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Abstract

This paper argues that social security enjoys wider political support than other welfare programs because: (i) retirees constitute the most homogeneous voting group, and (ii) the intragenerational redistribution component of social security induces low-income young to support this system. In a dynamically efficient overlapping generation economy with earnings heterogeneity, we show that, for sufficient income inequality and enough elderly in the population, a welfare system composed of a within-cohort redistribution scheme and an unfunded social security system represents the political equilibrium of a two-dimensional majoritarian election. Social security is sustained by a majority of retirees and low-income young; intragenerational redistribution by low-income young. Our model suggests that to assess how changes in inequality affect the welfare state, the income distribution should be decomposed by age groups.

Keywords: Social Security, Income Inequality, Subgame Perfect Structure Induced Equilibrium.

JEL Classification: H53, H55, D72.

1 Introduction

In most industrialized countries, social security represents the single largest item of social welfare expenditure, and a predominant component in the government budget. In 1992 the US social security system gathered almost 76% of all cash benefits[1] in transfers to the old, as opposed to 15.8% to unemployment, temporary disability benefits and workers' compensations and 8.2% to public assistance and supplemental security income (see Table 1).

Why does the largest social welfare scheme depend on the age of the recipients rather than on their income or wealth? And why do we observe so little income redistribution among individuals of the same age group? This paper argues that social security enjoys wider political support than other welfare programs for two reasons. First, the recipients of social security benefits, the retirees, constitute a homogeneous group, capable of clustering a large block of votes to support this program and to oppose others. Second, the intragenerational redistribution component of the social security system makes this program palatable to low-income young individuals, even when alternative redistribution schemes are available.

The main contribution of this paper is to show that a welfare state composed of a (large) social security system and a (small) income redistribution scheme constitutes the politico-economic equilibrium in a sequence of majoritarian elections when the economy displays sufficiently large labor income inequality, and there are enough elderly in the population. In this equilibrium, the social security system is supported by a voting majority composed of elderly and low-income young, whereas the income redistribution scheme only receives the votes of the low-income young.

It is hardly a new insight to relate the size of the welfare state to the degree of income inequality in the economy. Romer (1975), Roberts (1977), and Meltzer and Richard (1981) suggested that, in democracies,

[1] When also in-kind benefits are considered, the social security system still enjoys the largest share, 58.7%, of the Federal and State budget on all benefits excluding education (see Table 2).

more unequal income distributions induce larger redistribution policies. While we build on this idea, we introduce a further characterization of the agents, their age, to explain the contemporaneous existence of an income-based redistribution scheme and an age-based transfer scheme, the social security system.

Our analysis is motivated by two observations. First, we notice that a large proportion of the earning poor are indeed old individuals. Using 1992 US data, Díaz-Giménez, Quadrini, and Rios Rull (1997) find that respectively 63% and 28% of the individuals in the first and second earnings quintile are older than 65 years. We argue that these elderly voters may prefer an age-based to an income-based transfer scheme, therefore decreasing the support that income redistribution schemes are expected to enjoy among low-income individuals. Second, we emphasize the intragenerational redistribution component built in to many social security systems. In fact, many programs are known to redistribute both across and within cohorts, since contributions to the social security system are typically proportional to the labor income (up to a maximum), whereas benefits tend to be regressive. Boskin et al. (1987) and Galasso (2000) provide evidence supporting this view for the US. They show that, for a given cohort, low income families obtain larger internal rates of return from investing in social security than middle or high-income families. Like in Tabellini (1990), the existence of a within cohorts redistribution element in the social security system is crucial in our analysis, as social security becomes appealing to low-income young, even in the presence of other income redistribution schemes.

We use a dynamically efficient overlapping generation economy with storage technology. Young agents differ in their working ability, and therefore in their labor income. Old individuals do not work. The welfare state consists of two programs that have balanced budgets every period. An (intragenerational) income redistribution scheme taxes labor income and awards a lump sum transfer in young age, whereas an unfunded social security system imposes a payroll tax rate and pays a lump sum pension. The level of the two welfare programs, i.e., the income redistribution and the social security tax rates, are determined in a two-dimensional majority voting game by all agents alive at every election. This voting game has two important characteristics. First, because of the multidimensionality of the issue space, the existence of a Condorcet winner of the majority voting game is not guaranteed. Second, if an equilibrium exists, in absence of a commitment device over future policies, young voters have no incentive to support any intergenerational transfer scheme.

To overcome the former problem, we initially assume commitment over future social security policies, and concentrate on political equilibria induced by institutional restrictions, or structure-induced equilibria, as in Shepsle (1979). In our political system, the entire electorate has jurisdiction over the two issues (i.e., the two tax rates), but policy decisions have to be taken issue-by-issue. To deal with the latter feature of the game, we drop the assumption of commitment, and consider implicit contracts among successive generations, as Hammond (1975), and more recently Boldrin and Rustichini (2000), Cooley and Soares (1999), and Galasso (1999). To summarize, we introduce a notion of subgame perfect structure induced equilibrium, which combines the concept of structure induced equilibrium, introduced by Shepsle (1979), with the intergenerational implicit contract idea, originally presented by Hammond (1975).

We show that, if there is a sufficiently large proportion of elderly in the population and enough income inequality, then a welfare state composed of an income redistribution scheme and an unfunded social security system arises as the structure-induced equilibrium of the majority voting game. In this equilibrium, the social security system is voted by a majority of elderly and low-income young, whereas income redistribution only receives the support of the young voters whose labor income is below the average labor income in the economy.

The idea of a social security system which relies on the political support of low-income young and retirees dates back to Tabellini (1990). In his model, low-income, weakly altruistic agents vote for social security since the utility they derive from the pension their parents receive outweighs the direct cost of the social security tax, and an equilibrium with positive social security may arise. However, unlike in our model, this result is not robust to a more complete specification of the welfare state. And if an additional income redistribution scheme is introduced, the equilibrium disappears.

Our results are also relevant for the empirical literature on the relation between income inequality and the size of the welfare. Some studies, namely Tabellini (1990), Perotti (1996), Breyer and Craig (1997) and Lindert (1997), which have tried to establish some empirical regularities between various measures of income inequality and the size of the different welfare programs, have provided mixed evidence. This is consistent with the implication of our model that the effect on each individual welfare program depends on the magnitude of the change in income inequality, as well as on its specific impact on the income distribution. In particular, we argue that empirical studies should not focus on the overall income distribution, or its relevant statistics. This distribution should rather be decomposed by age groups, whenever age, rather than labor income, may represent the main component in the agents' decisions. Only then should a significant statistic be obtained by reaggregated voters' preferences according to age and income.

In an independent contribution, Lambertini and Azariadis (1998) analyze the contemporaneous existence of an intragenerational and an intergenerational redistribution instrument, in an alternative political setup that draws from Barron and Ferejohn (1989)'s legislative bargaining. Together with Tabellini (1990), we all share the result that social security is supported by a voting majority of old and low-ability young. However, unlike us, Lambertini and Azariadis (1998) find that the same voting majority also supports the intragenerational redistribution program[2]. Therefore, their model reproduces the traditional result that an increase in income inequality unambiguously rises the size of both programs of the welfare system (see e.g., Meltzer and Richard (1981)).

The paper proceeds as follows: Section 2 presents the model and

[2]In fact, in Lambertini and Azariadis (1998), the equilibrium intergenerational and intragenerational transfers represent the outcome of a bargaining process among three groups of agents, and thus both programs are constrained to be supported by the same minimum winning coalition.

the economic equilibrium. Section 3 develops the political system, and introduces our equilibrium concept. In section 4, we characterize the equilibria of the voting game. In section 5 and 6, we discuss the results and conclude.

2 The Model Economy

Consider an economy with overlapping generations and a storage technology. Every period two generations are alive, we call them "Young" and "Old". Population grows at a constant rate $\mu > 0$. It follows that in any given period t for every young there are $1/(1+\mu)$ old.

Agents work when young, and then retire in their old age. Consumption takes place in old age only. Young individuals differ in their working ability. Working abilities are distributed on the support $[\underline{e}, \overline{e}] \subset \Re_+$, according to the cumulative distribution function $G(.)$. An agent born at time t is characterized by a level of working ability and will therefore be denoted by $e_t \in [\underline{e}, \overline{e}]$. The distribution of abilities is assumed to have mean e_ϕ, and to be skewed, $G(e_\phi) > 1/2$, which implies a skewed labor income distribution, according to the available data from every country.

A production function transforms labor into the only consumption good, according to the worker's ability:

$$y(e_t) = e_t n(e_t) \tag{1}$$

where $n(e_t)$ represents the amount of labor supplied by the agent with ability e_t. A storage technology converts a unit of today's consumption good into $1+R$ units of tomorrow's good, $y_{t+1} = (1+R) y_t$. Since there are no outside assets or fiat money, all private intertemporal transfer of resources takes place through the storage technology. Then by assuming that $R > \mu$, we guarantee that the economy is dynamically efficient.

Agents value young age leisure and old age consumption[3] according to a log-linear utility function:

$$U\left(l_t, c_{t+1}^t\right) = \ln\left(l_t\right) + \beta c_{t+1}^t \tag{2}$$

where l is leisure, c is consumption, β represents the individual discount factor, subscripts indicate the calendar time and superscripts indicate the period when the agent was born.

Young agents face the usual trade off between labor, $n\left(e_t\right)$, and leisure, $l\left(e_t\right)$, since $n\left(e_t\right) = \bar{l} - l\left(e_t\right)$, where $\bar{l}\,(> 0)$ is the total amount of disposable time, which we assume to be equal across types. Young pay payroll taxes on their labor income, receive a transfer, and save their disposable income for old age consumption. Old agents have no economic decision to take as they consume their entire income. The life time budget constraint for an agent born at time t with ability e_t is then:

$$c_{t+1}^t = \left[e_t n\left(e_t\right)\left(1 - \tau_t - \sigma_t\right) + T_t\right]\left(1 + R\right) + P_{t+1} \tag{3}$$

where τ_t and σ_t are the income redistribution and the social security tax rates at time t, and T_t and P_{t+1} are respectively the young age transfer at time t, and the old age transfer at time $t+1$.

Young determine their labor supply by maximizing $U\left(l_t, c_{t+1}^t\right)$ with respect to $l\left(e_t\right)$ and subject to budget constraint (3). We assume that the individual discount factor is equal to the inverse of the interest factor, $\beta = 1/(1+R)$, so that the labor supply does not depend on the interest rate. The optimal labor supply for an ability type e_t agent is then:

$$n\left(e_t\right) = \max\left\{0, \bar{l} - \frac{1}{e_t\left(1 - \tau_t - \sigma_t\right)}\right\}. \tag{4}$$

[3] The assumption that consumption only takes place in old age guarantees the existence of a closed-form solution, but at the cost of abstracting from saving decisions. Boldrin and Rustichini (2000), Cooley and Soares (1998) and Galasso (1999) discuss the relevance of the saving decisions for the political sustainability of social security.

We assume that the labor supply is strictly positive for every type[4].

Because of the log-linearity of the utility function the labor supply is only affected by changes in the tax rates and not by changes in the transfers level. In this sense, income effects play no role, whereas taxes distort labor supply decisions. This largely simplifies the analysis, because it implies that today's labor supply is not affected by tomorrow's fiscal policies.

2.1 The Welfare System

We examine two social welfare instruments, an income redistribution system, and a social security (or pension) system.

