result is that, while low-skilled workers can only undertake simple tasks, high-skilled workers can undertake both complex and simple tasks, being therefore more easily matched. Further, Wolcott (2018) reports that they also have lower separation rates.

Replicating the results in our empirical exercise, our theoretical model is able to predict that an unexpected cut in interest rates *raises* labor income inequality by increasing the relative labor share for high-skilled workers (who are already richer to begin with). This effect is mainly driven by an increase in the wage for the high skilled, who also fare better in terms of employment rates. The key assumption behind this result is the CSC production function since it introduces a *dynamic demand amplification channel*. The initial increase in high-skill employment induced by demand pressures after the monetary expansion, makes complementary capital more productive, encouraging a further rise in investment demand which creates further demand pressures (CSC channel) which are absent in a model with a standard Cobb-Douglas (CD) production function. This amplification is magnified further under asymmetric SAM frictions between skilled and unskilled workers (SAM-asymmetry channel). In effect, by considering this asymmetry, there is an additional source of initial imbalance in relative labor demand which interacts with higher demand pressures. With a sufficiently high degree of price rigidity, the introduction of CSC on its own is enough to generate a sizeable rise in the skill premium in a symmetric SAM environment. However, its *interaction* with asymmetric SAM frictions leads to a much larger rise in labor incomes inequality for a relatively low degree of price stickiness. This interaction is worth stressing, given the relevance of SAM asymmetries in the labor market.

It is important to highlight that these results are not specific to monetary policy shocks but also apply to any other favourable aggregate demand shock. Yet, we focus here on monetary shocks because they are likely to have quantitatively larger effects. This is due to their relatively more favourable impact on capital demand (important in the CSC channel), compared to government spending or discount factor shocks which, in contrast to monetary shocks, tend to crowd out investment. Offsetting aggregate demand pressures that lead to a rise of the relative demand for skilled labor might be desirable for monetary policy as long as income inequality is a policy concern. From this respect strict inflation targeting, which is successful in stabilizing the economy even in the presence of cost-push shocks (which present monetary policy with a trade-off between inflation and demand stabilization) is the most promising monetary strategy. Yet, we acknowledge that the simplicity of the model renders it limited for optimal policy experiments.

This paper is part of the recently growing literature on monetary policy and inequality. Most of the existing studies combine an incomplete market Aiyagari-type heterogeneous agent framework with New Keynesian nominal rigidities, resulting in what is now referred to as HANK models (see, for example Kaplan, Moll and Violante (2018); Ravn and Sterk

(2018); Luetticke (2017)). Gornemann, Kuester and Nakajima (2012) use this framework, augmented by SAM frictions, to make unemployment risk endogenous to monetary policy. They show that, unlike in our model, contractionary monetary shocks are the ones that increase income inequality – via a rise in precautionary savings by poorer households which leads to a higher value of the assets held by the wealthy rich – and therefore have larger welfare costs than thought before. Yet, Gornemann, Kuester and Nakajima (2016) and other existing studies do not account for either CSC or asymmetric SAM frictions across skills. Instead, we abstract from the role of uninsured idiosyncratic risk and the wealth distribution, and rather focus on the earnings heterogeneity channel. Thereby our paper provides an alternative theoretical insight into a new transmission mechanism of monetary policy to labor income inequality which, to the best of our knowledge, is novel in this area of research.

Moreover, to the best of our knowledge, this is the first paper to bring direct evidence on the effects of monetary policy shocks on the skill premium and the relative employment of high vs. low-skilled workers, both at the aggregate level and at the industry level. Contrary to our findings, Coibion et al. (2017) (using local projections) find that contractionary monetary policy shocks (identified as in Romer and Romer (1998)) systematically increase inequality as rising unemployment falls disproportionately on low-income workers. In the Online Appendix we show that the differences in conclusions with Coibion et al. (2017) are due to the different measures of inequality used.

The rest of the paper is organized as follows. In Section 1.2 we motivate our further analysis by estimating the dynamic effects of monetary policy shocks on the skill premium and the relative employment in structural VAR (SVAR) models. Section 1.3 lays out the theoretical model, while Section 1.4 discusses our calibration strategy. Results and sensitivity analysis are presented and discussed in Section 1.5. A final section provides concluding remarks. An Appendix gathers detailed information on the construction of the labor market variables and on additional estimation and simulation results. An Online Appendix provides the detailed structure of the model and its calibration.

1.2 Monetary Policy and Labor Income Inequality: SVAR evidence

To motivate our research question, we start by identifying the impact of an expansionary monetary policy shock on the skill premium and the relative employment rates of high and low-skilled workers (*employment-rate ratio*) in a SVAR model. We construct time series of both gaps using the NBER extracts of the Current Population Survey (CPS)

Merged Outgoing Rotation Groups, including in the sample only individuals in working age 15-64 and excluding part-time workers, self-employed and military employees. CPS provides monthly information from 1979:1 until 2016:6 on the participants' employment status, level of education, weekly earnings, and weekly hours of work. We classify workers as high-skilled and less-skilled according to whether they have experienced some college or not. Employment is defined as number of monthly hours of work per employee times the number of salaried workers in each skill category. We obtain hourly wages for both types of workers by computing the ratio of weekly wages and the corresponding number of weekly hours worked in each group.

In spite of being seasonally adjusted, the derived wage and employment series from the CPS micro data turn out to be too volatile at the monthly frequency. In line with the optimal choice of lag length in the VAR (5 lags), we use a backward 5-month moving average to smooth the data. Notice that, since we want to use these series in a VAR, each variable (time series) has to be modeled as a function of past values of the series, which justifies the choice of a backward moving average instead of a centered one involving future values.[4] These smoothed wage series are then used to compute the skill premium, i.e. the ratio between the weighted average of hourly wages of the high-skilled and low-skilled workers. Consistent with the evidence of Castro and Coen-Pirani (2008), the unconditional correlation between the constructed skill premium and the unemployment rate is -0.07 in the raw data, indicating that the skill premium is slightly pro-cyclical in our sample, while the one between the relative employment ratio and the unemployment rate equals 0.21, pointing to counter-cyclicality of high-skilled employment.

As Ramey (2016) acknowledges, it is very hard to identify meaningful monetary policy shocks on samples that include recent decades. Ramey (2016) also shows that, irrespective of the sample period considered, the method used to identify shocks in monetary policy might create several puzzles, such as the price puzzle, i.e. an increase in inflation after a monetary contraction, or the fact that contractionary monetary policy shocks appear to be expansionary in the post 1980 period. As a result, we proceed sequentially meaning that, prior to looking at the effects of monetary policy on our two variables of interest, we search for specifications for the aggregate US economy (abstracting from the labor market variables) that produce meaningful responses of the remaining variables in the SVAR to a monetary policy shock. To recover meaningful monetary policy shocks in the data we use

[4]Admittedly, smoothing the series is not innocuous. In the Online Appendix, we present the original IRFs of the VAR with the original labor market series. The VAR with the raw series exhibits an initial spike at 2, yet the skill premium increases significantly for about 32 months also in this specification. Smoothing, thus, matters for the quantitative results (the impact effect is smaller when using the smoothed series but the dynamic responses last longer than with the raw data).

the IV-SVAR approach advocated by Mertens and Ravn (2012) and Stock and Watson (2018). The central idea of this estimation procedure is the use of external instruments for the structural shocks of interest in a VAR setting. Following Mertens and Ravn (2012), we use the extended time series of the Romer and Romer (2004) narrative/Greenbook shocks constructed by Wieland (2016) till 2007 as a noisy measure of the true shocks in the Federal funds rate.

We examine monthly data covering the sample period January 1980-December 2007. The sample stops in 2007 both to exclude the financial crisis and for the practical reason that the Romer and Romer (2004) narrative series are not available after that date.[5] Our SVAR consists of seven variables: the unemployment rate, the log of real wages for skilled workers, the skill premium, the skilled employment rate, the relative employment rate, the consumer price index inflation, and the federal funds rate (FFR). Notice that by including the individual variables for skilled workers in addition to the skill and employment gaps, we are also able to retrieve the responses of the variables corresponding to unskilled workers variables. Data for both the unemployment rate and the CPI for all urban consumers are drawn from the Bureau of Labor Statistics; data for the effective FFR are produced by the Board of Governors of the Federal Reserve System. All variables except for the FFR are seasonally adjusted. Using different information criteria (AIC, HQ and BIC), we include five lags of each variable in the VAR.

Figure 1.1 displays point estimates and 68 (darker bands) and 95 (lighter bands) percent confidence intervals for the impulse function responses (IRFs) of the baseline SVAR model to the identified monetary policy shock.[6] As far as the aggregate macroeconomic variables are concerned, the shock has expansionary effects after some delay. Following the unexpected interest-rate cut, unemployment falls persistently. The response of the CPI inflation is negative but mostly non significant, alleviating concerns about the price puzzle in our chosen specification (See Ramey (2016)). Regarding the labor market variables, the skill premium and the employment ratio increase significantly on impact and

[5] Miranda-Agrippino (2016) has extended those series until 2012. In the Online Appendix we show that our results are robust when the sample is extended up to that year. Results are also robust if we use the extended narrative series of Romer and Romer (2004) constructed by Coibion et al. (2017). The series of Wieland (2016) are one year shorter but, perhaps not surprisingly, results are very similar since the correlation between the two series for the rest of the observations is 0.99. Finally, we also consider a Cholesky ordering for the whole sample period 1980-2016, obtaining that the skill premium and relative employment for skilled workers also increase in this experiment. Yet, these results need to be taken with some caution since the zero-lower bound (ZLB) was binding in that sample and the monetary policy conduct before and after 2007 was subject to regime shifts.

[6] The Romer and Romer narrative series have an F-statistic of 42.8 in the first-stage regression, rejecting the null hypothesis of a weak instrument at the 5 percent significance level.

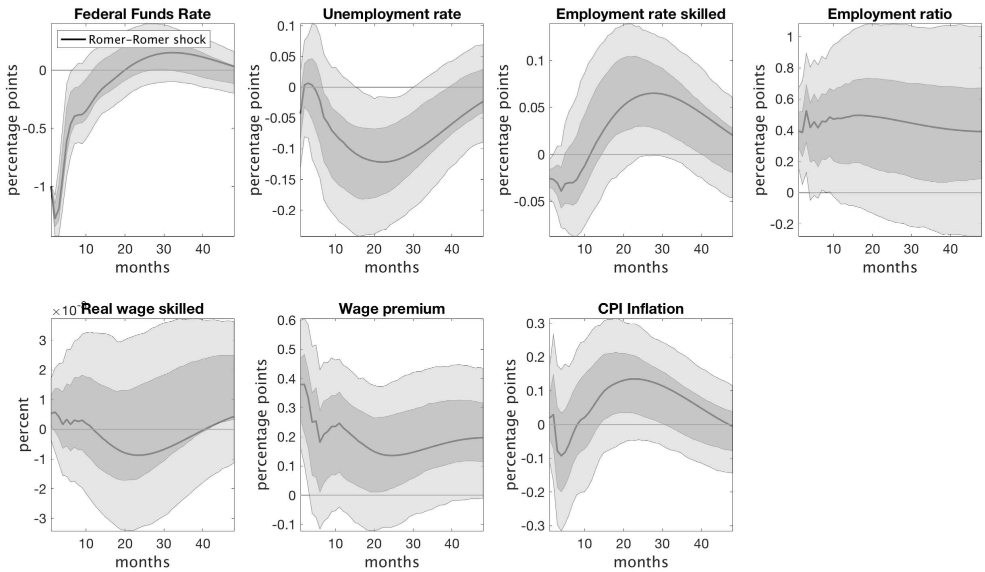

Figure 1.1: IRFs to a one percentage point unexpected reduction in the FF interest rate

they remain persistently above trend (at 68 percent confidence level), while the real wage and the employment rate of the high skilled also increase though the former is only significant on impact.[7] Overall, the reported IRFs suggest that inequality between high and low-skilled workers (in terms of wages and employment rates) is positively related to an unexpected cut in interest rates. At the peaks of the IRFs, the employment rate ratio increases by about 0.35 percentage points while the skill premium raises by around 0.4 percentage points.

In the Online Appendix we show that a monetary expansion robustly increases the wage premium and relative employment when we control for composition effects, and when we do not smooth the wage series. Moreover, given the discrepancy between our results and Coibion et al. (2017)'s, we repeat our exercise adopting the local projections methodology employed by Coibion et al. (2017) to identify the effects of a monetary policy contraction on inequality. We show that a monetary contraction decreases significantly the real wage of skilled and the wage premium when we use direct local projections to estimate the responses of those variables to such a shock. The differences in conclusions with Coibion et al. (2017) is due to the different series considered to measure inequality. While our focus is on the skill premium and relative employment, drawing data from the NBER extracts of the Current Population Survey (CPS), Coibion et al. (2017) construct measures of inequality using the Consumer Expenditure Survey (CEX) and report results for

[7] The response of the skilled real wage is sensitive to the specification used. For most of the alternative specifications considered in the Online Appendix, the real wage for skilled increases significantly after the monetary expansion.

inequality in labor income, total income, consumption and total expenditures. Furthermore, Coibion et al. (2017) acknowledge that their results are sensitive regarding earnings inequality. Indeed, for some of their empirical specifications, earnings inequality increases after a monetary expansion in accordance to the evidence presented here.

Next, in the different panels of Figure A.1 in Appendix A.2 we display the IRFs of the skill premium and relative employment in six sectors of the US economy: (1) Manufacturing, (2) Education and Health Services, (3) Agriculture, Mining and Transportation, (4) Wholesale and Retail Trade, (5) Professional Services, (6) Financial and Informational Services in that order.[8] The IRFs of the relative employment and the skill premium in the Manufacturing (Sector 1) and the Wholesale and Retail sector (sector 4) are the ones that mimic qualitatively the IRFs reported earlier for the aggregate data. For the remaining four sectors results are mixed. Hence, the evidence from the sectoral data points to the importance of CSC for the underlying mechanism behind the responses of the wage premium and relative employment to a monetary expansion.

In what follows we highlight the role of CSC and asymmetric SAM frictions in generating the pattern of responses observed for the labor market of skilled and less skilled workers both in Wholesale and Retail Trade and Manufacturing. Notice that these two sectors share several common features: (a) they represent industries which have a relatively high share of less-skill workforce, as in the aggregate ; (b) according to Blankenau and Cassou (2011), who classify industries in a similar way, the elasticity of substitution between low and high-skilled labor inputs and the rate of skill-biased technological progress is similar in these two sectors; (c) both underwent a massive restructuring and reallocation of activity in the 1990s, in parallel with intense technological advances (see, Foster et al. (2006)); (d) according to Sahin et al. (2014), they are characterized by similar degree of matching efficiencies in the labor market (using hires from the CPS, the estimates for industry-specific match efficiencies before 2007 are 0.38 in Wholesale and Retail Trade, and 0.42 in Manufacturing). Finally, using data from the American Community Survey, Rose (2017) confirms anecdotal evidence suggesting that a large fraction of job holders in the retail sector are overqualified despite the low share of the high skilled. For example, 22 percent of male and 18 percent of female workers hold a BA and work as retail salespersons, 6 percent of male laborers and freight, stock and material movers and 20 percent of female customer service have completed a BA degree, where all the aforementioned jobs have low or no-skill requirements. In sum, these two sectors exhibit characteristics which mimic better than the remaining ones those features which are relevant for our proposed

[8]Due to data limitations we were unable to dis-aggregate these sectors further. For ease of exposition we only present IRFs of the labor market variables, since the responses of the aggregate variables are similar to those displayed in Figure 1.1.

mechanism.

In view of these empirical findings, a theoretical model is presented in the next section to rationalize them, as well as to provide new insights about the interaction between monetary policy shocks and labor income inequality.

1.3 Model

Our model belongs to the family of New Keynesian DSGE models with SAM frictions in the labor market (Blanchard and Galí, 2010). The New Keynesian feature of nominal rigidities ensures that monetary policy has real effects on the macro-economy, while SAM frictions allow us to model unemployment. Heterogeneity in the population manifests itself along two dimensions: there are three different households types (high and low-skilled workers, and capital investor entrepreneurs, with no transitions among these three groups), as well as three different labor market status (employed, unemployed and inactive) endogenously governed by SAM frictions. Skill types differ in their labor market frictions (*"asymmetric" SAM*) (See, Brückner and Pappa (2012), and Pappa, Sajedi and Vella (2015)) and also in their role in production: high-skilled workers have a lower elasticity of substitution with capital than low-skilled workers do (*CSC*). Different households can trade with each other in a full set of state-contingent Arrow securities. This complete financial market setup provides perfect insurance against endogenous idiosyncratic unemployment risk *within* a given skill group, as well as against the asymmetric effects of aggregate shocks *across* different types, leading to constant *consumption* inequality. This assumption allows us to focus on cyclical fluctuations in labor *income* inequality.[9] Finally, we have an endogenous participation choice as in Ravn (2006), Brückner and Pappa (2012), Campolmi and Gnocchi (2016) and Christiano, Eichenbaum and Trabandt (2016).

Perfectly competitive intermediate good firms produce a homogeneous output by renting capital from entrepreneurs and the two types of labor from workers. Hiring and firing are subject to SAM frictions and wages are set by Nash bargaining. Intermediate output is then differentiated by monopolistically competitive retail firms who face Calvo-type nominal rigidities in the price of the final good. Final output is used for consumption,

[9] In the working paper version of this paper, Dolado, Motyovszki and Pappa (2018), we consider imperfect risk sharing (through a single risk free bond) *across* skill types against the asymmetric effects of aggregate shocks (while maintaining full risk sharing *within* a particular skill type against idiosyncratic unemployment risk). This leads to fluctuating consumption inequality between high and low-skilled workers. However, apart from the dynamics of consumption inequality, the results we present here are robust to the assumption of complete markets.

investment and (wasteful) government spending. Fiscal policy finances exogenous expenditures, unemployment benefits and production subsidies by lump-sum taxes. Monetary policy sets the short term nominal interest rate.

1.3.1 Labor market search and matching

As already mentioned, there are three different types of households: high and low-skilled workers and entrepreneurs, who all have constant masses φ^k, $k \in \{H, L, E\}$. We assume no transitions across those household types. In addition, the assumption of full insurance within a particular type allows us to model each type as a representative household splitting its time endowment between employment n_t^k, unemployment u_t^k, and inactivity (enjoying leisure) l_t^k. For simplicity, it is assumed that the entrepreneur types do not work, and only consume. The population size is normalized to one, i.e. $\sum_k \varphi^k = 1$.

$$1 = n_t^k + u_t^k + l_t^k \qquad k \in H, L \tag{1.1}$$

Intermediate good firms post vacancies v_t^k requiring different skills, which are then matched with unemployed job-searchers U_t^k according to the following matching technology:

$$m_t^k\left(v_t^k, U_t^k\right) = \psi^k \left(v_t^k\right)^{\varsigma} \left(U_t^k\right)^{1-\varsigma} \qquad k \in \{H, L\} \tag{1.2}$$

where ψ^k is the matching efficiency parameter for a k skilled unemployed. Aggregate measures of employment and unemployment are $N_t^k = \varphi^k n_t^k$ and $U_t^k = \varphi^k u_t^k$.

Labor market tightness θ_t^k, vacancy filling probabilities ν_t^k and hiring probabilities μ_t^k are defined as follows:

$$\theta_t^k = \frac{v_t^k}{U_t^k} \qquad k \in H, L \tag{1.3}$$

$$\nu_t^k = \frac{m_t^k}{v_t^k} \qquad k \in H, L \tag{1.4}$$

$$\mu_t^k = \frac{m_t^k}{U_t^k} = \psi^k \left(\theta_t^k\right)^{\varsigma} \qquad k \in H, L \tag{1.5}$$

An exogenous separation rate σ^k signals the fraction of employed workers losing their job, who then become unemployed. Unemployed agents either find a job, stay unemployed or exit the labor force. As a result, the transition dynamics between different labor market status can be expressed as:

$$N_{t+1}^k = (1 - \sigma^k) N_t^k + \underbrace{\mu_t^k U_t^k}_{m_t^k} \qquad k \in H, L \tag{1.6}$$

Participation in the labor force is chosen by a given-skilled household (from (1.1), we have: $1-l_t^k = u_t^k + n_t^k$). However, while the household can only decide to start *searching* for a job (going from inactive to unemployed), *getting* a job is constrained by search and matching frictions. Therefore, n_t^k are pre-determined (state) variables at time t, implying that the participation margin can only be adjusted through choosing u_t^k. Then, the choice of u_t^k can affect future employment through the hiring probabilities μ_t^k in (1.5). Similarly, the intermediate firm cannot decide directly how many workers to employ in a given period, but it can only affect future employment levels through its current posted vacancies v_t^k (as it also affects vacancy filling probabilities ν_t^k through labor market tightness θ_t^k in (1.4)). Once these choices are made by households and firms, and given the pre-determined levels of n_t^k, future flows into employment are governed by the laws of motion (1.6), which will act as constraints on the household's and firm's decision problems.

This also shows that there are two channels through which the different labor market status in our setup interact endogenously with the rest of the economy. One is the participation choice of the household through u_t^k, and the other one is the vacancy posting decision of intermediate firms v_t^k. Both take into account future desired levels of employment n_{t+1}^k, which in turn are subject to the constraints imposed by SAM frictions. The potential asymmetry in SAM frictions across skills $k \in \{H, L\}$ are captured by k-specific parameters σ^k, ψ^k.

1.3.2 Households

The three different household types (i.e. $k \in \{H, L, E\}$) exhibit some common features. They all maximize lifetime utility, which is a separably additive function of consumption c_t^k and leisure l_t^k. The intertemporal elasticity of substitution is $\frac{1}{\eta}$ for everyone. Lump sum taxes t^k are collected from the households by the government. The aggregate price level of final consumption goods is P_t.

The three households can trade with each other sequentially through complete spot financial markets, i.e. in a full set of one-period state-contingent claims $z_{t+1}^k(s^t, s_{t+1})$. These claims pay one unit of currency in $t+1$ if the particular state s_{t+1} occurs, and zero otherwise, and their time t nominal price is $q_{t,t+1}(s_{t+1}|s^t)$, given shock history s^t. Similar Arrow securities are also traded among individuals *within* a particular household type. This leads to full insurance against idiosyncratic income shocks stemming from endogenous SAM frictions-induced unemployment risk. This complete markets assumption allows us to model a continuum of potentially different consumers as single households representative of their types (akin to big families).[10]

[10]Notice that, since we have full insurance not only *within* a skill type but also *across* skill types,

Entrepreneurs

Entrepreneurs do not participate in the labor market, and for simplicity it is assumed that they derive no utility from leisure. In addition to trading in state contingent securities $z_{t+1}^{E}(s^t, s_{t+1})$, they can also save by investing in physical capital k_t[11]. Investment i_t also has to cover depreciation at rate δ and capital adjustment costs, the latter being governed by parameter, ω. Entrepreneurs then rent capital out to intermediate firms at a rental rate r_t. They own the firms in the economy, so they receive all profits as dividends d_t, and equity is not traded with workers. Finally, they maximize utility subject to their budget constraint and the capital law of motion.

$$\max_{\{c_t^E, i_t, k_{t+1}, z_{t+1}^E(s^t, s_{t+1})\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \, \frac{\left(c_t^E\right)^{1-\eta}}{1-\eta}$$

$$c_t^E + i_t + t_t^E + \frac{1}{P_t} \sum_{s_{t+1} \in \mathcal{S}} q_{t,t+1}(s_{t+1}|s^t) \; z_{t+1}^E(s^t, s_{t+1}) \leq r_t \, k_t + \frac{z_t^E(s^t)}{P_t} + d_t$$

$$i_t = k_{t+1} - (1-\delta)k_t + \frac{\omega}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 k_t$$

The solution to this problem is the standard Euler equation, $\left(c_t^E\right)^{-\eta} = \beta \; E_t\Big\{\left(c_{t+1}^E\right)^{-\eta} \frac{R_t}{\Pi_{t+1}}\Big\}$, and a no-arbitrage condition connecting the *ex ante* real interest rate $\frac{R_t}{E_t \Pi_{t+1}}$ to the return on capital (after depreciation and capital adjustment costs). Gross inflation is defined as $\Pi_t = P_t/P_{t-1}$, while $R_t = \left[\sum_{s_{t+1}} q_{t,t+1}(s_{t+1}|s^t)\right]^{-1}$ is the gross nominal interest rate of a risk-free nominal bond. For the full set of equilibrium conditions, we refer the reader to the Online Appendix.

Workers

Workers of each type maximize utility subject to the budget constraint and constraints on employment flows imposed by SAM frictions (1.6). Their problem can be summarized

we could have modeled the whole economy with a single representative household whose members have different skills. This would not change the dynamics in any significant way. However, we opted for making the distinction between household types, as this leads to a more "natural" distribution of steady state consumption corresponding to skill differences, rather than the arbitrary uniform allocation under a single household. This also leaves open the possibility of readily introducing incomplete markets *between* different skill types and some welfare analysis (for an attempt in this direction, see Dolado, Motyovszki and Pappa (2018)).

[11] Notice that all variables in the model are naturally functions of the shock history, to ease notation we are not making this explicit in the equations describing the model.

as follows:

$$\max_{\{c_t^k, z_{t+1}^k(s^t, s_{t+1}), u_t^k, n_{t+1}^k\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^k)^{1-\eta}}{1-\eta} + \Phi^k \frac{\left(1 - n_t^k - u_t^k\right)^{1-\xi}}{1-\xi} \right] \qquad k \in H, L$$

$$c_t^k + t_t^k + \frac{1}{P_t} \sum_{s_{t+1} \in \mathcal{S}} q_{t,t+1}(s_{t+1}|s^t)\ z_{t+1}^k(s^t, s_{t+1}) \leq w_t^k n_t^k + \varkappa^k u_t^k + \frac{z_t^k(s^t)}{P_t} \qquad k \in H, L$$

$$n_{t+1}^k = (1 - \sigma^k) n_t^k + \mu_t^k u_t^k \qquad k \in H, L$$

The elasticity of labor supply is influenced by ξ, while Φ^k governs the weight of the leisure of each skill type in their utility. Workers can trade in state-contingent Arrow securities $z_{t+1}^k(s^t, s_{t+1})$. Employed members of the household bring home a real wage w_t^k, while unemployed members get inflation-indexed unemployment benefits $\varkappa^k$ which are assumed to be time invariant.

The first-order conditions to the workers' problem can be found in the Online Appendix, which describes consumption-saving and labor supply decisions[12]. Due to complete financial markets, there is full insurance, i.e. consumption inequality across different skill types $k \in \{H, L\}$ does not fluctuate, but real wages, labor force participation and employment do in general move differently. Note, however, that even under complete markets there is consumption inequality since different households enjoy different consumption *levels* in the steady state (through different wages, rent, benefits, initial wealth).

1.3.3 Intermediate goods firms

A continuum of perfectly competitive firms produces a homogeneous intermediate good y_t, using high and less-skilled labor N_t^k and aggregate capital $K_t = \varphi^E k_t$ as inputs. Just like in the households' problem, N_t^k are state variables, given by matches and employment levels from the previous period. It is only next period's employment levels N_{t+1}^k which can be influenced by choosing how many vacancies v_t^k to post. This influence is subject to the same SAM frictions as in the case of the household, however, (1.6) is now reformulated by plugging in (1.4) to reflect how vacancies are affecting the number of matches.[13] Therefore,

[12] The law of motion for employment (1.6) is expressed here in *per capita* terms, i.e. divided by the mass of workers φ^k. Using laws of motion for employment (1.6) means that the household does not take the number of matches as given, but takes into account the effect of its unemployment decisions on matches, at least partially. It ignores, however, the full effect of its decisions on matches. In particular, its unemployment decisions also affect hiring probabilities μ_t^k through (1.5), which the household takes as given in the above formulation; see Brückner and Pappa (2012). The full effect would be taken into account only if we replaced (1.2) into (1.6), instead of using the formulation with hiring probabilities.

[13] Also, as in the case of the household, the firm does not take into account the full effect of its vacancy choices on the number of matches. In particular, it disregards the effect on vacancy filling probabilities through (1.4).

the firm's problem becomes dynamic:

$$V^F\left(N_t^H, N_t^L, \boldsymbol{s_t}\right) = \max_{K_t, N_{t+1}^H, N_{t+1}^L, v_t^H, v_t^L} x_t\, F\left(K_t, N_t^H, N_t^L\right) - r_t K_t - \sum_{k\in\{H,L\}} \left(w_t^k N_t^k + \kappa\, v_t^k\right) + \\ + E_t \Lambda_{t+1}\, V^F\left(N_{t+1}^H, N_{t+1}^L, \boldsymbol{s_{t+1}}\right)$$

$$N_{t+1}^k = (1-\sigma^k) N_t^k + \nu_t^k v_t^k \qquad k \in \{H, L\}$$

where $\Lambda_{t+1} = \beta\, (c_{t+1}^E / c_t^E)^{-\eta}$ is the stochastic discount factor of the entrepreneurs, reflecting the ownership of the firm. The real price of intermediate goods x_t is taken as given by the firm – this constitutes real marginal costs for retail firms. Posting vacancies has a unit cost of κ.

The production function is defined as

$$Y_t = F\left(K_t, N_t^H, N_t^L\right) = A_t \left[\phi \left[\lambda\, K_t^\gamma + (1-\lambda)(N_t^H)^\gamma\right]^{\frac{\alpha}{\gamma}} + (1-\phi)(N_t^L)^\alpha\right]^{\frac{1}{\alpha}} \tag{1.7}$$

Following Krusell et al. (2000) and Lindquist (2004), our baseline production function (1.7) is a nested CES composite of production factors, where we can separately control the elasticity of substitution between capital and high skilled labor, ϱ_{k,n^H} on the one hand, and between capital and low skilled labor ϱ_{k,n^L} on the other. The structure of the nesting implies that the elasticity of substitution between high and low-skilled labor must be the same as between capital and low-skilled labor $\varrho_{n^H,n^L} = \varrho_{k,n^L}$. The capital intensity of the "skilled input bundle" is controlled by λ, while ϕ represents the "skill intensity" of total production. The elasticities of substitution are governed by parameters γ and α, and can be defined as $\varrho_{k,n^H} = \frac{1}{1-\gamma}$, and $\varrho_{k,n^L} = \varrho_{n^H,n^L} = \frac{1}{1-\alpha}$. We restrict these elasticities to be positive in order to maintain strict quasi-concavity of the production function. This means that $\alpha, \gamma \leq 1$.

Capital-skill complementarity (CSC) is captured in the following way:

- $0 < \varrho_{k,n^H} < 1$ represents CSC ($\gamma < 0$, with larger absolute values corresponding to a higher degree of complementarity)
- $1 < \varrho_{k,n^L}$ shows the substitutability of less-skilled labor with the skilled inputs ($0 < \alpha \leq 1$ with larger values corresponding to a higher degree of substitution and $\alpha = 1$ meaning perfect substitutes)
- $\varrho = 1$ corresponds to a CD production function, with a unit elasticity of substitution ($\gamma, \alpha = 0$)

This follows from Koczan, Lian and Dagher (2017), who define production factors as complements whenever their elasticity of substitution is below unity. In such a case, a reduction in the relative price of one of the factors increases the income share of the other factor, and vice versa for substitutes. Lindquist (2004) uses a less strict definition: based on the following formula, he shows that as long as $1 \geq \alpha > \gamma$, a rise in the stock of capital will *ceteris paribus* raise the relative marginal product of skilled labor $\frac{F^H_{N,t}}{F^L_{N,t}}$, and this is what he calls *CSC effect.*[14] It also follows that a rise in low-skill employment relative to high-skill employment raises the marginal product of high-skilled workers, which is called the *relative supply effect.*

$$\frac{F^H_{N,t}}{F^L_{N,t}} = (1-\lambda)\frac{\phi}{1-\phi}\left[\lambda\left(\frac{K_t}{N^H_t}\right)^{\gamma} + (1-\lambda)\right]^{\frac{\alpha-\gamma}{\gamma}}\left(\frac{N^L_t}{N^H_t}\right)^{1-\alpha}$$

In order to see the effects of CSC relative to the case where it is absent, we can change parameters γ and α, but we also need a different benchmark production function, the structure of which allows for controlling the elasticity of substitutions separately between capital and any labor input, and between the two different types of labor.

$$\widetilde{F}\left(K_t, N^H_t, N^L_t\right) = A_t K^{\iota}_t \left[\varpi (N^H_t)^{\upsilon} + (1-\varpi)(N^L_t)^{\upsilon}\right]^{\frac{1-\iota}{\upsilon}} \tag{1.8}$$

where, as is well known, the assumed CD structure between capital and composite labor implies that these inputs are neither complements nor substitutes, and their income shares are constant, with ι denoting the share of capital. The two different types of labor are perfect substitutes when $\upsilon = 1$. Notice that, with equal intensity $\varpi = 0.5$, labor is basically homogeneous.

Finally, aggregate TFP in (1.7) follows an exogenous AR(1) process: $\ln A_t = \rho_a \ln A_{t-1} + \varepsilon^a_t$.

1.3.4 Wage bargaining

Workers and intermediate firms split the surplus from a match according to Nash-bargaining. Wages are negotiated separately on the high and low-skill labor markets.

$$\max_{w^k_t} \vartheta^k \ln\left(V^{E,k}_t\right) + (1-\vartheta^k)\ln\left(V^{F,k}_t\right) \qquad k \in H, L$$

$$V^{E,k}_t = \frac{\partial \mathcal{L}}{\partial n^k_t} = \lambda^{c,k}_t w^k_t - \Phi^k\left(l^k_t\right)^{-\xi} + (1-\sigma^k)\lambda^{n,k}_t$$

$$V^{F,k}_t = \frac{\partial V^F(N^k_t)}{\partial N^k_t} = x_t F^k_{N,t} - w^k_t + (1-\sigma^k)\frac{\kappa}{\nu^k_t}$$

[14] Unlike the work of Koczan, Lian and Dagher (2017) and Lindquist (2004), in our model SAM frictions establish a wedge between the wage and the marginal product of labor, but the two are still closely related. Therefore, the above argument can be applied to the skill premium as well.

where $V_t^{E,k}$ is the marginal value for the household of being employed, and $V_t^{F,k}$ is the value for the firm of a filled job. $\mathcal{L}$ is the Lagrangian of the household, V^F is the value function of the firm, while the weights ϑ^k represent the bargaining power of workers in each labor market.

The solution to this problem yields the real wage w_t^k:

$$w_t^k = \vartheta^k \left[x_t F_{N,t}^k + (1-\sigma^k)\frac{\kappa}{\nu_t^k} \right] + \frac{1-\vartheta^k}{(c_t^k)^{-\eta}} \left[\Phi^k \left(l_t^k \right)^{-\xi} - (1-\sigma^k)\lambda_t^{n,k} \right] \tag{1.9}$$

where $\lambda_t^{n,k}$ is the Lagrange-multiplier on the SAM-constraint (1.6) in the workers' problem.

1.3.5 Retail firms

We have a continuum $i \in [0,1]$ of monopolistically competitive retail firms, each of which buys $y_t(i)$ amount of the homogeneous intermediate good Y_t, and produces a differentiated product $y_t^r(i)$ with a linear technology, i.e. $y_t^r(i) = y_t(i)$. These differentiated products are then assembled to become final goods Y_t^r according to a CES aggregator:

$$Y_t^r = \left[\int_0^1 y_t^r(i)^{\frac{\epsilon-1}{\epsilon}}\, di \right]^{\frac{\epsilon}{\epsilon-1}} = \left[\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}}\, di \right]^{\frac{\epsilon}{\epsilon-1}} = Y_t$$

where the last equality follows from symmetry (each retailer buys exactly the same amount of intermediate $y_t(i)$), so that the r superscript can be dropped as there will be as much of the final goods as of intermediate goods. Finally, ϵ denotes the elasticity of substitution between different products.

Retail firms take as given the relative price x_t of the intermediate good, which is basically their real marginal cost. This depends neither on i (since intermediate goods are homogeneous, so retail firms are competitive *buyers*), nor on the amount of goods used (since all the retail firms are infinitesimally small). Due to differentiation, retailers have pricing power in setting the price of their own product $p_t(i)$, but take the aggregate price level P_t as given. The latter is defined as $P_t = \left[\int_0^1 p_t(i)^{1-\epsilon}\, di \right]^{\frac{1}{1-\epsilon}}$.

In setting their price, retailers are constrained by Calvo-type nominal rigidities, so that, in every given period, a fraction χ of them cannot adjust prices. The $(1-\chi)$ fraction of firms, who are able to adjust prices in a given period, will choose the new price, $(p_t^*(i))$, so as to maximize the real present value of expected future profits, taking into account nominal rigidities and also the price elastic demand of households.

$$p_t^*(i) \equiv \arg\max_{p_t(i)} \; E_t \sum_{s=0}^{\infty} \chi^s \, \Lambda_{t+s} \left[\frac{p_t(i)}{P_{t+s}} - (1-\tau)x_{t+s} \right] y_{t+s}(i)$$

$$y_{t+s}(i) = \left(\frac{p_t(i)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s}$$

where τ is a production subsidy used by the government to eliminate the static distortion coming from monopolistic competition. Due to symmetry across retailers, all of them will choose the same price $p_t^* \equiv p_t^*(i)$. The solution to this problem yields:

$$p_t^* = \underbrace{\frac{(1-\tau)\epsilon}{\epsilon-1}}_{(1-\tau)\mathcal{M}} E_t \; \frac{\sum_{s=0}^{\infty} \chi^s \; \Lambda_{t+s} \; y_{t+s}(i) \; \overbrace{P_{t+s}\, x_{t+s}}^{MC_t}}{\sum_{s=0}^{\infty} \chi^s \; \Lambda_{t+s} \; y_{t+s}(i)}$$

Calvo-rigidities imply that the evolution of the aggregate price level follows

$$P_t = \Big[(1-\chi)(p_t^*)^{1-\epsilon} + \chi \; P_{t-1}^{1-\epsilon}\Big]^{\frac{1}{1-\epsilon}}$$

1.3.6 Monetary and fiscal policies

Monetary policy sets short term nominal interest rates on risk-free bonds following a standard Taylor rule, reacting to inflation deviations form target and (potentially) also to the deviations of aggregate output from its steady state value (the latter denoted with upper-bar).

$$\frac{R_t}{\overline{R}} = \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\zeta^\pi} \left(\frac{Y_t}{\overline{Y}}\right)^{\zeta^y} e_t \tag{1.10}$$

where e_t captures a monetary policy shock which follows an AR(1) process $\ln e_t = \rho_R \ln e_{t-1} + \varepsilon_t^R$.

Fiscal policy involves an exogenous (and wasteful) government consumption G_t, a production subsidy τ to retailers, and inflation indexed unemployment benefits $\varkappa^k$, all of which are financed by lump-sum taxes T_t, so that the government runs a balanced budget in every period.

$$T_t = G_t + \tau x_t Y_t + \sum_{k\in\{H,L\}} \varkappa^k \, U_t^k \tag{1.11}$$

$$\ln G_t = (1-\rho_g)\ln(\Gamma\,\overline{Y}) + \rho_g \ln G_{t-1} + \varepsilon_t^g \tag{1.12}$$

where Γ is the steady state output share of government consumption. The distribution of lump-sum taxes is assumed to be equal, i.e. $t_t^k = T_t$ for $k \in \{H, L, E\}$, so that we have $T_t = \sum_k \varphi^k t_t^k = T_t \sum_k \varphi^k$.

1.3.7 Market clearing

Since households can only trade assets with each other, and not with the government or foreign agents, the markets for each Arrow security clear as follows:

$$\sum_{k\in\{E,H,L\}} \varphi^k z_{t+1}^k(s^t, s_{t+1}) = 0 \qquad\qquad \text{for } \forall \; s^t, s_{t+1} \in \mathcal{S}$$

Combining the budget constraints of the households and the government (and using the asset market clearing condition) we get the goods market clearing condition. Final output is used for consumption, investment, government expenditures and posting vacancies.

$$Y_t = C_t + I_t + G_t + \sum_{k \in \{H,L\}} \kappa^k v_t^k \tag{1.13}$$

where $C_t = \sum_{k \in (H,L,E)} \varphi^k c_t^k$ and $I_t = \varphi^E i_t$.

1.4 Calibration

To make our analysis comparable to the existing theoretical models, we consider the model period to be one quarter. Parameters are calibrated to match targets related to the steady-state values of participation and unemployment rates – separately for the high and low-skill labor markets. In doing this, we set values so that they track the pre-crisis averages for the U.S. As explained earlier, high-skilled workers are regarded to have some college education. According to this classification 21 percent of our households are high-skilled workers, 69 percent are low-skilled workers and the remaining 10 percent is the share of entrepreneurs in the economy. In terms of the model variables, the targets correspond to

$$partic^k \equiv \frac{N^k + U^k}{\varphi^k}$$
$$unemp^k \equiv \frac{U^k}{N^k + U^k}$$

Parameters Φ^k, ϑ^k for $k = H, L$, are calibrated so as to match the above targets. The exact values can be seen in the second panel of Table 3.1, with blue for targeted steady states and with red for calibrated parameters.

We assume symmetry in the matching elasticity ς for the matching functions, and in vacancy posting costs κ. The asymmetry in SAM frictions is captured by skill-specific parameters: we use the average quarterly values for the separation rates of high and low-skilled workers between 1979 and 2007 reported in Wolcott (2018) which, in line with Fallick and Fleischman (2004), results in $\sigma^H < \sigma^L$. Efficiencies are assumed to comply with $\psi^L < \psi^H$, in line with the evidence in Barnichon and Figura (2015), Wolcott (2018) and Eeckhout and Kircher (2018) that propose a theory of the labor market where firms choose both the size and quality of the workforce, and show that, in a competitive search equilibrium with large firms, high-skilled workers enjoy higher matching probabilities than less-skilled workers. Wolcott (2018), using the same definition as ours to classify low and high-skilled workers, reports a fall in the gap in labor tightness between the two groups

Parameters					
separation rate, H	σ^H	0.0245			
separation rate, L	σ^L	0.0562	capital intensity of skills	λ	0.3500
matching efficiency, H	ψ^H	0.7200	substitutability bw (N^H, K) and N^L	α	0.4000
matching efficiency, L	ψ^L	0.4550	capital-skill complementarity	γ	-0.4902
matching elasticity	ς	0.5000	capital adjustment costs	ω	4.0000
population weigth, H	φ^H	0.2100	depreciation rate	δ	0.0100
population weigth, L	φ^L	0.6900	discount factor	β	0.9900
population weigth, E	φ^E	0.1000	(inverse) intertemporal elasticity	η	2.0000
vacancy posting costs	κ	0.1300	labor supply elasticity parameter	ξ	4.0000
unemployment benefits, H	$\varkappa^H$	0.2875	elasticity of substitution bw goods	ϵ	6.0000
unemployment benefits, L	$\varkappa^L$	0.2875	nominal rigidities (Calvo)	χ	0.8000
TFP shock persistence	ρ_a	0.8500	st.st. output share of government	Γ	0.2000
fiscal shock persistence	ρ_g	0.7000	Taylor-coefficient on inflation	ζ^π	1.5000
monetary shock persistence	ρ_R	0.7000	Taylor-coefficient on output	ζ^y	0.0000
Parameters targeting st.st.			**Targeted steady states**		
utility weight of leisure, H	Φ^H	0.0516	participation rate, H	$partic^H$	0.6900
utility weight of leisure, L	Φ^L	0.2157	participation rate, L	$partic^L$	0.6600
bargaining power, H	ϑ^H	0.6955	unemployment rate, H	$unemp^H$	0.0280
bargaining power, L	ϑ^L	0.3740	unemployment rate, L	$unemp^L$	0.0780
production subsidy	τ	0.1667	real marginal costs	x	1.0000
skill intensity of production	ϕ	0.4273	wage premium	w^H/w^L	1.5306
Non targeted steady states					
market tightness, H	θ^H	1.3954	ratio of job finding rates	μ_t^H/μ_t^L	1.2803
market tightness, L	θ^L	2.1317			

Table 1.1: Parameters and selected steady state values. The 6 blue steady-state values are targeted by 6 red parameters.

of workers between 1979 and 2007. On average the tightness in the low-skill labour market during this period in the CPS data equals 2.13. The gap in tightness between the high and low-skilled depends on the definition of high-skill vacancies. Although not targeted, our steady state values for tightness match the estimates of Wolcott (2018) when the cutoff used to define high skilled vacancies in her sample equals 0.7. Also, these parameter values result in larger frictions for the low-skilled workers, making their steady-state hiring probabilities lower than those of the high skilled: $\overline{\mu}^L < \overline{\mu}^H$. As a result, the steady state ratio of job finding rates for high vs. low-skilled workers equals 1.28 in our calibration while in the CPS data this ratio equals 1.14 (see Wolcott (2018)). Similarly, vacancy filling probabilities are higher for high-skill vacancies, $\overline{\nu}^L < \overline{\nu}^H$. These values imply that it is relatively more costly for low-skilled households to increase their participation.

The non-equal share of different skill types in the population φ^k and our skill-specific steady- state targets for employment variables result in further asymmetries for calibrated parameters. In particular, given their lower participation rate, the weight of leisure (inactivity) in the households' utility function will be higher for low-skilled workers $\Phi^H < \Phi^L$ (Appendix A.3.5 also presents a calibration with $\Phi^H = \Phi^L$ and shows that this does not affect our main results). Furthermore, the bargaining power of these workers will be lower than that of high-skilled workers, $\vartheta^L < \vartheta^H$, implying that low skilled workers capture a smaller share of the surplus as wage. This is in line with the structural estimates of these parameters provided by Cahuc, Postel-Vinay and Robin (2006) using matched employer-employee data for France. The latter feature mitigates the relative costliness of low-skilled workers as the firm is able to capture a larger share of the surplus created by filling a low-skill vacancy. In addition, and given the wage premium, a similar real amount of unemployment benefits $\varkappa^k$ results in some asymmetry in the wage replacement rate $\varkappa^H/w^H < \varkappa^L/w^L$. Such asymmetry is supported by the data. According to Fischer (2017), income replacement rates differ across US states with the norm across states being a 40-60 percent replacement rate with a maximum ceiling, implying that effective replacement rates are likely to be lower for workers with higher earnings.

Heterogeneity across workers not only originates from the labor market, but it also has to do with their different role in production, as captured by CSC. We set elasticities of substitution based on the estimates provided by Krusell et al. (2000) for the proposed CSC production function. This means $\frac{1}{1-\gamma} = 0.67$ and $\frac{1}{1-\alpha} = 1.67$, which makes high-skill labor complementary to capital, while low-skill labor becomes substitute. Under our baseline parameterization we calibrate a steady-state skill premium of 53%, which corresponds to the average value in our data. Finally, the production subsidy is set to eliminate the static distortion coming from monopolistic competition $\tau = 1/\epsilon$ which makes the steady state real marginal cost (markup) x equal to one.

Other parameters are set to standard values in the literature. In the Online Appendix we further investigate the sensitivity of our results when we vary parameters– such as the labor supply elasticity, the degree of nominal rigidities, capital adjustment costs and the coefficients in the Taylor rule.

1.5 Theoretical results

We log-linearize the model around its deterministic steady-state, and compute IRFs to various shocks under different scenarios and parameterizations. The details of the calculation of the steady state can be found in the Online Appendix.

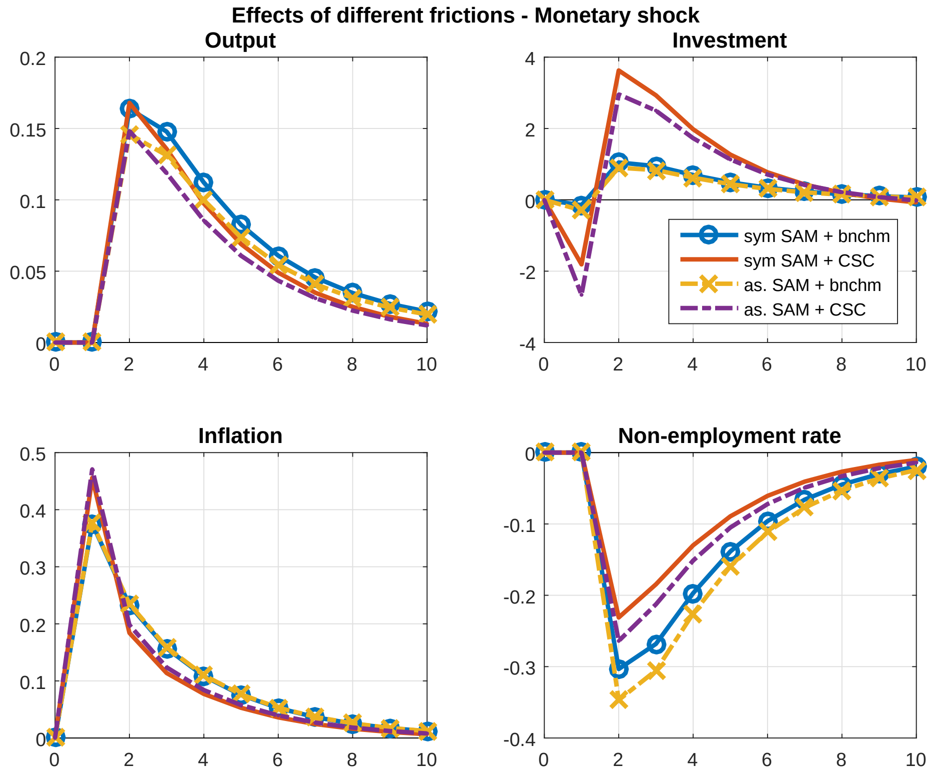

Figure 1.2: Effects of SAM asymmetry and CSC – aggregate variables

1.5.1 The effect of expansionary monetary policy shocks

An expansionary monetary policy shock (100 bp. cut in the annualized nominal interest rate) stimulates aggregate demand, which leads to expanding output and inflationary pressures (see purple dashed lines in Figure 1.2). Reacting to stronger demand, firms increase their demand for capital and labor, which leads to rising investment and higher employment, together with higher wages and a larger rent on capital (notice that the initial drop in capital investment is due to crowding out by higher investment into opening vacancies). What happens in our labor market with SAM frictions is that firms start posting more vacancies while households raise their labor market participation in response to better job finding prospects. However, employment cannot suddenly react much (i.e. it is not a jump variable) being subject to SAM frictions according to (1.6), which is why most of the adjustment takes place through higher wages. This is a standard result in SAM models. As demand pressures run up against SAM frictions, the surplus from a match increases a lot. In other words, firms are willing to agree to a much higher wage during the Nash bargaining, since they are compensated by higher revenues.

Heterogeneity in our labor market (asymmetric SAM) and different roles in production due to CSC imply that high and low-skilled workers will not experience the same increase in labor income as a result of the interest rate shock. Under our baseline scenario an expansionary monetary policy shock leads to a rise in the skill premium $\frac{w_t^H}{w_t^L}$ of about

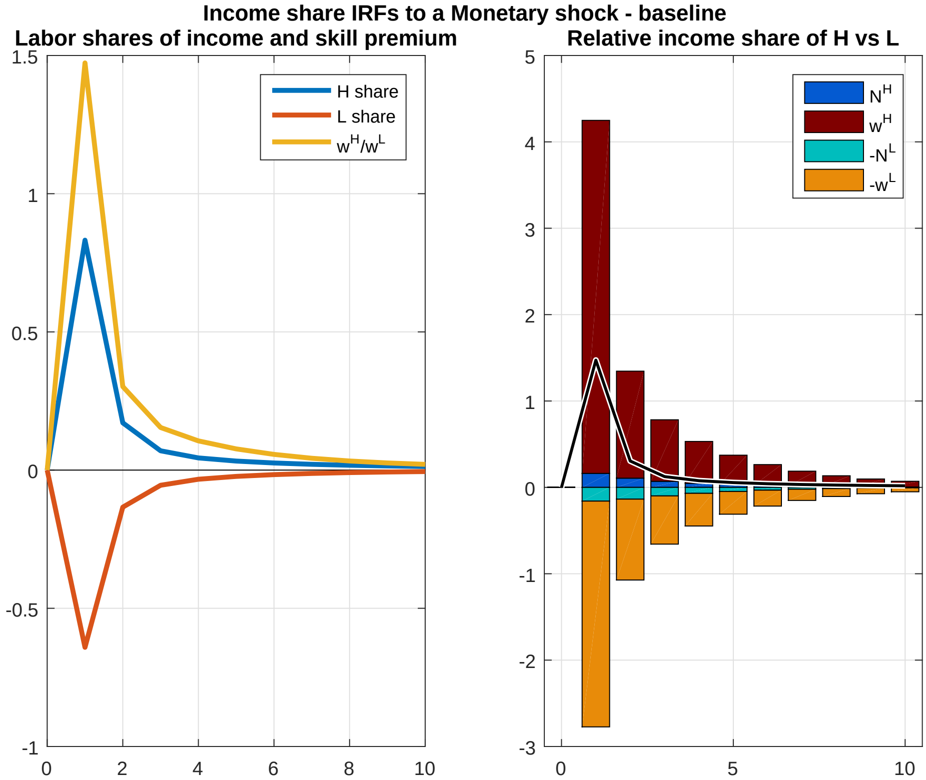

Figure 1.3: Income shares of labor types

1.5%. As shown in the left panel of Figure 1.3, this comes together with an increase in the income share $\frac{w_t^H N_t^H}{Y_t - \kappa v_t}$ for the high skilled at the expense of a *decreasing* income share for the low skilled. This means that the benefits of a monetary easing are not evenly distributed, with high-skilled workers getting *relatively* more of the increase in real income than do low-skilled workers – even though both types are better off in absolute terms. To the extent that the low skilled are poorer to begin with (as reflected by a steady-state skill premium of 53%), a monetary expansion raises labor income inequality. The right panel of Figure 1.3 shows that the rise in the relative income share of high skilled labor $\frac{w_t^H N_t^H}{w_t^L N_t^L}$ is driven mainly by an increase in wages, while changes in employment have a negligible effect.

