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Michael Pedersen

BADIA FIESOLANA, SAN DOMENICO (FI)

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European University Institute
Badia Fiesolana
I-50016 San Domenico (FI)
Italy

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Michael Pedersen†
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Abstract

This paper investigates whether the PPP holds between four metropolitan areas in the US applying a cointegrated VAR model. Three definitions form the basis of the empirical analysis. One relates the concept of underlying inflation to the sharing of common stochastic I(2) trends. The second is simply an econometric formulation of the PPP, whereas the third allows for adjustment of price levels. Evidence is found in favor of the same underlying inflation. Adjustment of price levels, however, has only taken place between the three geographically closer areas.

Keywords: PPP, Cointegrated VAR, common I(2) trends.
JEL classification: C32, C52, E31.

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†Economics Department, European University Institute, Florence, Italy. Email: michael.pedersen@iue.it.

1 Introduction

The present paper investigates the extent to which the Purchasing Power Parity (PPP) holds within the US. Evidence is found that prices are integrated of order 2 making it possible to take advantage of the rich structure of the cointegrated Vector Autoregressive (VAR) model for I(2) variables. This paper contributes to the existing literature by testing the PPP within an area with no trade barriers and where trade is associated with no risk of fluctuations in exchange rates. In this sense, it can also be considered as a benchmark case for what can be expected within the euro area with respect to the convergence of price levels.

Cassel (1922) was one of the first economists to pay attention to the question of whether or not goods are priced identically in different physical locations. He notes that "When two currencies have undergone inflation, the normal rate of exchange will be equal to the old rate multiplied by the quotient of the degree of inflation in the one country and in the other. There will naturally always be found deviations from this new normal rate, and during the transition period these deviations may be expected to be fairly wide.". Within the last two decades, in particular, extensive research has focused on how to measure and test the law of one price (LOP) between two identical goods traded at different physical locations. In the aggregated form this is known as the PPP. The development of strong econometric tools has intensified the research still further. Excellent reviews of literature and methods for empirical testing are given by Breuer (1994) and Froot and Rogoff (1995).

Much recent empirical testing of PPP has applied Johansen's (1991) econometric method to detect cointegration between non-stationary time series in a multivariate framework. Traditional studies focus on two or more countries with different currencies. These can be countries with either floating or fixed exchange rates. The present analysis, on the other hand, applies the Johansen technique to test whether the PPP holds within a currency union, i.e. an area with perfectly fixed exchange rates.

The data used here consists of monthly observations for the consumer price indices (CPI) for a period of 45 years from four metropolitan areas in the US. Based on evidence from earlier studies Froot and Rogoff (1995) note, among other things, the following: "... it is easier to reject the no-cointegration null across pairs of currencies that are fixed than across pairs that are floating."; "...tests based on CPI price levels tend to reject less frequently than tests based on WPIs."; and "... cointegration tests seem to yield much more reliable results when estimated over long sample periods...". Hence, a priori one might expect to find strong evidence in favor of PPP in the empirical analysis carried out in the present context.

As will be revealed, however, this is not the case.

To my knowledge this is the first paper to study PPP within a single country applying a multivariate cointegration framework allowing prices to be integrated of order 2. Several papers, however, investigated whether the LOP holds within a single country. Engle and Rogers (1996) investigate price differences between cities in America as well as in Canada. They find that the distance between cities matters in terms of relative prices, but geographical borders matter more. Parsley and Wei (1996) study 51 prices (traded and non-traded goods) from 48 US cities. They find evidence that the convergence rate to the PPP is higher than that which is typically found in cross-country studies. This rate, however, is slower the farther apart the cities are. Bayoumi and MacDonald (1998) apply a panel data framework and reach similar conclusions. They argue that relative price movements within countries are caused by real factors. Such results might throw doubt on the likelihood of finding strong evidence in favor of PPP in the present context. In a recent study Engle and Rogers (2001) surprisingly discover that variability in prices in the US are higher for traded than non-traded goods.

The rest of the paper is organized as follows: The next section contains a brief discussion of theoretical and empirical considerations with respect to the PPP theory in the context applied in this paper. A new definition of PPP which takes into account that price levels might adjust to each other is introduced and explained by a small economic model. In section 3, the econometric methods used in the empirical analysis in section 4 are discussed, while section 5 summarizes and concludes the analysis.

2 Theoretical and empirical aspects of PPP

In theory the price of a good should be the same even though it is traded in different physical locations. If this were not the case consumers would simply purchase the good where it is cheaper, forcing the producers to equalize prices. In reality, however, this is not the case for a number of reasons, such as cost of transportation, taxes etc. Rigidities in prices (such that they do not adjust as fast as exchange rates change) might also be a reason for the PPP not to hold in the short run. Yet prices should adjust in the long run to avoid arbitrage. Hence, PPP is considered a feature most likely to hold in the long run.

Let us first consider a simple general form of the parity in absolute form. Letting P^i be the price of a particular good - or more precisely a basket of goods - in country i and E^{ij} the exchange rate between country i and j (i.e. the price in country i's currency for one unit of country j's currency), the PPP between country i and j can be written as

$$P^i = P^j E^{ij}. \tag{1}$$

In the empirical analysis below it is investigated whether the PPP holds between four areas in the US. Since these use the same currency, we can simply set $E^{ij} = 1$. Letting small letters denote logarithm, the absolute PPP in the present context reads:

$$p^i = p^j. \tag{2}$$

According to (2) the price levels should be the same in all areas within the US. The relative form of the PPP states that the inflation rate should be the same in these areas:

$$\Delta p^i = \Delta p^j. \tag{3}$$

The relations (2) are (3) are based on theoretical relations and will only hold in specific cases. In general, there will exist a process h_t such that the following relation will hold:

$$p_t^i = p_t^j + h_t + const, \tag{4}$$

where $h_0 = 0$ and $const = p_0^i - p_0^j$. The relative version of PPP will hold if $h_t = 0$ for all t, and the absolute version holds if furthermore $const = 0$, i.e. if $p_0^i = p_0^j$. The term can be interpreted as the deviation from the PPP.

When testing theoretical formulations such as (2) and (3) empirically, a more flexible formulation might be needed.[1] This refers to the stochastic properties of the time series. We will allow for the prices to be integrated of order 1 or 2. Hence, it follows that $h_t \sim I(d)$, for $d = 0, 1, 2$. If $d = 0$ the PPP holds. Since indices are considered the condition $const = 0$ is not straightforward to interpret. It will be argued below that if price levels are adjusting towards each other, the process h_t might be interpreted in terms of the inflation rate. Hence, we get a relation on the form

$$p^i = p^j + \kappa \Delta p^i, \tag{5}$$

where κ is a coefficient which could be positive, negative or possible zero in which case (2) and (5) coincide.

The econometric implications of (3) and (5) depend on the properties of the time series for the prices. Some empirical evidence suggests that inflation is non-stationary, i.e. that price levels are integrated of order

[1] See also Haavelmo (1944) and Juselius (1995) for discussion of this point.

2. If this is the case and the impact from the $I(2)$ trend is the same, the properties of the series can be given interpretations related to the PPP. These are given in the three following (econometric) definitions:[2]

Definition 1 *Let us consider two time series for prices and let* $p_t^i, p_t^j \sim I(2)$. *If* p_t^i *and* p_t^j *cointegrate such that* $p_t^i - p_t^j \sim I(1)$, *that is* $\Delta p_t^i - \Delta p_t^j \sim I(0)$, *then we will say that the underlying inflation is the same in areas* i *and* j.

