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Risk-sharing, Enforceability, Information and Capital Structure

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Abstract

This paper, instead of focusing on agency cost, analyzes the role of risk-sharing under problems of enforceability (default) to explain the optimal determination of capital structure. Optimal contract structure presents equity and debt. Moreover, this paper accounts for both *(i)* equity simultaneously held by insider-entrepreneurs and outside investors as well as for *(ii)* entrepreneurs resorting to debt before investing 100 percent of their wealth in their ventures. These results provide a more accurate representation of reality within Optimal Security Design.

Keywords: debt contracts, capital structure, creditworthiness, enforceability, inside and outside equity, insurance, limited liability, private information, risk-sharing.
JEL classification numbers: D81, D82, G14, G22, G32.

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1 Introduction

So far, the literature on Optimal Security Design based on agency costs has focused only on a situation in which an entrepreneur needs to raise capital for a risky project. Thus, external funds, in the form of debt, equity or any other sort, are exogenously imposed. In all the models within this area, debt is optimal because its use ameliorates agency costs, regardless of whether these costs are interpreted as costly verification or as an inefficiency derived from the impossibility to write complete contracts. Furthermore, this theory of debt predicts an extreme inside participation rate: either the (risk neutral) insider does not resort to outside funding until he has invested 100 percent of his personal wealth or he (being risk averse) does not finance the project at all. In other words, these models can explain the co-existence of either debt and maximum inside equity— equity held by insider-entrepreneurs— or debt and outside equity— equity held by anyone with no direct role in the management of a project, but they have not yet provided the rationale for shares to be held by both insiders and outsiders.

However, we observe in reality that insiders, i.e. entrepreneur-managers, invest a relatively small, although positive, proportion of their wealth in their businesses. This is true for both closely and for widely held companies (see Harris and Raviv, 1991). In other words, evidence shows that projects are financed by both inside and outside funds even when the insider owns the resources required to finance the project himself.

The aim of these paper is to explain why entrepreneurs finance their projects only partially, or, more generally, why the use of inside and outside funds in the financing of a risky project is optimal. To avoid a trivial answer, both the entrepreneur and the outsider are endowed with the resources needed to finance the project on their own.

In contrast to most articles based on agency costs, this paper proposes risk-sharing and enforceability problems, in the form of limited liability or lack of creditworthiness, as the main determinants of capital structure. It examines the cash flows characterization of (comprehensive) contracts, and it borrows much from the Costly State Verification (CSV) literature, initiated by Townsend (1979). However, it differs from those models in the main assumptions: agents are risk averse and there is no costly verification regarding the project's returns. This paper also presents some similarities with models of Incomplete Contracting theory but it is far from them in spirit because there is no action to be taken and, hence, the allocation of control rights is unimportant.

The model is as follows. There is a risky investment project that is specific to an insider, an entrepreneur, in the sense that its returns cannot be generated without his cooperation. For simplicity, however, we ignore any action taken by the entrepreneur to generate them, that is, the returns are produced simply by his being in place. This indivisible investment project involves two dates: at the first one, when investment is undertaken, the returns are uncertain; at the second one, the return is realized and consumption takes place. To simplify, the investment requirements are assumed to be fixed, or, alternatively, only the optimal level of investment is discussed. In contrast to Hart and Moore (1989), the results of this paper are robust to the allocation of bargaining power, and, therefore, can be applied to both venture and well-functioning capital markets.

The issue is to study the contracting relationship between an entrepreneur-manager and an outsider, both of whom are risk averse, with the technological possibility of undertaking the above mentioned productive project. This is to say, how the project is to be financed and how the total wealth of this economy is to be shared.

As in Hart and Moore (1989) and Bolton and Scharfstein (1990), this environment leads to a nominal indeterminacy, in other words, to a multiplicity of optimal contracts. This is so because transfers can be done either at the first or at the second date.

The first step, therefore, is to show that the optimal allocation of risk can be attained by financing the project with both inside and outside funds. This result looks similar to traditional-capital-structure portfolio selection of a risk averse investor (in this case one with the ability to manage a project, that is, an entrepreneur) who decides optimally not to invest all his wealth in a single project (his own). However, we find that optimal risk sharing may take more extreme forms such as a contingent salary payment, by which the manager-employee does not finance any part of the project, and, thus, avoids the risk of losing any invested resources. In particular, the optimality of a positive but small inside participation rate in this model depends on both agents being risk averse and the project being very risky and costly. Above all, optimal portfolio selection, by taking the form of securities as exogenously given, does not shed any light on the more fundamental issue— which is addressed in section 4 of this paper— of why we observe inside and outside equity instead of other (a priori) possible contracts providing the same allocation of risk, for instance, a scheme of riskless debt plus insurance.

This paper further characterizes external funds depending on the information structure. It first assumes project returns to be verifiable without any cost, as in Chang (1992). Under this assumption, the optimal security is an equity. Alternatively, the paper assumes that only the lower return states (those in which the project fails) are verifiable. In this case, transfers can no longer be based on private-information (non verifiable) returns and a fixed cash flow payment, as in a debt contract, is imposed. The first result is new in explaining the optimality of inside and outside equity. The second one differs from most security-design articles based on agency cost because inside equity is combined with debt contracts even when the entrepreneur could afford to finance the project himself, thus signifying that there is no maximum inside participation.

In the second step, the indeterminacy is solved by imposing enforcement (non-default) constraints. This model assumes that the initial wealth of each agent is non-contractable, either because personal wealth is private information, non verifiable or is subject to limited liability by law. Therefore, the wealth of this economy is divided into contractable venture's returns and non-contractable personal endowments. This situation is similar to Hart and Moore (1989) and (1997), and Bolton and Scharfstein (1990), although they assume that the project's returns are non verifiable and, thus, introduce a certain verifiable asset. Because transfers at the second date cannot be paid from personal wealth, contracts such as riskless debt or insurance are non-enforceable, and hence, the indeterminacy is solved. Therefore, the optimal allocation of risk (under risk aversion with a risky and costly project) requires both parties to finance a part of the project.

The assumption that initial endowments are non-contractable is extreme. In reality, most contracts can be enforced at a cost. This cost can be thought of as the cost of acquiring information as in the CSV literature. In this case, the way to eliminate the 'verification' cost while achieving the optimal division of risk is to finance a part of the project and to rely on external funds in exchange of the project's returns, which, by assumption, are verifiable without cost. To put it simply, contracts involving an enforcement cost will never be used if there are other contracts that also sustain the optimal allocation without incurring such a cost. On the other hand, this enforcement cost can be interpreted in the light of the Incomplete Contracting literature as the cost of thinking, negotiating or writing new contracts, and, thus, as additional administrative or regulating costs. Since external funds in the form of standard debt and equity are necessary for entrepreneurs lacking sufficient resources to finance the project themselves, the theory of standardization (see Allen and Gale, 1994) affirms that such external funds will also be used when the entrepreneur does not lack these resources.

The remaining part of this section reviews the literature and suggests some future research. Section 2 describes the model under symmetric information. Section 3 characterizes the optimal allocation of event-contingent consumption, which proposition 2 in section 4 shows to be achievable by financing a part of the project with inside equity and relying on outside equity funds for the other part. Section 4 further explores enforcement problems to eliminate the nominal contractual indeterminacy. In particular, it shows that contracts such as a combination of riskless debt and insurance are non-default-proof at zero cost. Section 5 extends the model to the asymmetric information case to account for debt with a non-maximum inside participation rate. Finally, section 6 summarizes the results of the paper.

1.1 A review of the literature

This paper is inspired by Optimal Security Design based on agency costs. In both the Costly State Verification and the Incomplete Contracting literatures based on control rights, the optimality of maximum inside participation plus debt contracts (the project is not fully financed by the entrepreneur because he lacks the resources to do so) derives from the assumptions of risk neutrality and of agency costs. These costs take the form of verification costs in Townsend (1979) and Gale and Hellwig (1985), non-pecuniary penalties— such as loss of reputation and time spent in bankruptcy proceedings, costly "explaining" of poor results or search costs of a fired manager— in Diamond (1984), costs of liquidating a profitable project in Hart and Moore (1989) and (1997) and costs of withholding future finance in Bolton and Scharfstein (1990).

Since agents are risk neutral, the only scope of an optimal contract is to minimize the agency cost. In fact, both Townsend (1979) and Gale and Hellwig (1985) recognize the failure of their models to explain the optimality of debt under risk aversion. This cost is incurred when the entrepreneur fails to pay the amount contracted in exchange for the outside funds. Therefore, the entrepreneur himself optimally finances the risky project as far as he can in order to diminish the amount to be repaid and, hence, the probability of not being able to fulfil his obligations and incur the agency cost. Obviously, there would be neither an agency relation nor agency costs if the outsider, i.e. anyone with a non-direct role in the management of the risky project, did not participate or, in other words, if the entrepreneur-manager financed the project in its entirety. However, these models assume that the entrepreneur must resort to external funds, which take the form of debt optimally.

Debt is characterized by a fixed repayment if the entrepreneur is solvent, and by a declaration of bankruptcy if this payment is not satisfied, allowing the creditor-outsider to recoup as much as possible of his debt from the project's assets. In this event, control is allocated to the creditor. In the CSV literature, which focuses exclusively on cash flows, debt is optimal because it allows the reduction of verification costs by establishing non-contingent transfers and paying as much as possible in the low states to lessen the probability of default. The Incomplete Contracting literature focuses on the allocation of control rights, which debt assigns optimally. However, it explains the optimality of debt payments only partially. For instance, Bolton and Scharfstein are more concerned with how debt can be used strategically to influence competition in product markets than with a general characterization of debt, the payments of which are an (but not the unique) optimal contract.

The existence of agency costs that are incurred with a positive probability give rise to ex-post inefficiencies, and underinvestment in those models that allow the level of investment to be chosen. An exception is Diamond (1984) who endogenizes the cost and delivers a first-best solution. These ex-post inefficiencies present two problems. First, in the CSV models the threat of verification is not credible, and thus, the entrepreneur will not repay and the investor will not finance the project in first place. Therefore, a commitment technology , which is not described in these models, is necessary. Second, debt is no longer optimal when stochastic schemes are allowed. Random procedures can lessen the resource costs of verification or the liquidation inefficiencies while the threat of incurring the above mentioned agency costs induces honesty. In fact, the optimal contract with stochastic verification looks more like auditing than debt. Moreover, Hart and Moore (1997) have shown that the optimal contract with stochastic liquidation and uncertainty is no longer debt but an option-to-buy contract, which establishes contingent transfers.

On the contrary, neither does credibility nor the possibility of introducing stochastic schemes represent a problem in this model, because it delivers a first-best solution. Moreover, since it predicts an optimal level of investment, the assumption of a fixed investment requirement is non-restrictive.

Risk neutrality and agency costs lead to (i) maximum inside participation and (ii) a debt contract. Thus, in order to explain zero inside participation and outside equity, Chang (1992) assumes risk aversion on the part of the manager and no verification costs. Since the investor is assumed to be risk neutral, he optimally bears all the diversifiable risk of the firm. Furthermore, he assumes that the project's returns are diversifiable because they are verifiable without cost. Thus, the optimal financial contract must provide full insurance to the manager, i.e. the manager must receive a fixed compensation and the investor, the residual return. In short, the firm must be financed by public equity. Thereafter, he introduces a non-pecuniary agency cost, derived from optimal restructuring, to explain the use of debt to allocate control rights optimally. Thus, Chang explains the co-existence of debt and outside equity in large, widely-held corporations. However, he is unable to explain why there is also inside equity.

