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## Early Retirement

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# Early Retirement\*

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## Abstract

Generous early retirement provisions account for a large proportion of the drop in the labor force participation of elderly workers. The aim of this paper is to provide a political-economic explanation of the wide spread adoption of generous early retirement. We suggest that the political support for generous early retirement provisions relies on: (i) the existence of an initial, significant group of redundant elderly workers with incomplete working history, who are not entitled to an old age pension; and (ii) the policy persistence that this provision introduces by inducing low-ability workers to retire early. The majority which supports early retirement in a bidimensional voting game is composed of elderly with incomplete working history and low-ability workers; social security is supported by retirees and low-ability workers. A descriptive analysis of eleven OECD countries shows that early retirement provisions were adopted during the deindustrialization process, almost always, immediately after the first severe decrease in industrial employment.

**Keywords:** Social Security, Policy Persistence, Subgame Perfect Structure Induced Equilibrium.

**JEL Classification:** H53, H55, D72.

# 1 Introduction

Generous early retirement provisions are largely responsible for the dramatic drop in the labor force participation among middle age and elderly workers of the last thirty years (see Gruber and Wise (1999) and Blöndal and Scarpetta (1998)). The generosity<sup>1</sup> of these provisions has induced workers, in particular, low-educated ones, to retire early, thereby increasing the dependency ratio, and thus creating more financial distress to the social security system.

The aim of this paper is to provide a politico-economic explanation of the adoption of generous early retirement. Why did a majority of voters, in most industrialized countries, decide to award large pensions to middle aged workers with incomplete working history?

We suggest that the political support in favor of early retirement hinges on two crucial conditions. First, the appearance of a large group of redundant elderly workers with incomplete working history, who are not entitled to an old-age pension. The introduction of early retirement awards them a pension transfer. Second, the existence in the early retirement provision of an element of policy persistence. In fact, by inducing low-ability workers to retire early, this provision creates its own future constituency, since it gives rise to an endogenous group of workers with incomplete working history.

The main contribution of the paper is to demonstrate that under these two conditions, a social security system with early retirement arises, and is sustained, as a politico-economic equilibrium outcome of a dynamic majoritarian voting game. The voting majority which supports early retirement is composed of elderly with incomplete working history and low-ability young, who expect to retire early. The size of the social security system is determined by a voting majority of all retirees and low

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<sup>1</sup>Gruber and Wise (1999) identify two features of this provision, which display a strong correlation with the departure of elderly workers from the labor force: the early (and normal) retirement age, and the tax burden which is imposed on the labor income of the workers, after the early retirement age has been reached. Blöndal and Scarpetta (1998) provide additional evidence.

ability young. Although several studies have analyzed the economic determinants of the early retirement decisions (see among others, Feldstein (1974), Boskin y Hurd (1978), Diamond y Mirrless (1978), Lazear (1979), Crawford y Lilien (1981)), to our knowledge this is the first attempt to provide a theoretical explanation of the introduction of (generous) early retirement provisions<sup>2</sup>.

Our theoretical findings relate quite closely to the labor market situation at the time of the introduction of these provisions. The adoption of early retirement came during the process of deindustrialization, at its early stages in many European countries, and only after the oil shocks in the US and Canada. In particular, as discussed in section 2, all countries in Gruber and Wise's (1999) sample, with the exception of France, Japan, and Spain, introduced a generous provision<sup>3</sup> immediately after the first large reduction in industrial employment. This is in line with our view that the appearance of a large number of redundant elderly workers was crucial to gather the initial political support in favor of early retirement.

We introduce a dynamically efficient overlapping generations economy with storage technology. Young individuals, who are heterogeneous in their working ability, decide when to retire. Their labor income is endogenously determined by their retirement decision and by their initial ability. Old age retirement is mandatory. The social security system consists of a PAYG scheme. Young workers contribute a fixed proportion of their labor income to the system, and the proceedings are divided lump sum among the retirees. There exists an early retirement provision. Workers who exit the labor market at an early stage, i.e., with

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<sup>2</sup>Gruber and Wise (1999) put forward two possible explanations. They suggest that, in some instances, early retirements were introduced to induce elderly workers to exit the labor force, and thus to create employment for young workers. Alternatively, these provisions were adopted to accommodate a pre-existing decrease in the labor force participation, and thus to provide a soft landing for the mass of elderly workers who were already out of the labor force, or unable to find a job.

<sup>3</sup>Actuarially fair early retirements had previously been introduced in Belgium (1957) and in the US (1961). We abstract from these provisions, which fail to provide strong incentives to retire early, and therefore cannot be held responsible for the decrease in the labor force participation of the elderly.

an incomplete working history, are awarded an early retirement pension. Individuals who retire at mandatory age receive the full pension.

The social security system is determined in a bidimensional majoritarian voting game played by young and old agents. Voters cast a ballot over the payroll tax rate, which finances the social security system, and over the existence of an early retirement provision, which entitles agents with incomplete working history to a full pension. This political game displays two important features. First, because of the bidimensionality of the issue space, a Nash equilibrium of this majoritarian voting game may not exist. And second, in absence of a commitment device which restricts future policies, a social security system may not be politically sustainable. In fact, young workers may refuse to transfer resources to current retirees, as they have no guarantee to be rewarded with a corresponding pension in their old age.

To overcome the former problem, we initially analyze the voting game in a static setting, in which current voters can commit to future policies. As Shepsle (1979), we introduce a set of institutional restrictions which reduces the game to an issue-by-issue voting game, and thus concentrate on structure induced equilibria. To deal with the latter feature, we then replace commitment with the idea of an implicit contract<sup>4</sup> among successive generations. We look for structure induced equilibrium outcomes of the voting game with commitment which can be sustained as subgame perfect equilibrium outcomes of the game without commitment. To summarize, we introduce a notion of stationary subgame perfect structure induced equilibrium which combines the concept of structure induced equilibrium, introduced by Shepsle (1979), with the intergenerational implicit contract idea, originally presented by Hammond (1975).

The paper proceeds as follows: Section 2 analyzes the industrial employment at the time of the adoption of early retirement. Section 3 presents the economic model and the social security system, while section 4 introduces the voting game, the political institutions, and our notion of

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<sup>4</sup>See Hammond (1975), and more recently Cooley and Soares (1999), Galasso (1999), and Boldrin and Rustichini (2000).

equilibrium. Section 5 characterizes the politico-economic equilibria, and section 6 concludes. All formal definitions of the institutional restrictions, and all proofs are in the appendix.

## 2 Timing of Adoption of Early Retirement Provisions

Generous early retirement pensions were initially awarded in Europe in the 60s, and at the beginning of the 70s, often as disability pensions to those elderly workers who had been adversely affected by the labor market conditions (as in the Netherlands (1967), in Germany (1969), in Sweden (1970) and in the UK (1972)). A second round of adoptions, or modifications of the existing programs, took place after the 1974 oil shock. Table 1 reports the dates of adoption of the early retirement provisions, and their main characteristics, in all countries in Gruber and Wise's (1999) sample.

Since the mid60s, most industrialized countries have undergone a deep deindustrialization process, which has provoked large sectoral shifts in employment out of the industry and into services (see figures 1 to 3). Most of the job destruction has typically been born by unexperienced young and low-educated elderly workers<sup>5</sup>, to an extent that has largely depended on the institutional features of the labor market. Did this deindustrialization process lead to the build-up of a large mass of redundant elderly workers? To provide an answer to this question, we examine the total employment in industry (and services) in the countries included in Gruber and Wise's sample<sup>6</sup> for the years which immediately preceded the adoption of early retirement. A comparison of the timing of adoption of early retirement and of the contemporaneous behavior of the industrial

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<sup>5</sup>For example, Bartel and Sicherman (1993) show that an unexpected positive technological shock, which affects the skills required to perform a certain task, leads to early exit from the labor market of elderly workers.

<sup>6</sup>We use OECD data on total industrial employment in Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Spain, Sweden, the UK and the US from 1960 to 1990.

employment<sup>7</sup> (see figures 1 to 3, and table 1) indicates that in all countries, but France, Japan, and Spain<sup>8</sup>, the introduction of generous early retirements provisions has followed the first significant drop in industrial employment since 1960.

Figure 1 shows the industrial employment dynamics in France, Germany, Italy and Sweden. In Germany and Italy, the introduction of early retirement took place in 1969, after a severe two-years long slump in industrial employment, which started respectively in the third quarter of 1966 and in forth quarter of 1964. Sweden adopted the provision in 1970, following a decline in industrial employment, from 1966 to 1968 by 3.1% annual. Figure 2 displays the industrial employment in Belgium, the Netherlands, and the UK. The Netherlands were among the first to institute the provision (through the Disability Act) in 1967, after a slight decrease in employment in 1966, and a contemporaneous large reduction in 1967. In the UK, the adoption of early retirement came in 1972, during the steady decline in industrial employment, which had started in 1966. Belgium introduced a form of early retirement, which required the substitution of an elderly workers with a young unemployed, in 1976, a year after the begin of the decreasing trend in industrial employment. Finally, figure 3 shows the employment scenario in Japan, Spain, the US, and Canada. In the US, the “recalculation effect,” which discouraged early retirement, was substantially reduced (see Ippolito (1990)) in 1977. These changes came after the 1974 oil shock, which led to a two year long recession<sup>9</sup>. Finally, Canada adopted an early retirement provision only in 1984 (Quebec) and in 1987 (the rest of Canada), following the

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<sup>7</sup>Ideally, one would like to analyze the employment by sex, age, education and sector, to identify the labor market situation of those agents, who are more likely to benefit from early retirement, typically low educated, elderly male in manufacturing. Unfortunately, these data are not available for the period under consideration.

<sup>8</sup>Why were these three countries different? In France, generous early retirement provisions were introduced already in 1963, after a large drop in employment in mining and in the iron industry, two highly unionized sectors. Japan (1973) has always enjoyed one of the least generous provision (see Gruber and Wise (1999)); while Spain adopted a generous provision in 1976, during its transition to democracy.

<sup>9</sup>For an analysis of the effects of the 1974 oil shock on job reallocation in the US industrial sector see Davis and Haltiwanger (1999).

1982-83 spectacular drop in industrial employment.

### 3 The Economic Environment

We consider a two period overlapping generations model with storage technology. Every period two generations are alive: Young and Old. Population grows at a constant rate,  $n > 0$ . Young agents determine the length of their working period. Old age retirement is mandatory, and thus old agents do not work. All consumption takes place in old age, and agents only value young age leisure and old age consumption.

Agents are assumed to be heterogeneous in their working ability. Abilities are distributed on the interval  $[\underline{x}, \bar{x}] \subset \mathfrak{R}_+$ , according to a cumulative distribution function  $F(\cdot)$ , which is assumed to have mean  $\mu_x$ , and to be skewed,  $F(\mu_x) > 1/2$ .

Young agents decide when to exit the labor market. They may decide to work during the entire working period, i.e., until they reach mandatory retirement age, or they may retire early. Pension transfers, to be paid for the remaining period of their life, are awarded to all agents who have worked at least until minimum retirement age,  $\Theta < 1$ . However, the amount of the pension transfer an agent receives may differ according to the length of her actual working period. Let  $p_t$  be the pension awarded at time  $t$  to an old agent who retired at mandatory age (we will refer to  $p_t$  as the full pension). And let  $\Gamma_{t+1}^t$  be the percentage of the full pension transfer awarded at time  $t + 1$  to an old agent born at time  $t$ , then

$$\Gamma_{t+1}^t(\phi_t) = \Gamma_t^t(\phi_t) = \begin{cases} 0 & \text{if } \phi_t < \Theta \\ \alpha & \text{if } \Theta \leq \phi_t < 1 \\ 1 & \text{if } \phi_t = 1 \end{cases} \quad (1)$$

where subscripts indicate the calendar time and superscripts the period in which the agent was born,  $\phi_t \in [0, 1]$  represents the proportion of the working period she spent working,  $\Theta$  is the minimum length of the working period, or, analogously, the minimum retirement age to be eligible for a pension, and  $\alpha$  is the proportion of the full pension transfer to be

paid to an agent who retires early. In words, agents who work less than a proportion  $\Theta$  of their working period receive no pensions; agents who retire early obtain a share  $\alpha$  of the full pension during the remaining of their youth and in their old age (respectively  $(1 - \phi_t) \alpha p_t$  and  $\alpha p_{t+1}$ ) whereas agents retiring at the mandatory retirement age receive the full pension in their old age.

A production function converts the duration of the working period,  $\phi$ , into the only consumption good, according to the worker's ability,  $x$ :

$$y(\phi, x) = \phi x \quad (2)$$

A storage technology transforms a unit of today's consumption into  $1 + r$  units of tomorrow's consumption:  $y_{t+1} = (1 + r) y_t$ . All private intertemporal transfers of resources into the future are assumed to take place through this technology. Additionally, we assume that  $r > n$ , and thus that the economy is dynamically efficient.

Young agents have to decide the length of their working period,  $\phi$ , that is, whether they will retire early or at mandatory age. They pay a proportional tax on their labor income, and save all their resources for old age consumption through the storage technology. Old agents take no relevant economic decision; they simply consume all their wealth. The intertemporal budget constraint of a type  $x$  agent born at time  $t$  is thus:

$$c_{t+1}^t = \left( \phi_t x (1 - \tau_t) + (1 - \phi_t) \Gamma_t^t(\phi_t) p_t \right) (1 + r) + \Gamma_{t+1}^t(\phi_t) p_{t+1} \quad (3)$$

where  $\tau_t$  is the payroll tax rate which finances the pensions at time  $t$ , and  $p_t$  and  $p_{t+1}$  are respectively the full pension at time  $t$  and  $t + 1$ .

Agents value leisure in their working period and old age consumption, according to a separable utility function:  $U(\phi_t, c_{t+1}^t) = l(\phi_t) + \beta u(c_{t+1}^t)$ , where  $\beta$  is the individual time discount factor. We interpret the utility that an agent attaches to leisure as the utility associated to the free time which becomes available after an early exit from the labor market, i.e., after early retirement. If an agent decides to work, in equilibrium she will either retire at the minimum retirement age or at

mandatory age<sup>10</sup>, due to the relation between the length of the working period and the associated proportion of the full pension (see eq. 1). Thus, leisure,  $1 - \phi$ , will only take two values: 0 and  $1 - \Theta$ , and we can normalize the corresponding utilities to:  $l(1) = 0$  and  $l(\Theta) = v$ . Additionally, to make sure that every agent has an incentive to work, and that no agent will retire early in absence of an early retirement provision, we assume that  $l(\phi) = v < \underline{x}(1 - \Theta)$ ,  $\forall \phi \in [0, \Theta]$ .