It is not clear, however, what the source of the increase in inequality is. In order to separately identify the effects of asymmetric SAM frictions on the one hand, and CSC on the other, we construct a benchmark case with *symmetric* SAM frictions and a standard CD production function, where high and low-skill labor inputs are perfect substitutes (as defined by equation (1.8)). Then we add either SAM asymmetry only or CSC only, so as to compare the effect of each of these two features against our benchmark. Finally, we add both sources of heterogeneity together to retrieve our original (baseline) scenario.

The results of this exercise are shown in Figures 1.2 for the aggregate variables. Blue circled lines represent our symmetric benchmark scenario. The results after introducing

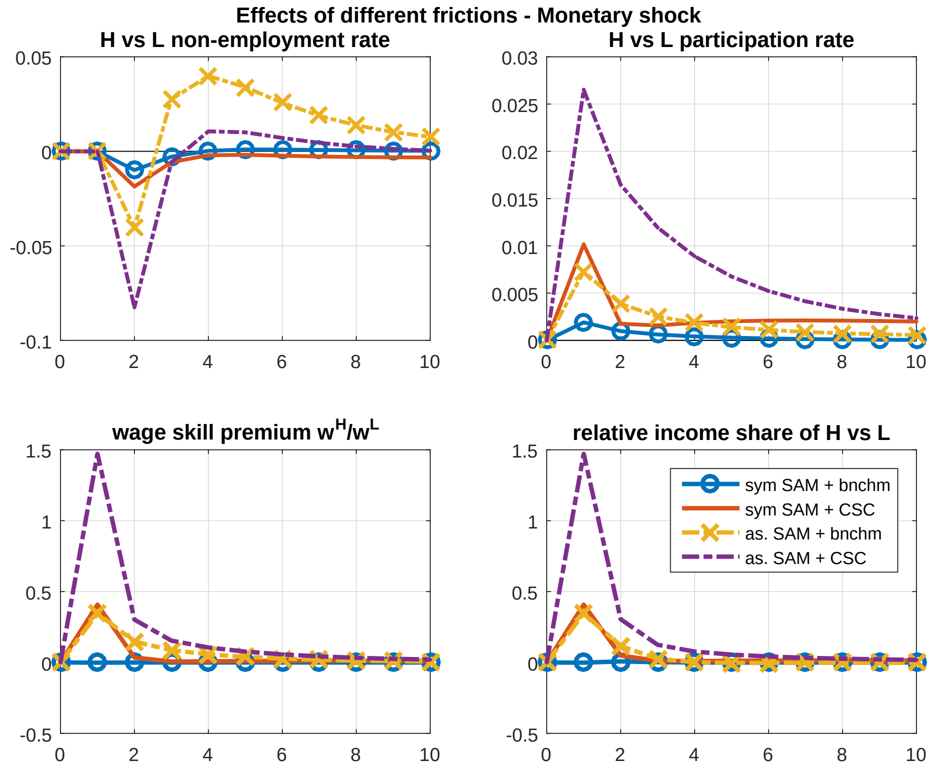

Figure 1.4: Effects of SAM asymmetry and CSC – relative variables

CSC are displayed in red solid lines. Changing the characteristics of the production function has an influence on the IRFs of aggregate variables. There is a somewhat smaller reduction of the non-employment rate $\frac{U_t+L_t}{N_t+U_t+L_t}$ and larger responses of investment when we assume a CSC production function. The effect of introducing only SAM asymmetries (and keeping the benchmark CD production function) is plotted with yellow crossed lines. IRFs of *aggregate* variables to expansionary monetary shocks are essentially identical to the benchmark (blue circled line) case, suggesting that labor market heterogeneity does not have significant consequences at the macro level when the benchmark production technology is CD.

In Figure 1.4 we depict relative measures between high and low-skilled workers. The relative income share of high-skilled workers increases in the presence of CSC (red solid line). Similarly, SAM asymmetries induce a comparable increase on the responses of high vs. low-skill *relative* variables (yellow crossed line). Both assumed asymmetries increase the skill premium roughly by 0.4%. The non-employment rate of high-skilled workers falls more relative to the case of no SAM asymmetries and their participation rate changes similarly in both scenarios. Yet, the magnitudes of these employment changes are small, implying that most of the rise in the relative labor income share of high-skilled workers is driven by the wage premium. Finally, we introduce CSC on top of SAM asymmetry, leading to responses which are plotted with purple dashed lines in Figure

1.4. The *interaction* of asymmetric SAM frictions with CSC *magnifies* the effect of the latter, raising the skill premium after an expansionary monetary shock by 1.5%. In other words, introducing CSC on its own, or SAM asymmetry alone leads to only a modest rise in the relative income share, while their interaction has a larger combined effect than the simple sum of their individual effects. We explore the underlying mechanism behind those responses in the next section.

1.5.2 Dissecting the mechanism

As we show in detail in the Online Appendix, log-linearizing the wage bargaining equation (1.9), and using "hats" to denote the log deviation of a variable from its steady state, i.e. $\widehat{f}_t = \log f_t - \log f$, we can express the log deviations of the real wage of each skill type, k, as:

$$\widehat{w}_t^k = \underbrace{\frac{\vartheta^k x F_N^k}{w^k}}_{\alpha_x^k} \widehat{x}_t + \underbrace{\frac{\vartheta^k x F_N^k}{w^k}}_{\alpha_{F_N}^k} \widehat{F}_{N,t}^k + \underbrace{\frac{1-\sigma^k}{w^k}\left[(1-\varsigma)\frac{\vartheta^k \kappa^k}{\nu^k} + \varsigma \frac{(1-\vartheta^k)\lambda^{n,k}}{\lambda^{c,k}}\right]}_{\alpha_\theta^k} \widehat{\theta}_t^k +$$

$$+ \underbrace{\frac{\eta(1-\vartheta^k)}{w^k}\left[\frac{\Phi^k (l^k)^{-\xi}}{\lambda^{c,k}} - (1-\sigma^k)\left(\frac{\lambda^{n,k}}{\lambda^{c^k}} + \frac{\varkappa^k}{\mu^k}\right)\right]}_{\alpha_c^k} \widehat{c}_t^k +$$

$$+ \underbrace{\xi \frac{(1-\vartheta^k)\ \Phi^k (l^k)^{-\xi}}{\lambda^{c,k}\ w^k}\left[\frac{(1-\sigma^k)}{\mu^k} - 1\right]}_{\alpha_l^k} \widehat{l}_t^k =$$

$$= \alpha_x^k\, \widehat{x}_t + \alpha_{F_N}^k\, \widehat{F}_{N,t}^k + \alpha_\theta^k\, \widehat{\theta}_t^k + \alpha_c^k\, \widehat{c}_t^k + \alpha_l^k\, \widehat{l}_t^k \tag{1.14}$$

Using equation (1.14) we can express the skill premium as:

$$\widehat{w}_t^H - \widehat{w}_t^L = \left(\alpha_x^H - \alpha_x^L\right)\widehat{x}_t + \left[\alpha_{F_N}^H\ \widehat{F}_{N,t}^H - \alpha_{F_N}^L\ \widehat{F}_{N,t}^L\right] + \left[\alpha_\theta^H\ \widehat{\theta}_t^H - \alpha_\theta^L\ \widehat{\theta}_t^L\right] +$$
$$+ \left[\alpha_c^H\ \widehat{c}_t^H - \alpha_c^L\ \widehat{c}_t^L\right] + \left[\alpha_l^H\ \widehat{l}_t^H - \alpha_l^L\ \widehat{l}_t^L\right] \tag{1.15}$$

Equation (1.15) enables us to decompose the dynamics of the skill premium into the contributions of the various factors which drive this gap in the face of an expansionary monetary policy shock. Naturally, all dynamic changes are ultimately caused by the exogenous shock itself. This exercise rather sheds light on the different *channels* through which the shock propagates and affects wages. In particular, from the firm's side (through labor demand and the firm's surplus) the skill premium dynamics is affected by: demand pressures, as captured by the real marginal cost of retailers $\widehat{x}_t$ (the real sales price for intermediate firms); dynamic changes of skill-specific marginal products of labor $\widehat{F}_{N,t}^k$; and the tightness of the respective labor markets $\widehat{\theta}_t^k$. From the household's side (through labor supply and workers' surplus) skill premium dynamics are affected again by labor market tightness, by differing wealth effects, captured by $\widehat{c}_t^k$, and by labor force participation $\widehat{l}_t^k$.

The results of this decomposition are shown in Figure 1.5, which depicts how movements in each of these five variables contribute to the dynamics (IRFs) of the real wage $\widehat{w}_t^k$, $k = H, L$ and the skill premium $\widehat{w}_t^H - \widehat{w}_t^L$. As can be observed, the dominant factor in driving the response of real wages is the rise in aggregate demand pressures, as represented by movements in the real marginal costs for retailers $\widehat{x}_t$ (blue bars). Intuitively, as expanding aggregate demand raises the relative price at which intermediate goods can be sold, the surplus from matching workers to jobs increases, some of which will be reflected in higher real wages.

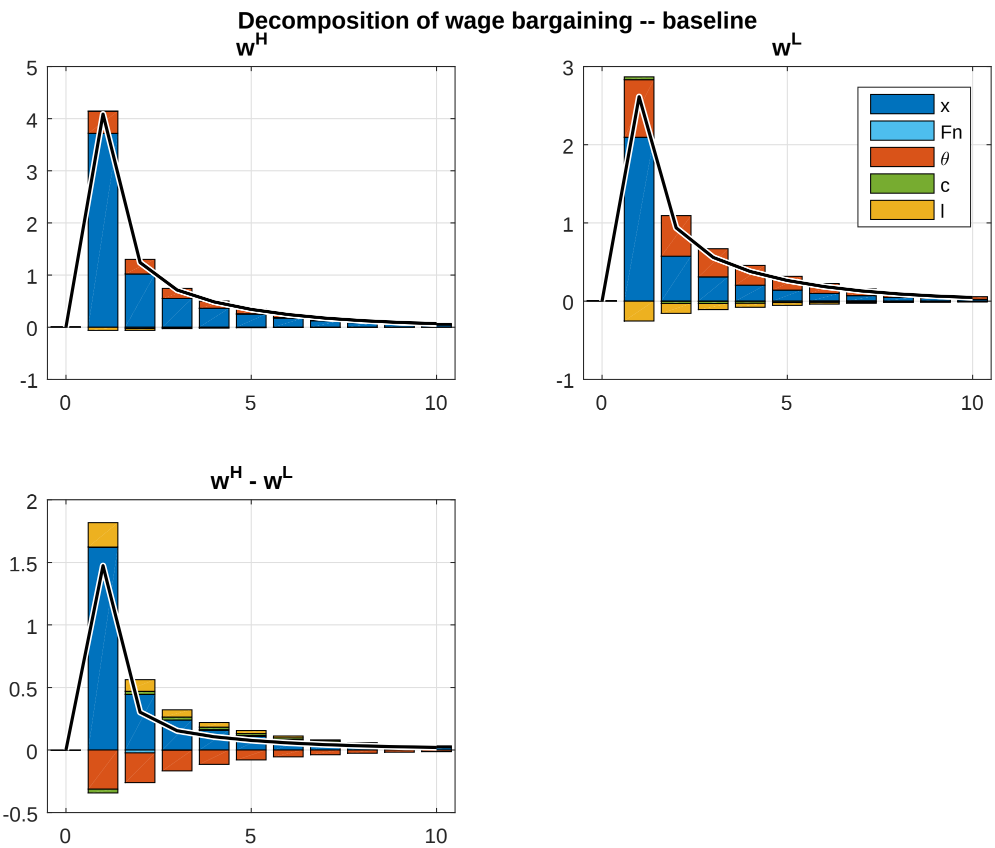

Figure 1.5: Decomposing real wage $\widehat{w}_t^k$ and wage premium $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics based on equations (1.14) and (1.15).

As regards the dynamics of the skill premium, the bottom left panel of Figure 1.5 shows that the rise in aggregate demand pressures is again the factor contributing the most in the increases of the skill premium after a monetary expansion. This suggests that the increase in this wage gap is achieved predominantly through changes in the firm's surplus, which lead to adjustments in labor demand. By contrast, changes in labor supply and in worker's surplus (as captured by $\alpha_c^k\ \widehat{c}_t^k$ and $\alpha_l^k\ \widehat{l}_t^k$) play a comparatively smaller role. Labor market tightness $\widehat{\theta}_t^k$ (through its effect on vacancy filling and job finding rates) contributes noticeably, but still by a much lower amount than aggregate demand pressures. Notice that labor-market tightness pushes real wages upwards but its impulse is higher for less-skilled than for high-skilled workers. This explains why tighter labor markets on their own somewhat *mitigate* the rise in the skill premium.

We next conduct a similar decomposition of the skill premium for our alternative scenarios (without asymmetric SAM and/or CSC) to identify the channels through which the combined introduction of both features operates. Once more, for all the alternative scenarios considered, the most important contributor in (1.15) is still the term associated with aggregate demand pressures: $\left(\alpha_x^H - \alpha_x^L\right) \widehat{x}_t$. What significantly differ across scenarios are the two components of this term: i) the responsiveness of the skill premium to *given* demand pressures $\left(\alpha_x^H - \alpha_x^L\right)$; and ii) the dynamic responses of demand, captured by $\widehat{x}_t$.

First, the responsiveness coefficient $\left(\alpha_x^H - \alpha_x^L\right)$ increases *ceteris paribus* with more SAM-asymmetry and/or with a CSC production function, as illustrated in the bottom right panel of Figure 1.6. The intuition is that a skill-intensive production structure, like our baseline CSC production function, raises the steady-state marginal product of high-skilled labor (and hence, the firm's surplus from skilled matches) relative to less-skilled labor, making firms tilt their hiring towards skilled workers. In the same vein, under asymmetric SAM frictions, firms prefer to hire workers with less frictions (the steady-state firm's surplus out of high-skilled jobs is larger). Lower matching efficiency in the unskilled sector makes it relatively more costly for firms to open low-skill vacancies and for households to enter this segment of the labor market. Likewise, the value of an unskilled match is relatively lower since the resulting job is more likely to be terminated and a subsequent match is less likely to take place. Notice that all of the above effects are due to differences in the steady-state values determining the responsiveness coefficients, and that the joint contribution of SAM asymmetry and CSC in this respect seems to be additive (bottom right panel in Figure 1.6).

However, *in addition* to the differences generated by steady-state properties of the model, CSC also introduces a *dynamic* demand amplification channel: apart from CSC making the skill premium more responsive to a *given* increase in demand pressures (through the coefficient $\alpha_x^H - \alpha_x^L$), it also makes the rise in aggregate demand pressures $\widehat{x}_t$ *themselves* stronger, as evidenced by the top left panel in Figure 1.6. In other words, the dynamics of marginal costs also depend crucially on the assumed production function: with CSC, the reaction of $\widehat{x}_t$ doubles in response to the same shock – and it does so independently of the assumed symmetry of SAM frictions.

The intuition for this result is that, with CSC, the initial increase in high-skilled employment makes complementary capital more productive, encouraging a further rise in

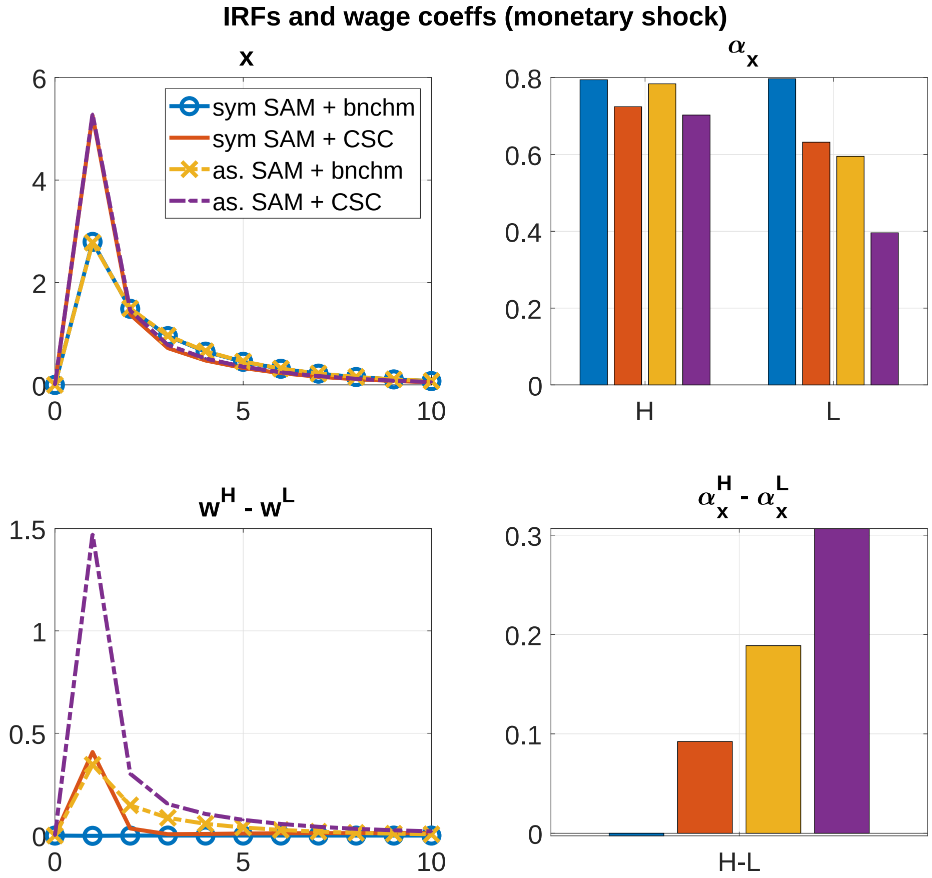

Figure 1.6: Comparing α_x^k and $\left(\alpha_x^H - \alpha_x^L\right)$, and $\widehat{x}_t$ and $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics across different scenarios.

investment demand which leads to a multiplier loop for aggregate demand (in a similar fashion to how a traditional Keynesian Cross multiplier works through consumption). This dynamic amplification is missing under a CD production function, where complementarity is not strong enough and the different types of labor are substitutes (even having SAM asymmetries wouldn't change this feature of CD production, since SAM frictions do not affect the marginal product of capital *dynamically*).[15]

Moreover, inspecting the skill premium dynamics helps understand why the dynamic amplification by CSC is magnified in an environment with asymmetric SAM frictions, relative to a symmetric SAM environment (see how differences between crossed yellow and

[15] In the Online Appendix we also include variable capital utilization in the model. In this case, both effective capital and investment expand on impact after the shock. Yet, the responses of real marginal costs and the respective steady state coefficients that determine the skill premium responses are very similar to those in Figures 1.5 and 1.6. The marginal product of labor plays a slightly bigger role (as more capital expansion makes complementary labor more productive), but this effect is much smaller than the role of aggregate demand pressures. This exercise shows that for the dynamic demand amplification channel to work, higher investment demand need not necessarily manifest itself in actually higher amounts of physical capital: demand *pressures* as evidenced by larger marginal costs are enough to engineer the CSC demand amplification channel.

dashed purple lines are larger than those between circled blue and solid red in the bottom left panel of Figure 1.6). This is due to the existence of another source of initial imbalance in relative labor demand to be multiplied by higher demand pressures (the one coming from SAM asymmetry in addition to CSC, and captured in the larger responsiveness coefficient). In sum, the dynamic amplification mechanism sheds light on why *introducing both SAM asymmetry and CSC leads to a more powerful effect than the simple sum of the two channels alone.*

Lastly, we analyze the responses of the variables of interest to other demand shocks (e.g. government spending shocks) and cost-push shocks in Appendix A.3. The main finding here is that these alternative shocks do not change the previous results qualitatively: an increase in aggregate demand pressures raises the skill premium. Moreover, the skill premium rises more under CSC due to the dynamic demand amplification mechanism, and asymmetric SAM magnifies the effect of this channel. Quantitative differences appear, however, depending on how investment reacts to these shocks: positive shocks to government spending increase the skill premium by around 0.4 percent, which is less than the corresponding rise after a monetary shock (see Figures A.2 and A.3 in the Appendix). This is explained by a substantial crowding out of investment through higher public consumption. As a result, the pronounced fall in the capital stock mitigates the advantage of using complementary high-skill labor, therefore muting the response of the skill premium.

1.5.3 Sensitivity analysis

Since both CSC and asymmetric SAM are governed by various parameters, in this section we check how each of them affects our baseline results.

Complementarity between capital and skilled labor

CSC is captured in our model through the elasticity of substitution between capital and high-skilled labor, $\frac{1}{1-\gamma}$. Figure 1.7 depicts responses of the key variables of interest when we vary this elasticity. Confirming our previous conclusions, a larger degree of CSC (i.e. lower elasticity of substitution, yellow dotted lines) favors high-skilled workers even more after an expansionary monetary shock. Looking at the wage dynamics decomposition and the term associated with aggregate demand pressures $\left(\alpha_x^H - \alpha_x^L\right) \widehat{x}_t$ we see that a higher complementarity manifests itself in this term less via differences in the responsiveness coefficient $\left(\alpha_x^H - \alpha_x^L\right)$, and more via larger increases of $\widehat{x}_t$ (see top left panel of Figure 1.7), implying that CSC mainly operates through the dynamic demand amplification channel rather than through steady-state differences in marginal products. [16] In response to expanding aggregate demand, the initial rise in high-skilled employment makes complementary capital more productive, inducing a further rise in investment demand and amplifying aggregate demand pressures x_t. Evidently, the increase in demand pressures is a positive function of CSC.

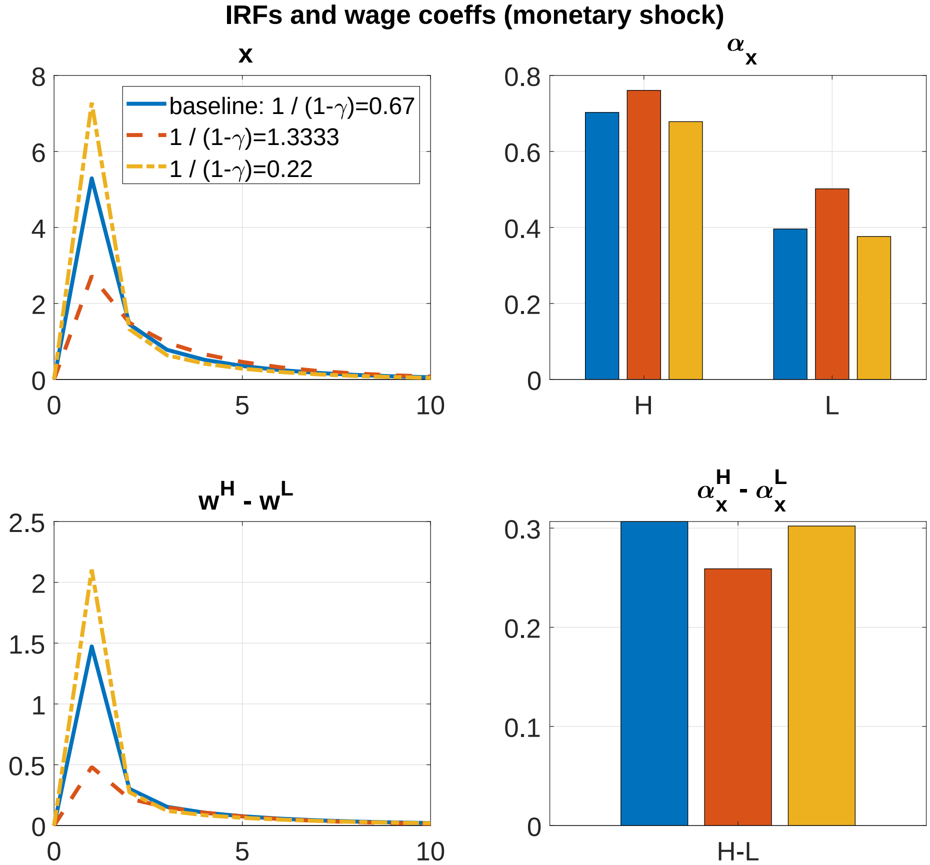

Figure 1.7: Comparing α_x^k and $\left(\alpha_x^H - \alpha_x^L\right)$, and $\widehat{x}_t$ and $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics across different scenarios.

Conversely, decreasing complementarity (red dashed lines) results in the opposite changes. Notice that in the latter scenario we still maintain the relation $\frac{1}{1-\alpha} = 1.67 > \frac{1}{1-\gamma} = 1.33$, which captures CSC in the sense defined by Lindquist (2004), even though capital and high-skillled workers are now substitutes (but less so than capital and unskilled workers are). If we move to the point where $\frac{1}{1-\gamma} > \frac{1}{1-\alpha}$, then the CSC channel would switch sign and it would actually *dampen* the increase in the relative labor income share.

The role of asymmetry in SAM-frictions

In our baseline model there are several sources of asymmetry in SAM frictions. High and low-skilled workers differ in terms of matching efficiencies $\psi^L < \psi^H$, as well as in job

[16] The fact that in the bottom right panel of Figure 1.6 the responsiveness coefficient is significantly higher under CSC than under the benchmark CD production function has more to do with the skill-intensity of the production structure than with differences in the elasticity of substitution $\frac{1}{1-\gamma}$ itself.

separation rates $\sigma^L > \sigma^H$ and bargaining power $\vartheta^L < \vartheta^H$. In order to gauge the relative importance of each of these asymmetries in driving our results, we repeat the exercise in Section 1.5.1 with the modification that adding "SAM asymmetry" now means allowing for only one of the asymmetries at a time, rather than all of them jointly.[17]

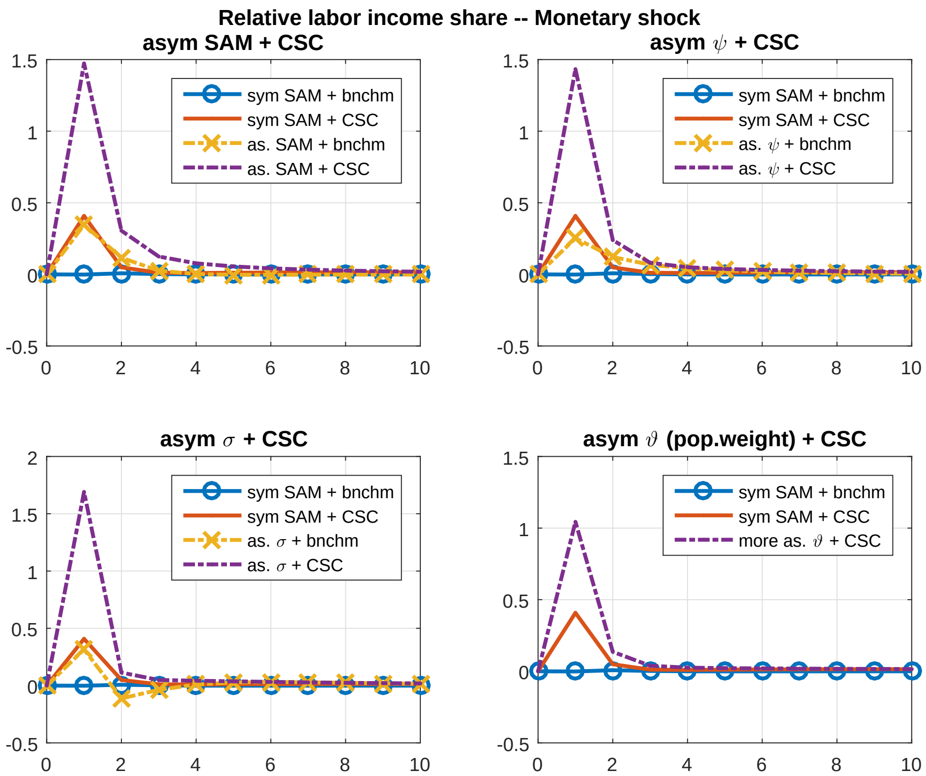

Figure 1.8: The effects of individual SAM asymmetries on the relative income share of high vs low skilled labor after an expansionary monetary policy shock

Figure 1.8 displays the results of this exercise, with the top left panel plotting the baseline scenario (same as the bottom right panel of Figure 1.4). Blue circled and red solid lines are the same across each panel as they capture scenarios with symmetric labor market frictions. As before, yellow crossed and purple dashed lines distinguish between production technology without or with CSC, respectively. Any type of labor market asymmetry intensifies the CSC amplification channel. Asymmetries in matching efficiency, ψ^k and separation rates, σ^k appear more important relative to asymmetries in the bargaining power, ϑ^k. When firms weigh the costs and benefits of hiring an additional high-skilled

[17] When making each of these frictions "asymmetric", we are using the calibration in Table 3.1. The only exception is the worker's bargaining power, ϑ^k, which is not a free parameter and it is already *asymmetric* in the otherwise "symmetric SAM + CSC" scenario ($\vartheta^H = 0.71 > \vartheta^L = 0.62$). This asymmetry stems from the fact that the surplus of high-skilled workers is higher with CSC. Given our calibration strategy, to obtain an *even more* asymmetric $\vartheta^H = 0.79 >> \vartheta^L = 0.57$ (purple dashed line) we have changed the original population weights to $\varphi^H = 0.1, \varphi^L = 0.8$. Without CSC and any other source of SAM-asymmetry, changing the population weights will not affect the bargaining power, so we cannot engineer a "more asymmetric ϑ^k + benchmark" scenario.

worker, the short-run reduction in high-skilled labor adjustment costs (stemming from their higher matching efficiency) brings benefits to firms, thereby increasing the relative demand of these workers. Similarly, lower separation rates increase the continuation value and hence the surplus of a high-skilled match. These arguments apply both under CSC and CD production functions. Hence, the qualitative pattern of our previous results is preserved even when one considers each of the three sources of SAM-asymmetry separately.

1.5.4 Different monetary policy strategies

Besides analyzing the effects of an expansionary monetary policy shock on labor income inequality, it could also be important to know how different kinds of *systematic* monetary policy strategies perform in response to other shocks that could drive cyclical fluctuations. Despite the fact that our assumption of complete financial markets does not provide the most realistic setup for optimal policy analysis, some comparison can still be made of how different policy regimes manage to smooth cyclical fluctuations in labor income inequality in the face of various shocks. Based on our previous finding that aggregate demand pressures are the most important driver of the skill premium, we would expect that a monetary policy rule which manages to stabilize demand fluctuations (i.e. close the output gap) will also do well in terms of preventing the distributional consequences of these shocks.

In a basic New Keynesian model, for shocks exhibiting the so called *divine coincidence*, the central bank does not face any trade-off between stabilizing inflation and the welfare-relevant output gap (Blanchard and Galí, 2007). In such cases, strict inflation targeting (IT) is the optimal policy, which also stabilizes aggregate demand. For models like ours, including labor market frictions, Blanchard and Galí (2010) and Ravenna and Walsh (2011) show that the divine coincidence vanishes, but they argue that delivering price stability remains very close to the optimal policy. Indeed, in our model strict IT performs best in terms of stabilizing aggregate demand and the skill premium. This is also the case in the face of various shocks (including cost-push shocks which introduce a trade-off between inflation and output gap stabilization). Strict IT dominates other Taylor rules in this respect also without CSC. The insight for this result is that a strict commitment to price stability helps the central bank manage inflation expectations more efficiently and improve the trade-off along the Phillips Curve, so that a given change in the inflation rate requires a smaller sacrifice in terms of output deviations. [18]

[18] In the Online Appendix we show that the ranking of different Taylor rules, with or without an explicit reaction to output stabilization depends on the presence of CSC in the face of cost-push shocks. An explicit output reaction can moderate the CSC dynamic demand amplification mechanism, mitigating the rise in the skill premium. With a standard production function, there is no such amplification to

1.6 Conclusions

In order to improve our understanding of the channels through which monetary policy affects labor income inequality, we have built a New Keynesian model with capital-skill complementarity (CSC) in the production function and asymmetric search and matching (SAM) frictions in the labor market between high and low-skilled workers. Our contribution here is to analyze a new mechanism through which monetary policy can affect labor income inequality: the assumption of a CSC production function leads to a *dynamic* demand amplification channel. Under a CSC production structure, the initial increase in high-skilled employment induced by the monetary expansion makes complementary capital more productive, encouraging a further rise in investment demand and creating a multiplier loop that favors high-skilled workers due to the more skill-intensive production structure. Asymmetric SAM frictions further enhance the relative demand of high-skilled workers, leading to considerably higher inequality. These findings are not qualitatively specific to monetary policy shocks but turn out to be similar for any other type of shocks that increase aggregate demand, although to a lesser extent. This is because unexpected cut in interest rates stimulates investment while, say, an unexpected expansionary fiscal shock crowds it out.

On the empirical front, we have shown that an expansionary monetary shock induces a significant rise in wage inequality. However, we have not tried to match model and theoretical IRFs in our analysis, the reason being that we wanted to analyze the basic intuition of our proposed mechanism through a rather simplified model that is not rich enough to enable such a match. Yet, through the reported sectoral evidence we have highlighted the relevance of CSC in delivering a rise in wage inequality after a monetary expansion. It is in those sectors characterized by high degree of CSC, such as Manufacturing and Wholesale and Retail Trade, where the skill premium and relative employment increase according to our theoretical predictions. Asymmetric SAM frictions are a realistic feature of the labor market and therefore we found it natural to include them in our analysis on the asymmetric effects of monetary shocks in the labor market.

Our findings are not to be necessarily taken as proposals that central banks should consider reacting to measures of inequality. Issues of inequality might be best dealt with by other policy areas led by elected officials. Nonetheless, it is worth being aware of the potential distributional consequences of monetary policy actions at business-cycle frequencies, even if it is not among the objectives in the mandate of central banks. That said, our main result that monetary easing increases labor income inequality should also be interpreted

mitigate, and an explicit output reaction worsens the trade-off along the Phillips curve, leading to more volatile responses to cost push shocks.

with caution. We were focusing only on one particular channel, namely the effect of CSC, while in reality the channels through which monetary policy affects inequality are more complex than what our model is capable of capturing. For a more complete picture and welfare analysis further analyses and different models are required.

References

Amaral, Pedro. 2017. “Monetary Policy and Inequality.” *Federal Resreve Bank of Cleveland Economic Commentary*, 2017(01).

Barnichon, Regis, and Andrew Figura. 2015. “Labor market heterogeneity and the aggregate matching function.” *American Economic Journal: Macroeconomics*, 7(4): 222–249.

Bell, Venetia, Michael Joyce, Zhuoshi Liu, and Chris Young. 2012. “The Distributional Effects of Asset Purchases.” *Bank of England Quarterly Bulletin*, 2012(Q3): 254–266.

Bernanke, Ben S. 2015. “Monetary Policy and Inequality.” *blog of Ben Bernanke*, 2015(June 1).

Blanchard, Olivier, and Jordi Galí. 2007. “Real Wage Rigidities and the New Keynesian Model.” *Journal of Money, Credit and Banking*, 39(1): 35–65.

Blanchard, Olivier, and Jordi Galí. 2010. “Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment.” *American Economic Journal-Macroeconomics*, 2(2): 1–30.

Blankenau, William F., and Steven P. Cassou. 2011. “Industry estimates of the elasticity of substitution and the rate of biased technological change between skilled and unskilled labour.” *Applied Economics*, 43(23): 3129–3142.

Brückner, Markus, and Evi Pappa. 2012. “Fiscal expansions, unemployment, and labor force participation: Theory and evidence.” *International Economic Review*, 53(4): 1205–1228.

Cahuc, Pierre, Fabien Postel-Vinay, and Jean-Marc Robin. 2006. “Wage bargaining with on-the-job search: Theory and evidence b.” *Econometrica*, 74(2): 323–364.

Campolmi, Alessia, and Ester Faia. 2015. “Rethinking Optimal Exchange Rate Regimes With Frictional Labor Markets.” *Macroeconomic Dynamics*, 19(05): 1116–1147.

Campolmi, Alessia, and Stefano Gnocchi. 2016. “Labor market participation, unemployment and monetary policy.” *Journal of Monetary Economics*, 79: 17–29.

Castro, Rui, and Daniele Coen-Pirani. 2008. “Why have aggregate skilled hours become so cyclical since the mid-1980s?” *International Economic Review*, 49(1): 135–185.

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. 2016. “Unemployment and Business Cycles.” *Econometrica*, 84(4): 1523–1569.

Coibion, Olivier, Yuriy Gorodnichenko, Lorenz Kueng, and John Silvia. 2017. “Innocent Bystanders? Monetary Policy and Inequality in the U.S.” *Journal of Monetary Economics*, 88(C): 70–89.

Dolado, Juan J, Gergő Motyovszki, and Evi Pappa. 2018. “Monetary Policy and Inequality Under Labor Market Frictions and Capital-Skill Complementarity.” *CEPR Discussion Paper Series*, , (DP12734).

Dolado, Juan J, Marcel Jansen, and Juan F Jimeno. 2009. “On-the-Job Search in a Matching Model With Heterogeneous Jobs and Workers.” *The Economic Journal*, 119(January): 200–228.

Eeckhout, Jan, and Philipp Kircher. 2018. “Assortative Matching With Large Firms.” *Econometrica*, 86(1): 85–132.

Fallick, Bruce C., and Charles A. Fleischman. 2004. “Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows.” *Federal Reserve Board - Finance and Economics Discussion Series*, , (2004-34).

Fischer, Georg. 2017. “The US Unemployment Insurance, a Federal-State Partnership: Relevance for Reflections at the European Level.” *IZA Policy Paper*, , (129).

Foster, Lucia, John Haltiwanger, and C J Krizan. 2006. “Market Selection, reallocation, and restructuring in the U.S. retail trade sector in the 1990s.” *Review of Economics and Statistics*, 88(4): 748–758.

Gornemann, Nils, Keith Kuester, and Makoto Nakajima. 2012. “Monetary Policy with Heterogeneous Agents.” *Federal Reserve Bank of Philadelphia Working Papers*, 12(21): 1–48.

Gornemann, Nils, Keith Kuester, and Makoto Nakajima. 2016. “Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy.” *Board of Governors of the Federal Reserve System International Finance Discussion Papers*, 2016(1167): 1–40.

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante. 2010. “Unequal we stand: An empirical analysis of economic inequality in the United States, 1967-2006.” *Review of Economic Dynamics*, 13(1): 15–51.

Kaplan, Greg, Benjamin Moll, and Giovanni Violante. 2018. “Monetary Policy According to HANK.” *American Economic Review*, 108(3): 697–743.

Koczan, Zsoka, Weicheng Lian, and Jihad Dagher. 2017. "Understanding the downward trend in labor income shares." *IMF World Economic Outlook*, 2017(April): 121–172.

Krusell, Per, Lee E. Ohanian, José-Víctor Ríos-Rull, and Giovanni L. Violante. 2000. "Capital-Skill Complementarity and Inequality: A Macroeconomics Analysis." *Econometrica*, 68(5): 1029–1053.

Lindquist, Matthew J. 2004. "Capital-Skill Complementarity and Inequality Over the Business Cycle." *Review of Economic Dynamics*, 7: 519–540.

Luetticke, Ralph. 2017. "Transmission of Monetary Policy with Heterogeneity in Household Portfolios." *University College London, mimeo*, 1–40.

Mertens, Karel, and Morten O Ravn. 2012. "The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States." *American Economic Review*, 103(4): 1212–1247.

Miranda-Agrippino, Silvia. 2016. "Unsurprising shocks: information, premia, and the monetary transmission." *Bank of England Staff Working Paper*, , (626).

Pappa, Evi, Rana Sajedi, and Eugenia Vella. 2015. "Fiscal consolidation with tax evasion and corruption." *Journal of International Economics*, 96(S1): S56–S75.

Ramey, Valerie A. 2016. "Macroeconomic shocks and their propagation." *NBER Working Paper*, w21978.

Ravenna, Federico, and Carl E Walsh. 2011. "Welfare-Based Optimal Monetary Policy with Unemployment and Sticky Prices: A Linear-Quadratic Framework." *American Economie Journal: Macroeconomics*, 3(2): 130–162.

Ravn, Morten O. 2006. "The Consumption-Tightness Puzzle." *NBER Working Paper Series*, w12421.

Ravn, Morten O, and Vincent Sterk. 2018. "Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach." *manuscript, UCL.*

Romer, Christina, and David Romer. 1998. "Monetary policy and the well-being of the poor." *NBER Working Paper Series*, , (w6793).

Romer, Christina D, and David H Romer. 2004. "A New Measure of Monetary Shocks: Derivation and Implications." *American Economic Review*, 94(4): 1055–1084.

Rose, Stephen J. 2017. "How Many Workers with a Bachelor's Degree Are Overqualified for Their Jobs?" *Urban Institute Research Report*, , (February).

Sahin, Aysegül, Joseph Song, Giorgio Topa, and Giovanni L. Violante. 2014. "Mismatch unemployment." *American Economic Review*, 104(11): 3529–3564.

Stock, James H., and Mark W. Watson. 2018. "Identification and Estimation of Dynamic Causal Effects in Macroeconomics Using External Instruments." *Economic Journal*, 128(610): 917–948.

Wieland, Johannes F. 2016. "Financial Dampening." *NBER Working Paper Series*, , (w22141).

Wolcott, Erin L. 2018. "Employment Inequality: Why Do the Low-Skilled Work Less Now?" *Middlebury College mimeo.*

Appendix A

A.1 Labor market data

The data for the wage premium and the relative employment ratios come from the NBER extracts of the Current Population Survey (CPS) Merged Outgoing Rotation Groups. We include in the sample only individuals in working age 15-64 and exclude part-time workers, self-employed and military employees. CPS provides monthly information from 1979:1 until 2016:6 on the participants' employment status, level of education, weekly earnings, and weekly hours of work. We classify workers as high-skilled and low-skilled according to whether they have experienced college or not. Low-skilled workers are defined as all those employed with a lower educational attainment. Defining high-skilled workers as those with college education using the NBER harmonization of education in CPS over time leads to difficulties in recovering skilled and unskilled workers at the sectoral level. To avoid inconsistencies in the definition of skilled and unskilled workers at the aggregate and at the sectoral level, we have opted to split workers depending on whether or not they have experienced some college.

A.1.1 Harmonization

Various variables in the Current Population Survey (CPS) are replaced over time to improve the survey instruments and adjust to changes of the labor market. In this section we illustrate how each relevant variable of this dataset was harmonized for our purposes.

Employment status. As it is commonly done when dealing with the CPS, three different variables are needed to construct an harmonized variable to distinguish employed, unemployed, and out of the labor force individuals. The harmonization is conducted as illustrated in the table below.

Education. The variable that captures the number of years in education is revised in 1992 to capture the different type of degrees workers may undertake. We conduct the

Harmonized variable	Original variables		
Education	ftpt79	lfsr89	lfsr94
from 1979 to 2016	*from 1979 to 1988*	*from 1989 to 1993*	*from 1994 to 2016*
Out of the Labor Force	0	5,7	5,7
Employed	1,2,4	1,2	1,2
Unemployed	3,5	3,4	3,4

standard harmonization in the literature that is presented below and only maintain two categories of workers with respect to education: those that have never experienced college (*No college education*), and the rest.

Harmonized variable	Original variables	
Education	gradeat	grade92
from 1979 to 2016	*from 1979 to 1991*	*from 1992 to 2016*
No college education	< 13	< 40
At least some college education	≥ 13	≥ 40

Industry. We focus our attention on six industries: (1) Manufacturing, (2) Education and Health Services, (3) Agriculture, Mining and Transportation, (4) Wholesale and Retail Trade, (5) Professional Services, (6) Financial and Informational Services. These industries capture relatively broad categories that all together represent about 80 percent of the labor force. We generate the industry group variable aggregating workers' industry sectors as recorded by the variable dind before 2000, and dind02 from the year 2000. The table below describes the extract grouping.

Harmonized variable	Original variables	
Industry	dind	dind02
from 1979 to 2016	*from 1979 to 1999*	*from 2000 to 2016*
(i) Manufacturing	5-20	5-23, 25-28
(ii) Education and Health Services	41-44	40-43
(iii) Agriculture, Mining and Transportation	1-3, 29, 46	1-3, 23
(iv) Wholesale and Retail Trade	32-33	21-22, 46
(v) Professional Services	45	36-39
(vi) Financial and Informational Services	24, 30, 34-37	25-35, 50

A.1.2 Series

We construct four types of series by aggregating nationally representative individual information from the repeated cross section of the CPS. The weighted averages for each skill group are calculated using the proper weights ERNWGT. These weights are computed each month such that, when applied, the resulting counts are representative of the national counts. Thus, this application of weights enables the results to be representative of the US population as a whole, instead of just the participants in the survey.

Employment Rate. This series is obtained as the share of employed workers in the labor force. We construct these series for any industry. We also construct these series both for any level of education and separately for each of our two levels of education. We use the employment rates for skilled workers in our VAR exercise.

Employment Ratios. This series is obtained as the ratio between employed skilled workers and employed unskilled workers. We construct these series both in the aggregate and for each industry separately.

Hourly Wage. This series is obtained by dividing weekly earnings (earnwke) by the number of hours per week that workers report to be usually working (uhourse). We construct these series both for any industry and for each industry separately. We also construct these series both for any level of education and separately for each of our two levels of education. The skill premium is obtained as the ratio of the two hourly wages. We construct these series both for any industry and for each industry separately.

A.1.3 Imputations

The hourly wage series that we construct at the industry level present some outlier observations. We drop these observations and replace them with in-sample predictions that we obtain using a Kalman filter. The table below lists all outlier observations that were replaced using this procedure.

