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Finding Evidence of Stock Market Integration Applying a CAPM or Testing for Common Stochastic Trends. Is There a Connection?

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Finding evidence of stock market integration applying a CAPM or testing for common stochastic trends. Is there a connection?*

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Abstract

In this paper it is demonstrates that if assets are priced according to Black's (1972) CAPM, then tests on the cointegrated VAR can reveal evidence for or against integration of financial markets. If the market portfolios cointegrate one-to-one and share the same deterministic long-run trend, the markets obey the law of one price. Furthermore, it is shown how the driving force of the prices can be found. Evidence from an empirical example suggests that the Danish and American stock markets are integrated because US stock prices drive those of Denmark.

Keywords: CAPM, market integration, common trends.
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1 Introduction

The following analysis aims at combining two directions in the existing literature of testing for stock market integration. It will be demonstrated how tests for integration assuming a capital asset pricing model (CAPM) and tests for common stochastic trends among stock price indices can be linked by restrictions on the cointegrated vector autoregressive (VAR) model. The source of the common trends is rarely discussed in the existing literature. The present paper takes up that discussion and shows how tests in the cointegrated VAR can provide information about it.

Relations between national financial markets have been of interest to researchers for more than a decade. Much effort has been made to find empirical evidence for or against links among stock markets in different countries, especially since the crash in stock markets in October 1987. Several studies of theoretical macroeconomic models have established many potential economic benefits from the integration of financial markets. The general consensus is that, because of better opportunities for risk sharing, the integration of financial markets can stimulate growth (see, for example, Pagano (1993) and Obstfeld (1989, 1998)). Indeed, much effort has been made at the political level in order to put legal conditions in place which, in turn, facilitate the integration of national financial markets. The European Union is an obvious example here (See Licht, 1997). This has led to the important question of whether or not financial markets have in fact become more integrated.

Various authors have suggested ways of testing for market integration. One direction of the literature argues (assumes) that markets are integrated if similar assets - i.e. assets with the same risk-adjusted payoff profile - are priced identically. A CAPM can be used to determine whether such assets have the same (theoretical) price. Another direction of research tests integration in terms of common stochastic trends (or cointegration) among international markets, which are measured by indices representing the whole market. The more markets are cointegrated - i.e. the fewer common stochastic trends the markets share - the stronger the evidence of integration.

In the present paper it is assumed that assets are priced according to a version of the CAPM developed by Black (1972). Furthermore, it is assumed that the data generating process (DGP) of the market portfolios can be described by a VAR model. With these assumptions in hand it is shown how evidence can be found for or against market integration simply by testing restrictions on the VAR model.

The rest of the paper is organized as follows: Section 2 discusses the concept of the integration of financial markets on the basis of proposed definitions in the financial literature. In section 3, the statistical model is set up and discussed followed by a discussion of the theoretical asset pricing model in section 4. The restriction on the statistical model, based on the CAPM, is treated in section 5, and section 6 provides a discussion of the source of the stochastic trends in stock prices. It is also demonstrated how a simple test on the cointegrated relations can reveal information of where the common trends come from. In section 7, an empirical example is given, and the analysis is summarized in section 8.

2 Integration of financial markets

In the financial literature a formal definition of market integration does not seem to exist. Nevertheless, many proposals have been put forward: see, for example, Jorion & Schwartz (1986); Wheatley (1988); Gultekin et al. (1989); Bekaert & Harvey (1995); Chen & Knez (1995); and Hardouvelis et al. (1999). In general there seems to be some consensus that two financial markets are considered integrated if assets with the same risk-adjusted return cash-flows are priced similarly. Some authors also refer to this as the law of one price. It follows from this that integration is related to convergence of risk aversion across markets in the sense that the difference between investors' degree of aversion on different markets narrows.

The question of integration is not only relevant in an international context. When considering national markets, tests of integration among, for example, IT-stocks and industrial stocks are also relevant.[1] In the empirical analysis which follows, the focus is on integration between stock markets in different countries, but a similar analysis could easily be conducted using data for different industries. In fact, it might be the case that if national markets are perfectly integrated, then investors might prefer to diversify their portfolio between industries rather than countries.[2]

In practice, many stocks are not traded at more than one (or occasionally a few) stock exchange(s), which complicates the testing integration. So, how can we even talk about stock markets being integrated and, furthermore, is it possible to test this at all? This leads to the question of how stock prices are determined. If, for example, stock prices in general are determined only by domestic fundamental factors, then an examination of convergence in the development of appropriate fundamentals could serve as a test for integration. In practice, however, there seems to be more factors involved than fundamental variables in terms of influencing stock prices. The October 1987 crash in the American stock

[1] Of course, comparing sectors in a national market is somewhat easier since fluctuations in exchange rates do not exist.

[2] This point was also noted by Hardouvelis et al. (1999).

market, which spread to many other countries despite the fact that the development in the fundamentals were very different, is an example of this. Furthermore, the sharp increase in US stock prices, which started in 1995, seems to have spread to other countries as well, as can be seen in Figure 1, which shows the development in real stock prices in the US, the UK and Germany.