The utility function is assumed to be linear in consumption:  $u(c_{t+1}^t) = c_{t+1}^t$ . This guarantees the young age decision, i.e., the length of working period, to be affected by the tax rates (substitution effect), but not by the level of the transfers (income effect), which only influences old age consumption. This assumption, as discussed in the next section, allows us to find an equilibrium of the voting game on the social security tax rate, even though preferences may not be single peaked. The assumption that consumption only takes place in old age is not innocuous, since it disregards a relevant element for social security: the saving decision<sup>11</sup>. Finally, we assume that the individual discount factor is equal to the inverse of the real interest factor,  $\beta = 1/(1+r)$ , so that the young decision over the length of the working period does not depend on the exogenous interest rate.

To summarize, agents decide the length of their working period by maximizing  $U(\phi_t, c_{t+1}^t)$  with respect to  $\phi_t$  subject to the budget constraint at equation 3. The following lemma characterizes this economic decision. All proofs are in the technical appendix.

**Lemma 1** *For a given tax rate  $\tau_t$ , and given proportions  $\alpha_t, \alpha_{t+1}$  of*

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<sup>10</sup>This retirement behavior, which is induced by the shape of the function  $\Gamma(\cdot)$ , is consistent with the evidence reported by Gruber and Wise (1999).

<sup>11</sup>In fact, the existence of a PAYG system induces changes in the factor prices of labor and capital, thereby affecting the saving decisions of the agents. In particular, the introduction of a PAYG social security system, by reducing the capital stock, may increase the real interest rate, decrease the wage rate, and thus modify the net wealth of the agents. Our model abstracts from these considerations, which are analyzed in Cooley and Soares (1998), Galasso (1999), and Boldrin and Rustichini (2000). See also Feldstein (1974) for the impact of the early retirement provision on the individual saving decisions.

the full pensions  $p_t$  and  $p_{t+1}$ , the economic decision of the agents can be summarized as follows:

$$\phi_t^*(x) = \begin{cases} \Theta & \text{if } x \leq x_t^R \\ 1 & \text{if } x > x_t^R \end{cases} \quad (4)$$

where

$$x_t^R = \frac{(1 - \Theta) \alpha_t p_t - \frac{1 - \alpha_{t+1}}{1+r} p_{t+1} + v}{(1 - \tau_t)(1 - \Theta)} \quad (5)$$

In words,  $x_t^R$  represents the ability level of an agent who is indifferent between retiring early and at the mandatory retirement age, as shown at eq. 4. Clearly, those with ability levels below the threshold,  $x < x_t^R$ , retire early, and the others at mandatory age. The endogenous threshold ability,  $x_t^R$ , and therefore the mass of early retirees, depends positively on the agents' valuation of their leisure,  $v$ , on the generosity of the early retirement provision,  $\alpha_t$  and  $\alpha_{t+1}$ , and of today's pension  $p_t$ , and on the current tax burden,  $\tau_t$ , and negatively on the generosity of the future pension's,  $p_{t+1}$  (see eq. 5). At this point, we can obtain the gross labor income for a type  $x$  young agent:

$$y_t^*(x, \tau) = \begin{cases} \Theta x & \text{if } x \leq x^R \\ x & \text{if } x > x^R \end{cases} \quad (6)$$

### 3.1 The Social Security System

We consider a pay as you go (PAYG) social security system, in which workers contribute a fixed proportion of their labor income to the system, and the proceedings are divided among old age and early retirees. A retired person receives a lump sum pension which may depend on the length of her working period, but not on her labor income. The system is assumed to be balanced every period, so that the sum of all awarded pensions has to equal the sum of all received contributions.

Due to the combination of a proportional labor income tax and of a lump sum pension, the system entails an element of within cohorts

redistribution, from the rich to the poor. As in Tabellini (1990) and in Conde-Ruiz and Galasso (1999), this feature is crucial, because it induces low ability young to support the social security system<sup>12</sup>.

The full pension transfer which balances the budget constraint is equal to:

$$p_t = \frac{\overbrace{(1+n) \int \phi_t(x) x dF(x)}^{\text{Tax Base}}}{\underbrace{\int \Gamma_t^{t-1}(\phi_{t-1}(x)) dF(x)}_{\text{Old Age Retirees}} + \underbrace{(1+n) \int (1-\phi_t(x)) \Gamma_t^t(\phi_t(x)) dF(x)}_{\text{Early Retirees}}} \tau_t. \quad (7)$$

By substituting in eq. 7 the economic decision of the agents at Lemma 3.1, we obtain

$$p_t = \frac{\left[1 - (1 - \Theta) L_F \left(F \left(x^R\right)\right)\right] (1+n) \mu_x}{1 - (1 - \alpha_t) F \left(x_{-1}^R\right) + (1+n) (1 - \Theta) \alpha_t F \left(x^R\right)} \tau_t \quad (8)$$

where  $\mu_x = \int_{\underline{x}}^{\bar{x}} x dF(x)$  is the mean ability in the economy,  $F \left(x^R\right)$  is the proportion of young who decides to retire early, and  $L_F \left(F \left(x^R\right)\right) = \left(\int_{\underline{x}}^{x^R} x dF(x)\right) / \mu_x$  represents the proportion of total ability owned by the early retirees.

Although in this model  $x^R$ , and thus  $F \left(x^R\right)$ , the mass of early retirees, is endogenous, at the beginning of our economy, at  $t = 0$ , there may exist a mass of old people with incomplete working history, who have not matured any right to a pension transfer. This represents the initial condition of the economy:

**Definition 1** *We call  $\epsilon \in [0, 1]$  the mass of old individuals with incomplete working history in the initial period,  $t = 0$ , who had not matured any right to a pension transfer. We will refer to this mass,  $\epsilon$ , as the initial condition of the economy.*

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<sup>12</sup>Evidence in favor of the existence of this within cohort redistribution can be found in Boskin et al. (1987) and Galasso (2000).

To summarize, in every period, the social security system can be characterized by a quadruple: the exogenous minimum retirement age, the payroll tax rate, the full pension, and the percentage of the full pension awarded to the early retirees,  $(\Theta, \tau, p, \alpha)$ . To simplify the analysis, we assume that early retirees are either awarded the full pension or nothing at all,  $\alpha \in \{0, 1\}$ . Since  $\alpha$  is determined by all electors in the voting game, this assumption amounts to restrict the choice over  $\alpha$  to whether to introduce the institution of a *generous* early retirement (which would pay the full pension) or not. The other dimension of the voting game, the payroll tax rate,  $\tau$ , is unrestricted,  $\tau \in [0, 1]$ , and thus agents can choose their most preferred size of the system. The next lemma shows an important implication of our assumption.

**Lemma 2** *For a given  $\tau_t = \bar{\tau}$  and for  $\alpha_t = \bar{\alpha} \forall t$ , if  $\epsilon \neq F(x_0^R)$ , the sequence of full pensions which balances the social security budget constraint in every period is a constant sequence,  $p_t = \bar{p} \forall t$ , if and only if  $\alpha = 1$ .*

A direct consequence of this lemma is that if (and only if) early retirees are awarded the full pension,  $\alpha = 1$ , then for any initial condition of the economy,  $\epsilon$ , a stationary social security system ( $\tau_t = \bar{\tau}$ ,  $\alpha_t = \bar{\alpha}$  and  $p_t = \bar{p} \forall t$ ) exists for different values of the stationary tax rate,  $\bar{\tau}$ . Moreover, if  $\alpha = 1$ , for any initial condition of the economy,  $\epsilon$ , the endogenous mass of elderly with incomplete working history,  $F(x^R)$ , reaches its steady state value in one period.

The following expression describes the relation between the full pension and the tax rate in the case of early retirement:

$$p(\tau, 1) = \frac{[1 - (1 - \Theta) L_F(F(x^R))](1 + n) \mu_x}{1 + (1 + n)(1 - \Theta) F(x^R)} \tau. \quad (9)$$

A rise in the tax rate has a direct, positive impact on the pension transfer, and an indirect negative effect, since it induces more early retirements, and thus increases the dependency ratio. The exact magnitude of these effects depends on the shape of the ability distribution function through

the endogenous mass of early retirees,  $F(x^R)$ , and their relative ability level,  $L_F(F(x^R))$ , and can be summarized by the elasticity of the full pension transfer to the tax rate:

$$\eta_{p(\tau,1),\tau} = \frac{\partial p(\tau,1)}{\partial \tau} \frac{\tau}{p(\tau,1)}. \quad (10)$$

When there is no early retirement provision, i.e.,  $\alpha_t = \bar{\alpha} = 0$ , if the tax rates are constant,  $\tau_t = \bar{\tau} \forall t$ , and there exists an initial mass of elderly people with incomplete working history, i.e.,  $\epsilon > 0$ , then the pension sequence will be constant, except in the initial period, when it will be larger than its constant value, that is,  $p_0 > p_t = \bar{p} \forall t$ . In fact, at  $t = 0$ , due to the existence of a mass of old individuals, who are not entitled to a pension, total contributions are divided among fewer old than in future periods, when no agents will retire early, since  $\alpha = 0$ , and thus every old will receive an old age pension. For a given tax rate,  $\tau$ , the constant pension levels are

$$p(\tau,0) = \begin{cases} (1+n)\mu_x\tau & \text{for } t > 0 \\ (1+n)\mu_x\tau/\epsilon & \text{for } t = 0 \end{cases} \quad (11)$$

Notice that, in this case, a rise in the tax rate induces an unambiguous increase in the pension transfer.

### 3.2 The Economic Equilibrium

The following definition introduces the economic equilibrium, given the values of the social security systems, which will be determined in the political game.

**Definition 2** For a given sequence  $\{\tau_t, \alpha_t, p_t\}_{t=0}^{\infty}$ , an early retirement age,  $\Theta$ , an exogenous interest rate,  $r$ , and the function  $\Gamma(\phi)$  defined in eq. 1, an economic equilibrium is a sequence of allocations,  $\{(\phi_t(x), c_{t+1}^t(x))\}_{x \in [\underline{x}, \bar{x}]}$ ,  $t=0, \dots, \infty$ , such that:

- i. In every period agents solve the consumer problem, i.e. every young individual maximizes her utility function  $U(\phi_t, c_{t+1}^t)$  with respect to  $\phi_t$ , and subject to eq. 3;
- ii. The social security budget constraint is balanced every period, i.e. eq. 7 holds;
- iii. The good market clears in every period: for every  $t$

$$\begin{aligned} \int c_t^{t-1}(x) dF(x) &= (1+r) \int (1-\tau_{t-1}) \phi_{t-1}(x) x dF(x) + \\ &(1+r) p_{t-1} \int (1-\phi_{t-1}(x)) \Gamma_{t-1}^{t-1}(\phi_{t-1}(x)) dF(x) + \\ &(1+n) p_t \int \Gamma_t^{t-1}(\phi_{t-1}(x)) dF(x) \end{aligned}$$

The utility obtained in an economic equilibrium by a type  $x$  young agent and by a type  $x$  old agent is represented respectively by the following indirect utility functions:

$$v_t^y(\tau_t, \alpha_t, \tau_{t+1}, \alpha_{t+1}, x) = \max \{v_t^{NR}(x), v_t^{WR}(x)\} \quad (12)$$

$$v_t^o(\tau_t, \alpha_t, x) = K_t(x) (1+r) + \Gamma_t(\phi_{t-1}(x)) p_t \quad (13)$$

where

$$v_t^{NR}(\tau_t, \alpha_t, \tau_{t+1}, \alpha_{t+1}, x) = (1-\tau_t)x + \frac{p_{t+1}}{1+r} \quad (14)$$

and

$$v_t^{WR}(\tau_t, \alpha_t, \tau_{t+1}, \alpha_{t+1}, x) = v + \Theta(1-\tau_t)x + (1-\Theta)\alpha_t p_t + \frac{\alpha_{t+1} p_{t+1}}{1+r}. \quad (15)$$

$v_t^{NR}(\tau_t, \alpha_t, \tau_{t+1}, \alpha_{t+1}, x)$  and  $v_t^{WR}(\tau_t, \alpha_t, \tau_{t+1}, \alpha_{t+1}, x)$  represents respectively the utility of a type  $x$  young individual when she retires at mandatory age and when she retires early, and  $K_t(x)$  is a constant which does not depend on past or current values of the social security system<sup>13</sup>.

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<sup>13</sup>Specifically,  $K_t(x) = \begin{cases} (1-\tau_{t-1})x & \text{if } \phi_{t-1}(x) = 1 \\ \Theta(1-\tau_{t-1})x + (1-\Theta)\alpha_{t-1}p_{t-1} & \text{if } \phi_{t-1}(x) = \Theta \end{cases}$

## 4 The Voting Game

The size and the composition of the social security system are determined through a political process which aggregates agents' preferences over the payroll tax rate,  $\tau \in [0, 1]$ , and over the existence of early retirement,  $\alpha \in \{0, 1\}$ . We consider a political system of majoritarian voting. Elections take place every period. All persons alive, young and old, cast a ballot over  $\tau$  and  $\alpha$ . However, since every agent has zero mass, no individual vote could affect the outcome of the election. To overcome this problem, we assume sincere voting.

Two features of this majoritarian voting game are worth noticing. First, because of the bidimensionality of the issue space,  $(\tau, \alpha)$ , a Nash equilibrium of this majoritarian voting game may not exist. Second, if no commitment device is available to restrict future policies, why would young people agree to transfer resources to current retirees, given that there is no guarantee that this young-to-old transfer policy will be kept in the future? To deal with these two features of the game, we introduce a notion of equilibrium which combines the concept of structure induced equilibrium, due to Shepsle (1979), with the idea of intergenerational implicit contract, introduced by Hammond (1975).

To analyze the possible lack of Nash equilibria induced by the bidimensionality of the issue space, we first consider the case of full commitment. Voters choose a constant sequence of the parameters of the social security system  $(\tau, \alpha)$ . Thus, the voting game becomes a bidimensional game in which  $\tau$  and  $\alpha$  are determined once and for all. It is well known in the political science literature<sup>14</sup> that multidimensionality of the issue space generates Condorcet cycles, unless very restrictive assumptions over the distribution of the agents' preferences apply, and a median in all directions exists<sup>15</sup>. Following Shepsle (1979), we choose to introduce some institutional restrictions to the voting game and thus

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<sup>14</sup>See, for example, Ordeshook (1986).

<sup>15</sup>See Conde-Ruiz and Galasso (1999) for a graphic interpretation of the Condorcet cycles in the context of a voting game over the size and composition of the welfare state.

to concentrate on structure induced equilibria. These institutional restrictions, which are presented in section 4.1, effectively transform the bidimensional election into an issue by issue voting game, in which a (structure induced) equilibrium always exists.