Series	Observations replaced with imputations
Hourly wage, some college, industry (i)	2016M10
Hourly wage, no college, industry (i)	2007M02
Hourly wage, some college, industry (ii)	2007M11
Hourly wage, no college, industry (ii)	2007M4,2007M5, 2016M10
Hourly wage, some college, industry (iii)	2005M10, 2009M1, 2009M3
Hourly wage, no college, industry (iii)	2010M6, 2011M11, 2012M9, 2016M4
Hourly wage, no college, industry (iv)	1993M12, 2005M8, 2008M1, 2015M9
Hourly wage, some college, industry (v)	2008M10
Hourly wage, no college, industry (v)	1993M7, 1995M1, 1997M9, 2014M6, 2016M10
Hourly wage, some college, industry (vi)	2012M12
Hourly wage, no college, industry (vi)	1992M8

A.2 Empirical IRFs at the Sectoral Level

(a) Manufacturing

(b) Education and Health Services

(c) Agriculture, Mining and Transportation

Figure A.1: IRFs of employment ratio and skill premium in different sectors to a one percentage point unexpected reduction in the FF interest rate

(d) Wholesale and Retail Trade

(e) Professional Services

(f) Financial and Informational Services

Figure A.1: IRFs of relative employment and skill premium in different sectors to a one percentage point unexpected reduction in the FF interest rate

A.3 Theoretical IRFs to different shocks

A.3.1 Expansionary government spending shock

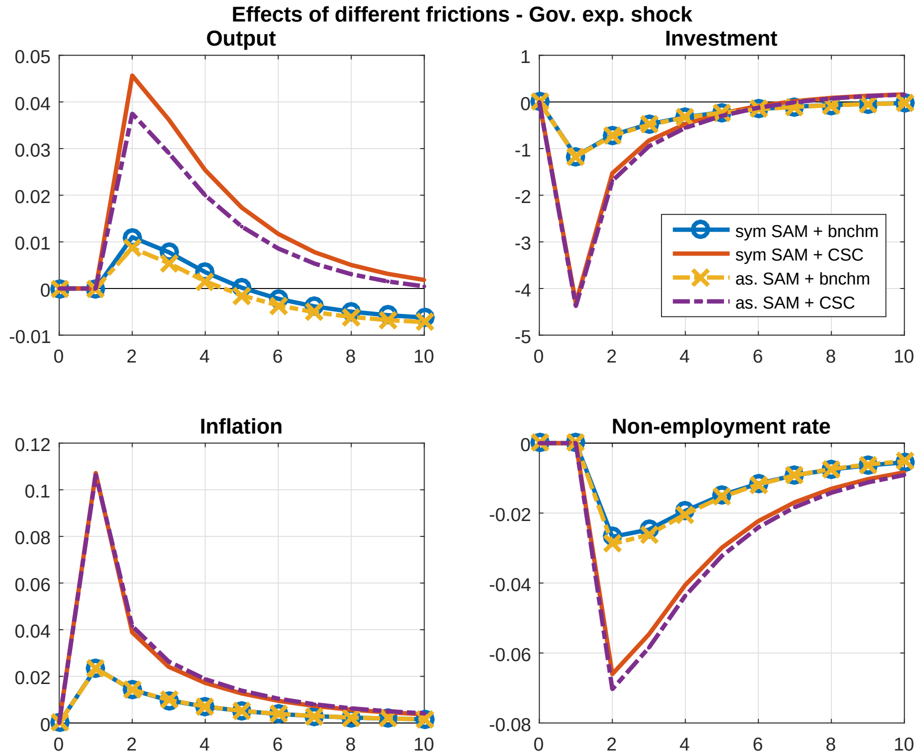

Figure A.2: IRFs after a 1% increase in G_t

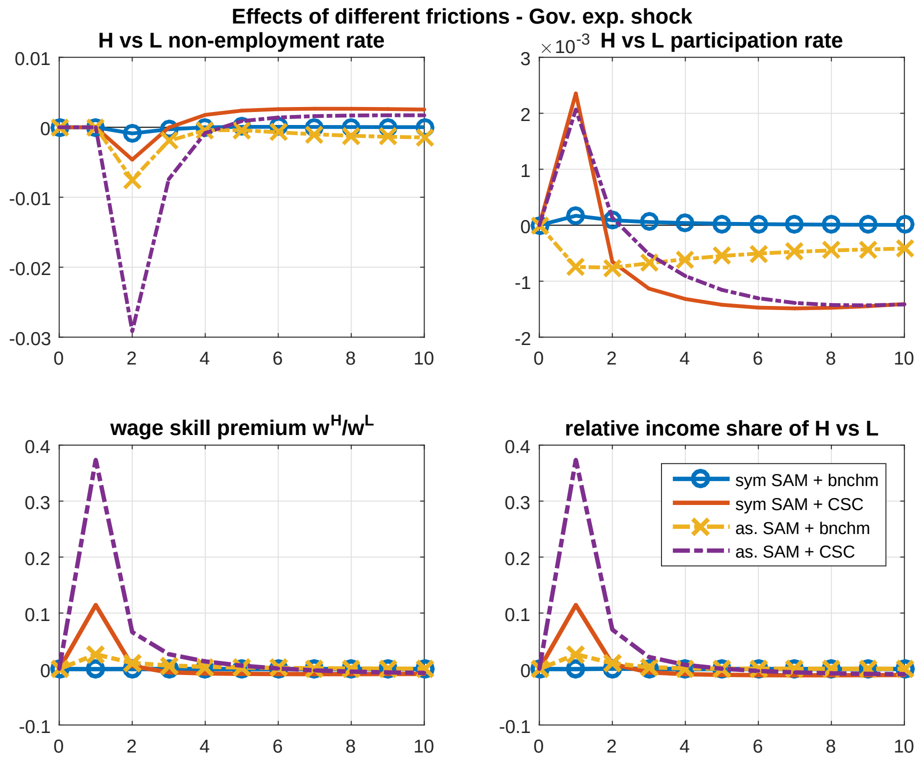

Figure A.3: IRFs after a 1% increase in G_t

A.3.2 Favorable (negative) cost-push shock

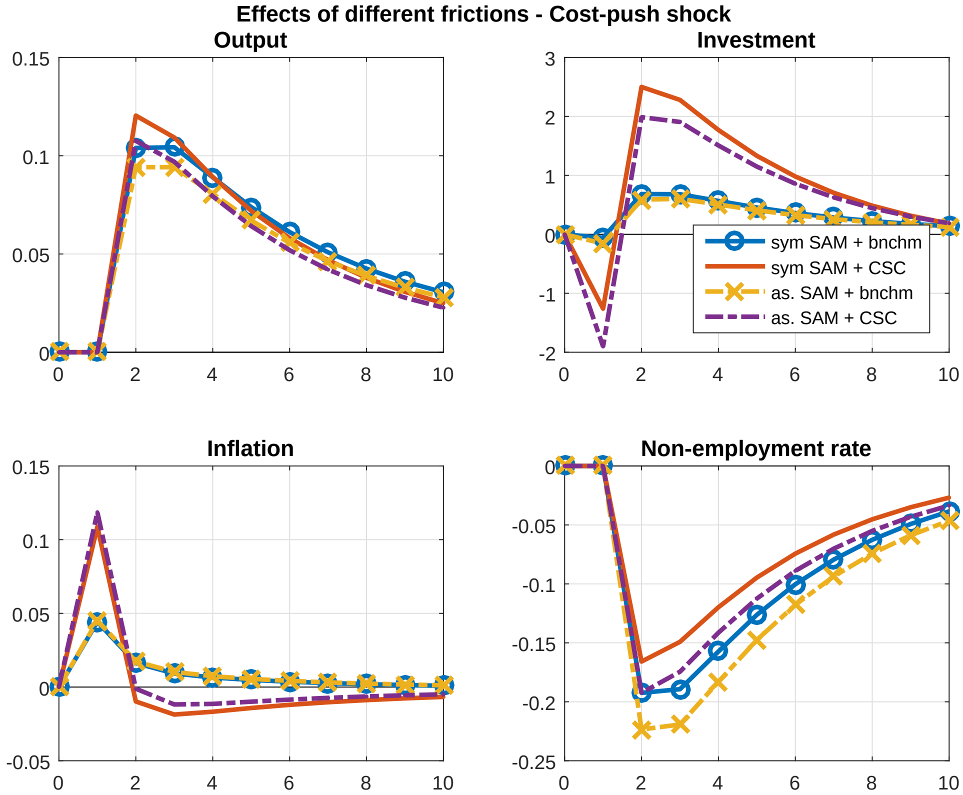

Figure A.4: IRFs after a 1% decrease in Ξ_t

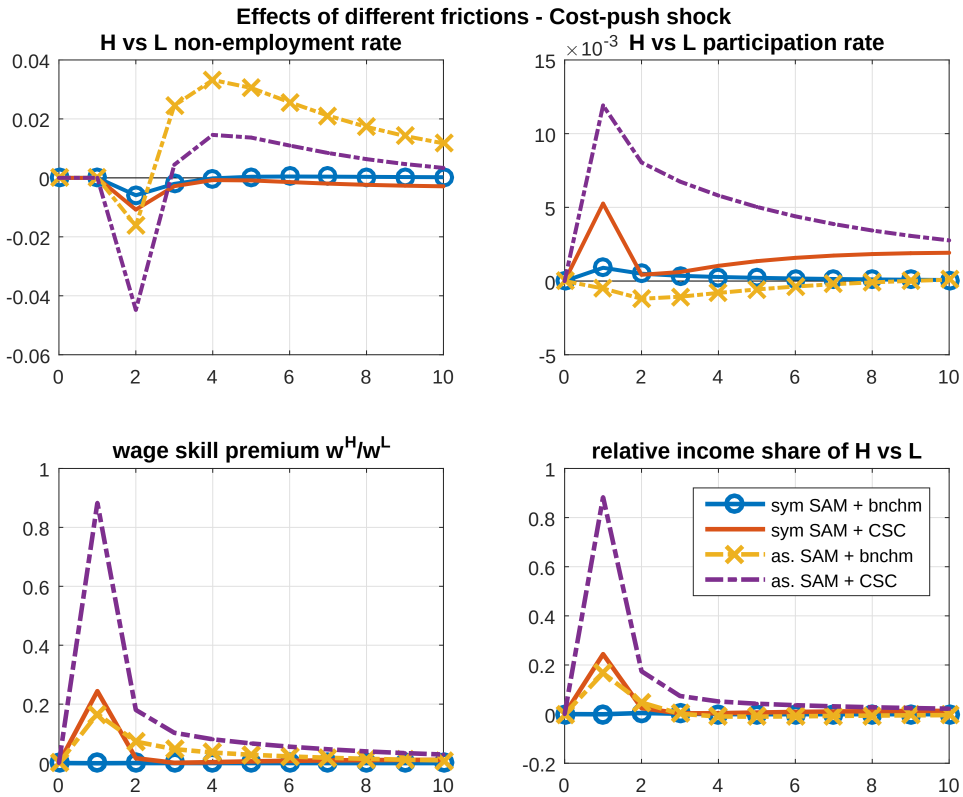

Figure A.5: IRFs after a 1% decrease in Ξ_t

A.3.3 Alternative steady state targets (monetary shock)

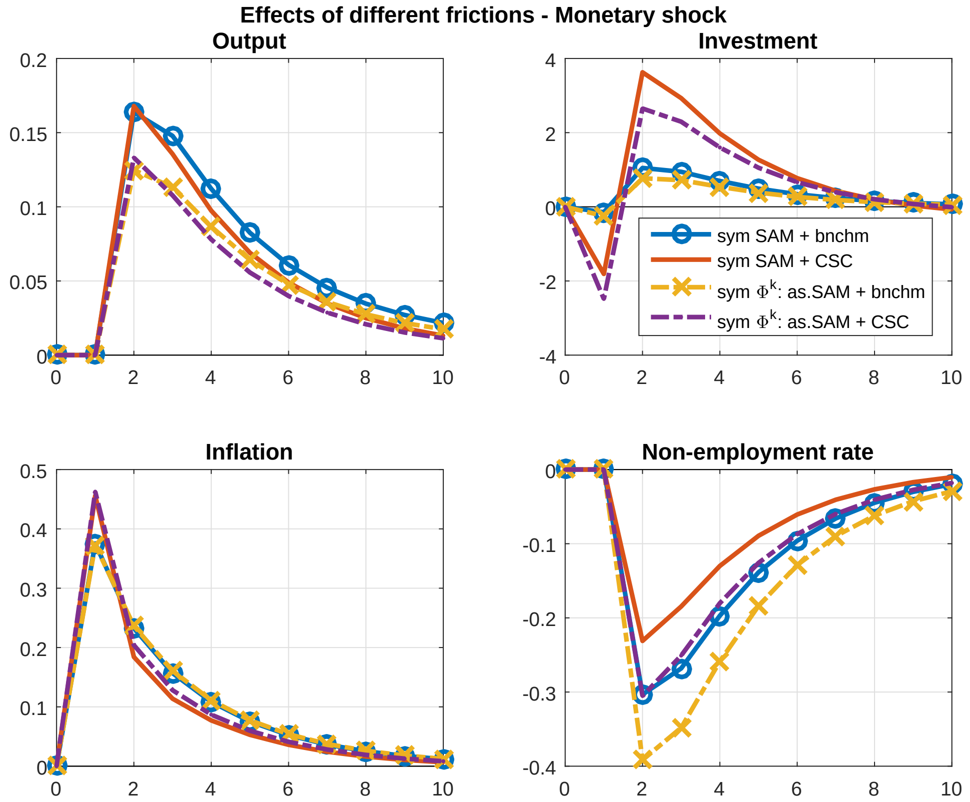

Figure A.6: IRFs after a 100 bp (annualized) cut in the policy rate

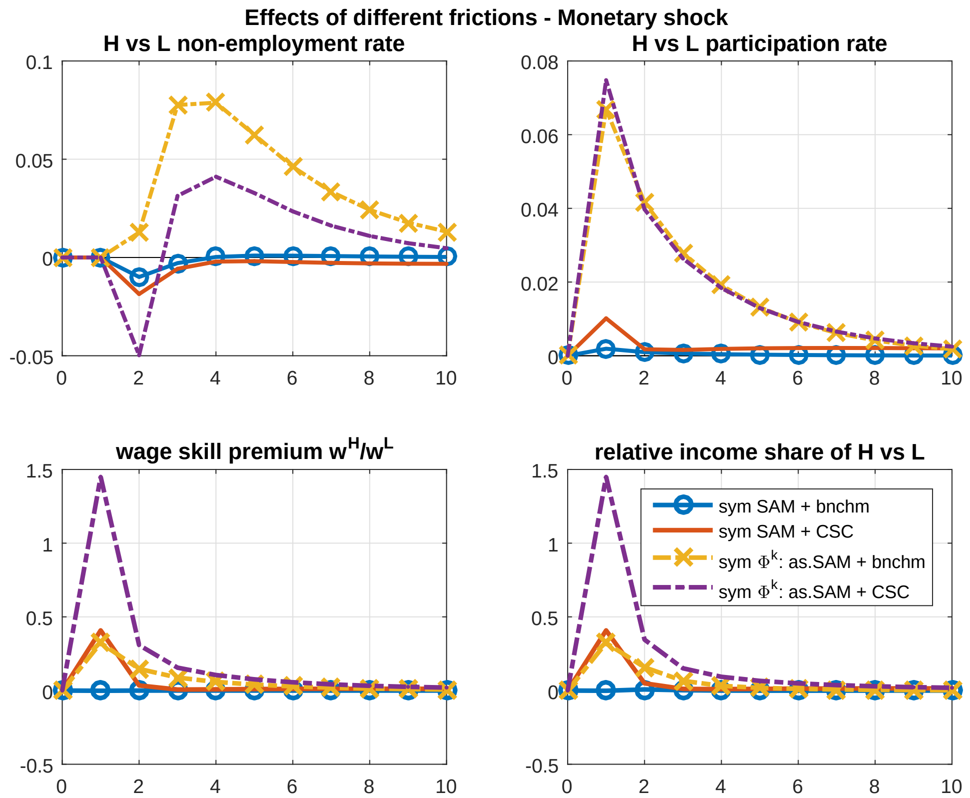

Figure A.7: IRFs after a 100 bp (annualized) cut in the policy rate

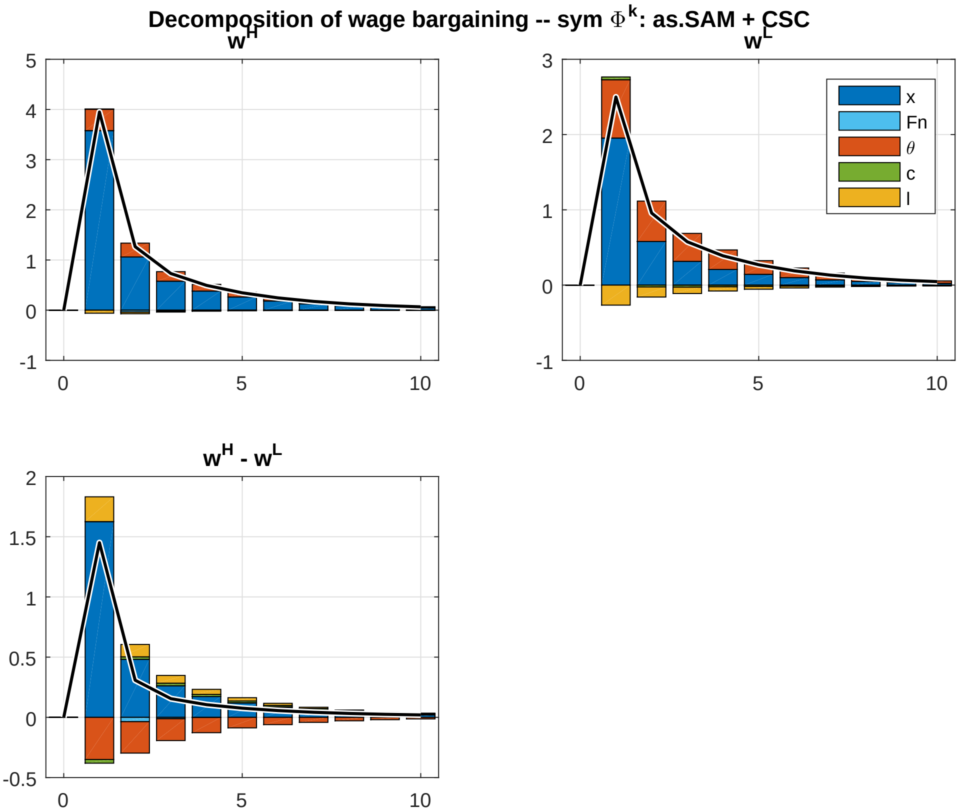

Figure A.8: Decomposing real wage $\widehat{w}_t^k$ and wage premium $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics, for alternative steady state targets.

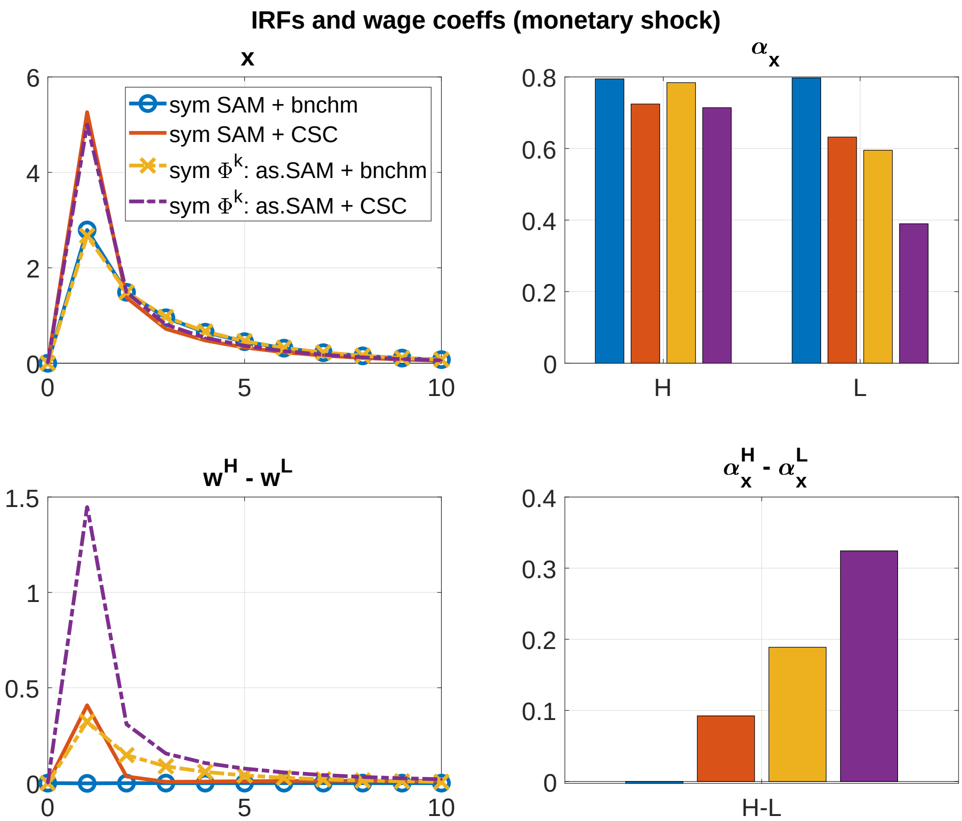

Figure A.9: Comparing α_x^k and $\left(\alpha_x^H - \alpha_x^L\right)$, and $\widehat{x}_t$ and $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics across different scenarios.

In order to check the robustness of our results to asymmetry in the baseline calibration of the labor preference parameter Φ^k, we have conducted this exercise with symmetric Φ^k which shows that, apart from slight quantitative differences, our main qualitative conclusions are unaffected. Asymmetry in the baseline calibration of Φ^k is not driving our dynamic results, and is only necessary to match skill-specific participation rate targets in the steady state.

The alternative calibration presented here differs from the baseline calibration in achieving $\Phi^H = \Phi^L = 0.05$ by setting the steady state target for low-skilled participation rate at $partic^L = 74.9\%$ (instead of 66% in the baseline), which also results in a steady state wage premium of $w^H/w^L = 1.6639$ (instead of 1.5306 in the baseline).

Comparing the above four figures to Figures 1.2, 1.4, 1.5, and 1.6 in the main text, it can be seen that our main conclusions are unchanged.

A.3.4 Favorable (negative) cost-push shock

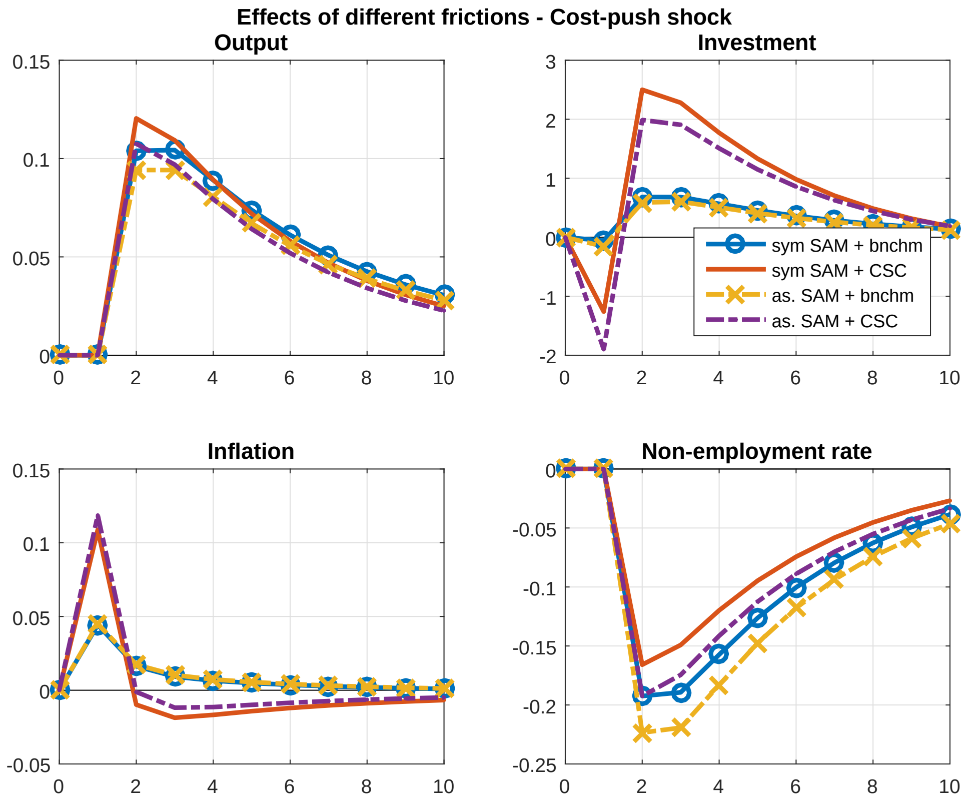

Figure A.10: IRFs after a 1% decrease in Ξ_t

A.3.5 Alternative steady state targets (monetary shock)

In order to check the robustness of our results to asymmetry in the baseline calibration of the labor preference parameter Φ^k, we have conducted this exercise with symmetric

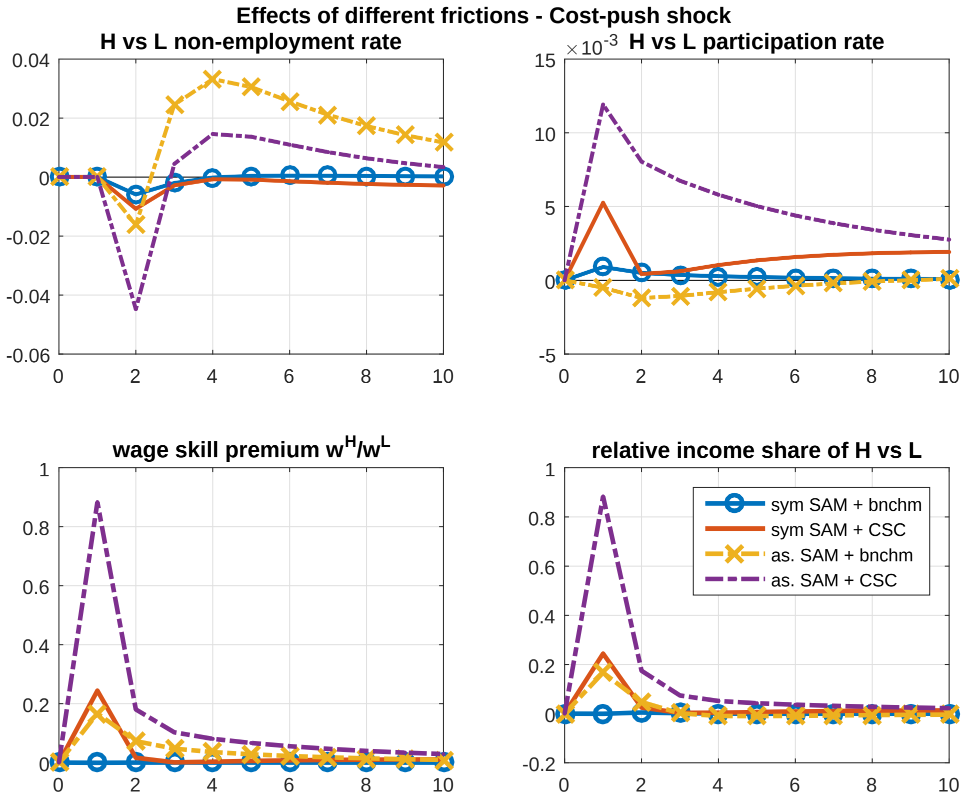

Figure A.11: IRFs after a 1% decrease in Ξ_t

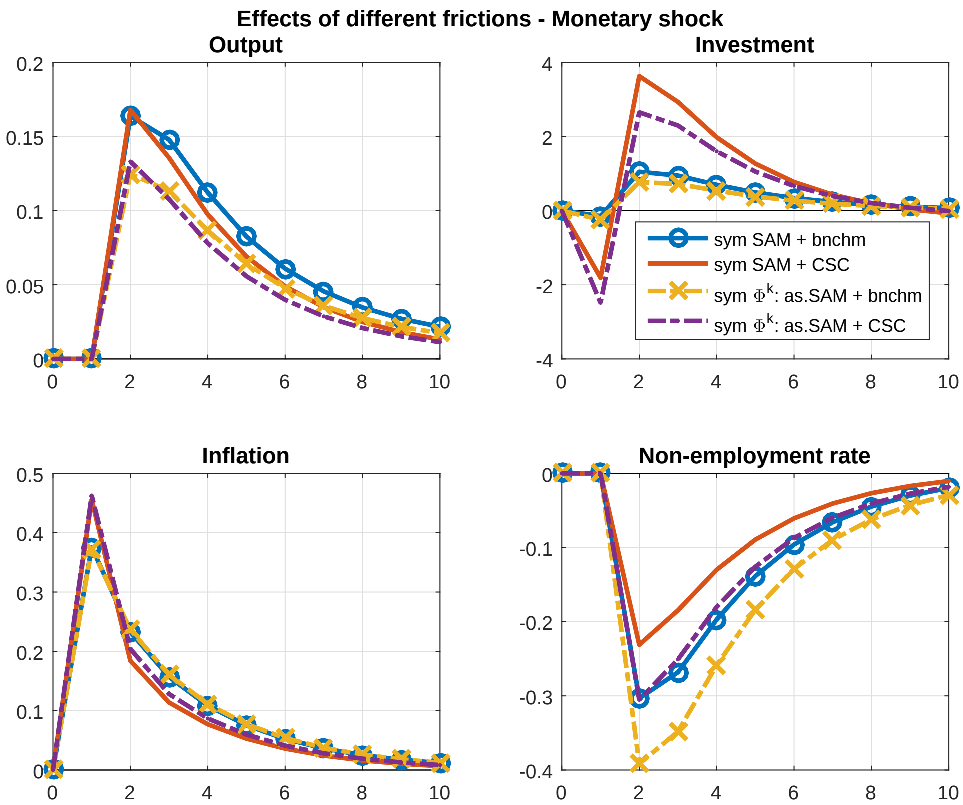

Figure A.12: IRFs after a 100 bp (annualized) cut in the policy rate

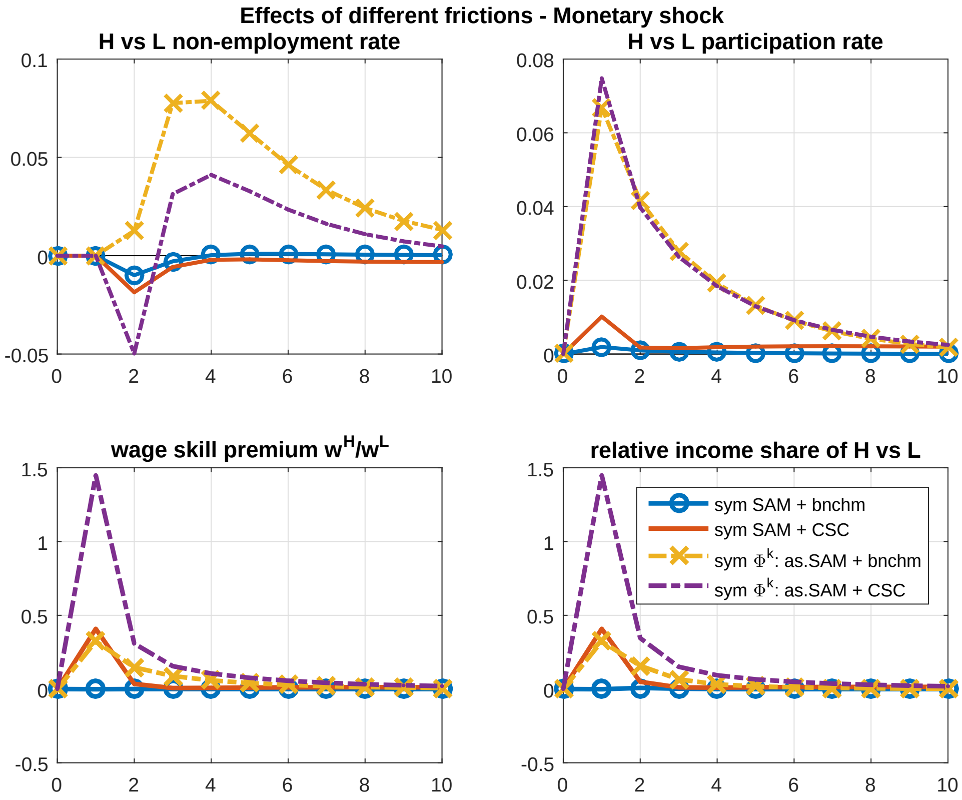

Figure A.13: IRFs after a 100 bp (annualized) cut in the policy rate

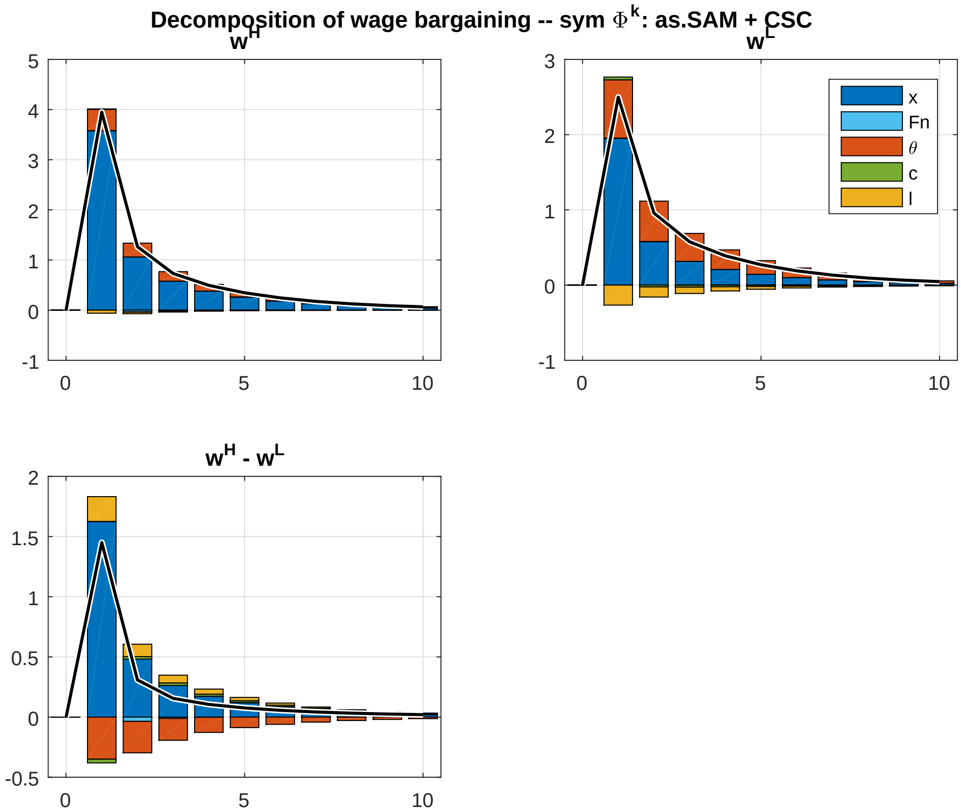

Figure A.14: Decomposing real wage $\widehat{w}_t^k$ and wage premium $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics, for alternative steady state targets.

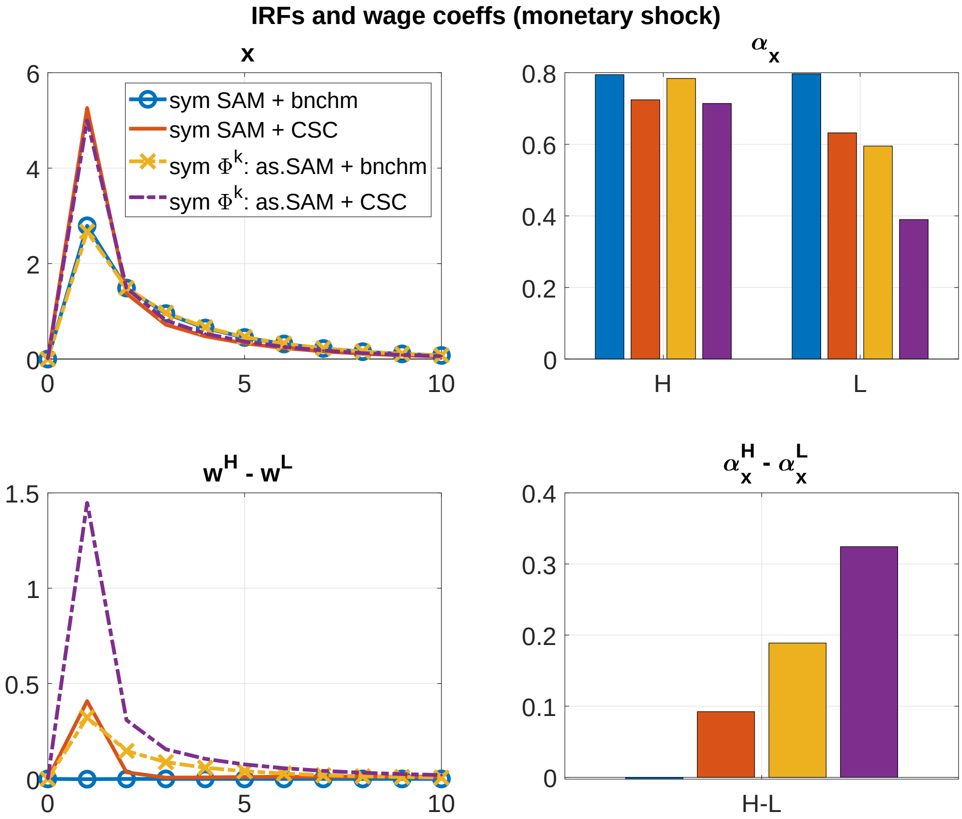

Figure A.15: Comparing α_x^k and $\left(\alpha_x^H - \alpha_x^L\right)$, and $\widehat{x}_t$ and $\widehat{w}_t^H - \widehat{w}_t^L$ dynamics across different scenarios.

Φ^k which shows that, apart from slight quantitative differences, our main qualitative conclusions are unaffected. Asymmetry in the baseline calibration of Φ^k is not driving our dynamic results, and is only necessary to match skill-specific participation rate targets in the steady state.

The alternative calibration presented here differs from the baseline calibration in achieving $\Phi^H = \Phi^L = 0.05$ by setting the steady state target for low-skilled participation rate at $partic^L = 74.9\%$ (instead of 66% in the baseline), which also results in a steady state wage premium of $w^H/w^L = 1.6639$ (instead of 1.5306 in the baseline).

Comparing the above four figures to Figures 1.2, 1.4, 1.5, and 1.6 in the main text, it can be seen that our main conclusions are unchanged.

Chapter 2

Monetary-Fiscal Interactions and Redistribution in Small Open Economies

2.1 Introduction

In response to the economic fallout from the covid-19 pandemic governments around the world are implementing recently unprecedented fiscal stimulus packages, insuring firms and households against the effects of the shock by cutting taxes and handing out transfers. As the resulting budget deficits lead to balooning public debt levels, the question of "how to pay for the stimulus" is asked increasingly often.

In most of the advanced economies real interest rates are low which makes the fiscal costs of public debt manageable: Blanchard (2019) pointed out that as long as safe real interest rates are below economic growth rates ($r < g$), current budget deficits need not be covered by tax increases in the future in order to keep debt-to-GDP ratios stable. Moreover, monetary policy shouldn't worry about the inflationary consequences of low interest rates since the *natural* or *neutral* interest rate is also low ($r^* \leq r$). This might be due to persistently low aggregate demand or secular stagnation caused by long-term structural trends, but the point is that in a world where $r^* \leq r < g$ applies, monetary policy can support fiscal expansion by keeping the costs of public debt low without having to worry about runaway inflation.

However, if and when *neutral* rates do rise above growth rates ($g < r^*$), monetary-fiscal policies will be presented with a dilemma. Either the central bank raises *actual* interest rates to ward off inflationary pressures, thereby forcing the fiscal authority to adjust the primary budget balance (in order to cover higher interest expenses and to stabilize public debt) – or an unresponsive monetary policy keeps interest rates low, tolerating higher inflation and essentially letting it erode the real value of nominal debt, without the need for fiscal policy to increase taxes.

In the terminology of Leeper (1991) the former regime can be characterized as an active monetary and passive fiscal policy mix (AM-PF), while the latter is the passive monetary and active fiscal policy mix (PM-AF).[1] In this framework the expectation of whether public debt will be paid for by taxes or by being "inflated away", already has an influence on the impact of fiscal stimulus today. Jacobson, Leeper and Preston (2019) argue that the success of Roosevelt's 1933 fiscal expansion was due to budget deficits not being backed by future taxes (made possible by monetary policy abandoning the gold standard), which prompted households to spend more of the windfall. Similarly, Bianchi, Faccini and Melosi (2020) show that if monetary and fiscal policies coordinate on an "emergency budget" which relies more on inflation than on costly fiscal adjustments to stabilize the resulting debt, the efficacy of fiscal stimulus is largely enhanced.

While for the time being advanced economies seem less pressed to face the dilemma of choosing between the AM-PF or PM-AF policy regimes, for emerging markets this might not be true. Even those able to borrow in their own currency do not have the privilige of issuing highly demanded reserve currencies, which means their rising debt ratios could lead to higher neutral interest rates due to more sensitive risk premiums. In other words, they are more likely to find themselves in a $g < r^*$ world. Of course, technically they can control interest rates in their own currencies but not following the risk premium would then result in exchange rate depreciation, passing through to higher inflation. The trade-off between higher inflation and fiscal adjustment is therefore more present in small open emerging economies, which is why the choice between AM-PF or PM-AF policy mixes seems even more relevant for them.

Apart from the question of what kind of monetary-fiscal policy mix should stabilize public debt, there is another important aspect of fiscal stimulus, in particular, its distribution across heterogeneous households. The breakdown of Ricardian equivalence in such an environment already renders fiscal decisions consequential, inducing another form of monetary-fiscal interactions even under an AM-PF regime (Kaplan, Moll and Violante,

[1] I.e. in AM-PF monetary policy *actively* manages the real interest rate to stabilize inflation, while fiscal policy *passively* adjusts the budget balance in order to stabilize public debt at the given interest rate. In PM-AF, fiscal *activism* means setting the path of primary budget balances independently of the need for debt stabilization, and instead having monetary policy *passively* accommodate government budgets by keeping interest rates low, tolerating higher inflation, and thereby ensuring a stable path for public debt. See discussion in Section 2.3.1.

2018). Targeting the same deficit-financed transfers towards households with a higher marginal propensity to consume (MPC) who spend most of their temporary income increases (rather than towards consumption-smoothing "Ricardian" agents) is shown to yield higher multipliers on output by Bayer et al. (2020).

In addition to targeting, financing also matters with household heterogeneity. Bilbiie, Monacelli and Perotti (2013) shows that whether the same transfer to high-MPC ("hand-to-mouth", HtM) households is financed by raising taxes on Ricardians during a balanced budget redistribution, or by selling public debt to Ricardians and running a budget deficit, influences the size of the output multiplier.[2] The reason is that while in the first scenario Ricardians are *paying* in full for the HtM transfers via a reduction in their lifetime income, in the latter they are just *lending* to HtM households via the government budget. The point is that financing decisions and public debt matters even under an AM-PF policy regime, due to the breakdown of Ricardian equivalence and household heterogeneity.

All the above arguments about fiscal redistribution are made within an AM-PF policy regime, while the discussion on potentially unbacked budget deficits (PM-AF) focuses on homogenous fiscal expansion in a representative agent setting. However, given the likely dilemma about public debt stabilization soon to be facing policymakers, it is of significant interest to explore how the redistributive features of fiscal stimulus play out under a PM-AF regime, and to see if redistribution interacts with the choice of monetary-fiscal policy mix. For this reason I build a small open economy Two Agent New Keynesian (TANK) model with monetary-fiscal interactions as in Leeper (1991). This allows me to analyse the distributional aspects of a fiscal stimulus under different policy regimes, while also accounting for open economy aspects that are relevant for emerging markets.

One of the main results concerns the relative importance across policy regimes of the targeting profile of fiscal transfers on the one hand, and whether they are balanced budget or deficit financed on the other hand. With an AM-PF policy mix, while public debt matters somewhat (to the extent that Ricardian equivalence fails), it is far more consequential how fiscal transfers are distributed across households. Targeting the same transfers more towards high-MPC agents increases the output multiplier to a much larger extent than deciding to finance a given transfer to high-MPC agents with public debt instead of taxes on Ricardians.[3] In other words, as long as hand-to-mouth households receive the same transfer, balanced budget redistributions provide almost as big stimulus as debt-financed

[2] To the extent that future taxes backing the public debt will not all be raised on Ricardian households, and to the extent that public debt is somewhat persistent.

[3] The latter decision would be completely inconsequential if future taxes backing public debt are all levied on Ricardian households. In this case, Ricardian equivalence holds, and the timing of taxes becomes irrelevant.

ones, and the arguments for deficit spending are not as strong. On the other hand, it is worth putting greater effort into the precise targeting of fiscal transfers, such that it reaches high-MPC households.

This is in contrast to the PM-AF policy regime, where targeting fiscal transfers towards high-MPC households matters much less than the size of the budget deficit *per se*. The essence of the transmission mechanism under this regime is that public debt does not entail future tax obilgations and therefore becomes nominal net wealth (Jacobson, Leeper and Preston, 2019), stimulating spending by bond holding Ricardians as well. In addition, an unresponsive monetary policy combined with the need of inflation to stabilize the real value of public debt results in falling real interest rates which also supports Ricardian consumption via intertemporal substitution. For these reasons, under PM-AF it is of much bigger importance whether a given transfer entails a budget deficit or not, relative to whom the transfer is targeted at, which is the opposite of the AM-PF regime's result. Cutting taxes on Ricardian households could be more stimulative as long as it is deficit financed, than giving the same transfer to hand-to-mouth agents during a balanced budget redistribution. Arguments for deficit spending are therefore much stronger with a PM-AF policy mix, i.e. if those deficits are unbacked by future tax revenues. At the same time, bothering about precise targeting is relatively less important.

The model yields other interesting results which, to the best of my knowledge, have not yet been discussed in the literature. Bilbiie (2008) shows that with a sufficiently high share of hand-to-mouth households interest rate increases can become expansionary ("inverted aggregate demand logic" or IADL), and an *inverted Taylor principle* can ensure a unique and stable dynamic equilibrium. I show that in a richer framework for monetary-fiscal interactions the inverted Taylor principle is not a necessary condition for equilibrium determinacy under IADL, and can be substituted by an active fiscal policy. In fact, in an open economy setting with sufficiently high external debt this is the only solution, as the inverted Taylor principle breaks down completely.

Open economy AM-PF models face a puzzle in the sense that they predict real appreciation following a fiscal stimulus, while empirical studies mainly detect real depreciation (Ravn, Schmitt-Grohé and Uribe, 2012; Monacelli and Perotti, 2010). This puzzle goes away with a PM-AF policy regime, where the real exchange rate depreciates after a fiscal expansion. This also changes the sign of the *expenditure switching* channel, meaning that instead of being crowded out, there's a beneficial effect on net exports as a result of relatively cheaper, more competitive domestic goods. Despite this, opening up the economy still reduces fiscal multipliers as some of the extra consumption spending now "leaks out" as imports, and this *expenditure changing* channel still dominates in the response of the trade balance. However, I show that this is not necessarily true if the rise in consumption is smaller due to a low share of HtM agents and/or if the price elasticity of trade is high enough, making expenditure switching dominate expenditure changing. This means that under a PM-AF regime there can be a constellation where the Mundell-Fleming prediction does not apply, i.e. that open economies need not face less effective fiscal multipliers compared to large closed economies.

This paper is most closely related to Bilbiie, Monacelli and Perotti (2013) who examine transfer multipliers and redistribution in a TANK model, and to Bayer et al. (2020) who consider in a HANK environment how targeting government transfers at high-MPC households during the covid-19 pandemic might affect multipliers. However, the above models feature closed economies and only look at an AM-PF policy mix. Regarding monetary-fiscal interactions, Bianchi, Faccini and Melosi (2020) and Jacobson, Leeper and Preston (2019) comes closest by analysing the PM-AF regime and unbacked emergency budgets, albeit in a closed economy setting with representative agents, which does not allow for studying redistribution across heterogeneous households. Finally, Leeper, Traum and Walker (2011) develop a medium-scale DSGE model which among its many features also includes hand-to-mouth households, PM-AF policy mix and open economy dimensions, however, they focus mostly on the size of government spending multipliers and not on redistribution via transfers.

Di Giorgio and Traficante (2018) build a two-country model to compare money-financed and debt-financed fiscal shocks. While money-financing (helicopter money) in their model can be thought of as analogous to the PM-AF regime studied here (see discussion in Section 2.3.1), it is not entirely the same. In addition, instead of utilizing a TANK model, they break Ricardian equivalence with a perpetual youth setup which prevents them from studying redistribution across households. Nevertheless, similarly to his paper's PM-AF poli9cy regime, their model also manages to predict real exchange rate depreciation after a money-financed tax cut. This is the same in Leith and Wren-Lewis (2008) who study the effects of monetary-fiscal interactions on equilibrium determinacy in their two-country OLG economy, and show that both policy branches can be active in one country, as long as monetary policy is passive in the other.

This paper is also part of a broader literature on TANK models,[4] and on monetary-fiscal

[4]Closed economy reference points include Bilbiie (2018), Bilbiie (2019), Debortoli and Galí (2018) and Broer et al. (2020), while the following also feature debt-financing for fiscal policy: Galí, López-Salido and Vallés (2007), Bilbiie and Straub (2004) and Cantore and Freund (2019), all with AM-PF policy mix. Open economy TANK is developed among others by Iyer (2017), Boerma (2014) and Cugat (2019), but with perfect international risk sharing and without a fiscal block.

interactions.[5] The vast literature on fiscal multipliers is also related,[6] however, their focus is mostly on government expenditures and not transfers, nor redistribution.

The rest of the paper is organized as follows. Section 2.2 describes the small open economy TANK model. Section 2.3 discusses how Ricardian equivalence and equilibrium determinacy are affected by household heterogeneity and monetary-fiscal interactions in this model. Section 2.4 presents the responses of the economy following an increase in fiscal transfers, and compares them across different policy regimes. Section 2.5 concludes.

2.2 Model

The model belongs to the family of New Keynesian small open economy models, as described in Corsetti, Dedola and Leduc (2010). It builds on the complete market model of Galí and Monacelli (2005) by adding hand-to-mouth households (Iyer, 2017) and introducing incomplete international financial markets (De Paoli, 2009). On the demand side of the economy a New Keynesian Cross is in operation, as in the closed economy two agent New Keynesian (TANK) model of Bilbiie (2019): λ fraction of households are excluded from financial markets, have unitary MPC and consume their current income (hand-to-mouth), while the rest (Ricardians) can smooth consumption intertemporally by saving/borrowing in a single, internationally traded bond and government debt.

The domestic economy faces a debt-elastic risk premium, effectively describing the asset supply of foreigners, ensuring stationary dynamics (Schmitt-Grohé and Uribe, 2003). Households consume both domestically produced and imported goods, the relative demand of which depends on the real exchange rate (as does export demand, too), which in turn affects the evolution of the trade balance and the external position of the economy, feeding back into the risk premium. The supply side of the economy consists of monopolistically competitive firms who are subject to nominal rigidities, and produce final goods with a linear production technology.

[5]The literature on the framework of active and passive policy rules is nicely summarized by Leeper and Leith (2016) and Sims (2013). Corsetti et al. (2019), Jarociński and Maćkowiak (2018) and Corsetti and Dedola (2016) point out the role of central banks to provide a monetary backstop to fiscal debt, in order to rule out self-fulfilling equilibria, especially in a liquidity trap, which is similar in nature to a PM-AF regime.

[6]Here are some of the papers which consider fiscal multipliers with unresponsive monetary policy in a liquidity trap: Woodford (2011), Eggertsson (2011) in closed economies, and Farhi and Werning (2016), Cook and Devereux (2013), Cook and Devereux (2019) in currency unions and open economies. Note, however, that importing monetary policy from abroad via an exchange rate peg should not be considered "passive" monetary policy in the sense used here, but instead it forces even harsher constraints on domestic policy.

Monetary policy either sets the short term nominal interest rate on local currency bonds, or controls the nominal exchange rate. Fiscal policy sets government expenditures and collects lump sum taxes from households, financing the potential budget deficit by issuing nominal debt. Taxes react to deviations of debt-to-GDP ratio from a target value. The distribution of taxes and transfers across households is decided by fiscal policy. Monetary-fiscal interactions are captured via the policy rules as in Leeper (1991).

2.2.1 Households

Hand-to-mouth households

There is a mass $0 \leq \lambda \leq 1$ of hand-to-mouth (HtM) households who are excluded from financial markets and cannot smooth consumption by saving/borrowing, but rather consume their income in every period. They solve the following static problem:

$$\max_{\check{C}_t, \check{N}_t} \quad E_t \left\{ \frac{\check{C}_t^{1-\sigma}}{1-\sigma} - \frac{\check{N}_t^{1+\varphi}}{1+\varphi} \right\}$$

$$P_t \, \check{C}_t = W_t \, \check{N}_t + \frac{\tau^D}{\lambda} P_t \Omega_t - P_t \check{T}_t \tag{2.1}$$

where P_t is the price of the consumption basket, $\check{C}_t$ is consumption by a HtM household, W_t is the nominal wage, $\check{N}_t$ is hours worked by a HtM household and $\check{T}_t$ are lump sum taxes paid by them to the government, which in turn redistributes τ^D fraction of aggregate profits Ω_t from firm owners towards HtM households. φ is the inverse Frisch elasticity of labor supply, while $1/\sigma$ is the intertemporal elasticity of substitution. The solution to this problem yields the labor supply condition of HtM households:

$$w_t \equiv \frac{W_t}{P_t} = \check{C}_t^{\sigma} \, \check{N}_t^{\varphi} \tag{2.2}$$

Ricardian households

A mass $1 - \lambda$ of households is Ricardian, as they are able to smooth consumption by saving and borrowing in international financial markets.

$$\max_{\widehat{C}_t, \widehat{N}_t, \widehat{B}_t, \widehat{B}_t^*} \quad E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\widehat{C}_t^{1-\sigma}}{1-\sigma} - \frac{\widehat{N}_t^{1+\varphi}}{1+\varphi} \right\}$$

$$P_t \, \widehat{C}_t + \frac{\widehat{B}_t}{1+i_t} + \frac{e_t \, \widehat{B}_t^*}{(1+i_t^*)\psi_t} \leq \widehat{B}_{t-1} + e_t \, \widehat{B}_{t-1}^* + W_t \, \widehat{N}_t + \frac{(1-\tau^D) P_t \Omega_t}{1-\lambda} - P_t \widehat{T}_t \tag{2.3}$$

where $\widehat{B}_t$ is a local currency (LCY) denominated *nominal* bond paying one unit of domestic currency on maturity. $\widehat{B}_t^*$ is a foreign currency (FCY) denominated bond paying one unit of foreign currency on maturity, which can be converted to domestic currency at a nominal exchange rate e_t.[7] The domestic household is subject to a risk premium ψ_t which it must pay on top of the risk-free foreign interest rate i_t^*. Ricardians own the firms in the economy and receive all profits Ω_t which are taxed at a rate τ^D, in additoin to which they also pay lump sum taxes $\widehat{T}_t$. The solution to the above problem yields:

$$w_t = \frac{W_t}{P_t} = \widehat{C}_t^{\sigma}\ \widehat{N}_t^{\varphi} \tag{2.4}$$

$$\frac{1}{1+i_t} = \beta\ E_t \left\{ \left[\frac{\widehat{C}_{t+1}}{\widehat{C}_t} \right]^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} \tag{2.5}$$

$$\frac{1+i_t}{\mathrm{E_t}\,\Pi_{t+1}} = \frac{1+i_t^*}{\mathrm{E_t}\,\Pi_{t+1}^*}\ \psi_t\ \frac{\mathrm{E_t}\,Q_{t+1}}{Q_t} \tag{2.6}$$

where (2.4) is the Ricardian labor supply condition, (2.5) is the Euler equation pricing LCY bonds and (2.6) is the real uncovered interest rate parity (UIP) condition signalling no-arbitrage between LCY and FCY bonds, and where $\Pi_t = P_t/P_{t-1}$ is gross CPI inflation and $Q_t = \frac{e_t P_t^*}{P_t}$ is the real exchange rate.

Portfolio choice is not modelled: given no-arbitrage between their expected returns, LCY and FCY bonds are perfect substitutes for the Ricardian household which should be indifferent between holding one or the other.[8] Therefore, these bonds are pinned down by the asset supply of foreigners and the government. We take look at two extreme scenarios. In our baseline setup only LCY-denominated bonds are traded internationally and FCY bonds are not (i.e. there is no *original sin*, and the domestic economy's holdings of FCY bonds B_t^* are restricted to be zero).[9] On the other hand, we can also consider the *currency mismatch* case where domestic households can borrow internationally only in FCY (original sin), and LCY bonds are restricted for domestic financial transactions with the government (see Section 2.2.5).

International risk-sharing

The rest of the world is modelled as a large economy which is populated by Ricardian households, solving a symmetric problem to the one above. The only difference is the absence of the risk premium ψ_t, so the foreign household faces the risk-free gross return $(1+i_t)/\psi_t$ on LCY-bonds, and $(1+i_t^*)$ on FCY bonds. Combining the resulting Euler

[7] Expressed as the local currency value of one unit of foreign currency, implying that an increase in e_t means a depreciation of the domestic currency.

[8] Taking into account different uncertainty around the ex post returns of LCY and FCY bonds would make the household prefer one or the other, but up to first order this makes no difference.

[9] In this case, the UIP no-arbitrage condition still applies, and follows from the foreign household's problem who has access to both assets and earns $(1+i_t)/\psi_t$ on the LCY bond, reflecting that it is relatively less risky than the domestic household.

equations with those of the domestic Ricardian household's (for the same assets) we arrive to the international risk-sharing condition:

$$\left[\frac{\mathrm{E_t}\,\widehat{C}_{t+1}}{\widehat{C}_t}\right]^{\sigma} = \left[\frac{\mathrm{E_t}\,C^*_{t+1}}{C^*_t}\right]^{\sigma}\ \psi_t\ \frac{\mathrm{E_t}\,Q_{t+1}}{Q_t} \tag{2.7}$$

(2.7) shows that due to incomplete markets there is only imperfect risk sharing, creating a less tight link between consumption and the real exchange rate than the Backus-Smith perfect risk sharing condition $\widehat{C}_t = \vartheta\ C^*_t Q_t^{\frac{1}{\sigma}}$ (which would keep the demand imbalance ϑ constant). There is still a link between foreign and domestic consumption growth, but only in expectation which does not hold *ex post*, and the real exchange rate will not fully absorb shocks to insure the domestic household against them (i.e. the demand imbalance ϑ_t will have inefficient deviations from its steady state level *ex post*).

The risk-premium ψ_t drives a further wedge between the countries. However, were it not for this *debt-elastic* risk-premium ψ_t, the demand imbalance ϑ_t between the two countries would follow a random walk, making the model dynamics non-stationary. This is a well-known problem in incomplete market open economy models, and introducing ψ_t also serves the purpose of getting around it by providing a feedback into the consumption-saving decision, and making assets an important state variable (Schmitt-Grohé and Uribe, 2003).

$$\mathrm{E_t}\left\{\frac{\widehat{C}_{t+1}}{C^*_{t+1}}\frac{1}{Q_{t+1}^{\frac{1}{\sigma}}}\right\} = \underbrace{\frac{\widehat{C}_t}{C^*_t}\ \frac{1}{Q_t^{\frac{1}{\sigma}}}}_{\vartheta_t} \neq \dots \neq \underbrace{\frac{\widehat{C}}{C^*}\ \frac{1}{Q_*^{\frac{1}{\sigma}}}}_{\vartheta}$$

On the aggregate economy level, market incompleteness is aggrevated by the fact that only $1-\lambda$ fraction of households can share risk internationally in any way: hand-to-mouth households are excluded from financial markets. I.e. even under complete markets, with Ricardian households having access to a full set of state-contingent securities, aggregate consumption would not be fully insured since $\widehat{C}_t \neq C_t$ (Iyer, 2017).