Definition 2 *Assume that either* $p_t^i, p_t^j \sim I(2)$ *or* $p_t^i, p_t^j \sim I(1)$ *and that* $p_t^i - p_t^j \sim I(0)$, *then we will say that the PPP holds.*

Definition 3 *Let* $p_t^i, p_t^j \sim I(2)$. *If* $p_t^i - p_t^j + \kappa \Delta p_t^i \sim I(0)$ *for some* $\kappa \neq 0$, *then we will say that the PPP with adjustment holds.*

Whereas the economic intuition of Definition 2 is quite apparent, further comments on Definitions 1 and 3 might be in place. In Definition 1 it is important to notice that the stochastic variation in the price levels is dominated by the $I(2)$ness. Hence, if the levels share the same stochastic $I(2)$ trend and the impact from this is the same, the development in the levels are similar from a stochastic point of view. This implies that the inflation rates are affected by the same impact from a common $I(1)$ trend and it will be referred to this as the underlying inflation is the same.[3] There is some relation between Definition 1 and the relative form of the PPP expressed in (3). In fact, Definition 1 can be thought of as the equivalence of the relative PPP when inflation is non-stationary. That the definition does not capture the economic concept of relative PPP is due to the fact that the price differential is $I(1)$ so that the two levels in principle could diverge. Note that for two price series to obey the relative PPP (in it's most strict form) it is required that $\Delta p_t^i - \Delta p_t^j \sim I(0)$ with mean zero.

To explain Definition 3 in greater detail, we consider two economies which seek to integrate their markets.[4] A measure for the degree of integration could be the difference of price levels: if markets for goods are perfectly integrated, prices should be the same. Assume for simplicity, that the price level in i is lower than in j. Hence, the problem is to

[2] The definitions are stated for the purpose of the present analysis such that the exchange rate is eliminated. They are, however, easy to extend to the case of areas with different currency, see Pedersen (2002).

[3] The underlying inflation rate should not be mixed up with the core inflation, which is the inflation rate when the most volatile components are taking out of the price index.

[4] The following discussion is motivated by Gregory et al. (1993) and Gregory (1994).

minimize $p_t^i - p_t^j$. This, however, has a cost, namely inflation. The problem can be formulated in terms of a linear-quadratic adjustment cost model:

$$\min_{\{p_s\}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\delta \left(p_s^i - p_s^j \right)^2 + (\Delta p_s^i)^2 \right], \tag{6}$$

where $\delta > 0$ is the relative weight between the benefit of integration on the cost of inflation, $0 < \beta < 1$ is a discount factor making today more important than tomorrow.[5] In this type of model the variable p_t^j is referred to as the tracking variable and it will be assumed that $p_t^j = x_t'\theta + e_t$, where x_t is a vector of forcing variables and $e_t \sim iid(0, \sigma_e^2)$. The information set at time t is $\mathcal{F}_t = \{e_t, p_{t-j}^i, x_{t-j+1}\}_{j=1}^{\infty}$. However, e_t is assumed not to be know to the investigating econometrician, whose information set $\mathcal{G}_t \subset \mathcal{F}_t$.

The Euler equation gives the first-order condition for optimum:

$$\Delta p_t^i = \beta E_t \Delta p_{t+1}^i - \delta(p_t^i - p_t^j). \tag{7}$$

Formula (7) constitutes a second-order difference equation and the characteristic polynomial for this is

$$\beta z^2 - (1 + \beta + \delta)z + 1 = 0. \tag{8}$$

This has two solutions: One larger than one and one smaller. Here we are only interested in the stable root, which will be named $\lambda(< 1)$. The solution of (7) is then given by

$$p_t^i = \lambda p_{t-1}^i + (1 - \lambda)(1 - \beta\lambda) E_t \sum_{s=t}^{\infty} (\beta\lambda)^{s-t} p_s^j \tag{9}$$

Two cases will now be considered. First, the one where the process for the tracking variable is integrated of order one and then where it is integrated of order two, i.e. $x_t \sim I(1)$ and $x_t \sim I(2)$. We consider for simplicity the case where x_t is a univariate process.

Case 1. $x_t \sim I(1)$. Let the stochastic process for x_t be given by

$$\Delta x_t = \varepsilon_t, \ \varepsilon_t \sim iid(0, \sigma_\varepsilon^2). \tag{10}$$

Then $E_t(x_{t+h}) = x_t$ for $h = 0, 1, ..., \infty$. It follows that $E_t \sum_{s=t}^{\infty} (\beta\lambda)^{s-t} p_s^j = \theta/(1 - \beta\lambda)x_t + e_t$.[6] Inserting in (9) gives us the so-called partial adjustment model

$$p_t^i = \lambda p_{t-1}^i + (1 - \lambda)\theta x_t + (1 - \beta\lambda)(1 - \lambda)e_t. \tag{11}$$

[5] In this particular model β can be expected to be very close to one.

[6] To obtain this the following relation has been applied

$$a + ak + ak^2 + ... + ak^n + ... = a/(1 - k) \ if \mid k \mid < 1.$$

Rewriting (11) gives us the error correction form (ECM):

$$\Delta p_t^i = (\lambda - 1)(p_{t-1}^i - \theta x_{t-1}) + (1-\lambda)\theta \Delta x_t + (1-\beta\lambda)(1-\lambda)e_t. \quad (12)$$

In order to see the cointegrating relations, (12) is reparametrized:[7]

$$p_t^i = \theta x_t - \frac{\lambda}{1-\lambda}\Delta p_t^i + (1-\beta\lambda)e_t. \quad (13)$$

Since the two last terms in (13) are stationary it follows that p_t and x_t cointegrate with the vector $(1, -\theta)$. Hence also p_t and p_t^* cointegrate with this vector.

Case 2. $x_t \sim I(2)$. We now consider the case were x_t is integrated of order two:

$$\Delta^2 x_t = \varepsilon_t \Leftrightarrow x_t = 2x_{t-1} - x_{t-2} + \varepsilon_t, \varepsilon_t \sim iid(0, \sigma_\varepsilon^2). \quad (14)$$

We have that $E_t(x_{t+h}) = (h+1)x_t - hx_{t-1} = x_t + h\Delta x_t$, for $h = 0, 1, ..., \infty$. We use this to find[8] $E_t \sum_{s=t}^{\infty} (\beta\lambda)^{s-t} p_s^j = \theta/(1-\beta\lambda) * x_t + \theta\beta\lambda/(1-\beta\lambda)^2 * \Delta x_t + e_t$, which is inserted in (9):

$$p_t^i = \lambda p_{t-1}^i + \theta(1-\lambda)x_t + \frac{\theta\beta\lambda(1-\lambda)}{1-\beta\lambda}\Delta x_t + (1-\lambda)(1-\beta\lambda)e_t. \quad (15)$$

Reorganizing (15) yields the ECM:

$$\Delta p_t^i = (\lambda - 1)(p_{t-1}^i - \theta x_{t-1}) + \frac{\theta(1-\lambda)}{1-\beta\lambda}\Delta x_t + (1-\beta\lambda)(1-\lambda)e_t. \quad (16)$$

Note that in (16) $p_{t-1}^i, x_{t-1} \sim I(2)$, $\Delta p_t^i, \Delta x_t \sim I(1)$, and $e_t \sim I(0)$. Again we reparametrize:

$$p_t^i = \theta x_t - \frac{\lambda}{1-\lambda}\Delta p_t^i + \frac{\beta\lambda}{1-\beta\lambda}\Delta x_t + (1-\beta\lambda)(1-\lambda)e_t. \quad (17)$$

From formula (17) it appears - since e_t is stationary - that cointegration from $I(2)$ to $I(1)$ exists such that $p_t^i - \theta x_t \sim I(1)$. Furthermore, $p_t^i - \theta x_t + \lambda/(1-\lambda)\Delta p_t^i - \beta\lambda/(1-\beta\lambda)\Delta x_t \sim I(0)$, which is a relation as in Definition 3 if $\theta = 1$. Since p_t^i and x_t share the same $I(2)$ trend, Δp_t^i and Δx_t share the same $I(1)$ trend and thus only one of the differences is needed to obtain stationarity.

Note that in definition 2 and 3 it will trivially hold that $\Delta p_t^i - \Delta p_t^j \sim I(0)$. Hence if the PPP holds (possibly with adjustment) then the underlying inflation rate in the areas must be the same. In other words, if prices are integrated of order 2 a necessary condition for the PPP (maybe with adjustment) to hold is that the areas have the same underlying inflation.

[7] In the present case it boils down to multiplying the equation with $1/(1-\lambda)$ and rearranging.

[8] See Appendix A.

3 The statistical model

3.1 The I(1) case

For the statistical analysis, consider an unrestricted VAR(k) model which in error correction form is written as:[9]

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + \varepsilon_t, \qquad (18)$$

where ε_t are identically independent distributed (iid) errors with mean zero and covariance matrix Ω. The data vector X_t consists of the four price indices such that $X_t = \{chi, ny, phil, la\}_t$.[10] The matrices Π, Φ and Γ_i are parameters to be estimated while D_t includes the deterministic terms of the model.