If the assumption of commitment is dropped, would the young still be willing to vote for a positive social security level? If young agents expect their current vote not to have any impact on future policies, they will vote for a zero social security tax rate, or they will incur in a net cost. However, young agents may believe that their current voting decision will influence future voters. In this case, as initially suggested by Hammond (1975), an implicit contract may arise among successive generations of voters, and young workers may agree to vote a pension to the current old as they expect to be rewarded in their old age by a corresponding pension.

In section 4.2, the assumption of commitment of future policies is replaced by the use of an implicit contract among successive generations of voters. We define the voting game and the stationary strategy profiles, which may support structure induced equilibrium outcomes of the voting game with commitment as subgame perfect equilibrium outcomes of the voting game without commitment. Like Cooley and Soares (1999) and Galasso (1999), we concentrate on stationary strategy profiles. This is because we want to generalize the structure induced equilibrium outcome obtained in a static environment (the game with commitment at steady state) to a dynamic environment, the game without commitment. Clearly, non stationary profiles, as in Azariadis and Galasso (1997) and Boldrin and Rustichini (2000), would give rise to additional equilibrium outcomes, which, however, would not be structure induced equilibrium outcomes of the static game with commitment. We call our notion of equilibrium a stationary subgame perfect structure induced equilibrium (SSPSIE).

## 4.1 Structure Induced Equilibria

In this section, we consider a majoritarian voting game with commitment at steady state. Voters determine the constant sequence of the payroll tax rate,  $\tau \in [0, 1]$ , and the existence of an early retirement provision, which would pay the full pension,  $\alpha \in \{0, 1\}$ . At steady state, the initial mass of old individuals with incomplete working history,  $\epsilon$ , is equal to its endogenous steady state value,  $F(x^R)$ . Therefore, the sequence of pensions,  $p$ , is constant, and the game of commitment, at steady state, collapses to a static bidimensional voting game over  $\tau$  and  $\alpha$ . Individual preferences over the two issues are represented by the indirect utility functions at equations 12 and 13. All individuals are assumed to vote sincerely.

To guarantee the existence of an equilibrium of this voting game, we follow Shepsle (1979) in defining a set of institutional restrictions, which determine how the political system aggregates individual preferences over the alternatives into a political outcome. An institution is composed of a committee structure, a jurisdictional system, an assignment rule, and an amendment control rule<sup>16</sup>. In Shepsle (1979), this political arrangement is intended to capture the policy making process in a representative democracy. By applying these institutional restrictions to our voting game, we implicitly assume that the representatives' preferences perfectly reflect the voters' ones.

The institutional restrictions we adopt consist of: (i) a *Committee of the Whole*, i.e., there exists only one committee, which coincides with the entire electorate; (ii) *Simple Jurisdictions*, i.e., every jurisdiction represents one dimension (or issue) in the space of alternatives; (iii) an *Assignment Rule*, which assigns every simple jurisdiction to the committee of the whole; and (iv) a *Germaneness Amendment Control Rule*, which establishes that only amendments to proposals which belong to the jurisdiction of the committee are accepted<sup>17</sup>.

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<sup>16</sup>See the Appendix for the formal definitions.

<sup>17</sup>In other words, if the committee is using its jurisdiction to deliberate a proposal over  $\alpha$ , then only amendments over  $\alpha$  will be accepted and viceversa.

Therefore, in our political system, every jurisdiction is assigned to the entire electorate, i.e., the entire electorate is allowed to make proposal to modify any of the two dimension in the issue space  $(\tau, \alpha)$ ; however, only separately, i.e., *issue by issue*. Simple jurisdictions and germaneness guarantee that alternatives are on the floor only issue by issue. This implication of our legislative restrictions is crucial to eliminate possible Condorcet cycles and to obtain a (structure induced) equilibrium.

As Shepsle (1979) [Theorem 3.1] showed, a sufficient condition for  $(\tau^*, \alpha^*)$  to be an equilibrium of the voting game induced by the legislative structure explained above is that  $\tau^*$  represents the outcome of a majority voting over the jurisdiction  $\tau$ , when the other dimension is fixed at its level  $\alpha^*$ , and viceversa. This theorem suggests that the equilibrium outcome can be found by calculating the median voter in both dimensions. In our voting game with commitment, however, individual preferences over constant tax rates, for a given  $\alpha$ , need not be single peaked, and thus the median voter theorem may not apply. The following example will illustrate this point. Let  $\alpha = 1$ , i.e., early retirees receive full pension benefits, and consider a young individual who does not plan to retire early, and opposes a small increase in the tax rate. The same agent could nevertheless agree to a larger increase in  $\tau$ , which would induce her to retire early and therefore to pay the income tax for a smaller period of time, and to receive pension for a longer period.

Despite this problem, however, our economic environment induces an ordering of individual preferences over the tax rate,  $\tau$ , for a given  $\alpha \in \{0, 1\}$  which is consistent with the use of the median voter (see Lemma A.7 in the appendix). In fact,

**Lemma 3** *The Nash equilibrium outcome of the static majoritarian voting game with commitment over  $\tau$ , for a given  $\alpha$ , is the tax rate preferred by a young individual of ability type  $x_{m\tau} = F^{-1}\left(\frac{n}{2(1+n)}\right)$ .*

The intuition is that individual preferences display hierarchical adherence, as defined by Roberts (1977). Young voters can be ordered according to their individual ability, whereas old voters always prefer a

higher tax rate than the young. Therefore, the median voter theorem applies. An analogous lemma applies to the voting game over  $\alpha$ :

**Lemma 4** *The Nash equilibrium outcome of the static majoritarian voting game with commitment over  $\alpha$ , for a given  $\tau$ , is the tax rate preferred by a young individual of ability type  $x_{m\alpha} = F^{-1}\left(\frac{2+n-2F(x^R)}{2(1+n)}\right)$ .*

In this case, the early retirement provision is preferred by old with incomplete working history and low-ability young. The order of preferences over the two issues is described in figure 4, and it will be discussed in details in section 4.1.

## 4.2 Subgame Perfection and Stationary Subgame Perfect Structure Induced Equilibria

In this section, the assumption of commitment is dropped, and we consider stationary voting strategies which may induce an implicit contract among successive generations of voters. We define the voting game, and then formalize our concept of equilibrium: the stationary subgame perfect structure induced equilibrium.

The public history of the game at time  $t$ ,  $h_t = \{(\tau_0, \alpha_0), \dots, (\tau_{t-1}, \alpha_{t-1})\} \in H_t$ , is the sequence of social security tax rates and early retirement parameters until  $t - 1$ , where  $H_t$  is the set of all possible history at time  $t$ . An action for a type  $x$  young individual at time  $t$  is a pair of social security tax rate and early retirement parameter,  $a_{t,x}^y = (\tau, \alpha) \in [0, 1] \times \{0, 1\}$ . Analogously, an action for a type  $x$  old individual at time  $t$  is  $a_{t,x}^o = (\tau, \alpha) \in [0, 1] \times \{0, 1\}$ . Thus, at time  $t$  every voter chooses a pair  $(\tau, \alpha)$ . We identify with  $a_t$  the action profile of all individuals (young and old) at time  $t$ :  $a_t = (a_t^y \cup a_t^o)$  where  $a_t^y = \bigcup_{x \in [\underline{x}, \bar{x}]} a_{t,x}^y$  and  $a_t^o = \bigcup_{x \in [\underline{x}, \bar{x}]} a_{t,x}^o$ .

A strategy for a type  $x$  young individual at time  $t$  is a mapping from the history of the game into the action space:  $s_{t,x}^y : h_t \rightarrow [0, 1] \times \{0, 1\}$ . Analogously, a strategy for a type  $x$  old individual at time  $t$  is  $s_{t,x}^o :$

$h_t \rightarrow [0, 1] \times \{0, 1\}$ . We denote with  $s_t$  the strategy profile played by all individuals at time  $t$ , i.e.,  $s_t = (s_t^y \cup s_t^o)$  where  $s_t^y = \bigcup_{x \in [\underline{x}, \bar{x}]} s_{t,x}^y$  and  $s_t^o = \bigcup_{x \in [\underline{x}, \bar{x}]} s_{t,x}^o$ .

For a given action profile at time  $t$ ,  $a_t$ , let  $(\tau_t^m, \alpha_t^m)$  be respectively the median of the distribution of tax rates, and the median of the distribution of the early retirement parameters. We call  $(\tau_t^m, \alpha_t^m)$  the outcome function of the voting game at time  $t$ . Notice that this outcome function corresponds to the structure induced equilibrium outcome of the voting game with commitment at steady state, described in the previous section.

The history of the game is updated according to the outcome function; at time  $t + 1$ :  $h_{t+1} = \{(\tau_0, \alpha_0), \dots, (\tau_{t-1}, \alpha_{t-1}), (\tau_t^m, \alpha_t^m)\} \in H_{t+1}$ .

For a given sequence of action profiles,  $(a_0, \dots, a_t, a_{t+1}, \dots)$ , and their corresponding realizations,  $((\tau_0, \alpha_0), \dots, (\tau_t, \alpha_t), (\tau_{t+1}, \alpha_{t+1}), \dots)$ , the expected payoff function for a type  $x$  young individual at time  $t$  is  $v_t^y(\tau_t, \alpha_t, \tau_{t+1}, \alpha_{t+1}, x)$ , according to eq. 12, and for a type  $x$  old agent is  $v_t^o(\tau_t, \alpha_t, x)$ , according to eq. 13.

Let  $s_{t|\hat{x}}^y = s_t^y / s_{t,\hat{x}}^y$  be the strategy profile at time  $t$  for all the young individuals except for the type  $\hat{x}$  young individual, and let  $s_{t|\hat{x}}^o = s_t^o / s_{t,\hat{x}}^o$  be the strategy profile at time  $t$  for all the old individuals except for the type  $\hat{x}$  old individual. At time  $t$ , the type  $\hat{x}$  young individual maximizes the following function:

$$V_{t,\hat{x}}^y \left( s_o, \dots, \left( s_{t|\hat{x}}^y, s_{t,\hat{x}}^y \right), s_t^o, s_{t+1}, \dots \right) = v_t^y \left( \tau_t^m, \alpha_t^m, \tau_{t+1}^m, \alpha_{t+1}^m, \hat{x} \right)$$

and a type  $\hat{x}$  old individual, at time  $t$ , maximizes the following function

$$V_{t,\hat{x}}^o \left( s_o, \dots, \left( s_{t|\hat{x}}^o, s_{t,\hat{x}}^o \right), s_t^y, s_{t+1}, \dots \right) = v_t^o \left( \tau_t^m, \alpha_t^m, \hat{x} \right)$$

where, according to our previous definition of the outcome function,  $(\tau_t^m, \alpha_t^m)$  and  $(\tau_{t+1}^m, \alpha_{t+1}^m)$  are, respectively, the median among the actions over the two parameters of the social security system played at time  $t$  and  $t + 1$ .

As previously discussed, our concept of equilibrium combines subgame perfection, and specifically the use of stationary strategies to support an intergenerational implicit contract, with the notion of structure induced equilibrium needed to overcome the bidimensionality problem. We thus define a stationary subgame perfect structure induced equilibrium of the voting game as follows:

**Definition 3 (SSPSIE)** *A voting strategy profile  $s = \{(s_t^y \cup s_t^o)\}_{t=0}^\infty$  is a Stationary Subgame Perfect Structure Induced Equilibrium (SSPSIE) if the following conditions are satisfied:*

- (i)  $s$  is a subgame perfect equilibrium.
- (ii) At every time  $t$ , the equilibrium outcome associated to  $s$  is a Structure Induced Equilibrium of the static game with commitment.
- (iii) In any period and for any history,  $h_t \in H_t$ , the sequence of equilibrium outcomes induced by  $s$  is constant.

## 5 Politico-Economic Equilibria

In this section, we apply the notion of stationary subgame perfect structure induced equilibrium (SSPSIE) to the voting game which determines the size and the composition of the social security system,  $(\tau, \alpha)$ . First, the structure induced equilibrium (SIE) outcomes of the static voting game with commitment are analyzed. We look at the SIE outcomes at steady state, and thus, as in Galasso (1999), we impose that the initial condition of the economy,  $\epsilon$ , is equal to its steady state value,  $F(x^R)$ . We, then, relax this assumption, and study the SSPSIE outcomes of the game with no commitment. In this dynamic environment, the initial condition,  $\epsilon$ , i.e. the initial mass of old agents with incomplete working history, becomes a crucial element to determine the introduction of the early retirement provision.

## 5.1 The Voting Game with Commitment

The institutional restrictions introduced in section 4.1 reduce the bidimensional voting game over  $(\tau, \alpha)$  to an issue by issue election. To obtain the equilibrium outcome of this election, we first need to calculate every elector's ideal point over the social security tax rate for every value of the early retirement parameter; and then over the early retirement parameter for every value of the tax rate. For every  $\alpha$ , we identify the median ideal  $\tau$ ; and for every  $\tau$ , we identify the median ideal  $\alpha$ . The points in which these two median functions cross represent structure induced equilibrium outcomes of the voting game (by Shepsle (1979), Thm 4.1). Notice that, since  $\alpha$  can only take two values,  $\alpha \in \{0, 1\}$ , we only have to calculate the median voter's (over the dimension  $\tau$ ) most preferred tax rate with and without early retirement, respectively  $\tau^{WR}$  and  $\tau^{NR}$ , and then to evaluate the median voter's (over  $\alpha$ ) indirect utility at  $\alpha = 0$  and  $\alpha = 1$  for the corresponding tax rates,  $\tau^{NR}$  and  $\tau^{WR}$ .

When voting on the tax rate, old agents always choose a higher tax rate<sup>18</sup> than any young, since they receive a lump sum transfer, and, unlike the young, they are no longer required to contribute to the system. In particular, they choose the tax rate that maximizes their pension transfer:  $\tau^o(\alpha) \in \arg \max_{\tau \in [0,1]} p_t(\tau, \alpha)$ .

Among the young, low ability types prefer a higher tax rate than high ability ones, because of the within-cohort redistribution component of the system. Agents can be ordered over this issue as shown in figure 4 (top panel), and the median voter over  $\tau$  will be a young individual with ability  $x_{m\tau} = F^{-1}(n/2(1+n))$ , as in Lemma 4.1.

If there is no early retirement provision,  $\alpha = 0$ , the median voter's optimization problem over  $\tau$  is linear, since an increase in the tax rate entails no distortion. Therefore, the median voter most preferred tax

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<sup>18</sup>Notice that, since we analyze SIE in which the economy starts at steady state, if  $\alpha = 0$ , then  $\epsilon = 0$ , i.e., there are no old agents with incomplete history; whereas if  $\alpha = 1$ , then  $\epsilon = F(x^R)$ . In both cases, every old agent votes for the same tax rate.

rate,  $\tau^{NR}$ , is the following:

$$\tau^{NR} = \begin{cases} 0 & \text{if } x_{m\tau} > \frac{1+n}{1+r}\mu_x \\ 1 & \text{if } x_{m\tau} \leq \frac{1+n}{1+r}\mu_x \end{cases} \quad (16)$$

where  $\frac{1+n}{1+r}$  can be interpreted as the relative performance of the social security system with respect to the other available saving technology. In words, if the median voter has a low enough ability level (as compared to the average ability), she will support the largest feasible system, whereas she will oppose any system if her ability level is large enough.

