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Productivity Growth, Bounded Marginal Utility,  
and Patterns of Trade

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**EUROPEAN UNIVERSITY INSTITUTE  
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## **Abstract**

The workhorse model of the New Trade Theory fails to explain four strong and central patterns of postwar trade data. These patterns are, first, the massive increase in trade volumes, second, the small fraction of traded varieties the average country imports, third the correlation between per capita income growth and trade growth, and fourth, the correlation between trade growth and growth in the number of source countries per imported good. The present paper shows that a small and reasonable change in the demand structure can reconcile the model with the data. It departs from standard theory by assuming that consumers derive bounded marginal utility from varieties. This implies that consumers purchase only the cheaper share of varieties and that expensive foreign varieties bearing high transport costs are not imported. Technological progress which increases per capita consumption of the varieties in the consumption basket decreases marginal utility derived from each of them and induces consumers to extend their consumption to more expensive varieties produced at more distant locations. This additional margin along which trade can expand induces a substantial increase in the trade share as productivity grows. Productivity change is thus identified as a joint determinant of trade shares, the number of source countries per good, and per capita income, explaining the trends and correlations in the data.

## **Keywords**

Trade Volume, Source Country

**JEL-classification:** F1, F4



# Productivity Growth, Bounded Marginal Utility, and Patterns of Trade

Philip Sauré<sup>1</sup>

## 1 Introduction

The impact of Krugman's (1980) workhorse model on trade theory can hardly be overestimated. As the most successful formalization of the New Trade Theory, it inspired research on economic agglomeration, trade agreements and economic development, sparked off the New Economic Geography and the literature on trade and heterogeneous firms, and hence reaches far beyond the analysis of the home market effect and intra-industry trade it originally set out to explain. The model is one of the building blocks of modern trade theory.

Yet recent literature has pointed out strong and important regularities of trade data that seem to indicate some limitations of the New Trade Theory. These regularities are, first, the massive growth of trade volumes, second, the tiny fraction of tradable varieties that countries tend to purchase, third, the parallel growth of per capita income and trade, fourth and finally, the strong correlation between trade growth and the growth in the number of source countries per imported good.

The spectacular rise in trade shares is well-known. After Krugman's (1995) account of the surprisingly divergent views concerning its causes, Baier and Bergstrand (2001) singled out tariff reductions as its most important determinant. Yi (2003) subsequently pointed out that the observed increases in world trade shares imply an excessive import elasticity in standard trade models (see also Bergoeing et al (2004) on this point). Substantial progress was made in explaining this "elasticity puzzle" as it was later labeled (see Yi (2003), Ruhl (2005), and Cuñat and Maffezzoli (2006)). However, the New Trade Theory remains unable to square the raise of trade volumes at reasonable import elasticities.

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Haveman and Hummels (1999) observed the second empirical regularity. Defining varieties as goods differentiated by production origin,<sup>2</sup> the authors report that "importers purchase a very small fraction of available varieties," while standard models predict that each country imports all varieties from all trade partners. The authors conclude that the data suggest that standard theory "considerably overstates the extent of specialization [...] or the degree to which consumers value varieties," and Anderson and van Wincoop (2003) acknowledge that their observation is an "important drawback of the existing theory."

The third observation concerns the nature of the rise in trade volumes. Figure 1 illustrates with US data and the aggregate world data of the period 1972 to 2000 that the growth of trade was paralleled by an increase of per capita income. Ventura (2005) puts forward that the data strongly suggest a positive relation between trade growth and growth in per capita income. The New Trade Theory, however, predicts that trade shares and productivity parameters are entirely orthogonal, so that any correlation of per capita income and trade variables remains outside its framework. Finally, Broda and Weinstein (2004) report a substantial expansion of the number of source countries per imported good for all of the 20 largest importers over the period 1972 to 2000. This increase reflects a dynamic aspect of the second observation above. Countries tend to purchase each of their imports from an increasing number of trade partners which they had previously excluded from the set of suppliers. Figure 2 illustrates that the expansion of the average number of source countries per good closely parallels the increase of the import share in the US data. Disaggregating these data by good categories (HTS and TSUSA) confirms this pattern: Figure 3 shows a strong correlation between changes in trade volumes and changes in the average number of countries from 1972 to 2000. Finally, relying on bilateral trade data of 188 countries by 4-digit SITC good categories, Figure 4 repeats this plot adding a country dimension. The graph exhibits the same strong correlation between changes in import volumes and in the origin margin. Once again, the New Trade Theory has nothing to say about the trend nor the observed correlations.

In sum, there are four strong and central patterns of international trade which the New Trade Theory fails to address thus leading one to the temptation to downgrade the model as an analytical instrument for trade theory. The present paper shows that this is not necessary. It argues that a small and realistic deviation from the standard demand structure goes a long way in alleviating the listed drawbacks. This paper's key assumption is that consumers' marginal utility derived from each variety is bounded. This twist in the demand structure implies that consumers buy only the cheaper fraction of varieties and exclude the too expensive ones from their consumption basket. Varieties from very distant destinations - the varieties that are expensive due to large trade costs - are therefore not purchased. This explains the

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<sup>2</sup>This is a common definition of variety. It departs from earlier definitions. The present paper follows Krugman (1980) and identifies a variety by the firm that produces it.



observation of void bilateral trade relations in some sectors. Moreover, the change in demand patterns implies that technological progress, when it is biased towards the marginal production cost, raises the number of source countries per good. The reason for this effect is slightly more subtle. A decreasing ratio of fixed costs over marginal cost induces a higher per capita consumption of varieties already consumed and marginal utility derived from each of them falls. At the same time marginal utility of varieties outside the consumption basket stays trivially constant. This shift of the ratio of marginal utility means that varieties outside the consumption basket become more attractive and consumers expand their consumption basket. Consequently, the number of source countries for a given import good rises.

As the consumption basket expands towards the more expensive foreign varieties, it goes together with reallocation of expenditure towards imported varieties. Thus, productivity growth becomes a joint determinant of trade shares and the number of source countries per good, which hence explains the observed correlation between both (Figures 2-4). Finally, the model generates the positive correlation of trade shares and per capita income since growth of marginal productivity positively affects both of them.

In this way, the workhorse model of the New Trade Theory, albeit slightly modified on the demand side, qualitatively explains the four empirical regularities the standard version failed to address. In order to evaluate to what extent it is able to alleviate the drawbacks quantitatively, a simple calibration exercise is performed. It shows that technological progress can account for about half of the observed increase in trade shares while the other half of the increase remains to be explained by the fall in trade costs, i.e. by reductions in tariffs and transport costs. The relatively modest reductions of US data then explain the data with a time-average of 3.6, which is still on the high side of estimates for elasticities (it peaks at an 8.8 in 1972 and is lowest in 2000 with a values of 1.7). Nevertheless, it constitutes substantial progress compared with the standard New Trade Model, which implies import elasticities well over 10 when explaining the trade shares with trade cost reductions only. In sum, the assumption of bounded marginal utility on the variety level thus goes a long way to alleviate the three shortcomings of the standard New Trade Model and brings it much closer to the data in these important dimensions.

The present paper is not the first to analyze the role of non-homothetic demand for trade shares. A number of authors have formalized and tested the Linder Hypothesis according to which bilateral trade volumes correlate positively with per capita income similarity due to the fact that per capita income determines the demand patterns and output structure of countries (see Thursby and Thursby (1987), Françoise and Kaplan (1996), and Hallak (2004)). Closer predecessors of the present paper are Markusen (1986) and Bergstrand (1990) who assume Stone-Geary preferences, so that consumers cover a minimum level of a homogenous, domestically produced good before demanding aggregates of imported varieties. The present model does

not impose such asymmetries on demand but assumes equal valuation of varieties for consumers, while the cost of transportation creates endogenous asymmetries in demand via its effects on consumer prices. In a recent paper that stands somewhat out of this literature Hummels and Lugovskyy (2005) use a Lancaster-type utility to analyze the role of market size and per capita income on the market structure and international trade. They predict - and empirically confirm - that "richer consumers will pay more for varieties closer matched to their ideal types". Consequently, higher per capita income markets are more segmented and own-price elasticities are lower, while the market size has opposite effects. In this respect, Hummels and Lugovskyy (2005) are very much in line with present paper, which takes the observed regularities of demand as a starting point and formulates them in reduced form to derive predictions concerning trade flows and the number of source countries per good.