To sum up, the optimal capital structure in Security Design models based on agency costs is determined by the inability of the entrepreneur to self-finance the risky project and depends on *(i)* the information/verification structure of the project's return and *(ii)* risk neutrality.[1]

Although this literature has contributed much to the understanding of (financial) contract schemes, it remains far from reality because risk neutrality is a very restrictive assumption. On the one hand, both a standard debt contract and equity provide insurance to some extent. Thus, risk aversion will naturally play an important role in explaining why they do so. On the other hand, risk neutrality is justified by perfect risk pooling. Hence, in any situation in which either the total portfolio is to be chosen or the available projects are neither independent nor large enough for the law of large numbers to apply, risk aversion will be the proper assumption about agents' preferences. Risk aversion is, indeed, the standard assumption for individual proprietorships, partnerships and closely-held corporations. These might not only characterize what in this model is described as the insider, but also apply to the numerous cases in which a private party (the outsider) provides credit and/or insurance in a venture capital market or through a microlending scheme, mostly used

[1]Other theories studying capital structure focus on asymmetric information, corporate control, the influence of financial structure on competition in the product/input market, and taxes. For an excellent review of both the Traditional Capital Structure and the Security Design approaches, see Harris and Raviv, 1992

in less developed economies. In addition, the hedging literature has recognized the appropriateness of risk aversion to model the on-the-margin optimal behaviour of large widely-held corporations when there is managerial compensation (Smith and Stulz, 1985) or proprietary information (De Marzo and Duffie, 1991).

Therefore, this paper assumes risk aversion for all parties. It finds that the irrelevance theorem of Modigliani and Miller applies to this risk aversion framework— in other words, that debt plus equity is one of the many optimal structures of capital— unless enforcement (non-default) constraints are imposed. In other words, enforcement problems in this model are playing a similar role to agency costs in the above mentioned literature to determine a unique optimal structure of capital.

This paper endows the entrepreneur with the resources needed to self finance the risky project with the aim to analyze the optimal proportion in which the insider finances the project. In order to explain the co-existence of shares held by insider-entrepreneurs and outsiders, it assumes risk aversion for both the insider-entrepreneur and the outsider, and follows Chang in assuming a verifiable stream of returns. An extension to study inside equity and debt goes in the opposite direction of assuming asymmetric information regarding returns, which cannot be verified at any cost. Yet, it should be noted that in both cases the insider partially participates in the funding of the project and problems of enforceability prevent other contracts from being optimal.

Outside equity, if feasible, is Pareto superior to debt since none of them incur any (verification-bankruptcy) costs and equity provides risk sharing in all the states, whereas a debt contract, subject to limited liability, provides insurance to the entrepreneur only in the lower states (when returns are very low). However, outside equity is not always incentive compatible: depending on the information structure, entrepreneurs will be able, or not, to commit themselves to pay the cash flow which equity requires. Thus, depending on the entrepreneurs' ability to commit to reveal honestly the true return or on the investors' ability to monitor without cost, either a debt or an equity contract will emerge optimally. A possible extension of the model is to enrich it by allowing the information structure to be chosen and, then explaining the simultaneous use of inside funds, debt and outside equity.

2 Description of the economy

2.1 The environment

2.1.1 Production Possibilities

There are two technologies in this economy: one riskless and the other risky but, on average, more productive. Both technologies involve two dates, 0 and 1. At the first date, investment is chosen; at the second one, the return is available. The former technology yields a riskless return $r \geq 1$ in the second stage for each unit of the only consumption good in which everything is measured. To simplify, let us normalize $r = 1$, so that wealth is transferred from *date* 0 to *date* 1 at a 1 to 1 rate. The venture needs both a manager to develop the project and k units of the single good,[2] and it yields a random return $s \in S = \{y, \underline{x}, \bar{x}\}$ with probability p_s, where

$$\bar{x} > \underline{x} >> y$$

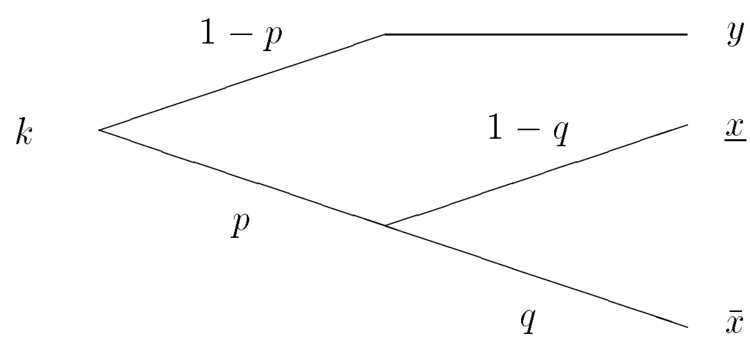
and

$$\sum_{s=y,\underline{x},\bar{x}} p_s \, s >> k > y \tag{1}$$

since the project is much more productive on average but it is risky. This stream of returns represents a situation in which a venture may either fail with probability $1 - p$ and a very low return y or succeed, in which case the return may be either very high, $\bar{x}$, with probability q or only high, $\underline{x}$. Therefore, $p_y = 1 - p$, $p_{\underline{x}} = p(1 - q)$ and $p_{\bar{x}} = pq$. Examples of this technology are any economic activity performed under the presence of some non-commercial risk of the following sort: *i*) a natural catastrophe, *ii*) a political breakdown, *iii*) a maritime disaster due to pirates, fire or a storm in long-distance commerce during Antiquity and the Middle

[2] Alternatively, k can be thought of as the optimal investment level.

Ages, etc. These events account for the 'very bad' state y, whereas the commercial risk that is inherent to any economic activity is represented by the states $\underline{x}$ and $\bar{x}$. In a more general framework, this technology can also be interpreted as any economic activity involving a cost of failure in the form of a loss of reputation or a liquidation cost.[3] It is worth noting that the realization of the return is completely exogenous: the manager does not have any ability to influence it, i.e. there is no moral hazard in the form of hidden actions.



2.1.2 Agents, Preferences and Endowments

There are two agents, indexed by the superscript $i = 1, 2$. Both of them are expected utility maximizers over their own date-1-consumption, with preferences represented by continuous, twice differentiable and strictly increasing utility functions.[4] In addition, let both agents be risk-averse, with a decreasing relative risk-aversion (DARA) coefficient, which is indeed the most realistic assumption about preferences. A further assumption is that utility functions exhibit constant relative risk-aversion (CRRA). Even though this is one of the most standard assumptions about preferences, it is more restrictive and, hence, will be carefully stated whenever it be used.[5]

[3] Simplicity is the only reason to assume a three value function of returns instead of a continuum one over the commercial states plus a jump to account for the non-commercial one; yet three states are the minimum number needed to account for the diversity in the contracts this paper deals with, as will become clear in subsection 2.2.

[4] For simplicity, agents only consume at *date* 1.

[5] The reason to assume this kind of linearity in preferences is to obtain optimal linear sharing rules. In this case, the optimal allocation can be sustained by an equity

$$U^i(c^i) \text{ with } U^{i'}(.) > 0, U^{i''}(.) < 0, DARA, \text{ for } i = 1, 2$$

Agent 1, the entrepreneur, has the inherent ability to manage the risky technology, i.e. he is the insider. He is initially endowed with $k^1 > k$ units of the single good. However, at one point this assumption will be withdrawn to illustrate what happens when the entrepreneur lacks the necessary resources to finance the venture, and, for simplicity, he will be assumed to have zero initial wealth in this case, $k^1 = 0$. Agent 2 is an outsider, i.e. someone with no direct role in the management of the project. He is endowed with $k^2 \geq k$ units of wealth at *date* 0.[6] Therefore, if the project is financed, the total endowment at *date* 1 in state s is $w_s = k^1 + k^2 - k + s$; otherwise the total endowment would be $k^1 + k^2$.[7] This is smaller on average than the former because of (1). Moreover, it is further assumed that the first inequality of (1) is big enough to satisfy

$$E[U^i(s)] > U^i(k) \; \forall i \tag{2}$$

2.1.3 Information Structure

A crucial assumption involves the information structure through the existence of private information regarding the initial endowments: each agent only knows his own endowment both at *date* 0 and 1. This is an extreme assumption that accounts in a very simple way for all enforceability problems preventing contract payments from being taken from these endowments.

contract, establishing linear transfers. However, the scope of this paper is not so much to explain the optimality of equity contracts as to identify the conditions under which other plausible contracts that also sustain the optimal allocation are not enforceable. Therefore, the general analysis allows for more flexible preferences.

[6] For the project to be financed, initial endowments must only fulfil $k^1 + k^2 > k$. However, since this paper focuses on the optimal participation rate in the funding of a project, it is convenient to assume that both agents could finance the project entirely on their own.

[7] Since agents consume only at the second date, the non-invested-in-the-venture endowments are transferred from the first to the second date by the riskless technology.

The information structure of the project is very simple, and will be modified in section 5. At *date* 0 both agents know the possible value of the venture's returns but are ignorant regarding which particular one will be realized at *date* 1. When the return is realized at *date* 1, both agents observe it without any cost. In short, there is neither adverse selection nor hidden information with regard to the technologies.[8]

In addition, there is a coercive power that enforces contracts conditioned on any publicly available information, so that there is no distinction between observability and verifiability.

2.1.4 Property Rights

The initial endowment of each agent belongs to himself. However, the ownership of the project is irrelevant to this analysis because, on the one hand, this study is robust to the allocation of property rights (it considers all levels of bargaining power subject to individual rationality), and, on the other hand, there is no action to be taken and consequently the allocation of control rights is unimportant.

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[8] To formalize the information structure, let us define some concepts. A state of nature $s \in S$ is a complete description of the exogenous uncertainty. An event is a subset of S. A partition of S is a collection of events such that the union of all its elements is S and its pairwise intersection is the null set. A partition of S describes the information revelation represented by an event tree.

Uncertainty in this economy is modeled by uncertain returns of the venture to be realized at *date* 1. Thus, states are identified with the returns $s \in S$. The information structure of both agents at *date* 0 is a partition S^{t_0} of S, where

$$S^{t_0} = \{S\} = \{\{y\}, \{\underline{x}\}, \{\bar{x}\}\}$$

since at *date* 0 agents only know the possible values of the venture's return; in short, at *date* 0 the information is common and imperfect.

At *date* 1, the information about the project is public and perfect, so that the information structure of agent i is the partition S^i of S, where

$$S^i = \{\{y\}, \{\underline{x}\}, \{\bar{x}\}\} \; for \; i = 1, 2$$

2.2 The problem

Given this model, the issue is to find, first, the optimal allocation of consumption and, second, a set of contracts that sustain this allocation.[9] These contracts can be viewed as the securities a firm issues, where the concept of a firm is borrowed from Jensen and Meckling (1976): a firm is a nexus of contracts between different parties with different interests. Thus, capital structure is determined endogenously as the set of contracts–securities that sustain the optimal allocation. In this paper, contracts–securities are exclusively characterized by the cash flows they specify.