If the early retirement provision exists,  $\alpha = 1$ , the median voter may either decide to retire early or at mandatory age. If she decides to retire early, her most preferred tax rate,  $\tau^{WR}$ , is implicitly defined by the following expression:

$$\Delta^{WR}\eta_{p(\tau,1),\tau} = 1 \quad (17)$$

where  $\eta_{p(\tau,1),\tau}$  is the elasticity of the pension to the tax rate, as defined in eq. 10, and

$$\Delta^{WR} = \frac{p(\tau,1)\left(1 - \Theta + \frac{1}{1+r}\right)}{\tau\Theta x_{m\tau}} \quad (18)$$

represents the ratio of lifetime discounted pension transfers to lifetime contributions for the median voter. The median voter determines the tax rate by equating the marginal disutility from the income tax to the marginal utility from the increase in the pension, where the elasticity measures both the direct positive effect, and the indirect negative effects of an increase in the tax rate over the pension.

If the median voter decides to work during her entire working period, her most preferred tax rate,  $\tau^R$ , is defined by the following expressions:

$$\Delta^R\eta_{p(\tau,1),\tau} = 1 \quad (19)$$

$$\text{where } \Delta^R = \frac{p(\tau,1)/(1+r)}{\tau x_{m\tau}} \quad (20)$$

which have an analogous interpretation as equations 17 and 18. Clearly, a necessary condition for  $\tau^{WR}$  and  $\tau^R$  to be positive is respectively that

$\Delta^{WR} > 1$  and  $\Delta^R > 1$ , i.e., that the net present value from social security has to be positive.

Finally, to determine her vote when  $\alpha = 1$ , the median voter has to compare the utility associated with voting  $\tau^{WR}$  and retiring early to the utility associated to voting  $\tau^R$  and retiring at mandatory age. For a given  $\alpha$ , the median voter's most preferred tax rate can be summarized as follows:

$$\tau_{x_{m\tau}}(\alpha) = \begin{cases} \tau^{WR} & \text{if } \alpha = 1 \text{ and } v^{WR}(\tau^{WR}, 1, x_{m\tau}) \geq v^{NR}(\tau^R, 1, x_{m\tau}) \\ \tau^R & \text{if } \alpha = 1 \text{ and } v^{WR}(\tau^{WR}, 1, x_{m\tau}) < v^{NR}(\tau^R, 1, x_{m\tau}) \\ \tau^{NR} & \text{if } \alpha = 0 \end{cases} \quad (21)$$

where, for constant sequences  $(\tau, \alpha)$ ,  $v^j(\tau, \alpha, x)$  with  $j = NR, WR$ , identifies the indirect utility function  $v_t^j(\tau_t, \alpha_t, \tau_{t+1}, \alpha_{t+1}, x)$  at equations 14 and 15.

Who is willing to support an early retirement provision? In every period, there may exist a mass of old agents with incomplete working history, who have not matured any right to an old age pension<sup>19</sup>. These agents clearly support the existence of early retirement,  $\alpha = 1$ . On the other hand, old individuals with complete working history oppose this provision, which would reduce their full (old age) pension. Among the young, low ability types would take advantage of the early retirement provision, and therefore they will support its institution, whereas high ability types will oppose it. Agents can thus be ordered over this issue as shown in figure 4 (bottom panel). By Lemma 4.2, the median voter over  $\alpha$  will be a young individual with ability:

$$x_{m\alpha} = F^{-1} \left( \frac{2 + n - 2F(x^R)}{2(1 + n)} \right) \quad (22)$$

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<sup>19</sup>At the beginning of the economy,  $t = 0$ , this mass is exogenous, and represents the initial condition of the economy,  $\epsilon \in [0, 1]$ . In the following periods,  $t = 1, 2, \dots$ , the fraction of young individuals,  $F(x^R)$ , who decide to exit the labor market with an incomplete working history is endogenous, and depends on the equilibrium outcome of the voting game. In this section, we assume to start off our economy at steady state, and thus we impose that  $\epsilon = F(x^R)$ .

To determine the median voter's ideal  $\alpha$ , for a given  $\tau$ , it is useful to identify a key threshold for the young ability level. For a given tax rate  $\tau$ , let  $\hat{x}(\tau)$  be the ability level of the young agent who is indifferent between voting for a social security system of size  $\tau$  with early retirement,  $(\tau, 1)$ , or without it,  $(\tau, 0)$ :

$$\hat{x}(\tau) \text{ s.t. } v^{WR}(\tau, 1, \hat{x}(\tau)) = v^{NR}(\tau, 0, \hat{x}(\tau)). \quad (23)$$

Clearly, agents with  $x < \hat{x}(\tau)$  will vote for a system with early retirement,  $(\tau, 1)$ , and viceversa<sup>20</sup>.

The median voter's most preferred  $\alpha$ , for a given  $\tau$ , can thus be summarized as follows:

$$\alpha_{x_{m\alpha}}(\tau) = \begin{cases} 1 & \text{if } x_{m\alpha} \leq \hat{x}(\tau) \\ 0 & \text{if } x_{m\alpha} > \hat{x}(\tau) \end{cases} \quad (24)$$

Additionally, for a given tax rate  $\tau$ , let  $\tilde{x}(\tau)$  be the ability level of the young agent who is indifferent between a social security of size  $\tau$  with early retirement,  $(\tau, 1)$ , in which case she would retire early, and no social security:

$$\tilde{x}(\tau) \text{ s.t. } v^{WR}(\tau, 1, \tilde{x}(\tau)) = v^{NR}(0, 0, \tilde{x}(\tau)) = \tilde{x}(\tau). \quad (25)$$

Agents with  $x < \tilde{x}(\tau)$  prefer a social security system with early retirement,  $(\tau, 1)$ , and viceversa. Finally, let define a threshold for the utility from leisure,  $v^*$ , as follows:

$$v^* = (1 - \Theta) \frac{1 + n}{1 + r} \mu_x \left[ 1 - \frac{\Theta(r - n)}{1 + (1 + n)(1 - \Theta)} \right]. \quad (26)$$

The next proposition is an application of Shepsle's (1979) main result to our voting game with commitment at steady state. A structure

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<sup>20</sup>Notice that the threshold ability which makes an agent indifferent between voting a system with or without early retirement is lower than the threshold ability which makes an agent indifferent between retiring early or at mandatory age,  $\hat{x}(\tau) < x^R(\tau)$ . This is because the individual economic choice over the time of retirement does not generate any distortion over the social security system whereas the political voting decision over whether or not to institute early retirement does.

induced equilibrium with (or without) early retirement provision exists if and only if the reaction functions of the median voters over  $\alpha$  and  $\tau$  cross at  $\alpha = 1$  ( $\alpha = 0$ ).

**Proposition 5** *In the voting game with commitment, at steady state: (i) for  $x_{m\tau} \in \left[\underline{x}, \frac{1+n}{1+r}\mu_x\right]$ , there exists a SIE with outcome  $(\tau^{NR} = 1, 0)$  if and only if  $v \leq v^*$ , and there exists a SIE with outcome  $(\tau^{WR}, 1)$  if and only if  $x_{m\alpha} \leq \hat{x}(\tau^{WR})$ ; (ii) for  $x_{m\tau} \in \left(\frac{1+n}{1+r}\mu_x, \bar{x}\right]$ , there always exists a SIE with outcome  $(\tau^{NR} = 0, 0)$ , and there exists a SIE with outcome  $(\tau^{WR}, 1)$  if and only if  $x_{m\alpha} \leq \hat{x}(\tau^{WR})$  and  $x_{m\tau} \leq \tilde{x}(\tau^{WR})$ .*

When the ability level of the median voter  $x_{m\tau}$  is sufficiently low to guarantee her a positive net present value from “investing” in social security (with no early retirement), a social security system, with or without early retirement provision, arises as a SIE outcome of the voting game, provided that the utility from leisure in the case of early exit from the labor market is not too large ( $v \leq v^*$ ). For higher ability levels,  $x_{m\tau} > \frac{1+n}{1+r}\mu_x$ , social security may only exist together with the early retirement provision. Notice that, for some parameter values, the voting game with commitment displays multiple equilibria, and two SIE outcomes,  $(\tau^{WR}, 1)$  and  $(\tau^{NR}, 0)$ , may arise.

This proposition suggests that the introduction of early retirement hinges on the median voter over  $\alpha$  being a low ability type,  $x_{m\alpha} \leq \hat{x}(\tau^{WR})$ , which in turn requires the mass of elderly with incomplete working history to be large in equilibrium. In other words, the institution of early retirement provision depends crucially on the incentives they provide to low-ability workers to retire early, i.e., on their *generosity*.

Notice that, since we concentrate on SIE outcomes at steady state, this proposition is formulated in terms of  $F(x^R)$ , rather than of  $\epsilon$ . In fact, we force  $\epsilon$  to be equal to  $F(x^R)$ . The initial condition,  $\epsilon$ , will, however, be crucial in the proposition which analyzes the (stationary subgame perfect structure induced) equilibrium outcomes of the game with no commitment.

## 5.2 The Voting Game without Commitment

We now turn to the analysis of the stationary subgame perfect structure induced equilibria. The idea is to generalize the results obtained in proposition 5.1 for the game with commitment at steady state to a game without commitment, in which the initial condition of the economy,  $\epsilon$ , could differ from its steady state, endogenous value,  $F(x^R)$ . Moving from a game with commitment to a game in which commitment is replaced by the idea of implicit contract does not affect the steady state analysis. The initial period,  $t = 0$ , need, on the other hand, to be studied. In fact, even if the conditions for a SIE with early retirement are satisfied at steady state (Prop 4.1), there may not be enough old with incomplete working history,  $\epsilon$ , to support early retirement at  $t = 0$ . In this case, the equilibrium outcome of the game with commitment would not carry over to the game without commitment. The next proposition identifies sufficient conditions for SIE of the voting game with commitment to be SSPSIE outcomes of the voting game without commitment.

**Proposition 6** *(i) Every pair  $(\tau^{WR}, 1)$  which constitutes an outcome of a SIE of the static voting game with commitment is a SSPSIE outcome of the voting game without commitment, if*

$$\epsilon \geq \min \left\{ \frac{2+n}{2} - (1+n)F(\hat{x}(\tau^{WR})), \frac{2+n}{2} - (1+n)F(\tilde{x}(\tau^{WR})) \right\}$$

*(ii) Every pair  $(\tau^{NR}, 0)$  which constitutes an outcome of a SIE of the static voting game with commitment is a SSPSIE outcome of the voting game without commitment.*

For a SIE outcome of the voting game with commitment to be a SSPSIE outcome of the game without commitment, we need to specify a voting strategy profile, or implicit contract, which supports this outcome, and represents a subgame perfect equilibrium of the game without commitment. A formal description of an equilibrium strategy profile is in the appendix. The proposition above quantifies the initial condition  $\epsilon$ , which is needed for a social security system with early retirement,  $(\tau^{WR}, 1)$ , to

be initially introduced. Specifically, at  $t = 0$ , there have to be enough old agents with incomplete working history,  $\epsilon$ , to guarantee that the ability of the initial median voter over  $\alpha$  is sufficiently small to induce her to prefer a social security system with early retirement  $(\tau^{WR}, 1)$  to one without it  $(\tau^{WR}, 0)$ , and to no social security at all  $(0, 0)$ . After one period, the endogenously determined mass of early retirees jumps to its steady state level, and the conditions for  $(\tau^{WR}, 1)$  to be a SIE outcome are sufficient to guarantee that  $(\tau^{WR}, 1)$  will be sustained in the game without commitment as well.

Notice that, as shown in proposition 5.1, for some range of parameters, the voting game with commitment displays multiple equilibria, and two SIE outcomes,  $(\tau^{WR}, 1)$  and  $(\tau^{NR}, 0)$ , may arise. In these cases, the initial condition of the economy,  $\epsilon$ , could be used to rule out  $(\tau^{WR}, 1)$  as a SSPSIE outcome of the game, according to condition (i) in proposition 5.2. This situation has an interesting interpretation. Consider two economies that have the same structure, i.e. same ability distribution function, population growth rate, and rate of return, but that differ in their initial stock of old agent with incomplete working history. Then, the economy with a large  $\epsilon$  could adopt the early retirement provision, whereas the other would implement a social security system with no early retirement. This example underlines the importance of the initial mass of elderly who exited the labor market with incomplete history in the introduction of the early retirement institution.

The next corollary contains the main result of the paper. It shows the sufficient conditions for a social security system with early retirement,  $(\tau^{WR}, 1)$ , to be sustained a SSPSIE outcome in a game without commitment.

**Corollary 7** *If (A)*

$\epsilon \geq \min \left\{ \frac{2+n}{2} - (1+n)F(\hat{x}(\tau^{WR})), \frac{2+n}{2} - (1+n)F(\tilde{x}(\tau^{WR})) \right\},$   
*(B)  $F(x^{WR}) \geq \frac{2+n}{2} - (1+n)F(\hat{x}(\tau^{WR}))$ , and (C)  $\frac{n}{2(1+n)} \leq F(\tilde{x}(\tau^{WR}))$ , there exists a SSPSIE in which the outcome is a constant sequence  $(\tau, \alpha) = (\tau^{WR}, 1)$ .*

Condition (A) guarantees that, at  $t = 0$ , there exists a large enough initial mass of old people with incomplete working history to politically sustain the introduction of the early retirement provision. Condition (B) guarantees that enough young agents will choose to retire early, once early retirement has been introduced, and will therefore be willing to support this institution. Condition (C) is only required in the extreme case in which  $x_{m\tau} < \frac{1+n}{1+r}\mu_x$ , that is, when a social security system with no early retirement provision would not be supported. It guarantees that enough old and low-ability young individuals support a social security system (with an early retirement provision) of size  $\tau^{WR}$ . These conditions depend on the shape of the ability distribution function. Ceteris paribus, a more unequal ability distribution leads to more early retirements, and thus makes it easier to sustain the system.

