

# Essays on Economics and Demography

Daniele Angelini

Thesis submitted for assessment with a view to  
obtaining the degree of Doctor of Economics  
of the European University Institute

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European University Institute  
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# Abstract

This thesis is a collection of three essays on the effect of demographic changes on the economy.

In Chapter 1, I empirically document a non-monotonic effect of an aging workforce on the adoption of ICT (new technology) and on productivity which I rationalize using a task-based OLG model. Initially, the aging of the population has a positive effect on productivity as it reduces the labor supply and increases the capital stock triggering the adoption of new (labor-saving) technologies. However, as young workers (with a comparative advantage in the use of new technologies) become scarce, further aging depresses the adoption of new technologies and reduces productivity. The model also provides policy recommendations regarding the optimal retirement age.

In Chapter 2, co-authored with Max Brès, we analyze the effect of a change in the age composition of consumers on sector aggregates. To identify the effect coming from the demand side of the economy, we use a shift-share IV approach and instrument the change in the age composition of demand with foreign demographics. We find that only middle-aged consumers are associated with higher prices, lower production, and lower productivity suggesting that the age composition of demand affects the economy through changes in the market structure.

In Chapter 3, co-authored with Max Brès, we propose a model with age-specific search costs and elasticity of substitution to highlight the mechanisms behind the empirical results in Chapter 2. The model shows that an increase in the share of middle-aged consumers (who have high search costs and low elasticity of substitution) leads to an increase in both within-sector and between-sectors competition, increasing prices and reducing production and productivity. To capture general equilibrium effects and substitution across sectors, we nest the calibrated sector-level model within a multi-sector general equilibrium framework. We find that the general equilibrium model substantially dampens the

sector-level effects due to lower substitution across sectors. Fitting the model using US demographic data, we find that in the period 1995-2004, as the share of middle-aged increased, the age demand channel contributed to a reduction in US GDP growth, while in the period 2005-2019, as the middle-aged grew old, the age demand channel had a positive effect on US GDP growth.



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# Chapter 1

## Workforce Aging and Technology Adoption

### 1.1 Introduction

Around the world, and especially in industrialized countries, populations are aging. The median age of the populations and the fraction of elderly people are climbing, while fertility rates are falling. An aging population has profound implications in the labor market as it changes the relative supply of inputs in the economy. In particular, aging affects the labor supply and the composition of the labor force. Figure [A.1](#) in Appendix [A.1](#) shows that in the last decades the working-age population has been consistently declining in all the major economies in Europe. This trend is shared by most of the developed countries in the world. The population aging process also affects the composition of the labor force. As we can see from Figure [A.2](#) in Appendix [A.1](#), from 1995 to 2015 the fraction of prime-aged workers (from 25 to 50 years old) in the EU15 countries has declined, while the fraction of old workers (50+ years old) has increased. The reduction in the working-age population reduces the labor supply and affects the capital stock through the saving decisions of the agents, while the change in the labor force composition affects the type of human capital in the economy. These effects influence the incentives firms have in introducing new technologies.

This paper aims to analyze how the change in the relative supply of labor inputs driven by the demographic process affects the adoption of new technologies and the consequent effects on productivity and wages. Recent literature has already analyzed the effect of

aging on the adoption of automated technology. In particular, [Acemoglu and Restrepo \(2018a\)](#) shows that an increase in the ratio of older to middle-aged workers is associated with greater adoption of automated technologies, while [Abeliansky and Prettnner \(2017a\)](#) shows that a reduction in the population growth is associated with an increase in robot adoption. However, different from these papers which consider an automated technology that allows human labor to be perfectly substituted with machines, I consider a new technology that still requires human labor to be operated highlighting the complementarity between technology and the different types of labor<sup>1</sup>. In this framework, I also analyze how the optimal retirement age changes as the population ages, and highlight how not taking into account the endogeneity of technology could lead to sub-optimal retirement policies.

In the empirical analysis, I consider Information and Communication Technologies (ICT) as a proxy for the new technology. I document a sector-level reversed-U shaped relation linking the share of old workers and the adoption of ICT. As the population ages and the aggregate labor supply reduces, producers have an incentive to adopt new (labor-saving) technologies. However, since old (50+ years old) and young workers (between 25 and 49 years old) differ in their human capital, with young workers having a comparative advantage in the use of the new technology, the scarcity of young workers reverses the relation. Therefore, the relation linking aging and new technology adoption is non-monotonic: adoption of new technology initially increases as producers respond to the relative scarcity of the aggregate labor supply, and then it decreases as the economy is constrained by the scarcity of young labor input.

To formally describe these mechanisms, I consider a (supply side) partial equilibrium model of technology adoption with two different technologies, new and vintage technology, and two types of workers, young and old. I assume that new technology is relatively labor-saving and that young workers have a comparative advantage in the use of new technologies. The demographic change is defined as an increase in the share of old agents in the economy who individually supply less labor with respect to young agents as they are partially in the working-age period and partially in the retirement period. This definition of demographic change captures the two effects in the labor market that we observe in the data: a reduction in the aggregate labor supply, and an increase in the share of

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<sup>1</sup>See [Chari and Hopenhayn \(1991\)](#).



old workers. I use a task-based approach allowing for endogenous shares of inputs that depend on the relative supply of labor inputs in the economy. This particular feature makes task-based models suitable to investigate the effects of changes in the composition of inputs in the economy driven by the demographic process or by retirement age policies. An aging population, indeed, reduces the aggregate labor supply increasing the relative availability of capital, and changes the age composition of the labor force increasing the relative labor supply of old workers. The interplay of these different effects on the labor market describes the non-linear dynamic of technological adoption as the population ages. An increase in the retirement age, instead, leads to both an increase in the aggregate labor supply and an increase in the relative labor supply of old workers with depressing effects on the adoption of new technologies.

I, then, consider a general equilibrium version of the model describing the consumers' side of the economy through a standard Diamond overlapping generation model (Diamond, 1965) in which the capital supplied to the economy is determined by the consumption and saving behavior of the agents. Results in this framework are consistent with the main findings in the partial equilibrium model and it overcomes the ever-decreasing effect of an aging population on total production. Indeed, while in the partial equilibrium model with fixed capital, an aging population reduces the aggregate inputs in the economy leading to a reduction in production, in the general equilibrium model with endogenous capital, the reduction in the aggregate labor is compensated by an increase in the capital stock in the economy. Finally, I identify the optimal retirement age policy and show that an efficient (up to an infinitesimal efficiency loss) retirement age can be reached in a decentralized economy.

This model is based on three main assumptions. I assume that: 1) new technology is labor-saving with respect to vintage technology; 2) young workers have a comparative advantage in the use of new technologies, and 3) older workers supply a lower amount of labor. Assuming that new technology is labor-saving is a natural assumption when thinking of technological progress as automation of tasks as in Acemoglu and Autor (2011) or Acemoglu and Restrepo (2018c). Moreover, a consistent part of the literature finds that technological progress generally reduces employment.<sup>2</sup> However, as argued

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<sup>2</sup>Frey and Osborne (2017) finds that about 47% of total US employment is at high risk of being substituted by ICT within the next 20 years. Similar results are those of Bowles (2014), who repeated the study for the European Union countries. Analyzing tasks within occupations, rather than occupations themselves, Bonin et al. (2015) argues that the potential job losses due to technological change are about

in Brynjolfsson and McAfee (2014) and Autor (2015) new technologies have different degrees of complementarity with the human labor depending on the tasks performed and the type of worker. I, therefore, assume that young workers are more complementary to new technology with respect to old workers as a large literature has shown that aging is related to slower information processing, lower learning aptitude toward new technologies (Weinberg, 2004), and lower problem-solving abilities in technology-rich environments (see Figure A.3 in Appendix A.1) implying that younger workers have a comparative advantage in the use of new technology (Dorn et al., 2009).<sup>3</sup> Finally, I assume that older workers supply less labor with respect to younger workers. This assumption can be motivated by recognizing that, while young people are fully in the working-age, the elderly are partially in the working-age and partially in the retirement period. We can relax this assumption in the general equilibrium framework as the accumulation of capital in the context of an aging population has similar effects of a reduced labor supply.

The rest of the paper is structured as follows: in the next paragraph, I present the contribution of this paper with respect to recent literature while, in Section 2, I document the non-linear and non-monotonic relation linking aging and adoption of ICT and other empirical results. In Sections 3 and 4, I present the partial equilibrium and the general equilibrium models and the predicted effects of an aging population on technological adoption, productivity, and production. In Section 5, I define the optimal retirement age policy. Finally, Section 6 concludes.

**Literature Review** This paper is related to some recent literature. The first branch of the literature I refer to is literature analyzing the implications of the introduction of new technologies on the labor market. Early works such as Autor et al. (2003), Goos and Manning (2007), and Autor and Dorn (2013) show evidence suggesting that automation of routine jobs has been associated with greater wage inequality and the decline of middle-skill occupations. More recently, Graetz and Michaels (2017) and Acemoglu and Restrepo (2017a) have estimated the effects of the adoption of robotics technology on employment and wages. Although complementary, my approach is quite different from this literature as I focus on the determinants of the adoption of new technologies depending on the

---

12% for Germany. Following the same approach, Arntz et al. (2016) finds that between 6 and 12% of occupations are at high risk of automation among the OECD countries.

<sup>3</sup>See also Börsch-Supan et al. (2005).

composition of the inputs supplied instead of the implications on the labor market of the adoption of new technologies.

Second, recent literature has analyzed the costs of an aging population at the macroeconomic level (Baldwin and Teulings, 2014); however, besides Abeliatsky and Prettnner (2017a), Acemoglu and Restrepo (2017a) and Acemoglu and Restrepo (2018a), not much research has been done yet on the impact of aging on technology adoption. Differently from Abeliatsky and Prettnner (2017a) who focus on the effect of the slowdown of population growth on the different types of capital, and from Acemoglu and Restrepo (2018a) who consider the change in the age composition of labor, I consider both of these effects. Although I use a similar approach, our results differ from those in Acemoglu and Restrepo (2018a) due to the different definition of technology; while they consider an automated technology replacing labor, I consider a labor-saving technology in which young workers have a comparative advantage. Moreover, I also introduce an overlapping generation model (Diamond, 1965) allowing for endogenous capital.

Finally, the conceptual approach builds on directed technological change literature that was introduced and developed in a series of papers by Acemoglu (1998, 2002, 2007, 2010), while the theoretical framework is based on task-based models (Zeira, 1998; Acemoglu and Zilibotti, 2001; Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018c) which allows for endogenous shares of inputs. This particular feature makes task-based models suitable to investigate the effects of changes in the composition of inputs in the economy driven by the demographic process.

## 1.2 Empirical Evidence

In this section, I analyze the relation linking aging with the adoption of technologies, productivity, and the evolution of the young-to-old wage ratio as the population ages.

### 1.2.1 Technology Adoption

As a proxy for technology adoption, I use investment in ICT measured as the share of capital allocated to hardware, software, and databases.<sup>4</sup> I use sector-level data from EU

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<sup>4</sup>See Appendix A.1 for the description of the variables.

KLEMS in the period between 1995 and 2015 for 10 Western European countries.<sup>5</sup> As a proxy for the age variable, I consider the ratio between workers above 50 and those between 25 and 49 using population and employment data from Eurostat. As we can see from Figure 1.1, the relation linking age (in the x-axis) and investment in ICT (y-axis) is not linear. In particular, for most of the countries, the relation appears to have a reversed-U shape. In order to test this hypothesis, I run the following quadratic model:

$$ICT_{s,t,c} = \beta_0 + \beta_1 Age_{s,t,c} + \beta_2 Age_{s,t,c}^2 + \Gamma X, \quad (1.1)$$

where  $X$  are time, sector, and country dummies. Sector dummies control for the different

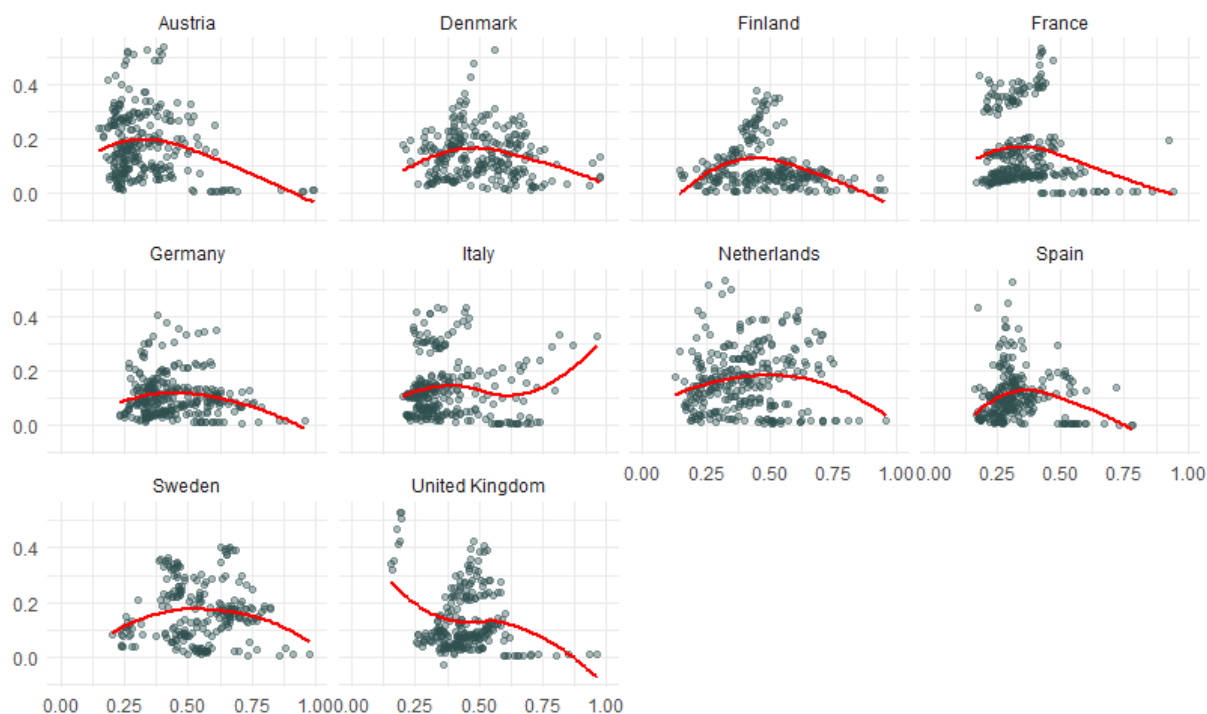


Figure (1.1) Raw relation between Age (ratio between workers above 50 and those between 25 and 50, on the x-axis) and Technological adoption (share of ICT investment, on the y-axis). EU KLEMS sector-level data (1995-2015). The curves are produced using local averaging.

use of ICT in the different sectors, time dummies control for the time trend in the use of ICT, while country dummies control for country fixed effects. Columns (1), (2), and (3) present the coefficient estimates of the linear model (i.e., the model without the quadratic term). Results of the linear model are not consistent across the different specifications. In particular, the richest specification with sector, time, and country fixed effects shows

<sup>5</sup>Austria, Denmark, Finland, France, Germany, Italy, Netherlands, Spain, Sweden, and the United Kingdom. We use countries for which we have complete data excluding Luxembourg.

no relation linking demography to ICT adoption. Once we introduce the quadratic term, instead, the estimated  $\beta_1$  is positive and the estimated  $\beta_2$  is negative and consistent for all the specifications considered suggesting a reversed-U shape relation between age and technology adoption. A possible caveat of using sector-level data is that as workers age, they may switch between sectors. As a robustness check addressing such a concern, I repeat the analysis using country-level observations. Table [A.1](#) in Appendix [A.1](#) shows that, in the richest specification with time and country fixed effects, the country-level results are consistent with the sector-level results.

Table (1.1) Relation between age and ICT investment. EU KLEMS sector-level data (1995-2015) for 10 Western European countries.

	<i>Dependent variable:</i>					
	ICT					
	(1)	(2)	(3)	(4)	(5)	(6)
Age	-0.093*** (0.024)	0.104*** (0.029)	0.044 (0.038)	0.245*** (0.084)	0.356*** (0.077)	0.357*** (0.093)
Age <sup>2</sup>				-0.302*** (0.072)	-0.221*** (0.063)	-0.246*** (0.067)
Time fixed effects		✓	✓		✓	✓
Sector fixed effects		✓	✓		✓	✓
Country fixed effects			✓			✓
Observations	2,800	2,800	2,800	2,800	2,800	2,800
R <sup>2</sup>	0.005	0.367	0.379	0.011	0.370	0.382
Adjusted R <sup>2</sup>	0.005	0.359	0.369	0.011	0.362	0.372

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

To analyze the contribution of the different investments which make up the ICT investment, I repeat the analysis separately considering Information Technology (IT), Communication Technology (CT), and Software and Database (Software and DB). Table [1.2](#) shows that the reversed-U shape relation linking age and ICT is driven almost exclusively by Software and DB technology. IT only marginally contributes to the shape of the relation, while CT negatively relates to age.

The impact of the workforce age structure on the different components of ICT depends on the different degrees to which these technologies are labor-saving and on the different levels of complementarity with different types of labor. In particular, the more the technology is labor-saving, the more it is adopted as the labor force reduces or the

Table (1.2) Relation between age and ICT investment and its different components: Information Technology (IT), Communication Technology (CT), Software and Database (Software & DB).  $ICT = IT + CT + \text{Software \& DB}$ . EU KLEMS sector-level data (1995-2015) for 10 Western European countries.

	<i>Dependent variable:</i>			
	ICT	IT	CT	Software & DB
	(1)	(2)	(3)	(4)
Age	0.357*** (0.093)	0.021* (0.012)	-0.026* (0.014)	0.362*** (0.090)
Age <sup>2</sup>	-0.246*** (0.067)	-0.017* (0.009)	0.012 (0.010)	-0.241*** (0.064)
Time fixed effects	✓	✓	✓	✓
Sector fixed effects	✓	✓	✓	✓
Country fixed effects	✓	✓	✓	✓
Observations	2,800	2,800	2,800	2,800
R <sup>2</sup>	0.382	0.551	0.346	0.314
Adjusted R <sup>2</sup>	0.372	0.543	0.336	0.303

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

capital is accumulated due to aging. Table [1.3](#) shows that, while IT is independent on the fraction of the working-age population, the amount of CT increases with the working-age population suggesting a certain degree of complementarity with the labor force. On the contrary, Software and DB technology increases with a reduction in the fraction of the working-age population suggesting a certain degree of substitutability with the labor force. As Software and DB technology can substitute labor, as the population ages and the labor supply declines relative to capital, the adoption of Software and DB technology, therefore, increases.

Software and DB technology, however, requires peculiar skills to be operated (programming and ICT knowledge in general). Since these skills are mostly held by young workers as they have a more recent education, the relation reverses as young workers become scarce due to aging. The interaction between the substitutability property of Software and DB (and ICT in general) with respect to labor and the complementarity with skills held by young workers explains the reversed-U shaped relation between age and Software and DB technology described in Table [1.2](#).

CT instead increases as the fraction of the working-age population increases (Table

1.2) meaning that it is complementary to the labor force. Therefore, as the population ages and the labor supply reduces, also investments in CT reduce (Table 1.2). However, CT and IT differ with respect to Software and DB technologies in terms of the skills that are required to be operated. In particular, while CT and IT require no particular skills to be used, Software and DB technology requires programming skills that are mostly held by young workers. This explains why aging has almost no effect on CT and IT investment, but it greatly affects Software and DB technology (Table 1.2).

Table (1.3) Relation between fraction of the working-age population (15-64) and ICT investment and its different components: Information Technology (IT), Communication Technology (CT), Software and Database (Software & DB). ICT = IT + CT + Software & DB. EU KLEMS country-level data (1995-2015) for 10 Western European countries.

	<i>Dependent variable:</i>			
	ICT	IT	CT	Software & DB
	(1)	(2)	(3)	(4)
Working-age share	-1.085*** (0.155)	-0.068 (0.061)	0.206*** (0.077)	-1.223*** (0.127)
Country fixed effects	✓	✓	✓	✓
Observations	206	206	206	206
R <sup>2</sup>	0.494	0.256	0.062	0.489
Adjusted R <sup>2</sup>	0.437	0.171	-0.046	0.431

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

## 1.2.2 Productivity

Recent literature has analyzed the effect of aging on productivity producing a large variety of results. A possible source of the mixed results could be the non-linear and possibly non-monotonic relation linking age and productivity.

As argued in Cardona et al. (2013) reviewing results and methods of 150 studies analyzing the relationship between ICT and productivity, there is solid evidence that in the last decades the increase in ICT investment has been a primary driver of productivity and output growth. This implies that since the relation linking aging and ICT adoption is not linear, also the relation between aging and productivity might mimic such non-linear

behavior. In order to test this hypothesis, I estimate the following quadratic model:

$$Productivity_{s,t,c} = \beta_0 + \beta_1 Age_{s,t,c} + \beta_2 Age_{s,t,c}^2 + \Gamma X. \quad (1.2)$$

Table 1.4 shows that, while the richest linear specification with sector, time, and country fixed effects shows no relation between demography and productivity (value-added per hour), in the quadratic model, the estimated  $\beta_1$  is significantly positive and the estimated  $\beta_2$  is significantly negative for all the specifications considered. These results are in line with the results found for ICT and suggest a reversed-U shape relation also between age and productivity.

Table (1.4) Relation between age and productivity (value-added per hour). EU KLEMS sector-level data (1995-2015) for 10 Western European countries.

	<i>Dependent variable:</i>					
	Productivity					
	(1)	(2)	(3)	(4)	(5)	(6)
Age	0.003*** (0.0004)	0.010*** (0.001)	0.0003 (0.001)	0.011*** (0.001)	0.020*** (0.001)	0.006*** (0.001)
Age <sup>2</sup>				-0.007*** (0.001)	-0.009*** (0.001)	-0.004*** (0.001)
Time fixed effects		✓	✓		✓	✓
Sector fixed effects		✓	✓		✓	✓
Country fixed effects			✓			✓
Observations	2,856	2,856	2,856	2,856	2,856	2,856
R <sup>2</sup>	0.014	0.353	0.601	0.026	0.368	0.604
Adjusted R <sup>2</sup>	0.014	0.346	0.595	0.025	0.361	0.597

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

### 1.2.3 Wages

As the population ages, young workers become scarcer in the economy. Under the assumption that young and old workers are different in terms of their complementarity with respect to technologies, we expect an increase in young workers' wages relative to old workers' wages as the population ages. Table 1.5, indeed, shows that there exists a positive relationship between the young-to-old wage ratio and aging. This relation is robust to the inclusion of sector, time and country fixed effects. To control for the heterogeneity



of the relevance of the age variable to determine wages across sectors, I also include the  $Age \times Sector$  fixed effects interaction term. Introducing this interaction term allows us to control for unobserved variables determining wages in the different sectors, such as experience. The relevance of the inclusion of the interaction term can be seen from the Table (1.5) Relation between age and young to old wage ratio. EU KLEMS sector-level data (2008-2015) for 26 European countries and the US.

	<i>Dependent variable:</i>				
	Young to Old Wage Ratio				
	(1)	(2)	(3)	(4)	(5)
Age	0.141*** (0.008)	0.217*** (0.011)	0.156*** (0.011)	0.227*** (0.010)	0.517*** (0.022)
Time fixed effects		✓	✓	✓	✓
Sector fixed effects		✓	✓	✓	✓
Country fixed effects			✓	✓	✓
Age $\times$ Sector fixed effects				✓	✓
Age $\times$ Country fixed effects					✓
Observations	4,554	4,554	4,554	4,554	4,554
R <sup>2</sup>	0.062	0.197	0.412	0.428	0.369
Adjusted R <sup>2</sup>	0.062	0.191	0.405	0.421	0.361

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

fact that the point estimate substantially increases and the estimation becomes slightly more precise (i.e., the standard error reduces). Results are also robust to the inclusion of the  $Age \times Country$  fixed effects interaction term which controls for the heterogeneity of the relevance of age to determine wages across countries. Also in this case the point estimate increases.

To summarize our empirical evidence, I find a reversed-U shape relation linking age and ICT investments. This relation is almost completely driven by the Software and DB component of ICT. Software and DB technology is labor-saving in the sense that it is negatively related to the fraction of the working-age population in the economy. Moreover, Software and DB technology requires peculiar skills mostly held by young workers to be used. Since the relation between age and ICT is driven by Software and DB technology, in the following theoretical model, I define the new technology as being labor-saving and complementary with young workers. I also find that the reversed-U shape relation linking age and ICT investment also holds for productivity. This does not come as a surprise as

the literature has well documented the positive and significant relation linking ICT and productivity. Aging also affects the relative wage of the different types of workers (young and old) as it changes the relative supply of the different labor inputs in the economy. In particular, I document a positive relationship between the aging and the young-to-old wage ratio suggesting that young and old workers are not perfect substitutes. In the rest of the paper, I formalize and rationalize these findings using a theoretical model.

## 1.3 Partial Equilibrium Model

In this section, a partial equilibrium version (with fixed capital) of the model is presented which allows us to characterize the impact of an aging population and of an increase in the labor participation of old workers on technological adoption, capital allocation, productivity, and factor prices. In the next paragraphs, I present the model environment, describe the supply side of the economy, the market-clearing conditions, and define the equilibrium.

### 1.3.1 Environment

I consider an economy producing a unique final good  $Y$  by combining a unit measure of tasks,  $y(i)$  with  $i \in [0, 1]$ , with unitary elasticity of substitution:

$$Y = \exp \left\{ \int_0^1 \ln y(i) \, di \right\}. \quad (1.3)$$

Each task can be produced either using *new* technology ( $y_n(i)$ ), *vintage* technology ( $y_v(i)$ ) or a linear combination of the two:

$$y(i) = \underbrace{\mu_n(i) \cdot \ell_n(i)^\alpha k_n(i)^{1-\alpha}}_{y_n(i)} + \underbrace{\mu_v(i) \cdot \ell_v(i)^\beta k_v(i)^{1-\beta}}_{y_v(i)}, \quad (1.4)$$

where  $\ell_n(i)$  ( $\ell_v(i)$ ) and  $k_n(i)$  ( $k_v(i)$ ) are labor and capital used in new (vintage) technology,  $\mu_n(i)$  ( $\mu_v(i)$ ) is the task-specific productivity of task  $i$  using new (vintage) technology, and  $\alpha$  ( $\beta$ ) is the output elasticity of labor using new (vintage) technology. Throughout, I impose the following assumptions:

**Assumption 1:** *The ratio  $\mu_n(i)/\mu_v(i)$  is decreasing in  $i$ .*

**Assumption 2:**  $\alpha < \beta$ .

Assumption 1 describes a productivity schedule in which new technology is more efficient in the production of low-indexed tasks, and vintage technology is more efficient in the production of high-indexed tasks. This productivity schedule implies that there exists a threshold task  $I \in [0, 1]$  such that only new technology is used to produce tasks  $i < I$ , and only vintage technology is used to produce tasks  $i \geq I$ . This implies that we can rewrite the task production function (1.4) of task  $i$  as:

$$y(i) = \begin{cases} y_n(i) & \text{if } i \in [0, I) \\ y_v(i) & \text{if } i \in [I, 1]. \end{cases}$$

The threshold  $I$  represents the fraction of tasks that are produced using new technology, and it can be interpreted as a measure of new technology adoption.

Assumption 2 states that the output elasticity of labor (or labor share) of the new technology task production function is smaller than the vintage technology one meaning that the new technology is labor-saving with respect to the vintage technology.

### 1.3.2 Producers

I recover the demand of each task by maximizing the net output with respect to task  $i$ :

$$\max_{\{y(i)\}} Y - \int_0^1 p(i) \cdot y(i) di. \quad (1.5)$$

where  $p(i)$  is price of task  $i$ . From problem (1.5) I get the demand for task  $i$ :

$$y(i) = \frac{Y}{p(i)}. \quad (1.6)$$

Producers of tasks are price takers. For tasks  $i \in [0, I)$ , they maximize profits taking the price of tasks,  $p(i)$ , the price of capital,  $R$ , and the wages,  $w_n$ , as given:

$$\max_{\{\ell_n, k_n\}} p(i) \cdot y_n(i) - w_n \cdot \ell_n(i) - R \cdot k_n(i). \quad (1.7)$$

Using the demand of task  $i$  in equation (1.6), and the FOC from problem (1.7), I get the following demands for labor and capital for tasks produced using new technology:

$$\ell_n(i) = \frac{\alpha Y}{w_n}, \quad (1.8)$$

$$k_n(i) = \frac{(1 - \alpha)Y}{R}. \quad (1.9)$$

Similarly, I recover the demands for labor and capital for tasks produced using vintage technology (i.e.,  $i \in [I, 1]$ ):

$$\ell_v(i) = \frac{\beta Y}{w_v}, \quad (1.10)$$

$$k_v(i) = \frac{(1 - \beta)Y}{R}. \quad (1.11)$$

### 1.3.3 Market Clearing Conditions

The population in the economy is normalized to one. Agents in the population can be either *young* or *old*. Therefore,  $N^y + N^o = 1$ , where  $N^y$  and  $N^o$  represent the fraction of young and old respectively. Moreover, I impose the following assumptions:

**Assumption 3:** *Young agents inelastically supply one unit of labor, while old agents supply  $\phi \in [0, 1]$  units of labor.*<sup>6</sup>

**Assumption 4:** *Young workers can use either technology, while old workers can only use vintage technology.*<sup>7</sup>

Following Assumption 3, the aggregate labor supply is:  $N^y + \phi N^o$ . Assumption 4 implies that the market clearing conditions in the labor market are the following:

$$\int_0^I \ell_n(i) di = \gamma \cdot N^y, \quad (1.12)$$

---

<sup>6</sup>The lower labor supply of old workers with respect to young workers is susceptible to either an intensive or an extensive margin interpretation. According to the intensive margin interpretation,  $\phi$  represents the effective units of labor supplied on average by each old person in the economy, while, according to the extensive margin interpretation,  $\phi$  represents the fraction of old people working supplying inelastically one unit of labor. In the latter case, we can, therefore, consider three types of agents: young workers, old workers, and old people not working (i.e., pensioners).