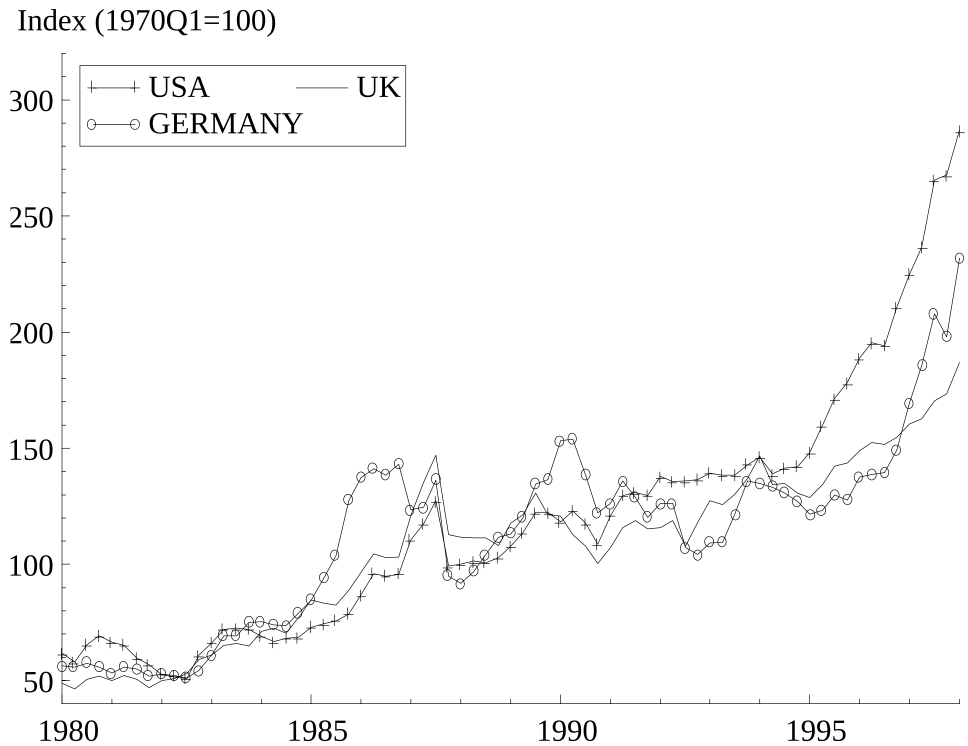

Figure 1. Real stock price indices. Quarterly data.

Testing integration empirically raises the difficult question of how to measure risk-adjusted return cash-flows. Several studies test integration by applying theoretical pricing models such as CAPM and APT (Arbitrage Pricing Theory). Both models are price assets in general equilibrium. In CAPM the rates of return on all risky assets are functions of their covariances with the market portfolio (the portfolio consisting of all assets in the market). In APT models the returns of risky assets are linear combinations of various factors that affect asset returns. Hence, APT is more general than CAPM and it can be shown that CAPM is a special case of APT.[3] In the analysis below, a version of the CAPM is used to illustrate the meaning of risk-adjusted prices, and to describe the link between cointegration among prices and market integration.

[3] The CAPM and the APT model are described in standard text books on financial theory such as Copeland & Weston (1988).

3 The statistical model

We consider the unrestricted VAR(k) model (k is the number of lags), which is written in error correction form:[4]

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu_0 + \mu_1 t + \varepsilon_t, \varepsilon_t \sim iid(0, \Omega), \qquad (1)$$

where X_t represents the data vectors of dimension p, for example consisting of stock price indices from different countries. The matrices Π, Γ_i, μ_0 and μ_1 include coefficients to be estimated. To simplify notation possible dummies are disregarded. The first thing to investigate is whether X_t is $I(1)$ and if any of the time series share the same stochastic trend(s). These hypotheses can be formulated as a reduced rank condition on Π:

$$H_1 : \Pi = \alpha\beta' \text{ has reduced rank } r < p,$$
$$H_2 : \alpha'_{\perp}\Gamma\beta_{\perp} \text{ has full rank } p - r,$$

where $\Gamma = I - \sum_{i=1}^{k-1} \Gamma_i$. The $_{\perp}$ notation indicates an orthogonal complement such that $\alpha'_{\perp}\alpha = 0$ and $\beta'_{\perp}\beta = 0$. The hypothesis H_2 is about the $I(1)$ space having full rank such that there are no stochastic $I(2)$ trends present. This hypothesis ensures that we have $p - r$ independent common stochastic trends besides r cointegration relations. If H_1 and H_2 are accepted the prices share at least one common trend.

A test for the number of common trends is developed by Johansen (1988, 1991). He shows how to apply Anderson's (1951) technique of reduced rank regression to form a likelihood ratio test, where the maximized likelihood functions are found by solving an eigenvalue problem. More precisely, by making the reduced rank regression we get p eigenvalues: $1 > \widehat{\lambda}_1 > ... > \widehat{\lambda}_p > 0$. The likelihood ratio (Trace) test for r cointegrating vectors (and hence $p - r$ common trends) is given by:

$$-2 \ln Q(r \mid p) = -T \sum_{i=r+1}^{p} \ln(1 - \widehat{\lambda}_i), \qquad (2)$$

where T is the number of observations. Johansen & Juselius (1990) derive the asymptotic distribution of the test and present critical values.

In the case of cointegration the moving average representation is, according to Granger's representation Theorem, given by:

$$X_t = C \sum_{i=1}^{t} (\varepsilon_i + \mu_0 + \mu_1 i) + C_1(L)(\varepsilon_t + \mu_0 + \mu_1 t) + A, \qquad (3)$$

[4] The approach used here was developed by Johansen (1988, 1991, 1996).

where $C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp$, A depends on the initial conditions and $C_1(L)$ satisfies the conditions given in Johansen's (1996) Theorem 4.2. Since $\alpha'_\perp \sum_{i=1}^{t} \varepsilon_i$ is the only non-stationary part of the process, this is defined as the common stochastic trends.[5] The matrix $\alpha'_\perp$ are the coefficients for the common trends and $\beta_\perp$ are the loadings from the common trends into the variables. The latter indicates to what extent the variables are affected by the trends.

