EUI DEPARTMENT OF ECONOMICS

Economies in transition: ageing, markups and place-based policies

Max Brès

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 11 May 2023

European University Institute
Department of Economics

Economies in transition: ageing, markups and place-based policies

Max Brès

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Researcher declaration to accompany the submission of written work
Department Economics - Doctoral Programme

I Max Brès certify that I am the author of the work Economies in transition: ageing, markups and place-based policies I have presented for examination for the Ph.D. at the European University Institute. I also certify that this is solely my own original work, other than where I have clearly indicated, in this declaration and in the thesis, that it is the work of others.

I warrant that I have obtained all the permissions required for using any material from other copyrighted publications.

I certify that this work complies with the Code of Ethics in Academic Research issued by the European University Institute (IUE 332/2/10 (CA 297).

The copyright of this work rests with its author. Quotation from it is permitted, provided that full acknowledgement is made. This work may not be reproduced without my prior written consent. This authorisation does not, to the best of my knowledge, infringe the rights of any third party.

I declare that this work consists of 28891 words.

Statement of inclusion of previous work:

I confirm that chapter 1 was jointly co-authored with Daniele Angelini and I contributed 50% of the work.

I confirm that chapter 2 was jointly co-authored with Philipp Kircher and David Koll and I contributed 33% of the work.

Signature and date:

03/05/2023

Max Brès

Abstract

In the first chapter, joint with Daniele Angelini, we analyze the effect of a change in the age composition of consumers on growth through competition. To identify the effect coming from the demand side of the economy, we instrument changes in the age composition of demand with unexpected foreign demographics shocks. We find that middle-aged consumers reduce competition. We rationalize our results in a general equilibrium heterogeneous agent model. Focusing on the USA, we estimate that changes in the age composition of consumers have contributed to a reduction in GDP growth of 8.7% in the period 1995-2004 and an increase of 10.3% in the period 2005-2019.

In the second chapter, joint with Philipp Kircher and David Koll, we estimate labor market responses to local sector-level shocks as proxies for place-based policies. We show that local policy maker could face a trade-off between subsidising low-growth sectors with large short-term employment benefits or subsidising high growth sectors with initially smaller but persistent labor market improvements.

Finally, in the third chapter, I present a model of consumer search that captures firms' endogenous growth along two margins: *customer capital* and product range. The model provides micro-foundations for the existence of within-firm heterogeneity in terms of prices, sales and markups. Consistent with recent empirical evidence, firms do not increase markups over their growth path; instead, I show that they achieve a higher surplus from each match by increasing their product range and, therefore, buyer's loyalty. Nevertheless, as a firm grows larger, its share of incumbent buyers increases and demand spreads more homogeneously across goods, mechanically reducing its average markup. I called this novel mechanism the *curse of the large firm*.

Acknowledgements

First and foremost, I am deeply grateful to my advisor Philipp Kircher for his guidance, humanity, and unwavering support. You have been a source of inspiration at every step of this journey, as a teacher for your inspiring lectures, as a researcher and supervisor for your enthusiasm in applying economic theory to policy questions, and as a human being who always made yourself available during good times and, especially, bad times. I am also deeply grateful to Tom Crossley, my second advisor, for his guidance. Despite our infrequent meetings, each piece of advice you gave me has proved to be very valuable. Furthermore, I would like to express my special gratitude to the jury members, Leena Rudanko and Anna Salomons, whose suggestions will undoubtedly contribute to turning each chapter of my thesis into much improved articles.

I also owe a huge debt to my co-authors. The first chapter is co-authored with Daniele Angelini. Thank you, Dani, for spending countless hours together solving models and sharing ideas, first in the gardens of Villa La Fonte, and later, sadly, on Zoom. The second chapter is co-authored with Philipp Kircher and David Koll. Thank you, David, for your enthusiasm and helping me improve as a researcher.

Many others indirectly contributed to making this work possible. I would like to particularly thank Nikolaj Broberg, Žymantas Budrys, and Sebastian Rast. I could not have hoped for better coworkers, partners in crime, and simply friends during the first years of the PhD and, hopefully, for many years to come. I am also grateful to the Pietrapiana crew - Chiara, Thibault, and Stella - for creating a peaceful and caring environment to come back to every evening after long days at the EUI. I am also grateful to those who counted during my time in Florence - My, Mustafa, David, Flavia, Lukas, Jaakko, Ola, Henning, Théo, and so many others.

I thank the Institute for International Economic Studies (IIES) at Stockholm University for becoming my second home away from Florence during Covid. I'm particularly grateful to members of the Imdb - Markus, Stefan, Philipp, and Gualtiero among others - for the many discussions, bringing light and humor to Tuesday mornings and hosting me as one of you. I am also grateful to Markus, Malin, Pierre, José, Marie-Pascale, Orsa, and everyone else at the Swedish Institute for Social Research (SOFI) for welcoming me into your thriving and inclusive environment during the last months of my PhD. I now know how fantastic a research institution can be.

I extend my heartfelt gratitude to my parents, Antoine and Béatrice, for their support and their contagious passion for research. Thank you, Elsa, for always reminding me to trust my choices and being a great example. Last but not least, I'm extremely grateful to my amazing family: Christine Alamaa, and my two PhD-girls, Marni and Thelma. Thank you, Christine, for sharing my expansive enthusiasm and deep doubts, and showing kindness, patience, and unconditional belief throughout this journey. I would have neither started nor achieved any of this without you. Thank you Marni and Thelma for always bringing smiles and calm even on the worst days.

Contents

1

The effect of a change in the age composition of consumers on the market structures and growth

Joint with Daniele Angelini [1]

Abstract This paper analyzes the effect of population aging on economic growth through its impact on competition in the goods market. As individuals age, they update their consumption basket less frequently and their opportunity cost of time drops after retirement. Hence, population aging changes the characteristics of demand, which, in turn, modifies market structures and aggregate trends. To identify the effect coming from the demand side of the economy, we instrument changes in the age composition of demand with unexpected foreign demographics shocks. We find that middle-aged consumers reduce competition relative to younger and older consumers, leading to a decline in average productivity. We rationalize our results in a general equilibrium heterogeneous agent model in which firms compete within and between varieties. Younger consumers increase competition between varieties due to a higher elasticity of substitution, while older consumers increase competition within varieties because of a lower opportunity cost of time. With a larger share of middle-aged consumers, demand shifts towards less productive firms and reduces the overall incentive to post low prices and improve

[1] Angelini: University of Konstanz

productivity. Our estimates indicate that this *age demand* channel resulted in an 8.7% reduction in US GDP growth from 1995-2004 and a 10.3% increase from 2005-2019.

1.1 Introduction

Consumption behaviors differ across age categories. For instance, in terms of their shopping effort, workers face a higher opportunity cost to search for goods, and young consumers substitute goods in their baskets more frequently. When population ages, firms adapt their strategies to a changing demand, which affects competition in the goods market and contribute to aggregate trends. In this paper, we propose to answer the following questions: how do demographic shocks affect sector equilibrium through demand? What has been the contribution of aging consumers to US economic growth?

First, we propose new empirical evidence that an aging demand has a non-monotonic effect on competition at the sector level. Young and old consumers increase competition, relative to middle-aged peers. Second, we develop a general equilibrium model with three types of agents (young, middle-aged, and old), which are heterogeneous in their search costs for prices and elasticity of substitution between varieties. Firms compete within and between varieties. Third, we estimate the model on US data to match our sector-level evidence and quantify the contribution of this *age demand* channel to recent US GDP growth.

Age is a strong determinant of consumption and purchasing behaviors. Older consumers spend more time shopping and eventually pay lower prices for given variety (Aguiar and Hurst, 2005, 2007) and develop a stronger attachment to brands and, therefore, compare fewer varieties (Bornstein, 2019; Lambert-Pandraud and Laurent, 2010; Lambert-Pandraud et al., 2005).[2] At the same time population ages quickly in the United-States. The share of young adults (25-44) has decreased by ten percentage points (going from 68% in 1995 to 58% in 2020), while the shares of middle-aged (45-64) and older people (65+) have steadily increased and will continue in the coming decades. Hence, current demographic transitions could transform the market structures with potentially significant macroeconomic consequences.

To estimate the net effects of an aging demand, we need to account for all behavioral differences across age categories. Indeed, taken individually, micro-empirical evidence extrapolate to ambiguous

[2] The marketing and psychology literature investigates other dimensions determining age-specific consumption behaviors. As consumers become older, their cognitive abilities decrease (Gutchess, 2011; Peters, 2011) influencing the way they process information and compare goods; the size of their social network shrinks (East et al., 2014). Furthermore, older consumers show a lower usage rate and ability to use ICT technologies to find information (Hargittai, 2001).

aggregate effects. For instance, competition could decrease across varieties because older consumers compare fewer varieties. Concomitantly, competition could increase within variety because those same consumers exert a higher shopping effort to find low prices. We need an empirical model which captures all the behavioral differences across age categories.

To estimate the macroeconomic consequences of aging consumers, we need to measure its effects on all the sectors in the economy. Final goods sectors represent only a small share of the US GDP.[3] But shocks to final goods sectors can propagate up the value chain to intermediate good sectors.[4] We need an empirical model which captures the direct and indirect effects of an aging demand.

In the first part of the paper, we estimate the effect of a change in the age composition of consumers on all sectors through direct and indirect sales. We face two main hurdles: first, the multitude and the simultaneity of channels through which a demographic change affects an economy; second, the predictability of demographic changes. To address the first one, we exploit the age composition of foreign demand as a wedge between the age compositions of the domestic economy and the one of total demand. We can then disentangle the *age demand* channel from essentially domestic ones, such as channels going through the labor supply, the saving rate, and public finance. Using a world input-output table, we obtain precise measures of the direct and indirect exposure of each sector in each country to the demand of each foreign trading partner through its value chain. To address the predictability of demographic changes, we consider revisions in official demographic predictions as a proxy for unpredicted demographic changes. We find that aging has a non-linear effect on competition. In particular, an increase in the share of middle-aged consumers reduces the aggregate value added produced and increases the average price and the profit share of a sector. These results are in contrast with the effects of standard demand shocks that are usually associated with variations in quantities and prices in the same direction. We interpret this negative co-movement and the increased profits as evidence that demographic shocks alter the market structures through demand and that middle-aged consumers reduce, on average, competition.

Our findings are the result of distinct mechanisms already analyzed in the literature. While young consumers are less loyal and increase competition across varieties (Bornstein, 2019), older consumers, instead, have more time to search for goods and increase competition within a variety (Aguiar and Hurst, 2007). We develop a model to reconcile these channels and match our sector-level evidence.

[3]The wholesale and retail sectors account for respectively only 6% and 6.2% of GDP in the US in 2021 (source: US Bureau of Labor Statistics)

[4]see for example Ferrari (2019) for evidence about the propagation of demand shocks upstream through the value chain.

Then, we nest the sector-level calibrated model into a general equilibrium framework to account for substitution across sectors and general-equilibrium effects. We finally estimate the contribution of the age demand channel on production growth.

Our model has three types of consumers: young, middle-aged, and old, and competition within and between varieties. Young and middle-aged consumers pay a higher search cost to observe prices in each market. Middle-aged and older consumers have a lower elasticity of substitution across goods. On the supply side, we consider a continuum of sectors. Within sectors, a finite number of firms produce a single variety, compete through prices, and choose the technology to adopt. A better technology materializes into higher labor productivity. The demographic process, defined as a change in the age composition of consumers, mainly affects the economy through a re-allocation of demand across firms. Firms respond endogenously by updating their pricing and technology strategies. An increase share of consumers with higher search costs (young and middle-aged) reduces competition among firms within the same sector. An increased share of consumers with lower elasticity of substitution (middle-aged and old) reduces between-sectors competition. Since middle-aged consumers have both higher search costs and lower elasticity of substitution, an increase in the share of the middle-aged leads to an overall reduction in competition. It materializes into higher average prices and profits and lower total production.

Finally, we use the general equilibrium model to assess the relevance of the age demand channel contribution to the US growth in the last decades. We estimate our model's parameters to replicate our empirical findings with sector-specific demographic shocks. We calibrate the remaining parameters to mimic the US economy in 1995. Aggregate demographic shocks to demand are significantly mitigated compared to sector specific ones. In the former case, consumers cannot substitute to or away sectors affected by the shock. We finally estimate that changes in the age composition of consumers have contributed to a reduction in GDP growth of 8.7% in the period 1995-2004 and an increase of 10.3% in the period 2005-2019, compared to a counterfactual in which the US demography would have remained unchanged. While the reduction in GDP between 1995-2004 is due to the increase in the share of middle-aged consumers, the increased GDP in the following period is due to the baby-boomer generation retiring and progressively contributing to increasing overall competition in the economy.

This paper adds to four strands of the literature. First, this work contributes to the strand of the literature which studies macroeconomic consequences of an aging demand. Most research focus on the non-durable retail sector and extrapolate micro-evidence about specific behavioral differences to aggregate effects. The closest to our paper, Bornstein (2019), mainly rely on retail and consumer purchase surveys. Instead, we consider sectors selling directly to consumers as well as sectors indirectly affected through their value chain. Hence, we can provide a direct empirical estimation of the effect of demographic changes through demand on all sectors in the economy. Considering all possible heterogeneity of consumption behaviors across age categories, we present a novel fact: aging has a non-linear effect on competition.

Second, a growing worry centers around the possible effects of an aging population on production and productivity trends through the labor market. In this direction, several recent studies have focused on the role of automated technologies (Abeliansky and Prettner, 2017; Abeliansky et al., 2020; Acemoglu and Restrepo, 2017, 2018; Gehringer and Prettner, 2019). However, the emphasis given by the literature on the effect of an aging workforce has overshadowed the economic impact through a complementary channel: the change in the age composition of consumers, what we call the *age demand* channel. We instead show that the demand side is at least as important to understand the effect of demographic transitions on firms and economy-wide outcomes.

Third, this work contributes to the strand of the literature studying search frictions on the goods market. Seminal contribution include Butters (1977), Varian (1980), Burdett and Judd (1983) and Stahl (1989). We build on the latter and introduce sequential search in a general equilibrium model with multiple varieties and endogenous technology.[5]

Finally, our paper contributes to the growing literature showing that shocks propagate through value chains. [6] We provide preliminary evidence that competition shocks can propagate upstream.

1.2 Empirical analysis

In this section, we present a novel method to identify sector-level demographic demand shocks using trade data. The demand that a sector can address consist of a domestic and a foreign component. While domestic demographic changes affect firms strategies though several direct and indirect channels, foreign demographic changes affects firms mainly through their sales. We build a shift-share instrument with unpredicted foreign demographic changes as the shifts, and ex-ante exposure to foreign final demands as the shares. We provide preliminary evidence that demographic demand shocks propagate upstream through the value-chain. Hence, the *age demand* channel affects all sectors in the economy. We find

[5]Additional works on the macroeconomic consequences of search frictions on the goods market include Kaplan and Menzio (2016), Albrecht et al. (2021),Bai et al. (2019), Kryvtsov and Vincent (2021) Sara-Zaror (2021) among others.

[6]For empirical evidence, see among others Acemoglu et al. (2016), Barrot and Sauvagnat (2016), Ferrari (2019), Carvalho et al. (2021), Boehm et al. (2019),Dhyne et al. (2021)

1.2.1 Data

To perform our empirical analysis, we use the World Input-Output Database (Timmer et al., 2015) which provides sector-level statistics on value-added, hours worked, price deflator, and input-output tables with information on trade links at the country-sector level. The WIOD covers the 27 EU countries, 13 other major countries in the world for the period from 1995 to 2009, and it provides estimates for the rest of the world.[7] The dataset provides information over 35 sectors.

Figure (1.1) The structure of a world input-output table

			Input use & value added							Final use			Total use
			Country 1			···	Country J			Country 1	···	Country J	
			Industry 1	···	Industry S	···	Industry 1	···	Industry S				
Intermediate	Country 1	Industry 1	Z_{11}^{11}	···	Z_{11}^{1S}	···	Z_{1J}^{11}	···	Z_{1J}^{1S}	F_{11}^{1}	···	F_{1J}^{1}	Y_1^1
		···	···	Z_{11}^{rs}	···	···	···	Z_{1J}^{rs}	···	···	···	···	···
		Industry S	Z_{11}^{S1}	···	Z_{11}^{SS}	···	Z_{1J}^{S1}	···	Z_{1J}^{SS}	F_{11}^{S}	···	F_{1J}^{S}	Y_1^S
inputs	···	···	···	···	···	Z_{ij}^{rs}	···	···	···	···	F_{ij}^{r}	···	Y_i^r
supplied	Country J	Industry 1	Z_{J1}^{11}	···	Z_{J1}^{1S}	···	Z_{JJ}^{11}	···	Z_{JJ}^{1S}	F_{J1}^{1}	···	F_{JJ}^{1}	Y_J^1
		···	···	Z_{J1}^{rs}	···	···	···	Z_{JJ}^{rs}	···	···	···	···	···
		Industry S	Z_{J1}^{S1}	···	Z_{J1}^{SS}	···	Z_{JJ}^{S1}	···	Z_{JJ}^{SS}	F_{J1}^{S}	···	F_{JJ}^{S}	Y_J^S
Value added			VA_1^1	···	VA_1^S	VA_j^s	VA_J^1	···	VA_J^S				
Gross output			Y_1^1	···	Y_1^S	Y_j^s	Y_J^1	···	Y_J^S	···			

Source: World Input Output Database

The structure of the input-output tables is represented in Figure 1.1. Each row represents the country-sector supplying the input, while the columns represent the country-sector pairs buying the inputs to be used in the production process. Each cell represents the amount traded in terms of value-added. The last columns, “Final use" and “Total use" represent the amount produced by each country-sector which is consumed by each country and the total production of the country-sector respectively.[8] Using the input-output tables allows us to capture the interdependence coming from the integrated production structure of the world's economies.

To recover the demographic variables, we use the UN World Population Prospects (WPP, 1996, 2006, and 2019 revisions) which provide estimates and projections in 1996, 2006, and 2019 of the age composition of the population in past, current and future years. We use these datasets to recover

[7] Besides the 27 EU countries, the WIOD dataset contains information for Australia, Brazil, Canada, China, Indonesia, India, Japan, South Korea, Mexico, Russia, Turkey, Taiwan, and the United States. In our analysis, we use all countries and the proxy for the rest of the world. Since the UN World Population Prospect does not provide population predictions for Taiwan, we assume no demographic change in Taiwan (we cannot directly remove the observation as we need the entire input-output matrix to estimate exposures).

[8] Several variables compose the “Final use" columns (i.e. “Final consumption expenditure by households", “Final consumption expenditure by non-profit organizations serving households (NPISH)", “Final consumption expenditure by the government", “Gross fixed capital formation", “Changes in inventories and valuables"). Coherently with the scope of our analysis, we consider only the “Final consumption expenditure by households" variable.

measures of the actual demographic change (WPP 2019), and of the unpredicted demographic changes (WPP 1996 and 2006).[9]

Reduced form

In order to estimate the effect of a change in the age composition of consumers on sector aggregates (price, production, and productivity), we use a long difference fixed effect specification as in Acemoglu and Restrepo (2018).

$$\Delta log Y_i^s = \beta_0 + \beta_1 \cdot \Delta AgeDemand_i^s + \beta_2 \cdot ForeignExposure_i^s + \gamma_i + \gamma_s + \epsilon_i^s, \tag{1.1}$$

The left-hand side variables are, in turn, the log difference in sector aggregate prices, production, and productivity between 1996 and 2006.[10] We use sector-specific value-added price deflator as a proxy for the average price. It measures prices from the perspective of the producers and thus, contrary to the consumer price index (CPI), does not depend on the basket of goods and services purchased by consumers. We use real value-added and the real value-added per hour as proxies for respectively production and productivity. We estimate the sector-level profit share following van Vlokhoven (2019). The regressor of interest is the change in the age composition of demand and it is defined as the difference between the average demographics weighted by the exposure of each country-sector to each country's demand between 1996 and 2006:

$$\Delta AgeDemand_i^s = \sum_j \xi_{i,j,2006}^s \cdot age_{j,2006} - \sum_j \xi_{i,j,1996}^s \cdot age_{j,1996}, \tag{1.2}$$

where $age_j \in \{young_j, middle\text{-}aged_j, old_j\}$. We define $young_j$ as the share of population between 25 and 44, relative to the total population between 25 and 79 in country j. Similarly, we define $middle\text{-}aged_j$ as the share of population between 45 and 64, and old_j as the share of population between 65 and 79. We exclude younger and older consumers (<25 and 80+) as we focus on independent consumers.[11]

[9] source: United Nations (2022)

[10] We consider the longest available time span in the WIOD 2013 release excluding the recession years (from 2007 on); we consider 1996 instead of 1995 (the first year available in the WIOD 2013 release) in order to consistently define the dependent variables across the different model specifications as the UN WPP was published in 1996 but not in 1995.

[11] Younger consumers (<25) might still have no income and depend on their parents regarding their consumption decisions; older consumers (80+) might delegate their consumption decision choices to someone else (think, for instance, of the older in nursing homes).

To control for the heterogeneous exposure to foreign demand, we include in the regression the exposure to foreign demand ($ForeignExposure_i^s$) at the initial period (1996) defined as the sum of the exposure of each country-sector with respect to foreign countries, i.e. $\sum_{j \neq i} \xi_{i,j,1996}^s$. Furthermore, in order to control for sector-specific and country-specific trends, we include country and sector dummies (γ_i and γ_s respectively). Sectors selling to the same foreign countries are exposed to the same shocks. Therefore, following Adão et al. (2019), we cluster standard errors by the main exposure country (i.e. the country's final demand to which each country-sector sells the highest share of its value-added). Our clustering partly mitigates concerns related to this correlation.[12]

Estimation of the country-sector exposure to demand

Countries are interconnected through the global value chains of production. This implies that to estimate the sector exposure to demand one needs to track down all the intermediate trades occurring from the initial producer to the final consumer, considering both direct and indirect sales.

Consider, for example, a German car producer using tires produced in Italy. Assume the German producer sells its cars in France. Considering solely direct sales, we would not capture the exposure of the Italian tire producer to French demand. This simple example shows, particularly for very interconnected countries, the need to account for indirect sales to retrieve the correct measure of final demand exposure. Following Ferrari (2019), we define the share of output of sector s in country i that is consumed by country j:

$$\xi_{i,j}^s \equiv \frac{F_{i,j}^s + \sum_r \sum_k a_{i,k}^{s,r} F_{k,j}^r + \sum_r \sum_k \sum_g \sum_m a_{i,k}^{s,r} a_{k,m}^{r,g} F_{m,j}^g + ...}{Y_i^s}, \tag{1.3}$$

where the first term in the numerator represents the output produced by sector s in country i which is directly sold for consumption in country j; the second term represents the fraction of output produced by sector s in country i sold to any producer r in any country k which is then consumed in country j. The same logic applies to higher order terms.

$$\left[\xi_{i,j}^s\right] = (I - A)^{-1} \cdot [f_{i,j}^s]. \tag{1.4}$$

We compute the exposure of each sector s in country i to country j making use of the Leontief inverse matrix $(I - A)^{-1}$. We retrieve the Leontief inverse matrix from the Leontief input-output model

[12] In this OLS model, we identify the main exposure country also taking into account the domestic economy. In the IV models in which we consider foreign demographic changes, instead, we consider only foreign countries.

relating production (Y) with final demand (F): $Y = AY + F$, where $A \equiv [a^{s,r}_{i,j}] \equiv [Z^{s,r}_{i,j}/Y^s_i]$ is the input-output matrix. The Leontief inverse matrix relates the exposure share matrix $[\xi^s_{i,j}]$ to the final demand matrix $[f^s_{i,j}] \equiv [F^s_{i,j}/Y^s_i]$ that shows the amount of output produced by sector s in country i consumed or invested in country j as a share of the total output produced by sector s in country i.[13]

Shift-Share IV

Demographic changes affect firms through several channels. Hence, changes in the age-composition of demand are necessarily correlated with, for example, the change in the age composition of workers. The latter is a strong predictor of technology adoption which influences sectors' aggregates such as price, production, and productivity (Acemoglu and Restrepo, 2018). In order to identify the *age demand channel*, we instead focus on foreign demand. While domestic demographic changes affect firms strategies though several direct and indirect channels, foreign demographic changes affects firms mainly through exports and intermediate trades along the global value chain

We use a shift-share IV approach. We follow the methodology described in Borusyak et al. (2018); Goldsmith-Pinkham et al. (2020) and Adão et al. (2019) which requires either exogenous shares (the exposure to foreign demand) or exogenous shifts (the change in foreign demographics). The exposure to foreign demand is likely endogenous as firms choose the sectors and the countries to trade. Instead, we identify exogenous shifts. Finally, to control for the simultaneity between firms exposure and the shifts, we fix exposure shares at the initial period of our panel.

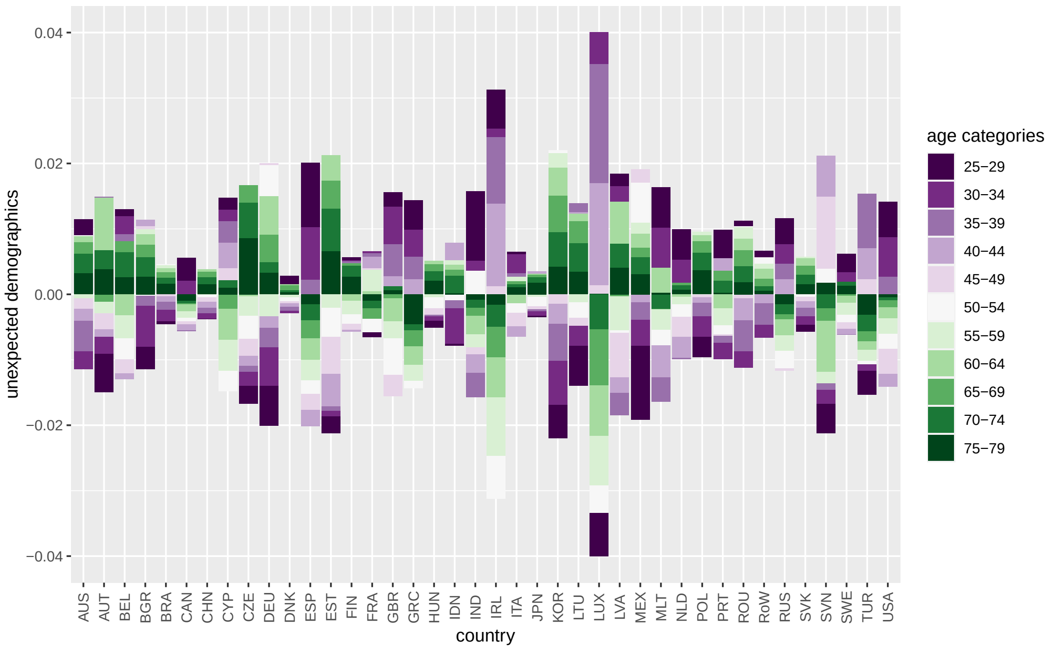
We consider two instruments. In the first model, we define the shifts as the actual demographic changes between 1996 and 2006. However, demographic changes are largely predictable. In the second model, we use the unpredicted demographics defined as the change in demographic expectations for the year 2006 between 1996 and 2006. Figure 1.2 shows the unpredicted demographics for the countries considered in the analysis and the proxy for the rest of the world (RoW). There are large variations in prediction errors across countries and age categories.[14]

Since consumers across different age categories have different preferences, as the age composition of consumers changes, the consumption basket of consumers changes as well. This could lead to a change in the consumption of domestic and foreign goods influencing the exposure of each country-sector to foreign demand. Think, for example, about older consumers that consume more health-related services

[13]The Leontief inverse matrix $(I - A)^{-1}$ has dimension $(S \times J)$ by $(S \times J)$; the exposure share matrix $[\xi^s_{i,j}]$ has dimension $(S \times J)$ by J; the input-output matrix A has dimension $(S \times J)$ by $(S \times J)$; the final demand matrix $[f^s_{i,j}] \equiv [F^s_{i,j}/Y^s_i]$ has dimension $(S \times J)$ by J where S and J represents the number of sectors and countries respectively.