<sup>7</sup>This assumption simplifies the analysis with respect to the general setup in which each type can use both types of technologies and young workers have a comparative advantage in using new technology.

$$\int_I^1 \ell_v(i) di = (1 - \gamma) \cdot N^y + \phi N^o, \quad (1.13)$$

where  $\gamma \in [0, 1]$  is the fraction of young workers employed in the production of tasks that are produced using new technology and it is endogenous. The left-hand side of equation (1.12) and of equation (1.13) are the aggregate labor demands for tasks produced with new and vintage technology respectively.

Given the technology used, it can be shown that the task labor demand is constant across tasks, i.e.,  $\ell_n(i) = \ell_n$  and  $\ell_v(i) = \ell_v$  (see Appendix A.2.4). This implies that the market clearing conditions (1.12) and (1.13) can be written as:

$$\ell_n = \frac{\gamma \cdot N^y}{I}, \quad (1.14)$$

$$\ell_v = \frac{(1 - \gamma) \cdot N^y + \phi N^o}{1 - I}. \quad (1.15)$$

Similarly, I consider the market clearing conditions for the capital market. Since  $k_n(i) = k_n$  and  $k_v(i) = k_v$  (see Appendix A.2.4), we can write:

$$k_n = \frac{\theta \cdot K}{I}, \quad (1.16)$$

$$k_v = \frac{(1 - \theta) \cdot K}{1 - I}, \quad (1.17)$$

where  $K$  is the capital stock and  $\theta \in [0, 1]$  is the fraction of capital allocated to tasks produced using new technology which is endogenous. The capital stock is taken as given (it will be endogenized via the household saving decision in Section 4).

### 1.3.4 Partial Equilibrium

Given the capital stock,  $K$ , the fraction of young (old) agents in the economy,  $N^y$  ( $N^o$ ), the demands for labor and capital for new and vintage technology, we can now characterize the equilibrium value of output, factor prices, the threshold task ( $I$ ), and the input allocation shares ( $\gamma$  and  $\theta$ ). At this point, it helps to distinguish between two different cases: the case in which the young labor input constraint (from now on, input constraint)

is slack (i.e.,  $0 < \gamma < 1$  meaning that young workers are employed in the production of tasks produced either with new or vintage technology), and the case in which the input constraint is binding (i.e.,  $\gamma = 1$  meaning that all young workers are employed in the production of tasks using new technology, while all old workers are producing tasks using vintage technology). In the situation in which the input constraint is slack, optimality requires that the wages of the workers employed in the production of tasks using new and vintage technologies are the same. This is because there is no scarcity of young workers and young workers are employed in tasks produced using either new or vintage technology. In this case, young and old workers are “as perfect substitutes”. In the situation in which the input constraint is binding, wages differ since workers are not perfect substitutes, and wages depend on the respective labor supplies.<sup>8</sup>

It can be shown that there exists a threshold value  $\hat{N}^o < 1$  such that for  $N^o < \hat{N}^o$ , the input constraint is slack ( $0 < \gamma < 1$ ), and for  $N^o \geq \hat{N}^o$ , the input constraint is binding ( $\gamma = 1$ ).<sup>9</sup> In other words, if the old population is relatively small, not all young workers use new technologies, while if the old population is relatively large, then all young workers are employed in the production of tasks produced using new technology.

From the factor demands described in equations (1.8), (1.9), (1.10), and (1.11), the market clearing conditions in equations (1.14), (1.15), (1.16), and (1.17), and the considerations above over wages, we recover the factor prices and the allocations of young workers ( $\gamma$ ) and capital ( $\theta$ ):

$$R = \frac{Y}{K}(1 - \varepsilon), \quad (1.18)$$

$$w^y = \begin{cases} \frac{Y}{L} \cdot \varepsilon & \text{if } N^o < \hat{N}^o \\ \frac{Y}{N^y} \cdot \alpha I & \text{if } N^o \geq \hat{N}^o, \end{cases} \quad (1.19)$$

$$w^o = \begin{cases} \frac{Y}{L} \cdot \varepsilon & \text{if } N^o < \hat{N}^o \\ \frac{Y}{\phi N^o} \cdot \beta(1 - I) & \text{if } N^o \geq \hat{N}^o, \end{cases} \quad (1.20)$$

$$\gamma = \begin{cases} \frac{L}{N^y} \cdot \frac{\alpha I}{\varepsilon} & \text{if } N^o < \hat{N}^o \\ 1 & \text{if } N^o \geq \hat{N}^o, \end{cases} \quad (1.21)$$

<sup>8</sup>See Appendix A.2.4 for the proof.

<sup>9</sup>The threshold age is defined as:  $\hat{N}^o = \frac{\beta(1-I)}{\phi\alpha I + \beta(1-I)}$ . We find  $\hat{N}^o$  by solving  $\gamma(\hat{N}^o) = 1$ .

$$\theta = \frac{(1 - \alpha)I}{1 - \varepsilon}, \quad (1.22)$$

where  $w^y(w^o)$  is the wage of young (old) workers;  $\varepsilon \equiv \alpha I + \beta(1 - I)$  is the labor share for  $N^o < \hat{N}^o$ ,  $(1 - \varepsilon)$  is the capital share for any level of  $N^o$ ; and  $L \equiv N^y + \phi N^o$  is the aggregate labor supply.<sup>10</sup> It can be shown that the aggregate production function can be rewritten as follows (see Appendix A.2.4):

$$Y = \begin{cases} A \cdot \left(\frac{L}{\varepsilon}\right)^\varepsilon \cdot \left(\frac{K}{1-\varepsilon}\right)^{1-\varepsilon} & \text{if } N^o < \hat{N}^o \\ A \cdot \left(\frac{N^y}{\alpha I}\right)^{\alpha I} \cdot \left(\frac{\phi N^o}{\beta(1-I)}\right)^{\beta(1-I)} \cdot \left(\frac{K}{1-\varepsilon}\right)^{1-\varepsilon} & \text{if } N^o \geq \hat{N}^o, \end{cases} \quad (1.23)$$

where:

$$A \equiv \exp \left\{ \int_0^I \ln \{ \alpha^\alpha (1 - \alpha)^{1-\alpha} \mu_n(i) \} di + \int_I^1 \ln \{ \beta^\beta (1 - \beta)^{1-\beta} \mu_v(i) \} di \right\}. \quad (1.24)$$

Equation (1.23) reflects the considerations discussed above regarding the different degrees of substitutability between young labor and old labor inputs in the situations of slack or binding input constraint. In the case of a slack input constraint ( $N^o < \hat{N}^o$ ), young and old labor are perfect substitutes and they enter the aggregate production function as a single input,  $L$ . In the case of a binding input constraint ( $N^o \geq \hat{N}^o$ ), young and old labor enter the aggregate production function as different inputs.

In this model, the factor shares are endogenous since they depend on the threshold task  $I$ . In particular, by allowing task producers to choose between technologies with different output elasticity of labor ( $\alpha$  and  $\beta$  for new and vintage technology, respectively), they optimally choose the technology depending on the supply of inputs. At the aggregate level, therefore, the supply of inputs endogenously determines the factor shares.

As the last step, to fully characterize the equilibrium, we need to pin down the threshold task  $I$ . This can be done by noting that in task  $i = I$ , a task producer should break even either using new or vintage technology. In other words, it must hold that the price of the task  $i = I$  is the same when using new or vintage technology:

$$p_n(I) = p_v(I), \quad (1.25)$$

---

<sup>10</sup>Since wages of young and old workers are the same as long as the input constraint is not binding, and wages of young (old) workers equal the wages of workers using new (vintage) technology, it holds that  $w^y = w_n$  ( $w^o = w_v$ ) as defined in equations (1.19) and (1.20).

where  $p_n(i)$  ( $p_v(i)$ ) is the price of task  $i$  when the task is produced using new (vintage) technology.<sup>11</sup>

### 1.3.5 Partial Equilibrium Analysis

In this section, I analyze how changes in the supply of labor inputs affect the labor market in terms of adoption of new technologies, allocation of capital to different technologies, and wages. Such changes in the input supplies can be motivated by an aging population which increases the supply of old workers, reduces the supply of young workers, and negatively affects the aggregate labor supply, and by policy shocks such as an increase in the retirement age that increases the labor supply of old workers and the aggregate labor supply.

Task-based models are particularly well suited to analyze changes in the supply of factors that have different degrees of complementarity with different technologies. Indeed, in this framework firms do not only adapt their demand of inputs depending on the relative supply of factors but also choose the technology to use. Since technologies differ in terms of the output elasticity of inputs, as the supply of factor changes and firms optimally choose the technology to adopt, the aggregate factor shares change as well with effects that standard models do not capture.

#### Aging analysis with fixed capital

I now consider the effects of an increase in the fraction of old people in the economy,  $\uparrow N^o$ . I define this increase in the share of the elderly in a fixed population simply as *aging*. I obtain the following result:

**Proposition 1.3.1.** *The relation linking aging and the adoption of new technologies,  $I$ , and aging and the share of capital allocated to new technologies,  $\theta$ , is positive for “low” levels of aging ( $N^o < \hat{N}^o$ ), and negative for “high” ( $N^o \geq \hat{N}^o$ ) levels of aging:*

$$\frac{dI}{dN^o} = \begin{cases} > 0 & \text{if } N^o < \hat{N}^o \\ < 0 & \text{if } N^o \geq \hat{N}^o, \end{cases} \quad (1.26)$$

---

<sup>11</sup> $p_n(i) = \mu_n(i)^{-1} \cdot \left(\frac{w_n}{\alpha}\right)^\alpha \cdot \left(\frac{R}{1-\alpha}\right)^{1-\alpha}$  and  $p_v(i) = \mu_v(i)^{-1} \left(\frac{w_v}{\beta}\right)^\beta \cdot \left(\frac{R}{1-\beta}\right)^{1-\beta}$ , i.e., the price of producing a task using either technology negatively depends on the technology task-specific productivity, and it is increasing in the average price of factors weighted by the respective shares. The derivation is straightforward using the FOC from the maximization problem (1.7).



$$\frac{d\theta}{dN^o} = \begin{cases} > 0 & \text{if } N^o < \hat{N}^o \\ < 0 & \text{if } N^o \geq \hat{N}^o. \end{cases} \quad (1.27)$$

Proof in Appendix [A.2.4](#).

Proposition [1.3.1](#) states that, as long as there are enough young people in the economy ( $N^o < \hat{N}^o$ ) and, therefore, the input constraint is not binding,  $\gamma \in (0, 1)$ , an aging population leads to an increase in the adoption of new technology and the share of capital allocated to new technologies. As population ages further ( $N^o \geq \hat{N}^o$ ) and the input constraint binds ( $\gamma = 1$ ), the relation reverses.

The idea is that aging reduces the aggregate labor supply leading to an increase in the price of labor with respect to the price of capital. Due to the increase in the cost of labor, if the labor constraint is not binding (i.e.,  $N^o < \hat{N}^o$ ), a producer close to the threshold task,  $I$ , will find it optimal to switch from the vintage technology which is labor-intensive to the new (labor-saving) technology. This would lead to an increase in the share of firms adopting new technologies ( $\uparrow I$ ) increasing the adoption of young labor in tasks produced using new technology. In this case, since the input constraint is not binding, workers are “as perfect substitutes” and producers can freely choose which type of technology to adopt. Finally, as the relative employment in new technologies increases, optimality requires an increase in the share of capital allocated to new technologies ( $\uparrow \theta$ ).

In the situation in which all the young workers are already using new technologies ( $\gamma = 1$  and  $N^o \geq \hat{N}^o$ ), the age composition of the labor force determines the technology to use. Therefore, as the population ages and the number of young labor shrinks, the adoption of new technology reduces ( $\downarrow I$ ) as well as the share of capital allocated to new technology ( $\downarrow \theta$ ). In this situation, indeed, as shown in equation [\(1.23\)](#), the different types of labor are imperfect substitutes.

Figure [1.2](#) shows the effect of aging on relevant economic variables. Wages for young and old are the same up to the point in which the input constraint binds (the dashed vertical line); as the scarcity of young labor hits the economy, wages diverge reflecting the higher productivity of young workers using new technology with respect to old workers using vintage technology. Since the labor supply reduces as the population ages, labor productivity increases monotonically if the input constraint is not binding due to the

fixed amount of capital. As the input constraint binds, two opposite forces determine the average productivity: on one side, productivity increases since capital is fixed and the labor supply is shrinking; on the other side, old workers will start producing tasks in which the vintage technology is not efficient due to the lack of young workers. The latter effect becomes stronger as the population ages, and it dominates when the fraction of old people in the economy is high enough. We, therefore, observe a reversed-U shape relation linking the share of the elderly and productivity. The effect of aging on production is unambiguously negative. Given a fixed amount of capital, an aging population reduces the labor supply and it necessarily leads to a reduction in production. Moreover, once aging starts reducing the average productivity, production reduces at a faster rate.

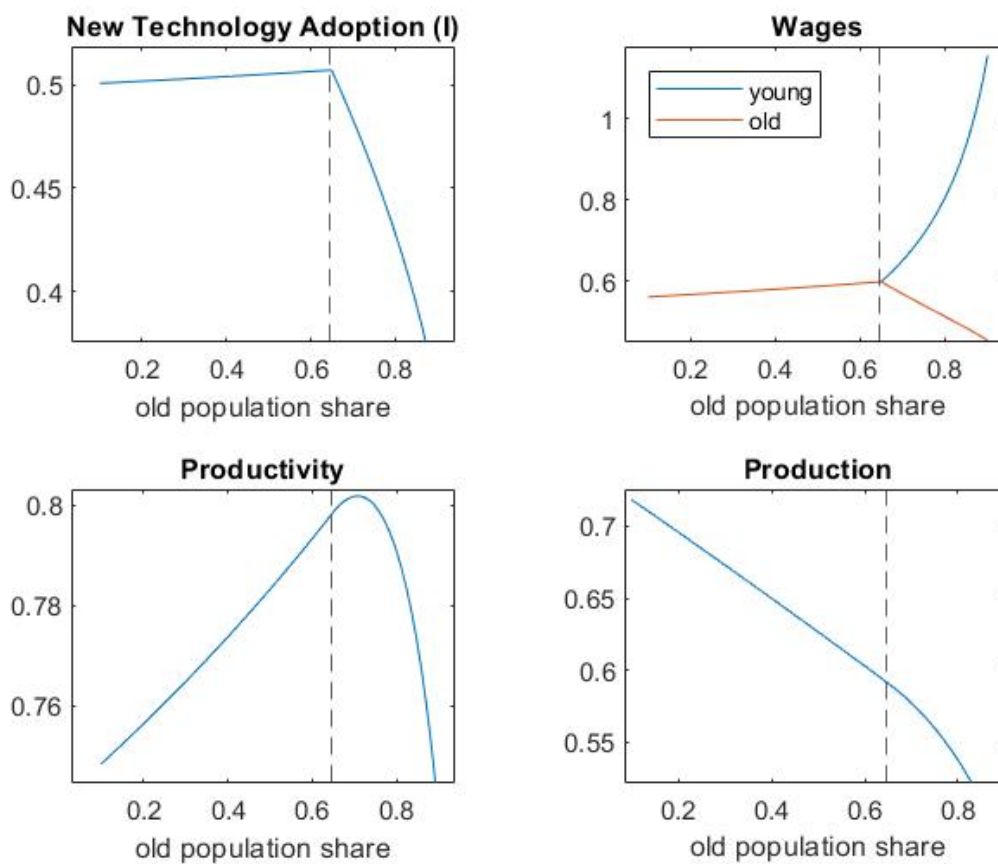


Figure (1.2) Aging analysis (share of old people in the economy on the  $x$  axis).  
 $(\alpha = 0.7, \beta = 0.8, K = 1, \phi = 0.6, \mu_n(i) = 1 - i, \mu_v(i) = i)$ .

### Retirement age analysis with fixed capital

I now consider the effect of an increase in  $\phi$  on technology adoption and on the allocation of capital. We can interpret an increase in  $\phi$  as an increase in the retirement age. I will, therefore, refer to  $\phi$  simply as the retirement age. I obtain the following result:

**Proposition 1.3.2.** *An increase in the retirement age,  $\phi$ , leads to a reduction in the adoption of new technologies,  $I$ , and a reduction in the share of capital allocated to new technologies,  $\theta$ :*

$$\frac{dI}{d\phi} < 0, \quad \frac{d\theta}{d\phi} < 0, \quad \forall N^o \in (0, 1). \quad (1.28)$$

Proof in Appendix [A.2.4](#).

Proposition [1.3.2](#) states that an increase in the retirement age reduces the use of new technologies and reduces the share of capital allocated to new technologies. For  $N^o < \hat{N}^o$ , an increase in the retirement age increases the aggregate labor supply which reduces the relative price of labor leading to an increase in the use of vintage (labor-intensive) technologies ( $\downarrow I$ ) and a reduction in the share of capital allocated to new technologies ( $\downarrow \theta$ ). For  $N^o > \hat{N}^o$ , all the young workers use the new technology, and all the old workers use the vintage technology. In this situation, the increase in the retirement age has a purely market size effect: since old labor has become relatively more abundant with respect to young labor, the adoption of vintage technology and the share of capital allocated to vintage technology increase.

Figure [1.3](#) shows that an increase in the retirement age leads to a reduction in the adoption of new technologies for any level of aging. Since the stock of capital is fixed, as the labor force increases following the policy, productivity reduces and so do the wages if the input constraint is not binding. As the scarcity of young workers hits the economy and the population ages, the gap between the wage of the young and the wage of the old widens. Although productivity reduces as the retirement age increases, production increases as capital is fixed and the labor supply increases following the policy.

The increase in the retirement age has also an effect on the aging threshold,  $\hat{N}^o$ :

**Corollary 1.3.2.1.** *An increase in the retirement age ( $\phi$ ) makes the input constraint binding for a lower level of aging:*

$$\frac{d\hat{N}^o}{d\phi} < 0 \quad (1.29)$$

Proof in Appendix [A.2.4](#)

Increasing the number of old workers increases the fraction of workers that is constrained in using vintage technologies. This means that a lower share of young workers will be employed in tasks produced using vintage technology leading to a binding constraint for lower levels of aging.

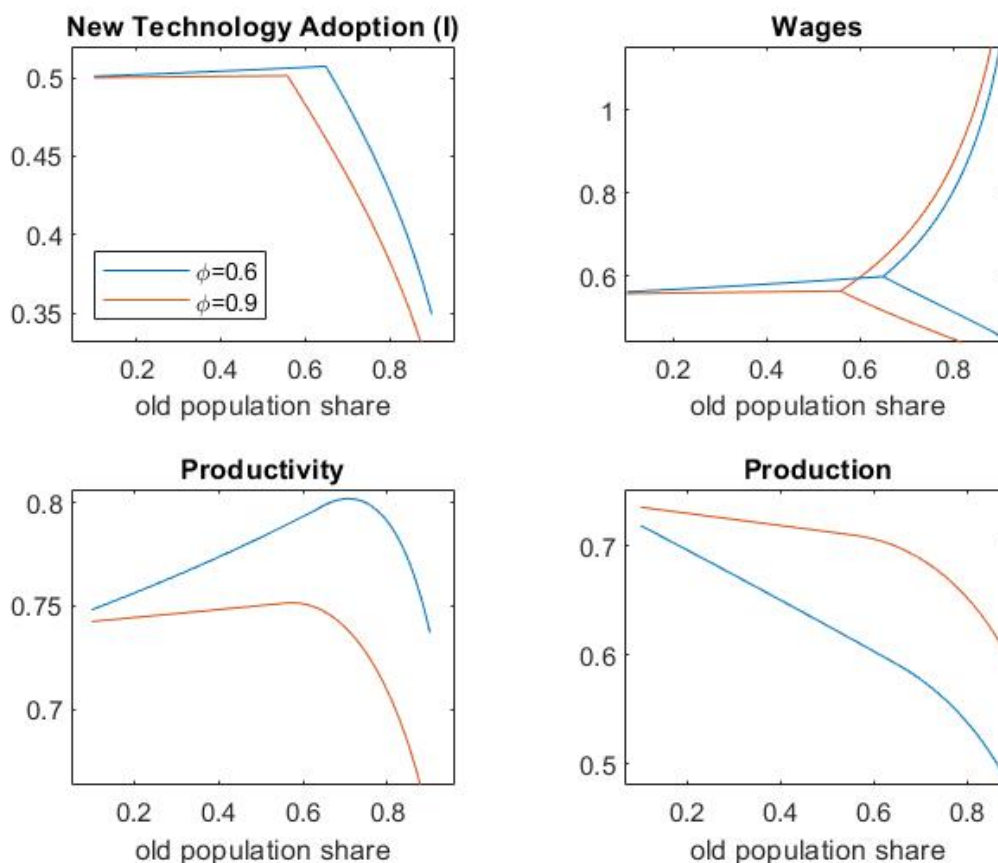


Figure (1.3) Retirement age analysis (share of old people in the economy on the  $x$  axis). ( $\alpha = 0.7$ ,  $\beta = 0.8$ ,  $K = 1$ , “low” retirement  $\phi = 0.6$ , “high” retirement  $\phi = 0.9$ ,  $\mu_n(i) = 1 - i$ ,  $\mu_v(i) = i$ ).

## 1.4 General Equilibrium Model

In this section, I introduce a simple Diamond overlapping generation model to describe the consumer side of the economy and to study the effect of aging in a general equilibrium framework. By modeling the labor supply and the saving decisions of the consumers, we will be able to endogenize the supply of capital and overcome the result in the partial equilibrium model of an ever-decreasing production as the population ages.

### 1.4.1 Consumption choice

I consider a representative consumer living two periods. In the first period agents are “young”, while in the second period, they are “old”. I define as young, prime-aged workers from 25 to 50 years old, and as old, people from 50 to 75. Therefore, each of the periods lasts for about 25 years. While young agents are fully in the working-age period, the old agents are partially in the working-age and partially in the retirement period. These considerations allow us to interpret  $\phi$  as the retirement age which we assume to be exogenously determined. A representative young agent at time  $t$  faces the following lifetime utility maximization problem:

$$\max_{\{C_t^y, C_{t+1}^o\}} U(C_t^y) + \rho \cdot U(C_{t+1}^o) \quad \text{s.t.} \quad (1.30)$$

$$C_t^y = w_t^y - S_t,$$

$$C_{t+1}^o = \phi w_{t+1}^o + (1 + r_{t+1}) \cdot S_t,$$

where  $\rho$  is the discount rate, and  $S_t$  are savings in period  $t$ . In period  $t$ , the young agent supplies one unit of labor and receives a salary  $w_t^y$  that can be either consumed or saved. In period  $t + 1$ , the old agent supplies  $\phi$  units of labor and gets a salary  $w_{t+1}^o$  per unit of labor supplied. Old agents consume everything by the end of the second period.  $r_{t+1}$  is the net price of capital in period  $t + 1$  with capital depreciation ( $\delta$ ) already deducted, i.e.,  $r_{t+1} = R_{t+1} - \delta$ . Since the model divides human life into two periods, each period is quite long (in historical time) and it is thus reasonable to assume that capital fully depreciates within the period ( $\delta = 1$ ) implying  $R_{t+1} = 1 + r_{t+1}$ .

In order to complete the description of the environment, we define the following relations linking the generations:

$$N_{t+1}^y = (1 + n) \cdot N_t^y, \quad (1.31)$$

$$N_{t+1}^o = N_t^y, \quad (1.32)$$

where  $n$  is the rate at which new generations reproduce. Equations (1.31) and (1.32) together imply that the old to young labor ratio is equal to:

$$\frac{N_t^o}{N_t^y} = \frac{1}{1+n}. \quad (1.33)$$

### 1.4.2 General Equilibrium

Assuming log preferences (i.e.,  $U(x) = \log(x)$ ), from the maximization problem (1.30) and the feasibility constraint,  $K_{t+1} = N_t^y \cdot S_t$ , we obtain:

$$k_{t+1} = \frac{\theta_{t+1}}{\gamma_{t+1}(1+\rho)(1+n)} \left( \rho \cdot w_t^y - \frac{\phi \cdot w_{t+1}^o}{R_{t+1}} \right), \quad (1.34)$$

where  $k_t$  is defined as the capital allocated per worker in new-technology  $\left( \frac{\theta_t K_t}{\gamma_t N_t^y} \right)$  in period  $t$ . Using equation (1.34) and the prices of inputs (1.19) and (1.20) from the supply side, we obtain an expression for the steady state capital such that:  $k_{t+1} = k_t = k$  (see Appendix A.2.1).

### 1.4.3 General Equilibrium Analysis

In what follows, I analyze the effect of aging and of an increase in the retirement age on the adoption of new technologies, and its effects on wages, productivity, and production with endogenous capital. I take a long-term perspective analyzing the effect of the demographic change on the steady-state equilibrium.

#### Aging analysis with endogenous capital

Figure 1.4 shows the effects of aging on the relevant economic variable. In particular, the effect of aging on the adoption of new technology, wages, and productivity is consistent with the effect found in the partial equilibrium model with fixed capital. However, by allowing capital to be endogenous, another effect shapes the results. As the population ages, more capital is accumulated in the economy since old consumers supply capital. The increase in the amount of capital in the economy further creates incentives to adopt new technologies as the new technologies are capital-intensive. In order to disentangle the effect of the capital increase on new technology adoption with respect to the effect coming from the declining labor supply as the population ages, I assume that both young

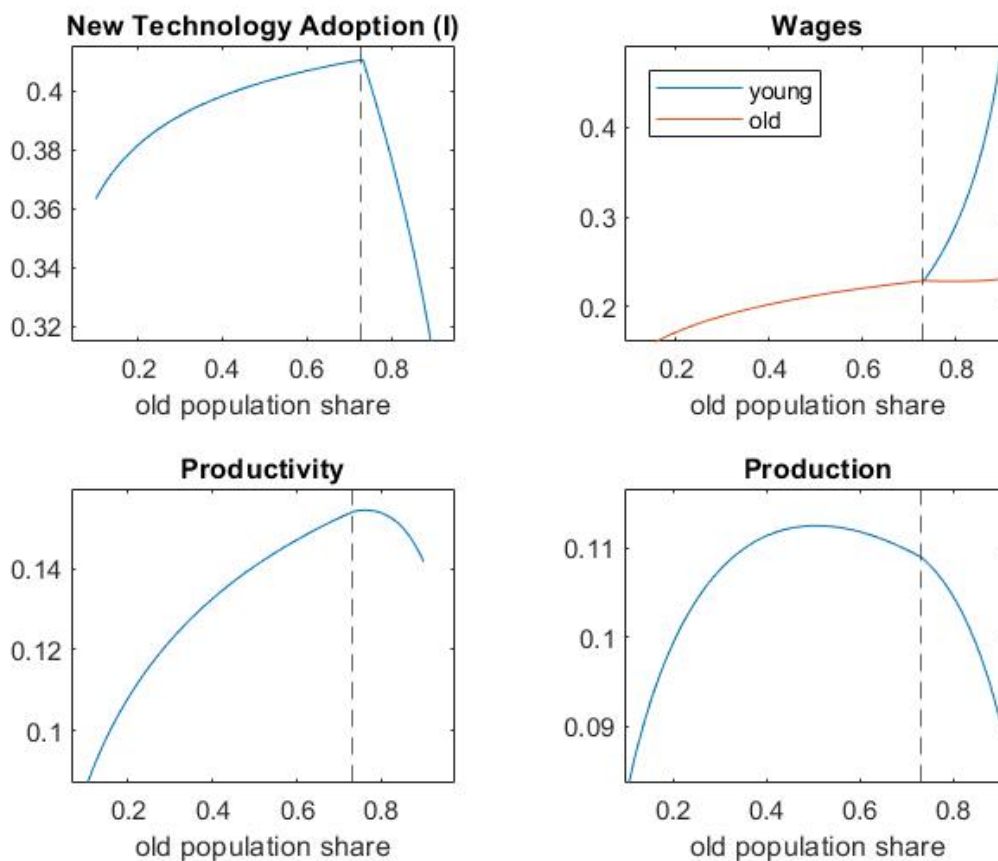


Figure (1.4) Aging analysis (share of old people in the economy on the  $x$  axis).  
 $(\alpha = 0.7, \beta = 0.8, \phi = 0.6, \rho = 0.96^{25} = 0.36, \mu_n(i) = 1 - i, \mu_v(i) = i)$ .

and old supply one unit of labor. As we can see in Figure [1.5](#), even in the case in which the reduction in the labor supply is sterilized ( $\phi = 1$ ), we still have a non-monotonic behavior of technological adoption and investment in new technology due to the effect going through the accumulation of capital.