Consumption baskets and demand functions

Both households consume a composite of Home produced C_t^H and Foreign produced (imported) C_t^F goods, with elasticity of substitution η between them. The import intensity is captured by α, which is a measure of openness: $(1-\alpha)$ represents home bias in consumption. $\alpha \to 0$ is the closed economy limit.

$$\check{C}_t = \left[(1-\alpha)^{\frac{1}{\eta}} (\check{C}_t^H)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (\check{C}_t^F)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2.8}$$

$$\widehat{C}_t = \left[(1-\alpha)^{\frac{1}{\eta}} (\widehat{C}_t^H)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (\widehat{C}_t^F)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2.9}$$

Solving the corresponding expenditure minimization problem gives us the folowing demand functions:

$$\check{C}_t^H = (1-\alpha) \left[\frac{P_t^H}{P_t} \right]^{-\eta} \check{C}_t \tag{2.10}$$

$$\widehat{C}_t^H = (1-\alpha) \left[\frac{P_t^H}{P_t} \right]^{-\eta} \widehat{C}_t \tag{2.11}$$

$$\check{C}_t^F = \alpha \left[\frac{P_t^F}{P_t} \right]^{-\eta} \check{C}_t \tag{2.12}$$

$$\widehat{C}_t^F = \alpha \left[\frac{P_t^F}{P_t} \right]^{-\eta} \widehat{C}_t \tag{2.13}$$

with the consumer price index (CPI) being a weighted average of the domestic producer price index (PPI) P_t^H and the import price index P_t^F:

$$P_t = \left[(1-\alpha)(P_t^H)^{1-\eta} + \alpha (P_t^F)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{2.14}$$

In turn, the imported good basket $C_t^F$is a composite of imports from particular countries C_t^j, $j \in [0,1]$, with elasticity of substitution γ between them: $\check{C}_t^F = \left[\int_0^1 (\check{C}_t^j)^{\frac{\gamma-1}{\gamma}} \, \mathrm{d}j \right]^{\frac{\gamma}{\gamma-1}}$. Solving the relevant expenditure minimization problem gives us the demand function $\check{C}_t^j = \left[\frac{P_{t,j}}{P_t^F} \right]^{-\gamma} \check{C}_t^F$, with the import price index $P_t^F = \left[\int_0^1 P_{t,j}^{1-\gamma} \, \mathrm{d}j \right]^{\frac{1}{1-\gamma}}$. A similar demand function applies to the Ricardian household.

Finally, each consumption basket is composed of differentiated goods $i \in [0,1]$ with elasticity of substitution ε between them: $\check{C}_t^j = \left[\int_0^1 \check{C}_t^j(i)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}i \right]^{\frac{\varepsilon}{\varepsilon-1}}$. Solving the relevant expenditure minimization problem gives us the demand demand function $\check{C}_t^j(i) = \left[\frac{P_{t,j}(i)}{P_{t,j}} \right]^{-\varepsilon} \check{C}_t^j$, with the price level of country j, expressed in LCY, being $e_{t,j} P_t^j = P_{t,j} = \left[\int_0^1 P_{t,j}(i)^{1-\varepsilon} \, \mathrm{d}i \right]^{\frac{1}{1-\varepsilon}}$. For $j = H$ we get demand for a Home produced good of variety i: $\check{C}_t^H(i) = \left[\frac{P_t^H(i)}{P_t^H} \right]^{-\varepsilon} \check{C}_t^H$, where the producer price index (PPI) is $P_t^H = \left[\int_0^1 P_t^H(i)^{1-\varepsilon} \, \mathrm{d}i \right]^{\frac{1}{1-\varepsilon}}$. Similarly for the Ricardian household.

For the foreign households in country j we can derive similar demand functions for the products of the Home country H:

$$C^F_{t,j} = \alpha \left[\frac{P^{F,j}_t}{P^j_t}\right]^{-\eta} C_{t,j} \tag{2.15}$$

$$C^H_{t,j} = \left[\frac{P^H_t}{e_{t,j} P^{F,j}_t}\right]^{-\gamma} C^F_{t,j} \tag{2.16}$$

$$C^H_{t,j}(i) = \left[\frac{P^H_t(i)}{P^H_t}\right]^{-\varepsilon} C^H_{t,j} \tag{2.17}$$

where $e_{t,j}$ is the bilateral exchange rate, $C_{t,j}$ indicate consumption of the foreign household in country j.

2.2.2 Exchange rates

The *effective* nominal exchange rate is defined as $e_t = \left[\int_0^1 e_{t,j}^{1-\gamma}\,\mathrm{d}j\right]^{\frac{1}{1-\gamma}}$. The *bilateral* real exchange rate is $Q_{t,j} = \frac{e_{t,j} P^j_t}{P_t}$, while the *effective* real exchange rate is defined as $Q_t = \left[\int_0^1 Q_{t,j}^{1-\gamma}\mathrm{d}j\right]^{\frac{1}{1-\gamma}}$, resulting in $Q_t = P^F_t / P_t$, using the definition for the import price index.

The Law of One Price holds for imports (but due to home bias, $\alpha \neq 1$, Purchasing Power Parity in terms of the CPI P_t does not apply):

$$P^F_t = e_t P^*_t \tag{2.18}$$

where P^*_t is the world price index in FCY. This also leads to the real effective exchange rate (REER) being:

$$Q_t = \frac{e_t P^*_t}{P_t} \tag{2.19}$$

Due to openness ($\alpha \neq 0$) there will be a wedge between the CPI and the PPI, which can be expressed in terms of the REER, by combining the CPI definition (2.14) with the REER definition (2.19) and the law of one price condition (2.18):

$$\frac{P_t}{P^H_t} = \left[\frac{1-\alpha}{1-\alpha\, Q_t^{1-\eta}}\right]^{\frac{1}{1-\eta}} \equiv h(Q_t) \tag{2.20}$$

2.2.3 Firms

Final good producers (retail firms)

Final good producer firms are perfectly competitive and they bundle together differentiated intermediate goods $Y_t(i)$, subject to the aggregation technology (2.21), taking as given aggregate demand Y_t, the PPI P_t^H, and individual prices $P_t^H(i)$:

$$\max_{Y_t(i)} \Big\{ P_t^H Y_t - \int_0^1 P_t^H(i) Y_t(i) \, \mathrm{d}i \Big\}$$
$$Y_t = \Big[Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \Big]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.21}$$

This yields the familiar demand function for an individual intermediate good $Y_t(i)$ which will be a constraint for the intermediate firm's problem.[10]

$$Y_t(i) = \left[\frac{P_t^H(i)}{P_t^H} \right]^{-\varepsilon} Y_t \tag{2.22}$$

Intermediate good firms

There is a continuum $i \in [0, 1]$ of monopolistically competitive firms producing differentiated intermediate goods $Y_t(i)$. They face a downward sloping demand curve from retailers (2.22) which depends on the elasticity of substitution ε between goods varieties. Intermediate goods firms are also subject to Calvo type nominal rigidities, whereby each period only a fraction $(1 - \theta)$ can reset their prices. They work with a linear production technology $Y_t(i) = A_t N_t(i)$, using only labor as an input. The firm receives a wage subsidy τ^w from the government which is financed by a lump sum tax T_t^s paid by the firm.

The problem of the firm is:

$$\max_{P_t^H(i)} \quad \sum_{k=0}^{\infty} \theta^k \underbrace{\frac{1}{\prod_{s=1}^{k}(1 + i_{t+s})}}_{\equiv \Psi_{t,t+k}} \Big[P_t^H(i)\, Y_{t+k}(i) - (1 - \tau^w) TC_{t+k}(i) - P_{t+k} T_{t+k}^s \Big]$$

$$Y_{t+k}(i) = \left[\frac{P_t^H(i)}{P_{t+k}^H} \right]^{-\varepsilon} Y_{t+k}$$

where $TC_t(i) = W_t N_t(i)$. This leads to the following optimal price decision which, due to symmetry, is the same for all firms who are able to reset their prices in a given period:

$$P_t^H(*) = \underbrace{\frac{\varepsilon(1 - \tau^w)}{\varepsilon - 1}}_{\mathcal{M}} \ \mathrm{E_t} \ \frac{\sum_{k=0}^{\infty} \theta^k \Psi_{t,t+k}\, Y_{t+k}(i)\, MC_{t+k}(i)}{\sum_{k=0}^{\infty} \theta^k \Psi_{t,t+k}\, Y_{t+k}(i)} \tag{2.23}$$

where $MC_t = W_t / A_t$ is the nominal marginal cost and $\Psi_{t,t+k} = \beta^k \left(\frac{\widehat{C}_{t+k}}{\widehat{C}_t} \right)^{-\sigma} \frac{1}{\Pi_{t,t+k}}$ is the stochastic discount factor of the Ricardian households, who own the firm. This shows that, when resetting their price $P_t^H(*)$ (potentially lasting for many periods), firms would like to achieve *on average* a desired markup $\mathcal{M}$ over marginal costs (which they could

[10] This is similar to the consumer's demand of domestically produced differentiated goods $C_t^H(i)$, but $Y_t(i)$ also contains exports

always achieve under flexible prices, but now price stickiness prevents them from doing so, resulting in a time-varying markup).

By the Calvo pricing scheme we have that aggregate PPI inflation and the optimal price decision are connected as:

$$\frac{P_t^H(*)}{P_t^H} = \left[\frac{1-\theta\left(\Pi_t^H\right)^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \tag{2.24}$$

Real marginal costs are the inverse of the *time-varying* markup

$$\begin{aligned} rMC_t &= \frac{MC_t}{P_t^H} = \\ &= \frac{W_t}{A_t P_t^H} = \frac{w_t}{A_t} h(Q_t) \end{aligned} \tag{2.25}$$

Aggregate production, profits and price dispersion

Aggregate labor is $N_t = \int_0^1 N_t(i)\ \mathrm{d}i$, which together with the retailer demand function (2.22) and the firm-level production technology gives us the aggregate production function:

$$Y_t\ \Xi_t = A_t\ N_t \tag{2.26}$$

where the price dispersion $\Xi_t = \int_0^1 \left[\frac{P_t^H(i)}{P_t^H}\right]^{-\varepsilon} \mathrm{d}i$ can be expressed recursively (using (2.24)) as:

$$\Xi_t \ = \left(\Pi_t^H\right)^{\varepsilon} \theta\ \Xi_{t-1} + (1-\theta)\left[\frac{1-\theta\left(\Pi_t^H\right)^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.27}$$

The (CPI-deflated) profits of the firm, using $\tau^w w_t N_t = T_t^s$, are:

$$\begin{aligned} \Omega_t &= \frac{P_t^H}{P_t} Y_t - (1-\tau^w) w_t N_t - T_t^s = \\ &= \frac{Y_t}{h(Q_t)} - w_t N_t = \frac{Y_t}{h(Q_t)}\left[1 - \ rMC_t\ \Xi_t\right] \end{aligned} \tag{2.28}$$

Setting the wage subsidy at $\tau^w = 1/\varepsilon$ makes the steady state markup $\mathcal{M} = 1$, getting rid of the static distortion coming from monopolistic competition. With the wage subsidy being financed by a tax levied on the firm (as in Bilbiie (2018)), this also results in zero steady state profits.[11]

[11] This leads to a symmetric steady state between HtM and Ricardian households (provided that steady state bond holdings $\widehat{B}$ are also zero), independently of profit redistribution τ^D.

2.2.4 Government policies

Monetary policy

Monetary follows a Taylor-type *instrument* rule:

$$\frac{1+i_t}{1+i} = \left(\frac{\Pi_t^H}{\Pi^H}\right)^{\phi^\pi} \left(\frac{Y_t}{\overline{Y}_t}\right)^{\phi^y} \left(\frac{e_t}{e_{t-1}}\right)^{\phi^e} v_t \tag{2.29}$$

$$\ln v_t = \rho_R \ln v_{t-1} + \epsilon_t^R \tag{2.30}$$

where $\overline{Y}_t$ is flexible price output when $\theta = 0$. This rule can be replaced by more extreme *targeting* policies:

- strict domestic inflation (or PPI) targeting: $\Pi_t^H = 1$
- exchange rate peg: $e_t/e_{t-1} = 1$
- strict inflation (CPI) targeting: $\Pi_t = 1$

Fiscal policy

The government spends only on domestically produced goods (perfect home bias). The public consumption good G_t is assembled from differentiated products i with the same retail technology as private consumption $G_t = \left[\int_0^1 G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \,\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$ which, after cost minimization, leads to a similar demand function as for the households (the relevant price index now being the domestic producer price index P_t^H due to perfect home bias):

$$G_t(i) = \left[\frac{P_t^H(i)}{P_t^H}\right]^{-\varepsilon} G_t \tag{2.31}$$

G_t follows an AR1 exogenous process $\ln G_t = (1-\rho_g)\ln(\Gamma Y) + \rho_g \ln G_{t-1} + \epsilon_t^g$, where $\Gamma = G/Y$ is the steady state GDP share of government spending.

The government levies a lump sum tax T_t^s on firms (as opposed to Ricardian households) which is used to finance a wage subsidy τ^w. This "sub-budget" is balanced every period: $T_t^s = \tau^w w_t N_t$. Setting $\tau^w = \frac{1}{\varepsilon}$ ensures efficient net markups in steady state $\mathcal{M} = \frac{\varepsilon(1-\tau^w)}{\varepsilon-1} = 1$, and per (2.28) also entails zero firm profits in steady state.

To finance public spending G_t the fiscal authority collects lump sum taxes T_t from households and issues nominal LCY government debt B_t^g at a discount of $(1+i_t)^{-1}$. The nominal government budget constraint is then:

$$P_t T_t + \frac{B_t^g}{1+i_t} = P_t^H G_t + B_{t-1}^g$$

$$T_t + \frac{b_t^g}{1+i_t} = [h(Q_t)]^{-1} G_t + \frac{b_{t-1}^g}{\Pi_t} \tag{2.32}$$

where we used (2.20) and defined $b_t^g \equiv B_t^g/P_t$ as the CPI-deflated real value of public debt. This equation demonstrates how surprise inflation Π_t can reduce the burden of already existing public debt stock b_{t-1}^g.[12]

A fiscal rule governs the endogenous reaction of taxes to outstanding public debt, while taxes can also be subject to exogenous (household specific) shocks. The parameter ϕ_B in the fiscal rule determines how much taxes adjust to stabilize the path of real government debt as a fraction of steady state GDP around a target level $\bar{b}^g$.

$$\frac{T_t - T}{Y} = \phi_B \left(\frac{b_{t-1}^g}{Y} - \bar{b}^g \right) - \left[\lambda \check{\varepsilon}_t^T + (1-\lambda) \hat{\varepsilon}_t^T \right] \tag{2.33}$$

The distribution of the aggregate tax burden across households is pinned down by individual tax rules as follows:

$$\frac{\check{T}_t - \frac{\phi}{\lambda} T}{Y} = \frac{\phi}{\lambda} \phi_B \left(\frac{b_{t-1}^g}{Y} - \bar{b}^g \right) - \check{\varepsilon}_t^T \tag{2.34}$$

$$\frac{\widehat{T}_t - \frac{1-\phi}{1-\lambda} T}{Y} = \frac{1-\phi}{1-\lambda} \phi_B \left(\frac{b_{t-1}^g}{Y} - \bar{b}^g \right) - \hat{\varepsilon}_t^T \tag{2.35}$$

where ϕ governs the degree of exogenous redistribution, showing what fraction of *expected* aggregate tax burden is levied on HtM households. $\phi = \lambda$ corresponds to the uniform taxation case. Taxes of each household can be subject to individual shocks as well, similarly to Bilbiie, Monacelli and Perotti (2013). These equations together characterize *exogenous* redistribution via the tax system. Combined with (2.33) they also imply the relationship: $T_t = \lambda \check{T}_t + (1-\lambda) \widehat{T}_t$.

The government also taxes the dividends of Ricardian households at a rate τ^D and redistributes the proceeds to HtM households as transfers (*endogenous* redistribution).

[12] In other words, the *ex post* real interest rate $(1+r_{t-1}) = \frac{1+i_{t-1}}{\Pi_t}$, which determines the real burden of public debt, can be reduced by surprise inflation. Put differently, surprise inflation can have revaluation effects on existing public debt. Expressing the change in the real *market value* of public debt:

$$\begin{aligned}
\frac{b_t^g}{1+i_t} &= \frac{G_t}{h(Q_t)} - T_t + \frac{b_{t-1}^g}{\Pi_t} = \\
&= \left[\frac{G_t}{h(Q_t)} - T_t \right] + (1+r_{t-1}) \frac{b_{t-1}^g}{1+i_{t-1}} \\
\frac{b_t^g}{1+i_t} - \frac{b_{t-1}^g}{1+i_{t-1}} &= \underbrace{\left[\frac{G_t}{h(Q_t)} - T_t \right]}_{\text{primary deficit}} + \underbrace{i_{t-1} \frac{b_{t-1}^g}{1+i_{t-1}}}_{\text{interest payment}} - \underbrace{\frac{\Pi_t - 1}{\Pi_t} b_{t-1}^g}_{\text{revaluation}}
\end{aligned}$$

2.2.5 Market clearing

Consumption aggregates

Aggregate consumption indices are the weighted sums of Ricardian and HtM household consumption:

$$C_t = \lambda \check{C}_t + (1-\lambda)\widehat{C}_t \tag{2.36}$$
$$C_t^H = \lambda \check{C}_t^H + (1-\lambda)\widehat{C}_t^H \tag{2.37}$$
$$C_t^F = \lambda \check{C}_t^F + (1-\lambda)\widehat{C}_t^F \tag{2.38}$$

From the above we can also create aggregated demand functions. Applying (2.37) for individual goods i, and using the individual demand functions of HtM and Ricardian agents from before we get domestic demand for a Home produced good i:

$$C_t^H(i) = \left[\frac{P_t^H(i)}{P_t^H}\right]^{-\varepsilon} \underbrace{\left[\lambda \check{C}_t^H + (1-\lambda)\widehat{C}_t^H\right]}_{C_t^H} \tag{2.39}$$

Then combining (2.37) + (2.10) + (2.11) + (2.36) we get domestic demand for Home produced goods:

$$C_t^H = (1-\alpha)\left[\frac{P_t^H}{P_t}\right]^{-\eta} \underbrace{\left[\lambda \check{C}_t + (1-\lambda)\widehat{C}_t\right]}_{C_t} \tag{2.40}$$

Combining (2.38) + (2.12) + (2.13) + (2.36) gives us the import demand of the domestic economy:

$$C_t^F = \alpha \left[\frac{P_t^F}{P_t}\right]^{-\eta} \underbrace{\left[\lambda \check{C}_t + (1-\lambda)\widehat{C}_t\right]}_{C_t} \tag{2.41}$$

Total external demand from all foreign countries faced by domestic exporters is derived by combining (2.15) + (2.16):

$$\begin{aligned} C_{t,*}^H &\equiv \int_0^1 C_{t,j}^H \,\mathrm{d}j = \\ &= \alpha \int_0^1 \left[\frac{P_t^H}{e_{t,j} P_t^{F,j}}\right]^{-\gamma} \left[\frac{P_t^{F,j}}{P_t^j}\right]^{-\eta} C_{t,j} \,\mathrm{d}j \end{aligned} \tag{2.42}$$

Goods market

Output of a domestic firm i is either consumed domesticly (privately or publicly) or exported abroad to countries $j \in [0,1]$. Using demand functions (2.39), (2.31) and (2.17)

$$Y_t(i) = C_t^H(i) + G_t(i) + \int_0^1 C_{t,j}^H(i)\mathrm{d}j =$$

$$= \underbrace{\left[\frac{P_t^H(i)}{P_t^H}\right]^{-\varepsilon} C_t^H}_{(2.39):\ C_t^H(i)} + \underbrace{\left[\frac{P_t^H(i)}{P_t^H}\right]^{-\varepsilon} G_t}_{(2.31):\ G_t(i)} + \int_0^1 \underbrace{\left[\frac{P_t^H(i)}{P_t^H}\right]^{-\varepsilon} C_{t,j}^H\,\mathrm{d}j}_{(2.17):\ C_{t,j}^H(i)} =$$

$$= \left[\frac{P_t^H(i)}{P_t^H}\right]^{-\varepsilon} \underbrace{\left[C_t^H + G_t + \underbrace{\int_0^1 C_{t,j}^H\,\mathrm{d}j}_{(2.42):\ C_{t,*}^H}\right]}_{\text{based on (2.22): } Y_t} \tag{2.43}$$

Applying (2.22), we see that aggregate Home output Y_t is either consumed domestically or exported. Plugging in domestic and external demand functions (2.40) and (2.42), goods market clearing will entail:

$$Y_t = C_t^H + G_t + C_{t,*}^H =$$

$$= \underbrace{(1-\alpha)\left[\frac{P_t^H}{P_t}\right]^{-\eta} C_t}_{C_t^H} + G_t + \underbrace{\alpha\int_0^1 \left[\frac{P_t^H}{e_{t,j}P_t^{F,j}}\right]^{-\gamma}\left[\frac{P_t^{F,j}}{P_t^j}\right]^{-\eta} C_{t,j}\,\mathrm{d}j}_{C_{t,*}^H} =$$

$$= \left[\frac{P_t^H}{P_t}\right]^{-\eta}\left[(1-\alpha)\,C_t \;+\; \alpha\left(\frac{P_t^H}{P_t}\right)^{\eta}\int_0^1 \left(\frac{P_t^H}{e_{t,j}P_t^{F,j}}\right)^{-\gamma}\left(\frac{P_t^{F,j}}{P_t^j}\right)^{-\eta} C_{t,j}\,\mathrm{d}j\right] + G_t =$$

$$= \underbrace{\left[\frac{P_t^H}{P_t}\right]^{-\eta}}_{\left[h(Q_t)\right]^{\eta}}\left[(1-\alpha)\,C_t \;+\; \alpha\int_0^1 \left(\frac{e_{t,j}P_t^{F,j}}{P_t^H}\right)^{\gamma-\eta}\left(\underbrace{\frac{e_{t,j}P_t^j}{P_t}}_{Q_{t,j}}\right)^{\eta} C_{t,j}\,\mathrm{d}j\right] + G_t =$$

$$= \left[h(Q_t)\right]^{\eta}\left[(1-\alpha)\,C_t \;+\; \alpha\int_0^1 \left(\underbrace{\frac{P_t^{F,j}}{P_t^j}}_{X_t^j}\;\underbrace{\frac{P_t^j e_{t,j}}{P_t^H}}_{X_{t,j}}\right)^{\gamma-\eta} Q_{t,j}^{\eta}\; C_{t,j}\,\mathrm{d}j\right] + G_t \tag{2.44}$$

Assuming symmetric foreign countries we substitute j notation with $*$, and use $P_t^{F,*} = P_t^*$. Furthermore, we impose foreign goods market clearing, treating the rest of the world as a closed economy $C_{t,*} = Y_t^*$. Then:

$$Y_t = \left[h(Q_t)\right]^{\eta}\left[(1-\alpha)\,C_t \;+\; \alpha\left[h(Q_t)Q_t\right]^{\gamma-\eta} Q_t^{\eta}\; Y_t^*\right] + G_t =$$

$$= \left[h(Q_t)\right]^{\eta}\left[(1-\alpha)\,C_t \;+\; \alpha\left[h(Q_t)\right]^{\gamma-\eta} Q_t^{\gamma}\; Y_t^*\right] + G_t \tag{2.45}$$

In an open economy with $\alpha \neq 0$ (2.45) is one of most important relationships governing aggregate demand. It shows how output is affected by domestic and external demand through C_t and Y_t^*, respectively (*expenditure changing* channel), and how the latter also depend on the real exchange rate Q_t through *expenditure switching* effects. Just like producers selling domestically, exporters face a downward sloping demand curve: a real depreciation makes exports more competitive boosting external demand, while it also makes imports more expensive causing substitution towards domestically produced goods. The real exchange rate Q_t is the most important international relative price and the sensitivity of aggregate demand to it is governed by elasticities η and γ.

Labor market

$$N_t = \lambda \check{N}_t + (1-\lambda)\widehat{N}_t \tag{2.46}$$

Asset market

Both LCY and FCY bonds are in zero net supply *globally*.

$$0 = \tilde{B}_t + \underbrace{(1-\lambda)\widehat{B}_t - B_t^g}_{\equiv B_t} \tag{2.47}$$

$$0 = \tilde{B}_t^* + \underbrace{(1-\lambda)\widehat{B}_t^*}_{\equiv B_t^*} \tag{2.48}$$

where $\tilde{B}_t, \tilde{B}_t^*$ denote foreigners' bond holdings, i.e. the opposite side of any domestic bond position must necessarily be taken by the foreign economy. Foreign asset demand is basically the mirror image of foreign asset supply ($-\tilde{B}_t, -\tilde{B}_t^*$), and as discussed in Section 2.2.1, in the absence of modelling the portfolio choice problem (which would yield domestic asset demand functions for LCY and FCY), the currency composition of the net domestic bond position (B_t and B_t^*) will be determined by foreign asset supplies.

We consider two extreme scenarios for asset supply. In our baseline setup only LCY-denominated bonds are traded internationally and FCY bonds are not (i.e. there is no *original sin*, and the domestic economy's holdings of FCY bonds B_t^* are restricted to be zero).

$$-\tilde{B}_t^* = \qquad B_t^* = 0 \tag{2.49}$$

On the other hand, we can also consider the *currency mismatch* case where domestic households can borrow internationally only in FCY (original sin), and LCY bonds are restricted for domestic financial transactions between Ricardian households and the government.

$$-\tilde{B}_t = \qquad B_t = 0 \tag{2.50}$$

Under both scenarios, international asset supply in the unrestricted currency is implicitly determined by the debt-elastic risk premium function (2.55) defined below.

2.2.6 External balance

Trade balance

The *CPI-deflated* real trade balance is defined as:

$$
\begin{aligned}
NX_t &= \frac{P_t^H}{P_t} C_{t,*}^H - \frac{P_t^F}{P_t} C_t^F \\
&= \frac{P_t^H}{P_t} (Y_t - G_t) - C_t \;=\; \frac{Y_t - G_t}{h(Q_t)} - C_t
\end{aligned}
\tag{2.51}
$$

which is the difference between CPI-deflated exports and imports.[13] Substituting into to the trade balance (2.51) the aggregate demand equation (2.45), aggregate consumption (2.36) and international risk sharing (2.7) for Ricardian consumption, we see that it is affected by the real exchange rate (e.g. a depreciation) through several channels:

- through the *expenditure switching* channel both domestic and foreign consumers substitute towards relatively cheaper Home goods, pushing NX_t upwards, governed by trade elasticities η and γ
- through the *terms-of-trade revaluation* channel, due to the CPI/PPI wedge $h(Q_t)$ which is increasing in Q_t through (2.20), *even if actual quantities do not change*, the CPI-deflated NX_t will drop as the same nominal export revenue from domestic goods is now worth less in terms of the consumption basket (which includes imported goods).
- through the *risk sharing* channel: even under incomplete markets the real exchange rate acts as a shock absorber partially insuring the ratio of cross-country consumption values between Ricardians and foreigners. This means that if the relative price of foreign consumption (Q_t) goes up, then Home Ricardians get to consume more. I.e. for given foreign output, weaker exchange rate allows higher aggregate Ricardian consumption in Home, some of which goes towards higher imports, pushing NX_t downwards, governed by the intertemporal elasticity of substitution $1/\sigma$.

[13] This can be verified by starting from the nominal trade balance and applying previous definitions:

$$
\begin{aligned}
\mathcal{NX}_t &= P_t^H (Y_t - G_t) - P_t C_t = \\
&= P_t^H \underbrace{(C_t^H + C_{t,*}^H)}_{(2.44):\ Y_t - G_t} - \underbrace{(P_t^H C_t^H + P_t^F C_t^F)}_{P_t C_t} = \\
&= P_t^H C_{t,*}^H - P_t^F C_t^F
\end{aligned}
$$

- through the *hand-to-mouth* channel (New Keynesian Cross): a higher share λ of HtM means that the average MPC is higher in the economy, leading to potentially higher "New Keynesian" multipliers for any demand shifter shock. Since a real depreciation boosts aggregate demand through (2.45), a larger λ can amplify this increase in output Y_t (to the extent that it doesn't limit the initial real depreciation too much, so the condition set out in Bilbiie (2019) that HtM income *overreacts* aggregate income must hold). Despite the higher output multplier however, consumption would increase even more as part of it goes towards imports, which is why this would mitigate the rise in the trade balance. Through a lower share of Ricardians, a higher λ would also weaken the risk-sharing channel.

In the special case of Galí and Monacelli (2005) with $\lambda = 0$ and $\sigma = \eta = \gamma = 1$ with a symmetric steady state of zero NFA, and complete international financial markets, all of these channels exactly offset each other, and the trade balance does not depend on the real exchange rate but stays zero at all times. Any deviation from this benchmark will make the trade balance react to the real exchange rate.[14]

Balance-of-payments

The Net Foreign Asset (NFA) position of the economy becomes an important state variable under incomplete markets, as it provides crucial feedback into the consumption-saving decision via the debt-elastic risk premium (in other words, foreign asset supply is a function of the domestic NFA position). The law of motion for the NFA position is governed by the Balance-of-Payments (BoP) equation which is derived by combining the budget constraints of domestic households (2.1) and (2.3) with the firm's profit equation (2.28)

[14]Under complete markets where (2.7) is replaced by $\widehat{C}_t = Q_t^{\frac{1}{\sigma}} Y_t^*$, doing the above substitutions leads to the following representation of the trade balance:

$$NX_t = \big[h(Q_t)\big]^{\eta-1}\Big\{(1-\alpha)\,\big[\lambda\check{C}_t + (1-\lambda)Q_t^{\frac{1}{\sigma}}Y_t^*\big] \;+\; \alpha\,\big[h(Q_t)\big]^{\gamma-\eta}\,Q_t^{\gamma}\,Y_t^*\Big\} - \Big[\lambda\check{C}_t + (1-\lambda)Q_t^{\frac{1}{\sigma}}Y_t^*\Big] =$$

$$= \underbrace{\Big\{\underbrace{(1-\alpha)\,\big[h(Q_t)\big]^{\eta}\big[h(Q_t)\big]^{-1}\big[\lambda\check{C}_t + (1-\lambda)Q_t^{\frac{1}{\sigma}}Y_t^*\big]}_{[h(Q_t)]^{-1}C_t^H} \;+\; \underbrace{\alpha\,\big[h(Q_t)Q_t\big]^{\gamma}\big[h(Q_t)\big]^{-1}\,Y_t^*}_{[h(Q_t)]^{-1}C_{t,*}^H}\Big\}}_{[h(Q_t)]^{-1}(Y_t-G_t)} - \underbrace{\Big[\lambda\check{C}_t + (1-\lambda)Q_t^{\frac{1}{\sigma}}Y_t^*\Big]}_{C_t} =$$

$$= \underbrace{\alpha\,\big[h(Q_t)Q_t\big]^{\gamma}\big[h(Q_t)\big]^{-1}\,Y_t^*}_{\text{real exports}} \;-\; \underbrace{\alpha\,Q_t^{-\eta}\,Q\,\big[\lambda\check{C}_t + (1-\lambda)Q_t^{\frac{1}{\sigma}}Y_t^*\big]}_{\text{real imports}}$$

With incomplete markets there is no such clean representation, but it also depends on the full future expected paths of foreign output $\{Y_t^*\}$ and the real exchange rate $\{Q_t\}$.

and the government budget constraint (2.32):

$$\left[\frac{b_t}{1+i_t} - \frac{b_{t-1}}{\Pi_t}\right] + \left[\frac{b_t^*}{(1+i_t^*)\psi_t} - \frac{b_{t-1}^*}{\Pi_t^*}\frac{Q_t}{Q_{t-1}}\right] = NX_t \tag{2.52}$$

where $b_t + b_t^* \equiv \frac{B_t}{P_t} + \frac{e_t B_t^*}{P_t}$ is the *face value* of the Net Foreign Asset (NFA) position of the economy (expressed in LCY and in CPI-deflated real terms). The Balance-of-Payments states that the change in NFA (the "Financial Account balance") must be equal to the net savings of the domestic economy (the "Current Account balance" which in turn is the sum of the trade balance and net interest income). A country that is producing more than it is consuming (i.e. saves) will lend the resulting savings to foreigners and accumulate claims on them.

In the baseline scenario there is no original sin, and international trade is financed by LCY bonds only. Applying (2.49) to the BoP equation (2.52) we get

$$\frac{b_t}{1+i_t} - \frac{b_{t-1}}{\Pi_t} = NX_t \tag{2.53}$$

This demonstrates how the ability to issue LCY debt (or save in LCY bonds) can allow surprise domestic inflation Π_t to reduce the real burden of already existing external debt stock $(-b_{t-1})$, as determinded by the *ex post* real interest rate $(1+r_{t-1}) = \frac{1+i_{t-1}}{\Pi_t}$, similarly to how it can ease the burden of public debt on the government.[15] Put differently, surprise inflation can have revaluation effects on the existing external debt stock.[16]

Notice how this makes monetary policy non-neutral even under flexible prices, as suprise inflation can affect the real trade balance and next period's real borrowing/saving needs b_t which in turn feeds back into the effective real interest rate through the risk premium $\psi_t(b_t)$. This introduces another important channel through which monetary policy affects the economy.[17]

[15]Manipulating (2.53) leads to:

$$\underbrace{\frac{b_t}{1+i_t}}_{NFA_t} = NX_t + \underbrace{\frac{1+i_{t-1}}{\Pi_t}}_{1+r_{t-1}}\underbrace{\frac{b_{t-1}}{1+i_{t-1}}}_{NFA_{t-1}}$$

$$\underbrace{\frac{b_t}{1+i_t} - \frac{b_{t-1}}{1+i_{t-1}}}_{FA_t} = \underbrace{NX_t + \overbrace{i_{t-1}\frac{b_{t-1}}{1+i_{t-1}}}^{\text{interest payment}} + \overbrace{\left[\frac{1}{\Pi_t} - 1\right] b_{t-1}}^{\text{revaluation}}}_{CA_t}$$

[16]Note that here NFA is defined as the real *market value* $\frac{b_t}{1+i_t}$ (as opposed to *face value* b_t) of the net bond position, and the Financial Account the change of this NFA position $FA_t = NFA_t - NFA_{t-1}$.

[17]Note the parallel with government debt. With passive monetary policy (in the Leeper (1991) sense) inflation would play a large role in real public debt stabilization which seems to carry over

This is not the case in the alternative scenario with original sin, when the small open economy can only borrow (or save) in FCY. Then, after inserting (2.49) into (2.52), the balance-of-payments will be:

$$\frac{b_t^*}{(1+i_t^*)\psi_t} - \frac{b_{t-1}^*}{\Pi_t^*}\frac{Q_t}{Q_{t-1}} = NX_t \tag{2.54}$$

In the case of FCY bonds the above described valuation effects can only happen via changes in the real exchange rate (or foreign inflation, which we treat here as fixed), which monetary policy cannot affect under flexible prices. In other words, FCY debt inherited from last period B_{t-1}^* cannot be inflated away by surprise domestic inflation, since it needs to be paid back in FCY, and under flexible prices higher inflation would just lead to an offsetting nominal depreciation (such that the real exchange rate does not change $\bar{Q} = \frac{\uparrow e_t P_t^*}{P_t \uparrow}$), and more LCY would be needed to pay back the same FCY amount. On the other hand, surprise real exchange rate fluctuations *can* cause valuation effects in the NFA position, potentially affecting the current account. Under sticky prices unexpected nominal exchange rate movements also suffice to achieve this, since they translate into real exchange movements.[18]

The difference between the LCY or FCY regimes, in terms of the dynamics of the real *market value* of NFA position, can be precisely captured by surprise nominal depreciation.[19]

to the open economy setting when it is the external debt of the whole economy instead of the government's which needs stabilizing. `Could a passive monetary policy regime substitute for the debt-elastic risk premium to play the role of making external debt stationary?` But even with active monetary policy it matters whether it fixes the nominal exchange rate, CPI inflation or just follows a flexible Taylor rule, since these imply different paths for inflation Π_t – just like in the absence of Ricardian equivalence when monetary policy matters also via its fiscal consequences.

[18]Manipulating (2.54), and applying the nominal UIP condition, we get:

$$\underbrace{\frac{b_t^*}{(1+i_t^*)\psi_t}}_{NFA_t^*} = NX_t + \frac{e_t/e_{t-1}}{\Pi_t}\, b_{t-1}^* =$$

$$= NX_t + \underbrace{\frac{(1+i_{t-1}^*)\,\psi_{t-1}\, e_t/e_{t-1}}{\Pi_t}}_{\frac{(1+i_{t-1})\,\nu_t}{\Pi_t} = (1+r_{t-1})\,\nu_t} \underbrace{\frac{b_{t-1}^*}{(1+i_{t-1}^*)\,\psi_{t-1}}}_{NFA_{t-1}^*}$$

$$\underbrace{\frac{b_t^*}{(1+i_t^*)\psi_t} - \frac{b_{t-1}^*}{(1+i_{t-1}^*)\psi_{t-1}}}_{FA_t} = \underbrace{NX_t + \overbrace{\left[(1+i_{t-1}^*)\psi_{t-1} - 1\right]\frac{b_{t-1}^*}{(1+i_{t-1}^*)\,\psi_{t-1}}}^{\text{interest payment}} + \overbrace{\left[\frac{e_t/e_{t-1}}{\Pi_t} - 1\right] b_{t-1}^*}^{\text{revaluation}}}_{CA_t}$$

[19]Note that in the symmetric equilibrium with zero steady state NFA, the first-order valuation effects coming from either higher inflation or real depreciation are zero, thereby making the FCY and LCY regimes identical.

Since the nominal uncovered interest rate parity does not necessarily hold ex post, generally we have an expectation error $\nu_t \equiv \frac{e_t}{\mathrm{E}_{t-1} e_t} \neq 1$ such that $(1+i_{t-1})\nu_t = (1+i^*_{t-1})\,\psi_{t-1}\,\frac{e_t}{e_{t-1}}$. Applying this to the balance-of-payments equations (2.53) and (2.54), after some manipulations (see in the footnotes), we get:

$$NFA_t = NX_t + (1+r_{t-1})\,NFA_{t-1}$$

$$NFA^*_t = NX_t + (1+r_{t-1})\,\nu_t\,NFA^*_{t-1}$$

In other words, the effective *ex post* real interest rate will be different in the two currencies due to this exchange rate expectation error ν_t.

Premium function

A debt-elastic premium function ψ_t represents the asset supply of foreigners which is a negative function of the economy's NFA position. The intuitive way to think about it is that if the domestic economy were to go deeper in debt (lower and negative b_t) than some exogenous tolerated ζ_t value, then foreigners would lend only at a higher interest rate.

The premium function depends on the *face value* of the NFA position $b_t + b^*_t$ (relative to GDP) which is determined by the consumption-saving decisions of the domestic economy as captured by the BoP equation (2.52):

$$\begin{aligned}\psi_t &= e^{-\delta\left(\frac{B_t+e_t\,B^*_t}{P^H_t Y_t}-\zeta_t\right)} = \\ &= e^{-\delta\left((b_t+b^*_t)\frac{h(Q_t)}{Y_t}-\zeta_t\right)} \qquad (2.55)\\ \zeta_t &= (1-\rho_\zeta)\zeta + \rho_\zeta\zeta_{t-1} + \epsilon^\zeta_t \qquad (2.56)\end{aligned}$$

where shocks to ζ_t are used to model "sudden stops", i.e. a sudden worsening of international lending conditions leading to a reversal of capital inflows and forcing the domestic economy to rapid external adjustment.

Apart from being used to capture sudden stops, the presence of ψ_t also serves the purpose of making the dynamics of our incomplete market economy stationary, and to pin down a unique steady state, as shown by Schmitt-Grohé and Uribe (2003). In the absence of idiosyncratic risk, assets would not feature in the consumption/saving decision of households (as governed by the Euler equation or international risk sharing condition) without the presence of ψ_t, and therefore nothing would anchor the NFA position of the economy (which is a result of past consumption/saving choices as pinned down by the balance-of-payments). This would not only make the effect of unexpected shocks permanent (making the demand imbalance across countries a random walk, as shown above), but would also prevent pinning down a unique steady state NFA position. The presence of ψ_t allows the asset position of the economy to feed back into consumption-saving decisions via risk-adjusted interest rates, and thereby rendering it stationary.

If foreign households also have a personal discount factor of β, then based on (2.6) the steady state risk premium consistent with a stationary equilibrium (i.e. one without real depreciation/appreciation) is $\psi = 1$, which means that the steady state NFA position is pinned down as $b\frac{h(Q)}{Y} = \zeta$.[20]

2.2.7 Dynamic equilibrium

Equilibrium is depicted in the above model by equations as listed in Appendix B.1.

2.3 Monetary-fiscal interactions, heterogeneity and openness

2.3.1 Active and passive monetary and fiscal policies

Monetary-fiscal interactions are modelled in the framework of Leeper (1991), via the monetary and fiscal policy rules (2.29) and (2.33). Depending on the policy reaction parameters ϕ^{π} and ϕ_B we can talk about "active" or "passive" policies. In a coordinated setting only one of the policy branches can be active, meaning that it can freely *lead* in pursuing a given objective while the other policy branch must passively *follow*, in a sense "subordinating" itself to the objective of the former.[21]

In a regime with active monetary and passive fiscal policy mix (AM-PM), the central bank actively manages the real interest rate to stabilize inflation (through affecting aggregate demand), while fiscal policy must passively adjust the primary budget balance to offset the monetary-induced changes in interest rates such that it ensures a stable path for public debt. In terms of the policy parameters (and assuming raising real interest rates is contractionary for aggregate demand) this policy regime is characterized by $\phi^{\pi} > 1$ and $\phi_B > 1 - \beta$, since these ensure a strong enough reaction of nominal interest rates to

[20] Notice that this does not really get around the problem of *endogenously* pinning down the steady state asset distribution. Conditional on the parameter ζ, the steady state NFA is determined, but ζ is still chosen arbitrarily. Without idiosyncratic risk and a borrowing constraint, however, there's no precautionary saving motive (at least up to second order) which would pin it down, so this choice is necessarily arbitrary.

[21] With both policies being passive (PM-PF) the price level is not pinned down uniquely, giving rise to multiple sunspot equilibria, while both policies being active (AM-AF) leads to conflict between them resulting in explosive dynamics (see Figure 2.1).

inflation (such that real rates move in the same direction), together with a strong enough reaction of fiscal surpluses to public debt[22] (upper right quadrant of Table 2.1 amd Figure 2.1).

	$\phi^\pi < 1$	$1 < \phi^\pi$
$\phi_B > 1-\beta$	PM-PF	AM-PF
$\phi_B < 1-\beta$	PM-AF	AM-AF

Table 2.1: Policy regimes in the "Keynesian" (non-IADL) region of the paramter space.

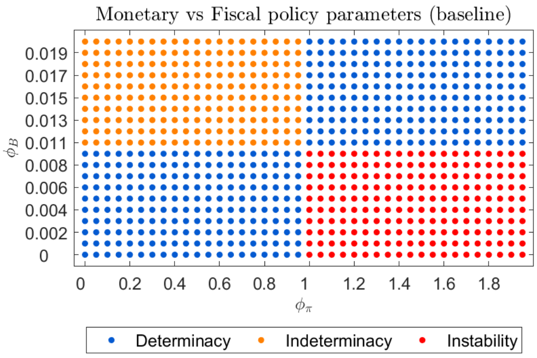

Figure 2.1: Model determinacy properties in the (ϕ^π, ϕ_B) plain, given other parameters at baseline values ($\lambda = 0.3, \alpha = 0.5, \varphi = 2$, i.e. non-IADL, Keynesian region).

In contrast, under a regime with passive monetary and active fiscal policy mix (PM-AF), instead of being constrained by the need for debt stabilization, fiscal policy is free to actively set the path of primary budget surpluses, while monetary policy must passively accommodate fiscal shocks by tolerating deviations from price stability and letting inflation adjust to revalue nominal public debt. Thereby, instead of the primary budget balance, inflation becomes the primary tool for public debt stabilization to which monetary policy is forced to subordinate its price stability objective. In terms of policy parameters this translates to $\phi^\pi < 1$ and $\phi_B < 1-\beta$, since these reflect relatively unresponsive interest rates to inflation, which help stabilize debt dynamics, enabling fiscal surpluses to react less to public debt.

[22]By looking at the log-linearized government budget constraint (ignoring G_t for simplicity, and tilde denoting linear deviations from steady state) and substituting in policy rules, real public debt becomes a mean reverting stationary process (which can be solved backward) precisely iff $\phi_B < 1-\beta$:

$$\tilde{b}^g_t = \beta^{-1}\tilde{b}^g_{t-1} + \bar{b}^g\left[i_t - \beta^{-1}\pi_t\right] - \beta^{-1}\tilde{T}_t$$

$$\tilde{b}^g_t = \beta^{-1}\left(1-\phi_b\right)\tilde{b}^g_{t-1} + \bar{b}^g\left[\phi_\pi - \beta^{-1}\right]\pi_t + \varepsilon^T_t$$

In other words, in the PM-AF regime monetary policy essentially helps keep the real burden of public debt manageable, creating fiscal space for the government to run larger primary deficits. In this sense the PM-AF regime in this model can be thought of as the analogue of "helicopter money" or money-financed fiscal stimulus. In both cases an unresponsive monetary policy accommodates the fiscal expansion by keeping interest rates low and "inflating away" or "monetizing" some of the nominal debt. The only difference is that in the latter case the monetary policy rule is defined not in terms of interest rate policy, but by a money supply rule. Instead of directly influencing interest rates to keep them low, the central bank prints money, which *in turn* will lead to lower interest rates (via the interaction of money demand and the increased money supply). As argued by Bianchi, Faccini and Melosi (2020), money is just a tool to deliver a given interest rate, and modelling it explicitly or just assuming the central bank is able to set the nominal interest rate makes no big difference to equilibrium dynamics.[23] What matters instead is whether monetary policy is conducted with price stability as the primary objective, or is subordinated to the needs of fiscal debt stabilization.

Rewriting the government budget constraint (2.32) in terms of the CPI-delfated real *market value* of public debt, we can see how debt dynamics depend on the primary budget balance, nominal interest rates and inflation.

$$\begin{aligned}\frac{b_t^g}{1+i_t} - \frac{b_{t-1}^g}{1+i_{t-1}} &= \left[\frac{G_t}{h(Q_t)} - T_t\right] + r_{t-1}\,\frac{b_{t-1}^g}{1+i_{t-1}} \\ &= \underbrace{\left[\frac{G_t}{h(Q_t)} - T_t\right]}_{\text{primary deficit}} + \underbrace{i_{t-1}\frac{b_{t-1}^g}{1+i_{t-1}}}_{\text{interest payment}} - \underbrace{\frac{\Pi_t - 1}{\Pi_t} b_{t-1}^g}_{\text{revaluation}}\end{aligned}$$

where $r_{t-1} = \frac{1+i_{t-1}}{\Pi_t}$ is the *ex post* real interest rate. The distinction between AM-PF and PM-AF can be captured by the relative roles of the above components. In an AM-PF

[23] There is a slight difference though, to the extent that money is non-interest bearing liability of the consolidated government – as opposed to bonds. Therefore, the decision whether to finance the budget deficit by issuing bonds or money (or whether to have the central bank buy up some of the bonds with newly printed money) does matter somewhat as far as the difference between interest rates on bonds and money is positive. In such a case monetary financing can generate some *seigniorage* revenue for the government. However, in most modern economies the monetary liabilities of the government (central bank reserves) do pay an interest rate similar to those of government bonds, which limits the scope for obtaining seizable seigniorage revenue. In any case, the significance of seigniorage pales in comparison to the distinction between AM-PF and PM-AF regimes which relates to the *objectives* of monetary policy (instead of the debt management policy of the consolidated government, trying to optimize the *composition* of its liabilities). Even though in the PM-AF regime of the above model the deficit is fully debt-financed, this debt can be thought of as the joint (money and bond) liabilities of the consolidated state, and the interest rate being the average interest on central bank reserves and government bonds.

regime debt stabilization mainly depends on the primary balance, while relatively stable inflation keeps debt revaluation in check, and actively managed interest rates could even force the hand of the fiscal authority to adjust the budget. In contrast, in a PM-AF regime the primary balance can be set completely exogenously, so stationarity of debt dynamics must be ensured by relatively unresponsive interest costs, and inflation providing more cushion for fiscal policy through revaluation. The latter channel is present because the government issues *nominal* bonds, whose real value can be eroded by *surprise* inflation. The joint effects of nominal interest rates and inflation are nicely summarized by the *ex post* real interest rate, reflecting the real burden of debt: from this perspective a PM-AF policy regime stabilizes public debt less via adjusting the primary government budget, and more via letting inflation move the *ex post* real interest rate.

However, there is a more discontinuous contrast between the two policy regimes than just quantitative differences in the relative roles of budget balances and inflation in stabilizing debt dynamics. As explained in Leeper and Leith (2016), under the AM-PF regime (as long as Ricardian equivalence holds), monetary-fiscal interactions are like a one way street, going from the central bank to the government. A rise in interest rates can force the government to adjust the budget balance, but inflation is completely insulated from how fiscal policy is conducted and fiscal imbalances are not relevant for inflation determination. Bianchi and Melosi (2019) call this "*Monetary and Fiscal Dischotomy*". By contrast, under the PM-AF regime this dichotomy breaks down, and inflation becomes a joint monetary-fiscal phenomenon, determined by the very need the stabilize the real value of public debt. This rhymes with the Fiscal Theory of the Price Level.

Of course, when Ricardian equivalence fails (e.g. due to household heterogeneity and fiscal redistribution), fiscal policy does matter even under the AM-PF regime. The timing of taxes via ϕ_B affects the disposable *current* income of high MPC hand-to-mouth households which then influences aggregate demand and inflation. These kind of monetary-fiscal interactions due to the breakdown of Ricardian equivalence, however, are of a fundamentally different nature than the one arising under a PM-AF regime. In fact, with a PM-AF policy mix Ricardian equivalence breaks down even in a representative agent model without redistributive fiscal policies *because of* the kind of monetary-fiscal interactions in this regime (as explained in the next section by the nominal wealth effect).

2.3.2 Ricardian equivalence

Strictly speaking, Ricardian equivalence means that the *timing* of taxes does not matter, i.e. that the debt stabilization decisions of fiscal policy are irrelevant. By the same token, evironments where Ricardian equivalence breaks down and fiscal policy matters more, present an obvious candidate for richer monetary-fiscal interactions which is why it is worth exploring when this occurs.

In terms of our model Ricardian equivalence translates to ϕ_B being irrelevant for equilibrium dynamics. While household heterogeneity and the presence of high-MPC hand-to-mouth agents seems to make it straightforward that in this TANK model Ricardian equivalence fails even in the AM-PF policy regime, there are some special conditions under which it still holds. Such a condition is that no taxes are paid by hand-to-mouth agents, i.e. that $\phi = 0$. In this case, optimizing and consumption-smoothing Ricardian households completely internalize the government's budget constraint and do not care about the time path of taxes or government bonds: they are the only holders of public debt which is exactly offset by the present value of their tax obligations (in other words, government bonds constitute zero net wealth for them).[24]

Whenever $\phi > 0$, HtM households also pay some of the taxes, and since they consume their current after-tax income every period, the timing of taxes ϕ_B obvioulsy matters, and Ricardian equivalence breaks down. Ricardian households are still optimizing lifetime income consumers so the time path of *their* taxes $\widehat{T}_t$ should not directly matter (as long as their present value is the same), and it is primarily the path of HtM taxes $\check{T}_t$ what matters for aggregate dynamics. But notice that with $\phi > 0$ government bonds become net wealth for Ricardians, as they hold all the public debt but are liable for only a $(1 - \phi)$ fraction of the present value of the offsetting tax burden. This net wealth essentially represents a loan from Ricardians to HtM households (who are otherwise shut out of financial markets) via the intertemporal government budget. The timing of taxes ϕ_B determines for how long Ricardians must hold this net wealth, i.e. how fast HtM will repay their share of the public debt to Ricardians. The more persistent public debt is (lower ϕ_B), the more

[24] Bilbiie, Monacelli and Perotti (2013) point out another important special case. Even if $\phi > 0$ (i.e. HtM households, for whom timing matters, are also taxed), with flexible prices $\theta = 0$ and with equal steady state consumption $\widehat{C} = \check{C}$ (or with fully inealstic labor supply $\varphi \to \infty$) this would still not affect the *aggregate* dynamics. Therefore the timing of taxes is also irrelevant and Ricardian equivalence still prevails in this limited sense (i.e. meaning only aggregate variables). While $\phi > 0$ means that government bonds *are* net wealth for Ricardians (as they hold all the public debt but are liable for only a $(1 - \phi)$ fraction of the present value of the offsetting tax burden), holding this net wealth crowds out precisely as much Ricardian consumption and leisure, as the increase in HtM consumption and leisure induced by their tax cut: income effects on labor supply exactly offset each other in this case. Although the timing of taxes ϕ_B certainly matters for the time path of distributional variables, it does not affect aggregate dynamics. **For Ricardian equivalence to break down also in the latter sense, (barring steady state consumption inequalities) nominal rigidities are crucial** as they introduce an additional negative income effect on Ricardians' labor supply via countercyclical profit variations. This will prompt them to work more than by which the HtM is willing to work less, supporting the aggregate expansion in output.

it crowds out Ricardian consumption (which in turn hurts HtM households too, as lower demand hurts their incomes).

The breakdown of Ricardian equivalence due to the above reasons of household heterogeneity will naturally induce some monetary-fiscal interactions. On the one hand, monetary policy has fiscal consequences via interest expenses and the revaluation of nominal public debt, which now have differing impact on the real economy depending on how fiscal policy is managing public debt and how it distributes taxes across households. On the other hand, as pointed out by Kaplan, Moll and Violante (2018), the decision of the fiscal authority whether to finance current public expenditures by raising taxes or by issuing debt, or how the tax burden is shared between households would no longer be inconsequential, which in turn affects monetary policy and could force the central bank to react. In other words, inflation would no longer be completely insulated from fiscal policy and the *Monetary-Fiscal Dichotomy* would break down even in an AM-PF policy regime.