The former is an intragenerational redistribution scheme which only affects young generations. In fact, all young persons benefit from a lump sum transfer, T_t, which is financed through a payroll tax, τ_t, on the labor income. Clearly, this system redistributes from rich (above mean income types) to poor (below mean income types) young. The latter scheme consists of a sequence of transfers from workers to retirees. Each worker contributes a payroll tax rate, σ_t, from her labor income, and every retiree receives a flat transfer, P_t. Every system is assumed to be individually balanced every period, so that its total expenditure has to be equal to the amount of collected taxed.

The budget constraint at time t for the income redistribution scheme is thus:

$$T_t = \tau_t \int_{\underline{e}}^{\overline{e}} e_t n\left(e_t\right) dG\left(e_t\right) \tag{5}$$

whereas the budget constraint for the social security system is

$$P_t = \sigma_t\left(1+\mu\right) \int_{\underline{e}}^{\overline{e}} e_t n\left(e_t\right) dG\left(e_t\right). \tag{6}$$

[4]This assumption amounts to imposing a restriction on the tax rates: $\tau_t + \sigma_t < 1 - 1/\bar{l}\underline{e}$.

By substituting the labor supply in (4) into (5) and (6), we obtain two new expressions for the welfare system budget constraints:

$$T_t(\tau_t, \sigma_t) = \tau_t \left[e_\phi \bar{l} - \frac{1}{1 - \tau_t - \sigma_t} \right] \tag{7}$$

and

$$P_t(\tau_t, \sigma_t) = \sigma_t (1 + \mu) \left[e_\phi \bar{l} - \frac{1}{1 - \tau_t - \sigma_t} \right]. \tag{8}$$

The young age lump sum transfer displays a Laffer curve with respect to the corresponding tax rate and depends negatively on the social security payroll tax rate. In fact, the social security tax rate induces a distortion, which contributes to decrease the average income in the economy and thus reduces the young-age benefits. Analogously, the lump sum pension displays a Laffer curve with respect to the corresponding tax rate and depends negatively on the income redistribution tax rate.

2.2 The Economic Equilibrium

The economic equilibrium can now be defined as follows

Definition 1 *For a given sequence of tax rates, $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$, and a given real interest rate, R, an economic equilibrium is a sequence of allocations, $\left\{ l(e_t), c_{t+1}^t(e_t) \right\}_{e_t \in [\underline{e}, \overline{e}]}^{t=0,\ldots,\infty}$, such that:*

- the consumer problem is solved for each generation, i.e., agents maximize $U\left(l_t, c_{t+1}^t\right)$ with respect to $l(e_t)$, subject to the restriction in eq.3;
- the welfare budget constraints are balanced every period, and thus equations 5 and 6 are satisfied; and

- the goods market clears every period:

$$\int_{\underline{e}}^{\overline{e}} c_t^{t-1}(e_{t-1})\, dG(e_{t-1}) =$$
$$(1+R)\int_{\underline{e}}^{\overline{e}} (1-\sigma_{t-1})\, e_{t-1} n(e_{t-1})\, dG(e_{t-1}) + \tag{9}$$
$$\sigma_t(1+\mu)\int_{\underline{e}}^{\overline{e}} e_t n(e_t)\, dG(e_t)\,.$$

The utility level obtained in an economic equilibrium at time t by an ability type e_t young and by an ability type e_{t-1} old agent can be expressed by their indirect utility functions. For the young:

$$v_t^t(\tau_t,\sigma_t,\tau_{t+1,}\sigma_{t+1},e_t) = -\ln e_t - 1 - \ln(1-\tau_t-\sigma_t) + e_t\bar{l}(1-\tau_t-\sigma_t)$$

$$+\tau_t\left[e_{\phi,t}\bar{l} - \frac{1}{1-\tau_t-\sigma_t}\right] + \sigma_{t+1}\frac{1+\mu}{1+R}\left[e_{\phi,t+1}\bar{l} - \frac{1}{1-\tau_{t+1}-\sigma_{t+1}}\right]. \tag{10}$$

For the old:

$$v_t^{t-1}(\tau_t,\sigma_t,e_{t-1}) = K(e_{t-1}) + \sigma_t\frac{1+\mu}{1+R}\left[e_{\phi,t}\bar{l} - \frac{1}{1-\tau_t-\sigma_t}\right] \tag{11}$$

where $K(e_{t-1})$ is a constant[5] which depends on the agent's type, but not on current or future tax rates.

These indirect utility functions characterize the young and old agents' preference relations over current (and future) tax rates. Notice that the old individuals' ability type scales their utility up or down, but does not affect their preferences over the tax rates. In other words, all old agents, regardless of their ability type, share the same preferences over welfare programs.

[5]Specifically, $K(e_{t-1}) = e_{t-1}\bar{l}(1-\tau_{t-1}-\sigma_{t-1}) - e_\phi\bar{l}\tau_{t-1} - \frac{1-\sigma_{t-1}}{1-\tau_{t-1}-\sigma_{t-1}}$.

3 The Voting Game

The amount of welfare expenditures, i.e., income redistribution and social security, is decided through a political process which aggregates the agents' preferences over the two tax rates. We consider a political regime of majority voting. Elections take place every period, and voters are all agents alive. At every election voters cast their ballots on the two current tax rates, τ_t and σ_t. However, since every agent has zero mass, no individual voter would affect the outcome of the election, and thus *any* individual voting strategy would be part of an equilibrium. To overcome this problem, we assume sincere voting.

This majority voting game displays two important features. First, due to the two-dimensionality of the issue space, (τ, σ), a Condorcet winner of the majority voting game may fail to exist. Second, if an equilibrium exists, would a majority of young (and old) individuals agree to transfer resources to current retirees when there is no guarantee that such policy will be carried on in the next periods? To deal with these two characteristics of the game, we introduce an equilibrium concept which applies the notion of subgame perfection to the concept of structure induced equilibrium, which was originally proposed by Shepsle (1979). In the remaining of this section, we discuss how our equilibrium concept allows us to successfully tackle these two issues. Section 3.1 and 3.2 provide a formal treatment.

To concentrate on the non-existence of Nash equilibria, we initially assume full commitment over the social security tax rate. Today's voters determine the current and future social security tax rates, however, they only set the current income redistribution tax rate[6]. The assumption of commitment effectively reduces the voting game to a static one, in which we can analyze how the two-dimensionality of the issue space may

[6]We choose to limit commitment to the social security policy only, because, due to its intergenerational nature, this is the only policy in which commitment can be achieved as an implicit contract among successive generations of voters. We will return to this point in section 4.3.

prevent a Nash equilibrium from arising[7]. The following example will illustrate this point.

Figure 1 displays the preferences of three representative voters, old, rich young, and poor young, as utility contours in the two-dimensional issue space[8], (τ, σ). Old voters clearly support a social security scheme, and oppose any income redistribution system, τ, which decreases the average income in the economy and does not award them any benefits. The pair of tax rates which maximizes their indirect utility (eq. 11), i.e., their bliss point, is thus $(\tau, \sigma) = (0, \sigma^*_{old} > 0)$; and their indifference contours are represented by the dashed curves. Rich young voters, i.e., voters whose income is above the mean income in the economy, dislike both welfare schemes, to which they are net contributors. Their bliss point is the origin, $(\tau, \sigma) = (0, 0)$; and their indifference contours are the dotted lines. A poor young, e.g., a young with ability $\underline{e}$, on the other hand, is a net recipient from both schemes. If she maximizes her indirect utility function with respect to the current tax rates, and under the assumptions that the decision of current social security tax rate binds the future tax rate, $\sigma_t = \sigma_{t+1}$, whereas the current income redistribution tax rate has no impact on future policies, her bliss point[9] is $(\tau, \sigma) = \left(\tau^*_{yp} > 0, \sigma^*_{yp} = 0\right)$. The poor young prefers to obtain her entire welfare transfer through the income redistribution benefit. Her preferences are represented by the continuous-line contours.

If these three agents were the only voters and had equal weights, no Nash equilibrium of the majority voting game would exist, and Condorcet cycles would arise. For example, in Figure 1, point b would be preferred to point a by the poor young and the old; c would be preferred to b by the old and the rich young; and finally rich and poor young would close the cycle moving from c to a. The same result would apply to the voting game played by the entire electorate, unless a median in all directions

[7]See Ordershook (1986) for an extensive review of these issues.

[8]Young voters' preferences are depicted under the assumption that both tax rates remain constant over time: $\tau_t = \tau_{t+1}$, and $\sigma_t = \sigma_{t+1}$.

[9]The condition for the bliss point to occur at $\sigma = 0$ is that $\left(e_{\phi,t} - \underline{e}\right)/e_{\phi,t} < \bar{l}e_{\phi,t} - 1$.

exists[10].

To overcome this well-known problem, we follow Shepsle (1979) in analyzing voting equilibria induced by institutional restrictions, i.e., structure-induced equilibria. In section 3.1, we discuss the set of institutional restrictions, which are needed to convert our two-dimensional election into a simultaneous *issue-by-issue* voting game, in which a (structure induced) equilibrium exists.

Suppose now that the assumption of commitment over future social security policies is dropped, would the young be willing to support a social security system? If young agents expect their voting behavior to have no relevance for future choices, they should vote for a zero social security tax rate, or else they would incur in a current labor tax with no future benefits. However, current electors may expect their voting decisions to have an impact on future policies. In this case, as Hammond (1975) initially suggested, an implicit contract among successive generations of voters may arise, in which today's young agree on a transfer to current retirees because they expect to be rewarded with a corresponding transfer in their old age. A failure to comply with the implicit contract is punished with no old age transfers.

In section 3.2, we follow this route. We drop the assumption of commitment over future social security policies, and we analyze the voting strategies, which are compatible with the arise and the sustainability of an implicit contract. Formally, we introduce a notion of (subgame perfect) equilibrium which requires subgame perfect equilibrium outcomes of the voting game without commitment to be structure induced equilibrium outcomes of the voting game with commitment.

To summarize, our notion of equilibrium applies the idea of subgame perfection to the concept of structure induced equilibrium, in the context of a dynamic, two-dimensional voting game. First, we study the structure induced equilibria of the two-dimensional voting game with commitment. Then, we replace the commitment device with an implicit contract, to analyze which structure induced equilibrium outcomes of the

[10]For a formal definition, see the appendix.

game with commitment are also subgame perfect equilibrium outcomes of the game without commitment. A formal definition of our equilibrium notion, which we call subgame perfect structure induced equilibrium (SP-SIE), is given in section 3.2.

3.1 Structure-Induced Equilibria

An institutional arrangement characterizes how the political system aggregates the individual preferences over the alternatives into a political outcome (τ^*, σ^*). The space of alternatives, or issues, is $(\tau, \sigma) \in R^2$ subject to $\tau + \sigma \leq 1 - 1/\bar{l}\underline{e}$, which we imposed in order for the labor supply of any young agent to be positive (see footnote 3). Individual preferences over the alternatives are derived from the indirect utility functions at equations 10 and 11. We assume that there exists perfect commitment over the social security policy, whereas no commitment is available on the income redistribution policy.

An arrangement is composed of a committee system, a jurisdictional arrangement, an assignment rule, and an amendment control rule. The committee system separates the electorate, E, in committees $\{C_j\}$. The jurisdictional arrangement, J, divides the issues (τ, σ) into jurisdictions $\{J_k\}$. Jurisdictions are then associated to committees, according to an assignment rule, $f : C_j \rightarrow J_k$. In this way, the political system assigns the decision over a subset of the issue space, e.g., a single issue, to a particular committee. Every committee is entitled to make a proposal to change the current value of the issue (the status quo) which falls into its jurisdiction. The amendment control rule determines how proposals can be further modified (amended) by the electorate before the final stage is reached, and the (possibly amended) proposal is then voted in a majority rule, pairwise comparison against the status quo by the entire electorate.