What kind of contract is needed to attain the optimal allocation of consumption and, in first place whether the establishment of a contract between both parties is necessary or not, depends on how agents' endowments at *date* 1 differ from the optimal consumption allocation. Let $c^1 := c^1_s = w^1_s - \tau_s$ and $c^2 := c^2_s = w^2_s + \tau_s$ denote the consumption of agent $i = 1, 2$ in the state $s \in S$ where w^i_s is the endowment of agent i at *date* 1 and $\tau_s \in \Re$ is the transfer that agent 1 gives to agent 2 at *date* 1. From (2) and all utility functions exhibiting DARA, it follows that the efficient allocation requires the funding of the, on average, more productive technology. Otherwise, the problem would be uninteresting: there would be no room for capital structure analysis and the problem would be limited to a simple pure exchange economy with no risk, where property rights regarding initial endowments would prevent transfers from being different from zero. Therefore, optimal transfers and endowments at *date* 1 are defined to finance the risky project; only if there is no enforceable contract sustaining the venture, will individual consumption be equal to the autarkic endowment k^i.

[9] The Revelation Principle and the work on mechanism design (see, among others, Harris and Townsend (1981) and Myerson (1982)) ensure that any contingent allocation that does satisfy the self-selection property (i.e. which is incentive compatible) can be achieved under a mechanism, in this case, a contract. This allows us to convert a problem of characterizing efficient contracts into a simpler one of characterizing efficient allocations, as in a pure exchange economy. Then, we only have to find a contract that sustains this optimal allocation and respects all the restrictions.

In this model, there is a technological asymmetry between the insider, who is by definition the only one who can manage the (first-best) venture, and the outsider, who cannot. Since the insider-entrepreneur is further endowed with the necessary resources to finance the venture himself, he could undertake the project alone, with an event-contingent consumption $c_s^1 = k^1 - k + s$. Thus, the outsider must compensate the entrepreneur properly in order to be 'accepted' to the project. Since the entrepreneur owns the resources required to finance the project in its entirety, the outsider must provide any advantage other than funds. One such advantage is risk-sharing.

Contracts might present a nominal indeterminacy because individual date-1-endowments w_s^i are not defined by property rights a priori when the venture is financed (and, by assumption, society as a whole is better off when the on average more productive technology is used). Optimal individual date-1-endowments depend on the contract itself in two ways: first, on the quantity invested in the project at *date* 0 and, second, on each agent's rights to the return from the venture, prior to transfers at *date* 1. With regard to the latter, we must insist that nominal ownership is irrelevant to this model; only final consumption matters. Therefore, for notational convenience, we assume that the project's returns first accrue to the entrepreneur, who directly delivers the investment requirement k.[10] In order to formalize this, let us define a contract formally.

Definition 1 *A contract (at date 0) for this economy specifies* (τ_0, τ), *where* $\tau_0 \in \Re$ *and* $\tau = (\tau_y, \tau_{\underline{x}}, \tau_{\bar{x}})' \in \Re^3$ *are respectively the transfers that agent* 1 *gives to agent* 2 *at date* 0 *and at date* 1 *when the state is* $s \in S = \{y, \underline{x}, \bar{x}\}$. *Transfers at date* 0 *are prior to the realization of the state and, therefore, are independent of it.*

[10]Nothing would be changed if the project's returns were given to the outsider, and consumption expressed as $c_s^1 = k^1 - \hat{\tau}_0 - \hat{\tau}_s$ and $c_s^2 = k^2 - k + s + \hat{\tau}_0 + \hat{\tau}_s$ with these new transfers to be understood as a contingent salary payment; the optimal transfers will still give rise to the same optimal allocation of consumption, with $\hat{\tau}_0 = k + \tau_0$ and $\hat{\tau}_s = -s + \tau_s$ corresponding to equation (3).

Thus, there might be a nominal indeterminacy, as in Hart and Moore (1989): a particular consumption allocation can be attained either through transfers at *date* 0 (altering the individual date-1-endowments w_s^i) or at *date* 1 .

$$c_s^1 = w_s^1 - \tau_s = k^1 - k - \tau_0 + s - \tau_s = k^1 - k + s - [\tau_0 + \tau_s] \qquad (3)$$

$$c_s^2 = w_s^2 + \tau_s = w^2 + \tau_s = k^2 + \tau_0 + \tau_s = k^2 + [\tau_0 + \tau_s]$$

Contracts defined in this way allow the outsider to play a role as either an investor or an insurer or both. In this respect, it is interesting to note the risk-sharing role that contracts providing funds for a project might play. For instance, any credit contract providing limited liability, as standard debt and equity, prevents both agents from losing more than the amount invested in the project, and thus provides some insurance to them.

Before proceeding, let us define certain common contracts for this economy. A contract (τ_0, τ) provides outside funds if $-k \leq \tau_0 < 0$. In this case, transfers at *date* 0 can be re-stated as $\tau_0 = -(1-\phi)k$, where $\phi \in [0, 1)$ is the inside participation rate. In the limit $\phi = 1$ and $\tau_0 = 0$ for maximum participation rate. We can think about $w_s^1 = (k^1 - \phi\, k)r + s$ and $w_s^2 = [k^2 - (1-\phi)k]r = w^2$— which is certain— as the agents' endowments at *date* 1 or as their wealth prior to transfers when agent 1 and 2 have financed the venture in a proportion ϕ and $1-\phi$. Transfers at *date* 1 define, therefore, the type of (outside) crediting contract that is signed. For instance,

Definition 2 *A riskless debt contract is characterized by* $-k \leq \tau_0 < 0$ *and* $\tau_y = \tau_{\underline{x}} = \tau_{\bar{x}}$.

Definition 3 *A debt contract is characterized by* $-k \leq \tau_0 < 0$, $\tau_y = y$ *and* $\tau_{\underline{x}} = \tau_{\bar{x}}$.

Definition 4 *An outside equity contract is characterized by* $-k \leq \tau_0 < 0$ *and* $\tau_s = a + b(s - \tau_s^{debt})$, *where* τ_s^{debt} *is the state-contingent-debt payment, and* $a \geq 0$ *and* $b > 0$. *When only outside equity is issued,* $\tau_s = a + bs$.

Both debt and (outside) equity provide limited liability to both the entrepreneur and the outsider. This, however, is not the case for the entrepreneur with a riskless debt contract, in which, as its name indicates, repayment is independent of the profitability of the venture.

Alternatively, an entrepreneur who finances the project in its entirety may either hold (inside) equity, that is $\tau_s = 0 \, \forall s \in S$, or hedge, say through an insurance contract.

Definition 5 *A pure insurance contract is characterized by either* $\tau_0 > 0$ *with at least one* $\tau_s < 0$ *(a premium insurance contract) or* $\tau_0 = 0$ *with at least one* $\tau_s < 0$ *and other* $\tau_s > 0$. *In brief, an insurance contract establishes negative transfers at date* 1, $\exists s : \tau_s < 0$.

3 Optimal risk sharing

The aim of this section is to characterize the set of Pareto optimal event-contingent consumption allocations. In other words, it establishes the optimal risk sharing rule that assign to each agent a part of the state-contingent total endowment.

The organization of this section is as follows. Subsection 3.1 defines the programming problem, the solution of which characterizes the set of efficient allocations. Subsection 3.2 rewrites the problem in order to reduce the number of variables in which it must be solved, and characterizes the optimal sharing rule. Subsection 3.3 further characterizes the optimal event consumption allocation by limiting the range of consumption to be individually rational.

This is important because certain kinds of contracts violate enforceability constraints for any individually rational consumption; but section 4 will deal with this.

3.1 The programming problem

The individually-rationality Pareto efficient allocations are defined by the set of event-contingent consumption that solve the following programming problem, and fulfil the relevant information and enforceability constraints

Program 1:

$$\max_{\{c_s^i\}_{s\in S}^{i=1,2}} \quad E[U^1(c_s^1)]$$

$$\text{s.t.} \quad E[U^i(c_s^i)] \geq U^i(k^i)\ \forall i \tag{4}$$

$$E[U^1(c_s^1)] \geq E[U^1(k^1 - k + s)] \tag{5}$$

$$c_s^i \geq 0\ \forall i, \forall s \tag{6}$$

$$\sum_i c_s^i \leq w_s\ \forall s \tag{7}$$

where c_s^i and w_s are the consumption of agent i and the total endowment of the economy at *date* 1 in the state $s \in S$ when the risky project has been undertaken. The optimality of using the, on average, more productive but risky technology is guaranteed by the ex-ante participation constraints (4), where the reservation value of each agent is given by the utility they would obtain by investing in the less productive technology.[11] Restriction (5) is agent 1 individually rational constraint, which is more restrictive than his ex-ante participation one. Restriction (6) is the ex-post participation constraints. Restriction (7) accounts for feasibility.

The ex-ante best alternative for agent 2 is to invest in the riskless technology, and, consequently, an optimal contract has to provide him

[11] Remember that optimal *date* 1 transfers are set to zero when only the riskless technology is used. This is because only voluntary exchange occurs, property rights give each agent's return to himself, and there is no insurance motive since endowments are certain.

with at least the same expected utility $U(k^2)$; otherwise, he will not sign the contract. For each particular point of the core, his bargaining-power, measured by $\overline{U}^2 \geq U^2(k^2)$, will take a particular value, for which the participation constraint of agent 2 must hold with equality by definition of program 1.

The ex-ante participation constraint of agent 1 is always satisfied in the optimum because he does always want to undertake the (first-best) project: $E[U^1(c_s^1)] \geq E[U^1(k^1 - k + s)] > U^1(k^1 - k + k) = U^1(k^1)$, where the first inequality derives from (5); the second one is true because of (2) for $i = 1$ and utility functions exhibit decreasing absolute risk aversion (DARA); and the equality is obtained from simple operating. Restriction (5) takes into account the technological and financial ability of the entrepreneur to undertake the project alone. Therefore, any individually rational contract must provide him with at least the same utility he would get by financing the project himself without any contract.

The non-negativity consumption constraints (6) can be viewed as unrestricted ex-post participation constraints. This is because, regardless of the information and the legal structures, a coercive institution cannot collect what is not there, so consumption is restricted to be non-negative.

Enforceability constraints

The solution of program 1 gives an unrestricted solution. On it, we should impose information and enforceability constraints. The information structure at *date* 1 regarding the realization of the state is common and perfect to both parties, and therefore, events are equivalent to states. Thus, no restriction should be imposed to program 1. With state contingent consumption (the number of events is equal to the number of states), agents are unrestricted in their wealth transfers across states, except, of course, by the ex-post participation constraints.[12]

[12] In a more financial terminology, the fact that event consumption can be chosen differently in any state implies that the optimal allocation defined as the optimal portfolio of a complete structure of elementary claims can be achieved. An elementary

On the other hand, ex-post participation constraints become more restrictive when there are problems of default. In the model, the information about each agent's initial endowments is private and, hence, event contingent consumption is lower bounded to be greater than $\underline{c}^i$, where $\underline{c}^1 = k^1 - k - \tau_0$ and $\underline{c}^2 = k^2 + \tau_0$. It is worth noting that $\underline{c}^i$ depends on the contract, more concretely on transfers at *date* 0. With enforceability problems both agents can decide to invest some non-negative amount in the venture, but they have to make the ex-post transfers depend only on the verifiable result of the risky project, that is, $0 \leq \tau_s \leq s \; \forall s$.