[14]Figure A.1 in the Appendix shows the same unpredicted demographics with aggregate age categories as defined in the empirical specification.

Figure (1.2) Unpredicted demographic changes.



Note: this figure shows the prediction revisions for the year 2006 between 1996 and 2006. For each age category we compute its share in the total population between 25 and 79 years old. The prediction revisions are computed according to equation (1.8) from different vintages of the UN World Population Prospects.

that are usually domestically produced. In this case, the effect of a change in the age composition of foreign demand would be confounded with the effect of a change in the exposure to foreign markets. However, we don't find any significant correlation between the two (see Table A.3 in the Appendix). This is probably due to our definition of exposure to demand that takes into account not only the direct exposure to demand but also all the intermediate trades along the value chain. Our methodology requires a milder assumption, consumption baskets can differ, only the inputs which compose the final goods in the consumption baskets should be similar across age categories.

Baseline regression In this second model, we build an instrument from foreign demographic changes. We, therefore, estimate the following first stage model:

$$\Delta AgeDemand_i^s = \alpha_0 + \alpha_1 \cdot \Delta ForeignAgeDemand_i^s + \alpha_2 \cdot ForeignExposure_i^s + \gamma_i + \gamma_s + \nu_i^s \tag{1.5}$$

where $\Delta ForeignAgeDemand_i^s$ is the shift-share instrument defined as the average change in demographics between 1996 and 2006 (the shifts) weighted by the exposure of each country-sector with respect to foreign demand at the initial period (the shares), i.e.:

$$\Delta ForeignAgeDemand_i^s \equiv \sum_{j \neq i} \xi_{i,j,1996}^s \cdot [age_{j,2006} - age_{j,1996}]\,. \tag{1.6}$$

As in equation (1.2), we define age_j as the share in country j of young (24-44), middle-aged (45-64), and old (65-79) in the population between 25 and 79. The rest of the controls are defined as for equation (1.1): $ForeignExposure_i^s$ is the sum of the exposures to foreign economies controlling for the heterogeneity of the exposure to foreign demand across country-sectors; γ_i and γ_s are the country and sector dummies respectively controlling for country and sector trends.[15]

In order to ensure the exogeneity of the instrument with respect to the dependent variables in this framework, we require **a)** the exogeneity of the shifts with respect to the dependent variables (which in our framework boils down to the assumption that foreign demographics is exogenous with respect to domestic price, production, and productivity), and **b)** the independence of the shifts with respect to the shares. In order to ensure the independence of the shifts with respect to the shares, it is standard in shift-share literature to use lagged shares which shields the results from contemporaneous co-movements of the shifts and the shares. However, in our framework, this strategy might not be enough to guarantee the independence between the shifts and the shares as demographic changes are highly predictable. It could be, therefore, that demographic expectations affect the lagged exposure shares. Although very small, we find, indeed, a correlation between the shifts and the shares as defined in this framework (see Table A.2 in the Appendix). Therefore, we build a second IV model in which, instead of considering the actual demographic changes as shifts, we consider the unpredicted demographics.

Unpredicted demographic changes In this second model, we build an instrument from *unpredicted* foreign demographic changes. We, therefore, estimate the following first stage model:

$$\Delta AgeDemand_i^s = \eta_0 + \eta_1 \cdot unpredictedForeignAgeDemand_i^s + \eta_2 \cdot ForeignExposure_i^s + \gamma_i + \gamma_s + \nu_i^s, \tag{1.7}$$

where $unpredictedForeignAgeDemand_i^s$ is the shift-share instrument defined as the average unpredicted foreign demographics (the shifts) weighted by the initial exposure of each country-sector with respect to foreign demand (the shares). In particular, we define unpredicted demographic shocks as the

[15] We exclude the domestic economy from the exposure shares. But when estimating a shift-share IV with incomplete shares, i.e. the sum of the shares does not sum up to 1 , it is necessary to control for the sum of the exposure shares (Borusyak et al., 2018)

prediction revision between 1996 and 2006 for the 2006 demographic compositions (equation 1.8).

$$unpredictedForeignAgeDemand_i^s \equiv \sum_{j \neq i} \xi_{i,j,1996}^s \cdot [age_{j,2006} - \mathbb{E}_{1996}(age_{j,2006})]. \tag{1.8}$$

By fixing exposure shares at the initial period and considering the change in expectations between 1996 and 2006 regarding demographics in 2006, we ensure that our demographic shocks are unpredicted and, therefore, uncorrelated with the exposure shares.

1.2.2 Results

In this section, we analyze the results of our empirical analysis. We show that an increased share of middle-aged consumers in a sector's demand decreases production and increases prices and profits. We interpret these co-movements a reduction in competition. It materializes as well into a drop in average productivity, possibly due to a re-allocation of the demand from more productive to less productive firms or, a lower incentive to invest in production technologies.

Prices We proxy the average price by the value-added price deflator. Our results suggest that middle-aged consumers are associated with an increase in prices, while older consumers are associated with a reduction in prices (Table 1.1). In particular, one standard deviation increase (2.20%) in the share of middle-aged consumers is associated with an increase in prices between 0.70% (OLS specification) and 2.28% (IV(2) specification) per year in the period between 1996 and 2006, while an increase in one standard deviation of older consumers (1.45%) is associated with a reduction in prices between 0.64% (OLS specification) and 2.10% (IV(2) specification) per year in the same period.[16]

Production We proxy total production by total real value-added. Our results suggest that there is a positive association between young and old consumers and production (although not significant for old

[16] We compute annualised changes in price using the following formula:

$$\pi_t = 100 \times \left(1 + \frac{\%\Delta y}{100}\right)^{\frac{1}{2006-1996}} - 100, \tag{1.9}$$

where $\%\Delta y$ is the cumulative percentage change in price (over the period 1996-2006) which is computed as:

$$\%\Delta y = 100 \cdot (e^{\hat{\beta}_1 \cdot \Delta x} - 1), \tag{1.10}$$

where $\hat{\beta}_1$ is the coefficient estimates for the different model specifications in Table 1.1, and Δx is set to be equal to one standard deviation in the domestic change between 1996 and 2006 of the share of the age category analyzed. Similarly, we estimate the average annual changes for production and productivity using the coefficient estimates in Tables 1.2 and 1.4.

Table (1.1) Price

	Δlog Price (1996-2006)								
		OLS			*IV(1)*			*IV(2)*	
share:					Foreign Exposure			Foreign Exposure	
shift:					ΔDemographics (1996-2006)			ΔExpectations (1996-2006)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ΔYoung Demand	1.034			−9.946			−12.859		
	(0.670)			(8.959)			(14.195)		
ΔMiddle-aged Demand		3.162***			4.247			10.212**	
		(0.750)			(2.895)			(4.440)	
ΔOld Demand			−4.437***			−4.843**			−14.597*
			(0.939)			(2.110)			(8.571)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R^2	0.884	0.885	0.885	0.859	0.885	0.885	0.845	0.877	0.875
Adjusted R^2	0.877	0.878	0.879	0.851	0.878	0.879	0.835	0.870	0.868

*p<0.1; **p<0.05; ***p<0.01

Note: OLS refers to the regression model in equation (1.1), IV(1) and IV(2) refers to the use of, respectively, foreign demographic change (equation 1.5) and unpredicted foreign demographic changes (equation 1.7) as instruments for total demographic changes. Our proxy for the average price is the value added deflator. Errors are clustered at the country to which each sector is the most exposed.

consumers) and a negative and significant relation between middle-aged and production (table 1.2). In particular, one standard deviation increase (2.20%) in the share of middle-aged consumers is associated with a reduction in production between 0.69% (OLS specification) and 1.81% (IV(2) specification) per year in the period between 1996 and 2006.

Profits We compute sector-level profit shares following van Vlokhoven (2019). Our results suggest that middle-aged consumers generally increase profits while older consumers reduce them (table 1.3)

.

Mechanism

We find strong evidence that middle-aged consumers are associated with an increase in prices and a reduction in production. These results mean that changes in the age composition of consumers are non-standard demand shocks. This negative co-movement and the negative effect on profits suggests instead that the *age demand* channel changes the competition framework. This is in line with Bornstein (2019) showing that, compared to young consumers, middle-aged consumers re-optimize their consumption basket less often making it more difficult for entrants to establish a customer base and, therefore, reducing competition in the market. However, in contract to his results, we also find that older consumers are associated with lower prices and higher production suggesting that an increase in

Table (1.2) Production

	Δ*log* Production (1996-2006)								
	OLS			*IV(1)*			*IV(2)*		
share:				Foreign Exposure			Foreign Exposure		
shift:				ΔDemographics (1996-2006)			ΔExpectations (1996-2006)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ΔYoung Demand	0.029			10.339			12.395*		
	(0.804)			(7.483)			(6.920)		
ΔMiddle-aged Demand		−3.143***			−3.647			−8.308***	
		(0.901)			(2.545)			(2.878)	
ΔOld Demand			0.838			2.685			19.627
			(1.135)			(4.131)			(20.029)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R^2	0.578	0.582	0.578	0.524	0.582	0.577	0.500	0.571	0.487
Adjusted R^2	0.553	0.557	0.553	0.496	0.557	0.552	0.470	0.546	0.457

*p<0.1; **p<0.05; ***p<0.01

Note: OLS refers to the regression model in equation 1.1, IV(1) and IV(2) refers to the use of, respectively, foreign demographic change (equation 1.5) and unpredicted foreign demographic changes (equation 1.7) as instruments for total demographic changes. Our proxy for total production is real value added. Errors are clustered at the country to which each sector is the most exposed.

Table (1.3) Profit share

	Δ*log* Profit share (1996-2006)								
	OLS			*IV(1)*			*IV(2)*		
share:				Foreign Exposure			Foreign Exposure		
shift:				ΔDemographics (1996-2006)			ΔExpectations (1996-2006)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ΔYoung Demand	1.171			−8.512			−109.172		
	(3.046)			(16.443)			(181.879)		
ΔMiddle-aged Demand		18.744***			21.105*			46.099**	
		(3.181)			(12.123)			(21.020)	
ΔOld Demand			−31.190***			−51.785***			−49.142***
			(3.984)			(7.967)			(8.965)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	917	917	917	917	917	917	917	917	917
R^2	0.269	0.297	0.318	0.260	0.297	0.297	−0.870	0.236	0.302
Adjusted R^2	0.205	0.237	0.259	0.196	0.236	0.236	−1.032	0.170	0.241

*p<0.1; **p<0.05; ***p<0.01

Note: OLS refers to the regression model in equation 1.1, IV(1) and IV(2) refers to the use of, respectively, foreign demographic change (equation 1.5) and unpredicted foreign demographic changes (equation 1.7) as instruments for total demographic changes. We compute profit shares following van Vlokhoven (2019). Errors are clustered at the country to which each sector is the most exposed.

the share of the older consumers increases competition. This result is consistent with Aguiar and Hurst (2007) showing that older consumers tend to face lower prices as they have a lower opportunity cost and, therefore, lower search costs. Our findings would be the results of those combined mechanisms.

Table (1.4) Productivity

	Δlog Productivity (1996-2006)								
	OLS			*IV(1)*			*IV(2)*		
share:				Foreign Exposure			Foreign Exposure		
shift:				ΔDemographics (1996-2006)			ΔExpectations (1996-2006)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ΔYoung Demand	0.955			12.792			28.810**		
	(0.869)			(13.172)			(14.553)		
ΔMiddle-aged Demand		−5.239***			−7.261			−19.170***	
		(0.968)			(5.083)			(3.726)	
ΔOld Demand			2.531**			11.732			46.126
			(1.226)			(8.593)			(48.222)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R^2	0.517	0.527	0.518	0.447	0.526	0.497	0.128	0.450	0.040
Adjusted R^2	0.488	0.499	0.490	0.414	0.498	0.467	0.077	0.418	−0.016

*p<0.1; **p<0.05; ***p<0.01

Note: OLS refers to the regression model in equation 1.1, IV(1) and IV(2) refers to the use of, respectively, foreign demographic change (equation 1.5) and unpredicted foreign demographic changes (equation 1.7) as instruments for total demographic changes. Our proxy for productivity is real value added per hours worked. Errors are clustered at the country to which each sector is the most exposed.

We also find that a decrease in competition materializes into a drop in average productivity (table 1.4). Lower competition reallocates demand from low to higher pricing firms. Assuming the existence of a correlation between price and productivity at the firm level, an increased share of middle-aged consumers would indeed worsen misallocation in the economy and reduce aggregate productivity. The trade literature provides extensive evidence about the link between competition and misallocation. Alternatively, lower competition could also reduce the incentive to invest in better production technologies. Those two channels are not mutually exclusive but could instead reinforce each others.

1.2.3 Robustness Checks

We perform a series of robustness checks. In particular, we run our baseline regression models changing the age category thresholds, [25-39, 40-59, 60-79] instead of [25-44, 45-64, 65-79]. We, then also consider a different dataset (the WIOD 2016 release) and estimate the baseline regression models in the period 2006-2014 instead of 1996-2006. Overall, the results are robust especially for price and quantities (tables A.7 and A.6 in the appendix).

We also consider the change in expectations regarding future demographics.

$$unpredictedForeignAgeDemand_i^s \equiv \sum_{j \neq i} \xi_{i,j,1996}^s \cdot \left[\mathbb{E}_{2006}\left(age_{j,t}\right) - \mathbb{E}_{1996}\left(age_{j,t}\right)\right], \qquad (1.11)$$

where $t \in \{2010, 2015, 2020\}$. Table A.5 shows that the results obtained considering the change in expectations with respect to future demographics are fully consistent with the results of the baseline IV(2) model. Indeed, we find that all the significant coefficient estimates in the baseline IV(2) model remain significant when considering expectations regarding future demographics. The point estimates, also, are very similar and declining as we consider demographic expectations further in the future. Intuitively, the economy responds stronger to demographic unpredicted changes that are closer in time.

1.3 Theory

To rationalize the empirical results, we build a multi-sector model general equilibrium model with agents heterogeneous in their age.

1.3.1 Sector-level Model

In this section, we study the sector-level effects of a change in the age composition of demand on the market structure, the allocation of demand, the firms' strategies, and the key macroeconomic variables such as price, production, and productivity.

The economy consist of a continuum of sectors each populated by a finite number of ex-ante identical profit-maximizing firms producing out of labor an homogeneous final good. Firms compete through prices and set their level of investment in a labor-saving technology. Young, middle-aged, and old consumers search for prices sequentially. Young and middle-aged consumers have higher search costs relative to older consumers, and middle-aged and old consumers have lower elasticity of substitution with respect to young consumers.

Optimal Consumer Search

The economy is populated by a fraction λ^Y of young consumers, a fraction λ^M of middle-aged consumers, and a fraction λ^O of old consumers such that $\lambda^Y + \lambda^M + \lambda^O = 1$. Young and middle-aged consumers have high search costs, while old consumers have low search costs. In particular, we assume that older consumers face zero search costs, i.e., $s^O = 0$, while young and middle-aged consumers pay $s^Y = s^M \equiv \bar{s} > 0$ to sequentially observe prices.

In each sector, consumers observe the first price for free. Consumers with positive search costs find it profitable to continue searching if the expected benefits from searching exceed the costs. Given the lowest previously observed price z, we define the consumer surplus of type $j \in \{Y, M\}$ of finding a price $p < z$ as:

$$CS^j(p;z) \equiv \int_p^z D^j(x)\,dx, \tag{1.12}$$

where we allow agents to have age-specific demand functions $D^j(x)$. In particular, we assume that agents have CES preferences so that the sector-level demand depends on the age-specific elasticity of substitution.[17] The expected consumer surplus for a young and a middle-aged consumer of randomly observing another price is then the integral of the consumer surplus over the price distribution $F(p)$:

$$ECS^j(z) \equiv \int_b^z CS^j(p;z)\,dF(p), \tag{1.13}$$

where b is the minimum price in the distribution.

Definition 1.3.1 (Reservation price and search rule)**.** *We define the reservation price for the consumer of type j, r^j, such that $ECS^j(r^j) = \bar{s}$ if $ECS^j(p)$ has a root, $r^j = +\infty$ otherwise. This implies the following search rule for consumer of type j:*

- *if $p \leq r^j$, the consumer of type j stops its search and purchase at price p;*
- *if $p > r^j$, the consumer of type j continues to search;*
- *if $p > r^j$ for all firms, the consumers of type j picks the lowest observed price.*

The optimal search rule for young and middle-aged consumers states that the consumer of type j continues searching if the lowest observed price, p, is greater than the type-specific reservation price r^j, and purchases if the observed price is lower than that. If all firms have prices exceeding r^j, then the consumer picks the lowest price observed.[18]

Old consumers pay a zero search cost, so they do not stop searching until they observe $p = b$.[19] However, as we show in a later section, the event $p = b$ has zero probability of occurring meaning that old consumers will observe all the prices of the N firms in each sector and purchase at the lowest price.

[17] We micro found the sector-level age-specific demands in the general equilibrium framework.

[18] As will become clear in what follows, the definition of the indifference case $z = r^j$ is not relevant since, as in equilibrium, the probability of a firm setting price equal to r^j is null.

[19] To rule out the monopolistic price equilibrium in which all firms set the price equal to the monopolistic price, we assume that old consumers stop searching only once they observe twice the lowest price b. Otherwise, firms could all charge the monopolistic price, and the old consumers would stop at the first observation since the lowest price is the monopolistic price. Since the old consumers stop searching only once they face the lowest price twice, the monopolist price equilibrium cannot be sustained since firms have a profitable deviation in marginally lowering the price to attract all the old buyers.

Firms

Production Technology In each sector, a finite number $N \geq 2$ of ex-ante identical firms produce a homogeneous good and compete through prices. Each firm produces goods using only labor ℓ with a constant return to scale technology:

$$y^s = \alpha(a) \cdot \ell, \tag{1.14}$$

with $\alpha(a)$ being the productivity of labor which depends on the amount of technological adoption, a, which is a choice variable of the firm. A higher technological adoption leads to higher productivity and reduces the marginal costs. In particular, we define marginal costs as:

$$c(a) \equiv \frac{w}{\alpha(a)} = w \cdot (\bar{a} - a), \tag{1.15}$$

where $\bar{a}$ is a technology parameter, w is the wage paid for one unit of labor, and $(\bar{a} - a)$ is the amount of labor required to produce one unit of good. This implies that a technology a reduces the unit labor requirement by a. We assume quadratic technological costs, i.e.:

$$z(a) = \frac{a^2}{\bar{z}}, \tag{1.16}$$

where $\bar{z}$ is a technology cost parameter.

Expected Demand In order to recover the expected demand by each firm in equilibrium, we consider symmetric Nash Equilibria defined similarly as Chen and Zhang (2011):

Definition 1.3.2 (Equilibrium Definition)**.** *A symmetric Nash Equilibrium is a vector $\{F(p), G(a), r^Y, r^M\}$, where $F(p)$ is the price distribution function in the support $[b, \bar{r}]$ and $G(a)$ is technology adoption distribution function. Given the reservation prices of young and middle-aged, and that other firms adopt $F(p)$ and $G(a)$, it is optimal for every firm to choose $F(p)$ and $G(a)$; and given $F(p)$ and $G(a)$, young and middle-aged consumers optimally search sequentially with reservation price r^Y and r^M respectively.*

The game does not have any pure-strategy equilibria and it is atomless on its support. Intuitively, if a measure of firms set the same price, a firm can increase profits by marginally undercutting this price in order to attract consumers with zero search costs that observe all prices, so that a price distribution with mass points cannot be optimal.

Lemma 1.3.3. *Given the NE-distribution of prices F, and the following condition:*

$$\begin{cases} \mathbb{E}\{\pi(p,F)\} > \frac{\lambda^M}{N} R^M(\hat{p}) - z\left(a(\hat{p})\right) & \text{if } r^Y \leq r^M \\ \mathbb{E}\{\pi(p,F)\} > \frac{\lambda^Y}{N} R^Y(\hat{p}) - z\left(a(\hat{p})\right) & \text{if } r^Y > r^M, \end{cases} \tag{1.17}$$

where $R^j(p) \equiv D^j(p)(p-c(a))$ is the revenues per buyer of type $j \in \{Y,M,O\}$, $D^j(p)$ is the demand of agent of type j, and $\hat{p} \in (min\{r^Y, r^M\}, max\{r^Y, r^M\}]$, then the support of the price distribution is bounded above by $\bar{r} = min\{r^Y, r^M, p^{monopoly}\}$.

Proof. in Appendix A.2.1. □

Lemma 1.3.3 sets the limit of the upper bound of the equilibrium distribution of prices which cannot be greater than the monopolistic price (a price above the monopolistic price can never be optimal) and it needs to be equal to the smallest between the reservation prices if $min\{r^Y, r^M\} < p^{monopoly}$. Indeed, a firm setting a price above $max\{r^Y, r^M\}$ would face a zero demand and makes zero profits which is not an equilibrium. Moreover, inequalities (1.17) set the condition under which there is not profitable deviation in setting a price $\hat{p} \in (min\{r^Y, r^M\}, max\{r^Y, r^M\}]$, i.e., a firm has no incentive in only targeting those agents with the highest reservation price as it would make lower profits than setting a price below or equal to $min\{r^Y, r^M\}$. Given the condition (1.17) and that $min\{r^Y, r^M\} < p^{monopoly}$, then the upper bound of the distribution is given by $min\{r^Y, r^M\}$ since a firm setting a price $\bar{r} < min\{r^Y, r^M\}$ would be able to sell to the same number of consumers by setting the price to $min\{r^Y, r^M\}$ which is closer to the monopolistic price that maximizes the profits. Similar condition can be found in **?**. We assume condition (1.17) to hold and verify numerically our conjecture ex-post.

Given Lemma 1.3.3, for a given price $p < p^{monopoly}$, each producer faces the following expected demand:

$$\mathbb{E}\{y^d(p)\} = \frac{\lambda^Y}{N} \cdot D^Y(p) + \frac{\lambda^M}{N} \cdot D^M(p) + \lambda^O \left[1 - F(p)\right]^{N-1} \cdot D^O(p). \tag{1.18}$$

The expected demand function is composed of three terms: the first two are the demands coming from young and middle-aged consumers, while the last one is the demand coming from older consumers. Since the upper limit of the equilibrium price distribution is $min\{r^Y, r^M\}$, young and middle-aged consumers randomly observe a single price, so they are equally assigned across firms and each firm expects λ^Y/N young and λ^M/N middle-aged consumers. Old consumers, instead, have zero search costs, observe all the prices, and purchase at the lowest pricing firm in each sector. $[1 - F(p)]^{N-1}$ is, indeed, the probability that a firm setting price p is the lowest pricing firm in the sector.

Profit Maximization Problem The timing of a firm's decisions runs as follows:

1. the firm chooses a strategy in terms of prices and technology adoption (p,a) given the strategies of the other firms and taking into account the expected demand;
2. agents search and the demand realizes;
3. the firm produces and sells.

The producers choose the price and the technology adoption level in order to maximize profits. We define the expected profits as:

$$\mathbb{E}\{\pi(p,F)\} \equiv \max_{\{a,p\}} \mathbb{E}\{y^d(p)\} \cdot (p - c(a)) - z(a). \tag{1.19}$$

Since in equilibrium the expected profits need to be the same (say π) for any price in the support of the price distribution, it must hold that:

$$\mathbb{E}\{\pi(p,F)\} = \pi \quad \forall\, p \in [b,\bar{r}]. \tag{1.20}$$

Proposition 1.3.4 (Equilibrium Technology Adoption)**.** *Given the optimality condition from the maximization problem (1.19), and the equilibrium condition (1.20), we get the equilibrium technology adoption:*

$$a^*(p) = \left[\left(\frac{p}{w} - \bar{a}\right)^2 + \bar{z}\pi\right]^{\frac{1}{2}} - \left(\frac{p}{w} - \bar{a}\right), \tag{1.21}$$

which is positive and decreasing in price.

Proof. in Appendix A.2.2 □

This proposition allows a first characterization of firms. First, firms always adopt a positive amount of technology; this holds also for firms setting the price equal to $\bar{r}$ that aim at attracting solely consumers observing one price. The model also predicts a negative relationship between price and productivity which implies a positive relationship between size (in terms of total sales) and productivity.

Equilibrium Price Distribution Characterization At price $\bar{r}$, $F(\bar{r}) = 1$ a firm is the highest pricing firm in the sector which means that it is not able to attract any old consumer as older buyers observe all prices and purchase at the lowest pricing firm. Therefore:

$$\mathbb{E}\{\pi(\bar{r},F)\} = \frac{\lambda^Y}{N} \cdot R^Y(\bar{r}) + \frac{\lambda^M}{N} \cdot R^M(\bar{r}) - z(a(\bar{r})). \tag{1.22}$$

Since in equilibrium all prices in the support of F give the same expected profits, it must hold that $\mathbb{E}\{\pi(\bar{r},F)\} = \mathbb{E}\{\pi(p,F)\}$ from which we get the equilibrium cumulative distribution of prices:

$$F(p) = 1 - \left[\frac{\lambda^Y\left(R^Y(\bar{r}) - R^Y(p)\right) + \lambda^M\left(R^M(\bar{r}) - R^M(\bar{p})\right)}{N \cdot \lambda^O R^O(p)} - \frac{z(a(\bar{r})) - z(a(p))}{\lambda^O R^O(p)}\right]^{\frac{1}{N-1}}. \tag{1.23}$$

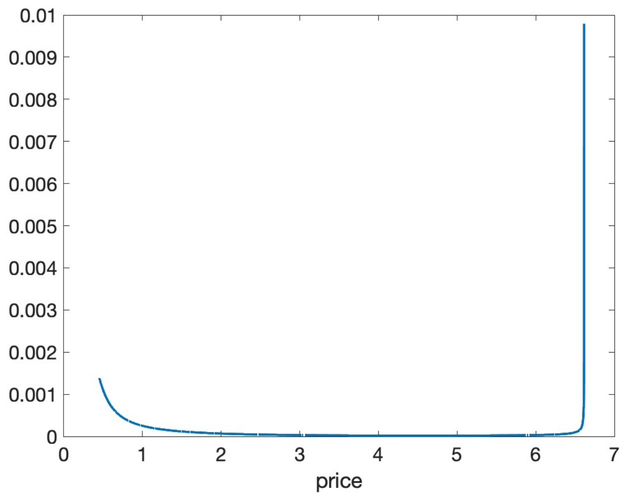
To complete the characterization of $F(p)$, we pin down the lower bound of the price distribution, b, using the fact that $F(b) = 0$.