In the unrestricted version of (18), X_t will in general be $I(0)$. The $I(1)$ and $I(2)$ models are nested in this one. The hypothesis that X_t is integrated of order one can be formulated as the double requirement that $\Pi = \alpha\beta'$ has reduced rank $r < p(= 4)$ and $\alpha'_\perp \Gamma \beta_\perp$ ($\Gamma = I - \sum_{i=1}^{k-1} \Gamma_i$) has full rank $p - r$.[11] The second part of the requirement ensures that the $I(1)$ space has full rank and hence that X_t is not $I(2)$.

A test for the number of common trends in the I(1) model was developed by Johansen (1988, 1991). The principle of the so-called Trace-test is to maximize the likelihood functions under the null-hypothesis and the alternative hypothesis by applying the technique of reduced rank regression of Anderson (1951). The likelihood functions are maximized by solving eigenvalues problems and a likelihood ratio test for the hypothesis of r cointegrating vectors against the alternative of p can then be formulated as:

$$-2 \ln Q(r \mid p) = -T \sum_{i=r+1}^{p} \ln(1 - \widehat{\lambda}_i), \qquad (19)$$

where $\widehat{\lambda}_i$ are the estimated eigenvalues and T is the total number of observations in the sample. The asymptotic distribution and critical values for the test statistic are derived by Johansen and Juselius (1990).

If there exists $p \times r$ matrices of full rank such that $\Pi = \alpha\beta'$ and $\alpha'_\perp \Gamma \beta_\perp$ has full rank, then the moving average representation for X_t is

[9] For a more formal and complete treatment of the procedures described below see, for example, the textbook by Johansen (1996).

[10] See section 4.1 for a detailed description of data.

[11] The notation $\perp$ indicates an orthogonal complement such that $\alpha'_\perp \alpha = 0$ and $\beta'_\perp \beta = 0$.

given by:[12]

$$X_t = C \sum_{i=1}^{t} (\varepsilon_i + \Phi D_i) + C(L)(\varepsilon_t + \Phi D_t) + A, \tag{20}$$

where $C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp$, $C(L)$ is a convergent power series such that the effect of a shock at time t will die out and thus will have no long-run effect, and A depends on initial conditions so that $\beta' A = 0$. Since $\alpha'_\perp \sum_{i=1}^{t} \varepsilon_i$ is the only non-stationary part of the process these are referred to as the common stochastic trends. Notice that the C matrix has reduced rank $p - r$. The matrix $\alpha'_\perp$ represent the coefficients for the common trends and $\beta_\perp$ represent the loadings from the common trends into the process. The term $(\alpha'_\perp \Gamma \beta_\perp)^{-1}$ is simply a normalization.

The moving average representation gives a nice intuitive understanding of test of hypotheses formulated on β and α in the case of $r = p - 1$ cointegration vectors. In this case both $\alpha_\perp$ and $\beta_\perp$ are $p \times 1$ vectors. When $r < p - 1$ it is a more complicated matter since $\alpha_\perp$ and $\beta_\perp$ have more than one column and hence it is the span of these columns which should be interpreted.

An interesting hypothesis on α could be whether it has any zero-rows. For $r = p - 1$ a zero-row for variable i in α implies zeros in all rows except for i in $\alpha_\perp$. This can be formulated as weak exogeneity of ΔX_{it} for β, see Engle et al. (1983).

The matrix $\beta_\perp$ represents the loadings from the common trends into the variables, i.e. the impact of the common trends. The dimension of $\beta_\perp$ is $p \times (p - r)$ and for the case $r = p - 1$ it is simply a $p \times 1$ vector. In many econometric analyses an interesting hypothesis β is for some kind of homogeneity between all variables, i.e. if the coefficients for every pair of variables sum to one. This hypothesis can also be formulated in terms of whether the variables are equally affected by the common trend. In other words, under this hypothesis the coefficients in $\beta_\perp$ are equal.

The techniques for testing hypotheses on α and β were developed by Johansen and Juselius (1990, 1992, 1994). A general hypothesis of no adjustment to long-run relations can be formulated as a linear restriction on the columns of α as $\alpha = H\psi$, where H is a $p \times m$ design matrix imposing $p - m$ restrictions and ψ includes $m \times r$ parameters to be estimated.

In the case of more than one cointegration vector, tests on β are often carried out in two steps. The first involves testing hypotheses on individual vectors. These can be formulated as $\beta = \{H\phi, \psi\}$, where H is a $p \times m$ design matrix imposing $p - m$ restrictions, ϕ is a $m \times 1$ matrix of

[12] According to Granger's representation Theorem. See Engle and Granger (1987).

free parameters and ψ is a $p \times (r-1)$ matrix of unrestricted coefficients. The second step is a joint test of hypotheses accepted in step one.

3.2 The I(2) case

For the error correction model in the I(2) case (18) is reparametrized in accelerations, velocity and levels:

$$\Delta^2 X_t = \Pi X_{t-1} - \Gamma \Delta X_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 X_{t-i} + \Phi D_t + \varepsilon_t. \quad (21)$$

The parameters to be estimated are Π, Γ, Ψ_i, and Φ. The data matrix X_t is $I(2)$ if $\Pi = \alpha\beta'$ has reduced rank $r < p$, $\alpha'_\perp \Gamma \beta_\perp = \xi\eta'$ has reduced rank $s_1 < p - r$ and the $I(2)$ space has full rank, i.e. $\alpha'_{\perp 2}\theta\beta_{\perp 2}$ has full rank.[13] In this case there are r (maybe multi or polynomial) cointegration vectors, s_1 common $I(1)$ trends and $s_2 = p - r - s_1$ common $I(2)$ trends. The process can be rotated to separate direct cointegrating, polynomial cointegrating and non-cointegrating directions. With the notation here $\beta_{\perp 2}$ is orthogonal to $(\beta, \beta_{\perp 1})$, $sp(\beta, \beta_{\perp 1}, \beta_{\perp 2}) = \mathbf{R}^p$ and $\beta = (\beta_0, \beta_1)$. The same notation holds with respect to α.

If $r > s_2$, $r - s_2$ vectors cointegrate directly from $I(2)$ to $I(0)$ in the β_0 direction. With the terminology of Engle and Granger (1987) these are $CI(2,2)$. There are s_2 vectors which multicointegrate - i.e. they require differences of data to obtain stationarity - from $I(2)$ to $I(0)$ in the β_1 direction. The associated stationary process for the multicointegrating relations are given by $\beta'_1 X_t + \kappa' \Delta X_t \sim I(0)$, and this relates to Definition 3. Furthermore, s_1 directions cointegrate from $I(2)$ to $I(1)$ - i.e. are $CI(2,1)$ - in the $\beta_{\perp 1}$ direction.

If $X_t \sim I(2)$, then $\Delta X_t \sim I(1)$ and the number of unit roots in the characteristic polynomial is $s_1 + 2s_2$. Notice that the difference of a stochastic $I(2)$ trend is $I(1)$, whereas s_1 is the number of "independent" $I(1)$ trends, i.e. those which are not associated with any $I(2)$ trends.

The moving average representation of an $I(2)$ process is given by:[14]

$$X_t = C_2 \sum_{s=1}^{t} \sum_{i=1}^{s} (\varepsilon_i + \Phi D_i) + C_1 \sum_{i=1}^{t} (\varepsilon_i + \Phi D_i) + C_2(L)(\varepsilon_t + \Phi D_t) + A + Bt, \quad (22)$$

where $C_2 = \beta_{\perp 2}(\alpha'_{\perp 2}\theta\beta_{\perp 2})^{-1}\alpha'_{\perp 2}$, $\beta' C_1 = -(\alpha'\alpha)^{-1}\Gamma C_2$ and $\beta'_{\perp 1} C_1 = -\overline{\alpha}'_{\perp 1}(I - \theta C_2)$. The terms A and B are functions of the initial conditions.[15] The interpretation of C_2 is equivalent to the C matrix in the

[13] The short-hand notation of Johansen (1996) is used: $\theta = \Gamma\overline{\beta}\overline{\alpha}'\Gamma + \sum_{i=1}^{k-1} i\Gamma_i$, where in general - here and in the following - $\overline{\alpha} = \alpha(\alpha'\alpha)^{-1}$.