These enforcement problems may derive from limited liability, lack of creditworthiness, the existence of private information about the initial endowments and so on and so forth. To make the analysis simpler, the model encompasses all these possibilities in the private information character of initial endowments k^i. For instance, if there is limited liability, the entrepreneur will not voluntary participate ex-post in any agreement providing him with a smaller allocation than his date-1 protected wealth $k^1 - k - \tau_0$, and, by law, he could not be forced not to default on his ex-ante transfer. The same applies when agents' personal wealth at date-1 is private information. In this case, the role of law in protecting private wealth is played by the unverifiable character of the personal wealth itself. Similarly too, there might be problems of creditworthiness in reality due to "people fleeing with other people's money" or investing in other non-diversifiable risky projects, with the possible result of a complete failure and the impossibility of fulfilling other obligations from their now inexistent private wealth.

claim is a security paying one unit of consumption if and only if a particular state is realized, and zero otherwise. Elementary claims are theoretical constructions that rarely have an exact counterpart in reality. Nevertheless, in reality there are contracts (complex securities) that to some extent perform the same wealth-transferring role as elementary claims. In a contracting economy, the optimal allocation is directly determined by the contract, instead of being achieved through the quantity of these elementary claims traded.

3.2 The optimal sharing rule

Applying the increasing monotonicity of the utility functions to the feasibility constraint (7), we obtain $c_s^1 = w_s - c_s^2$. The general program can thus be rewritten as,

$$\max_{\{c_s^2\}_{s\in S}} \quad E[U^1(w_s - c_s^2)]$$

$$\text{s.t.} \quad E[U^2(c_s^2)] = \overline{U}^2 \tag{8}$$

$$w_s - c_s^2 \geq \underline{c}^1 \tag{9}$$

$$c_s^2 \geq \underline{c}^2 \tag{10}$$

where individual rationality (and ex-ante participation) is satisfied by fixing $\overline{U}^2$ in equation (8) within a certain interval. Let η be the Lagrangian multiplier of restriction (8) and $\mu_s^i\ \forall s$ the ones of restriction (9) and (10) for $i = 1$ and $i = 2$ respectively.

The first order conditions (FOC) for interior points ($\mu_s^i = 0\ \forall i, \forall s$) establish the following relations where λ stands for the parameters of the model

$$\Phi(c_{\underline{x}}^2, c_{\bar{x}}^2; \lambda) = \frac{U^{2'}(c_{\underline{x}}^2)}{U^{2'}(c_{\bar{x}}^2)} - \frac{U^{1'}(w_{\underline{x}} - c_{\underline{x}}^2)}{U^{1'}(w_{\bar{x}} - c_{\bar{x}}^2)} = 0 \tag{11}$$

$$\Psi(c_y^2, c_{\underline{x}}^2; \lambda) = \frac{U^{2'}(c_y^2)}{U^{2'}(c_{\underline{x}}^2)} - \frac{U^{1'}(w_y - c_y^2)}{U^{1'}(w_{\underline{x}} - c_{\underline{x}}^2)} = 0 \tag{12}$$

From applying $U^{i'}(.) > 0$ and $U^{i''}(.) < 0$ to the FOC, it derives that optimal event contingent consumption must satisfy

$$c_{\bar{x}}^{i*} > c_{\underline{x}}^{i*} > c_y^{i*}\ \forall i \tag{13}$$

This sharing rule is very generic because the only assumption about utility functions is that they are monotonically increasing and concave; therefore, the optimal sharing rule specifies that both agents undertake a part and only a part of the total risk. If we further assume that utility functions exhibit a constant relative risk aversion (CRRA) coefficient—$U^i(c^i) = (c^i)^{\frac{1}{\rho}}$ with $\rho > 1$—, we obtain a linear sharing rule

$$\begin{aligned} c_s^{2*} &= \alpha + \beta\, s \; \forall s \\ c_s^{1*} &= \alpha^1 + \beta^1\, s \; \forall s \end{aligned} \tag{14}$$

where $\alpha = \beta(k^1 + k^2 - k) \geq 0$, $\alpha^1 = \frac{\alpha(1-\beta)}{\beta} \geq 0$, $\beta^1 = 1 - \beta \in [0,1]$ and $\beta \in [0,1]$ depends on η and ρ. In particular, $0 \leq \beta = \frac{1}{1+\eta^{\frac{\rho}{1-\rho}}} \leq 1$ because $\eta \geq 0$ by definition of a Kuhn-Tucker multiplier and $\rho > 1$ by assumption.

3.3 Optimal (individually rational) event-contingent allocations

The previous subsection characterizes the optimal sharing rule, thus, (14) defining the optimal event-contingent consumption for any interior point in the contract curve, since individual rationality has not yet been explicitly imposed. It is worth to note that β depends on η, and therefore the optimal allocation of event-contingent consumption varies according to the level $\overline{U^2}$ at which the expected utility of agent 2 is set up.

This subsection imposes two limiting values, each of which assigns all the bargaining power to one party subject to voluntary exchange, in order to further limit the (individually rational) range of optimal consumption.[13] This is mathematically accomplished by proposition 1, and graphed in figure 1.

[13] For each particular value of $\overline{U^2}$, there is a unique optimal allocation of event-contingent consumption (because the program is concave) and any individually rational event-contingent consumption must be between these two limiting levels (because the contract curve imposes a monotonic relationship between each agent consumption in different events).

(Insert figure 1 around here)

Figure 1 represents both a face of the Edgeworth boxes for a given value of $c^2_{\bar{x}}$ and for parameters λ_1 and λ_2. Since there are 3 events, the contract curve must be represented in a 3-dimensional Edgworth box. However, as we are looking at a binding restriction for only one event, we can abstract from the optimal relationship between $c^2_{\bar{x}}$ and $c^2_{\underline{x}}$ given by $\Phi(.) = 0$ and graph the optimal allocation in an intuitive 2-dimensional Edgworth box with coordinates $c^i_y, c^i_{\underline{x}}$. When the entrepreneur is initially endowed with $k^1 > k > 0$, there are more resources in the economy than when he does not have any initial wealth, $k^1 = 0$, and the Edgeworth box is larger. The reservation value of agent 2 remains the same and, consequently, so does his relevant indifference curve. Moreover, the contract curve, as will be proved, lies below that for $k^1 = 0$. The individually rational allocations belong to the interval $[BP^1, BP^2]$ on the contract curve, where point BP^i is the optimal allocation when agent i has all the bargaining power.

This further characterization of the optimal allocations is essential in order to assess what kind of contract sustains the optimal allocations and, in particular, whether a positive but small inside participation rate leads to the optimum, which is one of the aims of the paper. Moreover, restrictions (9) or (10) might be binding for all individually rational bargaining power levels in certain kinds of contracts. Restrictions (9) and (10) can be interpreted as providing limited liability to agent i up to a quantity $\underline{c}^i$, in other words, up to his non-verifiable initial endowments minus payments at *date* 0. These payments are part of the optimal contract and, therefore, enforcement constraints are restricting the form of optimal contracts. In particular, we are looking at the restrictions $c^i_y \geq \underline{c}^i \; \forall i$.

This subsection involves two steps. The first one assigns all the bargaining power to the outsider and, thus, restricts his optimal consumption in state y to be smaller than k^2 for all individually rational trade, meaning, $c^{2*}_y < k^2$. The second step explores the characterization of optimal consumption subject to the voluntary participation of the outsider. This is giving all the bargaining power to the entrepreneur to study the unique optimal allocation for $\overline{U^2} = U^2(k^2)$, and then noting that all individually rational levels of consumption are bounded by this limiting value: $c_y^{2*} > k^2 - k + y$. This last step is based on the assumption that an enforcement (ex-post participation) constraint is binding when the entrepreneur is forced to rely on external funds because he lacks the necessary resources to finance the venture himself ($k^1 = 0$).

Step 1: When the outsider has all the bargaining power, the optimal event consumption allocation lies on the indifference curve of agent 1 providing him with his minimum individually rational expected utility $E[U^1(k^1 - k + s)]$ (see point A in figure 1). Risk aversion leads to (13), and this applied to the previous utility level implies $c_y^{2*} \leq k^2$.

Step 2: For each value of agent 2's expected utility $\overline{U^2} \geq U^2(k^2)$ the unique optimal allocation depends on the parameter values $\lambda = (\rho, k^1, k^2, k, p, q, y, \underline{x}, \bar{x})$. González de Lara (1997) shows that for parameter values such as those of the model, namely, when the venture is risky ($p, q \in (0,1)$ and $y << \underline{x} << \bar{x}$) and costly ($k$ big in comparison with $k^1 + k^2$), and both agents are risk averse, the following assumption is satisfied.

Assumption 1. *When the entrepreneur lacks the wealth to finance his project in its entirety ($k^1 = 0$), the optimum is a binding allocation in which the multiplier associated with the restriction $w_y - c_y^2 \geq 0$ is positive.*

This assumption means that not only is agent 2 optimally consuming as much as possible in the state y, but that both the entrepreneur and himself would be better off if more resources in this state were allocated to the consumption of the investor. In other words, this assumption states that the optimum is a corner solution.

When $k^1 = 0$, the entrepreneur can neither finance the venture nor transfer wealth at *date* 0 to the outsider, who is, thus, an investor necessarily. Therefore, in the optimum $\tau_0 \geq -k$, and the enforceability restriction $w_y - c_y^2 \geq \underline{c}^1 = k^1 - k - \tau_0 \geq 0$ is binding whenever the former restriction is so. In other words, for a restricted set $R = \{(c_y^2, c_{\underline{x}}^2, c_{\bar{x}}^2) : w_s - c_s^2 \geq \underline{c}^1 \forall s, c_s^2 \geq \underline{c}^2 \forall s, E[U^2(c_s^2)] \equiv \overline{U^2} \geq U(k^2)$ and $\Phi(c_{\underline{x}}^2, c_{\bar{x}}^2; \lambda_1) = 0\}$, and $\Psi(c_y^2, c_{\underline{x}}^2; \lambda_1) > 0 \; \forall \, (c_y^2, c_{\underline{x}}^2, c_{\bar{x}}^2) \in R$, where λ_1 represents the set of parameter values with $k^1 = 0$. In particular, this hold for the (binding) solution $(\tilde{c_y^2}, \tilde{c_{\underline{x}}^2}, \tilde{c_{\bar{x}}^2})$, where $\tilde{c_y^2} = k^2 - k + y$, and $\tilde{c_{\underline{x}}^2}$ and $\tilde{c_{\bar{x}}^2}$ are given by (11) and (8) for each value of $\overline{U^2}$.

$$\Psi(\tilde{c_y^2}, \tilde{c_{\underline{x}}^2}; \lambda_1) > 0 \tag{15}$$

From the crucial assumption (15) and utility functions exhibiting DARA, it derives that the allocation $(\tilde{c_y^2}, \tilde{c_{\underline{x}}^2}, \tilde{c_{\bar{x}}^2})$ is not optimal for parameters λ_2 such that $k^1 > k$ either.

Lemma 1 $\Psi(\tilde{c_y^2}, \tilde{c_{\underline{x}}^2}; \lambda_2) > 0.$

Proof: $\frac{\partial \Psi(\tilde{c_y^2}, \tilde{c_{\underline{x}}^2}; \lambda)}{\partial k^1} > 0$. The operation of this derivative is very similar to that provided in the appendix for the asymmetric information case and, thus, it is left for the reader.