Sector-level Analysis

In this section, we describe the equilibrium price distribution and analyze how the model works highlighting the different mechanisms linking the age composition of consumers to the allocation of demand across firms, firms' strategies, and productivity. To simplify the analysis and highlight the mechanisms going through the search cost and elasticity of substitution heterogeneity, we assume that each agent faces the same budget constraint. We relax this assumption in the numerical analysis in which we calibrate the age-specific wealth using US data.[20] We assign values to the elasticity of substitution such that $\bar{\sigma} \equiv \sigma^Y > \sigma^M = \sigma^O \equiv \underline{\sigma}$ which is in line with the evidence in Bornstein (2019) showing that middle-aged and old consumers have similar consumption inertia which is significantly higher than the one of young consumers, and the evidence in Lambert-Pandraud et al. (2005) showing that middle-aged and old consumers show higher brand loyalty with respect to young consumers. We estimate those values later in the general equilibrium context. We assume that middle-aged consumers have a lower reservation price so that the upper bound of the price distribution is equal to the reservation price of middle-aged consumers, i.e., $\bar{r} = r^M$. Intuitively, since middle-aged consumers have a lower elasticity of substitution between goods produced in different sectors, once they observe a high price, they are more willing (relative to young consumers) to pay the search costs to observe another price. Young consumers would find instead optimal to reduce the consumption of the good with a high price

[20]Since there is an infinite number of sectors, the fraction of the wealth that is allocated to each sector is infinitesimal. Therefore, the budget constraint does not play any role at the sector level. The budget constraint will be, instead, endogenous and relevant in the general equilibrium framework as the total wealth in the economy will depend on the equilibrium profits, wage, and technology adoption costs.

Figure (1.3) Probability distribution of price strategies



and purchase more of another good without having to pay search costs. We verify numerically ex-post that reservation prices fulfil this order.

Equilibrium Price Distribution The equilibrium price distribution (Figure 1.3) of strategies is U-shaped as in Stahl (1989) and presents two larger density areas of strategies. The first one, an almost mass point at the upper bound of the price distribution, includes all the strategies focused on young and middle-aged buyers who randomly observe only one price. Firms that set the price close to the upper bound of the distribution attract few buyers, sell few products per buyer and use low-productivity technologies. These correspond to what is commonly named mom-and-pop shops.

The second density area, close to the minimum price, includes the strategies focused on old buyers that observe all prices and, therefore, purchase at the lowest price in the sector. These firms compete to attract more buyers, use high-productivity technology as they are able to spread technology costs over a large production.

Comparative statics We now analyze the effects of a demographic change in our economy. We keep, in turn, the share of different age categories constant in order to highlight the different mechanisms at play (see Appendix A.2.7 for the related graphical analysis) . Demographic changes usually affect the shares of all the age categories meaning that the actual effect of a demographic change is a mixture of the effects we depict in this analysis.

Increase in Middle-aged keeping Young constant We consider now an increase in the share of middle-aged consumers keeping the share of young consumers constant. The share of consumers with high search costs ($\lambda^Y + \lambda^M$) increases while the share of consumers with low elasticity of substitution ($\lambda^M + \lambda^O$) stays constant. Therefore, the effect on the economy coming from this particular demographic process is solely determined by the change in the share of consumers with high search costs. An increase in the share of consumers with high search costs affects the economy in two ways. First, it changes the allocation of demand across firms as a larger share of consumers will only observe one price and, therefore, will be randomly allocated to firms. This *allocation effect* mechanically allocates a higher demand to firms setting higher prices that have a low level of productivity leading to an average increase in prices, an average reduction in production, and an average reduction in productivity. Second, given the different allocation of demand, firms will experience a lower within-sector competition and will update their strategies favoring high price and low technology adoption strategies since a larger share of consumers will randomly pick a price. This *strategy effect*, therefore, leads to higher average prices, lower production, and lower productivity. Since the two effects go in the same direction, an increase in the share of middle-aged consumers keeping the share of young consumers constant leads to an average increase in price, and a reduction in production and productivity.

Increase in Middle-aged keeping Old constant An increase in the share of middle-aged consumers keeping the share of old consumers constant means that the share of young consumers reduces to fully compensate the increase in middle-aged, i.e., $\uparrow \lambda^M = \downarrow \lambda^Y$. This implies that the share of consumers with high search costs ($\lambda^Y + \lambda^M$) stays constant, while the share of consumers with low elasticity of substitution ($\lambda^M + \lambda^O$) increases. The effect on the economy coming from this type of demographic process is, therefore, determined only by the increase in the share of consumers with low elasticity of substitution. As the firms in the sector observe an increase in the share of consumers with low elasticity of substitution, they will exploit the lower between-sectors competition favoring higher prices and lower technology adoption strategies leading to an average increase in prices, and a reduction in production and productivity (in this case only the *strategy effect* is at play).[21]

Increase in Young keeping Middle-aged constant An increase in the share of young consumers keeping the share of middle-aged consumers constant implies a reduction in the share of old consumers such that $\uparrow \lambda^Y = \downarrow \lambda^O$. This implies an increase in the share of consumers with high search costs ($\lambda^Y + \lambda^M$) and a reduction in the share of consumers with low elasticity of substitution ($\lambda^M + \lambda^O$). While the increase in the share of consumers with high search costs leads to a reduction in the within-sector competition, the reduction in the share of consumers with low elasticity of substitution increases

[21] See Appendix A.2.7

the between-sectors competition. Since the reduction in the within-sector competition and the increase in the between-sectors competition have opposite effects on prices, production, and productivity, the net effect on those variables is uncertain.

1.3.2 General Equilibrium Model

We embed the sector-level model into a multi-sector general equilibrium framework which allows us to retrieve estimates of the effect of a demographic demand change taking into account general equilibrium effects and substitution across sectors. We first derive the consumers' demands for each sector solving the household utility maximization problem. Aggregating the sector-level demands, we then recover the aggregate demand and the aggregate labor demand.

Household Demand

Each household $j \in \{Y, M, O\}$ gets utility by consuming an aggregate consumption good, $U(C^j) = C^j$, composed by aggregating goods from sector $s \in [0, \mathcal{S}]$ with $\mathcal{S} = 1$, through a CES consumption aggregator. We assume that agents have age-specific preferences. In particular, we assume that agents differ in terms of their elasticity of substitution between goods across different sectors, σ_j. Therefore, agent j faces the following utility maximization problem:

$$max_{\{c_s^j\}}\, C^j \equiv \left[\int_s (c_s^j)^{\frac{\sigma_j - 1}{\sigma_j}} ds\right]^{\frac{\sigma_j}{\sigma_j - 1}} \tag{1.24}$$

$$\text{s.t.} \quad I^j = \widetilde{P}^j \cdot C^j, \tag{1.25}$$

where $I^j \equiv \phi^j \cdot W$ is the income of agent j and ϕ^j is the share of the total wealth (W) in the economy held on average by each agent j. The total wealth in the economy is endogenous and it is given by the sum of the total labor costs, total profits, and total technology costs. We can interpret technology costs as the capital income. $\widetilde{P}^j \equiv [\int_s (\widetilde{p}_s^j)^{1-\sigma_j} ds]^{\frac{1}{1-\sigma_j}}$ is the normalized price aggregator.[22] The price aggregators are age-specific as agents differ in terms of both their search costs and their elasticity of substitution. From the maximization problem (1.24), we obtain the sector-specific demand for agent of

[22]See Appendix A.2.4.

type j:

$$D^j(p) \equiv c_s^j = C^j \left(\frac{\widetilde{p}_s^j}{\widetilde{P}^j} \right)^{-\sigma_j}. \tag{1.26}$$

See Appendix A.2.5 for the derivation.

Aggregation and the General Equilibrium

The production in each sector fulfills the market clearing condition. Therefore, the aggregate production in the economy Q is simply the aggregation of the sector-level demand:

$$Q = \int_s \int_j (c_s^j) dj ds = \int_j \left(\frac{C^j}{\widetilde{P}^{j^{-\sigma_j}}} \int_s (\widetilde{p}_s^j)^{-\sigma_j} ds \right) dj. \tag{1.27}$$

Since all the sectors are identical, the conditional to type $j \in \{Y, M, O\}$ expected price in each sector is the same (i.e., $\int_s (\widetilde{p}_s^j)^{-\sigma_j} ds = \mathbb{E}^j[\widetilde{p}^{-\sigma_j}]$).[23] We can rewrite the aggregate production as:

$$Q = \sum_j \lambda^j \frac{C^j}{\widetilde{P}^{j^{-\sigma_j}}} \mathbb{E}^j[\widetilde{p}^{-\sigma_j}], \tag{1.28}$$

where $\mathbb{E}^j[\widetilde{p}^{-\sigma_j}]$ is the expectation over $\widetilde{p}^{-\sigma_j}$ given that the agent is of type j. In particular, since young and middle-aged consumers randomly observe a price, they face the same price distribution as the price strategy distribution of firms, i.e., $F^Y(p) = F^M(p) = F(p)$. Old consumers, instead, observe all prices and buy in each market at the lowest price which means that out of N independent draws from $F(p)$, they pick the lowest one. Therefore:

$$F^O(p) = 1 - (1 - F(p))^N, \tag{1.29}$$

$$f^O(p) = \frac{\partial F^O(p)}{\partial p} = N(1 - F(p))^{N-1} f(p), \tag{1.30}$$

where $f(p)$ is the probability distribution function of equilibrium prices.

In order to highlight the effect of a demographic change on the market structure through the demand channel, we fix the labor supply, i.e., $ALS = 1$. On the labor demand side, using the market clearing

[23] See Appendix A.2.6.

conditions at the sector level, we have that the aggregate labor demand is:

$$ALD = \int_s \int_j \frac{c_s^j}{\alpha_s^j}\, dj ds = \int_j \left(\frac{C^j}{\widetilde{P}^{j^{-\sigma_j}}} \int_s (\widetilde{p}_s^j)^{-\sigma_j} \cdot (\bar{a} - a^*)\, ds \right) dj. \tag{1.31}$$

As for the aggregate production, we can rewrite the aggregate labor demand as:

$$ALD = \sum_j \lambda^j \frac{C^j}{\widetilde{P}^{j^{-\sigma_j}}} \mathbb{E}^j \left[\widetilde{p}^{-\sigma_j} \cdot (\bar{a} - a^*) \right]. \tag{1.32}$$

Finally, the wage adjusts such that the aggregate labor demand equals the aggregate labor supply.

1.4 Numerical Analysis

In this section, we present our calibration strategy and numerically analyze how the economy reacts to demographic changes taking into account the general equilibrium effects and substitution across sectors. Finally, we compare the general equilibrium results with those of the sector-level model.

1.4.1 Calibration

We have 9 parameters in the general equilibrium model: a search cost parameter ($\bar{s}$) a technology parameter ($\bar{a}$), a technology cost parameter ($\bar{z}$), two elasticity of substitution parameters ($\underline{\sigma}, \bar{\sigma}$), the number of firms N, and three budget constraint parameters (ϕ^j for $j \in \{Y, M, O\}$). We calibrate the budget constraint parameters on US wealth data so that the income of agent j is equal to the average wealth of a consumer in the j age category in 1995.

The remaining parameters, $\{\bar{s}, \bar{a}, \bar{z}, \underline{\sigma}, \bar{\sigma}, N\}$, are particular to the model presented and we estimate them targeting the empirical results in Chapter 2. Since the empirical results in Chapter 2 are sector-level results, we use the sector-level model to target such relations. We, then, nest the calibrated sector-level model into the general equilibrium framework to recover estimates of the effect of a demographic change that also takes into account general equilibrium effects and substitution across sectors. We target coefficient estimates in our preferred specification, i.e., the *IV(2)* model specification. The estimated and calibrated parameters are in Appendix A.3.

1.4.2 Analysis

We analyze the effect of a demographic change on production for both the sector-level (SL) model and the general equilibrium (GE) model. The ratio of the effects (i.e., $\frac{GE}{SL}$) provides a measure of

Table (1.5) The relative effects of sector-specific and economy-wide demographic shocks on production

	Demographic change	Sector-level (SL)	General Equilibrium (GE)	Ratio ($\frac{GE}{SL}$)
Production	$\uparrow \lambda^M, \downarrow \lambda^Y$	-3.37	-0.52	0.16
	$\uparrow \lambda^M, \downarrow \lambda^O$	-4.05	-0.54	0.13
	$\uparrow \lambda^O, \downarrow \lambda^Y$	0.79	-0.041	-0.052

Note: The effect on production of a one-standard-deviation increase in the share of an age-category, i.e. $2.2\% \times \lambda^M$ for middle-aged consumers and $1.45\% \times \lambda^O$ for old consumers. Sector-level shocks replicate

the dampening or the amplifying effect of the multi-sector general equilibrium model. We focus our analysis on production for two reasons: first, our empirical evidence regarding the effect of the age demand channel on production is the most robust; second, we model the technology adoption in a way that allows us to keep the model tractable but might not capture all the relevant effects driving the change in productivity. Although the model predictions for productivity are qualitatively similar to those in our empirical estimation, they are substantially smaller than those we document empirically.

Table 1.5 shows the effect of a *clean* demographic change (i.e., an increase in one standard deviation increase in λ^i compensated by a reduction of λ^j where $i \neq j$ and $i, j \in \{Y, M, O\}$). Considering one standard deviation increase in the share of middle-aged consumers compensated by a reduction in young consumers, the sector-level model predicts a reduction in GDP growth of 3.37 percentage points, while the general equilibrium model predicts a reduction of 0.52 percentage points dampening the effect by 84%. Similarly, for an increase in the middle-aged at the expense of old consumers the general equilibrium model dampens the negative effect on GDP growth by 87%. The dampening effect is due to the fact that, when demographic changes affect a single sector consumers can substitute their consumption to or from this sector. Instead, in the general equilibrium model, demographic changes hits the entire economy and substitution effects are reduced. When we consider an increase in the share of old consumers compensated by a reduction in young consumers, we find a substantially smaller effect at the sector level with respect to the effect of an increase in middle-aged and virtually no effect in the general equilibrium model. The increased within-sector competition offesets the reduction of competition between-sectors.

Relating the empirical findings to the general equilibrium results We can now reinterpret the empirical results in Chapter 2 taking into account the general equilibrium effects and the correct substitution across sectors. We find that when considering an increase in the share of the middle-aged, the general equilibrium model dampens the estimates of the sector-level model as it embeds substitution effects across sectors. In order to interpret our empirical results taking into account the substitution across sectors, we pass them through the $\frac{GE}{SL}$ ratio that allows us to capture the dampening effect of the general equilibrium. In the empirical analysis, the underlining assumption was to analyze the effect of an increase in one of the age categories keeping the share of the other age categories constant. This means that the increase in one of the age categories was at the expense of both the others in such a way as to keep their ratio constant. In order to remain consistent with this assumption, we decompose the demographic change into two *clean* demographic shocks weighting the effect of the change in one age category by its relative initial share. Focusing on the middle-aged category, we estimate the effect of an increase in the share of the middle-aged on production (β^M_{GE}) taking into account substitution across sectors and general equilibrium effects as follows:

$$\beta^M_{GE} = \beta^M_{SL} \cdot \left[\frac{\lambda^Y}{\lambda^Y + \lambda^O} \cdot \left(\frac{GE}{SL} \right)^M_Y + \frac{\lambda^O}{\lambda^Y + \lambda^O} \left(\frac{GE}{SL} \right)^M_O \right], \tag{1.33}$$

where β^M_{SL} is the sector-level coefficient estimate capturing the effect of an increase in the share of middle-aged on production; $\frac{\lambda^Y}{\lambda^Y+\lambda^O}$ and $\frac{\lambda^O}{\lambda^Y+\lambda^O}$ are the weights for the effect coming from the reduction in young and old respectively; and $\left(\frac{GE}{SL}\right)^M_Y$ and $\left(\frac{GE}{SL}\right)^M_O$ are the general equilibrium to sector-level ratios resulting from an increase in middle-aged compensated by a reduction in young and old respectively as estimated in Table 1.5.[24] We find that $\beta^M_{GE} = -1.27$ which implies a cumulative reduction in GDP growth following one standard deviation increase (2.2 percentage points) in the middle-aged of around 2.76 percentage points (dropping from the 16.7 percentage points reduction estimated through the sector-level empirical model) in the period 1996-2006 or, equivalently, a per year reduction in GDP growth of 0.28 percentage points (dropping from the 1.81 percentage points reduction estimated through the sector-level empirical model).

1.4.3 US Exercise

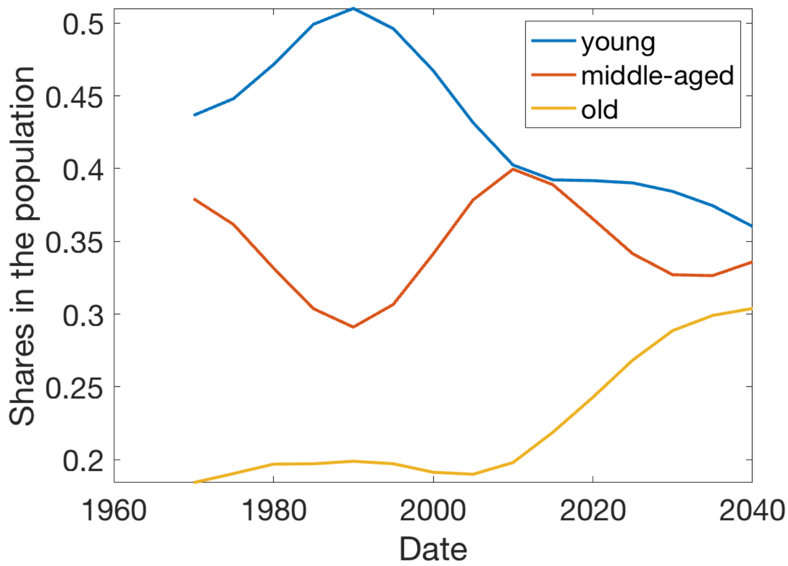
In this section, we look more precisely at the case of the US. Using our estimated general equilibrium model and demographic data about the age composition of the population, we compute the contribution of the demographic demand channel to the GDP growth trend in the US over the past 25 years.

US Demographics

The US demographic evolution has been largely determined by the baby boomer generation (those born between 1946 and 1965) who have represented by far the largest generational cohort in the twentieth

[24]We set $\beta^M_{SL} = -8.308$ as estimated in the *IV(2)* model specification in Chapter 2 and we use US demographic data in 1995 to estimate the weights. We set, therefore, $\frac{\lambda^Y}{\lambda^Y+\lambda^O} = 0.77$ and $\frac{\lambda^O}{\lambda^Y+\lambda^O} = 0.23$.

Figure (1.4) Demographic trends in the United States



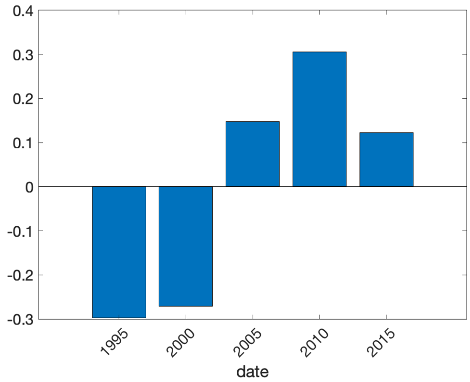
Note: age decomposition of the demography in the United-States of America, we define young as 25-44, middle-aged as 45-64 and old as 64-79 years old. Share are computed with total population between 25 and 79 as denominator. Source: UN WPP 2019.

century, and still today represent the second largest living cohort. Figure 1.4 shows how the shares of the young, middle-aged, and old population have evolved from the 1970s. As the baby boomers entered into the young age (25-44 as we have defined it), the share of young category started to increase. The increase continued up to the early 1990s as the baby boomers became middle-aged determining a sharp reduction in the share of the young population and an equivalent increase in the share of the middle-aged. The share of middle-aged people reached a peak in the early 2010s and started to decline as the baby boomers grew older. In the last decade, indeed, we observed a clear increase in the share of the older population which had remained constant in the previous decades. The prominent relevance of the baby boomer generation explains the relatively constant long-term trend in the middle-aged population and its large long-term fluctuations over the past and coming decades.

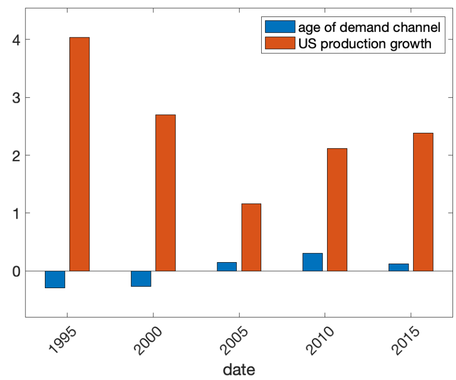
Results

We focus on the period between 1995 and 2019. Figure 1.5 panels (a) shows the 5-year average contribution of the age demand channel to GDP growth. We observe that in the period 1995-2004 the effect of the age demand channel on GDP growth was negative contributing to a reduction in GDP growth of around 8.7%. From Figure 1.4 panel (b), we, indeed, observe that the average GDP growth in that period was around 3.35%, while the age demand channel contributed to a reduction in GDP growth of around 0.29percentage points per year. This negative effect on GDP growth is mostly due to the increase in the share of the middle-aged in the period. In the period following 2005, as the share of the middle-aged declined and the share of the old kept increasing (i.e., as the baby boomers went from being middle-aged to old) the effect of the age demand channel reversed, having a positive effect on GDP growth. We find, indeed, that the contribution of the age demand channel in the period 2005-2019 was around 0.19 percentage points per year, contributing to an increase in GDP growth (1.87% on average in the period) of around 10.3%.

Figure (1.5)



(a) Production changes (age demand channel)



(b) Age demand channel and GDP growth

Note: Age demand channel compared with the US GDP growth in the period 1995-2019. Each bar represents the per year average effect of the age demand channel (5-years average) and the US 5-years average GDP growth measured in percentage points.

1.5 Conclusion

We provide novel evidence of the demographic determinant of the market structure: aging of the demand doesn't have a linear effect on competition. Based on micro evidence from the literature, we rationalize these findings into an heterogeneous agent model with competition over two margins: within and between markets. In a general equilibrium version of this model, we estimate a significant contribution of demographic trends to US GDP growth.

2

Future versus present labor markets: the trade-off of place-based policies

Joint with Philipp Kircher and David Koll [1]

Abstract In western economies, the structural decline of manufacturing sectors left formerly flourishing regions in distress. To counteract these developments many governments implemented place-based policies that can trigger large employment responses in depressed areas. But despite substantial resources allocated to those policies, local labor market conditions persist over time. In this paper, we estimate labor market responses to local sector-level shocks as proxies for place-based policies. We show that local policy maker could face a trade-off between subsidising low-growth sectors with large short-term employment benefits or subsidising high-growth sectors with initially smaller but persistent labor market improvements.

[1] Kircher: Cornell University & Université Catholique de Louvain. Koll: University of Mannheim & European University Institute. The authors gratefully acknowledge funding through the European Research Council Grant No. 818859. Access to confidential data, on which this work is based, has been made possible within a secure environment offered by CASD – Centre d'accès sécurisé aux données (Ref. 10.34724/CASD)

2.1 Introduction

In western economies, the structural decline of manufacturing sectors left formerly flourishing regions in distress. The Rust Belt in the US, the Ruhr valley in Germany as well as the Nord-Pas de Calais Mining Basin are among the most striking examples of this transformation. Regions that faced a structural decline of locally important sectors typically experienced a drop in private sector employment.[2]

To counteract these developments many governments implemented place-based policies that can trigger large employment responses in depressed areas (Austin et al., 2018). But despite substantial resources allocated to those policies, local labor market conditions persist over time.[3] So are these depressed areas trapped? Or does there exist a trade-off between current employment stabilisation and future employment growth?

We show in this paper that low-skilled workers represent a high share of the active population in depressed areas while growing sectors usually employ disproportionately more workers in high-skilled occupations. Policy makers then face a trade-off when designing place-based policies: First, they could *invest for the present* by supporting already present, *prevalent* sectors. This policy can have large short term effects: firms hire displaced local workers and thereby reduce regional unemployment immediately.[4] Nevertheless, these firms are also more likely to belong to sectors with little to no growth perspective. Hence, the policy might have no or even negative employment effects in the long run. Second, policy makers could *invest for the future* by attracting *new, promising* sectors which are expected to induce substantial long-term employment growth. Those firms have a labour structure with less overlap in terms of skills and occupations with the current local population and might need to attract workers from other areas. Hence, we expect the impact on local employment to be negligible in the short run. But investing into these sectors could change the fate of the area in the long run by creating lasting job opportunities.

In both cases, the redeployment of the labor force comes at a cost because human capital is at least partially sector-specific.[5] Workers would suffer from loss of human capital and therefore lower wages (Couch and Placzek, 2010; Neal, 1995). Firms would need to invest into the re-training of local workers or into attracting outside workers. The level of transferability of human capital from declining sectors becomes a key determinant of which sectors – the *prevalent* or the *promising* – to support.

[2]The structural decline could be due to, e.g., technological change, increased international competition, change in consumer taste, etc.

[3]see e.g. Bilal (2021); Kline and Moretti (2013); Kuhn et al. (2021).

[4]Alternatively, you can also think about this as retaining the existing jobs in the declining sectors.

[5]see e.g. Couch and Placzek (2010); Gathmann and Schönberg (2010); Kambourov and Manovskii (2009)

We attempt to shed some light upon the existence and strength of this trade-off by answering the following questions: Focusing on depressed areas, is the impulse response of local employment to a shock heterogeneous across sectors? Does the response differ for *prevalent* and *promising* sectors in the short versus long run? What are the implications for an efficient design of place-based policies?

In this paper, we estimate employment responses to positive shocks across sectors and population groups in French depressed areas. In the spirit of Moretti (2010), we use exogenous variation to inform us about the probable effects of place-based policies. As explained in more detail below, to identify potential place-based policies, we estimate as a proxy idiosyncratic variation in firm performance based on value added. The impulse responses look comparable across *prevalent* and *promising* sectors meaning that in principle the government might be able to mimic such shocks through tax breaks or subsidies with arguably similar effects. In practice, states already use value added produced as a fiscal base. In France, the *Cotisation sur la valeur ajoutée des entreprises* provides a noteworthy example.[6]

Our empirical strategy then consists of five steps. First, we identify *depressed* areas. Since we study labor markets, the relevant geographical entity for our analysis is the employment area. To obtain representative groups of areas, we rank them according to a measure of residualised unemployment rate which controls for the absolute level of employment. We define *depressed* and *prosperous* areas as respectively the bottom and top 25 percent.

Second, we define *prevalent* and *promising* sectors as sectors being over-represented in respectively *depressed* and *prosperous* areas. On average, *promising* sectors have a higher growth of employment and value added during the sample period, as well as a higher share of workers employed in high-skilled occupations. Hence, there exists a skill and occupation mismatch between the workforce present in *depressed* areas, which is predominantly lower skilled, and sectors which could provide a better economic outlook in the long run. In the following analysis, we focus on *depressed* areas because place-based policies are particularly focused on reviving those areas.

Third, using data about the universe of French establishments for the period 1994-2018 [7], we estimate idiosyncratic variation in firm performance from value added (Fagereng et al., 2018; Galaasen et al., 2020). We identify shock processes which are homogeneous across *prevalent* and *promising* sectors. This implies that both shock processes could be mimicked by a place-based policy.

[6]The policy consists of a tax of 0.25% on value added for firms with revenues above 500k euro and up to 0.75% for firms with revenues above 50m euro. Fiscal authorities could instead use this tax to discriminate firms by sector.

[7]We combine the employment data from the *Déclarations Annuelles de Données Sociales* (DADS) and the balanced sheet data from the FICUS, provided by the Insee and French Finance Ministry.

Forth, we aggregate firm-level idiosyncratic changes to exogenous sector-level shocks following Gabaix and Koijen (2020). The economic fate of large firms is a significant driver of aggregate fluctuations in the economy (di Giovanni et al., 2014). Locally, idiosyncratic shocks to those granular firms can provide exogenous variation to the relative performance of sectors.