Nominal wealth effect under the PM-AF regime

Everything drastically changes under the PM-AF policy regime when fiscal policy gives up debt stabilization ($\phi_B \approx 0$) and runs budget deficits unbacked by the present value of future primary surpluses. As already mentioned above, in this case Ricardian equivalence fails even in a RANK model (with only optimizing Ricardian agents, $\lambda = 0$), precisely *because of* the kind of monetary-fiscal interactions under this policy mix, and not due to household heterogeneity: even a simple debt-financed tax cut for Ricardian agents can set off large dynamic effects.

Jacobson, Leeper and Preston (2019) explain that this is due to the wealth effects of unbacked nominal debt issuance on aggregate demand: deficit-financed transfers to households today do not entail tax increases in the future, which prompts consumers to spend rather than save them. Government bonds become *nominal* net wealth. However, in equilibrium there's nobody to sell their windfall nominal bonds to in exchange for consumption goods, so it leads to a collapse in the real value of bonds via higher consumer price inflation.[25] At the same time, since output is demand determined with nominal rigidities, the rise in aggregate demand due to the nominal wealth effect, results in real

[25] Putting it another way, government bonds are still not *real* net wealth ex post, since the real intertemporal government budget constraint (which households internalize) must hold. As Bianchi and Melosi (2019) point out, since nominal public debt is no longer backed by the present value of future tax revenues, agents realize that the government will not be able to repay it with consumption goods in the future, therefore everyone wants to sell bonds in exchange for consumption goods, the price of which must go up to clear the market. And the fiscal deficit will have been paid for by the erosion of the real value of household assets: inflation tax instead of explicit taxation.

economic expansion.

The rise in the price level also ensures that the intertemporal *real* government budget constraint still holds: even though the present value of future primary surpluses has fallen as a result of the unbacked tax cut, the real value of outstanding debt has also been eroded by inflation. It is essentially this revaluation of already existing nominal assets via an "inflation tax" which ends up paying for the fiscal deficit in real terms – but in a way that (with rigid prices) generates a huge expansion in the meantime. For all this to work, monetary policy must passively accommodate rising inflation and let it "inflate away" or stabilize the real value of public debt in the spirit of the Fiscal Theory of the Price Level. The above discussion again underlines how an unbacked fiscal expansion can influence inflation, and how debt has monetary consequences in such an environment.

Exploring how openness ($\alpha > 0$) and the HtM amplification ($\lambda > 0$) via the New Keynesian Cross interacts with this PM-AF policy regime, as exchange rate movements influence inflation and income, including that of high-MPC HtM households, is an important objective of this paper.

2.3.3 Open economy New Keynesian Cross with redistribution

As in the closed economy TANK model of Bilbiie (2019), there is a New Keynesian Cross in operation. This gives the economy a more "Keynesian flavor" in the sense that the influence of monetary policy on aggregate demand operates more through *indirect* general equilibrium propagation on income, rather than mainly through *direct* intertemporal substitution in response to real interest rate changes (as in RANK models). The reason for this is that in presence of hand-to-mouth agents the average MPC of the economy rises, meaning that aggregate consumption will be more responsive to changes in current income than with only permanent income consumer Ricardian agents who smooth out temporary income changes. A higher average MPC implies a steeper planned expenditure (PE) curve, so whatever shifts aggregate demand, its effect will be multiplied through the effect on HtM consumption in a similar fashion than in the old Keynesian Cross analysis. In other words, the HtM channel can deliver amplification. The same channel manages to deliver positive fiscal multipliers on aggregate consumption.

However, as Bilbiie (2019) points out, it is not the mere addition of HtM agents (and the ensuing increase in average MPC) that delivers amplification, but an income distribution such that *their* income rises more than proportionally to aggregate income, which in turn depends on endogenous profit redistribution in their favor through τ^D, and the labor supply elasticity φ. Bilbiie (2019) refers to this as the *counter-cyclical inequality channel*. In a closed economy amplification through the HtM channel occurs if and only if the elasticity of HtM income to aggregate income is higher than unity: $\chi = 1+\varphi\left(1-\frac{\tau^D}{\lambda}\right) > 1$. Otherwise the smaller size of the direct effect due to HtM presence (the shift of the PE curve is decreasing in λ) will dominate the larger indirect effect coming from higher average MPC (the slope of the PE curve is increasing both λ and χ). As long as $\chi > 1$ is satisfied, income inequality is countercyclical and there is AD amplification of monetary policy as well as positive fiscal multipliers which increase in the share of HtM λ. Bilbiie (2019) shows that the interest elasticity of aggregate demand (in the aggregate Euler equation) is $-\frac{1}{\sigma}\frac{1-\lambda}{1-\lambda\chi}$.[26]

As pointed out by Broer et al. (2020), countercyclical profit variations are an important part of the New Keynesian transmission mechanism through inducing income effects on the labor supply of households who receive them. To the extent that there is *endogenous profit redistribuition* towards HtM households ($\tau^D > 0$), it is important also because its part of their current disposable income which they consume every period. Bilbiie (2019) shows that, since profits are countercyclical, τ^D can dampen the degree to which the income of HtM households *overreacts* aggregate income, potentially making it *underreact* (if $\tau^D > \lambda$).[27] This effect on the cyclicality of HtM income reduces the amplification through the New Keynesian Cross, originally coming from higher average MPC.

Bilbiie (2019) also considers *exogenous redistribution* by varying the ϕ share of aggregate taxes which fall on HtM households ($\phi = \lambda$ being the uniform taxation case): a higher ϕ mitigates the fiscal multiplier. However, he looks at *balanced budget* multipliers where taxes rise immediately to cover higher government expenditures. In contrast, the government budget inthis model can be in deficit which is financed fully by issuing debt – taxes adjust only later to service public debt, and in this setup ϕ will have a different effect, mainly via effecting Ricardian lifetime income, which can get multiplied via the New Keynesian Cross, also affecting HtM households (see Figure B.1 in the Appendix).

In our open economy setup these multipliers are mitigated as α increases, since some of the increase in consumption will be directed towards import goods. Boerma (2014) and Iyer (2017) shows that in a complete market small open economy with $\tau^D = 0$ we get $\chi = 1 + \varphi(1-\alpha)$. With incomplete markets, however, the conditions are likely to be different.[28] Due to imperfect international risk sharing, the real exchange rate is

[26] With $\chi = 1$ (e.g. with uniform profit distribution such that $\tau^D = \lambda$, or with $\varphi = 0$ infinitely elastic labor supply) the total effect in RANK and TANK models are identical, and it is only their decomposition into direct and indirect effect which changes, as also shown by Kaplan, Moll and Violante (2018). In other words, under acyclical income inequality the HtM-TANK channel cannot amplify the total effect.

[27] At the same time, more profit redistribution away from Ricardians reduces the income effect on their labor supply.

[28] See Appendix **??**.

decoupled from Ricardian consumption (in addition to also affecting HtM income), so it enters differently in the aggregate IS curve.

2.3.4 Equilibrium determinacy

Inverted Aggregate Demand Logic

Bilbiie (2008) shows that with a sufficiently high share of Hand-to-Mouth agents $\lambda^* < \lambda$, the interest elasticity of aggregate demand can change sign, i.e. decreasing real interest rates have contractionary effects. In a closed economy without endogenous profit redistribution ($\tau^D = 0$) he shows this threshold to be a decreasing function of the inverse Frisch elasticity of labor supply φ. Boerma (2014) generalizes this condition in a complete market open economy setting to get $\lambda^* = \frac{1}{1+\varphi(1-\alpha)}$, which shows that openness shrinks the "non-Keynesian" region where this *inverted aggregate demand logic* (IADL) applies.

The intutition is that falling real interest rates affect aggregate demand via three channels: i) intertemporal substitution induces Ricardian households to bring consumption forward, ii) a depreciating real exchange rate stimulates external demand for domestic goods, and iii) higher real wages erode firm profits, causing a negative income effect and thereby *hurting* the consumption for firm-owning Ricardian households (while also prompting them to work more). Real interest rate reductions become contractionary when the third channel dominates the previous two. This can happen with more inelastic labor supply (high φ) when real wages need to rise more to satisfy higher labor demand, thereby causing a shaper fall in firm profits; and/or when a given fall in profits is concentrated on a smaller fractions of firm-owning households (high λ). As Boerma (2014) explains, in a more open economy (higher α) this negative income effect now has to additionally offset expanding external demand (channel ii)) as well in order to reach the IADL region.

Figure B.11 in the Appendix shows the combinations of of λ, φ and α which constitute the IADL region of the paramter space in the model of this paper, corresponding to the points where Figure B.11 indicates model indeterminacy.[29] This is similar to the analogous figure in Boerma (2014).[30]

[29] In the IADL region the model will have multiple stable solutions (indeterminacy) under a monetary policy rule satisfying the Taylor-principle $\phi^\pi > 1$ (and fiscal rule with $\phi_B < 1 - \beta$). See explanation later.

[30] Although does not match it exactly probably due to different parametrization and imperfect international risk sharing in the current model. For example, trade elasticities η, γ can also infuence the IADL region as they determine the strength of real exchange rate effects on external demand, for given openness α.

Policy regimes, IADL and openness

The existence of a unique and stable dynamic equilibrium depends crucially on the specification of monetary and fiscal policies to rule out self-fulfilling expectations and to pin down the price level. Determinacy requires that only one of the policy branches be "active", while the other must remain "passive" (AM-PF or PM-AF). With both policies being passive (PM-PF) the equilibrium is not pinned down uniquely, while both policies being active (AM-AF) leads to conflict between them resulting in explosive dynamics. When monetary policy actively manages the real interest rate to keep inflation around its target (through affecting aggregate demand), then fiscal policy cannot rely on inflation to make public debt stationary, but instead must passively adjust the primary budget balance ($\phi_B > 1 - \beta$) otherwise debt would explode. In contrast, when fiscal policy actively ignores debt stabilization ($\phi_B < 1 - \beta$), then monetary policy must passively accommodate fiscal shocks by letting inflation adjust to revalue nominal public debt.

Consider first a debt stabilizing passive fiscal policy featuring $\phi_B > 1-\beta$. In the "Keynesian" (non-IADL) region of the model, with $\phi^y = \phi^e = 0$ in the monetary rule, a unique equilibrium requires the central bank to satisfy the Taylor-principle: $\phi^\pi > 1$ ensures that the effect of inflationary news about the future entails a rise in the real interest rate, dampening the effect of such news by constraining aggregate demand. The baseline scenario satisfies this AM-PF policy mix.

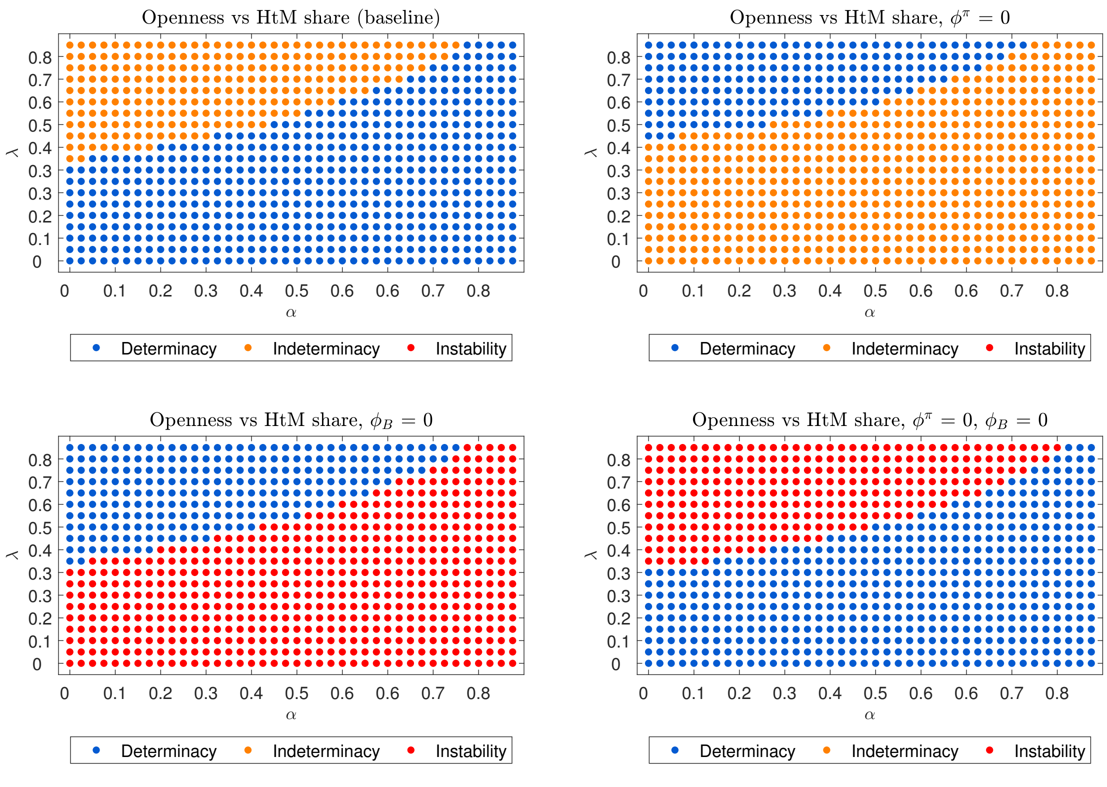

Figure 2.2: Model determinacy properties in the (α, λ) plain, for different policy regimes. Unless otherwise indicated, baseline parameters are $\phi^\pi = 1.5$ and $\phi_B = 0.2 > 1 - \beta$.

However, as we have seen above, in the IADL region of the parameter space a rise in the real interest rate is *expansionary*. Therefore, satisfying the Taylor principle would mean that that in response to inflationary (potentially unfounded) news the central bank would raise the real interest rate, which in the IADL region *stimulates* aggregate demand, further amplifying the initial inflationary shock (or validating the unfounded beliefs, enabling self-fulfilling sunspot equilibria), resulting in indeterminacy and multiple stable equilibria.

This is what can be seen in the top left panel of Figure 2.2: while the baseline policy specification satisfying the Taylor principle yields a unique equilibrium in the Keynesian region, in the northwest corner of high λ and low α combinations constituting the IADL region (for given φ), it yields indeterminacy. The solution to this, as proposed by Bilbiie (2008) and generalized by Boerma (2014), is the *inverted Taylor principle*. For instance, by making monetary policy completely unresponsive to inflation and setting $\phi^{\pi} = 0$, as in the top right panel of Figure 2.2, determinacy is ensured in the IADL region.

So far we looked at cases where fiscal policy is passive. However, **by allowing for a richer framework of monetary-fiscal interactions, I show that the inverted Taylor principle is not necessary to restore equilibrium determinacy in the IADL region.** The lower panels of Figure 2.2 represent an *active* fiscal policy which completely ignores debt stabilization ($\phi_B = 0$). In this case, keeping the baseline monetary policy rule, which satisfies the traditional Taylor principle $\phi^{\pi} > 1$, does in fact deliver a uniqe stable equilibrium in the IADL region (bottom left panel). In other words, **active fiscal policy can substitute the inverted Taylor principle under IADL.** Moreover, applying the inverted Taylor principle prescription of $\phi^{\pi} = 0$ when fiscal policy is active, is not only unnecessary but, instead of ensuring determinacy, would just lead to instability and explosive solutions in the IADL region (bottom right panel).

The reason for this can be found in the discussion on active and passive monetary and fiscal policies. A unique and stable equilibrium requires that we are in either the AM-PF regime or the PM-AF regime, while PM-PF yields indeterminacy, and AM-AF leads to instability. But notice that **what constitutes "active" monetary policy, changes in the IADL region** of high λ and/or low α. Since it is now real interest rate *decreases* that are contractionary, actively countering an inflationary shock requires the central bank to let the real interest rate fall – as opposed to raising it via the Taylor-principle. Therefore, $\phi^{\pi} = 0$ becomes the active monetary policy under IADL. This gives determinacy in combination with passive fiscal policy according to the inverted Taylor principle (top right panel), but leads to instability with active fiscal policy (bottom right panel). On the other hand, $\phi^{\pi} > 1$ will mean passive monetary policy in the IADL region, which results in indeterminacy together with passive fiscal policy (top left panel), but can still

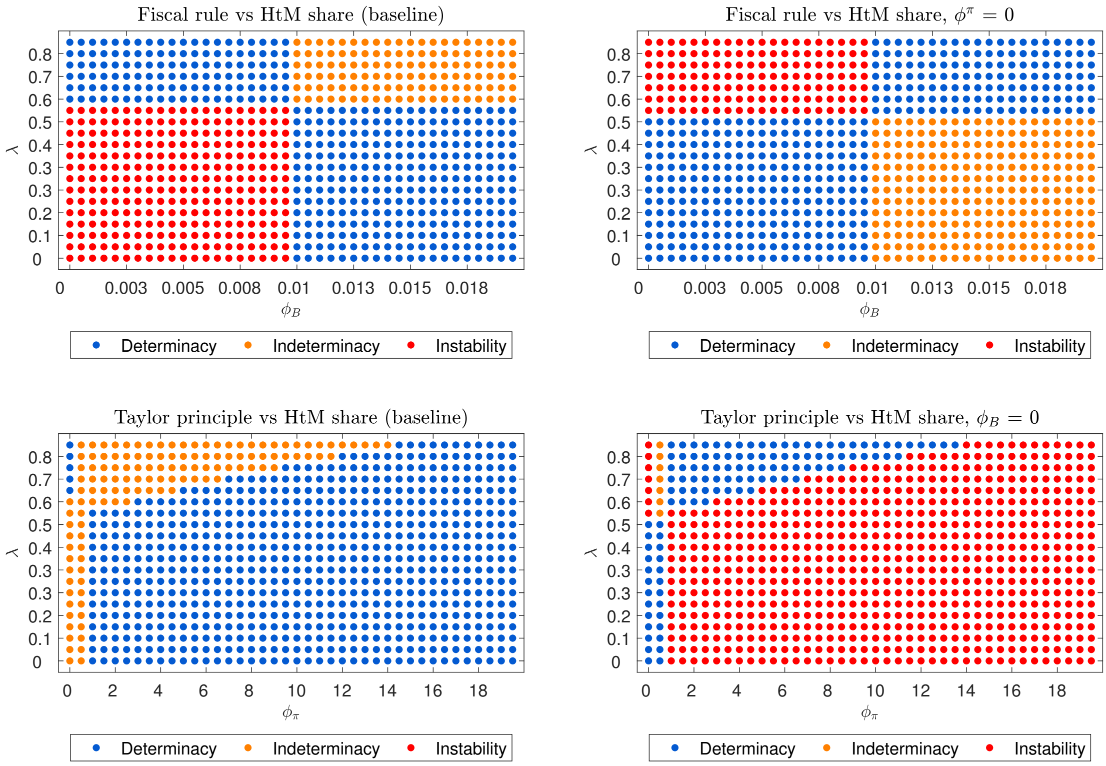

Figure 2.3: Model determinacy properties in the (ϕ_B, λ) and the (ϕ^π, λ) plains. Unless otherwise indicated, baseline parameters are $\phi^\pi = 1.5$ and $\phi_B = 0.2 > 1 - \beta$, with openness set at $\alpha = 0.5$.

deliver a unique equilibrium with fiscal activism (bottom left panel).

This is why the left column of Figure 2.2 is basically the flipside of the right column, precisely along the border of the IADL region. Depending on which side of that border we are, it differs whether $\phi^\pi > 1$ or $\phi^\pi = 0$ constitutes active monetary policy.

Figure 2.3 tells the same story from another perspective, exploring a continuum of values for ϕ^π or ϕ_B instead of just two discrete points. The upper panels show how the threshold for active fiscal policy lies at $\phi_B = 1 - \beta$ without being affected by the share of hand-to-mouth households λ.[31] The bottom panels show that with a Taylor rule that reacts to *current* inflation (as opposed to expected inflation) such as (2.29), monetary activism in the IADL region of high λ can not only be achieved by reducing ϕ^π to close to zero, but

[31] This is in contrast to Leith and Wren-Lewis (2008) who show in a perpetual youth model that having more non-Ricardian consumers raises the required degree of fiscal feedback ϕ_B which would make fiscal policy passive. According to their argument, in this case a more agressive fiscal response is necessary to avoid a debt interest spiral, since with non-Ricardian households higher debt can stimulate demand and inflation more (due to larger multipliers), which is offset by higher real interest rates from the part of an active monetary policy, which then raises interest expenditures on public debt. Instead of having hand-to-mouth agents, they introduce non-Ricardian households by making them finitely lived with a positive probability of death. Apparently this difference is crucial for their result, since the same amplification via the New Keynesian Cross should also exist in this model with HtM households, and yet the threshold for ϕ_B remains independent of λ.

also by *raising* ϕ^{π} above a sufficiently high threshold $\hat{\phi}^{\pi}$ which is increasing in HtM share λ. The threshold $\hat{\phi}^{\pi}$ is decreasing in openness, so making monetary policy active in this way requires harder efforts in closed economies.

Figure 2.4: Model determinacy properties in the (ϕ_B, ϕ^{π}) plain for different values of λ and α.

Another paramter which interacts with the threshold $\hat{\phi}^{\pi}$ is the passivity of fiscal policy ϕ_B. As Figure 2.4 shows, $\hat{\phi}^{\pi}$ is increasing in ϕ_B when λ is high enough. In other words, **when IADL applies, a more passive fiscal policy requires a more aggressive inflation reaction ϕ^{π} from the central bank to make monetary policy active.**[32] This interaction between the degree of monetary activism and fiscal passivism applies only in the IADL region (i.e. λ must be high enough, unlike in the baseline scenario in the top left panel). The trade-off is also much more relevant in open economies: as the bottom right panel shows, in a closed economy ($\alpha = 0$) the threshold $\hat{\phi}^{\pi}$ is already higher than realistic values, even for very low levels of ϕ_B.

Throughout the above anaylsis we looked at the case of a symmetric external steady state, with zero NFA position ($\zeta = 0$). However, another interesting result, applying solely in the open economy context, is that **with a sufficiently negative NFA position, the inverted Taylor principle under IADL fails.** Figure B.12 in the Appendix illustrates that with $\zeta << 0$ applying $\phi^{\pi} = 0$ cannot restore determinacy under IADL

[32]Of course, the inverted Taylor principle of applying $\phi^{\pi} \approx 0$ keeps working as well, and it is not affected by the degree of fiscal passivism ϕ_B. In addition, the above result only concerns Taylor rules which react to current, and not to expected inflation.

(top right panel), so the only remaining option is to keep $\phi^\pi > 1$ and switch to active fiscal policy instead (bottom left panel). It seems like in an IADL environment sufficiently high external debt can prevent $\phi^\pi = 0$ from making monetary policy active.[33] Running sensitivity analyses it can be confirmed that more fiscal passivity (higher ϕ_B) can help in pushing down the threshold for ζ where this phenomenon occurs, as if more aggressive debt stabilization by the government can address issues stemming from higher external indebtedness of the economy. Reducing openness has the same effect, as external debt becomes less relevant for the whole economy.

A very similar issue exists with regards to steady state public debt. Under $\phi^\pi = 0$ a larger $\bar{b}^g$ makes indeterminacy more likely, i.e. applying even at IADL values of λ where $\phi^\pi > 1$ also yields indeterminacy. This rhymes with the findings of Leith and von Thadden (2008) who pointed out the role of the fiscal steady state in the determinacy conditions of a model without Ricardian equivalence. They show that without referring to steady state public debt it is not possible to determine the degree of monetary and fiscal activism necessary for ensuring unique and stable equilibrium dynamics.

2.4 Responses to transfer shocks

2.4.1 Calibration

We are going to consider several fiscal transfer shocks to compare the dynamic responses in our model economy across different policy regimes. For the purposes of this exercise the parameters of the model are set such that we are in the non-IADL, Keynesian region where interest rate increases are contractionary. In particular, the share of HtM households λ is not too high given the labor supply elasticity φ and openness α, but still significant such that the New Keynesian Cross is visibly in operation.

As for the distributive characteristics of the tax system, in the baseline parametrization all households pay their fair share of *expected* aggregate taxes (uniform taxation), meaning that $\phi = \lambda$ must hold.[34] There is no profit redistribution ($\tau^D = 0$) so countercyclical profit variations will not mitigate multipliers. The wage subsidy is calibrated to offset static distortions due to monopolistic competition $\tau^w = \frac{1}{\varepsilon}$ which (together with the lump sum taxes T^s levied on firms) results in zero steady state firm profits.

In order to allow for a rather general setup, we look at the non-symmetric steady state

[33] Not only that, but as the bottom right panel indicates, it also seems to make fiscal policy passive even with $\phi_B = 0$, since indeterminacy implies we must be in a PM-PF policy regime.

[34] Note that this does not mean that *unexpected* shocks to taxes are also uniform. In fact, their heterogeneity will be important in some of the scenarios.

Parameters					
discount factor	β	0.99	openness	α	0.5
(inverse) Frisch-elasticity	φ	2	share of HtM households	λ	0.3
risk aversion	σ	1	risk-premium sensitivity	δ	0.1
elasticity of subs. bw H and F	η	1.5	steady state NFA-to-GDP	ζ	-0.25
elasticity of subs. bw countries	γ	1.5	share of government spending	Γ	0.2
elasticity of subs. bw varieties	ε	6	Calvo price rigidities	θ	0.75
Fiscal policy parameters			**Monetary policy parameters**		
debt stabilization (PF; AF)	ϕ_B	0.2; 0	Taylor inflation coeff (AM; PM)	ϕ^π	1.5; 0.2
public debt-to-GDP target	$\bar{b}^g$	0.6	PPI inflation target	$\bar{\Pi}^H$	1
tax distribution	ϕ	λ	Taylor output gap coeff.	ϕ^y	0
profit redistribution	τ^D	0	Taylor NEER coeff.	ϕ^e	0
wage subsidy	τ^w	$1/\varepsilon$			
Steady states					
Consumption, HtM	$\check{C}$	0.8577	Consumption, Ricardian	$\widehat{C}$	0.8610

Table 2.2: Parameters and selected steady state values.

with external indebtedness, i.e. the steady state NFA position ζ is negative. However, it is smaller in absolute value than public debt $\bar{b}^g$ which means that Ricardian households have positive net worth in steady state, subjecting them to surprise revaluation effects (via inflation and in case of FX-debt, also via the exchange rate). The interest income earned on this asset position results in some steady state consumption inequality, despite uniform taxation and zero profits. Trade is set to be slightly more price elastic than the Obstfeld-Cole case of $\sigma = \eta = \gamma = 1$ such that the expenditure switching channel is not completely offset, giving rise to variations in the trade balance.

The AM-PF and PM-AF policy mixes feature ad-hoc parameters ϕ_B and ϕ^π which capture the nature of the given policy regime. As monetary policy is not completely unresponsive under PM-AF, it lends the inflation process some persistency. Other parameters are set to standard values and are reported in Table 2.2.

2.4.2 AM-PF policy mix – the role of redistribution

In order to assess how fiscal transfers with different redistributive properties affect the economy in the presenece of household heterogeneity and public debt, we look at several different shock scenarios. We consider a persistent debt-financed tax cut amounting to one percentage point of steady state output, targeted either solely to HtM households, or spread out uniformly across all consumers, or focused solely on Ricardian agents. In addition, we also look at a (persistent) balanced budget within period redistribution from Ricardian households to Hand-to-Mouth agents. These exercises are summarized in Table 2.3. The "debt-financed uniform tax" cut and the "balanced budget redistribution" scenarios are analogous to those analysed by Bilbiie, Monacelli and Perotti (2013) in the context of their closed economy TANK model, in an AM-PF policy regime.

	HtM tax cut	uniform tax cut	Ricardian tax cut	BB redistribution
HtM shock: $\breve{\varepsilon}_t^T$	ϵ/λ	ϵ	0	ϵ
Ricardian shock: $\widehat{\varepsilon}_t^T$	0	ϵ	$\frac{1}{1-\lambda}\,\epsilon$	$\frac{-\lambda}{1-\lambda}\,\epsilon$
total: $(1-\lambda)\widehat{\varepsilon}_t^T + \lambda\,\breve{\varepsilon}_t$	ϵ	ϵ	ϵ	0

Table 2.3: Description of tax cut shocks for different scenarios (with total tax cut ε_t^T kept the same, except for the balanced budget redistribution). ϵ denotes a shock size which is one percentage point of steady state output on impact, and declines with persistence $\rho_T = 0.3$.

Consider first the case of an AM-PF policy mix, where monetary policy is actively responding to deviations of inflation from target and fiscal policy is committed to raise future primary budget surpluses for debt stabilization purposes. As the results in Figure 2.5 indicate,[35] **in the presence of household heterogeneity transfer multipliers of a debt-financed tax cut depend very much on whom they target. Tax cuts are much more effective if they are targeted at high MPC Hand-to-Mouth agents,** who consume all their current disposable income, relative to the case where every household gets the same transfer, and even more so relative to cutting the taxes only of consumption-smoothing Ricardians. This is in line with the findings of Bayer et al. (2020) in the context of their closed HANK model. In the present open economy setting ($\alpha > 0$), however, these impact multipliers are mitigated relative to a closed economy (see Figure B.5 in Appendix), as net exports are crowded out by public debt: i.e. some of the fiscal stimulus "leaks" out as import spending (*expenditure changing*), further encouraged by the appreciating real exchange rate (*expenditure switching*).

The New Keynesian Cross is in operation, providing amplification of transfer multipliers. Given that fiscal stimulus is expansionary, incomes rise which prompts HtM households to consume more, pushing income further upwards. However, as discussed earlier (see Section 2.3.3), it is not the mere presence of a $\lambda > 0$ share of HtM agents that manages to raise the average MPC in the economy, but also countercyclical income inequality meaning that HtM income overreacts aggregate income. This condition is satisfied in the baseline parametrization for *pre-tax* income ($\chi > 1$). With heterogeneous tax cuts, in terms of the *after-tax* income this is further amplified when the tax cut is focused on HtM, and is mitigated when it falls only on Ricardians. But even in the latter case, income inequality remains countercyclical and therefore amplification is present, the more so, the higher the λ share of HtM agents are (see Figure B.3 in Appendix).

[35] For all variables percentage (log) deviations from their steady state are shown, except for the trade balance NX_t, public debt b_t^g and taxes T_t, for which level (linear) deviations are plotted expressed as a percentage of steady state output.

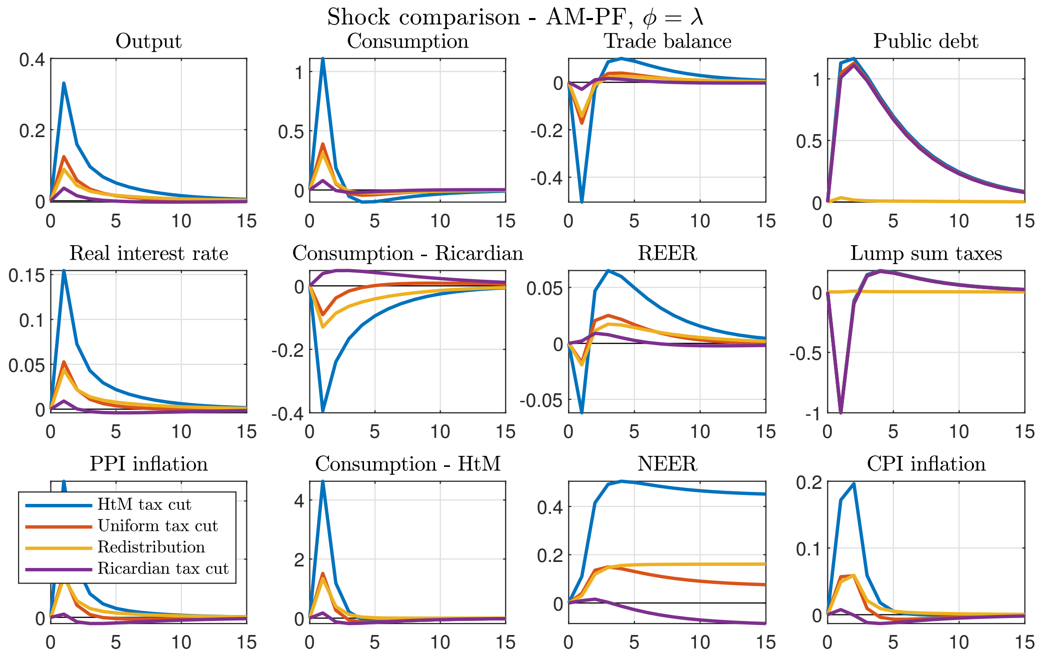

Figure 2.5: Shock comparison under AM-PF policy regime, with uniform tax distribution $\phi = \lambda$

Instead of comparing different scenarios about how the *same* budget deficit is distributed across households (as above, and by Bayer et al. (2020)), Bilbiie, Monacelli and Perotti (2013) compare the "debt-financed uniform tax cut" and the within period "balanced budget redistribution" scenarios. In this case HtM agents get the same tax cut in both scenarios, but while in the former it is financed by selling government bonds to Ricardians (to be paid off by future taxes), in the latter it is financed by raising taxes on Ricardians today. Both necessarily crowd out Ricardian consumption (facilitated by an active monetary policy raising the real interest rate), but the former implies a fiscal deficit and rising public debt, while the latter does not.

As discussed before in Section 2.3.2, with $\phi = 0$ Ricardian equivalence holds, meaning that the irrelevance of the timing of taxes translates into the irrelevance of what happens with Ricardian taxes altogether (since they are the ones paying all taxes). I.e. whether the needed funds for the HtM transfer are raised by taxing Ricardians today or by selling them debt today and swapping it for taxes later, should not make any difference. The only relevant factor for model dynamics should be the size of the HtM transfer, which is the same across the two scenarios, and public debt should not matter at all.

However, with $\phi = \lambda$ this is no longer true, since in this case it is not only the timing of Ricardians' taxes which differs between the two scenarios, but also their present value, making Ricardian lifetime income different – which is something even optimizing consumption-smoothers react to.[36] The debt-financed tax cut scenario does not involve

[36] The breakdown of Ricardian equivalence is perhaps best illustrated by looking at the pure Ricardian

any lifetime income redistribution (everybody is liable for a fair share of public debt), while the balanced budget redistribution by construction does, from Ricardians to HtM (equivalently to the $\phi = 0$ debt-financed tax cut, when Ricardians are liable even for the part of debt which finances the HtM transfer).[37] In response to this more adverse lifetime income profile, Ricardians cut their consumption back more, which explains the differences on impact *despite* the HtM transfer being the same.

To put it another way, when $\phi = 0$ (Ricardian equivalence), government debt is not net wealth, so its size does not matter: debt-financed tax cut and balanced budget redistribution yield the same dynamics (see top left panel of Figure 2.7). But **when $\phi \neq 0$ (Ricardian equivalence fails), government debt *is* net wealth, and its size *does* matter,**[38] making the two scenarios different (top right panel of Figure 2.7). Ricardians who hold this net wealth will have a better lifetime income position, so their consumption will be higher than without public debt (conditional on the same HtM transfer). This captures a kind of "redistribution via public debt" as explained by Bilbiie, Monacelli and Perotti (2013): Ricardians hold all the debt, but they are liable only for a $(1 - \phi)$ fraction of it.

Notice that in the balanced budget redistribution scenario, the sharper drop in Ricardian consumption, by hurting aggregate demand, also harms HtM incomes and consumption: more redistribution towards them is actually harmful for their consumption! The same argument can be illustrated by comparing debt-financed tax cut scenarios for different values of tax distribution ϕ (see Figure B.1 in the Appendix), where lower ϕ leads to a smaller rise in HtM consumption, and therefore smaller output multipliers, too.

The takeaway is that in the absence of Ricardian equivalence public debt matters, as it has redistributive consequences. However, it does not matter *much*. The differences between the debt-financed tax cut and balanced-budget redistribution scenarios are small[39]

tax cut scenario in Figure 2.5. Under Ricardian equivalence it should generate any dynamics as housholds just save all the tax windfall to pay debt off in the future. However, with $\phi > 0$ some of the debt financing this transfer to Ricardians will be paid by HtM agents, which is why it constitutes a lifetime income change for Ricardians, setting of a dynamics response.

[37]Notice that the balanced budget redistribution scenario can be equivalently rewritten as a debt financed uniform tax cut scenario with $\phi = 0$, since in this case Ricardian taxes do not matter. Then, the above comparison is equivalent to comapring two debt-financed tax cut scenarios with different values for ϕ. It also illustrates how ϕ captures the amount of redistribution via public debt.

[38]Ricardians must still buy all the public debt, but ϕ portion of it will not be backed by their future tax liabilities – it is like lending to HtM households via the givernment budget, and holding a real claim on them.

[39]To a lesser extent, but this still holds true for more agressive debt stabilization policy (higher ϕ_B) as illustrated in Figure B.2 in the Appendix. Bilbiie, Monacelli and Perotti (2013) show that as ϕ_B tends

compared to differences relative to scenarios where the size of HtM transfers changes. In other words, **while public debt matters somewhat under an AM-PF policy mix to the extent that Ricardian equivalence fails, far more important is how fiscal transfers are distributed across households, and in particular, to what extent the same budget deficit is targeted at high-MPC agents.** This result will not hold for with a PM-AF policy regime.

2.4.3 PM-AF policy mix – the role of public debt

Under a PM-AF policy mix, fiscal policy can run debt-financed budget deficits that are unbacked by the present value future tax revenues. The real burden of such unbacked debt is kept manageable by a passive monetary policy which keeps nominal interest rates unresponsive and tolerates deviations of inflation from target. This is similar to the unbacked fiscal deficit of Roosevelt in 1933 as described in Jacobson, Leeper and Preston (2019), or to the "emergency budget" advocated by Bianchi, Faccini and Melosi (2020). As discussed in Section 2.3.2, in such an environment Ricardian equivalence breaks down even in a RANK model due to the *nominal wealth effect*, which results in debt having important monetary consequences.

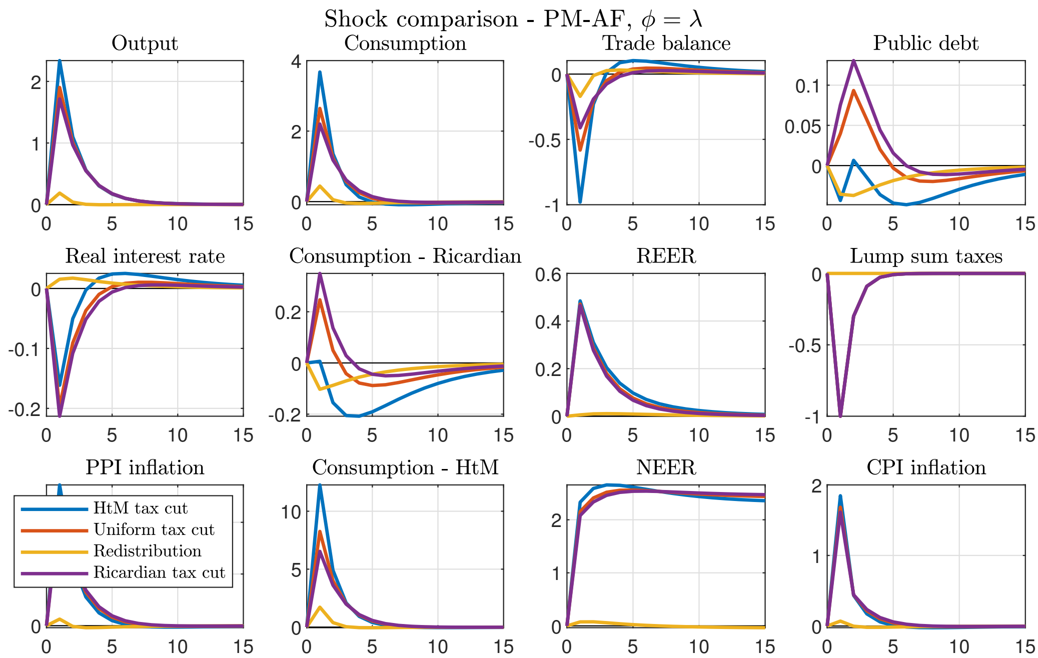

Figure 2.6: Shock comparison under PM-AF policy regime, with uniform tax distribution $\phi = \lambda$

For this reason, **with a PM-AF policy mix public debt *per se* will have a much more important role relative to the redistributive profile of fiscal transfers, than in the AM-PF regime.** As can be seen in Figure 2.6, transfer multipliers are still

to its passive fiscal policy lower bound of $1 - \beta$, the two scenarios essentially become equivalent.

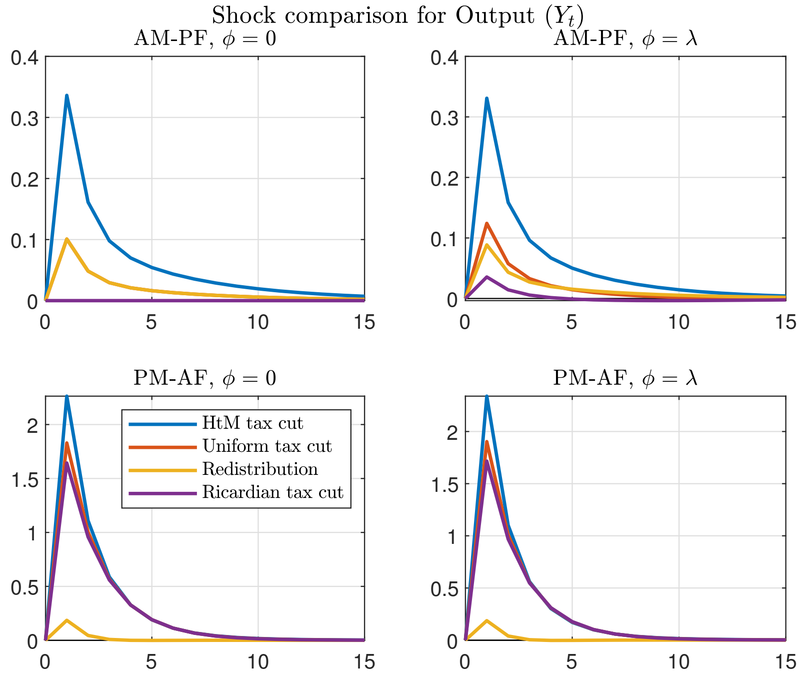

Figure 2.7: Shock comparison for output, across policy regimes and tax distribution ϕ (as in Table...)

influenced by how much of a given aggregate tax cut is targeted at high MPC households. But while in the AM-PF regime this was the dominant factor, now its significance pales in comparison to whether there is a budget deficit or not. In particular, HtM taxes drop by the same amount in the "debt-financed uniform tax cut" scenario as in the "balanced budget redistribution" scenario. But in the former they are deficit-financed, while in the latter the government budget stays in balance,[40] and this difference makes a much bigger impact on the transfer multiplier than altering the size of HtM transfers. This is in contrast to the AM-PF regime where financing played secondary role only (if any, conditional on $\phi > 0$), and the responses are more similar (mainly driven by the equality of HtM transfers between these two scenarios). Figure 2.7 facilitates this comparison.

Evidently, debt matters a lot in the PM-AF regime. But unlike in the AM-PF regime, it is not through its redistributive properties that this influence manifests itself. In other words, it is not that the redistributive properties of public debt become much more important with active fiscal policy. After all, debt is unbacked by future taxes so it shouldn't matter who *doesn't* pay those taxes (with $\phi_B = 0$ the previously important tax distribution parameter ϕ even drops out of the model's equilibrium conditions).[41] Instead, debt matters via its monetary consequences due to the *nominal wealth effect* (see Section 2.3.2 and Jacobson, Leeper and Preston (2019)). Inflation has to rise to erode real value of

[40] The trajectory of "lump sum taxes" T_t is equivalent to the primary budget surplus, given government spending G_t is unchanged.

[41] In Section 2.3.2 we even discussed how unbacked nominal government bonds are not real net wealth for their holders as they understand the government will not pay them back with consumption goods.

unbacked nominal debt, which is why it is the the size of the budget deficit *per se* that is important.[42]

The bottom line is the following. Under the AM-PF regime it made little difference whether a transfer to HtM households was financed by raising Ricardian taxes or by issuing debt, but it is of paramount importance with a PM-AF policy mix. While previously the distribution of transfers across households was the crucial factor, it now takes a back seat relative to the question of financing. **With passive, accommodative monetary policy a deficit-financed transfer can provide a much bigger output multiplier than a balanced budget redistribution.**

2.4.4 Transmission of fiscal shocks across policy regimes

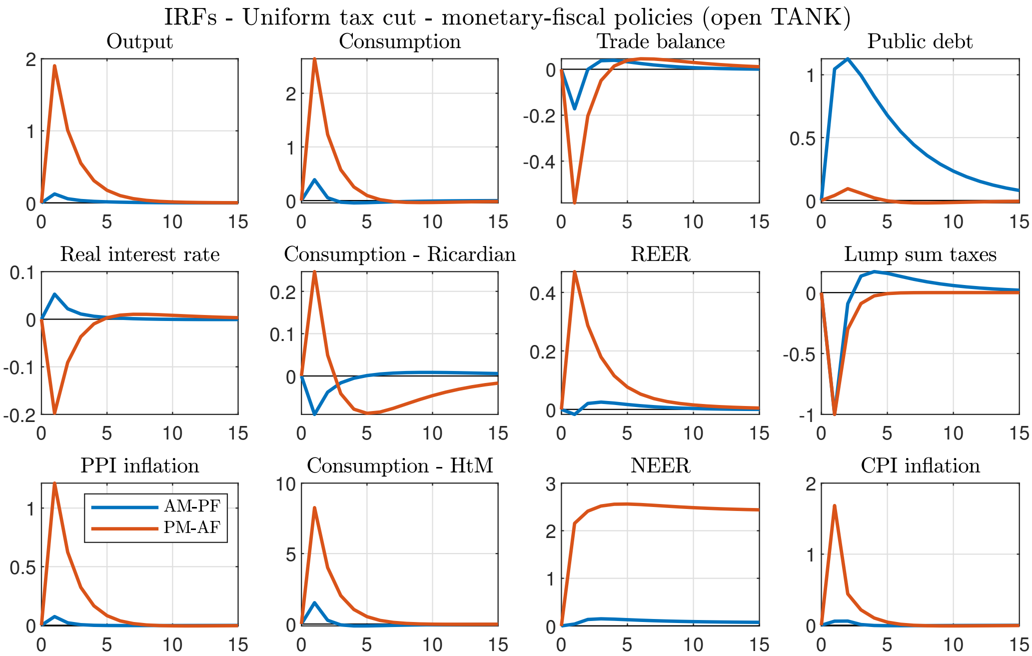

Figure 2.8: Impulse responses to a uniform tax cut

In other words, **it is with a PM-AF policy mix, that deficit-financing can really be potent.** Figure 2.8 compares the two policy regimes after a debt-financed uniform tax cut, illustrating the large difference not just in output multipliers but also in the transmission mechanism of the shock itself. With PM-AF, the need for inflation to stabilize the real value of public debt, coupled with relatively unresponsive nominal interest rates, leads to *falling* (as opposed to increasing) real interest rates. Via intertemporal substitutin channels this *crowds in* Ricardian consumption instead of the usual crowding out effect of

[42]There is *some* redistribution though via the revaluation of *already existing* public debt which is all held by Ricardians. Suprise inflation will reduce the real value of their assets, imposing on them a kind of "inflation tax" which ends up paying for the budget deficit in real terms.

public debt. Expanding Ricardian demand is also in line with the *nominal wealth effect* as they try to consume their new bonds which have no future tax obligation attached to them. These positive developments on aggregate demand are also beneficial for high-MPC HtM households, beyond the direct effect of their tax cut, since pre-tax incomes are further boosted by Ricardian spending. Higher HtM consumption then amplifies the expansion in output via the New Keynesian Cross. Despite taxes not rising in the future to offset the initial (persistent) budget deficits, the real value of public debt rises much less as a result of the higher price level. Inflating away public debt revalues the assets of Ricardian households, who suffer a negative wealth effect, in effect putting all the real burden of the fiscal stimulus on them in the form of on "inflation tax".

The open economy aspects of the different transmission mechanism under PM-AF are worth noting, too. The fall in the real interest rate makes the real exchange rate *depreciate* instead of the impact appreciation under AM-PF. This improved external competitiveness works towards *crowding in* net exports (*expenditure switching* effect due to trade elasticities η, γ), and thereby provides further stimulus. The trade balance, however, still moves deeper into deficit, as the import leakage out of a much higher consumption (*expenditure changing* effect, governed by openness α) dominates the weaker real exchange rate. **The weakening real exchange rate under the PM-AF policy regime following a fiscal stimulus is also more in line with empirical evidence** (see Monacelli and Perotti (2010), Ravn, Schmitt-Grohé and Uribe (2012)), which otherwise presents real appreciatrion-predicting AM-PF open economy models with a puzzle. In an open economy, subordinating monetary policy to the objectives of public debt stabilization and accommodating higher inflation also necessarily leads to a permanently weaker nominal exchange rate.

The choice of policy regime makes a difference even in the case when there is no deficit. Figure 2.9 compares the two policy mixes in the balanced budget redistribution scenario. The differences are much smaller in this case, underlining the fact that the main distinctive feature of PM-AF is how it handles unbacked fiscal deficits (which do not arise here). However, redistributing to high-MPC households necessarily sets off some aggregate demand effects under nominal rigidities, and absent an active monetary policy to stabilize the economy, something else must take its place: this something is the need to stabilize the real value of public debt.

Although there's no primary budget deficit, public debt will rise somewhat under AM-PF due to the active response of monetary policy, which reacts to aggregate demand expansion by raising real interest rates, that in turn raise interest expenditure on existing government

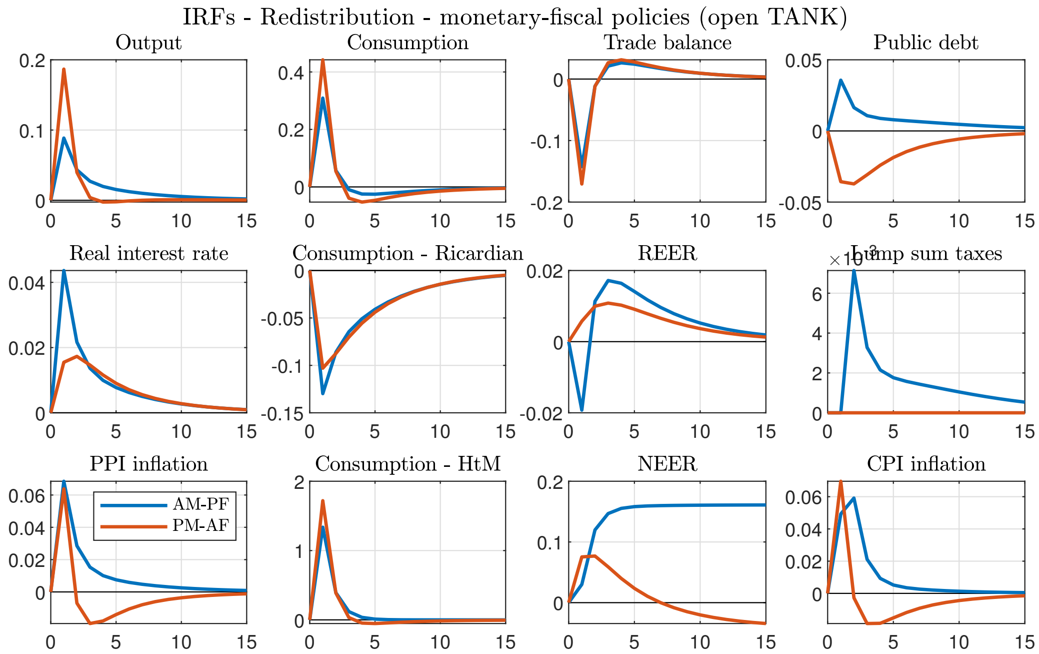

Figure 2.9: Impulse responses to a balance budget redistribution

debt.[43] A passive fiscal policy raises some taxes to cover this. However, with PM-AF the real value of pre-existing public debt *falls* on impact due to the inflationary effect of rising demand. This means that debt stabilization now requires rising real interest rates to push debt back up towards its steady state target. In the absence of a responsive central bank this can only be brought about by falling prices, i.e. deflation. Therefore, **both regimes will produce rising real rates in response to the demand expansion set off by redistribution. But while in AM-PF this is engineered by monetary policy raising the nominal interest rate in response to inflation, in PM-AF it happens via the need for deflation to stabilize public debt.** In this scenario the price level (and the nominal exchange rate) rise permanently in the AM-PF regime, while it is the PM-AF policy mix which preserves the value of local currency. Note that on balance, the output multiplier is larger with PM-AF.

2.4.5 Effect of open economy – sensitivity analysis

As we have seen, the response of the real exchange rate differs markedly across AM-PF and PM-AF policy mixes, which is why open economy dimensions can be important when comparing these different policy regimes.

[43] Up to first order, this effect only exists if steady state public debt is non-zero.

Openness and HtM share

The effects of any aggregate demand shock on output are normally mitigated in an open economy setting ($\alpha > 0$) since some of the increase in spending will "leak out" in the form of rising imports. This effect on the trade balance due to changes in domestic spending is called *expenditure changing*. In an AM-PF regime, to the extent that the stimulus leads to a real exchange rate appreciation, this effect is further aggrevated by a crowding ot of net exports due to a loss of external competitiveness (*expenditure switching* channel). This echoes the results of a simple Mundell-Fleming model where the effects of a fiscal expansion are completely offset by a deteriorating trade balance.