Therefore, the set of optimal allocations is not $(\tilde{c_y^2}, \tilde{c_{\underline{x}}^2}, \tilde{c_{\bar{x}}^2})$ but $\mathbf{c}^* = (\tilde{c_y^2} + \epsilon, \tilde{c_{\underline{x}}^2} - \delta_{\underline{x}}(\epsilon), c_{0\bar{x}}^2 - \delta_{\bar{x}}(\epsilon))'$, where each point of the core is given by a different values of ϵ.

Lemma 2 *There exists an $\epsilon > 0$, a $\delta_{\underline{x}}(\epsilon) > 0$ and a $\delta_{\bar{x}}(\epsilon) > 0$ such that all the restrictions are satisfied and $\Psi(\tilde{c_y^2} + \epsilon, \tilde{c_{\underline{x}}^2} - \delta_{\underline{x}}(\epsilon); \lambda_2) = 0$.*

Proof: The result follows from applying the sign of the following derivates to lemma 1. $\frac{\partial \Psi(c_y^2, c_{\underline{x}}^2; \lambda)}{\partial c_y^2} < 0$ and $\frac{\partial \Psi(c_y^2, c_{\underline{x}}^2; \lambda)}{\partial c_{\underline{x}}^2} > 0$. The proof is straightforward. Just derivate and note that $U'(.) > 0$ and $U''(.) < 0$. **QED**

Lemma 2 implies that the contract curve for interior points for parameters λ_2, such that $k^1 > k > 0$, lies below the contract curve for

λ_1, with $k^1 = 0$. The general proof for monotonocally increasing and concave utility functions exhibiting DARA is very similar to and simpler than that on the appendix for the asymmetric information structure. Here, only the proof for the more restrictive utility functions exhibiting CRRA is reported. In this case, (12) becomes $c^2_{\underline{x}} = \frac{w_{\underline{x}}}{w_y}\, c^2_y = \frac{k^1+k^2-k+\underline{x}}{k^1+k^2-k+y}\, c^2_y$. At any $c^2_{\bar{x}}$ and c^2_y fixed, the optimal $c^2_{\underline{x}}$ for parameter values λ_2 such that $k^1 > k > 0$ is smaller than that for parameter values λ_1 such that $k^1 = 0$ because $\frac{\partial\, c^2_{\underline{x}}}{\partial\, k^1} = \frac{w_y - w_{\underline{x}}}{(w_y)^2}\, c^2_y = -\frac{1}{(w_y)^2}\, c^2_y(w_{\underline{x}} - w_y) < 0$. The intuition behind this result is that, because of DARA, the indifference curves of the entrepreneur become flatter as he receives more initial wealth, while those of agent 2 are unaltered by any increase of agent 1's wealth.

Proposition 1 $k^2 - k + y < c^{2*}_y < k^2$

Proof: From lemma 2, which is based on *assumption 1*, it follows that $c^{2*}_y = k^2 - k + y + \epsilon$ with $\epsilon > 0$. This justifies the first inequality of proposition 1. The second one is proved in step 1.

4 Contracts that support the optimal allocation

So far, the concern of this paper has been directed to the optimal allocation of risk once the venture is financed. Looking at the allocation of event-contingent consumption obscures how this final allocation is achieved; it ignores the mechanism by which insurance is provided. The aim of this section is to find contracts that sustain the optimal event contingent allocations and fulfils all the restrictions. In order to do this, it is convenient to express feasibility and enforceability constraints in terms of the contract itself: $-k^2 \leq \tau_0 \leq k^1$ and

$$0 \leq \tau_s < s \;\forall s \in S$$

4.1 Optimal contracts, which satisfy enforcement constraints

Proposition 2 *The optimal allocation can be attained by self-financing a part of the venture* $\phi^* \in (0,1)$ *and raising external funds through outside equity for the rest of the capital requirements* $(1-\phi^*)\,k$. *In short,* $(-(1-\phi^*)\,k, a+b\,\mathbf{s})$ *is an optimal contract with* $\mathbf{s} = (y, \underline{x}, \bar{x},\ \phi^* \in (0,1)$, *and parameters* $a \geq 0$ *and* $0 < b < 1$. *Note that this contract satisfies feasibility and enforceability constraints.*

Proof: Matching equation (14) of subsection 3.2 with lemma 2 of subsection 3.3, we can express β as a function of ϵ

$$c_y^{2*} = \alpha + \beta\ y = k^2 - k + y + \epsilon$$

where $\alpha = \beta\,(k^1 + k^2 - k)$. For interpretational purposes, we can redefine $\epsilon = \gamma\,k$ with $\gamma \in (0,1)$ for proposition 1 to hold. From operating the above expression, it follows that

$$c_s^{2*} = \frac{k^2 - (1-\gamma)k + y}{k^1 + k^2 - k + y}\,(k^1 + k^2 - k) + \frac{k^2 - (1-\gamma)k + y}{k^1 + k^2 - k + y}\,s\ \forall s \quad (16)$$

A contract of outside equity providing funds in a proportion $1-\phi^*$ gives agent 2 an event contingent consumption of

$$c_s^2 = [k^2 - (1 - \phi^* k)] + a + b\,s\ \forall s \quad (17)$$

with $\phi^* \in (0,1)$, $a \geq 0$ and $b > 0$. Choosing the optimal participation rate $\phi^* = \gamma \in (0,1)$ and $b = \beta = \frac{[k^2-(1-\phi^*)k]r+y}{(k^1+k^2-k)r+y} \in (0,1)$ we can equate (16) to (17) to get $a = \frac{(k^1-\phi^* k)r}{(k^1+k^2-k)r+y}\,y > 0$ such that the ex-ante participation constraint is satisfied. Therefore, the optimal allocation of risk can be attained by an outside equity contract providing funds in a proportion $1-\gamma$ and paying contingent transfers at $date\,1$, $a+b\,s$.**QED**

Note that for different levels of bargaining-power, measured by $\epsilon = \gamma\, k$, the entrepreneur optimally self-finances the venture in different amounts, $\phi^* = \gamma$ and outside equity pays different date-1-transfers in the commercial states $\underline{x}, \bar{x}$ (a and b depends on ϕ^*). However, optimal outside equity transfers all the venture's return in the very bad state y to the outsider investor regardless of the individually rational level at which the expected utility of agent 2 is set up: $\tau_y^* = a + by = y$

This result is new in both the Costly State Verification and the Incomplete Contracting security design literatures. It explains, firstly, a positive although non-maximum inside participation rate and, secondly, the co-existence of inside and outside equity. The capital structure is driven by risk-sharing, instead of agency problems. Thus, whenever possible, the optimal external contract must give some insurance in all the states. One such contract is outside equity. However, under certain circumstances, an outside equity contract providing funds for the entirety of the venture gives too much insurance— it trades off too much consumption of the entrepreneur–manager in the good events for the increase it provides in the low event. This is why inside equity is issued, or rather, why the entrepreneur–manager finances a part of the venture himself and, hence, undertakes the risk of this part.

There is evidence suggesting that such circumstances might exist. Think about the extremely high cost that certain entrepreneurs must pay in order to have their projects financed. This cost is commonly high because repayment is uncertain: dividends in an equity contract vary with the risky performance of the firm (this is also the case for a debt contract, which exonerates the fixed payment if bankruptcy is declared). Often, entrepreneurs would be willing to commit themselves to a more stable stream of payments to reduce the high cost of external financing, i.e they would prefer to bear a greater part of the risk inherent to their firms.

4.2 A comparison with the traditional Finance literature

The second part of this section compares proposition 2 with the traditional Finance literature. A positive but small inside participation rate ($\phi^* \in (0, 1)$) has been explained in the Finance literature as the result of optimal portfolio selection when agents are risk averse: a manager who invests all of his wealth in a single firm (his own) will generally bear a welfare loss (if he is risk averse) because he is bearing more risk than necessary.

However, propositions 1 and 2 depend on the form of the contract curve, and hence on the parameter values. There might be some feasible parameter values with both agents risk averse for which the optimal allocation cannot be attained through the quantity invested by each party, but rather by a contract of insurance, plus a zero inside participation rate. With such a scheme, the outsider is not only funding the whole venture and, thus, undertaking the risk of losing his capital in the case of failure, but he is providing extra insurance to the manager in the form of, say, a (contingent) salary payment. This is the case for the contract curve CC' in figure 2. Therefore, risk aversion is a necessary but not a sufficient condition to explain a positive although small inside participation rate.

(Insert Figure 2 around here)

Figure 2 shows two different possible contract curves, CC and CC'. Their main difference is that while CC provides an optimal allocation establishing a consumption level for agent 2 in the state y greater than his endowment at *date* 1 w^2, CC' establishes a smaller one. Thus, the optimal allocation for CC' gives more insurance to the entrepreneur than the optimal allocation under CC. This is so regardless of the chosen level of inside funds ϕ^* and the kind of external funding contract that is signed: $w^2 = [k^2 - (1 - \phi^*)k]r \leq (k^2 - k)r$.[14]

[14] Since the Finance literature deals with the proportion an entrepreneur decides to invest in a project managed by him, contracts providing pure insurance at *date* 0 in the form of $0 < \tau_0 < -k$ are ruled out.

This difference is important because the optimal allocation under CC' requires a pure insurance contract (since the consumption of agent 2 in state y must be smaller than his endowment, some negative transfer in this state is needed) whereas the optimal allocation under CC can be achieved under an equity contract providing funds by both agents, as is showed in proposition 2. The optimal allocation under CC' can also be achieved through a financing contract with any participation rate plus a contract of insurance. In fact, there is a nominal indeterminacy concerning the contracts that support a particular allocation. However, if there are costs of hedging (hedging in the form of insurance contracts) the optimal contract will provide the lower level of insurance, and this is achieved through the zero inside participation rate. Moreover, an insurance contract is needed in any case; therefore, the argument of Finance explaining the optimal portfolio selection as a mixture of equity held by managers and outsiders is, at best, partial for this economy.

Above all, the traditional Finance argument— by taking the form of securities as exogenously given— does not shed any light to the more fundamental issue of why we observe inside and outside equity instead of other (a priori) possible contracts providing the same allocation of risk, for instance, a scheme of riskless debt plus insurance.

4.3 Other contracts sustaining the optimal allocation that are non-enforceable

Lastly, this subsection explores the role of other kinds of contracts to attain the optimal allocation under CC, i.e. under the assumption of a risky and costly venture to be undertaken by risk averse agents.