In an event-study type of regression, we estimate the dynamic response of employment to a positive performance shock. *Prevalent* sectors increase employment significantly in the short to medium run, with a maximum elasticity of hours to value added of 0.29 after four years. The effect then slowly fades out. Instead, *promising* sectors do not show a large response in the years directly after the shock but show a strong and significant impact six to seven years after the shock hit with a maximum elasticity of 0.43. This evidence supports our hypothesis that policy makers face a trade-off between investing into present or future labor markets.

This paper contributes to the large and quickly expanding literature about the optimal design and the evaluation of place-based policies. Closest to our paper, Kline and Moretti (2014) present evidence for heterogeneous responses of sectors to place-based policies. They emphasize the role agglomeration economies play in explaining the more persistent and amplifying effect of place-based policies on manufacturing compared to agriculture activities. Our paper instead highlights a novel determinant, namely the role of firms in different sectors – even within manufacturing – in terms of labor force composition and persistence of place-based policy interventions.

The structure of the paper is as follows. In Section 2.2, we discuss in a simple model the equivalence between a subsidy to value added and performance shocks. Next, in Section 2.3 we present our empirical framework. Finally, in Section 2.4 we discuss our main results. Section 2.5 concludes.

2.2 A simple model

Natural experiments of industrial policies are rare and difficult to identify. Therefore, in this paper we rely on idiosyncratic shocks to firms as a proxy for policy interventions. In this spirit, we illustrate in this section how an exogenous idiosyncratic shock to value added can be informative on the effects of a firm subsidy.

Let's define the profit maximization problem of a price-taking firm that enters the period with a given productivity $\bar{A}$ and a idiosyncratic shock to productivity μ, a stock of capital K and of labor L, price vector $p = (p_Y, p_K, w, p_I)$ for output Y and inputs (capital, labor and flexible inputs (I)), and a sales tax/subsidy $(\tau - 1)$:

$$\max_{K',L',I} V(\bar{A},\mu,K,L,p,\tau) = \pi(\bar{A},\mu,K,L,I,p,\tau) + \beta EV(\bar{A}',\mu',K',L',p',\tau') \tag{2.1}$$

$$\text{s.t. } \pi(\bar{A},\mu,K,L,I,p,\tau) = \tau p_Y Y - wL - p_K K - p_I I - C_K(K',K) - C_L(L',L)$$

$$Y = A \cdot F(L,K,I) \;\;,\;\; A = \bar{A}\cdot\mu$$

$$(\bar{A}',p') = M(\bar{A},p),\; \mu' = \hat{M}(\mu),\; \tau' = \tilde{M}(\tau)$$

where $V(\cdot)$ denotes the firm's value, $\pi(\cdot)$ denotes its current period profit comprising revenue minus costs of input minus adjustments costs, Y the output of the firm with $F(\cdot)$ being the production function, and $C_L(\cdot)$ and $C_K(\cdot)$ allow for adjustment costs in labor and capital and incorporate depreciation and exogenous labor losses.[8] The law of motion on productivity and prices is given by the (possibly stochastic) mapping M, which can incorporate aggregate conditions at the national, regional and industry level that affect the firm. Idiosyncratic firm shocks and subsidies are governed by possibly stochastic mappings $\hat{M}$ and $\tilde{M}$.

In this setting one can back out idiosyncratic shocks under appropriate assumptions. For example, if firm output is Cobb-Douglas in flexible inputs (i.e., $F(L,K,I) = \tilde{F}(L,K)I^{\gamma}$), its log-value-added is additively separable in the idiosyncratic shock. If other innovations to prices, baseline productivity, and general subsidies are shared by all firms in the same industry, time, or geography so that they can be absorbed by fixed effects, idiosyncratic shocks to value added identify idiosyncratic innovations in μ.[9]

Now consider for initial illustration a setting where the idiosyncratic firm shock is i.i.d. across time and normal subsidies/taxes are constant at some level τ. Compare two firms i and j with same vector $(\bar{A},K,L,p,\tau)$, but one has a idiosyncratic productivity draw μ_i that is larger than shock μ_j of the other, i.e. $\mu_i > \mu_j$. Consider the performance and behavior of these two firms, contemporaneously and in the future. Their difference is informative of the effects of a subsidy to firm j. If firm j were offered a one-time subsidy so that τ_j in this period reaches

$$\tau_j = \frac{\mu_i}{\mu_j}\tau, \tag{2.2}$$

[8]If there is an exogenous loss of capital $\delta_K K$ and labor $\delta_L L$, adjustment costs are often a function of the difference $K' - (1-\delta_K)K$ and $L' - (1-\delta_L)L$.

[9]Value added VA equals the value of output minus the total costs of flexible inputs, so $VA = p_Y \tau\mu\bar{A}\tilde{F}(L,K)I^{\gamma} - p_I I$. Using the first order condition to the firms optimization problem with respect to flexible inputs, we can write $\ln(VA) = \ln(1-\gamma) + \ln(p_Y) + \ln(\tau) + ln(\bar{A}) + \ln(\tilde{F}(L,K)) + \ln(\mu) + \gamma\ln(I)$. If the first four terms are shared by firms and absorbed by fixed effects and one controls for capital and labor, the remaining shocks enter through the last two terms. By the envelope theorem the last term is of second order, so shocks to $\ln(VA)$ approximately identify shocks to $\ln(\mu)$.

firm j would change its behavior and performance to that of firm i.

The proof is trivial because output and subsidies/taxes enter the profit function multiplicatively via the product of τ and μ. Firm i and j are identical except that $\tau\mu_i > \tau\mu_j$. Now firm j with a subsidy reaches $\tau_j\mu_j = \tau\mu_i$, and therefore faces exactly the same maximization problem and takes exactly the same decisions as firm i. So this opens the possibility to learn from firms that face different shocks about the behavior of a firm that receives subsidies. This can be explored not only at the firm level, but also in its effects on the entire sector.

Obviously idiosyncratic firm shocks might be correlated through time, so that a shock $\mu_i > \mu_j$ not only advantages firm i in the current period but also in future periods. In this case the difference in firm performance and behavior between firm i and firm j informs us about the reaction of firm j to a prolonged subsidy, namely a subsidy that shares a similar correlation over time. For example, consider idiosyncratic log-productivity that follows an AR(1)-process with $ln(\mu') = \alpha \ln(\mu) + \epsilon$ and $\alpha < 1$. Consider the difference in firm behavior and firm performance between firm i and j when $\mu_i > \mu_j$ all else equal. Now consider again only firm j, but faced with an initial subsidy according to equation (2.2), as well as deterministically declining future subsidies: let $s_i := \tau_i/\tau$ be a measure of the current subsidy relative to the time-invariant base τ, then $ln(s_i') = \alpha ln(s_i)$. This subsidy-schedule renders firm $j's$ product of subsidy and idiosyncratic productivity equal to that of firm i not only contemporaneously, but also in the next period and further on.

In summary, in this theory two otherwise identical firms which differ in their idiosyncratic shocks can serve as a proxy for a subsidy. A similar sequence of shocks requires the same sequence of subsidies. Hence, if we isolate a similar shock sequence for different firms, we can infer whether the associated similar subsidy sequence would have different effects on firm behavior. We note that this approach is just a first pass to this problem, and it might well be possible to write down firm problems where the equivalence above fails to hold. But in the absence of clear direct evidence on the effects of subsidies, this approach seems a promising first pass to studying this problem.

2.3 Empirical methodology

In this section, we first describe the data. Then, we identify the relevant area and sector groups. Finally, we present our method to estimate performance shocks from value added in *depressed* areas.

2.3.1 Data

Our main empirical investigation is based on a panel of establishments assembled from two French administrative data sources. First, we rely on the panel of firm-level tax data (FARE/FICUS) which covers balance sheet information for the universe of private sector firms in France. Second, through a unique firm identifier, we can then link to each firm detailed information about their workforce using the *Déclaration Annuelle de Données Sociales* (DADS), a cross-sectional worker-level data set which contains the social declarations of all private sector employees at the establishment level. Hence, we create a panel of establishments covering the period 1994-2018 with detailed information about establishment characteristics such as value added, capital, investments, etc as well as its workforce, e.g. number of employees, their occupations, hours worked.[10] We select the set of firms that report positive total hours worked, a positive amount of capital and positive value added.[11]

2.3.2 Selection of areas

We rank employment areas by their unemployment rate conditional on their employment size. *Depressed* and *prosperous* regions are defined as respectively the top and bottom employment areas according to this criteria. The geographical level of analysis is the *zone d'emploi*, a definition of employment areas by the INSEE, the French statistical office.[12] In the most recent partition in 2020, France is composed of 306 employment areas. We use the terminologies employment area, area and region interchangeably. Labor force size and unemployment rate are positively correlated across areas. To obtain size-representative groups of areas and avoid selecting the largest French metropolitan areas as *depressed* areas in our analysis, we rank areas according to a residualised unemployment rate u_c after controlling for the absolute level of employment:

$$U_c = \alpha + \beta \ln E_c + u_c \tag{2.3}$$

where c denotes the area indicator, U_c the unemployment rate, and E_c the aggregate number of hours worked. We estimate the regression on the cross section of employment areas in 2003, the earliest year

[10] As the tax data is at the firm level, we infer establishment-level balance sheet data for multi-establishment firms by weighting the firms' total financial amounts with the respective establishment's share of total hours worked within the firm.

[11] The data sources which we use in our analysis are: Insee (2012a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p, 2014a,b,c, 2015, 2016a,b, 2017, 2019, 2020, 2021a,b), Insee and Ministère Des Finances (DGFiP) (2013, 2014a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r, 2015, 2016, 2017, 2018, 2019, 2020, 2021)

[12] According to the INSEE, these areas are defined to maximize the number of labor market participants who live and work in the same area, with targets of 70% of such individuals and 25000 labor participants.

for which the Insee provides public information about unemployment rates at the employment area level. We define *depressed* and *prosperous* areas as those respectively above the 75th percentile and below the 25th percentile according to $\hat{u}_c$, the estimated residuals of equation (2.3).

Table (2.1) Summary statistics about employment areas

	total hours (Mio)		all sectors			manufacturing		
area group	mean	sd	wage	VA/hour	N	wage	VA/hour(K)	N
depressed	67.92	78.35	13.65	30.94	282418	15.08	30.84	34802
prosperous	65.54	72.88	14.43	38.14	224287	15.23	30.65	32358
others	109.12	388.19	15.64	52.78	763283	16.34	34.52	97136

Note: the summary statistics are computed based on the cross section of establishments reporting positive hours worked in 2003. Mean and standard deviation report respectively the average and standard deviation of the total number of hours worked in each employment area denoted in million. Manufacturing sectors are those numbered 10 to 36 in the 2-digit 2003 NAF revision 1. Variables are computed at the establishment level and averaged at the group of sector level, weighted by total hours. Wage is the total labor cost per hours worked. VA/hour is value added per hour worked. N is the number of establishments. For multi-establishment firms, we split value-added at the establishment level by the respective share of hours worked within the firm.

Table 2.1 provides summary statistics on three different categories of employment areas: *depressed*, *prosperous* and *other* areas. The former two groups consist of areas with similar employment size distributions with a mean of total hours between 65 and 68 and a comparable standard deviation. *Depressed* areas exhibit worse labor market indicators across all sectors resulting in a 5.5% lower average hourly wage and a 19% lower value added per hour.[13] Table 2.2 shows that the *depressed* areas also host less qualified jobs compared to *prosperous* areas with about 21% less high-skilled occupations and 27% less high-skilled technical occupations. These summary statistics are reassuring that our methodology correctly identifies depressed local labor markets with a lower skilled workforce.

Table (2.2) Occupation composition of employment areas

	all sectors		manufacturing	
area group	share of high skilled	share of high skilled technical	share of high skilled	share of high skilled technical
depressed	9.78	4.60	10.75	7.10
prosperous	12.36	6.34	13.89	9.49
others	15.81	7.09	15.71	9.16

Note: the summary statistics are computed based on the cross section of establishments reporting positive hours worked in 2003. We average area-level statistics across areas. High skilled is the occupation 3: "managers and highly intellectual professions" in the 1-digit 1982 PCS classification. High skilled technical are the occupations 34 and 38 in the 2-digit 1982 PCS classification; respectively "professors and scientific occupations" and "engineers and technical business managers". Manufacturing sectors are those numbered 10 to 36 in the 2-digit 2003 NAF revision 1. classification.

[13] We do not report results here, but these stylized fact are exacerbated after controlling for areas' sector composition.

2.3.3 Selection of sectors

We define *prevalent*, respectively *promising*, sectors as those being over-represented in *depressed*, respectively *prosperous*, areas. We consider 327 manufacturing sectors defined by the 4-digit sector classification *Nomenclature des Activités Française* (NAF). For each area group, we rank sectors by their employment share relative to the national average and select the top 20% largest sectors (rounded to 65 sectors).[14]

Table (2.3) Summary statistics about manufacturing sector groups

		national occupation composition (%)		national growth trends (%)		
sector group	N	high skilled	high skilled technical	hours	employees	value added
prevalent	20810	9.26	6.19	-23.41	-19.67	5.59
promising	49210	9.65	6.57	-12.76	-9.99	16.84
others	118895	11.23	6.73	-15.68	-14.47	14.09

Note: Summary statistics are computed based on the cross-section of firms reporting positive hours worked in 1998. High skilled is the occupation 3: "managers and highly intellectual professions" in the 1-digit 1982 PCS classification. High skilled technical are the occupations 34 and 38 in the 2-digit 1982 PCS classification; respectively "professors and scientific occupations" and "engineers and technical business managers". Variables are aggregated at the sector group level. Growth trends are the percentage growth of these aggregates computed between years 1998 and 2007. N is the number of establishments. For multi-establishment firms, we split value-added at the establishment level by the number of hours worked in each establishment. Manufacturing sectors are those numbered 10 to 36 in the 2-digit 2003 NAF revision 1.

In line with our original intuition, the summary statistics in Table 2.3 show that *prevalent* sectors employ a lower share of high-skilled workers and grow slower in terms of employment but also value added.

2.3.4 Identification of idiosyncratic shocks

We estimate idiosyncratic variation in firm performance from value added using an unbalanced panel of all French establishments for the period 1994-2018. We infer balance sheet data at the establishment level for multi-establishment firms by weighting the firms' total financial amounts with the respective establishment's share of total hours worked within the firm. Hence, in the following section, we use the terms establishment and firm interchangeably. As a robustness check, we confirm our results by running the analysis on the sample of single-establishment firms.[15]

We follow an existing literature which uses unexplained idiosyncratic variation in value added or performance to extract idiosyncratic firm shocks (e.g di Giovanni et al. (2014), Galaasen et al. (2020)).

[14] We compute shares at the start of our sample, in 1994. In case a sector fulfills the criteria for both groups, we exclude it from the rankings.

[15] We will report these results in a future version of the paper.

Hence, we identify shocks to firm performance as the residuals in regression (2.4), $\epsilon_{i,t}$:

$$\ln \text{VA}_{i,t} = \alpha_i + \alpha_{s(i),c(i),t} + \beta_1 \ln K_{i,t} + \beta_2 \ln W_{i,t} + \lambda' X_{i,t} + \epsilon_{i,t} \tag{2.4}$$

where i denotes an establishment, t denotes a year, $s(i)$ denotes an establishment's 2-digit sector and $c(i)$ its employment area. Hence, $\alpha_{s(i),c(i),t}$ denotes the sector-area-year fixed effect, α_i the establishment fixed effect, $VA_{i,t}$ the dependent variable – value added –, and $X_{i,t}$ is a vector of control variables exogenous to firms' decisions, in this case a second order polynomial of firm age. In addition, we control for the amount of capital $K_{i,t}$ and the wage bill $W_{i,t}$.

As a validation, we test the persistence and the heterogeneity of shocks across sectors. For that purpose, we regress firms' value added on their own shocks including leads and lags of the shocks. In addition, we include interaction terms for the promising sector and the other sectors.[16]

We find a strong and significant effect of the shock on value added on impact for firms in both, prevalent and promising, sector groups which fades out almost instantaneously (see figure 2.1b). Hence, past shocks (i.e. $k > 0$) have a negligible impact. In addition, future shocks do not have an effect on value added thereby emphasising that there seems to be no anticipation of future shocks. Looking at the differential response by sectors, we find that at every horizon the endogenous response of value added to the performance shock is not significantly different between *prevalent* and *promising* sectors (see figure 2.1a). It implies that in principle these shocks can be mimicked by a single subsidy schedule or tax breaks for both sector groups.[17]

In a robustness exercise, we control for possible auto-correlation of the idiosyncratic firm shocks to value added, $\epsilon_{i,t}$, by using in the following analysis the residuals of the regression $\epsilon_{i,t} = \alpha + \rho\epsilon_{i,t-1} + \xi_{i,t}$. Our results are robust to this specification.

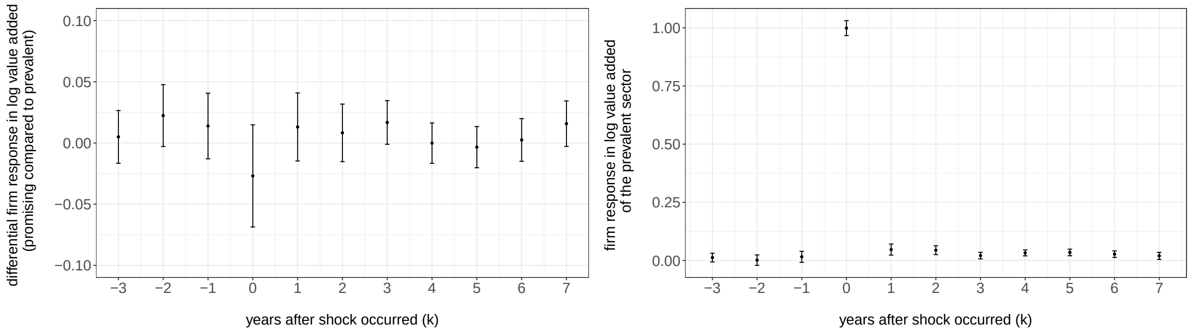
[16]In fact, we run the following regression:

$$\ln(VA_{i,t}) = \sum_{k=-3}^{7} (\beta_k + \delta_{k,pro}\, I_{i \in Pro} + \delta_{k,oth}\, I_{i \in oth}) \cdot \epsilon_{i,t-k} + \psi_{s(i),c(i),t} + \psi_i + u_{i,t} \tag{2.5}$$

where ψ denote the respective fixed effects and $I_{i \in pro}$ and $I_{i \in oth}$ indicate if the establishment is in the promising sector or among the other sectors.

[17]The regression results are also shown in the Appendix in Table B.3.

Figure (2.1) Firm-level responses to an idiosyncratic shock to value added



(a) Difference of promising vs. prevalent sectors **(b)** Baseline effect of prevalent sector

Notes: The figure 2.1a plots the coefficients $\delta_{k,pro}$ for $k \in \{-3, 7\}$ of the regression (2.5)*. Hence, it shows the difference in the average firm level response in value added by firms in the promising sectors compared to firms in the prevalent sectors due to an idiosyncratic shock. The figure 2.1b plots the coefficients β_k for $k \in \{-3, 7\}$ of the regression* (2.5)*. It shows the baseline average response of firms in the prevalent sector to an idiosyncratic shock in value added. Note that a negative number on the horizontal axis, i.e. $k < 0$, implies that the shock occurred k years after we observed the outcome of value added.*

2.4 Results

In this section, we estimate the heterogeneous response of sectors to performance shocks in *depressed* areas. We find that in the short-run *prevalent* sectors show a significantly stronger labor response compared to *promising* sectors. In the longer run around six to seven years after the shock, this appears to reverse.

2.4.1 Main regression

Our main empirical framework investigates the effect of regional sector-level shocks on regional, sector-level labor statistics. In Section 2.3.4, we estimated idiosyncratic innovations to value added at the firm level following Galaasen et al. (2020). Now, we aggregate these firm-level idiosyncratic changes to exogenous regional sector-level shocks following the granular instrumental variable approach of Gabaix and Koijen (2020).

The following equation (2.6) specifies the main empirical framework:

$$\ln(L_{S,c,t}) = \sum_{k=-3}^{7} (\beta_k + \gamma_{k,pro}\, I_{S=Pro} + \gamma_{k,oth}\, I_{S=oth}) \cdot \bar{\epsilon}_{S,c,t-k} + \psi_{c,t} + \psi_{S,t} + \psi_{c,S} + u_{S,c,t} \quad (2.6)$$

$S \in \{Pro, Pre, oth\}$ denotes the *promising*, *prevalent* or other sectors. $L_{S,c,t}$ is either the total number of hours worked or the total number of employees in sector x region x year cell. We define the weighted sum of innovations $\bar{\epsilon}_{S,c,t} \equiv \sum_{i \in \mathbb{N}_{S,c}} \xi_{i,t-3} \epsilon_{i,t}$, with innovations $\epsilon_{i,t}$, weights $\xi_{i,t-3} \equiv \frac{VA_{i,t-3}}{\sum_{i \in \mathbb{N}_{S,c}} (VA_{i,t-3})}$ and $\mathbb{N}_{S,c}$ the set of firms i in employment area c and sector group S.

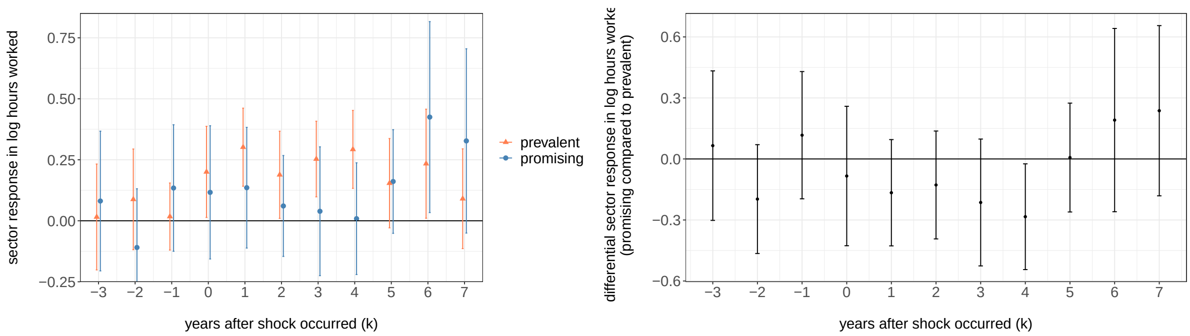
The first set of coefficients of interest is captured by β_k which yields the average effect on the outcome $L_{S,c,t}$ for promising sectors due to a regional sector-specific shock which took place k years prior to the current year t with $k \in \{-3, ..., 7\}$. Hence, $k < 0$ implies that the shock occurred k years after the outcome. In addition, the coefficients $\gamma_{k,pro}$ capture if the average response differs for *promising* sectors for different time horizons k.

Nevertheless, innovations estimated are potentially a mix of a common factor and an idiosyncratic shock, so we can rewrite $\epsilon_{i,t} \equiv \eta_{i,t} + \delta_{c(i),s(i),t}$ where $\eta_{i,t}$ denotes the idiosyncratic component and δ the common factor. Regressing directly on $\bar{\epsilon}_{S,c,t-k}$ would create endogeneity issues. To tackle this issue, we follow Gabaix and Koijen (2020) and build an instrument in the following way:

$$z_{S,c,t} \equiv \sum_{i \in \mathbb{N}_{S,c}} \xi_{i,t-3}\epsilon_{i,t} - \sum_{i \in \mathbb{N}_{S,c}} \frac{1}{n_{S,c}}\epsilon_{i,t} = \sum_{i \in \mathbb{N}_{S,c}} \epsilon_{i,t}\left(\xi_{i,t-3} - \frac{1}{n_{s,c}}\right) \tag{2.7}$$

$n_{S,c}$ denotes the cardinal of $\mathbb{N}_{S,c}$. $z_{S,c,t}$ is a weighted sum of idiosyncratic shocks, with positive weights for establishments larger than the average, and uncorrelated with the common factor. We use $z_{S,c,t}$ as our main instrument. [18]

Figure (2.2) Prevalent and Promising Sector Level Responses in Log Hours Worked



(a) Both sector responses **(b)** Differential effect in the response ($\gamma_{k,pro}$)

Notes: The figure 2.2a plots the coefficients β_k and $\beta_{k,pro} + \gamma_{k,pro}$ for $k \in \{-3, 7\}$ of the regression (2.6)*. Hence, it shows the average sector level response of the prevalent (β_k) and the promising ($\beta_{k,pro} + \gamma_{k,pro}$) sectors to a in log hours worked to a shock to value added using the GIV. The figure 2.2b plots the coefficients $\gamma_{k,pro}$ for $k \in \{-3, 7\}$ of the regression* (2.6)*. Hence, it shows the differential effect of a shock to value added on log hours worked in the promising sector compared to the prevalent sector as the baseline. Note that a negative number on the horizontal axis, i.e. $k < 0$, implies that the shock occurred k years after we observed the outcome.*

Figure 2.2 presents our main results.[19] First, focusing on the resulting coefficients for the prevalent sector in Figure 2.2a we observe that the employment in prevalent sectors increases in the short-run

[18]Following Gabaix and Koijen (2020), we run a series of robustness checks on our instrument which will be included in a future version of the paper.

[19]The respective regression tables can be found in the Appendix in Tables B.1 and B.2.

in response to a shock in value added. In fact, a 1% increase in value added translates into a 0.2% higher total hours worked instantaneously. This effect persists over time with its peak four years after the shock with an increase in total hours worked of about 0.3%. Then the effect slowly fades out. Hence, investing into the prevalent sector seems to boost employment in that sector in the short to medium term. In addition, we observe that shocks in the future ($k < 0$) do not translate into a rise in employment in the current period, i.e. β_k is not significantly different from zero for $k < 0$. Next, we can compare the responses of the promising sector compared to the prevalent sector by looking at the differential response depicted in Figure 2.2b, i.e. the estimates of $\gamma_{k,pro}$: first, we note that also the differential effect of the promising sector exhibits also no response to shocks that occur in the future, i.e. $\gamma_{k,pro}$ is not significantly different from zero for $k < 0$. In addition and more interestingly, we observe different employment responses to a contemporaneous or past shock ($k \geq 0$): we observe that a negative difference is building up over time leading to a 0.28% lower response in total hours worked in the promising sector four years after the shock which is significant at the 5% level. This difference shrinks in the following years leading to a positive, but not significant coefficient seven years after the shock.

Finally, Figure 2.2a also shows the estimated coefficients and significance levels of the entire effect for the promising sector (i.e. $\beta_{k,pro} + \gamma_{k,pro}$). [20] We find that the response in log hours worked of the promising sector to a shock in value added are not significantly different from zero for the first five years after a shock occurred. In contrast, we find that the employment responses become sizeable and significantly positive six years after the shock. We observe no significant pre-trends. These results show that investing into promising sectors today can lead to long-run sectoral employment gains. The estimated effects take into account the direct effect of firms hit by positive shocks, as well as spillovers to other firms within the same sector.

2.5 Conclusion

Does there exist a trade-off in place-based policies between stabilising current local employment and investing into sectors with more promising future employment growth perspectives? We show that sector-level employment in *prevalent* sectors expands in the short to medium term in response to a regional sector-level performance shock. The *promising* sectors only show a positive employment response six to seven years after the shock. This evidence supports our hypothesis that there exists a

[20] We also show the effects and its significance levels in the Appendix in regression Table B.2 in which we assume that the promising sector is the baseline sector.

trade-off between investing into present labor markets versus investing into future labor markets. We plan to investigate in more detail who benefits from those policies by decomposing the employment response between local and non-local workers, between job-to-job and out-of-unemployment transitions and between different skill levels. To that end, we plan to use a representative panel of the population covering 1/12 of the French population which provides information about both employment and unemployment spells of each individual. In addition, we plan to investigate how the different shocks affect total regional employment.