As originally proposed by Shepsle (1979), this institutional arrangement does not directly apply to elections, but rather describes the process of policy making decision by representatives in a legislature. Therefore, in adopting these institutional restrictions in our voting game, we are implicitly assuming that the elections select a group of representatives whose preferences exactly match the voters' preferences. In section 5, we discuss alternative ways of aggregating individual preferences, either through a model of electoral competition – probabilistic voting model – or through another post-election model – agenda setter model – and we compare the results.

The political system we adopt is characterized by the following arrangements[11]:

- Committee of the Whole: there exists only one committee, which coincides with the electorate, $C = \{E\}$;
- Simple Jurisdictions: each jurisdiction is a single dimension of the issue space, $J = \{\{\tau\}, \{\sigma\}\}$. In other words, one jurisdiction has the power to deliberate on the income redistribution tax rate, τ, and another one on the social security tax rate, σ.
- Every simple jurisdiction is assigned to the committee of whole, $f : E \rightarrow \{\{\tau\}, \{\sigma\}\}$.
- Germaneness Amendment Control Rule: amendments to the proposal are permitted only along the dimensions that fall in the jurisdiction of the committee. That is, if the proposal regards τ, only amendments on τ are permitted, and viceversa.

In this political system, the entire electorate has jurisdiction, i.e., it is entitled to make proposals, on the two issues; however, only separately, that is, issue by issue. As in Shepsle (1979), the adoption of simple jurisdictions and germaneness amendment rule converts our two-dimensional voting game into a simultaneous issue-by-issue voting game. This is needed to overcome the possible lack of a Condorcet winner of the two-dimensional majority voting game. No further restrictive jurisdictional arrangements are imposed. The choice of a committee of the whole, for example, guarantees that no subset of the electorate which constitutes a committee is effectively awarded gate-keeping power over

[11] See the appendix for a formal definition.

an issue. In fact, any such committee could block any alternative to the status quo which would be preferred by a majority of the electorate, but not by a majority of the members of this committee.

Since Shepsle (1979) [Theorem 4.1], we know that a sufficient condition for the existence of an equilibrium in a voting game played under the institutional arrangements described above is that voters' preferences are single peaked over the issue space, $(\tau, \sigma) \in R^2$.

To establish single-peakedness for our voters' preferences, it is useful to introduce some additional definitions[12]. We refer to the induced ideal point of a voter i in the j-th direction of the issue space (e.g., σ) as the point that maximizes voter i' s indirect utility function along the j-th dimension (σ), while the other issue (τ) is at its status quo value. An induced ideal point for voter i on a line in the issue space maximizes her utility function on this line with respect to both issues. Finally, we define preferences to be single-peaked over a line in the issue space as follows:

Definition 2 *Let* $X = \{x \mid x = \lambda y + (1-\lambda) z,\ y, z \in (\tau, \sigma),\ \lambda \in [0,1]\} \subseteq (\tau, \sigma)$ *be the line connecting two arbitrary points* y *and* z*, in the issue space,* (τ, σ)*. Preferences are single peaked on* X *if and only if, for all* $x \in X$ *and* $x \neq x^{*i}$*,* $u_i [\alpha x + (1-\alpha) x^{*i}] > u_i [\beta x + (1-\beta) x^{*i}]$ *whenever* $0 \leq \alpha < \beta \leq 1$ *and* x^{*i} *is the induced ideal point on* X*.*

In other words, voter i's preferences are single peaked over a line in the issue space if and only if, for any point on this line on one side of voter i's induced ideal point, points closer to the induced ideal point provide higher utility.

Recall that at time t the preferences of an ability type e_t young voter over τ_t, and $\sigma_t \,(= \sigma_{t+1})$ are described by eq. 10; while old voters' preferences are represented by eq. 11. Then we can state the following proposition.

Proposition 1 *Over the issue space* $(\tau, \sigma) \in R^2$*, old voters' preferences are single peaked; young voters' preferences are single peaked over a line in the issue space if* $N = (1+\mu) / (1+R) \geq 1/2$*.*

[12] Formal definitions are provided in the appendix.

To prove single peakedness over a line in the issue space for young voters we show that $N \geq 1/2$ is sufficient[13] to guarantee that their utility function is quasi concave in (τ, σ); then we apply Shepsle (1979) [Lemma 3.1] to deduce single peakedness. For old voters, since their utility is not concave, we directly apply the definition of single peakedness to eq. 11. A formal proof is provided in the appendix.

The next proposition characterizes the structure induced equilibrium we use.

Proposition 2 *Let X_j^* be the set of j-th components from the induced ideal points of all voters in the direction j from the status quo x^o. For one-dimensional (simple) jurisdictions, a germaneness rule for amendments, a committee of the whole, and single peaked preferences, x^o is a structure-induced equilibrium outcome if and only if, for all j, $x_j^o =$ median X_j^*.*

Proof: Since preferences are single peaked on any line in the issue space, if $x_j^o =$ median X_j^* then x^o defeats all points along the j-th dimension, by Black's median voter's theorem. Given simple jurisdictions and germaneness rule, issues are voted once at a time; and since x_j^o cannot be defeated by any point along any dimension j, then x^o is a structure induced equilibrium outcome, which proves sufficiency. Suppose now that x' is a structure induced equilibrium outcome, where $x' \neq x_j^o =$ median X_j^* $\forall j$ along some dimension i. Since we have a committee of the whole, then x' would always be defeated by x^o along the i-th dimension, which proves the proposition.

Notice that the necessary condition established in this proposition relies on the use of a committee of the whole. In fact, if a jurisdiction were to be assigned to a committee which is a strict subset of the electorate, then along that jurisdiction the committee could use its gate-keeping power to force the electorate to choose on a restricted issue space and thus structure-induced equilibrium outcomes other than x^o could arise.

[13] Since N can be interpreted as the performance of the social security system relative to the saving (storage) technology, the sufficient condition, $N \geq 1/2$, is not restrictive, since it only requires social security to yield at least half of the returns of the alternative technology, for the agent with average ability.

3.2 Subgame Perfection and Structure Induced Equilibria

In this section, the assumption of commitment over future social security policies is dropped. Voters can only determine current tax rates, although they may expect their vote to condition future voters' decisions. In this voting game with no commitment, we concentrate on a class of subgame perfect equilibria. Specifically, we introduce our equilibrium concept, that we call subgame perfect structure induced equilibrium, which requires the subgame perfect equilibrium outcomes of the game without commitment to be structure induced equilibrium outcomes of the voting game with commitment.

In order to formalize our notion of equilibrium, we first define the voting game with no commitment.

The sequence of social security and income redistribution tax rates until $t-1$ constitutes the public history of the game at time t, $h_t = \{(\tau_0, \sigma_0), ..., (\tau_{t-1}, \sigma_{t-1})\} \in H_t$, where H_t is the set of all possible history at time t.

An action for a type e young individual at time t is a pair of tax rates, $a^y_{t,e} = (\tau, \sigma) \in \Upsilon$, where

$$\Upsilon = \left\{ (\tau, \sigma) : \tau \in [0,1], \sigma \in [0,1], \tau + \sigma \leq 1 - 1/\bar{l}\underline{e} \right\}.$$

Analogously, an action for a type e old individual at time t is $a^o_{t,e} = (\tau, \sigma) \in \Upsilon$. We call a_t the action profile of all individuals (young and old) at time t: $a_t = (a^y_t \cup a^o_t)$ where $a^y_t = \bigcup_{e \in [\underline{e}, \overline{e}]} a^y_{t,e}$ and $a^o_t = \bigcup_{e \in [\underline{e}, \overline{e}]} a^o_{t,e}$.

For a type e young individual a strategy at time t is a mapping from the history of the game into the action space: $s^y_{t,e} : h_t \rightarrow \Upsilon$, and analogously for a type e old individual at time t: $s^o_{t,e} : h_t \rightarrow \Upsilon$. The strategy profile played by all individuals at time t is denoted by $s_t = (s^y_t \cup s^o_t)$ where $s^y_t = \bigcup_{e \in [\underline{e}, \overline{e}]} s^y_{t,e}$ and $s^o_t = \bigcup_{e \in [\underline{e}, \overline{e}]} s^o_{t,e}$.

At time t, for a given action profile, a_t, the pair (τ^m_t, σ^m_t) represents the medians of the distributions of tax rates. We take (τ^m_t, σ^m_t) to be the outcome function of the voting game at time t. This outcome function corresponds to the structure induced equilibrium outcome of the voting game with commitment, according to Proposition 2. The history of the game is updated according to the outcome function; at time $t+1$: $h_{t+1} = \{(\tau_0, \sigma_0), ..., (\tau_{t-1}, \sigma_{t-1}), (\tau_t^m, \sigma_t^m)\} \in H_{t+1}$.

For every agent, the payoff function corresponds to her indirect utility. Formally, for a given sequence of action profiles, $(a_0, ..., a_t, a_{t+1}, ...)$, and of corresponding realizations, $((\tau_0, \sigma_0), ..., (\tau_t, \sigma_t), (\tau_{t+1}, \sigma_{t+1}), ...)$, the payoff function for a type e young individual at time t is $v_t^t(\tau_t, \sigma_t, \tau_{t+1}, \sigma_{t+1}, e)$, as defined in eq. 10, and for a type e old agent is $v_t^{t-1}(\tau_t, \sigma_t, e)$, according to eq. 11.

Let $s_{t|\widehat{e}}^y = s_t^y / s_{t,\widehat{e}}^y$ be the strategy profile at time t for all young individuals except for type $\widehat{e}$, and let $s_{t|\widehat{e}}^o = s_t^o / s_{t,\widehat{e}}^o$ be the strategy profile at time t for all old individuals except for the type $\widehat{e}$. Then, at time t, a type $\widehat{e}$ young individual maximizes

$$V_{t,\widehat{e}}^t\left(s_o, ..., \left(s_{t|\widehat{e}}^y, s_{t,\widehat{e}}^y\right), s_t^o, s_{t+1}, ...\right) = v_t^t\left(\tau_t^m, \sigma_t^m, \tau_{t+1}^m, \sigma_{t+1}^m, \widehat{e}\right)$$

and a type $\widehat{e}$ old individual maximizes

$$V_{t,\widehat{e}}^{t-1}\left(s_o, ..., \left(s_{t|\widehat{e}}^o, s_{t,\widehat{e}}^o\right), s_t^y, s_{t+1}, ...\right) = v_t^{t-1}\left(\tau_t^m, \sigma_t^m, \widehat{e}\right)$$

where, according to our previous definition of the outcome function, (τ_t^m, σ_t^m) and $\left(\tau_{t+1}^m, \sigma_{t+1}^m\right)$ are, respectively, the medians among the actions over the two welfare programs tax rates played at time t and $t+1$.

As previously argued, to deal with the two-dimensionality of the issue space, and to allow for intergenerational implicit contracts to arise, our equilibrium concept combines subgame perfection with the notion of structure induced equilibrium. We can now define a subgame perfect structure induced equilibrium of the voting game as follows:

Definition 3 (SPSIE) *A voting strategy profile* $s = \{(s_t^y \cup s_t^o)\}_{t=0}^{\infty}$ *is a Subgame Perfect Structure Induced Equilibrium (SPSIE) if the following conditions are satisfied:*

- s is a subgame perfect equilibrium.
- At every time t, the equilibrium outcome associated to s is a Structure Induced Equilibrium of the static game with commitment.