Proposition 3 *The unrestricted optimal allocation can be attained by either funding the venture in its entirety by a contract of riskless debt* $(\phi = 0)$ *plus an insurance contract or by the entrepreneur financing the venture himself* $(\phi = 1)$ *plus the same insurance contract. In short, both* $(-k, k + t_s)$ *and* $(0, t_s)$ *are unrestricted optimal contracts with* $t_y < 0 < t_{\underline{x}} < t_{\bar{x}}$. *Note that neither of these contracts satisfies the enforceability* *constraints* $0 \leq \tau_s \leq s$ *because riskless debt imposes* $\tau_y = k > y$ *and an insurance contract,* $\tau_y = t_y < 0$*. Therefore, they are not optimal.*

Proof: Either if the entrepreneur self-financed the venture completely ($\phi = 1$) or if the investor did so ($\phi = 0$) in return for a risk-free loan, which was to be actually implemented with $\tau_y = k > y$, the consumption of the investor at *date* 1 would be k^2. If, then, an incentive-compatible insurance contract, $(t_y, t_{\underline{x}}, t_{\bar{x}})$, were undertaken, his consumption in the event s would become $c_s^2 = k^2 + t_s$. Let us define $(t_y, t_{\underline{x}}, t_{\bar{x}})$ such that

$$\mathbf{c}^* = \begin{bmatrix} k^2 - k + y + \epsilon \\ \\ k^2 - k + \tilde{t}_{\underline{x}} - \delta_{\underline{x}}(\epsilon) \\ \\ k^2 - k + \tilde{t}_{\bar{x}} - \delta_{\bar{x}}(\epsilon) \end{bmatrix} = k^2 \begin{bmatrix} 1 \\ \\ 1 \\ \\ 1 \end{bmatrix} + \begin{bmatrix} t_y \\ \\ t_{\underline{x}} \\ \\ t_{\bar{x}} \end{bmatrix}$$

Therefore,

$$t_y = y + \epsilon - k < 0 \ \text{ since } \epsilon < k - y \text{ from } c^{2*}y < k^2$$

$$t_{\underline{x}} = t_{\underline{x}0} - \delta_{\underline{x}}(\epsilon) - k > 0$$
$$t_{\bar{x}} = t_{\bar{x}0} - \delta_{\bar{x}}(\epsilon) - k > 0$$

QED

This is represented in figure 3. A scheme of riskless debt plus insurance moves from point B to A with unenforceable transfers $\tau_y > y$, and from A to the optimal allocation of consumption once again with unenforceable transfers $\tau_y < 0$. Point B lies on the vertical line $\phi = 0$, because $k^2 - k$ is the wealth of agent 2 at *date* 1 after having financed the project completely ($\phi = 0$), and prior to transfers. Point A lies on the vertical line $\phi = 1$, because k^2 is either the wealth of agent 2 prior to transfers at *date* 1 when the entrepreneur has financed the venture himself ($\phi = 0$), or after riskless debt transfers with $\phi = 0$ have been paid.

(Insert Figure 3 around here)

This does not mean that one of the agents cannot finance the project entirely; they can and they would be happy to do so (all the restrictions will be satisfied, including the participation constraints). However, there are other schemes with $\phi \in (0,1)$ that provide the same utility to one party and increase the 'happiness' of the other party;so, an extreme ϕ is not efficient when there are enforcement problems preventing transfers of the kind $y < \tau_y < 0$. In conclusion, if the initial wealth of both agents is subject to limited liability (for example because k^1 and k^2 are private information), the venture will optimally be financed by both of them ($\phi^* \in (0,1)$).

For this argument to work, the claims of both the entrepreneur and the investor must be protected by 'limited liability'. If, for instance, the entrepreneurs date-1 wealth were subject to limited liability but the wealth of the outsider at date 1 were not, an insurance contract would be enforceable. Yet, the cost of writing new contracts may justify the inside-outside funding scheme.

In fact, given that an outside equity contract is necessary to finance the venture when the manager lacks the required resources to do so, an insurance contract is redundant, since it can be attained by a linear combination of the existing contracts. If there is any cost of writing new contracts or of learning how to use them, agents will rely on the existing ones even when enforceability does not prevent riskless debt and insurance contracts from being used.

A very realistic extension of the model is to assume that k^1 and k^2 are not unverifiable but that enforcing a contract with transfers based on this private wealth involves a cost. For instance, in the Middle Ages there was a coercive power able to tax private wealth, and, therefore, to know the amount of this private wealth to a certain extent; however, seizing property was, and often it still is, costly for the coercive power. For instance, banks are still very reluctant to go through bankruptcy procedures for the various costs involved. Similarly, an insurance company paying from what in the model is called k^2, runs administrative cost greater than those of a capital market where equity is traded. Then, a capital structure with a small but positive participation rate can be explained by enforceability costs rather than by the impossibility to enforce other contracts sustaining the optimal allocation of risk.

5 An extension of the model

If, alternatively, there were asymmetric information regarding the project's returns, the relevant program would no longer be program 1, but a restricted version in which restrictions (18)-(19) are added to (4)-(7) to account for the entrepreneurs better information about the venture returns at *date* 1.

In the model, the state y is representing an extremely bad situation which, by its nature, is easily observable, and, by assumption, verifiable without any cost. We can think of a natural catastrophe or a political breakdown; in a more general framework, we can view state y as a firm sacking people, closing plants, restructuring and so on and so forth, which will, in any case, involve less effort (cost) to be verified than that required to distinguish returns of a well-functioning firm. Therefore, the information is asymmetric because while agent 1 is perfectly able to identify the true state (return), agent 2 can only distinguish whether the firm has failed (the state is y) or not; hence, $\{\underline{x}, \bar{x}\}$ is an event for agent 2. In other words, the information structure of agent i at *date* 1 is the partition S^i of S, where

$$S^1 = \{\{y\}, \{\underline{x}\}, \{\bar{x}\}\}$$

$$S^2 = \{\{y\}, \{\underline{x}, \bar{x}\}\}$$

Since the utility function of each agent is defined on event contingent consumption, an individual cannot consume different amounts across the states that comprise an event (see Radner, 1968). In fact, this is so because the model is a one-dimensional one (there is only one period of time and a single consumption good), so that incentive compatibility constraints are imposing fixed event-consumption (see Townsend, 1982):

$$c^2_{\underline{x}} = c^2_{\bar{x}} = c^2_x \tag{18}$$

This asymmetry in information is also influencing the participation constraints of agent 1, the entrepreneur. His ex-post participation constraint in the state $\bar{x}$ is a little bit more restrictive, since $\bar{x}$ is private information: $c^1_{\bar{x}} \geq \bar{x} - \underline{x} > 0$. Applying this to the feasibility constraint (7),

$$c^2_{\bar{x}} \leq k^1 + k^2 - k + \underline{x} = w_{\underline{x}} \tag{19}$$

This means that any contract establishing a transfer in the state $\bar{x}$ greater than $\underline{x}$—which by assumption is the minimum verifiable return—is not enforceable, because the coercive power will be unable to force the entrepreneur to repay an amount greater than that which is public information, and the entrepreneur will never gift wealth to the other party voluntarily.

5.1 Optimal event-consumption allocations with asymmetric information

The First Order Conditions for the program under asymmetric information defines one implicit function rather than two explicit functions.

$$\Upsilon(c^2_y, c^2_x; \lambda) = \frac{U^{2'}(c^2_y)}{U^{2'}(c^2_x)} - \frac{U^{1'}(w_y - c^2_y)}{q\,U^{1'}(w_{\bar{x}} - c^2_x) + (1-q)U^{1'}(w_{\underline{x}} - c^2_x)} = 0 \tag{20}$$

This makes the analysis more complex in mathematical terms, but the main results remain the same. In short, the optimal sharing rule (13) is partially modified to account for asymmetric information: $c^2_{\underline{x}} = c^2_{\bar{x}} = c^2_x > c^2_y$; and proposition 1 still holds. The proofs are reported in the appendix.

5.2 Contracts that sustain the optimal allocation of risk under asymmetric information

For the asymmetric information structure, lemma 5 in the appendix characterizes the optimal allocation by $(c_y^{2\phi^*}, c_{\underline{x}}^{2\phi^*}, c_{\bar{x}}^{2\phi^*})$ where $c_y^{2\phi^*} = \tilde{c_y^2} + \epsilon$ and $c_{\underline{x}}^{2\phi^*} = c_{\bar{x}}^{2\phi^*} = c_x^{2\phi^*} = \tilde{c_x^2} - \delta(\epsilon)$.

Before proceeding, let us just interpret the meaning of the allocation $(\tilde{c_y^2}, \tilde{c_x^2})$. This is the binding solution to the programming problem under parameter values λ_1, meaning that the entrepreneur lacks the resources to finance the venture $(k^1 = 0)$. The restricted optimum under this parameter's vector λ_1 for the asymmetric information structure can be viewed as a debt contract. Since the entrepreneur lacks initial wealth, he must rely on external funding, $\phi = 0$. Under this information structure, ex-post project's returns are the borrower's private information: outsiders cannot observe the ex-post profitability because the entrepreneur can appropriate some of the returns to himself. This implies that financial contracts cannot depend directly on this information $(c_{\underline{x}}^2 = c_{\bar{x}}^2)$, ruling out (outside) equity contracts. To complete the characterization of a debt contract, the assumption is fixing the consumption of the investor in state y at the maximum enforceable level or, equivalently, is fixing $\tau_y = y$. Therefore, the optimal allocation is $(\tilde{c_y^2}, \tilde{c_x^2}) = (k^2 - k + y, k^2 - k + \tilde{t_x})$ where $\tilde{t_x}$ is such that the participation constraint of agent 2 holds with equality.

Proposition 4 *The optimal allocation can be attained by self-financing a part of the venture $\phi^* \in (0,1)$ through a debt contract if the information structure at date 1 is asymmetric. In short $(-(1-\phi^*)k, y, t_x^{\phi^*}, t_x^{\phi^*})$ with $\phi^* \in (0,1)$ is an optimal contract.*

Proof: Just choose $\phi^* : \phi^* k = \epsilon = \gamma k$.

$$\begin{bmatrix} k^2 - k + y + \epsilon \\ \\ k^2 - k + \tilde{t_x} - \delta(\epsilon) \end{bmatrix} = \begin{bmatrix} k^2 - (1-\phi)k + y \\ \\ k^2 - (1-\phi)k + t_x^{\phi^*} \end{bmatrix} = (k^2 - (1-\phi)k) \begin{bmatrix} 1 \\ \\ 1 \end{bmatrix} + \begin{bmatrix} y \\ \\ t_x^{\phi^*} \end{bmatrix}$$

where the first equality derives from imposing $\epsilon = \phi^* k$, and calculating $\delta(\epsilon)$ such that the expected utility of the investor equals the relevant value $\overline{U^2}$ and $t_x^{\phi^*}$ is calculated from

$$q\,U^2[k^2 - k + \tilde{t_x} + \delta(\epsilon)] + (1-q)\,U^2[k^2 - k + y + \epsilon]$$
$$\equiv \overline{U^2} \equiv$$
$$q\,U^2[k^2 - (1-\phi)k + t_x^{\phi^*}] + (1-q)\,U^2[k^2 - (1-\phi)k + y]$$

QED

6 Summary

This paper explains *(i)* why entrepreneurs finance their ventures only partially— $\phi^* \in (0,1)$— and *(i)* why they rely on debt and equity.

First, the optimality of using outside funds to some extent, $(1-\phi^*) \in (0,1)$, derives from risk-sharing and enforcement problems. Capital structure is determined to allocate risk optimally between an insider-entrepreneur and an outsider, both of whom are risk averse. Thus, the outsider must provide some insurance. The reason why he does so by funding a part of the risky project, and, hence, undertaking the risk to lose his funds, instead of signing an insurance contract focuses on enforceability.

The difference between these two schemes lies in the assets from which transfers are paid and the timing of payments. With outside funds subject to limited liability (for instance, standard debt and outside equity) an outsider-investor gives some wealth 'today' for the promise of a (contingent) payment 'tomorrow' from the returns of the project; with an insurance contract, the entrepreneur must pay a tangible premium 'today' for a promise of compensation in certain contracted events 'tomorrow' from the outsider's personal wealth, or both parties exchange a promise to deliver some wealth depending on the state in the future.