3

Customer loyalty and firm dynamics over their intensive and extensive margins

Abstract Firms build their customer base through a lengthy process. A larger product range allows firms to increase customers' loyalty, but, in turn, firms risk reducing their aggregate productivity by selling goods they are worst at producing. In this paper, I develop a model of consumer search that captures firms' endogenous growth along two margins: *customer capital* and product range. The model provides micro-foundations for the existence of within-firm heterogeneity in terms of prices, sales and markups. Consistent with recent empirical evidence, firms do not increase markups over their growth path; instead, I show that they achieve a higher surplus from each match by increasing their product range and, therefore, buyer's loyalty. Nevertheless, as a firm grows larger, its share of incumbent buyers increases and demand spreads more homogeneously across goods, mechanically reducing its average markup. I called this novel mechanism the *curse of the large firm*.

3.1 Introduction:

Most firm provide multiple goods and increase their product range as they grow larger. Uber inc. provides a striking example; its original business consisted in offering a marketplace for car transportation, but later expanded into meals and grocery delivery as well as freight. All these products have in common localisation and matching technologies. The choice of products seems to be motivated both by the opportunity to increase the number of trades per customer, providing a fuller range of goods, as well as the capacity of partially replicating a core technology to the production of other services.

In this paper I claim that product diversification functions as a tool for firms to retain their buyers whenever their customers' preferences change. To investigate this mechanism, I develop a model of multi-product firm dynamics with endogenous customer loyalty which provides micro-foundations for the observed heterogeneity within and among firms over the growth path.

The EFIGE-survey provides indirect evidence about the relevance of this trade-off. [1]. The lack of demand is the most cited factor preventing growth, and broadening of the product range the third most cited factor of success. We know also from the literature that demand for goods grows slowly along their life-cycle [2].

The aim of this paper is to present a tractable model in which firms face a trade-off to diversification between productivity and growth. On one hand, expanding the product range beyond the *core product* decreases the aggregate productivity of a firm, as it starts selling goods it is less efficient at producing (De Loecker et al., 2016; Dhyne et al., 2017). At the same time, its presence in more markets increases its potential demand, as well as it improves the loyalty and, therefore, the surplus it can extract from each incumbent buyer.

I develop a framework which relies on three main assumptions. First, frictions on the good market take the form of coordination issues and capacity constraints. Firms and customers are never certain to trade because customers cannot coordinate their search; some firms receive demand which they cannot fulfil, while other do not meet any buyers. Firms can increase their chance to meet customers by recruiting sellers, with each of them having the possibility to carry on a limited number of transactions per period. Nevertheless, their decreasing efficiency makes the marginal cost of a transaction increasing in the number of sellers, and therefore make it optimal for firms to hire a limited number of them per time period. As a result firms grow slowly from their entry size to their optimal size.

Second, buyers experience preference shocks. It models the unpredictable shifts in an individual buyer's purchasing patters arising from potentially a large variety of changes, e.g. income and taste.

Finally, firms hold a core product. The core product is the original entry sector of a firm as well as the production in which it is the most efficient. The technology and organization are originally designed to the production and commercialization of a specific good. A firm can replicate its technology to the production of other goods, but at the costs of a lower efficient. This assumption relates to the work

[1]Source: The Survey on European Firms in a Global Economy, computations of the author. https://www.bruegel.org/publications/datasets/efige/

[2]see Fitzgerald et al. (2016) for evidence about tradable goods and Fitzgerald and Priolo (2018) for the retail sector

of Milgrom and Roberts (1990) about *flexible manufacturing*, used more recently in papers such as Eckel and Neary (2010). In my framework, the replication inefficiency materializes into lower labour productivity.

My contribution is three-fold. First, my framework reconciles the customer market literature [3] with the most recent micro evidences on firms' pricing. My model predicts that product specific markups are constant over the growth path of the firms, in line with evidence from Fitzgerald and Priolo (2018). Nevertheless, the invest/harvest mechanism described in most of the literature about customer market takes place here as well. Though instead of young firms sacrificing profit to attract buyer and charge higher markups in the future, firms make customers increasingly captive as they grow. The surplus of both firms and consumers increase as they can circumvent search frictions. My model highlights a *curse of the large firm*. As a the customer base grows larger, its share of incumbent customers increases and its demand distribution over its product range becomes flatter; in turn, aggregate markups drops.

I further provide a novel micro-foundation for the existence of markup heterogeneity within firms (De Loecker et al., 2016; Dhyne et al., 2017). Firms introduce new products to retain customers and markups heterogeneity arises from the necessity for firms to counteract taste dispersion of its customer and refocus demand to its most profitable good. Firms offer an imperfect pass-through of productivity. Buyers' loyalty plays an important role in firm decisions over their intensive and extensive margin.

Finally, I show that my model fits the five major empirical facts about multi-product firms dynamics. Most firms sell multiple products (Hottman et al., 2016). Positive correlation between sales and production efficiency (Dhyne et al., 2017). Positive correlation between markups and production efficiency (Anderson et al., 2018; De Loecker et al., 2016). The demand for a good grows slowly (Fitzgerald and Priolo, 2018). Prices and markups are constant over the growth process (Anderson et al., 2018; Fitzgerald and Priolo, 2018)

3.2 Model

I present a model of customer market with one-to-many directed search a la Moen (1997) and Kaas and Kircher (2015), in which firms expand over two dimensions: the customer capital and the product range. Customer loyalty arises endogenously from buyers' optimal decision between the opportunity costs to search on the market and contracts offered by the firm.

[3] see among others Phelps and Winter (1970),Klemperer (1995), Paciello et al. (2018) and Gilbukh and Roldan (2017)

3.2.1 Environment

Time is discrete. All agents are risk neutral and discount time with parameter β. The economy is populated with a fixed measure of buyers normalized to 1 and an endogenous measure N_f of incumbent firms. There is a fixed set of homogeneous, indivisible and perishable goods in the economy Ω of measure $|\Omega| = 1$ and ordered along a circle.

Buyers A buyer get utility u from purchasing a single unit of a single good. Each period, her preference varies according to an idiosyncratic preference shock. Two types of buyers coexist: active buyers who search on the market and inactive buyers who purchase the good from the same firm as previous period.

Firms A firm enters the economy at cost K with a core product $\hat{i}$ and labour productivity z. Firms produce solely from a single input, labour, with a convex production costs $C(n_i;\hat{i},z)$ s.t. $C_1 > 0$ and $C_3 < 0$, with C_i the derivative of C with respect to the i^{th} variable. Each technology is replicable to the production of other products $i \in \Omega$, but the further away a good is from the core product the lower the productivity at producing it, i.e. $C_2 < 0$. This assumption follows the work of Milgrom and Roberts (1990) about *flexible manufacturing*. We obtain with this assumption that, as in reality, competing firms in any given market have overlapping product ranges. Finally, firms pay firm-level head quarter costs c_{hq} and product-level fixed costs c_f each period. The product range ε defines a subset of the universe of products $\Omega_f \in \Omega$.

Contracts Firms decide about their optimal strategy in each good market in two ways: posting contracts and setting its marketing spending, i.e. the number of sellers it recruits. In each market i it participates, a firm cannot price discriminate and competes to attract customers by posting a single contract $x(p_i, s^+)$ which defines the price of the good this period p_i and the expected utility the firms commit to provide to its customers next period s^+. The sign "$+$" over any variable refers to its value next period. The unpredictability of agents' preference and the absence of price discrimination, implies that firm offer the future surplus even if it would offer different prices today. Firm set their marketing effort a_i at the product level, at convex cost $\kappa(a_i)$.

3.2.2 Good Markets

Markets are subject to search frictions: buyers cannot coordinate and firms face a capacity constraint; in practice, some buyers cannot be served while some firms are never visited.

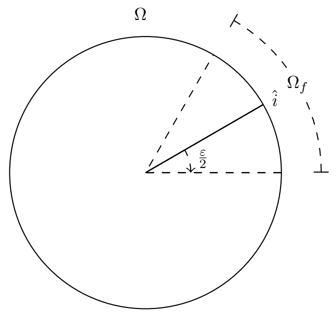


Figure (3.1) The extensive margin of a firm: its core product $\hat{\imath}$, its product set $\Omega_f \equiv [-\frac{\varepsilon}{2}, \frac{\varepsilon}{2}]$ and range $\varepsilon = |\Omega_f|$.

Market Utility Assumption Buyers are aware of all the contracts and direct their search at costs c_s towards the contract market providing the highest value $B = \max_x b(x)$ with $b(x)$ the value of searching on a given sub-market [4] providing contract $x(p, s^+)$.

$$b(x) = -c_s + \frac{\eta}{\theta(\eta)}\{u - p + \beta(1-\delta)s^+ + \beta\delta\mathbb{E}(b(x))\} + (1 - \frac{\eta}{\theta(\eta)})\beta\mathbb{E}(b(x))$$

where $\eta/\theta(\eta)$ is the probability a customer purchases the good, with θ the ratio between number of goods offered and the number of customers directing their search towards this sub-market and η the probability a good being bought.

Competition among sellers ensures that sub-markets are active if and only if they provide the highest utility $\hat{x} \in \operatorname{argmax} b(x)$. In expectation, buyers are indifferent between searching and staying inactive:

$$(1-\beta)B = -c_s + \underbrace{\frac{\eta}{\theta(\eta_i)}\{u - p + \beta(1-\delta)(s^+ - B)\}}_{\equiv \rho}$$

We obtain the so called market utility assumption, which defines the price as a function of the (inverse) matching probability and the promised surplus of being matched, i.e. the promised utility net of the value of searching.

$$p = u - \frac{\theta(\eta)}{\eta}\rho + \beta(1-\delta)\left(s^+ - B\right) \tag{3.1}$$

3.2.3 Retention

Continuation utility: An incumbent buyer has the possibility to keep trading with the same firm and, in that case, circumvent search frictions. In this framework incumbent buyers never refuse a contract,

[4] A sub-market consist of all the sellers offering this contract and the buyers directing their search towards it.

since each contract fulfils the market utility assumption and as such offers a higher utility than going back to the market [5].The firm has to set contracts which provide in expectation the utility promised in the previous period. The value of being matched to a firm with contract s:

$$s = \mathbf{E_i}\left\{\varphi(\varepsilon, \boldsymbol{x})\left[u - p_i + \beta\left((1-\delta)S^{+}) + \delta B\right)\right] + (1-\varphi(\varepsilon, \boldsymbol{x}))B\right\} \tag{3.2}$$

$\varphi(\varepsilon, \boldsymbol{x})$ is the loyalty rate of an incumbent buyer, a function of the quality of the contracts offered and the product range of the firm. The expectation is taken over the universe of products with p_i the price of set by the firm for good with distance i to the core product. The value is promised ex-ante, therefore it is the expected sum of the value from pursuing the relation and the value of searching for another firm, weighted by the retention rate.

$$s - B = \mathbf{E_i}\left\{\varphi(\varepsilon, \boldsymbol{x})\left[u - p_i - \rho + c_s + \beta(1-\delta)\left(S^{+} - B\right)\right]\right\}$$

I obtain a law of motion for the promised utility. The surplus of being matched is simply the probability of the relation to pursue times the expected flow utility of this period net of the opportunity cost of not searching this period, $\rho - c_s$, plus the expected utility of further pursuing the relation.

Participation Constraint I can replace the value of p_i using equation (3.1).

$$s - B = \mathbf{E_i}\left\{\varphi(\varepsilon, \boldsymbol{x})\left(\left(\frac{\theta(\eta_i)}{\eta_i}\rho\right) - \rho + c_s\right)\right\} \tag{3.3}$$

The participation constraint summarizes an intra-temporal condition which simply states that this period's surplus of the buyer equals the retention probability, $\varphi(\varepsilon, \boldsymbol{x})$, times the expected surplus offered by the firm this period [6], minus the opportunity costs of not searching on the market this period, i.e. $(1-\beta)B \equiv \rho - c_s$.

Proposition 3.2.1. *If the search costs c_s is low enough, the customer retention rate of a firm is solely a function of its product range. (1) A buyer, incumbent or searching, always accepts a contract respecting*

[5] Since the matching probability is strictly smaller than 1, $u - p_i + \beta(1-\delta)(s' - B_i) > \rho - c_s, \forall x$ s.t. x defines an active sub-market

[6] Rewriting (3.1), we obtain an expression for the expected value for a buyer of a contract: $S^B(\eta_i) \equiv (\theta(\eta_i)/\eta_i)\rho = u - p_i + \beta(1-\delta)\left(S^{+} - B\right)$

the Market Utility Assumption (2) A firm separate from all the buyers who want to buy a good it does not provide.

Proof in appendix C.2.1. □

From Proposition 3.2.1, I can rewrite the participation constraint as:

$$s - B = \varphi(\varepsilon)\left(\mathbf{E_i}\left\{\frac{\theta(\eta_i)}{\eta_i}\right\}\rho - \rho + c_s\right) \tag{3.4}$$

To summarize, the law of motion of a firm's incumbent buyers depends on the firm's strategy in each market; the quality of the contracts and the number of sellers, as well as the number of products it produces:

$$N^+ = \int_{\Omega_f} n_i di = \int_{\Omega_f} \frac{\varphi(\varepsilon)}{\varepsilon} N + a_i\eta_i di = \varphi(\varepsilon)N + \int_{\Omega_f} a_i\eta_i di \tag{3.5}$$

with n_i and $(\varphi(\varepsilon)/\varepsilon)N$ respectively the number of buyers and incumbent buyers per product. It highlights a novel trade-off a firm faces to increase the number of its incumbent buyers: producing more goods at higher fixed costs and lower aggregate productivity, or increasing the sales in each existent market at a lower profit (higher price and potentially lower marginal productivity of labour).

Block recursivity property Firms' and buyers' problems do not depend on the distribution of agents over the states (s, N). This *block resistivity* property arises as a result of directed search. Firms cannot price discriminate among buyers and a contract contains all the information necessary to forecast the expected utility an agents get from accepting it. As such, market tightness is a sufficient statistics for both sellers and buyers to take an optimal decision. Finally, the competitive nature of directed search ensures that the market utility assumption is fulfilled in any active sub-market. The inactive buyer's outside option is constant across good markets thanks to the free entry condition of firms in each market.

Timing A period of time follows a precise timing:

1. Firms enter the economy at cost K.
2. Firms decide about their product range $\Omega_f = [-\varepsilon/2; \varepsilon/2]$ and, in each market, the number of sellers they hire a_i and the contract they offer $\{x_i(p_i, s')\}_{i\in\Omega_f}$ to their new and incumbents customers.
3. Firms produce goods.

4. Buyers "wake up" and get to know about which product i they want to buy today at utility u and observe the contracts available.
5. If available from the firm it purchased last period she expresses her interest to the firm. Otherwise the firm and customer separate with the endogenous probability $\varphi(\varepsilon)$.
6. Buyers who did not trade last period and incumbent buyers who separated search on the market.
7. Trades take place.
8. Firms are randomly hit by an exit shock with probability δ.

3.2.4 Firms' problem:

Firms maximize the discounted sum of their flow profits. We obtain a value function $V(s,N;x)$ with x its productivity, s its surplus commitment, and N its incumbent buyers.

$$V(N,s;x) = \max_{\mathbf{p},\boldsymbol{\eta},\mathbf{n},\mathbf{a},s^+,\Omega_f,N^+} \int_{\Omega_f} \left[n_i p_i - C(n_i;\hat{i},z) - \kappa(a_i) - c_p\right] di - c_{hq} + \beta(1-\delta)V(N^+,s^+;x) \tag{3.6}$$

conditional on the Market utility assumption (3.1), the Law of Motion of the number of incumbent buyers (3.5) and the Promised utility constrain (3.4). The flow profit consist of the integral of the per product profits over the product range. The head-quarter cost ensures that it is optimal for a firm absent of every market to exit.

At the market level, firms have two means of increasing their sales: the price and the marketing intensity. Combining the first order conditions with respect to (a_i), (η_i) and the envelop condition with respect to (N), I obtain an intra-temporal within market condition:

$$(\theta'(\eta_i)\eta_i - \theta(\eta_i)\rho = \kappa'(a_i) \Leftrightarrow \frac{1-\sigma_{\eta,\theta}}{\sigma_{\eta,\theta}} S^B(\eta_i) = \frac{\kappa'(a_i)}{\eta_i} \tag{3.7}$$

with S^B the surplus of the buyer and $\sigma_{\eta,\theta}$ the matching function elasticity [7]. Firms set their matching probability and the number of seller in a given market such that the marginal costs of increasing sales

[7] We define $\sigma_{\eta,\theta} = \frac{\theta(\eta_i)}{\theta'(\eta_i)\eta_i}$ the matching function elasticity, and $S^B(\eta_i) = \frac{\theta(\eta_i)}{\eta_i}\rho$ the expected surplus of matching for the buyer, we then obtain:

$$\frac{\partial}{\partial \eta_i}\left(\frac{\theta(\eta_i)}{\eta_i}\rho\right) = \frac{\theta'(\eta_i)\eta_i - \theta(\eta_i)}{\eta_i^2}\rho = \left(\frac{\theta'(\eta_i)\eta_i}{\theta(\eta_i)}.\frac{\theta(\eta_i)}{\eta_i^2} - \frac{\theta(\eta_i)}{\eta_i^2}\right)\rho = \left(\frac{1-\sigma_{\eta,\theta}}{\sigma_{\eta,\theta}}S^B(\eta_i)\right)\frac{1}{\eta_i} = \frac{\xi(\eta_i)}{\eta_i}$$

using each margin are equal; the marginal cost of hiring an extra seller per unit sold equals the marginal change of the matching rate to an increase in the quality of the contract. The optimal level of the matching probability a_i is increasing in the marching probability η_i, which implies that in a given market a firm would concomitantly use both margins to increase its sales: lowering the price and increasing its marketing spendings.

Firms enter each period with an equal number of incumbent buyers in each market but hold a lower productivity in goods further from the core product. Combining the first order condition with respect to (η_i) for two goods: the core product $\hat{\imath}$ and $j \in \Omega_f$, I obtain an intra-temporal condition:

$$(\theta'(\eta_{\hat{\imath}}) - \theta'(\eta_j))\rho = C_1(n_i; j, z) - C_1(n_i; \hat{\imath}, z) \tag{3.8}$$

It defines the optimal matching probability in each market relative to the one at the core product and can be interpreted as the higher the relative marginal costs of production the lower the relative matching rate. Optimally, firms set a lower matching rate and a higher marginal cost in markets for goods further from their core competence

Proposition 3.2.2. *Firms offer an imperfect pass-through of its heterogeneous productivity across goods.*

(1) The further a good is from the core product, the higher the price and the lower the quantity sold.

(2) Under standard assumptions about the matching function, the markup a firm charges over its marginal costs is decreasing in the distance from its core product.

Proof in appendix C.2.2. □

These results are in line with the empirical facts (2) and (3) about the positive correlation between product efficiency and respectively sales and markups.

The absence of persistence in buyers' preference provides tractability as the distribution of incumbent buyers across the product range is not a state variable in the firms' recursive problem. I conjecture that this assumption is not key to my results. Indeed, introducing persistence in buyers' preference would further increase the incentive for firms to attract buyers at goods close to the core competence. In the limit case of full persistence, multi-product firms would behave across markets as independent single-product firms with heterogeneous productivity; I can then infer from Gilbukh and Roldan (2017) that the multi-product firm would charge higher markups on goods for which it has a higher productivity.

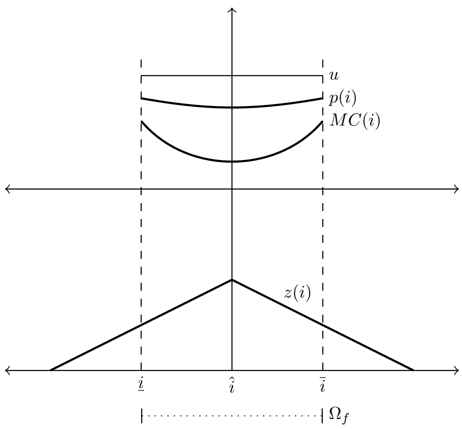


Figure (3.2) The within firm heterogeneity across markets i in terms of labour productivity $z(i)$, marginal costs $MC(i)$ and prices $p(i)$. The relative convexities of $MC(i)$ and $p(i)$ highlight the imperfect pass-through in prices of the decreasing productivitya along the product range, as well as the markups heterogeneity.

Multi-product firms also decide over their extensive margin, the product range. They differ from single-product firms because their market specific strategies encompass firm level determinants. Combining the first order conditions for (ε) and (η_i) taken at $i = \hat{\varepsilon}$ I obtain:

$$\begin{aligned} &\left(\varphi'(\varepsilon) - \frac{\varphi(\varepsilon)}{\varepsilon}\right) N \int_{-\varepsilon/2}^{\varepsilon/2} C_{i,1} di + C_\varepsilon + c_p \\ &= (\varphi'(\varepsilon)N + a_\varepsilon \eta_\varepsilon) C_{\varepsilon,1} + \underbrace{a_\varepsilon \eta_\varepsilon \left(\theta'(\eta_\varepsilon) - \frac{\theta(\eta_\varepsilon)}{\eta_\varepsilon}\right)}_{a_\varepsilon C_{\varepsilon,2}} \rho + \varphi'(\varepsilon) N (\theta'(\eta_\varepsilon)\rho - \rho + c_s) \end{aligned} \tag{3.9}$$

Firms decide upon how to attract buyers over their growth path. Using (3.3) at the next period, I obtain a dynamic condition relating the flow social surplus of attracting a buyer to the recruitment costs wedge between today and tomorrow:

$$u - C_{1,i} - \beta(1-\delta)\varphi(\varepsilon^+)(\rho - c_s) = \rho[\theta'(\eta_i) - \beta(1-\delta)\varphi(\varepsilon^+)\mathbf{E_j}(\theta'(\eta_j^+))] \tag{3.10}$$

It simply states that if a firm's marginal costs of production increases today, it would decrease its matching probability today relative to tomorrow.

Stationary competitive search equilibrium. Due to free entry of firms, no entrant makes positive profits on expectation:

$$\sum_{x \in \mathbb{X}} \pi(x) \int_{\Omega} \phi(i) V(0,x) di \leq K \quad \Leftrightarrow \quad \sum_{x \in \mathbb{X}} \pi(x) V(0,x) \leq K \tag{3.11}$$

with equality if entry is positive [8]. With $\phi(i)$ the uniform probability distribution of entering in market i and $\pi(x)$ the probability distribution of having productivity x.

Since the model is deterministic and all markets are identical by assumption, age is a sufficient information to define the optimal policy of a firm. Let's define B_0 the measure of firms entering the market in equilibrium, B_a the measure of firms of age a and N_f the measure of incumbent firms. Since the exit shock is exogenous and hits with exogenous probability δ, we know that the number of incumbent firms of age a is:

$$F_a = (1-\delta)^a F_0 \tag{3.12}$$

$$N_f = \sum_{a\geq 0, x\in\mathbb{X}} F_a = \frac{F_0}{1-\delta} \tag{3.13}$$

I can then define an optimal path $(N_a^x, \boldsymbol{\eta}_a^x, \boldsymbol{p}_a^x, \boldsymbol{a}_a^x, \varepsilon_a^x, s_a^{x+})$ which solely depends on the firm's age. Similarly, the number of potential buyers looking for consumption good i at contract $x_i(\eta_a, s_a^+)$ is $\theta(x_i(\eta_a, s_a^+))a_{i,a}$. We can then pin-down the equilibrium firm entry B_0 from the aggregate feasibility condition:

$$1 = \sum_{a\geq 0, u\in\mathbb{X}} (1-\delta)^a B_0 \int_{-\bar{\varepsilon_a^x}}^{\bar{\varepsilon_a^x}} \left(\frac{\varphi(\varepsilon_a^x)}{\varepsilon_a^x} N_a^x + \theta(x_i(\eta_a^x, s_a^{x,+}))a_{i,a}^x \right) di \tag{3.14}$$

Given the problem described above, we can define and characterize a stationary competitive search equilibrium.

Definition 3.2.3. *A stationary competitive search equilibrium in this framework is a list:* $(B_0, (N_a^x, \boldsymbol{\eta}_a^x, \boldsymbol{p}_a^x, \boldsymbol{a}_a^x, \varepsilon_a^x)_{a\leq 0, x\in\mathbb{X}})$ *with the following properties. Potential customers' consumption good search strategies maximize the utility:* (3.1) *holds for all contract* $x_i(p_a^{x,i}, s_a^{x,+})$. *Firms' customer recruitment policies are optimal:* $(N_a^x, \boldsymbol{\eta}_a^x, \boldsymbol{p}_a^x, \boldsymbol{a}_a^x, \varepsilon_a^x)$ *solves* (3.6). *There is free entry of firms:* (3.11) *and* $B_0 \leq 0$ *hold with complementary slackness. Aggregate resource feasibility* (3.14) *holds.*

Proposition 3.2.4. *The stationary competitive equilibrium is socially optimal.*

Proof in appendix C.2.3. □

[8] We could assume instead free entry in each market. The results would be unchanged in the stationary version of the model as expected profits in each market are identical

3.3 Numerical solution and simulation

3.3.1 Solution method

For the simulation I assume that production technology is the simplest concave production function with labour as single input, while frictions take the form of a Cob-Douglas search function. The parametrization and the functional form are explained in more details in appendix C.3.1. At this stage, the parametrization is mainly based on the literature, but the value of some parameters still has no meaningful origin. In a later iteration of the paper, I hope to estimate the model to target empirical moments. Finally, the numerical solution suffers from severe numerical approximation due to the absence of endogenous grid in the solution algorithm.

We proved that the stationary competitive equilibrium is socially optimal. Therefore, we solve the simpler social planner's problem described in appendix C.2.3. Using a Value Function Iteration algorithm, I obtain the policy functions $\left\{\{\eta^x_{i,a}, a^x_{i,a}\}_{i\in\Omega^x_{f,a}}, \varepsilon^x_a, N^x_a\right\}_{a\geq 0, x\in X}$ of the firm from maximizing the social value of the firm (C.6). I adjust the multiplier λ_P so that the free entry condition is exactly fulfilled. Finally, I pin-down the entry F_0 using the aggregate feasibility condition and the law of motion of firm size :

$$F_0 = \frac{1}{\sum_{a\geq 0}(1-\delta)^a \sum_x \pi(x)\left(\varphi(\varepsilon^x_a)N^x_a + \int_{\varepsilon^x_a}\theta(\eta^x_{i,a}).a^x_{i,a}di\right)}$$

That way, we can recover the distribution of firms.

3.3.2 Numerical results

The numerical results allow to get a wider perspective on some results. Firms slowly expand their product range as they grow (see figure 3.3), while the matching rate at the core product stays almost constant (see figure 3.4) [9]. As their customer base increases, firms can spread the fixed costs of producing an extra good over more buyers. This latter result is in line with the empirical fact (5).

