DEPARTMENT
OF ECONOMICS

Competition in Regulated Over-the-Counter Markets

Natalie Kessler

Thesis submitted for assessment with a view to
obtaining the degree of Doctor of Economics
of the European University Institute

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European University Institute
Department of Economics

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Abstract

A common thread throughout all three thesis chapters is the combined usage of theoretical models and empirical methods to study the effect of a regulatory shift on the (competitive) equilibrium outcomes.

In the first chapter, I analyze the effect of mandatory counterparty default insurance (central clearing) of over-the-counter (OTC) derivatives on aggregate financial risk exposure. I carefully model the competitive mechanisms in both the OTC derivatives and their insurance market. I show that the introduction of mandatory insurance empowers the for-profit central counterparty (CCP) to raise prices, wherefore only larger clients opt to additionally insure their derivatives (lower credit risk). Smaller clients instead exit the market and remain unhedged (higher market risk). I conclude with a model calibration and counterfactual policy evaluation for the EuroDollar FX derivatives market, showing that mandatory insurance increases aggregate financial risk.

In the second chapter, joined with Iman van Lelyveld and Ellen van der Woerd, we predict the reduction in short-sell constraints due to an out-ruling of exclusive security lending agreements (ESLAs). Broker-dealers intermediate stock lending between lenders with large portfolios and borrowers seeking to short sell. We study why some lenders commit to a single broker-dealer via exclusive security lending agreements (ESLAs) at the cost of foregoing profitable trades, and how this impacts aggregate lending. We provide a detailed market overview both on a transaction and counterparty-pair level. Gained insights inform a three-period representative lender model that rationalizes why ESLAs arise in equilibrium. After carefully evaluating the model fit, we predict the counterfactual trading volumes in the absence of existing ESLAs. We find that trading volumes would significantly increase, up to 8%.

In my final chapter, joined with Johannes Fischer, we study the impact of stress tests and complementary dividend regulations on equilibrium bank lending. Bank stress tests, regularly conducted to ensure stable lending, constitute a de facto constraint on balance sheets: equity must be sufficient to maintain current lending also in the future, even after absorbing severe loan losses.

We study the effects of such forward looking constraints in a representative bank model. More severe-stress test scenarios lead to lower dividends, higher equity levels, and universally lower, albeit less volatile, lending. We calibrate our model to large U.S. banks, subject to Federal Reserve stress tests, and compute the optimal, state-dependent severity of stress tests and implied capital buffers (up to 6% during normal times). Finally, we complement stress tests with three macro-prudential policies: the Covid-19 dividend ban, the counter-cyclical capital buffer (CCyB), and the proposed dividend prudential target (DPT). We find that combing stress tests with a dividend ban or DPT improves supervisor welfare equally. Due to its discontinuous nature, however, relaxing the CCyB falls short.

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Contents

Chapter 1

Mandatory Central Clearing and Financial Risk Exposure

Abstract I analyze the effect of mandatory counterparty default insurance (central clearing) of over-the-counter derivatives on aggregate financial risk. I carefully model the competitive mechanisms in both the OTC derivatives and their insurance market. I show that the introduction of mandatory insurance empowers the for-profit central counterparty (CCP) to raise prices, wherefore only larger clients opt to additionally insure their derivatives (lower credit risk). Smaller clients instead exit the market and remain unhedged (higher market risk). I conclude with a model calibration and counterfactual policy evaluation for the EuroDollar FX derivatives market, showing that mandatory insurance increases aggregate risk exposure.

1.1. Introduction

Central counterparties (CCPs) play an increasingly important role in financial risk mitigation by providing counterparty default insurance (central clearing) for over-the-counter (OTC) derivatives. OTC derivatives are bilateral contracts usually entered between large firms, hedge funds or pension funds (buyers) and banks or broker-dealers (sellers). They are used by buyers to hedge the market risk associated to assets worth more than $8 trillion USD globally (BIS, 2020).

Holding these OTC derivatives, however, exposes buyers to seller default risk, stemming both from the seller's total OTC sales and from other business lines. To ensure payments even in case of seller default, buyers may additionally insure their OTC derivative at a CCP. The CCP takes over the contracted transfer should the seller default.

The benefits of having counterparty default insurance were powerfully illustrated during the Lehman Brothers default in 2008. With $35 trillion of notional outstanding in OTC derivatives, Lehman Brothers defaulted on 5% of all global derivatives contracts at the time. However, only a small share were insured at a CCP against the default. And while the claims of buyers with insured derivatives were resolved within three weeks after the default, resolution of non-insured derivatives took several years (Cunliffe, 2018; Fleming and Sarkar, 2014).[1] Influenced by these events, increasing the use of counterparty default insurance became a global regulatory objective. And thus, at the 2009 Pittsburgh summit, the G20 leaders agreed to introduce mandatory counterparty default insurance of standardized OTC derivatives contracts.[2]

This regulatory change, not surprisingly, led to a significant increase in insured OTC derivatives. Further, sellers now post significantly more collateral both for their insured and non-insured OTC derivatives. Empirical evidence thus suggests that mandatory insurance was successful in lowering counterparty default risk exposure in the OTC market (Cominetta et al., 2019). However, buyers exposed to the new regulatory regime report both increased derivatives prices and insurance fees. Additionally, especially smaller buyers experience difficulties in accessing the OTC market altogether. Both the increased prices and limited access have since resulted in former market participants to either lower or cease their hedging activities altogether (BCBS et al., 2018; ESMA, 2019a). Therefore, the benefits of lower credit risk seems to have come at the cost of higher exposure to market risk for at least some buyers.

Research Agenda Focusing on the trade-off between credit risk and market risk, this paper analyzes the effect of mandatory counterparty default insurance of OTC derivatives. I start

[1] First payments were made only in 2012, coming at a loss (Cunliffe, 2018; Fleming and Sarkar, 2014).

[2] Mandatory counterparty default insurance was introduced as part of the Dodd Frank Act in the US (U.S. CFTC, 2019) and as part of the EMIR regulation in the EU (European Commission, 2019b); for the equivalent regulations in other countries see (BCBS et al., 2018))

by carefully modeling the competitive mechanisms in both the OTC derivatives market and their insurance market, describing how prices and traded quantities are determined. Here, I emphasize how a monopolistic for-profit CCP may also impact the equilibrium outcomes in the derivatives market, both before and after the introduction of mandatory insurance. In this setting, I then examine how the CCP's profit maximizing objective (dis-)aligns with the regulatory objective to mitigate risk exposure. This provides a rationale for why especially smaller market participants suffer from increased market risk exposure. Ultimately, I quantify whether this increase in market risk is justified by a sufficiently large decrease in seller credit risk that benefits not only this but also other financial markets.

For the purpose of providing these insights, I first develop a theoretical model that builds mostly on earlier works by Biais et al. (2012, 2016) and Huang (2019): risk-averse buyers purchase derivatives to hedge their exposure to market risk; risk-neutral derivatives sellers may strategically default; and a monopolistic for-profit CCP offers counterparty default insurance. However, to capture the downstream effects of a regime change more accurately, I relax the standard assumption that buyers only trade with a single (randomly assigned) seller. Instead, I assume that buyers can additionally trade with any other seller upon the payment of switching costs. Further, I assume that buyers are heterogeneous in their size, such that the switching costs have differentiated effects.

This combination of buyer heterogeneity and switching allows me to provide a rich set of new theoretical insights. A core contribution is to study how the for-profit CCP restricts direct access via a two-part tariff system. Here, previous models with homogeneous buyer-seller relationships are limited to assuming that all market participants have equal access to the CCP (Antinolfi et al., 2018; Capponi et al., 2018; Duffie and Zhu, 2011). In reality, only sellers that pay the fixed fee (clearing members) directly access the CCP, providing intermediation for other agents. To the best of my knowledge, this is the first paper to endogenenize the dual role of sellers as derivative counterparties and potential clearing members. Explicitly modeling this dual seller role allows me to study how the monopolistic for-profit CCP exploits its two-part tariff system under both voluntary and mandatory insurance to heavily influence the downstream derivatives market. I am thereby able to understand the heterogeneous effect of the regime shift on both buyers and sellers of different sizes, and their consequent risk exposure.

I complement the theoretical analysis with a calibration exercise to illustrate how these insights can be utilized to understand the effects of mandatory counterparty insurance in a specific derivatives market. For this purpose, I parameterize the model environment to the European EuroDollar FX derivatives market. Simulating the market equilibria under both voluntary and mandatory insurance, I quantify the effect of a regime switch on the CCP's profits, buyers' utilities, and sellers' profits and default probabilities. Given these equilibrium objectsw, I then derive the changes in the average buyer's credit and market risk exposure, and the average seller's default probability. Ultimately, comparing their relative magnitudes under both insurance regimes allows me to perform a counterfactual analysis of aggregate financial risk exposure.

Findings To capture the heterogeneous buyer size in an intuitive way, I will refer to buyers as either being small, medium sized or large throughout this paper. Similarly, I label sellers matched with small, medium sized and large buyers, as small, medium sized and large respectively. The model framework contains three stages: First, the CCP sets its two-part tariff system and sellers decide whether to become clearing members. Secondly, buyers and seller trade the derivative and mandatorily/voluntarily add a counterparty default insurance. In the third stage, the underlying asset uncertainty realizes, sellers may strategically default, and pay-offs are realized. Given this structure, the model is solved through backwards induction. And the consequent sub-game perfect Nash equilibrium (SPNE) characterizes the CCP's entry decision and two-part tariff, sellers' clearing membership choice, derivative and insurance prices, buyers' choice of seller (switching/no switching) and total sales.

I find that under mandatory insurance, the CCP's two-part tariff system introduces a unique size threshold: All smaller buyers and sellers exit the market; and all medium sized and larger sellers become clearing members and sell the bundle of derivative and insurance products. Because insurance is mandatory, the sellers are able to capture the buyers' entire utility gains through the derivatives and insurance price. The absence of utility gains implies for one that buyers are indifferent between participating in the market or not. It further implies that buyers are captive and never switch, as only realizing reservation utility does not warrant the payment of switching costs.

The market outcomes are quite different under voluntary counterparty default insurance. Here, buyers can hold the derivative also without insurance, which strengthens their negotiation stance and results in over-all higher buyer utility. Because buyers anticipate utility gains from (at least) the derivative, they also now consider switching to be worthwhile. And while the increased competition reduces derivatives prices (relative to mandatory insurance), this is not entirely bad news for sellers. Being able to offer only the derivative also reduces their pressure to become a clearing member: They are not automatically forced to exit the market, should they choose not to pay the fixed entrance fee. This reduction in CCP market power results in both smaller and medium sized buyers/sellers solely trading the derivative. Larger sellers are also here clearing members and trade the bundle of derivative and insurance. However, as I will show later in the simulation, under certain market characteristics, these sales might be insufficient to incentive CCP entry.

Not surprisingly, I therefore find that a monopolistic for-profit CCP strictly benefits from the increased market power under mandatory counterparty default insurance. Contrary to this, the effect on buyers is unambiguously negative, when their option of holding the derivative alone is removed. The effect on sellers, however, depends on size. Here, it is easy to see that small sellers strictly suffer under mandatory insurance: Instead of offering only the derivative, they exit the market. Contrary to this, larger sellers strictly benefit: They offer both the derivative and insurance under both regimes, but can charge higher prices under mandatory insurance due to the decreased negotiation stance of buyers. The effect on medium sized sellers is ambiguous and depends on market characteristics: Under mandatory insurance, they face additional costs of becoming clearing members; however, they can potentially off-set this by charging higher prices due to decreased buyer options.

Besides the differentiated expected profits/utilities under the two regimes, the paper also sets out to comment on the overall impact on financial risk exposure. Here, the theoretical analysis highlights three margins of change: buyers' credit risk exposure, buyers' market risk exposure, and a credit risk externality. For the first two, the model matches the originally anticipated effects of higher market risk and lower credit risk exposure: Under mandatory insurance some (smaller) buyers are fully exposed to their market risk, but because all derivatives sales are insured, there is no buyer exposed to credit risk. Under voluntary insurance, only large buyers and sellers insure and thus remove credit risk; small and medium sized buyers hold the derivative alone and are exposed to credit risk but not to market risk. The model also uncovers a third financial risk factor: Having no uninsured and more insured sales under the mandatory regime results in strictly lower seller default probabilities. This decrease will also benefit other financial markets, increasing overall financial stability.

All three channels, and thus also their aggregate effect, crucially depend on the density of small and medium sized relative to larger buyers and sellers. To highlight how regulators can apply these insights to a specific OTC derivatives (sub-)market, I conclude the analysis by calibrating the model environment to the EuroDollar FX derivatives market. Consequently, I perform a counterfactual simulation under both regimes. I show that this market is predominantly used by many, yet relatively small buyers. Simultaneously, the overall impact of this market on seller default risk is negligible. Thus regulators, correctly, refrain from mandating counterparty default insurance in this market.

Literature With this set of empirical and theoretical insights, I contribute to a small but growing literature analyzing the role that CCPs play in counterparty risk mitigation and overall financial stability. Led by Biais et al. (2012, 2016), early papers are exclusively theoretical and focus mainly on moral hazard effects of (voluntary) counterparty default insurance.[3] They show that access to central clearing creates disincentives for buyers and sellers of OTC derivatives to enter more secure trades. As Antinolfi et al. (2018) highlight, these adverse effects are compensated when the CCP reveals sufficient private information to counteract moral hazard. Commonly, these papers introduce mutually-owned CCPs funded by the sellers with the objective to mitigate risk

[3] See also Koeppl et al. (2012); Koeppl (2013)

via risk-sharing. More recently and especially since the introduction of mandatory insurance, independently owned for-profit CCPs were able to gain importance in the market (Huang, 2019).

Focusing on the impact of especially for-profits CCPs on systemic risk, the theoretical papers by, for example, Amini et al. (2013), Capponi and Cheng (2018) and Huang (2019) have since complemented the earlier research. These papers mostly abstract from moral hazard problems. Instead, they highlight that for-profit CCPs ultimately fail to internalize the risk mitigation object: They set fees and collateral requirements that are too high and low respectively to achieve the highest risk-mitigation possible. I build on these papers by introducing heterogeneity in the buyer size and show that the size of the externality does not affect all agents equally.

Further, I relax the commonly used assumption under which each buyer is randomly matched with exactly one seller, after which the buyer becomes captive (Antinolfi et al., 2018; Huang, 2019; Koeppl et al., 2012). Instead, I allow buyers to switch away from their matched seller, for which they incur a fixed switching cost.[4] This captures the role of established business relationships that rose in importance as the OTC market has become more regulated.[5] I thereby further contribute to the ongoing debate on the price setting mechanism in the OTC market. Early works by Duffie et al. (2005) and Perez Saiz et al. (2012) study previously dominant market frictions, such as physical distance and sequential search. Those have become less important with the introduction of simultaneous multilateral search technologies such as pricing platforms (Glebkin et al., 2022). I build on the latter and introduce both buyer size heterogeneity and on-boarding cost, motivated by more recent regulatory frictions including know-your-customer requirements. I am thereby able to provide novel theoretical insights on differential pricing in the OTC derivatives market; ultimately bringing me closer to the empirical literature.

Here, my paper most closely relates to the empirical study by Hau et al. (2021), which I also use as a source for data moments. They document derivative price discrimination given a

[4]The impact of switching cost frictions is previously mostly studied in the loan market. Please see Schwert (2018) for a detailed literature review on this.

[5]An important new friction are customer due diligence requirements. When on-boarding to new sellers, buyers are required to provide substantial documentation about their business lines, making it a costly and lengthy process to purchase from a new seller, where there are no established prior business relationships (European Commission, 2019a; ESMA, 2018).

range of buyer characteristics, including buyer size, in the EuroDollar FX OTC derivatives market. As this market is not yet subject to mandatory counterparty default insurance, their paper is limited to reporting the status quo. I provide a theoretical counterpart to their empirical findings. Subsequently calibrating my model and performing a structural exercise, allows me to also discuss the equilibrium outcomes under the counterfactual case of mandatory default insurance. Other, less related empirical studies are the papers by Eisfeldt et al. (2018) and Jager and Zadow (2021), respectively studying the impact of CCP exit and entry on other market participants. Both have in common that they take the voluntary insurance-policy regime as given, and do not comment on the counterfactual case.

1.2. The Model Environment

Model Overview There are three dates, $t = 0, 1, 2$, a large set of risk-averse buyers, a large set of risk-neutral sellers, and a monopolistic for-profit CCP. At $t = 0$, every seller (she) is matched with exactly one buyer (he).[6] Buyers are endowed with a heterogeneous number of risky assets.[7] The CCP (it) decides on a two-part tariff system and collateral requirements.[8] Subsequently, sellers may become clearing members by paying the fixed entry fee for access to the CCP.[9] At $t = 1$, all trades take place. Buyers and sellers trade a financial derivative product d used for hedging the asset risk. Here, buyers must pay switching costs when interacting with any other seller besides their initial match. Additionally, clearing members and their product d buyers may mutually agree to purchase product m. Provided by the CCP for a variable fee, product m insures buyers against seller default. At $t = 2$, all uncertainty resolves, payments defined by product d and m are made, and sellers may strategically default.[10] All agents share a common discount factor

[6] This is similar to Antinolfi et al. (2018); Huang (2019); Koeppl et al. (2012)

[7] This is a new model feature assumed explicitly to study the heterogeneous reaction given buyer size.

[8] The monopolistic CCP is modeled similar to Huang (2019); Capponi and Cheng (2018); Amini et al. (2013), but its fee structure is extended to a two-part tariff system with clearing members.

[9] Here, I explicitly assume that buyers cannot access the CCP directly. This is to reflect the reality that regulatory requirements in terms of size and financial due diligence are impossible to meet for buyers. Instead, clearing member sellers may intermediate on their behalf.

[10] The default decision of sellers and how collateral enters is modeled similar to Huang (2019).

that is assumed to be 1.

Table 1.1: Model Timeline

	$\mathbf{t=0}$	$\mathbf{t=1}$	$\mathbf{t=2}$
CCP	• Sets two-part tariff and collateral • Gets entry fee from clearing members	• Sells prd. m via clearing members • Collects variable fee and collateral	• Pays transfers to insured buyers with defaulting sellers
Sellers	• May become a clearing member	• Sell product d to buyers • Clr. mbs. may agree to product m	• Choose whether to default • Pay transfers if not defaulting
Buyers	• Endowed with a_b risky assets	• Buy product d to hedge assets • Buy product m to insure product d	• Get transfers given asset endowment and product choices

CCP The for-profit CCP is a monopolistic insurer of seller default risk, and providing insurance product m is its only (potential) business line.[11] The structure of product m, described in detail below, is designed by regulators; the CCP's complete profit maximization problem and entry decision are studied in detail in Section 1.4.. For now note that at $t = 0$, the CCP maximizes profit by choosing a two-part tariff system and collateral requirements for product m. I label the sellers, that obtain the right to access the CCP by paying the fixed entrance fee, as clearing members. I assume explicitly that non-clearing members have no access to the CCP and thus product m.[12] The CCP enters the market, when expecting positive profits from the entry fee, the variable fee and product m sales, and losses exceeding collateral upon clearing member default.[13] Here, it takes into account that other agents expect the CCP to default with probability zero.[14]

Sellers There exists a finite, but large set S of risk-neutral sellers.[15] There sellers are protected by limited liability and thus may (strategically) default at $t = 2$. Seller default is determined by their (un-)insured sales in this market and the realization of an exogenous income

[11] A for-profit CCP is prohibited by regulators to have other business lines. In the EU central clearing and CCPs are regulated in the European market infrastructure regulation (EMIR) and in the US by the Dodd Frank Act (European Commission, 2019b; U.S. CFTC, 2019)

[12] Note that previous versions of this paper additionally studied the case where non-clearing members could access the CCP via a clearing member that intermediates. However, this was never optimal in equilibrium and would not alter any of the below derived results. It was thus omitted to improve readability.

[13] See Jager and Zadow (2021) for an empirical paper studying the entry decision of CCPs into a market.

[14] Appendix 1.1.3. discusses this assumption in detail and provides a micro-foundation.

[15] By assuming S is large, the presence of a monopolistic seller is ruled out.

stream L.[16] L is assumed to be an i.i.d draw from a normal distribution with mean $\mu_L > 0$ and variance σ_L^2: $L \sim N(\mu_L, \sigma_L^2)$. Denote the profits of a seller s at time t with Π_s^t and the default probability with $D_s = Pr(\Pi_s^2 \leq 0)$. Then, a seller's total expected profits $\mathbb{E}_0\Pi$ take on the following functional form:[17]

$$\mathbb{E}_0\Pi = \Pi_s^0 + \mathbb{E}_0\Pi_s^1 + (1 - D_s)\mathbb{E}_0\left[\Pi_s^2 \mid \Pi_s > 0\right] + D_s \cdot 0. \tag{1.1}$$

As sellers are heterogeneous in their matched buyer size (more immediately below), their clearing membership decision is not uniform.[18] I denote the subset of clearing members with M, and their total expected profits with an additional subscript $_M$. Then for every seller s seller (not) choosing to become a clearing member, it holds that:

$$\forall s \in M: \quad \mathbb{E}_0\Pi_M \geq \mathbb{E}_0\Pi, \tag{1.2}$$

$$\forall s \notin M: \quad \mathbb{E}_0\Pi_m < \mathbb{E}_0\Pi. \tag{1.3}$$

Buyers There exists a large set B of risk-averse buyers with mean variance utility $u(x) = E(x) - \frac{\gamma}{2}Var(x)$, where x are the time-2 pay-offs and $\gamma > 0$ is the degree of risk-aversion.[19] At $t = 0$, each buyer b is endowed with a_b different risky assets. a_b is drawn from a discrete distribution $\mathcal{A}$ over positive integers with minimum value $\underline{a}$ and maximum value $\overline{a}$: $a_b \sim \mathcal{A}\{\underline{a}, \overline{a}\}$. The distribution $\mathcal{A}\{\underline{a}, \overline{a}\}$ is common knowledge, the realization of a_b is, however, only know to the buyer in question and the sellers.[20] Each of the a_b assets is of unit size and pays a gross return $1 + \tilde{r}$ at $t = 2$. Here, $1 + \tilde{r}$ is an i.i.d. drawn from a normal distribution with mean μ_r and variance σ_r^2:

[16] The introduction of other business lines is motivated by the fact that Lehman Brothers defaulted, despite having significant positive profits from their OTC business lines (Fleming and Sarkar, 2014).

[17] See Appendix 1.1.2. for a detailed discussion of $\mathbb{E}_0\Pi$ and$\mathbb{E}_0\Pi_m$. Here note that clearing members: at $t = 0$ pay the fixed fee; at $t = 1$ collect prices for product d and m sales at and post collateral, at $t = 2$ either default or collect L, profits from product d sales and receive back collateral. Non-clearing members: at $t = 1$ collect prices for product d, at $t = 2$ either default or collect L and profits from product d sales.

[18] Sellers matched with larger buyers sell higher quantities; allowing them to afford the fixed fee e_m.

[19] See for example Eisfeldt et al. (2020) for a similar approach.

[20] Hence, neither the CCP nor other buyers know this. This follows the narrative of OTC trading platforms. Here, buyers post their demand for hedging and sellers post prices. However, all resulting trades are private, bilateral contracts. Therefore, parties not directly involved in the competitive bidding or the final deal have no access to the terms of trade, which includes notional size.

$1+\tilde{r} \sim N(\mu_r, \sigma_r^2)$ with pdf $f(\cdot)$ and cdf $F(\cdot)$.[21] A buyer's per-asset reservation utility, denoted u_r, is thus:

$$u_r = \mu_r - \frac{\gamma}{2}\sigma_r^2. \tag{1.4}$$

Product d At $t = 1$, sellers offer product d to buyers to hedge their asset risk. Sellers can always provide product d at zero marginal cost and charge a price p_d. The product d specifies a transfer τ from seller to buyer paid at $t = 2$. τ is mean-preserving and thus a function of the underlying asset's realized return: $\tau = \mu_r - (1+\tilde{r}) \sim N(0, \sigma_r^2)$.[22] When evaluating product d, buyers must, however, additionally account for the possibility of the seller defaulting on τ: When seller default coincides with $\tau \leq 0$, bankruptcy laws require the buyer to pay τ regardless, leaving the buyer with μ_r; when seller default coincides with $\tau > 0$, the buyer will not receive the transfers and is left with the asset realization $1 + \tilde{r} < \mu_r$. Unable to determine a seller's true probability of defaulting on positive transfers, buyers instead form a prediction $\widehat{D}_s$.[23] $\widehat{D}_s$ is endogenously determined in equilibrium, and a function of L and the seller's anticipated equilibrium sales.[24] Denote with u_d a buyer's per-asset utility given $t = 2$ pay-offs x_d, emerging from hedging as single risky asset with a product d. Further, denote the pdf associated with the pay-offs x_d with $f_d(x_d)$. Then:[25]

$$u_d = E(x_d) - \frac{\gamma}{2}Var(x_d) \qquad \text{where} \qquad f_d(x_d) = \begin{cases} \widehat{D}_s f(x_d) & x_d \leq \mu_r \\ \widehat{D}_s \left(1 - F(\mu_r)\right) + (1 - \widehat{D}_s) & x_d = \mu_r \\ 0 & x_d > \mu_r \end{cases} . \tag{1.5}$$

[21] Introducing variable returns that are draw from a continuous distribution is an extension of existing works, such as Biais et al. (2012, 2016); Huang (2019)

[22] To the best of my knowledge this is the first paper that models transfers that fully insure over a continuum of asset realizations, thereby extending the frameworks with discrete asset state-space proposed in Biais et al. (2012, 2016); Perez Saiz et al. (2012); Huang (2019).

[23] A seller's total sales is unknown to buyers, but correctly anticipated in equilibrium.

[24] Note that τ inherits the i.i.d. property from the underlying asset, implying that transfers are uncorrelated within and across buyers, and independent from L.

[25] Please see Appendix 1.1.1. for the closed form expression of (1.5) in terms of model parameters.

Product m The realized product d sales may subsequently be insured against the seller default by combining it with a product m. Product m is provided by the CCP via a two-part tariff and collateral system, which is set at $t = 0$ subject to several regulatory constraints.[26] First, the CCP sets a fixed entrance fee e_m that is paid by sellers at $t = 0$ for the right to access the product m. Here, the CCP must set e_m such that there exist at least two clearing members.[27] Further, the CCP charges a non-discriminatory variable fee v_m for every product m. Here regulators require both product d counterparties to simultaneously and separately purchase product m at $t = 1$.[28] Finally, the CCP must set a strictly positive collateral requirement $g_m \geq \underline{g_m}$.[29]

Collateral g_m is collected from clearing members for every product m purchase at $t = 1$, and at $t = 2$ is either returned to non-defaulting clearing members or used to cover defaulting clearing members' transfers. Tying up liquidity in the form of collateral, clearing members face an opportunity cost δ for every unit posted.[30] To compensate for their incurred cost from agreeing to product m, clearing members ask their product d buyers for an additional price p_m, also paid at $t = 1$. Buyers are willing to pay (reasonable) v_m and p_m, as holding product m allows them to eliminate all risk: They expect the CCP to cover transfers, even if those exceed the defaulting clearing member's posted collateral.[31] A buyer's utility u_{dm} from combining a single risky asset with both a products d and m is thus:

$$u_{dm} = \mu_r. \tag{1.6}$$

[26] These restrictions are specified in the Dodd Frank Act (U.S. CFTC, 2019) and the EMIR regulation in the EU (European Commission, 2019b). For a global overview, please see (BCBS et al., 2018).

[27] With this, regulators rule out a monopoly in the intermediation market. Further, they ensure the CCP is exposed to more than one seller, thereby diversifying the CCP's exposure to seller default risk.

[28] To my knowledge, this is the first paper that carefully incorporates into the analysis that both counterparties of a product d need to agree to the purchase of product m.

[29] Throughout the paper, I perform comparative statics over this parameter and subsequently compare it to the currently required minimum collateral equal to the 5-day 99.5% value-at-risk in the simulation.

[30] This is motivated by an opportunity cost of capital that could else have been invested (Huang, 2019).

[31] Underlying this is the assumption that the CCP is not expected to default, even if collateral is insufficient to cover τ. See Appendix 1.1.3. for more details.

Switching Costs and Captive Consumers The initial random match between a buyer and a seller represents existing business relationships, and establishing new relationships is costly.[32] Therefore, buyers pay strictly positive switching costs C before trading product d with an unmatched seller.[33] These switching costs thus create incentives for buyers to purchase from their initially matched seller. Contrary to this, the risk aversion γ incentivize buyers to switch to the seller they believe to be the safest. Especially for small buyers', however, the per-asset switching cost C/a_b may exceed their total utility gain from switching for product d and (potentially) m to safer sellers. These buyers are consequently labeled captive consumers.[34]

Quantities, Prices and Competition I assume that a single asset can either be fully hedged and then insured or not at all, but never partially. However, I allow buyers to freely choose whether to purchase products d and m for none, some or all of their assets. The fraction of hedged and subsequently insured assets will depend on prices p_d and p_m. Here, I assume that all sellers compete over prices p_d in a Bertrand fashion. Additionally, clearing members set price p_m in a take-it-or-leave-it fashion: The product d seller makes a single offer without competition; the buyer subsequently chooses whether to accept or refuse.[35] I allow for sellers to price discriminate, such that p_d and p_m may depend on individual buyer characteristics, switching costs, the number of assets hedged/insured, and the buyer incurred portion of CCP fees.[36] Further, p_m is set after product d sales have realized and thus may additionally depend on p_d.

Imperfect Information and The Equilibrium Notion Agents are informed about all model elements unless specifically stated otherwise: The CCP does not observe the realized buyer sizes and the resulting matches, but only the underlying distribution $\mathcal{A}\{\underline{a}, \overline{a}\}$ and market size B.[37] Buyers neither observe the other buyers' realized sizes nor prices offered to them nor their choice

[32]There is a rich empirical banking literature documenting that informational frictions result in costly on-boarding procedures for new clients. See Schwert (2018) for a detailed discussion.

[33]Introducing switching cost is an extension to Biais et al. (2012), where buyers are captive once matched with a good or bad seller.

[34]See for example Armstrong and Vickers (2019) for a detailed analysis of captive consumers.

[35]This is because the risk-neutral clearing members must not only agree to the insurance, but can also refuse it, thus giving them the entire bargaining power.

[36]Recall, the CCP's fees and collateral are set in a monopolistic, non-discriminatory fashion.

[37]OTC contracts are private, bilateral trade agreements and therefore modeled as contracts under incomplete information (Acharya and Bisin, 2014; Antinolfi et al., 2018; Eisfeldt et al., 2018).

of seller.[38] Given this, I apply the notion of sub-game perfect Nash equilibrium (SPNE) with incomplete information; relying on backwards induction to derive all quantities, prices, fees, and collateral requirements.

Parameter Restrictions I assume that the following parameters are strictly positive: asset return mean μ_r and variance σ_r^2, minimum and maximum buyer sizes $\underline{a}$ and $\overline{a}$, mean μ_L and variance σ_L^2 of exogenous profits L, sellers' collateral cost δ, and switching cost C. Further, I assume that underlying agency frictions result in p_d and p_m being weakly positive.[39] Similarly, I assume that the CCP's entry fee e_m and variable fee v_m are weakly positive.

For the collateral requirements g_m, there exist two model implied thresholds. The first one, labeled g_m^*, denotes the collateral level required for seller default probabilities to strictly decrease in *insured* product d sales. The second one, labeled g_m^{**}, exceeds the first and denotes the collateral level required to induce that seller profits are a strictly increasing function in combined product d and m sales. For the remainder of the main analysis, and confirmed by the calibration exercise later, I assume that the regulatory collateral requirement $\underline{g_m}$ exceeds g_m^{**}. For completeness, Appendix 1.4. states the results also for the alternative case. The three thresholds are:

$$g_m^* = \frac{\mu_L \sigma_r^2}{2\sigma_L^2}, \qquad g_m^{**} = \frac{\mu_L \sigma_r^2}{2\sigma_L^2} + \frac{\sigma_r^2}{2\sigma_L}, \qquad \underline{g_m} > g_m^{**} > g_m^*. \tag{1.7}$$

1.3. Equilibrium Prices and Quantities at Time 1

Before diving into the analysis, I define the different types of equilibria that may arise at $t = 1$. For this purpose, denote a buyer's aggregate utilities from staying and switching with $U(a_b; stay)$ and $U(a_b; switch)$ respectively; and the total payments in either case with $P(a_b; stay)$ and $P(a_b; switch)$. All four terms are equilibrium objects and depend on: the buyer's asset endowment size, the equilibrium share of hedged and insured assets, the number of switching buyers,

[38]The latter two are important, as else buyers could infer the size of others from prices and seller choices.

[39]Underlying frictions may for example be that individual broker bonuses that depend on their $t = 1$ profits. Further, regulatory pressure may result financial institutions avoiding speculative losses.

and the switching buyers' choice of sellers. More in the following sections. For now note that, characterized by the buyers' choices of product d seller at $t = 1$, three types of equlibria may arise: a no switching equilibrium, a partial switching equilibrium and a fully switching equilibrium.

No Switching Equilibrium A no switching equilibrium is characterized by every buyer purchasing product d exclusively from his matched seller: Observing all equilibrium prices and C, for every buyer the utility of staying must exceed the utility from switching. Formally, the no switching equilibrium exists if:

$$U(a_b; stay) - P(a_b; stay) \geq U(a_b; switch) - P(a_b; switch) - C \qquad \forall b \in B. \tag{1.8}$$

Fully Switching Equilibrium In a fully switching equilibrium, all buyers find it optimal to switch for their product d purchase. And thus given all prices and C, for every buyer there exists at least one unmatched seller where the aggregate utility from switching exceeds the utility from staying. This is formalized in condition (1.9) below:

$$U(a_b; switch) - P(a_b; switch) - C > U(a_b; stay) - P(a_b; stay) \qquad \forall b \in B. \tag{1.9}$$

Partial Switching Equilibrium In a partial switching equilibrium, at least one buyer finds staying optimal and simultaneously at least one buyer prefers to switch sellers for product d.

I denote the subsets of staying and switching buyers as $B_{stay} \subset B$ and $B_{switch} = B \setminus B_{stay}$ respectively. Then buyers select into these subsets as follows:

$$U(a_b; stay) - P(a_b; stay) \geq U(a_b; switch) - P(a_b; switch) - C \qquad \forall b \in B_{stay}, \tag{1.10}$$

$$U(a_b; stay) - P(a_b; stay) < U(a_b; switch) - P(a_b; switch) - C \qquad \forall b \in B_{switch}. \tag{1.11}$$

Captive Consumers Recall that buyers vary in size due to their different number of risky assets a_b, yet face the same switching cost C. Especially for smaller buyers, having a high per-asset switching cost, switching may come at a total loss. These buyers are captive consumers, as they never consider switching. Define a buyers total reservation utility with $U_r = a_b u_r$. Then, a buyer is captive if:

$$U(a_b; switch) - C \leq U_r(a_b). \tag{1.12}$$

1.3.1. Mandatory Counterparty Default Insurance at Time 1

With these definitions in mind, I now derive the equilibrium outcome under mandatory counterparty default insurance at $t = 1$. Here, product d cannot be held alone, and buyers have two choices: buying the bundle from a clearing member or receiving their reservation utility. I start by analyzing the market outcome, when at least some clearing members offer the combination of product d and m bundle. Subsequently, I comment on the outcome without clearing members or when those never offer the product bundle.

1.3.1.1. Clearing Members Offering The Bundle

For this sub-section, I explicitly assume that clearing members exist and indeed are willing to offer the bundle of product d and m. I derive the buyers' choices of clearing member, the bought quantities and paid prices through backwards induction: First, I derive p_m, then p_d and the share of assets combined with bundle of product d and m, and finally the buyers' choice of seller.

Product m Prices To derive the price p_m, I assume that product d sales have realized. Then, the product d seller (always a clearing member) can take advantage of the fact that buyers cannot hold product d alone: A buyer can only choose between agreeing to the bundle at price p_m or remaining unhedged with reservation utility u_r. Exploiting this absence of alternatives, the seller sets p_m such that the buyer is just indifferent between holding the bundle of product d and m or remaining unhedged. As equation (1.13) illustrates, the sellers account for the buyer paying v_m to the CCP, and p_d to said seller for product d.

$$p_m = u_{dm} - u_r - v_m - p_d. \tag{1.13}$$

Price p_d and No Switching Because p_m is a linear function in p_d, any increase in p_d is compensated by a one-for-one decrease in p_m. Thus individually, p_m and p_d are not uniquely determined. In equilibrium, a seller may set any combination of p_m and p_d, where their sum is equal to the buyer's utility gains from holding both products. The total bundle price thus becomes:

$$p_d + p_m = u_{dm} - u_r - v_m. \tag{1.14}$$

Bundle Quantities & No Switching Equilibrium First note that given the total price (1.14), the clearing members' profits strictly increase in bundle sales.[40] And hence, they offers the bundle for all of a buyer's assets. Further note that the bundle price (1.14) fails to internalize any switching costs potentially paid for the product d purchases by unmatched buyers. A buyer considering to switch for product d, and anticipating p_m, thus expects a total utility $U_r - C$ — independently of the actually bought bundle quantities upon switching. However, both when staying with his matched seller or not purchasing any product at all, the buyer would instead receive his aggregate reservation utility U_r. Thus, all buyers are captive under mandatory counterparty default insurance and never switch.

***Proposition* 1.** *Under mandatory insurance, all buyers are captive. Hence, the no switching equilirbium is unique and characterized by:*

(i) Buyers, matched with a clearing member, purchasing the product d and m bundle from said seller at a bundle price:

$$p_d + p_m = u_{dm} - u_r - v_m. \tag{1.15}$$

(ii) Buyers, not matched with a clearing member, exiting the market.

1.3.1.2. A Market Without Clearing Members

The market outcome in the absence of any clearing member follows directly from property *(ii)* in *Proposition 1*: If there is no clearing member offering the bundle, then all buyers exit the

[40]This result depends on the parameter restrictions $\underline{g_M} > g_M^{**}$. The alternative is discussed in detail in the Appendix.

market and remain fully exposed to their market risk. And hence, the market experiences failure.

***Corollary* 1.** *Under mandatory insurance, the absence of any clearing member offering the product d and m bundle causes market failure.*

1.3.2. Voluntary Counter Party Default Insurance at Time 1

In this section, I derive the equilibrium outcome at $t = 1$ under the counterfactual case of voluntary counterparty default insurance. Here, buyers have three options: receive their reservation utility, hold product d as a stand-alone, or additionally add-on a product m. Again, I study two different scenarios: there exist clearing members offering product m, and there exists no clearing member offering product m.

1.3.2.1. Clearing Members Offering Add-on Product m

Assuming that there exist clearing members that offer product m, I again rely on backwards induction to derive the equilibrium outcomes: First, I assume that that product d sales have realized and derive p_m given p_d. Here, I account for the fact that product d can be held as a stand-alone. Then, I derive the prices p_d , buyers' choice of sellers and share of hedged risky assets. I conclude with comparative statics over switching cost and show how different levels of C impact the equilibrium outcome.

Product m Prices Buyers can always at least purchase product d as a stand-alone. Thus, the product d seller, if a clearing member, can at most charge the utility gains from adding product m on to product d. Setting p_m to capture all buyer surplus, the clearing member accounts for the buyers variable fee v_m paid to the CCP, but not the product d price p_d: p_d is paid regardless whether a product d combined with a product m or not, consequently entering both sides of the buyer's participation constraint and dropping out. Equation (1.16) formalizes this:

$$p_m = u_{dm} - u_d - v_m. \tag{1.16}$$

Given the price p_m, buyers extract no further utility surplus from purchasing product m. Additionally, the (sunk) switching costs paid upon product d purchase are not accounted for in p_m. From this, it immediately follows that buyers only compare the utility gains from switching for product d against the associated switching costs, when deciding whether to stay with their matched seller.

Product d Prices Knowing that the subsequent insurance decision does not influence a buyer's seller choice, I can now derive the product d prices. First, it can be shown that buyers always pay $p_d = 0$ upon switching. The market is large, and thus ex ante no unmatched seller has unique characteristics in the eyes of any buyer. Further, product d can be provided at zero marginal cost, yet all sellers' profits strictly increase in any product d sale.[41] Thus, standard Bertrand competition arguments apply and all unmatched sellers offer unrestricted access to product d at:

$$p_d(a_b; switch) = 0. \tag{1.17}$$

With all unmatched sellers offering product d for zero costs, buyers only consider switching to the clearing members with the highest anticipated product m sales from *other* buyers:[42] Insuring all their product d sales, and experiencing decreasing default rates in their total number of sales, implies them to be safest. I denote the associated utility from switching for product d to the largest clearing members with $u_d(a_b; switch)$. To deter switching, the matched seller must thus be able to set a positive price $p_d(a_b; stay)$ that, given per asset-switching cost C/a_b, makes that the buyer is just indifferent between staying or not. If not possible retain the matched buyer, competition drives down the price $p_d(a_b; stay)$ to zero. Thus, for matched buyers $p_d(a_b; stay)$ is:

$$p_d\left(a_b, stay\right) = \max\left\{C/a_b - \left[u_d\left(a_b; switch\right) - u_d\left(a_b; stay\right)\right],\ 0\right\}. \tag{1.18}$$

[41]Providing uninsured derivatives increases the variance of total seller profits. Protected by limited liability and strategic default, however, more so for positive than for negative realizations.

[42]Note that the belief over seller default probability $\widehat{D}$ is formed before any product m sale. Hence, the buyer is indifferent whether he himself is additionally offered the add-on. He, however, cares about the other product m sales given their reduction of seller default risk.

The price (1.18) applies if the buyer is non-captive and thus the matched seller indeed has to compete for him. For captive buyers, the per-asset switching costs exceeds the utility benefits from switching. With captive buyers never considering to switch, the matched sellers can utilize this by setting $p_d(a_b, captive)$ equal to the utility gains from product d:

$$p_d\left(a_b, stay\right) = \max\left\{C/a_b - \left[u_d\left(a_b; switch\right) - u_d\left(a_b; stay\right)\right],\ 0\right\} \tag{1.19}$$

***Lemma* 1.** *Under voluntary insurance, sellers always offer product d and additionally, clearing members always offer product m.*

1. *For product d, sellers charge:*

   (i) Their unmatched buyers a price:

$$p_d\left(a_b, switch\right) = 0, \tag{1.20}$$

   (ii) Their non-captive, matched buyer a price:

$$p_d\left(a_b, stay\right) = \max\left\{C/a_b - \left[u_d\left(a_b; switch\right) - u_d\left(a_b; stay\right)\right],\ 0\right\}, \tag{1.21}$$

   (iii) Their captive, matched buyer a price:

$$p_d(a_b, captive) = u_d\left(a_b; stay\right) - u_r. \tag{1.22}$$

2. *For product m, clearing members charge their product d buyers a price:*

$$p_m = u_{dm} - u_d - v_m. \tag{1.23}$$

Switching in Equilibrium Following *Lemma 1*, two properties common to all equilibria can be derived. First, all buyers choose the same seller and product combination for all their a_b assets. Given this, I can simplify the notation and let $U_d(a_b)$ denote the aggregate utility from a buyer's a_b assets. Then:

$$U_d(a_b; switch) = a_b \cdot u_d(a_b; switch), \tag{1.24}$$

$$U_d(a_b; stay) = a_b \cdot u_d(a_b; stay). \tag{1.25}$$

Here, it is important to note that $U_d(a_b; switch)$ endogenously depends on the behavior of other buyers in the market: The more *other* buyers switch to a certain clearing member, the lower the clearing member's default probability, the larger the benefits from also switching. From this endogeneity of seller default follows the second equilibrium property that all switching buyers choose the same clearing member. All other buyer choices cannot be sustained, as there is always at least one clearing member with more sales to whom to deviate to.

Whether none, some or all of the buyers switch depends both on C and $U_d(a_b; switch)$, given the (anticipated) behavior of all other buyers in the market. I will start with the level of C under which a no switching equilibrium arises. Quite intuitively, the largest buyers also have the largest gains from switching, as they have the most assets to hedge and insure. Simultaneously, all buyers face the same switching cost. Thus, a no switching equilibrium exists only if C is just equal to or exceeding the largest buyer's benefits from switching, conditional on no other buyer switching. The threshold that induces a no switching equilibrium is labeled with C_{NS}.

Proposition 2. *The no switching equilibrium exists if and only if C exceeds a threshold level C_{NS}. C_{NS} denotes the level of switching cost that makes the largest buyer, i.e. $a_b = \overline{a}$, just indifferent between switching and staying, conditional on nobody else switching:*

$$C_{NS} = U_d(\overline{a}; switch) - U_d(\overline{a}; stay). \tag{1.26}$$

The no switching equilibrium is contrasted by the fully switching equilibria. In each of them, even the smallest buyers must find switching optimal, conditional on all other buyers switching: The smallest buyers have the lowest aggregate utility gains from switching, but face the same switching costs. This implies that there exist a threshold $\underline{C}$, such that only if $C \leq \underline{C}$, also the smallest buyers switch. Here, all switching buyers choose the same clearing member in equilibrium. However, it does not matter which clearing member exactly they choose. And thus, there exist as many fully switching equilibria as there are clearing members.[43]

Proposition 3. *There exists as many fully switching equilibria as clearing members if and only if C is below a threshold level $\underline{C}$. $\underline{C}$ denotes the level of switching costs where the smallest buyers,*

[43] Recall that $\mid M \mid \geq 2$.

i.e. $a_b = \underline{a}$*, are just indifferent between staying and switching, conditional on all other buyers switching to the same clearing member:*

$$\underline{C} = U_d(\underline{a}; switch) - U_d(\underline{a}; stay). \tag{1.27}$$

For any level of C exceeding $\underline{C}$, the cost of switching outweigh the benefits for the smaller buyers. And thus only a fraction relatively larger buyers prefers switching. Here, it can be shown that for every level of $C > \underline{C}$, buyers are divided into the switching and non-switching fraction by a unique size threshold a_{PS}. For all buyers smaller than a_{PS}, the costs of switching outweigh the benefits. All buyers larger than a_{PS}, however, switch to the same clearing member. Note here that because now some buyers do not switch, this may result in clearing members with heterogeneous sales. And thus, while there exist multiple partial switching equilibria, there are not necessarily as many as clearing members.

***Proposition* 4.** *For every level of* $C > \underline{C}$*, there exists a unique buyer size threshold* a_{PS} *characterizing the partial switching equilibria: buyers of size* $a_b \leq a_{PS}$ *stay with their matched seller, and all buyers with size* $a_b > a_{PS}$ *switch to the same clearing member. Here,* a_{PS} *solves the following equality:*

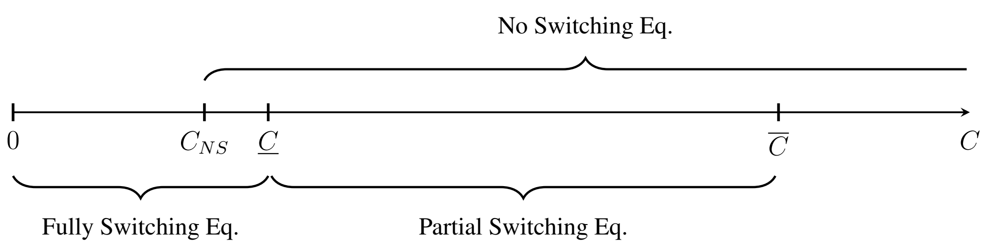
$$C = U_d\left(a_{PS}; switch\right) - U_d\left(a_{PS}; stay\right). \tag{1.28}$$

It can be shown that threshold size a_{PS} strictly increases in switching cost C. Thus there exists a threshold $\overline{C}$ for which $a_{PS} = \overline{a}$ and only the largest buyers switch. Any level of C exceeding $\overline{C}$ results in a no switching equilibrium.

***Corollary* 2.** *The set of switching sellers decreases with* C*, such that there exists a switching cost level* $\overline{C}$ *for which only the largest buyers of size* $\overline{a}$ *switch. For any higher level of* $C > \overline{C}$*, the no switching equilibrium is unique.*

Co-Existence of Equilibria Summarizing the results, there are three important thresholds on switching cost C that determine when the different types of equilibria (co-)exist: $\underline{C}$, $\overline{C}$, and C_{NS}. The thresholds $\underline{C}$ and $\overline{C}$ are exogenous to the market. They determine the existence of the fully and partial switching equilibria, which never co-exist. The threshold C_{NS} is endogenously determined at time 0 and is increasing in g_m.[44] At this stage, C_{NS} is taken as given and may be below, equal to or above $\underline{C}$, but is always below $\overline{C}$. Thus, when C_{NS} is lower than $\underline{C}$ the no switching equilibria may co-exist both with the fully and partial switching equilibria. When C_{NS} exceeds $\underline{C}$, only the no and partial equilibria may coexists. The existence and multiplicity of equilibria as a function of C are summarized in Figure 1.1 below.[45]

Figure 1.1: Existence of Equilibria When Product m is Traded



1.3.2.2. A Market Without Product m Sales

A market without any product m sales may arise for three reasons: the CCP chooses not to enter, there are no clearing members, clearing members exist but never offer product m. The equilibrium outcome is, however, not too different from above. For one, all sellers offer the stand-alone product d to all buyers for all their assets. And again, the market is large such that no unmatched seller is unique in the eyes of a single buyer. Hence, unmatched buyers charge $p_d(a_b; switch) = 0$. Further, sellers again charge their captive consumers their entire utility gains from product d.

The main difference is that now the utility from switching strictly decreases in the unmatched seller's total sales: a seller's default probability strictly increases in the volume of uninsured product d sales. Given this, a matched seller deters switching by setting a price such that her buyer is just indifferent between staying or switching to the seller with the lowest total sales. Put

[44] Higher collateral makes switching more profitable, implying a higher C to deter also the largest buyers.
[45] For a formal discussion, please see Appendix 1.2.2..

differently, the matched seller is able to charge a premium above per-asset switching cost equivalent to the utility losses experienced from switching to other sellers. Ultimately, this leads to an equilibrium with no switching, where the unmatched sellers with the lowest default probabilities are those matched with a buyer of size $\underline{a}$. Let $u_d(a_b; switch)$ denote the utility from switching to one of those sellers.

Proposition 5. *Under voluntary insurance and in the absence of clearing members, the no switching equilibrium is unique. Here, sellers always offer product* d *and charge:*

(i) Their unmatched buyers a price:

$$p_d\left(a_b, switch\right) = 0, \tag{1.29}$$

(ii) Their non-captive matched buyer a price:

$$p_d\left(a_b, stay\right) = C/n_b + u_d\left(a_b; stay\right) - u_d\left(a_b; switch\right) > 0, \tag{1.30}$$

(iii) Their captive matched buyer a price:

$$p_d(a_b, captive) = u_d\left(a_b; stay\right) - u_r. \tag{1.31}$$

1.4. CCP and Seller Choices at Time 0

I now turn to deriving the optimal $t = 0$ choices, and hence the SPNE, given the (anticipated) $t = 1$ market outcomes and the consequent realizations at $t = 2$. The equilibrium choices of agents at $t = 0$ realize in two stages: First, the CCP simultaneously decides on entry, the two-part tariff (membership fee e_m and the variable fee v_m), and the collateral requirements g_m. Then, observing the CCP's choices and anticipating sales, sellers decide whether to become a clearing member.

1.4.1. The CCP's Profit Maximization Problem

Recall that CCP is unaware of the realizations of a_b, and thus forms (rational) expectations $\mathbb{E}_0$ over the buyer-seller matches and consequent market outcomes at $t = 0$, $t = 1$ and $t = 2$.

Denote the associated CCP profits at time t with Π_C^t. Further, recall that I assume that the CCP is never expected to default. This assumption is discussed in detail in Appendix 1.1.3., where I show this to hold true for a sufficiently capitalized CCP. In that case, default does not enter the CCP maximization problem, who expects to bear all potential losses. And hence, the CCP chooses v_m, e_m, and g_m simultaneously to maximize the following constrained problem:

$$\mathbb{E}_0\Pi_C = \max_{e_m, v_m, g_m} \quad \mathbb{E}_0\Pi_C^0(e_m) + \mathbb{E}_0[\Pi_C^1(v_m, g_m) \mid M] \; + \; \mathbb{E}_0[\Pi_C^2(\tau, L, g_m) \mid M; Q_{dm}], \tag{1.32}$$

$$s.t.$$

$$|M(e_m)| \geq 2, \tag{1.33}$$

$$g_m \geq \underline{g_m}. \tag{1.34}$$

As indicated in equation (1.32), time zero profits Π_C^0 directly depend on the choice of e_m dictating the number of clearing members. Here, constraint (1.33) applies, stating that there must be at least two clearing members, i.e. the set M has a cardinality weakly greater than two. At $t = 1$, v_m and g_m determine total product m sales and thus profits Π_C^1. Here, sales are conditional on the sellers' clearing membership choices. Further, constraint (1.34) applies defining a lower bound of the collateral choice g_m, which must weakly exceed the regulatory minimum $\underline{g_m}$. At $t = 2$, the CCP may experience losses from covering the transfers τ of the defaulting clearing member(s) with insufficient collateral. Given the total sales Q_{dm} of each clearing members, these default losses thus depend on expected transfers τ, collateral g_m and exogenous profits L.

The CCP enters, whenever total expected profits $\mathbb{E}_0\Pi_C$ are weakly positive. In this context, note that closed form solutions of entry requirements, optimal fees and collateral are complex and depend on the relative size of model parameters. I therefore present only general results in the following subsections. Instead, I provide a full set of numerical solutions for a carefully calibrated set of parameters in Section 1.5..

