The Ex Post Accuracy of Subjective Beliefs: A New Measure and Decomposition

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Abstract

The choices, and hence outcomes, of students, consumers, or investors depend on their beliefs. Inaccurate beliefs lead to poor choices and undesirable outcomes. Recent attempts to elicit probabilistic expectations that capture the full distribution describing agents' beliefs provide an opportunity to study those beliefs. We propose a novel but intuitive measure of the average ex post accuracy of a group of economic agents' beliefs, and show how a decomposition of that measure captures different sources of inaccuracy: failure of rational expectations, subjective uncertainty, and aggregate shocks. We illustrate with application to the income expectations of college students and recent graduate in the BEREA panel survey.

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1 Introduction

The choices, and hence outcomes, of students, consumers, or investors depend on their beliefs. Inaccurate expectations lead to poor choices and undesirable outcomes. Probabilistic expectations data, which have recently become increasingly common, provide for the first time an opportunity to ask: how accurate are economic agents' actual beliefs?

Ex post, beliefs can be inaccurate for a number of reasons. Agents' beliefs may not be rational (in the formal sense of the Rational Expectations Hypothesis, see D'Haultfoeuille, Gaillac, and Maurel, forthcoming; Crossley et al., 2021). Alternatively, beliefs may be rational, but formed in a challenging environment. Economic agents may have little relevant information about future events, so that they experience considerable subjective uncertainty. Further, a population of agents with Rational Expectations may make large *average* errors if their outcomes are not independent (for example, if aggregate shocks are present.)

In this paper we propose a novel but intuitive measure of the average ex post accuracy of a group of economic agents' beliefs. This measure, which can be implemented on probabilistic expectations data, captures the multiple sources of ex post inaccuracy outlined above. In the next section of the paper we develop our measure of ex post accuracy of beliefs and show that it has a natural decomposition. We note how different components of the decomposition relate to different sources of inaccuracy, showing how it relates to rational expectations, to subjective uncertainty, and to dependence arising from aggregate shocks.

While previous research has focused primarily on ex ante rationality, differentiating between the broader set of inaccuracy sources that are present in the ex post accuracy measure is of importance for both research and policy. For example, if young people tend to have rational beliefs about future income but with considerable subjective uncertainty, then policy may seek to reveal additional information about factors, such as income, that create uncertainty when they are not completely observed. On the other hand, if beliefs tend to be irrational, then policy might seek to help agents use the information they have more effectively, for example by providing guidance about how known factors such as educational attainment or college major influence the distribution of future income.

We illustrate our proposed measures with application to the income expectations of college students and recent graduates in the Berea Panel Study (BPS).

Our proposed measure of ex post accuracy contributes to a growing literature on the use of probabilistic expectations data to study economic behaviour (see Manski, 2004; Delavande, 2008; Van der Klaauw, 2012; Manski, 2018). Our application of that measure to students' and young adults' beliefs about future income is particularly relevant to the literature on students' beliefs and expectations (for example Zafar, 2011; Arcidiacono, Hotz, and Kang, 2012; Stinebrickner and Stinebrickner, 2012), and on income expectations and subjective uncertainty about future income (for example Dominitz and Manski, 1996, 1997b,a; Guiso, Jappelli, and Terlizzese, 1992; Das and Donkers, 1999; Gong, Stinebrickner, and Stinebrickner, 2020).

2 A Measure and Decomposition of Ex Post Inaccuracy

Let Y_i denote the random variable associated with an outcome of interest. For each individual i in a sample, the realization of Y_i is denoted y_i. At an earlier time t, each individual i reports information characterizing the distribution describing her beliefs about y_i. We let Y_{it}^B denote the random variable whose distribution describes the subjective beliefs of individual i at time t about y_i.