The remainder of the paper is organized in the following way: Section 2 reviews a standard multi-country New Trade Model and briefly discusses the three drawbacks. Section 3 introduces the change in the demand structure, showing the progress that can be made there. Section 4 concludes.

## 2 The Standard New Trade Model

The present section aims to illustrate some implications of the Krugman (1980) model that stand in blatant contrast to the data. These implications are, first, that all countries consume all tradable goods and varieties from all countries at all times, second, that trade shares are independent of technologies and therefore of per capita income, and third, that trade shares react modestly to tariff reductions under realistic elasticities. To highlight these problems, the new trade model with a set of  $I$  different countries is briefly reviewed in the following.

**Demand.** Each country  $i \in I$  is populated with identical individuals who have the following Cobb-Douglas preference structure over a nontradable good  $D_i$  and a composite of tradable varieties  $C_i$

$$U_i = D_i^{1-\gamma} C_i^\gamma \quad (1)$$

The composite  $C_i$  aggregates the consumed quantities  $c_{ij}$  of the varieties  $j \in J$  which the individual values with constant elasticity of substitution

$$C_i = \int_J c_{ij}^{1-1/\varepsilon} dj \quad (2)$$

The constant share of expenditure on the nontradable  $D$  will be read as the local content of the average variety. By the Cobb-Douglas structure the expenditure share on  $D$  is constant and the local content is  $1 - \gamma$  throughout. When expenditure

in county  $i$  equals  $W_i$ , the constant expenditure shares further allow to write the sub-budget constraints on the composite of varieties as

$$\int_J q_{ij} c_{ij} dj \leq \gamma W_i \quad (3)$$

where  $q_{ij}$  is the consumer price of the variety  $j$  in country  $i$ . Finally, consumer optimization of the variety composite implies  $c_{ij} \sim q_{ij}^{-\varepsilon}$  so that the own-price demand elasticity is  $\varepsilon$ .

**Supply.** In each of the  $I$  countries the nontradable good  $D$  is produced competitively with a constant returns to scale technology

$$D = L_D \quad (4)$$

The tradable varieties are produced according to the increasing returns to scale technology

$$L_j = \alpha + \beta x_j \quad (5)$$

where  $L_j$  is labor and  $x_j$  is output of firm  $j$ . The positive parameter  $\alpha$  represents a setup or entry cost in terms of units of labor while  $\beta$  is the marginal unit labor requirement. There is an unlimited pool of potential entrants into the market and each active firm produces one variety. Firms engage in monopolistic competition, and free entry into production ensures that operating profits just cover the setup cost.

**Prices.** In the competitive nontradable sector prices trivially equal wages within each country. In the tradable sector, international trade costs drive a wedge between producer and consumer prices. In particular, it will be assumed that shipments from country  $k$  to country  $i$  require the payment of a gross iceberg-type transport cost  $\tau_{ki}$ . Consequently, in country  $i$  the consumer price of a variety  $j$  that is produced in country  $k$  is  $q_{ij} = \tau_{ik} p_{ij}$  where  $p_{ij}$  is the price net of transport cost firm  $j$  sets in country  $i$ . In order to maximize profits, monopolists charge the constant markup  $\varepsilon/(\varepsilon - 1)$  over marginal cost. As all firms located in a given country face the same production and pricing decisions, the prices of monopolists located in country  $k$  are all identical and can be indexed by the country of origin

$$p_k = \frac{\varepsilon}{\varepsilon - 1} \beta w_k \quad (6)$$

Note that the marginal production cost in country  $k$  is the product of marginal unit labor requirement  $\beta$  and the local wage  $w_k$ . With the monopolist prices (6) the relative consumer price of either pair of varieties equals the iceberg cost times the wage ratios. Thus, individual optimality condition for each country  $i$  requires

$$c_{ik} = c_{ii} (\tau_{ki} w_k / w_i)^{-\varepsilon} \quad (7)$$

This equation implies the well-known fact that all individuals consume all varieties in positive quantities.<sup>3</sup> This implies that whenever a country exports a good, it exports it to all countries worldwide, a prediction that stands in stark contrast to the trade patterns Haveman and Hummels (1999) report. In particular, the authors find that "importers purchase a very small fraction of available varieties" traded on the world market.

As a lemma of this finding, the number of source countries of the imported good equals the number of the rest-of-the-world countries and is, in particular, constant in the parameters of the model.<sup>4</sup> This constitutes the second contradiction to the data since, as Broda and Weinstein (2004) report, countries have substantially increased the number of source countries per imported good over the past decades.

**Equilibrium.** Regardless of prices and income, the expenditure on the non-tradable good is  $1 - \gamma$  so that, since prices are handed through as factor rewards, the equilibrium labor allocation in the nontradable sector equals  $(1 - \gamma)L_i$  in each country  $i$ . In the following description of the equilibrium, the trivial allocations in the nontradable sector will be neglected.

The equilibrium in the traded sector is determined by the sub-budget constraint, the resource constraint, and the trade balance for all countries  $i \in I$ . When writing  $n_k$  for the number of varieties produced in country  $k$ , and under the assumption that individuals spend all their labor income on consumption, the individual sub-budget constraint (3) in country  $i$  becomes

$$\sum_k n_k \tau_{ki} p_k c_{ik} = \gamma w_i \quad (8)$$

The resource constraint requires that within each country  $i$  the labor demand in the tradable sector equals  $\gamma L_i$ , which is the inelastically supplied amount of labor in that sector

$$n_i \left[ \alpha + \beta \sum_k L_k \tau_{ik} c_{ki} \right] = \gamma L_i \quad (9)$$

Finally, balanced trade of country  $i$  implies

$$n_i p_i \sum_k L_k \tau_{ik} c_{ki} = L_i \sum_k n_k c_{ik} \tau_{ki} p_k \quad (10)$$

Combining (9) and (10) leads to

$$n_i \alpha + \beta L_i \sum_k n_k c_{ik} \tau_{ki} (p_k / p_i) = \gamma L_i$$

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<sup>3</sup>The cases of zero or infinite wage in one country can readily be ruled out.

<sup>4</sup>This is of course true only under the implicit assumption that the number of countries in the world is constant. The increase in the number of nations that actually took place is, however, by far insufficient to explain the rise in the number of source countries per good.

and determines with (6) and (8) the number of active firms in each country

$$n_i = \gamma L_i / (\alpha \varepsilon) \quad (11)$$

which does not depend on whether and how much countries trade. Using the monopolist prices (6), the consumer optimality condition (7) and the number of firms (11), one can write the individual sub-budget constraint (8) as

$$c_{ii} \sum_k L_k (\tau_{ki} w_k / w_i)^{1-\varepsilon} = (\varepsilon - 1) \alpha / \beta \quad (12)$$

and the trade balance (10)

$$\sum_k c_{kk} L_k \tau_{ik}^{1-\varepsilon} (w_k / w_i)^\varepsilon = c_{ii} \sum_k L_k (\tau_{ki} w_k / w_i)^{1-\varepsilon}$$

The last two equations combine to

$$\sum_k \frac{L_k w_k \tau_{ik}^{1-\varepsilon} w_i^{-\varepsilon}}{\sum_m L_m (\tau_{mk} w_m)^{1-\varepsilon}} = 1 \quad (13)$$

for all  $i \in I$ . Up to normalization, this system determines the wages  $w_i$  and together with (6), (7), (11), and (12) describes the equilibrium.<sup>5</sup> Notice that wages depend only on transport costs, the relative country-sizes and the substitution elasticity. Consequently, the trade share of country  $i$

$$e_i = \gamma \left( 1 - \frac{n_i p_i c_{ii}}{w_i} \right) = \gamma \left( 1 - \frac{L_i}{\sum_k L_k (\tau_{ki} w_k / w_i)^{1-\varepsilon}} \right)$$

is independent of technology. This independence of technological progress and trade shares constitutes a third mismatch of the new trade model regarding the data. As Ventura (2005) points out, time series as well as cross section data of trade and per capita income strongly suggest a positive interconnection of productivity growth and trade shares. The model is unable to deliver such patterns.