1.4.2. Mandatory Counterparty Default Insurance

I start by assuming that the CCP has decided to enter and subsequently set a reasonable two-part tariff structure inducing some clearing membership. Then, when insurance is mandatory, sellers anticipate the no switching equilibrium to arise at $t = 1$. Here, all clearing members sell the bundle of product d and m for all their matched buyers' asset and only for those assets. Non-clearing members and their matched buyers, instead, exit the market. Given this, I first describe the sellers' total expected profits as a function of membership status and matched buyer size, and then the SPNE's general characteristics.

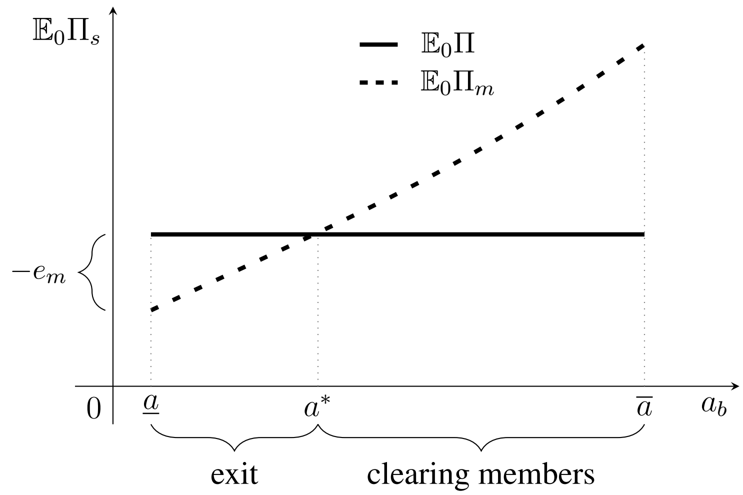
Sellers' Expected Profits Under mandatory insurance, the sellers compare profits from exiting the market ($\mathbb{E}_0\Pi$) to becoming a clearing member that only trades with her matched buyer ($\mathbb{E}_0\Pi_m$). The equations (1.35) and (1.36) state their respective functional forms:

$$\mathbb{E}_0\Pi = (1-D)\mathbb{E}_0[L \mid L > 0], \tag{1.35}$$

$$\mathbb{E}_0\Pi_M = -e_m + a_b\left(p_d + p_m - v_m - (1+\delta)\, g_m\right) + (1-D_M)\mathbb{E}_0\left[L + a_b(g_m - \tau) \mid L + a_b(g_m - \tau) > 0\right]. \tag{1.36}$$

Below, Figure 1.2 plots them over the space of matched buyer sizes a_b. Here, note that the profits from market exit are independent of matched buyer size and $\mathbb{E}_0\Pi$ is, thus, a flat line over all a_b. Instead, expected clearing member profits $\mathbb{E}_0\Pi_m$ are strictly increasing, and convex in matched buyer size.

Figure 1.2: Seller Profits Under Mandatory Counterparty Default Insurance



The CCP can influence the degree of convexity of $\mathbb{E}_0\Pi_M$, as well as the intersection with the y-axis, by setting different values for g_m, v_m and e_m. Here, increases in both g_m and v_m reduce the degree of convexity. Further, setting $e_m = 0$ implies that $\mathbb{E}_0\Pi_m(\underline{a})$ approximates $\mathbb{E}_0\Pi(\underline{a})$, while increase in e_m shifts the expected clearing member profits $\mathbb{E}_0\Pi_M$ downwards.

The No Switching SPNE Given the properties of $\mathbb{E}_0\Pi$ and $\mathbb{E}_0\Pi_m$, Figure 1.2 quite intuitively illustrates that there exists a unique size threshold a^* that divides sellers into clearing members and non-clearing members: For sellers with matched buyers smaller than a^*, leaving the market is profit maximizing. For sellers with matched buyers weakly larger than a^* becoming a clearing member and offering the product bundle is profit maximizing. Threshold a^*, thus, characterizes the SPNE with an active CCP under mandatory insurance.

Proposition 6. *Under mandatory counterparty default insurance, the presence of a CCP induces a SPNE with no switching at $t = 1$. The SPNE characterized by a size threshold $a^*(g_m, v_m, e_m)$, where:*

(i) Every seller matched with a buyer weakly larger than a^* becomes a clearing member and offers the product d and m bundle.

(ii) Every seller matched with a buyer smaller than a^* exits the market.

Market Unraveling Alternatively, the CCP may find that no combination of e_m, v_m and g_m simultaneously results in positive expected profits $\mathbb{E}_0\Pi_C$ and at least two clearing members. In that case, the CCP does not enter the market and the set of clearing members M is empty. It follows directly from *Proposition 6 (ii)* that, without the ability to sell an uninsured product d, all sellers exit the market. And thus, both the product d and m markets seize to exist.

***Corollary* 3.** *Under mandatory insurance, the absence of a CCP and/or clearing members induces a SPNE with market failure, where neither product d nor product m is traded.*

1.4.3. Voluntary Counterparty Default Insurance

I now derive the SPNE, when product m is voluntary. Again, I start by assuming that the CCP has entered and set reasonable fees to induce some clearing membership. Then, depending in C, agents may anticipate a no, fully or partial switching equilibrium respectively. Sellers decide again whether to become clearing members, but are now able to sell an uninsured product d.

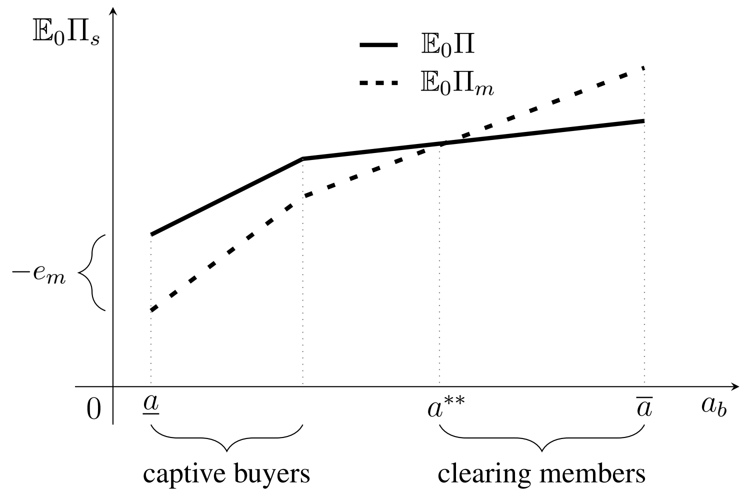
The No Switching SPNE In the no switching equilibrium all buyers stay with their matched seller and buy the product combination they are offered. The resulting profits of (non-)clearing members are stated in equations (1.37) and (1.38):

$$\mathbb{E}_0\Pi = a_b p_d + (1 - D)\mathbb{E}[L - a_b\tau \mid L - a_b\tau > 0], \tag{1.37}$$

$$\mathbb{E}_0\Pi_M = -e_m + a_b(p_d + p_m - v_m - (1+\delta)g_m)) + (1 - D_M)\mathbb{E}[L + a_b(g_m - \tau) \mid L + a_b(g_m - \tau) > 0]. \tag{1.38}$$

The respective functional forms are illustrated in Figure 1.3 below. Note that $\mathbb{E}_0\Pi$ is now strictly increasing in a_b, and thus non-clearing members can (and will) always sell product d to their matched buyer. Similarly to before, clearing members always additionally sell product m and $\mathbb{E}_0\Pi_m$ is strictly increasing in a_b. However, expected clearing member profits $\mathbb{E}_0\Pi_m$ are not globally convex anymore and only displays piece-wise convexity. This is due to kink, both $\mathbb{E}_0\Pi_m$ and $\mathbb{E}_0\Pi$ experience, where buyers move from being captive to non-captive.

Figure 1.3: Seller Profits in a No Switching SPNE under Voluntary Insurance



Again, the CCP can decrease the slope and intercept of $\mathbb{E}_0\Pi_m$ by increasing g_m/v_m and e_m respectively. The local convexity in both the section for captive and non-captive consumers is always preserved. Therefore, also here, there exists a unique size threshold a^{**}, where only the sellers matched with larger buyers become clearing members.

***Proposition* 7.** *Under voluntary insurance, the no switching SPNE with a CCP is characterized by a unique size threshold $a^{**}(g_m, v_m, e_m)$, where:*

(i) Every sellers matched with a buyer larger than a^{**} becomes clearing members to offer also product m.

(ii) Every seller matched with a buyer smaller than a^{**} offers only product d as non-clearing members.

Non-Existence of Fully Switching SPNE Instead, assume market participants anticipate a fully switching equilibrium at $t = 1$, where all buyers switch to the same clearing member. This clearing member is, thus, expected to post collateral for all B buyers' a_b assets. Because B is large, such levels of collateral result in a default probability of quasi zero. Therefore, this seller cannot extract any profits from her product m sales: u_d equals u_{dm} for $D_M \simeq 0$ and thus $p_m = u_{dm} - u_d = 0$. Additionally, the chosen clearing member charges $p_d(a_b; switch) = 0$, resulting in overall zero price charges from the product d and m sales. Yet, the seller would still have to pay cost δ for every unit of posted collateral . This results in strictly negative expected profits of a clearing member (see equation (1.39)). If instead the seller exits the market, she would still realize the strictly positive profits from L (see equation (1.40)).

$$\mathbb{E}_0\Pi_m = - e_m - Q_{dm}(v_m + \delta g_m) < 0 \qquad \forall e_m, v_v \geq 0, \tag{1.39}$$

$$\mathbb{E}_0\Pi = (1 - D)\mathbb{E}_0[L \mid L > 0] > 0. \tag{1.40}$$

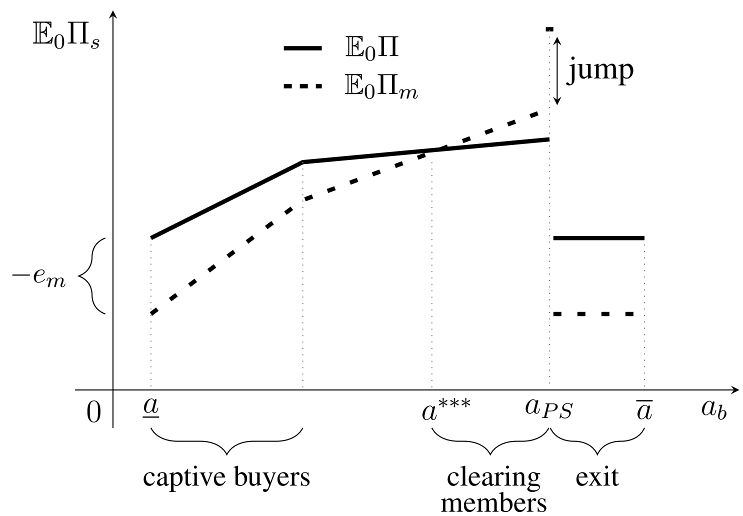
And thus, the positive expected profits from exiting the market exceed the strictly negative expected profits from becoming a clearing member. To incentivize sellers to become clearing members regardless, the CCP would need to charge negative fees, which is ruled out by assumption. However, it can be shown that allowing for negative fees would lead to strictly negative CCP profits. And thus, a for-profit CCP would in either case, rather exit than serving the market with an anticipated fully switching equilibrium. This violates the assumption of an active CCP.

Because the CCP does not serve the market, there exist no clearing members and sellers never offers product m. Hence, there does not exists any SPNE with CCP entry and fully switching under voluntary insurance. Instead, the no switching equilibrium without clearing members as the arises as the SPNE.

Proposition 8. *A SPNE with CCP entry and fully switching does not exist under voluntary insurance. Whenever anticipating all buyers to switch at $t = 1$, the CCP prefers to exit the market and a no switching SPNE without clearing members arises.*

The Partial Switching SPNE Finally, assume that a a partial switching equilibrium is expected to arise. Any partial switching equilibrium is characterized by an exogenous size threshold a_{PS}, where all larger buyers switch and all weakly smaller buyers stay. Given this, profits, and thus choices, of sellers with matched buyers strictly smaller than a_{PS} are identical to the no switching equilibrium described above. Instead, the sellers matched with buyers of size strictly larger than a_{PS} cannot retain their buyers. And thus, their profits equal those realized from market exit. The most interesting happens exactly at the threshold a_{PS}. Because $\mathbb{E}_0\Pi_m$ is strictly increasing in a_b, those seller must be clearing members. And not just any clearing members, but those with the highest anticipated sales. Therefore, all switching buyers only consider switching to them and their expected profits experience a jump. This is illustrated in Figure 1.4 below.

Figure 1.4: Seller Profits in a Partial Switching SPNE Under Voluntary Insurance



Summarizing the SPNE with partial switching, there exists thus a first threshold a^{***} that divides sellers into non-clearing and clearing members. Similarly to the other thresholds, a^{***}increases in v_m, g_m and e_m. However, this holds only until cut-off $a_{PS} \geq a^{***}$, determining the largest clearing members. One of those clearing members will ultimately attract all (larger) switching buyers $t = 1$, leading to the jump in expected profits. And hence, there exists as many partial switching SPNE as buyers of size a_{PS}. Finally, all large sellers exit the market as non-clearing members as it their matched buyers anticipated to switch.

Proposition 9. *Under voluntary insurance and intermediate switching costs $C \in [\underline{C}, \overline{C}]$, CCP entry induces as many partial switching SPNE as sellers of size a_{PS}. They all share a unique size-thresholds $a^{***}(g_m, v_m, e_m)$, where:*

(i) Sellers matched with buyers strictly smaller than a^{***} offer product d as non-clearing members.

(ii) Sellers matched with buyers between a^{***} and a_{PS} become clearing members and offer product d and m.

(iii) Sellers matched with buyers strictly larger than a_{PS} exits the market.

SPNE without a CCP As previously discussed when ruling out the fully switching SPNE, the absence of a CCP implies also the absence of any clearing members. And hence, a no switching SPNE arises with all buyers purchasing product d for all their risky assets from their matched seller.

***Lemma* 2.** *Under voluntary insurance, the absence of a CCP induces a no switching SPNE with sellers solely providing product* d *to their respective matched buyers.*

1.4.3.1. The Effect of a Regime Shift on Payoffs and Risk Exposures

Before comparing the outcomes under mandatory and voluntary insurance, I will briefly summarize them. Under mandatory insurance, the presence of a CCP induces a unique no switching equilibrium: Large sellers become clearing members to supply their matched buyers with the product bundle, while small sellers and their matched buyers exit. The absence of a CCP results in market failure. Under the voluntary insurance, the CCP only operates when at least a portion of buyers do not switch. Then larger buyers hold the bundle, while smaller buyers only hold product d. In the absence of a CCP, buyers never switch and hedge their a_b different assets with a product d provided by their matched seller. To maintain readability and relevance, the below comparisons assume CCP entry under mandatory insurance. In the counterfactual absence of a CCP, mandatory insurance triggers market failure and is thus never beneficial.

The Effect on the CCP Not surprisingly, the CCP is strictly better off under the mandatory insurance regime: Relatively lower-valued outside options and higher clearing member profits lead to more clearing members and increased total product m sales. To illustrate this, recall that a seller becomes a clearing member if:

$$\mathbb{E}_0\Pi_m \geq \mathbb{E}_0\Pi. \tag{1.41}$$

Now, assume that the CCP sets the same fees and collateral under both regimes. Under mandatory insurance, denoted with ^{M}, the seller's only alternative to clearing membership is to exit the market. Under voluntary insurance, denoted with ^{V}, the sellers can sell product d as a stand-alone, and thus realize strictly increasing profits also without offering product m:

$$\mathbb{E}_0\Pi^M < \mathbb{E}_0\Pi^V. \tag{1.42}$$

Simultaneously, clearing members can realize a higher profit under mandatory insurance. For one, buyers cannot longer hold uninsured derivatives, which decreases their bargaining power. Further, larger buyers are captive under mandatory insurance but non-captive under voluntary. Thus:

$$\mathbb{E}_0\Pi_m^M > \mathbb{E}_0\Pi_m^V. \tag{1.43}$$

Inequalities (1.42) and (1.43) jointly ensure that clearing membership is more profitable under mandatory insurance. Therefore, more sellers become clearing members and total product m sales increase. Additionally, the CCP is of course able to adjust fees and collateral, potentially extracting even more surplus.

The Effect on Sellers The effect of mandatory insurance on sellers' profits is not uniform and depends both on matched buyer size and the SPNE under voluntary insurance. For smaller buyers a regime switch implies losing their ability to sell uninsured derivatives and they, instead, exit the marker. Here, inequality (1.42) directly applies showing smaller buyers to be strictly worse off.

Contrary to this, large sellers strictly benefit from a regime shift. Their buyers have the lowest per-asset switching cost. Under voluntary insurance they are thus able to only charge relative low prices to retain customers, who might nevertheless switch away in the partial switching SPNE. Under mandatory insurance all buyers become captive, allowing the large sellers to always retain their matched buyers and to charge significantly higher prices. This results in the following profit comparison:

$$\mathbb{E}_0\Pi_m^M > \begin{cases} \mathbb{E}_0\Pi^V & \text{without CCP} \\ \mathbb{E}_0\Pi_m^V & \text{with CCP and no switching SPNE} \\ 0 & \text{with CCP and partial switching SPNE} \end{cases} \qquad \text{for large sellers} \tag{1.44}$$

The effect of a regime switching on medium sized sellers is, however, ambiguous. Under voluntary insurance, they sell only product d, while mandatory insurances forces them into clearing membership at price e_m not previously paid. However, their buyers now become captive, allowing them to extract more utility via $p_d + p_m$. How much more depends on the variable fee v_m set by the CCP. Thus, depending on model parameters dictating CCP choices, they may overall benefit or suffer:

$$\mathbb{E}_0\Pi_m^M \lesseqgtr \mathbb{E}_0\Pi^V \qquad \text{for medium sellers} \tag{1.45}$$

The Effect on Buyers Recall that under mandatory insurance, buyers are always left with their reservation utility: buyers matched with clearing members are captive and buyers not matched with clearing members exit the market. Under voluntary insurance, only small buyers are captive and left with their reservation utility. The smaller buyers are, thus, just indifferent between the two regimes:

$$u_{dm}^{M} - (p_{d}^{M} + p_{m}^{M}) = u_{r} = u_{d}^{V} + p_{d}^{V}. \tag{1.46}$$

All non-captive (larger) buyers always experience utility gains from their hedging with product d under voluntary insurance. These are thus strictly worse off when product m is mandatory:

$$u_{dm}^{M} - (p_{d}^{M} + p_{m}^{M}) = u_{r} < u_{d}^{V} + p_{d}^{V}. \tag{1.47}$$

***Corollary* 4.** *Regime change from voluntary to mandatory counterparty default insurances:*

(i) Makes the CCP strictly strictly better off.

(ii) Makes smaller sellers strictly, worse off, has ambiguous effects on medium-sized sellers, and makes large sellers strictly better off.

(iii) Has no effect on smaller buyers, but makes larger buyers strictly worse off.

The above *Corollary* concludes the micro-structure analysis of this market. I now turn to discussing how a regime shift between mandatory and voluntary insurance impacts the overall financial risk exposure. I start by reflecting on the trade-off between credit risk-exposure and market-risk exposure common to this market. I also argue how the model highlights a third risk-channel: the credit risk externality. As sellers become safer, their clients in other markets benefits and overall financial stability is improved.

Credit Risk Exposure The policy objective of mandatory counterparty default insurance is the reduction of buyer exposure to seller default (credit) risk. As the theoretical results highlight, this is indeed the case. Mandatory insurance eliminates all uninsured product d sales: Large and and medium sized sellers become clearing member and now offer insured sales to their buyers, smaller buyers and sellers exit the market. This implies an average credit risk exposure CR^{M} of zero under mandatory insurance:

$$CR^{M} = 0. \tag{1.48}$$

Under voluntary insurance, the buyers' average credit risk exposure varies with the different types of equilibria. In the absence of a CCP, a no switching equilibrium with full hedging and, thus, exposure to matched buyer credit risk arises. In the presence of a CCP, the SPNE with no switching and partial switching only smaller buyers up to thresholds a^{**} and a^{***} respectively are uninsured and exposed to credit risk. Denote the density of buyer size with $Pr(a_b)$ and recall that matched seller default probability is denoted with D_s. Then, the credit risk under voluntary insurance CR^V is:

$$CR^V = \begin{cases} \sum_{a_b=\underline{a}}^{\overline{a}} Pr(a_b) D_s a_b \sigma_r & \text{without CCP} \\ \sum_{a_b=\underline{a}}^{a^{**}} Pr(a_b) D_s a_b \sigma_r & \text{with CCP and no switching SPNE} \\ \sum_{a_b=\underline{a}}^{a^{***}} Pr(a_b) D_s a_b \sigma_r & \text{with CCP and partial switching SPNE} \end{cases} . \tag{1.49}$$

In all three cases, we have that mandatory insurance decreases credit risk as:

$$0 = CR^M < CR^V \tag{1.50}$$

Market Risk Exposure This decease in credit risk exposure following a regime switch to mandatory insurances comes at a cost: Buyers smaller than a^* remain unhedged due to exit and are thus fully exposed to their market risk. Depending on the size of the underlying asset variance σ_r^2, this might leave buyers with quite a substantial average market risk exposure MR^M:

$$MR^M = \sum_{a_b=\underline{a}}^{a^*} Pr(a_b) a_b \sigma_r. \tag{1.51}$$

Under voluntary insurance all buyers always at least hedge their asset risk with a product d. Hence, no buyer remains exposed to their market risk and:

$$MR^V = 0. \tag{1.52}$$

Seller Default Risk Both the credit risk decrease and the marker risk increase exclusively consider buyers average risk exposures and reflect back on the main trade-off mentioned in the introduction. However, the change in regimes also impacts sellers' default probabilities. Eliminating uninsured derivatives from seller balance sheets implies a decrease in default risk for both the smaller seller exiting and the medium sized sellers who become clearing members.[46] A substantial reduction in the default risk of small and medium sized sellers would benefit other financial market segments, where the same sellers operate. Thus, mandatory default insurance might result in an important default risk externality contributing to overall financial stability.

Overall Financial Risk All three above described measures of risk depend on: [1] the relative density of small and medium sized buyers, [2] the size of σ_r^2. To provide a quantitative assessment of the aggregate effect for any given derivatives class to be subjected to a regime shift, a model calibration over these parameters is, thus, necessary.

Corollary 5. *A regime switch to mandatory insurance decreases buyers' average credit risk exposure and increases buyers' average market risk exposure. The overall effect depends on the buyer size distribution and must be weighted against the decrease in sellers' default risk.*

1.5. Calibration and Counterfactual Policy Evaluation

This section illustrates how the above described model insights can be utilized for a counterfactual analysis of mandatory and voluntary counterparty default insurance for a specific OTC derivatives market. For this purpose, I parameterize the model environment to the European EuroDollar FX derivatives market. In this, I build on the analysis by Hau et al. (2021), who provide some data moments for parameterization. They also show that during this period, the average OTC FX derivative contract had a duration of 69 days, almost exactly one quarter. Therefore, I assume that the above described model reflects one quarter ahead trade choices. I normalize all variables to be denoted in millions of euros (€mn).

1.5.1. Parameterization

To calibrate the buyer size distribution at the core of this analysis, I relax the assumption that the buyer size distribution $\mathcal{A}\{\underline{a}, \overline{a}\}$ is discrete, and bounded from above and below. Instead, I assume that $\mathcal{A}[\underline{a}, +\infty)$ is continuous and only bounded from below (at just above zero). This allows me to estimate the functional form using simulated method of moments, relying on data moments provided by Hau et al. (2021)

[46] Note that for the latter, assumption $\underline{g_m} > g_m^{**}$ is required and shown to hold true in the next section.

for the period between April 1st, 2016 and March 31st, 2017. All other model parameters are calibrated using data from 2014Q1 to 2016Q1 to reflect that financial market participants traditionally use (public) historic data to inform their decision making. The model parameterization and methods are summarized in Table 1.2 and subsequently described in detail. If not interested in such, the reader may move directly to the counterfactual evaluation in Section 1.5.2..

Table 1.2: Model Parameterization Normalized to €mn

Parameter	Notation	Value	Method	Data Source
Buyer size	$a_b \sim Weibull(\lambda, k)$	$\lambda = 0.686$, $k = 0.689$	SMM	Hau et al. (2021)
Asset Return	$(1+\tilde{r}) \sim N(\mu_r, \sigma_r^2)$	$\mu_r = 1.012$, $\sigma_r = 0.095$	return of US corp. bonds and exchange rate volatility	St. Louis Fed (2021) Bundesbank (2021)
Risk Aversion	γ	$\gamma = 4.37$	-	Eisfeldt et al. (2020)
Seller profits	$L \sim N(\mu_L, \sigma_L)$	$\mu_L = 199.846$, $\sigma_L = 115.169$	avg., std.	S&P Global (2021)
Collateral Cost	δ	$\delta = 0.000636$	avg. EURIBOR	Bundesbank (2021)
Switching Costs	C	$C \in \{\underline{C},\ \overline{C},\ 2\overline{C}\}$	parameter implied	-

Buyers' Asset Size Distribution I estimate the buyer size distribution using a two-step simulated method of moments (SMM) estimation with 1000 Montecarlo draws in each step[47]. Here, I assume that the buyer size is drawn from a Weibull distribution with parameters λ and κ, as it is bounded below at zero and relatively free in shape.[48] As moments, I use the 10th, 25th, 50th and 75th percentile of notional outstanding of Euro/Dollar FX derivatives clients as stated in Hau et al. (2021).[49] I chose percentiles as moments, rather than mean or standard deviation, to ensure a good match at the lower and middle part of the size distribution. This is motivated by the theoretical analysis highlighting that results are driven mainly by changes in the market outcomes for small and medium sized buyers and their matched sellers. The estimation results are summarized in Table 1.3 below. Table 1.4 states the size-grid over which the simulation is ultimately performed, where for computantial reasons, I set the minimum size $\underline{a}$ to 0.001 and maximum size $\overline{a}$ to the 99th percentile. Figure 1.5 plots the resulting Weibull densities.

[47] See for example Evans (2018) for a description

[48] For robustness, I additionally tested the Pareto and exponential distribution, but both performed significantly worse in matching the data moments.

[49] Note that their percentiles are stated annually, wherefore I divide the total notional outstanding by four to proxy quarterly notional outstanding.

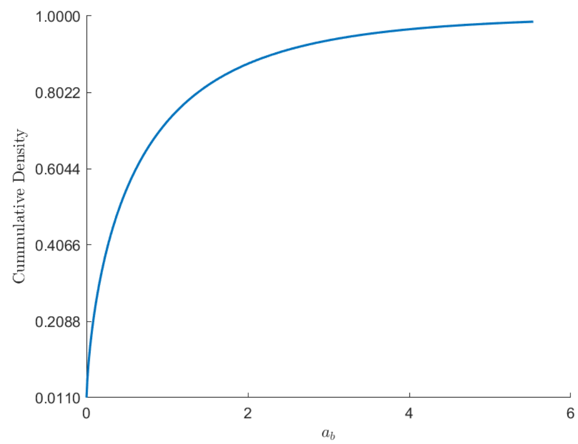
Table 1.3: Notional Outstanding (in €mn)

	p10	p25	p50	p75
Data Moments (Hau et al., 2021)	0.025	0.100	0.450	2.850
Simulated Moments (SMM) $a_b \sim Wbl(\lambda = 0.686, k = 0.689)$	0.020	0.091	0.357	0.989

Table 1.4: Buyer Size Grid for Simulation

	$\underline{a}$	$\overline{a}$	Steps
$a_b \in$	0.001	5.536	500

Figure 1.5: Simulated Buyer Size CDF



Risky Asset Returns For the Euro/Dollar FX derivatives market, the relevant asset volatility (for a given notional outstanding) is determined by the exchange rate. Assume that a buyer has invested 1€ in a U.S. $ denominated corporate bond with return r. Denote the EuroDollar exchange rate today and a quarter ahead with ξ_t and ξ_{t+q} respectively. Then this investment realizes the following risky return $\tilde{r}$:

$$(1+\tilde{r}) = (1+r)\frac{\xi_t}{\xi_{t+q}}. \tag{1.53}$$

To calculate r, I use the daily Moody's Seasoned Triple-A Rated Corporate Bond Yield (DAAA) time series for the period Q12014-Q12016 (St. Louis Fed, 2021). From the data, I first calculate the mean daily return, which I then transform into quarterly returns. For simplicity, I assume away all return volatility in r.[50] Then, the volatility of $\tilde{r}$ is determined solely by the exchange rate volatility and can be fully hedged away. I obtain the realized EuroDollar exchange rates ξ_t and ξ_{t+q} from the Bundesbank's statistical warehouse (Bundesbank, 2021). Then, I calculate $(1+\tilde{r})$ for the period 2014Q1 to including 2016Q1, setting $q = 63$.[51] I lose 14 quarterly-returns to public holidays. The calibrated parameters are summarized in Table 1.5 below.

Seller Profits To obtain the mean and volatility of seller profits, I use financial balance sheet and income statement data from S&P Global Market Intelligence from 2014Q1 to including 2016Q1. I limit

[50] Note here that any volatility in corporate bond returns due to firm default on the coupon can be separately insured using CDS swaps. Here, I focus exclusively on the exchange rate risk and the associated hedging.

[51] There are 63 days in a trading quarter.

the sample to those EU financial institutions that most commonly offer OTC derivatives: commercial banks, investment banks, brokers and capital markets service providers.[52] I exclude all entities with non-operating parent companies, missing net-income, and missing or negative common equity.[53] Further, I trim at the 1st and 99th percentile to exclude outliers due to extreme loss or profit shifting purely driven by accounting practices. I am thus left with 121 individual sellers and 776 observations.[54] Subsequently, I calculate the mean and variance of the net income variable. Finally, to obtain μ_L and σ_L, I add the sample-average of common equity to the mean.[55]. This correction is required because in reality financial institutions do not default when profits are negative, but when equity capital is not sufficient to capture losses.

Figure 1.6: Seller Profits (Data and Fitted)

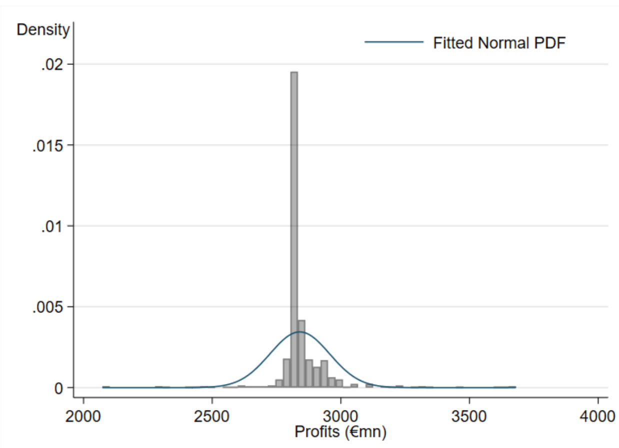


Table 1.5: Calibrated Asset Return

	Obs.	μ_r	σ_r
$1+\tilde{r}$	649	1.012	0.09492

Table 1.6: Calibrated Seller Profits

	Obs.	μ_L	σ_L
L	776	199.846	115.169

Collateral Cost For every insured OTC sale, the seller must post cash collateral. This money could have been invested elsewhere, obtaining market returns. Given the quarterly time-frame of the model, I use the EURIBOR three-month funds rate (daily quotations) as a relevant comparative investment opportunity.[56] Again, I compute the average quarterly return for the periods 2014Q1 to 2016Q1. This results in a

[52]I include the EU's small state affiliates Andorra, Faeroe Islands, Greenland, Gibraltar, Vatican, which are also subject to the EMIR. I, however, exclude Norway, Iceland and Liechtenstein, who only joined the EMIR agreement in July, 2017.

[53]Ideally, one would use common equity tier 1 levels. Unfortunately, this variable is only available for the largest 44 entities and would reduce the sample by an significant amount. Therefore, I rely on the more general common equity measure as the second best.

[54]Note that this is slightly less than the 204 FX derivatives dealers reported in Hau et al. (2021).

[55]Equity capital is by definition not normally distributed. Thus to preserve the normality of the net income variable, I add the mean equity only ex post.

[56]The time-series is obtained via the Bundesbank statistical warehouse and carries the serial number BBK01.ST0316.

δ equal to 0.000636.

Switching Cost The two switching cost thresholds $\underline{C}$ and $\overline{C}$, that determine the existence and uniqueness of different SPNE, are implied by the model parameters. Table 1.7 states their values and an additional level of C used for the analysis. The threshold C_{NS} is additionally determined by the CCP choice of g_m under voluntary insure. However, as I will show below, the CCP will not enter under voluntary insurance. And thus, C_{NS} plays no role in the further analysis and is omitted here.

Table 1.7: Schwitching Costs Thresholds (in €mn)

$\underline{C}$	$\overline{C}$	$2\overline{C}$
0.000002	0.010891	0.021782

1.5.2. Calibrated Equilibrium Outcomes

This subsection describes the simulated SPNE, given the calibrated market parameters. First, I briefly describe the simulation algorithm, after which I derive the SPNE under voluntary and mandatory counterparty default insurance. These SPNE are composed of: a CCP entry decision and the resulting fees; the sellers' membership choice, default probabilities prices and expected profits, and the buyers' expected utilities.

The Solution Algorithm I perform the following computational exercise: First, I take CCP entry as given. Then, I numerically solve for the equilibrium outcomes, including expected CCP profit, for a wide range of possible e_m, v_m and g_m combinations (see Table 1.8). Here, I rely on the functional forms derived in the theoretical analysis. Subsequently, I check whether there exist combinations for which entry leads to positive CCP profits. If not, I conclude that there is no CCP entry and derive the SPNE equilibrium absent of a CCP. If yes, I identify the CCP-profit maximizing combination of v_m, e_m and g_m, and, given these, derive the remaining equilibrium outcomes.

Table 1.8: Grid Space For Optimization

Grid	lowest value	highest value	steps
e_m	0	$\overline{a}\frac{\gamma}{2}\sigma_r^2$	200
v_m	0	$\frac{\gamma}{2}\sigma_r^2$	200
g_m	$\underline{g_m} = \sqrt{5/63} \cdot 2.576\sigma_r$	$10\sigma_r$	200

Table 1.9: Collateral Thresholds

$\underline{g_m}$	g_m^*	g_m^{**}
0.006538	0.000068	0.000107

Here note that I have applied the EMIR regulatory minimum collateral requirement of 99.5% five-day value-at-risk (European Commission, 2012; ESMA, 2021). As shown in Table 1.9 above, $\underline{g_m}$ is thus above both model implied collateral thresholds g_m^* and g_M^{**}. Thus, the initial parameter restriction $\underline{g_m} > g_M^{**}$ is validated and the above theoretical results accurately match the European regulatory framework.

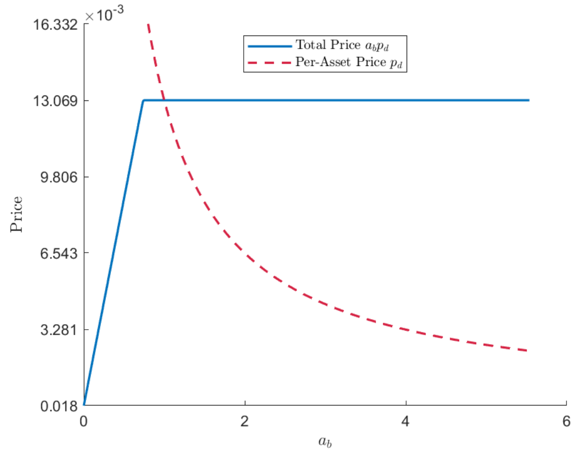
Market Outcome Under Voluntary Insurance In the years analyzed no CCP was willing to provide counterparty default insurance for the European OTC EuroDollar FX derivatives market (ESMA, 2019b). I confirm that my model is able to replicate this by checking that a CCP does not find entry profitable.[57] This lack of profits stems from the very low market risk underlying the exchange rate derivative (σ_r) relative to the variance in seller profits (σ_L). Thus, OTC trades only marginally contribute to the seller credit risk and counterparty default insurance provides little additional value for buyers. Providing only limited utility gains to be captured, but being exposed to high costs upon seller default, the CCP expects negative profits and decides not to enter. Here *Lemma 1* shows that price p_m is independent of the size C, such that this result holds for any level of $C > 0$. For illustrative purposes, I below show the SPNE outcome for $C = 2 \cdot \overline{C}$.

Recall from *Proposition 5* that, in the absence of a CCP and assuming $C > 0$, the SPNE with no switching is unique. And further that the SPNE is characterized by all smaller (captive) buyers paying a price p_d equal to their the utility surplus from buying product d. All larger (non-captive) buyers pay the switching cost plus a premium equal to their utility loss upon switching. Below, Figure 1.7a plots the buyers' total and per-asset price as a function of their size a_b. Figure 1.7b plots the resulting per-asset utility as a function of their size a_b, accounting for the paid price. In both graphs you can see a kink, at the size where buyers stop being captive.

[57]Note here that checking for CCP entry in the case of an anticipated no switching equilibrium is sufficient: Given the parameters, CCP profits are strictly higher in a no switching SPNE then in a partial switching SPNE.

Figure 1.7: Buyer Prices and Utilities (no CCP, $C = 2\overline{C}$)

(a) Total and Per-Unit Prices



(b) Buyers' Per-Asset Utility Post Payment

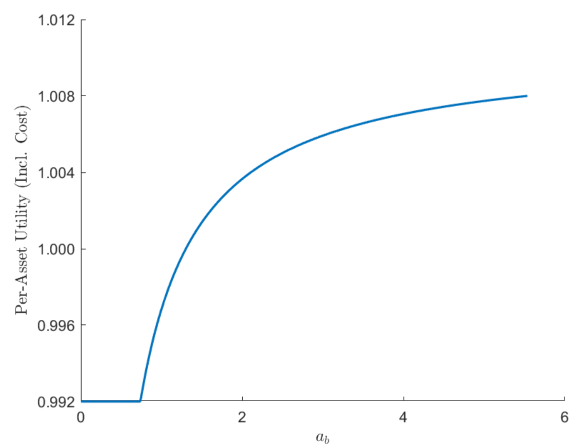
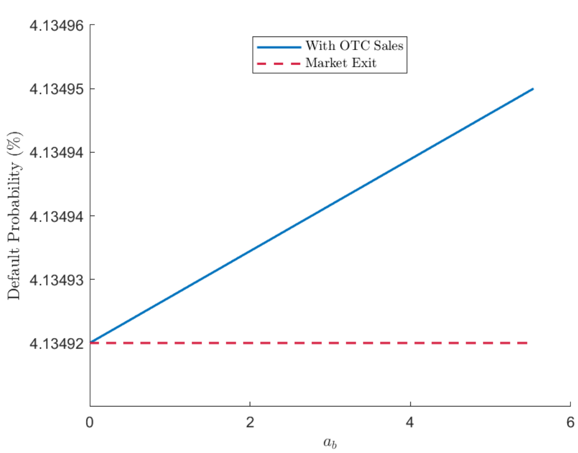


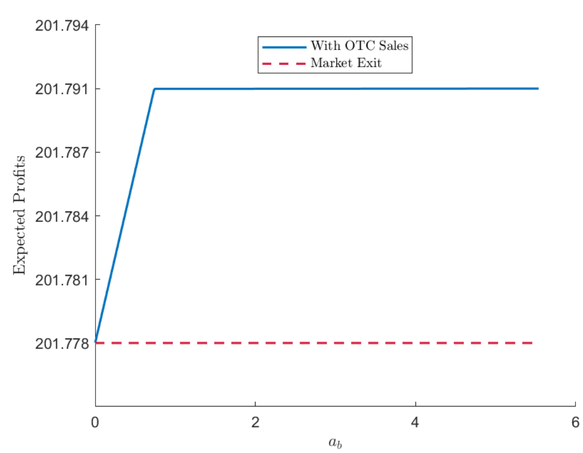
Figure 1.8 (below) plots the SPNE outcomes regarding sellers. Here, Figure 1.8a plots the sellers' default probabilities as a function of matched buyer size. As all hedges are uninsured, not surprisingly, the sellers' default probabilities increase with the matched buyer's size and thus sales. However, due to the small size of the EuroDollar FX OTC market, the effect is minute and around 0.005 basis points. For the same reason, the sellers' total profits also only marginally increase when serving the OTC market (Figure 1.8b). Notice here that, as discussion in Section 1.4.3., the profit function displays the kink, where buyers become non-captive.

Figure 1.8: Seller Default and Profits (no CCP, $C = 2\overline{C}$)

(a) Seller Default



(b) Seller Profits



Market Outcome Under Mandatory Insurance The *Corollary 4* concludes that the CCP strictly benefits from mandatory counterparty default insurance, as it can capture a higher surplus from buyers and sellers. My model simulation predicts that, would mandatory insurance be introduced in this market, this surplus is sufficient to incentive CCP entry (see Figure 1.9).[58] The CCP's profit maximizing choices of e_m, v_m and g_m are summarized in Table 1.10.

Figure 1.9: CCP Profits (Simulated)

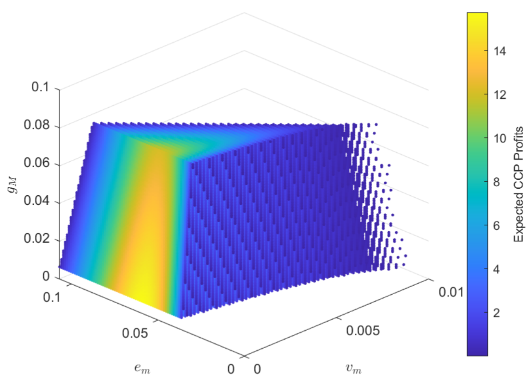


Table 1.10: Equilibrium Product m Prices

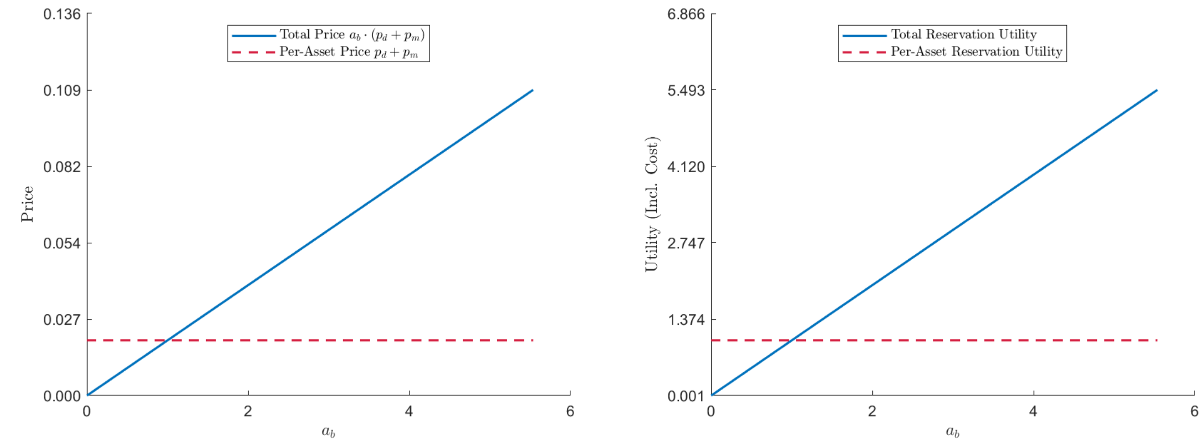
	Value
e_m	0.05944
v_m	0
g_m	0.006538
CCP Profits	15.72
Clearing Membership Rate (%)	7.215

Lemma 3 states that in a market subject to mandatory insurance and an active and an active CCP, buyers either leave the market or are charged their entire utility surplus from product d and m. In either case, they are always left with their reservation utility (see Figure 1.10).

[58]This is not an unlikely scenario. As the policy debate more recently moved to including more markets in mandatory insurance regime, a CCP has indeed already secured the (monopoly) right to serve the EuroDollar FX market. It has, however, not yet offered any actual insurance. To this date insurance remains voluntary ESMA (2019b).

Figure 1.10: Buyer Prices and Utilities under Mandatory Insurance

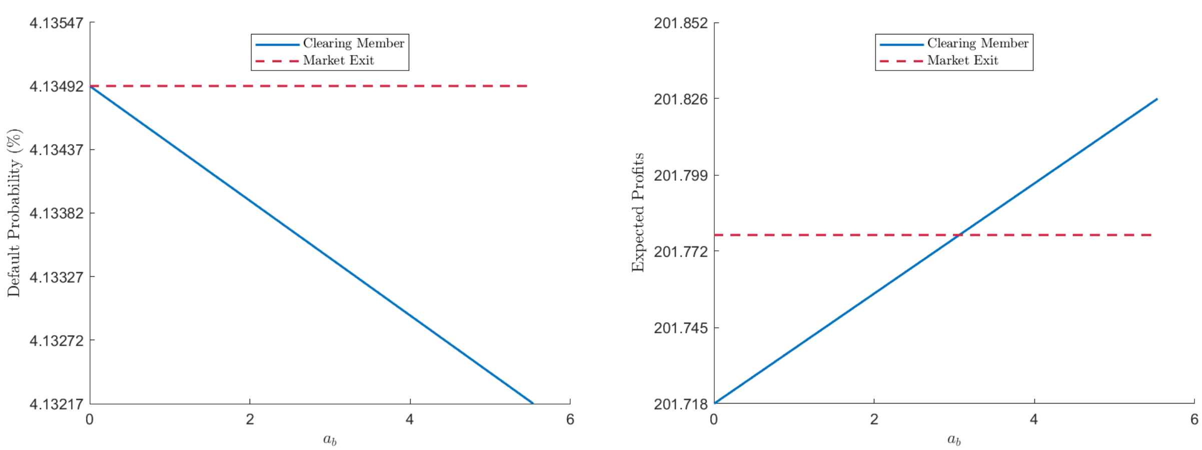
(a) Total and Per-Unit Prices (b) Buyers' Total Utility



Proposition 1 states, that because buyers gain no utility from trade, the no switching equilibrium is unique and all buyers are captive. Nevertheless, given the optimal CCP fees, the ability to extract the matched buyer's entire utility is not sufficient for all sellers to find it optimal to become clearing members. From *Proposition 6*, we know that only larger sellers find it optimal to become clearing members, while smaller sellers exit. Figure 1.11b confirms is partial clearing membership for the here simulated EuroDollar FX derivatives market. Ultimately, the model simulation predicts a clearing membership rate of 7.215% under a mandatory insurance regime (see Table 1.10).

Figure 1.11: Seller Default and Profits Under Mandatory Insurance

(a) Seller Default (b) Seller Profits



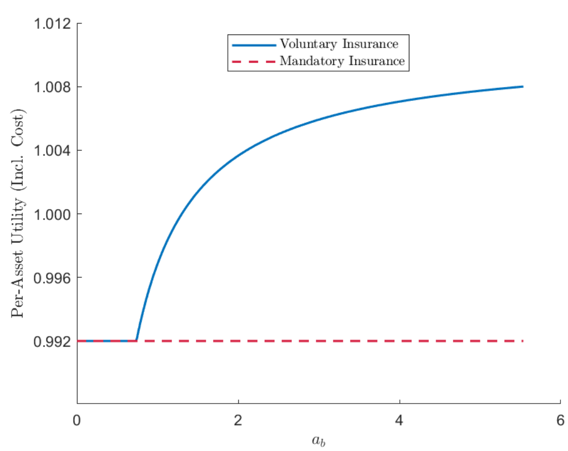
1.5.3. Counterfactual Comparison

Having derived the equilibrium outcomes under both regimes, I now turn to the counterfactual comparison. Mirroring the theoretical analysis, I considers first the effects on agents' and then on the overall financial risk, given the increase in buyers' marker risk-exposure, the decrease in buyers' credit-risk exposure and the credit risk-externality.

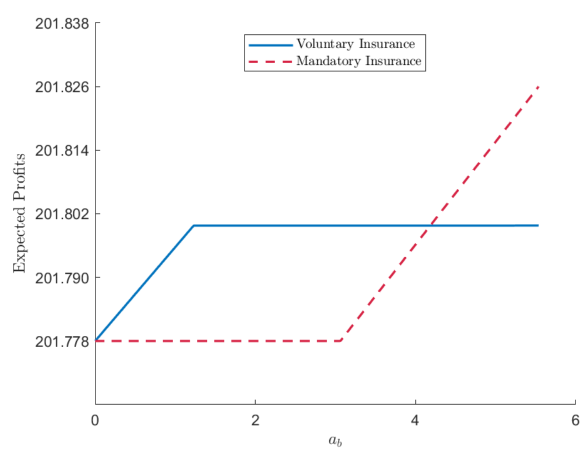
The Impact On Market Participants The impact on the CCP has already been briefly discussed above: under voluntary insurance it would not enter, while expecting a positive profit under mandatory insurance. Figure 1.12 (below) further confirms and quantifies that captive buyers are indifferent while non-captive buyers are strictly worse off under mandatory insurance. Thus the model calibration matches the theoretical result presented in *Corollary 4* with respect to the CCP and buyers.

Figure 1.12: Counterfactual Market Outcomes ($C = 2\overline{C}$)

(a) Buyers' Per-Asset Utility



(b) Sellers' Profits

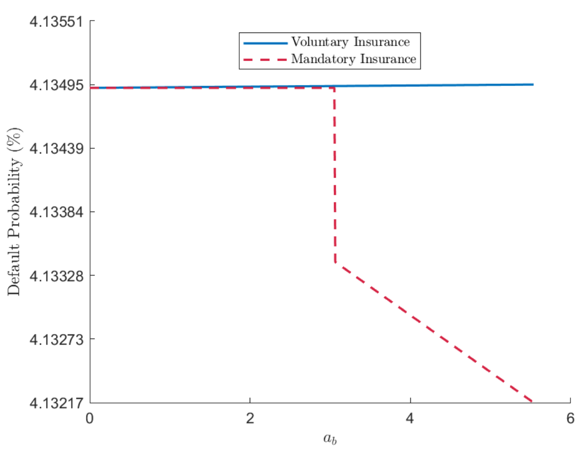


Corollary 4 also highlights that especially for medium sized sellers it depends on model parameters whether the ability to set higher prices under mandatory insurance offsets the cost of becoming clearing members. In this particular market, the underlying currency uncertainty σ_r^2 is relatively small, implying a high reservation utility and only low utility gains from insurance. Unable to set significantly higher prices, medium sized sellers therefore suffer from the introduction of mandatory counterparty default insurance (see Figure 1.12a).[59]

[59]Simulations confirm that, given the parameter space, this result holds for all levels of C.

Credit Risk Externality This negative impact on buyers, and small and medium sized sellers may be an acceptable cost to pay given a significant improvement in seller default risk. However, comparing the impact of the regime switch on seller default probabilities, this effect is negligible. Figure 1.13 plots the simulated default risk under both regimes, depending on the size of the seller's matched buyer. And the improvement in seller default probabilities is always less than 0.1 basis points. Accounting for the relatively higher density of small clients, the difference in average default probabilities under voluntary (D^V) and mandatory (D^M) insurance is even smaller (see equations (1.54) through (1.56)). Mandatory counterparty default insurance for EuroDollar FX derivatives would therefore only marginally impact the stability of the financial market as a whole.

Figure 1.13: Seller Default Risk Comparison



$$D^V = \int_{\underline{a}}^{\overline{a}} D_s^V w(a_b) da_b = 4.02808\ \% \tag{1.54}$$

$$D^M = \int_{\underline{a}}^{\overline{a_b}} D_s^M w(a_b) da_b = 4.02769\ \% \tag{1.55}$$

$$\Delta D = D^M - D^V = -0.00039\ \% \tag{1.56}$$

Buyers' Credit Risk Exposure Additionally, seller default probabilities of ca. 4.03% are insufficient to result in any meaningful credit risk exposure under voluntary insurance, even if all buyers trade insured derivatives (see *Proposition 5*). This comes from a yet their probability of joined seller default and total negative transfers is low. Equation (1.57) describes the average buyer's credit risk exposure under voluntary insurance, denoted with CR^V. Under mandatory insurance, buyers either exit the market or purchase the bundle of both the derivative and the insurance (see *Proposition 1*). Hence, no buyer is exposed to seller credit risk and the average credit risk CR^M is equal to zero. Given this, the decrease in exposure due to a potential regime shift from voluntary to mandatory insurance, denoted ΔCR, is also below.

$$CR^V = \int_{\underline{a}}^{\overline{a}} D_s^V a_b \sigma_r w(a_b) \mathrm{d}a_b = 0.00324 \tag{1.57}$$

$$CR^M = 0 \tag{1.58}$$

$$\Delta CR = CR^M - CR^V = -0.00324 \tag{1.59}$$

Buyers' Market Risk Exposure The simulation confirms *Lemma 1*: Under voluntary insurance all buyers hedge their asset with a product d. Thus, their exposure to market risk MR^V is zero (see Equation (1.60)). Under mandatory insurance, however, all buyers' matched with a non-clearing members exit the OTC market and remain fully exposed to market risk (see *Proposition 6*). Recall further that the size-threshold for clearing membership was denoted with a^*. Then equation (1.61) below states the average buyer's market risk exposure under mandatory counterparty default insurance MR^M. Having no exposure to market risk under voluntary, and some exposure under mandatory insurance, implies that the difference in marker risk exposure ΔMR is increasing with a potential regime shift.

$$MR^V = 0 \tag{1.60}$$

$$MR^M = \int_{\underline{a}}^{a^*} a_b \sigma_r w(a_b) \mathrm{d}a_b = 0.05570 \qquad \text{where} \quad a^* = 3.062 \tag{1.61}$$

$$\Delta MR = MR^M - MR^V = 0.05570 \tag{1.62}$$

Average Value-at-Risk The above described market risk and credit measures are the two components of the average buyer's 95th percentile value-at-risk, denoted 95% VAR (see equation 1.63). Relying on the values of MR and CR under both regimes, allows me to derive the relative change in the average buyer's 95% VAR following a shift to mandatory insurance. Denoted with $\%\Delta VAR$, this change in the average buyer risk-exposure is substantial with $1744.31\ \%$. As equation highlights (1.64), this is independent of the actually percentile VAR considered, as the multiplying factor enters both nominator and denominator and thus cancels out.

$$95\%\ VAR = 1.96 \cdot [MR + CR] \tag{1.63}$$

$$\%\Delta VAR = 100\frac{\Delta MR + \Delta CR}{MR^V + CR^V} = 1744.31\ \% \tag{1.64}$$

Summarizing Table 1.11 (below) summarizes the overall impact that the introduction of mandatory counterparty default insurance would have on the three risk measures. Accounting for the relative increase in market risk and the decrease in credit risk, the average buyers total risk exposure increases by $1744.31\ \%$. Simultaneously, the improvement in credit risk, benefiting other financial market segments, is negligible. Therefore, introducing mandatory counterparty insurance for EuroDollar FX OTC derivatives would go against the regulatory objective to decrease financial risk and thereby enhancing financial stability. And thus European supervisors have rightly so refrained from introducing it in this market.