Our interest is in examining the ex post accuracy of beliefs, in other words, some measure of how beliefs deviate from realized outcomes. For individual i, we propose the measure of ex post inaccuracy at time t, denoted Δ_{it}^B, given by:

$$\begin{aligned}
\Delta_{it}^B &= E_{Y_{it}^B}[(Y_{it}^B - y_i)^2] \qquad (1)\\
&= E_{Y_{it}^B}[((Y_{it}^B - \mu_{it}^B) - (\mu_{it}^B - y_i))^2]\\
&= E_{Y_{it}^B}[((Y_{it}^B - \mu_{it}^B)^2 + (\mu_{it}^B - y_i)^2 - 2(Y_{it}^B - \mu_{it}^B)(\mu_{it}^B - y_i))]\\
&= var_{Y_{it}^B}[Y_{it}^B] + (\mu_{it}^B - y_i)^2,
\end{aligned}$$

where $\mu_{it}^B = E_{Y_{it}^B}(Y_{it}^B)$. Δ_{it}^B is the belief-weighted mean squared ex post error in individual i's beliefs. The last line of Equation (1) shows that this measure can be written as the sum of individual i's uncertainty (measured by the individual variance of Y_{it}^B) and her squared ex post error (relative to her mean belief). Two points bear noting. First, a feature of this measure seen in the last line of 1 is that even if $\mu_{it}^B = y_i$ (the agent's mean belief is exactly realized), $\Delta_{it}^B > 0$ because of subjective uncertainty $var_{Y_{it}^B}[Y_{it}^B]$. Second, we are integrating over the *subjective* distribution of beliefs.

Aggregating Δ_{it}^B over i yields an aggregate measure of ex post inaccuracy of beliefs at time t, which we denote Δ_t^B:

$$\Delta_t^B = E_i\{E_{Y_{it}^B}[(Y_{it}^B - y_i)^2]\}. \tag{2}$$

Throughout, we use an i subscript whenever we take the expectation across individuals, and use the corresponding individual-specific random variable as the subscript whenever we take the expectation over the distribution of that random variable for an individual.

A better understanding of Δ_t^B comes from inserting Equation (1) into Equation (2):

$$\begin{aligned}\Delta_t^B &= E_i\{E_{Y_{it}^B}[(Y_{it}^B - y_i)^2]\} \\ &= E_i[var_{Y_{it}^B}[Y_{it}^B]] + E_i[(\mu_{it}^B - y_i)^2].\end{aligned} \tag{3}$$

In the usual mean squared error way, the second term can be written as the sum of a *cross-sectional* variance and a squared aggregate error:

$$\Delta_t^B = \underbrace{E_i[var(Y_{it}^B)]}_{\delta_{t,1}^B} + \underbrace{var_i(y_i - \mu_{it}^B)}_{\delta_{t,2}^B} + \underbrace{(\bar{y} - \bar{\mu}_t)^2}_{\delta_{t,3}^B}, \tag{4}$$

where $\bar{\mu}_t = E_i(\mu_{it}^B)$, and $\bar{y} = E_i(y_i)$.

Equation (3) shows that our measure of ex post inaccuracy, Δ_t^B, consists of three components, each of which is of interest. The first component $\delta_{t,1}^B \equiv E_i[var(Y_{it}^B)]$ measures the average amount of subjective uncertainty about y_i at time t. The second component $\delta_{t,2}^B \equiv var_i(y_i - \mu_{it}^B)$ is the cross-sectional variance of innovations $(y_i - \mu_{it}^B)$ and measures the dispersion of relevant new information received after t. If individuals' realizations are i.i.d., this is the individuals' "actual" uncertainty about y_i at time t. The third component, the squared aggregate error, $\delta_{t,3}^B \equiv (\bar{y} - \bar{\mu}_t)^2$, captures the size of the systematic ex post prediction error which can arise because of non-rational expectations or aggregate shocks.

3 Two Applications

3.1 Data

The Berea Panel Study (BPS) is a multi-purpose longitudinal survey project, which collected detailed information from students at Berea College in Kentucky throughout college and after college to about age 30. About 720 students answered a baseline survey in 2000 or 2001, administered immediately before the beginning of a student's freshman year. All students in the two entering cohorts who graduated were eligible for the annual post-college surveys that were administered in May/June until 2013, providing up to 9 years of post-college data. Details of high survey participation rates and evidence that the BPS data are of high quality is provided in, for example, Stinebrickner and Stinebrickner (2012); Gong, Stinebrickner, and Stinebrickner (2019); Crossley et al. (2021).