The cumulation of a deterministic trend is a quadratic trend. Since this is rarely (probably never) seen in economic time series this cumulation should be avoided. This can be achieved by restricting the trend to the cointegrating space. Formally this is done by decomposing μ_i $(i = 1, 2)$ such that $\mu_i = \alpha\rho_i + \alpha_\perp\gamma_i$. The restriction $\gamma_1 = 0$ is then imposed. The error correction model can then be rewritten:

$$\Delta X_t = \alpha \begin{pmatrix} \beta \\ \rho_1 \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + ... \tag{4}$$

It can be shown that the linear trend in the process is given by[6]

$$\tau_1 = C\alpha_\perp\gamma_0 + (C\Gamma - I_p)\beta(\beta'\beta)^{-1}\rho'_1. \tag{5}$$

Considering a process on the form $X_t = \tau_0 + \tau_1 t + stoch.\ terms$ implies that $E(\Delta X_t) = \tau_1$, which will be used later. Note that in the long-run relations, $\beta' X_t$, the coefficient for the deterministic trend is given by $\beta'\tau_1 = -\rho'_1$. Hence, a test on the long-run trend is simply a test about ρ_1.

From (3) it follows that a test of the same impact from the common trends should be performed on the C matrix. Tests should reveal whether the rows of C are identical. Since $(\alpha'_\perp \Gamma \beta_\perp)^{-1}$ is only a normalization, the test for identical rows in C can be performed as a test for identical loadings from the stochastic trends, i.e. a test for that $\beta_\perp$ is proportional to $(1, 1, ..., 1)'$. If the prices share only one common trend, $\beta_\perp$ will be a vector of p components. Alternatively we can perform the test on the β matrix. The test will be for one-to-one cointegration between the variables. For example, in the case with $p = 2$ the test can be formulated as the hypothesis:

$$\beta_\perp = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Longrightarrow \beta = \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \tag{6}$$

It is well known that in general β is not unique. In the case $p = 3$, the test of similar loadings from the common trends can be formulated

[5] See Johansen (1996) Definition 3.7.

[6] See Johansen (1996) exercise 6.1.

differently on β as illustrated below:

$$\beta_\perp = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Longrightarrow \beta = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix} \text{ or } \beta = \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{7}$$

Testing hypotheses formulated as (6) and (7) is straightforward using the procedure described in Johansen & Juselius (1994). Furthermore, testing hypotheses on β is standard procedure in software packages such as PCFIML (Doornik & Hendry, 1997) and CATS in RATS (Hansen & Juselius, 1995).

4 Black's CAPM

This paper will operate on the assumption that integration of markets is a long-run feature. It seems reasonable to believe that two markets remain integrated regardless of whether or not the integration is punctuated by short periods of divergence. Furthermore, when applying an equilibrium model to price assets, short-term divergence from the equilibrium prices is likely to occur in the data. Prices in the long run - or in equilibrium - should, however, be the same for similar assets traded on different markets if these are perfectly integrated.

The original version of the CAPM was - among others - developed by Sharpe (1963, 1964). In this model it is assumed that investors have the opportunity to invest in a risk-free asset, which gives a risk-free return. Black (1972) shows that the results of the standard model also apply if no such risk-free asset exists. Another asset (or portfolio) which is unrelated (zero-correlated) with the market in general can take the place of the risk-free asset.

For practical purposes there might be reasons not to consider investments in risk-free assets. One reason is that no unique definition of a risk-free asset exists. Empirical testing of the CAPM often uses a money market rate as a proxy for the risk-free rate of return. Whether this is really a risk-free return or not can be subject to discussion.

This issue will not be touched upon further in this analysis since it is doubtful whether investors really take into account the risk-free rate of return when trying to determine the price of a stock. Assuming that stocks are priced according to Black's CAPM has the advantage that one does not have to make any decision about the role of the risk-free asset. Hence, when looking for evidence of whether two different markets would price identical assets similarly, we need only consider the stochastic properties of the stock prices themselves and not take into account the developments in, say, money market rates.

The assumptions of the model are the following: (i) investors are risk-averse agents, who maximize end-of-period expected utility of wealth; (ii) they are price-takers with homogenous expectations of jointly normal distributed asset returns; (iii) assets are available in a fixed quantity and are all tradable and divisible; (iv) markets are frictionless and all information is free and available to all investors; (v) there are no market imperfections nor restrictions on short selling. As mentioned before, in the standard CAPM it is also assumed that a risk-free asset exists. In Black's version this is not the case. Instead there exists a portfolio with a return which is independent (zero-correlated) of that of the market portfolio. Examining a model with no risk-free asset allows us to investigate the integration of markets, in which all assets bear a risk. Black's version is more general than the standard one and has Sharpe's model as a special case.

The underlying assumption of the model is that an investor only demands additional return as compensation for any risk that is correlated with the market as a whole. This is referred to as systematic risk. For other risks, called unsystematic, the investor requires no compensation. The market as a whole is referred to by the market portfolio m. This is defined as the portfolio, which consists of all assets of the market held in proportion to their value weights. Hence, the proportion of asset i in the market portfolio is:

$$w_i = \frac{V_i}{\sum_{j=1}^{n} V_j}, \tag{8}$$

where V_i is the market value of asset i and n is the total number of assets. The idea is to measure the price of an arbitrary risky asset i as a price adjusted for the systematic risk.

Given the assumptions mentioned above, all investors will hold an efficient portfolio, i.e. a portfolio which minimizes the risk given the required rate of return or equivalently maximizes the rate of return given the risk the investor is willing to take. Hence, the set of all efficient portfolios is the solution to the problem

$$\begin{aligned} &\min_{W} W'\Sigma W (= \sigma^2) \\ s.t. \, (i) \quad & r^e = W'R^e \\ (ii) \quad & W'\mathbf{1} = 1 \\ (iii) \quad & r^e \geq r^e_{mvp}, \end{aligned} \tag{9}$$

where W is a $n \times 1$ vector consisting of the portfolio weights for each of the n assets, Σ is the covariance matrix of returns, r^e is the required expected return (a number), R^e is a $n \times 1$ vector containing the expected return of all assets, $\mathbf{1}$ is a $n \times 1$ vector of ones and r^e_{mvp} is the return of the minimum-variance-portfolio, i.e. the portfolio which solves the problem (9) disregarding the constraints (i) and (iii). In appendix A it is shown that the set of solutions to (9) disregarding (iii) graphically represents a hyperbola in the $\sigma - r^e$ space. This set is called the frontier. The constraint (iii) ensures that investors always choose a portfolio on the upper part - i.e. the efficient part - of the frontier.