Differently from the partial equilibrium model, we find that an aging population does not have a monotonic effect on production. Indeed, while in the partial equilibrium model with fixed capital, an aging population reduces the aggregate labor supply, in the general equilibrium model, the negative effect of a reduction in the labor supply on production is counteracted by the increase in capital supplied by old people. These two forces combined lead to a non-monotonic effect on production. Initially, as the population is relatively young and the capital stock in the economy is relatively scarce, the aging of the population leads to an increase in production as the marginal product of capital is high and the positive capital effect dominates over the negative labor effect. The marginal product of capital progressively reduces as the capital stock builds up explaining the

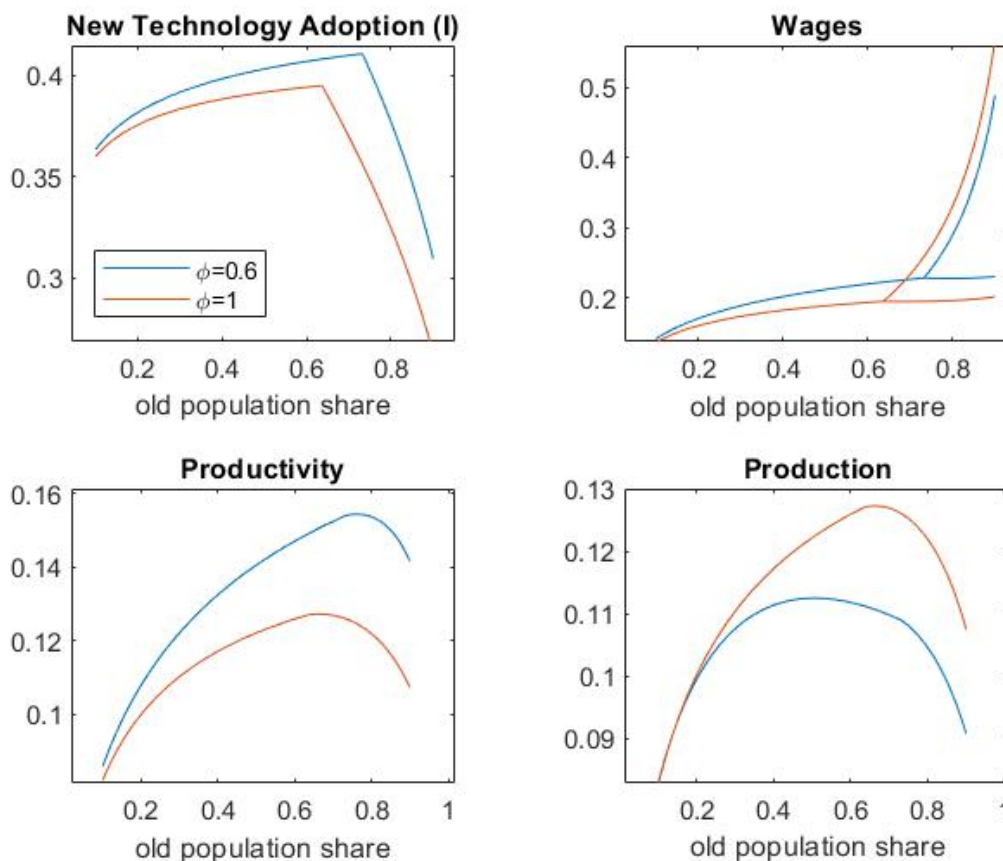


Figure (1.5) Retirement age analysis (share of old people in the economy on the  $x$  axis). ( $\alpha = 0.7, \beta = 0.8, \rho = 0.96^{25} = 0.36, \mu_n(i) = 1 - i, \mu_v(i) = i$ ).

increase of production and productivity at a progressively lower rate as the population ages. Finally, as the marginal product of capital reduces as the population ages, the negative labor effect dominates the capital effect leading to the non-monotonic effect on production.

### Retirement age analysis with endogenous capital

The effects of an increase in the retirement age are similar to those in the partial equilibrium model. Figure 1.5 shows that, as in the partial equilibrium model, an increase in the retirement age reduces the incentives to use new technology due to the increase in the labor supply. However, in the general equilibrium model, another effect contributes to the result. Indeed, the increase in the retirement age also reduces the need to save which depresses the capital accumulation and further reduces the adoption of new technology.



## 1.5 Optimal Retirement Age

In this section, I analyze the optimal retirement age defined as the retirement age maximizing the aggregate utility of the agents. I, therefore, compare the social planner outcome with the outcome of the competitive equilibrium in order to assess the efficiency of a decentralized policy.

### 1.5.1 Social Planner

The social planner with weights  $\{\rho_t^s\}_{t=0}^\infty$  (where  $\rho^s \in (0, 1)$ ) faces the following maximization problem:

$$\begin{aligned} \max_{\{C_t^y, C_{t+1}^o, K_{t+1}, \phi_{t+1}\}} \quad & \rho_0^s \rho (\log\{C_1^o\} - b \cdot \phi_1) + \sum_{t=1}^{\infty} \rho_t^s (\log\{C_t^y\} + \rho (\log\{C_{t+1}^o\} - b \cdot \phi_{t+1})) \\ \text{s.t.} \quad & Y_t = K_{t+1} + N_t^y \cdot C_t^y + N_t^o \cdot C_t^o, \end{aligned} \tag{1.35}$$

where  $b$  represents the disutility from working when old.<sup>12</sup>

**Proposition 1.5.1.** *The optimal retirement age,  $\phi^s \in (0, 1)$ , is given by:*

$$\phi^s = \begin{cases} \frac{\rho^s + \rho}{\rho b} \frac{\varepsilon(\phi^s)}{1 - \rho^s(1 - \varepsilon(\phi^s))} - (1 + n) & \text{if } N^o < \hat{N}^o \\ \frac{\rho^s + \rho}{\rho b} \frac{\beta(1 - I(\phi^s))}{1 - \rho^s(1 - \varepsilon(\phi^s))} & \text{if } N^o \geq \hat{N}^o. \end{cases} \tag{1.36}$$

Proof in Appendix [A.2.2](#).

We observe that  $\phi^s$  depends on the adoption of new technology which depends on the supply of inputs in the economy and, therefore, on the retirement age itself. This implies that failing to consider the endogeneity of technology would lead to sub-optimal retirement age policies. Since technology adoption is a firm-level decision, it cannot be expected from the social planner with limited information regarding firms' decisions to being able to set the optimal retirement age policy right. We, therefore, analyze whether an efficient equilibrium can be reached in a decentralized economy.

<sup>12</sup>We assume that the disutility from working when young is equal to zero. This can be interpreted as the fact that in this economy the young necessarily need to work as they have no savings yet, while elderly people who already have some savings can decide how much to work.

## 1.5.2 Competitive Equilibrium

In order to evaluate whether the optimal retirement age policy can be reached in a decentralized environment, I compute the competitive equilibrium retirement age solving the consumer utility maximization problem (see Appendix [A.2.3](#)).

**Proposition 1.5.2.** *The competitive equilibrium retirement age,  $\phi^* \in (0, 1)$ , is given by:*

$$\phi^* = \begin{cases} \frac{1+\rho}{\rho \cdot b} - R(\phi^*) & \text{for } N^o < \hat{N}^o(\phi^*) \\ \frac{1+\rho}{\rho \cdot b} \cdot \frac{\beta(1-I(\phi^*)) \cdot (1+n)}{(1+n) \cdot \beta(1-I(\phi^*)) + \alpha I(\phi^*) \cdot R(\phi^*)} & \text{for } N^o \geq \hat{N}^o(\phi^*) \end{cases} \quad (1.37)$$

which is equal to the social optimum in the limit case of  $\rho^s \rightarrow 1$ , and it is increasing in  $N^o$ .

Proof in Appendix [A.2.4](#).

Proposition [1.5.2](#) states that the decentralized equilibrium retirement age is the same as the social planner retirement age in the limit case in which the planner's weights  $\rho^s$  are equal to 1. However, since the social planner problem is not properly defined for  $\rho^s = 1$ , the competitive equilibrium is only efficient up to an infinitesimal efficiency loss, i.e.,  $\phi^* \approx \phi^s(\rho^s = 1 - \epsilon)$ , where  $\epsilon$  is arbitrarily "small". Moreover, the competitive equilibrium retirement age is increasing with aging. As the share of elderly agents increases, so does the supply of capital with negative effects on the interest rates. As the capital income reduces, agents optimally decide to supply more labor.

This result implies that under a fully funded or no-government pension system, allowing workers to choose their retirement age is efficient (up to an infinitesimal efficiency loss), and it dominates the equilibrium with fixed retirement age.

## 1.6 Conclusion

In this paper, I analyze the effect of an aging population on the adoption of new technologies, productivity, and wage dynamics. Empirically, I provide evidence of a non-monotonic relation linking aging and the adoption of new technology (proxied by ICT). In particular, I find that the relation linking demography and the adoption of new technology is positive for low levels of aging and negative for high levels of aging. In line with the idea

that, in the last decades, the increase in productivity has been driven by technological progress, I find a similar non-monotonic relation linking aging and productivity. An aging population also affects wages; I empirically find that an aging population is linked with an increase in the young-to-old wage ratio suggesting that young and old workers are not perfect substitutes.

I rationalize these findings using a theoretical model of technology adoption with two different technologies, new and vintage, and two types of workers, young and old. As I find that the relation linking aging and new technology is driven by Software and DB technology which is labor-saving and requires skills mostly held by young workers, I model the new technology as being labor-saving and complementary to young labor. An aging population has two effects on the labor force: it reduces the labor supply and changes the age composition of workers increasing the share of elderly workers. The reduction in the labor supply initially triggers the adoption of new (labor-saving) technologies. However, since the new technology requires young workers, once the young labor input becomes scarce, further aging reduces the adoption of new technology explaining the reversed-U shaped relation found in the data. The model is also able to replicate the reversed-U shaped relation linking aging and productivity, and wage dynamics.

Endogenizing capital through a standard OLG model, I introduce a further channel in the model going through the accumulation of capital as the population ages. Different from the model with fixed capital, the model with endogenous capital is consistent with a growing economy in the context of an aging population. This theoretical model embeds several channels through which the demographic process affects the adoption of technologies and productivity through the labor market highlighting the relevance of the complementarity between the different labor inputs with the different technologies.

Within this framework, I finally consider the optimal retirement age policy. I find that the optimal retirement age affects technological adoption which implies that failing to consider the endogeneity of technology would lead to sub-optimal retirement age policies. A solution to reach efficiency is to decentralize the retirement age decision.



# Chapter 2

## The effect of a change in the Age Composition of Consumers: a Shift-Share IV empirical approach

### 2.1 Introduction

Over the last 25 years, in the United States, the share of young (0-24) and young adults (25-44) has decreased by 10 percentage points (going from 68% in 1995 to 58% in 2020), while the proportion of middle-aged (45-64) and elderly people (65+) has steadily increased and it is predicted to further increase in the coming decades. Such trends are observable all around the world, in both developed and developing countries.

A growing worry about this phenomenon centers around the possible negative effects of an aging population on production and productivity through the labor market.<sup>1</sup> However, the emphasis given by the literature on the effect of an aging workforce has overshadowed the economic impact through a complementary channel: the change in the age composition of consumers, what we call the *age demand channel*. Age is, indeed, a strong determinant of consumption and purchasing behaviors. People across different age categories have different wealth, preferences, amount of time to devote to searching for goods and prices (Aguiar and Hurst, 2005, 2007), different attachment to brands and propensity

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<sup>1</sup>In this direction, several recent studies have focused on the role of automated technologies in counteracting the shrinking of the labor force and the change in the age composition of the workforce (Acemoglu and Restrepo, 2017b, 2018b; Abeliansky and Prettnner, 2017b; Gehringer and Prettnner, 2019; Abeliansky et al., 2020).

towards comparing brands (Lambert-Pandraud et al., 2005; Lambert-Pandraud and Laurent, 2010) or updating their consumption baskets (Bornstein, 2019).<sup>2</sup> Altogether, these heterogeneous consumption behaviors point towards the fact that the age composition of demand might be a strong determinant of the goods market structure. Given the large demographic transitions at play, the age demand channel might contribute significantly to economic outcomes.

In this paper, we aim to empirically estimate the effect of a change in the age composition of consumers on sector aggregates such as prices, production, and productivity. We face two main hurdles: first, the multitude and the simultaneity of channels through which a demographic change affects an economy; second, the predictability of demographic changes. To address the first, we use a shift-share IV approach and instrument the change in the age composition of demand with foreign demographics. We, therefore, exploit the natural wedge between the age compositions of demand and the age composition of the domestic country. This allows us to disentangle the age demand channel from other domestic channels such as the ones going through the labor market which directly affect the sector aggregates. Using world input-output matrices, we obtain precise measures of the direct and indirect exposure of each sector in each country to the demand of consumers from each foreign trading partner. To tackle the second issue, i.e., the predictability of demographic changes, we consider changes in demographic expectations to recover a measure of unexpected demographic changes.

We find that age categories have heterogeneous effects on sector aggregates through the age demand channel. In particular, we find that an increase in the share of middle-aged consumers increases prices, reduces production and productivity, while young and elderly consumers tend to affect those variables in the opposite direction. These results are quite in contrast with the effects of standard demand shocks that are usually associated with variations in quantities and prices in the same direction. We interpret this negative co-movement as evidence that demographic demand shocks alter the market structure, and that middle-aged consumers reduce, on average, competition, and, vice versa, young and old consumers increase competition.

Our findings can be interpreted as the result of distinct mechanisms already analyzed

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<sup>2</sup>The marketing and psychology literature investigates other dimensions such as the different levels of cognitive abilities influencing the way agents process information and compare goods (Gutchess, 2011; Peters, 2011), the different size of social networks (East et al., 2014), and the heterogeneity of usage rate and mastering of technologies to search and make purchases (Hargittai, 2001).

in the literature. While young consumers tend to increase competition in the market as they are less loyal to the brand as shown in [Bornstein \(2019\)](#), elderly consumers increase competition because they have more time to search for goods ([Aguiar and Hurst, 2007](#)). An increase in competition can, then, lead to an increase in productivity as empirically shown in [Symeonidis \(2008\)](#).

Our paper greatly differs from previous literature. Indeed, while [Aguiar and Hurst \(2007\)](#) focuses on the micro evidence linked to the age heterogeneity of consumers, we provide evidence of the effects of the age heterogeneity of consumers at the macro level. In particular, [Aguiar and Hurst \(2007\)](#) shows that elderly consumers have a lower opportunity cost of time which allows them to search more for goods and to face lower prices but they do not study the macro implications of age heterogeneity. We, instead, provide evidence that the macro effects of micro-heterogeneity across age categories are relevant as the demographic change affects the market structure and triggers supply side reactions with effects on sector aggregates. Moreover, while [Bornstein \(2019\)](#) shows that aging of the population leads to a reduction in competition because of the higher consumer inertia of middle-aged and old consumers, our results suggest that the effect of aging consumers on competition is not linear. Although the conclusions we draw are different with respect to [Bornstein \(2019\)](#), the results are not in contrast with its findings. Indeed, the baby boomer generation (born in the 1950s and 1960s) became middle-aged in the 1990s and 2000s reducing competition and increasing the markups in those years which is both in line with our results and the results in [Bornstein \(2019\)](#). Differently from [Bornstein \(2019\)](#), however, our results suggest that as the baby boomers grow old, the effect on competition reverses. Finally, previous literature analyzing the macro consequences of an aging population has mostly focused on the effect of aging on production and productivity through the labor market and the adoption of technology on the supply side of the economy ([Acemoglu and Restrepo, 2017b, 2018b](#); [Abeliansky and Prettnner, 2017b](#); [Gehring and Prettnner, 2019](#); [Abeliansky et al., 2020](#)). To the best of our knowledge, our paper is the first to provide empirical evidence of the effects of an aging population on aggregate variables through the demand side of the economy highlighting the relevance of this novel age demand channel.

The rest of the chapter is structured as follows: in the next section, we present the data used in the analysis; in Section 3, we explain the empirical methodology employed.

In Section 4, we present and discuss the results, while we perform some robustness checks in Section 5. Finally, Section 6, concludes.

## 2.2 Data

To perform our empirical analysis, we use the World Input-Output Database (WIOD, 2013 release) which provides sector-level statistics on value-added, hours worked, price deflator, and input-output tables with information on trade links at the country-sector level. The WIOD covers the 27 EU countries, 13 other major countries in the world for the period from 1995 to 2009, and it provides estimates for the rest of the world.<sup>3</sup> The dataset provides information over 35 sectors.

		Input use & value added								Final use			Total use
		Country 1				Country $J$				Country 1	...	Country $J$	
		Industry 1	...	Industry $S$	...	Industry 1	...	Industry $S$	...				
Intermediate inputs supplied	Country 1	Industry 1	$Z_{11}^{11}$	...	$Z_{11}^{1S}$	...	$Z_{11}^{1J}$	...	$Z_{11}^{1S}$	$F_{11}^1$	...	$F_{1J}^1$	$Y_1^1$
		...	...	$Z_{11}^{rS}$	...	...	$Z_{1J}^{rS}$	...	...	...	...	...	...
		Industry $S$	$Z_{11}^{S1}$	...	$Z_{11}^{SS}$	...	$Z_{11}^{S1}$	...	$Z_{11}^{SS}$	$F_{11}^S$	...	$F_{1J}^S$	$Y_1^S$
	...	...	...	...	$Z_{1j}^{rS}$	...	...	...	...	$F_{1j}^r$	...	$Y_1^r$	
Country $J$	Industry 1	$Z_{j1}^{11}$	...	$Z_{j1}^{1S}$	...	$Z_{j1}^{1J}$	...	$Z_{j1}^{1S}$	$F_{j1}^1$	...	$F_{jJ}^1$	$Y_j^1$	
	...	...	...	$Z_{j1}^{rS}$	...	...	$Z_{jJ}^{rS}$	...	...	...	...	...	
Industry $S$	$Z_{j1}^{S1}$	...	$Z_{j1}^{SS}$	...	$Z_{j1}^{S1}$	...	$Z_{j1}^{SS}$	$F_{j1}^S$	...	$F_{jJ}^S$	$Y_j^S$		
Value added		$VA_1^1$	...	$VA_1^S$	$VA_j^S$	$VA_1^J$	...	$VA_j^S$					
Gross output		$Y_1^1$	...	$Y_1^S$	$Y_j^S$	$Y_1^J$	...	$Y_j^S$					

Figure (2.1) World Input Output Table

The structure of the input-output tables is represented in Figure 2.1. Each row represents the country-sector supplying the input, while the columns represent the country-sector pairs buying the inputs to be used in the production process. Each cell represents the amount traded in terms of value-added. The last columns, “Final use” and “Total use”, represent the amount produced by each country-sector which is consumed by each country and the total production of the country-sector respectively.<sup>4</sup> Using the

<sup>3</sup>Besides the 27 EU countries, the WIOD dataset contains information for Australia, Brazil, Canada, China, Indonesia, India, Japan, South Korea, Mexico, Russia, Turkey, Taiwan, and the United States. In our analysis, we use all countries and the proxy for the rest of the world. Since the UN World Population Prospect does not provide population prospects for Taiwan, we assume no demographic change in Taiwan (we cannot directly remove the observation as we need the entire input-output matrix to estimate the exposure to demand measures).

<sup>4</sup>Several variables compose the “Final use” columns (i.e. “Final consumption expenditure by households”, “Final consumption expenditure by non-profit organizations serving households (NPISH)”, “Final consumption expenditure by the government”, “Gross fixed capital formation”, “Changes in inventories and valuables”). Coherently with the scope of our analysis, we focus on the “Final consumption expenditure by households” variable.



input-output tables allows us to capture the interdependence coming from the integrated production structure of the world's economies.

To recover the demographic variables, we use the UN World Population Prospects (WPP, 1996, 2006, and 2019 revisions) which provide population data and population projection estimates in 1996, 2006, and 2019 of the age composition of the population in future years. We use these datasets to recover measures of the actual demographic change (WPP 2019), and of the unexpected demographic changes (WPP 1996 and 2006).

The aggregated nature of our dataset limits our analysis as we can only estimate the average effects at the sector level. The dataset also lacks direct information regarding the age composition of demand at the sector level. However, by weighing our demographic variables by the exposure of each country-sector to the different countries, we obtain sector-level demographic measures.

## 2.3 Methodology

In this section, we present the empirical methodology. We start by showing how we estimate the exposure to the demand of each country-sector taking into account both the direct and indirect exposure, i.e. considering all the intermediate trades between the initial producer and the final consumer. We, then, present the OLS regression using the change in the age composition of demand as our regressor of interest defined as the average share of different age categories weighted by the country-sector specific exposure to domestic and foreign economies. Since the age composition of demand is necessarily correlated with the age structure of the workforce which directly affects sector aggregates, we adopt a shift-share IV approach instrumenting the change in the age composition of demand with the age composition of foreign demand. We, therefore, exploit the natural wedge existing between the age structure of consumers (which depends on both domestic and foreign demographics) and the age structure of workers (which only depends on domestic demographics) to disentangle the effect of a demographic change coming from the demand side of the economy. Finally, in order to ensure the independence between the shifts (the demographic variable), and the shares (the exposure to demand), we present a second shift-share IV model in which we consider unexpected demographics instead of actual demographic changes. Since the unit of observation of our analysis is at the country-

sector level, if not otherwise specified, the observations in all the regressions we present are weighted by the relative value-added of each country-sector.

### 2.3.1 Estimation of the country-sector exposure to demand

Countries are very interconnected as a large part of global trade, in terms of intermediate inputs, follows the global value chains of production. This implies that to estimate the sector exposure to demand one needs to track down all the intermediate trades occurring from the initial producer to the final consumer, considering both the direct and the indirect sales.

Consider, for example, a German car producer using tires produced in Italy. Assume the German producer sells its cars in France. By only considering the direct sale, we would not capture the exposure of the Italian tire producer to French demand. This simple example shows, particularly for very interconnected countries, the necessity of taking into account also indirect sales to retrieve the correct measure of final demand exposure. Taking into account those indirect flows also appears to be of primary importance as [Ferrari \(2019\)](#) shows that demand shocks magnify upstream through the global value chain.

We define the share of output of sector  $s$  in country  $i$  that is consumed by country  $j$  as in [Ferrari \(2019\)](#):

$$\xi_{i,j}^s \equiv \frac{F_{i,j}^s + \sum_r \sum_k a_{i,k}^{s,r} F_{k,j}^r + \sum_r \sum_k \sum_g \sum_m a_{i,k}^{s,r} a_{k,m}^{r,g} F_{m,j}^g + \dots}{Y_i^s}, \quad (2.1)$$

where the first term in the numerator represents the output produced by sector  $s$  in country  $i$  which is directly sold for consumption in country  $j$ ; the second term represents the fraction of output produced by sector  $s$  in country  $i$  sold to any producer  $r$  in any country  $k$  which is then consumed in country  $j$ . The same logic applies to higher order terms.

We compute the exposure of each sector  $s$  in country  $i$  to country  $j$  making use of the Leontief inverse matrix  $(I-A)^{-1}$ . We retrieve the Leontief inverse matrix from the Leontief input-output model relating production ( $Y$ ) with final demand ( $F$ ):  $Y = AY + F$ , where  $A \equiv [a_{i,j}^{s,r}] \equiv [Z_{i,j}^{s,r}/Y_i^s]$  is the input-output matrix. The Leontief inverse matrix relates the exposure share matrix  $[\xi_{i,j}^s]$  to the final demand matrix  $[f_{i,j}^s] \equiv [F_{i,j}^s/Y_i^s]$  that shows the

amount of output produced by sector  $s$  in country  $i$  consumed or invested in country  $j$  as a share of the total output produced by sector  $s$  in country  $i$ .<sup>5</sup> We, therefore, estimate the exposure share matrix using the following formula:

$$[\xi_{i,j}^s] = (I - A)^{-1} \cdot [f_{i,j}^s]. \quad (2.2)$$

### 2.3.2 OLS

In order to estimate the effect of a change in the age composition of consumers on sector aggregates (price, production, and productivity), we use a long difference fixed effect specification as in Acemoglu and Restrepo (2018b). We, therefore, estimate the following model:

$$\Delta \log Y_i^s = \beta_0 + \beta_1 \cdot \Delta \text{AgeDemand}_i^s + \beta_2 \cdot \text{ForeignExposure}_i^s + \gamma_i + \gamma_s + \epsilon_i^s, \quad (2.3)$$

where the left-hand side variables are, in turn, the log difference in sector aggregate prices, production, and productivity between 1996 and 2006.<sup>6</sup> We use the value-added price deflator as a proxy for prices which measures prices from the perspective of the producers and thus, contrary to the consumer price index (CPI), does not depend on the basket of goods and services purchased by the consumers. We use the real value-added and the real value-added per hour as proxies for production and productivity respectively. We use measures built on the value-added rather than on the gross output as the value-added measures better reflect the underlying technology of the processing industry and are in line with the definition of the shares built using the input-output tables showing the amount traded in terms of value-added. The regressor of interest is the change in the age composition of demand and it is defined as the difference between the average demographics weighted by the exposure of each country-sector to each country's demand

<sup>5</sup>The Leontief inverse matrix  $(I - A)^{-1}$  has dimension  $(S \times J)$  by  $(S \times J)$ ; the exposure share matrix  $[\xi_{i,j}^s]$  has dimension  $(S \times J)$  by  $J$ ; the input-output matrix  $A$  has dimension  $(S \times J)$  by  $(S \times J)$ ; the final demand matrix  $[f_{i,j}^s] \equiv [F_{i,j}^s/Y_i^s]$  has dimension  $(S \times J)$  by  $J$  where  $S$  and  $J$  represents the number of sectors and countries respectively.

<sup>6</sup>We consider the longest available time span in the WIOD 2013 release excluding the recession years (from 2007 on); we consider 1996 instead of 1995 (the first year available in the WIOD 2013 release) in order to consistently define the dependent variables across the different model specifications as the WPP dataset (that we use in the IV(2) specification) is available for the year 1996 but not for the year 1995.

between 1996 and 2006:

$$\Delta AgeDemand_i^s = \sum_j \xi_{i,j,2006}^s \cdot age_{j,2006} - \sum_j \xi_{i,j,1996}^s \cdot age_{j,1996}, \quad (2.4)$$

where  $age_j \in \{young_j, middle-aged_j, old_j\}$ . We define  $young_j$  as the share of population between 25 and 44, relative to the total population between 25 and 79 in country  $j$ . Similarly, we define  $middle-aged_j$  as the share of population between 45 and 64, and  $old_j$  as the share of population between 65 and 79. We exclude younger and older consumers (<25 and 80+) as we focus on independent consumers.<sup>7</sup>

To control for the heterogeneous exposure to foreign demand, we include in the regression the exposure to foreign demand ( $ForeignExposure_i^s$ ) at the initial period (1996) defined as the sum of the exposure of each country-sector with respect to foreign countries, i.e.  $\sum_{j \neq i} \xi_{i,j,1996}^s$ .

Finally, in order to control for sector-specific and country-specific trends, we include country and sector dummies ( $\gamma_i$  and  $\gamma_s$  respectively). In particular, sector dummies control for heterogeneous sector-specific technological developments, while country dummies control for institutional features at the country level.

### 2.3.3 Shift-Share IV

The biggest issue in the OLS model in section 2.3.2 is that the  $\Delta AgeDemand$  variable is necessarily correlated with the change in the age composition of workers. As argued in Acemoglu and Restrepo (2018b), indeed, the age composition of the labor force is a strong predictor of technology adoption that directly influences sector aggregates such as price, production, and productivity.

In order to identify the age demand channel (the effect of a demographic change going through the demand side of the economy), we instrument the  $\Delta AgeDemand$  variable in equation (2.3) with foreign demographics. Foreign demographics, indeed, affect the age structure of demand through exports and the intermediate trades along the global value chain (*relevance* of the instrument) without changing the age composition of the labor force. We, therefore, exploit the wedge between the demographic structure of de-

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<sup>7</sup>Younger consumers (<25) might still have no income and depend on their parents regarding their consumption decisions; older consumers (80+) might delegate their consumption decision choices to someone else (think, for instance, of the elderly in nursing homes).

mand (which depends on both domestic and foreign demographics), and the effect of a demographic change going through the labor market (depending only on domestic demographics) to disentangle the demand channel from other domestic channels (*exogeneity* of the instrument).

We estimate the instruments, using a shift-share IV approach. We follow the methodology described in [Borusyak et al. \(2018\)](#); [Goldsmith-Pinkham et al. \(2020\)](#) and [Adão et al. \(2019\)](#) requiring either exogenous shares (the exposure to foreign demand) or exogenous shifts (the change in the foreign demographics). Since the exposure to foreign demand is likely endogenous as firms choose the sectors and the countries to trade with, identification can be obtained through the exogeneity assumption of the shifts. However, when firms decide which market to link, expectations regarding future demographics are likely already in firms' information set. The usual strategy to avoid this concern is to use lagged shares. Following this strategy, we fix the shares at the initial period as in [Huneus \(2018\)](#) and [Ferrari \(2019\)](#).

We consider two different IV models. In the first model, IV(1), we define the shifts as the actual demographic changes between 1996 and 2006. However, demographic changes are largely predictable and an expected change in the age composition of foreign consumers might lead to an endogenous change even in the lagged shares. Therefore, in the second model, IV(2), we use the unexpected demographics defined as the change in demographic expectations for the year 2006 between 1996 and 2006. [Figure 2.2](#) shows the unexpected demographics for the countries considered in the analysis and the proxy for the rest of the world (RoW). The figure shows that there is large heterogeneity in prediction errors for most of the age categories guaranteeing sufficient variation in the shocks.<sup>8</sup>

A possible caveat in this approach is that, since consumers across different age categories have different preferences, as the age composition of consumers changes, the consumption basket of consumers changes as well. This could lead to a change in the consumption of domestic and foreign goods influencing the exposure of each country-sector to foreign demand. Think, for example, about elderly consumers that consume more health-related services that are usually domestically produced. In this case, the effect of a change in the age composition of foreign demand would be confounded with the effect of a change in the exposure to foreign markets. However, we find that there is no corre-

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<sup>8</sup>Figure [B.1](#) in the Appendix shows the same unexpected demographics with aggregate age categories as defined in the empirical specification.