As Yi (2003) demonstrates, the observed increases in world trade volumes imply an excessive import elasticity in conventional models. Two simple calibration exercises shall show in the following that the new trade model is no exception in that respect. First, a two-country version of the model is fed with data on population size and trade costs. Population data for the US and the ROW are taken from the ERS International Macroeconomic Data Set and the US tariff and the cif/fob measure of

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<sup>5</sup>For existence see the appendix. By summing up over  $i$  it is confirm to check that the system does not have full rank. The missing rank reflects the fact that the trade balance of the last country holds if all other countries have balanced trade.

transport costs from Feenstra et al (2002). Trade costs are assumed to be the sum of trade-weighted tariffs and the cif/fob measure (both from the US data) plus 40%. The latter number is the "border cost" estimated by Anderson and van Wincoop (2004). The elasticity of substitution  $\varepsilon$  and the parameter  $\gamma$  are used to calibrate the trade shares to the data at the initial and final period of the time interval considered. As Figure 4a shows, this is not possible: for the values  $(\varepsilon, \gamma) = (11.9, 1)$  the rise in the trade share is maximal; for higher elasticities, the initial trade share drops below the 4.5% observed in the data. The implied import elasticity<sup>6</sup> of this model takes an average of 11.5 with the maximum of 11.6 and a minimum of 11.47. At these excessive elasticities, the model is able to explain little less than 70% of the increase in the trade share. As Yi (2003) states, standard theory cannot explain the rise of the trade share without assuming excessive import elasticities and, in this particular case of the new trade model, even then it only partially matches the US import data between 1972 and 2000.

Second, a symmetric model of 20 equally-sized countries is fed with the trade cost data (the US population equals about 1/20 of the world population in the time interval considered). Each country has a set of 19 trade partners and faces a vector of bilateral trade costs. This vector of trade cost is assumed to be, up to reordering, identical across countries and is denoted by  $\tau = (\tau_1, \tau_2, \dots, \tau_{M-1})$ . Without loss of generality, assume that these  $\tau_m$  are ordered according to ascending values. Finally, the bilateral trade costs are a composite of tariffs  $t$  and transportation cost  $\delta$  and are assumed to take the linear form  $\tau_m = 1.4 + t + \delta (m/(M-1))^\sigma$ . For each period, the value  $\delta$  is chosen so that the cif/fob measure implied by the model coincides with the value implied by the data. Here again,  $\varepsilon$  and  $\gamma$  are chosen to calibrate the trade shares to the data at the initial and final period of the time interval considered. Figure 4b shows that with  $(\varepsilon, \gamma, \sigma) = (14.535, 1, 1.9)$  the multi-country model explains almost 90% of the observed increase in the trade share, yet still at a very high import elasticity, averaging 10.16 (the minimum and maximum are, respectively, 10.02 and 10.21). To understand where this improvement of fit stems from, notice that in presence of positive tariffs  $t > 0$ , any reduction in  $\delta$  reduces the transportation costs for varieties from more distant countries relatively more than those from close countries. Consequently, a reduction in  $\delta$  induces an increase in imports that is particularly pronounced for varieties from distant countries. This, in turn, implies an upward bias in the cif/fob measure. To meet that measure in the data, the parameter  $\delta$  must drop by more than the cif/fob measure in the data.

The major problem of this calibration is clearly the excessive import elasticity, which is needed to generate a substantial rise in trade shares in response to reductions in trade costs. Another, possibly less obvious, drawback stems from the fact that

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<sup>6</sup>The import elasticity does not equal  $\varepsilon$  since at positive trade shares a change of import prices leads to a partial allocation towards domestic goods. This effect tends to reduce the implied import elasticity. This reduction is larger for commodities with a higher the expenditure share; thus the implied import elasticity drops over time with a rising trade share.

the measured cif/fob measure implies the strong drop in the parameter  $\delta$ , which then accounts for about 80% of the rise in trade share. This dominant role of the drop in transport cost contradicts the finding of Baier and Bergstrand (2001) who estimate the effect of decreases in transport cost on bilateral trade volumes to be small relative to that of tariff cuts.

This section has shown that the multi-country version of the Krugman (1980) model mismatches the data in four important dimensions: first, it fails to explain the rise in trade volumes at reasonable elasticities; second, it predicts that all countries import all tradable varieties and, third, consequently fails to account for the rise in source countries per imported good; fourth, it predicts no response of trade shares to productivity and thus generates zero correlation between trade shares and per capita income. These shortcomings, however, do not imply that the model is misshaped to address those issues. As the following section will show, a reasonable change in the models structure suffices to set it back on track and make substantial progress in all four dimensions.

### 3 New Trade and Bounded Marginal Utility

This section will show that a mild change in the above setup goes a long way in alleviating the shortcomings of Krugman's (1980) workhorse model, illustrated in the previous section. The change in the setup concerns the demand structure, and will be introduced next.

**Demand.** As in the previous section, consumers derive utility from the two final goods  $D$  and  $C$  according to (1). Within the composite of tradables consumers love variety but, in contrast to the previous section, the marginal utility derived from each of the varieties is supposed to be bounded. This latter feature constitutes a major departure from standard literature and is central to the results below. Preference structures with these characteristics have been used to study topics in trade literature and it will be convenient to follow an important precursor and assume with Young (1991) that the composite of varieties takes the form

$$C_i = \int_J \ln(c_{ij} + 1) dj \quad (14)$$

where again  $c_{ij}$  are the consumed quantities of variety  $j$ . Again, with  $q_{ij}$  as the price of variety  $j$  in country  $i$  the demand curve of an individual located in market  $i$  is derived by maximizing utility (14) subject to the sub-budget constraint (3). Individual demand is then

$$c_{ij} = \max\{1/(\lambda_i q_{ij}) - 1, 0\} \quad (15)$$

where  $\lambda_i$  stands for the shadow price on the budget constraint.

Notice that for either two varieties consumed in positive quantities, the optimal consumption requires further

$$\frac{1 + c_{ij}}{1 + c_{il}} = \frac{q_{il}}{q_{ij}} \quad (16)$$

**Supply.** Production takes place as specified in the previous section. The non-tradable  $D$  is produced competitively at constant returns to scale (4) while varieties are produced with technology (5). Firms produce one variety each and engage in monopolistic competition in each market.

**Prices.** Here again, prices trivially equal the wages in the competitive non-tradable sector. In the tradable sector, firms can price-discriminate across different countries by assumption and consequently a firm  $j$  sets the price  $p_{ij}$  for market  $i$  in order to maximize its operating profits  $\pi_{ij}$  in that specific market. When firm  $j$  is located in country  $k$  and gross iceberg-type trade costs between country  $k$  and country  $i$  are  $\tau_{ki} \geq 1$ , the consumer price in country  $i$  of variety  $j$  is the  $q_{ij} = \tau_{ki}p_{ij}$ . Firm  $j$  that is in located in country  $k$  makes a profit in the market  $i$  that amounts to  $\pi_{ij} = L_i\tau_{ki}c_{ji}(p_{ij} - w_k\beta)$ . Here,  $L_i$  is the number of individuals in the market  $i$  and  $w_k$  is the wage in country  $k$ ; hence  $w_k\beta$  is the marginal cost of production. With local demand (15), firm  $j$ 's profits are thus

$$\pi_{ji} = L_i\tau_{ki} (1/(\lambda_i\tau_{ki}p_{ji}) - 1) (p_{ji} - w_k\beta) \quad (17)$$

Generic profit maximization thus implies

$$p_{ji} = \sqrt{w_k\beta/(\lambda_i\tau_{ki})} \quad (18)$$

for each market  $i$  separately. Here again, within each country firms are all identical and charge the same prices in a given market so that prices can be indexed by country of origin and country of destination.

Under free entry to production, total operating profits, i.e. the sum of a firm's profits in each market, are equal to setup costs.

### 3.1 Closed Economy ( $I = 1$ )

Assume that international transport costs are too high to allow any kind of trans-border trade. Countries are in autarky and can be analyzed by looking at one representative closed economy, within which transport costs are negligible ( $\tau_{ii} = 1$ ). Dropping country indices, firm  $j$  in the economy then sets its price according to (18). As all firms are identical in equilibrium, the shadow price on the sub-budget constraint satisfies  $1/\lambda = p_j(c_j + 1) \equiv p(c + 1)$  and the monopolist prices are

$$p = (c + 1)w\beta \quad (19)$$



Note that the monopolist markup over marginal cost is  $c + 1$  and the demand elasticity from (15) equals  $\varepsilon = (c + 1)/c$ . The specific demand structure assumed in (14) thus implies that the own-price elasticity of varieties depends on quantities consumed, is larger than unity and, in particular, unbounded. Consequently, in the context of the present paper, whose motivation partly rests on the "elasticity-puzzle", i.e. on the discrepancy between measured import elasticities and the observed rise in trade volumes, a calibration of the model is inevitable to assess the implied import elasticities at observed trade shares.