[14] See Johansen (1996) Theorem 4.6.

[15] The matrices A and B satisfy the conditions $(\beta, \beta_{\perp 1})' B = 0$ and $\beta' A - \overline{\alpha}\Gamma\overline{\beta}_{\perp 2}\beta'_{\perp 2} B = 0$.

$I(1)$ case. The matrix $\alpha_{\perp 2}$ are the coefficients to the common $I(2)$ trends and $\beta_{\perp 2}$ are the loadings. The C_1 matrix is a more complicated matter and does not have the same 'nice' interpretation.

Whereas tests for adjustment to long-run relations in the $I(1)$ model is simply a test on the α matrix, it is a more complicated matter in case of $I(2)$ness. Paruolo and Rahbek (1999) show that not only will this be a test for restrictions on α but also on $\alpha_{\perp 1}$ and $\beta_{\perp 1}$. The theory for testing hypothesis on $\beta_{\perp 1}$ and $\beta_{\perp 2}$ has been developed too (see Johansen (1997) and Paruolo (1998)).

3.3 Scenario analysis

When discussing price data there is some dispute between economists and econometricians. From a theoretical point of view, it is not possible for the inflation rate to be non-stationary in the long run. On the other hand much empirical research suggests that the inflation rate does indeed seem to be integrated of order one, i.e. the price acceleration is stationary when analyzing a sample which consists of course of less than infinitely many observations.[16]

Let us first consider the case where the prices are integrated of order 2. We consider a system with four variables: *chi*, *ny*, *phil*, and *la*. One implication of Definition 1 is that the four indices share the same $I(2)$ trend. In the case of one common $I(2)$ trend and, say, one (independent) common $I(1)$ trend, the system can be written as follows:

$$\begin{bmatrix} chi_t \\ ny_t \\ phil_t \\ la_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \sum_{s=1}^{t} \sum_{i=1}^{s} u_i + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \\ d_{41} & d_{42} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_i \\ \sum_{i=1}^{t} v_i \end{bmatrix} + stat, \qquad (23)$$

where in general u_i and v_i are functions of ε_{chi}, ε_{ny}, ε_{phil} and ε_{la}. The term *stat* refers to the stationary part of the process. Note that in this case we have $(r, s_1, s_2) = (2, 1, 1)$ which implies three unit roots in the characteristic polynomial but only two zero roots in the matrix Π. According to Definition 1, it is required that $c_1 = c_2 = c_3 = c_4$ if the underlying inflation is the same in the four areas. In this case the system can be transformed into the $I(1)$ space with no loss of information, i.e. the likelihood functions of the two systems will be approximately the same. A transformed system could look as follows:

$$\begin{bmatrix} chi_t - ny_t \\ chi_t - phil_t \\ chi_t - la_t \\ \Delta chi_t \end{bmatrix} = \begin{bmatrix} d_{11} - d_{21} & d_{12} - d_{22} \\ d_{11} - d_{31} & d_{12} - d_{32} \\ d_{11} - d_{41} & d_{12} - d_{42} \\ c_1 & 0 \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_i \\ \sum_{i=1}^{t} v_i \end{bmatrix} + stat. \qquad (24)$$

[16] See for example Juselius (1999).

In the transformed model interesting hypotheses can be formulated based on whether or not cointegration vectors consist of a price differential and the inflation rate according to Definition 3. As an example we look for cointegration between $chi_t - ny_t$ and Δchi_t. The combination $(1,0,0,\kappa)X_t$ will be stationary if $\kappa c_1 = -(d_{11} - d_{21})$ and $d_{12} = d_{22}$. The implication is that not only is the underlying inflation the same in the areas of Chicago and New York (Definition 1), but also the PPP with adjustment (Definition 3) holds. This is the case because, even though the price levels do not cointegrate directly to $I(0)$, they do multicointegrate. Hence, the price levels seems to have adjusted, perhaps towards a sustainable PPP level.

In the case where $X_t \sim I(1)$ the underlying inflation is determined by the $I(1)$ part of the process. Hence, areas have the same underlying inflation if they share one common $I(1)$ trend with the same impact. But in this case this follows trivially from the fact that the PPP (Definition 2) holds.

It should be clear that if the question of whether there is an $I(2)$ trend in data is in doubt (maybe there is an almost $I(2)$ trend), then it is better to treat the model as $I(2)$ in this case of examining multiple PPP. To illustrate this, let us - falsely - assume that $X_t \sim I(2)$. The next step after detecting the nominal trend is to find the impact on each of the variables and figure out if it is possible to make a transformation of the model into the $I(1)$ space. Even though the variables in fact were all $I(1)$, this transformation would still be valid if the PPP holds. In this case we would end up with a stationary system. Hence, if the transformation cannot be made it is indeed evidence that the PPP does not hold between the areas.

4 Empirical analysis

4.1 Data and misspecification tests

The LOP is a theory stating that two identical goods should have the same price regardless of where they are traded. Arbitrage in good markets will equalize prices, in the case of areas with different currency this could happen via the exchange rate. This of course makes no sense if the good is defined as an Arrow-Debreu good, which is - among other things - described by the time and location of the purchase.

In its aggregate form the LOP is referred to as the PPP. According to this all identical goods will be priced identically at different locations. A more flexible formulation of this - which might be useful when testing the PPP empirically - could state that some kind of geometric weighted average of prices should be the same at different locations. This formulation allows for different demands in different areas to have an influence. For example, it makes little sense to compare prices on winter clothes in Greenland and Florida, as the demands are very different. Furthermore, the more flexible formulation might also to a greater extent mirror consumer behavior. Often consumers do not bother to go shopping in two supermarkets, even though two different articles, say bread and wine, are cheaper in different markets. An economic argument for this behavior could be a kind of shoe-leather effect: 'it is not worth doing the extra walking to save a few pennies'.

How should the prices be aggregated then? This question does not have an obvious answer and is still an issue for discussion. One answer is to aggregate with the weights of the consumption. This has the advantage of comparing the cost of the consumption and in this sense a kind of 'cost-of-living', which takes account of the fact that geographical, cultural and other circumstances might require consumption of particular goods such as the example of winter clothes in Greenland. On the other hand, this could result in comparing prices on "apples" and "bananas", which is not appropriate when testing the PPP.

Hence, a discussion of the data used in empirical testing of LOP and PPP is certainly important. In the present analysis indices for the development in consumer prices are considered. More precisely, four indices from major metropolitan areas in the US are used on a monthly basis covering the period from 1953 to 1997, which gives a total of 45 years or 540 observations. The starting date of the period was chosen to avoid effects from World War II and to avoid using data from the beginning of the period which may be too imprecise (see below). The end-date is due to limitation in monthly observation in one of the series. The data was extracted from the US Bureau of Labor Statistic's (BLS) homepage[17] and is collected by local branches, as can be seen in Table 1.