Under a PM-AF regime, however, the real exchange rate *depreciates* which changes the sign of the expenditure switching channel. Now external demand expands and domestic households direct more of their consumption increase towards home produced goods as those become more competitive. The strength of this channel depends on trade price elasticities η and γ, and in the baseline calibration it is not strong enough to completely offset the expenditure changing channel (import leakage) which is now even more significant, given the much larger increase in consumption. On balance, thererfore, **opening up the economy still hurts the output multipliers even under the PM-AF policy mix.** This is illustrated in Figure 2.10 by comparing the red and purple lines.

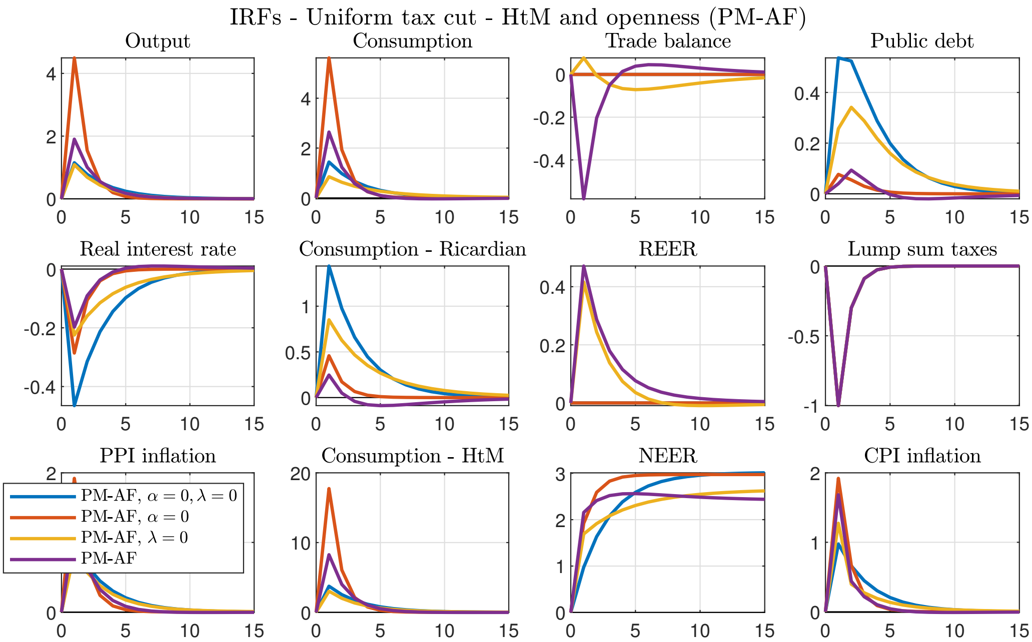

Figure 2.10: Impulse responses to a debt-financed uniform tax cut, in open/closed TANK/RANK models (given PM-AF policy mix). Unless otherwise indicated, baseline values $\alpha = 0.5$ and $\lambda = 0.3$ apply.

However, Figure 2.10 also shows that this result is almost non-existent in a RANK setting ($\lambda = 0$).[44] Without HtM households, the amplification via the New Keynesian Cross

[44]Under the AM-PF regime a RANK model would produce no dynamics at all in response to a tax

is almost completely muted, which results in a much smaller increase of consumption, thereby reducing import leakage. At the same time, the extent of real depreciation is similar, so the stimulative effects of expenditure switching can manage to roughly offset the smaller import leakage via the expenditure changing channel. This is why **in a RANK economy with PM-AF policy mix, opening up is not as harmful for multipliers,** if at all,[45] which runs contrary to the standard Mundell-Fleming type results.[46]

As noted above, the importance of the expenditure switching channel depends on how price elastic import demand for foreign goods (η) and external demand for domestic goods (γ) are. These trade elasticities influence how sensitively net exports react to real exchange rate movements which, in turn, move in different directions depending on the policy regime (appreciating in AM-PF, and depreciating in PM-AF). Since they amplify an expenditure switching channel with opposite sign, **increasing trade elasticities affects the output multiplier in opposite ways across different policy mixes, *mitigating* it in AM-PF, but *amplifying* it in PM-AF** (see Figures B.9 and B.10 in the Appendix).[47]

Price rigidities

It is also in relation to the expenditure switching channel that **in an open economy a sufficient amount of nominal rigidities are crucial for the PM-AF policy mix to yield higher multipliers than AM-PF.** Figure 2.11 shows how very flexible prices (low θ) result in such a sharp jump in inflation under the PM-AF regime that the real exchange rate appreciates instead of depreciating. Moreover, it does so to a larger extent than with AM-PF, crowding out net exports even more forcefully and hurting output so much that it not only *decreases*, but is actually going lower than with AM-PF. This is in contrast to our baseline with stickier prices (shown in Figure 2.8) where the PM-AF policy mix managed to achieve larger, and not smaller, output multipliers after a debt-financed fiscal expansion.

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cut, since Ricardian equivalence prevails. So comparing open and closed settings would not make any sense. But with a PM-AF policy mix that is no longer true due to the nominal wealth effect.

[45] With large enough η, γ price elasticities of trade, opening up in a RANK model (PM-AF regime) might even slightly increase the output multiplier, as the expenditure switching channel comes to dominate expenditure changing.

[46] See results for more values of α in the Appendix, in Figure B.6 (TANK model) and Figure B.7 (RANK model).

[47] A very similar pattern is observable regarding the currency denomination of external debt. With FX-debt (i.e. negative steady state NFA position) unexpected exchange rate movements on impact revalue outstanding liabilities in a way which, via the debt-elastic risk premium, amplify the original exchange rate movement, thereby strengthening the expenditure switching channel. See Figure B.8 in the Appendix.

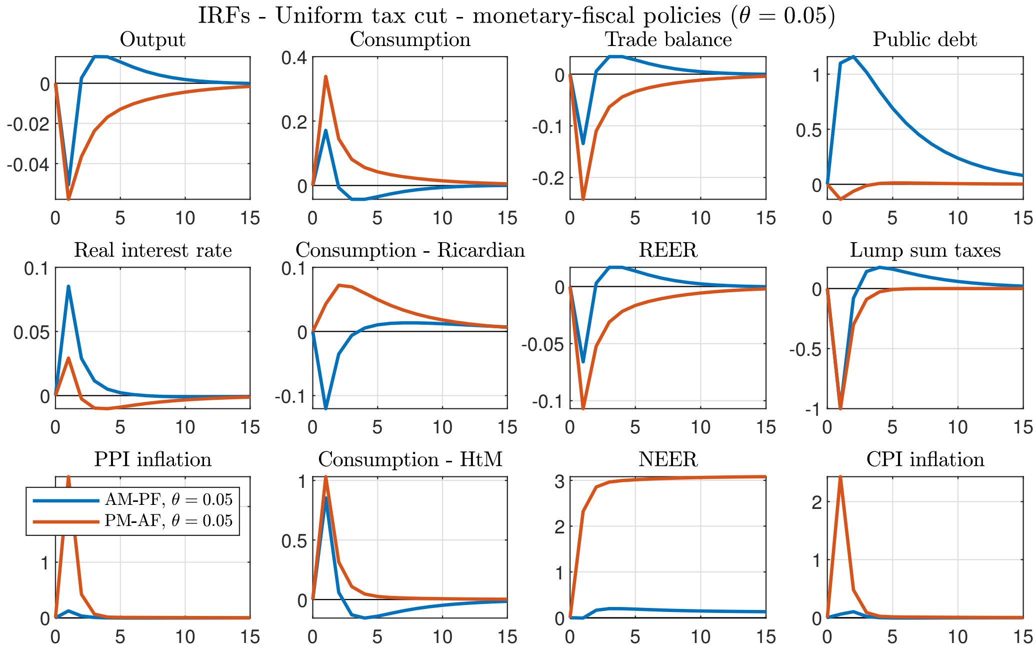


Figure 2.11: Impulse responses to a debt-financed uniform tax cut across policy regimes (very flexible prices $\theta = 0.05$)

Notice that this phenomenon only applies to open economies. While increasing price flexibility reduces the the effect of fiscal stimulus on aggregate demand in open and closed economies alike, as well as under both AM-PF and PM-AF regimes, it does not do so to the same extent. In a closed economy, absent the above described expenditure switching channel, the response of output would still remain positive even with $\theta \to 0$, and PM-AF would still yield weakly higher multipliers than AM-PF. Therefore, open economies with more flexible prices would not necessarily see as much gain from switching to a PM-AF regime than those with higher nominal rigidities.

2.5 Conclusion

In this paper I have explored a dilemma regarding the choice of the monetary-fiscal policy mix in open economies with heterogeneous households. After a fiscal disturbance, the central bank can either raise real interest rates to ward off inflationary pressures, which would force costly fiscal adjustment to stabilize public debt – or an unresponsive monetary policy could keep interest rates low, tolerating higher inflation and letting it erode the real value of nominal debt without fiscal policy having to raise taxes. The choice between these policy mixes affects the efficacy of the fiscal expansion already today and can interact with the distributive properties of the stimulus. Targeting fiscal transfers more towards high-MPC agents increases the output multiplier of a fiscal stimulus, while raising the degree of deficit-financing for these transfers also helps. One of the main results of this paper is that precise targeting is much more important under the AM-PF regime than the question of financing, while the opposite is the case with a PM-AF policy mix: then deficit-spending is crucial for the size of the multiplier, and targeting matters less.

Under the PM-AF regime fiscal stimulus entails a real exchange rate depreciation which might offset "import leakage" by stimulating net exports, if the share of hand-to-mouth households is low and trade is price elastic enough. Therefore, a PM-AF policy mix might break the Mundell-Fleming prediction that open economies have smaller fiscal multipliers relative to closed economies.

I also showed that the inverted Taylor principle is not a necessary condition for equilibrium determinacy under inverted aggregate demand logic, and can be substituted by an active fiscal policy. In fact, in an open economy setting with sufficiently high external debt this is the only solution, as the inverted Taylor principle breaks down completely.

This is a highly stylized model framework which, while forcefully illustrates the role of MPC heterogeneity, has its limitations in generating a full fledged distribution with much richer wealth heterogeneity. In addition, the model also abstracts from uninsured idiosyncratic uncertainty which is why it cannot capture precautionary saving motives. It would be especially interesting to see how precautionary saving affects results under the PM-AF regime where nominal assets of households can be subject to sudden revaluations, prompting them to rebuild their portfolios, potentially affecting aggregate outcomes, too. Introducing monetary-fiscal interactions into a full-fledged heterogeneous agent incomplete market (HANK) model could provide insights into these issues, and can be a promising avenue for further research.

References

Bayer, Christian, Benjamin Born, Ralph Luetticke, and Gernot J Müller. 2020. "The Coronavirus Stimulus Package: How large is the transfer multiplier?" *CEPR Discussion Paper*, , (DP14600).

Bianchi, Francesco, and Leonardo Melosi. 2019. "The dire effects of the lack of monetary and fiscal coordination." *Journal of Monetary Economics*, 104: 1–22.

Bianchi, Francesco, Renato Faccini, and Leonardo Melosi. 2020. "Monetary and Fiscal Policies in Times of Large Debt: Unity Is Strength." *NBER Working Papers*, , (w27112).

Bilbiie, Florin O. 2008. "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic." *Journal of Economic Theory*, 140(1): 162–196.

Bilbiie, Florin O. 2018. "Monetary Policy and Heterogeneity: An Analytical Framework." *CEPR Discussion Papers*, , (DP12601).

Bilbiie, Florin O. 2019. "The New Keynesian Cross." *Journal of Monetary Economics.*

Bilbiie, Florin O., and Roland Straub. 2004. "Fiscal policy, business cycles and labor-market fluctuations." *Magyar Nemzeti Bank Working Papers*, 2004(6).

Bilbiie, Florin O., Tommaso Monacelli, and Roberto Perotti. 2013. "Public Debt and Redistribution with Borrowing Constraints." *Economic Journal*, 123(566): 64–98.

Blanchard, Olivier. 2019. "Public Debt and Low Interest Rates." *American Economic Review*, 109(4): 1197–1229.

Boerma, Job. 2014. "Openness and the (inverted) aggregate demand logic." *DNB Working Paper*, , (436).

Broer, Tobias, Niels-Jakob Harbo Hansen, Per Krusell, and Erik Öberg. 2020. "The New Keynesian Transmission Mechanism: A Heterogeneous-Agent Perspective." *The Review of Economic Studies*, 87(1): 77–101.

Cantore, Cristiano, and Lukas B Freund. 2019. "Workers, Capitalists, and the Government: Fiscal Policy and Income (Re)Distribution." *mimeo - University of Cambridge.*

Cook, David, and Michael B. Devereux. 2013. "Sharing the burden: Monetary and fiscal responses to a world liquidity trap." *American Economic Journal: Macroeconomics*, 5(3): 190–228.

Cook, David E., and Michael B. Devereux. 2019. "Fiscal Policy in a Currency Union at the Zero Lower Bound." *Journal of Money, Credit and Banking*, 51(1): 43–82.

Corsetti, Giancarlo, and Luca Dedola. 2016. “The Mystery of the Printing Press: Monetary Policy and Self-Fulfilling Debt Crises.” *Journal of the European Economic Association*, 14(6): 1329–1371.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2010. “Optimal monetary policy in open economies.” In *Handbook of Monetary Economics*. Vol. 3. 1 ed., , ed. Benjamin M. Friedman and Michael Woodford, 861–933. Elsevier B.V.

Corsetti, Giancarlo, Luca Dedola, Marek Jarociński, Bartosz Maćkowiak, and Sebastian Schmidt. 2019. “Macroeconomic stabilization, monetary-fiscal interactions, and Europe’s monetary union.” *European Journal of Political Economy*, 57: 22–33.

Cugat, Gabriela. 2019. “Emerging markets, household heterogeneity, and exchange rate policy.” *mimeo - Northwestern University.*

Debortoli, Davide, and Jordi Galí. 2018. “Monetary Policy with Heterogeneous Agents: Insights from TANK models.” *mimeo, CREI and UPF.*

De Paoli, Bianca. 2009. “Monetary Policy under Alternative Asset Market Structures: the Case of a Small Open Economy.” *Journal of Money, Credit and Banking*, 41(7): 1301–1330.

Di Giorgio, Giorgio, and Guido Traficante. 2018. “Fiscal shocks and helicopter money in open economy.” *Economic Modelling*, 74: 77–87.

Eggertsson, Gauti B. 2011. “What Fiscal Policy Is Effective at Zero Interest Rates?” *NBER Macroeconomics Annual 2010*, 25(May): 59–112.

Farhi, Emmanuel, and Iván Werning. 2016. “Fiscal Multipliers: Liquidity Traps and Currency Unions.” In *Handbook of Macroeconomics*. Vol. 2, Chapter 31, 2417–2492.

Galí, Jordi, and Tommaso Monacelli. 2005. “Monetary Policy and Exchange Rate Volatility in a Small Open Economy.” *Review of Economic Studies*, 72(3): 707–734.

Galí, Jordi, David López-Salido, and J. Vallés. 2007. “Understanding the effects of government spending on consumption.” *Journal of the European Economic Association*, 5(1): 227–270.

Iyer, Tara. 2017. “Optimal Monetary Policy in an Open Emerging Market Economy.” *Federal Reserve Bank of Chicago Working Paper*, , (WP 2016-06).

Jacobson, Margaret M, Eric M Leeper, and Bruce Preston. 2019. “Recovery of 1933.” *NBER Working Papers*, , (w25629).

Jarociński, Marek, and Bartosz Maćkowiak. 2018. “Monetary-Fiscal Interactions and the Euro Area’s Malaise.” *Journal of International Economics*, 112: 251–266.

Kaplan, Greg, Benjamin Moll, and Giovanni Violante. 2018. “Monetary Policy According to HANK.” *American Economic Review*, 108(3): 697–743.

Leeper, Eric M. 1991. “Equilibria Under ’Active’ and ’Passive’ Monetary Policies monetary and fiscal policies.” *Journal of Monetary Economics*, 27: 129–147.

Leeper, Eric M., and Campbell Leith. 2016. “Understanding Inflation as a Joint Monetary-Fiscal Phenomenon.” In *Handbook of Macroeconomics*. Vol. 2, , ed. John B Taylor and Harald Uhlig, 2305–2415. Elsevier.

Leeper, Eric M, Nora Traum, and Todd B Walker. 2011. “Clearing Up the Fiscal Multiplier Morass.” *NBER Working Papers*, , (w17444).

Leith, Campbell, and Leopold von Thadden. 2008. “Monetary and fiscal policy interactions in a New Keynesian model with capital accumulation and non-Ricardian consumers.” *Journal of Economic Theory*, 140(1): 279–313.

Leith, Campbell, and Simon Wren-Lewis. 2008. “Interactions between monetary and fiscal policy under flexible exchange rates.” *Journal of Economic Dynamics and Control*, 32(9): 2854–2882.

Monacelli, Tommaso, and Roberto Perotti. 2010. “Fiscal Policy, the real exchange rate and traded goods.” *Economic Journal*, 120(544): 437–461.

Ravn, Morten O., Stephanie Schmitt-Grohé, and Martín Uribe. 2012. “Consumption, government spending, and the real exchange rate.” *Journal of Monetary Economics*, 59(3): 215–234.

Schmitt-Grohé, Stephanie, and Martin Uribe. 2003. “Closing small open economy models.” *Journal of International Economics*, 61(1): 163–185.

Sims, Christopher A. 2013. “Paper money.” *American Economic Review*, 103(2): 563–584.

Woodford, Michael. 2011. “Simple Analytics of the Government Expenditure Multiplier.” *American Economic Journal: Macroeconomics*, 3(January): 1–35.

Appendix B

B.1 Model equations

Hand-to-Mouth households optimize

(2.2): HtM labor supply $$w_t = \check{C}_t^{\sigma}\ \check{N}_t^{\varphi} \tag{B.1}$$

(2.1): HtM budget $$\check{C}_t = w_t\ \check{N}_t + \frac{\tau^D}{\lambda}\Omega_t - \check{T}_t \tag{B.2}$$

Ricardian households optimize

(2.4): Ricardian labor supply $$w_t = \widehat{C}_t^{\sigma}\ \widehat{N}_t^{\varphi} \tag{B.3}$$

(2.5): LCY bond Euler $$\frac{1}{1+i_t} = \beta\ E_t \left\{ \left[\frac{\widehat{C}_{t+1}}{\widehat{C}_t}\right]^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} \tag{B.4}$$

(2.7): int'l risk sharing $$\left[\frac{\widehat{C}_{t+1}}{\widehat{C}_t}\right]^{\sigma} = \left[\frac{Y^*_{t+1}}{Y^*_t}\right]^{\sigma} \psi_t\ \frac{Q_{t+1}}{Q_t} \tag{B.5}$$

(2.6): real UIP $$\frac{1+i_t}{\mathrm{E_t}\,\Pi_{t+1}} = \frac{1+i^*_t}{\mathrm{E_t}\,\Pi^*_{t+1}}\ \frac{\mathrm{E_t}\,Q_{t+1}}{Q_t}\ \psi_t \tag{B.6}$$

(2.55): risk premium $$\psi_t = e^{-\delta\left(b_t \frac{h(Q_t)}{Y_t} - \zeta_t\right)} \tag{B.7}$$

(2.53): balance-of-payments $$\frac{b_t}{1+i_t} - \frac{b_{t-1}}{\Pi_t} = NX_t \tag{B.8}$$

Firms optimize

(??): AS-1
$$\Theta_t = \widehat{C}_t^{-\sigma}\, Y_t\, rMC_t\, \mathcal{M}\xi_t \;+\; \theta\,\beta\,\mathrm{E_t}\left\{\frac{h(Q_t)}{h(Q_{t+1})}\left(\Pi_{t+1}^H\right)^{\varepsilon}\,\Theta_{t+1}\right\} \tag{B.9}$$

(??): AS-2
$$\Delta_t = \widehat{C}_t^{-\sigma}\, Y_t \;+\; \theta\,\beta\,\mathrm{E_t}\left\{\frac{h(Q_t)}{h(Q_{t+1})}\left(\Pi_{t+1}^H\right)^{\varepsilon-1}\,\Delta_{t+1}\right\} \tag{B.10}$$

(2.24): AS-3 (NKPC)
$$\frac{\Theta_t}{\Delta_t} = \left[\frac{1-\theta\left(\Pi_t^H\right)^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \tag{B.11}$$

(2.25): real marginal costs
$$rMC_t = \frac{w_t}{A_t}\, h(Q_t) \tag{B.12}$$

(2.26): production function
$$Y_t\,\Xi_t = A_t\, N_t \tag{B.13}$$

(2.27): price dispersion
$$\Xi_t = \left(\Pi_t^H\right)^{\varepsilon}\,\theta\,\Xi_{t-1} + (1-\theta)\left[\frac{1-\theta\left(\Pi_t^H\right)^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.14}$$

(2.28): profits
$$\Omega_t = \frac{Y_t}{h(Q_t)}\Big[1-\; rMC_t\;\Xi_t\Big] \tag{B.15}$$

Market clearing and accounting

(2.45): goods market
$$Y_t = \left[h(Q_t)\right]^{\eta}\Big[(1-\alpha)\,C_t \;+\; \alpha\,\left[h(Q_t)\right]^{\gamma-\eta}\,Q_t^{\gamma}\,Y_t^{*}\Big] + G_t \tag{B.16}$$

(2.46): aggregate labor
$$N_t = \lambda\check{N}_t + (1-\lambda)\widehat{N}_t \tag{B.17}$$

(2.36): aggregate consumption
$$C_t = \lambda\check{C}_t + (1-\lambda)\widehat{C}_t \tag{B.18}$$

(2.51): trade balance
$$NX_t = \left[h(Q_t)\right]^{-1}(Y_t - G_t) - C_t \tag{B.19}$$

Fiscal policy block

(2.32): government budget
$$T_t + \frac{b_t^g}{1+i_t} = [h(Q_t)]^{-1}G_t + \frac{b_{t-1}^g}{\Pi_t} \tag{B.20}$$

(2.33): fiscal rule
$$T_t - T = \phi_B\,(b_{t-1}^g - \bar{b}^g\,Y) - Y\,[\lambda\check{\varepsilon}_t^T + (1-\lambda)\hat{\varepsilon}_t^T] \tag{B.21}$$

(2.34): HtM taxes
$$\check{T}_t - \frac{\phi}{\lambda}T = \frac{\phi}{\lambda}\,\phi_B\,(b_{t-1}^g - \bar{b}^g\,Y) - \check{\varepsilon}_t^T Y \tag{B.22}$$

(2.35): aggr. taxes
$$T_t = \lambda\,\check{T}_t + (1-\lambda)\,\widehat{T}_t \tag{B.23}$$

Others

(2.20): CPI-PPI wedge
$$\frac{P_t}{P_t^H} = h(Q_t) = \left[\frac{1-\alpha}{1-\alpha\,Q_t^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{B.24}$$

(2.19): REER definition
$$Q_t = \frac{e_t P_t^*}{P_t} \tag{B.25}$$

(2.29): monetary policy
$$\frac{1+i_t}{1+i} = \left(\frac{\Pi_t^H}{\bar{\Pi}^H}\right)^{\phi^{\pi}}\left(\frac{Y_t}{\bar{Y}_t}\right)^{\phi^{y}}\left(\frac{e_t}{e_{t-1}}\right)^{\phi^{e}} v_t \tag{B.26}$$

(??): CPI inflation
$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{B.27}$$

(??): PPI inflation
$$\Pi_t^H = \frac{P_t^H}{P_{t-1}^H} \tag{B.28}$$

Exogenous processes

(2.30): monetary policy shock	$\ln v_t = \rho_R \ln v_{t-1} + \epsilon_t^R$	(B.29)
(2.56): sudden stop	$\zeta_t = (1-\rho_\zeta)\zeta + \rho_\zeta \zeta_{t-1} + \epsilon_t^\zeta$	(B.30)
(??): government spending	$\ln G_t = (1-\rho_g)\ln(\Gamma\, Y) + \rho_g \ln G_{t-1} + \epsilon_t^g$	(B.31)
cost push shock	$\ln \xi_t = \rho_\xi \ln \xi_{t-1} + \epsilon_t^\xi$	(B.32)
TFP process	$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_t^A$	(B.33)
foreign output	$\ln Y_t^* = (1-\rho_{Y^*})\ln Y^* + \rho_{Y^*} \ln Y_{t-1}^* + \epsilon_t^{Y^*}$	(B.34)
foreign prices	$P_t^* = 1$	(B.35)
tax shocks	$\check{\varepsilon}_t^T, \quad \widehat{\varepsilon}_t^T$	

which is (B.1)-(B.28) 28 equations for 28 endogenous variables, plus 7 exogenous processes (and 8 shocks):

- quantities: $Y_t, C_t, N_t, NX_t, b_t, \Omega_t, T_t, b_t^g$ (8)
- domestic prices: $\Pi_t, \Pi_t^H, rMC_t, \frac{W_t}{P_t}, \Xi_t, \Theta_t, \Delta_t$ (7)
- international prices: Q_t, e_t (2)
- interest rates: i_t, i_t^*, ψ_t (3)
- disaggregated variables: $\check{C}_t, \widehat{C}_t, \check{N}_t, \widehat{N}_t, \check{T}_t, \widehat{T}_t$ (6)
- definitions: P_t, P_t^H (2)
- exogeneous variables: $v_t, \zeta_t, G_t A_t, \xi_t, Y_t^*, P_t^*$ (7)

With FX debt use the following BoP and premium functions:

(??): risk premium FCY	$\psi_t = e^{-\delta\left(b_t^* \frac{h(Q_t)}{Y_t} - \chi_t\right)}$	(B.36)
(2.54): balance-of-payments FCY	$NX_t = \frac{b_t^*}{(1+i_t^*)\psi_t} - b_{t-1}^* \frac{Q_t}{Q_{t-1}}$	(B.37)

B.2 Steady state

Supply side

- zero inflation steady state: from the Taylor rule (B.26) we get $\Pi^H = \bar{\Pi}^H = 1$ (by setting the inflation target parameter $\bar{\Pi}^H = 1$)
- using the above $\Pi^H = 1$ in the 4 equations describing the optimal firm decision (B.9)-(B.12) (and substituting out rMC, Δ, Θ) we get the **firm's labor demand** equation:

$$\begin{aligned}
\Theta &= \widehat{C}^{-\sigma}\, Y\, rMC\, \mathcal{M} \;+\; \theta\, \beta\, \Theta \\
\Delta &= \widehat{C}^{-\sigma}\, Y \;+\; \theta\, \beta\, \Delta \\
\frac{\Theta}{\Delta} &= \left[\frac{1-\theta}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \\
rMC &= \frac{w}{A}\, h(Q) \\
\Rightarrow \qquad w &= \frac{A}{\mathcal{M}}\left[h(Q)\right]^{-1}
\end{aligned} \tag{AS}$$

  - I.e. real marginal costs are the inverse of the desired markup $rMC = 1/\mathcal{M}$

- via (B.14) price dispersion in the steady state $\Xi = 1$ (which also implies via (B.13) the steady state aggregate production $Y = A\, N$
- firm profits from (B.15) are then:

$$\begin{aligned}
\Omega &= \frac{Y}{h(Q)}\left[1 - \; rMC\;\Xi\right] = \\
&= \frac{Y}{h(Q)}\left[1 - \frac{1}{\mathcal{M}}\right]
\end{aligned}$$

Demand side

- government expenditures are exogenous from (B.31), $G = \Gamma\, Y$
- the **goods market clearing** condition (B.16) captures domestic and external demand

$$Y = \left[h(Q)\right]^{\eta}\Big[(1-\alpha)\, C \;+\; \alpha\, \left[h(Q)\right]^{\gamma-\eta}\, Q^{\gamma}\, Y^{*}\Big] + \Gamma\, Y \tag{AD}$$

International risk-sharing

- the **imperfect intrenational risk-sharing condition** (B.5) gives us the steady state risk-premium (since discount factors are the same at home and abroad $\beta = \beta^*$, and provided that we want to avoid steady state real depreciation/appreciation $\Delta Q \neq 0$)

$$\psi = 1$$

- $\psi = 1$, together with the risk premium function (B.7) pins down the steady state NFA position

$$\psi = 1 = e^{-\delta\left(b\frac{h(Q)}{Y} - \zeta\right)}$$
$$\Rightarrow \qquad b = \zeta Y \left[h(Q)\right]^{-1}$$

- that, in turn can be used in the **balance-of-payments** equation (B.8) (together with the $\Pi^H = 1$ result, the CPI-PPI wedge (B.24) giving us steady state CPI inflation, the LCY Euler (B.4) pinning down nominal interest rates, and the trade balance definition (B.19))

$$\frac{\Pi}{\Pi^H} = \frac{h(Q)}{h(Q)} = 1 \qquad \Rightarrow \qquad \Pi = \Pi^H = 1$$
$$\frac{1}{1+i} = \beta \left\{\frac{1}{\Pi}\right\}$$
$$NX = \left[h(Q)\right]^{-1} (Y - G) - C$$
$$\frac{b}{1+i} - \frac{b}{\Pi} = NX$$

$$\Rightarrow \qquad \frac{C}{Y}\left[h(Q)\right] = (1 - \Gamma) + (1-\beta)\,\zeta \tag{BoP}$$

<u>Fiscal block</u>

- the fiscal rule (B.21) pins down the steady state government debt (via the debt target parameter $\bar{b}^g Y$): $b^g = \bar{b}^g Y$

- the government budget constraint (B.20) is used to back out aggregate taxes (using previous results)

$$T + \frac{b^g}{1+i} = [h(Q)]^{-1} G + \frac{b^g}{\Pi}$$
$$T = Y\left[[h(Q)]^{-1}\Gamma + (1-\beta)\,\bar{b}^g\right]$$

- HtM taxes from (B.22) are then:

$$\check{T} = \frac{\phi}{\lambda} T = \frac{\phi}{\lambda} Y\left[[h(Q)]^{-1}\Gamma + (1-\beta)\,\bar{b}^g\right] \tag{F}$$

<u>Household heterogeneity</u>

- Without HtM households $\lambda = 0$ the 3 steady state relationships (AS), (AD) and (BoP) pin down the steady state of Y, C, Q (after substituting out w with the Ricardian (= aggregate) labor supply (B.3) and the production function (B.13)).

- However, with HtM households $\lambda \neq 0$ we will need to account for household heterogeneity.

- In particular, plugging in the characterization of fiscal redistribution (F) into the steady state **HtM budget** (B.2) (and using steady state profits from above):

$$\begin{aligned}\check{C} &= w\,\check{N} + \frac{\tau^D}{\lambda}\Omega - \check{T} = \\ &= w\,\check{N} + Y\left\{\frac{1}{h(Q)}\frac{\tau^D}{\lambda}\frac{\mathcal{M}-1}{\mathcal{M}} - \frac{\phi}{\lambda}\left[\frac{\Gamma}{h(Q)} + (1-\beta)\,\bar{b}^g\right]\right\}\end{aligned} \tag{HtM}$$

  - some of the profits potentially get redistributed to HtM households – provided there *are* steady state profits (i.e. $\tau^w \neq \frac{1}{\epsilon} \Rightarrow \mathcal{M} \neq 1$) and the government decides so ($\tau^D \neq 0$, i.e. endogenous redistribution)
  - HtM households must also pay their share ϕ of agrgegate taxes financing government expenditures and the interest expenditures on steady state government debt ($\phi \neq \lambda$ corresponds to exogenous redistribution)

The resulting minimal steady state system to solve is the following:

$$\text{(B.12)} \Rightarrow \text{(AS)}: \quad w = \frac{A}{\mathcal{M}}[h(Q)]^{-1} \tag{B.38}$$

$$\text{(B.16)} \Rightarrow \text{(AD)}: \quad (1-\Gamma)\,Y = [h(Q)]^{\eta}\Big[(1-\alpha)\,C \;+\; \alpha\;[h(Q)]^{\gamma-\eta}\;Q^{\gamma}\;Y^*\Big] \tag{B.39}$$

$$\text{(B.8)} \Rightarrow \text{(BoP)}: \quad \frac{C}{Y}[h(Q)] = (1-\Gamma) + (1-\beta)\,\zeta \tag{B.40}$$

$$\text{(B.17)}: \quad N = \lambda\check{N} + (1-\lambda)\widehat{N} \tag{B.41}$$

$$\text{(B.18)}: \quad C = \lambda\check{C} + (1-\lambda)\widehat{C} \tag{B.42}$$

$$\text{(B.2)} \Rightarrow \text{(HtM)}: \quad \check{C} = w\,\check{N} + Y\left\{\frac{1}{h(Q)}\frac{\tau^D}{\lambda}\frac{\mathcal{M}-1}{\mathcal{M}} - \frac{\phi}{\lambda}\left[\frac{\Gamma}{h(Q)} + (1-\beta)\,\bar{b}^g\right]\right\} \tag{B.43}$$

$$\text{(B.1)}: \quad w = \check{C}^{\sigma}\;\check{N}^{\varphi} \tag{B.44}$$

$$\text{(B.3)}: \quad w = \widehat{C}^{\sigma}\;\widehat{N}^{\varphi} \tag{B.45}$$

$$\text{(B.13)}: \quad Y = A\;N \tag{B.46}$$

which is a system of 9 equations in 9 variables: $Y, C, Q, w, N, \check{N}, \widehat{N}, \check{C}, \widehat{C}$

Normalize $Q = 1$ (implying $h(Q) = 1$ via (B.24)) and endogenize Y^* instead (which only shows up at (B.39)).

- from AS (B.38) + ($Q = 1$)

$$w = \frac{A}{\mathcal{M}} \tag{B.47}$$

- from BoP (B.40) + ($Q = 1$)

$$C = Y\,\Big[(1-\Gamma) + \zeta(1-\beta)\Big] \tag{B.48}$$

- from HtM budget (B.43) + HtM labor supply (B.44) + ($Q = 1$):

$$\check{N} = \left[\frac{w}{\check{C}^{\sigma}}\right]^{\frac{1}{\varphi}} \tag{B.49}$$

$$\begin{aligned}\check{C} &= w\left[\frac{w}{\check{C}^{\sigma}}\right]^{\frac{1}{\varphi}} + Y\left\{\frac{\tau^D}{\lambda}\,\frac{\mathcal{M}-1}{\mathcal{M}} - \frac{\phi}{\lambda}\left[\Gamma + (1-\beta)\,\bar{b}^g\right]\right\} = \\ &= w^{\frac{1+\varphi}{\varphi}}\check{C}^{-\frac{\sigma}{\varphi}} + Y\left\{\frac{\tau^D}{\lambda}\,\frac{\mathcal{M}-1}{\mathcal{M}} - \frac{\phi}{\lambda}\left[\Gamma + (1-\beta)\,\bar{b}^g\right]\right\}\end{aligned} \tag{B.50}$$

- from production fcn (B.46) + aggregate labor (B.41) + (B.49):

$$\frac{Y}{A} = \lambda\left[\frac{w}{\check{C}^{\sigma}}\right]^{\frac{1}{\varphi}} + (1-\lambda)\widehat{N}$$

$$\widehat{N} = \frac{Y/A - \lambda\left[\frac{w}{\check{C}^{\sigma}}\right]^{\frac{1}{\varphi}}}{1-\lambda} \tag{B.51}$$

- from aggregate consumption (B.42) + (B.48):

$$\widehat{C} = \frac{Y\left[(1-\Gamma) + \zeta(1-\beta)\right] - \lambda\check{C}}{1-\lambda} \tag{B.52}$$

- from Ricardian labor supply (B.45) + (B.52) + (B.51):

$$w = \left[\frac{Y\left[(1-\Gamma) + \zeta(1-\beta)\right] - \lambda\check{C}}{1-\lambda}\right]^{\sigma}\left[\frac{Y/A - \lambda\left(\frac{w}{\check{C}^{\sigma}}\right)^{\frac{1}{\varphi}}}{1-\lambda}\right]^{\varphi} \tag{B.53}$$

- given (B.47) $w = A/\mathcal{M}$, we can solve (B.50) and (B.53) for $\check{C}, Y$ which then can be used to recover all the other variables

The **full steady state solution** can be recovered as:

normalization (endog. Y^*)	$Q = 1$
	$\mathcal{M} = \frac{\varepsilon(1-\tau^w)}{\varepsilon - 1}$
(B.12) ⇒ (B.47) AS	$w = \frac{A}{\mathcal{M}}$
(B.3) ⇒ (B.53) Ricardian labor supply – `numerical`:	$Y = \bar{y}$
(B.2) ⇒ (B.50) HtM budget – `numerical`:	$\check{C} = \bar{c}$
(B.1) ⇒ (B.49) HtM labor supply	$\check{N} = \left[\frac{w}{\check{C}^{\sigma}}\right]^{\frac{1}{\varphi}}$
(B.13) ⇒ (B.46) production fcn	$N = \frac{Y}{A}$
(B.17) ⇒ (B.51) aggregate labor	$\widehat{N} = \frac{N - \lambda\check{N}}{1-\lambda}$
(B.8) ⇒ (B.48) BoP	$C = Y\left[(1-\Gamma) + \zeta(1-\beta)\right]$
(B.18) ⇒ (B.52) aggregate consumption	$\widehat{C} = \frac{C - \lambda\check{C}}{1-\lambda}$
(B.16) ⇒ (B.39) AD	$Y^* = \frac{(1-\Gamma)Y - (1-\alpha)C}{\alpha}$

(B.31) exog G	$G = \Gamma Y$
(B.19) trade balance	$NX/Y = (Y - C - G)/Y$
(B.5) int'l risk sharing	$\psi = 1$
(B.7) risk premium fcn	$b = \zeta Y$
(B.26) Taylor rule	$\Pi^H = \bar{\Pi}^H = 1$
(B.28) + normalization	$P^H = 1$
(B.24) CPI-PPI wedge	$P = 1$
(B.35) exog foreign prices	$P^* = 1$
(B.25) REER definition	$e = \frac{Q\,P}{P^*} = 1$
(B.27) inflation definition	$\Pi = 1$
(B.4) LCY Euler	$1 + r = \frac{1+i}{\Pi} = \frac{1}{\beta} \quad \Rightarrow \quad i = \frac{1-\beta}{\beta}$
(B.6) real UIP	$\frac{1+i^*}{\Pi^*} = \frac{1+i}{\Pi} = \frac{1}{\beta} \quad \Rightarrow \quad i^* = \frac{1-\beta}{\beta}$

(B.21) fiscal rule	$b^g = \bar{b}^g\, Y$
(B.20) gov. budget	$T = G + (1-\beta)\, b^g$
(B.22) HtM tax rule	$\check{T} = \frac{\phi}{\lambda} T$
(B.23) aggregate taxes	$\widehat{T} = \frac{T - \lambda \check{T}}{1-\lambda}$
(B.10)	$\Delta = \frac{\widehat{C}^{-\sigma} Y}{1-\theta\beta}$
(B.11)	$\Theta = \Delta$
(B.9)	$rMC = \frac{1}{\mathcal{M}}$
(B.14)	$\Xi = 1$
(B.15)	$\Omega = Y\left[1 - \frac{1}{\mathcal{M}}\right]$

which is 30 equations to pin down the steady state of 28 endogenous variables (with the steady state versions of (B.1)-(B.28)), plus 2 exogenous processes G, P^*.

Check if Walras' Law holds, i.e. whether aggregating budget constraints yields the BoP equation:

(B.2): $\lambda \left\{ \check{C} = w\, \check{N} + \frac{\tau^D}{\lambda}\Omega - \check{T} \right\}$

(2.3): $(1-\lambda) \left\{ \widehat{C} + \frac{\widehat{b}}{1+i} = \widehat{b} + w\, \widehat{N} + \frac{(1-\tau^D)}{(1-\lambda)}\Omega - \widehat{T} \right\}$

(B.20): $G + b^g = \frac{b^g}{1+i} + T$

$$\Rightarrow \quad \underbrace{\left[\lambda\check{C} + (1-\lambda)\widehat{C}\right]}_{C} + (\beta-1)\left[(1-\lambda)\widehat{b} - b^g\right] + G =$$

$$= \underbrace{w\,\underbrace{\left[\lambda\check{N} + (1-\lambda)\widehat{N}\right]}_{N}}_{Y} + \Omega - \underbrace{\left[\lambda\check{T} + (1-\lambda)\widehat{T}\right]}_{T} + T$$

$$(\beta-1)\underbrace{\left[(1-\lambda)\widehat{b} - b^g\right]}_{b} = \underbrace{Y - C - G}_{NX}$$

$$b = -\frac{NX}{1-\beta}$$

which is indeed consistent with (B.48): $C = Y\left[(1-\Gamma) + \zeta(1-\beta)\right]$, i.e. $\underbrace{C + G - Y}_{-NX} = (1-\beta)\underbrace{\zeta Y}_{b}$

B.3 Further figures

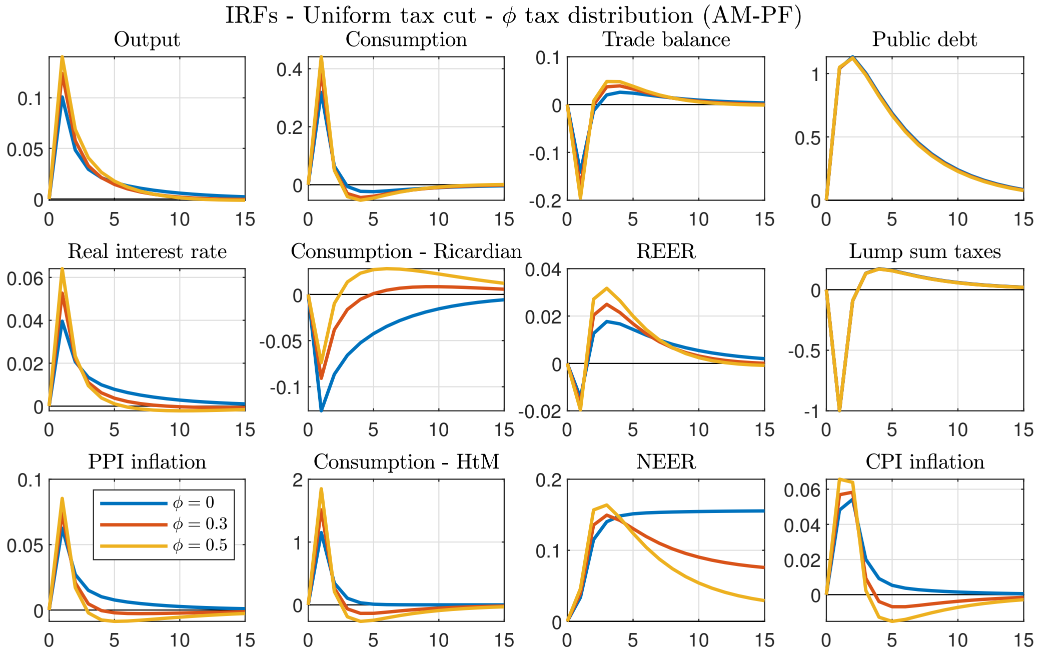


Figure B.1: Impulse responses to a uniform tax cut, for different tax distribution ϕ (AM-PF policy mix with $\phi_B = 0.2$)

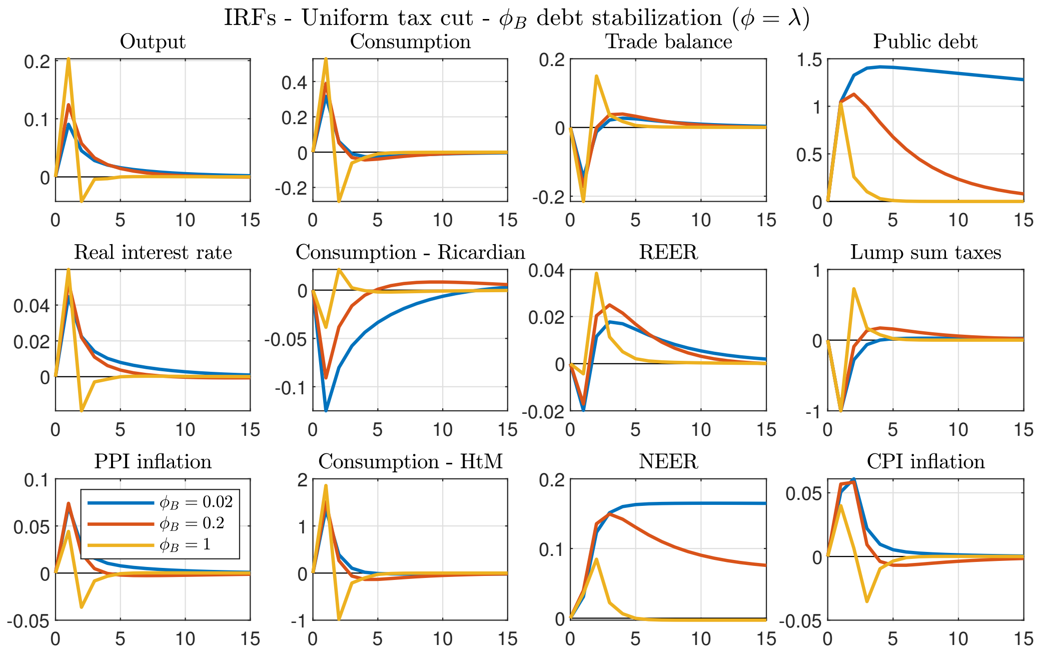


Figure B.2: Impulse responses to a uniform tax cut, for different debt stabilization ϕ_B (AM-PF policy mix, with uniform tax distribution $\phi = \lambda$)

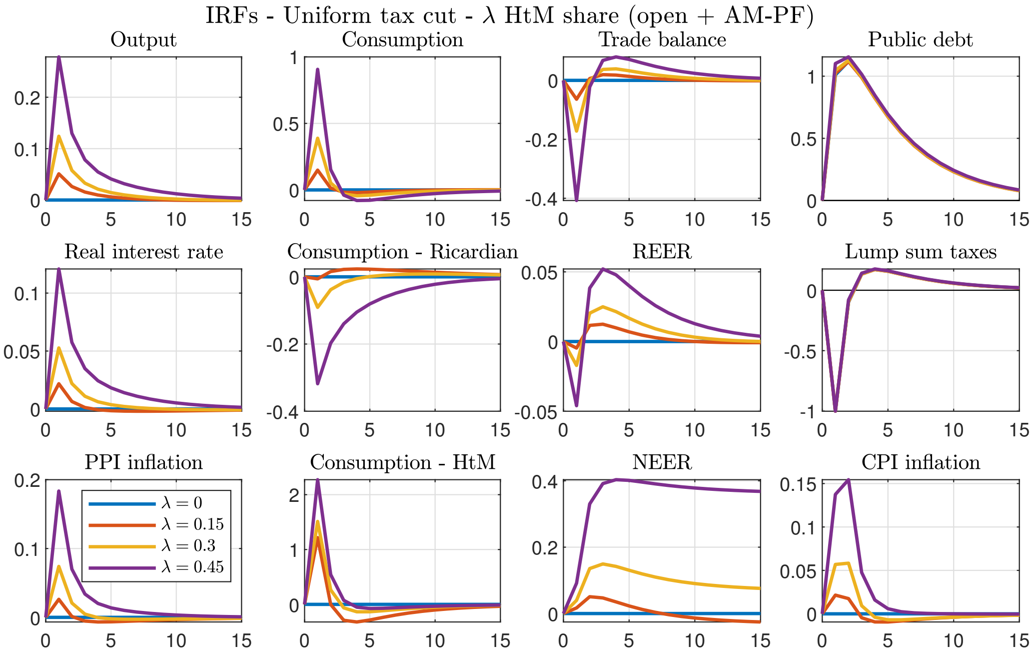


Figure B.3: Impulse responses to a debt-financed uniform tax cut, for different HtM shares λ (AM-PF policy mix)

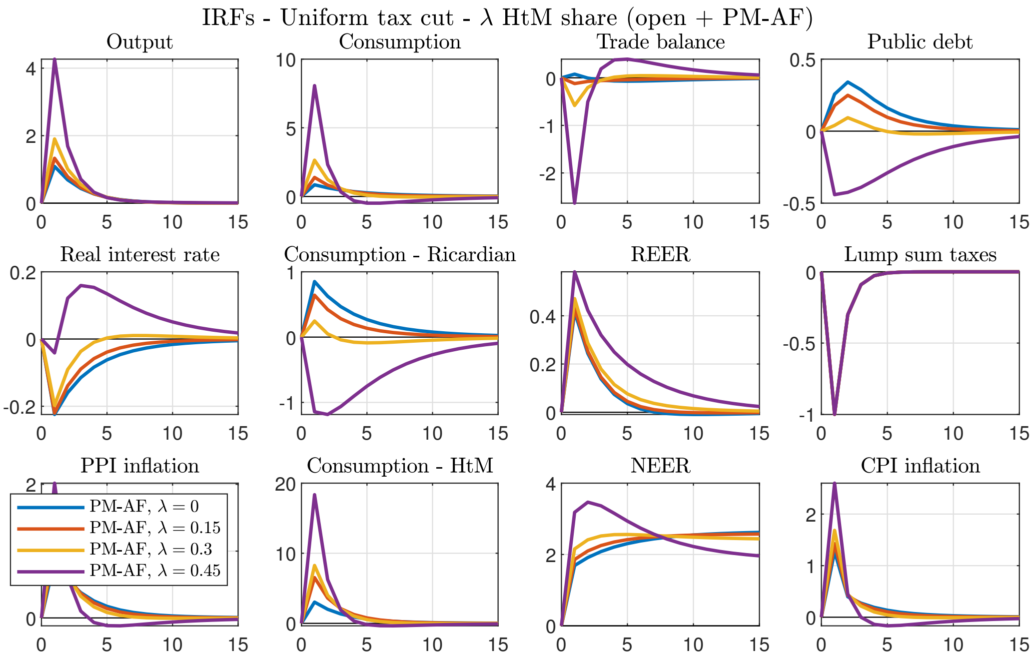


Figure B.4: Impulse responses to a debt-financed uniform tax cut, for different HtM shares λ (PM-AF policy mix)

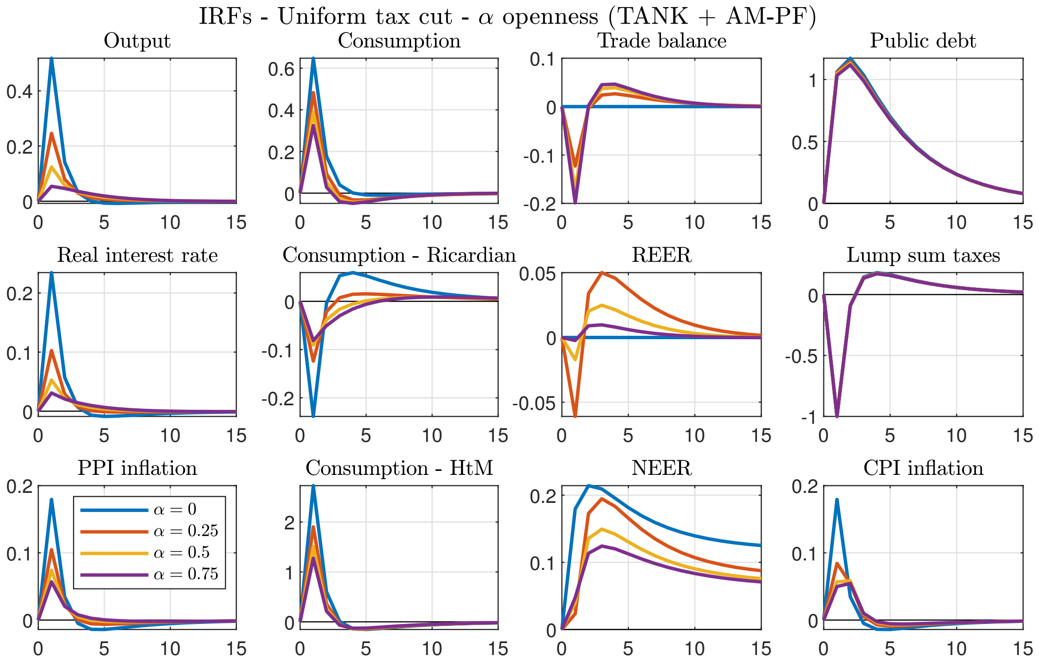


Figure B.5: Impulse responses to a debt-financed uniform tax cut, for different openness α (AM-PF policy mix)