Table 1.11: The Effect of Mandatory Counterparty Default Insurance

Credit Risk Exposure	Market Risk Exposure	Change in VAR (%)[60]	Credit Risk Externality
$\Delta CR = -0.00324$	$\Delta MR = 0.05570$	$\%\Delta VAR = 1744.31\ \%$	$\Delta D = -0.00039\ \%$

1.6. Conclusion

In this paper, I set out to understand the effect of a policy shift from voluntary to mandatory insurance of OTC derivatives on both the market equilibrium and the associated financial risk. For this purpose, I first carefully model the competitive environment in the markets of OTC derivatives and their insurance. Here, I analyze the SPNE under both regimes and derive which buyer purchases which products at which price from which seller. I pay special attention to how a for-profit CCP may influence not only the purchase of counterparty default insurance, but also to which extent buyers purchase the derivative at all. Subsequently, I compare the SPNE outcomes under both regimes and derive buyers' average exposure to

[60]This is calculated by $100 \cdot (\Delta MR + \Delta CR)/(MR^V + CR^V)$.

both market risk and credit risk.

This ultimately allows me to evaluate the theoretically predicted effects of a regime switch against the policy objective of overall financial risk reduction under mandatory counterparty default insurance. Here, I highlight in particular that the effectiveness of mandatory insurance in reducing risk exposure is determined by the trade-off between smaller buyers exiting and medium sized buyers additionally purchasing the insurance product. The overall effect depends on the relative density of smaller and medium sized buyers. I further uncover an additionally risk component to be consider: the potential spillover effect into other markets via overall reduced seller default risk. To highlight how these insights can be used for a concrete OTC derivatives class, I quantify these trade-offs through a calibration and simulation of the EuroDollar FX derivatives market.

The core limitation of the above analysis is the assumption that the market is served by a (monopolistic) for-profit CCP. And indeed, the largest derivatives markets are served by mostly monopolistic for-profit CCPs. However, for some smaller derivatives classes, the market sellers instead jointly found a mutualized CCP.[61] Participating in a mutually owned CCP exposes them directly to the default risk of other sellers, and thus the CCP might be able to internalize the risk mitigation objective of the regulation. However, new membership in such mutualized CCPs requires the approval of existing clearing members. Thus, original members may use the introduction of mandatory insurance to drive out market competitors. Ultimately, this might also lead to high market exit rates and again, significant increases in market risk exposure rates for buyers. A natural next steps would thus be to look into how the markets react (differently) to mandatory counterparty insurance, when served by a mutualized CCP. This might also provide insights why some markets are insured by for-profit CCPs and others by mutualized CCPs.

[61] See Appendix C in Huang (2019) for a list of for-profit and mutualized CCPs and ESMA (2019b) for an up-to-date overview over CCP licenses in the EU.

Chapter 2

Exclusive Security Lending Agreements

Abstract To sell stocks short, arbitrageurs borrow stocks from lenders through broker-dealers. We study why some lenders commit to a single broker-dealer via exclusive security lending agreements (ESLAs) at the cost of foregoing profitable trades, and how this impacts aggregate lending. First, we provide a detailed market overview both on a transaction and a counterparty-pair level. Gained insights inform a three-period representative lender model that rationalizes why ESLAs arise in equilibrium. After carefully evaluating the model fit, we predict the counterfactual trading volumes in the absence of existing ESLAs. We find that trading volumes would significantly increase, but at most by 8%.

2.1. Introduction

Sufficient security lending supply is crucial for short sales to ensure that the no arbitrage condition holds in our financial markets. Consequently, market disruptions due the lack of such supply, typically referred to as short-sale constraints, have been widely studied. Yet, their micro-foundation has received relatively little attention in the literature. In this paper, we study the common usage of exclusive security lending agreements (ESLAs) as one such driver of short-sale constraints.

The vast majority of ESLAs are entered between security portfolio holders (lenders) and broker-dealers (Kessler et al., 2022). With an ESLA, a lender grants a single broker-dealer the exclusive right to borrow from her portfolio against pre-determined terms. In return, the lender refrains from engaging with competing broker-dealers, even if this would result in favorable terms ex post or the security is not demanded by the ESLA holder. ESLAs are, thus, anti-competitive in nature and prohibit profitable transactions. In this paper, we assess the aggregate market inefficiency due to ESLA induced lending supply shortages.

For this, we first utilize the newly available confidential Security Transactions Financing Regulation (SFTR) data to provide a detailed overview of the lending contracts under and outside ESLAs with at least one European counterparty. Surprisingly, we find that under ESLAs lenders on average lend less frequently and at lower lending fees. Subsequently, we rationalize why ESLAs may nevertheless arise in equilibrium in a 3-period representative agent framework. We show that a lender grants an ESLA only when interacting with at most two broker-dealers. Matching reality, equilibrium lending under an ESLA occurs at a lower per-transaction fee and is less frequent. Instead, lenders are compensated via quantity-independent lump-sum transfers typical to over-the-counter (OTC) markets. Finally, we assess the aggregate frequency reduction in stock lending across all active ESLAs by calibrating our model and performing a counterfactual analysis. Assuming away all existing ESLAs, we predict a significant increase in total trading that is likely between 0-5.3% and at most 8%.

The counterfactual analysis is possible due to the granularity of the SFTR data, containing daily stock and flow updates from execution until maturity for every single equity lending transaction with at least one EU counterparty. Besides detailed information on standard contracting fields, such as lending fee, underlying stock, quantity and more, each observation contains an indicator whether the transaction was covered by an ESLA. This allows us to provide detailed transaction level insights on the difference in contract terms between those covered by ESLAs and those not. Further including counterparties' LEI codes, we are able to track individual market participants over the whole year of 2021. Aggregating their trades, we identify five types of market participants: lenders, broker-dealers, borrowers, traders and private clients. We find that over half of both the borrowers and lenders interact with a single broker-dealer. However, only a bit more than 4.5% of borrowers grant ESLAs while 35% of the lenders prefer such.[1]

Building on these empirical insights, we propose a three-period model populated by a representative lender and $N \geq 2$ competing broker-dealers. At $t = 0$, the lender is endowed with a large portfolio of different equity securities. Further, the broker-dealers are each endowed with uncertain ask-prices (borrowing demand) that are independently drawn from identical uniform distribution both on a security and

[1] A detailed analysis on the transactions by the other types of agents than lenders, borrowers and broker-dealers can be found in an accompanying market analysis by Kessler et al. (2022).

broker-dealer level.[2] Thus, broker-dealers are symmetrical *ex ante*. At $t = 1$, before the borrowing demand realizes, the broker-dealers choose whether to compete in offering an ESLA agreement that specifies a uniform security-level bid-price and a lump-sum transfer. At $t = 2$, the borrowing demand realizes: For each security, each broker-dealer draws an independent ask-price. If an ESLA was entered at $t = 1$, the ESLA-holding broker-dealer borrows every securities with a realized positive bid-ask-spread given the agreed-upon bid-price. The lump-sum transfer is paid regardless of actual borrowing demand.[3] If no ESLA was entered, either due to lack of offers or by rejection of the lender, the broker-dealers compete on a security-to-security level for borrowing via second-price auctions. Here, we assume that broker-dealers are unaware of each others realized ask-prices and, thus, posses private information when submitting their auction bids.[4]

To derive equilibrium prices, transaction quantities, and profits, we apply the notion of termination-proof sub-game perfect Nash equilibrium (SPNE).[5] First, we derive lender and broker-dealer profits assuming the second-price auctions are held for each security at $t = 2$. For the both the lender and broker-dealers these profits serve as ESLA participation constraints. The lender may find it optimal to reject the ESLA outright at $t = 1$ if auctions are more profitable. Instead, the ESLA-holding broker-dealer may choose to terminate the ESLA to trigger the auctions after observing ask-prices at $t = 2$. We assume that ESLAs are binding contracts, such that the termination mandates the broker-dealer to compensate the lender for losses and vice versa. With this mind, we derive the competitive uniform bid-price and lump-sum transfers, and the associated payoffs. Ultimately, this allows us to derive the broker-dealers' optimal ESLA offer and termination strategy.

For lenders with two or more broker-dealer connections, we find two types of SPNEs: an auctions SPNE and a competitive ESLA SPNE. A monopolistic ESLA offer never occurs in equilibrium. The auction SPNE always exists and, further, is unique for three or more broker-dealers: For $N \geq 4$ the lender always rejects ESLAs as she can benefit from the increased competition. For $N \leq 3$, the lender would always

[2]Having two or more broker-dealers with separate borrowing demand ensures that we simultaneously capture the core-periphery structure documented in the Data Section 2.3. and have excess borrowing demand (short sale constraints).

[3]This structure of the ESLA contract is inspired by the demand-boost theory of exclusive dealing by Calzolari et al. (2020).

[4]This is in alignment with both Duffie et al. (2014) and Babus and Kondor (2018), studying the implications of private information for competition in OTC market settings.

[5]The more common concept of renegotiation-proof equilibrium, see e.g. Segal and Whinston (2000), is not applicable in this setting as the lump-sum transfer ensures that one agent's gain results in an equivalent loss for the counterparty.

grant an ESLA as broker-dealers competing over the whole portfolio via ESLAs is more profitable than security-by-security competition by those broker-dealers with positive ask-prices. However, we find that for $N = 3$, the ESLA-holding broker-dealer is better of terminating due to quite low compensation payments. Only for $N = 2$, there exists an termination proof competitive ESLA SPNE. Here, the lender always grants an ESLA if offered one. Further, the compensation for termination is just equal to the broker-dealers profits gains from terminating. Therefore, the broker-dealer does not terminate. Finally, for $N = 1$, the lender is indifferent between granting ESLAs or not as the broker-dealer is always a monopolist extracting all gains of trade. In short, only lenders with one or two broker-dealers grant ESLAs.

After confirming the model fit, comparing predicted and observed lending fees, we study how granting ESLAs impact individual level transaction frequencies. We show theoretically that every ESLA-granting lender matched with two broker-dealers experiences a 50% lower trading volume in the competitive ESLA SPNE relative to the auction SPNE: Instead of lending when at least one of two broker-dealers draws a positive ask price, she lends when the ESLA holder alone draws a positive ask-price. For lenders with only one connected broker-dealer, there is no difference in trading quantity between granting an ESLA or not. Applying these counterfactual trading volumes to the lenders with ESLAs observed the data is challenging, as we do not observe the true number of broker-dealer connections (one versus two).

We overcome this by applying a bootstrapping strategy, where we fix the share of ESLA-granting lender with one versus two broker-dealers to be between $0 - 100\%$. For each share, we randomly assign each ESLA-granting lender either one or two broker-dealers in the counterfactual auction SPNE. Repeating this for 100,000 bootstraps, we obtain both a predicted increase in total trading volumes and the associated confidence intervals. For all shares, we predict a significantly larger trading volume for the counterfactual case of outlawing ESLAs. These trading volume gains increase linearly in the share of lenders with two counterparties and range between 0% and 5.3%. We conclude with discussing the caveat that the model assumes away the lenders' ability to purposely limit themselves to two instead of three broker-dealers to force the competitive ESLA equilibrium to arise. We, therefore, conclude with an upper estimate of increase trading volume in the counterfactual case of no ESLAs, assuming all ESLA-granting lenders would have three broker-dealers to compete for lending. We find that even in such case, the gain total market transaction is at most 8%.

Literature Review Our paper is foremost an addition to the existing literature on the pricing of security lending transactions with earliest works by D'Avolio (2002) and Duffie et al. (2002). The seminal paper by Duffie et al. (2002) studies the lending price formation in a search-and-bargaining model, where pessimists are over time matched with both lenders and optimists, thereby being able to short sale. For tractability purposes, they abstract from the role of broker-dealers intermediating between lenders and borrowers, and profiting from the bid-ask-spread. In our paper, we take a complementary approach and focus specifically on the interaction between lenders and broker-dealers. To maintain tractability in our framework, we in return simplify the interaction between broker-dealers and the ultimate borrowers (pessimists).

This brings us closer the recent paper by Colliard et al. (2021), who model the interaction between stock owners and dealers in a more general network model of an OTC market. They focus primarily on how dealers minimize portfolio inventory cost by trading the borrowed/bought stock with each other. Similar to related papers, such as Gofman (2014), they take the network structure as endogenously given. In our paper, we instead zoom in on a single representative lender with several broker-dealers where number of connections is still exogenously given. The lender, however, endogenously chooses the type of connection, being either ESLA or not, in equilibrium.

Allowing for the choice between bilateral trading (via ESLAs) and trading via centralized platforms (second-price auctions) resembles the set-ups described by Babus and Kondor (2018) and Dugast et al. (2019). We confirm their findings that less connected lenders prefer the bilateral ESLA option, while more connected lenders prefer the centralized auctions. By rationalizing why some lenders voluntarily engage with a single broker-dealer via ESLAs, we are able to provide a complementary channel to endogenously explain the core-periphery structure of dealer markets to those described by Neklyudov (2019) and Babus and Parlatore (2022).

Both our model choices regarding the centralized trading platform to host a second-price auction and competition in prices over ESLAs are motivated by the idea that broker-dealers have private information regarding their borrowing demand (Duffie et al., 2014; Babus and Kondor, 2018). Similar to Duffie et al. (2014) we introduce auction pricing after (private) demand uncertainty has realized. Unlike them, as they focus on sophisticated traders only, we do not deem it reasonable to assume that lenders acquire knowledge over time. We, thus, opt to introduce second-price auctions instead of double-auctions to clear the market. In our model, we have additional ex ante uncertainty over broker-dealers private demand. Similar in set-up to Babus and Kondor (2018), we follow their lead and allow for *ex ante* competition in prices over trading. We, however, abstract from quantity limitations on an individual security level and, rather, introduce price-competition over the entire portfolio in return for exclusive access.

Modeling the ESLA contracts in this fashion brings us close to the literature on exclusive contracting between retailers and manufacturers (Bernheim and Whinston, 1998; Calzolari and Denicolò, 2013, 2015; Mathewson and Winter, 1987).[6] Here, we deviate from the substitute goods assumption common to this literature. Instead, we consider exclusive contracting over an entire equity portfolio, where each stock poses a distinct good. Following Calzolari et al. (2020), we allow the broker-dealers to compete by setting both a per-transaction fee and a lump-sum transfers. We are able confirm their result that in equilibrium exclusive contracts (ESLAs) have a zero lending fee to boost demand and full profit-pass-through via lump-sum transfers. In both our and their paper, this demand-boosting leaves the buyer (broker-dealer) with exclusive rights worse of than in the competitive equilibrium.

2.2. Data

The paper is motivated by stylized facts uncovered in an exploration of the recently available data collection of all EU over-the-counter security financing transactions in the Securities Financing Transaction Regulation (SFTR). The data set contains ca. 100 contract fields and 800 enrichment fields for every single repo, margin lending, security buy-and-sell back, and securities or commodities lending transaction since mid 2020 with at least one EU counterparty. It thus, provides, a rich source of information for both policy makers and researchers alike.

Data Description In this paper, we solely focus on equity lending transactions, which are reported as part of the security and commodities lending subset of the SFTR data. A detailed description of the data set, the cleaning procedures and output generation process can be found in a complementary market analysis (Kessler et al., 2022). Simplified, the raw data contains both a daily stock and flow report from execution until maturity submitted by every counterparty registered in the EU for each of their individual transactions.

[6] See Armstrong and Wright (2007) for exclusive contracting by two-sided platforms, where the platform does not interpose as an intermediary and therefore does not become the direct counterparty to both the supply and demand side.

First, we combine all daily reports in a single observation. Here, we collect contract variables of particular interest: loan volume, quantity, prices and fees, collateral, realized maturity, and both counterparty LEIs (lending and borrowing side). Next, we limit our sample to transactions both entered and matured in 2021 to have consistent reporting standards throughout the sample. Subsequently, we drop all intra-group transactions, where the equity holder and the security receiver have the same parent LEI code. On the remaining transactions, we perform a series of quality checks. Here, we first exclude all transactions with obvious miss-reporting in the variables of interest to us: lending fee, loan value, quantity stock price and collateral type.[7] We then check for double reporting in cases, where both counterparties are registered in the EU. Here, we count the number of missing fields of interest that were mandatory to report and keep the better quality leg. Ultimately, we are left with 15,816,332 observations. Out of these, 13.89% are subject to ESLA agreements (see Table 2.1 below).

Table 2.1: Transaction Statistics by ESLA Status

	% of transactions	Avg. Loan Value (eur)	Avg. Stock Price (eur)	Avg. Quantity
With ESLA	13.89	127837.71	42.43	9080.11
Without ESLA	86.11	291252.68	46.77	19742.78

Counterparty Level Overview To gain a better understanding of who lends to whom and under which conditions, we assign each counterparty one of five labels: borrower, broker-dealer, lender, private client and trader. Types are assigned based on their aggregate trading patterns across all observations in our sample. All types, except private clients, are corporate entities that are identified by their LEI code. For borrowers and lenders, we observe that 99% of all their transactions are borrowing and lending, respectively. Broker-dealers and traders engage in both lending and borrowing (see Table 2.2 below). Here, we find that traders are typically smaller agents with less than 100 counterparties that tend to lend more than they borrow. Broker-dealers on the other hand have larger trading volumes, more than a 100 counterparties and tend to borrow more than they lend.

For private clients, typically natural persons not subject to reporting requirements, we only observe a client ID assigned by the reporting counterparty. As assigned client IDs vary across reporting agents, we have no further information to the identify the private individuals behind the transactions. Therefore, we

[7]We do not check for collateral value as it is not always mandatory to report.

Table 2.2: Lending and Borrowing Volumes

Type	Total Lending value (bil eur)	Total Borrowing Value (bil eur)	Ratio Lending-over-Borrowing
Broker-Dealer	1990.40	2867.86	0.69
Trader	590.09	383.49	1.54

can not observe a private investors' total trading volume across all counterparties. Here, our analysis is limited to the specific reporting agent and private client combination. However, the transaction volume involved is small relative to the total market size. For all other agents, we can report aggregated transaction statistics across all their counterparties (see Table 2.3 below).

Table 2.3: Trading Party Characteristics

Type	Nr. of Parties	Avg. Total Loan Value (mil eur)	Avg. Transaction Value (mil eur)	Avg. Nr. Transactions	Avg. Nr. of Counterparties
Borrower	469	2106.29	2.62	3305.13	1.54
Broker-Dealer	39	124570.89	1.23	423022.13	5859.36
Lender	6512	239.83	8.86	293.11	3.27
Private client	208214	0.54	0.01	51.73	1
Trader	442	2202.67	1.55	1893.40	6.07

Besides the obvious (and expected) differences in trading volume, Table 2.3 highlights a stark difference in the average number of trading parties given the type. Broker-dealers have on average more than 5500 different counterparties. Both lenders and borrowers are less connected, with on average 3.37 and 1.54 counterparties, respectively. Such core-periphery structure becomes quite apparent when studying the network visually in Figure 2.1. There, we have aggregated all borrowers, lenders and private clients with the same single counterparty as one trading party, and scaled the size of counterparties non-linearly to maintain confidentiality.

A noteworthy feature of the network, and a potential reason for fewer trading parties, is the presence of ESLAs. Indicated by the red connections, these can predominantly be found between lenders and broker-dealers. Upon further analysis, we find that 35% of all lenders grant an ESLA to a specific broker-dealer. 17% of all private clients agree to ESLAs, while only 4.5% of all borrowers do. Yet, the vast majority borrowers limit themselves to a single broker-dealer. Figure 2.2 illustrates these findings.

Transactions Between Lenders and Broker-Dealers A natural follow up question is to which extent transaction level characteristics differ when covered by an ESLA versus when not. As ESLAs are predominantly effectuated between lenders and broker-dealers, we focus exclusively on transaction be-

Figure 2.1: Network Plot of the EU Equity Lending Market

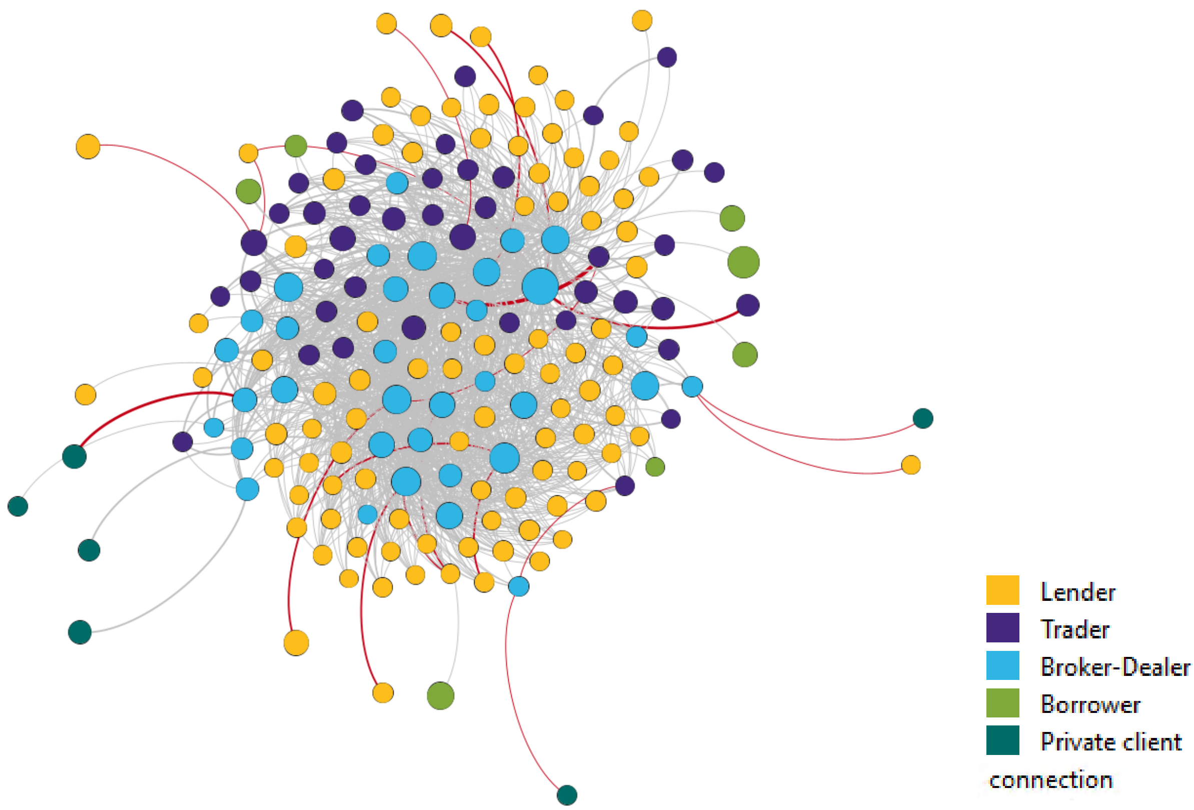
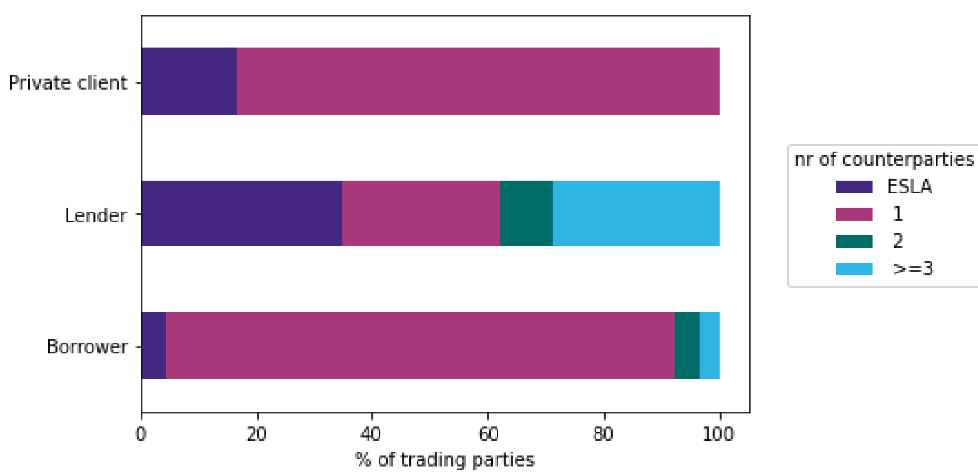


Figure 2.2: Relative Share of Trading Parties per Type



tween these two types from here on. Statistics on all other transactions can be found in the market overview by Kessler et al. (2022). An important contract characteristic, determining among others the fee structure, is the underlying type of collateral. Here, we observe three types of collateralization: none, basket, and cash.

Table 2.4: Collateral Usage

Collateral Type	Nr. Transactions	Total Loan (bil eur)	Avg. Loan Value (eur)	Avg. Quantity
Basket	1501566	1327.65	884176.0	52831.0
Cash	154771	36.27	234368.0	20006.0
None	195337	115.53	591424.0	27607.0

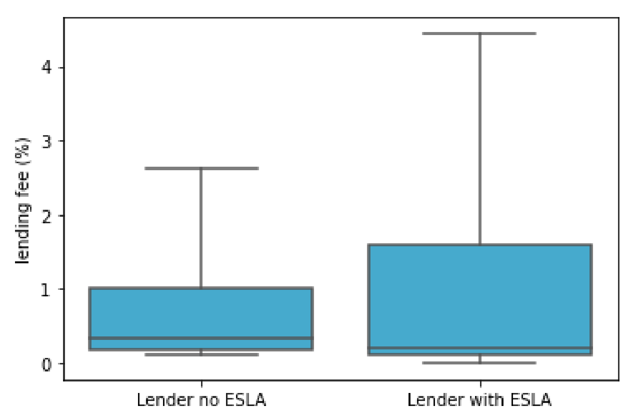
In the case of cash collateral, a net-rebate rate is reported and defines the difference between rebate rate paid by the lender for collateral re-use minus the lending fee paid by the borrower. Table 2.4 shows that only 10.5% of all lending transactions are cash-collateralized, making up 7.8% of the total transaction volume. Given the low market share, we abstract from further analysis of both the cash collateral and net-rebate rates in this paper.

Instead, we focus on the remaining transactions secured with either a collateral basket (89%) or no collateral (2.5%). In both cases, the borrowers pays the lender a fee, but no rebate rate is charged. Table 2.5 and Figure 2.3 below, illustrate, respectively, the mean and median of the lending fees. Here, we distinguish between transactions covered by ESLAs and those that are not. Note here, that the whiskers of the two boxplots in Figure 2.3 indicate the 10th and 90th percentiles to ensure confidentiality.

Table 2.5: Average Lending Fee by ESLA

	Avg. Lending Fee (%)
Lenders without ESLA	1.02
Lenders with ESLA	1.54

Figure 2.3: Lending Fee Distribution by ESLA



We can see that ESLAs on average yield higher lending fees. However, this is mainly driven by a longer upper tail of the distribution. As indicated by the horizontal line, ESLAs have an median lending fee just above zero that is below the median fee of transactions not covered by ESLAs. Naturally, this leads to the main research question of this paper: Why are lenders agreeing to ESLAs if they receive zero fees for the majority of transactions?

Empirical Take-Aways Before moving to the theoretical analysis, we briefly summarize the main insights that motivate our subsequent modeling choices. For one, ESLAs are predominately entered between lenders and broker-dealers. Further, even lenders without ESLAs trade only with a selected hand-full of broker-dealers and rarely more than three. In both cases, each individual lender makes up a small portion of any given broker-dealer's portfolio. Therefore, the model focuses on the interaction between a representative lender and a small number of broker-dealers, when studying the establishment of ESLAs.

Despite their low usage of ESLAs, 88% of all borrowers trade with only a single broker-dealer. We, therefore, assume that every broker-dealer enjoys an independent borrowing demand that is un-observable by others. Because traders predominantly lend and rarely connect with lenders, we abstract from their presence. Private clients are excluded for similar reasons. Finally, we refrain from utilizing fees as model input but rather verify our model by comparing the theoretically predicted and observed fee distributions.

2.3. Baseline Model: N Symmetric Broker-Dealers

2.3.1. Model Environment

We will start with describing an intuitive baseline environment to highlight the main competitive mechanism. We will add complexity in the subsequent sections for a richer set of insights. For now, there exists a risk-neutral lender that is endowed with an equity portfolio. Further, there exist N for-profit broker-dealers, each endowed with an independent and uncertain demand for equity borrowing. In a first period, and before lending demand realizes, the broker-dealers compete to enter an ESLA with the portfolio holder that specifies a uniform bid-price paid per security and a lump-sum transfer. In a second period, the broker-dealers each draw independent ask-prices for each security from identical uniform distributions. If an ESLA is granted, the chosen broker-dealer borrows all equities with a positive bid-ask-spread from the lender's portfolio. If no ESLA is entered, broker-dealers bid via second-price auctions on a security level to borrow those equities with positive ask-prices.

Table 2.6: Model Timing

	Broker-Dealers	Lender
$t = 0$	• Endowed with uncertain equity borrowing demand.	• Endowed with equity portfolio.
$t = 1$	• Anticipate their borrwing demand. • Compete for ESLA: – Uniform security bid-price. – Lump-sum transfer.	• Anticipates lending profits under and outside ESLA. • Decides whether to enter ESLA.
$t = 2$	• Demand uncertainty realizes. • Active ESLA: – ESLA holder borrows securities with positive bid-ask-spread and pays lump-sum transfer. – Other broker-dealers remain inactive. • No ESLA: – Broker-dealers bid security-by-security in a second price auction. – Highest bidder gets to borrow.	• Active ESLA: – Receives lump-sum transfers. – Receives uniform bid-price for all lend out securities. • No ESLA: – Offers each security via a second price auction. – Highest bidder gets to borrow.

Lender The risk-neutral lender (she) holds an portfolio of S equity securities of unit size, each indexed with s. We assume S to be large, reflecting the size of typical market participants, such as pension funds, hedge funds, large firms and small banks. At time $t = 0$, each security independently draws a market value v_s from a continuous distribution V. The lender holds the portfolio for the long run but is willing to lend it out to broker-dealers for positive bid-prices $b_s \geq 0$.

Broker-Dealers There exist $N \geq 2$ profit-maximizing broker-dealers, which we label with superscripts $n \in \{1, 2, \ldots, n, \ldots, N\}$. The broker-dealers intermediate lending between the portfolio holder and potential borrowers. Here, we abstract from a detailed analysis on the borrower side, and simply assume that for each security s each broker-dealer n draws an independent ask-price a_s^n from an identical uniform distribution at $t = 2$: $a_s^n \sim U(-a, a)$. To realize such ask-prices, they must borrow the security from the portfolio holder at competitive bid-price b_s^n (more below). For now, notice that broker-dealers are willing to trade any security with a positive bid-ask spread:

$$\pi_s^b = a_s^n - b_s^n > 0. \tag{2.1}$$

Exclusive Security Lending Agreements Anticipating their ask-prices, broker-dealers may offer competitive ESLAs to the lender at $t = 1$. Each offered ESLA specifies a uniform bid-price b_E^n to be paid for each borrowed security separately and a single lump-sum transfer T_E^n paid for the right to exclusively borrow. Being ex ante identical, broker-dealers simply compete in prices. Throughout the paper, we refer to the broker-dealer that has offered and was granted an ESLA as the (single) ESLA-holder. Then, the ESLA-holder borrows every security at $t = 2$, where:

$$\pi_s^n = a_s^n - b_E^n > 0. \tag{2.2}$$

Simultaneously, the lump-sum transfer T_E^n is paid regardless of the borrowed quantities. We assume that ESLAs are binding contracts, such that either non-payment of T_E^n or one-sided termination entitles the lender to compensation equal to the lost profits. Due to their legal complexity, ESLAs can neither be offered nor entered at $t = 2$.

Competitive Lending If no ESLA agreement is active at $t = 2$, all broker-dealers compete on a security-to-security level for borrowing. They observe their realized ask-price a_s^n, and subsequently set the bid-price b_s^n via a second-price auction (Vickrey auction), capturing that broker-dealers typically only observe their own ask-price and not the other's, yet possess sufficient market knowledge to avoid paying more than necessary.[8] With a small abuse of notation, we define $\max_{k \neq n} b_s^k$ as the largest value of all other k submitted bids or zero in the absence of such. For any given auction s and a bid b_s^n, a broker-dealer n's realized bid-ask-spread is, thus:

$$\pi_s^n = \begin{cases} a_s^n - \max_{k \neq n} b_s^k & b_s^n > \max_{k \neq n} b_s^k \geq 0 \\ 0 & \max_{k \neq n} b_s^k > b_s^n \geq 0 \\ 0 & b_s^n = \emptyset \end{cases} \cdot \tag{2.3}$$

Here, the payoff function (2.3) reflects that the lender requires an at least weakly positive bid-

[8]The consequent bids and payoffs are equivalent to those under the assumption of Bertrand competition prices, where broker-dealers observe all ask-prices.

price $b_s \geq 0$ from auction participants. And thus, broker-dealers refrain from bidding whenever they expect a negative bid-ask-spread:

$$b_s^n = \emptyset \quad \text{if} \quad a_s^n \quad \text{s.t.} \quad \mathbb{E}_2 \left[\pi_s^n < 0 \mid a_s^n \right] < 0 \quad \forall b_s^n \geq 0. \tag{2.4}$$

Equilibrium Notion We apply the notion of sub-game perfect Nash equilibrium (SPNE). We start by assuming that no broker-dealer holds an ESLA, deriving the auction bids and, consequently, lender and broker-dealer payoffs at $t = 2$. The lender payoffs serve as a sort of reservation utility: She can always refuse to enter any offered ESLA and lend on a security level to the highest bidder. Subsequently, we derive the ESLA terms offered at $t = 1$, accounting for whether only one or more broker-dealers offer the ESLA and the lender's participation constraint. Finally, we identify the SPNEs characterized by the broker-dealers' strategic choices of offering ESLAs and check their renegotiation proofness.

Broker-Dealer Default In the above described model, we implicitly assume away broker-dealer defaults that result in the non-returning of borrowed securities (in a hypothetical period $t = 4$). This is equivalent to assuming that correctly priced non-cash collateral is posted for every sale, generating the same expected lender pay-offs. Investigating potential collateral miss-pricing frictions in the security lending market is beyond the scope of this paper. The distinction between cash and non-cash collateral is necessary, as security lenders typically re-use the provided cash for (profitable) investments. The resulting returns shift the minimum bid necessary for a profitable deal to below zero. Because only a small share of transactions between lenders and broker-dealers is cash-collateralized, we abstract from such in this papers.

2.3.2. Deriving the SPNEs

Second-Price Auctions We first derive the optimal auction bids and payoffs in the absence of any ESLA. Here, broker-dealers decide on a security-by-security level whether to participate in the auction and, conditional on participation, what to bid. For participation, recall that the lender only accepts weakly positive bids while a broker-dealer requires a positive bid-ask spread. Combining the two conditions, a broker-dealer participates only, whenever he draws a weakly positive ask-price to avoid losses:

$$\pi_s^n = a_s^n - b_s^n \geq 0, \tag{2.5}$$

$$a_s^n \geq b_s^n \geq 0. \tag{2.6}$$

Whenever observing a positive ask-price $a_s^n \geq 0$, the broker-dealer bids the entire ask-price, as is standard in second-price auctions. This is the highest possible bid that still ensures a weakly positive spread π_s^n, while simultaneously maximizing the chances of winning. A broker-dealer's optimal bidding strategy is thus:

$$b_s^n = \begin{cases} a_s^n & a_s^n \geq 0 \\ \emptyset & \text{otherwise} \end{cases} . \tag{2.7}$$

To derive the broker-dealers' total payoffs from all S auctions, we rely on the law of large numbers (LLN): For a large number of auctions, the realized total payoffs are close the expected payoffs. For simplicity, we assume exact equality for now and discuss slight deviations together with renegotiations below. Further, we follow standard conventions and assume the probability of equal ask-prices to be zero: $Pr(a_s^n = \max_{k \neq n} b_s^k) = 0$. Finally,we let k denote the number of other participating broker-dealers out of the remaining $N-1$. Then, a broker-dealers total payoffs Π^n from all S auctions are:

$$\Pi^n = S Pr(a_s^n > 0) \sum_{k=0}^{N-1} Pr(k) Pr\left(a_s^n > \max_{k \neq n} b_s^k \geq 0\right) \mathbb{E}_1\left[a_s^n - \max_{k \neq n} b_s^k \mid a_s^n > \max_{k \neq n} b_s^k \geq 0\right] \tag{2.8}$$

$$= \frac{Sa}{2^N} \frac{2^{N+1} - N - 2}{N(N+1)} \tag{2.9}$$

The equation (2.8) contains S times the expected value of a single auction. Here, the first probability reflects the likelihood of participating. The summation accounts for the number k of participants out of $N-1$ other broker-dealers and $P(k)$ for the respective likelihoods. The third probability accounts for likelihood of winning when participating together with k others. Finally, the conditional expectation reflects the expected bid-ask-spread upon winning, reflecting that winning broker-dealer, only pays the second highest bid or nothing if he is the sole bidder. To arrive at the closed-form expression (2.9), note here that k follows a binomial distribution: with $p = 0.5$ each broker-dealer draws a positive ask-price and participates. Thus, after some manipulation, we can apply the binomial theorem. For completeness, recall that a denotes the maximum value of draw a_s^n.

It is crucial to notice that it requires at least two participating broker-dealers for the lender to make a positive profit in a single auction. Alternatively, the bid-price is set to zero. With a slight abuse of notation, let n denote the the index of the broker-dealer with the highest positive ask-price. Relying on the distributional properties of the ask-prices and LLN, the lender's payoffs Π^l from the S auctions at $t = 2$ are:

$$\Pi^l = S \sum_{n=2}^{N} Pr(n)\mathbb{E}_1 \left[\max_{k \neq n} b_s^k \mid n \right] \tag{2.10}$$

$$= \frac{Sa}{2^N} \frac{2^N(N-3) + N + 3}{N+1} \tag{2.11}$$

Similarly to broker-dealer profits above, payoff function (2.10) contains S times the pay-off from a single auction. Here, we again utilize that the number of bids follow a binomial distribution with n success (positive ask-prices) and account for the likelihood of such. Trivially, the expected bid-price from the auction is the one from the second highest of n draws. Again, the closed form expression (2.11) can be obtained by applying the binomial theorem after some manipulation. We can now summarize the subgame outcomes under S second-price auctions in Lemma 3,

***Lemma* 3.** *The broker-dealers' optimal bidding strategy in a single second-price auction for security s at time $t = 2$ is:*

$$b_s^n = \begin{cases} a_s^n & a_s^n \geq 0 \\ \emptyset & otherwise \end{cases}. \tag{2.12}$$

Aggregating the resulting payoffs over all S securities, the lender and broker-dealers realize the following respective total payoffs:

$$\Pi^l = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \qquad \Pi^n = \frac{Sa}{2^N}\frac{2^{N+1}-N-2}{N(N+1)} \quad \forall n \in N. \tag{2.13}$$

ESLA The lender compares the (expected) auction payoffs with the expected payoffs given the available ESLAs at $t = 1$. We denote all prices and payoffs associated with an ESLA with an additional subscript $_E$. We start with assuming that the lender has entered an ESLA with broker-dealer n against a promised uniform bid-price b^n_E that is identical for all securities and a lump-sum transfer T^n_E. Then, for a given security, the broker-dealer facilitates lending at $t = 2$, if:

$$\pi_s = a^n_s - b^n_E \geq 0. \tag{2.14}$$

Accounting for the likelihood of lending and size S of the portfolio, the lender's aggregate expected payoffs under an ESLA agreement are:[9]

$$\mathbb{E}_1\Pi^l_E = SPr(a^n_s > b^n_E)b^n_E + T^n_E = S\frac{a-b^n_E}{2a}b^n_E + T^n_E. \tag{2.15}$$

Expression (2.15) highlights that expected lender utility are monotonically increasing in T^n_E. They are, however, non-linear in the ESLA bid-price: A higher b^n_E increases the revenue from a single transaction, but reduces the probability of said transaction taking place. This non-linearity must be taken into account, when deriving the competitive ESLA terms. Further, we must account for whether only a single broker-dealer makes an ESLA offer or two or more broker-dealers compete over it.

We first study the case when two or more broker-dealers offer an ESLA. In the eyes of the lender, those broker-dealers are identical when competing over ESLAs at $t = 1$. Then, competition prices dictates that they make zero profits in equilibrium. Too see this, initially assume that a broker-dealer has been granted an the ESLA for a given bid-price b^n_E and T^n_E, yet is expected to make a profit. Then any other broker-dealer could offer the same bid-price but a slightly higher lump-sum transfers to attract the lender

[9] Again the LLN applies, such that expected and realized payoffs are equivalent.

instead. This applies for all bid-price and transfer combinations, where the ESLA holder makes a positive profit. Hence, for any given equilibrium bid-price, lump-sum transfers must ensure that the ESLA holder makes zero profits.

$$T_E^n = \mathbb{E}_1 \Pi_E^n(b_E^n; T_E^n = 0) = SPr(a_s^n > b_E^b)\mathbb{E}_1[a_s^n - b_E^n \mid a_s^n > b_E^n] = S\frac{(a - b_E^n)^2}{4a} \tag{2.16}$$

We then insert the equilibrium lump-sum transfer (2.16) into the lender profits. By equating the associated first-order-condition with respect to b_E^n to zero, we can show that $b_E^n = 0$ is the unique lender profit maximizing bid-price:

$$\mathbb{E}_1 \Pi_E^l = S\frac{a - b_E^n}{2a} b_E^n + S\frac{(a - b_E^n)^2}{4a} \tag{2.17}$$

$$\frac{\partial \mathbb{E}_1 \Pi_E^l}{\partial b_E^n} = -2\frac{S}{4a} b_E^d = 0 \tag{2.18}$$

Inserting $b_E^n = 0$ into the optimal lump-sum transfer leads to the following lender and broker-dealer profits:

$$\mathbb{E}_1 \Pi_E^l = \frac{Sa}{4} \qquad \mathbb{E}_1 \Pi_E^n = 0 \quad \forall n \in N \tag{2.19}$$

In a final step, we must check that expected lender profits in equation (2.17) are greater than or at least equal to the expected profits from the S auctions in (2.13):

$$\mathbb{E}_1 \Pi^l = \frac{Sa}{2^N} \frac{2^N(N-3) + N + 3}{N+1} \leq \frac{Sa}{4} = \mathbb{E}_1 \Pi_E^l \tag{2.20}$$

$$N \leq 3 \tag{2.21}$$

***Lemma* 4.** *A lender only excepts one of multiple ESLA offers whenever $N \leq 3$, in which case the optimal fees and expected profits are:*

$$b_E^n = 0 \qquad T_E^n = \mathbb{E}_1\Pi_E^l = \frac{Sa}{4} \qquad \mathbb{E}_1\Pi_E^n = 0 \quad \forall n. \tag{2.22}$$

Whenever $N \geq 4$, the lender strictly prefers engaging in the second-price auctions.

Instead, if only a single broker-dealer offers an ESLA, he acts a monopolists. He simply sets the profit maximizing combination of b_E^n and T_E^n that maximizes his profits constrained by the lender's (binding) participation constraint, And thus, the lender always participates:

$$\mathbb{E}_1\Pi_E^n = \max_{b_E^d, T_E^n} S\frac{(a - b_E^n)^2}{4a} - T_E^n \tag{2.23}$$

$$s.t.$$

$$\mathbb{E}_1\Pi_E^l = \mathbb{E}_1\Pi^l \tag{2.24}$$

Solving the broker-dealer's constraint maximization in (2.23) above, we find that setting $b_E^n = 0$ is again optimal. Further, optimal lump-sum transfer are set just such that the lender's participation constraint just binds given $b_e^n = 0$. This ensures the lender accepts the ESLA and all other broker-dealers are left empty handed. Lemma 5 below summarizes the bid-price, transfer and resulting lender and broker-dealer's payoffs, whenever only a single broker-dealer n offers.

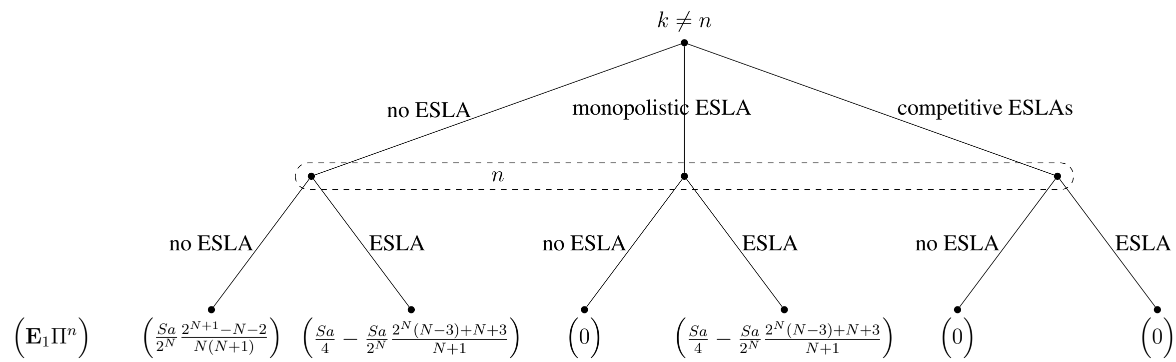
$$\mathbb{E}_1\Pi_E^l = T_E^n = \mathbb{E}_1\Pi^l \qquad \mathbb{E}_1\Pi_E^n = \frac{Sa}{4} - T_E^n \qquad \mathbb{E}_1\Pi_E^k = 0 \quad \forall k \neq n \in N \tag{2.25}$$

***Lemma* 5.** *If offered a single ESLA, the lender always accepts and the optimal fee and expected profits are:*

$$b_E^n = 0 \qquad T_E^n = \mathbb{E}_1\Pi_E^l = \mathbb{E}_1\Pi^l \qquad \mathbb{E}_1\Pi_E^n = \frac{Sa}{4} - T_E^n \qquad \mathbb{E}_1\Pi_E^k = 0 \quad \forall k \neq n \in N \tag{2.26}$$

Candidate SPNE Comparing the just derived payoffs, we now determine the broker-dealers' optimal ESLA offer strategy in equilibrium. For this, we compare a representative broker-dealer n's expected payoffs when (correctly) anticipating none versus at least one other broker-dealer to offers an ESLA. Such payoffs are summarized in Figure 2.4 below. Using this comparison, we derive three types candidate SPNEs characterized by their frequency of ESLA offers: auction SPNEs, single ESLA SPNEs, and multiple ESLA SPNEs. In the next paragraph, we check whether such candidate SPNEs are termination proof and, thus, truly sub-game perfect.

Figure 2.4: A Broker-Dealer's Payoffs given the Others' Choices



Let us start assuming that a broker-dealer n anticipates that no other broker-dealer $k \neq n$ has offered an ESLA (left branch of the tree). Conditional on that, we obtain the optimal broker-dealers choice from comparing the two left-most payoffs in Figure 2.4. After some manipulation, we reach the following two (in-)equalities:

$$\mathbb{E}_1[\Pi^n \mid \text{no ESLA}] > \mathbb{E}_1[\Pi^n_E \mid \text{no ESLA}] \qquad \forall\, N > 2 \tag{2.27}$$

$$\mathbb{E}_1[\Pi^n \mid \text{no ESLA}] = \mathbb{E}_1[\Pi^n_E \mid \text{no ESLA}] \qquad \text{if } N = 2 \tag{2.28}$$

Because all broker-dealers are symmetric, we can directly from (2.27) and (2.28) conclude that there, thus, exists an auction SPNE where no broker-dealer offers an ESLA agreement: Conditional on no other broker-dealer offering an ESLA, a single broker-dealer has no incentive to deviate an offer on. Further, combining this with the insights from Lemma 4, we know that the auction SPNE is unique for $N \geq 4$.

***Lemma* 6.** *There always exists a candidate auction SPNE. It is sole candidate SPNE for $N \geq 4$.*

Equality (2.28) we can see that for $N = 2$ a single broker-dealer n has no incentive to deviate from an ESLA conditional on the other broker-dealer not offering one. To confirm that such could reflect a single ESLA SPNE, we must further verify that conditional on broker-dealer n offering an ESLA, the initial broker-dealer indeed finds it optimal to refrain. Relying on symmetry, this can excluded by additionally checking at the middle branch in Figure 2.4. Here, we see that conditional on one other broker-dealer offering an ESLA as a monopolist, the second broker-dealer is striclty better off slightly underbidding the single ESLA offer and, de facto, making the same returns.

***Lemma* 7.** *There exists no candidate SPNE with a single broker-dealer offering an ESLA.*

This logic of underbidding can naturally be applied to all situations with at least two broker-dealers competing by making ESLA offers. This brings us to the final set of candidate SPNEs, where $N \leq 3$ and at least two broker-dealers offer ESLAs. Comparing the payoffs in the right branch of Figure 2.4, a broker-dealer is always indifferent between offering an ESLA on not, conditional on anticipating at least one other ESLA. Again relying on symmetry, this holds also for all other broker-dealers. Consequently, there exists at most four of such competitive ESLA SPNEs: one for each broker-dealer pair and one for all three borker-dealers. Recall from Lemma 4 that such multiple ESLA SPNEs cannot exist for $N \geq 4$, as there the lender rejects any ESLA.

***Lemma* 8.** *For $N \leq 3$, there exists several candidate competitive ESLA SPNEs, each characterized by either two or three broker-dealer competing via ESLAs.*

Terminations A common concern in the exclusive contracting literature is whether the candidate SPNE are renegotiation proof (Segal and Whinston, 2000). In this model, any sure gain from the broker-dealer(s) always results in a sure loss to the lender. Thus, mutual beneficial renegotiations are not possible.[10] Further, the auction SPNE is renegotiation proof by assumption, as no ESLA can be offered/entered at $t = 2$. A closely related and more relevant concept is that of termination proof SPNEs: No contract holder has an incentive to single-handedly terminate the contract. Because ESLA contracts always take the lender's participation constraint into account, the lender never has an incentive to terminate an ESLA in SPNE. For the ESLA holding broker-dealer recall that any ESLA termination requires the lender compensation of losses. And such, the ESLA holder does *not* terminate if:

[10]Note that this partially depends on the presence of lump-sum transfers. If ESLAs could only specify uniform lending fees, then lender profits are an u-shaped function in b_E^n. Thus, the profit maximizing lending fee leaves profits for the broker-dealer and renegotiation may be mutually beneficial ex post.

$$\mathbb{E}_1\Pi^n - \left(\mathbb{E}_1\Pi^l_E - \mathbb{E}_1\Pi^l\right) \leq \mathbb{E}_1\Pi^n_E. \tag{2.29}$$

Inequalities (2.30) and (2.30) consider payoff gains from termination when $N = 3$ and when $N = 2$ are respectively inserted in equation (2.29):

$$N = 3: \qquad \mathbb{E}_1\Pi^n - \left(\mathbb{E}_1\Pi^l_E - \mathbb{E}_1\Pi^l\right) = \frac{Sa11}{96} - \left(\frac{Sa}{4} - \frac{Sa3}{16}\right) > 0 = \mathbb{E}_1\Pi^n_E. \tag{2.30}$$

$$N = 2: \qquad \mathbb{E}_1\Pi^n - \left(\mathbb{E}_1\Pi^l_E - \mathbb{E}_1\Pi^l\right) = \frac{Sa}{6} - \left(\frac{Sa}{4} - \frac{Sa}{12}\right) = 0 = \mathbb{E}_1\Pi^n_E \tag{2.31}$$

As inequality (2.30) highlights, the multiple ESLA SPNE is not termination proof for $N = 3$: The ESLA holder finds it profit maximizing to terminate the contract, thereby triggering an auction SPNE. And thus, for $N \geq 3$ the auction SPNE is the unique termination proof SPNE. For $N = 2$, however, a multiple ESLA SPNE additionally exists. As equation (2.31) highlights, here the ESLA holder is just indifferent between entering the auctions and paying the punishment or not, and hence does not choose to terminate. In Appendix 2.3., we show that the ESLA holders has a strict preference for not terminating in case he *ex post* observes higher ask-prices than anticipated *ex ante*. For completeness note that there exists a mixed strategy equilibrium for $N = 2$, where each broker-dealer offers an ESLA with probability one-half.[11]

***Proposition* 10.** *There always exist a termination proof auction SPNE, which is unique for $N \geq 3$. For $N = 2$, there further exists termination proof SPNE with two competitive ESLA offers.*

Monopolistic Broker-Dealer For completeness, we also derive the SPNE with a single operating broker-dealer. For $N = 1$, the derivations are rather trivial as the broker-dealer is a monopolist and simply ensures that lender just participates. Hence, the lender makes zero profits from lending and the broker-dealer realizes the entire ask-price for every security where such is positive. An ESLA may be offered and granted, but neither changes profits nor fees. And, hence, an ESLA comes at no benefit to the lender.