Table 1. Description of data

Variable name	Metropolitan area	Region	Branch of BLS
chi	Chicago-Gary-Lake County	MW	Chicago
ny	New York-Northern N.J.-Long Island	NE	New York
phil	Philadelphia-Wilmington-Trenton	NE	Philadelphia
la	Los Angeles-Anaheim-Riverside	W	San Francisco

The indices - named CPI-U by the BLS - cover all urban consumers, which represents around 87 percent of the population. When using official indices covering long periods, problems may arise with respect to

[17] http://stats.bls.gov/.

the measurement. This is due to the fact that the base year of the indices from time to time is changed for ease of interpretation of the later observations. For the oldest data considered here, changes of base year has occurred four times. First, with 1947-49=100, then 1957-59=100, 1967=100, and latest 1982-84=100. The indices used in the present analysis have 1967 as base year. Since the indices are published with only one decimal, the accurateness of the changes in data in the beginning of the period - where the index number is relatively low - are less precise than for the later period. For example, the index from the Chicago area starts in January 1953 at 79.8 and ends in December 1997 at 486.5. An index change of 0.1 in the early period implies a monthly inflation of 0.125 per cent whereas the same change in the late period requires the prices to change only by 0.021 per cent over the month.[18] Logarithm of the data are illustrated in levels and differences in Figure 1.[19]

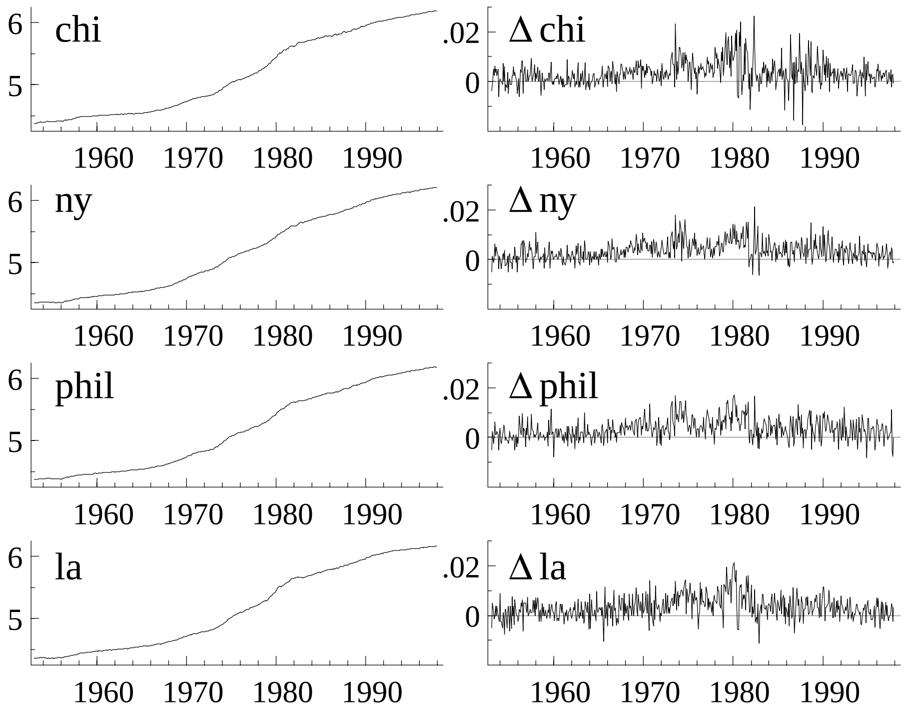


Figure 1. Logarithm of price indices in levels and differences.

So, what information can we get from an analysis of these indices? The indices contain information of prices for more than 2,000 articles.

[18]The estimations were also done using data with base year 1982-84=100. The outcome was more or less the same as the one reported in the next subsection, suggesting that the choice of base year might not be of great importance.

[19]Figures 1 and 2 were made using GiveWin and PcFiml (see Doornik and Hendry (1996, 1997)).

These are arranged in eight major groups: Food and Beverages; Housing; Apparel; Transportation; Medical Care; Recreation; Education and Communication; and Other Goods and Services.[20] Hence, they contain traded as well as non-traded goods. Furthermore, they include taxes which are directly related to the purchase such as sales and excise taxes. The indices, therefore, measure the price of a basket of consumption chosen by the average consumer. The baskets are, however, not the same in the areas according to Table 2, which means that a direct comparison of the indices should be made with some caution.

Table 2. Weights of components in the CPI-U, 1997

	chi	*ny*	*phil*	*la*
Food and beverages	15.283	16.493	17.199	16.213
Housing	40.227	43.336	39.446	43.141
Apparel	5.331	5.295	5.353	4.667
Transportation	17.959	14.127	16.450	17.021
Medical care	5.082	5.218	5.359	4.011
Recreation	5.648	5.234	6.339	5.731
Education and communication	5.623	6.074	5.318	5.311
Other goods and services	4.844	4.223	4.536	3.905

Source: BLS: 'Relative importance of components in the consumer price index, 1997.

Note: The area 'ny' covers the same cities as in Table 1. For the other areas changes there have been made. In this Table 'chi' covers Chicago-Gray-Kenosha, 'phil' covers Philadelphia-Wilmington-Atlantic City, and 'la' covers Los Angeles-Riverside-Orange County.

The numbers in Table 2 are for a specific year, namely 1997. The BLS conducts consumer surveys every year and the weights are updated accordingly. As appears from the table the weights are a bit different between the areas. Since the baskets mirror the consumers' choices between articles, which are made for given prices, the development in the indices does describe some kind of purchasing behavior which can be analyzed comparing the indices. One way to think about it is the following: If a consumer moves from one area to another, her/his consumption (i.e. the personal basket) is likely to change. For example, if a consumer moves from Chicago to New York the percentage of the entire consumption which is spent on Housing is likely to increase. Thus in this setting, testing PPP using consumer price data also tells us something about the degree to which consumers are mobile in order to equalize prices.

[20] For a more detailed description of the indices the reader is refered to BLS' homepage. Especially a look at "Frequently Asked Questions" is worthwhile.

Bases on prior testing, the cointegrated VAR considered includes a constant term restricted to the β-space. Hence the ECM reads

$$\Delta^2 X_t = \alpha(\beta' \rho_0') \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} - \Gamma \Delta X_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 X_{t-i} + \varepsilon_t. \qquad (25)$$

The econometric methods applied rely on the assumption that errors are *iid*. Recent research (Hansen and Rahbek, 1999), however, shows that the methods are robust to ARCH effects in the residuals. The main concern is whether there is evidence of autocorrelation in the errors. Investigation of the residuals indicates that a VAR model of order three ($k = 3$)[21] including four balanced impulse dummies and 12 centered seasonal dummies is sufficient to restore residuals with no autocorrelation. Lagrange-multiplier tests for non-autocorrelation of order one and four are accepted with test statistics $\chi^2(16) = 26.6$ ($p - value = 0.05$) and $\chi^2(16) = 16.6$ ($p - value = 0.41$).[22]

The four (balanced) impulse dummies are $d559$ (+1 in 1955:9; -1 in 1955:10; and 0 otherwise), $d658$ (+1 in 1965:8; -1 in 1965:9; and 0 otherwise), $d802$ (+1 in 1980:2; -1 in 1980:3; and 0 otherwise), and $d8710$ (+1 in 1987:10; -1 in 1987:11; and 0 otherwise).

4.2 Statistical analysis

The test for rank in the $I(2)$ model is a joint test for the number of cointegration vectors and the number of common stochastic trends. The low power of the test, however, has been demonstrated by Jørgensen (1998) and Johansen (2000). Hence, the final determination of the number of common $I(1)$ and $I(2)$ trends should not be based solely on the outcome of this test. A look at the number of unit roots in the companion matrix can help to provide insight into the properties of the process. These are reported in Table 3.

Table 3. Five largest roots of the companion matrix

Unrestricted	$r = 3$	$r = 2$	$r = 1$
1.00	1	1	1
0.99	1.00	1	1
0.97	0.96	1.00	1
0.93	0.92	0.92	0.99
0.63	0.63	0.63	0.63

[21] The choice of three lags was supported by Hannah-Quinn information criteria.

[22] All estimating and testing were performed using the program package CATS in RATS (see Hansen and Juselius, 1995). I(2) tests were performed using a routine developed by Clara Jørgensen.

In the unrestricted model the four largest roots are quite large (more than 0.9) whereas the fifth one is somewhat smaller. In fact, the largest root is unity. Restricting the model to three cointegration vectors raises the second root to one and the same is the case for $r = 2$. For $r = 1$ the fourth root is very close to one. In general, when restricting the number of unit roots in the process the following root raises towards unity which indicates $I(2)$ness as an $I(2)$ trend implies two unit roots. The fourth root seems to be quite stable, except for the case where it is the next root to the unit roots, which could suggest three unit roots and a root close to but less than one. Three unit roots are consistent with three $I(1)$ trends or one $I(2)$ trend and one $I(1)$ trend. From an economic point of view it might be reasonable to think that the indices might share a common nominal trend. The rank tests for the number of cointegration vectors and the number of common trends are given in Table 4.