Since all investors hold an efficient portfolio, the market portfolio is also efficient as this is just the sum of all portfolios. In fact, it can be shown that the set of efficient portfolios is convex and, hence that any linear combination of efficient portfolios is also efficient.[7]

As the existence of a portfolio uncorrelated with the market portfolio is essential for Black's CAPM, it will be shown formally that this does exist. The zero-correlation portfolio, which will be called z, fulfills $corr(r_m^e, r_z^e) = 0$, where r_m^e is the expected return of the market portfolio and r_z^e is the expected return of the z-portfolio. As demonstrated in appendix A, the necessary and sufficient conditions for the solution of (9) implies

$$W = g + hr^e \tag{10}$$

and (iii), where g and h are $n \times 1$ vectors defined in Appendix A. Using (10) the covariance between two arbitrary frontier portfolios, p and q, is given by[8]

$$cov(r_p^e, r_q^e) = W_p' \Sigma W_q = \frac{C}{D}\left((r_p^e - r_{mvp}^e)(r_q^e - r_{mvp}^e) + \frac{D}{C^2}\right), \tag{11}$$

where C and D are real numbers defined in Appendix A. Setting (11) equal to zero defines the unique zero-correlation portfolio corresponding to an arbitrary frontier portfolio:

$$r_z^e = r_{mvp}^e - \frac{D/C^2}{r_p^e - r_{mvp}^e}. \tag{12}$$

From (12) it follows that all frontier portfolios, except the minimum-variance portfolio, have a corresponding zero-correlation portfolio.[9] Particularly, the market portfolio has a corresponding z portfolio, assuming that $W_m \neq W_{mvp}$.

The idea of the CAPM is to form a portfolio consisting of the market portfolio m in proportion $1 - a$ and the rest in a risky asset i. This portfolio, p, has the following mean and variance of the return:

$$r_p^e = ar_i^e + (1 - a)r_m^e, \tag{13}$$

[7] See Huang and Litzenberger (1988) Chapter 3.

[8] See Appendix B.

[9] The zero-correlation portfolio for the market portfolio can also be found by solving problem (9), disregarding constraint (iii) and changing (i) to $W_p' \Sigma W_m = 0$.

$$\sigma_p^2 = a^2\sigma_i^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{im}, \tag{14}$$

where σ_{im} is the covariance of the returns of asset i and the market portfolio. Note that p can be thought of as an artificial portfolio. This follows from the fact that in equilibrium all portfolios are efficient and hence, as argued above, m is also efficient. As m consists of all assets, including i, a must be the excess demand of asset i. Now the idea is to investigate what happens when approaching equilibrium, i.e. when a approaches zero.

The ratio between the partial derivatives of the mean in (13) and the standard deviation in (14) for $a = 0$ gives us the slope of the frontier evaluated in market equilibrium:

$$\frac{\partial r_p^e/\partial a}{\partial \sigma_p/\partial a} = \frac{r_i^e - r_m^e}{(\sigma_{im} - \sigma_m^2)/\sigma_m} \text{ for } a = 0. \tag{15}$$

We apply the same trick and form an (artificial) portfolio consisting of the portfolios m and z. The expected return and variance of this are:

$$r_q^e = ar_z^e + (1-a)r_m^e, \tag{16}$$

$$\sigma_q^2 = a^2\sigma_z^2 + (1-a)^2\sigma_m^2. \tag{17}$$

Evaluating the slope of the risk-return trade-off in market equilibrium yields:

$$\frac{\partial r_q^e/\partial a}{\partial \sigma_q/\partial a} = \frac{r_m^e - r_z^e}{\sigma_m} \text{ for } a = 0. \tag{18}$$

In equilibrium (15) must hold between the market portfolio m and the risky asset i. At the same time (18) must hold between m and the zero-correlation portfolio z. Equalizing (15) and (18) and reorganizing gives us the expected return of i, expressed as a linear combination between the expected return of z and the expected return of m:

$$r_i^e = \left(1 - \frac{\sigma_{im}}{\sigma_m^2}\right) r_z^e + \frac{\sigma_{im}}{\sigma_m^2} r_m^e. \tag{19}$$

Black's model is also called a two-factor model, since we can determine the expected return of any risky asset by two factors. The ratio $\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$ is the quantity of risk in the model. Formula (19) is the expected risk-adjusted return of asset i. To find the risk-adjusted price, we apply the definition of the return:

$$p_{i0} = \frac{E(p_{ie})}{1 + (1-\beta_i)r_z^e + \beta_i r_m^e}, \tag{20}$$

where p_{i0} is the start-of-period price and $E(p_{ie})$ is the expected end-of-period price.

Evaluating (20) for two different markets can tell us something about the degree of integration between the markets. The idea is that we want to price two identical assets in two different markets. By identical assets are meant that the expected end-of-period prices are the same in both markets and the covariances with the market portfolios are equal. If the start-of-period prices are equal then the markets are perfectly integrated.

5 Integration and the restrictions on the VAR model

For simplicity, integration between only two markets is considered here. This can, however, easily be extended to consider three or more markets. From (20) it appears that equally risky assets will have the same price in market 1 and market 2 if the denominators are the same. Since the assets are assumed to be similar ($\sigma_{1,im} = \sigma_{2,im}$ and $E(p_{1,ie}) = E(p_{2,ie})$), this will occur if (i) the expected returns of the market portfolios are the same, $r^e_{1,m} = r^e_{2,m}$; (ii) the variances of the returns of the market portfolios are equal, $\sigma^2_{1,m} = \sigma^2_{2,m}$; and (iii) the expected returns of the zero-correlation portfolios are equal, $r^e_{1,z} = r^e_{2,z}$.