[11] In the mixed strategy SPNE, the uniform ESLA bid-price is always zero. Transfers are, however, conditional on the lenders total ESLA offers.

Remark 1. For $N = 1$, the lender is indifferent between being offered an ESLA or not, as the broker-dealer is a monopolist that always extracts all transaction surplus:

$$b_s = b_E = T_E = \Pi^l = 0 \qquad \Pi^n = \frac{Sa}{4}. \tag{2.32}$$

2.4. Model Fit

For we focus on $N \leq 2$ as only there ESLAs may arise and lenders with two-or-less counterparties are the most frequent. To verify the model fit with the data, we derive five testable hypotheses assuming a large number L of lenders. These are tested using the same data as described in the Data Section 2.2. above. Recall that the lending fees (F_s) are typically reported as percentages of total loan value (V_s). The loan value in return is the stock price (P_s) times the quantity (Q_s). To match this with the model assumptions, we transform the lending fee into a fee paid per-unit of borrowed stock:

$$b_s^* = \frac{F_s}{100} \frac{V_s}{Q_s}. \tag{2.33}$$

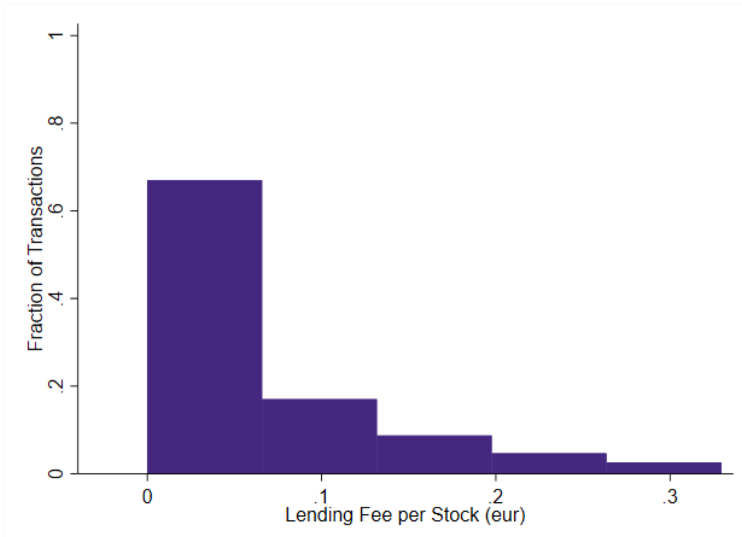
We will subsequently use these per-unit fees to test the fit of our predicted prices with those of the realized prices in the data. Here, we continuously ensure data confidentiality and would like to highlight that each reported statistic contains at least three lenders and broker-dealers each, and no two market participant make up more than 85%. All Figures are produced using data-points between the 10th and 90th percentile. Tables, where only aggregated statistics are shown, are derived on data-points between the 1st and 99th percentile to ensure outliers are not driving the results.

ESLAs We start considering the subset of ESLA granting lenders. From Lemma 3, we know that lenders with ESLAs should pay zero bid-prices. Of course, this simplifies reality as we assume in the model that lenders face zero marginal transaction costs. Softening the expectations slightly, we expect that lending fees should be close to zero under ESLAs.

H1 Per-unit lending fees of transactions under ESLAs should be close or equal to zero.

Testing *H1* statistically using conventional methods is not possible, as lending fees do not follow a normal distribution: They have both a clear cut off and mass point at zero. To verify *H1*, we instead rely on visual output. As Figure 2.5 below shows, the transaction fees under ESLA agreements are indeed very close to zero. In fact, 90% of all transactions are below five cents and all are less than 15 cents.

Figure 2.5: Distribution of the Per-Unit Fees under ESLA Agreements

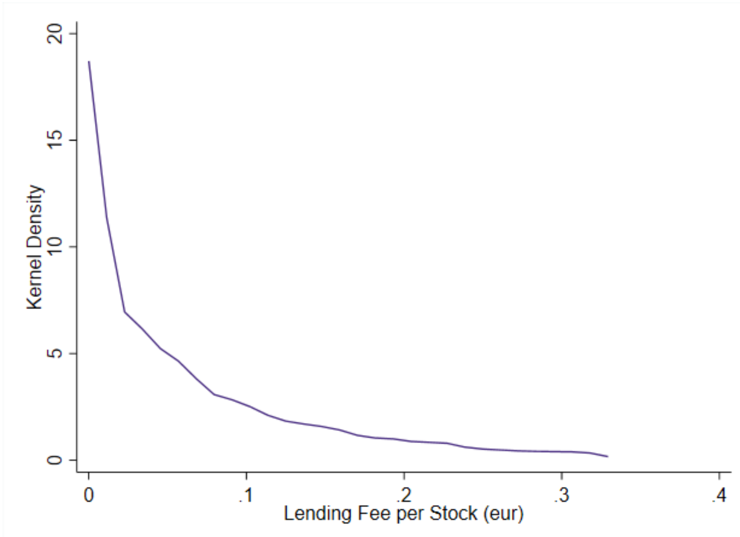


No ESLA In the absence of ESLAs, the realized fee distribution depends on the lenders' number of counterparties. Hypotheses H2 and and H3 below summarize the results for lenders with $N = 1$ and $N = 2$ respectively. We do not consider the case of lenders with three or more broker-dealers as predictions are cumbersome without any substantial additional insights. For $N = 1$, we predict that lending transactions have a zero lending fee as the single broker-dealer acts as a monopolist. Testing such hypothesis in the data is more challenging though, as we are only observing the realized number of counterparties and not the (hypothetically) available.

H2 *Outside ESLAs, transactions of lenders with a single broker-dealer have more often than not zero bid-prices.*

To verify *H2*, we estimate the Kernel density over all per-unit lending fees paid by lenders with a single broker-dealer connection. Displayed in Figure 2.6, we observe that zero or close to fees are most likely. However, we also see that a non-negligible portion of lenders indeed makes a profit. This could be due to non-zero lending costs or that their restriction to one broker-dealer is by choice. A lender requesting offers from multiple broker-dealers, but ultimately selecting one, allows her to nevertheless enjoy the benefits of (some) competition without paying e.g. on-boarding costs. Unfortunately, the data does show how many counterparties they considered ex ante.

Figure 2.6: Kernel Density of Per-Unit Fees without ESLA Agreements and $N = 1$



For lenders with two broker-dealers, obtaining the distribution of realized bid-prices, denoted with b_s^*, is more complex. It is derived from the likelihood that the second-largest price takes on a value (weakly) above zero when both broker-dealers experience a positive draw and exactly zero, when only one does. For ease of understanding, denote the other bid with b_s'. Below, equation (2.34) defines the resulting probability density function (pdf) for all observed bid-prices in its general form.

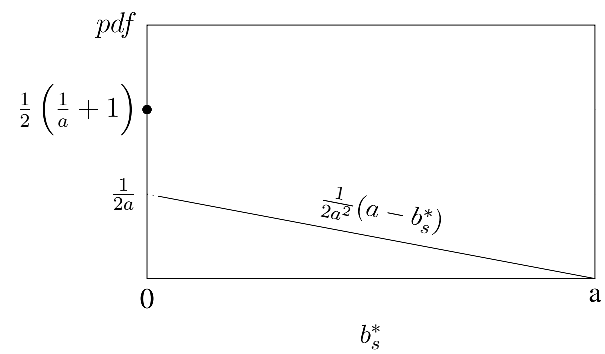
$$pdf(b_s^*) = \begin{cases} 2Pr(b_s^*)Pr(b_s^* < b_s') & b_s^* > 0 \\ 2Pr(b_s^*)Pr(b_s^* < b_s') + 2Pr(b_s' = \emptyset)Pr(b_s > 0) & b_s^* = 0 \end{cases} \tag{2.34}$$

Inserting the properties of the uniform ask-price distribution, we can obtain a closed form expression for the pdf of bid-prices. The density function, displayed in Figure 2.7a below, takes on the value $1/2(a^{-1} + 1)$ at exactly zero. For marginally increasing bid-prices, we observe an immediate jump downwards to $1/2a$. This jump reflects that a zero bid-price is more frequent relative to all other positive bid-prices due to the absence of second bids whenever one of the two broker-dealers draws a negative ask-price.

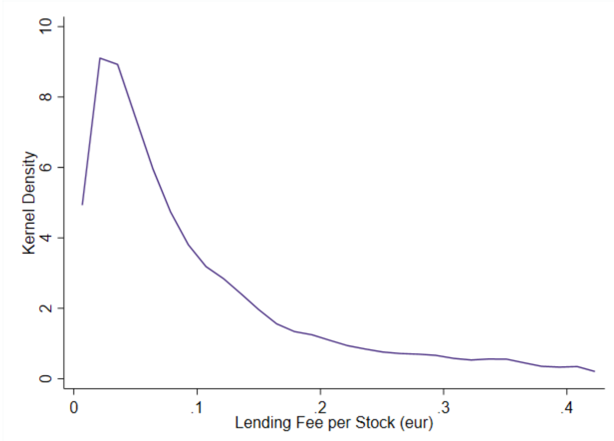
For every other value above zero, the density is a linearly decreasing function with a slope equal to $-1/2a^{-2}$. The hypotheses *H3* below summarizes such behavior.

H3 Trading with two broker-dealers, zero is the most frequent realized bid-price, followed by an immediate downward jump and consequent decrease in.

Figure 2.7: Bid-Price Distributions without ESLA and $N = 2$



(a) PDF of Bid-Prices



(b) Kernel Density ($N = 2$)

Notice here that *H3* intentionally does not describe the decrease in density for increasing fees as linear, albeit the theoretical model predicting this. Again, this is motivated by the fact that we only observe the realized number of broker-dealers but not the initially available. In reality, some lenders could access offers from three (or more) broker-dealers, but chose to engage with two for simplicity. This increases the density of fees above zero slightly. Figure 2.7b displays the estimated Kernel densities using the transactions by lenders which trade with only one broker-dealer. As a small caveat, we did not estimate a discrete Kernel with a jump at zero, but rather relied on the standard assumption of continuity common to available statistical packages. Nevertheless, we can clearly see that values just above zero indeed have the highest density. Further, the density declines for higher values. Such decline is, however, convex rather than linear. This is additional evidence that some lenders may choose to let several broker-dealers compete but ultimately only engage with one.

Stock Price Irrelevance Finally, our model predicts that fees are independent of the underlying securities value v_s, independent of ESLA stats. This stems from an implicit assumptions that the maximum perceived overvaluation is still below stock price value: $v_s - a > 0$. In reality, this of course not always the case as a stock can never be worth less than zero. Therefore, the maximum any speculator should ever be willing to pay is the stock value. For lenders with ESLAs, this limitation is not relevant due to the competition over the whole portfolio (*H4a*). However, for lending outside ESLAs, where competition happens on the security level, the stock value puts an upper bound on bid-prices. Hence, we should observe at least a moderately positive correlation between realized fees and stock prices (*H4b*).

H4a *Under ESLAs, lending fees should have a no correlation with stock prices.*

H4b *Outside ESLAs, per-unit lending fees should have a moderate and positive correlation with the underlying stock price.*

The statistics in Tables 2.7 and 2.8 test hypotheses *H4a* and *H4b*, respectively. As expected, we only find a close to zero, and even slightly negative, correlation between realized unit fees and underlying stock value for ESLA-covered transactions. We further confirm *H4b* and find a low and positive correlation between lending fees and stock values for transactions not covered by ESLAs.

Table 2.7: ESLA Fees and Stock Price

	Lending Fee (per Unit)	Stock Price
Lending Fee (per Unit)	1.000	
Stock Price	0.008**	1.000
	(0.01)	
Observations	104378	

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.8: No ESLA Fees and Stock Price

	Lending Fee (per Unit)	Stock Price
Lending Fee (per Unit)	1.000	
Stock Price	0.346***	1.000
	(0.00)	
Observations	1252041	

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

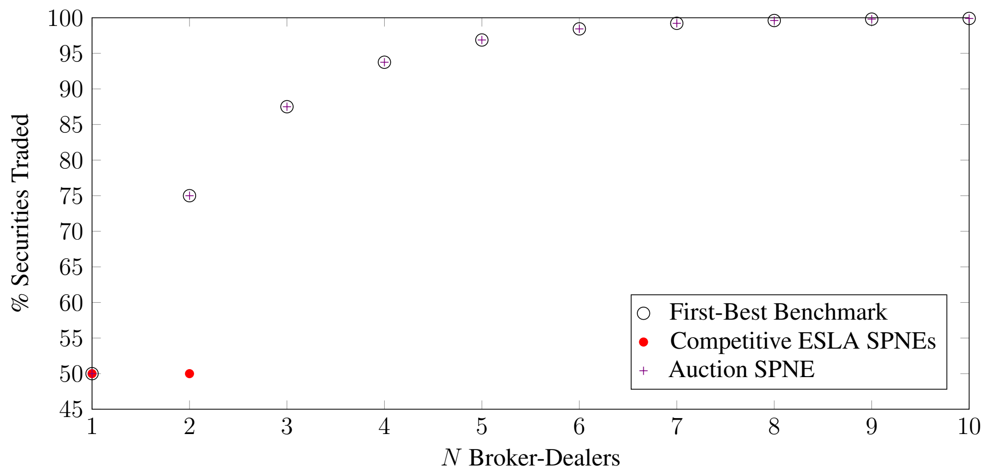
2.5. Market Efficiency

From a market efficiency perspective, all securities with at least one positive bid-ask-spread should be traded. The above model studies a single representative lender with N competing broker-dealers. Of course, the security lending market contains many of such lenders with varying size of N. To account for this, we first estimate market efficiency on a single lender level, and then appropriately aggregate across all lenders taking their number of connected broker-dealers into account.

2.5.1. Individual Level Inefficiency

For a single lender with N broker-dealers, the first-best benchmark implies that a security is traded with with a probability $1-0.5^N$. And for a large portfolio S, this aggregates to a total of $(1-0.5^N)*100$ percentage of securities traded. The hollow circles in Figure 2.8 below indicate such percentages as a function of N.

Figure 2.8: Percentage of Traded Securities in SPNE



The auction SPNE meets such first-best benchmark: Every broker-dealer with a weakly positive ask-prices participates in the auction and, thus, all securities with at least one realized ask-price of above 0 are traded. This is indicated by the blue crosses in Figure 2.8 overlapping exactly with the first-best. For $N = 2$, there additionally exists an ESLA SPNE, where both broker-dealers offer an ESLA, and a single one ultimately holds it. Upon realization, the ESLA holder borrows each security with probability one-half: the probability of a weakly positive ask-price. For the whole portfolio, this implies that 50% of all securities are traded, as indicated by the red dots in Figure 2.8. Here, it can easily be seen that the ESLA SPNE does not meet the first-best benchmark.

***Corollary* 6.** *The auction SPNE always meets the first-best benchmark.*

For $N = 2$, competitive ESLA SPNE has a 25 percentage points lower trading share relative to the first-best benchmark.

For $N = 1$, also the ESLA SPNE meets the first-best benchmark.

2.5.2. Aggregate Market Inefficiency

From a market efficiency perspective, we are interested in how much these individual reductions impact aggregate trading volumes. We, therefore, perform a counterfactual analysis by assuming away all ESLA agreements and predicting the aggregate increase in total lending. A challenge here is the measurement of lending frequency, as we of course do not observe lenders true portfolios. To nevertheless obtain a proxy, we use the number of distinct securities (ISINs) lent out by each lender and obtain total market size by the summation of such: 218,994.[12]

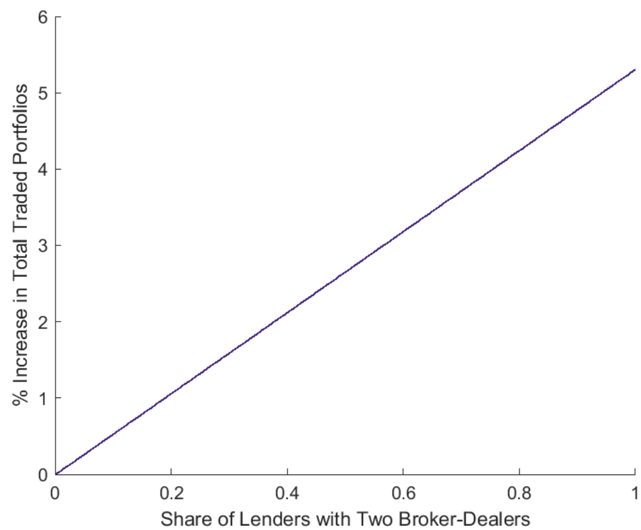
To obtain the counterfactual total lending, we predict how many ISINs each lender with an ESLA would have lent out in the absence of such: the same with one counterfactual broker-dealer, 50% more ISINs with two counterfactual broker-dealer connections. Difficult to overcome is the limitation that the model environment does not allow us to infer which of the lenders would have one or two broker-dealer connections. We, therefore, simply obtain predictions for different shares of lenders having one versus two broker-dealer connections. Then, for each share in the range 0 to 1, we use 100,000 Monte Carlo simulations in which we randomly assign lenders either one or two broker-dealers. To predict the increase in total traded portfolios, we compute the average across all simulations and its bootstrapped standard errors for the confidence intervals. A detailed description of the applied algorithm can be found in Appendix 2.2..

Figure 2.9 displays the average prediction across the 100,000 Monte Carlo draws. Bootstrapped confidence intervals are so narrow around the predictions that they would not be visible in the figure. They are, thus, omitted with the comment here that all predictions are significant. Standard errors can be found in Appendix 2.2.. We find that whenever at least some lenders with ESLAs would interact with two broker-dealers instead, the overall traded portfolios would increase significantly. Not surprisingly, the increase in traded portfolios is bigger the more lenders are assumed to have two broker-dealer connections in the counterfactual. However, even if all lenders with ESLAs had two connections, the increase in total traded portfolios would be around 5.3%. This is equivalent to adding 10,950 ISINs to traded portfolios in 2021, thereby relaxing their short sale constraints.

Conjecture 1. *In the counterfactual, ruling out ESLAs would result in 0-5.3% more stocks being traded.*

[12] Alternatively, one could use a lenders' number of transactions. However, this largely inflates trading volumes due to daily rolled over transactions. Here, we are mainly interested in how many different stocks are borrowed and not how frequently the same stock is rolled over.

Figure 2.9: Predicted Aggregate Inefficiency of ESLAs



We would like to acknowledge that the 5.3%. are a not the largest sensible estimate of the predicted enlargement in traded portfolios. Motivated by the results of Proposition 1 and Remark 1, we have assumed that ESLAs only arise in equilibrium when lenders have one or two connected broker-dealers. The model environment, however, takes a lender's number of connections as exogenously given. We could imagine a slightly modified model environment, where in a hypothetical period $t-1$, a lender decides to only onboard with two rather than three available broker-dealers to force the ESLA equilibrium to arise. This cannot be excluded in this model environment, as the following inequality shows:

$$\Pi^l_E = \frac{Sa}{4} > \frac{Sa3}{12} = \Pi^l(N=3). \tag{2.35}$$

From Figure 2.8 in the individual level analyses above, we know that lenders with connected with three broker-dealers lend out more than those with two broker-dealers, holding the portfolio size constant. Studying all potential combinations of how lenders can be assigned one, two or three broker-dealer counterfactual connections goes beyond the scope of the study. To nevertheless give some indication, we additional compute the predicted increase assuming that all ESLA granting lenders had three counterparties instead. Under this extreme scenario, we predict an 8% increase in traded portfolios or 17,520 distinct

ISINs.[13] Therefore, while an introduction endogenous lender and broker-dealer connections seem like a natural extension of this paper, the quantitative impact of such on the predictions of aggregate inefficiencies is likely small.

Conjecture 2. *A reasonable upper limit of the increase in stocks being traded due to an out-ruling of ESLA agreements is 8%.*

2.6. Conclusion

This paper has the objective to first answer why lenders grant ESLAs despite their anti-competitive nature and, subsequently, to which extent they impact market efficiency. To answer these questions, we first utilize the newly available and confidential SFTR data to provide a market overview. The insights are subsequently used to inform the model framework used to theorize under what conditions ESLAs would arise in equilibrium. Finally, we derive the counterfactual market outcome, should ESLAs be ruled out.

We show that in Europe, the equity lending market is populated by lenders, borrowers and private clients that predominantly interact with a small set of large broker-dealers and few smaller traders. Here, in particular lender and broker-dealer transactions are covered by ESLA agreements: 35% of all lenders grant a single broker-dealer exclusive access to their portfolio. While most borrowers only interact with a single broker-dealer, they rarely do so through ESLAs.

We use these market insights to inform a theoretical framework, where a representative lender has access to a small number of broker-dealers. The lender is endowed with a large equity portfolio, while the broker-dealers each are endowed with uncertain borrowing demand. Before the demand realizes, broker-dealers may compete for exclusive access to the lenders entire portfolio via ESLAs. If an ESLA is granted, only the holder gets to borrow those stocks, where she will make a (weakly) positive bid-ask-spread. If no ESLA was granted, broker-dealers compete on an equity level via second-price auctions.

The lender experiences a trade-off between higher *ex ante* competition over the entire portfolio via ESLAs but reduced lending *ex post* to only the holder, and higher *ex post* competition only in those equities that two or more broker-dealers demand. Due to the nature of price competition determining ESLA

[13]Note that because all lenders with ESLAs are assigned three broker-dealers, no bootstrapping of standard errors is possible, and hence no confidence intervals are obtained.

terms, the *ex ante* competitive benefits of ESLAs are identical for all $N \geq 2$. The benefits of the *ex post* competition, however, increases in the number of broker-dealers due to the increased likelihood of two or more positive demand draws. Ultimately, this leads to ESLAs only being granted by lenders with at most two broker-dealer connections.

In those cases, where an ESLA is granted, the model predicts a zero fee for every lending transaction. For all transactions not covered by ESLAs, the model predicts a price distribution that has a mass-point at zero, followed by an immediate jump downwards and subsequent linear decrease in frequency for higher lending fees. Those predictions are confirmed by carefully comparing them with the realized fee distributions observed in the data.

In a final step, we provide a set of estimates for the market in-efficiencies induced by ESLAs. We find that out-ruling ESLAs leads to an at most 5.3% increase in the European equity lending of stocks, even if all lenders with ESLA would have had two broker-dealer connections instead. We further acknowledge that in reality, the lender may well consciously choose how many connections to establish, thereby forcing the ESLA equilibrium to arise. Even allowing for this, we find at most an increase of 8% in a back-of-the-envelope type calculation. Thus, a natural extension is to expand the model to include an active choice of broker-dealer connections by the lender. Consequently calibrating the model environment would allow a full quantification of the market inefficiency also in that case.

Chapter 3

Optimal Severity of Stress-Test Scenarios

Abstract Stress tests constrain bank balance sheets: equity must be sufficient to maintain current lending also tomorrow, even after absorbing severe loan losses. We study such forward looking stress-test constraints in a three-period representative bank model, and show that they lead to lower dividends, higher equity levels, and universally lower, albeit less volatile, lending. Subsequently, we compare stress tests with several policy alternatives, such as the Covid-19 dividend ban, the counter-cyclical capital buffer (CCyB), and the dividend prudential target (DPT): while the first two perform well as complementary policies, a DPT is not welfare-improving for a supervisor seeking stable lending levels.

3.1. Introduction

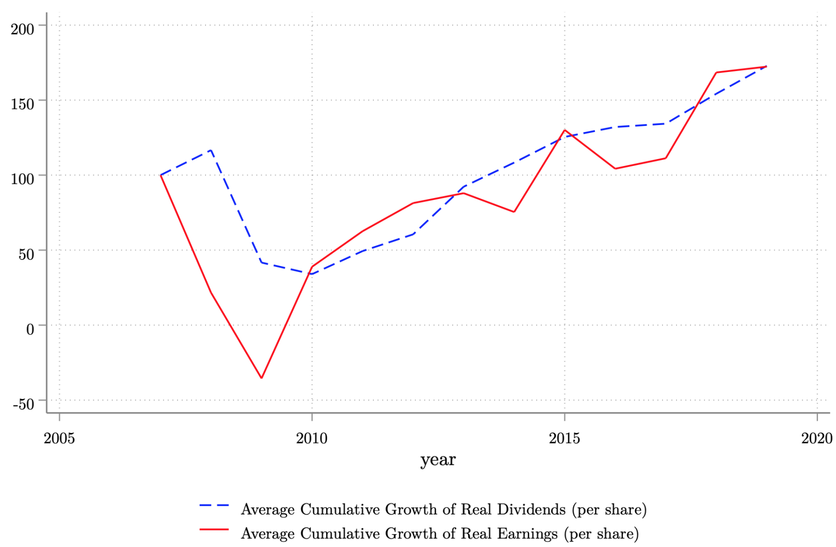
The financial crisis of 2008-09 has highlighted how crucial bank health is for economic stability and growth. To promote a safe and sound financial system going forward, supervisory authorities around the world have since introduced a wide range of new regulatory measures. As part of this policy package, the Federal Reserve (Fed), the European Central Bank (ECB), and many other authorities have begun to subject banks to regular, usually annual stress tests. The objective of stress tests is to ensure that banks are sufficiently capitalized to maintain their current lending activities even under severely adverse macroeconomic conditions in the future.[1] Banks found to be insufficiently capitalized in a hypothetical downturn are consequently restricted in their dividend payments: depending on the severity of violation, an increasing

[1] Thus, stress tests extend the existing macro-prudential framework by going beyond point-in-time-estimates.

amount of net-income must be retained to boost equity levels.[2]

This regulatory pressure on dividend payments clashes with the banks' apparent objective to generate stable dividends that compensate shareholders for their investments (Koussis and Makrominas, 2019; Larkin et al., 2017).[3] These dividends are paid from both accumulated equity and returns on assets that are financed via equity capital and debt. To keep dividends smooth across the business cycle, banks deplete capital reserves when facing negative earning shocks (see Figure 3.1). Unregulated, simultaneously maintaining stable dividend levels and minimum capital ratio requirements may lead to asset shrinkage during crisis periods. Thus, intuitively, supervisory restrictions on dividend payments via stress tests seem warranted to maintain equity capital and thereby to ensure lending to viable firms.

Figure 3.1: Cumulative Growth of BHC Earnings and Dividends (2007=100)



Note: Sample includes all banks that were registered as Bank Holding Companies (BHCs) in 2007 and were at any subsequent point subject to the stress tests of the Comprehensive Capital Analysis and Review (CCAR) regulatory framework.

This argument, however, ignores how banks might change their behavior in anticipation of stress-test constrained dividend payments. To the banks' risk-averse shareholders, a safe payment today is worth more than an expected equal amount tomorrow that is subject to uncertainty. To pass the stress tests, banks therefore may avoid cutting dividends and instead reduce lending levels. Hence, one must account for the

[2]A detailed description of the U.S. regulatory framework can be found in Appendix 2.4.. Similar restrictions exist in the European Union and China (Svoronos and Vrbaski, 2020).

[3]There is no shortage of potential explanations for banks' dividend smoothing policies, ranging from investor interests to managerial pay-out schemes directly linked to dividend stability (Lambrecht and Myers, 2012; Wu, 2018). In this paper we do not take a stand on the cause of this behaviour but rather take it as a given bank objective.

bank's margin of adjustment when evaluating the efficiency of stress tests. Thus far, the existing stress test literature provides little insights on *ex ante* dividend and lending choices by stress-tested banks, as it focuses mainly on the announcement effect of bank stress test results and the subsequent immediate stock-price responses (Beck et al., 2020; Goldstein and Leitner, 2018; Sahin et al., 2020).

Research Agenda In this paper, we therefore study the effect of a forward-looking stress-test constraint on banks' dividend policies, equity levels, and lending activities. To answer this, we build a partial equilibrium framework that characterizes these three bank choices, given different return realizations and varying tightness of the stress-test constraint. We model the stress-test as a forward-looking constraint on the bank's balance sheet choices similar to a minimum equity ratio, thus mirroring the frequency in which stress tests are performed by supervisors (and abstracting from strategic balance sheet adjustments). We then derive the optimal tightness of the stress-test constraint for a supervisor who seeks to maximize lending levels while avoiding lending volatility. For this derivation we partially rely on a calibration of our model for a quantitatively meaningful discussion. Finally, we investigate how stress tests perform relative to other policies, such as the Covid-19 dividend ban, the countercyclical capital buffer, and the dividend prudential target by Muñoz (2020) (banks must pay a punishment fee when dividends deviate from a regulatory target).

Theoretical Framework To illustrate the effects of a stress-test constraint on bank balance sheet choices, we propose a three-period, partial equilibrium framework. The model is populated by a supervisor with mean-variance welfare over bank lending and a representative investor with mean-variance preferences over dividends received from investments in said bank loans.[4] The objective of the supervisor is, thus, in conflict with objective of the investor: while the investor prefers high and stable dividends, the supervisor prefers high and stable lending. The environment is characterized by a single source of uncertainty: loan returns evolve over all three periods following an AR(1) process. The parameter space additionally contains an initial bank equity endowment, an interest rate on bank deposits, and an exogenously given minimum equity-to-loan ratio requirement.

In period 0, an initial loan return state realizes and the representative investor is endowed with the equity holding in the bank. Observing both, the supervisor decides on the tightness of the forward looking stress-test constraint, our key novelty in this paper, with the objective to maximize welfare. The

[4] Assuming mean-variance preferences introduces the above described bank preference for smooth dividends. Lambrecht and Myers (2012) provide a micro-foundation for such an objective function.

stress-test constraint will apply in period 1 and requires that the bank's retained equity is sufficient to absorb (simulated) severely adverse losses from the chosen lending levels without violating the minimum equity-to-loan ratio. In period 1, the bank observes the initial equity and an evolved loan return state. With the objective to maximize the shareholder's total expected dividends, the bank first decides how much equity to retain versus to pay out as period 1 dividends. The retained equity and additional external debt are used to invest in risky loans. Here, the degree of debt financing of loans is constrained by both the stress-test and the minimum equity-to-loan ratio constraint. In period 2, a further evolved loan return rate realizes and, together with last period's equity, lending, and debt choices, determines period 2 dividends. After paying out such to the investor, the bank ceases to exist.

Bank Choices First, we show that any meaningful stress test scenario results in a de facto increased minimum equity-to-asset ratio requirement. Hence, the forward looking stress-test constraint always binds before the minimum equity-to-loan ratio constraint. Moreover, the bank always lends as much as the stress-test constraint allows, given the level of optimal equity. The optimal equity follows a step function in return states: in bad return states no equity is retained as loans are very risky and investments not profitable; in medium states a portion of equity is retained for risky investments and a portion is paid out as dividends; only in high return states all inherited equity is retained to be fully invested in loans. Performing comparative statics over the stress-test constraint tightness (the severity of the adverse scenario) highlights the core supervisory trade-off: an increase in tightness leads to higher retained equity in (almost) all states of the world, but always reduces lending levels. At the same time, however, a tighter stress-test constraint leads to less volatile lending.

Optimal Tightness The underlying stochastic process together with the kinks in optimal lending and equity do not allow for a fully analytical expression of the optimal stress-test tightness. To nevertheless provide a quantitative estimate, we calibrate the model parameters using the balance sheet data of U.S. bank holding companies that are subject to the stress tests implemented by the Comprehensive Capital Analysis and Review (CCAR) regulatory framework.[5] We then numerically derive the ex-ante optimal tightness of the stress-test constraint that maximizes the supervisor's mean-variance preferences over expected lending. We find that the optimal tightness typically leads to additional capital buffers of $1\% - 9\%$, depending on the different initial return states and welfare weights: a supervisor more (less) concerned about the level

[5]See Appendix 2.4. for a detailed description of the regulatory environment.

than the volatility of lending imposes a looser (tighter) stress-test scenario; a supervisor in a higher (lower) initial return state imposes a looser (stricter) stress-test scenario. This numerical result closely matches the Federal Reserves' recently announced stress-test buffers for 2021 which are reported to lie between 2.5% to 7.5% (Federal Reserve Board, 2021), indicating that we are able to capture well the magnitude of bank balance sheet choices under stress tests.

Policy Extensions Utilizing the calibrated model, we first study bank choices when compliance with the stress-test constraint is voluntary and show that voluntary violation would often be optimal for stress-tested U.S. banks. We further use the model to evaluate several other policies in their ability to maintaining stable lending levels, acting both as complements and substitutes to stress tests. First, we investigate how a blanket dividend ban, as many supervisory agencies introduced at the beginning of the Covid-19 pandemic, impacts the lending of stress-tested banks. Here, we find that a ban successfully increases lending, but banks refrain from using as much debt financing as the stress-test constraint allows. Subsequently, we show that relaxing a counter-cyclical capital buffer (CCyB) increases lending in bad states. However, CCyB activation is less effective than the dividend ban and, when introduced on top of the ban, has no further effects.[6] We conclude by comparing the performance of the dividend prudential target (DPT) of Muñoz (2020) with that of stress tests. Here, we find that a DPT is a useful policy instrument to maximise lending levels. However, if a supervisor also cares about the volatility of lending, even an optimal DPT policy leads to substantially lower welfare than the stress-test constraint.

Literature Our paper primarily contributes to the stress test literature. Thus far, the bulk of papers in this literature is empirical and studies the information revealing mechanism of stress-tests and their immediate impact on stock prices (Bird et al., 2020; Morgan et al., 2014; Petrella and Resti, 2013; Quijano, 2014). Even though the outcomes of stress tests are to a large extent predictable (Ahnert et al., 2020), a range of studies has shown that the release of stress test results nonetheless provides valuable information. Among others, Flannery et al. (2017) and more recently Fernandes et al. (2020) identify positive abnormal equity returns and negative responses of CDS spreads in response to stress-test disclosure. However, this effect is heterogeneous across the business cycle (Sahin et al., 2020), across banks' risk-exposure (Flannery et al., 2017), and between those banks passing and those failing the stress test (Sahin et al., 2020). As a

[6]Here, we are thus able to provide an explanation for the current policy puzzle of unused CCyB buffers during the Covid-19 crisis (FSB, 2021).

result, the optimal disclosure policy of stress test results is not trivial: it depends non-linearly on a bank's capital gap (Goldstein and Leitner, 2018) and it is subject to a time inconsistency problem (Parlasca, 2021).

A small but growing empirical literature furthermore studies the change in lending levels following stress test announcements, thus going beyond the immediate disclosure effects of stress tests. Using U.S. loan-level data, Acharya et al. (2018), Cortés et al. (2020), and Doerr (2021) document that stress-tested banks reduce credit supply, especially to risky borrowers. However, it remains unclear whether this results in an aggregate decrease of credit supply or whether the decrease of stress-tested banks is offset by unaffected banks. Cappelletti et al. (2019) argue that the 2016 stress-testing exercise in the euro area similarly has led banks to increase their capital ratios by reducing their lending and risk-taking. Finally, Cornett et al. (2020) find that the banks subject to stress tests lower dividends significantly compared to non-tested banks. However, this behavior reverses completely afterwards, suggesting that stress-tested banks may be managing financial performance. Our paper provides the theoretical counterpart to these empirical analyses by rationalizing these findings in a partial equilibrium framework.

To the best of our knowledge, we are the first to explicitly model the forward looking stress-test constraint and thus theoretically study its impact on banks' joined decision over lending, equity, and dividend payments. The closest to our paper are Shapiro and Zeng (2019), who study how banks optimally risk-adjust their portfolio in response to stress tests, holding dividends, equity, and debt levels fixed. We complement their work by endogenising the banks' balance sheet choices while abstracting from portfolio risk-adjustments. For this purpose, we extend the banking model by Gollier et al. (1997), borrowing several elements from the dynamic banking literature. For our objective function, we rely on Lambrecht and Myers (2012), who provide a micro-foundation for the dividend smoothing behavior of banks. Further, we extend the uncertainty of the asset to span all three periods, by utilizing the AR(1) process describing loan returns in Bolton et al. (2020). To maintain tractability in the face of an evolving return state, we abstract from the possibility of bank default as originally studied in Gollier et al. (1997). The result is an easily extendable model that not only highlights the effect of stress tests, but allows us to study a range of complementary and substitute policy measures.

Overview The remainder of the paper is organized as follows. In Section 3.2., we describe the baseline model environment and state the bank's optimal dividend, equity, and lending choices. In Section 3.3., we calibrate the model to obtain a numerical value, consequently quantify the marginal responses of equity and lending to changes in the stress-test tightness, and finally numerically establish the optimal stress-test tightness. Section 3.4. addresses the possibility for banks to voluntary violate stress tests. In Section 3.5., we discuss several policy extensions, such as the Covid-19 dividend ban, the CCyB, and the dividend prudential target. Section 3.6. concludes and puts the theoretical and calibration exercise in perspective. The appendix contains a detailed description of the regulatory framework and all proofs.

3.2. Theoretical Analysis

The following section contains the representative bank problem and is structured in the two following sub-sections: Section 3.2.1. describes the baseline partial equilibrium framework inspired by the dynamic banking models of Bolton et al. (2020) and Lambrecht and Myers (2012), but modified to a three-period environment to allow for a tractable introduction of bank stress tests;[7] Section 3.2.2. subsequently derives the lending and equity choices by a stress-tested bank and, relying on this, Section 3.2.3. performs comparative statistics to study the response of equity and lending to the introduction of a stress test.

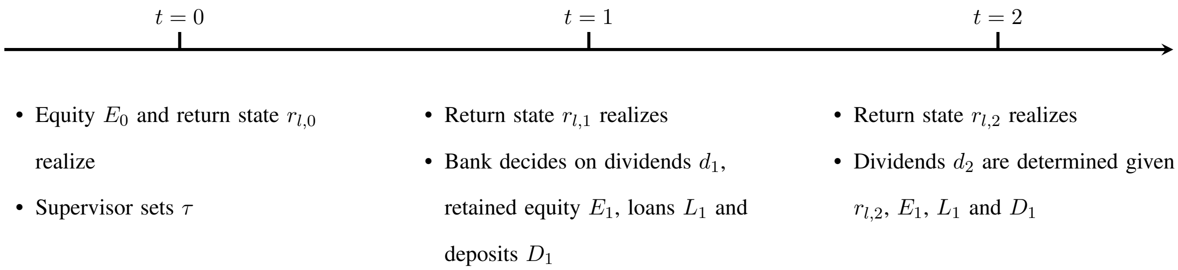
3.2.1. Three-period Model

The model is populated by a representative risk-averse investor owning a bank, or a representative bank for short, and a welfare-maximizing supervisor. Both agents live for three periods, denoted with $t = \{0, 1, 2\}$ respectively, and share a common discount factor β.[8] Each period t is characterized by the stochastic return on loans $r_{l,t}$ which follows an AR(1) process (more below). In period $t = 0$, an initial bank equity endowment $E_0 > 0$ and initial return state $r_{l,0}$ realize. Observing these, the supervisor decides on the optimal stress-test tightness τ. In period $t = 1$, the representative bank observes an evolved loan return $r_{l,1}$ and E_0, and decides how much of the inherited equity to pay out as dividends versus to retain

[7] We rely on the serially auto-correlated loan returns from Bolton et al. (2020), but abstract from bank default and investments in risk-free bonds for tractability, as these play a subordinate role in a three-period model, where the choice is only between consuming today versus tomorrow. Similarly to Lambrecht and Myers (2012), we further assume that deposit rates are fixed and we rely on their Proposition 1 that provides a micro-foundation for the bank objective function proposed here. Here, we take advantage of the fact that normally distributed future loan returns simplify their exponential utility function to mean-variance utility. We additionally include a supervisor constraining bank choices via stress tests.

[8] We make this assumption for simplicity but it does not affect the model outcomes. As will be discussed in more detail in Section 3.3.3., the supervisor has preferences only about the expected level and variance of lending in period $t = 1$. Therefore, there is no intertemporal trade-off for the supervisor that is influenced be the discount factor.

for loan investments. Here, the additional deposit financing of loans is constrained by both the stress test and a minimum equity-to-asset ratio requirement. In period $t = 2$, a further evolved loan return state $r_{l,2}$, together with inherited loan, deposit, and equity levels, determines the final dividend payment by the bank to the investor.



The Investor There exists a representative investor who is hand-to-mouth and subject to mean-variance utility $u(\cdot)$ from received time t dividends d_t.[9] We denote the resulting aversion to risk with γ, such that:

$$u(d_t) = \mathbb{E}[d_t] - \frac{\gamma}{2}\mathbb{VAR}[d_t] \tag{3.1}$$

The Bank's Balance Sheet The investor dividends are financed through an initial equity endowment E_0 in a representative bank. At time $t = 1$, the bank observes E_0, a loan return state $r_{l,1}$, and the two regulatory constraints (more below). Given these states, the bank first decides how much initial dividends d_1 to pay versus how much equity E_1 to retain.

$$d_1 = E_0 - E_1 \tag{3.2}$$

Subsequently, the bank additionally sources costly deposits D_1 at the exogenous interest rate r_d, to finance investments in the risky loans L_1:

[9] This assumption is micro-founded by Lambrecht and Myers (2012), who show that payout smoothing naturally arises when insiders are risk averse and/or subject to habit formation. Here, we rely on their result from Proposition 1 and directly model an objective function over dividends rather than over managerial rents subject to investor participation constraints.

$$L_1 = E_1 + D_1 \tag{3.3}$$

In period $t = 2$, a new loan return $r_{l,2}$ realizes, where we assume that the loan returns follow an AR(1) process:

$$r_{l,t} = \mu_l + \rho_l r_{l,t-1} + \sigma_l \epsilon_t \qquad \text{where} \quad \epsilon_t \sim \mathcal{N}(0,1), \quad \mu_l > r_d, \quad \rho_l \in (0,1) \tag{3.4}$$

Then the combined choices of equity E_1, deposits D_1, and lending L_2 determine dividends d_2. Accounting for the underlying AR(1) process and the loan return state $r_{l,1}$, this implies:

$$d_2 = r_{l,2} L_1 - r_d D_1 + E_1 \qquad \text{where} \quad d_2 \sim \mathcal{N}\left(\left(\mu_l + \rho_l r_{l,1}\right) L_1 - r_d D_1 + E_1,\ L_1^2 \sigma_l^2\right) \tag{3.5}$$

The Supervisory Constraints The choices of E_1, D_1, and L_1 are restricted by two supervisory constraints: a minimum equity-to-asset ratio constraint and a stress-test constraint. The first defines a minimum equity-to-asset ratio χ that effectively restricts the bank's debt financing of loans. Here, we assume that the minimum ratio χ is given exogenously.[10] For the choices E_1 and L_1 this implies:

$$\frac{E_1}{L_1} \geq \chi \tag{3.6}$$

The stress-test constraint is forward looking instead, and requires that the bank's available equity at time $t = 2$ cannot drop below χ even under a severely adverse loan return state realization $r_{l,2}$. Here, the expected available equity is the sum of the retained equity E_1 and next period profits $\Pi_2(\tau)$ simulated for stress-test scenario τ:

[10]This follows the narrative that global minimum capital standards, such at the Basel III requirements, are not quickly and easily adjustable by a national authority without severe costs. Furthermore, it allows us to focus on the effect of the forward looking constraint.

$$\Pi_2(\tau) = (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d D_1 \qquad \text{where} \quad \overline{\mu}_l = \frac{\mu_l}{1-\rho_l} \tag{3.7}$$

Here, $\overline{\mu}_l$ denotes the unconditional mean of the AR(1) process and τ defines the number of standard deviations below $\overline{\mu}_l$ that describe the adverse scenario of $r_{l,2}$. As τ defines the severity of the adverse scenario, we will refer to it as stress-test constraint tightness throughout the paper. For now, tightness $\tau > 0$ is taken as given and can be interpreted as a model parameter. In Section 3.3., we relax this assumption and explicitly determine the optimal τ numerically. With the definition of $\Pi_2(\tau)$ in mind, the stress-test constraint thus takes the following shape:

$$\frac{E_1 + \Pi_2(\tau)}{L_1} \geq \chi \tag{3.8}$$

The Bank's Optimization Problem The above described constraints complete the model environment and we now turn to the bank optimization problem in period $t = 1$. For this, we denote the investor's total utility from d_1 and d_2 with $U(d_1, d_2)$. The bank's optimization problem is thus:

$$U(d_1, d_2) = \max_{E_1,\, L_1} \quad d_1 + \beta\left[\mathbb{E}[d_2] - \frac{\gamma}{2}\mathbb{VAR}(d_2)\right] \tag{3.9}$$

$s.t.$

$$d_1 = E_0 - E_1 \tag{3.10}$$

$$L_1 = E_1 + D_1 \tag{3.11}$$

$$d_2 = r_{l,2}L_1 - r_d D_1 + E_1 \qquad \sim \mathcal{N}\Big((\mu_l + \rho_l r_{l,1})L_1 - r_d D_1 + E_1,\ \sigma_l^2 L_1^2\Big) \tag{3.12}$$

$$E_1 \geq \chi L_1 \tag{3.13}$$

$$E_1 + \Pi_2(\tau) \geq \chi L_1 \qquad \text{where} \quad \Pi_2(\tau) = (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d D_1 \tag{3.14}$$

$$L_1 \geq 0 \tag{3.15}$$

$$E_1 \in [0, E_0] \tag{3.16}$$

Here, Equations (3.10) - (3.12) are the bank's balance sheet constraints, Inequalities (3.13) and (3.14) denote the two supervisory constraints on equity, and Constraints (3.15) and (3.16) are the feasibility constraints on lending and equity.[11]

Parameter Restrictions For the AR(1) process on loan returns, we assume that $\mu_l > 0$, $\rho_l \in (0,1)$ and $\sigma_l > 0$. For the supervisory constraints, we assume $\chi \in (0,1)$ and $\tau > 0$. For the risk-aversion we assume that $\gamma > 0$. For the initial equity endowment, we assume that $E_0 >> 0$, reflecting that we are dealing with large banks. Finally, for the deposit rate, we assume that $r_d < \mu_l$ and $1 + r_d < 1/\beta$, jointly ensuring that debt financing of loans is desirable.[12]

3.2.2. The Bank's Optimal Choices

We now turn to solving the bank optimization, starting with simplifying the two supervisory constraints: the minimum equity-to-asset ratio (3.13) and the stress-test constraint (3.14). First, we use the budget constraint in (3.11) and the definition of $\Pi_2(\tau)$ to rearrange the stress-test constraint:

$$E_1 + (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d(L_1 - E_1) \geq \chi L_1 \tag{3.17}$$

$$E_1 \geq \frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} L_1 \tag{3.18}$$

Comparing this to the minimum equity-to-asset ratio constraint in (3.13), it is easy to see that, for sufficiently large τ, the stress-test constraint always binds first:

$$\frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} \geq \chi \tag{3.19}$$

$$\tau \geq \frac{\overline{\mu} - r_d(1 + \chi)}{\sigma_l} = \tilde{\tau} \tag{3.20}$$

And for τ below $\tilde{\tau}$, the minimum equity-to-asset ratio constraint binds first. In either case, the second constraint is binding exclusively in states where the first one is binding too.

[11] Constraint (3.15) implies that the bank cannot short-sell loans. In (3.16), the lower bound implies that the bank cannot debt-finance dividends and the upper bound rules out additional equity injections.

[12] The latter implies that shareholders are less patient than depositors and thus have a preference for debt-financing of loans. As Gollier et al. (1997) discuss, this is a necessary assumption for this type of banking models and thus commonly found. The alternatives with $1/\beta = 1 + r_d$ and $1/\beta < 1 + r_d$ would respectively imply that the Modigliani Miller theorem holds or that the bank exclusively equity-finances loans.

***Lemma* 9.** *There exists a stress-test tightness threshold $\tilde{\tau}$, such that :*

(i) If $\tau < \tilde{\tau}$, the minimum equity-to-asset ratio constraint always binds first.

(ii) If $\tau \geq \tilde{\tau}$, the stress-test constraint always binds first.

The results from *Lemma 9* allow us to generalize the bank optimization problem to nest both supervisory constraints in a single equity constraint:

$$E_1 \geq \chi(\tau) L_1 \qquad \text{where} \qquad \chi(\tau) = \begin{cases} \chi & \tau < \tilde{\tau} \\ \frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} & \tau \geq \tilde{\tau} \end{cases} \tag{3.21}$$

Relying on this, we then derive the bank's optimal equity, dividend, and lending choices as a function of $\chi(\tau)$. The proof is described in detail in Appendix 2.5., but follows a few very intuitive steps. First, it can be shown that, given the parameter assumptions, equity-financing loans is never desirable. Thus, the revised minimum equity constraint is always binding at the optimum. Denote the optimal loan level with L_1^*. Then this implies:

$$L_1^* = \frac{E_1}{\chi(\tau)} \tag{3.22}$$

This result can be substituted into the bank optimization problem to simplify it further. Temporarily ignoring the feasibility constraints on equity, equating the first-order-condition with respect to retained equity with zero, yields the following optimal equity level E_1^*:

$$E_1^* = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{3.23}$$

However, E_1 is feasibility-constrained from below at zero and from above at E_0. Inserting these bounds in the above Equation (3.23) and rearranging allows us to derive two thresholds $\underline{r_l}$ and $\overline{r_l}$:

$$\underline{r_l} = \frac{1}{\rho_l}\left[r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{3.24}$$

$$\overline{r_l} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{3.25}$$

Here, threshold $\underline{r_l}$ denotes the return state $r_{l,1}$ below which no equity is retained and $d_1^* = E_0$. $\overline{r_l}$ denotes the return threshold above which equity is fully retained and $E_1^* = E_0$. With this, the optimal choices are fully characterized for a given $\chi(\tau)$, and summarized in *Proposition 1*.

***Proposition* 11.** *A given constraint tightness τ, equity endowment E_1, and return state $r_{l,1}$ imply the following optimal bank choices:*

(i) If $r_{l,1} \leq \underline{r_l}$ all initial equity is paid out, such that:

$$d_1^* = E_0 \tag{3.26}$$

$$E_1^* = L_1^* = d_2^* = 0 \tag{3.27}$$

(ii) If $r_{l,1} \in (\underline{r_l}, \overline{r_l})$, some equity is paid out and some retained, such that:

$$E_1^* = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{3.28}$$

$$d_1^* = E_0 - E_1^* \tag{3.29}$$

$$L_1^* = \frac{E_1^*}{\chi(\tau)} \tag{3.30}$$

$$d_2^* = \frac{E_1^*}{\chi(\tau)}(r_{l,2} - r_d) + E_1^*(1 + r_d) \tag{3.31}$$

(iii) If $r_{l,1} \geq \overline{r_l}$, the initial equity is fully retained, such that:

$$E_1^* = E_0 \tag{3.32}$$

$$d_1^* = 0 \tag{3.33}$$

$$L_1^* = \frac{E_0}{\chi(\tau)} \tag{3.34}$$

$$d_2^* = \frac{E_0}{\chi(\tau)}(r_{l,2} - r_d) + E_0(1 + r_d) \tag{3.35}$$

It is important to note that the kinks of the lending function are not just outliers of the return distribution but are quantitatively important: For an initial equity level equal to the optimal equity level at the unconditional mean of the return process (i.e. $E_0 = E^{ss}(\tau)$), the full-retainment return level is exactly equal to the unconditional mean of the return process. To see this, first define the steady state equity level for a given stress-test tightness τ

$$E^{ss}(\tau) = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\bar{\mu}_l - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{3.36}$$

and substitute it into the full-retainment return level

$$\overline{r_l} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}\left(\frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\bar{\mu}_l - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]\right) + r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{3.37}$$

which simplifies to

$$\overline{r_l} = \frac{1}{\rho_l}\left(\bar{\mu}_l - \mu_l\right) = \bar{\mu}_l \tag{3.38}$$

Therefore, the bank will retain all its initial equity for all return states equal to or larger than the unconditional mean of the return process. The associated lending function will, thus, be also flat for all return states above the unconditional mean. This discontinuity prevents us from deriving a closed-form solution for the optimal stress-test tightness τ^* so that we rely on a numerically solution in Section 3.3.3. instead.

3.2.3. The Effect of Stress Tests

In this section, we analyze how E_1^* and L_1^* change when the supervisor decides to introduce a stress-test constraint by raising τ above $\tilde{\tau}$. For this purpose, we introduce two additional superscripts ^{e} and ^{s}, denoting the equilibrium outcomes under a binding minimum equity-to-asset ratio and a binding stress-test constraint, respectively.

First it can be shown that raising τ implies a higher $\chi(\tau) > \chi$, which consequently results in a higher no-retainment state $\underline{r_l}$. We thus have that:

$$\underline{r_{l,1}^s} > \underline{r_{l,1}^e} \tag{3.39}$$

An introduction of a τ above $\tilde{\tau}$ also implies that the full-retainment state is reached earlier:

$$\overline{r_{l,1}^s} < \overline{r_{l,1}^e} \tag{3.40}$$

This implies that at the low end of the return distribution, a stress-test constraint incentivizes banks to retain equity only in relatively better states. At the high end of the return state distribution, full retainment is reached already at relatively worse states. Complementing this, it can be shown that for all $r_{l,1}$ above $\underline{r_l}$ and below $\overline{r_l}$, the optimal retained equity E_1^* increases linearly in $r_{l,1}$ but with a steeper slope, the higher the τ:

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma \sigma_l^2}\rho_l \qquad\qquad \frac{\partial^2 E_1}{\partial r_{l,1} \partial \tau} = \frac{\rho_l}{(1 + r_d)\gamma \sigma_l} > 0 \tag{3.41}$$

Therefore, there exists a return state $\tilde{r} \in (\underline{r_{l,1}^e}, \overline{r_{l,1}^e})$, below (above) which a stress-test constrained bank retains less (more) equity than if it was constrained by the minimum-equity constraint only. Using Equation (3.23) we can characterize this threshold $\tilde{r}$ as:

$$\tilde{r}_l = \frac{1}{\rho_l}\left[r_d - \mu_l + \left(\chi(\tau) + \chi\right)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{3.42}$$

$$= \underline{r^s_{l,1}} + \frac{\chi}{\rho_l}\left(\frac{1}{\beta} - 1 - r_d\right) \tag{3.43}$$

Here, Equation (3.43) rearranges $\tilde{r}_l$ as a function of the no-retainment state, showing it to be only marginally higher. Thus, in most return states (and definitely the positive states) more equity is retained under stress tests. Figure 3.2a below illustrates this effect of a stress-test constraint on retained equity.