With the prices (19), the operating profit of the representative firm is

$$\pi = Lc(w\beta(c + 1) - w\beta) = w\beta Lc^2 \quad (20)$$

and the free entry condition  $\pi = w\alpha$  determines the quantities consumed per individual and variety

$$c = \sqrt{\frac{\alpha}{\beta L}} \quad (21)$$

Equation (21) shows that, quite intuitively, an increase in setup over marginal costs ( $\alpha/\beta$ ) increases per capita consumption of the average variety. At higher values of  $\alpha/\beta$ , varieties are relatively more costly to produce and are produced in lesser quantities so that the per capita consumption of each rises. Conversely, at increasing population size ( $L$ ), there is more demand for varieties, which increases the number of active firms and reduces per capita consumption of each single variety.

The equilibrium number of active firms,  $n$ , is finally pinned down by combining the conditions of labor market clearing ( $n[\beta x + \alpha] = \gamma L$ ) and goods market clearing ( $x = Lc$ ) within the tradable sector and is calculated to be

$$n = \frac{\gamma L}{\alpha + \sqrt{\alpha\beta L}} \quad (22)$$

Observe that the number of active firms is determined by technology parameters, the population and market size. The difference between the two latter parameters is noteworthy. Market size, defined as total expenditure on varieties ( $\gamma L$ ), enters the number of firms linearly, i.e. an increase in market size induces a proportional rise in the number of active firms. Population size, however, enters expression (22) separately and, holding the market size fixed, tends to reduce the number of firms. For an intuition of that finding remember that demand elasticity decreases with per capita consumption. Now compare two economies, the first with a double labor force but half the parameter  $\gamma$  of the second. Both economies spent the same total amount of resources to produce varieties, yet per capita expenditure of the first economy is half of the second. Now, *if* the number of firms was equal, this would imply that per capita consumption of each variety in the first economy is half of those in the second. This, in turn, would mean that demand elasticities are larger

in the first economy, and hence monopolist markups and profits are lower so that finally firms in the first economy made less profits than in the second. However, by the free entry condition, firms' total profits must be zero in each country, which means that the number of firms has to adjust: the first economy has less active firms in equilibrium. Thus, in addition to market size, per capita demand is a key determinant of the number of firms. This characteristic is very much unlike the one in the classical new trade model where, due to homothetic preferences, the market size matters exclusively (not per capita expenditure on varieties; compare (11)).

In the present model, the overall effect of an increase in population  $L$  on the number of firms is still positive, but, due to the demand effects discussed above, it is less than proportional. Consequently, an increase in population size induces a less than proportional drop of per capita consumption of the average variety.

### 3.2 Two Countries ( $I = 2$ )

Assume now that trade costs between two countries fall to a level where agents engage in international trade. To save notation, the variables of, say, the second of the countries will be marked by an asterisk. Individuals in one country can purchase varieties produced outside of their country when incurring an iceberg-type transport cost. In other words, for every unit of a variety to arrive at its destination,  $\tau > 1$  units of it have to leave the producer's country.

Firms maximize profits in each country separately, i.e. they can by assumption price discrimination across countries. Thus, the prices that firms charge in the domestic and foreign markets need to be distinguished. Let these prices be  $(p_d, p_f)$  for firms located in country 1 and  $(p_d^*, p_f^*)$  for firms located in country 2 and denote with  $(c_d, c_f)$  and  $(c_d^*, c_f^*)$  the quantities consumed in the respective country. Just as in the previous section, a firm's optimal price in the domestic market is

$$p_d = (c_d + 1)w\beta \quad \text{and} \quad p_d^* = (c_d^* + 1)w^*\beta \quad (23)$$

With these prices, firms make the profits

$$\pi_d = Lw\beta c_d^2 \quad \text{and} \quad \pi_d^* = L^*w^*\beta(c_d^*)^2 \quad (24)$$

in their respective domestic market. But in addition to activities in the domestic market, firms sell in the export market. A firm that is located in one country maximizes profits in the foreign market (see (17)) by setting its export prices according to (18). As the shadow price on country 2's budget constraint satisfies  $\lambda^* = p_d^*/(c_d^* + 1)$  and with (23) this leads to

$$p_f = \sqrt{\frac{w}{\tau w^*}} p_d^* \quad \text{and} \quad p_f^* = \sqrt{\frac{w^*}{\tau w}} p_d \quad (25)$$

for the export prices. Using now the optimality condition (16), the monopolist profits (17) in the foreign market can be written as<sup>7</sup>

$$\pi_f = L^* \tau (c_f^*)^2 w \beta \quad \text{and} \quad \pi_f^* = L \tau c_f^2 w^* \beta \quad (26)$$

The free entry conditions in both countries require then  $\pi_d + \pi_f = w\alpha$  in the first and  $\pi_d^* + \pi_f^* = w^*\alpha$  in the second country.

### 3.2.1 Two Symmetric Countries

It will prove instructive to look at the model under symmetry and to consider trade relations between two equally sized countries ( $L = L^*$ ). In this simple case wages equalize and monopolist prices in the domestic and foreign markets are

$$p_d = w\beta(c_d + 1) \quad \text{and} \quad p_f = \frac{p_d}{\sqrt{\tau}} \quad (27)$$

where the asterisk can be dropped because of symmetry. Optimal consumption choice (16) then implies

$$c_d + 1 = \sqrt{\tau}(c_f + 1) \quad (28)$$

whenever trade volumes are positive ( $c_f > 0$ ). The condition for trade to be positive is clearly  $c_d > \sqrt{\tau} - 1$ , or with (21)

$$\sqrt{\alpha/(\beta L)} > \sqrt{\tau} - 1 \quad (29)$$

In contrast to the standard model, finite tariffs can, if they are high enough, make bilateral trade volumes fall to zero. This is obviously a direct implication of the bounded marginal utility: when the ratio of marginal utilities exceeds the relative price, the more expensive variety drops out of the consumption bundle. While this cannot occur when marginal utilities approach infinity as quantities tend to zero it is well possible in the present setting.

Not surprisingly, higher tariffs tend to prevent trade flows between countries and may impede trade altogether. At the same time, a larger population  $L$  tends to impede trade since it increases the number of domestic firms, reduces per capita consumption of each variety and thus depresses the consumer's willingness to incur the transport costs to purchase an additional, foreign variety. Further, an increase in the ratio of setup costs and marginal cost ( $\alpha/\beta$ ) decreases the number of local firms, increases per capita consumption of each and thus makes the consumer willing to pay for more for foreign varieties.

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<sup>7</sup>Firms make less profit in foreign than in domestic markets for two reasons. First, and foremost, demand in the foreign market is lower since consumer prices, which bear additional transport costs, are higher. Second, a lower per capita demand induces a higher own-price elasticity and therefore depresses markups and profits in foreign markets. See also Hummels and Lugovskyy (2005) for a microfoundation and empirical evidence on this point.

The symmetric equilibrium is solved by using condition (28) to write the operating profits from the export market (26) as

$$\pi_f = L(c_d + 1 - \sqrt{\tau})^2 w \beta \quad (30)$$

Free entry to production requires that global operating profits just cover the setup cost

$$\pi = \pi_d + \pi_f = Lw\beta (c_d^2 + (c_d + 1 - \sqrt{\tau})^2) = w\alpha$$

which leads to

$$c_d = \frac{\sqrt{\tau} - 1}{2} + \sqrt{\frac{\alpha}{2\beta L} - \left(\frac{\sqrt{\tau} - 1}{2}\right)^2}$$

Each individual consumes  $c_d$  of a domestically produced variety, this quantity increases in the ratio of setup cost and marginal productivity ( $\alpha/\beta$ ).

The resource constraint in the tradable sector ( $n[\beta x + \alpha] = \gamma L$ ) and goods market clearing ( $x = L(c_d + \tau c_f)$ ) determine the number of active firms in both countries

$$n = \frac{\gamma L}{\alpha + \beta L \left( (\sqrt{\tau} + 1) \sqrt{\frac{\alpha}{\beta 2L} - \left(\frac{\sqrt{\tau} - 1}{2}\right)^2} - \frac{(\sqrt{\tau} - 1)^2}{2} \right)} \quad (31)$$

This expression can be shown to be increasing in the trade cost  $\tau$ . The positive relation between the level of trade cost and the number of active firms per country reflects the finding discussed in connection with equation (22). In a closed economy, the doubling of the labor force induces a less than proportional increase in the total number of firms. Now, a doubling of the labor force in a closed economy is equivalent to a move of a given country from autarky to full trade integration with an identical country. Thus, in such a situation the number of firms *per country* must fall. Finally, when the move from autarky to trade integration takes place via gradual trade cost reduction, the above equation shows that the drop in active firms per country is in fact monotonous.