Consider the prices of two market portfolios and assume that the development in these can be described by a VAR model. Furthermore, it is assumed that the prices are integrated of order one, $I(1)$, and cointegrate, i.e. they share the same stochastic trend. Hence, the system has $p = 2$ and the restriction $r = rank(\Pi) = 1$ is imposed. This can be tested applying the Trace test (2). The system of prices can then be written:

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \sum_{i=1}^{t} u_i + \begin{bmatrix} d_1 + k_1 \\ d_2 + k_2 \end{bmatrix} t + stat, \tag{21}$$

where $p_{i,t}$, $i = 1, 2$ is the logarithm of the series of prices for the market portfolios and $stat$ represents the stationary components of the process, which will be unimportant in the following. In general, $u_i = f(\varepsilon_{1i}, \varepsilon_{2i})$, i.e. some function of the residuals of the process such that $E(u_t) = 0$ and $var(u_t) = \sigma^2_u$. The coefficients d_1 and d_2 refer to that part of the deterministic trend which is not automatically eliminated in the cointegrating relation (the long-run trend), whereas k_1 and k_2 refer to the part which will disappear. The long-run trend is the important one here as we are considering pricing in a steady state. Hence, when relating this model to the cointegrated VAR, $(d_1, d_2)' = -\beta(\beta'\beta)^{-1}\rho_1$ such that $\rho_1 = -\beta'(d_1, d_2)'$ and $(k_1, k_2)' = \tau_1 + (d_1, d_2)'$. The coefficients c_i and $(d_i + k_i)$, $i = 1, 2$ are to be estimated. The returns of the two market portfolios are given by:

$$\begin{aligned} r_{1,t} &= \Delta p_{1,t} = c_1 u_t + d_1 + k_1 + ... \sim I(0), \\ r_{2,t} &= \Delta p_{2,t} = c_2 u_t + d_2 + k_2 + ... \sim I(0), \end{aligned} \tag{22}$$

which are stationary. Since the expected value of a stationary process is constant over time, the expected returns are:

$$\begin{aligned} r^e_{1,t} &= E(r_{1,t}) = d_1 + k_1, \\ r^e_{2,t} &= E(r_{2,t}) = d_2 + k_2. \end{aligned} \tag{23}$$

In the long run k_1 and k_2 are eliminated, i.e. $\beta'(k_1, k_2) = 0$, such that the expected returns of the two market portfolios are equal if the coefficients for the long-run trends are the same: $d_1 = d_2$. This implies that there will be no trend in the cointegrating relation. Hence, imposing the restriction of equal expected returns implies $\rho_1 = 0$ in (4).

For calculation of the variance of the returns it should be noted that, in the long run, the non-stationary part of the process dominates the stationary part with respect to the stochastic variation. The long-run variances of the returns are given by:

$$\begin{aligned} var_{LR}(r^e_{1,t}) &= c_1^2 var(u_t) = c_1^2 \sigma_u^2, \\ var_{LR}(r^e_{2,t}) &= c_2^2 var(u_t) = c_2^2 \sigma_u^2. \end{aligned} \tag{24}$$

Thus, the variation of the returns of the market portfolios are equal if $c_1 = \pm c_2$ and in particular if $c_1 = c_2$, which is the most natural case and the one investigated here. In other words, the impact from the common stochastic trend are the same on both variables, that is the prices cointegrate one-to-one. The restriction imposed on the cointegrating VAR is $sp(\beta) = sp(1, -1)$.

The return of the zero-correlation portfolios will by definition fulfill that $corr_{LR}(r^e_{i,t}, r^e_{iz,t}) = 0, i = 1, 2$. If $c_1 = c_2$ the expected return of the zero-correlation portfolios will be equal. To see this, notice that for $c_1 = c_2$, $r^e_{1,t}$ and $r^e_{2,t}$ are perfectly correlated in the long run: $corr_{LR}(r^e_{1,t}, r^e_{2,t}) = 1$. Hence $corr_{LR}(r^e_{1,t}, r^e_{2z,t}) = corr_{LR}(r^e_{2,t}, r^e_{1z,t}) = 0$. And as the zero-correlation portfolio is unique it must hold that $E(r^e_{1z,t}) = E(r^e_{2z,t})$ for $c_1 = c_2$.

The above discussion can be summed up as follows: If the time series of prices for two different stock markets have the same deterministic long-run trend and they cointegrate one-to-one, then two similar assets will be priced equally on both markets. In other words, the markets are integrated in the sense that two assets with the same risk profile will be priced identically. The hypothesis of integrated market can be formulated as $sp(\beta, \rho_1) = sp(1, -1, 0)$.

6 Where does the common trend come from?

This section focuses on the source of the common stochastic trend in the situation where markets are integrated. Here, finding the driving force behind stock prices is regarded as an empirical issue. By the driving force is meant the following: a variable, x_2 say, is driven by another variable, x_1 say, if the non-stationary part of x_2 is the cumulation of the errors of x_1 only.

In this situation, where we are interested in market integration, we might want to look for factors driving the larger market - which could be another (bigger) market - to be the driving force. For example, if one finds evidence of integration between two relatively small stock markets such as those of Denmark and Sweden, then the common stochastic trend might come from a larger market like Germany, for example. If this turns out to be the case, the next step could be to investigate what drives the German market. For example, one could include German GDP as a variable. If this is the (only) driving force of the prices, we should still find evidence of only one common trend.