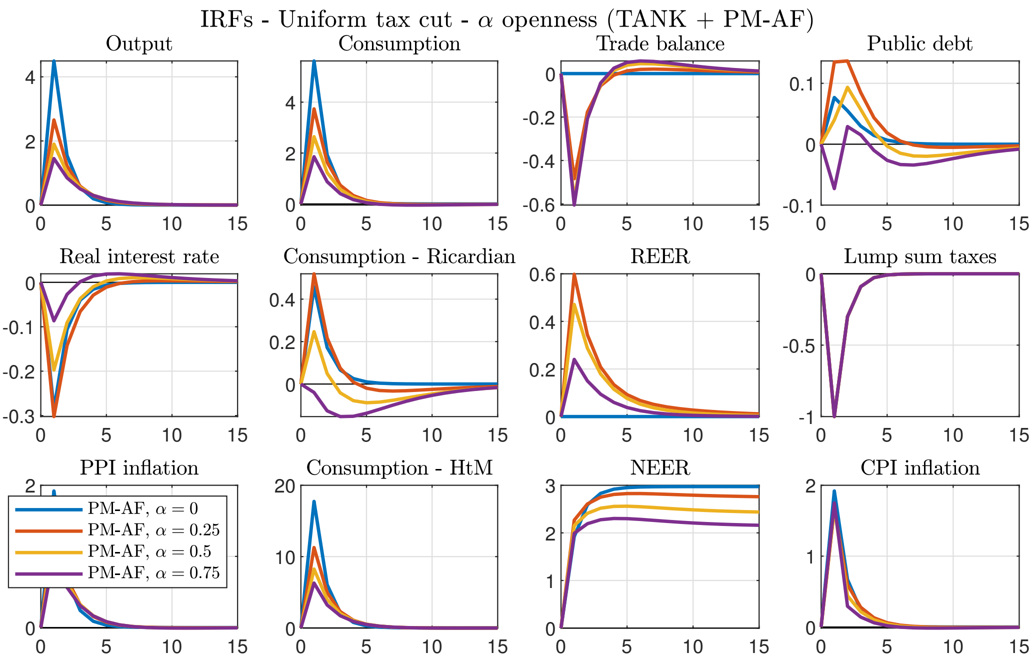


Figure B.6: Impulse responses to a debt-financed uniform tax cut, for different openness α (PM-AF policy mix)

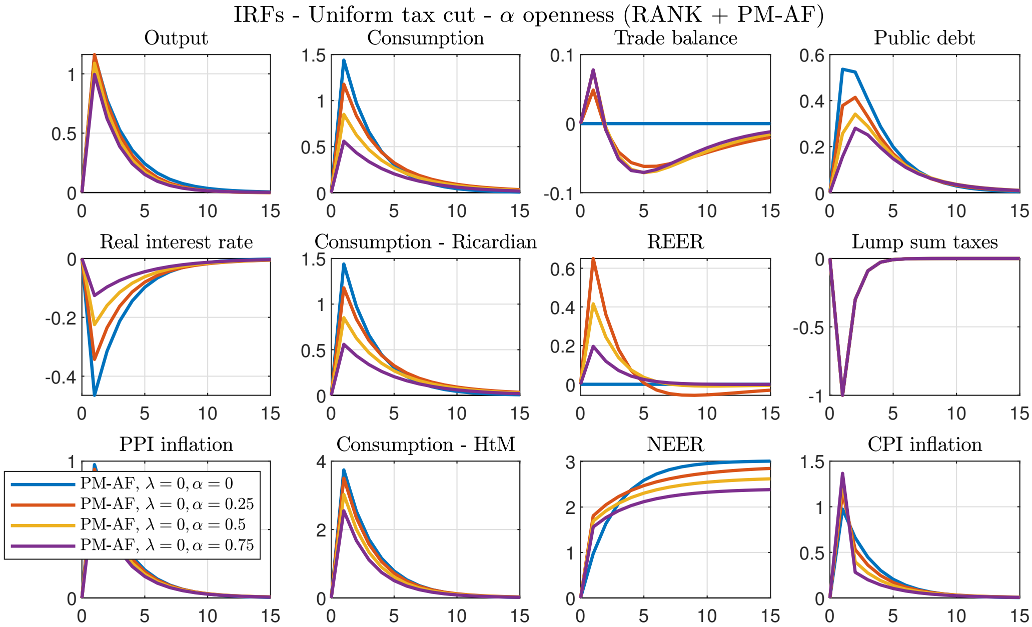


Figure B.7: Impulse responses to a debt-financed uniform tax cut, for different openness α (RANK model $\lambda = 0$, PM-AF policy mix)

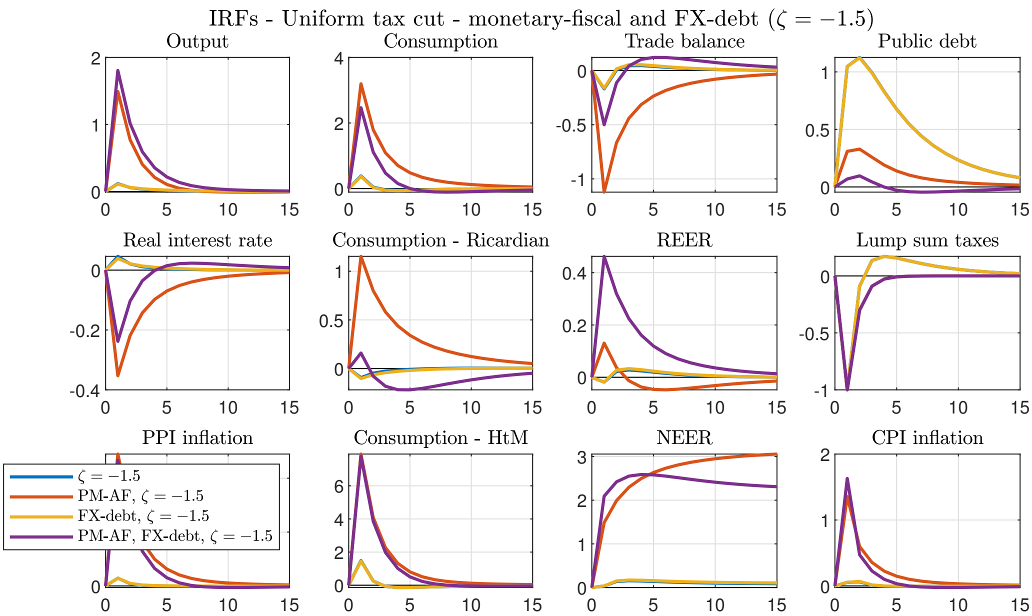


Figure B.8: Impulse responses to a debt-financed uniform tax cut, for different policy regimes and external debt denomination (NFA $\zeta = -1.5$)

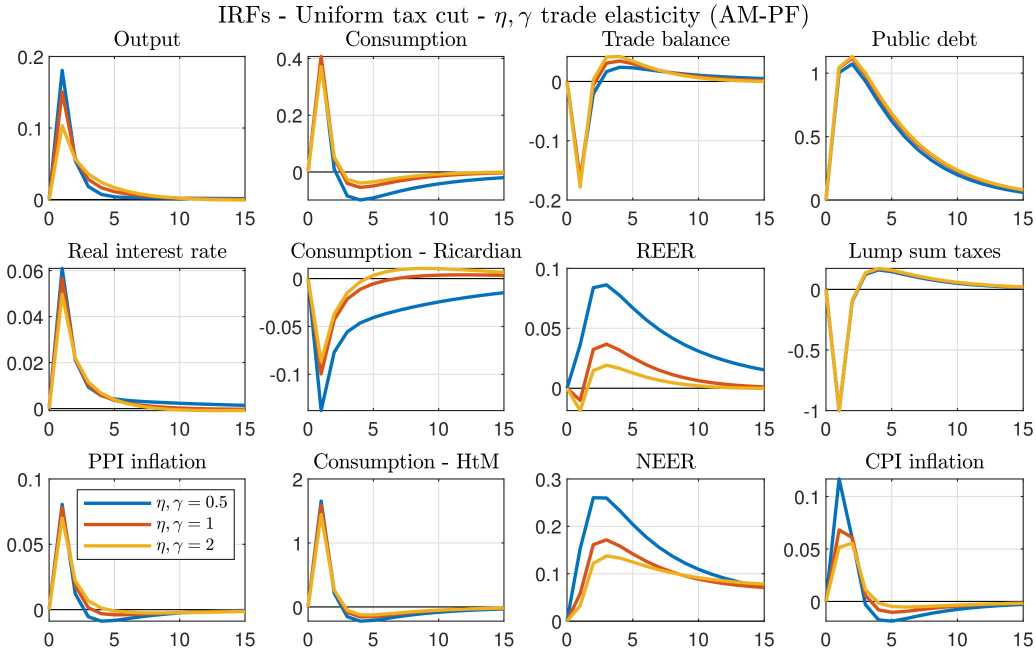


Figure B.9: Impulse responses to a debt-financed uniform tax cut, for different trade elasticities η, γ (AM-PF policy mix)

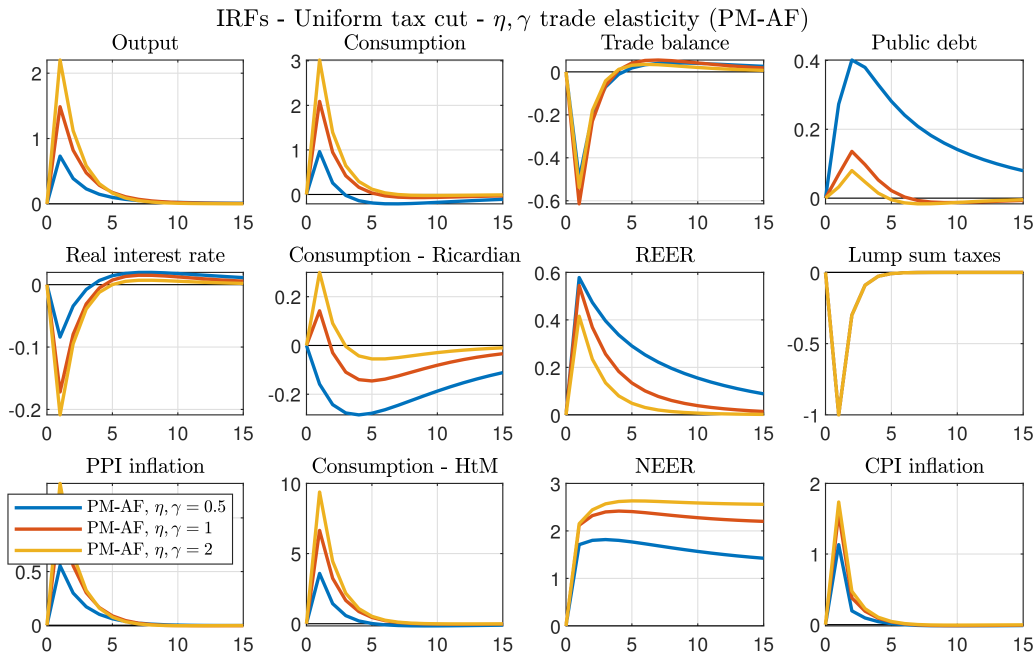


Figure B.10: Impulse responses to a debt-financed uniform tax cut, for different trade elasticities η, γ (PM-AF policy mix)

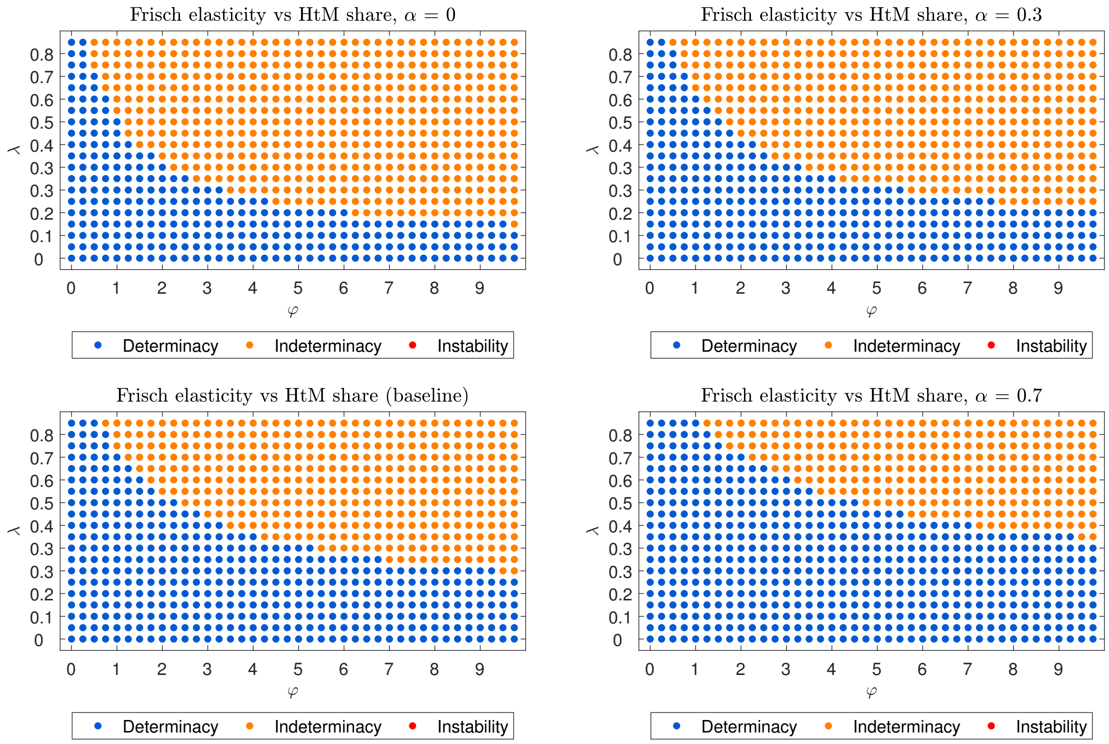


Figure B.11: Inverted Aggregate Demand frontier in the (φ, λ) plain, given differing degrees of openness α (baseline is $\alpha = 0.5$, with baseline policy specification (i.e. $\phi^{\pi} = 1.5$ and $\phi_B = 0.2 > 1 - \beta$.) and symmetric external steady state $\zeta = 0$).

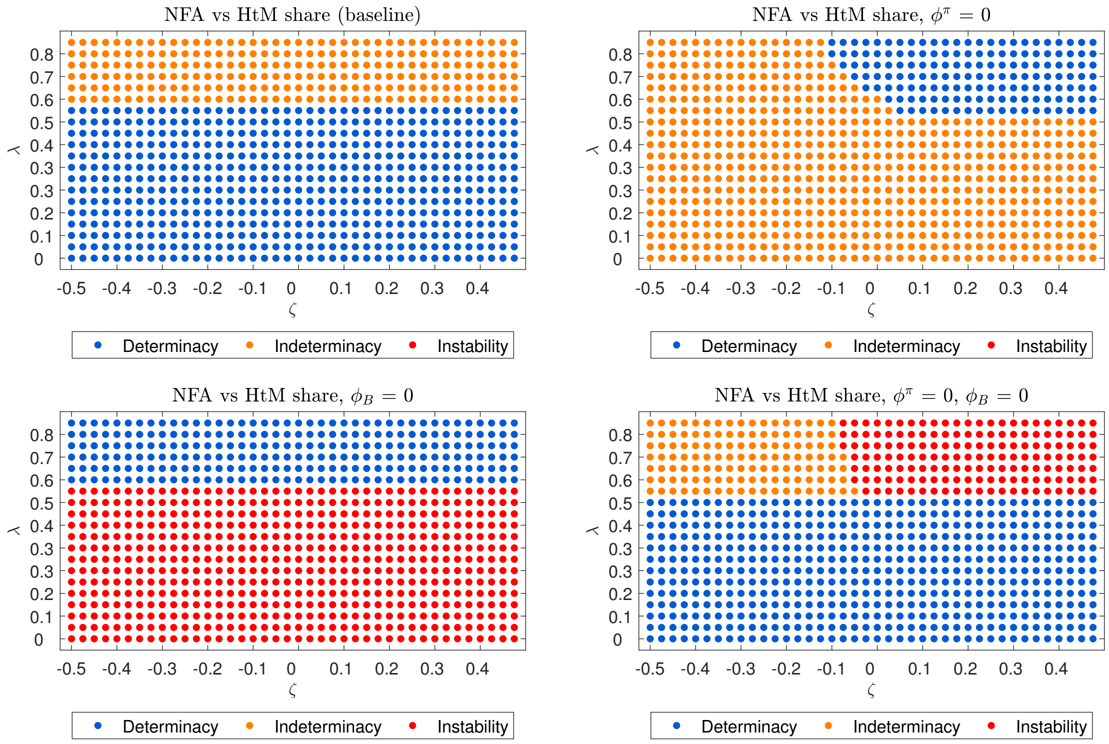


Figure B.12: Model determinacy properties in the (ζ, λ) plain, for different policy regimes. Unless otherwise indicated, baseline parameters are $\phi^{\pi} = 1.5$ and $\phi_B = 0.2 > 1 - \beta$ and we have $\alpha = 0.5$.

	HtM tax cut	uniform tax cut	Ricardian tax cut	BB redistribution
HtM shock: $\check{\varepsilon}_t^T$	ε	ε	0	ε
Ricardian shock: $\hat{\varepsilon}_t^T$	0	ε	$\varepsilon/(1-\lambda)$	$\frac{-\lambda}{1-\lambda}\varepsilon$
TOTAL: $(1-\lambda)\hat{\varepsilon}_t^T + \lambda\,\check{\varepsilon}_t$	$\lambda\,\varepsilon$	ε	ε	0

Table B.1: Description of tax cut shocks for different scenarios (with HtM tax shock $\check{\varepsilon}_t^T$ kept the same)

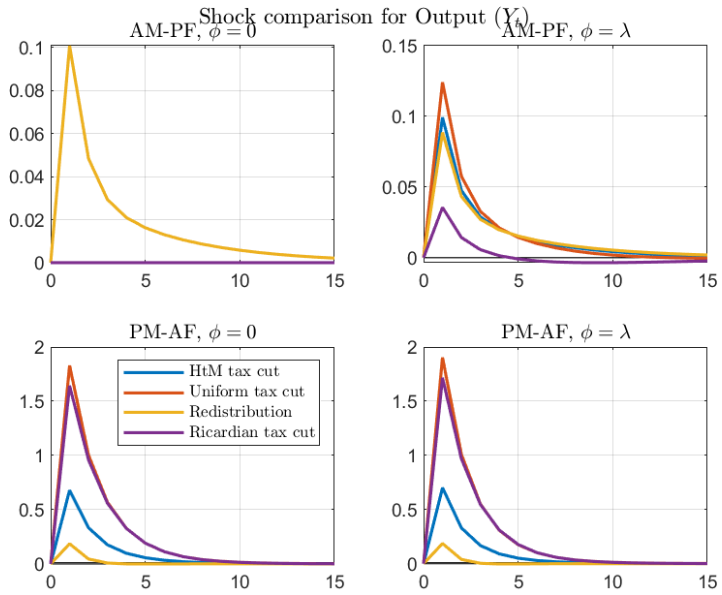


Figure B.13: Shock comparison for output, across policy regimes and tax distribution ϕ (with HtM tax shock $\check{\varepsilon}_t^T$ kept the same) as in Table B.1

Chapter 3

Weak Wage Recovery and Precautionary Motives after a Credit Crunch

joint with Łukasz Rachel and Rana Sajedi

3.1 Introduction

In several advanced economies the recovery from the Great Recession has been characterized by unusually weak wage growth. Even as the labor market tightened markedly and unemployment has fallen to pre-crisis lows, wage growth has failed to pick up, especially relative to previous recoveries. At the same time labor force participation rates have also fallen, productivity growth has been disappointing, while equity prices have rebounded very fast.

According to some explanations, like hysteresis or secular stagnation, the prolonged nature of the crisis or other secular trends might have changed the historic relationships between variables such as labor market slack and wage growth (somewhat akin to a flatter Phillips curve). We propose an alternative argument by combining labor market and financial frictions in an attempt to explain the above features of the recovery. While labor market developments seem especially central in driving the post-crisis recovery, the Great Recession is usually viewed as triggered by a financial shock. Therefore, the role of financial markets is potentially very important in our story.

We focus on how households' financial wealth might affect their labor market outcomes. Our hypothesis is that following a financial recession, when workers' balance sheets are still under deleveraging pressure, the utility loss associated with unemployment is larger than normally since options for consumption smoothing are more limited. Consequently, especially if they are already asset-poor, households are more desperate to keep existing or find new jobs which is why they are more likely to settle for lower wages in order to secure employment. On the other hand, this mechanism boosts the profitability of firms as their wage bill is falling which can account for rising equity prices and expanding employment.[1] Equity valuations might also be boosted by lower discount rates, as the extra saving desire pushes down real interest rates. Hence, the deleveraging pressure could also account for some of the fall (or the slow recovery) in estimated equilibrium interest rates throughout advanced economies.

To capture the above mechanism we propose a continuous time heterogeneous agent model (HACT) with incomplete financial markets and search-and-matching (SAM) frictions in the labor market, closely related to Krusell, Mukoyama and Sahin (2010). Households face uninsurable idiosyncratic income risk, endogenized through SAM frictions which drive flows in and out of employment. Financial frictions are captured by the combination of a borrowing constraint and the lack of complete financial markets (i.e. no full insurance against idiosyncratic uncertainty), while labor market frictions are captured through the SAM process, whereby unemployed agents searching for a job and unfilled vacancies posted by firms cannot find each other in a seamless manner, and some jobs may be terminated for exogenous reasons. Into this framework we introduce a credit crunch, modelled as a gradual tightening of the borrowing constraint, similarly to Guerrieri and Lorenzoni (2017), and look at the perfect foresight transition dynamics in the absence of aggregate uncertainty.

Our baseline results indicate that following a credit crunch wages fall despite a tightening labor market and expanding employment, while firm equity becomes more valuable, in line with the observed characteristics of the post-crisis recovery. In our model, apart from the usual precautionary saving behaviour, households can self-insure against idiosyncratic income risk also by settling for lower wages in order to secure a job and thereby avoid becoming borrowing constrained. This is the main transmission channel captured by the model: as households suddenly find themselves closer to the borrowing constraint, the increased precautionary motive drives them to accept lower wages in the bargaining process, while firms respond to this by posting more vacancies, leading to a tighter labour market and falling unemployment. Lower wages also mean that more of the surplus from the job stays with the firm which can boost profitability and explain higher equity prices.

[1] The effect on labor force participation is likely to be positive though as the deleveraging pressure shifts the labor supply curve out, even if falling wages mitigate the rise in participation.

However, lower discount rates play a larger part in this than higher profitability: the increased saving desire boosts the asset demand of households which depresses the real interest rate. If the deleveraging pressure is persistent enough, the above responses are more prolonged.

As for the other two features of the recovery, nameley weak productivity growth and lower participation rates, our model is more silent. In its baseline version we have fully inelastic labor supply and a linear production function, under which both participation and productivity stay constant. With decreasing returns to scale, however, we are able to generate falling productivity. This can be interpreted as a shortcut to modelling damage to the supply side of the economy following a prolonged recession.

Our paper combines two main strands of the literature. On the one hand, ours is a heterogeneous agent incomplete market model with idiosyncratic income risk in the Aiyagari-Bewley-Hugett tradition. On the other hand, it also features Diamond-Mortensen-Pissarides style search-and-matching frictions in the labor market. This combination leads to endogenous idiosyncratic risk. This paper is in no respect the first to make this combination: the main point of reference is Krusell, Mukoyama and Sahin (2010) to which our model is closest.[2] However, unlike Krusell, Mukoyama and Sahin (2010), we spell out our model in continuous time following the work of Bardóczy (2017), which allows us to exploit analytical and numerical advantages provided by the HACT (heterogeneous agent continuous time) methodology described in Achdou et al. (2017).

Another point of departure is that our model does not have productive capital. The only savings vehicle is a fixed supply of financial assets, so the economy as a whole cannot save more. We opt for this setup in order to avoid the expansionary effects of rising investment which would be the result of increased saving desire, and which we do not find compatible with the financial crisis. In this sense, the omission of capital is a shortcut to modelling other financial frictions and nominal rigidities, under which falling aggregate demand and more binding collateral constraints could impair investment as well.

Our focus is also different since we analyse the effects of a credit crunch in the HACT+SAM framework which, to our knowledge, has not been done yet. The modelled tightening of the borrowing constraint follows Guerrieri and Lorenzoni (2017) who work with an incomplete markets heterogeneous agent model. However, they do not endogenize idiosyncratic risk and do not model labor market frictions, which is key to our analysis. In addition, they use discrete time, while we cast the model in continuous time, incoropranting the

[2] Other well-known examples include Gornemann, Kuester and Nakajima (2012), and Ravn and Sterk (2018), although these models also feature nominal rigidities.

tightening borrowing constraint according to Mellior (2016).[3] On the other hand, we do not have endogenous labor supply choice which prevents us from modelling labor force participation. Guerrieri and Lorenzoni (2017) capture self-insurance through employment via an increased participation of low productivity workers (driven by stronger precautionary motives). The corresponding composition effect is the driving force behind the drop in average productivity, average labor income and a supply-side induced recession following the credit crunch. In contrast, our model with endogenous risk and an explicit modelling of wage bargaining generates economic expansion and tightening labor markets which we view as more in line with the later stages of the recovery. Nonetheless, introducing participation choice is high on our research agenda.

At this point we would like to emphasize that ours (like the others discussed above) is a real model with fully flexible prices. By omitting nominal rigidities, we ignore aggregate demand effects and focus on the supply side of the economy. One the one hand, this allows us to identify the pure downward contribution of the precautionary channel on wage dynamics during a deleveraging process, and tells us how results are likely to change *relative* to standard New Keynesian models of aggregate demand which imply a stronger positive comovement between wages and labor market tightness. On the other hand, we necessarily fail to capture the sharp drop in economic activity in the immediate aftermath of the credit crunch which we think of as an aggregate demand driven recession (unlike in Guerrieri and Lorenzoni (2017)), but this is not the focus of our paper. To the extent that aggregate demand can start to recover even while the deleveraging is still under way, our results can be relevant.

Nevertheless, introducing nominal rigidities in a HANK+SAM fashion is high on our research agenda, as aggregate demand is a crucial part of business cycle fluctuations in general, and financial crises in particular. Examples of these models include Gornemann, Kuester and Nakajima (2012) or Ravn and Sterk (2018).[4] As the latter point out, in a HANK+SAM model countercyclical income risk may arise which can *amplify* the effects of aggregate demand shocks like a deleveraging shock. In this case precautionary channels can also reinforce aggregate demand channels instead of working against them.

The rest of the paper is organized as follows. Section 2 describes the theoretical model.

[3] In short, our model can be viewed as a combination of Krusell, Mukoyama and Sahin (2010) and Guerrieri and Lorenzoni (2017), but cast in continuous time based on Bardóczy (2017) and Mellior (2016), and omitting productive capital.

[4] This family of models introduce SAM labor market frictions into a general HANK (Heterogeneous Agent New Keynesian) framework which is described in Kaplan, Moll and Violante (2017). HANK models usually involve aggregate uncertainty as well, which requires more advanced solution techniques as developed by Reiter (2009) and Ahn et al. (2017).

Section 3 discusses the results. A final section concludes.

3.2 Model

Our model is a continuous time heterogeneous agent model with incomplete markets (HACT) and search-and-matching (SAM) frictions in the labor market. Households can either be employed or unemployed. Matches between unemployed job searchers and vacancies posted by firms are created according to a matching function, and existing jobs are separated at an exogenous rate. While firms post vacancies (i.e. decide on their labor demand) with a view to maximize profits, households do not make a labor force participation decision: they have fully inelastic labor supply (meaning they always work or search for a job). Therefore, it is the vacancy posting decision of firms which is the sole endogenous driver of the SAM-process.

Households make consumption-saving decisions in the face of idiosyncratic uncertainty. Idiosyncratic income risk is embodied in the labor market status of households (being either employed or unemployed). Due to incomplete markets households cannot fully insure themselves against this risk so individual histories of shocks matter, and will give rise to an endogenous wealth distribution (an infinite dimensional object). A standard feature of such models is the emergence of precautionary savings whereby households try to self-insure.

The combination of the heterogeneous agent incomplete market setup with labor market frictions results in idiosyncratic uncertainty being endogenized through the SAM process which drives flows in and out of employment.[5] In addition, due to labor market frictions, each match creates a surplus which is to be shared among workers and firms during a wage bargaining process. Heterogeneity in wealth creates heterogeneity also in the relative value of employment which results in an endogenous wage schedule increasing in wealth (another infinite dimensional object).

There is no productive capital in the economy and the only savings vehicles are financial assets, so the economy as a whole cannot save more or less: agents can trade risk-free government-issued bonds only among themselves. Due to SAM-frictions, however, firms have positive profits even under perfect competition which yields non-zero equity values: shares in firms constitute another financial asset. Since the value of equity can fluctuate, the supply of assets is not completely fixed.

[5]One can think of this setup as a Bewley model with endogenous job finding rate, or equivalently, as a standard Diamond-Mortensen-Pissarides style search and matching model where workers can insure themselves against job loss through accummulating assets (precautionary savings).

The final good of the economy is produced with labor as the only input, and is exhasted by consumption and vacancy posting costs. The government finances unemployment benefits and interest payments by uniformly distributed lump sum taxes, which provide another source of insurance. There is no aggregate uncertainty. The economy does not have any nominal rigidities, so aggregate demand effects are absent.

Our model builds heavily on Bardóczy (2017), but leaves out capital accummulation and allows for a credit crunch.

3.2.1 Labor market

Households' income state is either employed or unemployed $s_t \in \{s^e, s^u\}$. The size of the population is normalized to one, so with u_t denoting the unemployment rate, aggregate employment is expressed as $1 - u_t$. Search-and-matching frictions in the labor market govern the dynamics of unemployment:

$$M(u_t, v_t) = u_t^{\eta}\, v_t^{1-\eta}$$

$$\lambda_t^f = \frac{M(u_t, v_t)}{v_t} = \theta_t^{-\eta} \tag{3.1}$$

$$\lambda_t^w = \frac{M(u_t, v_t)}{u_t} = \theta_t^{1-\eta} \tag{3.2}$$

$$\dot{u}_t = \sigma\,(1 - u_t) - \lambda_t^w\, u_t \tag{3.3}$$

where v_t is vacancies posted by firms, $\theta_t = v_t / u_t$ is the definition of labor market tightness, λ_t^f is the vacancy filling rate, λ_t^w is the job finding rate and σ is the exogenous job separation rate. The transition matrix for the *endogenous* idiosyncratic state is therefore:

$$\Lambda_t = \begin{bmatrix} -\sigma & \sigma \\ \lambda_t^w & -\lambda_t^w \end{bmatrix} \tag{3.4}$$

Notice that the endogeneity of the idiosyncratic income process is influenced solely through the vacancy posting decision (labor demand) of the firm, which affects labor market tightness and, in turn, job-finding rates. Since households do not make a labor force participation choice, they have no *direct* influence over the other determinant of labor market tightness, unemployment.[6]

3.2.2 Asset market

There are two types of financial assets in the economy: risk-free bonds issued by the government in a fixed supply B, and equity in the firm with a total market value of

[6] Although they do have *indirect* influence on λ_t^w through the wage bargaining process which affects firms' labor demand.

p. Households can also issue debt up to some borrowing constraint, however, it must be an asset for another household, so on the aggregate level the *net* supply of bonds is still B. Equity has positive value as labor market frictions result in positive firm profits: they share the surplus from a match with the worker, which drives a wedge between the marginal product of labor and the wage.

In the absence of aggregate uncertainty both assets are considered risk-free, therefore a no-arbitrage condition equalizes their returns.

$$r_t = \frac{d_t + \dot{p}_t}{p_t} \tag{3.5}$$

where r_t is the real return on bonds, and d_t is dividends paid by the firm. For the same reason, the household is indifferent between the two assets which is why we do not model portfolio choice but only the total value of a household's asset holdings defined as

$$a_t^i = \vartheta_t^i (B + p_t)$$

where ϑ_t^i is the share of a particular household i from total assets.

In the presence of idiosyncratic uncertainty the above setup means that financial markets are incomplete and full insurance against all contingencies is not possible.

3.2.3 Households

There is a continuum of households $i \in [0, 1]$, who make a consumption-saving choice in the face of idiosyncratic income risk. Their income state is either employed or unemployed $s_t \in \{s^e, s^u\}$ which is governed by a Poisson process determined by the SAM frictions as in Λ_t. Households derive no utility from leisure, so they have no endogenous labors supply (participation) choice: if offered a job, they work.[7] If employed, workers earn a wage according to a wage schedule $\omega_t(a_t)$ which depends on their wealth. Unemployed workers get unemployment benefits h from the government. All households pay lump sum taxes T_t to the government. In making their consumption-saving decision, households are subject to an exogenous borrowing constraint, meaning that their total assets cannot go below $\underline{a}$ (or the natural borrowing limit, if it is stricter).

[7]This also means that the law of motion for unemployment (3.3) is not a constraint in their problem. Current employment is a state variable in SAM-models, as it is determined by matching frictions. However, its next period value could be influenced through searching more intensively and increasing participation today, subject to (3.3). Without participation choice households cannot influence next period's chances of getting employment by increasing their search/unemployment today.

The sequential formulation of a household's problem is the following:

$$
\begin{aligned}
W(a_0, s_0) = \max_{c_t, \dot{a}_t} \quad & \mathbb{E}_0 \int_0^{\infty} e^{-\rho t}\, u(c_t)\, dt \\
& \dot{a}_t = y_t(a_t, s_t) + r_t\, a_t - c_t \\
& a_t \geq \max\left\{ \underline{a};\ -\frac{h}{r} \right\} \\
& s_t \in \{s^e, s^u\} \sim \text{Poisson } (\Lambda_t) \\
y_t(a_t, s_t) = & \begin{cases} \omega_t(a_t) - T_t & \text{if } s_t = s^e \\ h - T_t & \text{if } s_t = s^u \end{cases}
\end{aligned}
$$

where ρ is the personal discount rate. The change in assets $\dot{a}_t$ is interpreted as the flow savings of the household, which is pinned down buy the budget constraint.[8]

In continuous time it is convenient to write the above sequential problem recursively in the form of Hamilton-Jacobi-Bellman equations and boundary conditions. In doing so, we follow closely the HACT methodology as explained in Achdou et al. (2017), and as

[8] The reader can convince themselves that the return on total assets a_t is the same as on bonds due to the no-arbitrage condition with equity:

$$
\begin{aligned}
a_t &\equiv b_t + p_t \\
a_t^+ &= (1 + r_t \Delta t) b_t + p_t + \Delta p + d_t \Delta t \\
a_t^+ - a_t &= r_t \Delta t\, b_t + \Delta p + d_t \Delta t \\
\frac{a_t^+ - a_t}{\Delta t} &= r_t\, b_t + \underbrace{\frac{\Delta p}{\Delta t} + d_t}_{r_t\, p_t} \;=\; r_t\, a_t
\end{aligned}
$$

applied by Bardóczy (2017) to models with SAM frictions. [9]

$$\rho\, W_t(a,s^e) = \max_c \Big\{ u(c) + \partial_a W_t(a,s^e)\, \underbrace{\big[\omega_t(a) - T_t + r_t\, a - c\big]}_{\dot a_t} + \sigma\Big[W_t(a,s^u) - W_t(a,s^e)\Big] + \partial_t W_t(a,s^e)\Big\} \tag{3.6}$$

$$\rho\, W_t(a,s^u) = \max_c \Big\{ u(c) + \partial_a W_t(a,s^u)\, \underbrace{\big[h - T_t + r_t\, a - c\big]}_{\dot a_t} + \lambda_t^w\Big[W_t(a,s^e) - W_t(a,s^u)\Big] + \partial_t W_t(a,s^u)\Big\} \tag{3.7}$$

$$\partial_a W_t(\underline{a},s^e) \geq u'\Big(\omega_t(\underline{a}) + r_t\, \underline{a}\Big)$$
$$\partial_a W_t(\underline{a},s^u) \geq u'\Big(h + r_t\, \underline{a}\Big)$$

where $W_t(a,s)$ is the value function of households over the state space (a,s). The relevant state variables for the household in making their consumption/saving decision are the income state (being employed or unemployed) and financial wealth: (a,s). The FOC with respect to consumption and the budget constraint give us the optimal consumption and savings policies *as a function of* the value function's derivative (which is itself a function of the state variables).

$$c_t(a,s) = u'^{-1}\Big(\partial_a W_t(a,s)\Big) \tag{3.8}$$
$$\dot a_t(a,s) = y_t(a,s) + r_t\, a - c_t(a,s) \tag{3.9}$$

One of the advantages of using continuous time (apart from numerical and computational efficiencies) is that the borrowing constraint collapses into simple boundary conditions as above (because unlike in discrete time, it applies to a *state* variable rather than a control). More importantly, the first-order conditions (3.8) hold with equality *everywhere* in the state space (i.e. even at the borrowing constraint) – unlike in discrete time, where the Euler-equation is an inequality and will be slack whenever the borrowing constraint binds. In addition, the FOC is *static* which allows us to directly solve for the optimal consumption choice. In discrete time the FOC involves tomorrow's asset level (a choice

[9] Notice that the HJB equations are just the special versions for each income state s^e, s^u of the more general HJB formulation:

$$\rho\, W_t(a,s) = \max_c \Big\{ u(c) + \partial_a W_t(a,s)\, \underbrace{\big[y_t(a,s) + (r_t - \delta)a - c\big]}_{\dot a_t} + \sum_{s'\neq s} \lambda_{ss'}\big[W_t(a,s') - W_t(a,s)\big] + \partial_t W_t(a,s)\Big\}$$
$$\partial_a W_t(\underline{a},s) \geq u'\Big(y_t(\underline{a},s) + (r_t - \delta)\underline{a}\Big)$$

For a precise and detailed derivation of the HJB equations in continuous time problems we refer the interested reader to Achdou et al. (2017).

variable) which defines optimal consumption only implicitly, requiring costly root finding methods to solve it.[10]

Distribution

Due to incomplete markets different histories of idiosyncratic income shocks will lead to different asset levels for individual households, giving rise to a non-degenerate wealth distribution, an inifinite dimensional object which is an important component of the economy's aggregate state. Let $g_t(a,s)$ denote the density of the joint distribution of households over the asset-income state spce. Given the optimal consumption/saving choices $\dot{a}_t(a,s)$ from (3.9), which govern movements along the asset dimension, and the SAM-determined Poisson process for income Λ_t from (3.4), the dynamics of the distribution are described by the Kolmogorov Forward Equations: [11]

$$\partial_t g_t(a,s^e) = -\partial_a\Big[\dot{a}_t(a,s^e)\; g_t(a,s^e)\Big] - \sigma\; g_t(a,s^e) + \lambda_t^w\; g_t(a,s^u) \tag{3.10}$$

$$\partial_t g_t(a,s^u) = -\partial_a\Big[\dot{a}_t(a,s^u)\; g_t(a,s^u)\Big] - \lambda_t^w\; g_t(a,s^u) + \sigma\; g_t(a,s^e) \tag{3.11}$$

The density $g_t(a,s)$ naturally integrates to one, and since the population size is also normalized to one, the mass of employed and unemployed households give use the employment and unemployment rates, respectively.

$$\sum_{s\in\{s^e,s^u\}} \int_{\underline{a}}^{\infty} g_t(a,s)\; da = \underbrace{\int_{\underline{a}}^{\infty} g_t(a,s^e)\; da}_{1-u_t} + \underbrace{\int_{\underline{a}}^{\infty} g_t(a,s^u)\; da}_{u_t} = 1$$

[10]To see these differences, it might be helpful to compare continuous time and discrete time FOCs:

$$\begin{aligned} u'(c) &= W_a(a,y_j) \\ u'(c) &\geq \beta\, \mathbb{E}_{y'}\, W_a(a',y') \\ &\geq \beta \sum_{k=1}^{J} Pr(y_k|y_j)\; W_a\Big(\underbrace{y_j + (1+r)a - c}_{a'(a,y_j)},\; y_k\Big) \end{aligned}$$

[11]For a detailed derivation of the KFE equation we again refer to Achdou et al. (2017). The general KFE is:

$$\partial_t g_t(a,s) = -\partial_a\Big[\dot{a}_t(a,s)\; g_t(a,s)\Big] - g_t(a,s)\sum_{s'}\lambda_{ss'} + \sum_{s'}\lambda_{s's}\; g_t(a,s')$$

3.2.4 Firms

Firms produce the output of the economy using labor as the only input according to the following production technology:

$$
\begin{aligned}
z_t\, F(N_t) &= z_t\, N_t^{1-\alpha} \\
\frac{z_t\, F(N_t)}{N_t} &= z_t\, N_t^{-\alpha} = z_t\,(1-u_t)^{-\alpha}
\end{aligned}
$$

where z_t denotes TFP and the last line expresses per-capita production, i.e. the output corresponding to a single job, if matched with a worker. Within a job there is no intensive margin: the worker either works full hours or nothing. $\alpha = 0$ corresponds to a linear production technology, and CRS production. With $\alpha > 0$ we have decreasing returns to scale and also diminishing marginal product of labor, which means that the marginal product of an additional job will depend on the aggregate level of employment.

Firms create jobs by posting vacancies v_t at a fixed cost of ξ. Each job commands a wage $\omega_t(a)$ which depends on the wealth of the worker. Due to SAM frictions the firm solves a dynamic problem: current employment is a state variable, so it is only future employment which can be influenced by current vacancy posting decisions, subject to the dynamics imposed by SAM frictions in (3.3). Therefore, the labor demand choice is implicit in the vacancy posting decision.

We can write up the firm's dynamic problem recursively, using HJB equations involving value functions for a *single* filled job $J_t(a)$ and an unfilled vacancy V_t. Profits are discounted at a rate r_t (which is the relevant alternative cost for households who own the firm). In their profit maximiziation problem firms take into account that the state can change in the next instant, i.e. the job might be separated at a rate σ and an unfilled vacancy might get filled at a rate λ_t^f as well as the employed worker's asset position might change at a rate of their savings policy $\dot{a}_t(a, s^e)$ which would alter the wage payable to them. The resulting HJB equations are:

$$
\begin{aligned}
r_t\, J_t(a) &= \Big[z_t\,(1-u_t)^{-\alpha} - \omega_t(a)\Big] + \partial_a J_t(a)\; \dot{a}_t(a, s^e) + \sigma \Big[V_t - J_t(a)\Big] + \partial_t J_t(a) \\
r_t\, V_t &= -\xi + \lambda_t^f \int_{\underline{a}}^{\infty} J_t(a)\, \frac{g_t(a, s^u)}{u_t}\, da
\end{aligned}
$$

Due to free entry the value of opening a new vacancy must be zero in equilibrium. Therefore the labor demand decision of the firm is embedded in the condition that

$$
V_t = 0 \tag{3.12}
$$

i.e. the firm will post vacancies until their value drops to zero, which fills in the role of a

FOC with respect to v_t. Plugging this optimality condition back into the HJBs, we get:[12]

$$(\sigma + r_t)\, J_t(a) = \Big[z_t\,(1-u_t)^{-\alpha} - \omega_t(a) \Big] + \partial_a J_t(a)\; \dot{a}_t(a, s^e) + \partial_t J_t(a) \tag{3.13}$$

$$\xi = \lambda_t^f \int_{\underline{a}}^{\infty} J_t(a)\, \frac{g_t(a, s^u)}{u_t}\, da \tag{3.14}$$

Due to matching frictions there will be a wedge between the marginal product of labor and the real wage. Therefore, even in the presence of free entry the firms has positive profits which are paid out as dividend to the household. If $\pi_t(a)$ denotes the ex-vacancy profits of the firm, corresponding to a particular job held by a worker with asset level a and earning wage $\omega(a)$, then aggregate dividends d_t are determined as:

$$\pi_t(a) = z_t\,(1-u_t)^{-\alpha} - \omega_t(a) \tag{3.15}$$

$$d_t = \int_{\underline{a}}^{\infty} \pi_t(a)\; g_t(a, s^e)\; da - \xi v_t \tag{3.16}$$

3.2.5 Wage setting

The surplus from a match is shared between the worker and the firm according to some bargaining process, with β denoting the bargaining power of the worker. We explore two types of bargaining. Under Nash-bargaining the wage schedule is the solution to the problem

$$\omega_t(a) = \arg\max_{w} \Big[\widetilde{W}_t(a, s^e, w) - W(a, s^u) \Big]^{\beta} \Big[\widetilde{J}(a, w) \Big]^{1-\beta} \tag{3.17}$$

Alternatively, we can have egalitarian bargaining, where the surplus is shared according to

$$(1-\beta)\Big[W_t(a, s^e) - W_t(a, s^u) \Big] = \beta\, J_t(a)$$

As Bardóczy (2017) shows, in both cases continuous time allows for a closed form solution for the wage schedule.[13]

The main point here is the emergence of a wage schedule $\omega_t(a)$ which depends positively on the wealth of the worker – as opposed to a single wage being paid to every worker, which is the case in standard SAM models with complete markets. This is due to the fact

[12]Notice that the HJB equation for jobs (3.13) is not a maximization (as the only decision with respect to vacancies has already been taken): it is just an expression to derive the value coming from a filled job $J_t(a)$ which can later be used in the wage bargaining process.

[13]For the details of derivation, we refer to Bardóczy (2017). $\omega_t(a)$ is extracted from $W_t(a, s^e)$ and $J_t(a)$ when we solve the above problems. The "tilde" value functions include an arbitrary wage w, which coincide with the actual value function when using the optimal wage schedule $w = \omega(a)$. So $\widetilde{W}_t(a, s^e, \omega(a)) = W_t(a, s^e)$, while $\widetilde{J}_t(a, \omega(a)) = J_t(a)$, and $W_t(a, s^u)$ does not depend on the wage.

that the relative value of the worker's outside option, i.e. of turning down the job offer and staying unemployed, depends on their wealth. With less assets to rely on to smooth consumption (or equivalently being closer to the borrowing constraint) becoming/staying unemployed makes a much bigger difference than with sufficient wealth. Therefore, the worker is more eager for the job and is willing to accept lower wages in order to avoid getting closer to the borrowing constraint. In this sense, accepting lower wages in order to secure employment is a form of self-insurance against idiosyncratic uncertainty: on the one hand, it is a substitute for precautionary savings, but on the other hand it is also a means to secure higher income which in turn allows for more precautionary savings. This mechanism is key in our model, which drives wage dynamics following a tightening of the borrowing constraint.

3.2.6 Government

The government sustains a stable debt of B. It finances unemployment benefits and interest payments on its debt by collecting lump-sum taxes from the households.[14] The government's budget constraint is therefore:

$$r_t\, B = T_t - u_t\, h \tag{3.18}$$

3.2.7 Market clearing

Asset market clearing means that demand for financial assets from the households equals the total supply of financial assets which are a fixed supply B of government bonds and equity in the firms, valued at p_t:

$$\sum_{s \in \{s^e, s^u\}} \int_{\underline{a}}^{\infty} a\ g_t(a, s)\ da = B + p_t \tag{3.19}$$

Equilibrium in the asset market will be achieved by the adjustment of the real interest rate r_t which influences saving decisions and therefore the asset demand of households (reflected in the asset distribution $g_t(a, s)$).[15]

Labor market clearing is already implicit in the formulation of the model, as laid out above. The goods market should automatically clear by Walras' law, once the other

[14] We maintain the possibility of no government, in which case $u_t\, h$ is to be interpreted as home production by unemployed agents, while $r_t\, B$ is interest income earned on foreign assets (which is equivalent to the trade deficit).

[15] In fact, a changing interest rate also affects asset supply through influencing equity valuation as the present value of future dividends change.

markets clear.[16]

$$C_t + \xi v_t = \underbrace{z_t\,(1-u_t)^{-\alpha}}_{Y_t}$$

This is the resource constraint of the economy which shows that final output is exhausted by consumption and vacancy posting costs.[17]

3.2.8 Equilibrium

Equations (3.1) to (3.19) (together with the boundary conditions incorporating the borrowing constraint) describe the equilibrium of the model . The equilibrium consists of 19 variables which are:

- a set of quantities $u_t, v_t, T_t, \lambda_t^f, \lambda_t^w, d_t, \pi_t(a)$ – (also defining tightness $\theta_t = v_t/u_t$),
- a set of value functions $W_t(a, s^e), W_t(a, s^u), J_t(a), V$,
- a set of policy functions $c_t(a, s), \dot{a}_t(a, s)$,
- distributions over assets and employment $g_t(a, s^e), g_t(a, s^u)$

[16] This can be verified by aggregating the individual budget constraints of households and the government.

$$c_t(a,s) + \dot{a}_t(a,s) = y_t(a,s) + r_t\, a$$

$$C_t + [\dot{B} + \dot{p}_t] = \int_{\underline{a}}^{\infty} \omega_t(a)\, g_t(a,s^e)\, da + h \underbrace{\int_{\underline{a}}^{\infty} g_t(a,s^u)\, da}_{u_t} - T_t \;+\; r_t\,(B + p_t)$$

$$C_t = \int_{\underline{a}}^{\infty} \omega_t(a)\, g_t(a,s^e)\, da + \underbrace{u_t\, h\; - T_t + r_t B}_{=0 \text{ by } (3.18)} + \underbrace{r_t\, p_t - \dot{p}_t}_{d_t \text{ by } (3.5)}$$

$$C_t = \int_{\underline{a}}^{\infty} \omega_t(a)\, g_t(a,s^e)\, da + \underbrace{\int_{\underline{a}}^{\infty} \pi_t(a)\, g_t(a,s^e)\, da - \xi v_t}_{d_t \text{ by } (3.16)} =$$

$$= \int_{\underline{a}}^{\infty} \omega_t(a)\, g_t(a,s^e)\, da + \int_{\underline{a}}^{\infty} \underbrace{\Big[z_t(1-u_t)^{-\alpha} - \omega_t(a)\Big]}_{\pi_t(a) \text{ by } (3.15)} g_t(a,s^e)\, da - \xi v_t$$

$$= \int_{\underline{a}}^{\infty} \omega_t(a)\, g_t(a,s^e)\, da + \Big[z_t(1-u_t)^{-\alpha}\Big] \underbrace{\int_{\underline{a}}^{\infty} g_t(a,s^e)\, da}_{1-u_t} - \int_{\underline{a}}^{\infty} \omega_t(a)\, g_t(a,s^e)\, da - \xi v_t$$

$$= z_t(1-u_t)^{1-\alpha} - \xi v_t$$

[17] In the case of no government, final output would include home production in addition to private firm production. There would also be a trade deficit term as the interest income earned on foreign assets $r_t\, B$ would allow for higher domestic absorbtion than total output.

- a set of prices $r_t, p_t, \omega_t(a)$
- transition probabilities between employment states Λ_t

3.3 Results

3.3.1 Solution method

We model the credit crunch as a gradual tightening in the effective borrowing constraint, which essentially follows Guerrieri and Lorenzoni (2017). A slight technical difference in our case, due to computational convenience, is that the terminal borrowing constraint immediately jumps from $\underline{a} = a_1$ to its new value $\underline{a} = a_x$, but households who find themselves in the newly inadmissible region $a_1 \leq a < a_x$ are required to save their way towards the tighter borrowing limit only gradually (for those already above a_x, the new constraint is immediately binding). This is achieved by imposing positive savings in this region of *at least* Δa, provided that the non-negativity of consumption is not violated.[18,19]

$$\dot{a}_t(a,s) \geq \begin{cases} \min\left\{\Delta a,\ y_t(a,s) + r_t\, a\right\} & \text{for } \forall\ a_1 \leq a < a_x \\ \min\left\{0,\ y_t(a,s) + r_t\, a\right\} & \text{for } a = a_x \end{cases}$$

We ignore aggregate uncertainty, and we look at the perfect foresight transition dynamics between the two stationary equilibria: the initial one featuring the original borrowing constraint a_1, while the terminal one having a tighter borrowing constraint a_x. During the transition we impose the above rule for positive savings in the inadmissible region. This is similar to an "MIT type" of unexpected shock, which can be interpreted as the announcement of the new constraints.

In solving our HACT model, we follow Achdou et al. (2017) and use their finite difference scheme of upwinding to discretize the Hamilton-Jacobi-Bellman and Kolmogorov Forward equations. This procedure is very convenient in continuous time as the discretized versions

[18]Over time this condition will make sure that no households remain in $a_1 \leq a < a_x$ as they save themselves away from this region of state space. However, following Mellior (2016), we can completely guarantee this by imposing a stricter condition in the final period or terminal stationary equilibrium. This requires that in case there are still some households in the inadmissible region, they immediately jump to the new constraint by saving at least $\dot{a}_T(a,s) \geq a_x - a$ instead of Δa.

[19]Notice that these conditions translate into appending the boundary conditions accompanying the HJB equations of the household as follows:

$$\partial_a W_t(a,s) \geq \begin{cases} u'\Big(\max\left\{y_t(a,s) + r_t\, a - \Delta a,\, 0\right\}\Big) & \text{for } \forall\ a_1 \leq a < a_x \\ u'\Big(\max\left\{y_t(a_x,s) + r_t\, a_x,\, 0\right\}\Big) & \text{for } a = a_x \end{cases}$$

of both the HJB and KF equations will feature the same $\mathbf{A}_t$ matrix which describes transition rates within the (discretized) idiosyncratic state space. Continuous time means that this transition matrix is extremely sparse (only neighboring states can be reached within an instant of time) which allows for speedy computations. For details of this method we refer the reader to Achdou et al. (2017).

We introduce SAM frictions into the HACT framework based on Bardóczy (2017) who implements Krusell, Mukoyama and Sahin (2010) in continuous time. We incorporate the credit crunch into the HACT setting by appending the state constraints and boundary conditions as inspired by Mellior (2016). The details of our numerical algorithm can be found in the Appendix to this paper.

3.3.2 Calibration

Parameters			
γ	1.00	χ	1.10
ρ	0.05	σ	0.15
α	0.00	η	0.72
z	1.00	β	0.72
B	0.50	h	0.30
		ξ	0.199
Steady states	initial		terminal
$\mathbb{E}[\omega(a)]$	0.9576	$\mathbb{E}[\omega(a)]$	0.9576
u	0.1118	u	0.1113
θ	1.3294	θ	1.3537
p	0.1866	p	0.1874
r	0.0435	r	0.0412

Table 3.1: Parameters and selected steady state values

The utility function is CRRA, where $\gamma = 1$ corresponds to log utility $u(c_t) = \log c_t$.