With per capita consumption and the number of firms established, one can now turn to the volume of trade. The import share, defined as nominal imports over nominal GDP, is calculated with equations (27) and (28) to be

$$e = \gamma \frac{nLp_f\tau c_f}{nLp_d c_d + nLp_f\tau c_f} = \gamma \frac{\sqrt{\frac{2\alpha}{\beta L(\sqrt{\tau} - 1)^2} - 1} - 1}{2\sqrt{\frac{2\alpha}{\beta L(\sqrt{\tau} - 1)^2} - 1}} \quad (32)$$

in the two countries. This expression is decreasing in the country size ( $L$ ) and increasing in the ratio of setup cost and marginal productivity ( $\alpha/\beta$ ). For an intuition of this result note that a smaller population  $L$  and a larger ratio  $\alpha/\beta$  tend to

increase per capita consumption of the domestic variety. Further remember that, as discussed in connection with equation (29), demand for foreign varieties falls to zero when domestic per capita consumption is too high. Thus, starting from very low trade volumes, any parameter change that increases per capita consumption of domestic varieties must induce a more than proportional increase in per capita consumption of a foreign variety. In fact, the choice of the demand structure (14) implies that increases in per capita consumption of foreign varieties are over-proportionate along the whole range of parameters. (Take derivatives of equation (28) to see that  $d\ln(c_d) < d\ln(c_f)$ ). As prices stay constant in the symmetric equilibrium, this means that the expenditure share on foreign varieties grows.

Notice in particular that improvements of marginal productivity lead to increases in the trade share.

This symmetric model can be brought to the data. The population of the US and the rest of the world (ROW) are supposed to be of equal size. Both labor forces evolve according to US population growth. US per capita income is taken as a proxy for the technological change, which is moreover assumed to affect marginal productivity only (i.e. US per capita income  $\sim 1/\beta$ ). Trade costs are assumed to be the sum of tariffs and the cif/fob values plus 40%. The initial value of  $\beta$  and the trade share  $\gamma$  are jointly used to calibrate the import shares to the data at the initial and final period. With  $(\gamma, \beta) = (0.49, 15)$  the fit of the trade share is satisfactory. The implied import elasticity takes an average value of 6.8 with a maximum of 14.8 at the start of the period and a minimum of 3.9 at the end of it. This can be read as an improvement of the fit compared to the standard New Trade Model. The average implied import elasticity, however, is by far larger than the target of 2 to 3 Yi (2003) argues to be reasonable. The next paragraphs shall explore to what extent the imposed symmetry can be blamed for this failure.

### 3.2.2 Two Asymmetric Countries

In the general case of two asymmetric countries the monopolists' prices in the domestic and foreign markets are given, respectively, by (23) and (25). The full set of prices is then

$$\begin{aligned} p_d &= w\beta(c_d + 1) & p_f &= \sqrt{\frac{w}{\tau w^*}} p_d^* \\ p_d^* &= w^*\beta(c_d^* + 1) & p_f^* &= \sqrt{\frac{w^*}{\tau w}} p_d \end{aligned} \quad (33)$$

Provided that trade volumes are positive, optimal consumer choice (16) then implies

$$c_d + 1 = \sqrt{\tau w^*/w}(c_f + 1) \quad \text{and} \quad c_d^* + 1 = \sqrt{\tau w/w^*}(c_f^* + 1) \quad (34)$$

The condition for trade volumes to be positive is then  $c_f, c_f^* > 0$  or, with (21) and (34)

$$(\alpha/(\beta L) + 1)(\alpha/(\beta L^*) + 1) > \tau \quad (35)$$

Finally, the monopolists' operating profits in the respective markets are as in (24) and (26)

$$\begin{aligned}\pi_d &= w\beta Lc_d^2 & \pi_f &= w\beta L^*\tau(c_f^*)^2 \\ \pi_d^* &= w^*\beta L^*(c_d^*)^2 & \pi_f^* &= w^*\beta L\tau c_f^2\end{aligned}\quad (36)$$

so that the free entry conditions become

$$Lc_d^2 + L^*\tau(c_f^*)^2 = \alpha/\beta \quad \text{and} \quad L^*(c_d^*)^2 + L\tau c_f^2 = \alpha/\beta \quad (37)$$

The resource constraints in both countries require

$$n[(Lc_d + L^*\tau c_f^*)\beta + \alpha] = L \quad \text{and} \quad n^*[(L^*c_d^* + L\tau c_f)\beta + \alpha] = L^* \quad (38)$$

Finally, making use of the monopolists' prices (33) and imposing the trade balance  $nL^*\tau p_f c_f^* = n^*L\tau p_f^* c_f$  leads to

$$nL^*(c_d^* + 1)c_f^* = n^*L(c_d + 1)c_f \quad (39)$$

Now, equations (38) and (39) can be combined to eliminate the number of firms  $n$  and  $n^*$

$$\frac{(c_d^* + 1)c_f^*}{(c_d + 1)c_f} = \frac{Lc_d + L^*\tau c_f^* + \alpha/\beta}{L^*c_d^* + L\tau c_f + \alpha/\beta} \quad (40)$$

while eliminating relative wages in (34) leads to

$$(c_d + 1)(c_d^* + 1) = \tau(c_f + 1)(c_f^* + 1) \quad (41)$$

The four equations (37), (40), and (41) jointly determine the quantities  $c_d$ ,  $c_f$ ,  $c_d^*$ , and  $c_f^*$ . This is a nonlinear system which has no analytic solution and needs to be solved numerically. One can, however, show the following

**Proposition 1** *The system (37), (40), and (41) has a unique solution  $(c_d, c_f, c_d^*, c_f^*)$ .*

**Proof.** See appendix. ■

Given these quantities  $c_d$ ,  $c_f$ ,  $c_d^*$ , and  $c_f^*$ , the terms of trade are determined by (34)

$$\frac{w}{w^*} = \frac{c_f + 1}{c_d + 1} \frac{c_d^* + 1}{c_f^* + 1}$$

and equations (38) deliver the number of firms active in either country, which finally determines the trade share of the first country

$$e = \gamma \frac{n^* p_f^* \tau c_f}{n p_d c_d + n^* p_f^* \tau c_f} = \gamma \frac{n^* \sqrt{\tau w^*/w} c_f}{n c_d + n^* \sqrt{\tau w^*/w} c_f}$$

This asymmetric model can be brought to the data. Assume there are two countries with relative labor forces corresponding to those of the US and the ROW.

Technology ( $1/\beta$ ) is proxied by real US per capita income and trade cost is assumed to be the sum of US tariffs and the cif/fob values plus 40%. The initial value of  $\beta$  and the trade share  $\gamma$  are jointly used to calibrate the import shares to the data at the initial and of the period. The results, however, do not differ much from the calibration in the previous symmetric model. The mean of the implied elasticity is about the same as before (6.9) while its variance is higher with the maximum value of 24 and a minimum of 2.9. In sum, introducing asymmetry to the model does not contribute to a solution of the "elasticity puzzle".

### 3.3 A Symmetric Multicountry World ( $I > 2$ )

Take now an economy with  $I$  countries, which are all identical in terms of labor force. Pairs of countries differ regarding the respective transportation cost they face when engaging in bilateral trade, but to reap the virtues of symmetry assume finally that countries are symmetric in terms of potential trade partners. In particular, the vector of gross iceberg trade cost  $\tau_i = (\tau_{i1}, \tau_{i2}, \dots, \tau_{iI})$  is, up to reordering, identical across countries  $i$  (i.e. for all  $i, k \in I$  there is a permutation  $\xi : I \rightarrow I$  so that  $\xi(\tau_i) = \tau_k$ ). As all parameters that govern demand and supply are identical across countries, producer prices and wages equalize throughout countries.