Corollary 7. *Raising τ above $\tilde{\tau}$ leads to more retained equity in almost all states of the world.*

Figure 3.2: Minimum Equity-to-asset Ratio Versus Stress-test Constraint

(a) Retained Equity Levels (b) Lending Volume

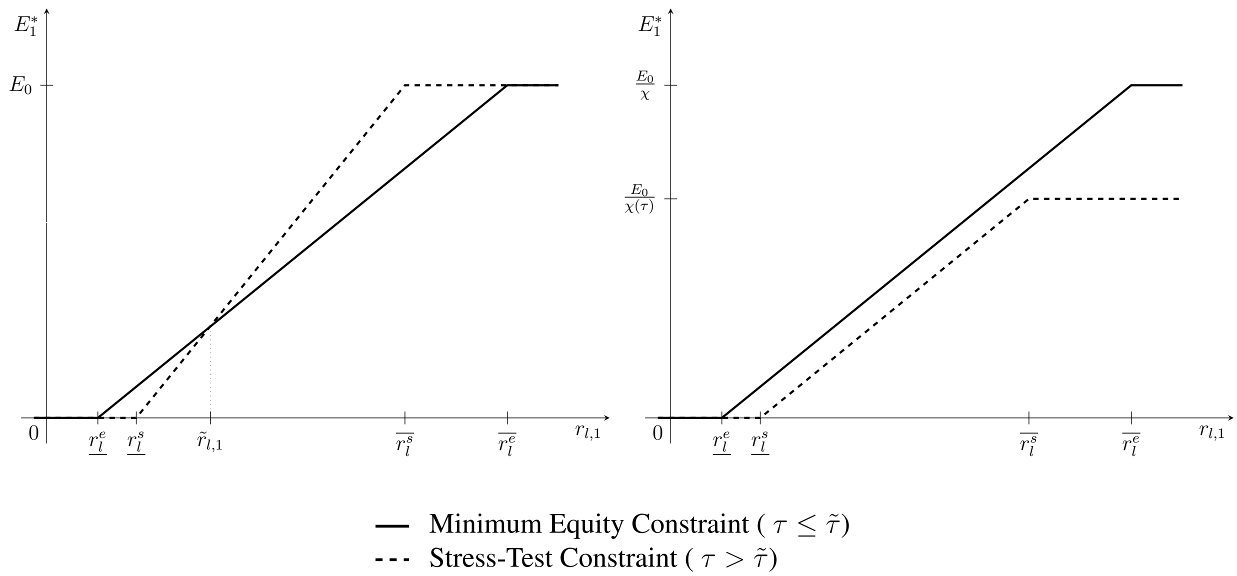


Figure 3.2b complements the comparison, by illustrating the effect of the stress-test constraint on lending. Here, we can see that the higher retained equity levels between $\tilde{r}_{l,1}$ and $r^s_{l,1}$ never translate into higher lending volumes. The extra equity is lower than the equity level that would be required to maintain the same level of lending under the tighter equity ratio constraint which is implied by the stress-test constraint. Thus:

$$L_1^{*,s} < L_1^{*,e} \quad \forall r_{l,1} > \underline{r_{l,1}^e} \tag{3.44}$$

Furthermore, the volatility of lending also decreases under the stress-test constraint, given that equity retainment starts only at a relatively better state but the full-retainment state is reached earlier.

***Corollary* 8.** *Raising τ above $\tilde{\tau}$ implies strictly lower but less volatile lending.*

3.3. Calibration & Optimal Stress-test Tightness τ

We now turn to the supervisory choice of τ in period 0 and the resulting impact on lending and equity levels. Since this analysis requires a realistic model calibration, we start by discussing our model calibration in Section 3.3.1.. We then use the calibrated model to quantify the marginal effects of adjusting the stress-test tightness τ on lending and equity in Section 3.3.2.. In a final step, we compute the optimal choice of τ in Section 3.3.3..

3.3.1. Model Calibration

To provide a quantitative estimate of the optimal τ, we calibrate our model with two sets of parameters (see Table 3.1).

The first set of parameters (Panel A. of Table 3.1) consists of the discount factor, the risk aversion parameter, and the minimum equity-to-asset ratio. We pick a discount factor β equal to 0.99, which corresponds to an annualized real interest rate of 1%. We take the risk aversion parameter from Eisfeldt et al. (2020) and set it to 4.37. Furthermore, we take a minimum equity-to-asset ratio of 7% as given.

The second set of parameters (Panel B. of Table 3.1) describes the loan return process as well as the return on deposits. For these parameters we use balance sheet data of U.S. Bank Holding Companies with more than \$10bn in assets between 2009 - 2019 (i.e. banks subject to CCAR stress tests) to calibrate the parameters of the loan return process as well as the return on deposits.

To calibrate the return process, we follow De Nicolò et al. (2014) and estimate an AR(1) process on the mean excess return on assets. We use the excess return over the risk-free interest rate to make sure that return movements are not driven by movements in the risk free interest. We compute the excess return on assets as the ratio of the interest and non-interest revenues to lagged assets (items *bhcp4000* and *bhck2170* respectively in the FR Y–9C reports) minus the 1-year Treasury rate. We then add this excess return to our implied (time-invariant) risk-free rate $1/\beta - 1$ to arrive at the mean of the return process. The calibrated return process has a mean of 1.02% with a standard deviation of 0.52% and a autocorrelation of $\rho_l = 0.62$, which implies an unconditional mean return of 2.66%.

Table 3.1: Calibration

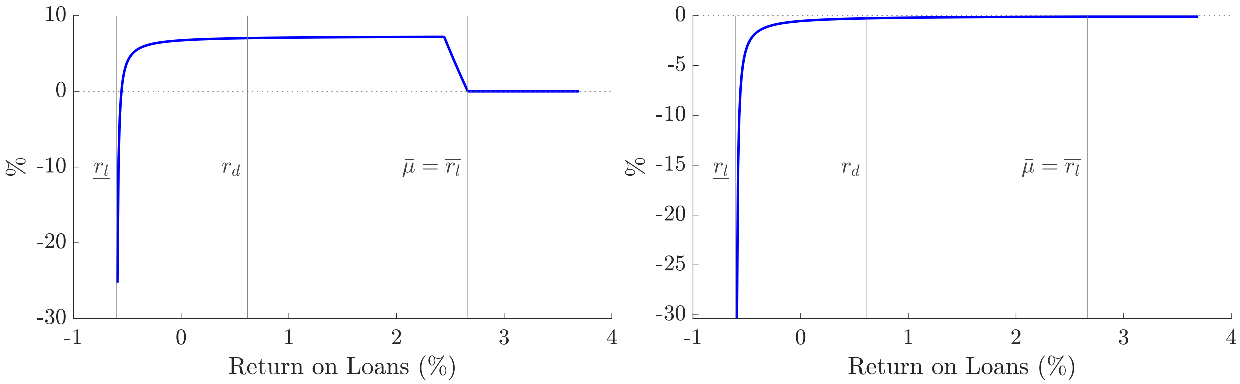
Description	Parameter	Value
A. Parameters assumed / obtained from literature		
Discount Factor	β	0.99
Risk Aversion	γ	4.370
Minimum Equity-to-asset Ratio	χ	0.07
B. Parameters obtained from data		
Mean Return of Risky Asset (%)	μ_l	1.02
AR(1) of Risky Asset	ρ_l	0.62
SD of Risky Asset (%)	σ_l	0.52
Lending Spread (%)	$1/\beta - 1 - r_d$	0.39
Return on Deposits (%)	r_d	0.62

To calibrate the deposit rate r_d, we again start by eliminating the movements of the risk-free rate and first estimate the deposit spread. We compute the deposit spread as the mean difference between the 1-year Treasury rate and the mean deposit rate, given by the ratio of interest paid on deposits (the sum of items *bhckhk03*, *bhckhk04*, *bhck6761*, and *bhck4172*) to lagged deposits (the sum of items *bhdm6631*, *bhdm6636*, *bhfn6631*, *bhfn6636*). We then subtract this deposit spread from our implied risk free rate $1/\beta - 1$ to arrive at the deposit rate. Over our sample period, bank deposits yielded on average 0.39 percentage points less than the 1-year Treasury rate, yielding a return on deposits of 0.62% for our implied risk free rate of 1%.

3.3.2. Effect of Stress Tests on Equity and Lending

To illustrate the effect of stress tests, we use the calibrated model and plot the marginal responses of equity and loan levels (in %) to a unit increase in the tightness of the stress-test constraint τ in Figure 3.3. It is clear that the effect of a higher stress-test constraint is highly non-linear in the state of the business cycle, i.e. the return state.

Figure 3.3: Marginal Response of Equity and Lending to a Unit Increase in τ



(a) Marginal Response of Equity (%)

(b) Marginal Response of Lending (%)

Following an increase of the stress-test tightness τ, equity (left panel) is lower for very bad states of the world due to an increased no-retainment threshold (see Equation (3.43)). However, for most of the return realizations below the unconditional mean return $\bar{\mu}_l$, equity is higher following the increase of τ. For return realizations above $\bar{\mu}_l$ the increase of τ does not lead to higher equity retainment, since banks retain all of their equity either way.

Lending volumes (right panel) are affected by changes of both retained equity as well as the minimum equity constraint in response to an increase of the stress-test tightness τ. For all return states below the unconditional mean return $\bar{\mu}_l$, an increase in τ reduces lending because the the increase in the minimum equity constraint offsets the increase in retained equity. For all return states above the unconditional mean return $\bar{\mu}_l$, retained equity is unchanged but the increased minimum equity constraint leads to lower lending. However, this effect is marginal because a unit increase in τ increases the implied equity constrained only by $\sigma_l/1+r_d$.

This demonstrates that in all but very bad states of the world, the increase of τ can weakly enhance the safety of banks, but this unequivocally comes at the cost of lower lending levels, as the right panel shows. This reduction in lending, however, approaches zero as the return realisations increase.

3.3.3. The Supervisory Choice of τ

We now investigate how a supervisor optimally sets the severity of simulated losses used in the stress test (i.e. the number of standard deviationsτ below the mean return $\bar{\mu}$) with the objective to ensure stable lending levels.[13] Here, Corollary 8 highlights the supervisory trade-off between reduced but consequently less volatile lending. To capture this trade-off, we assign the welfare weight $\omega \geq 0$ to the expected variance of optimal bank lending L_1^*. Then, observing E_0 and $r_{l,0}$, the supervisor solves:

$$\max_{\tau} \quad \mathbb{E}[L_1^* \mid r_{l,0}, E_0] - \omega \mathbb{VAR}_0[L_1^* \mid r_{l,0}, E_0] \tag{3.45}$$

s.t.

$$\chi(\tau) \in [\chi, 1) \tag{3.46}$$

where

$$r_{l,1} \leq \underline{r_l}: \quad L_1^* = 0 \tag{3.47}$$

$$r_{l,1} \in (\underline{r_l}, \overline{r_l}): \quad L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma \sigma_l^2} \tag{3.48}$$

$$r_{l,1} \geq \overline{r_l}: \quad L_1^* = \frac{E_0}{\chi(\tau)} \tag{3.49}$$

Equations (3.47) to (3.49) in Section 2.6. show that the supervisor anticipates a rectified normally distributed L_1^* with upper and lower bounds: (3.47) states that below $\underline{r_l}$, lending L_1^* is set to zero; (3.48) implies that between $\underline{r_l}$ and $\overline{r_l}$ lending is normally distributed with $\mathcal{N}\left(\mu_{L_1}, \sigma_{L_1}^2\right)$; (3.49) states that above $\overline{r_l}$, lending is set to $E_0/\chi(\tau)$.

To identify the optimal stress-test tightness τ^*, we utilize our parameterization from Section 3.3.1. and computationally maximize the supervisor's welfare directly, subject to the respective constraints. As argued previously, the fact that loans follow a two-sided rectified distribution prevents us from deriving a closed-form expression for the optimal stress-test tightness so that we solve this problem numerically. Since the results depend to a large degree on the amount of initial equity E_0, we first define the steady state level E_1^{ss} in the absence of stress tests as

[13] Note that this supervisory objective is taken directly from the Federal Reserve Board (2020c).

$$E_1^{ss} = \frac{\chi}{\gamma \sigma_l^2} \left[\bar{\mu}_l - r_d - \chi \left(\frac{1}{\beta} - 1 - r_d \right) \right] \tag{3.50}$$

and fix the initial equity endowment E_0 at this level to ensure comparable results.

To examine the supervisor's decision in more detail, we compute the optimal τ^* for i) different relative welfare weights ω and ii) different realizations of the period 0 return $r_{l,0}$. In particular, we consider four possible scenarios for $r_{l,0}$: one severe crisis scenario where $r_{l,0} = -0.43\%$ (a $6 * \sigma_l$ movement), one mild slowdown where $r_{l,0} = 1.62\%$ (a $2 * \sigma_l$ movement), one median case where $r_{l,0} = 2.66\%$, and one upswing scenario where $r_{l,0} = 3.69\%$ (a $2 * \sigma_l$ movement). For each of these initial return realizations we compute the optimal stress-test tightness for a supervisor who does not care about lending volatility (i.e. $\omega = 0$), a supervisor who cares as much about lending volatility as about lending levels (i.e. $\omega = 1$), a supervisor who dislikes lending volatility as much as the investor (i.e. $\omega = \gamma/2$), and a supervisor who dislikes lending volatility twice as much as the investor (i.e. $\omega = \gamma$).

Table 3.2 below states the resulting, numerically derived optimal stress-test tightness τ^*, the implied minimum equity to asset ratio $\chi(\tau)^*$ (see Equation (3.21)), and the associated supervisory welfare for the different environments. Based on the implied welfare for the respective τ^*, it is clear that the supervisory welfare function is increasing in the initial return realization $r_{l,0}$ and decreasing in the weight given to the variance of loans. The supervisor therefore optimally sets $\tau^* = 4.05$ such that $\chi(\tau^*) = \chi$ when she does not derive any disutility from the variance of loans (i.e. when $\omega = 0$) in order to maximize the level of loans. However, as ω increases and she derives more disutility from the variance of loans, she optimally sets a higher τ^* to reduce that variance. We furthermore note that τ^* increases less in ω for higher realisations of $r_{l,0}$.

In general, a supervisor who cares about both the level and the volatility of lending finds stress test capital buffers in the range of 1% to 9% to be optimal. This matches well the Federal Reserve's publicly announced stress-test buffers, reported to be between 2.5% to 7.5% in the 2021 CCAR report (Federal Reserve Board, 2021). This indicates that we are able to capture well both the mechanism behind and the magnitude of bank balance sheet choices under stress tests.

Table 3.2: Optimal Stress-test Tightness and Supervisor Welfare

Welfare Weight	Optimal Tightness τ^*	$\chi(\tau^*)$	Welfare
	$r_0 = \bar{\mu} - 6 * \sigma_l = -0.43\%$		
$\omega = 0$	4.05	0.07	72.05
$\omega = 1$	104.91	0.75	1.96
$\omega = \gamma/2$	153.88	0.84	-15.34
$\omega = \gamma$	185.57	1.00	-42.53
	$r_0 = \bar{\mu} - 2\sigma_l = 1.63\%$		
$\omega = 0$	4.05	0.07	138.52
$\omega = 1$	16.31	0.13	80.71
$\omega = \gamma/2$	19.06	0.15	72.78
$\omega = \gamma$	21.80	0.16	66.11
	$r_0 = \bar{\mu} = 2.66\%$		
$\omega = 0$	4.05	0.07	162.96
$\omega = 1$	9.16	0.10	115.93
$\omega = \gamma/2$	10.53	0.10	108.25
$\omega = \gamma$	11.81	0.11	101.84
	$r_0 = \bar{\mu} + 2\sigma_l = 3.69\%$		
$\omega = 0$	4.05	0.07	172.49
$\omega = 1$	5.14	0.08	150.60
$\omega = \gamma/2$	5.96	0.08	143.02
$\omega = \gamma$	6.70	0.08	136.70

Note: This table shows the results of computationally maximizing the supervisor's welfare, subject to the respective constraints (see Equation (3.45)-(3.49)). We rely on the calibration from Section 3.3.1. to derive the optimal stress-test tightness τ, the implied minimum equity to asset ratio $\chi(\tau^*)$ (see Equation (3.21)) and the associated supervisor welfare for different supervisory welfare weights ω and realizations of the initial return state $r_{l,0}$.

3.4. Voluntary Stress-test Violation

In our baseline model environment, banks can neither violate the minimum equity-to-asset ratio nor the stress-test constraint. The U.S. stress test framework, however, allows for voluntary violation of the stress-test constraint, albeit automatically triggering a (partial) ban on dividend payments (see Appendix 2.4. for details). This violation allows the bank to invest up to a binding minimum-equity-to-asset ratio constraint instead. In this section, we investigate when a bank might find it optimal to purposely violate the stress-test constraint. For simplicity, we assume that this immediately triggers a total ban on dividend payments in that period. Then, voluntary violation implies the following equalities:

$$d_1 = 0 \tag{3.51}$$

$$E_1 = E_0 \tag{3.52}$$

$$D_1 = L_1 - E_0 \tag{3.53}$$

Inserting these equalities in the original maximization problem results in the following revised bank objective:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{3.54}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi}\right] \tag{3.55}$$

Because full retainment implies sub-optimally high equity levels, the bank no longer chooses to equity-finance as little as possible. The upper feasibility limit in (3.55) reflects this, where now χ applies instead of $\chi(\tau)$. Quite intuitively, the upper feasibility limit is binding in very high return states above a threshold $\overline{r_{l,1}^V}$, where the bank would like to invest more in loans than the minimum equity requirements allow. Hence:

$$L_1^{*V} = \frac{E_0}{\chi} \qquad \forall r_{l,1} \geq \overline{r_l^V} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + r_d - \mu_l\right] \tag{3.56}$$

On the contrary, the lower feasibility limit is binding in bad return states, where the bank would like to invest nothing but must at least invest E_0. This applies to all return states below threshold $\underline{r_{l,1}^V}$:

$$L_1^{*V} = 0 \qquad \forall r_{l,1} \leq \underline{r_l^V} = \frac{1}{\rho_l}\left[\sigma_l^2 E_0 + r_d - \mu_l\right] \tag{3.57}$$

In between the two return thresholds, the bank equity-finances loans with a share strictly above χ but below one. The optimal loan level is determined by the first-order-condition of the objective function (3.54), when both feasibility constraint multipliers are zero. For $r_{l,1}$ above $\underline{r_{l,1}^V}$ and below $\overline{r_{l,1}^V}$, this implies an optimal lending:

$$L_1^{*V} = \frac{\mu_l - \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \qquad \forall r_{l,1} \in \left(\underline{r_{l,1}^V},\ \overline{r_{l,1}^V}\right) \tag{3.58}$$

To derive when voluntary violation is optimal, we must compare the total shareholder utility from voluntary violation, denoted with $U^V(d_1, d_2)$, to the one from the baseline analysis, denoted $U(d_1, d_1)$.

Total utility under voluntary violation:

$$r_{l,1} < \underline{r_l^V}: \quad U^V(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{3.59}$$

$$r_{l,1} \in [\underline{r_l^V}, \overline{r_l^V}]: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right) L_1^{*V} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*V}\right)^2 + (1 + r_d) E_0\right] \tag{3.60}$$

$$\text{where} \quad L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{3.61}$$

$$r_{l,1} > \overline{r_l^V}: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi^2} + E_0(1 + r_d)\right] \tag{3.62}$$

Total utility under compliance (baseline):

$$r_{l,1} < \underline{r_l}: \quad U(d_1, d_2) = E_0 \tag{3.63}$$

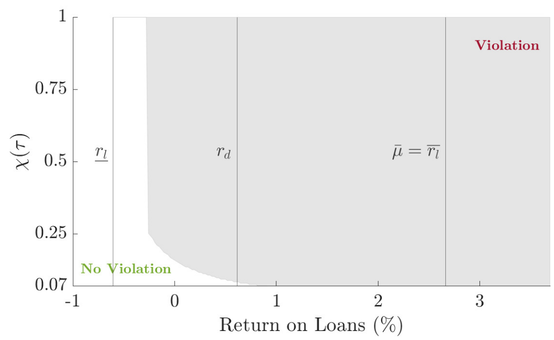
$$r_{l,1} \in \left[\underline{r_l}, \overline{r_l}\right]: \quad U(d_1, d_2) = E_0 - E_1^* + \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right) L_1^* - \frac{\gamma\sigma_l^2}{2}\left(L_1^*\right)^2 + E_1^*(1 + r_d)\right] \tag{3.64}$$

$$\text{where} \quad L_1^* = \frac{E_1^*}{\chi(\tau)} = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1 - 1/\beta + r_d)}{\gamma\sigma_l^2} \tag{3.65}$$

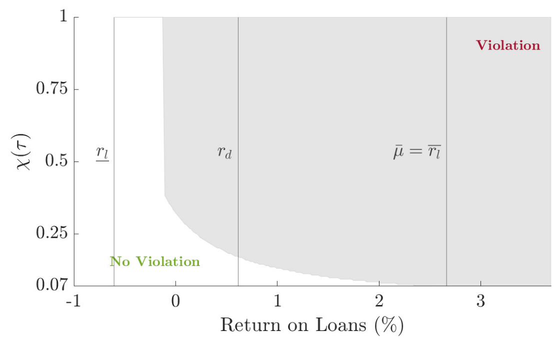
$$r_{l,1} > \overline{r_l}: \quad U(d_1, d_2) = \beta \left[\left(\mu_l + \rho_l r_{l,1} - r_d \right) \frac{E_0}{\chi(\tau)} - \frac{\gamma \sigma_l^2}{2} \left(\frac{E_0}{\chi(\tau)} \right)^2 + E_0(1 + r_d) \right] \tag{3.66}$$

To prove when $U^V(d_1, d_2)$ exceeds $U(d_1, d_1)$ is cumbersome, as the sizes of return thresholds $\underline{r_l^V}$ and $\overline{r_l^V}$ relative to $\underline{r_l}$ and $\overline{r_l}$ strongly depend on the initially inherited equity E_0 relative to other model parameters. Hence, a large number of different utility functions would have to be compared to cover all cases. Instead, we provide insights for a meaningful parameter space and numerically study the voluntary violation decision for large US banks, given our calibration. Figure 3.4 (below) illustrates when a bank violates the stress tests voluntarily for the above presented calibration and three levels of initial equity as a function of the steady state equity level E_1^{ss}.

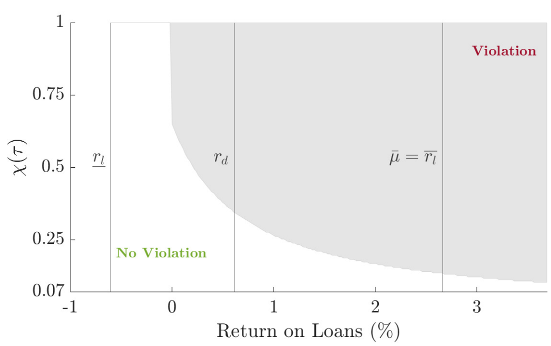
Figure 3.4: Optimal Choice of Stress-test Violation



(a) $E_0 = 0.5 \cdot E_1^{SS}$



(b) $E_0 = E_1^{SS}$



(c) $E_0 = 2 \cdot E_1^{SS}$

Each of the Panels 3.4a - 3.4c has the continuum of loan returns $r_{l,1}$ on the x-axis and the range of possible stress-test-implied minimum equity-to-loan ratio requirements on the y-axis. The gray shaded areas indicate when the bank finds it optimal to voluntarily violate the stress-test constraint. Here, we can see that this is generally the case for higher $\chi(\tau)$ and higher return states $r_{l,1}$. This should come as no surprise: the higher $\chi(\tau)$, the lower the total loans a stress-test compliant bank may issue and the more it can increase the loan capacity by voluntarily violating. Further, expanding loan capacity is more attractive in good states of the world, where risky loan investment is desirable. On the contrary, exposing (sub-optimally high) equity levels to risky loans in bad states by violating the stress-test constraint is not desirable. Therefore, the desirability of violation also decreases with the size of the initial equity endowment.

Remark 2. For U.S. stress tested banks, violation is optimal for higher tightness τ, higher loan return states $r_{l,1}$, and lower initial equity E_0.

It should be noted, however, that the voluntary violation of the stress test constraint in our model does not incur any costs over and beyond the restriction on dividend payouts, such as financial market stigma or increased supervisory scrutiny. This explains why in reality, unlike our model would predict, large banks almost never violate the stress-test constraint.

3.5. Policy Extensions

Bank stress tests are typically seen as a complementary measure to a rich set of additional prudential policies. To understand their relative effectiveness in stabilizing lending, we extend the model to include two currently utilized policy tools: the Covid-19 dividend ban and the counter-cyclical capital buffer (CCyB). Additionally, we provide a welfare comparison of stress tests to the dividend prudential target proposed by Muñoz (2020). In the latter, dividends are regulated directly, but less intensely than in an outright ban.

3.5.1. Covid-19 Dividend Restrictions

At the onset of the Covid-19 crisis, several jurisdictions introduced either an outright ban on dividend payments or a strong recommendation to stop payments temporarily (Beck et al., 2020). The goal was to boost equity and thereby counteract the procyclicality of lending. Here, we abstract from any moral suasion frictions between supervisors and banks, and analyze the effect of a dividend ban on bank lending levels.[14] An outright ban on dividends implies full equity retainment, such that:

$$d_1 = 0 \tag{3.67}$$

$$E_1 = E_0 \tag{3.68}$$

$$D_1 = L_1 - E_0 \tag{3.69}$$

Inserting these into the bank's original optimization problem results in a revised maximization similar to the one under voluntary violation:

$$U^B(d_1, d_2) = \max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{3.70}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi(\tau)}\right] \tag{3.71}$$

However, here the stress-test constraint still applies and determines the upper bound of loan investments in (3.71). It thus acts as a feasibility constraint for the revised bank maximization problem. Again, the lower and upper feasibility bounds on L_1 imply two return thresholds denoted with $\underline{r_l^B}$ and $\overline{r_l^B}$ respectively:

$$\underline{r_l^B} = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \qquad \overline{r_l^B} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)} E_0 + r_d - \mu_l\right] \tag{3.72}$$

Unlike in the baseline model, however, the two thresholds determine the share of debt financing instead of the degree of equity retainment: for return states below $\underline{r_{l,1}^C}$, the bank fully equity-finances L_1 now equal to E_0. Intuitively, in these bad return states, the shareholder would prefer to liquidate the bank but this is prevented by the dividend ban. Thus the only remaining option is to invest the existing equity in

[14] This is without loss of generality. As Beck et al. (2020) show, most European banks did indeed stop dividend payments following the ECB's recommendation.

loans.

$$L_1^{*B} = E_0 \qquad \forall\, r_{l,1} \leq \underline{r_l^B} \tag{3.73}$$

For intermediate return states, the bank sets an optimal loan level L_1^{*B} that requires a share of equity financing strictly below one but strictly above $\chi(\tau)$. Intuitively, in these return states the shareholder would actually prefer some dividends in period 1 but this is prevented by the dividend ban. At the same time, the loans are still relatively risky, limiting the attractiveness of investing in them. Thus the bank utilizes all its equity, but does not lever up as much as it could. In this case, the level of lending is:

$$L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \qquad \forall r_{l,1} \in \left(\underline{r_l^B}, \overline{r_l^B}\right) \tag{3.74}$$

For high return states above $\overline{r_l^B}$, the bank debt-finances as much as possible given E_0 and $\chi(\tau)$, where the stress-test constraint now becomes the upper feasibility limit:

$$L_1^{*B} = \frac{E_0}{\chi(\tau)} \qquad \forall r_{l,1} \geq \overline{r_l^B} \tag{3.75}$$

Comparing the optimal lending of a bank with free reign over the dividend payments with the one subject to a ban, we show that for all return states below $\overline{r_l}$, the lending is higher under the latter. Only for return states above $\overline{r_l}$ is the feasibility constraint on total lending binding under both regimes and and thus lending identical.[15]

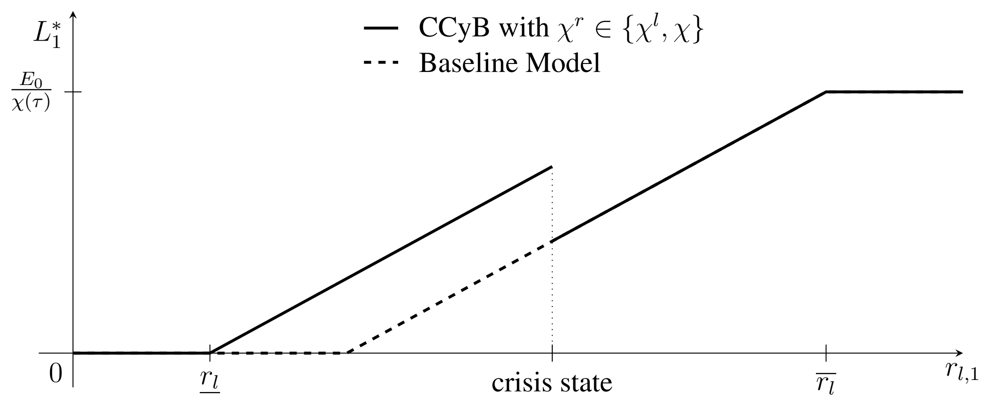
***Proposition* 12.** *A dividend ban leads to strictly higher lending during crises.*

[15]Note here that for the formal proof, we account for the fact that the thresholds $\underline{r_l}$ may be above or below $\underline{r_l^B}$. However, $\overline{r_l^B}$ is always below $\overline{r_l}$.

3.5.2. Counter-Cyclical Capital Buffer

A complementary policy tool to the dividend ban is the relaxation of the counter-cyclical capital buffer (CCyB) during times of crises. In the baseline model, we have assumed a constant χ that is state-independent. Instead, a CCyB implies a state-dependent χ^r that takes on a value $\chi^l < \chi$ for low return states. This relaxes the stress-test constraint in bad states via a reduction in $\chi(\tau)$. Relying on insights from Section 3.2.2., we know that this triggers an increase in lending and lowers the return thresholds below which no equity is retained. Figure 3.5 below illustrates this.

Figure 3.5: The Impact of CCyBs on Lending



Relaxing a CCyB is often combined with other crisis measures, such as the Covid-19 dividend ban discussed above. Two natural question are thus, how both compare in their ability to increase lending during a crisis, but also how effective is a joint introduction of both. Here, it can be shown that lending under a dividend ban is strictly higher than under a relaxed CCyB. Intuitively, the main driver of lower loan levels in bad return states is equity withdrawal, which is not adequately addressed by relaxing the CCyB. Furthermore, the CCyB actually has no additional effect once a dividend ban is put in place. A bank subject to a ban already holds sub-optimally high equity and debt-finances less than allowed. Therefore, a relaxed CCyB does not change the optimal loan levels when activated on top of a dividend ban during a crisis.

***Proposition* 13.** *Introduced individually, a relaxed CCyB increases lending during crises. However, the CCyB is less effective than a dividend ban and, if introduced additionally, has no further effect.*

We are thus able to provide an explanation for the recent policy puzzle regarding banks not using their CCyB buffers to finance lending during the Covid-19 crisis (FSB, 2021): additionally relaxing of

CCyBs simply does not impact lending choices of already dividend restricted banks.

3.5.3. Dividend Prudential Target

Finally, we discuss a substitute regulatory approach to stress tests: the dividend prudential target (DPT). Initially suggested by Muñoz (2020), the DPT restricts dividends directly by encouraging retainment in bad states and pay-outs in good states. It, thereby, attempts to directly offset the banks' dividend smoothing behavior to avoid capital depletion in bad states and reduce the pro-cyclicality of lending. In a first step, a DPT defines an ideal dividend pay-out – usually the pay-out made by an unrestricted bank in steady-state. We follow this tradition and evaluate our baseline model at the unconditional mean $\bar{\mu_l}$ of the AR(1) process. The dividends in steady-state, denoted with d_1^{SS}, take on the following value:

$$d_1^{SS} = E_1^{SS} + \bar{\mu}\frac{E_1^{SS}}{\chi} - r_d\left(\frac{E_1^{SS}}{\chi} - E_1^{SS}\right) - E_1^{SS}, \tag{3.76}$$

$$= \bar{\mu_l}\frac{E_1^{SS}}{\chi} - r_d\left(\frac{E_1^{SS}}{\chi} - E_1^{SS}\right), \tag{3.77}$$

$$= \left[\frac{\bar{\mu_l} - r_d}{\chi} + r_d\right]\frac{\chi}{,}\gamma\sigma_l^2\left[\bar{\mu_l} - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right]. \tag{3.78}$$

Consequently, a state-dependent target dividend level d_1^T is defined that increases in the return state. The goal is to incentive more payouts in good and less payouts in bad states, thereby stabablizing both retained equity and lending.

Here, we opt for the simplest possible option by scaling d_1^{SS} with the factor $r_{l,1}/\bar{\mu_l}$. This choice ensures that the target pay-out increases in return states and is exactly equally to the steady-state level in steady state:

$$d_1^T = \frac{r_{l,1}}{\bar{\mu_l}}d_1^{SS}. \tag{3.79}$$

Consequently, any (squared) deviations in dividend payouts d_1 from the target d_t^T are punished with a cost κ:

$$\frac{\kappa}{2}\left(d_1 - d_1^T\right)^2 \tag{3.80}$$

$$\frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS}\right)^2 \tag{3.81}$$

The cost κ is set by the supervisor at $t = 0$ and, similar to Jermann and Quadrini (2012), accounts for both fines to be paid and reputation costs from non-compliance. It is taken as given by the bank at $t = 1$ and enters the optimization problem in the following fashion:

$$\begin{aligned} U(d_1, d_2) = \quad & \max_{L_1, E_1} \quad E_0 - E_1 - \frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS}\right)^2 \\ & \qquad + \beta E_1 (1 + r_d) + \beta\left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma \sigma_l^2}{2} L_1^2\right] \end{aligned} \tag{3.82}$$

$$s.t.$$

$$\lambda_1 : \qquad L_1 \in \left[E_1, \frac{E_1}{\chi}\right] \tag{3.83}$$

$$\lambda_2 : \qquad E_1 \in [0, E_0] \tag{3.84}$$

An important feature to note from condition (3.83) in the maximization problem is that the optimal choice of E_1 impacts directly the feasibility constraints of L_1. We thus need derive both the optimal equity and lending choices under the DPT, taking this co-dependency into account. We, nevertheless, start by deriving the optimal equity level assuming away its impact on lending. After taking the FOC condition with respect to E_1, equating it to zero, and checking feasibility, we get the following constrained-optimal equity levels:

$$E_1^* = 0 \qquad \forall\, r_{l,1} \leq r_l^* = \frac{\bar{\mu}_l}{d_1^{SS}} \frac{1}{\kappa}\left(\beta\left(1 + r_d\right) - 1\right), \tag{3.85}$$

$$E_1^* = \frac{1}{\kappa}\left(\beta\left(1 + r_d\right) - 1\right) + E_0 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} \qquad \forall\, r_{l,1} \in (r_l^*, r_l^{**}] \tag{3.86}$$

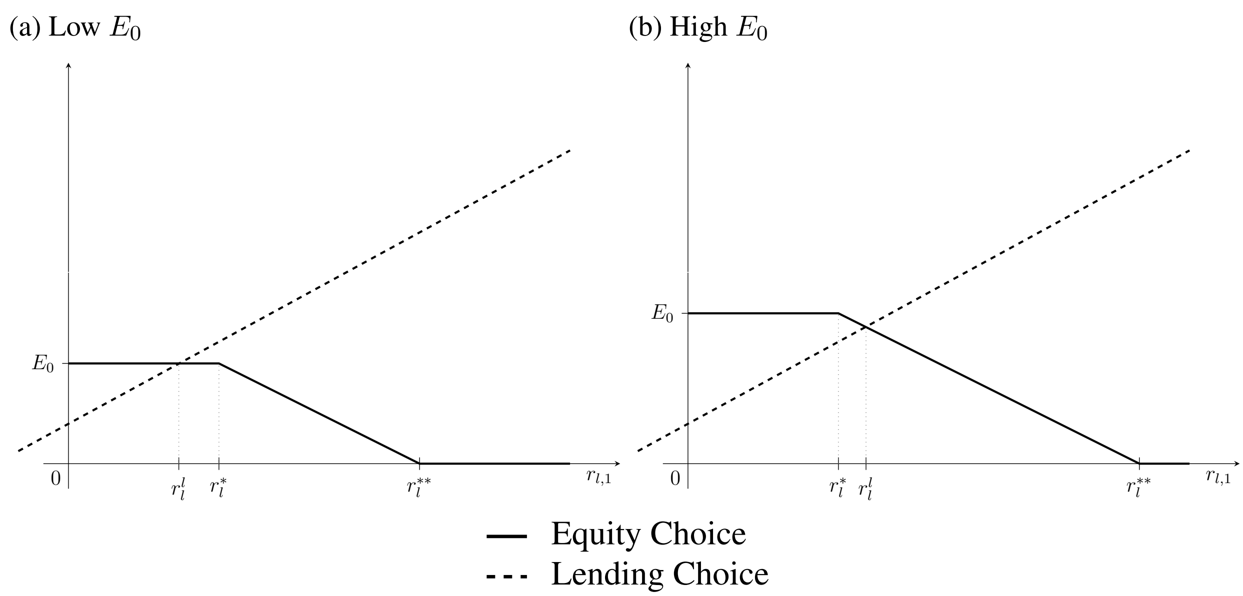
$$E_1^* = 0 \qquad \forall\, r_{l,1} > r_l^{**} = \frac{\bar{\mu}_l}{d_1^{SS}}\left[\frac{1}{\kappa}\left(\beta\left(1 + r_d\right) - 1\right) + E_0\right] \tag{3.87}$$

Here, we would immediately like to point out that equity now behaves quite differently than under stress tests: more equity is retained in bad states and less in good. This also impacts the optimal lending. Abstracting from feasibility constraints, taking the FOC with respect to L_1 and consequently equating it to zero yields the following optimal lending level:

$$L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2}. \tag{3.88}$$

The Figure 3.6 illustrates both the optimal equity described in equations (3.85)-(3.87) and the unconstrained optimal lending in (3.88). Here, it is immediately visible that L_1^* in (3.88) is not feasible for low return states $r_{l,1}$, where the bank would ideally like to lend out less than the equity it would like to retain.

Figure 3.6: Optimal Equity for Unrestricted Lending under the DPT



Here, we can observe two cases: For low E_0 the feasibility constraint only binds for return states below the full-retainment state (Figure 3.6a); for high E_0, the feasibility constraint already binds above the full retainment state (Figure 3.6b). The threshold level on initial equity $\overline{E_0}$ is:

$$\overline{E_0} = \frac{\rho_l \bar{\mu}_l}{\gamma \sigma_l^2 d_1^{SS}} \frac{1}{\kappa} \big(\beta\left(1+r_d\right)-1\big) + \frac{\mu_l - r_d}{\gamma \sigma_l^2} \tag{3.89}$$

We denote the return state below which the lower feasibility limit on L_1^* binds with r_l^l. For the case of low $E_0 \leq \overline{E_0}$ it can be shown, after some cumbersome re-arranging, that the bank is not willing to reduce the equity level in any return state below r_l^l to relax the lower limit on lending. Hence, the optimal equity choice is as defined as:

$$\text{If} \quad E_0 \leq \overline{E_0}:$$

$$r_l^l = \frac{\gamma \sigma_l^2 E_0 - \mu_l + r_d}{\rho_l} \tag{3.90}$$

$$L_1^* = E_1^* = E_0 \quad \forall\, r_{l,1} \leq r_l^l. \tag{3.91}$$

.

For the case with high $E_0 \geq \overline{E_0}$, the bank does find it optimal to take the impact on lending into account, when deciding how much to retain in low return states. Here, we find that for all states below r_l^l, the bank solves a slightly revised optimization problem, where $E_1 = L_1$. Taking again FOCs with respect to E_1 and equating it to zero allows us to derive a slightly different optimal equity below r_l^l (see equation (3.94)). The bank can of course only retain that much equity as long as it is below E_0. Even for high E_0, the upper-feasibility constraint is eventually binding below return states r_l^{ll}:

$$\text{If} \quad E_0 \geq \overline{E_0}:$$

$$r_l^{ll} = \frac{\bar{\mu}_l}{\beta \rho_l \bar{\mu}_l - \kappa d_1^{SS}} \Big[\beta \gamma \sigma_l^2 E_0 + 1 - \beta(1+\mu_l)\Big] \tag{3.92}$$

$$r_l^l = \frac{\bar{\mu}_l}{\rho_l \bar{\mu}_l + \gamma \sigma_l^2 d_1^{ss}} \left[\frac{\gamma \sigma_l^2}{\kappa}\big(\beta(1+r_d)-1\big) + \gamma \sigma_l^2 E_0 + r_d - \mu_l\right] \tag{3.93}$$

$$L_1^* = E_1^* = \frac{1}{\kappa + \beta \gamma \sigma_l^2}\left[-1 + \kappa E_0 - \kappa \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} + \beta(1+\mu_l+\rho_l r_{l,1})\right] \qquad \forall\, r_{l,1} \in (r_l^{ll}, r_l^l] \tag{3.94}$$

$$L_1^* = E_1^* = E_0 \qquad \forall\, r_{l,1} \leq r_l^{ll}. \tag{3.95}$$

Regardless whether E_0 is high or low, the banks preferred lending level will ultimately violate the minimum equity-to-asset ratio in high return states. This is the case for all return states above a threshold r_l^h. From there on-wards the bank takes into account that (costly) retainment allows for more (profitable) lending. Nevertheless, there exists a threshold r_l^{hh} above which the bank never retains any equity no matter how profitable lending would be:

$$r_l^h = \frac{\chi \bar{\mu}_l}{\chi \rho_l \bar{\mu}_l + \gamma \sigma_l^2 d_1^{SS}} \left[\frac{\gamma \sigma_l^2}{\chi \kappa} \left(\beta(1 + r_d) - 1 \right) + \frac{\gamma \sigma_l^2}{\chi} E_0 - \mu_l + r_d \right], \tag{3.96}$$

$$r_l^{hh} = \frac{\bar{\mu}_l \chi}{\kappa d_1^{SS} \chi - \bar{\mu}_l \beta \rho_l} \left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta \frac{\mu_l - r_d}{\chi} \right]. \tag{3.97}$$

Inserting $L_1 = E_1/\chi$ and, again, taking FOCs yields the following optimal lending and equity in high return states.

$$E_1^* = \chi L_1^* = \frac{\chi^2}{\chi^2 \kappa + \beta \gamma \sigma_l^2} \left[-1 + \kappa E_0 - \kappa \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} + \beta(1 + r_d) + \beta \frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi} \right] \quad \forall\, r_{l,1} \in (r_l^h, r_l^{hh}] \tag{3.98}$$

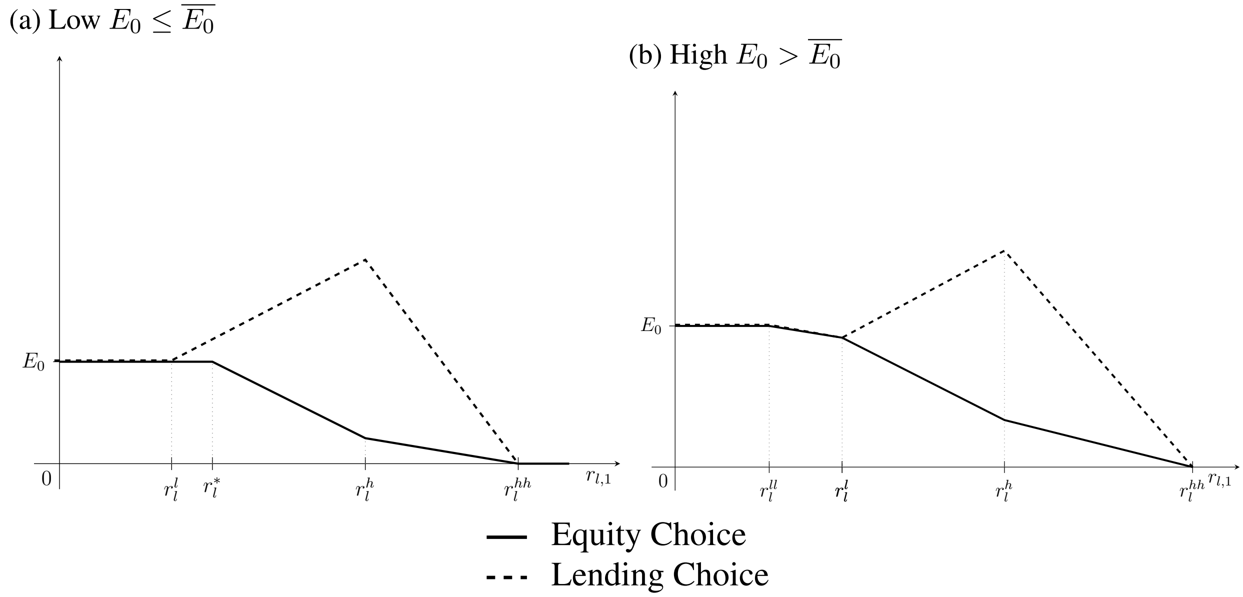
$$E_1^* = \chi L_1^* = 0 \qquad \forall\, r_{l,1} > r_l^{hh} \tag{3.99}$$

Summarizing the just derived solutions is a bit cumbersome and, we believe, not very informative for the reader. Therefore, we rather display the functional forms of L_1^* and E_1^* for both the low and high initial equity case in Figure 3.7 below. For the full set of analytical expressions on the return thresholds, the reader is kindly asked to refer to the Appendix 2.7.4..

It can be seen in Figure 3.7 that a DPT results in a hump shaped policy function over the state-space for both equity and lending. The punishment parameter κ influences the mean and variance of both by affecting equity choices directly and, furthermore, affecting the threshold interest rate levels. Recall that the supervisory authority sets κ in period 0 with the objective to stabilize lending, putting welfare weight ω on the expected lending variance.

Unfortunately, a full closed-form characterisation of the mean and variance of lending is cum-

Figure 3.7: Feasibility-Constrained Optimal Equity and Lending under the DPT



bersome and provides few general insights. We therefore again immediately rely on the calibrated model (assuming $E_0 = E^{ss}$) to derive the optimal κ and resulting supervisory welfare.[16] Again, we derive the optimal κ for a range of different initial return states $r_{l,1}$ and welfare weights ω (see Appendix 2.7.4.).

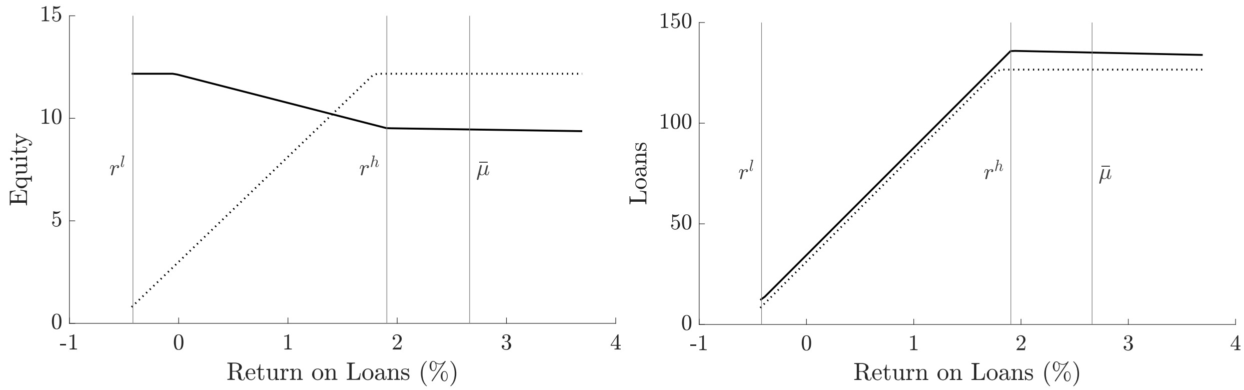
For intuition, we first plot the resulting policy functions for equity and loans under the optimal $\kappa^* = 0.07$ that maximizes the supervisor's welfare function (see Equation (3.45)), assuming that the initial return realization is at the unconditional mean of the process (i.e. $r_{l,0} = \bar{\mu}$) and that the supervisor cares equally about the level and the variance of lending (i.e $\omega = 1$). The left panel of Figure 3.8 shows that, relative to the stress-test framework, retained equity under the DPT is higher and lower for low and return states, respective. The DPT, thus, successfully addresses pro-cyclical retainment of equity and dividend smoothing.

Further, the right panel shows that, for low, intermediate and moderately high return states, the bank uses this equity to lever up more than under the under the stress-testing. Only for very higher return states, substantially above 4%, the dividend prudential target leads to lower levels than the stress-testing framework.

Unfortunately, the DPT's higher lending lending levels increase more in bad states than in good

[16]When numerically maximizing the supervisor's welfare function we impose that the supervisor cannot set κ in such a way that $r_l^{hh} < r_l^l$. That way loans would be set to zero for basically all loan return states which would of course minimize the volatility of lending. This that loans cannot be zero resembles the standard Inada condition.

Figure 3.8: Optimal Policies under Stress Tests and a Dividend Prudential Target

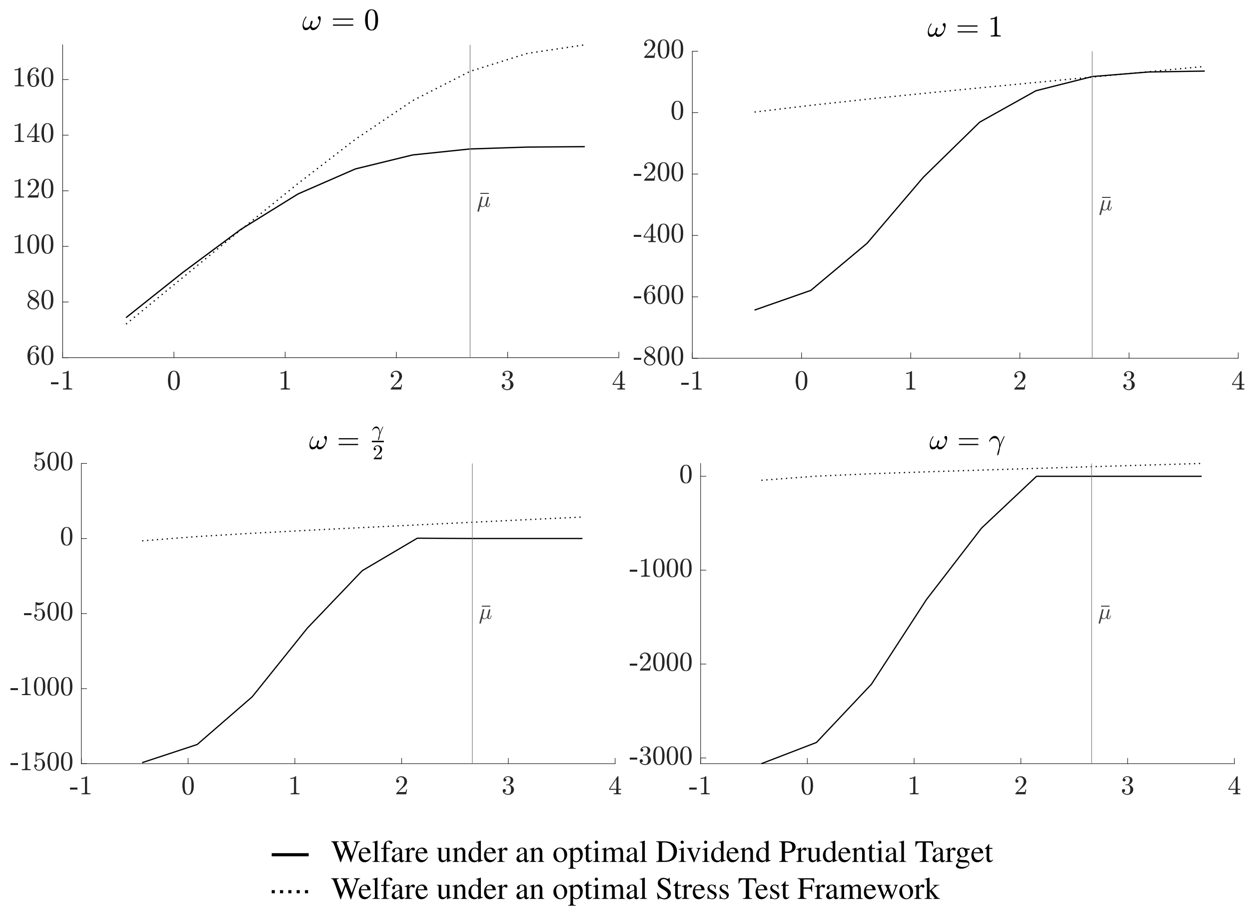


— Policies under an optimal Dividend Prudential Target
····· Policies under an optimal Stress Test Framework

states. And, hence, while the mean in lending is increased with a DPT, so is the variance.This is reflected in the welfare comparison between the dividend prudential target and the stress-testing framework. Figure 3.9 by plotting the supervisor's welfare under each policy framework given the optimal severity of tress test scenarios τ^* and dividend deviation punishment parameter $\kappa *$, respectively. Figure 3.9 plots the associated welfare for different realizations of the initial return on loans $r_{l,0}$ and different degrees of risk aversion ω.