### 5.3 Discussion

The first two conditions in corollary 5.3 have a very appealing economic interpretation. The former suggests that, in order for an early retirement provision to be initially introduced, a large number of elderly individuals with incomplete working history has to be redundant. We believe that this represents a fair description of the dominant scenario in many industrialized countries at the time of the initial adoption of the early retirement provisions. Since the late 60s, in fact, these countries have experienced a process of deindustrialization, which has provoked large sectoral shifts in employment (see figures 1 to 3). The timing and the magnitude of this process have largely differed across countries, possibly in response to differences in the initial mix of productions and in the labor market institutions (as trade unions density, existing labor market protections, etc.). However, in section 2 we showed that in most countries the adoption of early retirement occurred after an initial large reduction in total industrial employment, and that this decreasing trend has continued over the years. Since low ability elderly workers were among the most affected by the reduction in employment, these data are in line with our view that the early retirement provisions were introduced to provide

a pathway to accommodate the redundant, low ability, elderly workers out of the labor force and into retirement.

The latter condition suggests that the sustainability of the early retirement provision requires this institution to induce a large number of early retirements among the future generations. This result is related to the recent literature on policy persistence. As in Coate and Morris (1999), in our politico-economic equilibrium (with early retirement), the introduction of the policy, i.e., the institution of early retirement, induces the low ability young agents to undertake certain actions to benefit from the policy. These actions, notable the use of the early retirement provision, are crucial to create a new (endogenous) group of elderly with incomplete working history, and thus to guarantee the future sustainability of the policy. The institution of early retirement creates its own future constituency by inducing people to retire early.

Condition (B) also suggests that the early retirement provision induces a large proportion of workers – mainly low educated ones – to retire early. In fact, over the last two decades, most of the large share of early retirees have been low ability workers. Table 2 shows that, for male workers aged from 55 to 64, retirements are lower among college educated people, and reach the highest level among individual with less than primary education. This pattern is shared by several countries, with particularly large share of low-ability early retirees in Belgium, France and Italy.

## 5.4 Extensions

In our model, the political decisions over the social security system are divided into two jurisdictions,  $(\alpha, \tau)$ , and the third variable which defines a social security system, the full pension,  $p$ , is residually determined to balance the budget constraint. We refer to this political system as a  $\tau$ -legislature. Clearly, we could have analyzed a  $p$ -legislature, in which  $\alpha$  and  $p$  were directly determined through the political process, and  $\tau$  was residually obtained through the budget constraint. This modeling choice is not innocuous. In fact, in the  $\tau$ -legislature we use, the introduction of

an early retirement provision affects all individuals through a reduction in the full pension; whereas in a  $p$ -legislature, the cost of introducing this institution is entirely beard by the workers, through an increase in the tax rate. As a result, in a  $p$ -legislature, the old with complete working history are unaffected by the institution of early retirement; moreover, the cost of the early retirement provision is contemporaneous to its introduction, as the young are immediately required to pay higher taxes.

We choose to concentrate on a  $\tau$ -legislature in order to have a model in which the results would not heavily rely on the existence of an indifference relation over a relevant set of alternatives for a large set of voters, i.e., the old age retirees' decisions over the early retirement institution. Additionally, a  $\tau$ -legislature is able to account for the political relevance of an initial mass of elderly individuals in the introduction of the early retirement provision, which we identified as a crucial component of the establishment of this institution.

## 6 Conclusions

Generous early retirement provisions exacerbate the financial distress of current unfunded social security systems by increasing the dependency ratio. In fact, by inducing early exits from the labor market, these provisions reduce the number of workers, and thus of contributors to the social security system, while increasing the number of retirees, and thus of recipients from the system.

In a simple model which reproduces these characteristics, we analyzed the political determinants which may lead to the adoption of early retirement. The main message of this paper is that the initial introduction and the long run sustainability of early retirement provisions requires a large initial shock and some degree of policy persistence. Specifically, the initial adoption of this institution relies heavily on the existence of an initial stock of elderly people who exited the labor market with an incomplete working history, and who, therefore, are not entitled to an old age pension. The adoption of early retirement awards them a pension.

The long run political sustainability of this institution is based on the existence of a large number of (low-ability) workers, who, after the early retirement institution has been introduced, and thanks to the incentives it produces, decide to benefit from this provision, and retire early.

We relate the existence of an initial mass of elderly workers with incomplete working history to the initial stages of the deindustrialization process. In eight of the eleven countries analyzed by Gruber and Wise (1999), early retirement provisions were adopted immediately after the first severe reduction in industrial employment since 1960. This large drop in employment was mainly born by unexperienced young and low-ability elderly workers. Did early retirement represent a measure to reduce unemployment among the young, through a direct substitution of elderly workers with young unemployed? We believe not, since the large increase in youth unemployment came only later, in the mid70s; although we think that later modifications of the provisions may have been intended to serve this purpose.

In our view, early retirement provisions, often by mean of disability schemes, were meant to provide financial support to those middle aged, low ability workers who became redundant before having matured the right to an old age pension.

An complementary argument can be found in Caballero and Hammour (1998 and 1999). They argue that after the vigorous growth of the 1950s and 1960s, which fueled increasing profits rates, in the late sixties Europe experienced a period of tensions and strikes. This was due to the action of the labor movements, which tried to increase the share of the production appropriated by the labor factor. In this context, early retirement provisions may be seen as one of the instruments of redistribution from capital to labor.

We believe that the relevance of the institutional push in favor of the labor factor has to be combined with the existence of redundant elderly workers to explain the adoption of early retirement. In our view, the institutional push identified by Caballero and Hammour (1998 and 1999) helped to build up the political momentum for the introduction of early retirement, which then took place when the deindustrialization process

had contributed to made enough low-ability elderly workers redundant. This institutional differences may help to explain why most European countries immediately responded to reductions in industrial employment by instituting early retirement, whereas in the US and Canada, the adoption of a generous provision came only after the oil shocks.

Our model suggests that the long run political sustainability of the early retirement provision is due to its persistence. By creating strong incentives for the current low-ability young workers to retire early in the future, early retirement creates its own future political constituency. Does this imply that we will never get rid of this provision? We believe not. In a companion paper (Conde-Ruiz and Galasso (2000)), we show that as the population becomes older, and the dependency ratio exceeds a certain threshold, early retirement will eventually lose its political sustainability and be abandoned.

Finally, our model contradicts a well established result, based on unidimensional voting models (see Meltzer and Richard (1981)), that more unequal societies adopt larger redistributive systems. In our bidimensional voting model, this implication may break down. A more unequal economy with a large initial mass of elderly with incomplete working history may introduce early retirement and have a lower tax rate than a less unequal economy, which has less initial elderly with incomplete working history, and therefore does not adopt the provision.

## Table 1: Early Retirement

	Year of Adoption
France	1963, 1972 – 78(UB,YE), 1983
Netherlands	1967(DT), 1976(UB,FP)
Italy	1969, 197?(DT)
Germany	1969(DT), 1972(UB), 1984(YE)
Sweden	1970(DT), 1971(FP), 1976
UK	1972(DT), 1977(YE), 1981(UB)
Japan	1973
Belgium	1957(AF), 1976(YE)
Spain	1967(AF), 1976
US	1961(AF), 1977, 70s(FP)
Canada	1984(only in Quebec), 1987

Programs Characteristics: AF = actuarially fair; YE = pension to an elder worker in exchange for the employment of a young worker; DT = Disability Transfers awarded to elderly workers according to labor market conditions; UB = Unemployment Benefits as a bridge program toward old age pensions; Firms' Pension Plans (FP).

Sources: Gruber and Wise (1999), OECD Employment Outlook 1992.

\* Disability Insurance Act; \*\* Home Responsibility Protection

## Table 2

Share of Retirees among Male Workers aged 55-64 by Level of Education in 1995

	No further Education	Vocational Education	Third Level Education
Belgium	53.4%	57.6%	36.9%
France	51.1%	47.6%	28.9%
Italy	44.7%	47.4%	22.2%
Netherlands	56.8%	48.2%	40.8%
UK	24.1%	20.6%	21.4%
Germany	29.2%	28.5%	21.6%
Spain	24.9%	26.9%	21.6%

Source: Blöndal and Scarpetta (1998)

# A Appendix

## A.1 Structure Induced Equilibrium: Definitions

*The Political System:* Our political system describes a decision-making institution which has  $1 + 1/(1 + n)$  members, which form the electorate,  $E$ . The space of alternatives is a compact subset of  $\mathfrak{R}^2$ :  $(\tau, \alpha)$  s.t.  $\tau \leq 1$  and  $\alpha \in \{0, 1\}$ . Institutional arrangements differ along three dimensions: (a) committee structure; (b) jurisdiction structure; and (c) amendment structure. The first two structures follow from the definitions below.

**Definition 4 (Committee)** *The family of sets  $C = \{C_j\}$  is a committee system if it covers the entire electorate  $E$ . Then the committee  $C = \{E\}$  is the Committee of the Whole.*

**Definition 5 (Jurisdiction)** *Let  $B = \{b_1, b_2\}$  be the orthogonal basis for  $R^2$  where  $b_i$  is the unit vector for the  $i$ -th dimension. The family of set  $J = \{J_k\}$  is a jurisdictional arrangement if it covers  $B$ . Then  $J = \{\{b_1\}, \{b_2\}\}$  is a Simple Jurisdiction.*

Additionally, call  $f$  the function which associate a jurisdiction with a committee,  $f : C \rightarrow J_k$ . In our system  $f : E \rightarrow \{\{b_1\}, \{b_2\}\}$ , that is  $f^{-1}(b_1) = f^{-1}(b_2) = E$ .

To define an amendment structure we need to introduce the notions of status quo,  $x^o$ , and of proposal. A status quo,  $x^o$ , represents the previous agreed level on both dimensions of the issue space. For example, at time  $t$ ,  $\{x_1^o, x_2^o\} = \{\tau_{t-1}, \alpha_{t-1}\}$ .

**Definition 6 (Proposal)** *A proposal,  $x$ , is a change in  $x^o$  restricted to a single jurisdiction. The set of proposal available to the committee of the whole is*

$$g(E) = \{x \mid x = x^o + \lambda_i b_i, b_i \in f(E)\} \subseteq R^2 \text{ with } \lambda_i \in \mathfrak{R} \forall i.$$

**Definition 7 (Amendment Control Rule)** For any proposal  $x \in g(E)$ , the set  $M(x) \subseteq \mathfrak{R}^2$  consists of the modifications  $E$  may make in  $x$ .  $M(x)$  is said to be an amendment control rule. An amendment control rule is a Germaneness rule if  $M(x) = \{x' \mid x'_i = x_i^o \text{ if } x_i = x_i^o\}$ .

**Definition 8 (Vulnerability)** In our political system, the status quo,  $x^o$ , is vulnerable if there exists a proposal,  $x$ , and an amendment,  $x'$ , such that  $x \in g(E) \cap C(x, x^o)$  and  $x' \in C(x', x) \cap C(x', x^o)$ .

Where  $C(x, y)$  is the collective choice function, which in our political system is represented by the majoritarian voting.

**Definition 9 (Structure Induced Equilibrium)** The status quo,  $x^o$ , is a structure induced equilibrium (SIE) if and only if it is invulnerable.

## A.2 Technical Appendix

### A.2.1 Proof of Lemma 3.1

Since the function  $\Gamma_t^t(\phi_t) = \Gamma_{t+1}^t(\phi_t)$  is discrete, individuals will either retire at the minimum retirement age,  $\phi_t = \Theta$ , or at mandatory age,  $\phi_t = 1$ . For a type  $x$  young, the utility level from working during the entire working period,  $\phi_t(x) = 1$ , is equal to  $(1 - \tau_t)x + \frac{p_{t+1}}{1+r}$ ; whereas the utility from retiring at the minimum retirement age,  $\phi_t(x) = \Theta$ , is:  $\Theta(1 - \tau_t)x + v + (1 - \Theta)\alpha_t p_t + \frac{\alpha_{t+1}p_{t+1}}{1+r}$ . Since  $p_t$  and  $p_{t+1}$  are lump sum, it is easy to see that, for given parameters of the social security system,  $(\tau_t, \alpha_t, p_t, \alpha_{t+1}, p_{t+1})$ , the ability level which make an agent indifferent between retire at mandatory age or earlier is:

$$x_t^R = \frac{(1 - \Theta)\alpha_t p_t - \frac{1 - \alpha_{t+1}}{1+r} p_{t+1} + v}{(1 - \tau_t)(1 - \Theta)}.$$

Thus, young agents with ability type  $x \leq x^R$  will retire early, at  $\phi_t(x) = \Theta$ , whereas agents with ability type  $x > x^R$  will work for the entire working period<sup>21</sup>,  $\phi_t(x) = 1$ , which proves the lemma. ■

### A.2.2 Proof of Lemma 3.3

Suppose that  $\epsilon \neq F(x_0^R)$ ,  $\tau_t = \bar{\tau} \forall t$  and  $\alpha_t = \bar{\alpha} \forall t$

- 1.- ( $\rightarrow$ ) For  $\alpha = 1$ , notice that by eq. 5  $x^R = x^R(p_t)$ , and  $p_t = g(x^R(p_t)) \forall t$  where  $g(x^R(p_t)) = \frac{[1 - (1 - \Theta)L_F(F(x^R(p_t)))](1+n)\mu_x \bar{\tau}}{1 + (1+n)(1 - \Theta)F(x^R(p_t))}$  by eq. 8. Let define  $p'$  s.t.  $x^R = \bar{x}$ . Notice that  $g(x^R(0)) = (1+n)\mu_x \bar{\tau} > 0$  and  $g(x^R(p))$  is decreasing and continuous on the interval  $p \in [0, p']$  (because  $\frac{\partial f(x^R(p))}{\partial p} = \frac{\partial f(x^R(p))}{\partial x^R} \frac{\partial x^R(p)}{\partial p} \leq 0$ , in fact  $\frac{\partial f(x^R(p))}{\partial x^R} \leq 0$  and  $\frac{\partial x^R(p)}{\partial p} \geq 0$ ). Moreover,  $g(x^R(p))$  is constant for  $p > p'$ ,  $g(x^R(p)) = \frac{(\Theta(1+n)\mu_x \bar{\tau})}{(1 + (1+n)(1 - \Theta))}$ . Thus, the expression

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<sup>21</sup>We assume that individuals who are indifferent between early retirement and retirement at mandatory age will retire early.

$p = g(x^R(p))$  has an unique fixed point for  $\forall t$ , which proves sufficiency.