The bounded marginal utility from varieties implies, just like in the two-country model, that there is a threshold on the transportation cost  $\bar{\tau}$  above which there is no bilateral trade. The defining condition for this threshold is derived from demand (15) and satisfies

$$c_{ii} = \sqrt{\bar{\tau}} - 1 \quad (42)$$

Obviously, the set of country 1's trade partners consists of those countries with whom bilateral trade is least expensive.

Consider now, say, country 1 and denote by  $M$  the set of countries it engages in bilateral trade with ( $M$  includes the country 1 itself). With monopolist prices parallel to (27), the optimal consumer choice requires

$$c_{11} + 1 = \sqrt{\tau_{1k}}(c_{1k} + 1) \quad k \in M \quad (43)$$

The profit a firm  $j$  located in country 1 makes in the market  $k \in M$  is parallel to (30) and the free entry condition now becomes

$$\pi_1 = \sum_{k \in M} \pi_{1k} = \sum_{k \in M} Lw\beta (c_{11} - (\sqrt{\tau_{1k}} - 1))^2 = w\alpha \quad (44)$$

Writing the shorthand

$$T_{M,1} = \sum_{k \in M} (\sqrt{\tau_{1k}} - 1) \quad \text{and} \quad T_{M,2} = \sum_{k \in M} (\sqrt{\tau_{1k}} - 1)^2$$

condition (44) leads to an expression for per capita consumption of the domestic varieties:

$$c_{11} = \frac{T_{M,1}}{M} + \sqrt{\frac{\alpha}{\beta LM} + \left(\frac{T_{M,1}}{M}\right)^2} - \frac{T_{M,2}}{M} \quad (45)$$

Here, in slight abuse of notation,  $M$  stands for the set of trade partners and for the number of trade partners at the same time. Note that  $M$  is an endogenous variable that eventually depends on the schedule of bilateral trade costs. Formally, one has a problem of circular definition, i.e.  $M$  is defined as

$$M = \{k \in I \mid \sqrt{\tau_{1k}} < c_{11} + 1\} \quad (46)$$

while  $c_{11}$  depends on  $M$  itself. One can show, however, the following

**Proposition 2** *The set  $M$  is nonempty and uniquely defined by (46).*

**Proof.** Assume wlog that the elements of the vector  $\tau_1$  are ordered according to ascending size. Then note that  $\tau_{1l} \leq \tau_{1k}$  implies  $k \in M \Rightarrow l \in M$ . Consequently, any solution to (46) must be of the form  $\{1, 2, \dots, n\}$  for one  $n \in I$ . Finally, define  $c_n$  by (44) under  $M = \{1, 2, \dots, n\}$ . Observe that  $c_n$  is decreasing in  $n$ , so that the sequence defined by  $m_n = \max \{k \in I \mid \sqrt{\tau_{1k}} < c_n + 1\}$  is decreasing in  $n$ . By construction the set  $\{1, \dots, n\}$  with  $n$  satisfying  $m_n \geq n$  and  $m_{n+1} \leq n + 1$  solves (46)). As  $m_n$  is a monotonically falling sequence, there is maximal one such  $n$  and since  $\{1\} \in M$ , the set  $M$  is nonempty. ■

With the equilibrium per capita quantities of domestic varieties well defined, equations (43), (44), and the free entry condition ( $n [\alpha + \beta L (\sum_{k \in M} \tau_{1k} c_{1k})] = \gamma L$ ) determine the number of active firms

$$n = \frac{\gamma L}{\alpha + \beta L \left[ (T_{M,1} + M) \sqrt{\frac{\alpha}{\beta LM} + \left(\frac{T_{M,1}}{M}\right)^2} - \frac{T_{M,2}}{M} + \frac{(T_{M,1})^2}{M} - T_{M,2} \right]} \quad (47)$$

Finally, using again consumers' optimality condition (43) and (45), the trade share of country 1 can be expressed as

$$e = 1 - \frac{np_{11}c_{11}}{np_{11} \sum_{k \in M} \sqrt{\tau_{1k}} c_{1k}} = 1 - \frac{1}{M} \left( 1 + \frac{T_{M,1}}{\sqrt{\frac{\alpha M}{\beta L} + (T_{M,1})^2 - MT_{M,2}}} \right) \quad (48)$$

Equations (46) and (48) determine the representative country's number of trade partners and its trade share, the two key trade parameters which the present model aims to explain.



The set of a country's trade partners  $M$  depends on the model's parameters and may be any subset of the full set of countries  $I$  satisfying  $\{1\} \subset M$ . Just as in the two-country world, trade costs can, if they are too high, impede bilateral trade. Intuition from the two-country model above suggests that the inclination to pay for foreign varieties increases with the per capita consumption of the domestic varieties, as decreasing marginal utility from domestic varieties raises consumer's willingness to pay for the more expensive foreign ones. As population size  $L$  decreases and the ratio  $\alpha/\beta$  increases per capita consumption of a domestic variety, one can conjecture that these parameters drive the contraction or expansion of the set of trade partners.

The main variable of interest, the trade share (48) is equally expected to rise in the ratio  $\alpha/(\beta L)$  not only since per capita consumption of foreign varieties increase relatively more than domestic varieties, but also because the set of trade partners expands. The dimension of trade partners - or source countries - constitutes an extra margin along which trade volumes expand and amplifies the first effect.

The above considerations regarding the number of trade partners and the trade share prove right and the impact of population size and technology on these variables is summarized in the following

**Proposition 3** *The trade share  $e$  and the number of trade partners  $M$  increases in  $\alpha/(\beta L)$ .*

**Proof.** Note first that for any change in the set  $M$  condition (42) must be satisfied for the newcomer (dropout). Thus, the trade share as  $e = 1 - \frac{c_{11}}{M c_{11} - T_{M,1}}$  is continuous at any change in the set  $M$ . By this observation and (45) it is sufficient to prove that the equilibrium  $c_{11}$  is increasing in  $\alpha/(\beta L)$ . By (45) this is trivially the case whenever  $M$  is constant.

For any change in the set  $M$  consider wlog an increase in  $M$ . Define  $m$  as the index of the lowest bilateral iceberg trade cost outside the set of trade partners, i.e.  $m = \operatorname{argmin}_{k \in I \setminus M} \{\tau_{1k}\}$ . Whenever  $M$  increases condition  $\sqrt{\tau_{1m}} = c_{11} + 1$  must hold and equation (44) is satisfied for both of the two sets  $M$  and  $M \cup \{m\}$ , implying that  $c_{11}$  is continuous. As the set of changes in  $M$  is countable, this proves that  $c_{11}$  is increasing in  $\alpha/(\beta L)$ . ■

Proposition 3 shows that the rise in trade share and the expansion of the set of trade partners are jointly driven by productivity growth at the margin. When the effect of this joint determinant is strong, it can induce a strong positive correlation between the rises in trade volumes and the number of source countries per good as exhibited by the data (see Figures 3 and 4). Moreover, it implies the common dynamics between per capita income and trade volumes that Ventura (2005) points to. Remember that the standard model of the New Trade Theory failed to address any of these patterns of the trade data, so that the results presented in Proposition 3 constitute a considerable advance towards reconciling the theory with the data.

The sole departure from the standard setup was to assume that consumers derive only bounded marginal utility from varieties at zero consumption levels. Up to this point, however, the results remain purely qualitative and to complete the picture it is necessary to know how the model performs quantitatively, i.e. to evaluate whether it can jointly address the level and the rise in trade shares, the number of source countries per good and per capita income at reasonable import elasticity.