As the comparison of supervisory welfare between stress tests and the DPT show, the former is in a better position to maximize the supervisor's welfare in almost all circumstances. Only when supervisor cares about the level of loans alone (i.e. $\omega = 0$) and $r_{l,0}$ is relatively low, a dividend prudential target can increase welfare by forcing banks to hold on to more equity, resulting in higher expected loan levels. In all other circumstances stress tests increase the supervisor's welfare. Therefore, stress tests are overall a much better tool to generate stable lending than a DPT.

Figure 3.9: Welfare under Optimal Stress Tests and a Dividend Prudential Target



3.6. Conclusion

Bank stress tests have become an increasingly important policy tool designed with the intent to ensure stable lending and, thereby, to foster financial stability. In this paper, we derive the optimal bank balance sheet choices anticipating subsequent stress testing. Here, we explicitly model the forward-looking constraint that stress tests place on the bank's degree of debt financing: equity capital levels should be sufficient to maintain current lending tomorrow, even after absorbing severe losses from said lending. We find that stress tests influence the banks' joint decision over (retained) equity, dividends, and lending. Here, we document the core supervisory trade-off: the more severe the assumed losses, the lower are both expected lending and lending volatility.

To quantitatively assess how such a trade-off plays out in practice, we calibrate our model to the U.S. banks subject to the CCAR stress tests. We derive the optimal stress-test tightness (severity of the adverse scenario) and the implied stress test capital buffer. We find that a supervisor who prefers to maximize lending levels while minimize lending volatility finds stress test capital buffers in the range of 1% to 9% to be optimal. This matches well the Federal Reserves' publicly announced stress-test buffers, reported to be between 2.5% to 7.5% in the 2021 CCAR report (Federal Reserve Board, 2021). This indicates that we are able to capture well both the mechanism behind and the magnitude of bank balance sheet choices under stress tests.

Next, we turn to placing the stress test framework in the wider net of prudential policies. Here, we highlight in particular how stress tests may be complemented with a dividend ban and/or the relaxation of the CCyB in a crisis period. We find that separately introduced, both relax lending of stress-tested banks in bad states of the world. They can, thus, be utilized to dampen the stress-test induced decrease in lending during downturns. However, CCyB activation is less effective than the dividend ban and, when introduced on top of the ban, has no further effects. We are thus able to rationalize why the relaxation of the CCyB during the onset of the Covid-19 pandemic had no measurable effect on lending by stress-tested banks subject to the dividend bans (FSB, 2021).

We conclude the paper by studying a hypothetical substitute policy: the dividend prudential target. Contrary to stress tests regulating equity levels, the dividend prudential target directly regulates bank dividend payments by introducing a quadratic cost function for deviations from a target. This discourages both equity extraction in bad states and excessive leveraging in good states. We find, however, that such a policy mainly increases the mean of lending but fails to reduce volatility. It is thus not welfare-improving for a supervisor seeking stable lending levels.

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Appendix A

Appendix to Chapter 1

1.1. Agent Specifications

1.1.1. Buyers' Utility Function

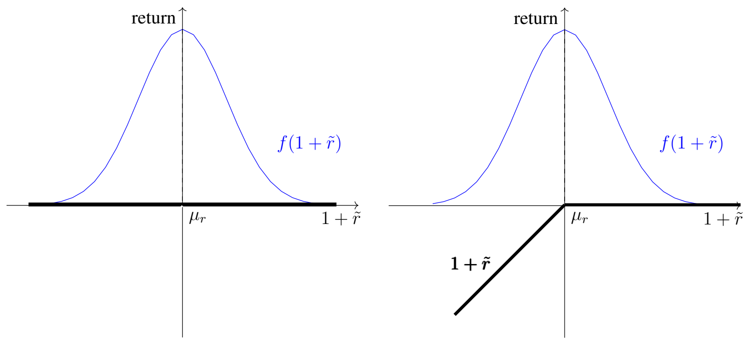
Utility from product d In $t = 1$ a buyer b can purchase product d from a seller s, which specifies transfers $\tau = \mu_r - (1 + \tilde{r})$. Assuming rational expectations, s has an estimated default probability of $\hat{D}_s$. In the case of no seller default, a buyer of product d is left with $1 + \tilde{r} + \tau = \mu_r$ (see Figure A.1a). In the case of seller default, it is important to distinguish whether the buyer is in-the-money or out-of-the-money. When the seller defaults while being in-the-money, bankruptcy laws require the buyer to honor his commitments. Thus, the buyer is still left with μ_r whenever default occurs for $1 + \tilde{r} > \mu_r$. In the case of seller defaulting out-of-the-money (where $1 + \tilde{r} \leq \mu_r$), a buyer does not receive the promised positive transfers. The buyer is therefore left with the asset realization $1 + \tilde{r}$ (see Figure A.1b).

Thus transfers τ have the following probability density function:

$$f(\tau) = \begin{cases} \widehat{D}_s f(1+\tilde{r}) & 1+\tilde{r} \leq \mu_r \\ \widehat{D}_s(1-F(\mu_r)) + (1-\widehat{D}_s) & 1+\tilde{r} = \mu_r \\ 0 & 1+\tilde{r} > 0 \end{cases} . \tag{A.1}$$

Figure A.1: Asset Return with product d

(a) No seller default: Pr= 1-$\widehat{D}_s$ (b) Seller default: Pr=$\widehat{D}_s$



Given this, I can compute the expected return from purchasing product d to hedge asset risk:

$$\mathbb{E}[1+\tilde{r}+\tau] = F(\mu_r)\widehat{D}_s\mathbb{E}[1+\tilde{r} \mid \tilde{r} \leq \mu_r] + \mu_r\left[\widehat{D}_s(1-F(\mu_r)) + (1-\widehat{D}_s)\right], \tag{A.2}$$

$$= \mu_r - \frac{\widehat{D}_s}{2}\sigma_r\frac{\sqrt{2}}{\sqrt{\pi}}. \tag{A.3}$$

Similarly, the variance is:

$$\mathbb{E}[1+\tilde{r}+\tau]^2 = \mu_r^2 - \widehat{D}_s\mu_r\sigma_r\frac{\sqrt{2}}{\sqrt{\pi}} + \frac{\widehat{D}_s^2}{4}\sigma_r^2\frac{2}{\pi}, \tag{A.4}$$

$$\mathbb{E}[(1+\tilde{r}+\tau)^2] = F(\mu_r)\widehat{D}_s\mathbb{E}[(1+\tilde{r})^2 \mid 1+\tilde{r} \leq \mu_r] + \mu_r^2\left[\widehat{D}_s(1-F(\mu_r)) + 1 - \widehat{D}_s)\right], \tag{A.5}$$

$$\mathbb{E}[(1+\tilde{r})^2 \mid 1+\tilde{r} \leq \mu_r] = \mathbb{VAR}[1+\tilde{r} \mid 1+\tilde{r} \leq \mu_r] + E[1+\tilde{r} \mid 1+\tilde{r} \leq \mu_r]^2, \tag{A.6}$$

$$= \sigma_r^2(1-\frac{2}{\pi}) + \mu_r^2 - 2\mu_r\sigma_r\frac{\sqrt{2}}{\sqrt{\pi}} + \sigma_r^2\frac{2}{\pi}, \tag{A.7}$$

$$= \mu_r^2 + \sigma_r^2 - 2\mu_r\sigma_r\frac{\sqrt{2}}{\sqrt{\pi}}, \tag{A.8}$$

$$\mathbb{E}[(1+\tilde{r}+\tau)^2] = 0.5\widehat{D}_s\sigma_r^2 - \widehat{D}_s\mu_r\sigma_r\frac{\sqrt{2}}{\sqrt{\pi}} + \mu_r^2, \tag{A.9}$$

$$\mathbb{VAR}[1+\tilde{r}+\tau] = \mathbb{E}[(1+\tilde{r}+\tau)^2] - \mathbb{E}[1+\tilde{r}+\tau]^2 = \frac{\widehat{D}_s}{2}\sigma_r^2(1-\frac{\widehat{D}_s}{\pi}). \tag{A.10}$$

And thus, the utility of hedging a single asset from seller s is:

$$u_d = \mu_r - \frac{\widehat{D}_s}{2}\sigma_r\frac{\sqrt{2}}{\sqrt{\pi}} - \frac{\gamma}{2}\frac{\widehat{D}_s}{2}\sigma_r^2(1 - \frac{\widehat{D}_s}{\pi}). \tag{A.11}$$

Utility from product m Conditional on having purchased product d, buyers may purchase product m from one of the clearing members. Product d and product m are not required to be bought from the same seller. Due to a combination of CCP structure and regulations, the CCP has an assumed default probability of zero (see section 1.1.3. for more details). The utility from product d purchased from seller s with product m bought from seller s' is thus:

$$u_{dm} = \mu_r. \tag{A.12}$$

Prices The above described utilities u_d and u_m are net of prices. Further, denote the per-unit price that a buyer pays for product d and product m with p_d and p_m respective.

1.1.2. Sellers' Profit Function

Recall that a seller's total product d sales are denoted Q_d. Additionally, denote Q_{dm} as the number of a seller's own product d sales that are additionally combined with a product m, such that necessarily $Q_{dm} \leq Q_d$. Further, denote a clearing member's total quantity of product m sales without involvement in the underlying product d as Q_m. Practically speaking Q_m denotes the number of trades where a clearing member purely acts as a contact point [1] T denotes the total transfers from al Q_{dm} sales of product m. Finally, for readability purposes, I anticipate that sellers will charge all their buyers the same prices p_d and p_m in equilibrium (proven to hold below). Then the sellers' total expected profits are:

$$\forall s \notin M: \qquad \mathbb{E}_0\Pi = \quad 0 \quad + \mathbb{E}_0[\Pi_s^1] + (1 - D_s)\,\mathbb{E}_0\left[\Pi_s^2 \mid \Pi_s^2 > 0\right], \tag{A.13}$$

where

$$\Pi_s^1 = Q_d p_d + Q_m(-p_m - v_m - (1+\delta)g_m), \tag{A.14}$$

[1]Note that theoretically a seller $s \in M$ also has the option to only sell product d and the buyer buys product m from another seller $s' \in M$. Again product m must also be bought by seller $s \in M$ for v_m and collateral g_m must be posted. I, however, account for this possibility in the following analysis where I explicitly rule out for this to be optimal.

$$\Pi_s^2 = L - T + Q_m g_m \quad \sim N(\mu_L, Q_d\sigma_r^2 + \sigma_r^2), \tag{A.15}$$

$$D_s = D(Q_d) = Pr(\Pi_s^2 \leq 0) \tag{A.16}$$

$$\forall s \in M: \quad \mathbb{E}_0\Pi_m = -e_m + \mathbb{E}[\Pi_s^1] + (1 - D_s)\,\mathbb{E}\left[\Pi_s^2 \mid \Pi_s^2 > 0\right], \tag{A.17}$$

where

$$\Pi_s^1 = Q_d p_d + Q_{dm}\left[p_m - v_m - (1+\delta)g_m\right] + Q_m p_m, \tag{A.18}$$

$$\Pi_s^2 = Q_{dm} g_m + L - T \quad \sim N(Q_{dm} g_m, Q_d\sigma_r^2 + \sigma_r^2), \tag{A.19}$$

$$D_s = D(Q_d, Q_{dm}) = Pr(\Pi_s^2 \leq 0). \tag{A.20}$$

Here, I would like to highlight that p_m enters negatively in the profit function of non-clearing members (see equation (A.14)) and positively in the profit function of clearing members (see (A.18)). This is because a non-clearing member must pay for intermediation services for each of her insured product d sales. Clearing members instead provide intermediation services both for their own product d sales (entering in Q_{dm}) and for other sellers' product d sales (entering in Q_m).

Properties of Expected Profits The properties of the profit functions for $t = 0$ and $t = 1$ are straight forward and more elaboration is thus omitted here. Below a few properties of the expected $t = 2$ profits $(1 - D_s)\,\mathbb{E}\left[\Pi_s^2 \mid \Pi_s^2 > 0\right]$ are derived. For this purpose, I define a helpful set of general variables.

Define: $t = 2$ profits as $Y \sim N(\mu_Y, \sigma_Y^2)$ with pdf $f(.)$ and cdf $F(.)$.

Define: $Y = \sigma_Y Z + \mu_Y$ where $Z \sim (0, 1)$.

Denote: $\varphi(.)$ and $\Phi(.)$ the pdf and cdf of the standard normal distribution $N(0, 1)$.

Define: X as the strategic default threshold, such that $x = \frac{X - \mu_Y}{\sigma_Y}$.

Property 1. Using the above definitions, expected profits $(1 - D_s)\mathbb{E}_0\left[\Pi_2 | \Pi_2 > 0\right]$ can be rewritten as $\left(1 - F(X)\right)\mu_Y + \sigma_Y^2 f(X)$:

$$D_s = F(0), \tag{A.21}$$

$$\mathbb{E}_0\left[\Pi_2 | \Pi_2 > 0\right] = \mu_Y + \sigma_Y \frac{\sigma_Y f(0)}{1 - F(0)} \tag{A.22}$$

$$(1 - D_s)\mathbb{E}_0\left[\Pi_2 | \Pi_2 > 0\right], = \left(1 - F(0)\right)\mu_Y + \sigma_Y^2 f(0). \tag{A.23}$$

Property 2. Define $\mu_Y(z)$ as a function of an equilibrium object z. Assume σ_Y^2 is independent of z. Then, the time-2 profit's FOC with respect to z is $\frac{\partial \mu_Y(z)}{\partial z}\big(1-F(0)\big)$:

$$\mathbb{E}_0\left[\Pi_2|\Pi_2>0\right] = \big(1-F\left(X\right)\big)\,\mu_Y(z) + \sigma_Y^2 f(X), \tag{A.24}$$

$$\frac{\partial\left[\big(1-F\left(X\right)\big)\,\mu_Y(z) + \sigma_Y^2 f(X)\right]}{\partial z} = \frac{\partial\big(1-F\left(X\right)\big)}{\partial z}\mu_Y(z) + \big(1-F\left(X\right)\big)\frac{\partial \mu_Y(z)}{\partial (z)} + \sigma_Y^2\frac{\partial f(X)}{\partial z} \tag{A.25}$$

$$\frac{\partial F(X)}{\partial z} = \frac{\partial \frac{1}{2}\left[1+erf\left(\frac{X-\mu_Y(z)}{\sigma_Y\sqrt{2}}\right)\right]}{\partial \mu_Y(z)} \tag{A.26}$$

$$\rightarrow \quad erf(-x) = -erf(x) \tag{A.27}$$

$$\frac{\partial F(X)}{\partial z} = \frac{\partial \frac{1}{2}\left[1-erf\left(\frac{\mu_Y(z)-X}{\sigma_Y\sqrt{2}}\right)\right]}{\partial z} \tag{A.28}$$

$$= -\frac{1}{2}\frac{\partial erf\left(\frac{\mu_Y(z)-X}{\sigma_Y\sqrt{2}}\right)}{\partial z} \tag{A.29}$$

$$= -\frac{\partial \mu_Y(z)}{\partial z}\frac{1}{2}\frac{1}{\sqrt{2}\sigma_Y}\frac{2}{\sqrt{\pi}}e^{\frac{-(\mu_Y(z)-x)^2}{2\sigma_Y^2}} \tag{A.30}$$

$$= -\frac{\partial \mu_Y(z)}{\partial z}f(X) \tag{A.31}$$

$$\frac{\partial\big(1-F\left(X\right)\big)}{\partial \mu_Y(z)}\mu_Y(z) = \frac{\partial \mu_Y(z)}{\partial z}f(X)\mu_Y(z) \tag{A.32}$$

$$\sigma_Y^2\frac{\partial f(X)}{\partial z} = \frac{\partial \mu_Y(z)}{\partial z}\sigma_Y^2\frac{2(X-\mu_Y(z))}{\sigma_Y^2 2}\frac{1}{\sqrt{2\sigma_Y^2\pi}}e^{\frac{-(X-\mu_Y(z))^2}{\sigma_Y^2 2}} \tag{A.33}$$

$$= \frac{\partial \mu_Y(z)}{\partial z}(X-\mu_Y(z))\frac{1}{\sqrt{2\sigma_Y^2\pi}}e^{\frac{-(X-\mu_Y(z))^2}{\sigma_Y^2 2}} \tag{A.34}$$

$$= \frac{\partial \mu_Y(z)}{\partial z}(X-\mu_Y(z))f(X) \tag{A.35}$$

$$\frac{\partial\big(1-F\left(X\right)\big)\mathbb{E}\left[Y|Y>X\right]}{\partial\partial z} = \frac{\partial \mu_Y(z)}{\partial z}\left[f(X)\mu_Y(z) + \big(1-F\left(X\right)\big) + (X-\mu_Y(z))f(X)\right] \tag{A.36}$$

$$= \frac{\partial \mu_Y(z)}{\partial z}\left[Xf(X) + \big(1-F\left(X\right)\big)\right] \tag{A.37}$$

$$\rightarrow \quad X = 0 \tag{A.38}$$

$$\frac{\partial\big(1-F\left(X\right)\big)\mathbb{E}\left[Y|Y>X\right]}{\partial z} = \frac{\partial \mu_Y(z)}{\partial z}\big(1-F(0)\big) \tag{A.39}$$

Property 3. Assume that both $\mu_Y(z)$ and $\sigma_Y(z)$ are endogenous functions of an equilibrium object z. Then:

$$\frac{\partial(1-D_s)\mathbb{E}_0\left[\Pi_2|\Pi_2>0\right]}{\partial z} = \frac{\partial\left(1-F\left(0\right)\right)\mu_Y(z)+\sigma_Y(z)^2 f(0)}{\partial z} \tag{A.40}$$

$$= f(0)\sigma_Y(z)\frac{\partial\sigma_Y(z)}{\partial z}+\left(1-F(0)\right)\frac{\partial\mu_Y(z)}{\partial z} \tag{A.41}$$

The derivations are lengthy, yet relatively straightforward. Hence, they are omitted. The following intermediate results, however, proof useful for analysis that follows:

$$\frac{\partial f(X)}{\partial z} = -\frac{1}{\sigma(z)}\frac{\partial\sigma(z)}{\partial z}f(0)+f(0)\left(\frac{-\mu_Y(z)}{\sigma_Y(z)^2}\right)\left[\frac{\partial\mu_Y(z)}{\partial z}-\frac{\mu_Y}{\sigma_Y(z)}\frac{\partial\sigma_Y(z)}{\partial z}\right], \tag{A.42}$$

$$\frac{\partial F(0)}{\partial z} = -f(0)\left[\frac{\partial\mu_Y(z)}{\partial z}+\frac{(-\mu_Y)}{\sigma_Y(z)}\frac{\partial\sigma_Y(z)}{\partial z}\right]. \tag{A.43}$$

Property 4. Assume that solely $\sigma_Y(z)$, but not μ_Y, is a function of an equilibrium object z. Then:

$$\frac{\partial F(0)}{\partial z} = -f(0)\left[\frac{(-\mu_Y)}{\sigma_Y(z)}\frac{\partial\sigma_Y(z)}{\partial z}\right] \tag{A.44}$$

The Collateral Thresholds A first model implied collateral threshold is one that ensures collateral g_m to be large enough such that the seller default probability (of a clearing member) actually decreases in insured sales. Hence, central clearing in the OTC market makes sellers actually less likely to default. Here, using the just derived properties this requires a minimum collateral:

$$\frac{\partial D_M}{\partial Q_{dm}} = -d_m\left[g_m-\frac{(g_m Q_{dm}+\mu_L)\sigma_r^2}{2(Q_{dm}\sigma_r^2+\sigma_L^2)}\right]<0 \tag{A.45}$$

$$\longrightarrow$$

$$g_m-\frac{(g_m Q_{dm}+\mu_L)\sigma_r^2}{2(Q_{dm}\sigma_r^2+\sigma_L^2)}>0 \qquad \forall Q_{dm}\geq 0 \tag{A.46}$$

$$g_m > \frac{\mu_L\sigma_r^2}{2\sigma_L^2} = g_m^* \tag{A.47}$$

Notice that this automatically implies that the probability density of default, denoted d_M, is decreasing in insured sales:

$$\frac{\partial d_M}{\partial Q_{dm}} = -\frac{\sigma_r^2}{2(Q_{dm}\sigma_r^2+\sigma_L^2)}d_M - d_M\frac{(Q_{dm}g_m+\mu_L)}{Q_{dm}\sigma_r^2+\sigma_L^2}\underbrace{\left[g_m - \frac{(g_mQ_{dm}+\mu_L)\sigma_r^2}{2(Q_{dm}\sigma_r^2+\sigma_L^2)}\right]}_{+} < 0 \tag{A.48}$$

For completeness, note that in Appendix 1.2. I derive a second threshold g_m^{**} that is required for seller profits to be strictly increasing in product m sales:

Both terms on the right-hand-side decrease in a_b, such that a necessary level of g_m to ensure convexity it:

$$g_m > \frac{\sigma_r^2}{2\sigma_L} + \frac{\mu_L\sigma_r^2}{2\sigma_L^2} > g_m^{**} > g_m^*. \tag{A.49}$$

For the main text, I commonly assume that the regulator sets a minimum collateral requirement $\underline{g_m}$ above g_m^{**} such that central clearing is always both risk-reducing *and* profitable. For completeness, Appendix 1.4. studies the case, where the latter is not met.

1.1.3. The CCP

The CCP's Profit Maximization Problem Recall that CCP is unaware of the realizations of a_b, and thus forms (rational) expectations $\mathbb{E}_0$ over the buyer-seller matches and consequent market outcomes at $t=0$, $t=1$ and $t=2$. Denote the associated CCP profits at time t with Π_C^t. Further, recall that I assume that the CCP is never expected to default. This assumption is discussed in detail in Appendix 1.1.3., where I show this to hold true for a sufficiently capitalized CCP. In that case, default does not enter the CCP maximization problem, who expects to bear all potential losses. And hence, the CCP chooses v_m, e_m, and g_m simultaneously to maximize the following constrained problem:

$$\mathbb{E}_0\Pi_C = \max_{e_m,v_m,g_m} \quad \mathbb{E}_0\Pi_C^0(e_m) + \mathbb{E}_0[\Pi_C^1(v_m,g_m) \mid M] \;+\; \mathbb{E}_0[\Pi_C^2(\tau,L,g_m) \mid M; Q_{dm}], \tag{A.50}$$

$$s.t.$$

$$|M(e_m)| \geq 2, \tag{A.51}$$

$$g_m \geq \underline{g_m}. \tag{A.52}$$

To fully characterize the different profit elements, I introduce an additional set of notations. Denote as $\bar{M}$ the expected number of clearing members: $\mathbb{E}[| M |]$. Further, denote with $\bar{Q}_{dm}$ the expected bundle sales of a clearing member. Finally, let $\mathbf{1}$ denote an indicator function that takes on the value one, when $\mathbb{E}_0\Pi_m$ exceeds $\mathbb{E}_0\Pi$ for a seller with a matched buyer of size a_b. Recall that the pdf of a_b is denoted with $a(a_b)$. Then:

$$\bar{M} = S \sum_{a_b=\underline{a}}^{\bar{a}} a(a_b)\mathbf{1}_{\mathbb{E}_0\Pi_m(a_b)>\mathbb{E}_0\Pi(a_b)}, \tag{A.53}$$

$$\bar{Q}_{dm} = \mathbb{E}[Q_{dm} \mid s \in M], \tag{A.54}$$

$$\mathbb{E}_0\Pi_C^0 = \bar{M}e_m, \tag{A.55}$$

$$\mathbb{E}_0\Pi_C^1 = \bar{M}\bar{Q}_{dm}2v_m, \tag{A.56}$$

$$\begin{aligned}\mathbb{E}_0\Pi_C^2 = &\bar{M}Pr(-T + \bar{Q}_{dm}g_m \leq 0 \;\&\; -T + \bar{Q}_{dm}g_m + L \leq 0) \\ &\cdot \mathbb{E}_0[-T + \bar{Q}_{dm}g_m \mid \bar{Q}_{dm}(-\tau + g_m) \leq 0 \;\&\; -T + \bar{Q}_{dm}g_m + L \leq 0],\end{aligned} \tag{A.57}$$

$$\text{where} \quad -T + \bar{Q}_{dm}g_m \sim N(\bar{Q}_{dm}g_m, \bar{Q}_{dm}\sigma_r^2) \quad \text{and} \quad L \sim N(\mu_L, \sigma_L^2). \tag{A.58}$$

Here, equation A.53 derives the expected number of clearing members that depends on both a seller's expected profits given a_b and the density of a_b. Equation A.54 defines the expected number of sales given clearing membership. Equation A.55 defines the CCP profits at $t = 0$, constituting the entrance fee times the expected number of clearing members. Equation A.56 defines the $t = 1$ profits, which are the number of sales times two times the variable fee for each clearing member. Finally, A.57 defines the CCP's expected losses upon default. Due to the independence assumption, they are simply $\bar{M}$ times the losses of a single clearing member. Here, note that the CCP only expects losses when two (co-dependent) conditions are met: the clearing member defaults and total transfers exceed total collateral. Given the nature of the underlying normal distributions, no further closed form can be obtained for time-2 losses. However, they can be numerically computed using the following double integrals:

$$Pr(-T + \bar{Q}_{dm}g_m \leq 0 \;\&\; -T + \bar{Q}_{dm}g_m + L \leq 0) = \int_{-\infty}^{0} f_{\bar{Q}_{dm}(-\tau+g_m)}(x) \int_{-\infty}^{-x} f_L(y)\mathrm{d}y\mathrm{d}x, \tag{A.59}$$

$$\mathbb{E}[-T + \bar{Q}_{dm}g_m \mid \bar{Q}_{dm}(-\tau + g_m) \leq 0 \;\&\; \bar{Q}_{dm}(-\tau + g_m) + L \leq 0]$$
$$= \frac{1}{Pr(-T + \bar{Q}_{dm}g_m \leq 0 \;\&\; -T + \bar{Q}_{dm}g_m + L \leq 0)} \int_{-\infty}^{0} x f_{-T+\bar{Q}_{dm}g_m}(x) \int_{-\infty}^{-x} f_L(y)\mathrm{d}y\mathrm{d}x. \quad \text{(A.60)}$$

CCP Default Mechanism Notice that the just described maximization problem is a simplified version of reality, as it assumes that the CCP is expected to never default. As a for-profit entity, the CCP may also strategically default at $t = 2$. Due to its central role in the market, default would have significant consequences for all parties involved. For the buyers with defaulting sellers, a defaulting CCP would mean that their product m carries little to no value. Additionally, a defaulting CCP is not able to (fully) repay initially non-defaulting sellers their collateral. This may cause cascading seller defaults.

To limit the default risk of the CCP, regulators impose several restrictions. I have previously discussed two of them: the CCP must collect collateral from sellers;[2] and must have more than one clearing member. Both ensure that the CCP is less exposed to any individual seller's default. Two additional measures are minimum CCP capital requirements K and a clearly defined default mechanism.[3] The latter minimizes organizational frictions by determining in which order resource are used to cover a defaulting seller's transfers (see Figure A.2).

Figure A.2: The CCP's Default Mechanism

1. The defaulting seller's positive transfers.
2. The collateral of the defaulting seller only.
3. Its own capital K (set by regulators).

\- default threshold

4. Collateral of the non-defaulting sellers.

To model the CCP's full default waterfall goes beyond the scope of this paper. However, from Figure A.2 it can quite intuitively be induced that cascading defaults can be limited by reducing individual

[2] I abstract from the possibility that the CCP asks sellers to additionally provide an lump-sum collateral payment ex ante. This is without loss of generality, because the true size of a clearing member is unknown to the CCP. Therefore, charging a lump sum independent os size is in expectations equivalent to setting a higher a higher g_m for the average sales of a clearing member.

[3] The here presented default mechanisms is a simplification of the real world and taken from Huang (2019). For a detailed theoretical analysis of CCPs default waterfalls, please see Duffie and Zhu (2011).

seller default via higher g_m and sufficient K to cover all defaulting sellers' losses exceeding collateral. Given the nature of normal distributions, a very bad realization may always happen such that cascading defaults are triggered. For sufficiently high K and the minimum $\underline{g_m}$ this occurs, however, with a quasi-zero default probability. Here, recent stress test of CCPs conducted by ESMA (2022) show that CCPs are indeed not expected to default due to credit risk but, if at all, due to organizational risk.

1.2. Proofs Section 1.3. – Time 1 Outcomes

1.2.1. Proofs Section 1.3.1. Mandatory Counterparty Default Insurance

For this section, note that every product d must be combined with a product m. If no product m is added, product d is nullified. Thus, buyers only consider to purchase from sellers that form the subset M. All remaining sellers exit the market per assumption.

Sketch of Proof for *Proposition 1*:

1. The proof starts by deriving price p_m.

1.1. To derive p_m, recall that it is set in a take-it-or-leave-it-fashion: the buyer agrees or declines the offer. Thus, p_m is set such that the buyer's participation constraint is just binding.

1.2. Further recall that product d cannot be held alone, as product m is mandatory. Then, a realized product d seller sets p_m such that the buyer is just indifferent between holding the product d and m bundle or nothing at all:

$$u_{dm} - p_d - p_m - v_m = u_r, \tag{A.61}$$

$$p_m = u_{dm} - p_d - v_m - u_r. \tag{A.62}$$

2. From this, it can be shown that no buyer switches.

2.1. The buyer is left with his reservation utility:

$$u_{dm} - p_d - p_m - v_m = u_{dm} - p_d - v_m - (u_{dm} - p_d - v_m - u_r) = u_r. \tag{A.63}$$

2.2. Note that p_m never accounts for the switching cost paid for the product d purchase. Then a buyer not switching is left with total utility U_r. This is the case when: the seller offers the bundle for some assets only, the seller offers the bundle for all asset, or the matched seller never offers any product. A switching buyer will however always expect a total utility $U_r - C$. Thus buyer's never switch.

2.3. It follows immediately that buyers matched with non-clearing members exit the market together with them.

3. Anticipating a bit the order of the main text, note that *Corollary 1* immediately follows from point 2.3. If all sellers exit, so do all buyers. And hence, we have market failure.

4. Given the price (A.62), then p_d can be derived. Looking at equation A.62, it becomes clear that any increase in p_d translates into a one-for-one decrease in p_m and thus the seller is indifferent between either. She simply sells a **bundle** of both products at prices $p_d + p_m = u_{dm} - u_r - f - \frac{C}{a_b}$.

5. Given the bundle price, I can derive the properties of clearing member profits $Pi_M^1 + \mathbb{E}_0\Pi_m^2$ as a function of bundle quantities $Q_d = Q_{dm}$. Here note that, given the absence of switching, a clearing member sells at most a_b: $Q_{dm} \in [0, a_b]$.

5.1.The clearing members' profits $\mathbb{E}_1\Pi_m$ are sum of two functions: a linear and a non-linear component:

$$\mathbb{E}_1\Pi_m = \underbrace{-e_m}_{\text{intercept}} + \underbrace{Q_{dm}\left(p_d + p_m(1+\delta)g_m\right)}_{\text{linear in } Q_{dm}} + \underbrace{(1 - D_M)(g_m Q_{dm} + \mu_L) + d_M(Q_{dm}\sigma_r^2 + \sigma_L^2)}_{\text{non-linear in } Q_{dm}}. \tag{A.64}$$

This is easily confirmed, when looking at the FOC wrt. Q_{dm}:

$$\frac{\partial \Pi_m}{\partial Q_{dm}} = \underbrace{p_d + p_m - v_m - (1+\delta)g_m}_{\text{constant}} + \underbrace{(1 - D_m)g_m + d_m\frac{\sigma_r^2}{2}}_{\text{dependent on } a_b}. \tag{A.65}$$

5.2. The the slope-value of linear component falls into three categories:

$\rightarrow$ 5.2.1. $p_d + p_m - v_m - \delta g_m > 0$.

$\rightarrow$ 5.2.2. $p_d + p_m - v_m - \delta g_m = 0$

$\rightarrow$ 5.2.3. $p_d + p_m - v_m - \delta g_m < 0$

5.3. The non-linear component of $\mathbb{E}_1 \Pi_m$ is strictly increasing in Q_{dm}. Depending on g_m however, it is either strictly convex ($g_m \geq \underline{g_m} \geq g_m^{**}$) or locally concave/convex ($g_m \geq\in [g_m^* g_m^{**}$), but never strictly concave.

5.3.1.The non-linear component has a strictly positive slope:

- $(1 - D_M)$ denotes the probability of non-default and is naturally between zero and one,

- I have assumed $g_m > \frac{\mu_L^2 \sigma_r^2}{2\sigma_L^2}$, which are all positive parameters,

- d_m denotes the pdf at point of default and is thus by definition positive, as is σ_r^2.

$$\longrightarrow \quad (1 - D_m) g_m + d_m \frac{\sigma_r^2}{2} > 0. \tag{A.66}$$

5.3.2. Using properties derived in Section 1.1.2., I can show that the second derivative of the non-liear part is:

$$\frac{\partial (1 - D_M) g_m + \frac{\sigma_r^2}{2} d_m}{\partial Q_{dm}} = d_m \left[g_m - \frac{\sigma_r^2}{2} \frac{g_m Q_{dm} + \mu_L}{Q_{dm} \sigma_r^2 + \sigma_L} \right]^2 - d_m \frac{\sigma_r^4}{4(Q_{dm} \sigma_r + \sigma_L)} \tag{A.67}$$

5.3.3. For this second-order-condition to be strictly positive, implying convexity, we must have:

$$d_m \left[g_m - \frac{\sigma_r^2}{2} \frac{g_m Q_{dm} + \mu_L}{Q_{dm} \sigma_r^2 + \sigma_L} \right]^2 - d_m \frac{\sigma_r^4}{4(Q_{dm} \sigma_r + \sigma_L)} > 0 \tag{A.68}$$

$$g_m > \frac{\sigma_r^2 \sqrt{Q_{dm} \sigma_r^2 Q_{dm} + \sigma_L^2}}{Q_{dm} \sigma_r^2 + 2\sigma_L^2} + \frac{\mu_L \sigma_r^2}{Q_{dm} \sigma_r^2 + 2\sigma_L^2} \tag{A.69}$$

Both terms on the right-hand-side decrease in Q_{dm}, such that a necessary level of g_m to ensure convexity it:

$$g_m \geq \frac{\sigma_r^2}{2\sigma_L} + \frac{\mu_L \sigma_r^2}{2\sigma_L^2} > g_m^{**} \tag{A.70}$$

5.3.4. Contrary, to be a strictly concave function, it must be that:

$$g_m < \frac{\sigma_r^2\sqrt{Q_{dm}\sigma_r^2 Q_{dm} + \sigma_L^2}}{Q_{dm}\sigma_r^2 + 2\sigma_L^2} + \frac{\mu_L \sigma_r^2}{Q_{dm}\sigma_r^2 + 2\sigma_L^2} \tag{A.71}$$

Given that again the right-hand-side is decreasing in Q_{dm}, it must hold for very large Q_{dm}:

$$g_m < \lim_{Q_{dm}\to\infty} \frac{\sigma_r^2\sqrt{Q_{dm}\sigma_r^2 Q_{dm} + \sigma_L^2}}{Q_{dm}\sigma_r^2 + 2\sigma_L^2} + \lim_{Q_{dm}\to\infty} \frac{\mu_L \sigma_r^2}{Q_{dm}\sigma_r^2 + 2\sigma_L^2} = 0 \tag{A.72}$$

However, I have assumed that $g_m > \frac{\mu_L \sigma_r^2}{2\sigma_L} = g_m^*$. Thus strict concavity of the non-linear part can be ruled out.

5.3.5. By logic of completion it must thus be that for levels of $g_m \in (\frac{\mu_L \sigma_r^2}{2\sigma_L}, \frac{\sigma_r^2}{2\sigma_L} + \frac{\mu_L \sigma_r^2}{2\sigma_L^2}] = [g_m^*, g_m^{**}]$, the non-linear part is concave for low Q_{dm} but convex for large Q_{dm}; yet in any case strictly increasing.

5.3.6. Note that in all g_m cases the non-linear function becomes approximately linear and equal to g_m, as Q_{dm} approaches infinity:

$$\lim_{Q_{dm}\to\infty} \left[(1 - D_m)\, g_m + \frac{\sigma_r^2}{2} d_m \right] = g_m \left(1 - \lim_{Q_{dm}\to\infty} D_m \right) + \frac{\sigma_r^2}{2} \lim_{Q_{dm}\to\infty} d_m \tag{A.73}$$

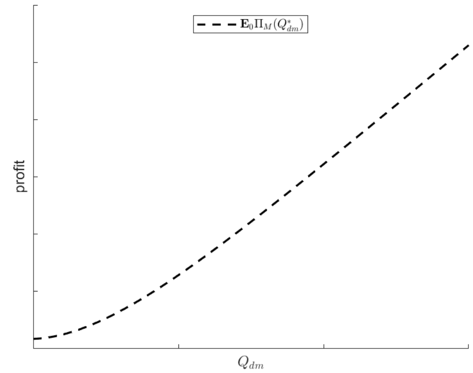
$$= g_m (1 - 0) + 0 = g_m. \tag{A.74}$$

(A.75)

5.4. Recall that, I have assumed that $g_m \geq \underline{g_m} \geq g_m^{**}$. And thus, in the main analysis, the non-linear component in $\mathbb{E}_1 \Pi_m$ is a strictly convex and increasing function. The contrary case is analyzed in Appendix 1.4..

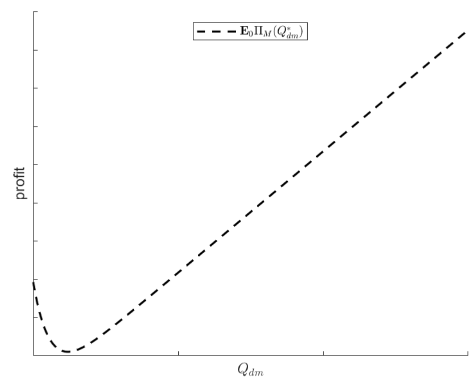
6. With this, I can show that $\mathbb{E}_1 \Pi_m$ is also strictly convex. This implies that, conditional on being a CM, a seller always offers the insurance. For this, I turn to combining the different cases from 5.3. and 5.4, characterizing all the possibly functional forms of $\mathbb{E}_1 \Pi_m$.

6.1. Assume that the linear part has a (weakly) positive slope (5.2.1. and 5.2.2.). Then, the sum of strictly increasing convex function (non-linear part) and strictly/weakly increasing quasi-convex function (linear part) results in a strictly increasing, convex function ($\mathbb{E}_1 \Pi_m$).



6.2. Assume that linear part has a strictly decreasing slope.

6.2.1. Recall that the nonlinear component approximates g_m for large Q_{dm}. Hence, if $g_m > -p_d - p_m - v_m + (1+\delta)g_m$, $\mathbb{E}_0\Pi_m$ is decreasing for low Q_{dm} but is ultimately strictly increasing and positive. For large Q_{dm}, $\mathbb{E}_1\Pi_m$ asymptotically approaches a slope of $p_d + p_m - v_m - \delta g_m > 0$.



Because $\mathbb{E}_0\Pi_m$ is strictly decreasing in Q_{dm}, this would imply for CMs matched with buyers of low a_b, that those do not offer any bundle, yet become clearing members. This is ruled out by assumption of this section: sellers become clearing members anticipating to offer the bundle. To not reach any contradiction, I will verify that this is indeed the case in the $time - 1$ section.

6.2.2. If $g_m < -p_d - p_m - v_m + (1+\delta)g_m$, then the $\mathbb{E}_1\Pi_m$ is always strictly decreasing (figure trivial, therefore omitted).

7. Recall from 2. that under mandatory insurance, the no switching equilibrium is unique. Combining the just derived results, the we conclude with stating that for $g_m > g_m^{*}*$, and conditional on being a clearing member, buyers hedge and insure all of their assets. All non-clearing members exit the

market together with their buyers. Recall, that it remains to be verified that, expecting not to hedge and insure any assets, the seller does not become a CM (point 6.2.1.).

This concludes the proofs for Section 1.3.1.. For proof of the Corollary, please see point 3. in *Proposition 1*.

1.2.2. Proofs Section 1.3.2. Voluntary Counter Party Default Insurance

Recall that under voluntary insurance, the product d can be held also without bundling it with a product m. Given this, I start by deriving p_m as stated in Lemma 1.

Sketch of proof for Lemma 1

1. Take a buyer that has bought product d from $s \in M$ at price p_d with the intention to buy product m.

2. Again, the seller can set a take-it-or-leave-it offer, [4] The difference to mandatory insurance however is, that the buyer now may choose to forego the insurance and hold product d as a stand-alone. Thus, the seller can only capture all utility up until $u_d - p_d$. Accounting for the buyer's other expenses associated with product m this implies:

$$u_{dm} - p_d - p_m - v_m = u_d - p_d, \tag{A.76}$$

$$p_m = u_{dm} - u_d - v_m. \tag{A.77}$$

3. Note for completeness that seller s will never allow the buyer to access the CCP via another clearing member. For that the buyer would additionally incur switching cost. Entering the buyer's participation constraint for product m, this will lower the utility gains the product d selling clearing member can extract. Hence, it is not profit maximizing to allow b to use another clearing member to access the CCP.

4. Then it remains to be shown that clearing members always offer product m to all their product d buyers regardless of size.

4.1. For this, assume buyers have consistent beliefs $\hat{D}$, such that in equilibrium $\hat{D}_s$ is correctly anticipated and shared by all buyers (confirmed later).

4.2. Assume that Q_d sales have realized. Then, any variation in expected profits solely depend

[4]The veto powers comes from regulators requiring that product m is bought by both counterparties, or it won't come into effect at all.

on changes in Q_{dm}.

5. Using results from Appendix 1.1.2., the slope of seller profits as a function of Q_{dm} for a given Q_d is:

$$\frac{\partial \Pi_s^1 + (1-D_s)\mathbb{E}_1[\Pi_s^2 \mid \Pi_s^2 > 0]}{\partial Q_{dm}} = p_m - v_m - (1+\delta)g_m + g_m(1-D_M), \tag{A.78}$$

$$= u_{dm} - u_d - 2v_m - (\delta + D_M)g_m. \tag{A.79}$$

6. Here it is important to note that any marginal change in Q_{dm} is not observable to buyers, such that the seller takes the buyers' $\hat{D}$ as given for this FOC.

6.1. Consequently, $u_{d-} - u_d - v_m - \delta g_m$ remain unchanged.[5]

6.2.Further, D_M is strictly decreasing in Q_{dm} when holding Q_d constant. This is because the seller posts additional collateral $g_m > g_m^*$ for every product m sale while at the same time keeping risk-exposure constant.

7. Given this, we again have three cases.

7.1. If $u_{dm} - u_d - v_m - \delta g_M < 0$, then the CM never offers product m to any buyer.

7.2. If $u_{dm} - u_d - v_m - \delta g_m - D_M(Q_{dm} = 0) > 0$. Then, the CM offers product m to any buyer, indenpendent of size.

7.3. If $u_{dm} - u_d - v_m - \delta g_m - D_M(Q_{dm} < 0) < 0$ and $u_{dm} - u_d - v_m - \delta g_m > 0$, then the expected profits first decrease in total product m sales but are ultimately strictly increasing. Then, a seller insures all product ds, conditional on having amassed sufficient sales. In this case, it is independent whether these sales come from one or several buyers. Not having amassed the required amount of product d sales, he simply insures none.

8. Summarizing points 7.1.-7.2., a clearing member either offers the product m to all or none of her product d sales. And thus, clearing members do not discriminate between buyers regarding product m.

9. It follows immediately from p_m that buyers choose sellers only based on the utility from product d:

9.1. The utility from holding product d alone is: $u_d - p_d$.

9.2. Recall that $p_m = u_{dm} - u_d - v_m$. Then utility minus prices from holding the bundle is:

[5]Nevertheless, to be verified that this (take-as-given) $\hat{D}$ is consistent ex ante from a buyer's point of view. This is formally addressed in *Remark 2*.

$u_{dm} - p_m - p_d - v_m = u_d - p_d$

9.3. Following this, all buyers' choice of seller solely depends on the utility u_d relative to the price p_d and per asset switching cost C/a_b.

10. This allows me to derive the price p_d and buyers choice of seller. Before doing so, please note the following about buyer's beliefs $\widehat{D}_s$ about seller default.

10.1. First recall, that I have assumed that S is large and sellers compete in prices over product d.

10.2. Second, it can be shown that sellers find it strictly profit maximizing to sell product d for any $p_d \geq 0$ and always offer product d for all of a buyers assets.

10.2.1. This follows directly from the sellers' right to strategically default on negative profits at $t = 2$. Using properties of the sellers' profit functions as derived in section 1.1.2., it can be shown that the expected profits are strictly increasing in the number of units of product d sold:

$$\frac{\partial \mathbb{E}_0 \Pi^s}{\partial Q_d} = p_d + \frac{\sigma_r^2}{2} f(0) > 0 \quad \forall p_d \geq 0. \tag{A.80}$$

10.2.2. Note here, that the seller takes the buyers' beliefs about their default probability as given. I.e. given an associated buyer belief of their default probability, sellers find it always profitable to sell additional derivatives. It is later to check that these buyer beliefs are ex ante correct.

10.3. Any equilibrium, where a buyer wants to insure only a fraction of the assets cannot be sustained in Bertrand competition. 10.3.1. To sustain such an equilibrium, p_d must be strictly above zero. Denote the fraction of hedged assets with x

$$x \in [0, 1), \tag{A.81}$$

$$x a_b (u_d - p_d) + (1 - x) a_b u_r - \mathbf{1}C > a_b (u_d - p_d) - \mathbf{1}C, \tag{A.82}$$

$$x (u_d - p_d) + (1 - x) u_r > u_d - p_d, \tag{A.83}$$

$$(1 - x) p_d + (1 - x) u_r > (1 - x) ud, \tag{A.84}$$

$$p_d > u_d - u_r > 0, \tag{A.85}$$

$$p_d > 0 \tag{A.86}$$

10.3. Sellers, however, have an incentive to deviate from any price $p_d > 0$, if this implies they gain new buyers. This is because they make strictly positive profits at $p_d = 0$. Hence any equilibrium, where a buyer insures only a fraction of assets after observing p_d cannot be sustained. Because there is always at least one seller charging $p_d = 0$.

11. Given this, it follows almost directly that all unmatched sellers charge a $p_d(a_b; switch) = 0$ to their non-matched buyers to incentivize them to switch.

11.1. Here recall that a large market implies no unmatched seller is ex ante unique in the eyes of buyers: there exist at least one other seller with the same sized matched buyers.

11.2. Recall that offering product d at $p_d = 0$ is still strictly profit maximizing.

11.3. Thus standard Bertrand competition arguments apply and all unmatched sellers charge $p_d(a_b, switch) = 0$.

12. Then, note that all buyers consider switching to the same seller and derive a per-asset utility $u_d(a_b, switch)$ from that.

12.1 Recall that I have assumed that there exist clearing members that are selling product m. Further, conditional on that, I have shown that they insure all their product d buyers asset.

12.2. Further recall that their default probability reduces in total product m sales.

12.3. A risk-averse buyer therefore prefers switching to the clearing member(s) they expect to have highest product m sales to other buyers.

12.4. All buyers have the same information set and preference. Any SPNE is thus characterized by the buyers' anticipating (correctly) the clearing members with the most product m sales. I denote this anticipated utility from switching with $u_d(a_b, switch)$.

13. Contrary to this, the utility from staying is denoted with $u_d(a_b; stay)$. Then, recall any other seller charges $p_d(a_b, switch) = 0$. Then, the matched seller charges a price $p_d(a_b; stay)$ that just makes the buyer indifferent between switching or not:

$$u_d(a_b; stay) - p_d(a_b; stay) = u_d(a_b, switch) - C/a_b, \tag{A.87}$$

$$p_d(a_b; stay) = C/a_b - \left[u_d(a_b, switch) - u_d(a_b; stay)\right]. \tag{A.88}$$

Any higher price for sure makes the buyer switch and cannot be profit maximizing for the matched seller. Recall that even for $p_d = 0$ selling product d is profitable. For any lower price however, the seller would leave profits on the table.

14. This price is not sufficient to deter switching if the gains from switching exceed the switching costs. In that case $p_d(a_b, stay)$ would be negative, violating the assumption restriction pf $p_d \geq 0$. Then the matched buyer would charge $p_d(a_b, stay) = 0$ such that:

$$p_d(a_b; stay) = \max\{C/a_b - \left[u_d(a_b, switch) - u_d(a_b; stay)\right]\ ;\ 0\}. \tag{A.89}$$

15. Similarly, it may also be that the gains from switching are not enough to ever entice switching. This is the case when;

$$u_d(a_b; switch) - C < u_r. \tag{A.90}$$

In that case, the matched seller is a de-facto monopolist can can charge:

$$p_d(a_b; stay) = u_d(a_b; stay) - u_r. \tag{A.91}$$

Given the above results, I will derive some general properties of the equilibria:

1. All buyers purchase the same combination of assets from the same seller.

1.1. From the Lemma above, we know that buyers are always offered product d at a competitive price that induces either their staying with their matched buyer or switching.

1.2. We further know that, when purchasing product d from a clearing member, the buyer is either offered product m for all or none of his assets. He accepts this in either case, as p_m is such that the buyer is always indifferent between holding the product or not.

1.3. And hence, buyers always hedge all assets, and insure either all or none of them.

2. Then note that set of sellers to which buyers consider to switch can be restricted to the set of clearing members M.

2.1. Before we know that once a seller is a clearing member, the seller always finds it optimal to sell the counterparty default insurance to all its buyers (or none).

2.2. For the same amount of buyers switching to a clearing member relative to a non-clearing member, they have a lower default probability due to $g_m \geq g_m^*$. Therefore, the buyers considering switching anticipates, that the clearing member will post collateral g_m for at all of her other buyers. [6]

3. Further, all buyers switch to the same clearing member. Proof by contradiction:

3.1 Assume not, such that majority share of the switching buyers has switched to sellers s^* and the remaining buyers have switched to s^{**}, where $s^*, s^{**} \in M$. Every buyer that found it profitable to switch to s^{**} finds it even more profitable to switch to s^*.

3.2 This is because switching costs are the same for every buyer whether they switch to s^* or s^{**}. However, the gains of trade from insurance increase the more buyers purchase from the same seller as $\partial\widehat{D}/\partial Q_{dm} > 0$.

Using results from section 1.1.2.:

$$\frac{\partial F\left(0, \mu\left(\text{\# of insurance sales}\right), \sigma^2\right)}{\partial \text{\# of insurance sales}} = -g_m f(0) < 0 \quad for \quad g_m > 0. \tag{A.92}$$

3.3 Hence, all buyers switching to the smaller clearing member have an incentive to switch to the bigger clearing member instead. Only if all buyers switch to the same clearing member do they not haven an incentive to deviate.

4. Thus, it just remains to be shown whether given C none, some or all buyers switch.

I will start with the fully switching equilibrium. Sketch of proof for ***Proposition*** *3* :

1. In a fully switching equilibrium, it must be that a matched seller s will not be able to hold his matched buyer b even for a price $p_d(a_b, stay) = 0$. As the switching cost frictions are not large enough to deter switching, standard price competition arguments apply and all sellers charge $p_d(a_b; switch) = 0$.

2. Then, we know from the general properties above that in any fully switching SPNE all buyers switch to the same clearing member.

3. Further, a buyer b is offered derivative product d at price $p_d(a_b; scwitch) = 0$ from that seller s^* (and all other sellers). Consequently,

4. The conditions (1.9) and (1.10) can be rewritten, such that for every b switching to $s^* \in M$ is

[6] The buyer can anticipate that they will also be offered the default insurance from any clearing member. However, recall that this seller will capture all utility gains from the insurance. The buyer considering to switch is thus only considering which seller offers the highest utility gains, when purchasing the derivative *without* insurance.

optimal if:

$$u_d(a_b; stay) \leq u_d(a_b; switch) - \frac{C}{a_b} \quad \forall a_b, \tag{A.93}$$

$$C \leq a_b \left[u_d(a_b; switch) - u_d(a_b; stay)\right] \quad \forall a_b. \tag{A.94}$$

5. Then note that in a fully switching equilibrium, all buyers switch to the same clearing member and that this clearing member posts collateral for every sale. Having assumed that $g_m > g_m^*$ implies that this clearing members default probability decreases with every sale. Further, having assumed the market to be large, implies $u_d(a_b; switch)$ is equal to $u_{dm} = \mu_r$. This is because for the large sales volume, D_M asymptotically approximates zero.

6. It can be shown that the right-hand-side of inequality (A.94) is strictly increasing in a_b.

$$\frac{\partial a_b \left[u_d(a_b; switch) - u_d(a_b; stay)\right]}{\partial a_b} = \frac{\partial a_b \left[u_{dm} - u_d(a_b; stay)\right]}{\partial a_b}, \tag{A.95}$$

$$= \underbrace{u_{dm} - u_d(a_b; stay)}_{+} - a_b \underbrace{\frac{\partial u_d(a_b; stay)}{\partial a_b}}_{-}. \tag{A.96}$$

6.1. The first term in the FOC (A.96) is positive, as for a risk-averse buyer the utility in the absence of default risk exceeds the utility from buying product d from a risky seller with a positive probability of default.