Table 4. Rank test for the joint hypothesis $Q(s_1, r)$

$p-r$	r	$Q(s_1,r)$				$Q(r)$
4	0	880.0	599.9	355.8	142.3	115.2
		111.6	*90.3*	*72.7*	*59.5*	*49.9*
3	1		507.5	263.1	48.5	44.5
			67.0	*51.4*	*40.2*	*31.9*
2	2			198.9	**12.2**	**12.0**
				33.2	*23.6*	*17.8*
1	3				**3.8**	**2.8**
					11.1	*7.5*
$s_2 = p - r - s_1$		4	3	2	1	0

Note: Numbers in italics are 90% quantiles from Paruolo (1996) Table A1 and Johansen (1996) Table 15.2.

Applying the technique of Pantula (1989) of starting from the most restricted hypothesis, the first hypothesis to be accepted is the case with two cointegration vectors ($r = 2$); one $I(2)$ trend ($s_2 = p - r - s_1 = 1$) and one $I(1)$ trend ($s_1 = 1$). This is consistent with three unit roots, and will be the choice for the continuation of the analysis. Hence, the model is restricted to $(r, s_1, s_2) = (2, 1, 1)$.

To gain further insight into the $I(2)$ part of the process, the estimates of the loadings for the $I(2)$ trend: $\widetilde{\beta}_{\perp 2} = \beta_{\perp 2}(\alpha'_{\perp 2}\theta\beta_{\perp 2})^{-1}$ and the coefficient for the trend: $\alpha_{\perp 2}$ are considered. The estimates are given in Table 5.

Table 5. Loadings and coefficients for the $I(2)$ trend

Variable	$\widehat{\widehat{\beta}}_{\perp 2}$	$\widehat{\alpha}_{\perp 2}$
chi	1	-0.1187
ny	0.9948	0.2215
phil	0.9845	-0.1645
la	0.9806	0.0173

Note: The estimates are normalized on $\widehat{\widehat{\beta}}_{\perp 2,1}$.

It is striking to note that the loadings into the variables are very similar. This is in line with economic intuition that the prices should share the same nominal trend and hence that the underlying inflation is the same in the areas (Definition 1). Indeed this indicates that it should be possible to transform the model to the $I(1)$ space by imposing restrictions of price homogeneity between the prices. The test for $sp(\beta_{\perp 2}) = sp(1,1,1,1)$ or alternatively that

$$sp(\beta, \beta_{\perp 1}) = sp \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

is $\chi^2(1)$ distributed, and with a test statistic very close to zero the hypothesis is strongly accepted.[23]

The unit root consistent with price homogeneity between all prices is imposed and we consider a system with the data vector $\widetilde{X}_t = [p_1, p_2, p_3, \Delta chi]_t$, where $p_1 = chi - ny$; $p_2 = chi - phil$; and $p_3 = chi - la$. This should give approximately the same likelihood function and we should lose no information about the price processes (see, for example, Juselius, 1996). To check if the errors in the transformed model are still *iid*, misspecification tests for autocorrelation of order one and four are performed. Both tests for non-autocorrelation are accepted with test statistics $\chi^2(16) = 22.9$ $(p - value = 0.12)$ and $\chi^2(16) = 17.9$ $(p - value = 0.33)$.

To perform a simple check on whether the $I(2)$ has indeed been removed, rank tests are performed on the transformed data set. The results are given in Table 6.

[23] The hypothesis was tested using Clara Jørgensen's $I(2)$ program with Hans Christian Kongstad's extensions. See Kongsted (1998).

Table 6. Rank test for the joint hypothesis $Q(s_1, r)$

$p-r$	r	$Q(s_1, r)$				$Q(r)$
4	0	1334.3	824.2	554.0	325.1	116.2
		111.6	*90.3*	*72.7*	*59.5*	*49.9*
3	1		739.9	458.7	237.9	**30.9**
			67.0	*51.4*	*40.2*	*31.9*
2	2			356.5	131.3	**8.3**
				33.2	*23.6*	*17.8*
1	3				126.5	**2.9**
					11.1	*7.5*
$s_2 = p - r - s_1$		4	3	2	1	0

Note: See Table 4.

Evidence from the tests reported above suggests that indeed the $I(2)$ trend has been removed. The Trace test indicates $r = 1$ cointegration relation but with $r = 2$ as a borderline case. The four largest roots in the companion matrix in the unrestricted model are all real: 0.99; 0.98; 0.96; 0.68. The question is whether or not 0.96 is a unit root. Consistent with the results from the $I(2)$ analysis $r = 2$ is chosen for the further analysis. This choice seems to be appropriate, also given the evidence from the sensitivity analysis reported later. The data in the transformed system are illustrated in Figure 2.

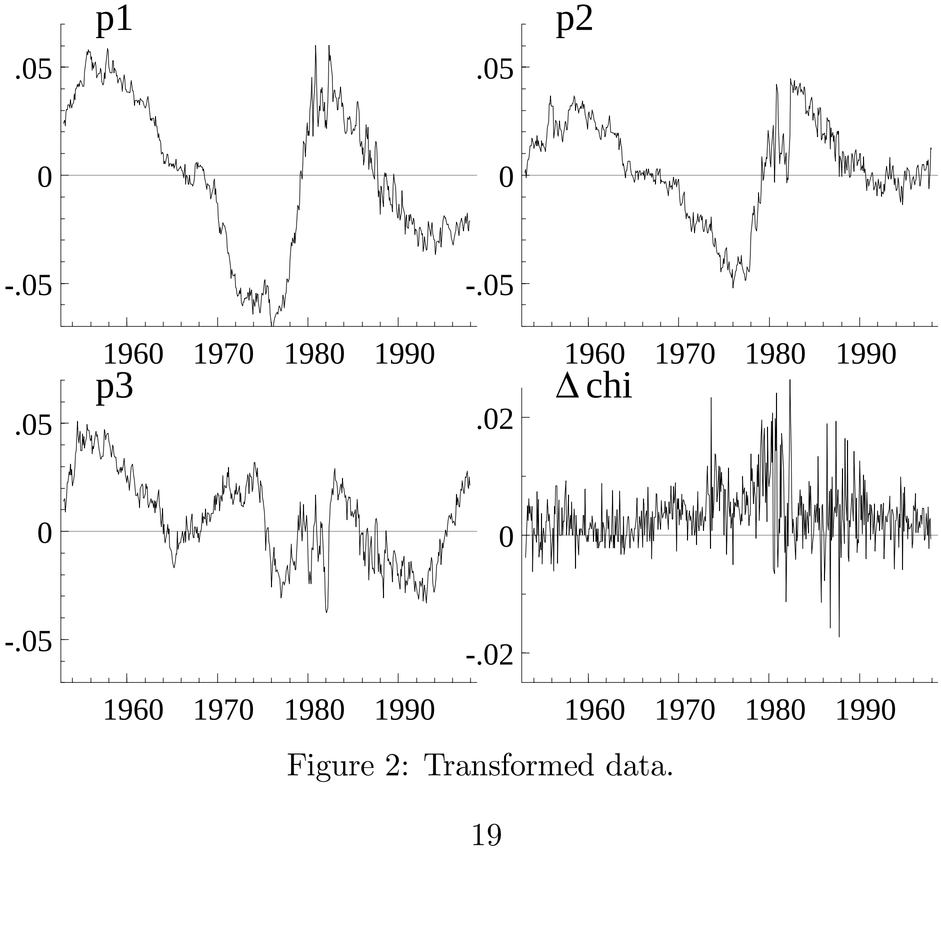


Figure 2: Transformed data.

Notice that the transformation could be made using any other area as numeraire. In this case the inflation rate from the numeraire area would be included in the system. This was tried and did not alter the results to a great extent as reported in the sensitivity analysis in section 4.3.

To identify the cointegrating space, hypotheses concerning Definitions 2 and 3 as well as hypotheses regarding long-run exclusion, stationarity and adjustment to long-run relations are tested. The first four hypotheses tested concern exclusion from the cointegrating space. These are reported in Table 7.