To this end the model will be calibrated to the US trade shares. An important parameter in the calibration is the import elasticity to tariffs, which is defined as

$$\mathcal{E} = -(1+t) \frac{d}{dt} \ln(Q)$$

where  $Q$  stands for the total imported quantity. With the optimal consumer choice (43), the monopolist prices (33), and the number of firms (47) this is

$$Q = n \sum_{M \setminus \{1\}} c_{1k} = \left( \frac{(c_{11} + 1) \sum_{M \setminus \{1\}} \frac{1}{\sqrt{\tau_{1k}}} - (M - 1)}{\alpha + \beta L [c_{11}(T_{M,1} + M) - T_{M,2} - T_{M,1}]} \right)$$

Now, how does this model square the data? To evaluate its performance a parallel exercise to that of the standard New Trade Model in section 2 is performed. The symmetric model of 20 identical economies is fed with population and trade cost data of the US between 1972 and 2000 (the US population equals about 1/20 of the world population in the time interval considered). Each country has a set of 19 potential trade partners and faces a vector of bilateral trade costs, which is assumed to be, up to reordering, identical across countries and is denoted by  $\tau = (\tau_1, \tau_2, \dots, \tau_{M-1})$ . The  $\tau_m$  are ordered according to ascending values. Finally, the bilateral trade costs are a composite of tariffs  $t$  and transportation cost  $\delta$  and are assumed to take the linear form  $\tau_m = 1.4 + t + \delta (m/(M - 1))^\sigma$ . For each period, the value  $\delta$  is chosen so that the cif/fob measure implied by the model coincides with measure from the data. The parameter  $\gamma$  and the initial marginal productivity  $\beta_{1972}$  are chosen to calibrate the trade shares to the data at the initial and final period of the time interval considered. Figure 5b shows that with  $(\gamma, \beta_{1972}, \sigma) = (0.15, 14.39, 1.9)$  the multi-country model follows the time series satisfactorily with about the same success as the two-country model. A big improvement, however, concerns the implied import elasticity, which now assumes a time-average of 3.7 with a peak of 8.8 at the start of the period and a minimum of 1.75 at the end of it. While the variance is high, the average import elasticity is close to the interval [2,3], which Yi (2003) puts forward as a realistic range.

The reason for this substantial improvement is the extension of trade along the dimension of source countries. Driven by improvements of marginal productivity ( $1/\beta$ ), consumers purchase not only larger quantities from previous trade partners (as in the two-country model) but open new trade relations and extend the number of source countries. Figure 5b illustrates these dynamics and compares the predictions

of the model to that of the data (values in 1972 are normalized). This additional margin along which trade expands amplifies the impact of technological change, which therefore explains a larger part of the increase in trade shares. The remaining part to be accounted for by the reductions in trade cost therefore shrinks, and the implied import elasticity falls.

As in the standard New Trade Model with 20 countries, the measured cif/fob values and the implied "real" transport cost  $\delta$  differ considerably. The reason for this is now twofold. First, as in the standard model, an exogenous drop in  $\delta$  induces consumers to increase their purchases of foreign varieties relatively more. Second, and in addition, the model with bounded marginal utility predicts that increases in marginal productivity induce consumers to buy from more distant locations. Both of these effects tend to increase the expenditure on transportation, and induce an upward bias of the conventional cif/fob measure of trade cost from the 'real' values so that the implied latter ones fall by more than the data suggest.<sup>8</sup> Despite this second amplifying effect that stems from the technological change, the fraction of the rise in trade shares explained by the drop in transportation is now 44%, which is still larger than what the estimations of Baier and Bergstrand (2001) imply (around one fourth, about a third of the impact that tariffs have), yet much below the 80% the standard New Trade Model of the previous section implied.

In sum, the model can explain two strong and important trends of trade data at reasonable parameter values: first, the massive growth of trade shares at a modest fall of trade costs and second the small fraction of trade relations that the average country engages in. It suggests technological progress as the core determinant of both variables and thereby generates two additional patterns of international trade data: first, the correlation between trade shares and per capita income and second, the correlation between the increases of trade volumes and the number of source countries.

Two clarifying remarks are necessary. The first one concerns the nature of productivity growth, which was assumed to affect manufacturing efficiency only and leave entry cost unchanged. The model's direct implication that stronger increasing returns (i.e. a higher ratio of setup cost over marginal cost) imply larger trade volumes is confirmed empirically in a cross section analysis by Harrigan (1994). A time-series analysis, however, that justifies this central assumption and could lend support to the hypothesis of a strong causal effect of productivity growth on trade volumes is still to be performed. The second remark concerns previous attempts to evaluate the role of non-homothetic preferences on trade volumes. These effects are generally estimated to be small (see Bergstrand (1990) and Bergoeing and Kehoe

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<sup>8</sup>Given that these effects are strong, conventional measures tend to substantially understate the drop in trade costs. It may be worth analyzing these systematic measurement problems in detail in empirical paper. In particular, an evaluation of their role in estimates as in Baier and Bergstrand (1999) seems in order. However, such an analysis is outside the scope of the present paper.

(2003) among others). However, these quantitative studies are consistently based on Stone-Geary type preferences between a homogeneous good and a composite of varieties. Consequently, they suffer from the drawback of the Krugman (1980) model in a slightly modified version: any country with positive trade share imports all tradable varieties from all other trading countries. Thus, these models rule out the existence of a non-trivial fraction of imported varieties over world tradables and thereby cannot explain an endogenous expansion of the set of trade partners. The difference between the performance of the two-country and the multi-country model has made clear, however, that this expansion of the set of trade partners contributes significantly to the success of the calibration. This qualitative difference may explain the strong discrepancy between the findings of previous work and the present paper. It finally suggests that future empirical studies should include the effect that stems from an expansion of the set of trade partners.

### 3.4 An Asymmetric Multicountry World

This final generalization of the model considers the multi-country model under asymmetric country size and trade costs. Denote country  $i$ 's vector of gross iceberg trade cost with  $\tau_i = (\tau_{i1}, \tau_{i2}, \dots, \tau_{iI})$ . For the sake of generality, assume that productivity may differ across countries, i.e.  $\alpha$  and  $\beta$  are indexed by  $i$ . Profit maximization then leads to the monopolist prices

$$p_{ki} = (c_{ik} + 1)\beta_k w_k$$

so that under individual optimization (16) implies

$$\frac{c_{ij} + 1}{c_{ik} + 1} = \sqrt{\frac{\tau_{ji} w_j \beta_j}{\tau_{ki} w_k \beta_k}} \quad (49)$$

whenever  $c_{ij}$  and  $c_{ik}$  are positive. Note that consumption of domestic varieties is not necessarily positive, as relative prices can vary dramatically with wage differentials, which, in turn, are determined by relative country sizes, trade costs and technology.

Further, the set of countries country  $i$  imports from ( $M_i$ ) does not need to coincide with the set of countries it exports to ( $E_i$ ). To define both sets, write first

$$i_o = \operatorname{argmin}_{k \in I} \{\tau_{ki} w_k \beta_k\}$$

so that varieties from  $i_o$  are the cheapest in country  $i$ . Then write

$$M_i = \left\{ k \in I \mid c_{i_o i} + 1 > \sqrt{\tau_{ki} w_k \beta_k / (\tau_{i_o i} w_{i_o} \beta_{i_o})} \right\} \quad (50)$$

and

$$E_i = \left\{ k \in I \mid c_{k_o k} + 1 > \sqrt{\tau_{ik} w_i \beta_i / (\tau_{k_o k} w_{k_o} \beta_{k_o})} \right\} \quad (51)$$

With the monopolist prices  $p_{ik} = (c_{ki} + 1)\beta w_i$  of firms in country  $i$  selling in market  $k$ , generic profits from market  $k$  are still

$$\pi_{ik} = L_k \tau_{ik} \beta_i w_i c_{ki}^2$$

so that the free entry condition in country  $i$  remains

$$\sum_{k \in E_i} L_k \tau_{ik} c_{ki}^2 = \alpha_i / \beta_i \quad (52)$$

The resource constraint in country  $i$  requires

$$n_i \left[ \alpha_i + \beta_i \sum_{k \in E_i} L_k \tau_{ik} c_{ki} \right] = L_i \quad (53)$$

and finally the trade balance is  $n_i \sum_{k \in E_i} L_k \tau_{ik} p_{ik} c_{ki} = L_i \sum_{k \in M_i} n_k \tau_{ki} p_{ki} c_{ik}$  or

$$\frac{\sum_{k \in E_i} L_k \tau_{ik} (c_{ki} + 1) c_{ki}}{\alpha_i / \beta_i + \sum_{k \in E_i} L_k \tau_{ik} c_{ki}} w_i = \sum_{k \in M_i} \frac{L_k \tau_{ki} (c_{ik} + 1) w_k c_{ik}}{\alpha_k / \beta_k + \sum_{m \in E_k} L_m \tau_{km} c_{mk}}$$

With (52) this leads to

$$w_i = \sum_{k \in M_i} w_k \frac{L_k \tau_{ki} (c_{ik} + 1) c_{ik}}{\sum_{m \in E_k} L_m \tau_{km} (c_{mk} + 1) c_{mk}} \quad (54)$$

The system defined by (49) - (54) jointly determine the wages  $\{w_i\}_{i \in I}$ , the consumed quantities  $\{c_{ik}\}_{i, k \in I}$  the number of active firms in each country  $\{n_i\}_{i \in I}$  and the sets of supplied and supplying countries  $\{E_i\}_{i \in I}$  and  $\{M_i\}_{i \in I}$ .