This paper assumes that personal wealth is either non verifiable or verifiable at a cost. Thus, insurance contracts (and riskless debt) either cannot take place or are not used because there are other contracts providing the same (optimal) allocation of risk without incurring such a cost. Moreover, the timing separation between the *quid* and the *quo* might give rise to problems of creditworthiness, which can be interpreted in the light of incomplete contracting. There is no doubt that promises can be broken. Therefore, it is natural to think that optimal contracts trade off the possibility of default by each party by making the promised payment as little as possible while sharing the risk efficiently. The way to achieve this in the economy considered in this paper is to make both parties finance a part of the project and, thus, reduce future payments.

However, risk aversion for both parties is not a sufficient condition for a scheme with inside and outside funds to be optimal. In fact, optimal risk-sharing might take a more extreme form such as a salary payment, by which the 'insider-employee' does not finance any part of the project, and, thus, avoids the risk of losing any invested resources. This is the case when the venture is not very risky and costly.

Second, outside funds optimally take the form of debt or (outside) equity depending on the information structure regarding the venture's returns, i.e., depending on entrepreneurs' ability to commit to reveal honestly the true return or on investors' ability to monitor without cost. Outside equity, if feasible, is Pareto superior to debt since no one incurs any (verification-bankruptcy) cost and equity provides risk sharing in all the states, whereas a debt contract, subject to limited liability, provides insurance to the entrepreneur only in the lower states (when returns are very low). However, outside equity is not always incentive compatible. A possible extension of the model is to enrich it by allowing the information structure to be chosen and, thus, explaining the simultaneous use of inside funds, debt and outside equity.

These results should also serve as a warning. When intuitively plausible economic environments are carefully specified, it turns out that many types of contracts do not arise optimally. One goal of this paper is to clarify the assumptions that lead to the actual use of a particular contract.

A Characterization of optimal event-contingent consumption under asymmetric information

The First Order Conditions for the program under asymmetric information defines the following implicit function, which must hold with equality for interior points.

$$\Upsilon(c_y^2, c_x^2; \lambda) = \frac{U^{2'}(c_y^2)}{U^{2'}(c_x^2)} - \frac{U^{1'}(w_y - c_y^2)}{q\, U^{1'}(w_{\bar{x}} - c_x^2) + (1-q)U^{1'}(w_{\underline{x}} - c_x^2)} = 0 \quad (21)$$

Lemma 3 $c_{\underline{x}}^2 = c_{\bar{x}}^2 = c_x^2 > c_y^2$

For each $c_y^2 \geq \underline{c_y^2}$, the *Implicit Function Theorem* ensures the existence of an implicit function φ such that $c_x^2 = \varphi(c_y^2, \lambda) := \varphi(c_y^2)$ and $\Upsilon(c_y^2, \varphi(c_y^2), \lambda) = 0$. It can be shown that $\varphi'(.) > 0$, that is $\varphi(c_y^2)$ is monotonically increasing in c_y^2. It can also be shown that, for interior points of the contract curve, $\varphi(c_y^2) > c_y^2$. This is so because of risk aversion.

Proposition 5 $k^2 - k + y < c_y^{2*} < k^2$

The last inequality follows from **step 1** in subsection 3.3. The first one, from the equivalent of assumption 1, lemma 1 and 2 in **step 2** of the same subsection.

Assumption 1 under asymmetric information means that for a restricted set $R = \{(c_y^2, c_{\underline{x}}^2, c_{\bar{x}}^2) : w_s - c_s^2 \geq \underline{c_s^1}, c_s^2 \geq \underline{c_s^2}, E[U^2(c_s^2)] \equiv \overline{U^2} \geq U(k^2) \text{ and } c_{\underline{x}}^2 = c_{\bar{x}}^2 = c_x^2\}$,

$$\Upsilon(c_y^2, c_x^2; \lambda_1) > 0 \,\forall\, (c_y^2, c_x^2) \in R$$

where λ_1 represents the set of parameter values with $k^1 = 0$. In particular this hold for the (binding) solution $(\tilde{c_y^2}, \tilde{c_x^2})$, where $\tilde{c_y^2} = (k^2 - k)r + y$ and $\tilde{c_x^2} = \frac{1}{p}[\overline{U^2} - (1-p)U^2(\tilde{c_y^2})]$

$$\Upsilon(\tilde{c_y^2}, \varphi(\tilde{c_y^2}, \lambda_1); \lambda_1) > 0 \tag{22}$$

From the crucial assumption (22,) it derives that the allocation $(\tilde{c_y^2}, \tilde{c_x^2})$ is not optimal for parameters λ_2 such that $k^1 > k$.

Lemma 4 $\Upsilon(\tilde{c_y^2}, \tilde{c_x^2}); \lambda_2) > 0$.

Proof: By assumption, $\Upsilon(\tilde{c_y^2}, \tilde{c_x^2}); \lambda_1) > 0$, and

$$\begin{aligned}
\frac{\partial \Upsilon(c_y^2, c_x^2; \lambda)}{\partial k^1} &= -\frac{U^{1''}(w_y - c_y^2)[p\,U^{1'}(w_{\bar{x}} - c_x^2) + (1-p)U^{1'}(w_{\underline{x}} - c_x^2)]}{[p\,U^{1'}(w_{\bar{x}} - c_x^2) + (1-p)U^{1'}(w_{\underline{x}} - c_x^2)]^2} \\
&+ \frac{U^{1'}(w_y - c_y^2)[p\,U^{1''}(w_{\bar{x}} - c_x^2) + (1-p)U^{1''}(w_{\underline{x}} - c_x^2)]}{[p\,U^{1'}(w_{\bar{x}} - c_x^2) + (1-p)U^{1'}(w_{\underline{x}} - c_x^2)]^2} \\
&= \frac{R^1(w_y - c_y^2)\,U^{1'}(w_y - c_y^2)}{[p\,U^{1'}(w_{\bar{x}} - c_x^2) + (1-p)U^{1'}(w_{\underline{x}} - c_x^2)]} - \frac{R^1(w_x - c_x^2)\,U^{1'}(w_y - c_y^2)}{[p\,U^{1'}(w_{\bar{x}} - c_x^2) + (1-p)U^{1'}(w_{\underline{x}} - c_x^2)]} = \\
&= \frac{U^{1'}(w_y - c_y^2)}{[p\,U^{1'}(w_{\bar{x}} - c_x^2) + (1-p)U^{1'}(w_{\underline{x}} - c_x^2)]}\left[R^1(w_y - c_y^2) - R^1(w_x - c_x^2)\right] > 0
\end{aligned}$$

where the first equality derives from simple derivation; the second one from applying the definition of absolute risk aversion coefficient, $R(z) = -\frac{U''(z)}{U'(z)}$, and defining $R^1(w_x - c_x^2) = \frac{[p\,U^{1''}(w_{\bar{x}} - c_x^2) + (1-p)U^{1''}(w_{\underline{x}} - c_x^2)]}{[p\,U^{1'}(w_{\bar{x}} - c_x^2) + (1-p)U^{1'}(w_{\underline{x}} - c_x^2)]}$ to make the notation more compact; the third inequality derives from operating and the inequality from $U'(.) > 0$, $U''(.) < 0$ and the utility function exhibiting DARA, where

$$w_y - c_y^2 < w_{\underline{x}} - c_x^2 < w_x - c_x^2$$

The first part of this last expression holds because agents are risk averse, so that the optimum gives some insurance to both parties. The last inequality derives from the definition of $R^1(w_x - c_x^2)$ and DARA. **QED**

Therefore, the optimal allocation is not $(\tilde{c_y^2}, \tilde{c_x^2})$ but $(\tilde{c_y^2} + \epsilon, \tilde{c_x^2} - \delta(\epsilon))$.

Lemma 5 *There exists an* $\epsilon > 0$ *and a* $\delta(\epsilon) > 0$ *such that all the restrictions are satisfied and* $\Upsilon(\tilde{c}_y^2 + \epsilon, \tilde{c}_x^2 - \delta(\epsilon); \lambda_2) = 0$

Proof: The result is as that of lemma 2.

(Insert figure 1' around here)

Figure 1', as figure 1, represents both the Edgeworth boxes for parameters λ_1 and λ_2. Since the information asymmetry is imposing $c^2_{\underline{x}} = c^2_{\bar{x}} = c^2_x$, we have two instead of three independent variables and the analysis can be graphed in a two dimensional picture, with a trick: agent 2's consumption is two state contingent, but agent 1's consumption is three state contingent; thus, agent 1's consumption has two different origins in the Edgeworth box.

Lemma 6 *The contract curve for interior points, defined by* $\varphi(c_y^2, \lambda)$ *for parameters* λ_2*, such that* $k^1 > k > 0$*, lies below the contract curve for* λ_1*, with* $k^1 = 0$*:* $\varphi(c_y^2, \lambda_2) < \varphi(c_y^2, \lambda_1)\ \forall c_y^2$

Proof: The *Implicit Function Theorem* also gives the first order comparative statics of any parameter on $\varphi(c_y^2)$, for any c_y^2 at a solution. In particular,

$$\frac{d\,\varphi(c_y^2, \lambda)}{d\,k^1} = -\frac{\frac{\partial \Upsilon(c_y^2, c_x^2; \lambda)}{\partial k^1}}{\frac{\partial \Upsilon(c_y^2, c_x^2; \lambda)}{\partial c_x^2}} < 0\ \forall c_y^2 \text{ in the restricted set} \tag{23}$$

since $U'(.) > 0$, $U''(.) < 0$ and the utility functions exhibit decreasing absolute risk aversion (DARA). **QED**