2.- ( $\leftarrow$ ) If  $p_t = \bar{p} \forall t$  then, because  $\tau_t = \bar{\tau} \forall t$  and  $\alpha_t = \bar{\alpha} \forall t$ , by eq. 5  $x_t^R = x^R \forall t$ . Using eq. 8, we have that

$$p_0 = \frac{\left[1 - (1 - \Theta) L_F(F(x^R))\right] (1 + n) \mu_x \bar{\tau}}{1 - (1 - \alpha) \epsilon + (1 + n) (1 - \Theta) F(x^R) \bar{\tau}}$$

$$p_1 = \frac{\left[1 - (1 - \Theta) L_F(F(x^R))\right] (1 + n) \mu_x \bar{\tau}}{1 - (1 - \alpha) F(x^R) + (1 + n) (1 - \Theta) F(x^R) \bar{\tau}}$$

then, since by assumption  $\epsilon \neq F(x_0^R)$ ,  $p_0 = p_1$  implies  $\alpha = 1$ , which completes the proof.

### A.2.3 Lemma A.7

To establish the following lemma, we need to introduce some notation. Let  $R^y(x, \alpha)$  be the weak preference relation over  $\tau \in [0, 1]$ , given  $\alpha$ , for a type  $x$  young agent,  $P^y(x, \alpha)$  be the strict preference relation over  $\tau \in [0, 1]$ , given  $\alpha$ , for a type  $x$  young agent, and  $I^y(x, \alpha)$  be the indifference relation over  $\tau \in [0, 1]$ , given  $\alpha$ , for a type  $x$  young agent.

**Lemma 8** *At steady state and with commitment, for a given  $\alpha$ , if  $\tau_1 > \tau_2$  than, for all  $x$ :*

$$(A) \quad \tau_1 I^y(x, \alpha) \tau_2 \longrightarrow \begin{cases} \tau_2 R^y(x', \alpha) \tau_1 & \text{for all } x' > x \\ \tau_1 R^y(x', \alpha) \tau_2 & \text{for all } x' < x \end{cases}$$

$$(B) \quad \tau_1 P^y(x, \alpha) \tau_2 \longrightarrow \tau_1 P^y(x', \alpha) \tau_2 \text{ for all } x' < x.$$

$$(C) \quad \tau_2 P^y(x, \alpha) \tau_1 \longrightarrow \tau_2 P^y(x', \alpha) \tau_1 \text{ for all } x' > x.$$

**Proof.**

Remember that  $\alpha \in \{0, 1\}$ . If  $\alpha = 0$ , then nobody will retire early,  $\phi(x) = 1 \forall x$ , and the proof is straightforward.

In the case of  $\alpha = 1$ , let us define  $\phi(x, \tau)$  as the length of actual working period of type  $x$  agent, given a tax rate  $\tau$ . Let us begin with part

$$(A) \quad \tau_1 I^y(x, 1) \tau_2 \longrightarrow \begin{cases} \tau_2 R^y(x', 1) \tau_1 & \text{for all } x' > x \\ \tau_1 R^y(x', 1) \tau_2 & \text{for all } x' < x \end{cases} .$$

Consider the following three cases:

$$i) \quad \phi(x, \tau_1) = \phi(x, \tau_2) = 1.$$

In this case, since we have  $\tau_1 I^y(x, 1) \tau_2$ , then using the indirect utility function, we have that

$$(\tau_2 - \tau_1) x + \frac{1}{1+r} (p_1 - p_2) = 0 \quad (27)$$

which implies  $p_1 > p_2$ . By Lemma 3.1 we know that no individual with ability level  $x' > x$  will retire early for  $\tau_1$  or  $\tau_2$ , (i.e.  $\phi(x', \tau_1) = \phi(x', \tau_2) = 1$ ), and clearly  $\tau_2 R^y(x', 1) \tau_1$ .

For  $x' < x$  and again using Lemma 3.1 there are three cases: (a)  $\phi(x', \tau_1) = \phi(x', \tau_2) = 1$ , then clearly  $(\tau_2 - \tau_1) x' + \frac{(p_1 - p_2)}{1+r} > 0$  which implies  $\tau_1 R^y(x', 1) \tau_2$ . (b)  $\phi(x', \tau_1) = \phi(x', \tau_2) = \Theta$ , then by eq. 27,

$$\Theta (\tau_2 - \tau_1) x' + \left( (1 - \Theta) + \frac{1}{1+r} \right) (p_1 - p_2) > 0$$

which implies  $\tau_1 R^y(x', 1) \tau_2$ , since now  $x'$  pays less taxes and receives a larger transfers over the life time than  $x$ . (c)  $\phi(x', \tau_1) = \Theta$  and  $\phi(x', \tau_2) = 1$ , in section (a) we showed that if the individual with ability  $x' < x$  does not retire early, then  $\tau_1 R^y(x', 1) \tau_2$ ; clearly, if she now prefers to retire early for  $\tau = \tau_1$  it will be because her utility is larger, thus her order of preferences will be  $\tau_1 R^y(x', 1) \tau_2$ . We showed that  $\tau_1 R^y(x', 1) \tau_2$  for all  $x' < x$ .

ii)  $\phi(x, \tau_1) = \phi(x, \tau_2) = \Theta$ . In this case, the proof is analogous to case i).

iii)  $\phi(x, \tau_1) = \Theta$  and  $\phi(x, \tau_2) = 1$ .

Using the indirect utility function, we have that

$$\begin{aligned} & (\Theta(1 - \tau_1) - (1 - \tau_2))x + \\ & \left( \frac{1}{1+r} + (1 - \Theta) \right) p_1 + v - \left( \frac{1}{1+r} \right) p_2 = 0 \end{aligned} \quad (28)$$

By definition of indifference, if a type  $x$  were to work all her working life for  $\tau_1$  ( $\phi(x, \tau_1) = 1$ ), she would be worst off with  $\tau_1$  than with  $\tau_2$ , that is:

$$(\tau_2 - \tau_1)x + \frac{p_1 - p_2}{1+r} \leq 0 \quad (29)$$

Analogously, if she were to retire early for  $\tau_2$ , ( $\phi(x, \tau_2) = \Theta$ ), she would be worst off with  $\tau_2$  than with  $\tau_1$ , that is

$$\Theta((\tau_2 - \tau_1))x + \left( \frac{1}{1+r} + (1 - \Theta) \right) (p_1 - p_2) \geq 0 \quad (30)$$

By Lemma 3.1, for  $x' > x$  we have two cases: (a)  $\phi(x', \tau_1) = \Theta$  and  $\phi(x', \tau_2) = 1$ , in which case we clearly have  $\tau_2 R^y(x', 1) \tau_1$ , by eq. 28; (b)  $\phi(x', \tau_1) = 1$  and  $\phi(x', \tau_2) = \Theta$ , in which case by inequality 29,  $\tau_2 R^y(x', 1) \tau_1$ . Again, by Lemma 3.1 for  $x' < x$  we have two cases: (a)  $\phi(x', \tau_1) = \Theta$  and  $\phi(x', \tau_2) = 1$ , and thus by eq. 28  $\tau_1 R^y(x', 1) \tau_2$ ; and (b)  $\phi(x', \tau_1) = \phi(x', \tau_2) = \Theta$ , which, by inequality 30 implies  $\tau_1 R^y(x', 1) \tau_2$ . Notice that the left hand sides of equations 28, 29, and 30 are all decreasing in  $x$ .

(B)  $\tau_1 P^y(x, 1) \tau_2 \longrightarrow \tau_1 P^y(x', 1) \tau_2$  for all  $x' < x$ .

Consider three cases:

i)  $\phi(x, \tau_1) = \phi(x, \tau_2) = 1$ .

By definition of strict preferences we have:

$$(\tau_2 - \tau_1)x + \frac{1}{1+r} (p_1 - p_2) > 0. \quad (31)$$

By Lemma 3.1, for  $x' < x$ , we have three cases: (a) for  $\phi(x', \tau_1) = \phi(x', \tau_2) = 1$ , by inequality 31, it is easy to see that  $\tau_1 P^y(x', 1) \tau_2$ ;

(b) for  $\phi(x', \tau_1) = \phi(x', \tau_2) = \Theta$ , since the type  $x'$  young pays less taxes and receives more life time transferences than a type  $x$ , we have

$$(\tau_2 - \tau_1) x' + \left( \frac{1}{1+r} + (1 - \Theta) \right) (p_1 - p_2) > 0 \quad (32)$$

and thus  $\tau_1 P^y(x', 1) \tau_2$ ; (c)  $\phi(x', \tau_1) = \Theta$  and  $\phi(x', \tau_2) = 1$ , in (a) we showed that if a type  $x'$  does not retire early for  $\tau_1$ , she strictly prefers  $\tau_1$  to  $\tau_2$ ; if she now retire early for  $\tau_1$  it is because her utility is larger, and thus she still prefers  $\tau_1$  to  $\tau_2$ , i.e.,  $\tau_1 P^y(x', 1) \tau_2$ .

ii)  $\phi(x, \tau_1) = \phi(x, \tau_2) = \Theta$ .

By definition of strict preference, we have

$$\Theta (\tau_2 - \tau_1) x + \left( \frac{1}{1+r} + (1 - \Theta) \right) (p_1 - p_2) > 0. \quad (33)$$

By Lemma 3.1, a type  $x' < x$  young will also retire early for  $\tau_1$  and  $\tau_2$ , and thus  $\tau_1 P^y(x', 1) \tau_2$ .

iii)  $\phi(x, \tau_1) = \Theta$  and  $\phi(x, \tau_2) = 1$ .

By definition of strict preference, we have

$$(\Theta (1 - \tau_1) - (1 - \tau_2)) x + \left( \frac{1}{1+r} + (1 - \Theta) \right) p_1 + v - p_2 > 0 \quad (34)$$

By Lemma 3.1, for  $x' < x$ , there are two cases: (a)  $\phi(x', \tau_1) = \Theta$  and  $\phi(x', \tau_2) = 1$ , in which case by the inequality 34, we have that  $\tau_2 P^y(x', 1) \tau_1$ ; (b)  $\phi(x', \tau_1) = \phi(x', \tau_2) = \Theta$ . Notice that since  $\tau_1 P^y(x, 1) \tau_2$ , then there will exist an  $\xi > 0$ , such that,  $(\tau_1, p_1 - \xi) I^y(x, 1) (\tau_2, p_2)$ , that is eq. 34 can be transformed into the following expression:

$$(\Theta (1 - \tau_1) - (1 - \tau_2)) x + \left( \frac{1}{1+r} + (1 - \Theta) \right) (p_1 - \xi) + v - p_2 = 0 \quad (35)$$

Then, if a type  $x$  agent would retire early for  $\tau_2$ ,  $\phi(x, \tau_2) = \Theta$  (which provides less utility than mandatory age retirement), we have

$$\Theta (\tau_2 - \tau_1) x + \left( \frac{1}{1+r} + (1 - \Theta) \right) ((p_1 - \xi) - p_2) > 0 \quad (36)$$

and this inequality will hold for every  $x' < x$ , thus  $\tau_1 P^y(x', 1) \tau_2$ .

(C)  $\tau_2 P^y(x, 1) \tau_1 \longrightarrow \tau_2 P^y(x', 1) \tau_1$  for all  $x' > x$ . The proof is analogous to the proof of case (B). ■

#### A.2.4 Proof of Lemma 4.1

**Proof.** Using Lemma A.7, it is easy to see that of  $x_{m\tau}$  displays a strict preference between two tax rates, then a majority of the electorate will have the same preference. Notice that old individual will always prefer the tax rate which maximizes their pension. ■

#### A.2.5 Proof of Lemma 4.2

**Proof.** At steady state there is a fraction  $F(x^R) \in [0, 1]$  of the old population with incomplete working history who supports the institution of the early retirement. Then, the support of an additional mass of  $(2 + n - 2F(x^R))/2$  individuals from the young generation is needed to obtain  $\alpha = 1$  by majority rule.

A type  $x$  young individual, for a given tax rate,  $\tau$ , we vote for  $\alpha = 1$ , if and only if

$$-x(1 - \tau)(1 - \Theta) + p(\tau, 1) \left( \frac{1}{1+r} + (1 - \Theta) \right) - \frac{p(\tau, 0)}{1+r} + v \geq 0 \quad (37)$$

or she will vote for  $\alpha = 0$ . Therefore, agents with ability types

$$x \leq \frac{p(\tau, 1) \left( \frac{1}{1+r} + (1 - \Theta) \right) - \frac{p(\tau, 0)}{1+r} + v}{(1 - \tau)(1 - \Theta)} = \hat{x}$$

will prefer  $\alpha = 1$ . Then, the ability type  $x_{m\alpha}$  individual such that  $(1 + n) F(x_{m\alpha}) = (2 + n - 2F(x^R))/2$ , will be the median voter on the jurisdiction  $\alpha$  for a given  $\tau$  at steady state. ■

#### A.2.6 Proof of proposition 5.1

By Shepsle (1979) Thm 4.1, a necessary and sufficient condition for a SIE to exist is that the two (median) reaction functions at equations 21 and 24 cross.

Part (i):  $x_{m\tau} \in \left[ \underline{x}, \frac{1+n}{1+r} \mu_x \right]$ .

Consider the SIE  $(\tau^{NR} = 1, 0)$ . For  $\alpha = 0$ ,  $\tau^{NR}$  is the median's ideal by definition of  $\tau^{NR}$  (see eq. 16), and it is equal to 1. For  $\tau = \tau^{NR} = 1$ , the median  $x_{m\alpha}$  prefers  $\alpha = 0$  to  $\alpha = 1$  if  $x_{m\alpha} > \hat{x}(\tau^{NR})$ . Using the definition at eq. 23, and after some simple algebra, this condition can be written as  $v < v^*$ , which proves the first part of (i).

Consider now the SIE  $(\tau^{WR}, 1)$ . First notice that  $\tau^{WR} \geq \tau^R$ , where  $\tau^{WR} \in \arg \max_{\tau \in [0,1]} v^{WR}(\tau, 1, x_{m\tau})$  and  $\tau^R \in \arg \max_{\tau \in [0,1]} v^{NR}(\tau, 1, x_{m\tau})$ , because if  $x_{m\tau}$  retires early, she will prefer a (weakly) higher tax rate. For  $\alpha = 1$ , the median voter  $x_{m\tau}$  votes for  $\tau^{WR}$  (rather than for  $\tau^R$ ) if  $v^{WR}(\tau^{WR}, 1, x_{m\tau}) \geq v^{NR}(\tau^R, 1, x_{m\tau})$ . For  $\tau = \tau^{WR}$ , the median over  $\alpha$ ,  $x_{m\alpha}$ , votes for  $\alpha = 1$  if  $x_{m\alpha} \leq \hat{x}(\tau^{WR})$ , that is, if  $v^{WR}(\tau^{WR}, 1, x_{m\alpha}) \geq v^{NR}(\tau^{WR}, 0, x_{m\alpha})$ . Notice that, since  $x_{m\alpha} \geq x_{m\tau}$ , then by lemma A.7 this last condition implies that  $v^{WR}(\tau^{WR}, 1, x_{m\tau}) \geq v^{NR}(\tau^{WR}, 0, x_{m\tau})$ . Moreover,  $v^{NR}(\tau^{NR}, 0, x_{m\tau}) > v^{NR}(\tau^{WR}, 0, x_{m\tau})$ , because  $\tau^{NR} = 1 = \arg \max_{\tau \in [0,1]} v^{NR}(\tau, 0, x_{m\tau})$ , and  $v^{NR}(\tau^{WR}, 0, x_{m\tau}) \geq v^{NR}(\tau^R, 0, x_{m\tau})$  because  $\tau^{NR} = 1 > \tau^{WR} \geq \tau^R$ . Finally,  $v^{NR}(\tau^R, 0, x_{m\tau}) \geq v^{NR}(\tau^R, 1, x_{m\tau})$ , since we are comparing indirect utilities for the case in which  $x_{m\tau}$  does not retire early. Thus,  $x_{m\alpha} \leq \hat{x}(\tau^{WR})$  implies  $v^{WR}(\tau^{WR}, 1, x_{m\tau}) \geq v^{NR}(\tau^R, 1, x_{m\tau})$ , i.e.  $x_{m\tau}$  votes for  $\tau^{WR}$ , which completes the proof of part (i).