4 Politico-Economic Equilibria

In this section, we study the subgame perfect structure induced equilibrium (SPSIE) outcomes of the voting game, to determine the size and the composition of the welfare state, (τ, σ). We do this in two steps. First, the structure induced equilibrium (SIE) outcome of the static voting game with commitment over the social security policy is characterized in Proposition 4.1. We, then, relax this assumption, and study the SPSIE outcome of the game with no commitment in Proposition 4.2.

In particular, in the next two sections we calculate the median among the induced ideal points of all voters over the two issues, τ and σ, in the voting game with commitment. In other words, we first calculate every elector's ideal over the income redistribution tax rate for every given social security tax $\tau(\sigma)$, and then over the current and future social security tax rate for every given income redistribution tax rate $\sigma(\tau)$. For each σ we identify the median ideal for τ. For each τ we identify the median ideal for σ. These median functions intersect at (τ^*, σ^*), which by Proposition 3.3 is a structure induced equilibrium.

4.1 Voting on the Income Redistribution Tax Rate

A quick look at eq. 11 reveals that old generations oppose any income redistribution transfer schemes, since, due to the distortionary taxation, they reduce the average income in the economy, and thus decrease their pension benefits, while they do not provide the old with any transfer. In fact, the maximization of eq. 11 with respect to τ yields $\tau^*_{old} = 0$ for any positive value[14] of σ.

[14] For $\sigma = 0$ the old's indirect utility does not depend on τ. Since the old are indifferent, we assume that $\tau^*_{old}(\sigma = 0) = 0$.

Young generations, on the other hand, may benefit from this intragenerational transfer scheme, depending on their ability and on the resulting income. For a given social security tax rate, $\sigma_t \,(= \sigma_{t+1})$, an ability type e_t young at time t would choose her most preferred income redistribution tax rate $\tau^*_{e,t}(\sigma)$ by maximizing her indirect utility in eq. 10 with respect to τ_t. The first order condition of this problem yields:

$$(e_\phi - e_t)\,\bar{l} - \frac{\tau_t}{(1 - \tau_t - \sigma_t)^2} = 0.$$

And thus the optimal income redistribution tax rate, for a given σ, is

$$\tau^*_{e,t}(\sigma) = \max\left\{0,\ 1 - \sigma_t + \frac{1 - \sqrt{1 + 4\,(e_\phi - e_t)\,\bar{l}\,(1 - \sigma_t)}}{2\,(e_\phi - e_t)\,\bar{l}}\right\}. \tag{12}$$

Unsurprisingly, the young's most preferred income redistribution tax rate is decreasing in their income. Above average income type would vote for $\tau^* = 0$, together with the old. Poor, i.e., below average income, $e_t < e_\phi$, young vote for positive tax rates.

When voting on the income redistribution tax rate, $\tau(\sigma)$, agents can thus be ordered according to their age and income, as shown at figure 2a. Since the old generation represents a minority of the total electorate[15], the median voter on the income redistribution tax rate, hereby intragenerational median voter, is the type-$m\tau$ young agent, who divides the electorate in halves, i.e., such that

$$G(e_{m\tau}) = \frac{2 + \mu}{2\,(1 + \mu)}. \tag{13}$$

Finally, if the median voter's ability is below the average ability, $e_{m\tau} < e_\phi$, then $\tau^*_{m\tau}(\sigma) > 0$, according to 12.

[15] Even if we adjust for voting participation rates, retirees are still a minority, although a large and powerful one, see Mulligan and Sala-i-Martin (1999).

4.2 Voting on the Social Security Tax Rate

The old have again a simple choice. Since they are no longer required to contribute to the system, they vote for the social security tax rate that maximizes their current transfer, see eq. 11. For a given income redistribution tax rate, τ, the first order condition of their optimization problem is

$$e_{\phi}n\left(e_{\phi}\right)=\frac{\sigma_t}{\left(1-\tau_t-\sigma_t\right)^2} \tag{14}$$

where $n\left(e_{\phi}\right)$ represents the average labor supply in the economy, see eq. 4. Their most preferred social security tax rate is thus:

$$\sigma^*_{old}\left(\tau\right)=1-\tau_t-\sqrt{\frac{1-\tau_t}{e_{\phi}\bar{l}}}. \tag{15}$$

Because of the assumption of commitment over social security policies, the voting decision of an ability type e_t young individual amounts to maximizing her indirect utility, eq. 10, with respect to the current and future social security tax rate: $\sigma_t=\sigma_{t+1}=\sigma$, and for given values of the current and future income redistribution tax rates, τ_t and τ_{t+1}. The first order condition yields:

$$e_t n\left(e_t\right)=\frac{\partial T_t}{\partial\sigma}+\frac{\frac{\partial P_{t+1}}{\partial\sigma}}{1+R} \tag{16}$$

We impose $\tau_t=\tau_{t+1}=\tau$ to restrict our analysis to steady states. Eq. 16 can then be rewritten as

$$Ne_{\phi}n\left(e_{\phi}\right)-e_t n\left(e_t\right)=\frac{\tau+N\sigma}{\left(1-\tau-\sigma\right)^2} \tag{17}$$

where $N=\left(1+\mu\right)/\left(1+R\right)$ can be interpreted as the performance of the social security system relative to the saving (storage) technology. The optimal social security tax rate for a young type e_t, given the income redistribution tax rate, τ, is then

$$\sigma^*_{e,t}\left(\tau\right)=\max\left\{0,1-\tau+\frac{1-\sqrt{1+4\bar{l}\left(N+\tau\left(1-N\right)\right)\left(e_{\phi}N-e_t\right)}}{2\bar{l}\left(e_{\phi}N-e_t\right)}\right\}. \tag{18}$$

This optimal tax rate, $\sigma^*_{e,t}(\tau)$, is clearly decreasing in the young income type, e_t, because of the within-cohort income redistribution that this scheme achieves through a combination of a proportional income tax, σ, and a lump sum old age transfer, P. In particular, for sufficiently small values of the income redistribution tax rate, $\tau \leq (1-N)/(2-N)$, only those voters whose pre-tax labor income is below a fraction N of the pre-tax average labor income in the economy, $e_t n(e_t) < N e_\phi n(e_\phi)$ (with $N < 1$), will vote for a positive social security tax; whereas richer young will oppose the scheme.

A look at equations 14 and 17 reveals that the old always vote for a larger social security tax than the poorest young, and, therefore, than any young. In fact, unlike the young, the old do not make any contribution to the system. Voters' preferences over social security can easily be ordered according to age, and income, as depicted at figure 2b. The median voter on the social security tax rate is the type-$m\sigma$ young who divides the electorate in halves:

$$G(e_{m\sigma}) = \frac{\mu}{2(1+\mu)}. \tag{19}$$

In other words, the median in the distribution of actions played by old and young voters is $\sigma^*(\tau) = \sigma^*_{e,t}(\tau)$ with $e_t = e_{m\sigma}$.

4.3 The Equilibria

In sections 4.1 and 4.2, we analyzed the voters' decisions over the two welfare schemes: we determined the decisive or median voter for each issue, $e_{m\tau}$ and $e_{m\sigma}$, and we calculated their most preferred tax rates, $\tau^*_{m\tau}(\sigma)$ and $\sigma^*_{m\sigma}(\tau)$. Equations 12 and 18 can indeed be interpreted as reaction functions: for a given value of the social security (income redistribution) tax rate, eq. 12 (18) pins down the income redistribution (social security) tax rate chosen by the median voter $e_{m\tau}$ ($e_{m\sigma}$). Therefore, by Proposition 3.3 the (structure-induced) equilibrium outcomes of this voting game correspond to the points where these functions cross.

It is now useful to introduce a measure of the relative ability of the two median voters, $\Delta_\tau = (e_\phi - e_{m\tau})\bar{l} = n(e_\phi)e_\phi - n(e_{m\tau})e_{m\tau}$ and

$\Delta_\sigma = (Ne_\phi - e_{m\sigma})\,\bar{l}$. Notice that, while Δ_τ simply measures the difference between the average labor income in the economy and the intragenerational median voter's labor income, what is relevant in Δ_σ is the difference between the average ability in the economy weighted by the relative performance of the social security system, N, and the social security median voter's ability. This is to take into account that social security is an inferior redistributive scheme for the young, due to its inefficiency in transferring resources into the future. Finally, let Δ be equal to $\Delta_\tau(1-N) - \Delta_\sigma$, and $\widehat{\Delta}_\tau$ to $\Delta_\tau - (1-N)\left(1+\sqrt{1+4\Delta_\tau}\right)/2$. The next proposition characterizes the structure-induced equilibrium outcome of the voting game with commitment.

Proposition 3 *There exists a unique structure-induced equilibrium of the voting game with commitment over the social security policies, with outcome* (τ^*, σ^*), *such that*

(I) if $\Delta_\tau \leq 0$ and $\Delta_\sigma \leq -(1-N)$, then $\tau^* = 0$ and $\sigma^* = 0$;

(II) if $\Delta_\tau \leq 0$ and $\Delta_\sigma > -(1-N)$, then $\tau^* = 0$ and $\sigma^* = 1 + \frac{1-\sqrt{1+4N\Delta_\sigma}}{2\Delta_\sigma} > 0$;

(III) if $\Delta_\tau > 0$ and $\Delta_\sigma \leq \widehat{\Delta}_\tau$, then $\tau^* = 1 + \frac{1-\sqrt{1+4\Delta_\tau}}{2\Delta_\tau} > 0$ and $\sigma^* = 0$;

(IV) if $\Delta_\tau > 0$ and $\Delta_\sigma > \widehat{\Delta}_\tau$, then

$$\begin{aligned} \tau^* &= \Delta_\tau \frac{1 - 2N\Delta - \sqrt{1-4N\Delta}}{2\Delta^2} > 0 \\ \sigma^* &= 1 - N - \tau^*\left(2 - N - \frac{\Delta_\sigma}{\Delta_\tau}\right) > 0. \end{aligned} \tag{20}$$

A proof is provided in the appendix. This proposition links the relative ability of the two median voters to the equilibrium welfare state. For sufficiently low levels of income inequality, case I, in equilibrium there are no welfare programs. In case II, the intragenerational median voter's ability is above the mean ability, while the social security median voter's ability is sufficiently low, and thus only the social security system is adopted. This case may arise in an economy with moderate overall income inequality and a large proportion of old voters, or in an economy where the high degree of labor income inequality is mainly due to a large share of retirees. Case III, on the other hand, presents a distribution of income with large inequality in the intragenerational voting, but only small inequality in the social security voting, and thus leads to an equilibrium with income redistribution transfers only. This case may correspond to a young, highly unequal society. Finally, for sufficiently high income inequality, case IV, the equilibrium outcome corresponds to a complete welfare state. Figure 3 illustrates the reaction functions and the equilibrium in case IV, when both systems arise.

This proposition suggests that to fully appreciate the relation between a welfare system and the labor income inequality in the economy, we need to analyze the underlining income distribution by age groups, since age, rather than income, may be the main determinant in some agents' voting decision. Therefore, the overall income distribution needs to be separated in age groups and than recomposed, as shown in Figures 2a and 2b, to take account of the income inequality as well as of the age.