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FIGURE 1

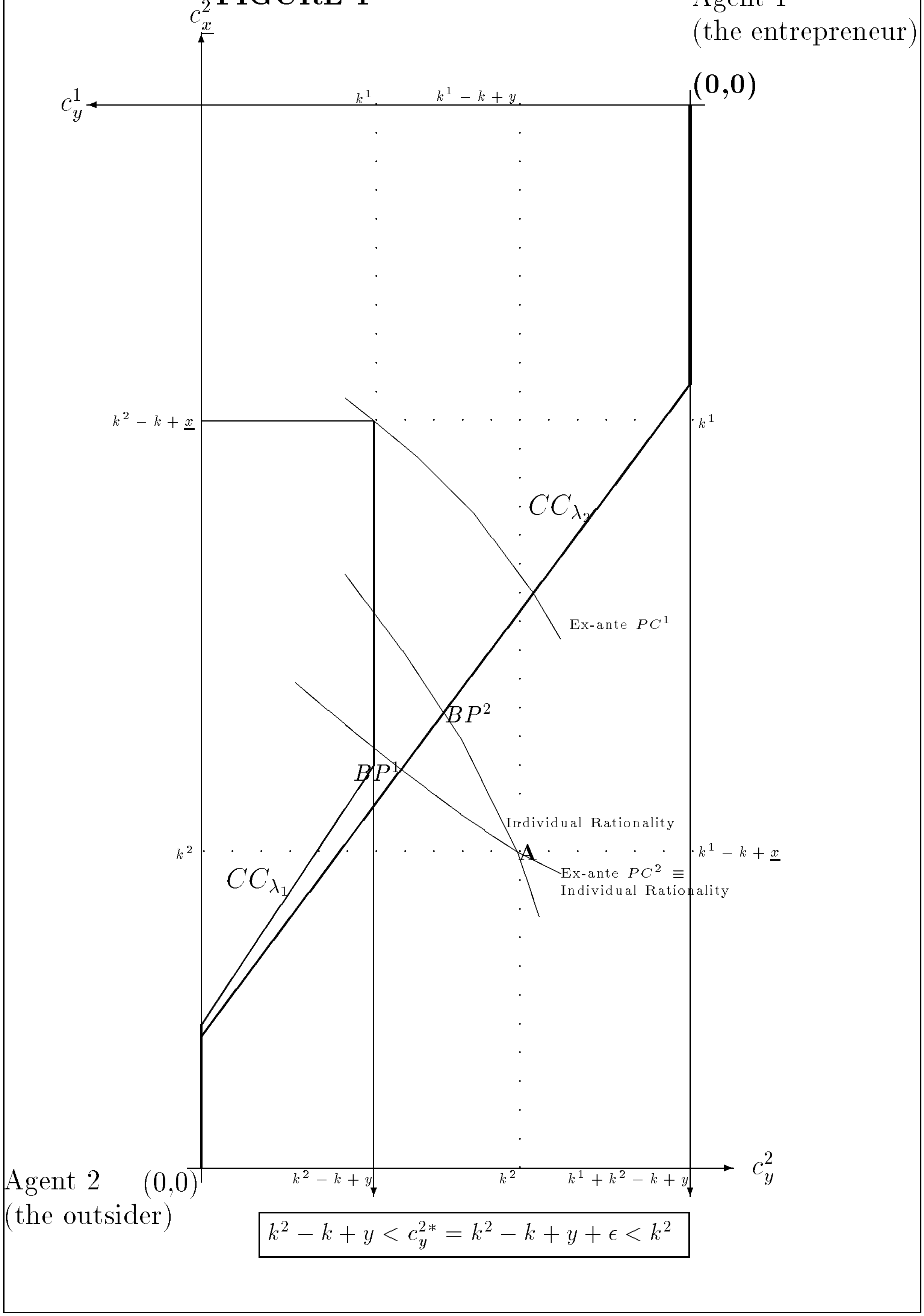

Agent 1
(the entrepreneur)
(0,0)
$c_{\underline{x}}^2$
c_y^1
k^1
$k^1 - k + y$
$k^2 - k + \underline{x}$
k^1
CC_{λ_2}
Ex-ante PC^1
BP^2
BP^1
Individual Rationality
k^2
A
$k^1 - k + \underline{x}$
Ex-ante PC^2 ≡
Individual Rationality
CC_{λ_1}
Agent 2
(the outsider)
(0,0)
$k^2 - k + y$
k^2
$k^1 + k^2 - k + y$
c_y^2
$k^2 - k + y < c_y^{2*} = k^2 - k + y + \epsilon < k^2$

FIGURE 2

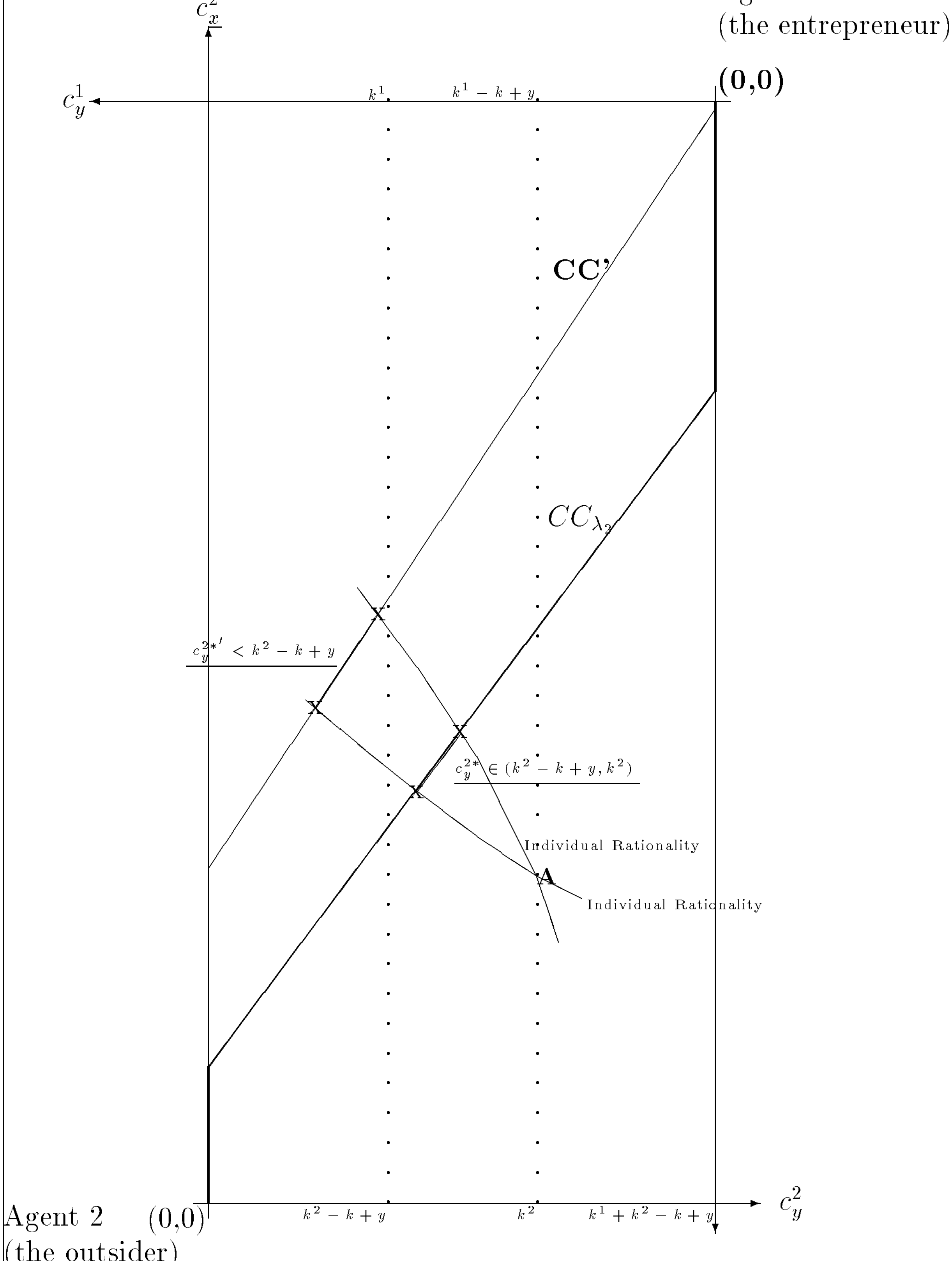
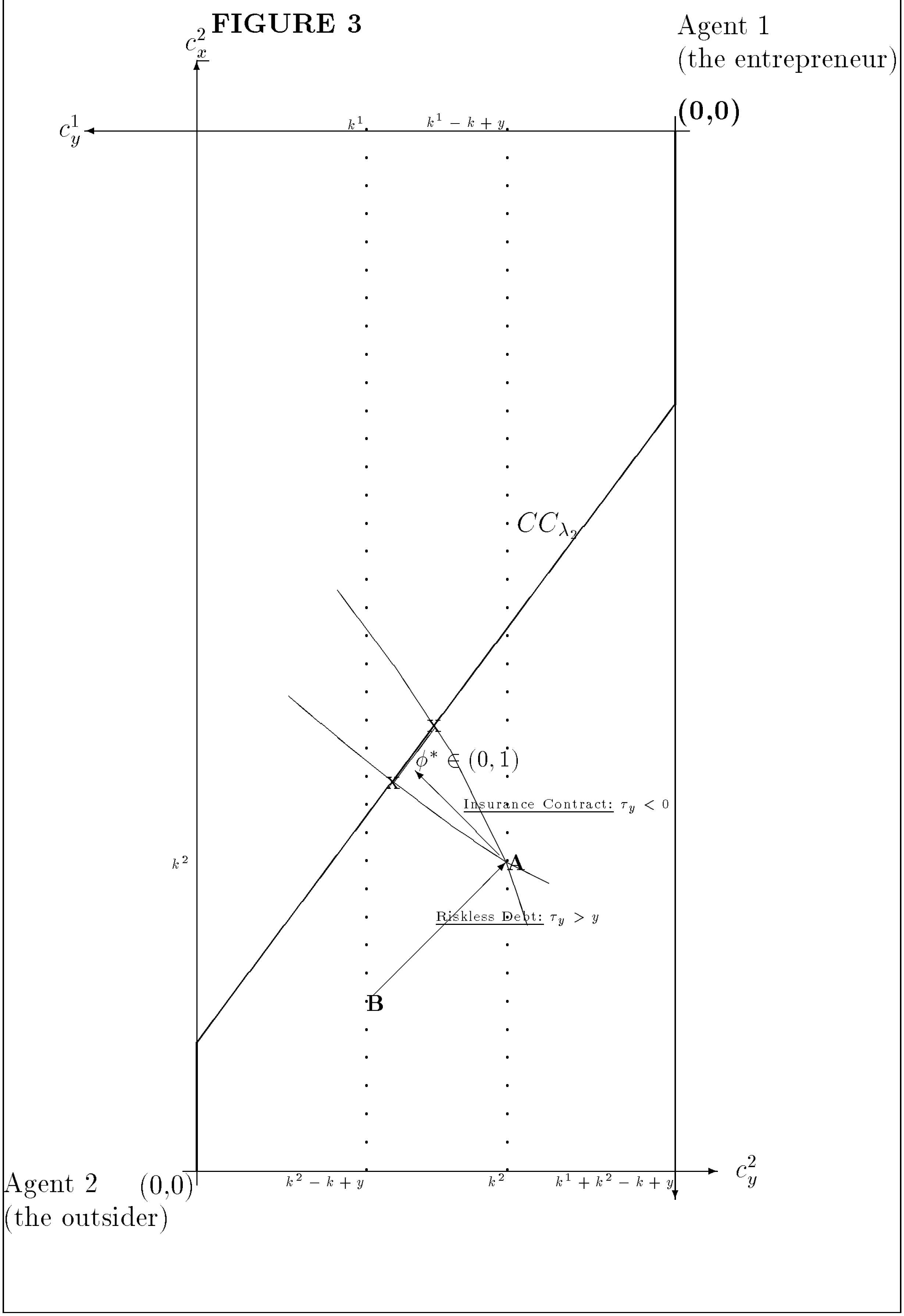

Agent 1
(the entrepreneur)
(0,0)
c_x^2
c_y^1
k^1
$k^1 - k + y$
CC'
CC_{λ_2}
X
X
X
X
$c_y^{2*'} < k^2 - k + y$
$c_y^{2*} \in (k^2 - k + y, k^2)$
Individual Rationality
A
Individual Rationality
Agent 2
(0,0)
(the outsider)
$k^2 - k + y$
k^2
$k^1 + k^2 - k + y$
c_y^2

FIGURE 3



c_x^2
Agent 1
(the entrepreneur)
(0,0)
c_y^1
k^1
$k^1 - k + y$
CC_{λ_2}
$\phi^* \in (0,1)$
Insurance Contract: $\tau_y < 0$
k^2
A
Riskless Debt: $\tau_y > y$
B
Agent 2
(0,0)
(the outsider)
$k^2 - k + y$
k^2
$k^1 + k^2 - k + y$
c_y^2