6.2. The second term in the FOC (A.96) is overall positive. $u_d(a_b; stay)$ is decreasing in the number of assets a buyer has, because it is formed before the seller would insure the assets at the CCP. And any uninsured product d sales increases the variance of seller profits, but not the mean. And thus ultimately increases default probability. Multiplied however, by a minus one implies it enters overall positively in the slope.

6.3. It is thus both necessary and sufficient to show that condition A.94 holds for the smallest buyer in the market. This is because the smallest buyer has the lowest utility gains from switching relative to the switching cost C. Nevertheless, switching must still be optimal for this buyer.

7. Applying the law of large numbers, the smallest buyer in the market is of size $\underline{a}$. Thus, for fully equilibrium to exists, it is both necessary and sufficient that:

$$C \leq \underline{a} \left[u_d(\underline{a}; switch) - u_d(\underline{a}; stay) \right], \tag{A.97}$$

$$\leq U_d(\underline{a}; switch) - U_d(\underline{a}; stay). \tag{A.98}$$

8. Then finally recall that the set of clearing members M contains at least two sellers. Ex ante, buyers are indifferent to which clearing member to switch to. Therefore, there exist $\mid M \mid$ fully switching equilibria.

The fully switching equilibrium is contrasted by the no switching equilibrium, where all buyers stay with their matched seller. Sketch of proof for ***Proposition 2*** :

1. Assuming that the number of buyers is large results in the expected value of the sample maximum converging to the distribution maximum: $\mathbb{E}[\max a_b] \to a_b$.

2. Recall that I have shown above that: clearing members offer product m, those clearing members with the most sales are safest, these are never monopolists and always charge p_d, and thus buyers only consider switching to them. Finally, the utility from switching to the safest clearing member is denoted with $u_d(a_b, switch)$,

3. Then the no switching equilibrium exists if, each seller can set a price $p_d(a_b; stay) \geq 0$ and still deter switching.

$$C \geq a_b \left[u_d(a_b; switch) - u_d(a_b; stay) \right] \qquad \forall a_b \tag{A.99}$$

4. Here note again that the right-hand side is increasing in a_b:

$$\frac{\partial a_b \left[u_d(a_b; switch) - u_d(a_b; stay) \right]}{\partial a_b} = \underbrace{u_d(a_b; switch) - u_d(a_b; stay)}_{+} + a_b \underbrace{\left[\frac{\partial u_d(a_b; switch)}{\partial a_b} - \frac{\partial u_d(a_b; stay)}{\partial a_b} \right]}_{+}. \tag{A.100}$$

4.1. The first term above is quite trivially positive. The clearing member, anticipating her buyer not to switch and offering product m to that buyer, is safer. Thus a risk-averse buyer has a higher utility from purchasing product d upon switching.

4.2. The second term is not so straight forward and math is omitted here due to complexity. But it can be shown that:

4.2.1. Both $u_d(a_b; switch)$ and $u_d(a_b; switch)$ decrease in a_b, as the default probability increases in uninsured sales.

4.2.2. $u_d(a_b; switch)$ decreases by less than $u_d(a_b; stay)$. This follows from the fact that the clearing member has a higher mean

from the collateralized product m sales. Using expression (A.44)), therefore CMs default probability increases less. Therefore, the risk-averse buyer's utility decreases less.

4.2.3. Then both $u_d(a_b; switch)$ and $u_d(a_b; switch)$ decreasing, but $u_d(a_b; stay)$ decreasing more, implies that the difference between the two derivatives is positive.

4.3. Thus, a positive plus a positive term is positive. And the right-hand-side strictly increases in a_b

5. Therefore it is both necessary and sufficient for the no switching equilibrium to exists, when even the largest buyers of size $\overline{a}$ do not switch, conditional on nobody else switching. Thus the above condition reduces to:

$$C \geq U_d(\overline{a}; switch) - U_d(\overline{a}; stay). \tag{A.101}$$

6. The no-switching equilibrium is the unique equilibrium if there does not exists any other belief system under which switching is the best response.

6.1. In the most extreme belief system, a buyer of size $\overline{a}$, expects all other buyers to switch to the same clearing member: This buyer as the lowest per asset switching costs; the lowest utility from staying with his matched buyer (due to the high volume of uninsured sales) and would expect a perfectly safe clearing member in a large market.

6.2. Hence, even in the most extreme belief system, switching is never optimal if:

$$C \geq U_{dm}(\overline{a}; switch) - U_d(\overline{a}; stay). \tag{A.102}$$

6.3. I label the threshold for which the equation just holds with $\overline{C}$.

Then, I can show that for every $C \in [\underline{C}, \overline{C}]$ a fraction of smaller buyers stay and a fraction of larger buyers switches. This defines a partial switching equilibrium.

Sketch of Proof for *Proposition 4*

1. The partial equilibria can ever only be characterized by a continuum of buyers below a threshold n_c not switching and a continuum of buyers above the threshold n_C switching. The proof follows through exclusion.

1.1. An alternative candidate equilibrium is, where some smaller and some larger buyers switch, but there is a discontinuity in-between. However, this equilibrium cannot be sustained. In point 4. for the proof of Proposition 2, I have shown that for a given number of switching buyers, the benefits of switching strictly increase in size. Thus, if the larger and the smaller buyers find switching optimal, so would the medium sized buyers in the middle. This would violate the assumption that (correctly anticipated), there is a break in the switching buyers' size. Thus, this candidate equilibrium can be ruled out.

1.2. Following a similar logic (omitted as identical to the above point), any partial equilibrium, where some smaller buyers switch and some larger buyers not, can be ruled out.

1.3. Thus, any partial switching equilibrium must have a unique threshold, where all larger buyers switch to the same seller and all smaller buyers stay with their matched seller.

2. Then, the size threshold n_C is thus the buyer, who is just indifferent between switching or not, anticipating all larger buyers to switch:

$$C = U_d(a_c; switch) - U_d(a_c; stay). \tag{A.103}$$

3. Here, notice that the subset of switching buyers endogenously adjusts: n_C solves the above equation, where switching is just optimal given that all other buyers of size n_c switch.

No Sellers Offering Product m This paragraph completes the analysis of voluntary insurance by derive the equilibrium, when no sellers offer product m.

1. First note, that no seller offers product m implies that no seller posts collateral. Therefore, any additional product d increases the seller's default probability: a product d only increases the variance of seller profits, but the mean remains constant at $\mu_L > 0$ (see equation A.44).

2. Second note that given this, buyer's utility from switching decreases in the total product d

sales from the unmatched seller. Therefore, b buyer's consider to switch only to the sellers with the lowest total sales.

3. Then it can be shown that the no switching equilibrium exists.

3.1. Again, similar arguments as above apply and all none-matched buyers charge $p_d(a_b; switch) = 0$.

3.2. Then, in the no switching equilibrium, the seller with the lowest sales is the one matched with a buyer of size $\underline{a}$. The associated utility from switching to that seller is $u_d(a_b; switch)$.

3.3. Anticipating (correctly) that no other buyer switches to a matched buyer's seller, he expects a utility $u_d(a_b; stay) > u_d(a_b; switch)$. This is because the matched seller has totally less sales, than if the buyer would switch.

3.4. Given this $p_d(a_b; switch) = 0$ and C, it can be shown that the matched buyer can always charge a price $p_d(a_b; stay) > 0$ that deters switching:

$$u_d(a_b; stay) - p_d(a_b; stay) = u_d(a_b; switch) - C/a_b \tag{A.104}$$

$$p_d(a_b; stay) = \underbrace{C/a_b}_{+} + \underbrace{u_d(a_b; stay) - u_d(a_b; switch)}_{+} > 0 \tag{A.105}$$

3.5. Then recall from earlier, that for any $p_d(a_b; stay) \geq 0$, the seller will offer the product d and find it profitable.

3.6. Thus, neither buyers nor sellers have an incentive to deviate and the no switching equilibrium exists.

3.7. Note for completeness, that in case a buyer is captive, he is still charged $p_d(a_b; stay) = u_d(a_b; stay) - u_r$.

4. Further, it can be shown that this no switching equilibrium is unique.

4.1 First, I exclude any equilibrium where two or more buyers purchase from the same seller. Two or more buyers purchasing from the same seller implies both that: at least one seller has zero sales and at least one buyer pays switching costs C. Then, the buyer, who has to pay switching costs C has an incentive to deviate to the seller without sales. He would have to pay C in either case. However, the utility from purchasing product d from a seller without any other sales strictly exceeds the one from staying with the seller with additional buyers. Hence, he has an incentive to deviate.

4.2. This leaves us with alternative equilibria, where at least some buyers switch but all switching buyers end up with an individual seller.

4.2.1 From price-setting above we know that all sellers set $p_d(a_b; switch) = 0$.

4.2.2. Then to introduce such equilibrium, the sellers charge matched buyers:

$$p_d(a_b; stay) > u_d(a_b; switch) - C/a_b. \tag{A.106}$$

Through such high price, they trigger their matched buyer to another seller (without staying buyers).

4.2.3. Such price in (A.106), however, is not optimal for the seller matched with buyer b. He could simply reduce p_d to:

$$p_d(a_b; stay) = u_d(a_b; switch) - C/a_b. \tag{A.107}$$

This would allow the seller to retain his matched buyer and keep the switching buyer.

4.2.4. Of course, as per point 4.1. above this cannot be an equilibrium either.

Summarizing The Voluntary Insurance Results Unlike under mandatory insurance, there exists a multiplicity of equilibria under voluntary insurance. Remark formally summarizes them.

Remark 3.

If M and $S_{dm} \cup M_{dm}$ are non-empty subsets and $C_{NS} \leq \underline{C}$, then for:

(i) $C \in [0, C_{NS})$ only the fully switching equilibria exist,

(ii) $C \in [C_{NS}, \underline{C}]$ the fully switching equilibria and the no switching equilibrium coexist,

(iii) $C \in (\underline{C}, \overline{C}]$ the partial switching equilibria and no switching equilibrium coexist ,

(iv) And $C > \overline{C}$ the no switching equilibrium is unique.

If M and $S_{dm} \cup M_{dm}$ are non-empty subsets and $C_{NS} > \underline{C}$, then for:

(v) $C \in [0, \underline{C}]$ only the fully switching equilibria exist,

(vi) $C \in (\underline{C}, C_{NS})$ only partial switching equilibria exist,

(vii) $C \in [C_{NS}, \overline{C}]$ the partial switching equilibria and no switching equilibrium coexist,

(viii) And $C > \overline{C}$ the no switching equilibrium is unique.

1.3. Proofs Section 1.4. – Time 0 Outcomes

1.3.1. Proofs for Section 1.4.2.: Mandatory Insurance

Recall from Section 1.3. that the no switching equilibrium is unique under mandatory insurance and, hence, $Q_{dm} = a_b$. The sellers, thus, compare the following two expected profits, when deciding whether to become a CM $t = 0$:

$$\mathbb{E}_0\Pi = (1-D)\mathbb{E}_0[L \mid L > 0] = (1-D)\mu_l + d(a_b\sigma_r^2 + \sigma_L^2), \tag{A.108}$$

$$\mathbb{E}_0\Pi_m = -e_m + a_b \cdot (p_d + p_m - (1+\delta)g_m) + (1-D_M)(a_b g_m + \mu_L) + d_M(a_b\sigma_r^2 + \sigma_L^2). \tag{A.109}$$

Given these, one can show that the SPNE is a unique no switching equilibrium with large sellers becoming clearing members and small sellers exiting (together with their buyers).

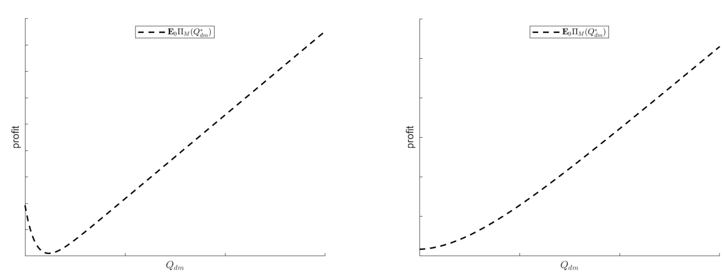
Sketch of proof for *Proposition 6*:

1. Note that $\mathbb{E}_0\Pi$ is a constant function at $D(Q_d = 0)\mu_L + d(Q_d = 0)\sigma_L^2$.

2. Note that for $Q_{dm} = Q_d = 0$, $\mathbb{E}_0\Pi_m$ is equal to $\mathbb{E}_0\Pi - e_m$, i.e., it intersects the y-axis e_m units below.

3. Further, recall the functional forms of $\mathbb{E}_0\Pi_m$:

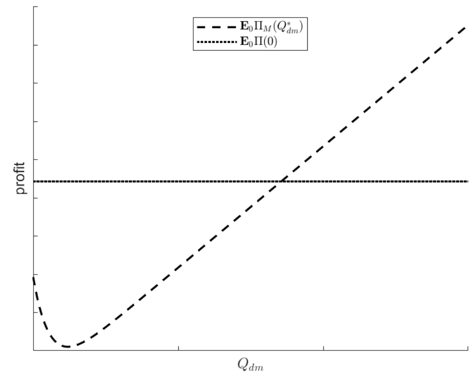
Figure A.3: Different Shapes of Expected Clearing Member Profits when $g_m > g_m^{**}$



4. It becomes immediately clear that intersecting the y-axis (weakly below) $\mathbb{E}_0\Pi$, the clearing member profits cross the profits from exiting at most once.

5. Pivoting back to the proof of *Proposition 1*, here I now consider the case where $\mathbb{E}_0\Pi_m$ has a local minimum in Q_{dm} before increasing and ultimately becoming positive. In that case it can easily be

seen that $\mathbb{E}_0\Pi$ crosses the expected clearing member profits only once. And more importantly in the upward sloping part. Consequently, conditional on being a CM, a seller offer the bundle to all assets of a buyer. This concludes the proofs under mandatory counterparty default insurance.



1.3.2. Proofs for Section 1.4.3.: Time 0 Equilibrium under Voluntary Insurance

In this sub-section I derive the SPNE under voluntary insurance anticipating the different outcomes at $t = 1$.

No Switching Equilibrium I start with all agents anticipating a no switching equilibrium at $t = 1$. The resulting expected profits of (non-) clearing members are thus:

$$\mathbb{E}_0\Pi_m = - e_m + a_b(p_d + p_m - v_m(1+\delta)g_m) + (1 - D_M)(a_b g_m + \mu_L) + d_M(a_b\sigma_r^2 + \sigma_L^2) \quad \text{(A.110)}$$

$$\mathbb{E}_0\Pi = a_b p_d + (1 - D) \cdot \mu_L + d(a_b\sigma_r^2 + \sigma_L^2) \quad \text{(A.111)}$$

Where:

$$p_m = u_{dm} - u_d(a_b; stay) - v_m \quad \text{(A.112)}$$

$$\text{captive buyers:} \quad p_d = u_d(a_b; stay) - u_r \quad \text{(A.113)}$$

$$\text{non-captive buyers:} \quad p_d = C/a_b - \left[u_d(a_b; switch) - u_d(a_b; stay)\right] \quad \text{(A.114)}$$

Sketch of proof for *Proposition 7*

1. Recall from earlier that profits $\mathbb{E}_0\Pi_0$ are always strictly increasing in total buyer size and thus sellers always offer at least product d. However, these profits depend on whether buyers are captive or not:

1.1. for captive buyers, $\mathbb{E}_0\Pi_0$ has the following slope in Q_d:

$$\frac{\partial \mathbb{E}\Pi}{\partial Q_d} = u_d(a_b; stay) + a_b - u_r \tag{A.115}$$

1.2. For non captive buyers, the slope is:

$$\frac{\partial \mathbb{E}\Pi}{\partial Q_d} = C + u_d(a_b; stay) - u_d(a_b; switch) \tag{A.116}$$

1.3. Recall that buyers are captive as long as: $u_d(a_b; switch) - C/a_b < 0$. These sellers thus extract the entire consumer surplus.

1.4. This is not longer possible, once buyers move from being captive to non-captive. Here, sellers can extract less utility from their matched buyers as they take into account that those may switch.

1.5 Hence, sellers experience a kink in their profit function exactly at the size threshold in a_b, where buyers become non-captive for a given C.

2. Now, I will turn to clearing member profits.

2.1. The clearing members matched with captive consumers can realize the same functional prices as under mandatory insurance. Even though CCP fees might vary under the two regimes, the general properties of the function remain the same.

2.2. For sellers matched with non-captive consumers, profits are still a strictly convex function:

$$\frac{\partial \mathbb{E}_0\Pi_m}{\partial Q_{dm}(Q_d)} = \underbrace{u_{dm} - u_d(a_b; switch) - v_m - (1+\delta)g_m + (1-D_m)g_M}_{linear part} + \underbrace{(1-D_m)g_m + \frac{\sigma_r^2}{2}d_m}_{convex part} \tag{A.117}$$

(A.118)

2.3. However, the overall slope is lower, as:

$$u_{dm} - u_d(a_b; switch) < u_{dm} - u_r \tag{A.119}$$

$$u_r < u_d(a_b; switch) \tag{A.120}$$

Thus, while maintaining local convexity, $\mathbb{E}_0\Pi_m$ also experience as kink at the local threshold.

3. Note that assuming reasonable values of v_m and g_m, this local convexity is sufficient to ensure a single crossing and unique solution for the threshold.

Fully Switching Equilibrium Instead for low C, the fully switching equilibria are anticipated at $t = 1$. Sketch of proof for*Proposition 8*:

0. Without loss of generality, assume $e_m = 0$.

1. Recall that I have assumed that there exists a clearing member indeed offering the product d plus m to all buyers. Then, it can be shown that $u_{dm} = u_d$.

1.1. For B large, $\widehat{D} = 0$ as $\frac{\partial D}{\partial Q_{dm}} < 0$ for $g_m > g_m^*$.

1.2. Because $\widehat{D}_M = D_M = 0$, we have $u_{dm} = u_d = \mu_r$.

2. Then, assuming $p_m \geq 0$, and $u_{dm} = u_d$, it must be that $v_m = 0$. For any higher $v_m > 0$, the buyer would not agree to product m as: $u_{dm} - u_d - v_m = -v_m < 0$.

3. Given this, it naturally follows that $p_m = 0$. Further, recall from *Proposition 3* that $p_d(a_b; switch) = 0$.

4. Assume that a seller expects to be the ultimately chosen clearing member. Then, offering product d and m results in strictly negative profits for $\delta > 0$. Denote the average buyer's expected size with $\hat{a}$. Then:

$$\mathbb{E}_1\Pi_m = B\hat{a}[p_d + p_m - (1+\delta)g_m] + (1-0)B\hat{n}g_m + 0 \cdot (B\hat{a}\sigma_r^2 + \sigma_L^2) \tag{A.121}$$

$$= -B\hat{a}\delta g_m < 0 \tag{A.122}$$

The probability of becoming the CM with all sales is one over the number of CMs: $\frac{1}{|M|}$.

5. Anticipating not to be the ultimately chosen clearing member, the seller expects the following profits from exiting the market.

$$\mathbb{E}_0\Pi = (1 - D(0))\mu_L + \sigma_l^2 d(0) \tag{A.123}$$

6. Then, comparing the pay-offs of being a clearing member and not are:

$$-\frac{1}{\mid M \mid} B\hat{n}\delta g_m + \frac{1}{\mid M \mid}[(1 - D(0))\mu_L + \sigma_l^2 d(0)] < \tag{A.124}$$

$$-B\hat{n}\delta g_m < (\mid M \mid -1)(1 - D(0))\mu_L + \sigma_L^2 d(0) \tag{A.125}$$

7. As the expected profits of becoming a clearing member are always below the expected profits from exiting the market, the subsets M is empty.

8. Unable to capture any clearing members, the CCP refrains from entering.

The Partial Switching Equilibrium Sketch of proof for *Proposition 9*:

1. The properties of the profit functions for all sellers with non-switching buyers are identical as the ones under the no switching equilibrium:

1.1. $\mathbb{E}_0\Pi$ intersects the y-axis above $\mathbb{E}_0\Pi_m$, but increases less in matched buyer size. Ultimately, they cross — upon which all larger sellers become clearing members.

1.2. The sellers with switching buyers cannot generate any sales. And thus, their profits are equal to market exit (minus e_m if exiting as a clearing member).

2. A reason they cannot generate any sales is due to all buyers switching to the seller with the largest non-switching buyers of size a_{PS}.

2.2. These sellers are clearing members, and thus insures her buyer's product m sales.

3. Any other equilibrium cannot be sustained.

3.1. Assume that only large sellers with non-switching buyers become clearing members. Then, one of them would attract all switching buyers and make a positive profit.

However, then a seller of with a buyer of size $a_P S$ has an incentive to deviate by becoming a clearing member. This way, she will for sure attract all buyers of size larger a_{PS}, because including her matched buyer, she has more total sales. And given the convex nature of $\mathbb{E}_0\Pi_m$, we know that offering additional the product bundles is optimal.

3.2. Assume only sellers, with matched buyers that are smaller than a_{PS} become clearing members cannot be optimal. If those find it optimal to pay e_m to sell the bundle to their matched buyer and potentially other (switching) buyers, then any larger seller also finds it optimal. This argument can be continued until a_{PS} is reached.

3.3 Thus, only equilibrium, where sellers matched with buyers of size a_{PS} become clearing members can be sustained. With equal probabilities, one of those sellers consequently attract all switching buyers at $t = 1$. In expectations, these sellers thus experience a jump in their profit function.

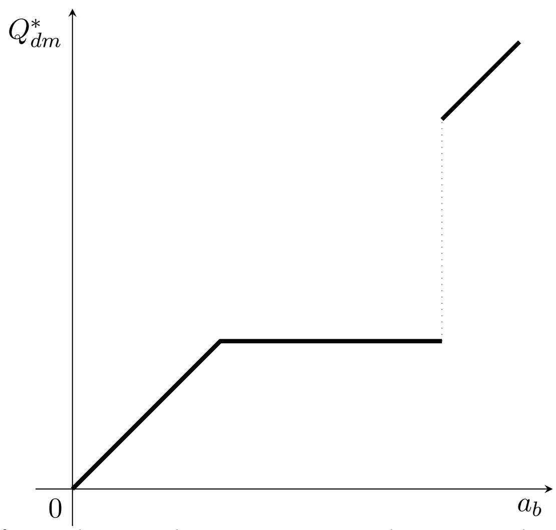
1.4. The SPNE When $g_m \leq g_{**}$

The main analysis focuses on the SPNE for the case where $\underline{g_m} > g_m^{**}$, which automatically also restricts the CCP's collateral choice g_m to that range. While this restriction is justified by the calibration exercise, this section nevertheless provides the theoretical results under the alternative: when the CCP sets $g_m \leq g_m^{**}$, supported by $\underline{g_m} \leq g_m^{**}$. Here, I maintain the assumption that $\underline{g_m} \geq g_m^{*}$, i.e. the regulator ensures that *insured* OTC derivatives increase the stability of the financial market.

1.4.1. Mandatory Insurance

Time 1 Outcomes The functional forms of prices p_m and p_d are unaffected by the level of collateral. Hence, buyers are still indifferent between holding the bundle or exiting the market. Further, the no switching equilibrium is still unique. However, for g_m weakly smaller than g_m^{**}, clearing member profits non-monotonically increase in bundle sales: for small and large buyers, the clearing member profits strictly increase in bundle quantities; medium sized buyers will only be offered to insure a share of their a_b assets.

Figure A.4: Equilibrium Bundle Quantities Q^*_{dm}
when $g_m \in (g^*_m,\ g^{**}_m]$



$$g^*_m = \frac{\mu_L \sigma^2_r}{2\sigma^2_L} \tag{A.126}$$

$$g^{**}_m = \frac{\mu_L \sigma^2_r}{2\sigma^2_L} + \frac{\sigma^2_r}{2\sigma_L} > g^*_m \tag{A.127}$$

***Proposition* 14.** *Under mandatory insurance, the no switching equilirbium is unique and characterized by a bunlde price $p_d + p_m = u_{dm} - u_r - v_m$. However, for $g_m \in [g^*_m, g^{**}_m]$, only small and large clearing members offer the bundle for all their matched buyer's assets, while medium sized buyers offer to hedge and insure only a fraction. Buyers not matched with a clearing member exit the market.*

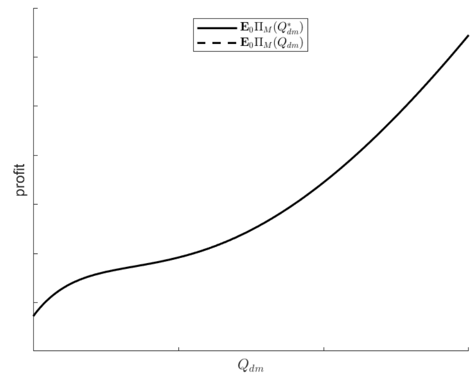
Sketch of proof:

1. The proof for bundle pries $p_d + p_m$ is omitted due to repetition. Please see Section 1.2. above for derivation.

2. Recall that $\mathbb{E}_1\Pi_m$ has two parts: a linear part in Q_{dm} and a non-linear part in Q_{dm}. Here, assuming $g_m \in (g^*_m, g^{**}_m]$ implies that the non-linear part is strictly increasing but concave for lower and convex only for higher Q_{dm}.

3. Given the properties og these two components, $\mathbb{E}_1\Pi_m$ can take on these functional forms:

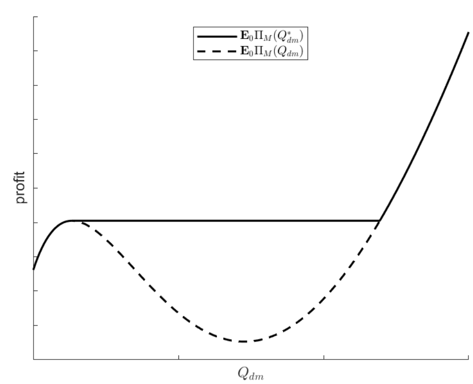
3.1. Assume that the linear part has a positive slope. Then $\mathbb{E}_1\Pi_m$ is strictly increasing, but preserves its concavity for low Q_{dm}.



3.2.Assume that the liner part has a negative slope.

3.2.1. Here, for $g_m > -p_d - p_m - v_m + (1+\delta)g_m$, $\mathbb{E}_1\Pi_m$ is increasing for low Q_{dm} until a local maximum, then decreasing for intermediate Q_{dm} until a local minimum, but ultimately strictly positive and for large Q_{dm} asymptotically approaches a slope of $p_d + p_m - v_m - \delta g_m > 0$. See the dashed line in Figure A.5 below.

Figure A.5: Profit Function

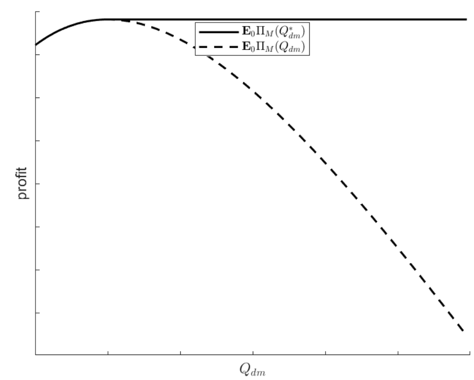


However clearing members are free to insure only a fraction $Q^*_{dm} \leq Q_{dm}$ of the possible sales available to them. And thus, given the no switching equilibrium, sellers matched with intermediate the realized profits have a flat line for intermediate sales.

Here notice that any increase in v_m increases the flat line both to the left and right side: it lowers the linear part, while leaving the non-linear part unaffected. Hence, the overall slope is decreased at all Q_{dm} And hence, if v_m is sufficiently low me move into case 5.4.1. below. The reverse however also holds true such that if v_m is low enough, the linear part has a positive slope and we move to case 5.1.

3.2.2. For $g_m < -p_d - p_m + v_m + (1+\delta)g_m$, the function $\mathbb{E}_1\Pi_m$ may increasing for low Q_{dm} until a local maximum is reached (almost immediately after 0), but consequently strictly decreases until it approximates a slope $p_d + p_m - v_m - \delta g_m < 0$. See dashed line in Figure A.6 below.

Figure A.6: Clearing member profits under low collateral and high variable fees



However again, the CM may offer to only partially hedge and insure their buyers' assets. And thus, the optimal number of sales Q_{dm} becomes a flat line after the maximum.

3.2.3. For $g_m << -p_d - p_m + v_m + (1+\delta)g_m$, the function $\mathbb{E}_1\Pi_m$ is strictly decreasing, even for low Q_{dm}. Figure omitted as trivial.

Time 0 Outcomes Recall from the time 1 outcomes that, for small $g_m \leq g_m^{**}$, medium sized clearing members may insure only a fraction of the buyers' assets. This is because a clearing member's total expected profits are non-monotonically increasing: $\mathbb{E}_0\Pi_m$ has a flat part with zero slope for intermediate values of a_b. Given this, it can easily be seen that Proposition 6 still holds here: there exists a unique size threshold a^* determining clearing membership.

Figure A.7: Different Shapes of Expected Clearing Member Profits when $g_m \in [g_m^*, g_m^{**}]$

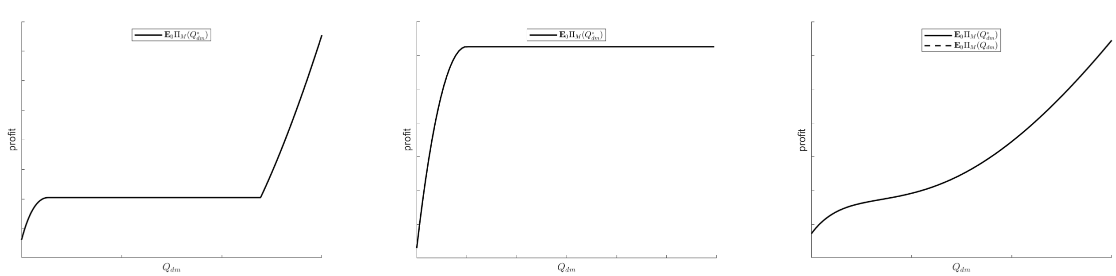
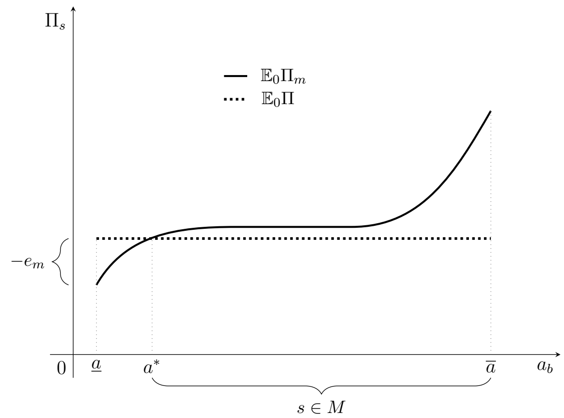


Figure A.8: The SPNE Under Mandatory Insurance When $g_m \leq g_M^{**}$



1.4.2. Voluntary Insurance

Time 1 Outcomes The time-1 outcomes under voluntary insurance do not depend whether g_m exceed g_m^{**} or not. For a more detailed analysis of this, please see the Section above For here just note that this is for the following reasons:

1. All sellers offer at least product d. Further, the buyers are however indifferent whether they are offered product m.

2. The beliefs of switching buyers are formed before any actual sales realize and also shared: all buyers switch to the same seller.

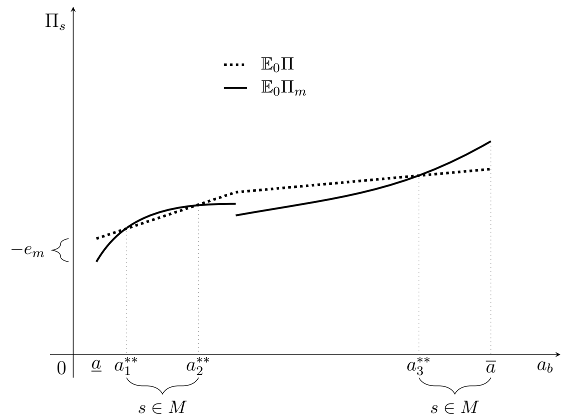
3. For a given belief set, any CM either clears all or none of her product d sales.

4. These beliefs must be ex ante correct. And thus, any equilibrium with switching must have a clearing member insuring all product d sales.

Time 0 Outcomes & No Switching If $g_m \in (g_m^*, g_m^{**}]$, the functional form of $\mathbb{E}_0\Pi_m$ becomes quite complex. Thus, the Proposition 7 does not hold any more, where a unique clearing membership threshold exists.

***Proposition* 15.** *Under voluntary insurance, the SPNE with a CCP, no switching at $t = 1$ and a collateral level $g_m \in (g_m^*, g_m^{**}]$, then multiple size thresholds define clearing membership and smaller clearing members may exist.*

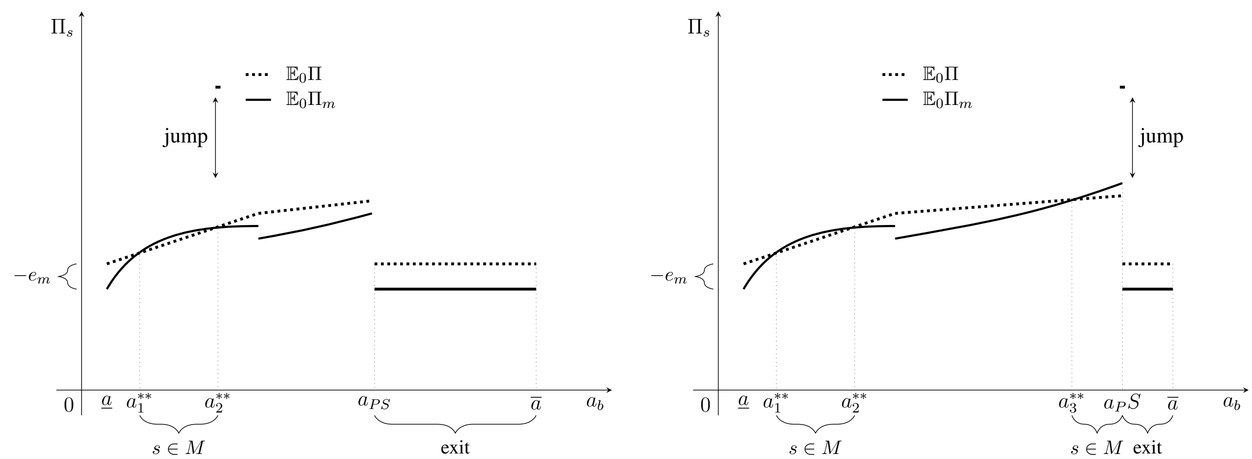
Figure A.9: The No Switching SPNE Under Voluntary Insurance When $g_m \in (g_m^*, g_m^{**}]$



Time 0 Outcomes & Fully Switching The results under fully switching are identical to the ones in the main text.

Time 0 Outcomes & Partial Switching Given the above described functional forms for $\mathbb{E}_0\Pi_m$ when $g_m \leq g_m^{**}$, the partial switching equilibrium is still characterized by large sellers exiting the market. It is, however, not longer given that the seller with the largest staying buyer becomes the clearing member of choice. AS is illustrated in the two Figures below, we might have now that the largest seller does not become a clearing member. Instead, a smaller seller might become a clearing member, where all buyers switch to. Fully characterizing the equilibrium in such case goes beyond the scope of this paper, and hence, full proofs are omitted. I would simply like the reader to take away that for intermediate switching cost and low collateral, who exactly ends up as clearing member is not so straight forward. And hence, regime shift effects are hard to predict.

Figure A.10: The Partial SPNE Under Voluntary Insurance When $g_m \in (g_m^*, g_m^{**}]$



(a) Case 1

(b) Case 2

Appendix B

Appendix to Chapter 2

2.1. Proofs

This section derives the results presented in the paper in consecutive order, starting with Lemma 1 deriving the optimal auction bid price.

Lemma 1 *The broker-dealers' optimal bidding strategy in a single second-price auction for security s at time $t = 2$ is:*

$$b_s^n = \begin{cases} a_s^n & a_s^n \geq 0 \\ \emptyset & otherwise \end{cases}. \tag{B.1}$$

Aggregating the resulting payoffs over all S securities, the lender and broker-dealers realize the following respective total payoffs:

$$\Pi^l = \frac{Sa}{2^N} \frac{2^N(N-3)+N+3}{N=1} \qquad \Pi^n = \frac{Sa}{2^N} \frac{2^{N+1}-N-2}{N(N+1)}. \tag{B.2}$$

Sketch of proof.

0. For this proof, we assume to have arrived at $t = 2$ and no ESLA has been entered.

1. Recall that all a_s^n independent draws from the same uniform distribution with a mean at zero: $a_s^n \sim U(-a, a)$.

1.1. Thus, the probability of drawing a positive/negative value in a single draw is 0.5.

1.2. The probability of having at least one out of N draws being positive is equal to one minus the probability of N negative draws. Relying on the symmetry of the uniform distribution around zero, this is equal to $1 - 0.5^N$.

1.3. Conditional on observe two positive draws, each broker-dealer is equally likely to have the higher draw.

2. Next, recall that a risk-neutral lender only provides the security, when receiving a weakly positive bid-price: $b_s \geq 0$.

This lower limit on the price results in broker-dealers only participating when $a_s^n \geq 0$, as they would realize the a loss otherwise:

$$\pi_s^n = a_s^n - b_s^n \geq 0 \tag{B.3}$$

$$a_s^n \geq b_s^n \geq 0. \tag{B.4}$$

.

Therefore, the optimal bidding strategy whenever $a_s^n < 0$ is abstinence and $b_s^n = \emptyset$.

3. If $a_s^n \geq 0$, truthful bidding is the optimal strategy.

3.1. First, assume that all other broker-dealers $k \neq n$ draw an negative ask-price $a_s^k < 0$ and thus $b_s^k = \emptyset$. Then, n is indifferent between truthful bidding, overbidding and underbidding. In either case, he would always pay zero and make a profit $\pi_s^n = a_s^n - 0$.

3.2. Second, assume that at least one other broker-dealer k participates in the auction bidding a generic $b_s^k < a_s^n$. Here, the broker-dealer find truthful bidding optimal. Conditional on winning, n is indifferent between truthful bidding or bidding slightly less: He would win, pay b_s^k regardless, and make a positive return. However, underbidding below b_s^k is not optimal, as he would lose and forego a profitable transaction. Bidding truthfully maximizes the chance of winning without risking any losses or extra cost.

3.3 Finally, assume that n participates in the auction bidding a generic $b_s^k < a_s^n$. Then the broker-dealer n is indifferent between truthful bidding and underbidding: He would lose regardless making zero profits. With sufficient overbidding, the broker-dealer would win the auction. However, because $b_s^k < a_s^n$, he would make a loss. This is less optimal than the zero profits from truthful bidding.

3.4. With the same arguments applying for all other (symmetric) broker-dealers. We can conclude that the optimal bidding strategy is:

$$b_s^n = \begin{cases} a_s^n & a_s^n \geq 0 \\ \emptyset & otherwise. \end{cases} \tag{B.5}$$

4. With the optimal bids in mind, we now turn to calculating the lender's expected lender profits.

4.1. First note that the lender's profits from a single auction depend on the number of broker-dealers with a positive bid-price. Each broker-dealer independently draws a positive bid-price with probability one half. The total number of positive bid-prices thus follows a Bernoulli distribution with N trials and $p = q = 0.5$.

4.2. Second recalls that the lender receives either the second highest bid, whenever two or more broker-dealers bid and zero otherwise. To capture this, denote the second-highest bid with $\max_{k \neq n} b_s^k \mid n$. With a slight abuse of notation, this operator automatically takes on the value zero, if strictly less than two broker-dealers participate.

4.3. Next, we rely on the distributional properties of the ask-prices and that S is large. This allows us to apply the law of large numbers stating that expected and realized lender profits are approximately equal. Thus, the lender's profits Π^l from the S auctions at $t = 2$ are:

$$\Pi^l = S \sum_{n=2}^{N} Pr(n) \mathbb{E}_1 \left[\max_{k \neq n} b_s^k \mid n \right] \tag{B.6}$$

$$= S \sum_{n=2}^{N} \binom{N}{n} 0.5^N a \frac{n-1}{n+1} = S 0.5^N a \left[\sum_{n=2}^{N} \binom{N}{n} \frac{n}{n+1} - \sum_{n=2}^{N} \binom{N}{n} \frac{1}{n+1} \right] \tag{B.7}$$

$$=S0.5^N a \left[\frac{N^2+3N-2^{N+2}+4}{2\,(N+1)} - \frac{(N-1)\left(N-2\left(2^N-1\right)\right)}{2(N+1)} \right] \tag{B.8}$$

$$=\frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1}. \tag{B.9}$$

4.4. The total expected lender profits from S auctions with $N=2$ are:

$$\mathbb{E}_1\Pi^l = SPr\left(a_s^n \& a_s^n > 0\right)\frac{a}{3} = \frac{Sa}{12}. \tag{B.10}$$

4.4. Notice that for lender's slight deviations between expected and realized profits to not matter for the model outcome. When the deviations realize at $t=2$, entering an ESLA is no longer possible. And hence, the lender is stuck with the auctions.

5. Next, we derive the seller profits from all S auctions, again we rely on the law of large numbers (LLN): For a large number of auctions, the realized total payoffs are close the expected payoffs. For simplicity, we assume exact equality for now and discuss slight deviations in Appendix 2.3..

5.1. First, we denote the broker-dealer, whose perspective we take on, with n. Further, we let $k \neq n$ denote the index of the highest bidder of all other participating broker-dealers out of the remaining $N-1$. Thus, $k \leq N-1$.

5.2. Next, recall that every broker-dealer draws his own ask-price. We follow standard conventions and assume the probability of equal ask-prices (and thus bid-prices) to be zero: $Pr(a_s^n = b_s^k) = 0$.

5.3. Then recall that a broker-dealer n expects a negative draw and zero profits with probability one half. With probability one half, he expects a positive draw. In that case, the number of other broker-dealers k that also have a positive ask-price draw, and thus participate, is again determined by the Bernoulli distribution.

The conditional likelihood of being the highest of $k=1$ draws is $1/(k=1)$.

5.4. Next, we let $\max_{k\neq n} b_s^k$ again denote the second highest bid, or zero in the absence of such. Conditional on winning, the broker-dealer then expects to make a profit between his realized ask-price and the second highest bid. Following the uniform distribution, the (expected) second highest bid out of $k+1$

bids is:

$$\mathbb{E}[\max_{k\neq n} b_s^k \geq 0] = \frac{k}{k+2}. \tag{B.11}$$

Similarly, the n's expected ask-price, conditional on beating the second highest bid, is equal to having the highest of $k+1$ draws:

$$\mathbb{E}_1\left[a_s^n \mid a_s^n > \max_{k\neq n} b_s^k \geq 0\right] = \frac{k+1}{k+2}. \tag{B.12}$$

5.5. Combining all above mentioned properties, a broker-dealer's expected profits are thus:

$$\Pi^n = SPr(a_s^n > 0)\sum_{k=0}^{N-1} Pr(k)Pr\left(a_s^n > \max_{k\neq n} b_s^k \geq 0\right)\mathbb{E}_1\left[a_s^n - \max_{k\neq n} b_s^k \mid a_s^n > \max_{k\neq n} b_s^k \geq 0\right] \tag{B.13}$$

$$= S0.5\sum_{k=0}^{N-1}\begin{pmatrix} N-1 \\ k \end{pmatrix} 0.5^{N-1}\frac{1}{k+1}a\left[\frac{k+1}{k+2} - \frac{k}{k+2}\right] \tag{B.14}$$

$$= \frac{Sa}{2^N}\frac{2^{N+1}-N-2}{N(N+1)}. \tag{B.15}$$

5.5. In the case of two broker-dealers, total broker-dealer profits over all S auctions are:

$$\Pi^b = \frac{Sa}{8} + \frac{Sa}{24} = \frac{Sa}{6}. \tag{B.16}$$

Next, we move on to the proof for Lemmas 2 and 3. Before doing so, we briefly discuss the general properties of the lender's under ESLAs. For this purpose, assume the lender has granted an ESLA with a given uniform bid-price b_E^n. Then, the lending happens with probability:

$$Pr(a_s^n > b_E^n) = 1 - Pr(a_s^n < b_E^n) = 1 - \frac{b_E^n + a}{2a} = \frac{a - b_E^n}{2a} \tag{B.17}$$

Given this, the lender expects a total payoff:

$$\mathbb{E}_1 \Pi^l = S(Pr(a_s^n > b_E^n) b_E^n + T_E^d = S \frac{a - b_E^n}{2a} b_E^n + T_E^d \tag{B.18}$$

The can similary dervie the ESLA holders expected profits and show that thez are strictly decreasing in both b_E^n and T_E^d:

$$\mathbb{E}_1 \Pi_E^d = S \cdot Pr(a_s^n > b_E^n) \mathbb{E}_1 \left[a_s^n - b_E^n \mid a_s^n > b_E^n \right] - T_E^d \tag{B.19}$$

$$= S \cdot \frac{a - b_E^n}{2a} \frac{a - b_E^n}{2} - T_E^d \tag{B.20}$$

$$\frac{\partial \mathbb{E}_1 \Pi_E^d}{\partial b_E^n} = S \frac{b_E^n - a}{2a} < 0 \tag{B.21}$$

$$\frac{\partial \mathbb{E}_1 \Pi_E^d}{\partial T_E^d} = -1 \tag{B.22}$$

Lemma 2 *A lender only excepts one of multiple ESLA offers whenever $N \leq 3$, in which case the optimal fees and expected profits are:*

$$b_E^n = 0 \qquad T_E^n = \mathbb{E}_1 \Pi_E^l = \frac{Sa}{4} \qquad \mathbb{E}_1 \Pi_E^n = 0 \quad \forall n. \tag{B.23}$$

Whenever $N \geq 4$, the lender strictly prefers engaging in the second-price auctions.

Sketch of Proof:

1. We start by establishing that both broker-dealers offer a bid-price and transfer combination that maximizes lender profit while neither resulting in losses nor gains for the broker-dealers. Any other combination prices cannot be sustained.

1.1. Assume that one broker-dealer sets a bid-price and transfer combination that allows him to make strictly positive profit. Then, any of the other broker-dealers can offer the same bid-price and marginally higher lump-sum transfer. This would allow him to attract the lender instead, and still making a positive, albeit slightly lower, profit.

1.2. Only if all broker-dealers make zero profit in equilibrium has no broker-dealer an incentive to deviate. Alternatively, every broker-dealer can simply attract the lender by offering part of said profit via a higher lump-sum transfer.

2. Next, we show that that there exists a single b_E^n and T_E^d combination that maximizes lender profits.

2.1. First note that the lump-sum transfer T_E^d must necessarily equate the chosen broker-dealer's expected profits to zero given the offered bid price b_E^n:

$$\mathbb{E}_1 \Pi_E^d = S \cdot \frac{a - b_E^n}{2a} \frac{a - b_E^n}{2} - T_E^d = 0 \tag{B.24}$$

$$T_E^d = S \cdot \frac{a - b_E^n}{2a} \frac{a - b_E^n}{2} \tag{B.25}$$

2.2. Inserting this into the lender's profit function, we can see that the lender profits are highest at $b_E^n = 0$:

$$\mathbb{E}_1 \Pi_E^l = \max_{b_E^n} S \frac{a - b_E^n}{2a} b_E^n + S \cdot \frac{a - b_E^n}{2a} \frac{a - b_E^n}{2} \tag{B.26}$$

$$\frac{\partial \mathbb{E}_1 \Pi_E^l}{\partial b_E^n} = -2 \frac{S}{4a} b_E^n = 0 \tag{B.27}$$

$$b_E^n = 0 \tag{B.28}$$

2.2. Inserting the solution back into the broker-dealers zero profit condition yields the the following transfers:

$$T_E^d = S \cdot \frac{a - b_E^n}{2a} \frac{a - b_E^n}{2} = \frac{Sa}{4} \tag{B.29}$$

3. Having derived optimal bid-price and lump-sum transfer, we can calculate the optimal broker-dealer lender profits: 3.1. Trivially, both broker-dealers make zero profits. This is regardless whether they are granted the ESLA or not.

$$\mathbb{E}_1\Pi_E^n = 0 \quad \forall n \tag{B.30}$$

3.2. The lender then expects (and realizes due to LLN) the following profit:

$$\mathbb{E}_1\Pi_E^l = T_E^d = S \cdot \frac{a - b_E^n}{2a}\frac{a - b_E^n}{2} = \frac{Sa}{4}. \tag{B.31}$$

4. Finally, notice that the auction pay-offs serve as a participation constraint for the lender. She may always reject an ESLA offer to try her luck at those. Hence, ESLAs are only every accepted then expected payoffs exceed those from the S auctions:

$$\mathbb{E}_1\Pi_E^l \geq \mathbb{E}_1\Pi^l \tag{B.32}$$

$$\frac{Sa}{4} \geq \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \tag{B.33}$$

$$3 \geq N. \tag{B.34}$$

Hence, lenders accept competitive ESLAs only if connected with less than four broker-dealers.

For completeness, we also derive optimal fee terms in case of a single ESLA offer. Notice that those will be ruled out as potential SPNE.

Lemma 3*If offered a single ESLA, the lender always accepts and the optimal fee and expected profits are:*

$$b_E^n = 0 \qquad T_E^n = \mathbb{E}_1\Pi_E^l = \mathbb{E}_1\Pi^l \qquad \mathbb{E}_1\Pi_E^n = \frac{Sa}{4} - T_E^n \qquad \mathbb{E}_1\Pi_E^k = 0 \quad \forall k \neq n \in N \tag{B.35}$$

1. We start by assuming only one broker-dealer offering an ESLA and correctly anticipating the other broker-dealer not to.

2. Being a monopolist, the broker-dealer then sets the lowest combination of bid-price and transfer possible that still motivate the lender to grant the ESLA. Hence, he equates the lenders participation constraint:

$$S\frac{a-b_E^n}{2a}b_E^n + T_E^d = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \tag{B.36}$$

$$T_E^d = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} - S\frac{a-b_E^n}{2a}b_E^n \tag{B.37}$$

2.3. Inserting (B.37) into the sellers profit maximization problem yields:

$$\mathbb{E}_1\Pi_E^d = \max_{b_E^n \geq 0} S\cdot\frac{a-b_E^n}{2a}\frac{a-b_E^n}{2} - \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} + S\frac{a-b_E^n}{2a}b_E^n \tag{B.38}$$

And the FOC wrt. b_E^n is equal to:

$$\frac{\partial \mathbb{E}_1\Pi_E^d}{\partial b_E^n} = -\frac{S}{a4}2b_E^n = 0 \tag{B.39}$$

$$b_E^n = 0. \tag{B.40}$$

2.4. An optimal bid-price b_E^n implies a lump sum transfer:

$$T_E^d = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \tag{B.41}$$

2.5. The respective total payoffs are thus:

$$\mathbb{E}_1\Pi_E^l = \frac{Sa}{12} \tag{B.42}$$

$$\mathbb{E}_1 \Pi_E^d = \frac{Sa}{4} - \frac{Sa}{2^N} \frac{2^N(N-3)+N+3}{N+1} \tag{B.43}$$

3. Because the transfers take the participation constraint into account, the lenders always accept.

4. For the broker-dealers, the profits from ESLA decrease in N. Here, offering a single ESLA is only optimal as long it is better than the auction payoffs.

$$\mathbb{E}_1 \Pi_E^d = \frac{Sa}{4} - \frac{Sa}{2^N} \frac{2^N(N-3)+N+3}{N+1} \geq \frac{Sa}{2^N} \frac{2^{N+1}-N-2}{N(N+1)} \tag{B.44}$$

$$N \leq 2 \tag{B.45}$$

Given the just derived pay-offs in the Lemmas 1-3, we now derive a set of candidate SPNEs.

Lemma 4 There always exists a candidate auction SPNE. It is sole candidate SPNE for $N \geq 4$.

Sketch of proof:

1. From Lemma 1, we know that both the lender and all broker-dealers make a strictly positive profit from the auctions. Hence, no one has an incentive to not participate, conditional on all other agents participating.

2. From Lemma 2, we know that the lender rejects any competitive ELSAs for $N \geq 4$.

3. From Lemma 3, we know that no monopolistic ESLA is offered for $N \geq 3$.

4. Combining all three, there always exists a candidate auction SPNE and that it is the sole candidate for $N \geq 4$.

Lemma 5 *There exists no candidate SPNE with a single broker-dealer offering an ESLA.*

Sketch of proof:

1. From Lemma 3, we know that a single broker-dealer only makes a monopolistic ESLA offer for $N = 2$. In all other cases, enticing the lender to participate is too costly.

2. For $N = 2$, however, the other broker-dealer has no incentive to actually refrain from offering an ESLA. Not offering an ESLA results in zero profits for the competitor. As argued in Lemma 2, the other broker-dealer could alternatively offer a slightly higher lump-sum transfer, winning the lender over and still making a profit.

3. As Lemma 2 highlights, such logic can be continued ultimately leading to the zero profit

Bertrand paradox typically found in price competition settings.

Lemma 6 *For $N \leq 3$, there exists several competitive ESLA SPNEs, each characterized by either two or three broker-dealer competing via ESLAs.*

Sketch of proof:

1. From Lemma 2, we know that the lender rejects any competitive ELSAs for $N \geq 4$, but accepts those for $N \leq 3$.

2. Here, conditional on some or all other broker-dealers making a competitive ESLA, a single ESLA is indifferent between making also a competitive ESLA offer or not: he would be left with zero profits in either case.