The evolution of the markups is heterogeneous (see figure 3.5), if constant for the core products it significantly decreases for the more margin products. Therefore, the mean markups (blue) decreases both due to the product range expansion and the decreasing markups at the marginal products. I name this phenomena the *curse of the large firm.* As a the customer base grows larger, a firm focuses less

[9] As we can see in C.1, firms do not grow further than 0.6

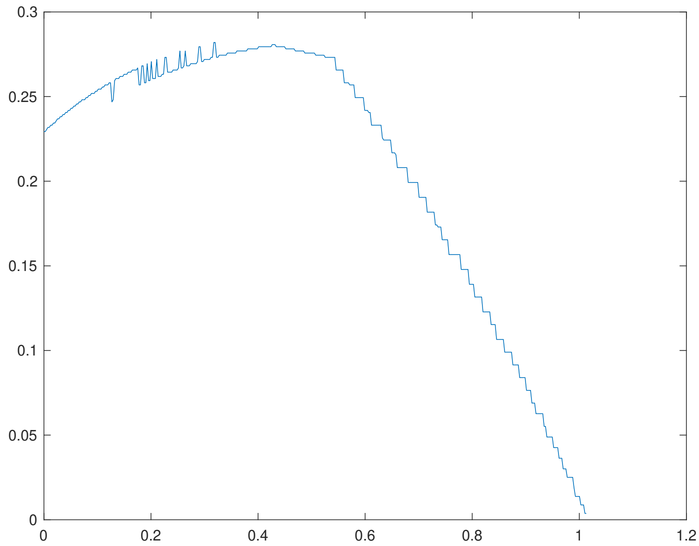


Figure (3.3) The optimal product range ε (in abscissa) as a function for a firm size

on attracting new buyers, and therefore, has greater difficulty to re-centre demand to its most efficient products; its demand distribution over its product range is flatter increasing mechanically its average marginal costs leading to a reduction of its mean markup. This result shows that it could be potentially important to account for the multi-product dimension when studying markups dynamics empirically.

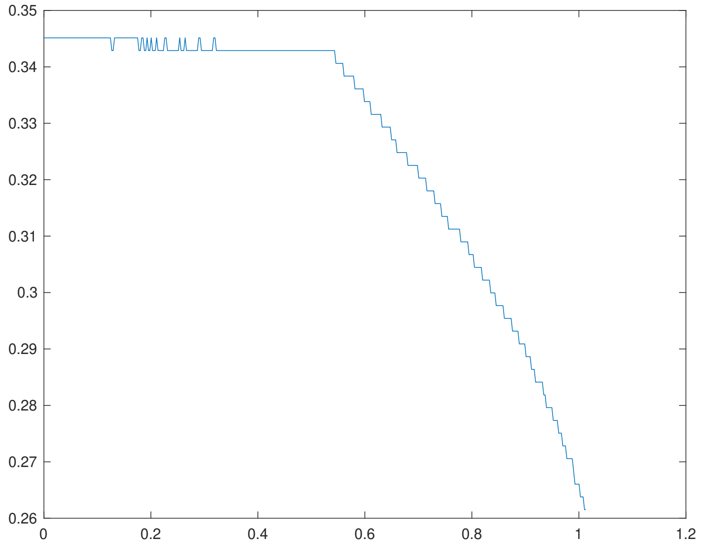


Figure (3.4) The optimal matching probability at the core product $\eta_{\hat{i}}$ as a function for a firm size

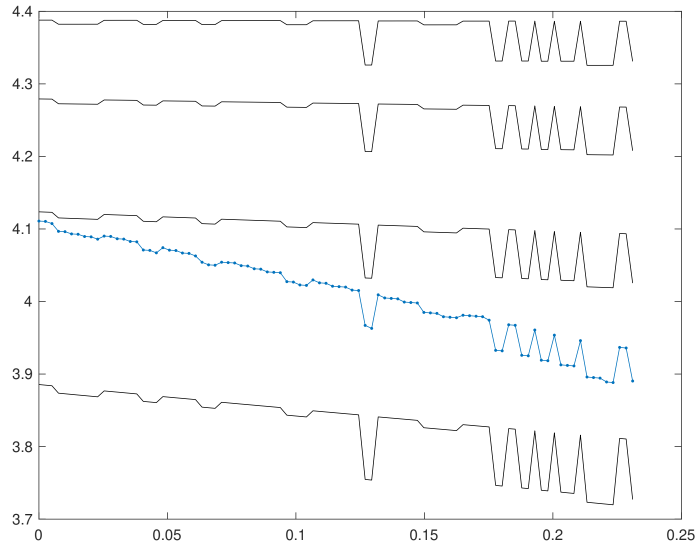


Figure (3.5) Markups over the product range (core to margin from top to down) and aggregate markups (blue) as a function of a firm's size

3.4 Conclusion

I presented a model of firm dynamics over two dimension: the product range and the customer capital. I proved and showed using a numerical solution that the model could fit stylized facts about within and across firm heterogeneity. A little explored area of research is the pass through of idiosyncratic firm level productivity shocks on multi-product firms, especially the potential heterogeneous pass-through arising from the heterogeneous trade-offs faced by firms of different sizes and product range.

References

Abeliansky, A. and Prettner, K. (2017). Automation and Demographic Change. SSRN Scholarly Paper ID 2959977, Social Science Research Network, Rochester, NY.

Abeliansky, A. L., Algur, E., Bloom, D. E., and Prettner, K. (2020). The future of work: Meeting the global challenges of demographic change and automation. *International Labour Review*, 159(3):285–306.

Acemoglu, D., Akcigit, U., and Kerr, W. (2016). Networks and the Macroeconomy: An Empirical Exploration. *NBER Macroeconomics Annual*, 30:273–335.

Acemoglu, D. and Restrepo, P. (2017). Secular Stagnation? The Effect of Aging on Economic Growth in the Age of Automation. *American Economic Review*, 107(5):174–179.

Acemoglu, D. and Restrepo, P. (2018). Demographics and Automation. Working Paper 24421, National Bureau of Economic Research.

Adão, R., Kolesár, M., and Morales, E. (2019). Shift-Share Designs: Theory and Inference*. *The Quarterly Journal of Economics*, 134(4):1949–2010.

Aguiar, M. and Hurst, E. (2005). Consumption versus Expenditure. *Journal of Political Economy*, 113(5):919–948.

Aguiar, M. and Hurst, E. (2007). Life-Cycle Prices and Production. *American Economic Review*, 97(5):1533–1559.

Albrecht, J., Menzio, G., and Vroman, S. (2021). Vertical Differentiation in Frictional Product Markets.

Anderson, E., Rebelo, S., and Wong, A. (2018). Markups Across Space and Time. Working Paper 24434, National Bureau of Economic Research.

Austin, B., Glaeser, E., and Summers, L. (2018). Jobs for the Heartland: Place-Based Policies in 21st-Century America. *Brookings Papers on Economic Activity*, pages 151–232.

Bai, Y., Rıos-Rull, J.-V., and Storesletten, K. (2019). Demand Shocks as Technology Shocks.

Barrot, J.-N. and Sauvagnat, J. (2016). Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks *. *The Quarterly Journal of Economics*, 131(3):1543–1592.

Bilal, A. (2021). The Geography of Unemployment. *NBER Working Papers*, page 69.

Boehm, C. E., Flaaen, A., and Pandalai-Nayar, N. (2019). Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tōhoku Earthquake. *The Review of Economics and Statistics*, 101(1):60–75.

Bornstein, G. (2019). ENTRY AND PROFITS IN AN AGING ECONOMY: THE ROLE OF CONSUMER INERTIA. page 82.

Borusyak, K., Hull, P., and Jaravel, X. (2018). Quasi-Experimental Shift-Share Research Designs. Technical Report w24997, National Bureau of Economic Research, Cambridge, MA.

Burdett, K. and Judd, K. L. (1983). Equilibrium Price Dispersion. *Econometrica*, 51(4):955.

Butters, G. R. (1977). Equilibrium Distributions of Sales and Advertising Prices. *Review of Economic Studies*, page 28.

Carvalho, V. M., Nirei, M., Saito, Y. U., and Tahbaz-Salehi, A. (2021). Supply Chain Disruptions: Evidence from the Great East Japan Earthquake*. *The Quarterly Journal of Economics*, 136(2):1255–1321.

Chen, Y. and Zhang, T. (2011). Equilibrium price dispersion with heterogeneous searchers. *International Journal of Industrial Organization*, 29(6):645–654.

Couch, K. A. and Placzek, D. W. (2010). Earnings Losses of Displaced Workers Revisited. *The American Economic Review*, 100(1):572–589.

De Loecker, J., Goldberg, P. K., Khandelwal, A. K., and Pavcnik, N. (2016). Prices, Markups, and Trade Reform. *Econometrica*, 84(2):445–510.

Dhyne, E., Kikkawa, A. K., Mogstad, M., and Tintelnot, F. (2021). Trade and Domestic Production Networks. *The Review of Economic Studies*, 88(2):643–668.

Dhyne, E., Petrin, A., Smeets, V., and Warzynski, F. (2017). Multi Product Firms, Import Competition, and the Evolution of Firm-product Technical Efficiencies. Working Paper 23637, National Bureau of Economic Research.

di Giovanni, J., Levchenko, A. A., and Mejean, I. (2014). Firms, Destinations, and Aggregate Fluctuations. *Econometrica*, 82(4):1303–1340.

East, R., Uncles, M. D., and Lomax, W. (2014). Hear nothing, do nothing: The role of word of mouth in the decision-making of older consumers. *Journal of Marketing Management*, 30(7-8):786–801.

Eckel, C. and Neary, J. P. (2010). Multi-Product Firms and Flexible Manufacturing in the Global Economy. *The Review of Economic Studies*, 77(1):188–217.

Fagereng, A., Guiso, L., and Pistaferri, L. (2018). Portfolio Choices, Firm Shocks, and Uninsurable Wage Risk. *The Review of Economic Studies*, 85(1):437–474.

Ferrari, A. (2019). Global Value Chains and the Business Cycle. page 78.

Fitzgerald, D., Haller, S., and Yedid-Levi, Y. (2016). How Exporters Grow. Technical Report w21935, National Bureau of Economic Research, Cambridge, MA.

Fitzgerald, D. and Priolo, A. (2018). How Do Firms Build Market Share? Working Paper 24794, National Bureau of Economic Research.

Gabaix, X. and Koijen, R. S. J. (2020). Granular Instrumental Variables. *NBER Working Papers*, page 98.

Galaasen, S., Jamilov, R., Juelsrud, R., and Rey, H. (2020). Granular Credit Risk. Working Paper 27994, National Bureau of Economic Research.

Gathmann, C. and Schönberg, U. (2010). How General Is Human Capital? A Task-Based Approach. *Journal of Labor Economics*, 28(1):1–49.

Gehringer, A. and Prettner, K. (2019). LONGEVITY AND TECHNOLOGICAL CHANGE. *Macroeconomic Dynamics*, 23(4):1471–1503.

Ghironi, F. and Melitz, M. J. (2005). International Trade and Macroeconomic Dynamics with Heterogeneous Firms. *The Quarterly Journal of Economics*, 120(3):865–915.

Gilbukh, S. and Roldan, P. (2017). Firm Dynamics and Pricing Under Customer Capital Accumulation. SSRN Scholarly Paper ID 2909347, Social Science Research Network, Rochester, NY.

Goldsmith-Pinkham, P., Sorkin, I., and Swift, H. (2020). Bartik instruments: What, when, why, and how. *American Economic Review*, 110(8):2586–2624.

Gourio, F. and Rudanko, L. (2014). Customer Capital. *The Review of Economic Studies*, 81(3):1102–1136.

Gutchess, A. H. (2011). Cognitive psychology and neuroscience of aging. In *The Aging Consumer*, pages 25–46. Routledge.

Hargittai, E. (2001). Second-level digital divide: Mapping differences in people's online skills. *arXiv preprint cs/0109068*.

Hottman, C. J., Redding, S. J., and Weinstein, D. E. (2016). Quantifying the Sources of Firm Heterogeneity. *The Quarterly Journal of Economics*, 131(3):1291–1364.

Insee (2012a). Déclarations annuelle de données sociales : Fichier Postes - 1993.

Insee (2012b). Déclarations annuelle de données sociales : Fichier Postes - 1994.

Insee (2012c). Déclarations annuelle de données sociales : Fichier Postes - 1995.

Insee (2012d). Déclarations annuelle de données sociales : Fichier Postes - 1996.

Insee (2012e). Déclarations annuelle de données sociales : Fichier Postes - 1997.

Insee (2012f). Déclarations annuelle de données sociales : Fichier Postes - 1998.

Insee (2012g). Déclarations annuelle de données sociales : Fichier Postes - 1999.

Insee (2012h). Déclarations annuelle de données sociales : Fichier Postes - 2000.

Insee (2012i). Déclarations annuelle de données sociales : Fichier Postes - 2001.

Insee (2012j). Déclarations annuelle de données sociales : Fichier Postes - 2002.

Insee (2012k). Déclarations annuelle de données sociales : Fichier Postes - 2003.

Insee (2012l). Déclarations annuelle de données sociales : Fichier Postes - 2004.

Insee (2012m). Déclarations annuelle de données sociales : Fichier Postes - 2005.

Insee (2012n). Déclarations annuelle de données sociales : Fichier Postes - 2008.

Insee (2012o). Déclarations annuelle de données sociales : Fichier Postes - 2009.

Insee (2012p). Déclarations annuelle de données sociales : fichier Postes - 2010.

Insee (2014a). Déclarations annuelle de données sociales : Fichier Postes - 2006.

Insee (2014b). Déclarations annuelle de données sociales : Fichier Postes - 2007.

Insee (2014c). Déclarations annuelle de données sociales : Fichier Postes - 2011.

Insee (2015). Déclarations annuelle de données sociales : Fichier Postes - 2012.

Insee (2016a). Déclarations annuelle de données sociales : Fichier Postes - 2013.

Insee (2016b). Déclarations annuelle de données sociales : Fichier Postes - 2014.

Insee (2017). Déclarations annuelle de données sociales : Fichier Postes - 2015.

Insee (2019). Base Tous Salariés : Fichier Postes - 2016.

Insee (2020). Base Tous Salariés : Fichier Postes - 2017.

Insee (2021a). Base Tous Salariés : Fichier Postes - 2018.

Insee (2021b). Base Tous Salariés : Fichier Postes - 2019.

Insee and Ministère Des Finances (DGFiP) (2013). Fichier approché des résultats d'Esane - 2011.

Insee and Ministère Des Finances (DGFiP) (2014a). Fichier approché des résultats d'Esane - 2008.

Insee and Ministère Des Finances (DGFiP) (2014b). Fichier approché des résultats d'Esane - 2009.

Insee and Ministère Des Finances (DGFiP) (2014c). Fichier approché des résultats d'Esane - 2010.

Insee and Ministère Des Finances (DGFiP) (2014d). Fichier approché des résultats d'Esane - 2012.

Insee and Ministère Des Finances (DGFiP) (2014e). Fichier de comptabilité unifié dans SUSE - 1994.

Insee and Ministère Des Finances (DGFiP) (2014f). Fichier de comptabilité unifié dans SUSE - 1995.

Insee and Ministère Des Finances (DGFiP) (2014g). Fichier de comptabilité unifié dans SUSE - 1996.

Insee and Ministère Des Finances (DGFiP) (2014h). Fichier de comptabilité unifié dans SUSE - 1997.

Insee and Ministère Des Finances (DGFiP) (2014i). Fichier de comptabilité unifié dans SUSE - 1998.

Insee and Ministère Des Finances (DGFiP) (2014j). Fichier de comptabilité unifié dans SUSE - 1999.

Insee and Ministère Des Finances (DGFiP) (2014k). Fichier de comptabilité unifié dans SUSE - 2000.

Insee and Ministère Des Finances (DGFiP) (2014l). Fichier de comptabilité unifié dans SUSE - 2001.

Insee and Ministère Des Finances (DGFiP) (2014m). Fichier de comptabilité unifié dans SUSE - 2002.

Insee and Ministère Des Finances (DGFiP) (2014n). Fichier de comptabilité unifié dans SUSE - 2003.

Insee and Ministère Des Finances (DGFiP) (2014o). Fichier de comptabilité unifié dans SUSE - 2004.

Insee and Ministère Des Finances (DGFiP) (2014p). Fichier de comptabilité unifié dans SUSE - 2005.

Insee and Ministère Des Finances (DGFiP) (2014q). Fichier de comptabilité unifié dans SUSE - 2006.

Insee and Ministère Des Finances (DGFiP) (2014r). Fichier de comptabilité unifié dans SUSE - 2007.

Insee and Ministère Des Finances (DGFiP) (2015). Fichier approché des résultats d'Esane - 2013.

Insee and Ministère Des Finances (DGFiP) (2016). Fichier approché des résultats d'Esane - 2014.

Insee and Ministère Des Finances (DGFiP) (2017). Fichier approché des résultats d'Esane - 2015.

Insee and Ministère Des Finances (DGFiP) (2018). Fichier approché des résultats d'Esane - 2016.

Insee and Ministère Des Finances (DGFiP) (2019). Fichier approché des résultats d'Esane - 2017.

Insee and Ministère Des Finances (DGFiP) (2020). Statistique structurelle annuelle d'entreprises issue du dispositif ESANE - 2018.

Insee and Ministère Des Finances (DGFiP) (2021). Statistique structurelle annuelle d'entreprises issue du dispositif ESANE - 2019.

Kaas, L. and Kircher, P. (2015). Efficient Firm Dynamics in a Frictional Labor Market. *American Economic Review*, 105(10):3030–3060.

Kambourov, G. and Manovskii, I. (2009). Occupational Specificity of Human Capital*. *International Economic Review*, 50(1):63–115.

Kaplan, G. and Menzio, G. (2016). Shopping Externalities and Self-Fulfilling Unemployment Fluctuations. *Journal of Political Economy*, 124(3):771–825.

Klemperer, P. (1995). Competition when Consumers have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics, and International Trade. *The Review of Economic Studies*, 62(4):515–539.

Kline, P. and Moretti, E. (2013). Place Based Policies with Unemployment. *American Economic Review*, 103(3):238–243.

Kline, P. and Moretti, E. (2014). Local Economic Development, Agglomeration Economics, and the Big Push: 100 Years of Evidence from the Tennessee Valley Authority. *The Quarterly Journal of Economics*, 129(1):275–332.

Kryvtsov, O. and Vincent, N. (2021). The Cyclicality of Sales and Aggregate Price Flexibility. *Review of Economic Studies*, 88(1):334–377.

Kuhn, M., Manovskii, I., and Qiu, X. (2021). The Geography of Job Creation and Job Destruction. Working Paper 29399, National Bureau of Economic Research.

Lambert-Pandraud, R. and Laurent, G. (2010). Why do Older Consumers Buy Older Brands? The Role of Attachment and Declining Innovativeness. *Journal of Marketing*, 74(5):104–121.

Lambert-Pandraud, R., Laurent, G., and Lapersonne, E. (2005). Repeat Purchasing of New Automobiles by Older Consumers: Empirical Evidence and Interpretations. *Journal of Marketing*, 69(2):97–113.

Milgrom, P. and Roberts, J. (1990). The Economics of Modern Manufacturing: Technology, Strategy, and Organization. *The American Economic Review*, 80(3):511–528.

Moen, E. R. (1997). Competitive Search Equilibrium. *Journal of Political Economy*, 105(2):385–411.

Moretti, E. (2010). Local Multipliers. *American Economic Review*, 100(2):373–377.

Neal, D. (1995). Industry-Specific Human Capital: Evidence from Displaced Workers. *Journal of Labor Economics*, 13(4):653–677.

Paciello, L., Pozzi, A., and Trachter, N. (2018). PRICE DYNAMICS WITH CUSTOMER MARKETS: Price Dynamics with Customer Markets. *International Economic Review*.

Peters, E. (2011). Aging-related changes in decision making. In *The Aging Consumer*, pages 97–124. Routledge.

Phelps, E. S. and Winter, S. G. (1970). Optimal price policy under atomistic competition. *Microeconomic foundations of employment and inflation theory*, pages 309–337.

Sara-Zaror, F. (2021). Expected Inflation and Welfare: The Role of Consumer Search. *SSRN Electronic Journal*.

Stahl, D. O. (1989). Oligopolistic Pricing with Sequential Consumer Search. *The American Economic Review*, 79(4):700–712.

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R., and de Vries, G. J. (2015). An Illustrated User Guide to the World Input–Output Database: The Case of Global Automotive Production. *Review of International Economics*, 23(3):575–605.

United Nations (2022). *World Population Prospects 2022: Summary of Results*. Statistical Papers - United Nations (Ser. A), Population and Vital Statistics Report. United Nations.

van Vlokhoven, H. (2019). Estimating the Cost of Capital and the Profit Share.

Varian, H. R. (1980). A Model of Sales. *The American Economic Review*, 70(4):651–659.

A

Appendix to chapter 1

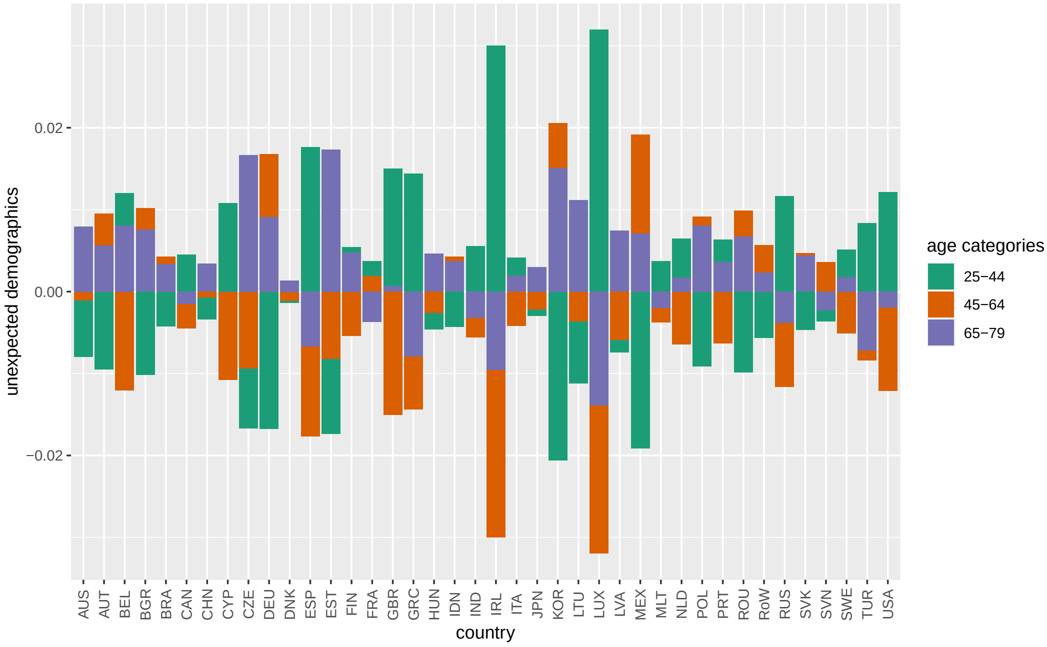
A.1 Empirical analysis

A.1.1 Summary statistics

Table (A.1) Summary statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Δlog Price	1,319	0.563	0.848	−1.661	0.109	0.670	3.776
Δlog Production	1,319	0.346	0.489	−2.885	0.091	0.566	3.177
Δlog Productivity	1,319	0.298	0.474	−2.318	0.026	0.530	3.841
Initial Foreign Exposure	1,319	0.390	0.260	0.0001	0.174	0.574	1.000
ΔYoung	1,319	0.033	0.022	−0.015	0.015	0.051	0.079
ΔMiddle-aged	1,319	−0.025	0.022	−0.078	−0.040	−0.007	0.026
ΔOld	1,319	−0.008	0.015	−0.044	−0.019	0.001	0.020
$\Delta\mathbb{E}$ Young	1,319	0.001	0.011	−0.021	−0.007	0.006	0.032
$\Delta\mathbb{E}$ Middle-aged	1,319	−0.004	0.007	−0.020	−0.008	0.001	0.012
$\Delta\mathbb{E}$ Old	1,319	0.003	0.007	−0.014	−0.002	0.007	0.017
ΔYoung Demand OLS	1,319	−0.030	0.018	−0.104	−0.041	−0.016	0.077
ΔMiddle-aged Demand OLS	1,319	0.024	0.018	−0.026	0.013	0.034	0.078
ΔOld Demand OLS	1,319	0.007	0.011	−0.046	−0.0003	0.014	0.051
ΔYoung Demand IV(1)	1,319	0.014	0.010	0.0000	0.006	0.020	0.059
ΔMiddle-aged Demand IV(1)	1,319	−0.010	0.008	−0.059	−0.015	−0.004	0.0000
ΔOld Demand IV(1)	1,319	−0.004	0.003	−0.024	−0.005	−0.001	0.006
ΔYoung Demand IV(2)	1,319	−0.0001	0.001	−0.013	−0.001	0.0003	0.008
ΔMiddle-aged Demand IV(2)	1,319	−0.001	0.001	−0.007	−0.001	−0.0001	0.003
ΔOld Demand IV(2)	1,319	0.001	0.001	−0.001	0.0003	0.001	0.010

Figure (A.1) Unpredicted demographic changes aggregated in three age categories.



Note: this figure shows the prediction revisions for the year 2006 between 1996 and 2006. For each age category we compute its share in the total population between 25 and 79 years old. The prediction revisions are computed according to equation (1.8) from different vintages of the UN World Population Prospects.

A.1.2 Tests

Shift-share correlation Test

Table (A.2) Shift-share correlation test

	Share				
	Correlation	95% Confidence Interval	t-stat	df	p-value
ΔYoung	0.047	[0.038 0.055]	10.895	54243	< 2.2e-16
ΔMiddle-aged	−0.033	[−0.042, −0.025]	−7.769	54243	8.022e-15
ΔOld	−0.019	[−0.028, −0.011]	−4.522	5424	6.148e-06

Note:

Correlation between demographic changes and foreign exposure

Table (A.3) Correlation between domestic demographic change and the change in foreign exposure. Changes are between 1996 and 2006.