The BPS was one of the first longitudinal studies to use probabilistic expectations questions to elicit, for each respondent, information about the full distribution describing beliefs (Stinebrickner and Stinebrickner, 2012). The study elicited each student's beliefs about future (post-college) income annually, and then subsequently also collected each student's actual realized earnings. The BPS is unique in allowing a comparison of probabilistic expectations and outcomes for an extended period. The survey questions eliciting beliefs about future income while respondents were in school are of the "percentile" format. The in-school survey questions elicited the minimum, the maximum, and the three quartiles of the distribution describing a student's beliefs about own annual income at three different ages (see the Online Appendix for exact question wordings). Below we focus on beliefs about own income at age 28.[1]

The survey questions eliciting post-college beliefs about future income differ in that they ask about family income and in that they use the "probability" format. The post-college survey questions elicited the perceived probability that a respondent's annual family income in the next year would fall in each of ten income categories and the perceived probability that a respondent's annual family income in five years would fall in each of ten income categories. As with the in-school expectations questions, respondents were instructed to take into account the possibility of changes in job situations. Given the family nature of the survey question, respondents were also asked to take into account the possibility that the number of workers in the family might change due to, for example, marriage and the birth of children.

Annual own and spousal income are constructed from a question that gave respondents flexibility over whether earnings were reported for an hourly, weekly, monthly, or yearly

[1] We exclude observations for which the mean of the belief distribution is more than three sample standard deviations away from the cross-sectional sample mean of this object. Similarly we exclude observations for which the standard deviation of the belief distribution is more than three sample standard deviations away from the cross-sectional sample mean of this object.

period. Annual family income is then constructed by adding up own and spousal income. All beliefs and realizations are deflated by the CPI to 2000 dollars. We exclude observations for which realized income is more than three sample deviations away from the cross-sectional sample mean of realized income.

For each pair of beliefs and realizations, we compute the inaccuracy measure for the students for whom both beliefs information and income realizations, y_i, are available. As detailed in the Online Appendix, fitting the elicited belief data to a step-wise uniform distribution yields values of $E(Y_{it}^B)$ and $var(Y_{it}^B)$ for each student. These objects along with realizations y_{it} allow us to calculate the three components of ex post inaccuracy measure, $\delta_{t,1}^B$, $\delta_{t,2}^B$, and $\delta_{t,3}^B$. Then, the inaccuracy measure Δ_t^B, can be obtained from Equation (3).

3.2 Application 1: In College Beliefs about Own Income at Age 28

In the first application, we examine how the ex post inaccuracy of college students' beliefs about their own income at age 28, Y_{it}^B, evolve during college. These beliefs were elicited annually from the time of entrance (10-years ahead of age 28) to the end of college (6-years ahead of age 28).

Figure 1 depicts the time pattern of the ex post inaccuracy measure and its three components. The blue line shows that Δ_t^B decreases, and therefore the accuracy of students' income beliefs improves, in a monotonic fashion during college. A further examination of the other lines indicates that this is also true for all three components of Δ_t^B. Between college entrance (10-year ahead) and the end of college (6-year ahead), the measure of aggregate subjective uncertainty, $\delta_{t,1}^B$, drops by about 40%, from 123.6 to 72.3. Similarly, we find that the measure of aggregate actual uncertainty, $\delta_{t,2}^B$, decreases during college by roughly 60%, from 442.9 at college entrance to 178.3 at the end of college. However, most striking is the purple line in Figure 1, which shows that the systematic bias about y_i, as characterized by $\delta_{t,3}^B$, falls by about 90% during college. At college entrance, students overestimate Y_i by $\sqrt{324.1} = 18.00$ thousand dollars. At the end of college, this overestimation is only $\sqrt{35} = 5.92$ thousand dollars.

There could be at least two forces that contribute to the substantial improvement of the accuracy of students' income beliefs during college. First, students can learn about income-influencing factors such as GPA, major, and labor market conditions during college. This could lead to a resolution of uncertainty (which corresponds to a decrease in $\delta^B_{t,1}$ and $\delta^B_{t,2}$). Moreover, as the time horizon becomes shorter, there would be fewer income-influencing factors about which the students could have biased beliefs, which results in a reduction in the squared mean bias, $\delta^B_{t,3}$. Second, over time, students might become better at using existing information to predict future income. As a result, the accuracy of students' income beliefs could improve, even conditional on students having the same information.