In what follows I investigate whether one of the two markets considered drives the system. Formulated another way, I want to discover whether the common stochastic trend is coming from one of the variables already included in the system. This can be formulated as a test on the coefficients for the common stochastic trends, i.e. on $\alpha_\perp$. The moving average representation (3) for the case $(p,r)=(2,1)$ can be written

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} = \begin{bmatrix} \widetilde{\beta}_\perp^1 \\ \widetilde{\beta}_\perp^2 \end{bmatrix} \begin{bmatrix} \alpha_\perp^1 \; \alpha_\perp^2 \end{bmatrix} \begin{bmatrix} \sum_{i=1}^t \varepsilon_{1,i} \\ \sum_{i=1}^t \varepsilon_{2,i} \end{bmatrix} + ..., \tag{25}$$

where $\widetilde{\beta}_\perp = \beta'_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}$. If the driving force of the system is x_1, say, the restriction on the VAR is $sp(\alpha_\perp) = sp(1,0)$ or formulated on α, $sp(\alpha) = sp(0,1)$. As with tests on the β vectors, tests on the α vectors are likelihood ratio and is a standard test in the software packages mentioned above. For a description of the procedure see Johansen (1996) Chapter 8.

7 An empirical example

In this example I investigate whether the Danish and the US stock markets are integrated, examining data from the post Bretton Wood period. A priori one might expect a big market like the American one to have considerable influence on a small one like that of Denmark. It is unlikely, however, that there is any reciprocal effect.

The source for the data is IMF's *International Financial Statistic (IFS)* and for a further description the reader is referred to the *IFS* manual. The stock price indices (ser. 62) do not cover the entire markets but are used as proxies for the market portfolios. To obtain real prices the stock prices are deflated with the consumer prices indices (ser. 64). The Danish data are converted to US dollars using the average exchange rate (ser. AF). The series are in logarithms and cover quarterly data from 1976Q1 to 1998Q4. The data are illustrated in levels and differences in Figure 2. A first look at these could suggest that the series are $I(1)$.

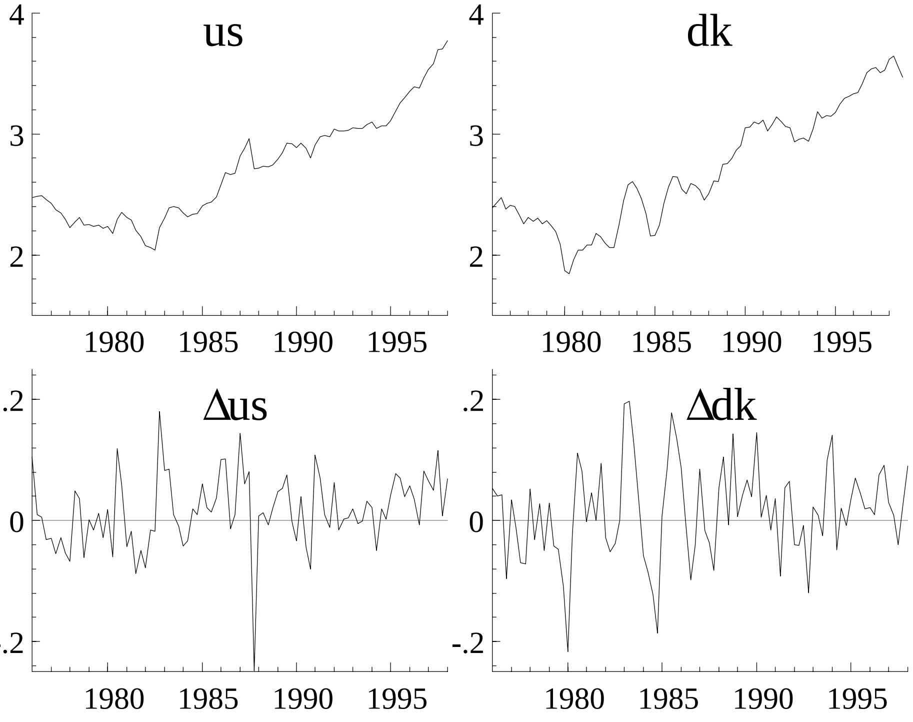

Figure 2. Data in levels and differences.

A VAR with 2 lags[10] is applied and misspecification tests indicate no autocorrelation and no ARCH in the residuals. There is only small evidence of normality, which is mainly due to the US data. Since normality is not crucial for the result obtained below, the model is considered well specified.

The critical values for the test of $rank(\Pi)$ depends on the specification of the deterministic terms. The first model considered includes a trend restricted to the cointegration space given in (4) This model will

[10] The number of lags was determined by considering successive F-test from 5 lags and down as well as looking at the information criteria of Schwarz and Hannan-Quinn. All indications were in favor of 2 lags.

be named $H^*(r)$.[11] With a 10% significance level we can accept the hypothesis of one cointegrating vector. Accepting $rank(\Pi) = r = 1$ means restricting a root of 0.91 in the companion form matrix to one. It is tested whether the trend should be included in the cointegrated space, i.e. whether the long-run expected returns of the market portfolios are equal. The hypothesis is formulated as:

$$\begin{array}{rcccl} & us & dk & trend & \\ \mathcal{H}_1 : (& * & * & 0 &) \in sp(\beta). \end{array}$$

The test is accepted with test statistic $\chi^2(1) = 3.96$ ($p - value = 0.05$). Hence the model is altered to include an unrestricted constant and is named $H_1(r)$. The outcome of the Trace test is a bit lower than the 90% critical quantile but with the two largest eigenvalues in the companion form matrix being 1.02 and 0.65 we accept the hypothesis of one cointegration vector. The hypothesis of restricting the constant to the β-space is accepted with $p - value = 0.22$ using the test statistic given in Theorem 6.3 in Johansen (1996) and the model is altered accordingly to $H_1^*(r)$. Also in this model evidence is in favor of $r = 1$ and the hypothesis of zero-coefficient for the constant is accepted with test statistic $\chi^2(1) = 0.33$ ($p - value = 0.56$). The acceptance of this hypothesis is due to the fact that the two indices have almost the same initial values. Hence, from the analysis of the deterministic terms it is concluded that the appropriate model in which to test hypotheses on β and a is without any deterministic terms. This model is named $H_2(r)$.

The Trace tests in $H_2(r)$ are given in Table 1 and are in favor of $r = 1$ but the hypothesis of two stationary relations is a borderline case. Imposed the restriction $r = 1$ seems reasonable as the second largest root in the companion form matrix is 0.65.