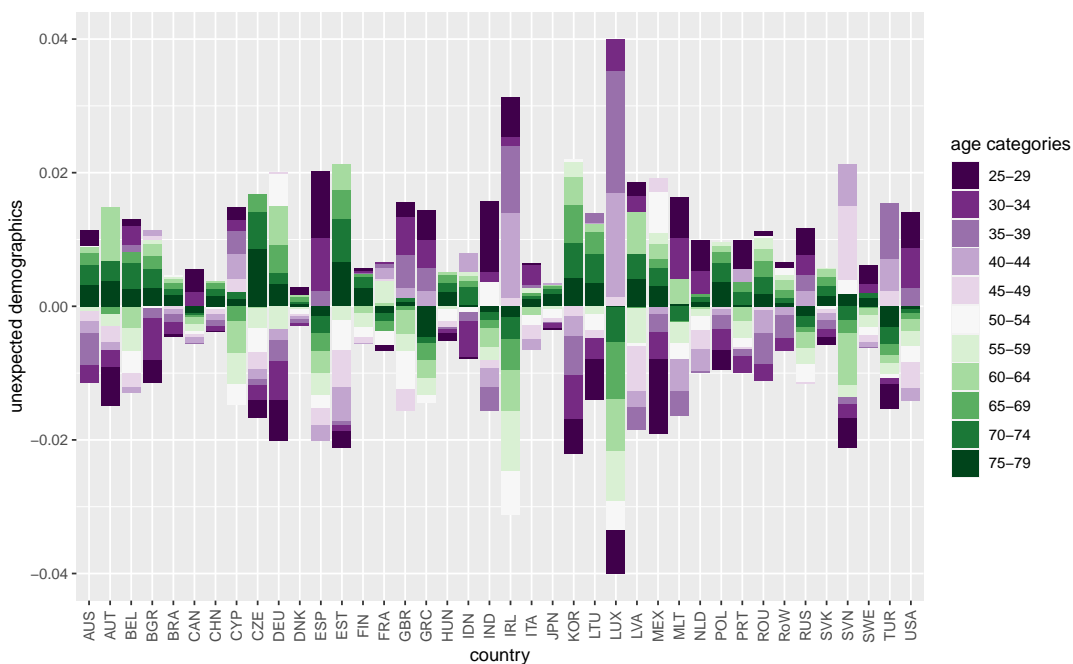


Figure (2.2) Unexpected demographics (5-years age categories).

lation between the change in domestic demographics and the change in the exposure to foreign demand (see Table [B.3](#) in the Appendix). This is probably due to our definition of exposure to demand that takes into account not only the direct exposure to demand but also all the intermediate trades along the value chain. Indeed, what is required in our analysis is that the consumption basket in terms of inputs used to produce the final goods (instead of the final goods consumption basket) does not vary across age categories. The fact that there is no correlation between domestic demographics and exposure to foreign demand provides indirect evidence of that.

Finally, we cluster standard errors by the main exposure country (i.e. the country to which each country-sector exports the most taking into account both the direct and the indirect exposure). As pointed by [Adão et al. \(2019\)](#), not clustering the errors would lead to over-rejection of the coefficient estimates. The over-rejection problem arises because the regression residuals are most likely correlated in clusters as country-sectors selling to the same countries are subject to the same shocks. By clustering with respect to the main exposure country, we control, at least partially, for such a correlation.

#### IV(1)

In this first IV model, we instrument the  $\Delta AgeDemand$  variable in equation [\(2.3\)](#) with a measure of foreign demographic change. We, therefore, estimate the following first stage

model:

$$\Delta AgeDemand_i^s = \alpha_0 + \alpha_1 \cdot \Delta ForeignAgeDemand_i^s + \alpha_2 \cdot ForeignExposure_i^s + \gamma_i + \gamma_s + \nu_i^s \quad (2.5)$$

where  $\Delta ForeignAgeDemand_i^s$  is the shift-share instrument defined as the average change in demographics between 1996 and 2006 (the shifts) weighted by the exposure of each country-sector with respect to foreign demand at the initial period (the shares), i.e.:

$$\Delta ForeignAgeDemand_i^s \equiv \sum_{j \neq i} \xi_{i,j,1996}^s \cdot [age_{j,2006} - age_{j,1996}]. \quad (2.6)$$

As in equation (2.4), we define  $age_j$  as the share in country  $j$  of young (24-44), middle-aged (45-64), and old (65-79) in the population between 25 and 79. The rest of the controls are defined as for equation (2.3):  $ForeignExposure_i^s$  is the sum of the exposures to foreign economies controlling for the heterogeneity of the exposure to foreign demand across country-sectors;  $\gamma_i$  and  $\gamma_s$  are the country and sector dummies respectively controlling for country and sector trends.<sup>9</sup>

In order to ensure the exogeneity of the instrument with respect to the dependent variables in this framework, we require **a**) the exogeneity of the shifts with respect to the dependent variables (which in our framework boils down to the assumption that foreign demographics is exogenous with respect to domestic price, production, and productivity), and **b**) the independence of the shifts with respect to the shares. In order to ensure the independence of the shifts with respect to the shares, it is standard in shift-share literature to use lagged shares which shields the results from contemporaneous co-movements of the shifts and the shares. However, in our framework, this strategy might not be enough to guarantee the independence between the shifts and the shares as demographic changes are highly predictable. It could be, therefore, that demographic expectations affect the lagged exposure shares. Although very small, we find, indeed, a correlation between the shifts and the shares as defined in this framework (see Table B.2 in the Appendix). Therefore, we build a second IV model in which, instead of considering the actual demographic

<sup>9</sup>When estimating a shift-share IV with incomplete shares (as in our case in which we exclude the domestic economy) such that the sum of the shares does not sum up to 1, it is necessary to control for the sum of the exposure shares (in our case  $ForeignExposure_i^s$ ). As argued in Borusyak et al. (2018), not controlling for the sum of the exposure shares would lead to biased results.

changes as shifts, we consider the unexpected demographics.

## IV(2)

In this second IV model, we instrument the  $\Delta AgeDemand$  variable in equation (2.3) with a measure of the unexpected foreign demographics. We, therefore, estimate the following first stage model:

$$\Delta AgeDemand_i^s = \eta_0 + \eta_1 \cdot UnexpectedForeignAgeDemand_i^s + \eta_2 \cdot ForeignExposure_i^s + \gamma_i + \gamma_s + \nu_i^s, \quad (2.7)$$

where  $UnexpectedForeignAgeDemand_i^s$  is the shift-share instrument defined as the average unexpected foreign demographics (the shifts) weighted by the initial exposure of each country-sector with respect to foreign demand (the shares). In particular, we define the unexpected foreign demographics as the difference between the actual demographics in 2006 and the expectations in 1996 regarding demographics in 2006. Therefore:

$$UnexpectedForeignAgeDemand_i^s \equiv \sum_{j \neq i} \xi_{i,j,1996}^s \cdot [age_{j,2006} - \mathbb{E}_{1996}(age_{j,2006})]. \quad (2.8)$$

By fixing the shares at the initial period (1996) and considering the change in expectations between 1996 and 2006 regarding demographics in 2006, we remove the predicted part of demographics in 1996 making the shifts and the shares mechanically independent one another. This provides the grounds for a causal identification of the effects on the variable of interest (Borusyak et al., 2018).

## 2.4 Results

In this section, we analyze the results of our empirical analysis. We present, in turn, the results of a change in the age composition of demand on prices, production, and productivity across the different specifications presented above. Then, in the context of the IV(2) model specification, we consider a change in expectations regarding future demographics.



### 2.4.1 Prices

Table 2.1 shows the effect of a change in the age composition of demand on prices defined as the value-added price deflator of the three different models we have specified above. Columns (1), (2), and (3) report the coefficient estimates of the OLS model specification for young, middle-aged, and old consumers respectively. We find that an increase in the share of middle-aged consumers is associated with a significant increase in prices, while an increase in the share of elderly consumers is associated with a significant reduction in prices.

Table (2.1) Price

<i>share:</i> <i>shift:</i>	$\Delta \log$ Price (1996-2006)								
	<i>OLS</i>			<i>IV(1)</i>			<i>IV(2)</i>		
	(1)	(2)	(3)	Foreign Exposure $\Delta$ Demographics (1996-2006)			Foreign Exposure $\Delta$ Expectations (1996-2006)		
$\Delta$ Young Demand	1.034 (0.670)			-9.946 (8.959)			-12.859 (14.195)		
$\Delta$ Middle-aged Demand		3.162*** (0.750)			4.247 (2.895)			10.212** (4.440)	
$\Delta$ Old Demand			-4.437*** (0.939)			-4.843** (2.110)			-14.597* (8.571)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R <sup>2</sup>	0.884	0.885	0.885	0.859	0.885	0.885	0.845	0.877	0.875
Adjusted R <sup>2</sup>	0.877	0.878	0.879	0.851	0.878	0.879	0.835	0.870	0.868

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Columns (4) to (6) report the results for the IV(1) specification in which actual demographic changes are considered. We find that the coefficient estimate for middle-aged consumers is similar to the estimate of the OLS model. However, the estimate is not significant. The coefficient estimate for the elderly consumers in the IV(2) model is, instead, significant and virtually identical to the estimate in the OLS model.

Columns (7) to (9) report the coefficient estimates for the IV(2) specification in which the unexpected demographics is considered. Consistently with the other model specifications, we find that middle-aged consumers are associated with higher prices, while elderly consumers are associated with lower prices. Differently from the OLS and the IV(1) specifications, the coefficients in the IV(2) model are substantially higher in magnitude indicating the possibility that there is an endogenous response in the shares to predicted demographics. We also find that young consumers appear to be associated with lower

prices. However, standard errors are large and the coefficient estimate is not significant.

These results generally suggest that middle-aged consumers are associated with an increase in prices, while elderly consumers are associated with a reduction in prices. In particular, one standard deviation increase (2.20%) in the share of middle-aged consumers is associated with an increase in prices between 0.70% (OLS specification) and 2.28% (IV(2) specification) per year in the period between 1996 and 2006, while an increase in one standard deviation of elderly consumers (1.45%) is associated with a reduction in prices between 0.64% (OLS specification) and 2.10% (IV(2) specification) per year in the same period.<sup>10</sup>

## 2.4.2 Production

Table 2.2 shows the effect of a change in the age composition of demand on production defined as the real value-added. Columns (1) to (3) report the coefficient estimates for the OLS model specification. We find a negative and significant effect of an increase in the share of middle-aged consumers on production, while the coefficient estimates for young and old consumers are not significantly different from zero.

Columns (4) to (6) report the coefficient estimates for the IV(1) model specification. Although the point estimate for middle-aged consumers is similar to that of the OLS specification, standard errors increase and the coefficient estimate is not significantly different from zero. We also find that the coefficient estimates for young and old increase in magnitude relative to the OLS estimates suggesting a positive association between the share of young and old consumers and production. However, due to large standard errors, the estimates are not significant.

Columns (7) to (9) report the coefficient estimates for the IV(2) model specification. Consistently with the other models, an increase in the share of middle-aged consumers is

<sup>10</sup>We have computed the average annual changes in prices using the following formula:

$$100 \times \left( 1 + \frac{\% \Delta y}{100} \right)^{\frac{1}{2006-1996}} - 100, \quad (2.9)$$

where  $\% \Delta y$  is the cumulative percentage change in price (over the period 1996-2006) which is computed as:

$$\% \Delta y = 100 \cdot (e^{\hat{\beta}_1 \cdot \Delta x} - 1), \quad (2.10)$$

where  $\hat{\beta}_1$  is the coefficient estimates for the different model specifications in Table 2.1 and  $\Delta x$  is set to be equal to one standard deviation in the domestic change between 1996 and 2006 of the share of the age category analyzed. Similarly, we estimate the average annual changes for production and productivity using the coefficient estimates in Tables 2.2 and 2.3.

Table (2.2) Production

<i>share: shift:</i>	$\Delta \log$ Production (1996-2006)								
	OLS			IV(1) Foreign Exposure $\Delta$ Demographics (1996-2006)			IV(2) Foreign Exposure $\Delta$ Expectations (1996-2006)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\Delta$ Young Demand	0.029 (0.804)			10.339 (7.483)			12.395* (6.920)		
$\Delta$ Middle-aged Demand		-3.143*** (0.901)			-3.647 (2.545)			-8.308*** (2.878)	
$\Delta$ Old Demand			0.838 (1.135)			2.685 (4.131)			19.627 (20.029)
Initial Foreign Exposure Country & Sector FE	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R <sup>2</sup>	0.578	0.582	0.578	0.524	0.582	0.577	0.500	0.571	0.487
Adjusted R <sup>2</sup>	0.553	0.557	0.553	0.496	0.557	0.552	0.470	0.546	0.457

Note:

\*p&lt;0.1; \*\*p&lt;0.05; \*\*\*p&lt;0.01

associated with a reduction in production. The coefficient estimate is highly significant as the coefficient becomes substantially more negative. Moreover, we find a positive and significant association between young consumers and production. We also find a positive association between old consumers and production. However, although the point estimate is large, the estimate is not significant due to the large standard error.

These results suggest that there is a positive association between young and old consumers and production (although not significant for old consumers) and a negative and significant relation between middle-aged and production. In particular, one standard deviation increase (2.20%) in the share of middle-aged consumers is associated with a reduction in production between 0.69% (OLS specification) and 1.81% (IV(2) specification) per year in the period between 1996 and 2006.

### 2.4.3 Productivity

Table [2.3](#) shows the effect of a change in the age composition of demand on productivity defined as the real value-added per hour worked. Columns (1) to (3) report the coefficient estimates of the OLS model specification. We find that the coefficient estimate for middle-aged consumers is negative and significant, while it is positive and significant for old consumers.

Columns (3) to (6) report the coefficient estimates for the IV(1) specification. Consistently with the OLS model specification, the coefficient estimates for middle-aged (old)

Table (2.3) Productivity

<i>share:</i> <i>shift:</i>	$\Delta \log$ Productivity (1996-2006)								
	<i>OLS</i>		<i>IV(1)</i>			<i>IV(2)</i>			
	(1)	(2)	(3)	Foreign Exposure $\Delta$ Demographics (1996-2006)		Foreign Exposure $\Delta$ Expectations (1996-2006)			(9)
$\Delta$ Young Demand	0.955 (0.869)			12.792 (13.172)			28.810** (14.553)		
$\Delta$ Middle-aged Demand		-5.239*** (0.968)			-7.261 (5.083)			-19.170*** (3.726)	
$\Delta$ Old Demand			2.531** (1.226)			11.732 (8.593)			46.126 (48.222)
Initial Foreign Exposure Country & Sector FE	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R <sup>2</sup>	0.517	0.527	0.518	0.447	0.526	0.497	0.128	0.450	0.040
Adjusted R <sup>2</sup>	0.488	0.499	0.490	0.414	0.498	0.467	0.077	0.418	-0.016

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

consumers show a negative (positive) association with productivity. However, due to an increase in the standard errors, the coefficient estimates are not significant.

Columns (7) to (9) show the coefficient estimates for the IV(2) model specification. Consistently with the OLS and the IV(1) models, middle-aged consumers are associated with a reduction in productivity and the coefficient estimate is highly significant. Differently from the OLS and IV(1) models, the point estimate is substantially higher. Consistently with the IV(1) model, we also find that young consumers are associated with higher productivity. Differently from the IV(1) model, the coefficient estimate is significant due to an increase in the point estimate. Moreover, consistently with both the OLS and the IV(1) models, the coefficient estimate for old consumers is positive. However, it is not significant due to the large standard error.

These results generally suggest a positive association between young and old consumers with productivity (although not significant for old consumers), and a negative association between middle-aged consumers and productivity. In particular, we find that one standard deviation increase (2.20%) in the share of middle-aged consumers is associated with a reduction in productivity between 1.15% (OLS specification) and 4.14% (IV(2) specification) per year in the period between 1996 and 2006.

Table (2.4) IV(2) model with demographic expectations for years 2010, 2015, 2020

<i>expectations for year:</i>	2010			2015			2020		
	$\Delta \log$ Price (1996-2006)								
$\Delta$ Young Demand	-12.110 (13.658)			-11.340 (13.157)			-10.553 (12.649)		
$\Delta$ Middle-aged Demand	9.279** (4.095)			9.615** (4.056)			7.511** (3.096)		
$\Delta$ Old Demand			-14.041* (7.770)			-13.001* (7.342)			-13.960** (6.400)
Initial Foreign Exposure Country & Sector FE	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R <sup>2</sup>	0.849	0.879	0.876	0.853	0.878	0.878	0.856	0.882	0.876
Adjusted R <sup>2</sup>	0.840	0.872	0.869	0.844	0.871	0.871	0.848	0.875	0.869
	$\Delta \log$ Production (1996-2006)								
$\Delta$ Young Demand	12.069* (6.748)			11.483* (6.442)			11.771* (6.549)		
$\Delta$ Middle-aged Demand	-7.849*** (2.252)			-7.249*** (2.096)			-5.340*** (1.496)		
$\Delta$ Old Demand			17.847 (18.286)			19.134 (20.468)			18.880 (19.576)
Initial Foreign Exposure Country & Sector FE	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R <sup>2</sup>	0.504	0.573	0.504	0.511	0.575	0.492	0.507	0.580	0.495
Adjusted R <sup>2</sup>	0.475	0.548	0.475	0.482	0.550	0.462	0.478	0.555	0.465
	$\Delta \log$ Productivity (1996-2006)								
$\Delta$ Young Demand	27.689* (14.283)			26.073* (13.457)			24.078* (12.597)		
$\Delta$ Middle-aged Demand	-17.808*** (3.523)			-16.267*** (3.404)			-10.423*** (3.738)		
$\Delta$ Old Demand			41.492 (43.199)			43.904 (47.284)			39.166 (40.524)
Initial Foreign Exposure Country & Sector FE	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R <sup>2</sup>	0.159	0.465	0.136	0.201	0.479	0.088	0.249	0.517	0.181
Adjusted R <sup>2</sup>	0.110	0.433	0.086	0.154	0.448	0.034	0.205	0.488	0.132

Note:

\*p&lt;0.1; \*\*p&lt;0.05; \*\*\*p&lt;0.01

### 2.4.4 Change in expectations regarding future demographics

In the setup of the IV(2) model, we now consider the change in expectations regarding future demographics. In particular, we consider a change in expectations between 1996 and 2006 regarding demography in the year 2010, 2015, and 2020. We, therefore, define the instrumental variable as:

$$UnexpectedForeignAgeDemand_i^s \equiv \sum_{j \neq i} \xi_{i,j,1996}^s \cdot [\mathbb{E}_{2006}(age_{j,t}) - \mathbb{E}_{1996}(age_{j,t})], \quad (2.11)$$

where  $t \in \{2010, 2015, 2020\}$ . Table 2.4 shows that the results obtained considering the change in expectations with respect to future demographics are fully consistent with the results of the baseline IV(2) model. Indeed, we find that all the significant coefficient estimates in the baseline IV(2) model remain significant when considering expectations regarding future demographics. The point estimates, also, are very similar and declining as we consider demographic expectations further in the future. Intuitively, this means that the economy responds stronger to demographic unexpected changes that are closer in time.

### 2.4.5 Discussion

To summarize our results, we find significant effects of the age composition of consumers on sector aggregates such as prices, production, and productivity. In particular, we find strong evidence that middle-aged consumers are associated with an increase in prices and a reduction in both production and productivity. Although not all the specifications considered are significant, the point estimates for young and old consumers suggest that these consumers tend to be associated with lower prices and higher production and productivity.

These results mean that a change in the age composition of consumers leads to a non-standard demand shock. Indeed, while standard demand shocks are characterized by a positive co-movement between prices and quantities, we observe a negative co-movement between these variables. This negative co-movement suggests that a change in the age composition of consumers can act as a competition shock altering the market structure. In particular, our results suggest that an increase in the share of middle-aged consumers increases competition in the market which leads firms to produce less and increase the prices. This is in line with Bornstein (2019) showing that, compared to young consumers,

middle-aged consumers re-optimize their consumption basket less often making it more difficult for entrants to establish a customer base and, therefore, reducing competition in the market.

Our results, however, contrast with the conclusion of [Bornstein \(2019\)](#) in which it is suggested that an aging society leads to lower competition. Indeed, we find evidence that elderly consumers are associated with lower prices and higher production suggesting that an increase in the share of the elderly consumers increases competition leading firms to produce more and to sell at lower prices. This result is consistent with the findings in [Aguilar and Hurst \(2007\)](#) showing that elderly consumers tend to face lower prices as they have a lower opportunity cost and, therefore, lower search costs.

We believe, therefore, that our findings are the results of different mechanisms. Relative to middle-aged consumers, young consumers (who quickly update their consumption basket and are less loyal to brands) tend to increase competition in the market ([Bornstein, 2019](#)). Elderly consumers, as well, tend to increase competition in the market because of their lower opportunity cost of time ([Aguilar and Hurst, 2007](#)) and search costs. Middle-aged consumers who, instead, are more loyal to the brand compared to young consumers and have a higher opportunity cost of time compared to elderly consumers, tend to reduce competition. This results in higher prices and lower production as the share of middle-aged consumers increases. The change in the market structure also affects productivity. Using a quasi-natural experiment, indeed, [Symeonidis \(2008\)](#) provides evidence of a negative effect of collusion (which can be interpreted as a reduction in competition) on labor productivity.

## 2.5 Robustness Checks

In this section, we perform a series of robustness checks. In particular, we run our baseline regression models changing the age category thresholds, [25-39, 40-59, 60-79] instead of [25-44, 45-64, 65-79]. We, then also consider a different dataset (the WIOD 2016 release) and estimate the baseline regression models in the period 2006-2014 instead of 1996-2006.

### 2.5.1 Age definition

When we consider different age specifications changing the definition of young (25-39), middle-aged (40-59), and old (60-79), although less significant, results are generally robust. Tables [B.5](#), [B.6](#), and [B.7](#) in the Appendix show the results for prices, production and productivity respectively. Although not significant, the coefficient estimates for the OLS and the IV(1) models have the same sign of the estimates of the respective baseline models with the baseline age definitions.<sup>[11](#)</sup>

The coefficient estimates for the IV(2) are consistent with those of the baseline model. In particular, middle-aged consumers are associated with higher prices, lower production, and lower productivity. While for both prices and productivity the middle-aged coefficient estimates are significant, for production the estimate is not significantly different from zero. However, the point estimate is negative and similar to the estimate in the baseline model, while for young and old the point estimate is positive as in the baseline specification.

### 2.5.2 WIOD 2016 dataset

As a further check, we consider a different dataset, i.e., the WIOD 2016 release which covers the period between 2000 and 2014. As the WIOD 2013 release, this dataset provides sector-level statistics on value-added, hours worked, price deflator, and input-output tables. Although these two datasets are quite similar, they cannot be merged into one as the WIOD 2016 covers a larger set of countries and a larger number of sectors, and it lacks some variables that are in the WIOD 2013.<sup>[12](#)</sup> While in the baseline analysis we consider the period between 1996 and 2006, as a check, we consider the complementary period between 2006 and 2014.

Tables [B.8](#), [B.9](#), and [B.10](#) in the Appendix show the results for prices, production and productivity respectively. The coefficient estimate in the IV(1) model shows, consistently with the baseline regression model, that an increase in middle-aged consumers is associated with a positive and significant effect on prices and a negative and significant effect on

<sup>11</sup>The only different sign is for the young age category in the OLS model analyzing the effect on production. While in the baseline model it is slightly positive, in the model with different age definitions it is slightly negative. Nevertheless, in both cases, the estimates are not significantly different from zero.

<sup>12</sup>We cannot subset the set of countries that are both the WIOD 2013 and WIOD 2016 datasets as, in order to build the exposure to demand measures, we need the complete input-output matrix.



production. The coefficient estimates in the IV(2) model for middle-aged consumers also show a positive association with prices and a negative relation with production. However, these results are not significant due to large standard errors. Results on productivity are generally not significant. The non-significance of the coefficient estimates, especially in the IV(2) regressions, are probably due to the shorter period of time considered which implies smaller unexpected demographics shocks.

## 2.6 Conclusion

In this paper, we provide empirical evidence of the effect of a change in the age composition of consumers on sector aggregates such as price, production, and productivity.

In order to estimate a measure of the age composition of demand, we construct a measure of the exposure of each country-sector to the demand of the domestic and foreign economies taking into account all the intermediate trades between the initial producer and the final consumer along the global value chain. However, such an estimate of the age composition of demand is necessarily correlated with the age composition of the labor force which directly affects sector aggregates. In order to avoid this issue, we exploit the wedge between the age structure of consumers (which depends on both domestic and foreign demographics) and the age structure of the labor force (which only depends on domestic demographics) instrumenting the age composition of demand with the age composition of foreign demand. We estimate the age composition of foreign demand using a shift-share approach using as shares the exposure of each country-sector to foreign demand, and as shifts the actual foreign demographic change and the unexpected demography alternatively. Using the unexpected demography allows us to overcome the predictability of demographic changes making the shifts and the shares mechanically independent, which provides the ground for a causal interpretation of the results.

We find that in the period between 1996 and 2006 only middle-aged consumers are associated with higher prices (one standard deviation increase in middle-aged consumers leading to an increase in prices between 0.70% and 2.28% per year), lower production (between 0.69% and 1.81% per year), and lower productivity (between 1.15% and 4.14% per year). Our results are robust to demographic expectations for future years, and to different age specifications. Using a different dataset, we also consider a different time

period (2006-2014) and we find that our results (although less significant) are consistent for prices and production but do not provide significant insights for productivity. This might be due to the short time period considered.

We believe that these results are driven by different mechanisms relying on the different purchasing behavior of consumers across different age categories which alter the market structure in terms of competition. In particular, middle-aged consumers who have a higher opportunity cost of time with respect to elderly consumers, and are more loyal with respect to younger consumers, tend to have a negative effect on competition which leads firms to increase prices and reduce the quantity produced. Finally, the change in the market structure also affects productivity by affecting the demand faced by firms and their strategies.

We investigate these possible mechanisms within a theoretical general equilibrium model with heterogeneous consumers in terms of both search costs and preferences in Chapter 3. By calibrating the sector-level theoretical model on the partial equilibrium results presented in this chapter and nesting the sector-level model within a multi-sector general equilibrium framework, we estimate the actual effect of a change in the age composition of consumers taking also into account substitution across sectors and general equilibrium effects.

# Chapter 3

## The effect of a change in the Age Composition of Consumers: a Theoretical Framework

### 3.1 Introduction

Demographic changes affect economies through demand because consumption behaviors vary across age categories. Measuring directly the effects of this *age demand* channel is challenging because of the multiplicity of simultaneous channels through which demography affects the economy. However, it is crucial to quantify its macroeconomic implications because of the important demographic transformations at play around the world. While in Chapter 2, we provide partial equilibrium sector-level empirical evidence, in this chapter, we develop and estimate a theoretical framework to quantify the effects of the change in the age composition of demand at the macro level. Indeed, since demographic changes hit the entire economy and not just one sector, a direct macro-level interpretation of the estimated effects in our empirical analysis would not correctly capture substitution across sectors. We, therefore, provide a more sensible method to estimate the economic consequences of economy-wide demographic changes. First, we build a sector-level model and estimate its parameters to replicate the empirical evidence in Chapter 2; then, we nest the sector-level model into a general equilibrium framework to estimate the dampening or amplifying effect of the multi-sector general equilibrium. Finally, we fit the general equilibrium model using US demographic data to capture the contribution of the change

in the age composition of consumers on US GDP growth in the period 1995-2019.

Consumption behaviors across age categories differ mainly along two dimensions: the ability to find the lowest price for a given good and the willingness to switch between goods. Regarding the former, [Aguiar and Hurst \(2007\)](#) shows that the price of purchase for a given good increases with the opportunity cost of time and, since older consumers have a lower opportunity cost of time, the price at which they purchase tends to be smaller. Using the American Time Use Survey (ATUS) dataset, we consider hourly wages as a measure for the outside option. We observe that it is roughly constant for agents between 25 and 59 years old, starts declining for the age category between 60 and 64, and drops for agents above 65 (see [Figure C.1](#) in [Appendix C.1](#)). Consistently, we observe that shopping time sharply increases after age 60, which coincides with the usual retirement age (see [Figure C.2](#) in [Appendix C.1](#)). Regarding the latter source of heterogeneity, i.e., the willingness to adjust the basket of goods, a large piece of literature points at the higher loyalty of older consumers ([Lambert-Pandraud et al., 2005](#); [Lambert-Pandraud and Laurent, 2010](#)) which leads middle-aged and old consumers to update their consumption basket less often ([Bornstein, 2019](#)).<sup>1</sup>

To capture these heterogeneities, we develop a model with three types of consumers: young, middle-aged, and old; and competition over two dimensions: within and between markets. Young and middle-aged buyers pay a relatively high search cost with respect to old consumers to sequentially observe prices in each market, while middle-aged and old buyers have a lower elasticity of substitution across goods with respect to young consumers. On the supply side of the economy, we consider a continuum of sectors. Within each sector, a finite number of firms produce a homogeneous good, compete oligopolistically through prices, and choose the technology to adopt. The demographic process, defined as a change in the age composition of consumers, mainly affects the economy through re-allocations of demand across firms to which firms respond by updating their strategies. An increase in the share of consumers with high search costs (young and middle-aged) reduces competition among firms within the same sector, while an increase in the share of consumers with low elasticity of substitution (middle-aged and old) reduces competition between sectors. Since middle-aged consumers have both high search costs and low elasticity of substitution, an increase in the share of the middle-aged leads to

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<sup>1</sup>These behaviors could arise indistinctly from preferences being revealed as consumers gain experience ([Khan et al., 2020](#)) or aging-related cognitive impairment ([Gutchess, 2011](#); [Peters, 2011](#)).

an overall reduction in competition. The lower competition, then, leads to an average increase in prices, a reduction in production, and a reduction in technological adoption which negatively affects productivity.