Part (ii):  $x_{m\tau} \in \left( \frac{1+n}{1+r} \mu_x, \bar{x} \right]$ .

Consider the SIE  $(\tau^{NR} = 0, 0)$ . The proof is analogous to part (i) for the SIE  $(\tau^{NR} = 1, 0)$ , except that now for  $\tau = \tau^{NR} = 0$ , the median voter over  $\alpha$ ,  $x_{m\alpha}$ , always prefers  $\alpha = 0$  to  $\alpha = 1$ .

Consider the SIE  $(\tau^{WR}, 1)$ . For  $\alpha = 1$ , the median voter  $x_{m\tau}$  votes for  $\tau^{WR}$  if  $x_{m\tau} \leq \tilde{x}(\tau^{WR})$ , i.e., if  $x_{m\tau} \leq v^{WR}(\tau^{WR}, 1, x_{m\tau})$ . For  $\tau = \tau^{WR}$ , the median voter over  $\alpha$ ,  $x_{m\alpha}$ , votes for  $\alpha = 1$  if  $x_{m\alpha} \leq \hat{x}(\tau^{WR})$ , which completes the proof.

### A.2.7 Proof of proposition 5.2

By definition of SSPSIE, we only need to show that a voting strategy profile, whose associated equilibrium outcome is a Structure Induced Equilibrium of the game with commitment, is a subgame perfect equilibrium.

**Part A: SIE with outcome  $(\tau^{WR}, 1)$ .** There are three cases to analyze: (i)  $x_{m\tau} \in [\underline{x}, \frac{1+n}{1+r}\mu_x]$ , and  $\hat{x}(\tau^{WR}) \leq \tilde{x}(\tau^{WR})$ ; (ii)  $x_{m\tau} \in [\underline{x}, \frac{1+n}{1+r}\mu_x]$ , and  $\hat{x}(\tau^{WR}) > \tilde{x}(\tau^{WR})$ ; and (iii)  $x_{m\tau} \in (\frac{1+n}{1+r}\mu_x, \bar{x}]$ . For simplicity we drop the argument in  $\hat{x}$  and  $\tilde{x}$ .

Case (i):  $x_{m\tau} \in [\underline{x}, \frac{1+n}{1+r}\mu_x]$ , and  $\hat{x} \leq \tilde{x}$  (or equivalently  $x_{m\alpha} \in [\underline{x}, \frac{1+n}{1+r}\mu_x]$ )

Let define the following sets of realization of the history of the game:

$$H_t^{0,0} = \{h_t \in H_t \mid (\tau_s = 0, \alpha_s = 0) \ s = 0, \dots, t-1\}$$

and

$$H_t^{\tau,1} = \{h_t \in H_t \mid \exists t_0 \in \{0, 1, \dots, t-1\} : (\tau_s = 0, \alpha_s = 0) \ \forall s < t_0, \\ (\tau_s = \tau^{WR}, \alpha_s = 1) \ \forall s \geq t_0\}$$

notice that  $H_t^{0,0} \cap H_t^{\tau,1} = \emptyset$ .

Consider the following voting strategy profile:

a) for  $x \in [\underline{x}, \tilde{x}]$

$$s_{t,x}^y = \begin{cases} (\tau^{WR}, 1) & \text{if } h_t \in H_t^{0,0} \cup H_t^{\tau,1} \\ (0, 0) & \text{if } h_t \in H_t / \{H_t^{0,0} \cup H_t^{\tau,1}\} \end{cases}$$

b) for  $x \in (\tilde{x}, \bar{x}]$

$$s_{t,x}^y = \begin{cases} (\tau_x^y, 0) & \text{if } h_t \in H_t^{0,0} \cup H_t^{\tau,1} \\ (0, 0) & \text{if } h_t \in H_t / \{H_t^{0,0} \cup H_t^{\tau,1}\} \end{cases}$$

where  $\tau_x^y \in \arg \max_{\tau \in [0,1]} \{v^{WR}(\tau, 1, x)\}$ .

c)  $\forall x$ , if the agent has a complete working history,  $\phi_{t-1} = 1$ ,

$$s_{t,x}^0 = (\tau^o(0), 0) \quad \forall h_t \in H_t$$

d)  $\forall x$ , if the agent has an incomplete working history,  $\phi_{t-1} = \Theta$ ,

$$s_{t,x}^0 = (\tau^o(1), 0) \quad \forall h_t \in H_t.$$

In this simple model the steady state is reached in one period. It is thus easy to see that a sufficient condition for this strategy to support  $(\tau^{WR}, 1)$  as a subgame perfect equilibrium outcome is that the median voter over  $\alpha$  is less than  $\tilde{x}$  at  $t = 0$  and at steady state:  $x_{m\alpha,0} \leq \tilde{x}$  and  $x_{m\alpha} \leq \tilde{x}$ . Since  $(\tau^{WR}, 1)$  is the outcome of a SIE, by proposition 5.1, and by definition of case i),  $x_{m\alpha} \leq \hat{x} \leq \tilde{x}$ . At  $t = 0$ , by Lemma 4.2  $x_{m\alpha,0} \leq \tilde{x}$  if and only if  $\epsilon \geq \frac{2+n}{2} - (1+n)F(\tilde{x}) = \epsilon^*(\tilde{x})$ .

Case (ii):  $x_{m\tau} \in \left[ \underline{x}, \frac{1+n}{1+r}\mu_x \right]$ , and  $\hat{x} > \tilde{x}$  (or equivalently  $x_{m\alpha} \in \left( \frac{1+n}{1+r}\mu_x, \bar{x} \right]$ )

Let define the following sets of realization of the history of the game:

$$H_t^0 = \{h_t \in H_t \mid \tau_s = 0, s = 0, \dots, t-1\}$$

and

$$H_t^\tau = \{h_t \in H_t \mid \exists t_0 \in \{0, 1, \dots, t-1\} : \tau_s = 0 \quad \forall s < t_0, \tau_s = \tau^{WR} \quad \forall s \geq t_0\},$$

notice that  $H_{t-1}^0 \cap H_{t-1}^\tau = \emptyset$ .

Consider the following voting strategy profile

a) for  $x \in [\underline{x}, \tilde{x}]$

$$s_{t,x}^y = \begin{cases} (\tau^{WR}, 1) & \text{if } h_t \in H_t^0 \cup H_t^\tau \\ (0, 1) & \text{if } h_t \in H_t / \{H_t^0 \cup H_t^\tau\} \end{cases}$$

b) for  $x \in (\tilde{x}, \hat{x}]$

$$s_{t,x}^y = \begin{cases} (0, 1) & \text{if } h_t \in H_t^0 \cup H_t^\tau \\ (0, 0) & \text{if } h_t \in H_t / \{H_t^0 \cup H_t^\tau\} \end{cases}$$

c) for  $x \in (\hat{x}, \bar{x}]$

$$s_{t,x}^y = (0, 0) \quad \forall h_t \in H_t$$

For the old individuals, the strategy is identical to case (i).

A sufficient condition for this strategy to support  $(\tau^{WR}, 1)$  as a subgame perfect equilibrium outcome is that the median voter over  $\alpha$  is less than  $\hat{x}$ , at  $t = 0$  and at steady state:  $x_{m\alpha,0} \leq \hat{x}$  and  $x_{m\alpha} \leq \hat{x}$ . Since  $(\tau^{WR}, 1)$  is the outcome of a SIE, by proposition 5.1,  $x_{m\alpha} \leq \hat{x}$ . At  $t = 0$ , by Lemma 4.2  $x_{m\alpha,0} \leq \hat{x}$  if and only if  $\epsilon \geq \frac{2+n}{2} - (1+n)F(\hat{x}) = \epsilon^*(\hat{x})$ .

$$\text{Case (iii): } x_{m\tau} \in \left( \frac{1+n}{1+r}\mu_x, \bar{x} \right]$$

First notice that, using the definitions of  $\hat{x}(\tau)$  and  $\tilde{x}(\tau)$  at equations 23 and 25, we obtain that  $\forall \tau \geq 0$ :

$$\begin{aligned} \hat{x}(\tau) &\geq \tilde{x}(\tau) \quad \text{if } \hat{x}(\tau) \geq \frac{1+n}{1+r}\mu_x \\ \hat{x}(\tau) &\leq \tilde{x}(\tau) \quad \text{if } \hat{x}(\tau) \leq \frac{1+n}{1+r}\mu_x \end{aligned}$$

By proposition 5.1,  $(\tau^{WR}, 1)$  is the outcome of a SIE if and only if  $x_{m\alpha} \leq \hat{x}(\tau^{WR})$  and  $x_{m\tau} \leq \tilde{x}(\tau^{WR})$ . Since  $x_{m\tau} \leq x_{m\alpha}$ , then  $\hat{x}(\tau^{WR}) \geq x_{m\alpha} \geq x_{m\tau} > \frac{1+n}{1+r}\mu_x$  implies that  $\hat{x}(\tau^{WR}) > \tilde{x}(\tau^{WR})$ .

Consider the same voting strategy profile as in case (ii). A sufficient condition for this strategy to support  $(\tau^{WR}, 1)$  as a subgame perfect equilibrium outcome is that: (a) the median voter over  $\alpha$  is less than  $\hat{x}$  at  $t = 0$  and at steady state:  $x_{m\alpha,0} \leq \hat{x}$  and  $x_{m\alpha} \leq \hat{x}$ ; and (b) the median voter over  $\tau$  is less than  $\tilde{x}$  at  $t = 0$  and at steady state:  $x_{m\tau,0} \leq \tilde{x}(\tau^{WR})$  and  $x_{m\tau} \leq \tilde{x}(\tau^{WR})$ . Conditions (a) are the same as in the case ii), whereas conditions (b) are satisfied by proposition 5.1. In fact, since  $(\tau^{WR}, 1)$  is a SIE, at steady state  $x_{m\tau} \leq \tilde{x}(\tau^{WR})$ . Moreover, by Lemma 4.1, the median voter over  $\tau$ ,  $x_{m\tau}$ , does not depend on the initial condition, and thus  $x_{m\tau} = x_{m\tau,0} \leq \tilde{x}(\tau^{WR})$ .

Finally, notice that for  $\hat{x} \leq \tilde{x}$ ,  $\epsilon^*(\hat{x}) \geq \epsilon^*(\tilde{x})$ , and for  $\hat{x} > \tilde{x}$ ,  $\epsilon^*(\hat{x}) < \epsilon^*(\tilde{x})$ . Therefore, conditions for the strategy profiles at part (i), (ii), and (iii) to constitute a subgame perfect equilibrium of the game without

commitment, with associated outcome  $(\tau^{WR}, 1)$ , can be summarized as follows:

$\epsilon \geq \min \left\{ \frac{2+n}{2} - (1+n)F(\hat{x}), \frac{2+n}{2} - (1+n)F(\tilde{x}) \right\}$ , which proves the part (i) of the proposition.

**Part B: SIE with outcome  $(\tau^{NR}, 0)$**

Consider the following voting strategy profile:

$$s_{t,x}^y = \begin{cases} (\tau_x^y, 0) & \text{if } h_t \in H_t^0 \cup H_t^\tau \\ (0, 0) & \text{if } h_t \in H_{t-1} / \{H_t^0 \cup H_t^\tau\} \end{cases}$$

whereas the strategy for the old individuals is identical to Part A. The proof is trivial, since in every period the median voter in the dimension  $\tau$ , has the same ability:  $x_{m\tau}$ , and  $\tau^{NR} = \arg \max_{\tau \in [0,1]} v^{NR}(\tau, 0, x_{m\tau})$ . ■

**A.2.8 Proof of corollary 5.3**

Notice that condition (B) implies that  $x_{m\alpha} \leq \hat{x}(\tau^{WR})$ , and condition (C) implies that  $x_{m\tau} \leq \tilde{x}(\tau^{WR})$ . Thus, by proposition 5.1, conditions (B) and (C) imply that  $(\tau^{WR}, 1)$  is a SIE of the voting game with commitment. By proposition 5.2, condition (A) guarantees that  $(\tau^{WR}, 1)$  is also a SSPSIE of the voting game without commitment. ■

## References

- [1] AZARIADIS, C. and V. GALASSO (1997) "Fiscal Constitutions and the Determinacy of Intergenerational Transfers". *Universidad Carlos III de Madrid Working Paper #71-97*.
- [2] BARTEL, A. and N. SICHERMAN (1993) "Technological Change and Retirement Decisions of Older Workers" *Journal of Labor Economics*, 11(1).
- [3] BECKER, G. (1975) "Human Capital. A theoretical and Empirical Analysis, with Special Reference to Education". The University of Chicago Press, Ltd., London.
- [4] BLÖNDAL, S. and S. SCARPETTA (1998) "The Retirement Decision in OECD Countries", *OECD Working Paper AWP 1.4*.
- [5] BOLDRIN, M. and A. RUSTICHINI (2000) "Equilibria with Social Security", forthcoming *Review of Economic Dynamics*.
- [6] BOSKIN, J. M. and M. D. HURD (1978) "The Effect of Social Security on Early Retirement", *Journal of Public Economics*, 10, 361-377.
- [7] BOSKIN, M.J., L.J. KOTLIKOFF, D.J. PUFFERTT and J.B. SHOVEN (1987), "Social Security: A Financial Appraisal across and within Generations", *National Tax Journal*, 40
- [8] CABALLERO, R.J and M.L. HAMMOUR (1998) "The Macroeconomics of Specificity", *Journal of Political Economy* 106 (4).
- [9] CABALLERO, R.J and M.L. HAMMOUR (1999) "The Limits of Special Interest", mimeo.
- [10] COATE, S. and S. MORRIS (1999), "Policy Persistence", *American Economic Review* 89 (5).
- [11] CONDE-RUIZ, J.I. and V. GALASSO (1999), "Positive Arithmetic of the Welfare State", *CEPR Discussion Paper # 2202*.