Table 1. Trace tests

H_0 for rank	Eigenvalues of Π	Trace	Asymp. 95% quant.
r=0	0.12	**15.44**	12.21
r=1	0.04	4.13	4.14

The first hypothesis to be tested in the model $H_2(1)$ is whether the common trend has the same impact on both markets. The hypothesis is formulated as

$$\begin{array}{rccl} & us & dk & \\ \mathcal{H}_2 : sp(& 1 & -1 &) = sp(\beta). \end{array}$$

[11] The name of this model, as well as the following considered, are the same as in Johansen (1996).

With statistic $\chi^2(1) = 1.48$ ($p - value = 0.22$) the hypothesis is accepted suggesting that the stock markets of the US and Denmark are indeed integrated. To examine the source of the stochastic trend, the following hypothesis is tested:

$$\begin{matrix} & us & dk & \\ \mathcal{H}_3 : (& 0 & * &) \in sp(\alpha). \end{matrix}$$

The test is clearly accepted with test statistic $\chi^2(1) = 1.30$ ($p - value = 0.25$), indicating that the development in the Danish stock market is determined by the American stock market. The joint hypothesis $\{\mathcal{H}_2, \mathcal{H}_3\}$ was accepted with $\chi^2(2) = 1.69$ ($p - value = 0.43$) and the restricted error correction model with t-values is given by:

$$\begin{pmatrix} \Delta us \\ \Delta dk \end{pmatrix}_t = \begin{pmatrix} 0 \\ \underset{(3.034)}{0.155} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}' \begin{pmatrix} us \\ dk \end{pmatrix}_{t-1} + \ldots.$$

The conclusion of the empirical analysis is that the US and Danish stock markets do seem to be integrated. That is, assets with identical risk-adjusted payoff profile have the same price on both markets. The integration seems to be caused by the fact that Danish stock prices follow those of America.

To see to what extent the role of the deterministic terms affected the conclusion, hypotheses on β and α were also tested in all of the three other models: $H^*(r)$, $H_1(r)$, and $H_1^*(r)$. In all cases the hypotheses of integrated markets and no adjustment of the US market were accepted, supporting the conclusion in the present analysis.

8 Summary of results

The main question analyzed in this paper can be formulated in the following manner. If we assume that a risky asset is priced according to the CAPM, is it then possible to find evidence for or against market integration, by looking for common stochastic trends in the asset prices? This paper has demonstrated that the answer is "yes".

In the financial literature there is no common accepted definition of when two markets are integrated. However, there seems to be some consensus about the following: If the law of one price holds between two markets then they can be considered integrated. The present analysis shows that if the market portfolios cointegrate one-to-on and share the same long-run trend, then the LOP will hold.

To avoid taking a position on the issue of equalization of risk-free returns across markets, Black's version of the CAPM is used. It is assumed that a portfolio with no correlation with the market portfolio exists. Using this model the price of an arbitrary risky asset is found. This tells us what conditions need to be fulfilled in order for two markets to be integrated. It is then demonstrated how these conditions can be interpreted in terms of restrictions imposed on the VAR model.

An issue which seems to have been somewhat neglected in the literature so far is the question of where the stochastic trends in prices come from. It is demonstrated how tests on the cointegrated VAR model can reveal information about this.

An empirical example is provided in order to illustrate the issues discussed. Evidence from this suggests that the American and the Danish stock markets are integrated. The reason for the integration seems to be that the Danish prices follow the American.

The CAPM, which was used to illustrate the concept of risk adjusting, is a one period model. It could be interesting to see if similar results apply when assets are priced with a dynamic model. Further empirical investigation should be undertaken to study the degree to which national stock markets are integrated. This issue might be of particular interest in the context of countries in the European Union. Furthermore, the actual timing of integration could be investigated making recursive analyses of the common stochastic trends. These issue will be left for future research.

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9 Appendix A

We consider the problem (9) disregarding (iii). The problem can be solved using several methods. Here it is solved applying the Lagrange method. The Lagrangian is

$$\pounds(W, \lambda_1, \lambda_2) = W'\Sigma W - \lambda_1(W'R^e - r^e) - \lambda_2(W'\mathbf{1} - 1), \tag{26}$$

where $\lambda_1, \lambda_2 > 0$. The necessary and sufficient first order conditions are:

$$\frac{\partial \pounds}{\partial W} = 2\Sigma W - \lambda_1 R^e - \lambda_2 \mathbf{1} = \mathbf{0}, \tag{27}$$

$$\frac{\partial \pounds}{\partial \lambda_1} = W'R^e - r^e = 0, \tag{28}$$

$$\frac{\partial \pounds}{\partial \lambda_2} = W'\mathbf{1} - 1 = 0. \tag{29}$$

Using (27) and (28) and (27) and (29) we find

$$r^e = \frac{1}{2}\lambda_1(R^{e\prime}\Sigma^{-1}R^e) + \frac{1}{2}\lambda_2(R^{e\prime}\Sigma^{-1}\mathbf{1}), \tag{30}$$

$$1 = \frac{1}{2}\lambda_1(\mathbf{1}'\Sigma^{-1}R^e) + \frac{1}{2}\lambda_2(\mathbf{1}'\Sigma^{-1}\mathbf{1}). \tag{31}$$

To simplify notation we define[12]

$$A = \mathbf{1}'\Sigma^{-1}R^e, \; B = R^{e\prime}\Sigma^{-1}R^e,$$
$$C = \mathbf{1}'\Sigma^{-1}\mathbf{1}, \quad D = BC - A^2.$$

The formulas (30) and (31) form a system of two equations and two unknown variables. This can be solved with respect to the Lagrange multipliers:

$$\lambda_1 = 2\frac{Cr^e - A}{D}, \tag{32}$$

$$\lambda_2 = 2\frac{B - Ar^e}{D}. \tag{33}$$

Inserting (32) and (33) in (27) yields the optimal portfolio:

$$W = \frac{1}{D}\left[B(\Sigma^{-1}\mathbf{1}) - A(\Sigma^{-1}R^e)\right] + \frac{1}{D}\left[C(\Sigma^{-1}R^e) - A(\Sigma^{-1}\mathbf{1})\right] r^e, \tag{34}$$

which is formula (10) with $g = \frac{1}{D}\left[B(\Sigma^{-1}\mathbf{1}) - A(\Sigma^{-1}R^e)\right]$ and $h = \frac{1}{D}\left[C(\Sigma^{-1}R^e) - A(\Sigma^{-1}\mathbf{1})\right].$

[12] The short-hand notation of Huang & Litzenberger (1988) is applied.