In order to capture general equilibrium effects and the actual substitution across sectors, we estimate the parameters in the sector-level model targeting the empirical results in Chapter 2. We nest the estimated sector-level model into a general equilibrium framework in which we endogenize the sector-level age-specific demands of the agents. Then, comparing the sector-level model and the general equilibrium model results, we estimate the contribution of the general equilibrium effects and substitution across sectors. Considering an increase in the share of middle-aged consumers, we find that the general equilibrium model dampens the sector-level model results on production by 84-87% depending on whether the increase in middle-aged consumers is compensated by a reduction in young or old ones. We, finally, relate our empirical findings in Chapter 2 to our model taking into account the general equilibrium dampening effect and estimate that an increase in one standard deviation in the middle-aged has a negative effect on GDP growth of 0.28 percentage points per year in the period 1996-2006 or, equivalently, a cumulative negative effect of 2.76 percentage points.

In the last part of the paper, we fit the general equilibrium model with US demographic data. We find that in the period 1995-2004, due to the increase in the share of middle-aged consumers, the age demand channel has reduced GDP growth by around 0.29 percentage points per year contributing an 8.7% reduction in the actual US GDP growth in the period. Instead, in the period 2005-2019, as the baby boomers went from being middle-aged to old, the age demand channel has increased GDP growth by around 0.19 percentage points per year contributing to an increase in GDP growth of 10.3%.

In this paper, we contribute to recent literature analyzing the effect of aging on GDP growth. While previous literature has mostly focused on the effect of a change in the age structure of workers, we observe that a change in the age composition of consumers also has significant effects on key economic variables.<sup>2</sup> We, therefore, propose a model highlighting a novel channel through which demography affects the economy, i.e., the *age demand* channel.

This model is able to reconcile the micro evidence studied in Aguiar and Hurst (2007)

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<sup>2</sup>See Maestas et al. (2016); Acemoglu and Restrepo (2017b, 2018b); Abeliatsky and Prettnner (2017b); Abeliatsky et al. (2020).

showing that only elderly consumers face lower prices, with our macro evidence showing that both young and old consumers tend to drive down prices compared to middle-aged consumers. In our framework, indeed, even if young consumers have high search costs and, therefore, individually buy at relative high prices with respect to old consumers (as in [Aguiar and Hurst \(2007\)](#)), at the macro-level, they drive down prices (as empirically shown in Chapter 2) as young consumers increase the between-sectors competition because of the higher elasticity of substitution. This model, therefore, allows us to distinguish between the within-sector and the between-sectors competition offering an explanation regarding the apparently contrasting results in [Aguiar and Hurst \(2007\)](#) and [Bornstein \(2019\)](#). Indeed, while [Aguiar and Hurst \(2005\)](#) shows that older consumers face lower prices, [Bornstein \(2019\)](#) argues that an aging population reduces competition. Our model reconciles these findings assuming age-specific heterogeneities in both search costs and elasticity of substitution. The lower search costs for old consumers, indeed, allow them to individually face lower prices, while the lower elasticity of substitution of middle-aged and old reduces the between-sectors competition as their shares increase. Moreover, in line with an important part of the industrial economics literature, heterogeneity in search costs allows us to capture the largely documented widespread price dispersion found in the data ([Baylis and Perloff, 2002](#); [Lach, 2002](#); [Baye et al., 2004](#)).<sup>3</sup>

The remainder of the paper is structured as follows: in Section 2, we present the three agents sector-level model and highlight the mechanism through which the change in the age composition of consumers affects the economy; in Section 3, we present the general equilibrium model. In Section 4, we describe our calibration strategy and perform the numerical analysis. In Section 5, we fit the calibrated model with US demographic data and analyze the effect of the demographic change over time in the US. Finally, Section 6 concludes.

## 3.2 Sector-level Model

In this section, we study the sector-level effects of a change in the age composition of demand on the market structure, the allocation of demand, the firms' strategies, and the key macroeconomic variables such as price, production, and productivity.

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<sup>3</sup>Theoretical contributions on price dispersion include: [Salop and Stiglitz \(1977\)](#); [Reinganum \(1979\)](#); [Varian \(1980\)](#); [Burdett and Judd \(1983\)](#); [Benabou \(1993\)](#) and [Chen and Zhang \(2011\)](#).

The sector-level model is a sequential search model based on Stahl (1989) with three consumer types: young, middle-aged, and old. We assume that young and middle-aged consumers have higher search costs relative to elderly consumers, and middle-aged and old consumers have lower elasticity of substitution with respect to young consumers. On the supply side of the model, we consider a continuum of sectors and a finite number of identical firms in each sector. Firms in the same sector produce a homogeneous good. Firms only use labor to produce and make strategic choices in terms of prices and adoption of technology. Technology allows firms to reduce the amount of labor required to produce one unit of good, it reduces marginal costs, and increases productivity. Technology costs are fixed costs that are convex in the amount of technology adopted and do not depend on the quantity produced.

We model the change in the age composition of consumers as the change in the share of young, middle-aged, and old consumers in the economy. In particular, we consider three cases spanning any demographic process: an increase in the share of middle-aged consumers keeping the share of young constant; an increase in the share of middle-aged consumers keeping the share of old consumers constant; and an increase in the share of young consumers keeping the share of middle-aged consumers constant. An increase in the share of middle-aged consumers with high search costs and relatively low elasticity of substitution has a negative effect on competition either because it reduces the competition within the sectors (if the share of old reduces) or because it reduces the competition between sectors (if the share of young reduces). An increase in the share of young consumers keeping the share of middle-aged consumers constant reduces the within-sector competition because it increases the share of consumers with high search costs (young and middle-aged) and increases the between-sectors competition because it increases the share of consumers with high elasticity of substitution (young) as well. In this case, the effect on the overall competition level and on prices, production, and productivity is, therefore, ambiguous. These theoretical insights are in line with the empirical findings in Chapter 2 in which we show that only middle-aged consumers are associated with higher prices, lower production, and lower productivity.

We now present the sector-level model analyzing how agents optimally search and the profit maximization problem faced by the firms. Then, we characterize the equilibrium price distribution and perform a comparative statics analysis to show the effects of

demographic changes on the economy.

### 3.2.1 Optimal Consumer Search

The economy is populated by a fraction  $\lambda^Y$  of young consumers, a fraction  $\lambda^M$  of middle-aged consumers, and a fraction  $\lambda^O$  of old consumers such that  $\lambda^Y + \lambda^M + \lambda^O = 1$ . Young and middle-aged consumers have high search costs, while old consumers have low search costs. In particular, we assume that elderly consumers face zero search costs, i.e.,  $s^O = 0$ , while young and middle-aged consumers pay  $s^Y = s^M \equiv \bar{s} > 0$  to sequentially observe prices.

In each sector, consumers observe the first price for free. Consumers with positive search costs find it profitable to continue searching if the expected benefits from searching exceed the costs. Given the lowest previously observed price  $z$ , we define the consumer surplus of type  $j \in \{Y, M\}$  of finding a price  $p < z$  as:

$$CS^j(p; z) \equiv \int_p^z D^j(x) dx, \quad (3.1)$$

where we allow agents to have age-specific demand functions  $D^j(x)$ . In particular, we assume that agents have CES preferences so that the sector-level demand depends on the age-specific elasticity of substitution.<sup>4</sup> The expected consumer surplus for a young and a middle-aged consumer of randomly observing another price is then the integral of the consumer surplus over the price distribution  $F(p)$ :

$$ECS^j(z) \equiv \int_b^z CS^j(p; z) dF(p), \quad (3.2)$$

where  $b$  is the minimum price in the distribution.

**Definition 3.2.1** (Reservation price and search rule). *We define the reservation price for the consumer of type  $j$ ,  $r^j$ , such that  $ECS^j(r^j) = \bar{s}$  if  $ECS^j(p)$  has a root,  $r^j = +\infty$  otherwise. This implies the following search rule for consumer of type  $j$ :*

- if  $p \leq r^j$ , the consumer of type  $j$  stops its search and purchase at price  $p$ ;
- if  $p > r^j$ , the consumer of type  $j$  continues to search;

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<sup>4</sup>We micro found the sector-level age-specific demands in the general equilibrium framework.



- if  $p > r^j$  for all firms, the consumers of type  $j$  picks the lowest observed price.

The optimal search rule for young and middle-aged consumers states that the consumer of type  $j$  continues searching if the lowest observed price,  $p$ , is greater than the type-specific reservation price  $r^j$ , and purchases if the observed price is lower than that. If all firms have prices exceeding  $r^j$ , then the consumer picks the lowest price observed.<sup>5</sup>

Old consumers pay a zero search cost, so they do not stop searching until they observe  $p = b$ .<sup>6</sup> However, as we show in a later section, the event  $p = b$  has zero probability of occurring meaning that old consumers will observe all the prices of the  $N$  firms in each sector and purchase at the lowest price.

### 3.2.2 Firms

**Production Technology** In each sector, a finite number  $N \geq 2$  of ex-ante identical firms produce a homogeneous good and compete through prices. Each firm produces goods using only labor  $\ell$  with a constant return to scale technology:

$$y^s = \alpha(a) \cdot \ell, \quad (3.3)$$

with  $\alpha(a)$  being the productivity of labor which depends on the amount of technological adoption,  $a$ , which is a choice variable of the firm. A higher technological adoption leads to higher productivity and reduces the marginal costs. In particular, we define marginal costs as:

$$c(a) \equiv \frac{w}{\alpha(a)} = w \cdot (\bar{a} - a), \quad (3.4)$$

where  $\bar{a}$  is a technology parameter,  $w$  is the wage paid for one unit of labor, and  $(\bar{a} - a)$  is the amount of labor required to produce one unit of good. This implies that a technology  $a$  reduces the unit labor requirement by  $a$ . We assume quadratic technological costs, i.e.:

$$z(a) = \frac{a^2}{\bar{z}}, \quad (3.5)$$

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<sup>5</sup>As will become clear in what follows, the definition of the indifference case  $z = r^j$  is not relevant since, as in equilibrium, the probability of a firm setting price equal to  $r^j$  is null.

<sup>6</sup>To rule out the monopolistic price equilibrium in which all firms set the price equal to the monopolistic price, we assume that old consumers stop searching only once they observe twice the lowest price  $b$ . Otherwise, firms could all charge the monopolistic price, and the old consumers would stop at the first observation since the lowest price is the monopolistic price. Since the old consumers stop searching only once they face the lowest price twice, the monopolist price equilibrium cannot be sustained since firms have a profitable deviation in marginally lowering the price to attract all the old buyers.

where  $\bar{z}$  is a technology cost parameter.

**Expected Demand** In order to recover the expected demand by each firm in equilibrium, we consider symmetric Nash Equilibria defined similarly as [Chen and Zhang \(2011\)](#):

**Definition 3.2.2** (Equilibrium Definition). *A symmetric Nash Equilibrium is a vector  $\{F(p), G(a), r^Y, r^M\}$ , where  $F(p)$  is the price distribution function in the support  $[b, \bar{r}]$  and  $G(a)$  is technology adoption distribution function. Given the reservation prices of young and middle-aged, and that other firms adopt  $F(p)$  and  $G(a)$ , it is optimal for every firm to choose  $F(p)$  and  $G(a)$ ; and given  $F(p)$  and  $G(a)$ , young and middle-aged consumers optimally search sequentially with reservation price  $r^Y$  and  $r^M$  respectively.*

As argued by [Varian \(1980\)](#) and [Stahl \(1989\)](#), the game does not have any pure-strategy equilibria and it is atomless on its support. Intuitively, if a measure of firms set the same price, a firm can increase profits by marginally undercutting this price in order to attract consumers with zero search costs that observe all prices, so that a price distribution with mass points cannot be optimal.

**Lemma 3.2.3.** *Given the NE-distribution of prices  $F$ , and the following condition:*

$$\begin{cases} \mathbb{E}\{\pi(p, F)\} > \frac{\lambda^M}{N} R^M(\hat{p}) - z(a(\hat{p})) & \text{if } r^Y \leq r^M \\ \mathbb{E}\{\pi(p, F)\} > \frac{\lambda^Y}{N} R^Y(\hat{p}) - z(a(\hat{p})) & \text{if } r^Y > r^M, \end{cases} \quad (3.6)$$

where  $R^j(p) \equiv D^j(p)(p - c(a))$  is the revenues per buyer of type  $j \in \{Y, M, O\}$ ,  $D^j(p)$  is the demand of agent of type  $j$ , and  $\hat{p} \in (\min\{r^Y, r^M\}, \max\{r^Y, r^M\}]$ , then the support of the price distribution is bounded above by  $\bar{r} = \min\{r^Y, r^M, p^{monopoly}\}$ .

*Proof.* in Appendix [C.2.1](#). □

Lemma [3.2.3](#) sets the limit of the upper bound of the equilibrium distribution of prices which cannot be greater than the monopolistic price (a price above the monopolistic price can never be optimal) and it needs to be equal to the smallest between the reservation prices if  $\min\{r^Y, r^M\} < p^{monopoly}$ . Indeed, a firm setting a price above  $\max\{r^Y, r^M\}$  would face a zero demand and makes zero profits which is not an equilibrium. Moreover, inequalities [\(3.6\)](#) set the condition under which setting a price  $\hat{p} \in (\min\{r^Y, r^M\}, \max\{r^Y, r^M\}]$

is not profitable, i.e., a firm has no incentive in only targeting those agents with the highest reservation price as it would make lower profits than setting a price below or equal to  $\min\{r^Y, r^M\}$ . Given the condition (3.6) and that  $\min\{r^Y, r^M\} < p^{\text{monopoly}}$ , then the upper bound of the distribution is given by  $\min\{r^Y, r^M\}$  since a firm setting a price  $\bar{r} < \min\{r^Y, r^M\}$  would be able to sell to the same number of consumers by setting the price to  $\min\{r^Y, r^M\}$  which is closer to the monopolistic price that maximizes the profits. Similar condition can be found in Chen and Zhang (2011). We assume condition (3.6) to hold and verify numerically our conjecture ex-post.

Given Lemma 3.2.3 for a given price  $p < p^{\text{monopoly}}$ , each producer faces the following expected demand:

$$\mathbb{E}\{y^d(p)\} = \frac{\lambda^Y}{N} \cdot D^Y(p) + \frac{\lambda^M}{N} \cdot D^M(p) + \lambda^O [1 - F(p)]^{N-1} \cdot D^O(p). \quad (3.7)$$

The expected demand function is composed of three terms: the first two are the demands coming from young and middle-aged consumers, while the last one is the demand coming from elderly consumers. Since the upper limit of the equilibrium price distribution is  $\min\{r^Y, r^M\}$ , young and middle-aged consumers randomly observe a single price, so they are equally assigned across firms and each firm expects  $\lambda^Y/N$  young and  $\lambda^M/N$  middle-aged consumers. Old consumers, instead, have zero search costs, observe all the prices, and purchase at the lowest pricing firm in each sector.  $[1 - F(p)]^{N-1}$  is, indeed, the probability that a firm setting price  $p$  is the lowest pricing firm in the sector.

**Profit Maximization Problem** The timing of a firm's decisions runs as follows:

1. the firm chooses a strategy in terms of prices and technology adoption  $(p, a)$  given the strategies of the other firms and taking into account the expected demand;
2. agents search and the demand realizes;
3. the firm produces and sells.

The producers choose the price and the technology adoption level in order to maximize profits. We define the expected profits as:

$$\mathbb{E}\{\pi(p, F)\} \equiv \max_{\{a, p\}} \mathbb{E}\{y^d(p)\} \cdot (p - c(a)) - z(a). \quad (3.8)$$

Since in equilibrium the expected profits need to be the same (say  $\pi$ ) for any price in the support of the price distribution, it must hold that:

$$\mathbb{E}\{\pi(p, F)\} = \pi \quad \forall p \in [b, \bar{r}]. \quad (3.9)$$

**Proposition 3.2.4** (Equilibrium Technology Adoption). *From the maximization problem (3.8), and the equilibrium condition (3.9), we get the equilibrium technology adoption:*

$$a^*(p) = \left[ \left( \frac{p}{w} - \bar{a} \right)^2 + \bar{z}\pi \right]^{\frac{1}{2}} - \left( \frac{p}{w} - \bar{a} \right), \quad (3.10)$$

which is positive and decreasing in price.

*Proof.* in Appendix C.2.2 □

This proposition allows a first characterization of firms. First, firms always adopt a positive amount of technology; this holds also for firms setting the price equal to  $\bar{r}$  that aim at attracting solely consumers observing one price. The model also predicts a negative relationship between price and productivity which implies a positive relationship between size (in terms of total sales) and productivity.

**Equilibrium Price Distribution Characterization** At price  $\bar{r}$ , a firm is the highest pricing firm in the sector which means that it is not able to attract any old consumer as elderly buyers observe all prices and purchase at the lowest pricing firm, i.e.,  $F(\bar{r}) = 1$ . Therefore:

$$\mathbb{E}\{\pi(\bar{r}, F)\} = \frac{\lambda^Y}{N} \cdot R^Y(\bar{r}) + \frac{\lambda^M}{N} \cdot R^M(\bar{r}) - z(a(\bar{r})). \quad (3.11)$$

Since in equilibrium all prices in the support of  $F$  give the same expected profits, it must hold that  $\mathbb{E}\{\pi(\bar{r}, F)\} = \mathbb{E}\{\pi(p, F)\}$  from which we get the equilibrium cumulative distribution of prices:

$$F(p) = 1 - \left[ \frac{\lambda^Y (R^Y(\bar{r}) - R^Y(p)) + \lambda^M (R^M(\bar{r}) - R^M(p))}{N \cdot \lambda^O R^O(p)} - \frac{z(a(\bar{r})) - z(a(p))}{\lambda^O R^O(p)} \right]^{\frac{1}{N-1}}. \quad (3.12)$$

To complete the characterization of  $F(p)$ , we pin down the lower bound of the price distribution,  $b$ , using the fact that  $F(b) = 0$ .

### 3.2.3 Sector-level Analysis

In this section, we describe the equilibrium price distribution and analyze how the model works highlighting the different mechanisms linking the age composition of consumers to the allocation of demand across firms, firms' strategies, and productivity. To simplify the analysis and highlight the mechanisms going through the search costs and elasticity of substitution heterogeneity, we assume that each agent faces the same budget constraint. We relax this assumption in the numerical analysis in which we calibrate the age-specific wealth using US data.<sup>7</sup> We assign values to the elasticity of substitution such that  $\bar{\sigma} \equiv \sigma^Y > \sigma^M = \sigma^O \equiv \underline{\sigma}$  which is in line with the evidence in Bornstein (2019) showing that middle-aged and old consumers have similar consumption inertia which is significantly higher than the one of young consumers, and the evidence in Lambert-Pandraud et al. (2005) showing that middle-aged and old consumers show higher brand loyalty with respect to young consumers. We estimate those values later in the general equilibrium context. We assume that middle-aged consumers have a lower reservation price so that the upper bound of the price distribution is equal to the reservation price of middle-aged consumers, i.e.,  $\bar{r} = r^M$ . Intuitively, since middle-aged consumers have a lower elasticity of substitution between goods produced in different sectors, once they observe a high price, they are more willing (relative to young consumers) to pay the search costs to observe another price. Young consumers would find instead optimal to reduce the consumption of the good with a high price and purchase more of another good without having to pay the search cost. We verify numerically ex-post that reservation prices fulfil this order.

#### Equilibrium Price Distribution

The equilibrium price distribution (Figure 3.1) of strategies is U-shaped as in Stahl (1989) and presents two larger density areas of strategies. The first one, an almost mass point at the upper bound of the price distribution, includes all the strategies focused on young and middle-aged buyers who randomly observe only one price. Firms that set the price close to the upper bound of the distribution attract few buyers, sell few products per buyer

<sup>7</sup>Since there is an infinite number of sectors, the fraction of the wealth that is allocated to each sector is infinitesimal. Therefore, the budget constraint does not play any role at the sector level. The budget constraint will be, instead, endogenous and relevant in the general equilibrium framework as the total wealth in the economy will depend on the equilibrium profits, wage, and technology adoption costs.

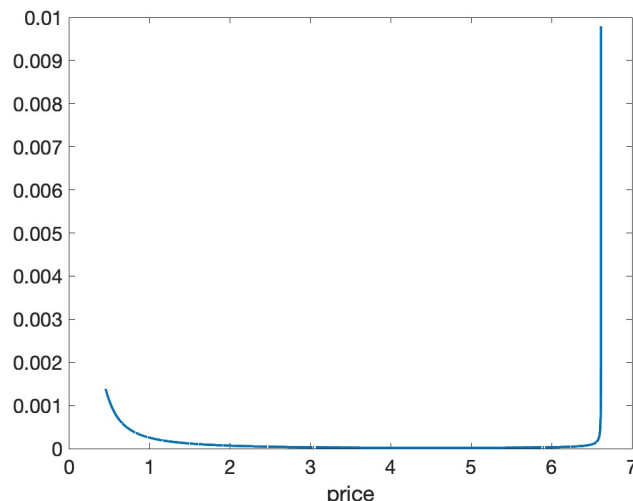


Figure (3.1) Price Distribution Function

and use low-productivity technologies. These correspond to the so-called “mom-and-pop” shops.

The second density area, close to the minimum price, includes the strategies focused on old buyers that observe all prices and, therefore, purchase at the lowest price in the sector. These firms compete to attract more buyers, use high-productivity technology as they are able to spread technology costs over a large production.

### Comparative statics

We now analyze the effects of a demographic change in our economy. We keep, in turn, the share of different age categories constant in order to highlight the different mechanisms at play (see Appendix [C.2.7](#) for the related graphical analysis). Demographic changes usually affect the shares of all the age categories meaning that the actual effect of a demographic change is a mixture of the effects we depict in this analysis.

**Increase in Middle-aged keeping Young constant** We consider now an increase in the share of middle-aged consumers keeping the share of young consumers constant meaning that the share of old consumers reduces to compensate for the increase in the middle-aged share, i.e.,  $\uparrow \lambda^M = \downarrow \lambda^O$ . The share of consumers with high search costs ( $\lambda^Y + \lambda^M$ ) increases while the share of consumers with low elasticity of substitution ( $\lambda^M + \lambda^O$ ) stays constant. Therefore, the effect on the economy coming from this particular demographic process is solely determined by the change in the share of consumers with high search costs. An increase in the share of consumers with high search costs affects the

economy in two ways. First, it changes the allocation of demand across firms as a larger share of consumers will only observe one price and, therefore, will be randomly allocated to firms. This *allocation effect* mechanically allocates a higher demand to firms setting higher prices that have a low level of productivity leading to an average increase in prices, an average reduction in production, and an average reduction in productivity. Second, given the different allocation of demand, firms will experience a lower within-sector competition and will update their strategies favoring high-price and low-technology adoption strategies since a larger share of consumers will randomly pick a price. This *strategy effect*, therefore, leads to higher average prices, lower production, and lower productivity. Since the two effects go in the same direction, an increase in the share of middle-aged consumers keeping the share of young consumers constant unambiguously leads to an average increase in price, and a reduction in production and productivity.

**Increase in Middle-aged keeping Old constant** An increase in the share of middle-aged consumers keeping the share of old consumers constant means that the share of young consumers reduces to fully compensate the increase in middle-aged, i.e.,  $\uparrow \lambda^M = \downarrow \lambda^Y$ . This implies that the share of consumers with high search costs ( $\lambda^Y + \lambda^M$ ) stays constant, while the share of consumers with low elasticity of substitution ( $\lambda^M + \lambda^O$ ) increases. The effect on the economy coming from this type of demographic process is, therefore, determined only by the increase in the share of consumers with low elasticity of substitution. As the firms in the sector observe an increase in the share of consumers with low elasticity of substitution, they will exploit the lower between-sectors competition favoring higher prices and lower technology adoption strategies leading to an average increase in prices, and a reduction in production and productivity (in this case only the *strategy effect* is at play).

**Increase in Young keeping Middle-aged constant** An increase in the share of young consumers keeping the share of middle-aged consumers constant implies a reduction in the share of old consumers such that  $\uparrow \lambda^Y = \downarrow \lambda^O$ . This implies an increase in the share of consumers with high search costs ( $\lambda^Y + \lambda^M$ ) and a reduction in the share of consumers with low elasticity of substitution ( $\lambda^M + \lambda^O$ ). While the increase in the share of consumers with high search costs leads to a reduction in the within-sector competition, the reduction in the share of consumers with low elasticity of substitution increases the between-sectors

competition. Since the reduction in the within-sector competition and the increase in the between-sectors competition have opposite effects on prices, production, and productivity, the net effect on those variables is uncertain.

### 3.3 General Equilibrium Model

We now embed the sector-level model into a multi-sector general equilibrium framework which allows us to retrieve estimates of the effect of a demographic demand change taking into account general equilibrium effects and substitution across sectors. We first derive the consumers' demands for each sector solving the household utility maximization problem. Aggregating the sector-level demands, we then recover the aggregate demand and the aggregate labor demand.

#### 3.3.1 Household Demand

Each household  $j \in \{Y, M, O\}$  gets utility by consuming an aggregate consumption good,  $U(C^j) = C^j$ , composed by aggregating goods from sector  $s \in [0, \mathcal{S}]$  with  $\mathcal{S} = 1$ , through a CES consumption aggregator. We assume that agents have age-specific preferences. In particular, we assume that agents differ in terms of their elasticity of substitution between goods across different sectors,  $\sigma_j$ . Therefore, agent  $j$  faces the following utility maximization problem:

$$\max_{\{c_s^j\}} C^j \equiv \left[ \int_s (c_s^j)^{\frac{\sigma_j-1}{\sigma_j}} ds \right]^{\frac{\sigma_j}{\sigma_j-1}} \quad (3.13)$$

$$\text{s.t.} \quad I^j = \tilde{P}^j \cdot C^j, \quad (3.14)$$

where  $I^j \equiv \phi^j \cdot W$  is the income of agent  $j$  and  $\phi^j$  is the share of the total wealth ( $W$ ) in the economy held on average by each agent  $j$ . The total wealth in the economy is endogenous and it is given by the sum of the total labor costs, total profits, and total technology costs. We can interpret technology costs as the capital income.  $\tilde{P}^j \equiv \left[ \int_s (\tilde{p}_s^j)^{1-\sigma_j} ds \right]^{\frac{1}{1-\sigma_j}}$  is the normalized price aggregator.<sup>8</sup> The price aggregators are age-specific as agents differ in terms of both their search costs and their elasticity of substitution. From the

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<sup>8</sup>See Appendix [C.2.4](#).



maximization problem (3.13), we obtain the sector-specific demand for agent of type  $j$ :

$$D^j(p) \equiv c_s^j = C^j \left( \frac{\tilde{p}_s^j}{\tilde{P}^j} \right)^{-\sigma_j}. \quad (3.15)$$

See Appendix C.2.5 for the derivation.

### 3.3.2 Aggregation and the General Equilibrium

The production in each sector fulfills the market clearing condition. Therefore, the aggregate production in the economy  $Q$  is simply the aggregation of the sector-level demands:

$$Q = \int_s \int_j (c_s^j) dj ds = \int_j \left( \frac{C^j}{\tilde{P}^j^{-\sigma_j}} \int_s (\tilde{p}_s^j)^{-\sigma_j} ds \right) dj. \quad (3.16)$$

Since all the sectors are identical, the conditional to type  $j \in \{Y, M, O\}$  expected price in each sector is the same (i.e.,  $\int_s (\tilde{p}_s^j)^{-\sigma_j} ds = \mathbb{E}^j[\tilde{p}^{-\sigma_j}]$ ).<sup>9</sup> We can rewrite the aggregate production as:

$$Q = \sum_j \lambda^j \frac{C^j}{\tilde{P}^j^{-\sigma_j}} \mathbb{E}^j[\tilde{p}^{-\sigma_j}], \quad (3.17)$$

where  $\mathbb{E}^j[\tilde{p}^{-\sigma_j}]$  is the expectation over  $\tilde{p}^{-\sigma_j}$  given that the agent is of type  $j$ . In particular, since young and middle-aged consumers randomly observe a price, they face the same price distribution as the price strategy distribution of firms, i.e.,  $F^Y(p) = F^M(p) = F(p)$ . Old consumers, instead, observe all prices and buy in each market at the lowest price which means that out of  $N$  independent draws from  $F(p)$ , they pick the lowest one. Therefore:

$$F^O(p) = 1 - (1 - F(p))^N, \quad (3.18)$$

$$f^O(p) = \frac{\partial F^O(p)}{\partial p} = N(1 - F(p))^{N-1} f(p), \quad (3.19)$$

where  $f(p)$  is the probability distribution function of equilibrium prices.

In order to highlight the effect of a demographic change on the market structure through the demand channel, we fix the labor supply, i.e.,  $ALS = 1$ .<sup>10</sup> On the labor demand side, using the market clearing conditions at the sector level, we have that the

<sup>9</sup>See Appendix C.2.6.

<sup>10</sup>We present a sketch of a model in which we allow the aggregate labor supply to be age-dependent in Appendix C.4.

aggregate labor demand is:

$$ALD = \int_s \int_j \frac{c_s^j}{\alpha_s^j} dj ds = \int_j \left( \frac{C^j}{\tilde{P}^{j-\sigma_j}} \int_s (\tilde{p}_s^j)^{-\sigma_j} \cdot (\bar{a} - a^*) ds \right) dj. \quad (3.20)$$

As for the aggregate production, we can rewrite the aggregate labor demand as:

$$ALD = \sum_j \lambda^j \frac{C^j}{\tilde{P}^{j-\sigma_j}} \mathbb{E}^j [\tilde{p}^{-\sigma_j} \cdot (\bar{a} - a^*)]. \quad (3.21)$$

Finally, the wage adjusts such that the aggregate labor demand equals the aggregate labor supply.

## 3.4 Numerical Analysis

In this section, we present our calibration strategy and numerically analyze how the economy reacts to demographic changes taking into account the general equilibrium effects and substitution across sectors. Finally, we compare the general equilibrium results with those of the sector-level model.