We can now generalized the results obtained at proposition 4.1 to the voting game with no commitment:

Proposition 4 *Every pair (τ^*, σ^*), which constitutes a structure induced equilibrium outcome of the voting game with commitment over the social security policies, is a subgame perfect structure induced equilibrium outcome of the game with no commitment.*

The idea of the proof, which is provided in the appendix, is very simple. It is easy to see that old agents support social security, and poor young support income redistribution. Moreover, low ability young individual, who would vote for a positive social security level in the game with commitment, will also be willing to enter an implicit contract among successive generations of voters to sustain social security. To illustrate this point, consider the structure induced equilibrium at case IV in Proposition 4.1, where both tax rates are positive ($\sigma^* > 0$, $\tau^* > 0$). Here, very low ability type young would prefer more income redistribution, $\tau' > \tau^*$, and less social security, $\sigma' < \sigma^*$, since the former is an inefficient redistributive program. However, even if they could affect the voting outcome[16], it is easy to see that they would not be able to change it in the desired direction. In fact, any individual, whose ability is below the median voter's ability, could decrease a tax rate (or both), by voting $\sigma = 0$ or $\tau = 0$ (or $\sigma = \tau = 0$), and thus reducing the median tax rate. However, she would not be able to increase the median tax rate, since she is already voting a tax rate larger than the median (voter's) tax rate. A similar reasoning applies to the other voters, and to all cases in Proposition 4.1.

4.3.1 An Example of Welfare System

To obtain a flavor of the result, we parameterize our simple model to the US economy. Every period corresponds to 25 years. The returns on social security are measured by the product of the real wage growth factor and the population growth factor. We set the annual real wage growth rate and the annual population growth rate respectively equal to 2% and 1.5% to match respectively the growth rates of output per hours worked, and of the labor force in the last forty years. The performance of the other saving schemes is indicated by the real rate of return over the same period, which we set equal to 6.4%, according to the average real return from the S&P Composite over the last hundred years. It follows that the performance of the social security system relative to other saving schemes, N, is equal to 0.5, which indicates that social security pays out, on average, 50% less than private savings over the lifecycle.

The degree of income inequality is summarized by the relative ability of the two median voters, $e_{m\tau}$ and $e_{m\sigma}$. We rank the voters according to their ability and age, as in figures 2a and 2b, and then we use the 1992 Survey of Consumer Finances (SCF) data on earning inequality, and the 1992 Presidential election participation rates by age and income, to calculate the ratio of the intragenerational and the social security me-

[16] Remember that every agent has zero mass.

dian voter ability to the mean ability[17]. They turn out to be respectively $e_{m\tau}/e_{\phi} = 0.99$ and $e_{m\sigma}/e_{\phi} = 0.66$. The mean ability in the economy, e_{ϕ}, is normalized to 1. The total amount of disposable time, $\bar{l}$, is set equal to 2.87 in order for the average (daily) working time to equal 8/14. The relative ability of the two median voters are thus $\Delta_{\tau} = 0.0287 > 0$ and $\Delta_{\sigma} = -0.4592 \left(> \widehat{\Delta}_{\tau} = -0.48526\right)$, additionally $\Delta = 0.473$. According to Proposition 4.1, this situation corresponds to case IV, and the associated equilibrium welfare system should thus be composed of positive income redistribution and social security tax rates. In fact, they turn out to be respectively $\tau^* = 0.019$ and $\sigma^* = 0.168$, which imply a total expenditure in direct income redistribution transfers equal to 11.3% of the total social security expenditure. These results are in line with the US welfare system, since the (employee-employer) social security tax rate is 14.4%, and, as shown in table 1, in 1992 the income redistribution transfers (i.e., public assistance and SSI) constituted 10.8% of the transfers to the old.

4.4 Equilibrium Tax Rates and Income Inequality

In this section we concentrate on a welfare system composed of both income redistribution and social security schemes, the most frequent situation, and analyze the effects of changes in income inequality on the equilibrium tax rates. Simple comparative statics show that ceteris paribus an increase in the ability of the intragenerational median voter shifts up the associated reaction function, $\tau^*_{m\tau}(\sigma)$, and thus increases the equilibrium income redistribution tax rate while decreasing the social security tax rate. Analogously, an increase in the social security median voter's ability or in the relative performance of the social security system as saving scheme, N, shifts up the other reaction function, $\sigma^*_{m\sigma}(\tau)$, increases the social security tax rate, and reduces the income redistribution one.

These results, however, are not sufficient to characterize how a change in labor income inequality would affect the equilibrium tax rates.

[17] See the appendix B for a description of the data and of the procedure to calculate the median voters' ability.

An increase, for example, in inequality would presumably tend to decrease both median voters abilities with respect to the mean ability in the economy, i.e., Δ_τ and Δ_σ would increase, and thus would shift both reaction functions in the same direction. The analysis of the consequences on the equilibrium tax rates of such changes represents the object of the next proposition.

First, we decompose the changes in the equilibrium tax rates into the effects due to the variation in the intragenerational median voter's ability ($d\Delta_\tau$) and in the social security median voter's ability ($d\Delta_\sigma$):

$$\begin{aligned} d\tau^* &= \underset{(+)}{\frac{\partial \tau^*}{\partial \Delta_\tau}}\, d\Delta_\tau + \underset{(-)}{\frac{\partial \tau^*}{\partial \Delta_\sigma}}\, d\Delta_\sigma \\ d\sigma^* &= \underset{(-)}{\frac{\partial \sigma^*}{\partial \Delta_\tau}}\, d\Delta_\tau + \underset{(+)}{\frac{\partial \sigma^*}{\partial \Delta_\sigma}}\, d\Delta_\sigma. \end{aligned} \tag{21}$$

As previously noted, the direct effect of a change in the median voter's relative ability is positive, whereas the crossed effects are negative. The following lemma[18] establishes another useful result.

Lemma 5 *For an interior solution of the voting game,* $(\tau^* > 0, \sigma^* > 0)$,

$$\left| \frac{\partial \tau^*}{\partial \Delta_\sigma} \right| \leq \left| \frac{\partial \sigma^*}{\partial \Delta_\sigma} \right|.$$

The absolute value of the direct effect of a change in the social security median voter's ability is larger than the absolute value of the indirect effect. Finally, let $\eta_{\tau^*,\Delta_\sigma} = \frac{\partial \tau^*}{\partial \Delta_\sigma} \frac{\Delta_\sigma}{\tau^*}$ be the elasticity of the equilibrium income redistribution tax rate to changes in Δ_σ, and $\eta_{\sigma^*,\Delta_\sigma} = \frac{\partial \sigma^*}{\partial \Delta_\sigma} \frac{\Delta_\sigma}{\sigma^*}$ be the elasticity of the equilibrium social security tax rate to changes in Δ_σ. We can now state the following proposition, which we prove in the appendix:

Proposition 6 *For an interior equilibrium of the voting game and for positive changes of* Δ_τ *and* Δ_σ $(d\Delta_\tau > 0,\ d\Delta_\sigma > 0)$*, the following holds:*

[18] A proof is in the appendix.

i) $d\tau^* \geq 0$ and $d\sigma^* \leq 0$ if $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \leq \left|\frac{\tau^*/\sigma^*}{\eta_{\sigma^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$.

ii) $d\tau^* \geq 0$ and $d\sigma^* \geq 0$ if $\left|\frac{\tau^*/\sigma^*}{\eta_{\sigma^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right| \leq \left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \leq \left|-\frac{1}{\eta_{\tau^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$.

iii) $d\tau^* \leq 0$ and $d\sigma^* \geq 0$ if $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \geq \left|-\frac{1}{\eta_{\tau^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$.

In other words, if the increase in income inequality induces a percentage increase in the measure Δ_σ of the social security median voter's ability which is sufficiently smaller than the percentage increase induced in Δ_τ (case i), then the income redistribution tax rate will increase and the social security tax rate will decrease. The opposite happens for percentage increases in Δ_σ sufficiently larger than Δ_τ (case iii). For changes in Δ_σ and Δ_τ of comparable magnitude (case ii), both tax rates increase. A numerical example will help to appreciate the magnitudes of the changes in Δ_σ and Δ_τ which lead to the three cases.

Example 4.5: We use the values of the equilibrium welfare system we parametrized to the US economy in the previous section. The changes in the tax rates can be decomposed as at eq. 21, and evaluated at $\tau^* = 0.019$ and $\sigma^* = 0.168$:

$$\begin{aligned} d\tau^* &= 0.728 * d\Delta_\tau - 0.134 * d\Delta_\sigma \\ d\sigma^* &= -2.165 * d\Delta_\tau + 3.008 * d\Delta_\sigma. \end{aligned}$$

Therefore, an increase in the social security tax rate associated with a decrease in the income redistribution tax rate, case (iii), occurs when the percentage change in the social security median voter's relative ability is larger than 33.9% of the corresponding change in the intragenerational median voter's relative ability, $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \geq 0.339$. Case (i), $d\tau^* \geq 0$ and $d\sigma^* \leq 0$, occurs for $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \leq 0.045$, whereas case (ii), $d\tau^* \geq 0$ and $d\sigma^* \geq 0$, takes place for intermediate values: $0.045 \leq \left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \leq 0.339$.

5 Discussion of the Results

The idea that a social security system may rely on the political support of low-income young and retirees was first formulated by Tabellini (1990). In his overlapping generation model, heterogeneous (in income), weakly altruistic agents[19] vote every period on the social security level. Young voters do not expect their decision to influence future policy outcomes. Nevertheless, because of their weak altruism, low-income young support social security, since the utility associated to their parents receiving a pension is larger than the direct (utility) cost of the tax. With sufficient income inequality, an equilibrium with social security arises. This equilibrium, however, is not robust to changes in the specification of the welfare system. In particular, if a fiscal policy that achieves income redistribution within cohorts is introduced, the equilibrium disappears.

Our paper generalizes this result to a more complete welfare system. In our model, social security may co-exist with an income redistribution program in the political equilibrium of a two-dimensional voting game. The intuition is straightforward. Due to the existence of a within-generation redistribution component in the social security system, low income young are willing to support both welfare schemes, although they would prefer pure income redistribution to social security. For the retirees, on the other hand, age represents the main determinant in their voting decision. They contribute their voting block to promote social security and to prevent intragenerational income redistribution schemes from being adopted. Therefore, they help to shape the two winning majorities. On social security, the majority is composed of retirees and poor young, and the decisive, or median, voter is a low income young, see Figure 2.b; whereas on income redistribution the decisive, or median, voter is a young agent with a higher labor income, see Figure 2.a. In this sense, the retirees' uniform voting behavior contributes to create a wedge between the abilities of the two decisive voters, which is crucial to obtaining an equilibrium welfare system composed of both schemes.

[19] Young altruism is towards their parents is weak, since they are not willing to give them a direct transfer of resources.

The same intuition applies to the analysis of the effects of a change in the overall labor income distribution on the equilibrium tax rates. The final result depends on the impact that the change in income inequality has on the wedge between the two decisive voters, as characterized in Proposition 4.4.

Our results are robust to changes in the specification of the welfare state and of the voting game. Consider a comprehensive income redistribution program, which imposes a proportional tax on all incomes (earnings, transfers, and pensions) and pays a lump sum transfer to all agents (young and old). Unlike in the previous specification, the retirees would now support positive levels of income redistribution, provided that the social security tax rate is sufficiently small. In fact, the smaller are their pensions, the smaller is their tax bill, and the total distortion induced by the income redistribution tax rate on their pensions. Although this problem has no closed-form solution, it is straightforward to show that, if we parametrize the economy as in the example 4.3.1, there still exists an equilibrium welfare system[20] with positive values of income redistribution, $\tau^* = 0.07$, and social security tax rates, $\sigma^* = 0.09$.