3. As all broker-dealers are symmetric, this holds for every broker-dealer. Hence, an candidate SPNE with competitive ESLAs exist.

4. Note, here for $N = 2$ is requires both broker-dealers making a competitive ESLA. For $N = 3$, it only requires two out of three making one.

Finally, we turn to test whether our candidate SPNEs are termination proof. This requires that the ESLA holder has no incentive to terminate the ESLA at $t = 2$, compensating the lender for potential losses.

Proposition 1

1. By assumption, the auction SPNE is termination proof. Once all agents have arrived at $t = 2$, no further ESLA can be offered and/or accepted. Hence, arising at $t = 1$ it remains unchallenged at $t = 2$.

2. Assume now an ESLA was granted at $t = 1$. The ESLA holder can terminate the contract at $t = 2$, triggering the auction SPNE. However, he must compensate the lender for the profit losses due to breach of contact. The ESLA holder thus terminates if:

$$\mathbb{E}_1\Pi^n - \left(\mathbb{E}_1\Pi^l_E - \mathbb{E}_1\Pi^l\right) \leq \mathbb{E}_1\Pi^n_E. \tag{B.46}$$

3. Let us start by assuming $N = 3$ and insert this into (B.46). It can easily be shown that the inequality is violated:

$$\frac{Sa11}{96} - \left(\frac{Sa}{4} - \frac{Sa3}{16}\right) > 0 = \mathbb{E}_1\Pi_E^n. \tag{B.47}$$

This implies that for $N = 3$, the ESLA holder terminates the ESLA and triggers the auction SPNE instead.

4. Inserting $N = 2$ into (B.46) instead yields in an equality:

$$\frac{Sa}{6} - \left(\frac{Sa}{4} - \frac{Sa}{23}\right) = 0 = \mathbb{E}_1\Pi_E^n. \tag{B.48}$$

Hence, for $N = 2$, the equilibrium is termination proof.

5. Just to provide some intuition, the punishment for termination decreases faster than the benefits from termination. And hence, for $N = 3$, the ESLA holder gains slightly less from termination than for $N = 2$, but pays substantially less punishment.

Finally, we conclude with studying the case of a single connected broker-dealer and $N = 1$.

Remark 1 *For $N = 1$, the lender is indifferent between being offered an ESLA or not, as the broker-dealer is a monopolist that always extracts all transaction surplus:*

$$b_s = b_E = T_E = \Pi^l = 0 \qquad \Pi^n = \frac{Sa}{4}. \tag{B.49}$$

Sketch of proof:

1. Starting with the auctions at $t = 2$. Given that there is no competitive bid, the broker-dealer always pays a zero bid prices and realities the whole -ask price as a profit for every borrowed security. This leads to:

$$\Pi^n = SPr(a_s^n > 0)\mathbb{E}_1[a_s^n \mid a_s^0] = \frac{Sa}{4} \tag{B.50}$$

$$\Pi^l = 0. \tag{B.51}$$

2. Because the auction profits serve as the lenders outside value, the broker-dealer can offer an ESLA with zero bid-price and lump-sum transfer. The profits of both agents remain the same.

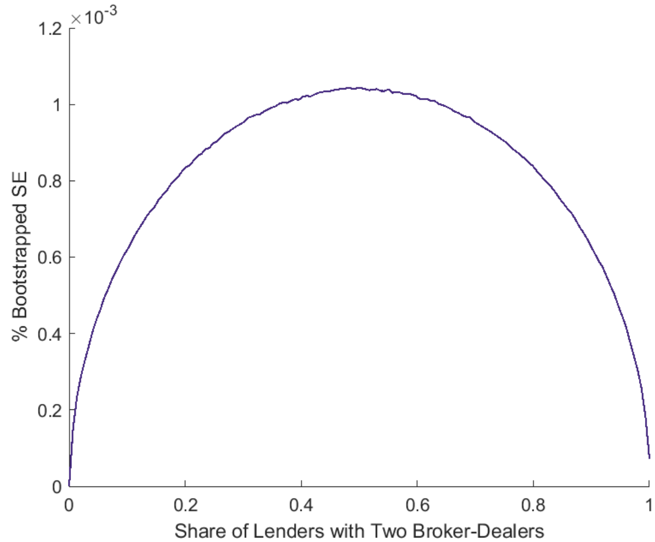
2.2. Computing the Aggregate In-Efficiency

In this section, we describe our algorithm to compute the predicted increase in trading due to the counterfactual assumption of outlawing ESLAs. In a first step, we calculate the portfolio size of each lender in our data. Here, we decided to measure portfolio size by the number of distinct ISINs underlying the transactions. An alternative could have been to use the number of transactions or trading volume. However, such measures would significantly inflate the market size as some transactions are rolled over daily over an extended horizon. But de facto, no new equity has exchanged hands. With this, we can derive an initial total market size as the sum of all portfolios.

Next, we divide the lenders into those with ESLAs and those without. We would like to acknowledge that in the data, we observe a small number of lenders that enter an ESLA with their main broker-dealer but trade a small part of their portfolio with others. As we do not observe the exact contract terms here, we act conservatively and assume those behave equivalently to those lenders with multiple counterparties and no ESLA. The portfolios of the lenders without ESLAs are temporarily put aside. On those with ESLAs, we perform the following bootstrapping algorithm :

1. We define a grid of 100 values denoted x falling in equal steps between zero and one: $x \in [0, 1]$. Each x represents the share of lenders having two versus one connected broker-dealers in the counterfactual.
2. For each share x defined in the grid, we repeat for 100,000 times:
   (a) Draw a random number from a standard uniform distribution for each lender.
   (b) Sort the lenders by their draw.
   (c) Assign the first x share of lenders two counterfactual broker-dealers and the remaining $1 - x$ one.
   (d) For those, where we assign two broker-dealers, we follow Corollary 1 and predict a 50% increase in the traded portfolio.
   (e) We compute the counterfactual aggregate portfolio by summing the counterfactual portfolios of

Figure B.1: Confidence Intervals around the predicted increase



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the ESLA holders and the actual portfolios of the non-ESLA holders.

(f) We obtain the percentage increase in aggregate portfolio relative to the true total.

3. From the 100,000 bootstraps, we calculate the average predicted increase in total portfolio and the bootstrapped standard errors. Wih both, we can obtain the 5th and 95th confidence intervals.

Figure 2.4 in the main text shows the predicted increases in traded portfolios, but confidence intervals are omitted due to their small size. Figure B.1 below plots the 5th to 95th percentile confidence range around the mean:

$$upper = + 1.96 \cdot std, \tag{B.52}$$

$$lower = - 1.96 \cdot std. \tag{B.53}$$

Here, we would like to point out that, as expected, the standard errors increase for medium shares. Here, there is simply higher variation between the bootstraps when assigning one versus two counterparties at random. Nevertheless, the standard errors always remain well below 0.02.

2.3. Extension: Ex Post Asymmetric Broker-Dealer

In the baseline model, the LLM ensures that all broker-dealers are both *ex ante* and *ex post* identical. In this extension, we check whether the ESLA SPNE is still termination proof, when the holder (unexpectedly) enjoys positively biased ask-price draws and, thus, on average can borrow more. Specifically, we assume that the cumulative probability of obtaining a draw above 0 for is ϵ above one-half. Denote such broker-dealer with biased draws with superscript b. Then:

$$Pr(a_s^b > 0) = 0.5 + \epsilon \qquad\qquad Pr(a_s^b \leq 0) = 0.5 - \epsilon. \tag{B.54}$$

To maintain tractability, we assume that conditional on having a positive or negative draw, the probability density is still uniform. Given this, the refined probability density function becomes:

$$Pr(a_s^b) = \begin{cases} \frac{0.5-\epsilon}{a} & a_s^b \leq 0 \\ \frac{0.5+\epsilon}{a} & a_s^b > 0 \end{cases}. \tag{B.55}$$

Assume that a broker-dealer arrives at $t = 2$ and observes the above described biased draws. In alignment with the private information assumption underlying second-price auctions, we assume that neither the lender nor the other broker-dealers are aware of such bias. Given other's oblivion, the expected profits from biased draws in the S auctions are:

$$\Pi^b = SPr(a_s^b > 0) \sum_{k=0}^{N-1} Pr(k) Pr\left(a_s^b > \max_{k \neq b} b_s^k \geq 0\right) \mathbb{E}_1 \left[a_s^b - \max_{k \neq b} b_s^k \mid a_s^b > \max_{k \neq b} b_s^k \geq 0\right] \tag{B.56}$$

$$= \frac{Sa(0.5+\epsilon)}{2^{N-1}} \frac{2^{N+1} - N - 2}{N(N+1)} \tag{B.57}$$

Because within the interval $[0, a]$ the ask-price a_s^b is still uniformly distributed, the expected pay-off from an auction, conditional on participation, is not affected. However, the broker-dealers likelihood of participating in an auction has been increased by ϵ. Hence, the biased broker-dealer'expected auction profits can alternatively be described as:

$$\Pi^b = (1 + 2\epsilon)\Pi^n. \tag{B.58}$$

Now assume that we are in the ELSA SPNE and broker-dealer b was the one originally assigned the ESLA. Bid-price and transfers are thus:

$$b^b_E = 0 \qquad T^b_E = \frac{Sa}{4}. \tag{B.59}$$

Here, again the biased draw increases the likelihood of a positive ask-price but not the expected ask-price conditional on being positive. Given this, the broker-dealer's profits under an ESLA become:

$$\Pi^b_E = SPr(a^b_s > 0)\mathbb{E}[a^b_s \mid a^b_s > 0] - \frac{Sa}{4} = \frac{aS\epsilon}{2}. \tag{B.60}$$

To check whether the ESLA SPNE remains termination proof, we must again account for the compensation that the broker-dealer must pay the lender upon termination:

$$\Pi^b_E \geq \Pi^b - (\Pi^l_E - \Pi^l) \tag{B.61}$$

$$\frac{\epsilon Sa}{2} \geq \frac{Sa}{6} + \frac{Sa\epsilon}{3} - \left(\frac{Sa}{4} - \frac{Sa}{12}\right) \tag{B.62}$$

$$\epsilon \geq 0. \tag{B.63}$$

From inequality B.36, we can conclude that when observing just a slightly biased draw, the broker-dealer strictly prefers not to terminate.

2.4. Regulatory Framework

Following the financial crisis 08/09, the Federal Reserve Board (FED) was mandated to perform two complementary stress tests: the Comprehensive Capital Analysis and Review (CCAR) and the Dodd-Frank Act stress testing (DFAST). The CCAR is a forward-looking exercise and assesses bank holding companies' (BHC) capital adequacy accounting for individual dividend payment plans. Banks with assets of 10$bn and above are required to take part in the CCAR. The DFAST takes the last three quarters' dividend policy as given and mainly focuses on the sufficiency of loss-absorbing capital (Federal Reserve Board, 2020c). Banks with assets of 250$bn and above are required to take part in the DFAST. For the purpose of this study (apart from the calibration), we focus on the CCAR stress test framework, which is described in detail in the following paragraphs.

CCAR Stress Test As part of the CCAR stress test, the FED calculates the individual BHCs' capitalization under three scenarios: baseline, supervisory adverse, supervisory severely adverse. Here, they account for the BHCs' proposed future dividend payments and capital repurchase plans. Subsequently, the FED decides whether to approve a BHC's planned capital actions by compare the post-stress capital levels under the severely adverse scenarios to the minimum capital requirements plus surcharges (Berrospide and Edge, 2019; Federal Reserve Board, 2019b).

Minimum Capital Requirements From 2009-2013, all stress-test eligible BHCs were subject to a minimum tier 1 common ratio of 5%. In 2014, all banks with at least $250 billion total assets or more than $10 billion foreign asset exposure were subject to a 4% minimum common equity tier 1 ratio (CET1) instead. The remaining banks continued to be subject to the 5% minimum tier 1 common ratio for one more year. From 2015 onward, all BHCs were subject to a 4.5% minimum common equity tier 1 ratio (Federal Reserve Board, 2015, 2016). This change in minimum capital measures was part of the phase-in of the Basel III framework, which also introduced additional capital surcharges.

Capital Surcharge BHCs identified as globally systemically important banks (G-SIB) are subject to additional minimum risk-adjusted capital measures of 1%-3.5%. From 2014 to 2016, the Basel Committee on Banking Supervisisons capital add-on is applied. Since 2017, the maximum of the surcharges calculated under the Basel capital framework and the Federal Reserve Board's assessment methodology titled "Method II" applies (Office of Financial Research, 2021). Additionally, a 2.5% conservation buffer was phased in from 2016-2019 (Federal Reserve Board, 2013, 2014). For our sample period, the banks are not subject to any countercyclical capital buffer (Federal Reserve Board, 2019a).

Supervisory Power over Dividend Payments Stress-test eligible BHCs are prohibited from any dividend payments and share repurchases until the FED has approved of the capital distribution plan in writing. As mentioned above, such approval is based on the stress test performance and follows in three steps. First, the FED performs an initial set of stress tests given the BHCs' original dividend payout plan. The resulting (preliminary) stress-test results are communicated to the BHC. All BHCs, both insufficiently and sufficiently capitalized, are allowed once to submit an adjusted capital plan (Berrospide and Edge, 2019; Federal Reserve Board, 2019b).

Then either the original or, if submitted, adjusted capital plan forms the base for the FED's payout policy interventions. Capital levels below the minimum tier 1 common ratio or CET1 (plus G-SIB surcharge) respectively, automatically trigger a payout ban. A violation of the capital conservation buffer automatically results in dividend payments to be restricted to a percentage of net income (see Table B.1). Sufficient capital levels do not result in automatic restrictions. The Fed, however, reserves the right to require a BHC to reduce or cease all capital distributions if it felt that the weaknesses in the BHC's capital planning warranted such a response (Federal Reserve Board, 2014). Thus BHCs may feel supervisory pressure especially when close to but not yet violating their respective minimum capital requirements.

Table B.1: Maximum Dividend to Net-income Ratio Given CET1

CET1	Maximum Pay-out Ratio
$< 5.125\%$	0%
$5.125\% - 5.75\%$	20%
$5.75\% - 6.375\%$	40%
$6.375\% - 7\%$	60%
$> 7\%$	no limitations

Source: BIS (2019)

Recent Developments In 2020, the Federal Reserve Board decided to replace the 2.5% capital conservative buffer by an individual stress test buffer for each BHC (Federal Reserve Board, 2020b,a). This falls outside our sample period.

2.5. Proofs for Section 3.2.

2.5.1. Solving the Bank's Optimization Problem

1. We start by defining dividend payments at $t = 1$ and $t = 2$.

$$d_1 = E_0 - E_1 \tag{B.64}$$

$$d_2 = L_1 r_{l,2} - r_d D_1 + E_1 \quad \sim N(\mu, \sigma^2) \tag{B.65}$$

$$\text{where} \quad \mu = (\mu_l + \rho_l r_{l,1}) L_1 - r_d D_1 + E_1 \tag{B.66}$$

$$\text{and} \quad \sigma^2 = \sigma_l^2 L_1^2 \tag{B.67}$$

Further note that D_1 is perfectly determined by E_1 and L_1 via the budget constraint:

$$D_1 = L_1 - E_1 \tag{B.68}$$

Finally, note that plugging this into the stress-test constraint yields:

$$\chi L_1 \leq E_1 + L_1(\bar{\mu_l} - \tau\sigma_l - r_d(L_1 - E_1) \tag{B.69}$$

$$(\chi - \bar{\mu_l} + \tau\sigma_l + r_d)L_1 - (1 + r_d)E_1 \leq 0 \tag{B.70}$$

2. Using the above stated equations and standard properties of a normal distributions, allows us to reduce the bank optimization problem to:

$$U(d_1, d_2) = \max_{E_1, L_1} E_0 - E_1 + \beta \left[L_1(\mu_l + \rho_l r_{l,1} - r_d(L_1 - E_1) + E_1 - \frac{\gamma \sigma_l^2}{2} L_1^2) \right] \tag{B.71}$$

$$s.t.$$

$$\lambda_1 : \quad \chi L_1 - E_1 \leq 0 \tag{B.72}$$

$$\lambda_2 : \quad (\chi - \bar{\mu_l} + \tau\sigma_l + r_d)L_1 - (1 + r_d)E_1 \leq 0 \tag{B.73}$$

$$\lambda_3: \qquad E_1 - E_0 \leq 0 \tag{B.74}$$

$$\lambda_4: \qquad E_1 - L_1 \leq 0 \tag{B.75}$$

$$\lambda_5: \qquad E_1 \geq 0 \tag{B.76}$$

We denote the multipliers associated with constraints (B.72)- (B.76) with λ_1 through λ_5 respectively.

3. Before taking any first order conditions, two comments on the constraints.

3.1. Notice that multipliers λ_3 and λ_5 can never be simultaneously be positive. They describe each their own corner solution: full retainment of equity and no retainment of equity.

3.2. Depending on τ, either minimum-equity and stress-test test constraint binds first. The other one consequently only binds in states in which the first one is already binding.

We start by rearranging the stress-test constraint:

$$\frac{(\chi - \bar{\mu}_l + \tau\sigma_l + r_d)}{(1 + r_d)} L_1 \leq E_1 \tag{B.77}$$

Then notice that the multiplier in front of L_1 in the above equation is determined fully by model parameters and does not depend on equilibrium choices. Further, it enters multiplicatively into the constraint in the same fashion as χ.

Then, logically, the stress-test constraint binds first whenever:

$$\frac{(\chi - \bar{\mu}_l + \tau\sigma_l + r_d)}{(1 + r_d)} \geq \chi \tag{B.78}$$

$$\tau \geq \frac{r_d\chi + \bar{\mu}_l - r_d}{\sigma_l} = \tau^* \tag{B.79}$$

And in reverse logic, the minimum equity constraint binds first, whenever $\tau < \tau^*$. This concludes the proof for *Lemma 1*.

4. The above described result of 3.2. allows us actually to combine the two supervisory constraints in the following fashion:

$$\chi(\tau) = \begin{cases} \chi & \tau < \tau^* \\ \frac{r_d\chi + \bar{\mu_l} - r_d}{\sigma_l} & \tau \geq \tau^* \end{cases} \tag{B.80}$$

And the revised constraint, which nests both cases, is:

$$\chi(\tau)L_1 \leq E_1 \tag{B.81}$$

5. Then, we start solving the simplified maximization problem by assuming the bank has choosen a feasible level $E_1 \in [0, E_0]$. Taking E_1 as given reduces the bank optimization problem to:

$$U(E_0 - E_1, d_2) = \quad E_0 - E_1 + \beta E_1(1 + r_d) + \max_{L_1} \beta \left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2} L_1^2 \right] \tag{B.82}$$

$$s.t.$$

$$\lambda_{1+2}: \qquad \chi(\tau)L_1 - E_1 \leq 0 \tag{B.83}$$

$$\lambda_4: \qquad -L_1 + E_1 \leq 0 \tag{B.84}$$

Then, the FOC wrt to L_1 becomes:

$$(\mu_l + \rho_{1,l} r_{l,1}) - r_d - \gamma\sigma_l^2 L_1 - \lambda_{1+2}\chi(\tau) + \lambda_4 = 0 \tag{B.85}$$

6. We now discuss the different cases for the multipliers. Here, notice that λ_{1+2} and λ_4 can never bind simultaneously: one would bind if the bank would like to set significantly lower L_1 than E_1 and one would bind if the bank would like set significantly higher than E_1/χ.

6.1. With this in mind, we start with (temporarily) ignoring both constraints. Then, the optimal loan level is:

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{B.86}$$

6.2. Then for a given E_1, logically there exists a lower threshold level $r^*_{l,1}$ for which investing $L_1 = E_1$ is optimal. And for all lower levels, the bank would like to set $L_1 < E_1$ but cannot due to its constraint choice.

Following a similar logic there exist a second threshold $r^{**}_{l,1}$, for which the bank would like to invest E_1/χ units into loans. And for any higher level, it would like to invest more, but cannot due to the minimum equity constraint.

6.3. However, as we will see later, these two thresholds are not really playing a core role, because E_1 is chosen by the bank and not taken as given. Here, it is important to take away from Equation (B.86) that any interior solution of L_1 without either constraints binding is independent of the level of equity E_1.

7. Lets start with assuming that $\lambda_{1+2} = \lambda_4 = 0$. This implies that the bank indeed finances some loans, but that these loans are more equity-financed than strictly required.

7.1. Recall then that L_1 is independent of E_1 and thus, the optimal level of E_1 can be chosen by the following optimization problem:

$$U(d_1, d_2) = \max_{E_1} E_0 - E_1 + \beta(1 + r_d)E_1 \tag{B.87}$$

$$s.t.$$

$$\lambda_3 : \quad E_1 - E_0 \leq 0 \tag{B.88}$$

Abstracting for now from constraint λ_3 this implies a FOC wrt E_1:

$$-1 + \beta(1 + r_d) \tag{B.89}$$

Relying on parameter assumptions, it can be shown that this FOC is always negative:

$$-1 + \beta(1 + r_d) < 0 \tag{B.90}$$

$$(1 + r_d) \leq \frac{1}{\beta} \quad \text{True by assumption} \tag{B.91}$$

Hence, any interior solution with only partial debt-financing cannot be sustained. Any solution with positive loan levels is characterized by $E_1 = \chi(\tau)L_1$.

8. With this in mind, we can now derive the optimal equity level E_1 by solving the following maximization problem:

$$U(d_1, d_2) = \max_{E_1} E_0 - E_1 + \beta \left[\frac{E_1}{\chi(\tau)}(\mu_l + \rho_l r_{l,1}) - \frac{\gamma\sigma_l^2}{2\chi(\tau)^2} E_1^2 - r_d \frac{1-\chi(\tau)}{\chi(\tau)} E_1 + E_1 \right] \tag{B.92}$$

$s.t.$

$$\lambda_4 : \quad E_1 - E_0 \leq 0 \tag{B.93}$$

$$\lambda_5 : \quad -E_1 \leq 0 \tag{B.94}$$

8.1. Again, we will for now ignore the two feasibility constraints. Then the FOC wrt E_1:

$$-1 + \beta \left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - \frac{\gamma\sigma_l^2}{\chi(\tau)^2} E_1 - r_d \frac{1-\chi(\tau)}{\chi} + 1 \right] = 0 \tag{B.95}$$

$$E_1^* = \frac{\chi(\tau)^2}{\gamma\sigma_l^2} \left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - r_d \frac{1-\chi(\tau)}{\chi} + 1 - \frac{1}{\beta} \right] \tag{B.96}$$

8.2. Now recall that an constraint solution requires $E_1 \leq E_0$. This holds up until:

$$\frac{\chi(\tau)^2}{\gamma\sigma_l^2} \left[\frac{\mu_l + \rho r_{l,1}}{\chi(\tau)} - r_d \frac{1-\chi(\tau)}{\chi} + 1 - \frac{1}{\beta} \right] \geq E_0 \tag{B.97}$$

$$r_{l,1} \geq \frac{1}{\rho_l} \left[\frac{\gamma\sigma_l^2}{\chi(\tau)} E_0 + \chi(\tau) \left(\frac{1}{\beta} - 1 \right) + r_d(1-\chi(\tau)) - \mu_l \right] = \overline{r_l} \tag{B.98}$$

Or in other words, for any level of $r_{l,1}$ exceeding the threshold $\overline{r_l}$ equity is fully retained and invested in loans. The optimal bank choices and (expected) dividends are thus:

$$E_1^* = E_0 \tag{B.99}$$

$$L_1^* = \frac{E_0}{\chi(\tau)} \tag{B.100}$$

$$d_1^* = 0 \tag{B.101}$$

$$\mathbf{E}[D_1^*] = E_0 \left[\frac{\mu_l + \rho_l r_{l,1}}{\chi\tau} - r_d \frac{(1-\chi(\tau))}{\chi(\tau)} + 1 \right] \tag{B.102}$$

8.3. A similar logic can be applied for the lower bound such that:

$$\frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l+\rho r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi} + 1 - \frac{1}{\beta}\right] \leq 0 \tag{B.103}$$

$$r_{l,1} \leq \frac{1}{\rho_l}\left[\chi(\tau)\left(\frac{1}{\beta}-1\right) + r_d(1-\chi(\tau)) - \mu_l\right] = \underline{r_l} \tag{B.104}$$

Or put differently, for any realized stated $r_{l,1}$ weakly below $\underline{r_l}$ no equity is retained. The bank's equilibrium choices and (expected) dividends are thus:

$$L_1^* = E_1^* = D_1^* = 0 \tag{B.105}$$

$$d_1 = E_0 \tag{B.106}$$

8.4. For intermediate levels $r_{l,1} \in (\underline{r_l},\ \overline{r_l})$ and interior solution exists with:

$$E_1^* = \frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l+\rho_l r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi(\tau)} + 1 - \frac{1}{\beta}\right] \tag{B.107}$$

$$L_1^* = \frac{E_1^*}{\chi(\tau)} \tag{B.108}$$

$$d_1^* = E_0 - E_1^* \tag{B.109}$$

$$\mathbf{E}[D_1^*] = E_1^*\left[\frac{\mu_l+\rho_l r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi(\tau)} + 1\right] \tag{B.110}$$

2.5.2. Comparative Statics Over τ

We now compare an environment where $\tau < \tilde{\tau}$ such that $\chi(\tau) = \chi$ with an environment, where $\tau > \tilde{\tau}$ such that $\chi(\tau \geq \tau) > \chi$.

1. We start by showing that that $\underline{r_l^s} < \underline{r_l^{n,e}}$.

$$\underline{r_l^s} < \underline{r_l^{n,e}}\chi\left(\frac{1}{\beta}-1-r_d\right) < \qquad \chi(\tau)\tilde{\tau} < \tau \tag{B.111}$$

2. Further, we can show that $\overline{r_l^s} > \overline{r_l^{n,e}}$:

$$\overline{r_l^s} > \overline{r_l^{n,e}} \tag{B.112}$$

$$\frac{\gamma\sigma_l^2}{\chi}E_0 + \chi\left(\frac{1}{\beta} - 1 - r_d\right) > \frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) \tag{B.113}$$

$$\gamma\sigma_l^2 E_0\left(\frac{1}{\chi} - \frac{1}{\chi(\tau)}\right) > (\chi(\tau) - \chi)\left(\frac{1}{\beta} - 1 - r_d\right) \tag{B.114}$$

Notice that the right hand side is a term very close to zero, and thus the inequality holds true under the assumption that $E_0 >> 0$.

3. With this, we know the upper and lower feasibility implied thresholds for equity and thus lending. Now, we turn to the slope of the optimal equity and lending policies.

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma\sigma_l^2}\rho_l \tag{B.115}$$

$$\frac{\partial^2 E_1}{\partial r_{l,1}\partial\chi(\tau)} = \frac{\rho_l}{\gamma\sigma_l^2} > 0 \tag{B.116}$$

3.1. It can be shown that E_1^* increases linearly in $r_{l,1}$:

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma\sigma_l^2}\rho_l \tag{B.117}$$

And confirming the relative return state bounds, it can be shown that the slope is steeper, the higher is τ:

$$\frac{\partial^2 E_1}{\partial r_{l,1}\partial\chi(\tau)} = \frac{\rho_l}{\gamma\sigma_l^2} > 0 \tag{B.118}$$

This implies that under a stress-test constraint, the bank starts to retain equity only in relatively higher states, but once started, it reaches full retainment earlier. Naturally, there exists a threshold $\tilde{r}$ for which the two equity functions intersect.

3.2. Turning to the loans, one can show that $L_1^{*,s} < L_1^{*,e}$. Her,e we first start with the loan rates implying $E_1 < E_0$. Then:

$$L_1^{*,s} < L_1^{*,e} \tag{B.119}$$

$$-\chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) < \chi\left(\frac{1}{\beta} - 1 - r_d\right) \tag{B.120}$$

$$\chi < \chi(\tau) \tag{B.121}$$

$$\tilde{\tau} < \tau \tag{B.122}$$

Now, we consider the high return states inducing $E_1^* = E_0$:

$$L_1^{*,s} < L_1^{*,e} \tag{B.123}$$

$$\frac{E_0}{\chi(\tau \geq \tau)} < \frac{E_0}{\chi} \tag{B.124}$$

$$\chi < \chi(\tau) \tag{B.125}$$

$$\tilde{\tau} < \tau \tag{B.126}$$

We omit the proof for the variance of lending here due to its complexity here, and discuss it in detail during the supervisory problem. We would nevertheless like to highlight here, that lending L_1^* follows a rectified normal distribution with a lower and an upper bound. By increasing τ (above $\tilde{\tau}$), we bring the bounds closer together, thus reducing the variance of the overall distribution.

2.6. The Optimal Tightness τ

In this section, we derive the optimal supervisory choice under two different objective functions. To maintain tractability, we will assume that the realization of return states above $r_{l,1}^{f,s}$ are very low probability events for large banks with sufficient equity stocks. Thus, loan levels are fully characterized. Let us denote the optimal lending in the absence of feasibility constraints with L_1^x, where:

$$L_1^x = \frac{1}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{B.127}$$

$$L_1^x \sim N(\mu_x, \sigma_x^2) \tag{B.128}$$

$$\mu_x = \frac{1}{\gamma\sigma_l^2}\left[\mu_l + \rho_l(\mu_l + \rho_l r_{l,0}) - r_d - \chi(\tau)(1/\beta - 1 - r_d)\right] \tag{B.129}$$

$$\sigma_x^2 = \left(\frac{\rho_l}{\gamma\sigma_l}\right)^2 \tag{B.130}$$

The optimal bank lending L_1^* thus takes the following step-function.

$$L_1^* = \begin{cases} 0 & L_1^x < 0 \\ L_1^x & 0 \geq L_1^x \geq \frac{E_0}{\chi(\tau)} \\ \frac{E_0}{\chi(\tau)} & L_1^x > \frac{E_0}{\chi(\tau)} \end{cases} \tag{B.131}$$

2.7. Additional Proofs

2.7.1. Proofs for Voluntary Violation

Voluntary violation of the stress-test constraint implies a ban on dividends and, thus, the following equalities:

$$d_1 = 0 \tag{B.132}$$

$$E_1 = E_0 \tag{B.133}$$

$$D_1 = L_1 - E_0 \tag{B.134}$$

With this, the optimization problem reduces to:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{B.135}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi}\right] \tag{B.136}$$

Here note that the upper feasibility limit is now determined by χ and not anymore $\chi(\tau)$.

Ignoring the two feasibility constraints for now, the FOC and the consequent optimal lending level are:

$$\mu_l + \rho_l - r_d - \gamma\sigma_l^2 L_1 = 0 \tag{B.137}$$

$$L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{B.138}$$

Recall that L_1^{*V} is bounded above by the minimum asset-to-equity ratio constraint which allows us to derive a threshold $\overline{r_l^V}$. Similarly, in this business model L_1 can never be below E_0, allowing us a lower threshold $\underline{r_l^V}$

$$\overline{r_l^V} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + r_d - \mu_l\right] \tag{B.139}$$

$$\underline{r_l^V} = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \tag{B.140}$$

With this in mind, it remains to be shown when the total utility exceeds the one of complying to the stress-test constraint. The resulting total utility from violation is:

$$r_{l,1} < \underline{r_l^V}: \quad U^V(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{B.141}$$

$$r_{l,1} \in [\underline{r_l^V}, \overline{r_l^V}]: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right) L_1^{*V} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*V}\right)^2 + (1 + r_d)E_0\right] \tag{B.142}$$

$$\text{where} \quad L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{B.143}$$

$$r_{l,1} > \overline{r_l^V}: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi^2} + E_0(1 + r_d)\right] \tag{B.144}$$

This, we have to compare to the following aggregate utilities from complying:

$$r_{l,1} < \underline{r_l}: \quad U(d_1, d_2) = E_0 \tag{B.145}$$

$$r_{l,1} \in \left[\underline{r_l}, \overline{r_l}\right]: \quad U(d_1, d_2) = E_0 - E_1^* + \beta \left[\left(\mu_l + \rho_l r_{l,1} - r_d \right) L_1^* - \frac{\gamma \sigma_l^2}{2} \left(L_1^* \right)^2 + E_1^*(1 + r_d) \right] \tag{B.146}$$

$$\text{where} \quad L_1^* = \frac{E_1^*}{\chi(\tau)} = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1 - 1/\beta + r_d)}{\gamma \sigma_l^2} \tag{B.147}$$

$$r_{l,1} > \overline{r_l}: \quad U(d_1, d_2) = \beta \left[\left(\mu_l + \rho_l r_{l,1} - r_d \right) \frac{E_0}{\chi(\tau)} - \frac{\gamma \sigma_l^2}{2} \left(\frac{E_0}{\chi(\tau)} \right)^2 + E_0(1 + r_d) \right] \tag{B.148}$$

To derive when violation would be optimal, one must compare the appropriate utilities given the return state $r_{l,1}$. A challenge here is that $\underline{r_l^V} \lesseqgtr \underline{r_l}$ and $\overline{r_l^V} \lesseqgtr \overline{r_l}$, depending on E_0:

$$\underline{r_l^V} \lesseqgtr \underline{r_l} \tag{B.149}$$

$$\frac{1}{\rho_l} \left[\gamma \sigma_l^2 E_0 + r_d - \mu_l \right] \lesseqgtr \frac{1}{\rho_l} \left[\chi(\tau) \left(\frac{1}{\beta} - 1 - r_d \right) + r_d - \mu_l \right] \tag{B.150}$$

$$E_0 \lesseqgtr \frac{\chi(\tau)}{\gamma \sigma_l^2} \left(\frac{1}{\beta} - 1 - r_d \right) \tag{B.151}$$

$$\overline{r_l^V} \lesseqgtr \overline{r_l} \tag{B.152}$$

$$\frac{1}{\rho_l} \left[\frac{\gamma \sigma_l^2}{\chi} E_0 + r_d - \mu_l \right] \lesseqgtr \frac{1}{\rho_l} \left[\frac{\gamma \sigma_l^2}{\chi(\tau)} E_0 + \chi(\tau) \left(\frac{1}{\beta} - 1 - r_d \right) + r_d - \mu_l \right] \tag{B.153}$$

$$E_0 \lesseqgtr \frac{\chi \chi(\tau)^2}{(\chi(\tau) - \chi) \gamma \sigma_l^2} \left(\frac{1}{\beta} - 1 - r_d \right) \tag{B.154}$$

Without further restrictions on E_0, a closed-form proof is a cumbersome comparison of all possible combinations for the different functional forms that the utilities may take. As this provides little additional insight without restricting the parameter space, we refrain from doing so. Instead, we show when voluntary violation is optimal for the above calibrated parameters and several different values of E_0. Please refer to the main text for results.

2.7.2. Covid-19 Dividend Ban

Sketch of proof for Proposition 12.

1. A ban on bank dividend payments implies the following equalities:

$$d_1 = 0 \tag{B.155}$$

$$E_1 = E_0 \tag{B.156}$$

$$D_1 = L_1 - E_0 \tag{B.157}$$

2. As the stress-test constraint is still binding, the optimization problem reduces to:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{B.158}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi(\tau)}\right] \tag{B.159}$$

3. Temporarily ignoring the two feasibility constraints, taking the FOC and equating it to zero yields the following optimal lending level:

$$\mu_l + \rho_l - r_d - \gamma\sigma_l^2 L_1 = 0 \tag{B.160}$$

$$L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}. \tag{B.161}$$

4. Now, we turn to the upper feasibility limit on L_1^{*B} determined by the stress-test-implied minimum asset-to-equity ratio constraint. This allows us to derive a threshold $\overline{r_l^B}$:

$$L_1^{*B} \geq \frac{E_0}{\chi(\tau)} \tag{B.162}$$

$$r_{l,1} \geq \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + r_d - \mu_l\right] = \overline{r_l^B} \tag{B.163}$$

Similarly, in this business model L_1 can never be lower than E_0, allowing us to define the lower threshold $\underline{r_l^B}$

$$L_1^{*B} \leq E_0 \tag{B.164}$$

$$r_{l,1} \leq \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] = \underline{r_l^B} \tag{B.165}$$

5. Then, the total utility under the Covid-19 dividend ban, denoted with $U^B(d_1, d_2)$, becomes:

$$r_{l,1} < \underline{r_l}: \quad U^B(d_1,d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{B.166}$$

$$r_{l,1} \in [\underline{r_l}, \overline{r_l}]: \quad U^B(d_1,d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)L_1^{*B} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*B}\right)^2 + (1 + r_d)\,E_0\right] \tag{B.167}$$

$$\text{where} \quad L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{B.168}$$

$$r_{l,1} > \overline{r_l}: \quad U^B(d_1,d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi(\tau)^2} + E_0\,(1 + r_d)\right] \tag{B.169}$$

6. We are left with showing that $L_1^* < L_1^{*,B}$:

6.1. Assume a realized $r_{l,1}$ in the range $(-\infty,\ \min\{\underline{r_l};, \underline{r_l^B}\}]$. Then:

$$L_1^* < L_1^{*B} \tag{B.170}$$

$$0 < E_0 \tag{B.171}$$

6.2. Assume a realized return in the range $(\underline{r_l},\ \underline{r_l^B}]$. Then:

$$L_1^* < L_1^{*B} \tag{B.172}$$

$$\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} < E_0 \tag{B.173}$$

$$r_{l,1} < \frac{1}{\rho_l}\left(\gamma\sigma_l E_0 - \mu_l + r_d + \chi(\tau)(1/\beta - 1 - r_d)\right) \tag{B.174}$$

$$< \underline{r_l^B} + \frac{1}{\rho_l}\chi(\tau)(1/\beta - 1 - r_d) \tag{B.175}$$

Which holds true by assumption.

6.3. Assume a realized return $r_{l,1}$ in the range $(\underline{r_l}^B,\ \underline{r_l}]$. Then:

$$L_1^* < L_1^{*B} \tag{B.176}$$

$$0 < \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{B.177}$$

$$\underline{r_l^B} - \frac{\gamma\sigma_l^2 E_0}{\rho_l} < r_{l,1} \tag{B.178}$$

Which holds true by assumptions.

6.4. Assume a realized $r_{l,1}$ in the range $\Big(\max\{\underline{r_l}, \underline{r_l^B}\},\ \min\{\overline{r_l}, \overline{r_l^B}\}\Big]$. Then:

$$L_1^* < L_1^{*B} \tag{B.179}$$

$$\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} < \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{B.180}$$

$$-\chi(\tau)(1/\beta - 1 - r_d) < 0 \tag{B.181}$$

Which holds true by parameter assumption.

6.5. Assume a realized $r_{l,1}$ in the range $(\overline{r_l},\ \overline{r_l^B})$. Then:

$$L_1^* < L_1^{*B} \tag{B.182}$$

$$\frac{E_0}{\chi(\tau)} < \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{B.183}$$

$$\overline{r_l} - \frac{1}{\rho_l}\chi(\tau)(1/\beta - 1 - r_d) < r_{l,1} \tag{B.184}$$

Which holds true by assumption.

6.6. Assume a realized $r_{l,1}$ in the range $(\overline{r_l^B},\ \overline{r_l})$. Then:

$$L_1^* < L_1^{*B} \tag{B.185}$$

$$\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} < \frac{E_1}{\chi(\tau)} \tag{B.186}$$

$$r_{l,1} < \overline{r_l} \tag{B.187}$$

This holds true by assumption.

6.7. Finally, assume a realized return state $r_{l,1} \in [\max\{\overline{r_l},\ \overline{r_l^B}\},\ +\infty$. Then:

$$L_1^* = L_1^{*B} \tag{B.188}$$

$$\frac{E_1}{\chi(\tau}) = \frac{E_1}{\chi(\tau)} \tag{B.189}$$

2.7.3. Proof for CCyB

Proof omitted due to its triviality. Please see the main-text for results.

2.7.4. Dividend Prudential Target

The steady state of our model is characterized by the unconditional mean $\bar{\mu}_l$ and implies a dividend of:

$$d_1^{SS} = E_1^{SS} + \bar{\mu}\frac{E_1^{SS}}{\chi} - r_d\left(\frac{E_1^{SS}}{\chi} - E_1^{SS}\right) - E_1^{SS} \tag{B.190}$$

$$= \bar{\mu}\frac{E_1^{SS}}{\chi} - r_d\left(\frac{E_1^{SS}}{\chi} - E_1^{SS}\right) \tag{B.191}$$

$$= \left[\frac{\bar{\mu} - r_d}{\chi} + r_d\right]\frac{\chi}{\gamma\sigma_l^2}\left[\bar{\mu} - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right]. \tag{B.192}$$

Given this, a state-dependend dividend prudential targest is introduced:

$$d_1^T = \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} \tag{B.193}$$

Any deviations from the target are punished with the following fine:

$$\frac{\kappa}{2}\left(d_1 - d_1^T\right)^2 \tag{B.194}$$

$$\frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS}\right)^2 \tag{B.195}$$

This results in the following revised optimization problem:

$$\begin{aligned} U(E_0 - E_1, d_2) = \quad & \max_{L_1, E_1} \quad E_0 - E_1 - \frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS}\right)^2 \\ & +\beta E_1(1 + r_d) + \beta\left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2} L_1^2\right] \end{aligned} \tag{B.196}$$

$$s.t.$$

$$\lambda_1 : \qquad L_1 \in \left[E_1, \frac{E_1}{\chi}\right] \tag{B.197}$$

$$\lambda_2 : \qquad E_1 \in [0, E_0] \tag{B.198}$$

1. We start by ignoring the feasibility constraints on L_1 and derive the optimal equity.

1.1. The FOC with respect to equity yields the following optimal equity levels:

$$\frac{\partial U(d_1, d_2)}{\partial E_1} = -1 - \frac{\kappa}{2}\left(-2E_0 + 2\frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} + 2E_1\right) + \beta(1 + r_d) = \tag{B.199}$$

$$E_1 = \frac{1}{\kappa}\big(\beta(1 + r_d) - 1\big) + E_0 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} \tag{B.200}$$

1.2. The equity in equation (B.200) is the unconstrained equity level and decreases in $r_{l,1}$. Hence, we know that for low $r_{l,1}$ below a threshold r_l^*, the upper feasibility limit binds:

$$E_1 \geq E_0 \tag{B.201}$$

$$r_{l,1} \leq r_l^* = \frac{\bar{\mu}_l}{d_1^{SS}} \frac{1}{\kappa}(\beta(1 + r_d) - 1). \tag{B.202}$$

1.3. Similarly, the equity level is constrained below at zero:

$$E_1 \leq 0 \tag{B.203}$$

$$r_{l,1} \geq = r_l^{**} = \frac{\bar{\mu}_l}{d_1^{SS}} \left[\frac{1}{\kappa}(\beta(1 + r_d) - 1) + E_0\right]. \tag{B.204}$$

2. The above derived thresholds on equity ignore that the equity choice may relax feasbility constraints on lending. They are nevertheless necessary for a complete proof.

3. Next, assume that a feasible E_1 has been chosen and thus the bank is left with the optimal lending choice. Here, we can rely on results from the bank section and now for a given level E_1, the bank chooses:

$$L_1 = E_1 \qquad \forall r_{l,1} \leq r_l^l = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_1 + r_d - \mu_l\right] \tag{B.205}$$

4. Notice that, unlike equity, lending increases in $r_{l,1}$. Hence, for low return states bank would lend out less than feasible and vice versa. Unconstrained, optimal lending is:

$$L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}. \tag{B.206}$$

5. Let us start with the upper feasibility limit. When is lending larger than optimal E_1/χ.

5.1. First, we assume that L_1 is already constrained below r_l^{**}:

$$L_1 \geq \frac{E_1^*}{\chi}, \tag{B.207}$$

$$r_{l,1} \geq r_l^h = \frac{\bar{\mu}\chi}{\chi\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}} \left[\frac{\gamma\sigma_l^2}{\chi} \left(\frac{1}{\kappa} \left(\beta(1+r_d) - 1 \right) + E_0 \right) + r_d - \mu_l \right]. \tag{B.208}$$

5.2. Next, we verify that indeed $r_h^< r_l**$:

$$r_l^h \leq r_l^{**} \tag{B.209}$$

$$\frac{\bar{\mu}\chi}{\chi\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}} \left[\frac{\gamma\sigma_l^2}{\chi} \left(\frac{1}{\kappa} \left(\beta(1+r_d) - 1 \right) + E_0 \right) + r_d - \mu_l \right] < \frac{\bar{\mu}_l}{d_1^{SS}} \left[\frac{1}{\kappa} (\beta(1+r_d) - 1) + E_0 \right] \tag{B.210}$$

$$0 < \frac{\chi\rho_l\bar{\mu}_l}{d_1^{SS}} \left[\frac{1}{\kappa} (\beta(1+r_d) - 1) + E_0 + \frac{\mu_l - r_d}{\chi} \right]. \tag{B.211}$$

5.3. we can then conclude that for all levels above r_l^h retaining more equity relaxes the upper feasibility constraint on lending.

6. Taking this into account, we define an alternative optimization problem for high return states above r_l^h, where $L_1 = E_1/chi$,

6.1. Next, we derive the revised FOC wrt. E_1 that assumes $L_1 = E_1/\chi$:

$$-1 - \frac{\kappa}{2} \left(-2E_0 + 2\frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} + 2E_1 \right) + \beta(1+r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi} - \beta\frac{\gamma\sigma_l^2}{\chi^2} E_1 = 0, \tag{B.212}$$

$$\kappa E_1 + \beta\frac{\gamma\sigma_l^2}{\chi^2} E_1 = -1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1+r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi}, \tag{B.213}$$

$$E_1 = \frac{\chi^2}{\chi^2\kappa + \beta\gamma\sigma_l^2} \left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1+r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi} \right] \tag{B.214}$$

The optimal equity level E_1 above r_l^h is strictly decreasing in $r_{l,1}$. Eventually, as $r_{l,1}$ increases it will meet the lower feasibility limit on E_1 of zero once again. The threshold return state r_l^{hh} is:

$$0 = \frac{\chi^2}{\chi^2\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS} + \beta(1 + r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi}\right], \tag{B.215}$$

$$\kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS} - \frac{\beta\rho_l}{\chi}r_{l,1} = \left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta\frac{\mu_l - r_d}{\chi}\right], \tag{B.216}$$

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l\beta\rho_l}\left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta\frac{\mu_l - r_d}{\chi}\right]. \tag{B.217}$$

7. Next, we turn to the lower feasibility limit on lending. Here we can distinguish two cases: L_1 intersects with E_1 below and above r_l^*. These two cases are determined by a threshold on E_0:

$$L_1^* \leq E_1 * \tag{B.218}$$

$$\frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \leq \frac{1}{\kappa}\big(\beta(1 + r_d) - 1\big) + E_0 - \frac{r_{l,1}}{\bar{\mu}_l}d_1^{SS}, \tag{B.219}$$

$$r_{l,1} \leq r_l^l = \frac{\bar{\mu}_l}{\rho_l\bar{\mu}_l + \gamma\sigma_l^2 d_1^{SS}}\left[\frac{\gamma\sigma_l^2}{\kappa}(\beta(1 + r_d) - 1) + \gamma\sigma_l^2 E_0 + r_d - \mu_l\right], \tag{B.220}$$

$$r_l^l \leq r_l^*, \tag{B.221}$$

$$E_0 \geq \frac{\rho_l\bar{\mu}_l}{\gamma\sigma_l^2 d_1^{SS}}\frac{1}{\kappa}(\beta(1 + r_d) - 1) + \frac{\mu_l - r_d}{\gamma\sigma_l^2} = \overline{E_0}. \tag{B.222}$$

8. We first study the case, where $r_l^l \geq r_l^*$ as $E_0 \geq \overline{E_0}$. Here, any reduction in equity allows the bank to relax the lower feasibility limit.

8.1. Accounting for $E_1 = L_1$ in the optimization problem, we obtain the following FOC fir equity:

$$-1 - \frac{\kappa}{2}\left(-2E_0 + 2\frac{r_{l,1}}{\bar{\mu}_l}d_1^{SS} + 2E_1\right) + \beta(1 + r_d) + \beta(\mu_l + \rho_l r_{l,1} - r_d) - \beta\gamma\sigma_l^2 E_1 = 0 \tag{B.223}$$

$$\kappa E_1 + \beta\gamma\sigma_l^2 E_1 = -1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1}) \tag{B.224}$$

$$E_1 = \frac{1}{\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] \tag{B.225}$$

8.2. Mirroring this, for low $r_{l,1}$, the upper feasibility limit of E_1 not exceeding E_0 applies:

$$E_0 \geq \frac{1}{\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] \tag{B.226}$$

$$r_{l,1} \leq= r_{l,1}^{ll} = \frac{\bar{\mu}_l}{\beta\rho_l\bar{\mu}_l - \kappa d_1^{SS}}\left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1 + \mu_l)\right] \tag{B.227}$$

9. Next, we study the case where $E_0 \leq \overline{E_0}$ and thus, $r_l^l \leq r_l^*$. Here, again the bank could relax the feasbility limit on L_1 by retaining more in equity states below r_l^l. That this is not optimal can easily be shown by the fact that:

$$r_l^l \leq r_l^{ll} \tag{B.228}$$

$$\frac{\bar{\mu}_l}{\rho_l\bar{\mu}_l + \gamma\sigma_l^2 d_1^{SS}}\left[\frac{\gamma\sigma_l^2}{\kappa}(\beta(1 + r_d) - 1) + \gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \leq \frac{\bar{\mu}_l}{\beta\rho_l\bar{\mu}_l - \kappa d_1^{SS}}\left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1 + \mu_l)\right] \tag{B.229}$$

$$E_0 \leq \frac{\rho_l\bar{\mu}_l}{\gamma\sigma_l^2 d_1^{SS}}\frac{1}{\kappa}(\beta(1 + r_d) - 1) + \frac{\mu_l - r_d}{\gamma\sigma_l^2} = \overline{E_0} \tag{B.230}$$

Because the above inequality (B.230) holds by assumption, we have that the bank never finds it optimal to pay out more equity to reduce lending.

10. For a given κ, assume that:

10.1.

$$E_0 \geq \overline{E_0} \tag{B.231}$$

Then, whenever we are in a very low return state $r_{l,1} \leq r_l^{ll}$, we have:

$$r_l^{ll} = \frac{\bar{\mu}_l}{\beta\rho_l\bar{\mu}_l - \kappa d_1^{SS}} \left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1+\mu_l)\right] \tag{B.232}$$

$$E_1 = E_0 = L_1 \tag{B.233}$$

For low return states, where $r_{l,1} \in (r_l^{ll}, r_l^l]$, whe have:

$$r_l^l = \frac{\bar{\mu}_l}{\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}} \left[\gamma\sigma_l^2 \left(\frac{1}{\kappa}\big(\beta(1+r_d) - 1\big) + E_0\right) + r_d - \mu_l\right] \tag{B.234}$$

$$E_1 = \frac{1}{\kappa + \beta\gamma\sigma_l^2} \left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] \tag{B.235}$$

$$L_1 = E_1 \tag{B.236}$$

For intermediate return states $r_{l,1} \in (r_l^l, r_l^h]$, we have that :

$$r_l^h = \frac{\bar{\mu}\chi}{\chi\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}} \left[\frac{\gamma\sigma_l^2}{\chi}\left(\frac{1}{\kappa}\big(\beta(1+r_d) - 1\big) + E_0\right) + r_d - \mu_l\right] \tag{B.237}$$

$$E_1 = \frac{1}{\kappa}\big(\beta(1+r_d) - 1\big) + E_0 - \frac{r_{l,1}}{\bar{\mu}_l} d_1^{SS} \tag{B.238}$$

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{B.239}$$

For high return states, where $r_{l,1} \in (r_l^h, r_l^{hh}]$, we have that:

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l\beta\rho_l} \left[-1 + \kappa E_0 + \beta(1+r_d) + \beta\frac{\mu_l - r_d}{\chi}\right] \tag{B.240}$$

$$E_1 = \frac{\chi^2}{\chi^2\kappa + \beta\gamma\sigma_l^2} \left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1+r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi}\right] \tag{B.241}$$

$$L_1 = \frac{E_1}{\chi} \tag{B.242}$$

And finally, for very high return states, where $r_{l,1} > r_l^{hh}$, we have:

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l\beta\rho_l}\left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta\frac{\mu_l - r_d}{\chi}\right] \tag{B.243}$$

$$E_1 = L_1 = 0 \tag{B.244}$$

10.2. If we have $E_0 \leq \overline{E_0}$, then the bank no longer retains less equity in low return states. For very low return states $r_{l,1} \leq r_l^l$, the optimal lending is thus:

$$r_l^l = \frac{\gamma\sigma_l^2 E_0 - \mu_l + r_d}{\rho_l} \tag{B.245}$$

$$L_1 = E_0. \tag{B.246}$$

For intermediate return states, $r_{l,1} \in [r_l^l, r_l^h]$, the lending choice is unrestricted and:

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}. \tag{B.247}$$

Similar to case 10.2, for high return states, where $r_{l,1} \in (r_l^h, r_l^{hh}]$, we have that:

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l\beta\rho_l}\left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta\frac{\mu_l - r_d}{\chi}\right] \tag{B.248}$$

$$E_1 = \frac{\chi^2}{\chi^2\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS} + \beta(1 + r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi}\right] \tag{B.249}$$

$$L_1 = \frac{E_1}{\chi} \tag{B.250}$$

And again, for very high return states, where $r_{l,1} > r_l^{hh}$, we have:

$$r_l^{hh} = \frac{\bar{\mu}\chi}{\kappa d_1^{SS}\chi - \bar{\mu}_l\beta\rho_l}\left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta\frac{\mu_l - r_d}{\chi}\right] \tag{B.251}$$

$$E_1 = L_1 = 0, \tag{B.252}$$