### 3.4.1 Calibration

We have 9 parameters in the general equilibrium model: a search cost parameter ( $\bar{s}$ ) a technology parameter ( $\bar{a}$ ), a technology cost parameter ( $\bar{z}$ ), two elasticity of substitution parameters ( $\underline{\sigma}, \bar{\sigma}$ ), the number of firms  $N$ , and three budget constraint parameters ( $\phi^j$  for  $j \in \{Y, M, O\}$ ). We calibrate the budget constraint parameters on US wealth data so that the income of agent  $j$  is equal to the average wealth of a consumer in the  $j$  age category in 1995.

The remaining parameters,  $\{\bar{s}, \bar{a}, \bar{z}, \underline{\sigma}, \bar{\sigma}, N\}$ , are particular to the model presented and we estimate them targeting the empirical results in Chapter 2. Since the empirical results in Chapter 2 are sector-level results, we use the sector-level model to target such relations. We, then, nest the calibrated sector-level model into the general equilibrium framework to recover estimates of the effect of a demographic change that also takes into account general equilibrium effects and substitution across sectors. We target coefficient estimates in our preferred specification, i.e., the  $IV(2)$  model specification. The estimated and calibrated parameters are in Appendix [C.3](#).

Table (3.1) The effect on production (percentage) of a one-standard-deviation ( $2.2\% \times \lambda^M$  for middle-aged consumers and  $1.45\% \times \lambda^O$  for old consumers) increase in  $\lambda^i$  at the expense of  $\lambda^j$  where  $i \neq j$  and  $i, j \in \{Y, M, O\}$ .

	Demographic change	Sector-level (SL)	General Equilibrium (GE)	Ratio ( $\frac{GE}{SL}$ )
Production	$\uparrow \lambda^M, \downarrow \lambda^Y$	-3.37	-0.52	0.16
	$\uparrow \lambda^M, \downarrow \lambda^O$	-4.05	-0.54	0.13
	$\uparrow \lambda^O, \downarrow \lambda^Y$	0.79	-0.041	-0.052

### 3.4.2 Analysis

We analyze the effect of a demographic change on production for both the sector-level (SL) model and the general equilibrium (GE) model. The ratio of the effects (i.e.,  $\frac{GE}{SL}$ ) provides a measure of the dampening or the amplifying effect of the multi-sector general equilibrium model. We focus our analysis on production for two reasons: first, our empirical evidence regarding the effect of the age demand channel on production is the most robust; second, we model the technology adoption in a way that allows us to keep the model tractable but might not capture all the relevant effects driving the change in productivity. Indeed, although the model predictions for productivity are qualitatively similar to those in our empirical estimation (right sign of the effect), they are substantially smaller than those we document empirically.

Table [3.1](#) shows the effect of a *clean* demographic change (i.e., an increase in one standard deviation in  $\lambda^i$  compensated by a reduction of  $\lambda^j$  where  $i \neq j$  and  $i, j \in \{Y, M, O\}$ ). Considering one standard deviation increase in the share of middle-aged consumers compensated by a reduction in young consumers, the sector-level model predicts a reduction in GDP growth of 3.37 percentage points, while the general equilibrium model predicts a reduction of 0.52 percentage points dampening the effect by 84%. Similarly, for an increase in the middle-aged at the expense of old consumers the general equilibrium model dampens the negative effect on GDP growth by 87%. The dampening effect is due to the fact that, while in the sector-level model the demographic change only occurs at the sector level and the substitution effects are at their highest, in the general equilibrium model, the demographic change hits the entire economy and the substitution effects are reduced. When we consider an increase in the share of old consumers compensated by a reduction in young consumers, we find a substantially smaller effect at the sector level with respect to the effect of an increase in middle-aged and virtually no effect in the general equilibrium model. This is because while an increase in the share of old con-

sumers (who have zero search costs and purchase at the lowest price within each sector) increases the within-sector competition with a positive effect on production, a reduction in the share of young consumers (who have a higher elasticity of substitution) reduces the between-sectors competition with a negative effect on production. The two effects, therefore, offset each other leading to a small net effect at the sector level, and an almost zero effect at the general equilibrium level.

**Relating the empirical findings to the general equilibrium results** We can now reinterpret the empirical results in Chapter 2 taking into account the general equilibrium effects and the correct substitution across sectors. We find that when considering an increase in the share of the middle-aged, the general equilibrium model dampens the estimates of the sector-level model as it reduces substitution effects across sectors. In order to interpret our empirical results taking into account the substitution across sectors, we pass them through the  $\frac{GE}{SL}$  ratio that allows us to capture the dampening effect of the general equilibrium. In the empirical analysis, the underlining assumption was to analyze the effect of an increase in one of the age categories keeping the share of the other age categories constant. This means that the increase in one of the age categories was at the expense of both the others in such a way as to keep their ratio constant. In order to remain consistent with this assumption, we decompose the demographic change into two *clean* demographic shocks weighting the effect of the change in one age category by its relative initial share. Focusing on the middle-aged category, we estimate the effect of an increase in the share of the middle-aged on production ( $\beta_{GE}^M$ ) taking into account substitution across sectors and general equilibrium effects as follows:

$$\beta_{GE}^M = \beta_{SL}^M \cdot \left[ \frac{\lambda^Y}{\lambda^Y + \lambda^O} \cdot \left( \frac{GE}{SL} \right)_Y^M + \frac{\lambda^O}{\lambda^Y + \lambda^O} \left( \frac{GE}{SL} \right)_O^M \right], \quad (3.22)$$

where  $\beta_{SL}^M$  is the sector-level coefficient estimate capturing the effect of an increase in the share of middle-aged on production;  $\frac{\lambda^Y}{\lambda^Y + \lambda^O}$  and  $\frac{\lambda^O}{\lambda^Y + \lambda^O}$  are the weights for the effect coming from the reduction in young and old respectively; and  $\left( \frac{GE}{SL} \right)_Y^M$  and  $\left( \frac{GE}{SL} \right)_O^M$  are the general equilibrium to sector-level ratios resulting from an increase in middle-aged compensated by a reduction in young and old respectively as estimated in Table 3.1<sup>11</sup>

<sup>11</sup>We set  $\beta_{SL}^M = -8.308$  as estimated in the *IV(2)* model specification in Chapter 2 and we use US demographic data in 1995 to estimate the weights. We set, therefore,  $\frac{\lambda^Y}{\lambda^Y + \lambda^O} = 0.77$  and  $\frac{\lambda^O}{\lambda^Y + \lambda^O} = 0.23$ .

We find that  $\beta_{GE}^M = -1.27$  which implies a cumulative reduction in GDP growth following one standard deviation increase (2.2 percentage points) in the middle-aged of around 2.76 percentage points (dropping from the 16.7 percentage points reduction estimated through the sector-level empirical model) in the period 1996-2006 or, equivalently, a per year reduction in GDP growth of 0.28 percentage points (dropping from the 1.81 percentage points reduction estimated through the sector-level empirical model).

## 3.5 US Exercise

In this section, we look more precisely at the case of the US. Using our estimated general equilibrium model and demographic data about the age composition of the population, we compute the contribution of the demographic demand channel to the GDP growth trend in the US over the past 25 years.

### 3.5.1 US Demographics

The US demographic evolution has been largely determined by the baby boomer generation (those born between 1946 and 1965) who have represented by far the largest generational cohort in the twentieth century, and still today represent the second largest living cohort. Figure [3.2](#) shows how the shares of the young, middle-aged, and old population have evolved since the 1970s. As the baby boomers entered into the young age (25-44), the share of young category started to increase. The increase continued up to the early 1990s as the baby boomers became middle-aged (45-64) determining a sharp reduction in the share of the young population and an equivalent increase in the share of the middle-aged. The share of middle-aged people reached a peak in the early 2010s and started to decline as the baby boomers grew older (65-79). In the last decade, indeed, we observed a clear increase in the share of the older population which had remained constant in the previous decades. The prominent relevance of the baby boomer generation explains the relatively constant long-term trend in the middle-aged population and its large long-term fluctuations over the past and coming decades.

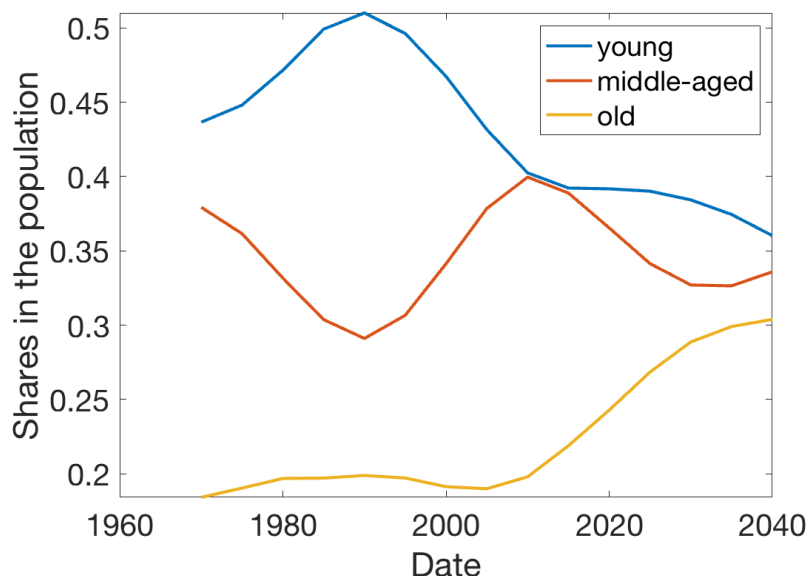
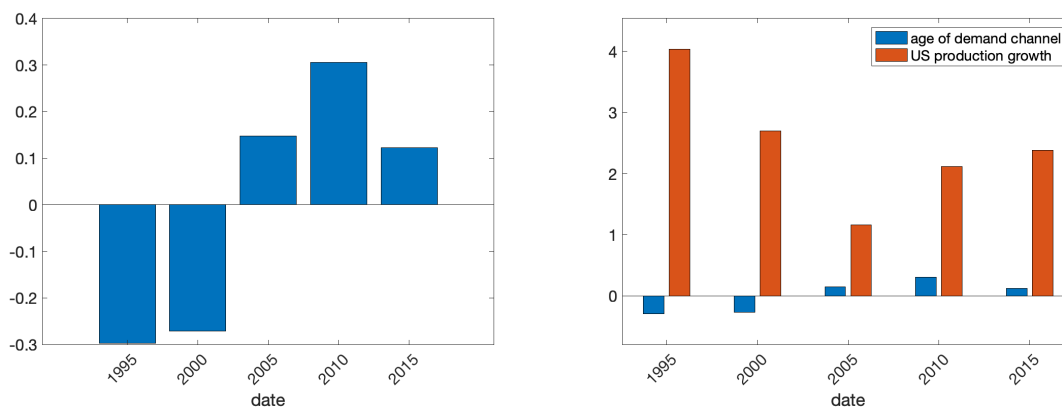


Figure (3.2) Age decomposition of the demography in the United States of America, we define young as 25-44, middle-aged as 45-64 and old as 65-79 years old. The expected demand is the average age composition over the next 10 years. Source: UN WPP 2019.

### 3.5.2 Results

We focus on the period between 1995 and 2019. Figure 3.3 panels (a) shows the 5-year average contribution of the age demand channel to GDP growth. We observe that in the period 1995-2004 the effect of the age demand channel on GDP growth was negative contributing to a reduction in GDP growth of around 8.7%. From Figure 3.2 panel (b), we, indeed, observe that the average GDP growth in that period was around 3.35%, while the age demand channel contributed to a reduction in GDP growth of around 0.29 percentage points per year. This negative effect on GDP growth is mostly due to the increase in the share of the middle-aged in the period. In the period following 2005, as the share of the middle-aged declined and the share of the old kept increasing (i.e., as the baby boomers went from being middle-aged to old) the effect of the age demand channel reversed, having a positive effect on GDP growth. We find, indeed, that the contribution of the age demand channel in the period 2005-2019 was around 0.19 percentage points per year, contributing to an increase in GDP growth (1.87% on average in the period) of around 10.3%.

(a) Age demand channel ( $\Delta$  production)

(b) Age demand channel and GDP growth

Figure (3.3) Age demand channel compared with the US GDP growth in the period 1995-2019. Each bar represents the per year average effect of the age demand channel (5-years average) and the US 5-years average GDP growth measured in percentage points.

### 3.6 Conclusion

This paper provides a theory explaining the empirical findings in Chapter 2 relating the age composition of demand to the market structure and to key economic variables, as well as highlighting the importance of the heterogeneities across different age categories. The various dimensions of heterogeneity across the different age categories characterize the aging of the population as a complex process. Our empirical analysis, indeed, shows that only the middle-aged are associated with higher prices, lower production, and lower productivity, suggesting that the aging of the population does not have a linear effect on the economy. This implies that standard representative-agent or two-agent models may fail to predict the effects of the upcoming large demographic changes. We, therefore, build a search on the goods market sector-level model with three types of agents (young, middle-aged, and old) who differ along two dimensions: their search costs and elasticity of substitution.

In order to estimate the macroeconomic effects of a change in the age composition of consumers, we first estimate the sector-level model using our sector-level empirical results and then we nest the estimated model within a multi-sector general equilibrium framework. We estimate that the general equilibrium model dampens the effect of an increase in the share of the middle-aged on production by 84-87%. Fitting the general equilibrium model with US demographic data, we find that in the period 1995-2004, as

the share of the middle-aged increased, the age demand channel contributed to reducing GDP growth by 0.29 percentage points per year (i.e., 8.7% of the average annual US GDP growth). Instead, in the period 2005-2019, as the middle-aged grew old, the age demand channel contributed to an increase in GDP growth by 0.19 percentage points per year (i.e., 10.3% of the average annual US GDP growth) suggesting that the age demand channel is a relevant determinant of macroeconomic outputs.



# Appendix A

## Appendix Chapter 1

### A.1 Empirical Evidence Appendix

#### A.1.1 Graphs

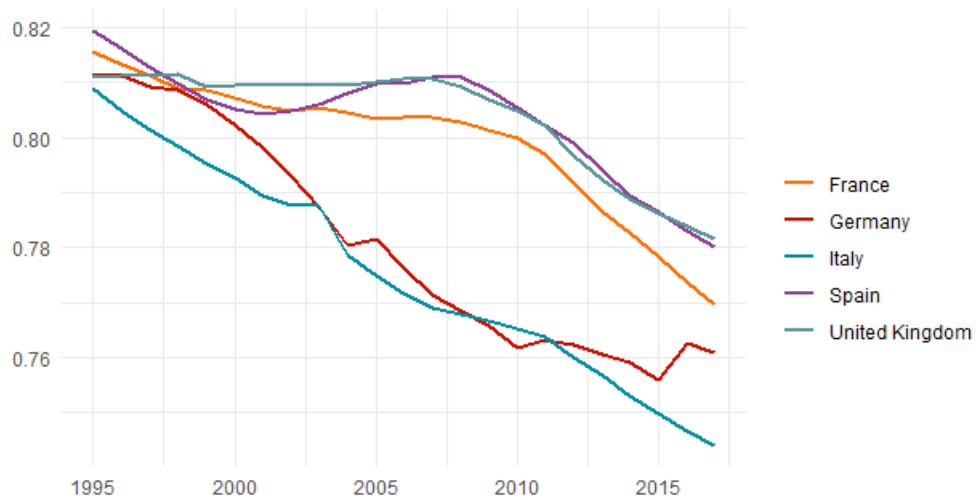


Figure (A.1) Working-age population share (15-65) among the biggest European economies (Ilostat).

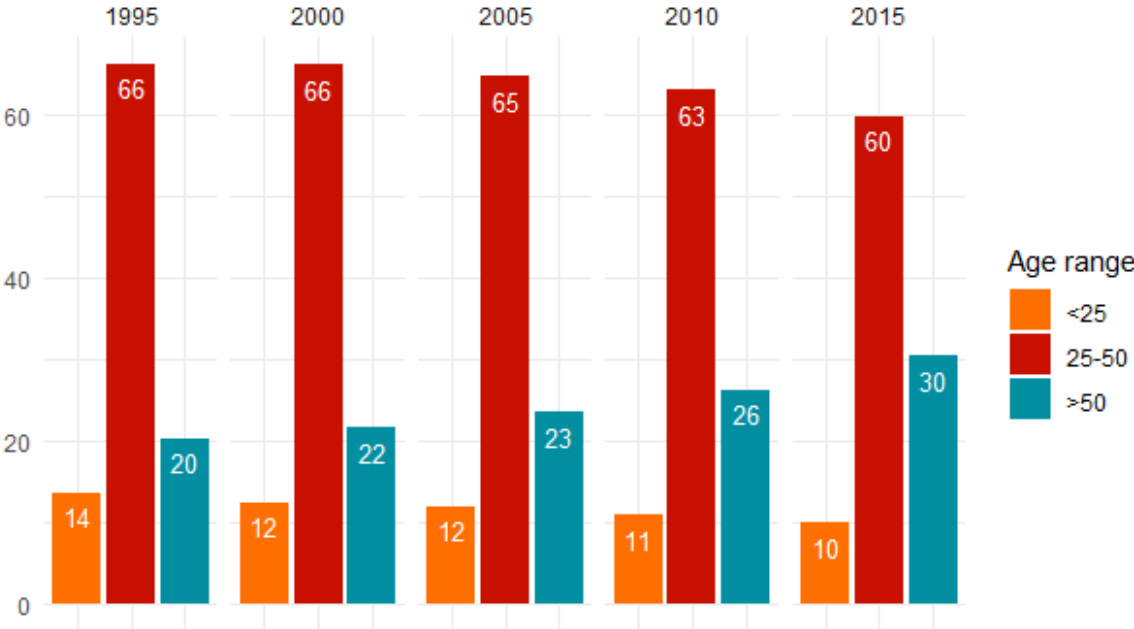


Figure (A.2) Labor market composition in EU15 countries (Eurostat).

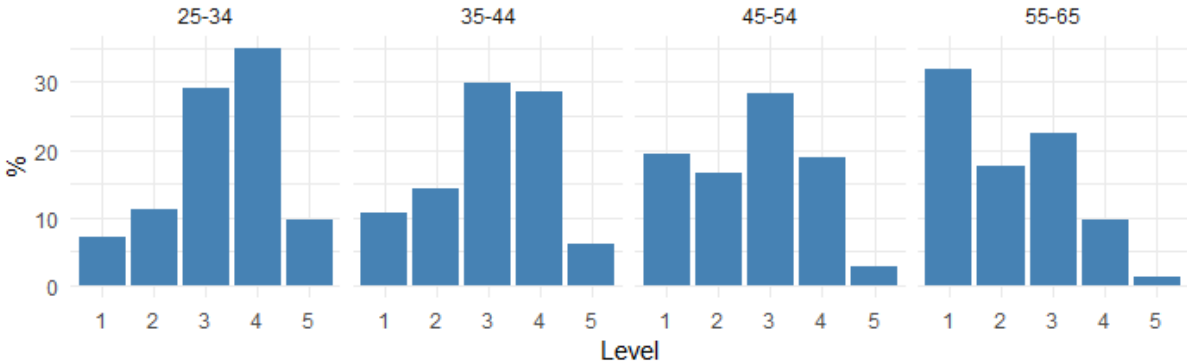


Figure (A.3) Problem-solving ability in technology-rich environments in OECD countries (PIAAC).

### A.1.2 Data and Variables

As a proxy for investments in new technologies (the dependent variable), I consider the fraction of ICT capital stock using data from EU KLEMS. As a proxy for the aging variable, I use population and employment data from Eurostat. While EU KLEMS investment data are already harmonized in accordance with the industry classification NACE Rev.2, data from Eurostat use NACE Rev.1.1 classification for data until 2007, and NACE Rev.2 classification for data from 2008. To harmonize the different NACE classifications for the Eurostat data, I used the correspondence table proposed by [Perani et al. \(2015\)](#) which is reported below. I consider the period from 1995 to 2015 for the countries for which sector-level data are available in the period considered, i.e., Austria, Germany, Denmark, Spain, Finland, France, Italy, Luxembourg, Netherlands, Sweden, and United Kingdom. The value-added data used to define productivity at the sector-level are taken from the EU KLEMS dataset as well.

The ICT investment variable is constructed in the following way:

$$ICT_{s,t,c} = \frac{IT_{s,t,c} + CT_{s,t,c} + Soft\_DB_{s,t,c}}{total\_GFCF_{s,t,c} - RD_{s,t,c}}$$

where  $IT_{s,t,c}$ ,  $CT_{s,t,c}$ ,  $Soft\_DB_{s,t,c}$ ,  $total\_GFCF_{s,t,c}$ , and  $RD_{s,t,c}$  are computer hardware, telecommunication equipment, computer software and databases investments, total investments, and R&D investments respectively in at time  $t$ , in sector  $s$ , in country  $c$ . The age variable is defined as the ratio between the number of workers aged 50 or older and those aged between 25 and 49 (prime-aged workers) for each sector, time and country. The productivity variable is defined as the value-added per hour worked.

### A.1.3 Tables

Table (A.1) Relation between age and ICT investment. EU KLEMS country-level data (1995-2015) for 10 Western European countries.

	<i>Dependent variable:</i>					
	ICT					
	(1)	(2)	(3)	(4)	(5)	(6)
Age	0.201*** (0.022)	0.102*** (0.028)	0.090*** (0.027)	0.120 (0.186)	-0.023 (0.193)	0.268*** (0.080)
Age <sup>2</sup>				0.099 (0.225)	0.151 (0.233)	-0.217** (0.092)
Time fixed effects		✓	✓		✓	✓
Country fixed effects			✓			✓
Observations	206	206	206	206	206	206
R <sup>2</sup>	0.298	0.402	0.921	0.299	0.404	0.924
Adjusted R <sup>2</sup>	0.295	0.334	0.908	0.292	0.332	0.910

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table (A.2) Correspondence table (Perani et al. (2015)) used to harmonize NACE Rev.1 with NACE Rev.2 classification.

NACE Rev.1.1		NACE Rev.2	
Section	Description	Section	Description
A, B	Agriculture, Hunting and Forestry (A), Fishing (B)	A	Agriculture, Forestry and Fishing
C	Mining and quarrying	B	Mining and quarrying
D	Manufacturing	C	Manufacturing
E	Electricity, gas and water supply	D, E	Electricity, gas, steam and air conditioning supply (D), Water supply, sewerage, waste management and remediation activities (E)
F	Construction	F	Construction
G	Wholesale and retail trade: repair of motor vehicles, motorcycles and personal and household goods	G	Wholesale and retail trade; repair of motor vehicles and motorcycles
H	Hotels and restaurants	I	Accommodation and food service activities
I	Transport, storage and communications	H, J	Transportation and storage (H), Information and communication (J)
J	Financial intermediation	K	Financial and insurance activities
K	Real estate, renting and business activities	L, M, N	Real estate activities (L), Professional, scientific and technical activities (M), Administrative and support service activities (N)
L	Public Administration and defence; compulsory social security	O	Public administration and defence; compulsory social security
M	Education	P	Education
N	Health and social work	Q	Human health and social work activities
O	Other community, social and personal services activities	R, S	Arts, entertainment and recreation (R), Other service activities (S)
P	Activities of private households as employers and undifferentiated production activities of private households	T	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use
Q	Extraterritorial organizations and bodies	U	Activities of extraterritorial organizations and bodies

Table (A.3) Relation between Age and IT investment. EU KLEMS sector-level data (1995-2015) for 10 Western European countries.

<i>Dependent variable:</i>						
IT						
	(1)	(2)	(3)	(4)	(5)	(6)
Age	0.003 (0.003)	0.023*** (0.004)	0.0004 (0.005)	0.072*** (0.012)	0.059*** (0.010)	0.012 (0.011)
Age <sup>2</sup>				-0.063*** (0.010)	-0.033*** (0.009)	-0.010 (0.008)
Sector fixed effects		✓	✓		✓	✓
Time fixed effects		✓	✓		✓	✓
Country fixed effects			✓			✓
Observations	3,272	3,272	3,272	3,272	3,272	3,272
R <sup>2</sup>	0.0002	0.392	0.546	0.012	0.395	0.546
Adjusted R <sup>2</sup>	-0.0001	0.386	0.540	0.011	0.388	0.540

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table (A.4) Relation between Age and CT investment. EU KLEMS sector-level data (1995-2015) for 10 Western European countries.

<i>Dependent variable:</i>						
CT						
	(1)	(2)	(3)	(4)	(5)	(6)
Age	-0.022*** (0.003)	-0.015*** (0.004)	-0.012** (0.005)	-0.024** (0.010)	-0.046*** (0.011)	-0.022* (0.012)
Age <sup>2</sup>				0.002 (0.009)	0.028*** (0.009)	0.008 (0.009)
Sector fixed effects		✓	✓		✓	✓
Time fixed effects		✓	✓		✓	✓
Country fixed effects			✓			✓
Observations	3,272	3,272	3,272	3,272	3,272	3,272
R <sup>2</sup>	0.015	0.180	0.361	0.015	0.182	0.361
Adjusted R <sup>2</sup>	0.015	0.171	0.352	0.015	0.173	0.352

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table (A.5) Relation between Age and investment in Software and DB. EU KLEMS sector-level data (1995-2015) for 10 Western European countries.

	<i>Dependent variable:</i>					
	Software & DB					
	(1)	(2)	(3)	(4)	(5)	(6)
Age	-0.037* (0.020)	0.148*** (0.024)	0.046 (0.033)	0.258*** (0.066)	0.454*** (0.062)	0.325*** (0.079)
Age <sup>2</sup>				-0.272*** (0.058)	-0.280*** (0.053)	-0.222*** (0.057)
Sector fixed effects		✓	✓		✓	✓
Time fixed effects		✓	✓		✓	✓
Country fixed effects			✓			✓
Observations	3,272	3,272	3,272	3,272	3,272	3,272
R <sup>2</sup>	0.001	0.299	0.321	0.008	0.305	0.324
Adjusted R <sup>2</sup>	0.001	0.291	0.311	0.007	0.297	0.314

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

## A.2 Model Appendix

### A.2.1 Steady state capital

I define  $y \equiv \frac{\theta Y}{\gamma N^y}$  and rewrite the equations as follows (omitting the time subscript for simplicity):

$$w^y = \begin{cases} \frac{y\gamma\varepsilon(1+n)}{\theta(1+n+\phi)} & \text{for } N^o < \hat{N}^o \\ \frac{y\alpha I}{\theta} & \text{for } N^o \geq \hat{N}^o \end{cases} \quad (\text{A.1})$$

$$w^o = \begin{cases} \frac{y\gamma\varepsilon(1+n)}{\theta(1+n+\phi)} & \text{for } N^o < \hat{N}^o \\ \frac{y\beta(1-I)(1+n)}{\phi\theta} & \text{for } N^o \geq \hat{N}^o \end{cases} \quad (\text{A.2})$$

$$y = \begin{cases} A \left( \frac{(1+n+\phi)\theta}{(1+n)\gamma\varepsilon} \right)^\varepsilon \left( \frac{k}{1-\varepsilon} \right)^{1-\varepsilon} & \text{for } N^o < \hat{N}^o \\ A \left( \frac{\theta}{\alpha I} \right)^{\alpha I} \left( \frac{\phi\theta}{\beta(1-I)(1+n)} \right)^{\beta(1-I)} \left( \frac{k}{1-\varepsilon} \right)^{1-\varepsilon} & \text{for } N^o \geq \hat{N}^o \end{cases} \quad (\text{A.3})$$

$$\gamma = \begin{cases} \frac{1+n+\phi}{1+n} \cdot \frac{\alpha I}{\varepsilon} & \text{for } N^o < \hat{N}^o \\ 1 & \text{for } N^o \geq \hat{N}^o \end{cases} \quad (\text{A.4})$$

$$R = \frac{y}{k}(1-\varepsilon). \quad (\text{A.5})$$

Substituting the equations above into the condition [\(1.34\)](#), in steady state I find:

$$k_{ss} = \begin{cases} \left[ \frac{\rho A \left( \frac{1+n+\phi}{1+n} \right)^\varepsilon \left( \frac{\theta}{\gamma} \right)^\varepsilon \left( \frac{\varepsilon}{1-\varepsilon} \right)^{1-\varepsilon}}{(1+n+\phi)(1+\rho)+\varepsilon\phi(1-\varepsilon)^{-1}} \right]^{\frac{1}{\varepsilon}} & \text{for } N^o < \hat{N}^o \\ \left[ \frac{\rho\alpha I A \left( \frac{\theta}{\alpha I} \right)^{\alpha I} \left( \frac{\phi\theta}{\beta(1-I)(1+n)} \right)^{\beta(1-I)} (1-\varepsilon)^{\varepsilon-1}}{(1+n)(1+\rho)+(1+n)\beta(1-I)(1-\varepsilon)^{-1}} \right]^{\frac{1}{\varepsilon}} & \text{for } N^o \geq \hat{N}^o. \end{cases} \quad (\text{A.6})$$