Would the results change if we adopt a political structure which induces sequential voting? Again, the answer is no. Suppose that elections take place in two rounds. Agents first determine the social security tax rate, σ, and then the income redistribution tax rate, τ. In the first round, voters realize that their decision over σ has a negative effect over τ, because a larger social security system crowds out the level of income redistribution. Since the median voter over σ is a low-income young, she favors a large income redistribution program. Thus, she will vote for a low σ, not to jeopardize the future decision over τ. Graphically, her reaction function $\sigma(\tau)$ is closer to the origin than in the issue-by-issue (simultaneous) voting (see Figure 3). An analog of Proposition 4.1 can thus be derived to provide the conditions under which a welfare state

[20] Notice that for this parametrization, there also exist two other equilibria of the game, one with zero income redistribution and positive social security: $\tau^* = 0$, $\sigma^* = 0.25$; and the other with positive income redistribution and no social security: $\tau^* = 0.36$, $\sigma^* = 0$.

composed of both programs represents a politico-economic equilibrium of this sequential voting[21].

In an independent contribution, Lambertini and Azariadis (1998) analyze a welfare system, composed of intragenerational and intergenerational transfers, to account for the rapid expansion in the government redistributive expenditure of the last decades. Although their work focuses on similar aspects, their results differs from ours in at least two major respects. In their politico-economic equilibrium, both programs are supported by the same voting majority composed of low-ability young and retirees. As a consequence, since we identify a different political support for the income redistribution program, also the implications of an increase in income inequality on the size of the welfare programs are different. In Lambertini and Azariadis (1998), the traditional implication holds, and the size of both programs rises. Our insight is that this is not necessarily true as age, rather than income, may be the crucial component in agents' decisions.

Another difference with Lambertini and Azariadis (1998) lies in the political game. Their post-electoral political system follows Baron and Ferejohn's (1989) legislative bargaining model with closed amendments rule. One of the three existing groups (old, skilled and unskilled young) is randomly chosen to be the agenda setter, and thus to make a policy proposal, which is then voted against the status quo at simple majority. Depending on the status quo, different agenda setters will form different minimum winning coalitions, and thus give raise to different compositions of the welfare state. They calibrate their model to the 1996 US economy and suggests that, when the unskilled young are selected to propose a policy, they enter a coalition with the old to support an equilibrium with positive intragenerational and intergenerational transfers.

An alternative way of aggregating individual preferences when the issue space is multidimensional is through a model of electoral competi-

[21] A numerical example parametrized to the US economy as in the previous section provides qualitatively similar results. The equilibrium social security tax rate is 12.3%, as opposed to 16.8% in the simultaneous issue-by-issue voting, and the income redistribution tax rate is 2%, as opposed to 1.9%.

tion, the probabilistic voting model[22]. In this pre-electoral voting model, candidates commit to an electoral program, which in our setting corresponds to a pair of tax rates (τ, σ). Voters care about the indirect utility associated to these electoral platforms. Additionally, they have idiosyncratic ideological preferences over the candidates. This individual ideology is distributed according to a distribution function, which may vary across group of individuals, such as low or high ability young, and old. On average, individuals are ideologically neutral. Candidates choose their platforms, which in equilibrium turn out to be identical, to maximize the expected probability of being elected.

It is easy to see that, if all groups share the same degree of ideology, i.e., the distributions are identical across groups, the candidates' optimization problem coincides with maximizing a utilitarian utility function. In our setting, the corresponding politico-economic equilibrium would display no income redistribution and positive social security. To obtain an equilibrium welfare state with positive levels of both programs, we would need to assume that the low-income young are more ideologically homogeneous, i.e., their distribution is more concentrate around the mean, than, say, high ability young. In this case, the candidates would optimally place their (identical) programs closer to the ideal point of the low ability young, and would thus choose a positive level of income redistribution[23].

6 Concluding Remarks

Why does the largest US welfare program select its recipients by their age, rather than by their earnings or wealth? In contrast to previous literature, we suggest that a welfare system composed of a (large) PAYG social security program and an income redistribution scheme may represent the political equilibrium of a voting game played by successive gen-

[22]See Persson and Tabellini (2000) for a survey of all these voting models.

[23]Clearly, if the high ability young are more ideologically homogenous, the equilibrium income redistribution is zero, and social security would be reduced.

eration of voters. In particular, the social security system is supported by a majority of retirees and low-income young.

Two features are crucial to this result: the political power of the old, which derives from their “extreme” and uniform voting behavior; and the intragenerational redistribution component of the social security system. Unlike the young and the middle age, the elderly constitute a fairly homogeneous group. They are old, and they have zero (when retired) or low labor earnings, although they may largely differ in their wealth. This homogeneity makes them a uniform electoral block when voting on redistribution issues: they all like social security, and they all may or may not support different forms of income-based redistribution. Since they are able to cluster and shift a large amount of votes, the elderly play a crucial role in shaping the two winning coalitions, as shown in Figure 2.

The existence of a within generation redistribution element in the social security system, on the other hand, induces low-income young to support the social security, even in the presence of other income redistribution schemes. This factor has often been overlooked by the social security literature. Its relevance for the political viability of a system is, however, crucial, as recent social security reforms have shown. In fact, most reformed systems have maintained an element of within generational redistribution, sometimes as a new, separate program financed through general taxation.

The common wisdom in the politico-economic literature has been that income inequality is positively related to government welfare transfers. The empirical evidence are, however, mixed. Tabellini (1990) and Perotti (1996) has provided evidence in support of a negative and significant relation between a joint measure of welfare and social security transfers, and income inequality. In Lindert (1997), higher inequality decreases all social expenditures[24]. We argue that the effect on each indi-

[24]Tabellini measures income inequality by the ratio between the pre-tax income received by the top 20% and by the bottom 40% of the population. Perotti uses share of income in the third and forth quintile. Lindert identifies inequality as the sum of the natural logaritms of the upper income gap and of the lower income gap. The

vidual welfare program depends not only on the magnitude of the change in income inequality, but also on its specific impact on the income distribution. In particular, our model suggests that, when analyzing pension transfers, the overall income distribution should be decomposed by age groups. This is because age, rather than labor income, may be the main component in the agents' decisions. Only then, should a measure of inequality be constructed according to income and age, as shown in figure 2.

Unfortunately, data availability on measures of labor income inequality by age group is very limited. The International Labor Office (ILO) provides one observation, in the mid 80s, of few measures of labor income dispersion for people aged 25 to 54, and for the overall population in 15 OECD countries. In the appendix, we use these few observations to relate the expenditures in old age pension and income redistribution to the explanatory variables suggested by our model. Interestingly, we find that old age pensions are strongly positively correlated to the share of elderly (aged 60+) in the population, and to a measure of the relative performance of the pension system, N. After controlling for these effects[25], in our 15 observations, the correlation between old age pensions and different measures of income inequality is always very weak. However, this correlation increases, and delivers the expected sign, when, as suggested by our theory, we move from a measure of after tax overall income inequality to a measure of pre-tax income inequality among people aged 25 to 54. We find this encouraging. As more data on labor income dispersion by age group become available, we believe that these simple considerations should be taken into account in future empirical studies.

upper income gap is the ratio of the average income for the top fifth to that for the third fifth. The lower income gap is the comparable measure between the third and the fifth quintiles.

[25]Income redistribution transfers turn out to be strongly positively correlated to the share of elderly (aged 65+) in the population, and negatively to different measures of income inequality, both overall and by age group.

Table 1

1992 Cash Benefits in billions of $

OASDHI	284.3
Railroad Ret.	7.3
Public Employees Ret.	103.7
Veterans' Pensions	16.5
Unemployment Ben.	37.3
Temporary Disability	4.0
Workers' Compens.	44.1
Public Assistance	22.4
SSI	22.3
TOTAL	541.9

Table 2

1992 Cash and In-Kind Benefits

	Federal	State
OASDHI	416.6	-
Railroad Ret.	58.2	45.1
Public Employees Ret.	7.7	-
Veterans' Pensions	16.5	-
Unemployment Benef.	9.9	31.2
Workers Comp	3.2	40.9
Public Aid	138.7	69.2
Medicaid	32.0	37.8
Housing	17.9	2.7
TOTAL	700.7	226.9
Education	20.2	272.0

Table 3

Participation Rates by Income
18-64 Year Old Voting Population

Income (I)	1992
$I < \$5,000$	30.4 %
$\$5,000 < I < \$9,999$	34.5 %
$\$10,000 < I < \$14,999$	40.1 %
$\$15,000 < I < \$19,999$	50.6 %
$\$20,000 < I < \$24,999$	59.8 %
$\$25,000 < I < \$34,999$	68.4 %
$\$35,000 < I < \$49,999$	75.6 %
$\$50,000 < I$	79.7 %
Income not Reported	54.9 %
All Incomes	60.8 %

A Appendix

The Political System: Our political system describes a decision-making institution which has $1 + 1/(1+\mu)$ members: the electorate, E. The space of alternatives is a compact subset of $\Re^2$: (τ, σ) $s.t.$ $\tau + \sigma \leq 1$. And there exists a complete, transitive binary preference relation $\underset{i}{\geq}$ over all alternatives $x, y \in \Re^2$, $\forall i \in E$, and represented by $v_i : \Re^2 \to \Re$. Institutional arrangements differ along three dimensions: (a) committee structure; (b) jurisdiction structure; and (c) amendment structure. The first two structures follow from the definitions below.

Definition 4 (Committee) *Call the family of sets $C = \{C_j\}$ a committee system if it covers the entire electorate E. Then the committee $C = \{E\}$ is the Committee of the Whole.*

Definition 5 (Jurisdiction) *Let $B = \{b_1, b_2\}$ be the orthogonal basis for $\Re^2$ where b_i is the unit vector for the i-th dimension. The family of set $J = \{J_k\}$ is a jurisdictional arrangement if it covers B. Then $J = \{\{b_1\}, \{b_2\}\}$ is a Simple Jurisdiction.*

Additionally, call f the function which associate a jurisdiction with a committee, $f : C \to J_k$. In our system $f : E \to \{\{b_1\}, \{b_2\}\}$ or $f(E) = \{\{b_1\}, \{b_2\}\}$.

To define an amendment structure we need to introduce the notions of status quo, x^o, and of proposal. A status quo, x^o, represents the previous agreed level on both dimensions of the issue space. For example, at time t, $\{x_1^o, x_2^o\} = \{\tau_{t-1}, \sigma_{t-1}\}$.

Definition 6 (Proposal) *A proposal, x, is a change in x^o restricted to a single jurisdiction. The set of proposal available to the committee of the whole is*

$$g(E) = \{x \mid x = x^o + \lambda_i b_i, b_i \in f(E)\} \subseteq \Re^2 \text{ with } \lambda_i \in \Re \ \forall i.$$

Definition 7 (Amendment Control Rule) *For any proposal $x \in g(E)$, the set $M(x) \subseteq \Re^2$ consists of the modifications E may make in x. $M(x)$ is said to be an amendment control rule. An amendment control rule is a Germaneness rule if $M(x) = \{x' \mid x'_i = x^o_i \text{ if } x_i = x^o_i\}$.*

Definition 8 (Induced Ideal Point) *For a status quo $x^o = (x^o_1, x^o_2)$ and a jurisdiction b_j, the induced ideal point in the j-th direction for $i \in E$ is $x^{*i} = \left(x^{*i}_j, x^o_{-j}\right)$ where $x^{*i}_j = \arg\max u_i\left(x_j, x^o_{-j}\right)$. Then, x^{*i} is the induced ideal point on an arbitrary set X if $u_i(x)$ is maximized on X at $x = x^{*i}$.*

Definition 9 (Median in all Directions) *In a two-dimensional issue space (τ, σ), $\widehat{x} = (\widehat{\tau}, \widehat{\sigma})$ is a median in all direction if any line passing through $\widehat{x}$ divided the issue space in two areas each one containing half of the electorate's ideal points.*

Proof of Proposition 3.2 (I) Young voters' preferences are represented by eq. 10. We derive eq. 10 with respect to τ_t and $\sigma\,(= \sigma_t = \sigma_{t+1})$, and then we impose the stationarity condition $\tau = \tau_t = \tau_{t+1}$ to obtain the following Hessian matrix:

$$\begin{matrix} -\frac{1+\tau-\sigma}{(1-\tau-\sigma)^3} & -\frac{2\tau}{(1-\tau-\sigma)^3} \\ -\frac{2\tau}{(1-\tau-\sigma)^3} & \frac{1-3\tau-\sigma-2N+2N\tau}{(1-\tau-\sigma)^3} \end{matrix}$$

Simple algebra shows that, if $N \geq \frac{(1-\tau_t-\sigma_t)^2}{2[1-\sigma_t-\tau_t(\tau_t-\sigma_t)]}\left(\leq \frac{1}{2}\right)$, this matrix is semi definite negative. Then young voters' preferences are quasi concave and, by Shepsle (1979) Lemma 3.1, they are single peaked.