Table 7. Tests for exclusion from stationary relations

	p_1	p_2	p_3	Δchi	$const.$			p-value
$\mathcal{H}_1$: (	0	*	*	*	*	) $\in sp(\beta)$	$\chi^2(2) = 15.56$	[0.00]
	0	*	*	*	*			
$\mathcal{H}_2$: (	*	0	*	*	*	) $\in sp(\beta)$	$\chi^2(2) = 17.88$	[0.00]
	*	0	*	*	*			
$\mathcal{H}_3$: (	*	*	0	*	*	) $\in sp(\beta)$	$\chi^2(2) = 5.85$	[0.05]
	*	*	0	*	*			
$\mathcal{H}_4$: (	*	*	*	0	*	) $\in sp(\beta)$	$\chi^2(2) = 79.32$	[0.00]
	*	*	*	0	*			

It appears that p_3, i.e. the price differential between the Chicago and the Los Angeles areas, are not significant in the long-run relations. The hypothesis of exclusion of the constant was also tested and rejected with test statistic $\chi^2(2) = 57.02$ ($p - value = 0.00$).

Tests of hypotheses regarding the PPP (Definition 2) and (constant) stationarity of the inflation rate are given in Table 8. The hypotheses $\mathcal{H}_8 - \mathcal{H}_{10}$ are hypotheses about the PPP between areas not including Chicago. All of the hypotheses are rejected.

Table 8. Tests for the PPP and stationary inflation rate

	p_1	p_2	p_3	Δchi	$const.$			p-value
$\mathcal{H}_5$: (	1	0	0	0	*	) $\in sp(\beta)$	$\chi^2(2) = 19.41$	[0.00]
$\mathcal{H}_6$: (	0	1	0	0	*	) $\in sp(\beta)$	$\chi^2(2) = 16.68$	[0.00]
$\mathcal{H}_7$: (	0	0	1	0	*	) $\in sp(\beta)$	$\chi^2(2) = 15.50$	[0.00]
$\mathcal{H}_8$: (	1	-1	0	0	*	) $\in sp(\beta)$	$\chi^2(2) = 16.71$	[0.00]
$\mathcal{H}_9$: (	1	0	-1	0	*	) $\in sp(\beta)$	$\chi^2(2) = 19.06$	[0.00]
$\mathcal{H}_{10}$: (	0	1	-1	0	*	) $\in sp(\beta)$	$\chi^2(2) = 18.06$	[0.00]
$\mathcal{H}_{11}$: (	0	0	0	1	*	) $\in sp(\beta)$	$\chi^2(2) = 6.24$	[0.04]

Hypotheses regarding the PPP with adjustment (Definition 3) are given in Table 9. These are tested with and without a constant in the relations. The presence of a constant could imply that the indices start at different levels. Note, however, that even though the indices start at the same level convergence can take place if the "true" price levels are different.

Table 9. Tests for the PPP with adjustment

		p_1	p_2	p_3	Δchi	$const.$			p-value
$\mathcal{H}_{12}:$	(	1	0	0	*	0	$) \in sp(\beta)$	$\chi^2(2) = 19.46$	[0.00]
$\mathcal{H}_{12}^c:$	(	1	0	0	*	*	$) \in sp(\beta)$	$\chi^2(1) = 0.11$	[0.73]
$\mathcal{H}_{13}:$	(	0	1	0	*	0	$) \in sp(\beta)$	$\chi^2(2) = 16.75$	[0.00]
$\mathcal{H}_{13}^c:$	(	0	1	0	*	*	$) \in sp(\beta)$	$\chi^2(1) = 0.95$	[0.33]
$\mathcal{H}_{14}:$	(	0	0	1	*	0	$) \in sp(\beta)$	$\chi^2(2) = 15.27$	[0.00]
$\mathcal{H}_{14}^c:$	(	0	0	1	*	*	$) \in sp(\beta)$	$\chi^2(1) = 4.74$	[0.03]
$\mathcal{H}_{15}:$	(	1	-1	0	*	0	$) \in sp(\beta)$	$\chi^2(2) = 14.47$	[0.00]
$\mathcal{H}_{15}^c:$	(	1	-1	0	*	*	$) \in sp(\beta)$	$\chi^2(1) = 0.09$	[0.76]
$\mathcal{H}_{16}:$	(	1	0	-1	*	0	$) \in sp(\beta)$	$\chi^2(2) = 17.85$	[0.00]
$\mathcal{H}_{16}^c:$	(	1	0	-1	*	*	$) \in sp(\beta)$	$\chi^2(1) = 2.43$	[0.12]
$\mathcal{H}_{17}:$	(	0	1	-1	*	0	$) \in sp(\beta)$	$\chi^2(2) = 18.02$	[0.00]
$\mathcal{H}_{17}^c:$	(	0	1	-1	*	*	$) \in sp(\beta)$	$\chi^2(1) = 5.68$	[0.02]

Hypotheses of the PPP with adjustment are accepted between the areas $chi - ny$, $chi - phil$, $ny - phil$ and $ny - la$ when allowing for a constant in the stationary relation. The hypotheses are rejected for $chi - la$ and $phil - la$. All in all this indicates that when testing the PPP distance matters which, is in line with economic intuition and the findings in the studies of LOP mentioned in the introduction. Furthermore, adjustment of price levels seems to have taken place between the areas of Chicago, New York and Philadelphia but not with the level in the Los Angeles area. There does, however, seem to be some adjustment between prices in New York and Los Angeles, but it should be kept in mind that p_3 is not significant in the stationary relations.

Finally, it is tested whether the variables adjust to the long-run relations. This seem to be the case for all of them as can be seen in Table 10.

Table 10. Tests for adjustment to long-run relations

	Δp_1	Δp_2	Δp_3	$\Delta^2 chi$			p-value
$\mathcal{H}_{18}$: (	0 0	$*$ $*$	$*$ $*$	$*$ $*$	) $\in sp(\alpha)$	$\chi^2(2) = 11.86$	[0.00]
$\mathcal{H}_{19}$: (	$*$ $*$	0 0	$*$ $*$	$*$ $*$	) $\in sp(\alpha)$	$\chi^2(2) = 23.79$	[0.00]
$\mathcal{H}_{20}$: (	$*$ $*$	$*$ $*$	0 0	$*$ $*$	) $\in sp(\alpha)$	$\chi^2(2) = 10.49$	[0.00]
$\mathcal{H}_{21}$: (	$*$ $*$	$*$ $*$	$*$ $*$	0 0	) $\in sp(\alpha)$	$\chi^2(2) = 53.59$	[0.00]

The complete cointegration space is identified as the joint hypothesis $\{\mathcal{H}_{12}^c, \mathcal{H}_{13}^c\}$, which in turn coincides with $\mathcal{H}_3$ since it is just a matter of normalization. The identified space implies the following relations:[24]

$$\begin{aligned} chi &= ny - \underset{(4.390)}{41.212}\,\Delta chi + \underset{(0.019)}{0.144} \\ chi &= phil - \underset{(2.256)}{22.396}\,\Delta chi + \underset{(0.010)}{0.082}\,. \end{aligned} \tag{26}$$

4.3 Sensitivity analyses

When making empirical analyses using econometric models, one always has to make some choices with respect to the model specification. Tests can help but it is rarely the case that it is absolutely clear how the model should be specified. This section contains a discussion of the sensitivity of the results regarding the number of cointegration relations and common $I(2)$ trends with respect to inclusion of the impulse dummies, the chosen lag length, the sample period and the transformation of the system with the Chicago area as numeraire. Also the sensitivity with respect to exclusion of p_3 in the transformed model - given $r = 2$ - is discussed.

First, however, a brief remark regarding the deterministic term. Prior to the analysis reported in this paper, the same experiments were conducted using a model allowing for a linear trend in the cointegrating relations and an unrestricted constant. Evidence was found in favor of one common $I(2)$ trend, one $I(1)$ trend and two cointegration relations. A transformation of the system similar to the one in the present analysis eliminated the $I(2)$ trend and the Trace statistics for the transformed system revealed clear evidence of two cointegrating relations. Furthermore, a χ^2 test strongly accepted the exclusion of p_3. The two identified stationary relation were similar to (26) and the trend was not significant

[24]Numbers in brackets are standard errors.

in any of them. A test for restricting the constant to the β-space was strongly accepted with $p - value = 0.35$.