### A.2.2 Social Planner Problem

The social planner maximizes the following problem:

$$\max_{\{C_t^y, C_{t+1}^o, K_{t+1}, \phi_{t+1}\}} \rho_0^s \rho (\log\{C_1^o\} - b \cdot \phi_1) + \sum_{t=1}^{\infty} \rho_t^s (\log\{C_t^y\} + \rho (\log\{C_{t+1}^o\} - b \cdot \phi_{t+1})) \quad (\text{A.7})$$

$$\text{s.t.} \quad Y_t = K_{t+1} + N_t^y \cdot C_t^y + N_t^o \cdot C_t^o$$



I rewrite the resource constraint as follows:

$$\frac{\gamma_t}{\theta_t} y_t = (1+n) \frac{\gamma_{t+1}}{\theta_{t+1}} k_{t+1} + C_t^y + \frac{C_t^o}{1+n} \quad (\text{A.8})$$

where  $y_t \equiv \frac{\theta_t Y_t}{\gamma_t N_t^y}$  and  $k_{t+1} = \frac{\theta_{t+1} K_{t+1}}{\gamma_{t+1} N_{t+1}^y}$ . I get the following FOC:

$$\frac{\rho_t^s}{C_t^y} = \lambda_t \quad (\text{A.9})$$

$$\frac{\rho_{t+1}^s \rho}{C_{t+1}^o} = \frac{\lambda_{t+1}}{1+n} \quad (\text{A.10})$$

$$\frac{\lambda_t(1+n)}{\lambda_{t+1}} = \frac{dy_{t+1}}{dk_{t+1}} \implies \frac{\lambda_t(1+n)}{\lambda_{t+1}} = \frac{y_{t+1}}{k_{t+1}} (1 - \varepsilon_{t+1}) \quad (\text{A.11})$$

$$\rho_t^s \rho b = \underbrace{-\lambda_t \left( \frac{d \left( \frac{\gamma_{t+1}}{\theta_{t+1}} (1+n) k_{t+1} \right)}{d\phi_{t+1}} \right)}_{=0} + \lambda_{t+1} \frac{d \left( \frac{\gamma_{t+1}}{\theta_{t+1}} y_{t+1} \right)}{d\phi_{t+1}}, \quad (\text{A.12})$$

where  $\lambda_t$  is the Lagrange multiplier of the resource constraint. From conditions (A.9), (A.10) and (A.11), I get the Euler equation:

$$\frac{C_{t+1}^o}{\rho \cdot C_t^y} = \frac{y_{t+1}}{k_{t+1}} (1 - \varepsilon_{t+1}). \quad (\text{A.13})$$

From condition (A.9) in period  $t+1$ , and condition (A.10), I get:

$$\frac{\rho_{t+1}^s \rho}{C_{t+1}^o} = \frac{\rho_{t+1}^s}{(1+n) C_{t+1}^y}. \quad (\text{A.14})$$

For a steady state to exist, weights must be of the form:  $\rho_t^s = (\rho^s)^t$  and  $\rho^s \in (0, 1)$ . Using this condition in equation (A.14), I get the following steady state condition:

$$\frac{C^o}{\rho \cdot C^y} = \frac{1+n}{\rho^s}. \quad (\text{A.15})$$

Using the steady state Euler equation and condition (A.15), I obtain:

$$\rho^s \frac{y}{k} (1 - \varepsilon) = 1 + n. \quad (\text{A.16})$$

Substituting conditions (A.15) and (A.16) into the steady state resource constraint, I obtain:

$$\frac{C^o}{1+n} = y \frac{\gamma}{\theta} \frac{\rho}{\rho^s + \rho} (1 - \rho^s (1 - \varepsilon)). \quad (\text{A.17})$$

Recovering  $\lambda_{t+1}$  from equation (A.10), and substituting into equation (A.12), I get:

$$\frac{b \cdot C_{t+1}^o}{1+n} = \frac{d \left( \frac{\gamma_{t+1}}{\theta_{t+1}} y_{t+1} \right)}{d\phi_{t+1}}. \quad (\text{A.18})$$

Considering equation (A.18) in steady state and condition (A.17), I get:

$$y \frac{\gamma}{\theta} \frac{b \cdot \rho}{\rho^s + \rho} (1 - \rho^s (1 - \varepsilon)) = \frac{d \left( \frac{\gamma}{\theta} y \right)}{d\phi}, \quad (\text{A.19})$$

where:

$$\frac{d \left( \frac{\gamma}{\theta} y \right)}{d\phi} = \begin{cases} \frac{\gamma}{\theta} \frac{y\varepsilon}{1+n+\phi} & \text{if } N^o < \hat{N}^o \\ \frac{1}{\theta} \frac{y\beta(1-I)}{\phi} & \text{if } N^o \geq \hat{N}^o. \end{cases} \quad (\text{A.20})$$

From equations (A.19) and (A.20), I finally obtain:

$$\phi^s = \begin{cases} \frac{\rho^s + \rho}{b\rho(1-\rho^s(1-\varepsilon))} \cdot \varepsilon - (1+n) & \text{if } N^o < \hat{N}^o \\ \frac{\rho^s + \rho}{b\rho(1-\rho^s(1-\varepsilon))} \beta(1-I) & \text{if } N^o \geq \hat{N}^o. \end{cases} \quad (\text{A.21})$$

### A.2.3 Competitive Equilibrium

An agent faces the following life time utility maximization problem:

$$\max_{\{C_t^y, C_{t+1}^o, \phi_{t+1}\}} \ln(C_t^y) + \rho [\ln(C_{t+1}^o) - b \cdot \phi_{t+1}] \quad \text{s.t.} \quad (\text{A.22})$$

$$C_t^y = w_t^y - S_t$$

$$C_{t+1}^o = \phi_{t+1} w_{t+1}^o + R_{t+1} S_t.$$

From problem (A.22), I get the following FOC:

$$C_{t+1}^o = \rho R_{t+1} C_t^y \quad (\text{A.23})$$

$$b = \frac{w_{t+1}}{C_{t+1}^o}. \quad (\text{A.24})$$

Rearranging, in steady state, I get:

$$\phi^* = \begin{cases} \frac{1+\rho}{\rho^b} - R & \text{for } N^o < \hat{N}^o \\ \frac{1+\rho}{\rho^b} \frac{\beta(1-I)(1+n)}{\beta(1-I)(1+n)+R\alpha I} & \text{for } N^o \geq \hat{N}^o. \end{cases} \quad (\text{A.25})$$

## A.2.4 Proofs

1. I want to show that:  $\ell_j(i) = \ell_j$  and  $k_j(i) = k_j$  for  $j \in \{n, v\}$ . Consider task  $i \in [0, I]$ ; from the  $\text{FOC}_\ell$ , I get that:  $\ell_n(i) = p(i)y(i) \cdot \alpha / w_n$ . Due to the Cobb-Douglas structure of the aggregate production function (1.3),  $p(i)y(i)$  is constant in  $i$  implying that  $\ell_n(i) = \ell_n$ . Same reasoning applies for tasks  $i \in (I, i]$  implying that  $\ell_v(i) = \ell_v$ . By substituting  $\ell_n$  and  $\ell_v$  into the  $\text{FOC}_k$  I also get that  $k_n(i) = k_n$  and  $k_v(i) = k_v$ .
2. I want to show that, if the input constraint is slack (i.e.,  $\gamma \in (0, 1)$ ), then optimality requires that wages are the same for all workers. Assume otherwise: consider the situation in which workers using new technology earn a higher salary with respect to workers using vintage technology. Since labor markets are competitive, this implies that workers using new technology are more productive. In the situation in which the input constraint is slack, the marginal task producer would benefit by switching from vintage to old technology. The process continues until the wages in new technology and vintage technology are the same; similar reasoning applies in the situation in which wages in vintage technology tasks are higher than those in new technology tasks.
3. I want to show that the aggregate output  $Y$  can be rewritten as in equation (1.23). Using the demand of task  $y(i) = Y/p(i)$ , I can rewrite output as:  $Y = \exp \left\{ \int_0^1 \ln Y - \ln p(i) di \right\}$ ; and, therefore, as:  $\int_0^1 \ln p(i) di = 0$ . By substituting into the latter equation, the equation for task price coming from the FOC of the maximization problem (1.7), I get equation (1.23).
4. **Proof of Proposition 1.3.1** I want to show inequalities (1.26), and (1.27) in Proposition 1. Using the Implicit Function Theorem (IFT), I can write  $\frac{dI}{dN^o} = -\frac{\partial F / \partial N^o}{\partial F / \partial I}$  where  $F \equiv p_n(I) - p_v(I) = 0$ . While  $\frac{\partial F}{\partial I} < 0 \forall N^o$ ,  $\frac{\partial F}{\partial N^o}$  is positive for

$N^o < \hat{N}^o$ , and negative for  $N^o \geq \hat{N}^o$  verifying (1.26). Using the chain rule, I can write  $\frac{d\theta}{dN^o} = \frac{\partial\theta}{\partial I} \cdot \frac{\partial I}{\partial N^o}$ . Since  $\frac{\partial\theta}{\partial I}$  is always positive,  $\frac{dI}{dN^o}$  and  $\frac{d\theta}{dN^o}$  have always the same sign verifying (1.27).

5. **Proof of Proposition 1.3.2** The proof runs similarly as the one for Proposition 1.3.1 using the IFT, see point 4

6. **Proof Corollary 1.3.2.1** It is straightforward by applying the chain rule, see point 4

7. **Proof of Proposition 1.5.2** Using the FOC from the social planner problem (A.7), I get the following relation:  $\rho^s \cdot \frac{dY}{dK} = 1 + n$  (equation (A.16)), or  $\rho^s \cdot R(\phi^s) = 1 + n$ . As  $\rho^s \rightarrow 1$ , then  $R(\phi^s) \rightarrow 1 + n$ . Substituting into equation (1.36), for  $\rho^s = 1$ , I obtain:

$$\phi^s = \begin{cases} \frac{1+\rho}{\rho \cdot b} - R(\phi^s) & \text{for } N^o < \hat{N}^o(\phi^s) \\ \frac{1+\rho}{\rho \cdot b} \cdot \frac{\beta(1-I(\phi^s)) \cdot (1+n)}{(1+n) \cdot \beta(1-I(\phi^s)) + \alpha I(\phi^s) \cdot R(\phi^s)} & \text{for } N^o \geq \hat{N}^o(\phi^s) \end{cases} \quad (\text{A.26})$$

which is equivalent to equation (1.37) meaning that  $\phi^* = \lim_{\rho^s \rightarrow 1} \phi^s(\rho^s)$ . To show that the competitive equilibrium retirement age is increasing in  $N^o$  for  $\phi^* \in (0, 1)$ , I rewrite the competitive equilibrium  $\phi^*$  as the social planner optimum evaluated at  $\rho^s = 1$ :

$$\phi^* = \begin{cases} \frac{1+\rho}{\rho \cdot b} - (1+n) & \text{for } N^o < \hat{N}^o(\phi^s) \\ \frac{1+\rho}{\rho \cdot b} \cdot \frac{\beta(1-I)}{\alpha I + \beta(1-I)} & \text{for } N^o \geq \hat{N}^o(\phi^s). \end{cases} \quad (\text{A.27})$$

For  $N^o < \hat{N}^o$ , an increase in  $N^o$  reduces  $n$  which increases  $\phi^*$ . For  $N^o \geq \hat{N}^o$ , we rewrite:

$$G = \phi^* - \frac{1+\rho}{\rho \cdot b} \cdot \frac{\beta(1-I)}{\alpha I + \beta(1-I)}. \quad (\text{A.28})$$

Using the IFT, we obtain:

$$\frac{d\phi^*}{dN^o} = - \frac{\frac{\partial G}{\partial N^o}}{\frac{\partial G}{\partial \phi^*}} = - \frac{\frac{\partial G}{\partial I} \frac{\partial I}{\partial N^o}}{\frac{\partial G}{\partial \phi^*}}. \quad (\text{A.29})$$

Since  $\frac{\partial G}{\partial \phi^*} > 0$ ,  $\frac{\partial G}{\partial I} > 0$  and  $\frac{\partial I}{\partial N^o} < 0$  (since  $N^o \geq \hat{N}^o$ ), we obtain that  $\frac{d\phi^*}{dN^o} > 0$ .

# Appendix B

## Appendix Chapter 2

### B.1 Summary statistics

Table (B.1) Summary statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
$\Delta \log$ Price	1,319	0.563	0.848	-1.661	0.109	0.670	3.776
$\Delta \log$ Production	1,319	0.346	0.489	-2.885	0.091	0.566	3.177
$\Delta \log$ Productivity	1,319	0.298	0.474	-2.318	0.026	0.530	3.841
Initial Foreign Exposure	1,319	0.390	0.260	0.0001	0.174	0.574	1.000
$\Delta$ Young	1,319	0.033	0.022	-0.015	0.015	0.051	0.079
$\Delta$ Middle-aged	1,319	-0.025	0.022	-0.078	-0.040	-0.007	0.026
$\Delta$ Old	1,319	-0.008	0.015	-0.044	-0.019	0.001	0.020
$\Delta \mathbb{E}$ Young	1,319	0.001	0.011	-0.021	-0.007	0.006	0.032
$\Delta \mathbb{E}$ Middle-aged	1,319	-0.004	0.007	-0.020	-0.008	0.001	0.012
$\Delta \mathbb{E}$ Old	1,319	0.003	0.007	-0.014	-0.002	0.007	0.017
$\Delta$ Young Demand OLS	1,319	-0.030	0.018	-0.104	-0.041	-0.016	0.077
$\Delta$ Middle-aged Demand OLS	1,319	0.024	0.018	-0.026	0.013	0.034	0.078
$\Delta$ Old Demand OLS	1,319	0.007	0.011	-0.046	-0.0003	0.014	0.051
$\Delta$ Young Demand IV(1)	1,319	0.014	0.010	0.0000	0.006	0.020	0.059
$\Delta$ Middle-aged Demand IV(1)	1,319	-0.010	0.008	-0.059	-0.015	-0.004	0.0000
$\Delta$ Old Demand IV(1)	1,319	-0.004	0.003	-0.024	-0.005	-0.001	0.006
$\Delta$ Young Demand IV(2)	1,319	-0.0001	0.001	-0.013	-0.001	0.0003	0.008
$\Delta$ Middle-aged Demand IV(2)	1,319	-0.001	0.001	-0.007	-0.001	-0.0001	0.003
$\Delta$ Old Demand IV(2)	1,319	0.001	0.001	-0.001	0.0003	0.001	0.010

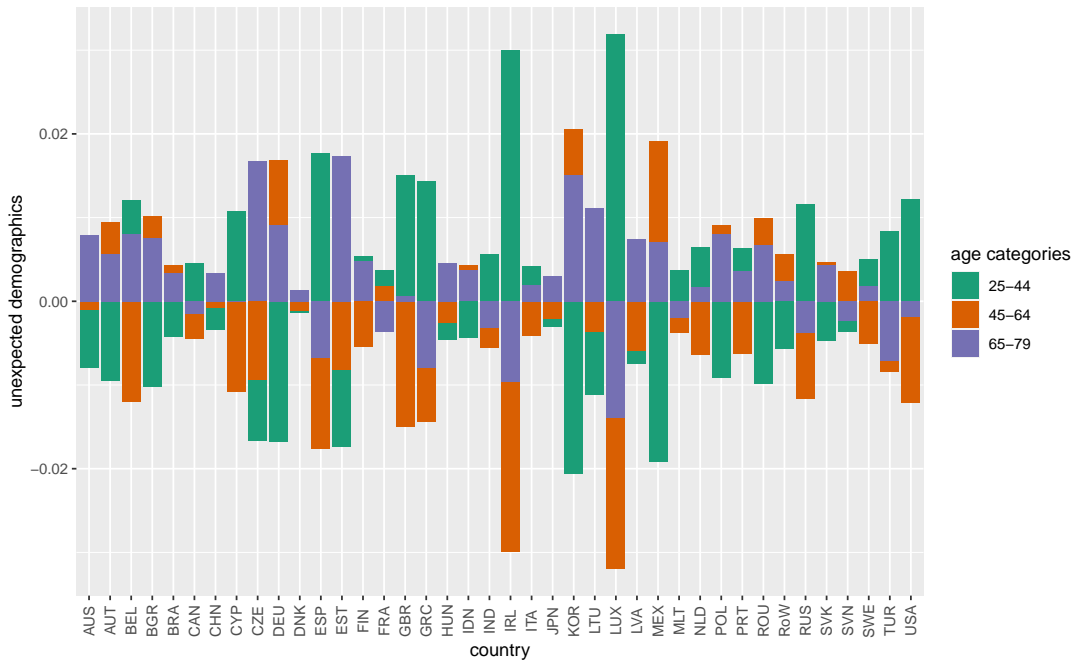


Figure (B.1) Unexpected demographics (aggregate age categories).

## B.2 Tests

### B.2.1 Shift-share correlation Test

Table (B.2) Shift-Share correlation test (change in actual demographics)

	Share			
	Correlation	95% Confidence Interval	t-stat	p-value
$\Delta$ Young	0.047	[0.038 0.055]	10.895	< 2.2e-16
$\Delta$ Middle-aged	-0.033	[-0.042, -0.025]	-7.769	8.022e-15
$\Delta$ Old	-0.019	[-0.028, -0.011]	-4.522	6.148e-06

## B.2.2 Demographics and Foreign Exposure correlation

Table (B.3) Correlation between domestic demographic change and the change in foreign exposure. Changes are between 1996 and 2006.

	$\Delta$ Foreign Exposure					
	non-weighted observations			weighted observations		
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta$ Young	-0.631 (1.184)			-0.679 (0.568)		
$\Delta$ Middle-aged		0.448 (0.840)			0.482 (0.403)	
$\Delta$ Old			-1.544 (2.895)			-1.661 (1.388)
Country & Sector FE	✓	✓	✓	✓	✓	✓
Observations	1,354	1,354	1,354	1,354	1,354	1,354
R <sup>2</sup>	0.256	0.256	0.256	0.545	0.545	0.545
Adjusted R <sup>2</sup>	0.214	0.214	0.214	0.519	0.519	0.519

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

## B.2.3 F Tests

Table (B.4) F tests

<i>share:</i> <i>shift:</i>	F test					
	IV(1)			IV(2)		
	Foreign Exposure $\Delta$ Demographics (1996-2006)			Foreign Exposure $\Delta$ Expectations (1996-2006)		
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta$ Young Demand	14.613			29.492		
$\Delta$ Middle-aged Demand		158.21			91.941	
$\Delta$ Old Demand			88.834			17.346

## B.3 Robustness Checks

### B.3.1 Different age categories

Table (B.5) Robustness check: Price (Baseline with different thresholds for the age categories)

<i>share:</i> <i>shift:</i>	$\Delta \log$ Price (1996-2006)								
	OLS			IV(1)			IV(2)		
	(1)	(2)	(3)	Foreign Exposure $\Delta$ Demographics (1996-2006)			Foreign Exposure $\Delta$ Expectations (1996-2006)		
$\Delta$ Young Demand (25-39)	2.221 (1.794)			-2.853 (4.155)			-32.921 (54.798)		
$\Delta$ Middle-aged Demand (40-59)		1.073 (1.693)			1.528 (2.975)			12.636*** (2.484)	
$\Delta$ Old Demand (60-79)			-2.640 (2.065)			-0.532 (4.231)			-28.398 (42.007)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R <sup>2</sup>	0.884	0.884	0.884	0.880	0.884	0.884	0.657	0.854	0.788
Adjusted R <sup>2</sup>	0.878	0.877	0.878	0.873	0.877	0.877	0.637	0.845	0.776

Note:

\*p&lt;0.1; \*\*p&lt;0.05; \*\*\*p&lt;0.01

Table (B.6) Robustness check: Production (Baseline with different thresholds for the age categories)

<i>share:</i> <i>shift:</i>	$\Delta \log$ Production (1996-2006)								
	OLS			IV(1)			IV(2)		
	(1)	(2)	(3)	Foreign Exposure $\Delta$ Demographics (1996-2006)			Foreign Exposure $\Delta$ Expectations (1996-2006)		
$\Delta$ Young Demand (25-39)	-0.922 (1.545)			5.773 (4.682)			28.929 (31.540)		
$\Delta$ Middle-aged Demand (40-59)		-1.320 (1.510)			-2.935 (2.308)			-7.706 (5.629)	
$\Delta$ Old Demand (60-79)			0.354 (1.812)			0.544 (2.217)			32.870 (66.508)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R <sup>2</sup>	0.578	0.579	0.578	0.557	0.577	0.578	0.164	0.556	0.190
Adjusted R <sup>2</sup>	0.554	0.554	0.553	0.531	0.553	0.553	0.115	0.530	0.143

Note:

\*p&lt;0.1; \*\*p&lt;0.05; \*\*\*p&lt;0.01



Table (B.7) Robustness check: Productivity (Baseline with different thresholds for the age categories)

<i>share:</i> <i>shift:</i>	$\Delta \log$ Productivity (1996-2006)								
	<i>OLS</i>			<i>IV(1)</i>			<i>IV(2)</i>		
	(1)	(2)	(3)	Foreign Exposure $\Delta$ Demographics (1996-2006)			Foreign Exposure $\Delta$ Expectations (1996-2006)		
$\Delta$ Young Demand (25-39)	0.228 (1.980)			1.815 (11.079)			73.831 (84.261)		
$\Delta$ Middle-aged Demand (40-59)		-2.704 (1.742)			-2.083 (6.014)			-22.647* (12.542)	
$\Delta$ Old Demand (60-79)			1.180 (2.100)			4.130 (4.481)			76.944 (157.982)
Initial Foreign Exposure Country & Sector FE	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R <sup>2</sup>	0.516	0.520	0.517	0.515	0.520	0.514	-1.950	0.302	-1.544
Adjusted R <sup>2</sup>	0.488	0.492	0.488	0.487	0.492	0.485	-2.123	0.261	-1.694

Note:

\*p&lt;0.1; \*\*p&lt;0.05; \*\*\*p&lt;0.01

### B.3.2 WIOD 2016

Table (B.8) WIOD2016 (2006-2014): Price

<i>share:</i> <i>shift:</i>	$\Delta \log$ Price (2006-2014)								
	<i>OLS</i>			<i>IV(1)</i>			<i>IV(2)</i>		
	(1)	(2)	(3)	Foreign Exposure $\Delta$ Demographics (2006-2014)			Foreign Exposure $\Delta$ Expectations (2006-2014)		
$\Delta$ Young Demand	-1.012* (0.571)			-6.395 (4.572)			36.183 (101.687)		
$\Delta$ Middle-aged Demand		0.775 (0.630)			6.657** (3.079)			17.916 (26.304)	
$\Delta$ Old Demand			0.382 (0.824)			-7.508 (10.969)			-5.428 (8.909)
Initial Foreign Exposure Country & Sector FE	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓	✓ ✓
Observations	2,231	2,231	2,231	2,230	2,230	2,230	2,230	2,230	2,230
R <sup>2</sup>	0.592	0.592	0.592	0.575	0.575	0.574	-0.220	0.450	0.582
Adjusted R <sup>2</sup>	0.574	0.573	0.573	0.556	0.556	0.555	-0.277	0.425	0.563

Note:

\*p&lt;0.1; \*\*p&lt;0.05; \*\*\*p&lt;0.01

Table (B.9) WIOD2016 (2006-2014): Production

<i>share:</i> <i>shift:</i>	$\Delta \log$ Production (2006-2014)								
	OLS			IV(1) Foreign Exposure $\Delta$ Demographics (2006-2014)			IV(2) Foreign Exposure $\Delta$ Expectations (2006-2014)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\Delta$ Young Demand	0.083 (0.673)			12.364 (9.861)			-18.526 (23.099)		
$\Delta$ Middle-aged Demand		-2.041*** (0.741)			-7.216** (2.805)			-16.169 (11.371)	
$\Delta$ Old Demand			2.901*** (0.969)			3.201 (2.293)			-1.054 (3.884)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	2,231	2,231	2,231	2,230	2,230	2,230	2,230	2,230	2,230
R <sup>2</sup>	0.592	0.594	0.594	0.529	0.585	0.594	0.446	0.524	0.591
Adjusted R <sup>2</sup>	0.574	0.575	0.575	0.507	0.565	0.575	0.420	0.502	0.572

Note:

\*p&lt;0.1; \*\*p&lt;0.05; \*\*\*p&lt;0.01

Table (B.10) WIOD2016 (2006-2014): Productivity

<i>share:</i> <i>shift:</i>	$\Delta \log$ Productivity (2006-2014)								
	OLS			IV(1) Foreign Exposure $\Delta$ Demographics (2006-2014)			IV(2) Foreign Exposure $\Delta$ Expectations (2006-2014)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\Delta$ Young Demand	0.236 (0.663)			0.419 (8.604)			6.616 (17.835)		
$\Delta$ Middle-aged Demand		-0.947 (0.731)			0.273 (1.747)			-3.167 (10.749)	
$\Delta$ Old Demand			1.843* (0.956)			-0.927 (7.032)			-4.522 (3.518)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	2,231	2,231	2,231	2,230	2,230	2,230	2,230	2,230	2,230
R <sup>2</sup>	0.449	0.450	0.450	0.449	0.449	0.448	0.425	0.447	0.439
Adjusted R <sup>2</sup>	0.424	0.424	0.425	0.424	0.423	0.422	0.399	0.422	0.413

Note:

\*p&lt;0.1; \*\*p&lt;0.05; \*\*\*p&lt;0.01

# Appendix C

## Appendix Chapter 3

### C.1 American Time Use Survey (ATUS)

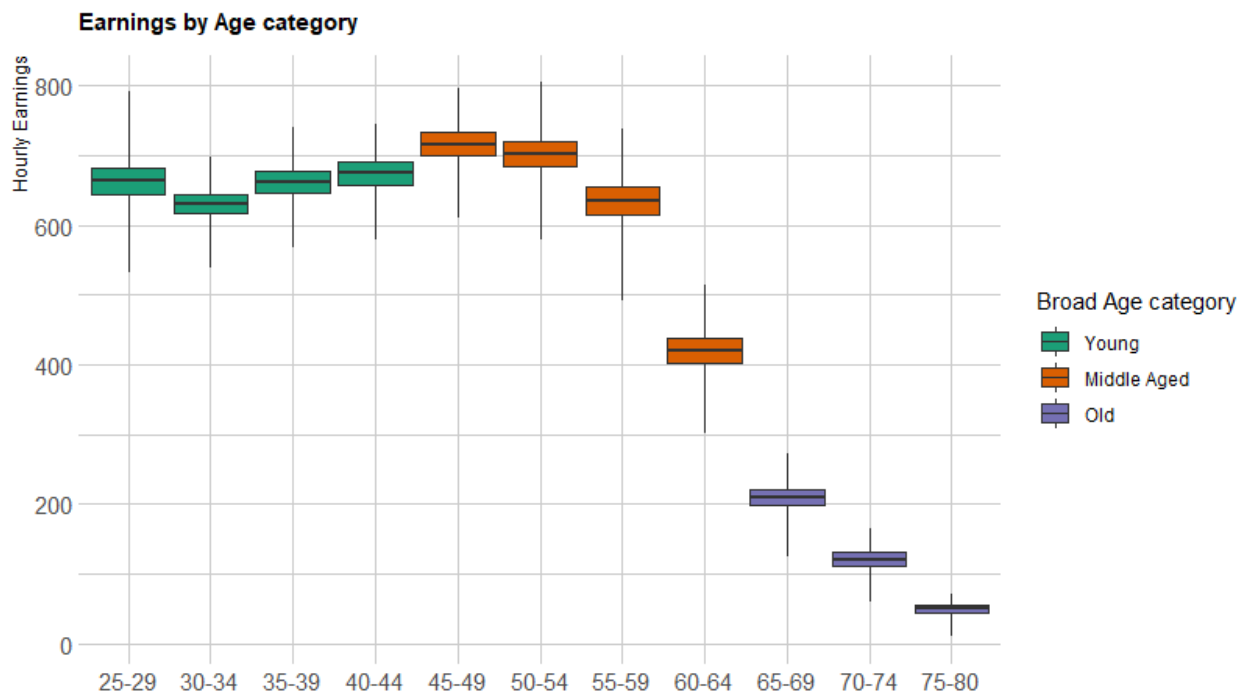


Figure (C.1) Hourly Earnings across Age categories

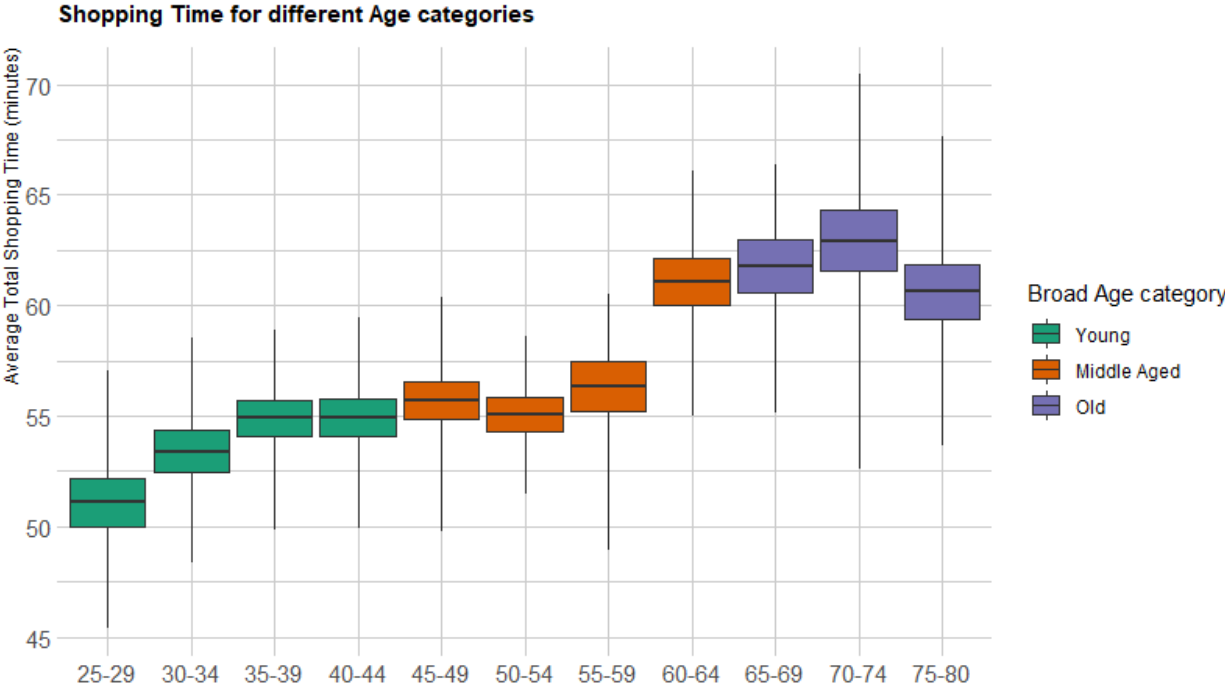


Figure (C.2) Average Shopping Time (minutes per day) across Age categories

## C.2 Model Appendix

### C.2.1 Proof of Lemma 3.2.3

*Proof.* Given  $F(p)$  defined in  $[b, \bar{r} \equiv \min\{r^Y, r^M, p^{monopoly}\}]$ , to show that the proposed is an equilibrium, we need to show that, given the reservation prices and the monopolistic price, and given that other firms choose  $F(p)$ , there is not a profitable deviation in setting a price above  $\bar{r}$ . For any  $p \in [b, \hat{r}]$ , the firm's expected profits are:

$$\mathbb{E}\{\pi(p, F)\} = \frac{\lambda^Y}{N} R^Y(p) + \frac{\lambda^M}{N} R^M(p) + \lambda^O (1 - F(p))^{N-1} R^O(p) - z(a(p)). \quad (\text{C.1})$$

Since in equilibrium, it must hold that  $\mathbb{E}\{\pi(p, F)\} = \mathbb{E}\{\pi(\bar{r}, F)\}$ , we can rewrite:

$$\mathbb{E}\{\pi(p, F)\} = \frac{\lambda^Y}{N} R^Y(\bar{r}) + \frac{\lambda^M}{N} R^M(\bar{r}) - z(a(\bar{r})). \quad (\text{C.2})$$

A firm setting a price above  $\max\{r^Y, r^O\}$  would make zero profits. But a firm setting a price  $\hat{p} \in (\bar{r}, \max\{r^Y, r^O\}]$  would still attract consumers with the highest reservation price, with expected profit:

$$\begin{cases} \mathbb{E}\{\pi(\hat{p}, F)\} = \frac{\lambda^M}{N} R^M(\hat{p}) - z(a(\hat{p})) & \text{if } r^Y \leq r^M \\ \mathbb{E}\{\pi(\hat{p}, F)\} = \frac{\lambda^Y}{N} R^Y(\hat{p}) - z(a(\hat{p})) & \text{if } r^Y > r^M. \end{cases} \quad (\text{C.3})$$

Imposing  $\mathbb{E}\{\pi(p, F)\} > \mathbb{E}\{\pi(\hat{p}, F)\}$ , we obtain:

$$\begin{cases} \mathbb{E}\{\pi(p, F)\} = \frac{\lambda^Y}{N} R^Y(r^Y) + \frac{\lambda^M}{N} R^M(r^Y) - z(a(r^Y)) > \frac{\lambda^M}{N} R^M(\hat{p}) - z(a(\hat{p})) & \text{if } r^Y \leq r^M \\ \mathbb{E}\{\pi(p, F)\} = \frac{\lambda^Y}{N} R^Y(r^M) + \frac{\lambda^M}{N} R^M(r^M) - z(a(r^M)) > \frac{\lambda^Y}{N} R^Y(\hat{p}) - z(a(\hat{p})) & \text{if } r^Y > r^M. \end{cases} \quad (\text{C.4})$$

Now, we want to show that under condition (C.4),  $\bar{r} = \min\{r^Y, r^M, p^{monopoly}\}$ .