	Δ Foreign Exposure					
	non-weighted observations			weighted observations		
	(1)	(2)	(3)	(4)	(5)	(6)
ΔYoung	−0.631			−0.679		
	(1.184)			(0.568)		
ΔMiddle-aged		0.448			0.482	
		(0.840)			(0.403)	
ΔOld			−1.544			−1.661
			(2.895)			(1.388)
Country & Sector FE	✓	✓	✓	✓	✓	✓
Observations	1,354	1,354	1,354	1,354	1,354	1,354
R^2	0.256	0.256	0.256	0.545	0.545	0.545
Adjusted R^2	0.214	0.214	0.214	0.519	0.519	0.519

Note: *p<0.1; **p<0.05; ***p<0.01

F Tests

Table (A.4) F tests

	F test					
	IV(1)			*IV(2)*		
share:	Foreign Exposure			Foreign Exposure		
shift:	ΔDemographics (1996-2006)			ΔExpectations (1996-2006)		
	(1)	(2)	(3)	(4)	(5)	(6)
ΔYoung Demand	14.613			29.492		
ΔMiddle-aged Demand		158.21			91.941	
ΔOld Demand			88.834			17.346

Note:

A.1.3 Robustness Checks

Prediction revision of future demographic changes

Table (A.5) IV with unpredicted demographic changes for years 2010, 2015, 2020

expectations for year:	2010			2015			2020		
	Δlog Price (1996-2006)								
ΔYoung Demand	−12.110			−11.340			−10.553		
	(13.658)			(13.157)			(12.649)		
ΔMiddle-aged Demand		9.279**			9.615**			7.511**	
		(4.095)			(4.056)			(3.096)	
ΔOld Demand			−14.041*			−13.001*			−13.960**
			(7.770)			(7.342)			(6.400)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R^2	0.849	0.879	0.876	0.853	0.878	0.878	0.856	0.882	0.876
Adjusted R^2	0.840	0.872	0.869	0.844	0.871	0.871	0.848	0.875	0.869
	Δlog Production (1996-2006)								
ΔYoung Demand	12.069*			11.483*			11.771*		
	(6.748)			(6.442)			(6.549)		
ΔMiddle-aged Demand		−7.849***			−7.249***			−5.340***	
		(2.252)			(2.096)			(1.496)	
ΔOld Demand			17.847			19.134			18.880
			(18.286)			(20.468)			(19.576)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R^2	0.504	0.573	0.504	0.511	0.575	0.492	0.507	0.580	0.495
Adjusted R^2	0.475	0.548	0.475	0.482	0.550	0.462	0.478	0.555	0.465
	Δlog Productivity (1996-2006)								
ΔYoung Demand	27.689*			26.073*			24.078*		
	(14.283)			(13.457)			(12.597)		
ΔMiddle-aged Demand		−17.808***			−16.267***			−10.423***	
		(3.523)			(3.404)			(3.738)	
ΔOld Demand			41.492			43.904			39.166
			(43.199)			(47.284)			(40.524)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R^2	0.159	0.465	0.136	0.201	0.479	0.088	0.249	0.517	0.181
Adjusted R^2	0.110	0.433	0.086	0.154	0.448	0.034	0.205	0.488	0.132

*p<0.1; **p<0.05; ***p<0.01

Note:

Different age categories

Table (A.6) Robustness check: Price: baseline with different thresholds for the age categories

	Δlog Price (1996-2006)								
	OLS			*IV(1)*			*IV(2)*		
share:				Foreign Exposure			Foreign Exposure		
shift:				ΔDemographics (1996-2006)			ΔExpectations (1996-2006)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ΔYoung Demand (25-39)	2.221			−2.853			−32.921		
	(1.794)			(4.155)			(54.798)		
ΔMiddle-aged Demand (40-59)		1.073			1.528			12.636***	
		(1.693)			(2.975)			(2.484)	
ΔOld Demand (60-79)			−2.640			−0.532			−28.398
			(2.065)			(4.231)			(42.007)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R^2	0.884	0.884	0.884	0.880	0.884	0.884	0.657	0.854	0.788
Adjusted R^2	0.878	0.877	0.878	0.873	0.877	0.877	0.637	0.845	0.776

*p<0.1; **p<0.05; ***p<0.01

Note:

Table (A.7) Production: baseline with different thresholds for the age categories

	Δlog Production (1996-2006)								
	OLS			*IV(1)*			*IV(2)*		
share:				Foreign Exposure			Foreign Exposure		
shift:				ΔDemographics (1996-2006)			ΔExpectations (1996-2006)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ΔYoung Demand (25-39)	−0.922			5.773			28.929		
	(1.545)			(4.682)			(31.540)		
ΔMiddle-aged Demand (40-59)		−1.320			−2.935			−7.706	
		(1.510)			(2.308)			(5.629)	
ΔOld Demand (60-79)			0.354			0.544			32.870
			(1.812)			(2.217)			(66.508)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R^2	0.578	0.579	0.578	0.557	0.577	0.578	0.164	0.556	0.190
Adjusted R^2	0.554	0.554	0.553	0.531	0.553	0.553	0.115	0.530	0.143

*p<0.1; **p<0.05; ***p<0.01

Note:

Table (A.8) Productivity: baseline with different thresholds for the age categories

	Δlog Productivity (1996-2006)								
	OLS			*IV(1)*			*IV(2)*		
share:				Foreign Exposure			Foreign Exposure		
shift:				ΔDemographics (1996-2006)			ΔExpectations (1996-2006)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ΔYoung Demand (25-39)	0.228			1.815			73.831		
	(1.980)			(11.079)			(84.261)		
ΔMiddle-aged Demand (40-59)		−2.704			−2.083			−22.647*	
		(1.742)			(6.014)			(12.542)	
ΔOld Demand (60-79)			1.180			4.130			76.944
			(2.100)			(4.481)			(157.982)
Initial Foreign Exposure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Country & Sector FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353	1,353
R^2	0.516	0.520	0.517	0.515	0.520	0.514	−1.950	0.302	−1.544
Adjusted R^2	0.488	0.492	0.488	0.487	0.492	0.485	−2.123	0.261	−1.694

*p<0.1; **p<0.05; ***p<0.01

Note:

A.2 Model Appendix

A.2.1 Proof of Lemma 1.3.3

Proof. Given $F(p)$ defined in $[b, \bar{r} \equiv min\{r^Y, r^M, p^{monopoly}\}]$, to show that the proposed is an equilibrium, we need to show that, given the reservation prices and the monopolistic price, and given that other firms choose $F(p)$, there is not a profitable deviation in setting a price above $\bar{r}$. For any $p \in [b, \hat{r}]$, the firm's expected profits are:

$$\mathbb{E}\{\pi(p,F)\} = \frac{\lambda^Y}{N} R^Y(p) + \frac{\lambda^M}{N} R^M(p) + \lambda^O (1 - F(p))^{N-1} R^O(p) - z(a(p)). \tag{A.1}$$

Since in equilibrium, it must hold that $\mathbb{E}\{\pi(p,F)\} = \mathbb{E}\{\pi(\bar{r},F)\}$, we can rewrite:

$$\mathbb{E}\{\pi(p,F)\} = \frac{\lambda^Y}{N} R^Y(\bar{r}) + \frac{\lambda^M}{N} R^M(\bar{r}) - z(a(\bar{r})). \tag{A.2}$$

A firm setting a price above $max\{r^Y, r^O\}$ would make zero profits. But a firm setting a price $\hat{p} \in (\bar{r}, max\{r^Y, r^O\}]$ would still attract consumers with the highest reservation price, with expected profit:

$$\begin{cases} \mathbb{E}\{\pi(\hat{p},F)\} = \frac{\lambda^M}{N} R^M(\hat{p}) - z(a(\hat{p})) & \text{if } r^Y \leq r^M \\ \mathbb{E}\{\pi(\hat{p},F)\} = \frac{\lambda^Y}{N} R^Y(\hat{p}) - z(a(\hat{p})) & \text{if } r^Y > r^M. \end{cases} \tag{A.3}$$

Imposing $\mathbb{E}\{\pi(p,F)\} > \mathbb{E}\{\pi(\hat{p},F)\}$, we obtain:

$$\begin{cases} \mathbb{E}\{\pi(p,F)\} = \frac{\lambda^Y}{N} R^Y(r^Y) + \frac{\lambda^M}{N} R^M(r^Y) - z\left(a(r^Y)\right) > \frac{\lambda^M}{N} R^M(\hat{p}) - z\left(a(\hat{p})\right) & \text{if } r^Y \leq r^M \\ \mathbb{E}\{\pi(p,F)\} = \frac{\lambda^Y}{N} R^Y(r^M) + \frac{\lambda^M}{N} R^M(r^M) - z\left(a(r^M)\right) > \frac{\lambda^Y}{N} R^Y(\hat{p}) - z\left(a(\hat{p})\right) & \text{if } r^Y > r^M. \end{cases} \tag{A.4}$$

Now, we want to show that under condition (A.4), $\bar{r} = min\{r^Y, r^M, p^{monopoly}\}$.

- $\bar{r} \leq max\{r^Y, r^M\}$ since a firm setting a price above $max\{r^Y, r^M\}$ earns zero profits;
- given that $p^{monopoly} > min\{r^Y, r^M\}$, then $\bar{r} \geq min\{r^Y, r^M\}$ since a firm setting a price $\bar{r} < min\{r^Y, r^M\}$ would be able to sell to the same number of consumers by setting the price to $min\{r^Y, r^M\}$ which is closer to the monopolistic price that maximizes the profits which implies that $\bar{r} \in [min\{r^Y, r^M\}, max\{r^Y, r^M\}]$;

- finally, under condition (A.4), it is not profitable to only focus on the consumer type with the highest reservation price and set a price in between the two reservation prices is not a profitable deviation.

□

In particular, in the case in which $p^{monopoly} > max\{r^Y, r^M\}$ then the most profitable deviation in $(min\{r^Y, r^M\}, max\{r^Y, r^M\}]$ is $max\{r^Y, r^M\}$ as it brings the price closer to the monopolistic price keeping the same share of consumers (i.e., the ones with the highest reservation price). In this case, condition (A.4) is fulfilled either if the share of consumers with the highest reservation price is low enough or if the difference between the reservation prices is small which in turn depends on the demand functions of the agents. In particular, since we assume that agents have different elasticity of substitution, given the share of consumer types with the highest reservation price, condition (A.4) is fulfilled if the elasticity of substitution of young and middle-aged are not too different.

A.2.2 Proof of proposition 1.3.4

Proof. Using the FOC from problem (1.19), we obtain an expression for the production:

$$y = \frac{2a}{w\bar{z}}. \tag{A.5}$$

Substituting equation (A.5) into the equilibrium profit condition (1.20) objective function, we obtain the following polynomial of order 2:

$$a^2 + 2a\left(\frac{p}{w} - \bar{a}\right) - \bar{z}\pi = 0, \tag{A.6}$$

which has two solutions:

$$a_1 = \left[\left(\frac{p}{w} - \bar{a}\right)^2 + \bar{z}\pi\right]^{1/2} - \left(\frac{p}{w} - \bar{a}\right),$$
$$a_2 = -\left[\left(\frac{p}{w} - \bar{a}\right)^2 + \bar{z}\pi\right]^{1/2} - \left(\frac{p}{w} - \bar{a}\right).$$

Since $a_2 < 0$, the unique solution for the investment is $a^*(p) = a_1$ which is decreasing in prices:

$$\frac{\partial a^*}{\partial p} = \frac{1}{w}\left[\frac{\frac{p}{w} - \bar{a}}{\sqrt{\left(\frac{p}{w} - \bar{a}\right)^2 + \bar{z}\pi}} - 1\right] < 0 \tag{A.7}$$

□

A.2.3 Equilibrium Profits

Since in equilibrium all prices in the support of F give the same expected profits π, it holds that $\mathbb{E}\{\pi(\bar{r},F)\} = \pi$. A firm setting the reservation price $\bar{r}$ is the highest pricing firm in the sector and $F(\bar{r}) = 1$. Therefore, it holds that:

$$\mathbb{E}\{\pi(\bar{r},F)\} = \frac{\lambda^Y}{N} \cdot R^Y(\bar{r}) + \frac{\lambda^M}{N} \cdot R^M(\bar{r}) - z(a(\bar{r})). \tag{A.8}$$

Substituting the optimal technology adoption (equation (1.21)) in equation (A.8), we obtain the following polynomial in π:

$$0 = -\pi + \frac{\lambda^Y}{N} R^Y(\bar{r}) + \frac{\lambda^M}{N} R^M(\bar{r}) - z(a(\bar{r})). \tag{A.9}$$

The polynomial has three solutions:

$$\begin{aligned}
\pi_1 &= -\frac{(\bar{r}-\bar{a}w)^2}{w^2\bar{z}};\\
\pi_2 &= \frac{\left[\lambda^Y D^Y(\bar{r}) + \lambda^M D^M(\bar{r})\right] \cdot \left[4N \cdot (\bar{r}-\bar{a}w) + w^2\bar{z} \cdot \left(\lambda^Y D^Y(\bar{r}) + \lambda^M D^M(\bar{r})\right)\right]}{4N^2};\\
\pi_3 &= -\frac{\left[\lambda^Y D^Y(\bar{r}) + \lambda^M D^M(\bar{r})\right] \cdot \left[4N \cdot (\bar{r}-\bar{a}w) + w^2\bar{z} \cdot \left(\lambda^Y D^Y(\bar{r}) + \lambda^M D^M(\bar{r})\right)\right]}{4N^2}.
\end{aligned}$$

Under the condition that $\bar{r} - \bar{a}w > 0$, π_1 and π_3 are negative. $\bar{r} - \bar{a}w > 0$ means that the reservation price is larger than the marginal costs of a firm which does not invest in the labor saving technology. Let's assume that $\bar{r} - \bar{a}w \leq 0$, then a firm setting its price at $\bar{r}$ would make zero or negative profits, which implies that $\bar{r}$ is not in the equilibrium support of F, a contradiction. This implies that the unique solution for profits is: $\pi = \pi_2$.

A.2.4 Price Normalization

We follow Ghironi and Melitz (2005) and normalize the price aggregator by setting the average price in the economy as the numeraire. We, therefore, divide the sector-level price of agent $j \in \{Y, M, O\}$ by the average price in the economy, and define:

$$\widetilde{p}_s^j \equiv \frac{p_s^j}{P_{AVR}}, \tag{A.10}$$

where

$$P_{AVR} = \lambda^Y \cdot P^Y + \lambda^M \cdot P^M + \lambda^O \cdot P^O. \tag{A.11}$$

We define the normalized price aggregator for agent j, $\widetilde{P}^j$, as:

$$\widetilde{P}^j \equiv \left[\int_s (\widetilde{p}_s^j)^{1-\sigma} ds \right]^{\frac{1}{1-\sigma}}, \tag{A.12}$$

which is also equal to:

$$\widetilde{P}^j \equiv \frac{P^j}{P_{AVR}}. \tag{A.13}$$

A.2.5 Derivation of the sector-specific demand

The consumer's problem is to maximize utility subject to the budget constraint:

$$\max_{c_s^j} \left[\int_s c_s^{j\,\frac{\sigma_j - 1}{\sigma_j}} \; ds \right]^{\frac{\sigma_j}{\sigma_j - 1}} \tag{A.14}$$

$$\text{s.t. } \widetilde{P}^j C^j \equiv \int_s \widetilde{p}^j(s) \cdot c_s^j \;\; ds = I^j \tag{A.15}$$

The first order conditions for a good j:

$$\left[\int_s c_s^{j\,\frac{\sigma_j - 1}{\sigma_j}} \; ds \right]^{\frac{1}{\sigma_j - 1}} c_s^{j\,-\frac{1}{\sigma_j}} - \theta \cdot \widetilde{p}^j(s) = 0, \tag{A.16}$$

where θ is the Lagrangian multiplier linked to the constraint (A.15). We combine the FOCs for two distinct products s and t:

$$c_s^j = \left(\frac{\widetilde{p}^j(s)}{\widetilde{p}^j(t)} \right)^{-\sigma_j} \cdot c_t^j \tag{A.17}$$

Replacing c_s^j in the budget constraint, we obtain the expenditure in a given market t:

$$\widetilde{p}^j(t) \cdot c_t^j = I^j \cdot \left(\frac{\widetilde{p}^j(t)}{\widetilde{P}^j} \right)^{1-\sigma} \tag{A.18}$$

Using the condition $I^j = \widetilde{P}^j C^j$, we then obtain the demand from each product:

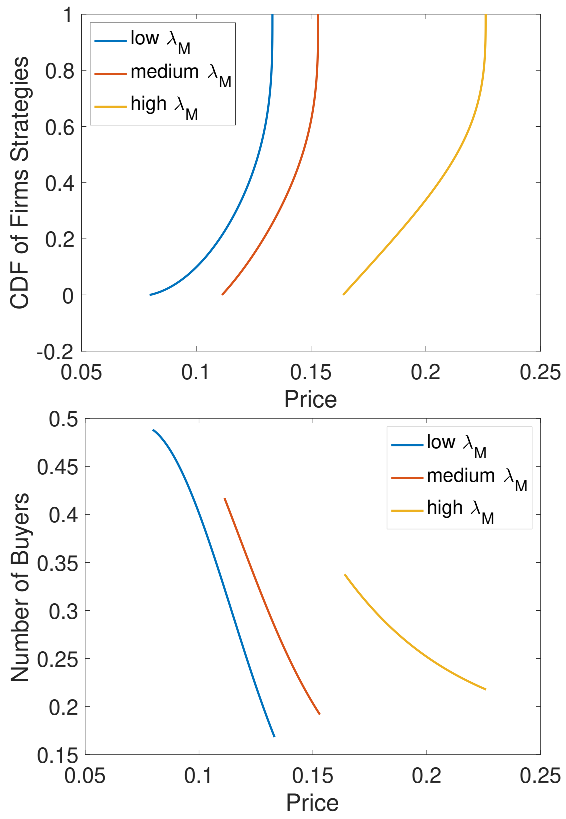
$$c_t^j = D(\widetilde{p}^j(t)) = C^j \left(\frac{\widetilde{p}^j(t)}{\widetilde{P}^j}\right)^{-\sigma}. \quad \text{(A.19)}$$

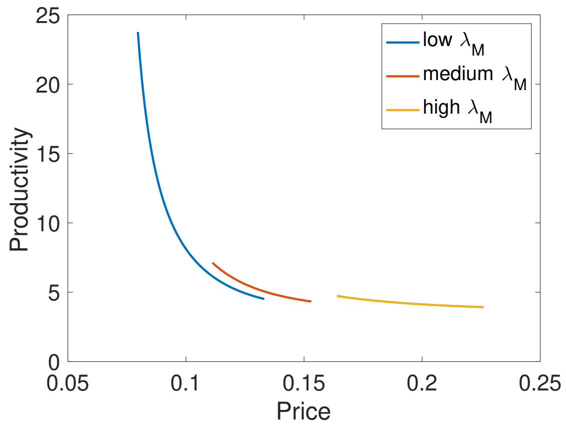
A.2.6 General Equilibrium Aggregation

Let's define $g(p_s^j)$ a function of the random variable $p_s^j \sim F^j(p)$. Since all the sectors are identical and agents face the same conditional to type j price distribution in every sector, by the law of large number we have that: $\int_s g(p_s^j)ds = S \cdot \frac{\int_s g(p_s^j)ds}{\mathcal{S}} \approx \mathcal{S} \cdot \mathbb{E}^j(g(p_s^j)) = \mathcal{S} \cdot \int_b^r g(p_s^j)dF^j(p)$. In our case, for $\mathcal{S} = 1$, we can write: $\int_s (\widetilde{p}_s^j)^{-\sigma_j} ds = \mathbb{E}^j[\widetilde{p}^{-\sigma_j}]$.

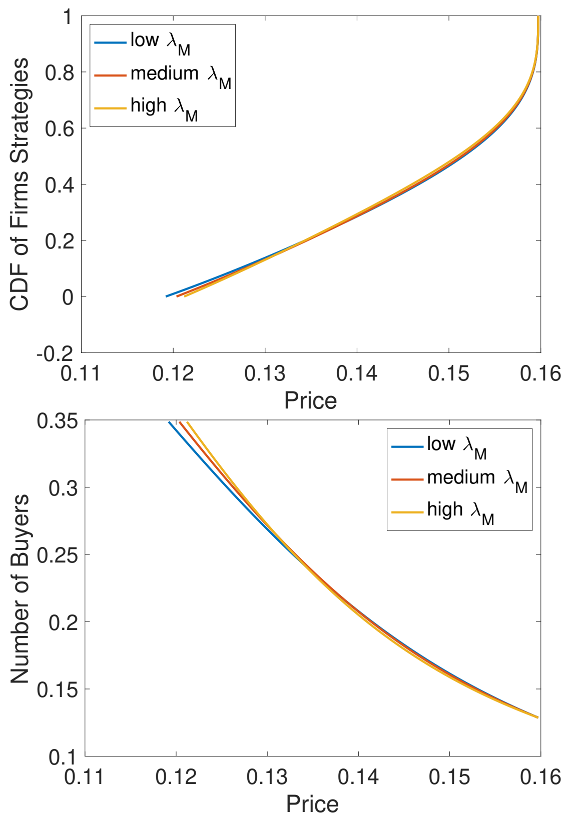
A.2.7 Sector-level comparative statics

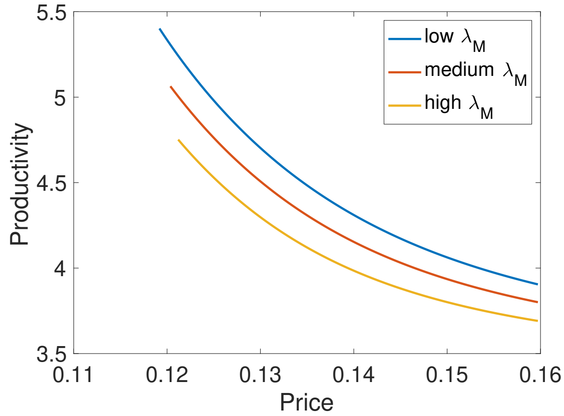
Increase in Middle-aged keeping Young constant





Increase in Middle-aged keeping Old constant





A.3 Calibration and estimation

The model is calibrated to the US economy in 1995. The share of each demographic group λ_i in the population between 25 and 79 years old comes from the UN World Population Prospect of 1996. We calibrate the share of total capital income, i.e. profits from firms and technology costs, captured by each consumer in age category i from the US average worth by age category in 1995 (Survey of Consumer Finances, 1995). An underlying and simplifying assumption is that returns on capital and relative portfolio compositions are identical across age categories.

$$\phi^i = \frac{\lambda_i wealth_i}{\sum_i \lambda_i wealth_i} \tag{A.20}$$

with $wealth_i$ the average wealth of age-category i in the data.

Table (A.9) Calibrated parameters

	Description	Calibration
ϕ^Y	share of wealth held on average by young	0.22
ϕ^M	share of wealth held on average by middle-aged	0.46
ϕ^O	share of wealth held on average by old	0.32
λ^Y	demographic share of young	0.48
λ^M	demographic share of middle-aged	0.3
λ^O	demographic share of old	0.22

Parameters are estimated targeting the point estimate of average price and total production responses to an increase in the share of young, middle-aged, and old. These 6 moments allows to estimate 6 parameters of the model. We assume that an increase in the share of an age category i, δ_{λ_i} , comes from each of the two other age categories in proportion to their share in the economy, i.e.:

$$\delta_{\lambda_j} \equiv -\frac{\lambda_j}{1-\lambda_i}\delta_{\lambda_i} \text{ , with } \; j \neq i \tag{A.21}$$

Our target function is the root mean square error between the model-implied moments and the data.

Table (A.10) Estimated parameters

	Description	Estimates
$\bar{a}$	technology parameter	0.75
$\bar{z}$	technology cost parameter	1.14
$\bar{s}$	search cost parameter	0.61
$\underline{\sigma}$	elasticity of substitution (young)	1.37
$\bar{\sigma}$	elasticity of substitution (middle-aged and old)	0.82
N	number of firms in each sector	4.00

Table (A.11) Targeted moments

	Targeted moments	Data	Model
$\frac{dp}{d\lambda^Y}$	effect of young on prices	-2.76	-1.36
$\frac{dp}{d\lambda^M}$	effect of middle-aged on prices	2.28	2.45
$\frac{dp}{d\lambda^O}$	effect of old on prices	-2.10	-0.76
$\frac{dQ}{d\lambda^Y}$	effect of young on production	2.74	1.66
$\frac{dQ}{d\lambda^M}$	effect of middle-aged on production	-1.82	-2.82
$\frac{dQ}{d\lambda^O}$	effect of old on production	2.89	0.84

B

Appendix to chapter 2

B.1 Additional regression tables

Table (B.1) Groups of sectors' employment response to a shock in value added

	log(hours worked)	log(employment)
β_{-3}	0.0156	0.0197
	(0.1107)	(0.1090)
β_{-2}	0.0878	0.0790
	(0.1053)	(0.1018)
β_{-1}	0.0174	0.0184
	(0.0701)	(0.0673)
β_0	0.2004**	0.1833*
	(0.0953)	(0.1008)
β_1	0.3017***	0.2928***
	(0.0816)	(0.0838)
β_2	0.1885**	0.1776*
	(0.0912)	(0.0929)
β_3	0.2531***	0.2273***
	(0.0791)	(0.0803)
β_4	0.2926***	0.3038***
	(0.0815)	(0.0829)
β_5	0.1541	0.1539
	(0.0934)	(0.0930)
β_6	0.2340**	0.2057*
	(0.1140)	(0.1158)
β_7	0.0904	0.0789
	(0.1043)	(0.1055)
$\gamma_{-3,oth}$	-0.0430	-0.0562
	(0.1406)	(0.1359)
$\gamma_{-3,pro}$	0.0654	0.0669
	(0.1875)	(0.1864)
$\gamma_{-2,oth}$	-0.0130	-0.0046
	(0.1564)	(0.1451)
$\gamma_{-2,pro}$	-0.1973	-0.2100
	(0.1365)	(0.1336)
$\gamma_{-1,oth}$	0.1608	0.1668
	(0.1341)	(0.1257)
$\gamma_{-1,pro}$	0.1168	0.1019
	(0.1596)	(0.1560)
$\gamma_{0,oth}$	-0.1016	-0.0767
	(0.1482)	(0.1446)
$\gamma_{0,pro}$	-0.0839	-0.0518
	(0.1749)	(0.1797)
$\gamma_{1,oth}$	-0.1605	-0.1558
	(0.1539)	(0.1488)
$\gamma_{1,pro}$	-0.1661	-0.1436
	(0.1333)	(0.1321)
$\gamma_{2,oth}$	-0.1305	-0.1256
	(0.1411)	(0.1357)
$\gamma_{2,pro}$	-0.1279	-0.1155
	(0.1353)	(0.1314)
$\gamma_{3,oth}$	-0.1238	-0.1073
	(0.1216)	(0.1181)
$\gamma_{3,pro}$	-0.2140	-0.1703
	(0.1591)	(0.1514)
$\gamma_{4,oth}$	-0.3450***	-0.3574***
	(0.1171)	(0.1143)
$\gamma_{4,pro}$	-0.2840**	-0.3033**
	(0.1326)	(0.1310)
$\gamma_{5,oth}$	-0.1790	-0.1887*
	(0.1131)	(0.1104)
$\gamma_{5,pro}$	0.0069	0.0183
	(0.1365)	(0.1379)
$\gamma_{6,oth}$	-0.2650**	-0.2430*
	(0.1230)	(0.1229)
$\gamma_{6,pro}$	0.1910	0.2088
	(0.2299)	(0.2255)
$\gamma_{7,oth}$	-0.2210*	-0.2086
	(0.1274)	(0.1260)
$\gamma_{7,pro}$	0.2371	0.1988
	(0.2135)	(0.2167)
Observations	3,131	3,131
R^2	0.97714	0.97731
Within R^2	0.07179	0.06928
area-year fixed effects	✓	✓
sector-area fixed effects	✓	✓
sector-year fixed effects	✓	✓

Note: *** 1%-level ** 5%-level * 10%-level
Standard errors are clustered at the area-year, sector-area, and sector-year level.