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

Time is continuous and $t = 1$ in the model corresponds to one year. The personal discount factor of $\rho = 0.05$ corresponds to an annual 5% equilibrium real interest rate under complete markets. With incomplete markets the precautionary saving motive depresses it to 4.35%. And even further with the tightening of the borrowing constraint to 4.12%.

In the baseline parametrization we use egalitarian bargaining (Nash bargaining results in a flatter wage schedule). The Hosios condition is satisfied as the worker's bargaining power

β equals the matching elasticity with respect to unemployment η. Production technology is linear and constant returns to scale ($\alpha = 0$).

3.3.3 Baseline credit crunch

In our baseline scenario we look at the effects of tightening the borrowing constraint from $a_1 = -2$ to $a_x = -1.44$ with gradual steps of $\Delta a = 0.07$ required (i.e. this is a tightening of $x = 8$ gridpoints). As Figure 3.1 illustrates, this causes a rightward shift in the asset demand curve $\sum_{s\in\{s^e,s^u\}} \int_{\underline{a}}^{\infty} a \; g_t(a, s; r_t) \; da$, reflecting an increased saving desire by households.[20] One the one hand, the stricter borrowing constraint directly forces asset-poor households to save more as they need to gradually deleverage and get out of the newly inadmissible asset region $a_1 \leq a < a_x$. On the other hand, all households suddenly find themselves closer to the borrowing constraint which increases precautinary saving motives throughout the wealth distribution, but especially for poorer households.

Despite the rise in aggregate asset demand, with a fixed supply B of government bonds the economy as a whole cannot actually save more, so the increased saving desire needs to be discouraged by a lower real interest rate which is where asset market equilibrium is restored. However, asset supply is not completely fixed since the value of equity is a negative function of the discount rate $p_t = \frac{d_t + \dot{p}_t}{r_t}$, which would mitigate the asset shortage and the fall in r_t. It turns out though, that firm profitability actually worsens a little bit (discussed later), shifting asset supply to the left and counteracting most of the discount rate effect. Equity prices still rise, but not by much, which is why aggregate savings are essentially unchanged.

Looking at the distribution of assets instead of aggregate measures, in Figure 3.2 we see that the direct deleveraging pressure and rise in precautionary saving motives are strongest for asset-poor households who are relatively close to the borrowing constraint: the lower segments of the net wealth distribution are emptied out gradually as those households rebuild their balance sheets. Since the economy as whole cannot increase its savings this

[20]Note that household asset demand, i.e. saving desire is a positive function of the interest rate for intertemporal reasons. In the formula for aggregate asset demand the interest rate enters through its effect on the wealth distribution $g_t(a, s; r_t)$.

Precautionary saving motives are due to incomplete markets and are captured by the asset demand curve's distance from the complete market benchmark (a vertical line at the borrowing constraint and a horoznital line at the personal discount rate ρ). With sufficient amount of assets above the borrowing constraint asset demand converges to its complete markets version. The higher degree of idiosyncratic risk there is, the further away is the curve from the complete market case. The closer we are to the borrowing constraint, the more the increased precautionary saving motive depresses the interest rate below ρ.

must be offset by dissaving from the part of richer households, and it is exactly what we see in the right tail of the distribution: wealthy agents are incentivized to reduce their asset holdings by a lower equilibrium interest rate. For the above reasons, instead of seeing a rightward shift in the asset distribution, it rather becomes more concentrated following the credit crunch.

Under SAM frictions these asset market developments have an effect on the labor market as well. As discussed in the model description in Section 3.2.5, the wage schedule $\omega_t(a)$ arising from wage bargaining is a positive function of wealth. More precisely, it is a positive function of the *distance* from the borrowing constraint: it reflects the relative value of employment to the outside option (unemployment) which is much larger when there is a more limited room for consumption smoothing. As households suddenly find themselves closer to a tighter borrowing constraint, they become keener to secure a job and are willing to accept lower wages as a form of self-insurance. This channel works parrallel to and is also a substitute for precautionary savings, and is driven by the same factors.

In line with the above argument, and as Figure 3.3 demonstrates, the wage schedule shifts down for poorer workers, immediately reaching its new steady state.[21] During the

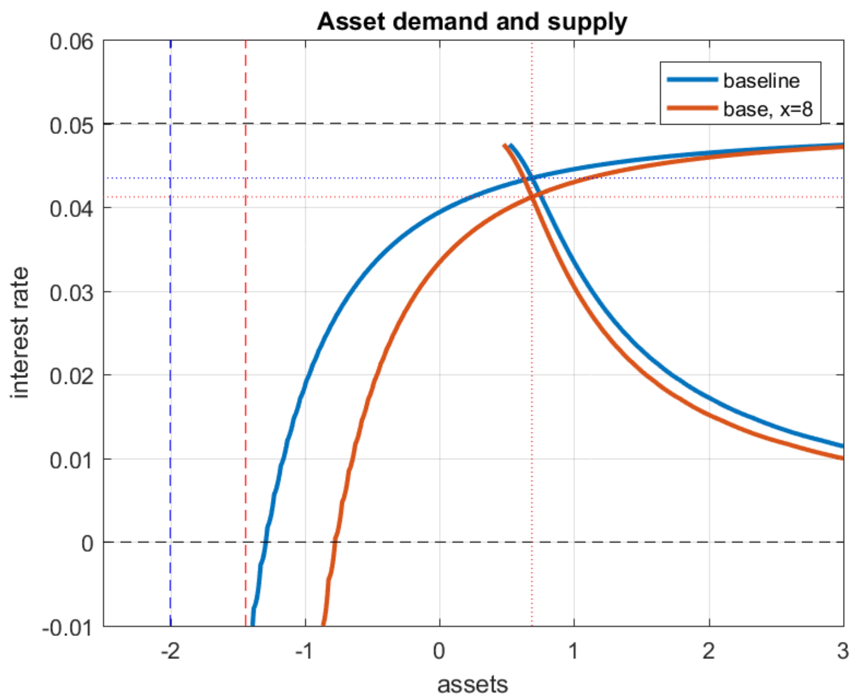
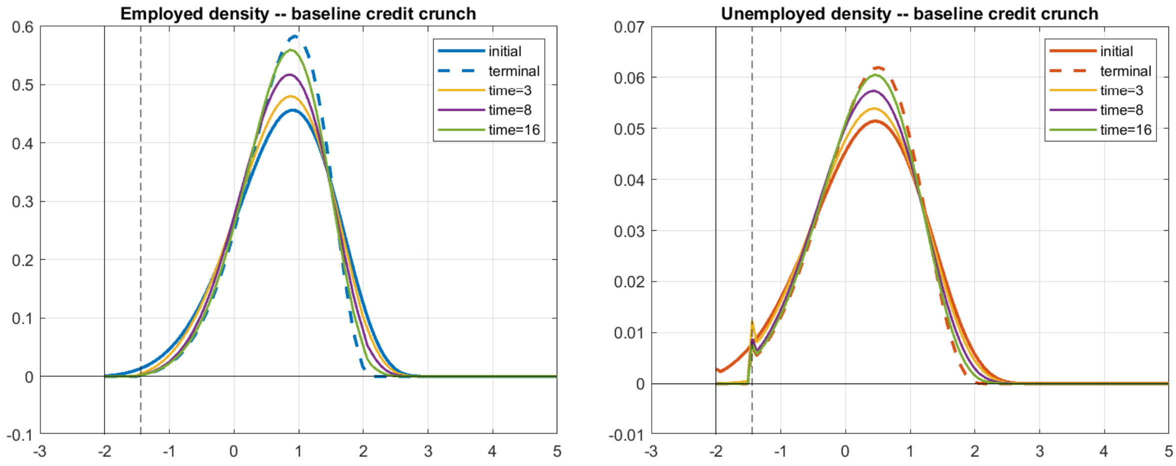


Figure 3.1: Asset demand $\sum_s \int_{\underline{a}}^{\infty} a \ g_t(a, s; r_t)\, da$ and asset supply $B + \frac{d_t + \dot{p}_t}{r_t}$. Blue lines correspond to the initial equilibrium, while red lines depict the terminal equilibrium after the credit crunch. Vertical dashed lines show the tightening in the borrowing constraint. The upper black dashed line is the interest rate (and asset demand) under complete markets, ρ. Dotted lines trace out equilibrium interest rates and assets.

[21] The kink in the new wage schedule at the new borrowing constraint is due to the fact that below this



(a) Wealth distribution of employed

(b) Wealth distribution of unemployed

Figure 3.2: Evolution of wealth distributions after a credit crunch

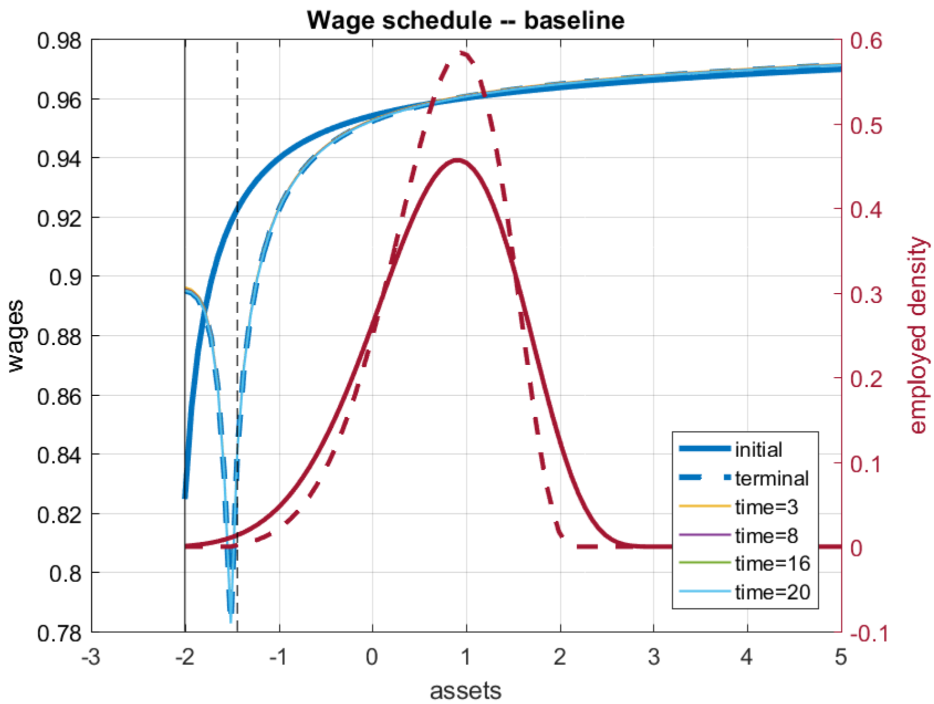


Figure 3.3: Evolution of the wage schedule after a credit crunch. Deep purple solid and dashed lines depict the initial and terminal density function for employed workers (rhs), respectively.

transition while there are still some mass of workers in these regions, this puts downward

level of assets we require at least a constant amount of saving Δa. This causes a kink in the consumptions policy function of unemployed agents who would have had negative savings in this region. This carries over to the consumptions policy of employed agents (who are interested in their relative position to the unemployed state) which in turn affects their wage schedule. If instead the minimum deleveraging requirement would be an increasing function of the shortfall from the new tighter borrowing constraint, this kink could be smoothed, but we opted for a constant Δa for everybody to allow for a more gradual adjustment for more "underwater" households. See Appendix C.2

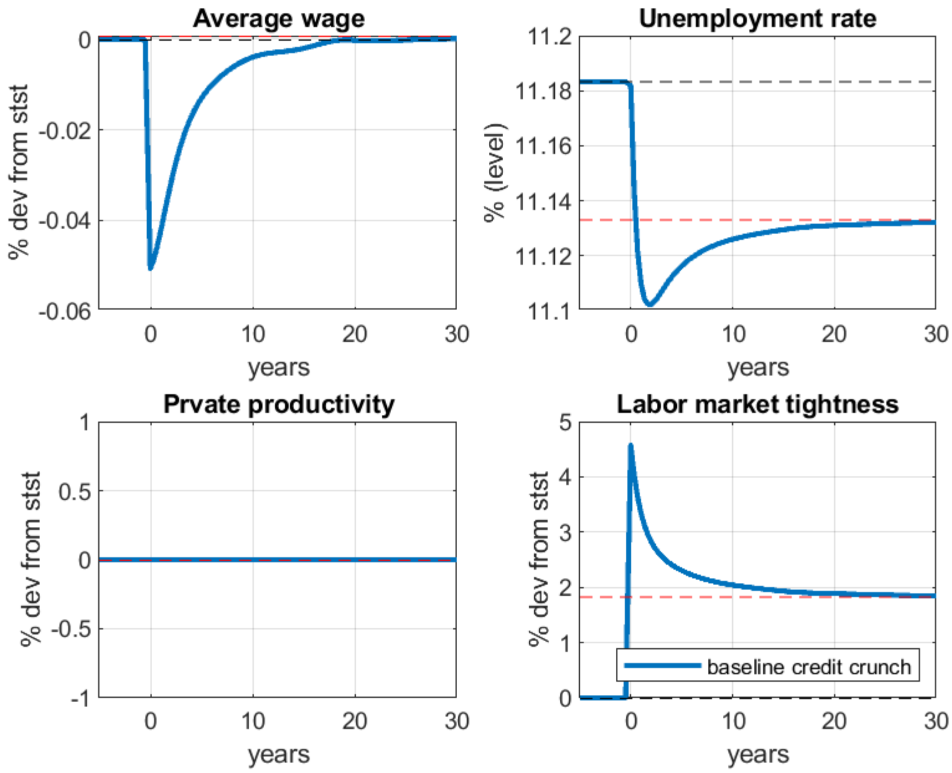


Figure 3.4: Impulse responses after a tightening of the borrowing constraint – labor market

pressure on the average wage as well, but due to the deleveraging pressure there will eventually be fewer agents here and the average wage will recover.

The latter point can be seen in the top left panel of Figure 3.4. The average wage falls on impact after the credit crunch, but as the deleveraging process goes on and agents rebuild their balance sheets, they also move up along the suddenly lower wage schedule, leading to a recovery in the average wage. All along the transition, however, wages stay lower than they would have in the absence of the credit crunch, and this difference is entirely due to the interaction of increased precautionary motives and labor market frictions, the main channel in our argument for explaining weak wages. By chaining this impulse response to one arising from a standard model of aggregate demand without this channel, we can demonstrate why we see both a sharper fall in wages on impact as well as a weaker wage recovery.

Figure 3.4 also shows that falling wages occur against the backdrop of falling unemployment and a tightening labor market, in line with the puzzling characteristics of the post-crisis recovery outlined in the Introduction. Lower wages prompt firms to open more vacancies and the tightening labor market leads to more matches, higher job finding rates and therefore falling unemployment. The combination of lower wages and higher employment might point towards a positive labor supply shock, which is true in the sense

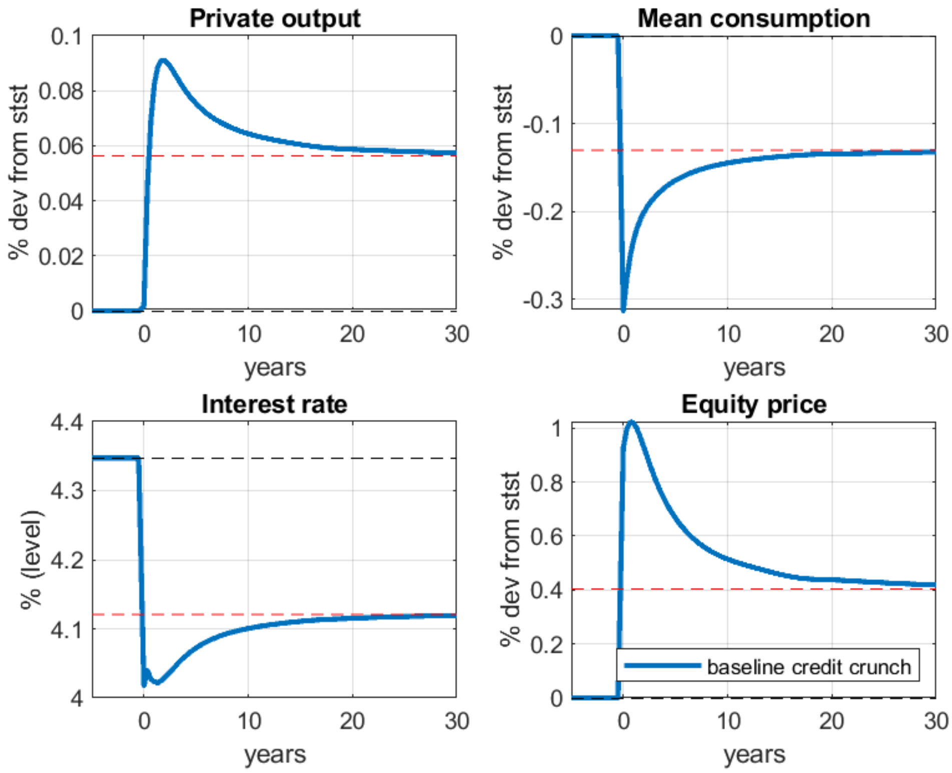


Figure 3.5: Impulse responses after a tightening of the borrowing constraint – goods and asset markets

that the change comes from the household's side as they are more desperate for jobs. But recall, that our model does not have endogenous labor supply choice and household decisions are only reflected through the wage bargaining process. In addition, a classic labor supply shock would not result in a tightening labor market.

Observing Figure 3.4 we can notice that while the wage reverts back to its original steady state, labor market tightness and employment settle at permanently higher levels. This part is due to the rise in the firm's labor demand, explained by other factors than wages. In particular, the fall in the equilibrium real interest rate (which acts as the firm's discount rate) raises the present value of future profits which prompts the firm to expand production, post more vacancies and hire more labor.

We can see these effects in Figure 3.5. The increase in savings desire after the credit crunch depresses the real interest rate which makes equity more valuable. The expansion in hiring raises private output. We see the overshooting pattern which is characteristic of the credit crunch also in Guerrieri and Lorenzoni (2017). However, unlike in their setup, the credit crunch in our case is expansionary. Recall that both models talk about the supply side of the economy, and ignore the obvious negative effects of the credit crunch on aggregate demand which undoubtedly entails a recession. Given the sign of the *deviations* from a standard model of aggregate demand, we view our results as more in line with characteristics of the post-crisis recovery, i.e. strong rebound in economic activity and employment together with disappointing wage dynamics.

Figure 3.5 also shows falling average consumption which can be surprising at first sight, given roughly unchanged aggregate savings and higher output. The reason is that higher vacancy posting costs crowd out consumption, despite a rise in production.[22] This is also behind the fact that firm dividends actually fall despite higher revenues and a lower wage bill (resulting in the earlier discussed leftward shift in the asset supply curve). Essentially, the firm reinvests the higher surplus coming from increased profitability into future jobs by posting more vacancies.

3.3.4 Sensitivity analysis

By introducing a non-linear production technology $\alpha > 0$ we are also able to capture falling productivity following the credit crunch. Decreasing returns to scale in the production function can be interpreted as a modelling shortcut to persistent damage of the supply side of the economy due to a prolonged period of stagnation. In this case lower productivity also results in lower steady state wages (see Figure 3.6).

3.4 Conclusion

In this paper we have developed a continuous time heterogeneous agent model with search-and-matching frictions in the labor market (HACT+SAM) and analysed the effects of a credit crunch. The combination of labor market frictions and precautionary motives (stemming from incomplete financial markets and endogenous idiosyncratic risk) provides a channel to explain the weak post-crisis wage recovery against the backdrop of tightening labor markets and falling unemployment. As the borrowing constraint tightens for households, they increase their self-insurance attemps: apart from precautionary savings this can be achieved by accepting employment even at lower wages. Allowing for decreasing returns to scale in production we can also capture the lacklustre productivity

[22]In the current model setup there is another reason: in the baseline parametrization, there is no government. This means that instead of unemployment benefits from the government, unemployed households engage in home production which falls as more of them get a job, thereby subtracting from private output. In addition, interest payments on bonds B do not come from the government (financed through taxes) but from abroad, earned on foreign assets. The fall in the interest rate therefore permits a smaller trade deficit $r_t B$, or equivalently, the improving current account also contributes toward crowding out consumption. The reason for ommiting government is that introducing involves some numerical difficulties, but this does not change the main message of our model. The resource constraint is now: $C_t + \xi v_t - r_t B = Y_t + hu_t$

Figure 3.6: Impulse responses after a tightening of the borrowing constraint – labor market

performance.

Our model ignores nominal rigidities and therefore is unable to capture aggregate demand effects which are undoubtedly an important part of business cycle fluctuations in general, and credit crunch scenarios in particular. Nevertheless, our paper points out a channel which can contribute to a deeper understanding of post-crisis wage dynamics and can optentially explain the weaker positive co-movement of wages and aggregate demand observed in the data. That said, introducing nominal rigidities is high on our research agenda, as well as accounting for an endogenous labor force participation choice.

References

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. 2017. “Income And Wealth Distribution In Macroeconomics: A Continuous-Time Approach.” *Princeton, mimeo.*

Ahn, SeHyoun, Greg Kaplan, Benjamin Moll, Thomas Winberry, and Christian Wolf. 2017. “When Inequality Matters for Macro and Macro Matters for Inequality.” *NBER Macroeconomics Annual.*

Bardóczy, Bence. 2017. “Labor-Market Matching with Precautionary Savings.” *mimeo*, 1–16.

Eggertsson, Gauti B., and Paul Krugman. 2012. “Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach.” *Quarterly Journal of Economics*, 127(3): 1469–1513.

Gornemann, Nils, Keith Kuester, and Makoto Nakajima. 2012. “Monetary Policy with Heterogeneous Agents.” *Federal Reserve Bank of Philadelphia Working Papers*, 12(21): 1–48.

Guerrieri, Veronica, and Guido Lorenzoni. 2017. “Credit Crises, Precautionary Savings, and the Liquidity Trap.” *Quarterly Journal of Economics*, 1427–1467.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante. 2017. “Monetary Policy According to HANK.” *Working Paper*, 1–61.

Krusell, Per, Toshihiko Mukoyama, and Aysegül Sahin. 2010. “Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations.” *Review of Economic Studies*, 77: 1477–1507.

Mellior, Gustavo. 2016. “Credit crunch on a Huggett-Poisson-HACT economy.”

Ravn, Morten O, and Vincent Sterk. 2018. “Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach.” *manuscript, UCL.*

Reiter, Michael. 2009. “Solving heterogeneous-agent models by projection and perturbation.” *Journal of Economic Dynamics and Control*, 33(3): 649–665.

Appendix C

C.1 Numerical algorithm

Based in large part on Bardóczy (2017), but without capital (and iterating instead over the interest rate). Details of the upwinding scheme and finite difference method used in HACT models are described by Achdou et al. (2017), with special attention to their Online Appendix. The credit crunch (tightening of the borrowing constraint) is incorporated into the upwinding scheme similarly as suggested by Mellior (2016).

C.1.1 Stationary equilibrium

Set up a discrete grid for $a \in \{a_i\}_{i=1}^{I}$

1. start iterating over $\ell = 1, 2, \ldots$ – outer loop

2. guess tightness θ^1 – update $\theta^{\ell+1}$ based on (3.14) in step 8

   using the matching function $M(u, v) = \chi u^\eta v^{1-\eta}$, and the definition $\theta = \frac{v}{u}$

$$\lambda^f = \chi\theta^{-\eta} \tag{3.1}$$

$$\lambda^w = \chi\theta^{1-\eta} \tag{3.2}$$

$$u = \frac{\sigma}{\sigma + \lambda^w} \tag{3.3}$$

$$v = \theta\, u$$

3. guess the interest rate $r^1 = 0.9\,\rho$ – update $r^{\ell+1}$ based on (3.19) in step 9 [1]

$$T = r\,B + u\,h \tag{3.18}$$

[1] ρ is the complete market interest rate. With incomplete markets there will be extra precautionary saving motive, and the incomplete market interest rate will be depressed downwards.

4. guess wage schedule $\omega^1(a_i) = \omega_i = \beta[z(1-u)^{-\alpha}]$ – update $\omega^{\ell+1}(a_i)$ based on (3.17) in step 10

$$y_{is} = \begin{cases} \omega_i - T & \text{if } s = s^e \\ h - T = (1-u)h - rB & \text{if } s = s^u \end{cases}$$

$$\pi(a_i) = \pi_i = z(1-u)^{-\alpha} - \omega_i \tag{3.15}$$

5. solve the worker's problem – first inner loop over $\imath$
   (a) iterations $\imath = 1, 2, \ldots$ over the worker's HJB: guess $\mathbf{W}^1 = \left\{\frac{u\left(y_{is}+r\,a_i\right)}{\rho}\right\}_{\forall is}$ – update $W^{\imath+1}$ at the end of step 5c)
   (b) discretize the HJB equation (3.6), (3.7) over the state space $\{a_i\} \times \{s^e, s^u\}$ by the finite difference method and the upwinding scheme (also using (3.8), (3.9)). The state constraints are appended depending on how tighter the new borrowing constraint is than the original one ($x \geq 1$) – build the $\mathbf{A}^\imath$ matrix[2]

$$\partial_a W_{is}^F \equiv \begin{cases} u'(y_{Is} + r\,a_I) & \text{for } i = I \\ \frac{W_{i+1,s} - W_{is}}{\Delta a} & \text{otherwise} \end{cases} \qquad \partial_a W_{is}^B \equiv \begin{cases} \frac{W_{is} - W_{i-1,s}}{\Delta a} & \text{otherwise} \\ u'(y_{is} + r\,a_i - (x-i)\Delta a) & \text{for } i \leq x \end{cases}$$

$$c_{is}^F = u'^{-1}\left(\partial_a W_{is}^F\right) \qquad \dot{a}_{is}^F = y_{is} + r\,a_i - c_{is}^F$$
$$c_{is}^B = u'^{-1}\left(\partial_a W_{is}^B\right) \qquad \dot{a}_{is}^B = y_{is} + r\,a_i - c_{is}^B$$
$$c_{is}^0 = y_{is} + r\,a_i - (x - \min\{i, x\})\Delta a$$

$$c_{is} = \mathbb{I}_{\{0<\dot{a}_{is}^F\}}\, c_{is}^F + \mathbb{I}_{\{\dot{a}_{is}^B<0\}}\, c_{is}^B + \mathbb{I}_{\{\dot{a}_{is}^F\leq 0\leq \dot{a}_{is}^B\}}\, c_{is}^0$$
$$\dot{a}_{is} = y_{is} + r\,a_i - c_{is}$$
$$\rho W_{is} = u(c_{is}) + \frac{W_{i+1,s} - W_{is}}{\Delta a}\,[\dot{a}_{is}]^+ + \frac{W_{is} - W_{i-1,s}}{\Delta a}\,[\dot{a}_{is}]^- + \sum_{s'\neq s} \lambda_{ss'}\left[W_{is'} - W_{is}\right]$$
$$\rho W_{is} = u(c_{is}) - \frac{[\dot{a}_{is}]^-}{\Delta a} W_{i-1,s} + \left(\frac{[\dot{a}_{is}]^- - [\dot{a}_{is}]^+}{\Delta a}\right) W_{is} + \frac{[\dot{a}_{is}]^+}{\Delta a} W_{i+1,s} + \sum_{s'\neq s} \lambda_{ss'}\left[W_{is'} - W_{is}\right]$$
$$\rho\,\mathbf{W}^\imath = \mathbf{u}(\mathbf{W}^\imath) + \mathbf{A}(\mathbf{W}^\imath;\,\mathbf{r})\,\mathbf{W}^\imath$$

[2] The terms in the $\mathbf{A}$ matrix are defined as $[\dot{a}_{is}]^+ = \max\{\dot{a}_{is}, 0\}$ and $[\dot{a}_{is}]^- = \min\{\dot{a}_{is}, 0\}$. This is in contrast to Mellior (2016) and Achdou et al. (2017) where it is defined as $[\dot{a}_{is}]^+ = \max\{\dot{a}_{is}^F, 0\}$. This difference does not matter for $x = 1$ (no tightening in the borrowing constraint), but it does for any other $x > 1$. Under the definition of Mellior (2016) the $\mathbf{A_s}$ matrix would only contain zeros in the rows corresponding to the inadmissible region $i < x$ whenever $\dot{a}_{is}^F < 0 < \dot{a}_{is}^B = (x-i)\Delta a$ while there we require positive actual savings $\dot{a}_{is} > 0$.

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_e} & \mathbf{0} \\ \mathbf{0} & \mathbf{A_u} \end{bmatrix} + \begin{bmatrix} -\sigma\,\mathbf{I} & \sigma\,\mathbf{I} \\ \lambda^w\,\mathbf{I} & -\lambda^w\,\mathbf{I} \end{bmatrix}; \quad \mathbf{A_s} = \begin{bmatrix} \frac{\dot{a}_{1s}^- - \dot{a}_{1s}^+}{\Delta a} & \frac{\dot{a}_{1s}^+}{\Delta a} & 0 & 0 & \cdots & 0 \\ -\frac{\dot{a}_{2s}^-}{\Delta a} & \frac{\dot{a}_{2s}^- - \dot{a}_{2s}^+}{\Delta a} & \frac{\dot{a}_{2s}^+}{\Delta a} & 0 & \cdots & 0 \\ 0 & -\frac{\dot{a}_{3s}^-}{\Delta a} & \frac{\dot{a}_{3s}^- - \dot{a}_{3s}^+}{\Delta a} & \frac{\dot{a}_{3s}^+}{\Delta a} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & -\frac{\dot{a}_{Is}^-}{\Delta a} & \frac{\dot{a}_{Is}^- - \dot{a}_{Is}^+}{\Delta a} \end{bmatrix}$$

(c) update the value function $\mathbf{W}^{\imath+1}$ using the implicit method, and go back to step 5b)

$$\frac{\mathbf{W}^{\imath+1} - \mathbf{W}^{\imath}}{\Delta} + \rho\,\mathbf{W}^{\imath+1} = \mathbf{u}^{\imath} + \mathbf{A}^{\imath}\,\mathbf{W}^{\imath+1}$$
$$\mathbf{W}^{\imath+1} = \left[\left(\frac{1}{\Delta} + \rho\right)\mathbf{I} - \mathbf{A}^{\imath}\right]^{-1}\left(\mathbf{u}^{\imath} + \frac{1}{\Delta}\mathbf{W}^{\imath}\right)$$

(d) after convergence save optimal value funtion, and consumption and savings policies (and keep the final $\mathbf{A}$ matrix):

$$\mathbf{W} = \mathbf{W}^{\imath} \qquad c_{is} = c_{is}^{\imath} \qquad \dot{a}_{is} = \dot{a}_{is}^{\imath}$$

6. calculate stationary distribution

it is basically an eigenvalue problem (imposing $\sum_s \sum_i g_{is} = 1$), solving the discretized KF equation which involves the same $\mathbf{A}$ matrix as the discretized HJB equation

$$(3.10), (3.11) \qquad 0 = \mathbf{A}^T\mathbf{g}$$

7. solve the firm's problem – second inner loop over $\jmath$

(a) discretize the job HJB equation (3.13) over the state space $\{a_i\}_{i=1}^{I}$ by the finite difference method and the upwinding scheme – ending up with the <u>*already calculated*</u> $\mathbf{A_e}$ matrix (saving is exogenous to the firm and we already applied the upwinding scheme for the household)

$$(3.13) \qquad (\sigma + r)\,J_i = \pi_i + \frac{J_{i+1} - J_i}{\Delta a}\,[\dot{a}_{ie}]^+ + \frac{J_i - J_{i-1}}{\Delta a}\,[\dot{a}_{ie}]^-$$
$$(\sigma + r)\,J_i = \pi_i - \frac{[\dot{a}_{ie}]^-}{\Delta a}J_{i-1} + \left(\frac{[\dot{a}_{ie}]^- - [\dot{a}_{ie}]^+}{\Delta a}\right)J_i + \frac{[\dot{a}_{ie}]^+}{\Delta a}J_{i+1}$$
$$(\sigma + r)\,\mathbf{J} = \pi + \mathbf{A_e}\,\mathbf{J}$$

(b) iterations $\jmath = 1, 2, \ldots$ over the firm's job HJB: guess $\mathbf{J}^1 = \left\{\frac{\pi(a_i)}{\sigma + r}\right\}_{\forall i}$ – update $J^{\imath+1}$ at the end of step 7c)

(c) update $\mathbf{J}^{\jmath+1}$ according to the implicit method, and go back to step 7b)

$$\frac{\mathbf{J}^{\jmath+1} - \mathbf{J}^{\jmath}}{\Delta} + (\sigma + r)\mathbf{J}^{\jmath+1} = \pi^{\jmath} + \mathbf{A_e}\,\mathbf{J}^{\jmath+1}$$
$$\mathbf{J}^{\jmath+1} = \left[\left(\frac{1}{\Delta} + \sigma + r\right)\mathbf{I} - \mathbf{A_e}\right]^{-1}\left(\pi^{\jmath} + \frac{1}{\Delta}\mathbf{J}^{\jmath}\right)$$

(d) after convergence save the optimal value function $\mathbf{J} = \mathbf{J}^{\jmath}$

8. evaluate free entry condition (vacancy decision)

$$FE = -\xi + \lambda^f \, \mathbf{J}^T \, \frac{\mathbf{g_u}}{u} \Delta a \tag{3.14}$$

with $FE > 0$ firms are too profitable, so update with a higher guess for labor market tightness (in the outer loop)

$$\theta^{\ell+1} = \theta^{\ell} + \Delta\theta \cdot FE^{\ell}$$

9. check asset market clearing

$$d = \pi^T \, \mathbf{g_e} \, \Delta a - \xi v \tag{3.16}$$

$$p = \frac{d}{r} \tag{3.5}$$

$$AD = \mathbf{a}^T [\mathbf{g_e} + \mathbf{g_u}] \, \Delta a - B - p \tag{3.19}$$

with $AD > 0$ there is an excess demand for assets (equivalently, too much saving), so the interest rate should be decreased. Update with bisection:

$$r^{\ell+1} = \begin{cases} \frac{r^{\ell}+r^{\ell}_{min}}{2} & \text{if } AD^{\ell} > 0 \\ \frac{r^{\ell}+r^{\ell}_{max}}{2} & \text{if } AD^{\ell} < 0 \\ r^{\ell} & \text{if } AD^{\ell} \approx 0 \end{cases} \qquad r^{\ell+1}_{min} = \begin{cases} r^{\ell}_{min} & \text{if } AD^{\ell} > 0 \\ r^{\ell} & \text{if } AD^{\ell} < 0 \end{cases} \qquad r^{\ell+1}_{max} = \begin{cases} r^{\ell} & \text{if } AD^{\ell} > 0 \\ r^{\ell}_{max} & \text{if } AD^{\ell} < 0 \end{cases}$$

10. update the wage schedule $\omega^{\ell+1}(a_i)$ from the bargaining equation (3.17)

11. use the updates
    - $\theta^{\ell+1}$ based on step 8,
    - $r^{\ell+1}$ based on step 9,
    - $\omega^{\ell+1}(a_i)$ based on step 10,

    and go back to step 2 – stop if both AD and FE are small enough

C.1.2 Transition dynamics

- embed the iterations for the HJB and KFE equations along the time path n into an outer loop ℓ over a triplet of guesses $(r_n, \theta_n, \hat{p}_1)$ $\forall n$
  - iterate HJB equations *backward* in time
  - iterate KFE equations *forward* in time (previously did not iterate)
  - keep all steps, as they are now correspond to the time path
- equity price on impact $\hat{p}_1$ needs to be guessed as the unexpected shock changes future profits and revalues the assets

1. discretize the state space
   - the employment state is already discrete $s \in \{s^e, s^u\}$
   - asset grid $a \in \{a_i\}_{i=1}^{I}$ with Δa
   - time grid $t \in \{t_n\}_{n=1}^{N}$ with $\{\Delta t_n\}_{n=1}^{N-1}$ non-uniform steps
2. set initial and terminal conditions $\mathbf{g_1}, \mathbf{W_N}, \mathbf{J_N}, \omega_N(a_i)$ (e.g. potentially different stationary equilibria)
3. specify exogenous time path for TFP process

$$z_n = 1 + (z_0 - 1)\, e^{-\nu t_n}$$

4. start iterating over $\ell = 1, 2, \ldots$ – outer loop
5. guess time path $\forall n$ for tightness θ_n^1 – update $\theta_n^{\ell+1}$ from the final step

$$\lambda_n^f = \chi \theta_n^{-\eta}$$
$$\lambda_n^w = \chi \theta_n^{1-\eta}$$

unemployment is recovered by iterating *forward* on the differential equation (3.3). Both implicit and explicit iterations work but explicit is more stable (and maintains no change on impact for unemployment, which is a state variable).

$$\text{expl:} \quad \frac{u_{n+1} - u_n}{\Delta t_n} = \sigma - (\lambda_n^w + \sigma) u_n \qquad u_{n+1} = \Delta t_n\, \sigma + \Big[1 - \Delta t_n\, (\lambda_n^w + \sigma)\Big] u_n$$

$$\text{impl:} \quad \frac{u_{n+1} - u_n}{\Delta t_n} = \sigma - (\lambda_{n+1}^w + \sigma) u_{n+1} \qquad u_{n+1} = \frac{u_n + \Delta t_n\, \sigma}{1 + \Delta t_n\, (\lambda_{n+1}^w + \sigma)}$$

6. guess time path $\forall n$ for the interest rate r_n^1 – update $r_n^{\ell+1}$ from the final step

$$T_n = r_n B + u_n\, h$$

7. solve the worker's <u>and</u> firm's problem *simultaneously* – first inner loop *backward* over n

(a) iterations $n = N, N-1, N-2, \ldots 1$: Instead of initial guesses, start from the terminal conditions $\mathbf{W}_N, \mathbf{J}_N,\ \omega_N(a_i)$ - and keep all steps! Not until convergence, but until reaching t_1.

$$y_{is}^n = \begin{cases} \omega_n(a_i) - T_n & \text{if } s = s^e \\ h - T_n = (1-u_n)h - r_n B & \text{if } s = s^u \end{cases}$$

$$\pi_n(a_i) = \pi_{i,n} = z_n(1-u_n)^{-\alpha} - \omega_n(a_i)$$

(b) from the worker's discretized time-dependent HJB equation build the time-dependent $\mathbf{A}_n$ matrix (for $n = N$ this should just give back the terminal $\mathbf{A}_N$)

$$\partial_a W_{is}^{F,n} \equiv \begin{cases} u'(y_{Is}^n + r^n\, a_I) & \text{for } i = I \\ \frac{W_{i+1,s}^n - W_{is}^n}{\Delta a} & \text{otherwise} \end{cases} \qquad \partial_a W_{is}^{B,n} \equiv \begin{cases} \frac{W_{is}^n - W_{i-1,s}^n}{\Delta a} & \text{otherwise} \\ u'(y_{is}^n + r^n\, a_i) & \text{for } i = x \\ u'(y_{is}^n + r^n\, a_i - \Delta a) & \text{for } i < x \end{cases}$$

$$c_{is}^{F,n} = u'^{-1}\left(\partial_a W_{is}^{F,n}\right) \qquad \dot{a}_{is}^{F,n} = y_{is}^n + r^n\, a_i - c_{is}^{F,n}$$

$$c_{is}^{B,n} = u'^{-1}\left(\partial_a W_{is}^{B,n}\right) \qquad \dot{a}_{is}^{B,n} = y_{is}^n + r^n\, a_i - c_{is}^{B,n}$$

$$c_{is}^{0,n} = y_{is}^n + r^n\, a_i - \mathbb{I}_{\{i<x\}} \Delta a$$

$$c_{is}^n = \mathbb{I}_{\{0<\dot{a}_{is}^{F,n}\}}\, c_{is}^{F,n} + \mathbb{I}_{\{\dot{a}_{is}^{B,n}<0\}}\, c_{is}^{B,n} + \mathbb{I}_{\{\dot{a}_{is}^{F,n} \le 0 \le \dot{a}_{is}^{B,n}\}}\, c_{is}^{0,n}$$

$$\dot{a}_{is}^n = y_{is}^n + r^n\, a_i - c_{is}^n$$

$$\rho W_{is}^n = u(c_{is}^n) + \frac{W_{i+1,s}^n - W_{is}^n}{\Delta a}\, [\dot{a}_{is}^n]^+ + \frac{W_{is}^n - W_{i-1,s}^n}{\Delta a}\, [\dot{a}_{is}^n]^- + \sum_{s' \neq s} \lambda_{ss'}^n \left[W_{is'}^n - W_{is}^n\right] + \frac{W_{is}^{n+1} - W_{is}^n}{\Delta t_n}$$

$$\rho W_{is}^n = u(c_{is}^n) - \frac{[\dot{a}_{is}^n]^-}{\Delta a} W_{i-1,s}^n + \left(\frac{[\dot{a}_{is}^n]^- - [\dot{a}_{is}^n]^+}{\Delta a}\right) W_{is}^n + \frac{[\dot{a}_{is}^n]^+}{\Delta a} W_{i+1,s}^n + \sum_{s' \neq s} \lambda_{ss'}^n \left[W_{is'}^n - W_{is}^n\right] + \frac{W_i}{}$$

$$\rho\, \mathbf{W}_n = \mathbf{u}(\mathbf{W_n}) + \mathbf{A}(\mathbf{W_n};\, \mathbf{r_n})\, \mathbf{W}_n + \frac{\mathbf{W}_{n+1} - \mathbf{W}_n}{\Delta t_n}$$

$$\mathbf{A}_n = \begin{bmatrix} \mathbf{A_{e,n}} & \mathbf{0} \\ \mathbf{0} & \mathbf{A_{u,n}} \end{bmatrix} + \begin{bmatrix} -\sigma\, \mathbf{I} & \sigma\, \mathbf{I} \\ \lambda_n^w\, \mathbf{I} & -\lambda_n^w\, \mathbf{I} \end{bmatrix};$$

$$\mathbf{A_{s,n}} = \begin{bmatrix} \frac{[\dot{a}_{1s}^n]^- - [\dot{a}_{1s}^n]^+}{\Delta a} & \frac{[\dot{a}_{1s}^n]^+}{\Delta a} & 0 & 0 & \cdots & 0 \\ -\frac{[\dot{a}_{2s}^n]^-}{\Delta a} & \frac{[\dot{a}_{2s}^n]^- - [\dot{a}_{2s}^n]^+}{\Delta a} & \frac{[\dot{a}_{2s}^n]^+}{\Delta a} & 0 & \cdots & 0 \\ 0 & -\frac{[\dot{a}_{3s}^n]^-}{\Delta a} & \frac{[\dot{a}_{3s}^n]^- - [\dot{a}_{3s}^n]^+}{\Delta a} & \frac{[\dot{a}_{3s}^n]^+}{\Delta a} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & -\frac{[\dot{a}_{Is}^n]^-}{\Delta a} & \frac{[\dot{a}_{Is}^n]^- - [\dot{a}_{Is}^n]^+}{\Delta a} \end{bmatrix}$$

(c) calculate the emphprevious period's value function $\mathbf{W}_{n-1}$ from the worker's HJB equations, using the implicit method

$$\rho \mathbf{W}_{n-1} = \mathbf{u}_n + \mathbf{A}_n \mathbf{W}_{n-1} + \frac{\mathbf{W}_n - \mathbf{W}_{n-1}}{\Delta t_{n-1}}$$
$$\left[\left(\frac{1}{\Delta t_{n-1}} + \rho\right)\mathbf{I} - \mathbf{A}_n\right]\mathbf{W}_{n-1} = \mathbf{u}_n + \frac{1}{\Delta t_{n-1}}\mathbf{W}_n$$

(d) calculate the previous period's value function $\mathbf{J}_{n-1}$ from the firm's HJB equations, using the implicit method

$$(\sigma + r_n)\mathbf{J}_{n-1} = \pi_n + \mathbf{A}_{\mathbf{e},\mathbf{n}} \mathbf{J}_{n-1} + \frac{\mathbf{J}_n - \mathbf{J}_{n-1}}{\Delta t_{n-1}}$$
$$\left[\left(\frac{1}{\Delta t_{n-1}} + \sigma + r_n\right)\mathbf{I} - \mathbf{A}_{\mathbf{e},\mathbf{n}}\right]\mathbf{J}_{n-1} = \pi_n + \frac{1}{\Delta t_{n-1}}\mathbf{J}_n$$

(e) given the worker and firm value functions $\mathbf{W}_{n-1}, \mathbf{J}_{n-1}$, calculate the previous period's wage schedule $\omega_{n-1}(a_i)$ from the bargaining equation (Nash or egalitarian)

(f) at each step n (!) keep the original value functions and optimal policies as well as the wage schedule, the profit and the $\mathbf{A}$ matrix

$$\mathbf{W}_n \quad \mathbf{J}_n \quad \omega_n(a_i) \quad c_{is}^n \quad \dot{a}_{is}^n \quad \pi_n(a_i) \quad \mathbf{A}_n$$

(g) go back to step 7a) with the freshly calculated $\mathbf{W}_{n-1}$ $\mathbf{J}_{n-1}$ $\omega_{n-1}(a_i)$ and start the $n-1$-st step, until reaching $n = 1$

8. guess impact equity price $\hat{p}_1^1$ to revalue assets – update $p_1^{\ell+1}$ from the final step

- on impact p_1 from the initial condition jumps to $\hat{p}_1$ as a result of the shock – due to change to the firm's future profits
- each agent sees their assets revalued on impact, without being able to do anything

$$\hat{a}_i = a_i\left[1 + \frac{\hat{p}_1 - p_1}{B + p_1}\right] \qquad \hat{g}_1(\hat{a}_i, s) = g_1(a_i, s)$$

  so the distribution shifts to a rescaled grid, but stays the same

- we want to work with the original grid $\{a_i\}$, so we interpolate the new distribution onto the original grid

$$\hat{g}_1(a_i, s) = \texttt{pchip}\Big(\hat{a}_i, \underbrace{\hat{g}_1(\hat{a}_i, s)}_{g_1(a_i,s)};\ a_i\Big)$$

- $\hat{\mathbf{g}}_1$ needs to integrate to one, so some rescaling might be needed

9. calculate time path for the distribution – second inner loop *forward* over n

(a) iterations $n = 1, 2, 3, \ldots N-1$: start from the <u>*revalued* initial</u> distribution $\hat{\mathbf{g}}_1$ - and keep all steps! Not until convergence, but until reaching t_N.

(b) calculate next period's distribution $\mathbf{g}_{n+1}$ from the discretized KF equation, involving the same $\mathbf{A}_n$ matrix (iterating using the implicit method)

$$\frac{\mathbf{g}_{n+1} - \mathbf{g}_n}{\Delta t_n} = \mathbf{A}_n^T \, \mathbf{g}_{n+1}$$
$$\mathbf{g}_{n+1} = \left[\mathbf{I} - \Delta t_n \, \mathbf{A}_n^T\right]^{-1} \mathbf{g}_n$$

(c) at each step keep the original distribution $\mathbf{g}_n$ and go back to step 9a) with the freshly calculated $\mathbf{g}_{n+1}$ and start the $n+1$-st step, until reaching $n = N$

10. calculate time path for vacancies (and check unemployment!)

$$u_n = \sum_{i=1}^{I} \sum_{s=s^u} g_{is}^n \, \Delta a$$
$$v_n = \theta_n \, u_n$$

we already have u_n from iterating on the law of motion (3.3) from step 5. But for large $\{\Delta t_n\}$ integrating from the distribution instead is more precise, so unless $\alpha \neq 0$, it is better to use this approach.

11. evaluate free entry condition

$$FE_n = -\xi + \lambda_n^f \, \mathbf{J}_n^T \, \frac{\mathbf{g_u}_n}{u_n} \, \Delta a$$

12. check asset market clearing

we also need the time path for the equity price (iterated implicitly backwards from terminal p_N based on no arbitrage $p_n \, r_n = d_n + \frac{p_{n+1} - p_n}{\Delta t_n}$) – this will lead to a $\tilde{p}_1$

$$d_n = \pi_n^T \, \mathbf{g_e}_n \, \Delta a - \xi \, v_n$$
$$p_{n-1} = \left[\frac{1}{\Delta t_{n-1}} + r_{n-1}\right]^{-1} \left[d_{n-1} + \frac{p_n}{\Delta t_{n-1}}\right]$$
$$AD_n = \mathbf{a}^T \left[\mathbf{g_e}_n + \mathbf{g_u}_n\right] \Delta a - B - p_n$$

13. use the updates

with decreasing sequences $\{\Delta_{rn}, \Delta_{\theta n}\}_n$ to help convergence

$$\hat{p}_1^{\ell+1} = \Delta_p \, \tilde{p}_1^{\ell} + (1 - \Delta_p) \, \hat{p}_1^{\ell}$$
$$r_n^{\ell+1} = r_n^{\ell} - \Delta_{rn} \cdot \Delta AD_n^{\ell} \qquad \forall n$$
$$\theta_n^{\ell+1} = \theta_n^{\ell} + \Delta_{\theta n} \cdot FE_n^{\ell} \qquad \forall n$$

where ΔAD_n^{ℓ} is alternating forward or backward difference.

Then go back to step 5 – stop if all elements in the triplet $\left[\theta_n, r_n, \hat{p}_1,\right]$ have converged

C.2 Additional figures

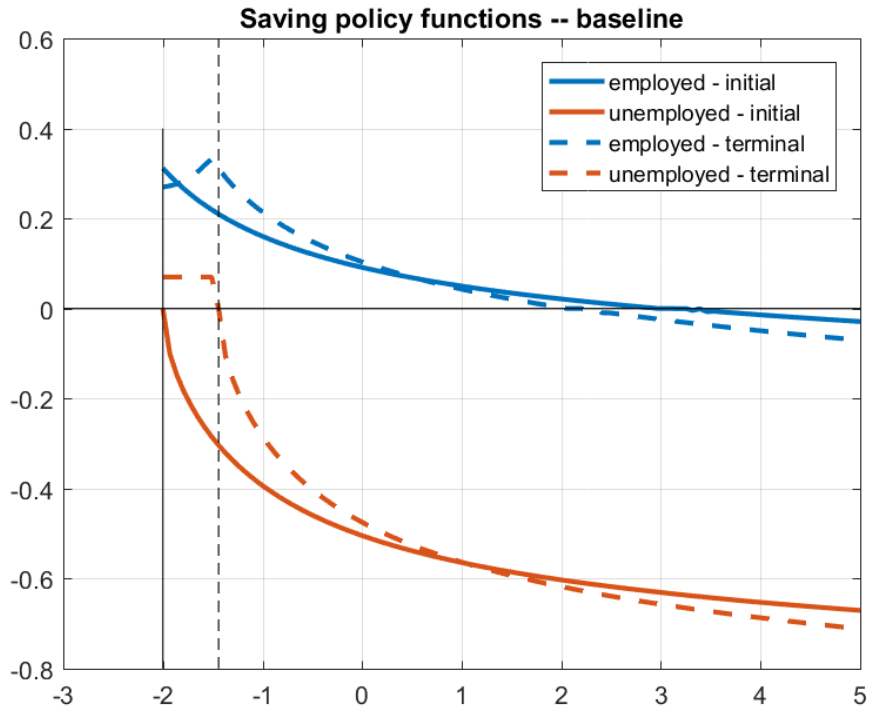


Figure C.1: Savings policy functions

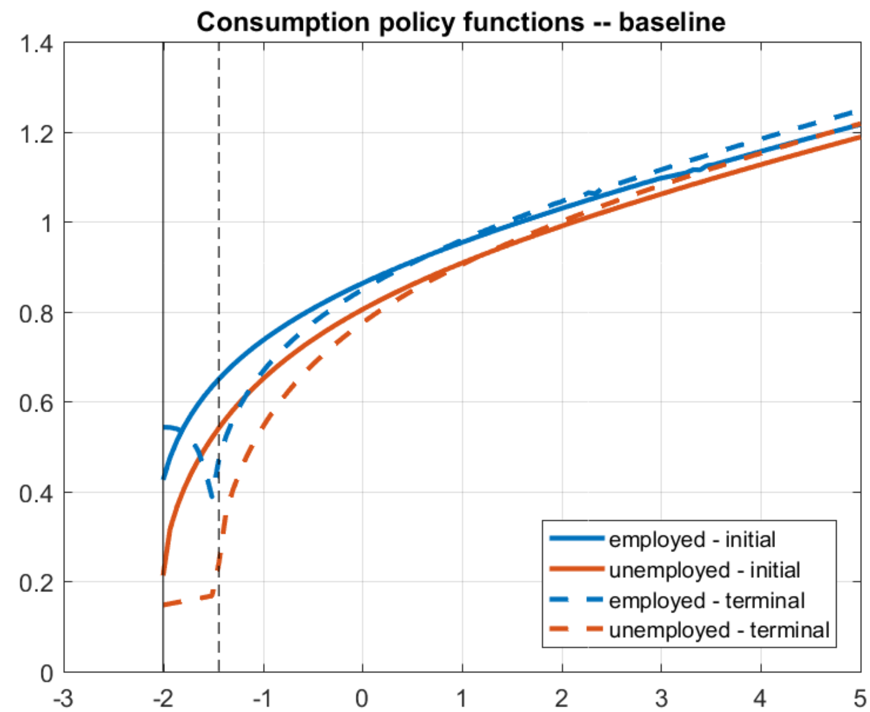


Figure C.2: Consumption policy functions