- $\bar{r} \leq \max\{r^Y, r^M\}$  since a firm setting a price above  $\max\{r^Y, r^M\}$  earns zero profits;
- given that  $p^{monopoly} > \min\{r^Y, r^M\}$ , then  $\bar{r} \geq \min\{r^Y, r^M\}$  since a firm setting a price  $\bar{r} < \min\{r^Y, r^M\}$  would be able to sell to the same number of consumers by setting the price to  $\min\{r^Y, r^M\}$  which is closer to the monopolistic price that maximizes the profits which implies that  $\bar{r} \in [\min\{r^Y, r^M\}, \max\{r^Y, r^M\}]$ ;

- finally, under condition (C.4), it is not profitable to only focus on the consumer type with the highest reservation price and set a price in between the two reservation prices is not a profitable deviation.

□

In particular, in the case in which  $p^{monopoly} > \max\{r^Y, r^M\}$  then the most profitable deviation in  $(\min\{r^Y, r^M\}, \max\{r^Y, r^M\}]$  is  $\max\{r^Y, r^M\}$  as it brings the price closer to the monopolistic price keeping the same share of consumers (i.e., the ones with the highest reservation price). In this case, condition (C.4) is fulfilled either if the share of consumers with the highest reservation price is low enough or if the difference between the reservation prices is small which in turn depends on the demand functions of the agents. In particular, since we assume that agents have different elasticity of substitution, given the share of consumer types with the highest reservation price, condition (C.4) is fulfilled if the elasticity of substitution of young and middle-aged are not too different.

## C.2.2 Proof of proposition 3.2.4

*Proof.* Using the FOC from problem (3.8), we obtain an expression for the production:

$$y = \frac{2a}{w\bar{z}}. \quad (\text{C.5})$$

Substituting equation (C.5) into the equilibrium profit condition (3.9) objective function, we obtain the following polynomial of order 2:

$$a^2 + 2a \left( \frac{p}{w} - \bar{a} \right) - \bar{z}\pi = 0, \quad (\text{C.6})$$

which has two solutions:

$$a_1 = \left[ \left( \frac{p}{w} - \bar{a} \right)^2 + \bar{z}\pi \right]^{1/2} - \left( \frac{p}{w} - \bar{a} \right),$$

$$a_2 = - \left[ \left( \frac{p}{w} - \bar{a} \right)^2 + \bar{z}\pi \right]^{1/2} - \left( \frac{p}{w} - \bar{a} \right).$$

Since  $a_2 < 0$ , the unique solution for the investment is  $a^*(p) = a_1$  which is decreasing in prices:

$$\frac{\partial a^*}{\partial p} = \frac{1}{w} \left[ \frac{\frac{p}{w} - \bar{a}}{\sqrt{(\frac{p}{w} - \bar{a})^2 + \bar{z}\pi}} - 1 \right] < 0 \quad (\text{C.7})$$

□

### C.2.3 Equilibrium Profits

Since in equilibrium all prices in the support of  $F$  give the same expected profits  $\pi$ , it holds that  $\mathbb{E}\{\pi(\bar{r}, F)\} = \pi$ . A firm setting the reservation price  $\bar{r}$  is the highest pricing firm in the sector and  $F(\bar{r}) = 1$ . Therefore, it holds that:

$$\mathbb{E}\{\pi(\bar{r}, F)\} = \frac{\lambda^Y}{N} \cdot R^Y(\bar{r}) + \frac{\lambda^M}{N} \cdot R^M(\bar{r}) - z(a(\bar{r})). \quad (\text{C.8})$$

Substituting the optimal technology adoption (equation (3.10)) in equation (C.8), we obtain the following polynomial in  $\pi$ :

$$0 = -\pi + \frac{\lambda^Y}{N} R^Y(\bar{r}) + \frac{\lambda^M}{N} R^M(\bar{r}) - z(a(\bar{r})). \quad (\text{C.9})$$

The polynomial has three solutions:

$$\begin{aligned} \pi_1 &= -\frac{(\bar{r} - \bar{a}w)^2}{w^2\bar{z}}; \\ \pi_2 &= \frac{[\lambda^Y D^Y(\bar{r}) + \lambda^M D^M(\bar{r})] \cdot [4N \cdot (\bar{r} - \bar{a}w) + w^2\bar{z} \cdot (\lambda^Y D^Y(\bar{r}) + \lambda^M D^M(\bar{r}))]}{4N^2}; \\ \pi_3 &= -\frac{[\lambda^Y D^Y(\bar{r}) + \lambda^M D^M(\bar{r})] \cdot [4N \cdot (\bar{r} - \bar{a}w) + w^2\bar{z} \cdot (\lambda^Y D^Y(\bar{r}) + \lambda^M D^M(\bar{r}))]}{4N^2}. \end{aligned}$$

Under the condition that  $\bar{r} - \bar{a}w > 0$ ,  $\pi_1$  and  $\pi_3$  are negative.  $\bar{r} - \bar{a}w > 0$  means that the reservation price is larger than the marginal costs of a firm which does not invest in the labor saving technology. Let's assume that  $\bar{r} - \bar{a}w \leq 0$ , then a firm setting its price at  $\bar{r}$  would make zero or negative profits, which implies that  $\bar{r}$  is not in the equilibrium support of  $F$ , a contradiction. This implies that the unique solution for profits is:  $\pi = \pi_2$ .

### C.2.4 Price Normalization

We follow [Ghironi and Melitz \(2005\)](#) and normalize the price aggregator by setting the average price in the economy as the numeraire. We, therefore, divide the sector-level price of agent  $j \in \{Y, M, O\}$  by the average price in the economy, and define:

$$\tilde{p}_s^j \equiv \frac{p_s^j}{P_{AVR}}, \quad (\text{C.10})$$

where

$$P_{AVR} = \lambda^Y \cdot P^Y + \lambda^M \cdot P^M + \lambda^O \cdot P^O. \quad (\text{C.11})$$

We define the normalized price aggregator for agent  $j$ ,  $\tilde{P}^j$ , as:

$$\tilde{P}^j \equiv \left[ \int_s (\tilde{p}_s^j)^{1-\sigma} ds \right]^{\frac{1}{1-\sigma}}, \quad (\text{C.12})$$

which is also equal to:

$$\tilde{P}^j \equiv \frac{P^j}{P_{AVR}}. \quad (\text{C.13})$$

### C.2.5 Derivation of the sector-specific demand

The consumer's problem is to maximize utility subject to the budget constraint:

$$\max_{c_s^j} \left[ \int_s c_s^j \frac{\sigma_j - 1}{\sigma_j} ds \right]^{\frac{\sigma_j}{\sigma_j - 1}} \quad (\text{C.14})$$

$$\text{s.t. } \tilde{P}^j C^j \equiv \int_s \tilde{p}^j(s) \cdot c_s^j ds = I^j \quad (\text{C.15})$$

The first order conditions for a good  $j$ :

$$\left[ \int_s c_s^j \frac{\sigma_j - 1}{\sigma_j} ds \right]^{\frac{1}{\sigma_j - 1}} c_s^j^{-\frac{1}{\sigma_j}} - \theta \cdot \tilde{p}^j(s) = 0, \quad (\text{C.16})$$

where  $\theta$  is the Lagrangian multiplier linked to the constraint [\(C.15\)](#). We combine the FOCs for two distinct products  $s$  and  $t$ :

$$c_s^j = \left( \frac{\tilde{p}^j(s)}{\tilde{p}^j(t)} \right)^{-\sigma_j} \cdot c_t^j \quad (\text{C.17})$$



Replacing  $c_s^j$  in the budget constraint, we obtain the expenditure in a given market  $t$ :

$$\tilde{p}^j(t) \cdot c_t^j = I^j \cdot \left( \frac{\tilde{p}^j(t)}{\tilde{P}^j} \right)^{1-\sigma} \quad (\text{C.18})$$

Using the condition  $I^j = \tilde{P}^j C^j$ , we then obtain the demand from each product:

$$c_t^j = D(\tilde{p}^j(t)) = C^j \left( \frac{\tilde{p}^j(t)}{\tilde{P}^j} \right)^{-\sigma}. \quad (\text{C.19})$$

### C.2.6 General Equilibrium Aggregation

Let's define  $g(p_s^j)$  a function of the random variable  $p_s^j \sim F^j(p)$ . Since all the sectors are identical and agents face the same conditional to type  $j$  price distribution in every sector, by the law of large number we have that:  $\int_s g(p_s^j) ds = S \cdot \frac{\int_s g(p_s^j) ds}{S} \approx \mathcal{S} \cdot \mathbb{E}^j(g(p_s^j)) = \mathcal{S} \cdot \int_b^t g(p_s^j) dF^j(p)$ . In our case, for  $\mathcal{S} = 1$ , we can write:  $\int_s (\tilde{p}_s^j)^{-\sigma_j} ds = \mathbb{E}^j[\tilde{p}^{-\sigma_j}]$ .

### C.2.7 Sector-level comparative statics

#### Increase in Middle-aged keeping Young constant

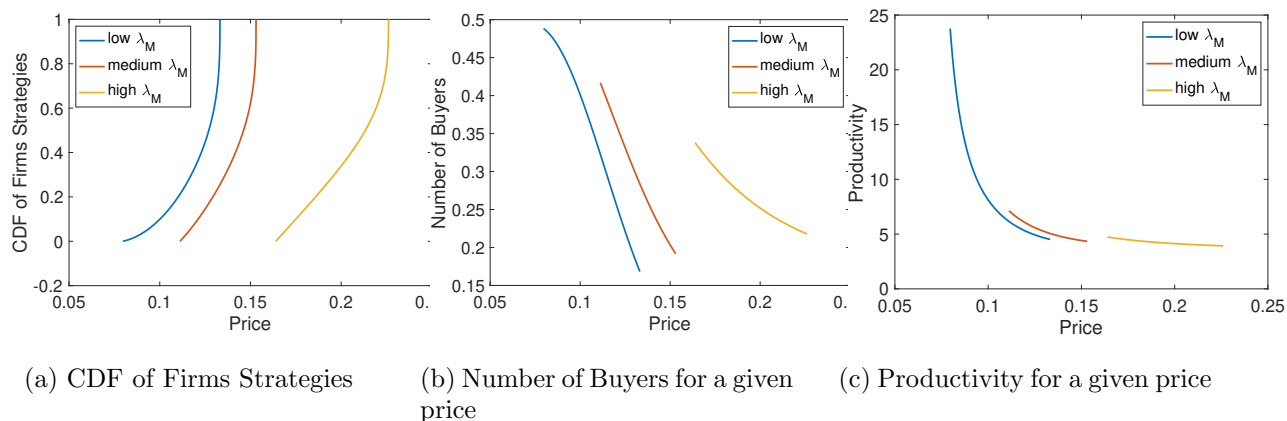


Figure (C.3) Sector-comparative static (the increase in the share of middle-aged consumers is compensated by a reduction in the share of old ones). As the share of middle-aged increases, (a) the CDF shifts right meaning higher prices on average; (b) the distribution of number of buyers becomes flatter (*allocation effect*); (c) firms increasingly adopt low-productivity strategies (*strategy effect*).

#### Increase in Middle-aged keeping Old constant

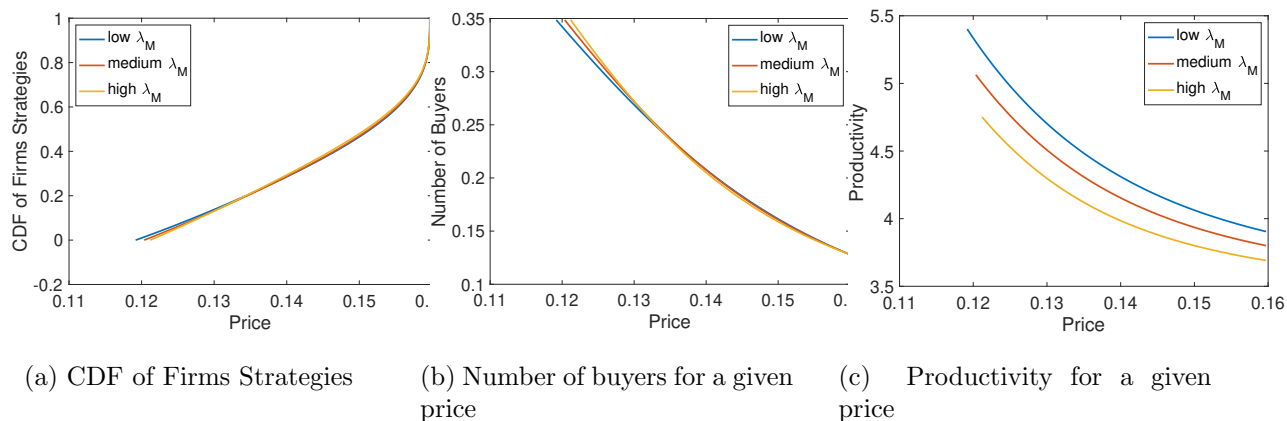


Figure (C.4) Sector-comparative static (the increase in middle-aged consumers is compensated by a reduction in the share of young ones). As the share of middle-aged increases, (a) the CDF does not move significantly; (b) the number of buyers distribution across prices does not varies appreciably (i.e., no *allocation effect*); (c) firms increasingly adopt low-productivity strategies (*strategy effect*).

### C.3 Calibration and estimation

Table (C.1) Calibrated parameters

	Description	Calibration
$\phi^Y$	share of wealth held on average by young	0.22
$\phi^M$	share of wealth held on average by middle-aged	0.46
$\phi^O$	share of wealth held on average by old	0.32
$\lambda^Y$	demographic share of young	0.48
$\lambda^M$	demographic share of middle-aged	0.3
$\lambda^O$	demographic share of old	0.22

Table (C.2) Estimated parameters

	Description	Estimates
$\bar{a}$	technology parameter	0.75
$\bar{z}$	technology cost parameter	1.14
$\bar{s}$	search cost parameter	0.61
$\underline{\sigma}$	elasticity of substitution (young)	1.37
$\bar{\sigma}$	elasticity of substitution (middle-aged and old)	0.82
$N$	number of firms in each sector	4.00

Table (C.3) Targeted Moments

	Targeted moments	Data	Model
$\frac{dp}{d\lambda^Y}$	effect of young on prices	-2.76	-1.36
$\frac{dp}{d\lambda^M}$	effect of middle-aged on prices	2.28	2.45
$\frac{dp}{d\lambda^O}$	effect of old on prices	-2.10	-0.76
$\frac{dQ}{d\lambda^Y}$	effect of young on production	2.74	1.66
$\frac{dQ}{d\lambda^M}$	effect of middle-aged on production	-1.82	-2.82
$\frac{dQ}{d\lambda^O}$	effect of old on production	2.89	0.84

The model is calibrated to the US economy in 1995. The share of each demographic group  $\lambda_i$  in the population between 25 and 79 years old comes from the UN World Population Prospect of 1996. We calibrate the share of total capital income, i.e. profits from firms and technology costs, captured by each consumer in age category  $i$  from the US average worth by age category in 1995 (Survey of Consumer Finances, 1995). An underlying and simplifying assumption is that returns on capital and portfolio compositions are identical across age categories.

$$\phi^i = \frac{\lambda_i \text{wealth}_i}{\sum_i \lambda_i \text{wealth}_i} \quad (\text{C.20})$$

with  $\text{wealth}_i$  the average wealth of age-category  $i$  in the data.

Parameters are estimated targeting the point estimate of average price and total production responses to an increase in the share of young, middle-aged, and old. These 6 moments allows to estimate 6 parameters of the model. We assume that an increase in the share of an age category  $i$ ,  $\delta_{\lambda_i}$ , comes from each of the two other age categories in proportion to their share in the economy, i.e.:

$$\delta_{\lambda_j} \equiv -\frac{\lambda_j}{1 - \lambda_i} \delta_{\lambda_i}, \text{ with } j \neq i \quad (\text{C.21})$$

Our target function is the root mean square error between the model-implied moments and the data.

## C.4 Extension: Demand and Labor market channels

In this section, we present a sketch of an extension of the model characterizing the labor market by assuming that the aggregate labor supply depends on the demographic characteristics of the agents in the economy. This allows us to analyze the effect of a demographic change on production and productivity also taking into account the labor market channel. We assume that only young and middle-aged agents work. Therefore, differently from the baseline model in which the labor supply is fixed and equal to 1, the aggregate labor supply in this framework is:

$$ALS = \lambda^Y + \lambda^M. \quad (\text{C.22})$$

An increase in the share of elderly people, therefore, negatively affects the labor supply.<sup>1</sup> This characterization of the labor market also allows for more precise modeling of the budget constraints as only the young and middle-aged would earn a salary. This characterization of the labor market would allow us to analyze the effect of a demographic change taking into account both the demand side and the supply side channels. Moreover, this modeling of the budget constraint would allow us to analyze the welfare effects of a demographic change.

**Analysis** An increase in the share of elderly people (aging), reduces the aggregate labor supply. The reduction of the labor supply triggers two opposite effects on productivity. On one side, keeping the volume of production constant, a decrease in the labor supply leads to an increase in wages. Since labor becomes more costly, firms will adopt more labor-saving technology with a positive effect on productivity. On the other side, a reduction of the labor supply reduces the volume of production as firms produce only using labor. This reduces the adoption of technology as firms have a lower production to spread the cost of technology, leading to a negative effect on productivity.

The effect of an aging population in this model on productivity is, therefore, the net effect of the effect going through the demand channel, and opposite effects from the labor market. A closer look at these competing effects is beyond the scope of this paper and left for future research.

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<sup>1</sup>In a more refined version of the model, we could also assume different degrees of complementarity between technology and workers in different age categories as in [Acemoglu and Restrepo \(2018b\)](#).



# Bibliography

- Abeliansky, A. and Prettner, K. (2017a). Automation and demographic change.
- Abeliansky, A. and Prettner, K. (2017b). Automation and Demographic Change. SSRN Scholarly Paper ID 2959977, Social Science Research Network, Rochester, NY.
- Abeliansky, A. L., Algur, E., Bloom, D. E., and Prettner, K. (2020). The future of work: Meeting the global challenges of demographic change and automation. *International Labour Review*, 159(3):285–306.
- Acemoglu, D. (1998). Why do new technologies complement skills? directed technical change and wage inequality. *The Quarterly Journal of Economics*, 113(4):1055–1089.
- Acemoglu, D. (2002). Directed technical change. *The Review of Economic Studies*, 69(4):781–809.
- Acemoglu, D. (2007). Equilibrium bias of technology. *Econometrica*, 75(5):1371–1409.
- Acemoglu, D. (2010). When does labor scarcity encourage innovation? *Journal of Political Economy*, 118(6):1037–1078.
- Acemoglu, D. and Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of labor economics*, volume 4, pages 1043–1171. Elsevier.
- Acemoglu, D. and Restrepo, P. (2017a). Robots and jobs: Evidence from us labor markets.
- Acemoglu, D. and Restrepo, P. (2017b). Secular Stagnation? The Effect of Aging on Economic Growth in the Age of Automation. *American Economic Review*, 107(5):174–179.

- Acemoglu, D. and Restrepo, P. (2018a). Demographics and automation. Technical report, National Bureau of Economic Research.
- Acemoglu, D. and Restrepo, P. (2018b). Demographics and Automation. Working Paper 24421, National Bureau of Economic Research.
- Acemoglu, D. and Restrepo, P. (2018c). The race between man and machine: Implications of technology for growth, factor shares, and employment. *American Economic Review*, 108(6):1488–1542.
- Acemoglu, D. and Zilibotti, F. (2001). Productivity differences. *The Quarterly Journal of Economics*, 116(2):563–606.
- Adão, R., Kolesár, M., and Morales, E. (2019). Shift-Share Designs: Theory and Inference\*. *The Quarterly Journal of Economics*, 134(4):1949–2010.
- Aguiar, M. and Hurst, E. (2005). Consumption versus Expenditure. *Journal of Political Economy*, 113(5):919–948.
- Aguiar, M. and Hurst, E. (2007). Life-Cycle Prices and Production. *American Economic Review*, 97(5):1533–1559.
- Arntz, M., Gregory, T., and Zierahn, U. (2016). The risk of automation for jobs in oecd countries.
- Autor, D. (2015). Why are there still so many jobs? the history and future of workplace automation. *Journal of economic perspectives*, 29(3):3–30.
- Autor, D. H. and Dorn, D. (2013). The growth of low-skill service jobs and the polarization of the us labor market. *American Economic Review*, 103(5):1553–97.
- Autor, D. H., Levy, F., and Murnane, R. J. (2003). The skill content of recent technological change: An empirical exploration. *The Quarterly journal of economics*, 118(4):1279–1333.
- Baldwin, R. and Teulings, C. (2014). Secular stagnation: facts, causes and cures. *London: Centre for Economic Policy Research-CEPR*.



- Baye, M. R., Morgan, J., and Scholten, P. (2004). Price dispersion in the small and in the large: Evidence from an internet price comparison site. *The Journal of Industrial Economics*, 52(4):463–496.
- Baylis, K. and Perloff, J. M. (2002). Price dispersion on the internet: Good firms and bad firms. *Review of industrial Organization*, 21(3):305–324.
- Benabou, R. (1993). Search market equilibrium, bilateral heterogeneity, and repeat purchases. *Journal of Economic Theory*, 60(1):140–158.
- Bonin, H., Gregory, T., and Zierahn, U. (2015). Übertragung der studie von frey/osborne (2013) auf deutschland. Technical report, ZEW Kurzexpertise.
- Bornstein, G. (2019). Entry an profits in an aging economy: the role of inertia. *Unpublished*, page 82.
- Börsch-Supan, A., Hank, K., and Jürges, H. (2005). A new comprehensive and international view on ageing: introducing the ‘survey of health, ageing and retirement in europe’. *European Journal of Ageing*, 2(4):245–253.
- Borusyak, K., Hull, P., and Jaravel, X. (2018). Quasi-experimental shift-share research designs. Technical report, National Bureau of Economic Research.
- Bowles, J. (2014). The computerization of european jobs.
- Brynjolfsson, E. and McAfee, A. (2014). *The second machine age: Work, progress, and prosperity in a time of brilliant technologies*. WW Norton & Company.
- Burdett, K. and Judd, K. L. (1983). Equilibrium Price Dispersion. *Econometrica*, 51(4):955.
- Cardona, M., Kretschmer, T., and Strobel, T. (2013). Ict and productivity: conclusions from the empirical literature. *Information Economics and Policy*, 25(3):109–125.
- Chari, V. V. and Hopenhayn, H. (1991). Vintage human capital, growth, and the diffusion of new technology. *Journal of political Economy*, 99(6):1142–1165.
- Chen, Y. and Zhang, T. (2011). Equilibrium price dispersion with heterogeneous searchers. *International Journal of Industrial Organization*, 29(6):645–654.

- Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5):1126–1150.
- Dorn, D. et al. (2009). This job is” getting old”: Measuring changes in job opportunities using occupational age structure. *American Economic Review*, 99(2):45–51.
- East, R., Uncles, M. D., and Lomax, W. (2014). Hear nothing, do nothing: The role of word of mouth in the decision-making of older consumers. *Journal of Marketing Management*, 30(7-8):786–801.
- Ferrari, A. (2019). Global Value Chains and the Business Cycle. page 78.
- Frey, C. B. and Osborne, M. A. (2017). The future of employment: how susceptible are jobs to computerisation? *Technological forecasting and social change*, 114:254–280.
- Gehring, A. and Prettnner, K. (2019). Longevity and technological change. *Macroeconomic Dynamics*, 23(4):1471–1503.
- Ghironi, F. and Melitz, M. J. (2005). International Trade and Macroeconomic Dynamics with Heterogeneous Firms. *The Quarterly Journal of Economics*, 120(3):865–915.
- Goldsmith-Pinkham, P., Sorkin, I., and Swift, H. (2020). Bartik instruments: What, when, why, and how. *American Economic Review*, 110(8):2586–2624.
- Goos, M. and Manning, A. (2007). Lousy and lovely jobs: The rising polarization of work in Britain. *The review of economics and statistics*, 89(1):118–133.
- Graetz, G. and Michaels, G. (2017). Is modern technology responsible for jobless recoveries? *American Economic Review*, 107(5):168–73.
- Gutchess, A. H. (2011). Cognitive psychology and neuroscience of aging. In *The Aging Consumer*, pages 25–46. Routledge.
- Hargittai, E. (2001). Second-level digital divide: Mapping differences in people’s online skills. *arXiv preprint cs/0109068*.
- Huneus, F. (2018). Production Network Dynamics and the Propagation of Shocks. page 97.

- Khan, I., Hollebeek, L. D., Fatma, M., Islam, J. U., and Riiivits-Arkonsuo, I. (2020). Customer experience and commitment in retailing: Does customer age matter? *Journal of Retailing and Consumer Services*, 57:102219.
- Lach, S. (2002). Existence and persistence of price dispersion: an empirical analysis. *Review of economics and statistics*, 84(3):433–444.
- Lambert-Pandraud, R. and Laurent, G. (2010). Why do Older Consumers Buy Older Brands? The Role of Attachment and Declining Innovativeness. *Journal of Marketing*, 74(5):104–121.
- Lambert-Pandraud, R., Laurent, G., and Lapersonne, E. (2005). Repeat Purchasing of New Automobiles by Older Consumers: Empirical Evidence and Interpretations. *Journal of Marketing*, 69(2):97–113.
- Maestas, N., Mullen, K. J., and Powell, D. (2016). The effect of population aging on economic growth, the labor force and productivity. Technical report, National Bureau of Economic Research.
- Perani, G., Cirillo, V., et al. (2015). Matching industry classifications. a method for converting nace rev. 2 to nace rev. 1. Technical report.
- Peters, E. (2011). Aging-related changes in decision making. In *The Aging Consumer*, pages 97–124. Routledge.
- Reinganum, J. F. (1979). A simple model of equilibrium price dispersion. *Journal of Political Economy*, 87(4):851–858.
- Salop, S. and Stiglitz, J. (1977). Bargains and ripoffs: A model of monopolistically competitive price dispersion. *The Review of Economic Studies*, 44(3):493–510.
- Stahl, D. O. (1989). Oligopolistic Pricing with Sequential Consumer Search. *The American Economic Review*, 79(4):700–712.
- Symeonidis, G. (2008). The effect of competition on wages and productivity: evidence from the united kingdom. *The review of Economics and Statistics*, 90(1):134–146.
- Varian, H. R. (1980). A model of sales. *The American economic review*, 70(4):651–659.

Weinberg, B. A. (2004). Experience and technology adoption.

Zeira, J. (1998). Workers, machines, and economic growth. *The Quarterly Journal of Economics*, 113(4):1091–1117.