A full characterization of this general case is beyond the scope of this paper and is left for future research.

## 4 Conclusion

This paper has shown that a small and realistic twist in the demand structure goes a long way in explaining four strong and important empirical regularities that the workhorse model of the New Trade Theory fails to explain. These empirical patterns are the massive rise in trade shares, the small fraction of traded varieties countries import, the correlation between trade growth and per capita growth, and the parallel rise in the number of source countries per imported good and trade volumes, and finally the correlation between trade growth and per capita growth. By the paper's key assumption marginal utility derived from each variety is bounded. This implies that varieties whose transport is very costly drop out of the individual's consumption basket. Technological progress that induces a higher per capita consumption of

those varieties already consumed decreases the marginal utility derived from them and makes consumers expand their consumption basket towards more expensive, foreign varieties. Therefore, such technological progress drives up the number of source countries per good, the trade share, and per capita income.

## A Appendix

**Existence of a solution to system (13).** Notice first that the system

$$\sum_k \frac{L_k w_k \tau_{ik}^{1-\varepsilon} w_i^{-\varepsilon}}{\sum_m L_m (\tau_{mk} w_m)^{1-\varepsilon}} = 1 \quad (\text{A1})$$

is homogeneous of degree zero in the wages and a potential solution can be normalized by setting  $w_1 = 1$ .

Now confirm that each term on the LHS of (A1) is decreasing in  $w_i$  and increasing in  $w_k$   $k \neq i$  and that  $\text{LHS} \rightarrow 0$  ( $w_i \rightarrow \infty$ ) and  $\text{LHS} \rightarrow \infty$  ( $w_i \rightarrow 0$ ) and hence for every vector  $(w_2, \dots, w_{i-1}, w_{i+1}, \dots, w_I) \in \mathbb{R}_+^{I-1}$  there is a unique solution

$$w_i = \varphi_i(w_2, \dots, w_{i-1}, w_{i+1}, \dots, w_I)$$

satisfying (A1). Moreover, the function  $\varphi_i : \mathbb{R}_+^{I-2} \rightarrow \mathbb{R}_+$  is strictly increasing in all arguments.

Now write  $\bar{\tau} = \max_{ij} \{\tau_{ij}\}$ ,  $\bar{w} = \max_i \{w_i\}$ ,  $\hat{w} = \min_i \{w_i\}$  and  $\lambda_i = L_i / \sum_k L_k$  and check that

$$w_i^\varepsilon \leq \bar{\tau}^{\varepsilon-1} \frac{\sum_k L_k w_k}{\sum_k L_k w_k^{1-\varepsilon}} \leq \bar{\tau}^{\varepsilon-1} \frac{\sum_k L_k w_k}{L_1} \leq \bar{\tau}^{\varepsilon-1} \frac{\bar{w}}{\lambda_1}$$

and

$$w_i^\varepsilon \geq \bar{\tau}^{1-\varepsilon} \frac{\sum_k \lambda_k w_k}{\sum_k \lambda_k w_k^{1-\varepsilon}} \geq \bar{\tau}^{1-\varepsilon} \lambda_1 \hat{w}^{\varepsilon-1}$$

or

$$\varphi_i(w_2, \dots, w_{i-1}, w_{i+1}, \dots, w_I) \leq \bar{\tau}^{(\varepsilon-1)/\varepsilon} (\bar{w}/\lambda_1)^{1/\varepsilon}$$

and

$$\varphi_i(w_2, \dots, w_{i-1}, w_{i+1}, \dots, w_I) \geq \bar{\tau}^{(1-\varepsilon)/\varepsilon} (\lambda_1 \hat{w}^{\varepsilon-1})^{1/\varepsilon}$$

This implies

$$\varphi_i : \left[ \lambda_1 / \bar{\tau}^{\varepsilon-1}, \bar{\tau} / \lambda_1^{1/(\varepsilon-1)} \right]^{I-2} \rightarrow \left[ \lambda_1 / \bar{\tau}^{\varepsilon-1}, \bar{\tau} / \lambda_1^{1/(\varepsilon-1)} \right]$$

Therefore, when  $P_i$  is the projection on the  $i$ -th component, the function  $\phi$  defined as

$$\phi = \begin{bmatrix} P_2 \\ \vdots \\ P_{I-1} \\ \varphi_I \end{bmatrix} \circ \begin{bmatrix} P_2 \\ \vdots \\ \varphi_{I-1} \\ P_I \end{bmatrix} \circ \dots \circ \begin{bmatrix} \varphi_2 \\ P_3 \\ \vdots \\ P_I \end{bmatrix} \left( \begin{matrix} w_2 \\ w_3 \\ \vdots \\ w_I \end{matrix} \right)$$

satisfies  $\phi : \left[ \lambda_1 / \bar{\tau}^{\varepsilon-1}, \bar{\tau} / \lambda_1^{1/(\varepsilon-1)} \right]^{I-1} \rightarrow \left[ \lambda_1 / \bar{\tau}^{\varepsilon-1}, \bar{\tau} / \lambda_1^{1/(\varepsilon-1)} \right]^{I-1}$  and is strictly increasing in all arguments, as the functions  $\varphi_i$  are so. Thus,

$$\phi(\lambda_1 / \bar{\tau}^{\varepsilon-1}, \dots, \lambda_1 / \bar{\tau}^{\varepsilon-1}) \geq (\lambda_1 / \bar{\tau}^{\varepsilon-1}, \dots, \lambda_1 / \bar{\tau}^{\varepsilon-1})$$

holds component by component. Applying  $\phi$  to both sides leads to  $\phi^2(\lambda_1/\bar{\tau}^{\varepsilon-1}, \dots) \geq \phi(\lambda_1/\bar{\tau}^{\varepsilon-1}, \dots)$  and by induction to

$$\phi^{m+1}(\lambda_1/\bar{\tau}^{\varepsilon-1}, \dots, \lambda_1/\bar{\tau}^{\varepsilon-1}) \geq \phi^m(\lambda_1/\bar{\tau}^{\varepsilon-1}, \dots, \lambda_1/\bar{\tau}^{\varepsilon-1})$$

so that, as each component of  $\phi^m$  is bounded above, the monotone components of  $\phi^{m+1}(\lambda_1/\bar{\tau}^{\varepsilon-1}, \dots, \lambda_1/\bar{\tau}^{\varepsilon-1})$  converge to  $(w_2^*, \dots, w_l^*)$ . This proves the existence of a fixpoint of  $\phi$ , which by construction is a solution to (A1).

**Existence and Uniqueness of a solution to system (37), (40), and (41).** First notice that there is nothing to show when (35) is violated. Now use (37) to write

$$c_f = \sqrt{\frac{\alpha}{\beta L \tau} - \frac{L^*}{L \tau} (c_d^*)^2} \quad \text{and} \quad c_f^* = \sqrt{\frac{\alpha}{\beta L^* \tau} - \frac{L}{L^* \tau} c_d^2}$$

with the function  $c_f(c_f^*)$  decreasing in  $c_d^*$  (in  $c_d$ ). Then rewrite (41) as

$$\frac{c_d + 1}{c_f^* + 1} - \tau \frac{c_f + 1}{c_d^* + 1} = 0$$

The LHS is increasing in  $c_d$  and in  $c_d^*$ . so that  $c_d = g(c_d^*)$  is a decreasing function. Use (35) to check that  $c_f > 0$  at  $c_d \rightarrow \alpha/(\beta L)$ . Finally, write (40) as

$$(c_d + 1) \left( L \frac{c_d}{c_f^*} + L^* \tau \right) + \frac{\alpha}{\beta c_f^*} - \left\{ (c_d^* + 1) \left( L^* \frac{c_d^*}{c_f} + L \tau \right) + \frac{\alpha}{\beta c_f} \right\} = 0$$

By the above observations the LHS is decreasing in  $c_d$  with  $\text{LHS} \rightarrow \infty$  as  $c_d \rightarrow \alpha/(\beta L)$  and  $\text{LHS} \rightarrow -\infty$  as  $c_d^* \rightarrow \alpha/(\beta L^*)$ . Thus, there is a unique solution for  $c_d$  and, solving backward, for  $c_d^*$  and finally for  $c_f$  and  $c_f^*$ .

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