Let us take a look at the graphical representation of the so-called frontier, which is defined as the set of portfolios W which has the smallest possible variance $W'\Sigma W = \sigma^2$ given the expected return $W'R^e = r^e$. Using (34) we can find

$$\sigma^2 = \frac{C}{D}(r^e - \frac{A}{C})^2 + \frac{1}{C} \Longleftrightarrow \frac{(\sigma - 0)^2}{\left(\sqrt{1/C}\right)^2} - \frac{(r^e - A/C)^2}{\left(\sqrt{D}/C\right)^2} = 1, \qquad (35)$$

which is a hyperbola in the $\sigma - r^e$ space with center in $(\sigma, r^e) = (0, \frac{A}{C})$ and asymptotes $r^e = \frac{A}{C} \pm \sqrt{\frac{D}{C}}\sigma$.

The minimum variance portfolio (*mvp*) is defined as the portfolio with the lowest possible variance, i.e. the portfolio with $(\sigma, r^e) = (\sqrt{\frac{1}{C}}, \frac{A}{C})$.

10 Appendix B

In this appendix it is demonstrated that the covariance between the rates of return of any two frontier portfolios, p and q, can be expressed as in (11). The covariance is defined as $cov(r_p^e, r_q^e) = W_p'\Sigma W_q$, with W_p and W_q given in (10). Using (10) we find

$$(g + hr_p^e)'\Sigma(g + hr_q^e) = g'\Sigma g + h'\Sigma g r_p^e + g'\Sigma h r_q^e + h'\Sigma h r_p^e r_q^e. \qquad (36)$$

We apply the definitions of g and h given in appendix A and consider each of the terms separately:

$$\begin{aligned} g'\Sigma g &= \left[\tfrac{B}{D}(\Sigma^{-1}\mathbf{1}) - \tfrac{A}{D}(\Sigma^{-1}R^e)\right]' \left[\tfrac{B}{D}\mathbf{1} - \tfrac{A}{D}R^e\right] \\ &= \tfrac{B^2C}{D^2} - \tfrac{A^2B}{D^2} - \tfrac{A^2B}{D^2} + \tfrac{A^2B}{D^2} \\ &= \tfrac{B^2C - A^2B}{D^2} = \tfrac{B}{D}, \end{aligned} \qquad (37)$$

$$\begin{aligned} h'\Sigma g r_p^e &= \left[\tfrac{C}{D}(\Sigma^{-1}R^e) - \tfrac{A}{D}(\Sigma^{-1}\mathbf{1})\right]' \left[\tfrac{B}{D}\mathbf{1} - \tfrac{A}{D}R^e\right] r_p^e \\ &= \left(\tfrac{ABC}{D^2} - \tfrac{ABC}{D^2} - \tfrac{ABC}{D^2} + \tfrac{A^3}{D^2}\right) r_p^e \\ &= \left(\tfrac{A^3 - ABC}{D^2}\right) r_p^e = -\tfrac{A}{D} r_p^e, \end{aligned} \qquad (38)$$

$$\begin{aligned} g'\Sigma h r_q^e &= \left[\tfrac{B}{D}(\Sigma^{-1}\mathbf{1}) - \tfrac{A}{D}(\Sigma^{-1}R^e)\right]' \left[\tfrac{C}{D}R^e - \tfrac{A}{D}\mathbf{1}\right] r_q^e \\ &= \left(\tfrac{ABC}{D^2} - \tfrac{ABC}{D^2} - \tfrac{ABC}{D^2} + \tfrac{A^3}{D^2}\right) r_q^e \\ &= \left(\tfrac{A^3 - ABC}{D^2}\right) r_q^e = -\tfrac{A}{D} r_q^e, \end{aligned} \qquad (39)$$

$$\begin{aligned} h'\Sigma h r_p^e r_q^e &= \left[\tfrac{C}{D}(\Sigma^{-1}R^e) - \tfrac{A}{D}(\Sigma^{-1}\mathbf{1})\right]' \left[\tfrac{C}{D}R^e - \tfrac{A}{D}\mathbf{1}\right] r_p^e r_q^e \\ &= \left(\tfrac{C^2B}{D^2} - \tfrac{A^2C}{D^2} - \tfrac{A^2C}{D^2} + \tfrac{A^2C}{D^2}\right) r_p^e r_q^e \\ &= \left(\tfrac{C^2B - A^2C}{D^2}\right) r_p^e r_q^e = \tfrac{C}{D} r_p^e r_q^e. \end{aligned} \qquad (40)$$

The terms are collected and with some algebra we find

$$\begin{aligned} cov(r_p^e, r_q^e) &= \tfrac{B}{D} - \tfrac{A}{D} r_p^e - \tfrac{A}{D} r_q^e + \tfrac{C}{D} r_p^e r_q^e \\ &= \tfrac{C}{D} \left(r_p^e - \tfrac{A}{C} \right) \left(r_q^e - \tfrac{A}{C} \right) + \tfrac{1}{C}, \end{aligned} \tag{41}$$

which is the same as (11) since $r_{mvp}^e = \frac{A}{C}$ and $\frac{C}{D}\frac{D}{C^2} = \frac{1}{C}$.