- [12] CONDE-RUIZ, J.I. and V. GALASSO (2000), “The Macroeconomics of Early Retirement”, mimeo.
- [13] COOLEY T.F., and J. SOARES (1999) “A Positive Theory of Social Security Based on Reputation”, *Journal of Political Economy*, vol.**107**, no.1, 135-160.
- [14] CRAWFORD, V. P., and D. M. LILIEN (1981) “Social Security and the Retirement Decision”, *Quarterly Journal of Economics*, **95**, 505-529.
- [15] DAVIS, S.J, and J. HALTIWANGER (1999) “Sectoral Job Creation and destruction Responses to Oil Price Changes”, NBER w.p. #7095.
- [16] DIAMOND, P.A. and J.A. MIRRLESS (1978) “ A Model of Social Insurance with Variable Retirement”, *Journal of Public Economics* **10**, 295-336.
- [17] FELDSTEIN, M. (1974) “Social Security, induced retirement, and Aggregate Capital Accumulation”, *Journal of Political Economy*, **82(5)**, 905-926.
- [18] GALASSO, V. (2000), “The US Social Security: A Financial Appraisal for the Median Voter.”, *CEPR Discussion Paper # 2546*.
- [19] GALASSO, V. (1999), “The US Social Security System: What Does Political Sustainability Imply?”, *Review of Economic Dynamics*, **2**, 698-730.
- [20] GRUBER, J. and D. WISE (eds.) (1999). *Social Security and Retirement Around the World*, University of Chicago Press, Chicago.
- [21] HAMMOND P. (1975) “Charity: Altruism or Cooperative Egoism” in “Altruism, Morality and Economic Theory” (E.S. Phelps, Ed.), Russell Sage Foundation, New York.
- [22] IPPOLITO, R.A. (1990) “Toward Explaining Earlier Retirement After 1970” *Industrial and Labor Relations Review*, 48 (5).

- [23] LAZEAR, E.P. (1979) “Why is There Mandatory Retirement”, *Journal of Political Economy*, 87(6), 1261-1284.
- [24] MELTZER, A.H., and S.F. RICHARD (1981), “A Rational Theory of the Size of Government”, *Journal of Political Economy*, 89 (5).
- [25] OECD (1992), *Employment Outlook*.
- [26] ORDERSHOOK, P.C. (1986), “Game Theory and Political Theory”, Cambridge University Press.
- [27] ROBERTS, K.W.S, (1977), “Voting over Income Tax Schedules”, *Journal of Public Economics*, **8**.
- [28] SHEPSLE, K.A. (1979), “Institutional Arrangements and Equilibrium in Multidimensional Voting Models”, *American Journal of Political Science*, **23** (1).
- [29] TABELLINI, G. (1990), “A Positive Theory of Social Security”, NBER *Working Paper* no. 3272.

Figure 1: Employment in Industry 1960-90

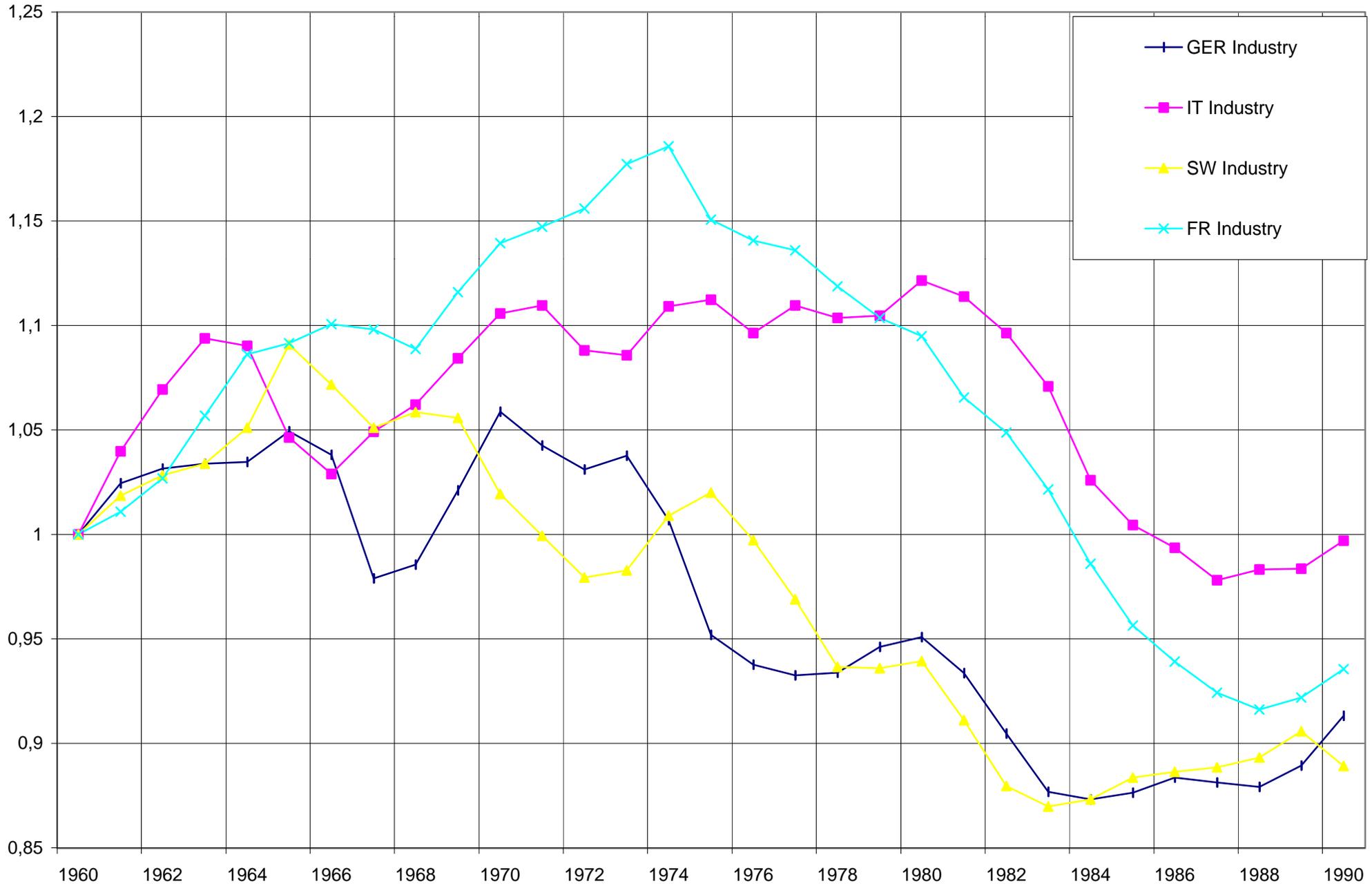


Figure 2: Employment in Industry 1960-90

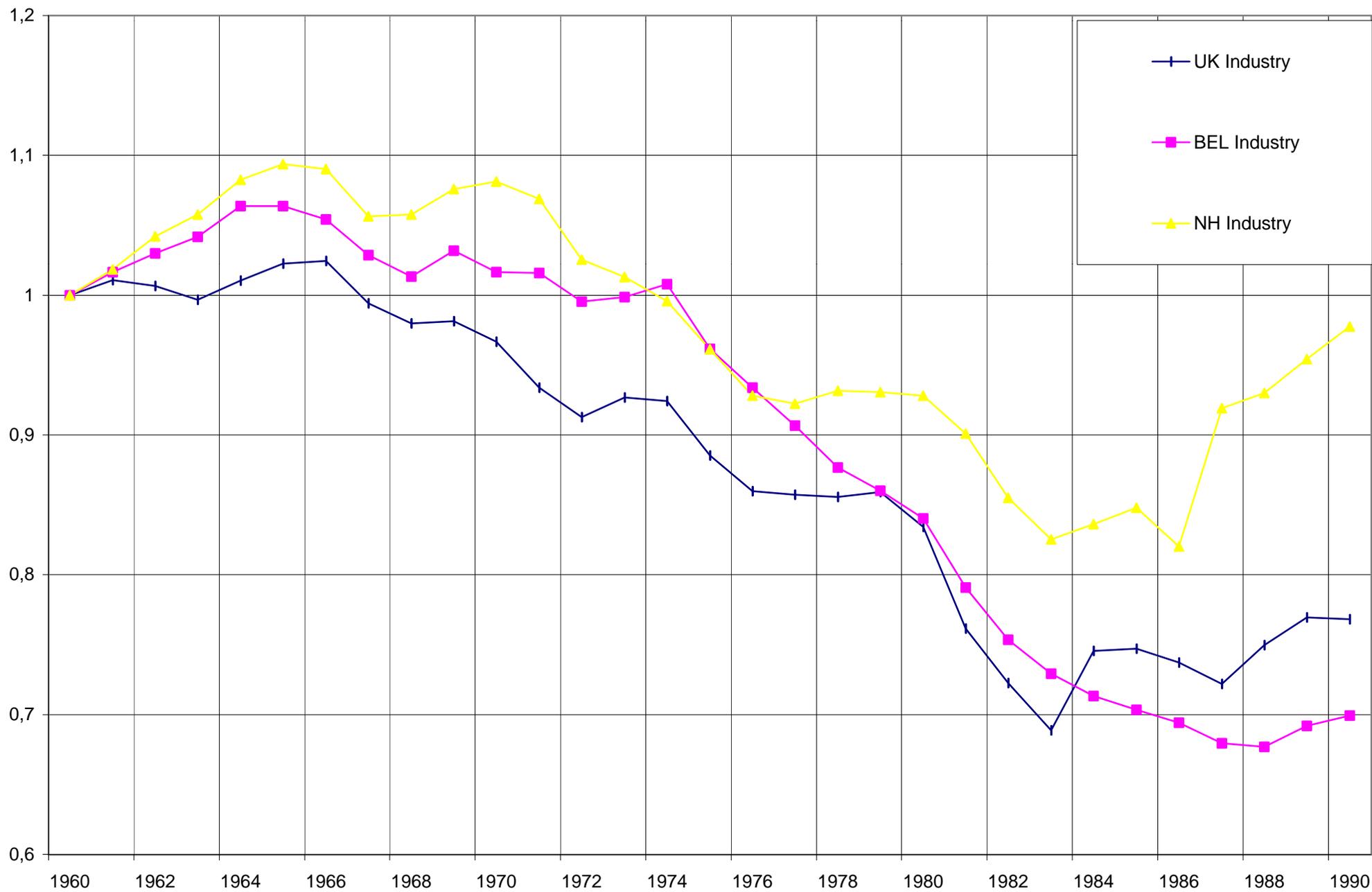


Figure 3: Employment in Industry 1960-90

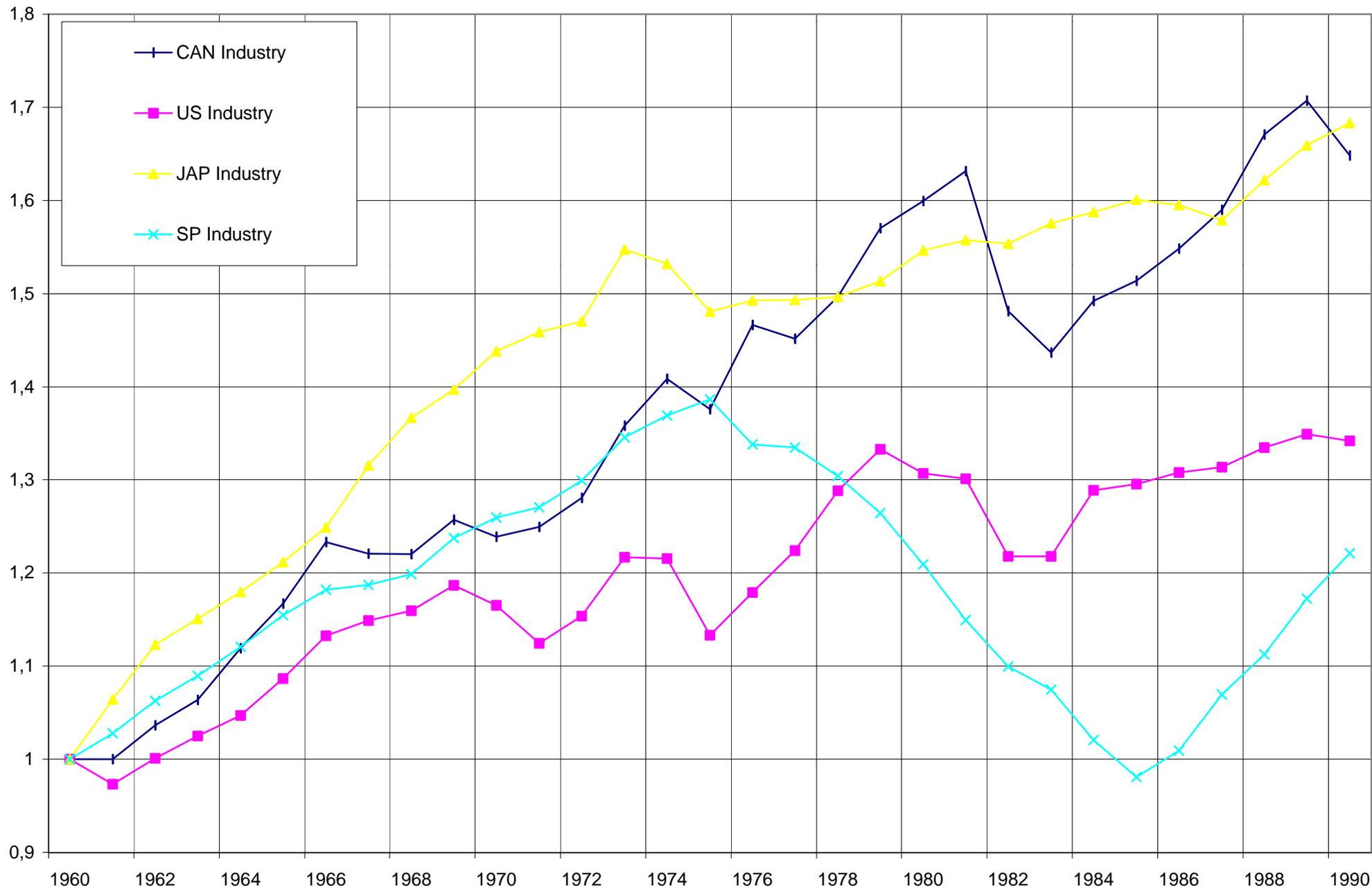


Figure 4: Voting on Social Security and ER