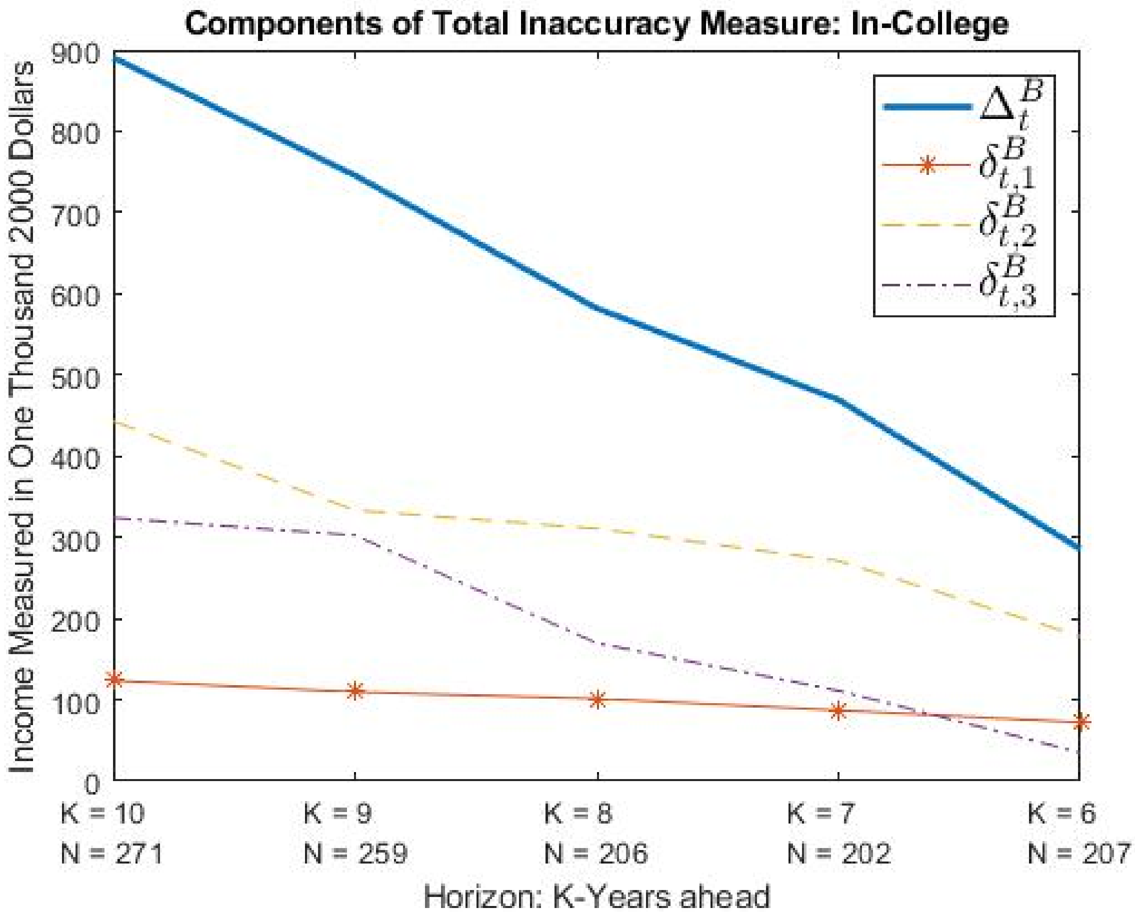

Figure 1: Decomposition of the Inaccuracy Measure: In-College

The sample size N is reported on the x-axis of the figure.

3.3 Application 2: Post College Beliefs about 1-Year ahead Family Income

In the second application, we examine the ex post inaccuracy of college graduates' 1-year ahead beliefs about their annual family income, Y^B_{it}, during the early post-college period.