Table (B.2) Groups of sectors' employment response to a shock in value added

	log(hours worked)	log(employment)
β_{-3}	0.0810	0.0866
	(0.1461)	(0.1463)
β_{-2}	-0.1095	-0.1310
	(0.1228)	(0.1202)
β_{-1}	0.1343	0.1204
	(0.1323)	(0.1267)
β_0	0.1165	0.1315
	(0.1393)	(0.1436)
β_1	0.1356	0.1492
	(0.1262)	(0.1244)
β_2	0.0607	0.0621
	(0.1055)	(0.0985)
β_3	0.0390	0.0570
	(0.1347)	(0.1283)
β_4	0.0086	0.0005
	(0.1168)	(0.1120)
β_5	0.1610	0.1722
	(0.1084)	(0.1086)
β_6	0.4250**	0.4145**
	(0.1997)	(0.1966)
β_7	0.3274*	0.2777
	(0.1927)	(0.1961)
$\gamma_{-3,oth}$	-0.1085	-0.1231
	(0.1562)	(0.1547)
$\gamma_{-3,pre}$	-0.0654	-0.0669
	(0.1875)	(0.1864)
$\gamma_{-2,oth}$	0.1843	0.2053
	(0.1471)	(0.1383)
$\gamma_{-2,pre}$	0.1973	0.2100
	(0.1365)	(0.1336)
$\gamma_{-1,oth}$	0.0440	0.0648
	(0.1603)	(0.1552)
$\gamma_{-1,pre}$	-0.1168	-0.1019
	(0.1596)	(0.1560)
$\gamma_{0,oth}$	-0.0177	-0.0249
	(0.1681)	(0.1707)
$\gamma_{0,pro}$	0.0839	0.0518
	(0.1749)	(0.1797)
$\gamma_{1,oth}$	0.0056	-0.0122
	(0.1718)	(0.1670)
$\gamma_{1,pre}$	0.1661	0.1436
	(0.1333)	(0.1321)
$\gamma_{2,oth}$	-0.0026	-0.0102
	(0.1402)	(0.1307)
$\gamma_{2,pre}$	0.1279	0.1155
	(0.1353)	(0.1314)
$\gamma_{3,oth}$	0.0902	0.0630
	(0.1581)	(0.1483)
$\gamma_{3,pre}$	0.2140	0.1703
	(0.1591)	(0.1514)
$\gamma_{4,oth}$	-0.0610	-0.0541
	(0.1329)	(0.1280)
$\gamma_{4,pre}$	0.2840**	0.3033**
	(0.1326)	(0.1310)
$\gamma_{5,oth}$	-0.1858	-0.2070*
	(0.1219)	(0.1204)
$\gamma_{5,pre}$	-0.0069	-0.0183
	(0.1365)	(0.1379)
$\gamma_{6,oth}$	-0.4560**	-0.4518**
	(0.1988)	(0.1956)
$\gamma_{6,pre}$	-0.1910	-0.2088
	(0.2299)	(0.2255)
$\gamma_{7,oth}$	-0.4581**	-0.4074*
	(0.2019)	(0.2035)
$\gamma_{7,pre}$	-0.2371	-0.1988
	(0.2135)	(0.2167)
Observations	3,131	3,131
R^2	0.97714	0.97731
Within R^2	0.07179	0.06928
area-year fixed effects	✓	✓
sector-area fixed effects	✓	✓
sector-year fixed effects	✓	✓

Note: *** 1%-level ** 5%-level * 10%-level
Standard errors are clustered at the area-year, sector-area, and sector-year level.

Table (B.3) Firms' value added response to an idiosyncratic performance shock

	log(value added)	log(hours worked)	log(employment)
β_{-3}	0.0123	0.0425***	0.0378***
	(0.0097)	(0.0109)	(0.0093)
β_{-2}	0.0013	0.0325***	0.0209**
	(0.0114)	(0.0118)	(0.0105)
β_{-1}	0.0156	0.0607***	0.0645***
	(0.0121)	(0.0129)	(0.0109)
β_0	0.9987***	0.0470**	0.0985***
	(0.0164)	(0.0191)	(0.0125)
β_1	0.0467***	0.0985***	0.0917***
	(0.0122)	(0.0145)	(0.0119)
β_2	0.0442***	0.0872***	0.0721***
	(0.0097)	(0.0117)	(0.0114)
β_3	0.0208***	0.0494***	0.0379***
	(0.0072)	(0.0107)	(0.0093)
β_4	0.0324***	0.0526***	0.0432***
	(0.0065)	(0.0106)	(0.0093)
β_5	0.0344***	0.0356***	0.0405***
	(0.0072)	(0.0103)	(0.0098)
β_6	0.0270***	0.0250**	0.0219**
	(0.0072)	(0.0109)	(0.0103)
β_7	0.0196**	0.0238**	0.0365***
	(0.0078)	(0.0118)	(0.0100)
$\delta_{-3,oth}$	0.0097	0.0004	-0.0096
	(0.0105)	(0.0125)	(0.0108)
$\delta_{-3,pro}$	0.0050	-0.0231*	-0.0161
	(0.0110)	(0.0139)	(0.0113)
$\delta_{-2,oth}$	0.0228*	0.0151	0.0135
	(0.0123)	(0.0141)	(0.0121)
$\delta_{-2,pro}$	0.0224*	0.0167	0.0240*
	(0.0129)	(0.0155)	(0.0133)
$\delta_{-1,oth}$	0.0169	0.0043	-0.0126
	(0.0133)	(0.0163)	(0.0129)
$\delta_{-1,pro}$	0.0139	0.0069	-0.0099
	(0.0137)	(0.0173)	(0.0137)
$\delta_{0,oth}$	-0.0602***	-0.1510***	-0.0733***
	(0.0202)	(0.0298)	(0.0159)
$\delta_{0,pro}$	-0.0269	-0.0622**	-0.0511**
	(0.0213)	(0.0300)	(0.0204)
$\delta_{1,oth}$	-0.0029	-0.0007	-0.0115
	(0.0137)	(0.0180)	(0.0145)
$\delta_{1,pro}$	0.0131	0.0173	0.0126
	(0.0142)	(0.0193)	(0.0155)
$\delta_{2,oth}$	-0.0104	-0.0178	-0.0106
	(0.0106)	(0.0143)	(0.0133)
$\delta_{2,pro}$	0.0083	0.0076	0.0203
	(0.0120)	(0.0168)	(0.0151)
$\delta_{3,oth}$	0.0059	-0.0035	-0.0005
	(0.0083)	(0.0132)	(0.0110)
$\delta_{3,pro}$	0.0168*	0.0087	0.0156
	(0.0091)	(0.0144)	(0.0116)
$\delta_{4,oth}$	0.0015	0.0032	0.0019
	(0.0075)	(0.0123)	(0.0107)
$\delta_{4,pro}$	-9.63×10^{-5}	-0.0077	0.0047
	(0.0084)	(0.0136)	(0.0115)
$\delta_{5,oth}$	-0.0082	0.0055	-0.0079
	(0.0081)	(0.0119)	(0.0110)
$\delta_{5,pro}$	-0.0034	0.0153	0.0059
	(0.0086)	(0.0133)	(0.0121)
$\delta_{6,oth}$	-0.0052	0.0102	0.0020
	(0.0081)	(0.0126)	(0.0117)
$\delta_{6,pro}$	0.0025	0.0146	0.0183
	(0.0089)	(0.0144)	(0.0131)
$\delta_{7,oth}$	0.0073	0.0142	-0.0032
	(0.0087)	(0.0134)	(0.0113)
$\delta_{7,pro}$	0.0158*	0.0176	0.0027
	(0.0095)	(0.0146)	(0.0127)
Observations	130,026	130,026	129,338
R^2	0.99245	0.97789	0.98081
Within R^2	0.66912	0.04085	0.04767
establishment fixed effects	✓	✓	✓
year-area-sector fixed effects	✓	✓	✓

Note: *** 1%-level ** 5%-level * 10%-level
Standard errors are clustered at the establishment and year-area-sector level.

C

Appendix to chapter 3

C.1 Detailed recursive firm problem:

$$V(N,s;x) = \max_{\mathbf{p},\boldsymbol{\eta},\mathbf{n},\mathbf{a},s^+,\Omega_f,N^+} \int_{\Omega_f} n_i p_i - C(n_i;\hat{\imath},z) - \kappa(a_i) - c_p di - c_{hq} + \beta(1-\delta)V(N^+,s^+;x)$$

$$N^+ = \int_{\Omega_f} n_i di \qquad \text{(Law of motion)}$$

$$n_i = \frac{\varphi(\varepsilon)}{\varepsilon} N + a_i \eta_i \qquad \text{(Per product customers)}$$

$$p_i = u - \frac{\theta(\eta_i)}{\eta_i}\rho + \beta(1-\delta)\left(s^+ - B\right) \qquad \text{(MUA)}$$

$$s = \varepsilon\left[u - \mathbf{E_i}(p_i) + \beta((1-\delta)s^+ + \delta B)\right] + (1-\varepsilon)B \qquad \text{(Promised utility)}$$

$$\Omega_f = [-\bar{\varepsilon},\bar{\varepsilon}] \quad \text{with } \bar{\varepsilon} = \varepsilon/2$$

I can replace the price using the Market utility assumption equation, as well as the next period number of incumbent buyers using the law of motion. I obtain:

$$V(N,s;x) = \max_{\boldsymbol{\eta},\boldsymbol{a},s^+,\varepsilon} \int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} \left[\left(\frac{\varphi(\varepsilon)}{\varepsilon} N + a_i \eta_i\right)\left(u - \frac{\theta(\eta_i)}{\eta_i}\rho + \beta(1-\delta)\left(s^+ - B\right)\right)\right.$$

$$-C\left(\frac{\varphi(\varepsilon)}{\varepsilon}N+a_i\eta_i;i,z\right)-\kappa(a_i)-c_p di \qquad subappendix$$
$$-c_{hq}+\beta(1-\delta)V\left(\varphi(\varepsilon)N+\int_{-\varepsilon/2}^{\varepsilon/2}\eta_i a_i di, s^+;x\right)$$

$$\text{s.t.,}\quad s-B=\frac{\varphi(\varepsilon)}{\varepsilon}\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}\frac{\theta(\eta_i)}{\eta_i}\rho di-\varphi(\varepsilon)(\rho-c_s)\quad,\quad(\lambda)\qquad\text{(Promised Utility)}$$

with λ the Lagrange multiplier of the promised utility condition. I obtain the following first order conditions:

$$(\eta_i):\qquad p(\eta_i,s^+)-\frac{(\frac{\varphi(\varepsilon)}{\varepsilon}N+a_i\eta_i)}{a_i}.\frac{\xi(\eta_i)}{\eta_i}-C_{1,i}+\beta(1-\delta)V_1^+ +\frac{\lambda}{a_i}.\frac{\varphi(\varepsilon)}{\varepsilon}.\frac{\xi(\eta_i)}{\eta_i}=0$$

$$(a_i):\qquad p(\eta_i,s^+)-C_{1,i}+\beta(1-\delta)V_1^+ -\frac{\kappa'(a_i)}{\eta_i}=0$$

$$(s^+):\qquad \int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}(\frac{\varphi(\varepsilon)}{\varepsilon}N+\eta_i a_i)\beta(1-\delta)di+\beta(1-\delta)V_2^+\Leftrightarrow V_2^+=-N^+$$

$$\begin{aligned}(\varepsilon):\qquad &\frac{\varphi'(\varepsilon)\varepsilon-\varphi(\varepsilon)}{\varepsilon^2}N\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}p(\eta_i,s^+)-C_{i,1}di\\ &+\lambda\left[\varphi'(\varepsilon)\left(\mathbf{E_i}\left(\frac{\theta(\eta_i)}{\eta_i}\right)\rho-(\rho-c_s)\right)-\frac{\varphi(\varepsilon)}{\varepsilon}\left(\mathbf{E_i}\left(\frac{\theta(\eta_i)}{\eta_i}\right)\rho-\frac{\theta(\eta_{\bar{\varepsilon}})}{\eta_{\bar{\varepsilon}}}\right)\right]\\ &+\underbrace{\left(\frac{\varphi(\varepsilon)}{\varepsilon}N+\eta_{\bar{\varepsilon}}a_{\bar{\varepsilon}}\right)p(\eta_{\bar{\varepsilon}},s^+)-C\left(\frac{\varphi(\varepsilon)}{\varepsilon}N+\eta_{\bar{\varepsilon}}a_{\bar{\varepsilon}},\bar{\varepsilon}\right)-c_p}_{\text{Flow profit from the marginal good}}+\beta(1-\delta)(\varphi'(\varepsilon)N+\eta_{\bar{\varepsilon}}a_{\bar{\varepsilon}})V_1^+=0\end{aligned}$$

And envelop conditions:

$$(N):\qquad V_1=\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}p(\eta_i,s^+)-C_{1,i}di+\beta(1-\delta)\varphi(\varepsilon)V_1^+=\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}}p(\eta_i,s^+)-C_{1,i}+\beta(1-\delta)V_1^+di$$

$$(s):\qquad V_2=-\lambda$$

C.2 Proofs

C.2.1 Proposition 1: Retention rate

Proof. 1. Since $\theta(\eta_i)/\eta_i > 1$ and under the condition that the firm is active, $\varphi(\varepsilon, \boldsymbol{x}) > 0$, (3.3) ensures that $s > B$, which means that firms always offer contracts with strictly higher value than searching; buyers have no incentive *ex-ante* to break the relation and look for another firm to trade with. $(\theta(\eta_i)/\eta_i).\rho > \rho$ further imply that buyers never refuse a contract; they have no incentive *ex-post* to break the relation.

2. An incumbent buyer separate from a firm if it does not provide the good she wants if in expectation she does not gain from waiting a period without consuming to preserve its link with the seller:

$$0 + \beta((1-\delta)S^+ + \delta B) \leq B$$

$$\Leftrightarrow c_s + \beta(1-\delta)(S^+ - B) \leq \rho$$

Using (3.3), we obtain a condition which is firm specific:

$$c_s \leq \frac{1 + \beta(1-\delta)\mathbf{E_i}\left(\varphi(\varepsilon^+, \boldsymbol{x}^+)\left(1 - \frac{\theta(\eta_i^+)}{\eta_i^+}\right)\right)}{1 + \beta(1-\delta)\mathbf{E_i}(\varphi(\varepsilon^+, \boldsymbol{x}^+))}\rho$$

The wider the range of goods a firm sells and the higher the expected surplus offered in the future, the lower the incentive for its incumbent buyers to separate if the good she wants to consume is not available. The former increases the probability to trade again in the future while the latter relates to the surplus expected from each trade. As $\frac{\theta(\eta_i^+)}{\eta_i^+}$ is not bounded above, I cannot ensure the existence of $c_s > 0$ such that this condition is fulfilled in any case. This condition needs to be checked ex-post in the numerical solution.

□

C.2.2 Proposition 2: Heterogeneity

. 1. We take $i, j \in \Omega_f$, two markets in which a firm produces good, such that $|i - \hat{\imath}| < |j - \hat{\imath}|$, which means that i is closer to the core competence of the firms than j is.

Existence and uniqueness: $C_{j,1} + \theta'(\eta_j)$ is a continuous function of η_j as the sum of two continuous functions in η_j. Since the policy function $a(\eta_j)$ is increasing in η_j and N is a state variable, the number of buyers in the market j, i.e. n_j, is increasing in η_j. Therefore, $C_{j,1}+\theta'(\eta_j)$ is strictly increasing in η_j.

Characterisation: Let assume $\eta_i=\eta_j$, then $C_1(N+\eta_i a(\eta_i),i) > C_1(N+\eta_i a(\eta_i),j)$ since the firm has lower production efficiency at j than i. It implies that the RHS of (3.8) is positive and strictly larger than the LHS.

Now, let η_j decrease while keeping η_i fixed, the LHS strictly increases while the RHS strictly decreases. So if a solution exists it is unique and such that $\eta_i>\eta_j$, which directly implies that $n_i>n_j$. According to (3.1), the price is a function of the promised utility, which is constant across markets, and decreasing in $\theta(\eta_i)/\eta_i$ and therefore increasing in η_i; implying $p_i>p_j$.

2. From the first order condition with respect to (η_i) for a given market i, we know that:

$$p(\eta_i,s^+)-C_{1,i}=\xi(\eta_i)-\beta(1-\delta)V_1^+$$

Taking the log we can obtain an expression for the markup:

$$\Rightarrow mk_i \equiv \log(p(\eta_i,s^+)-C_{1,i}) = \log\left(\xi(\eta_i)-\beta(1-\delta)V_1^+\right) = \log\left(\xi(\eta_i)-\beta(1-\delta)\varphi(\varepsilon^+)\mathbf{E_j}(\xi(\eta_j^+))\right)$$

The markup a firm set in a market i is a function of the difference between the demand sensitivity to price in this given market and the discounted expected demand sensitivity across markets next period.

As above we take $i,j\in\Omega_f$, two markets in which a firm produces good, such that $|i-\hat{i}|<|j-\hat{i}|$. We know from the above result that $\eta_i>\eta_j$ and therefore:

$$mk_i-mk_j=\log\left(\frac{\xi(\eta_i)-\beta(1-\delta)(\varepsilon^+)\mathbf{E_j}(\xi(\eta_j^+))}{\xi(\eta_j)-\beta(1-\delta)(\varepsilon^+)\mathbf{E_j}(\xi(\eta_j^+))}\right)>0$$

□

C.2.3 Proposition 3: Efficiency

Proof. The planner's state variables are: $\sigma^+=(N_a^+,F_a^+)_{a\geq 1}$

$$S(\sigma)=\max_{F_0,\{\{\eta_{i,a}^x,a_{i,a}^x\}_{i\in\varepsilon_a^x},\varepsilon_a^x\}_{a\geq 0}}\sum_{a\geq 0}N_a\int_{\Omega_f}u.n_{i,a}^x-C(n_{i,a}^x,i)-\kappa(a_{i,a}^x)di-KF_0+\beta S(\sigma') \quad \text{(C.1)}$$

$$\text{s.t. } N_0^x = 0\,, \quad N_{a+1}^{x,+} = \varphi(\varepsilon_a^x) N_a^x + \int_{\varepsilon_a^x} \eta_{i,a}^x a_{i,a}^x di\,, \quad n_{i,a}^x = \frac{\varphi(\varepsilon_a^x)}{\varepsilon_a^x} N_a^x + \eta_{i,a}^x a_{i,a}^x$$

$$\text{s.t. } F_{a+1}^{x,+} = (1-\delta) F_a^x$$

The aggregate condition is identical considering the span of one firm as below, or the span of all the firms in a given market, since firms, given age and productivity, have the same strategy. Indeed, the share of firms of age a and productivity x present in market $\hat{i}$ is ε_a^x, which is exactly the product range of a firm with core product $\hat{i}$. Given age and productivity, the strategy of a firm in a given market $\hat{i}$ is solely dependent on the distance of the market from their core product. Therefore, a firm with core product j , s.t. $|\hat{i}-j| = \Delta$ has the same strategy as a firm with core product $\hat{i}$ in a market i , s.t. $|\hat{i}-i| = \Delta$. The aggregate condition boils down to:

$$\text{s.t. } 1 \geq \sum_{a\geq 0} F_a \sum_x \pi(x) \left(\varphi(\varepsilon_a^x) N_a^x + \int_{\varepsilon_a^x} \theta(\eta_{i,a}^x).a_{i,a}^x di \right)\,, \qquad (\lambda_P)$$

For a matter of simplicity, I assume here, as in the numerical solution that $\varphi(\varepsilon_a^x) = \varepsilon_a^x$.

FOCs:

$$\begin{aligned}(F_0): \qquad & \sum_x \pi(x) \int_{\varepsilon_a^x} u.\eta_{0,i}^x a_{0,i}^x - C(\eta_{0,i}^x a_{0,i}^x, i) - \kappa(a_{i,0}^x) - c_f di - K + \beta(1-\delta) S_2(\sigma^+) \\ & - \lambda_P \sum_x \pi(x) \left(\int_{\varepsilon_0^x} \theta(\eta_{0,i}^x) a_{0,i}^x di \right) = 0\end{aligned}$$

$$(\eta_{a,i}^x): \qquad F_a \pi(x) \left(a_{a,i}^x.u - a_{a,i}^x.C_1 \right) + \beta a_{a,i}^x S_1(\sigma^+) - \lambda_P F_a \pi(x) \theta'(\eta_{a,i}^x) a_{a,i}^x = 0$$

$$(a_{a,i}^x): \qquad F_a \pi(x) \left(\eta_{a,i}^x.u - \eta_{a,i}^x.C_1 - \kappa'(a_{a,i}^x)) \right) + \beta \eta_{a,i}^x S_1(\sigma^+) - \lambda_P F_a \pi(x) \theta(\eta_{a,i}^x) = 0$$

$$(\varepsilon_a^x): \qquad F_a \pi(x)(u.n_{\varepsilon_a^x,a}^x - C(n_{\varepsilon_a^x,a}^x, \varepsilon_a^x) - \kappa(a_{\varepsilon_a^x,a}^x) - c_f) + \beta(N_a^x + \eta_{a,\varepsilon_a^x}^x a_{a,\varepsilon_a^x}^x) S_1(\sigma^+)$$

$$-\lambda_P F_a \pi(x)(N_a^x + \theta(\eta_{a,\varepsilon_a^x}^x) a_{a,\varepsilon_a^x}^x) = 0$$

Envelop conditions:

$$(\mathrm{F}_a): \qquad S_2(\sigma) = \sum_x \pi(x) \int_{\varepsilon_a^x} u.n_{i,a}^x - C(n_{i,a}^x, i) - \kappa(a_{i,a}^x) - c_f di$$
$$+\beta(1-\delta)S_2(\sigma^+) - \lambda_P \sum_x \pi(x) \left(\varepsilon_a^x N_a^x + \int_{\varepsilon_a^x} \theta(\eta_{a,i}^x) a_{a,i}^x di\right)$$

$$(N_a^x): \qquad S_1(\sigma) = F_a \pi(x) \int_{\varepsilon_a^x} u - C_1 di + \beta \varepsilon_a^x S_1(\sigma^+) - \lambda_P F_a \pi(x) \varepsilon_a^x$$

We can combine the FOCs w.r.t. $(\eta_{a,i}^x)$ and $(a_{a,i}^x)$:

$$\kappa'(a_{a,i}^x) = \lambda_P \left(\theta'(\eta_{a,i}^x).\eta_{a,i}^x - \theta(\eta_{a,i}^x)\right) \tag{C.2}$$

We can as well combine the FOCs w.r.t. $(a_{a,i}^x)$ at $i = \varepsilon_a^x$ and (ε_a^x), using some computations, we obtain:

$$\left(\frac{\theta(\eta_{\varepsilon_a^x}^a)}{\eta_{\varepsilon_a^x}^a} - 1\right)\lambda_P = \frac{(N_a^x + \eta_{a,\varepsilon_a^x}^x a_{a,\varepsilon_a^x}^x)}{N}\left[\frac{C(n_{\varepsilon_a^x,a}^x, \varepsilon_a^x) + \kappa(a_{\varepsilon_a^x,a}^x) + c_f}{n_{\varepsilon_a^x,a}^x} - \left(C_1 + \frac{\kappa'(a_{\varepsilon_a^x,a}^x)}{\eta_{\varepsilon_a^x,a}^x}\right)\right] \tag{C.3}$$

Combining the FOC w.r.t $(\eta_{a,i}^x)$ and the Envelop condition for (N_a^x):

$$u - C_1 - \beta(1-\delta)\varphi(\varepsilon^+)\lambda_P = \theta'(\eta_i) - \beta(1-\delta)\varphi(\varepsilon^+)\mathbf{E_i}(\theta(\eta_i^+))\lambda_P \tag{C.4}$$

Combining the FOC w.r.t. (F_0) and the recursive formulation obtained from the envelop condition w.r.t. (F_a), we obtain the efficient entry condition:

$$K = \sum_{a \geq 0} (\beta(1-\delta))^a \sum_x \pi(x) \int_{\varepsilon_a^x} u.n_{i,a}^x - C(n_{i,a}^x, i) - \kappa(a_{i,a}^x) - c_f - \lambda_P \left[N_a^x + \theta(\eta_{a,i}^x) a_{a,i}^x\right] di \tag{C.5}$$

Fixing $\lambda_P = \rho$, one obtains the decentralized equilibrium. Therefore, we can conclude that the model is efficient.

It is then equivalent to solve the decentralized problem (3.6) and maximizing the social value of firms G:

$$G(N) = \max_{\varepsilon,\{\eta_i,a_i\}_{i\in\varepsilon}} \int_{\Omega_f} n_i u - C(n_i,i) - \kappa(a_i) - c_f - \lambda_P \left(\frac{\varphi(\varepsilon)}{\varepsilon} N + \theta(\eta_i) a_i\right) di + \beta(1-\delta) G(N^+) \tag{C.6}$$

$$n_i = \frac{\varphi(\varepsilon)}{\varepsilon} N + \eta_i a_i$$

$$N^+ = \int_{\Omega_f} n_i di = \varphi(\varepsilon) N + \int_{\Omega_f} \eta_i a_i di$$

□

C.3 Assumptions for simulation

C.3.1 Functional forms:

I assume that the costs function consists of the labour costs of production

$$C(n_i, i; z) = w.l(n_i, i; z)$$

with w the unit labour cost and l_i the units of labour used in production of product i.

We assume the simplest concave production function:

$$n_i = z_i l_i^{\phi} \; ; \; z(i) = z - \mu i \Rightarrow l(n_i, i; z) = \left(\frac{n_i}{z - \mu i}\right)^{\frac{1}{\phi}}$$

with n_i the quantity produced, z_i the product specific productivity, μ the factor of technological specificity and i the distance between a product and the firm's core product.

Functional form for extensive margin costs:

$$\kappa(a_i) = \frac{a_i^{\alpha}}{\kappa_0}$$

We can then define the marginal costs:

$$C_{i,1} = \frac{w}{\phi(z-\mu.i)}\left(\frac{n_i}{z-\mu i}\right)^{\frac{1-\phi}{\phi}} \quad ; \quad C_{i,2} = \frac{\alpha}{\kappa_0}a_i^{\alpha-1}$$

Following Gourio and Rudanko (2014), I assume a CD matching function $\eta(\theta) = \xi_{MF}\theta^{\gamma}$, which implies the following inverse matching function:

$$\theta(\eta_i) = \left(\frac{\eta_i}{\xi_{MF}}\right)^{\frac{1}{\gamma}} \quad ; \quad \theta'(\eta_i) = \frac{1}{\xi_{MF}\gamma}\left(\frac{\eta_i}{\xi_{MF}}\right)^{\frac{1-\gamma}{\gamma}}$$

From the functional forms assumed above and (3.7) I obtain a closed form solution for the policy function $a(\eta_i)$:

$$a(\eta_i) = \left[\left(\frac{\eta_i}{\xi_{MF}}\right)^{\frac{1}{\gamma}}\frac{1-\gamma}{\gamma}.\frac{\rho}{\alpha}\right]^{\frac{1}{\alpha-1}}$$

C.3.2 Parametrization:

The parametrization follows broadly the work of Gourio and Rudanko (2014) and Kaas and Kircher (2015). I guessed the other parameter so that the results make economic sense, though I expect to estimate the model in a later version:

Parameter	Symbol	Value	Source
Annual discount rate:	β	0.95	(G&R)
Matching function elasticity	γ	0.11	(G&R)
Matching function coefficient	ξ_{MF}	0.15	(G&R)
Sellers' costs concavity parameter	α	2	(G&R)
Inverse sellers' costs scalling factor	κ_0	2	(G&R)
Cost of search	c_s	0.1	(Guess)
Wage	w	40	(Guess)
Buyer's utility	u	15	(Guess)
Entry costs	K	5	(Guess)
Head quarter costs	c_{hq}	0.1	(Guess)
Per product fixed cost	c_f	2	(Guess)
RTS parameter of production	ϕ	0.7 (or 1)	(K&K)
Labour productivity	z	5	(Guess)

C.4 Numerical solution

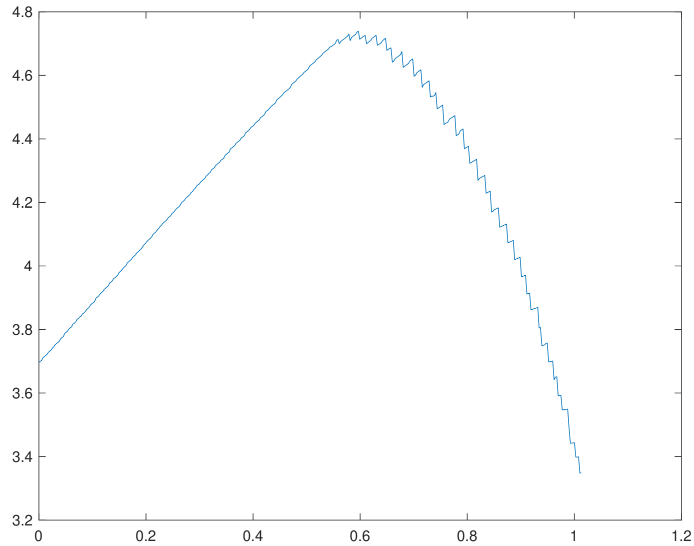


Figure (C.1) Social value of firm G as a function of its customer base N