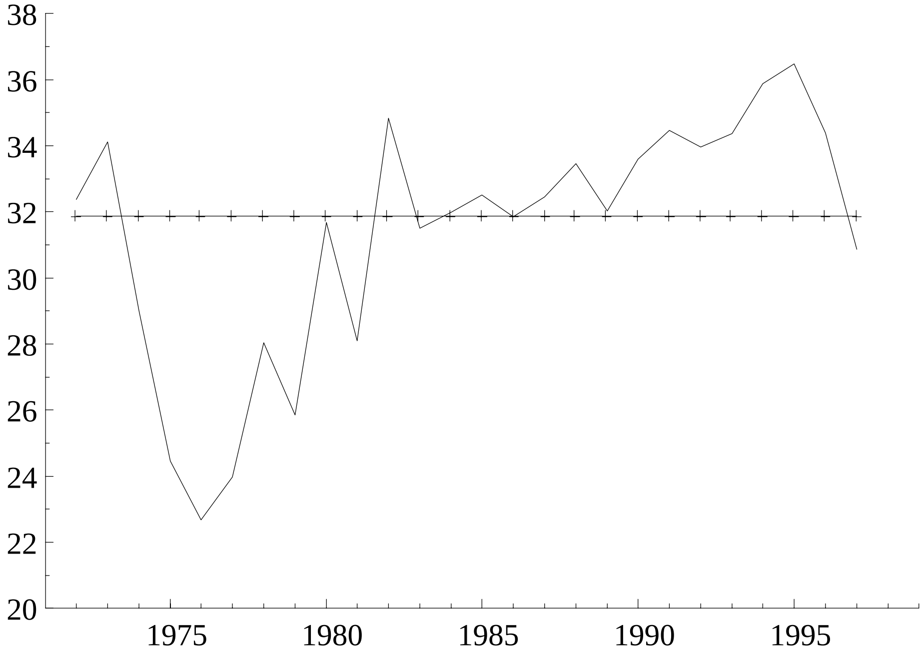
Dummies. It should be noted that without the inclusion of the four dummies in the system the Lagrange-Multiplier tests for non-autocorrelation of order one were rejected in the original as well as in the transformed system. Hence, it is doubtful whether the residuals are in fact *iid*. Having mentioned this, the inclusion of dummies does not change anything with respect to the conclusions. The first hypothesis accepted in the $I(2)$ case is for $(r, s_1, s_2) = (2, 1, 1)$. In the transformed system the Trace statistic for the hypothesis of more than one cointegration relation rises slightly but is still a borderline case. The hypothesis of exclusion of p_3 is accepted with $p - value = 0.05$.

Lags. The model was estimated with $k = 4, 5, 6$ lags and the relevant hypotheses were tested. Also with respect to the number of lags the results seem to be quite robust. In all cases the $I(2)$ analyses provided evidence of $(r, s_1, s_2) = (2, 1, 1)$. The Trace tests in the $I(1)$ model seem to decrease the more lags were included, and the relevant statistics were still borderline cases. For $k = 4$, the hypothesis for excluding p_3 from the stationary relations was rejected with $P - value = 0.03$ but was accepted for $k = 5, 6$.

The choice of *chi* as numeraire. The model was estimated three times using each of the areas as transformation variable. In all cases rank tests indicated that the transformations removed the $I(2)$ness from the system. When using *ny* and *phil* as numeraire, the results were quite robust, i.e. the trace tests indicated that $r = 2$ was a borderline case and in both cases the variable including *la* could be tested out of the cointegration space. In the case where *la* was used as numeraire the test for $r = 2$ was clearly accepted. None of the variables were insignificant, which indicate that when making the transformation from the $I(2)$ to the $I(1)$ space similar to the one made in this analysis, one should carefully choose which variable to use as numeraire.

Sample period. The final sensitivity analysis performed here is with respect to the sample period. The analysis is made by fixing an initial period of 20 years and then adding one year of observations recursively. Hence, 25 point estimates were made. Furthermore, the model was estimated for the last 26 years of the sample only, i.e. from 1972 to 1997. Rank tests for the joint hypothesis of the number of cointegration relations and the number of $I(2)$ trends are performed successively following the same principle as for the $I(1)$ case of Hansen and Johansen (1999). In all cases but two - the periods ending in 1979 and 1980 - the rank tests indicated that one common $I(2)$ trend is present in the data. In the latter two cases the test statistics were borderline cases.

The recursively performed Trace tests for the hypothesis of $r = 2$ in the transformed model are given in Figure 3.



Note: The "plus"-line indicates 90% critical value.

Figure 3. Recursive Trace tests for the hypothesis of $r = 2$.

In 14 cases the outcome of the Trace test supports the choice of two cointegration relations, and especially when the latter period is included in the sample the test seems supportive. This indicates that the choice of $r = 2$ in the transformed model is indeed appropriate, although it should be mentioned that the Trace statistic when estimating the model for the last 26 years is a borderline case with a value a bit lower than the critical. With respect to exclusion of p_3 the hypothesis is accepted in 17 out of the 25 cases in the recursive analysis and also when estimating the model only for the latter period.

All in all the results obtained in this paper seem to be quite robust with respect to the specification of the model and the chosen sample period.

5 Conclusion

Does the PPP hold within the US? The evidence from the analysis in this paper indicates that when allowing for adjustment of price levels, it does hold between areas which are geographically close to each other but not between areas far apart.

The empirical analysis is made on the basis of three introduced definitions. Whereas one of them - although not formally defined - has been applied in more empirical studies the other two are new (to the best of my knowledge) in the literature. Definition 1 introduces the concept of sharing underlying inflation. This is the case if price indices share a common $I(2)$ trend with the same impact. Definition 3 allows for price levels to adjust and is related to the concept of multicointegration in the cointegrated VAR model for $I(2)$ variables. It can be interpreted in terms of optimizing agents (policy makers) seeking integration of markets.

The price indices analyzed here seem to share one common $I(2)$ trend with the same impact on all variables. Hence, it is concluded that the underlying inflation has been the same in the areas of Chicago, New York, Philadelphia and Los Angeles. Furthermore, evidence is found in favor of two cointegration vectors. These can be identified as relations between the areas of Chicago and New York and between Chicago and Philadelphia. This suggests that price levels in these areas have adjusted to each other whereas the price level in the Los Angeles area seems to be more independent from the others. Hence, in accordance with other studies of the LOP and PPP, the evidence is that distance matters. A comprehensive sensitivity analysis reveals that the results are quite robust with respect to the specification of the statistical model.

The present analysis can be considered a benchmark case for what can be expected to happen with respect to PPP within the euro area. It suggests that there is reason to believe that the underlying inflation might be the same in the long run. On the other hand, there is no reason to expect that the PPP will hold between all the countries even though price levels can be expected to adjust between countries close to each other. Whether structural differences between the US and the euro area will lead to other conclusions is an issue which will be left for future research.

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6 Appendix A

Claim 4 *Let $p_t^* = \theta x_t + e_t$ with $E_t(e_t) = e_t$, $E_t(e_{t+h}) = 0$ $(h = 1, ..., \infty)$, and $E_t(x_{t+h}) = x_t + h\Delta x_t$, for $h = 0, 1, ..., \infty$. Then it holds that $E_t \sum_{s=t}^{\infty} (\beta\lambda)^{s-t} p_s^* = \theta/(1-\beta\lambda) * x_t + e_t + \theta\beta\lambda/(1-\beta\lambda)^2 * \Delta x_t$*

Proof. *Using the formula in footnote 6 we find that*

$$E_t \sum_{s=t}^{\infty} (\beta\lambda)^{s-t} p_s^* = e_t + \frac{\theta}{1-\beta\lambda} x_t + \theta\beta\lambda\Delta x_t + 2\theta(\beta\lambda)^2 \Delta x_t + ... \quad (27)$$

looking apart from the first two terms and applying the formula in footnote 6 on the remaining terms we find

$$\left(\frac{\theta\beta\lambda}{1-\beta\lambda} + \frac{\theta(\beta\lambda)^2}{1-\beta\lambda} + \frac{\theta(\beta\lambda)^3}{1-\beta\lambda} + ...\right)\Delta x_t = \frac{\theta}{1-\beta\lambda}\left(\frac{1}{(1-\beta\lambda)} - 1\right)\Delta x_t = \frac{\theta\beta\lambda}{(1-\beta\lambda)^2}\Delta x_{t.} \quad (28)$$

■