(II) Old voters preferences (eq. 11) are not concave in (τ, σ), thus we will establish single peakedness using definition A.7. Let $\tau = p + q\sigma$ be a line in the issue space (τ, σ). By definition A.6, the induced ideal point for old voters on this line is

$$\left(\tau^* = p + \frac{q\left(1 - p - \sqrt{\frac{1-p}{e_\phi \bar{l}}}\right)}{1+q}, \sigma^* = \frac{1 - p - \sqrt{\frac{1-p}{e_\phi \bar{l}}}}{1+q}\right),$$

for $p<1$, and $\left(\tau^*=0,\ \sigma^*=-\frac{p}{q}\right)$, for $p>1$ and $q<-1$ (Clearly for $p>1$ and $q>-1$, then $\tau+\sigma>1$). By definition A.7, single peakedness requires that

$$\begin{aligned} & u\left[\alpha\tau'+(1-\alpha)\tau^*,\alpha\sigma'+(1-\alpha)\sigma^*\right]> \\ & u\left[\beta\tau'+(1-\beta)\tau^*,\beta\sigma'+(1-\beta)\sigma^*\right], \end{aligned} \tag{22}$$

$\forall(\tau',\sigma')$ s.t. $\tau'=p+q\sigma'$, whenever $0\leq\alpha<\beta\leq 1$. To verify eq. 22, we substitute the values of $(\tau^*,\ \sigma^*)$ into

$$u\left[\alpha\tau'+(1-\alpha)\tau^*,\alpha\sigma'+(1-\alpha)\sigma^*\right],$$

and then derive it with respect to. α. The sign of this derivative is equal to

$$\begin{array}{ll} (1+q)\,\alpha\left[(1+q)\,\alpha\,(\sigma'-\sigma^*)-2\sqrt{\frac{1-p}{e_\phi\bar{l}}}\right] & \text{for } p<-1 \\ \left(\sigma'+\frac{p}{q}\right)\left[e_\phi\bar{l}-\frac{1-p}{\left[1+\frac{p}{q}-\alpha(1+q)\left(\frac{p}{q}+\sigma'\right)\right]^2}\right] & \text{for } p>-1 \end{array}.$$

This sign is negative in both cases for $\alpha<1$, which implies that the inequality at eq. 22 holds, and that old voters preference are single peaked.

Proof of Proposition 4.1 Using equations 12 and 18, it is easy to show that these reaction functions cross only once in the simplex $\tau+\sigma\leq 1$ at (τ^*,σ^*). This is the only point which represents the median among the induced ideal point along both dimensions, τ and σ, and thus by Proposition 3.3 (τ^*,σ^*) is the only structure induced equilibrium.

If $\Delta_\tau\leq 0$ and $\Delta_\sigma\leq-(1-N)$, the reaction functions 12 and 18 are only defined on the simplex $\tau+\sigma\leq 1$ at $(\tau=0,\sigma=0)$. If $\Delta_\tau\leq 0$ and $\Delta_\sigma>-(1-N)$, then $\tau^*_{m\tau}(\sigma)=0$, and thus it crosses the reaction function 18 on the σ axis at $\sigma^*=1+\left(1-\sqrt{1+4N\Delta_\sigma}\right)/2\Delta_\sigma>0$.

To find the condition for an interior solution, case iv, notice that for $\Delta_\tau > 0$ both reaction functions are negatively sloped, and that $\tau^*_{m\tau}(\sigma)$ has a higher intercept on the vertical, σ, axis than $\sigma^*_{m\sigma}(\tau)$. Since both reaction functions are continuous, if $\sigma^*_{m\sigma}(\tau)$ crosses the horizontal, τ, axis to the right of $\tau^*_{m\tau}(\sigma)$ there exists a political equilibrium of the voting game for $\tau^* = \Delta_\tau \left(1 - 2N\Delta - \sqrt{1 - 4N\Delta}\right) / 2\,\Delta^2$ and $\sigma^* = 1 - N - \tau^*\left(2 - N - \left(\Delta_\sigma / \Delta_\tau\right)\right)$. The condition for the reaction function $\sigma^*_{m\sigma}(\tau)$ to cross the horizontal axis to the right of $\tau^*_{m\tau}(\sigma)$ is that $\Delta_\sigma > \widehat{\Delta}_\tau = \Delta_\tau - (1 - N)\left(1 + \sqrt{1 + 4\Delta_\tau}\right)/2$. If, on the other hand, $\Delta_\sigma < \widehat{\Delta}_\tau$, then $\sigma^*_{m\sigma}(\tau)$ will cross the horizontal, τ, axis to the left of $\tau^*_{m\tau}(\sigma)$, and thus the equilibrium will be on the horizontal, τ, axis at $\tau^* = 1 + \left(1 - \sqrt{1 + 4\Delta_\tau}\right)/2\Delta_\tau$.

Proof of Proposition 4.2 Suppose (τ^*, σ^*) is a structure induced equilibrium outcome of the voting game with commitment over the social security policies. Let us define the following realization of the public history of the game:

$$H_t^0 = \{h_t \in H_t |\, \sigma_k = 0,\ k = 0, ..., t-1\}$$

and

$$\begin{aligned} H_t^\sigma = \{ & h_t \in H_t |\, \exists\, t_0 \in \{0, 1, ..., t-1\} : \sigma_t = 0\ \forall t < t_0 \\ & \text{and } \sigma_t = \sigma\ \forall t \geq t_0\} \end{aligned}$$

notice that $H_t^0 \cap H_t^\sigma = \emptyset$.

Consider the following strategy $s = \left(s^y_{t,e}, s^o_{t,e}\right)$, for a type e young:

i) if $e \leq e_{m\sigma}$

$$s^y_{t,e} = \begin{cases} \left(\tau_{t,e}(\sigma^*), \sigma^*\right) & \text{if} \quad h_t \in H_t^0 \cup H_t^\sigma \\ \left(\tau_{t,e}(0), 0\right) & \text{if} \quad h_t \in H_t / \{H_t^0 \cup H_t^\sigma\} \end{cases}$$

ii) if $e > e_{m\sigma}$

$$s^y_{t,e} = \begin{cases} \left(\tau_{t,e}(\sigma^*), \sigma_{t,e}(\tau^*)\right) & \text{if} \quad h_t \in H_t^0 \cup H_t^\sigma \\ \left(\tau_{t,e}(0), 0\right) & \text{if} \quad h_t \in H_t / \{H_t^0 \cup H_t^\sigma\} \end{cases}$$

and for an old individual

$$s_{e,t}^{o} = \quad (0, \sigma_{old}(\tau^{*})) \quad \text{if} \quad h_t \in H_t$$

where $\sigma_{t,e_{m\sigma}}(\tau^{*})$ is defined in eq. 12; $\tau_{t,e_{m\tau}}(\sigma^{*})$ in eq. 18, and $\sigma_{old}(\tau^{*})$ in eq. 15.

Since by definition of SIE, $\sigma^{*} = \sigma_{t,e_{m\sigma}}(\tau^{*})$, $\tau^{*} = \tau_{t,e_{m\tau}}(\sigma^{*})$, it is easy to see that:

$$\begin{aligned} \tau_{t,e}(\sigma^{*}) &\geq \tau^{*} \ \forall \ e \leq e_{m\tau}, \\ \sigma_{t,e}(\tau^{*}) &\geq \sigma^{*} \ \forall \ e \leq e_{m\sigma} \end{aligned}$$

Moreover, since the outcome function of the voting game at time t is the median in every dimension of the distribution of actions, (τ_t^m, σ_t^m), it is easy to see that the previous strategy profile $\left(s_{t,e}^{y}, s_{t,e}^{o}\right)$ constitute a subgame perfect equilibrium of the voting game with no commitment, with equilibrium outcome (τ^{*}, σ^{*}).

Proof of Lemma 4.3 From equation 20, $\frac{\partial \sigma^{*}}{\partial \Delta_\sigma} = \frac{\tau^{*}}{\Delta_\tau} - \left(2 - N - \frac{\Delta_\sigma}{\Delta_\tau}\right) \frac{\partial \tau^{*}}{\partial \Delta_\sigma}$ and $\frac{\partial \tau^{*}}{\partial \Delta_\sigma} = \frac{2}{\Delta}\tau^{*} + \frac{N\Delta_\tau}{\Delta^2}\left(1 - \frac{1}{\sqrt{1-4N\Delta}}\right)$. Since $(\partial \tau^{*}/\partial \Delta_\sigma) \leq 0$ and $(\partial \sigma^{*}/\partial \Delta_\sigma) \geq 0$, thus it is sufficient to show that $-(\partial \tau^{*}/\partial \Delta_\sigma) \leq (\partial \sigma^{*}/\partial \Delta_\sigma)$, which can be done from the previous two expressions and using some simple algebra.

Proof of Proposition 4.4 To prove part (iii), we use the decomposition at eq. 21 to write $d\tau^{*} \leq 0$ and $d\sigma^{*} \geq 0$ as $\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau} \geq -\frac{(\partial \tau^{*}/\partial \Delta_\tau)}{(\partial \tau^{*}/\partial \Delta_\sigma)}$ and $\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau} \geq -\frac{(\partial \sigma^{*}/\partial \Delta_\tau)}{(\partial \sigma^{*}/\partial \Delta_\sigma)}$. From equation 20, $\frac{\partial \tau^{*}}{\partial \Delta_\tau} = \frac{\tau^{*}}{\Delta} - (1-N)\frac{\partial \tau^{*}}{\partial \Delta_\sigma} \geq 0$ and $\frac{\partial \sigma^{*}}{\partial \Delta_\tau} = -\frac{\tau^{*}}{\Delta_\tau} - (1-N)\frac{\partial \sigma^{*}}{\partial \Delta_\sigma} \leq 0$. Substituting these derivatives in the previous inequality we obtain: $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \geq \left|-\frac{1}{\eta_{\tau^{*},\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$ and $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \geq \left|\frac{\tau^{*}/\sigma^{*}}{\eta_{\sigma^{*},\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$. Since by Lemma 3.5, $|1/\eta_{\tau^{*},\Delta_\sigma}| \leq |\tau^{*}/\sigma^{*} * \eta_{\sigma^{*},\Delta_\sigma}|$, then $d\tau^{*} \leq 0$ and $d\sigma^{*} \geq 0$ for $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \geq \left|-\frac{1}{\eta_{\tau^{*},\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$ which prove part (iii). We skip the proof of part (i) and (ii), which is analogous to part (iii).