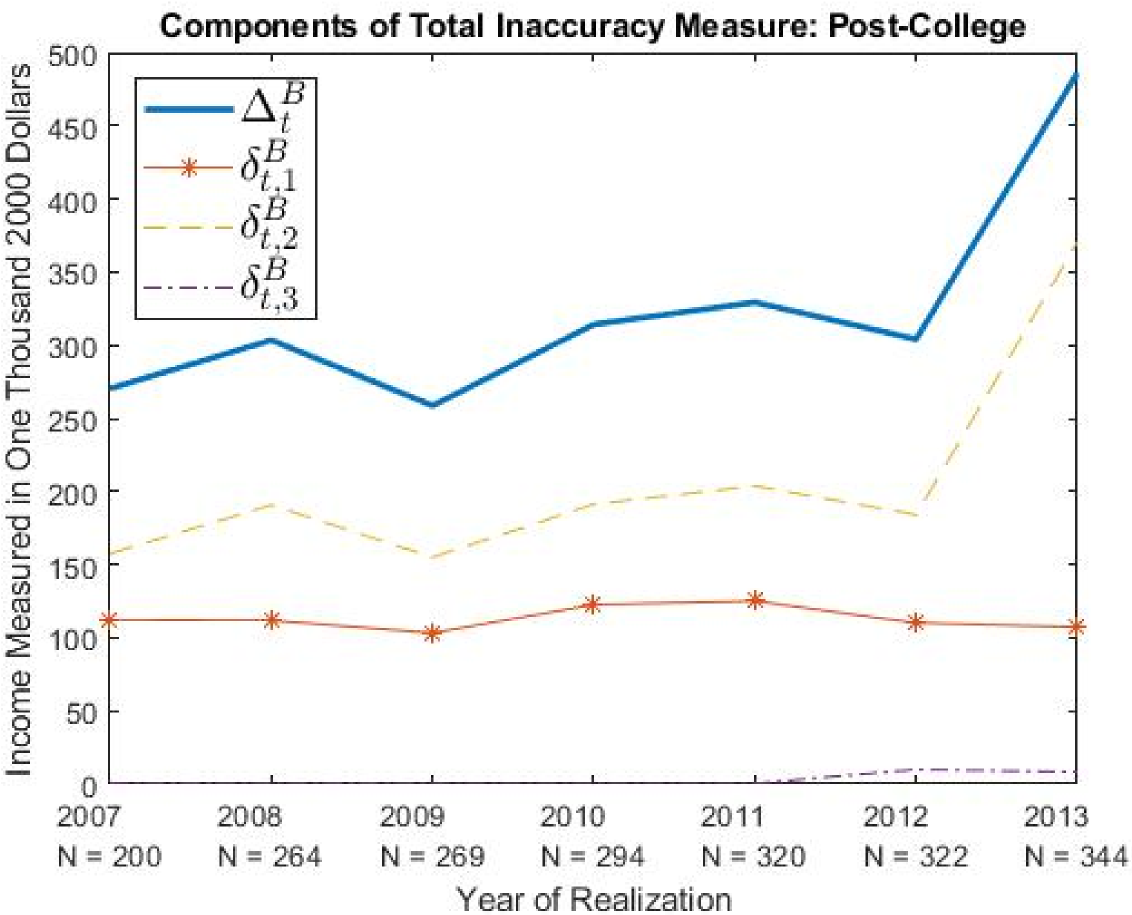

Figure 2: Decomposition of the Inaccuracy Measure: Post-College

The sample size N is reported on the x-axis of the figure.

These beliefs were elicited from 2006 to 2012.

Figure 2 shows the ex post inaccuracy measure and its three components for 1-year-ahead beliefs about family income realized in 2007-2013. A major distinction between Application 1 and Application 2 is that the time horizon between belief-elicitation and income realization is fixed at one year in Application 2. Hence, learning about income-influencing factors is likely not a major contributor to the time pattern of the ex post inaccuracy measure in Application 2. Total ex post inaccuracy is stable through the post college period until 2013, when there is a sharp increase driven entirely by an increase in aggregate actual uncertainty.

4 Discussion

The three individual components of the decomposition we present above ($\delta_{t,1}^B$, $\delta_{t,2}^B$, and $\delta_{t,3}^B$) are useful beyond their role in the computation and understanding of total ex post inaccuracy (Δ_t^B). In a related paper (Crossley et al., 2021) we show that they provide the basis for one of several new tests of the Rational Expectations Hypothesis. Hence, this decomposition of Δ_t^B also builds a link between the ex post accuracy and ex ante rationality of subjective beliefs.

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