B Appendix

In this appendix the 1992 Survey of Consumer Finances (SCF) data are used to analyze some feature of the US income distribution. The data unambiguously show that earnings, income, and wealth are unequally distributed across US families. In particular, the density functions of these distributions are skewed, as they display a fat lower tail, many poor, and a thin upper tail, few rich. For example, the ratio of median to mean was equal to 0.60 for earnings, to 0.58 for income, and to 0.28 for wealth. Other inequality indicators present the same picture.

Consider the ordering in the social security voting, as shown at figure 2b. The mass of retirees is placed on the left of the income distribution: they vote for a larger tax rate than the poorest young. When adjusted for the (1992 Presidential) election participation rates, they represent 19% of the actual voters. To account for the young voters, we drop the retirees (person older than 65 year) from the sample, and adjust the new earning distribution for the election participation rates. Since turnout rates are not available by earnings group, we need to combine earnings distribution data with the participation rates by income shown at table 3. Given the high correlation between income and earnings found on 1992 data by Díaz-Giménez, Quadrini, and Rios Rull (1997), who report a coefficient of 0.928, we assume that there are no permutations between the income and the earnings distributions. In other words, voters with low (high) earnings are associated to voters with low (high) income and to the corresponding participation rates. After the weights of the earnings distribution have been adjusted for the different participation rates, we obtain that the ratio of the social security median voter's earnings to the mean earnings in the economy is equal to 0.66.

In the case of intragenerational redistribution the ordering of the voters is shown at figure 2a. The retirees now vote against the transfer and are placed on the right of the income redistribution. Since in 1992 almost 13% of the elderly would receive intragenerational transfers, we subtract these individuals from the retirees and add them to the poor young. In fact, since, unlike in our theoretical model, they actually received a benefits, they would presumably have voted in favor of this policy. This makes the proportion of elderly voters in the voting population drop to 16.5%. We then adjust the earning distribution to account for young voters only, as described above, and calculate the ratio of the intragenerational redistribution median voter's earnings to the mean earnings in the economy, which is equal to 0.99.

C Appendix

In this appendix, we discuss the data, and the methodology we use to relate the expenditures in old age pension and income redistribution to the explanatory variables suggested by our model.

¿From an OECD (1995) report, we obtained ILO measures of labor income dispersion for people aged 25 to 54, and for the overall population in 15 OECD countries. The World Bank database described in Deininger and Squire (1996) provides measures of overall income inequality. The countries, and the years (first year ILO, second year WB) the data refer to are: Australia (1985-86, 1989), Belgium (1988, 1992), Canada (1987, 1991), Finland (1987, 1991), France (1984, 1984), Germany (1984, 1984), Ireland (1987, 1987), Italy (1986, 1991), Luxembourg (1985, 1985), the Netherlands (1987, 1991), Norway (1979, 1991), Sweden (1987, 1990), Switzerland (1982, 1982), UK (1986, 1991), and US (1986, 1991). The ILO measures are the ratio of the labor income of the individual in the 25 centile to the labor income of the median individual for people aged 25 to 54, and for the overall population, which we denotes respectively as $P25$ and $PT25$. The WB measure we use is the ratio of the income share of individuals in the 2nd quintile to the income share of individuals in the 4th quintile for the overall population: PT.

Data on the expenditures in old age pension and income redistribution as percentage of the GDP are obtained from the OECD Social Expenditure Database. These are average over the 1980-96 period. Income redistribution transfer include family cash benefits, family service, housing benefits and other contingences. The shares of elderly (aged

60+, and 65+) in the population are taken from World Bank data for the 1990. A measure of the relative performance of the pension system, N, represents our own calculation using Miles (1997) data on average real returns over 1962-94 and GNP real growth over 1961-94.

Column 2 of table C.1 displays the results of a simple OLS regression of the ratio of the income redistribution to GNP over the share of elderly (aged 65+) in the population, $Pop65$, and over a measure of income inequality, $PT25$. Both regressors are significant at a 5% level. $PT25$ has the expected sign, more income inequality (lower $PT25$) leads to higher transfers. The positive sign of $Pop65$ suggests that elderly people do care about income redistribution programs as well.

In column 3, 4, and 5, we regress the ratio of the Old Age Pensions to GNP over the share of elderly (aged 60+) in the population, $Pop60$, over our measure of the relative performance of the pension system, N, and over a measure of income inequality, respectively PT, $PT25$, and $P25$. Two regressors, $Pop60$ and N, have the expected sign, and are almost always significant at a 5% level. However, no measure of income inequality is significant. In fact, in column 3, where we use the measure of after tax overall income inequality, PT, even the sign is wrong. If, according to our theory we look at pre-tax income inequality among people aged 25 to 54, $P25$ (column 5) the regressor is still not significant, but with the right sign. The results of the regression with $PT25$ are reported in column 4.

Table C.1

	Intra	OAI	OAI	OAI
$Pop65$	$.3709^{*}$			
	(.1327)			
$PT25$	$-.0867^{*}$		$-.0051$	
	(.0358)		(.0545)	
$Const$	3.300	-6.0394	-4.9367	-3.5107
	(2.7518)	(3.2394)	(3.9177)	(1.4943)
$Pop60$		$.4362^{*}$	$.4778^{*}$	$.4953^{*}$
		(.1682)	(.1599)	(.1627)
N		3.1565	3.4986^{*}	3.6152^{*}
		(1.5095)	(1.5684)	(1.4943)
PT		3.4348		
		(5.6032)		
$P25$				$-.0319$
				.0709
R^2	50.25%	64.27%	63.08%	63.72%
$N.obs.$	15	15	15	15

Standard Errors in parenthesis. * denotes 5% significance level.

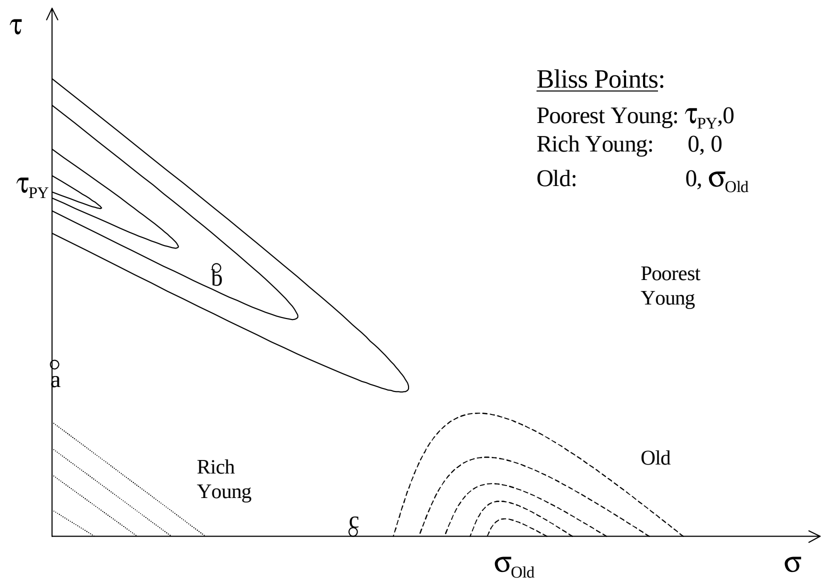

Grafico1

Voting on Intragenerational Transfers and Social Security

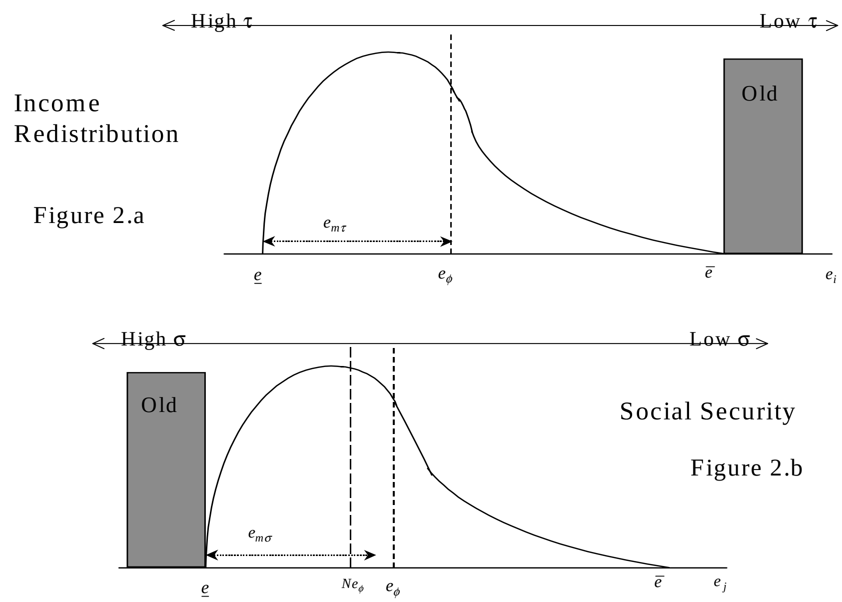

Grafico 2

Equilibrium with Income Redistribution and Social Security

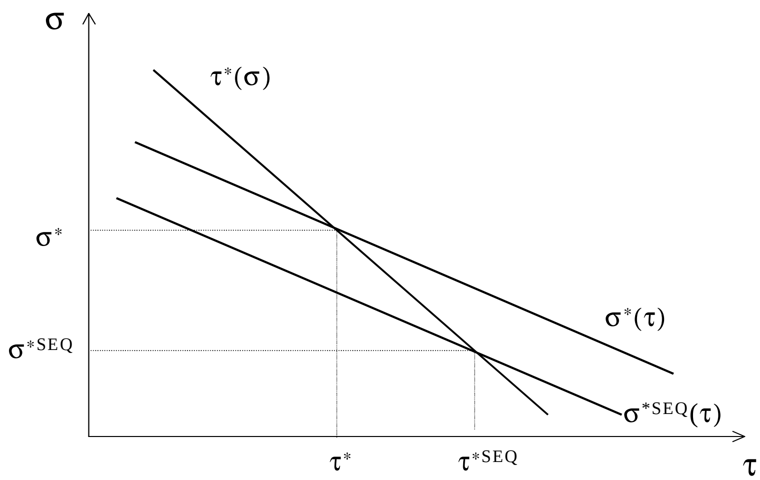

Grafico 3

Equilibria with Income Redistribution and Social Security

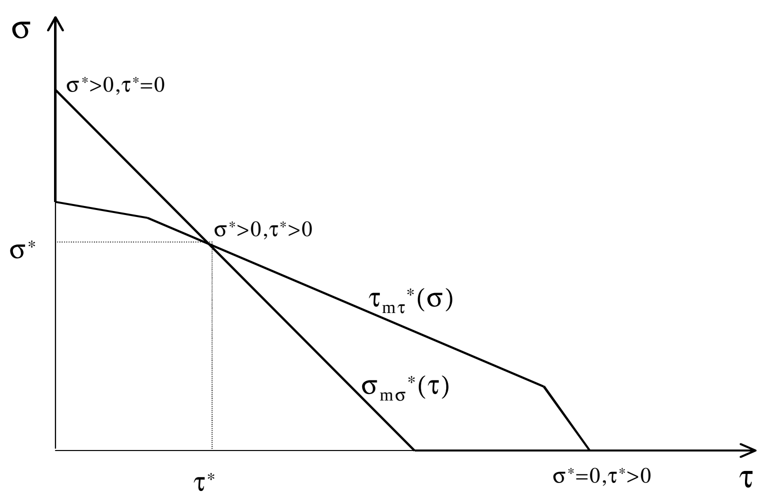

Grafico 4

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