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Shadow Economy, Rent-Seeking Activities and the Perils of Reinforcement of the Rule of Law[1]

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Abstract

An economy is considered where a possibility to seek rents (a particular case of this activity is corruption) exists along with production. A producer is able to hide part of his output from both bribery and taxation. It is shown that the presence of a shadow sector has different effects in economies with high and low rent-seeking.

As expected, in the economy with low corruption the direct law enforcement is beneficial for growth, and reduces the shadow sector. However, in the highly corrupt economy, combating the shadow economy reduces output and increases corruption, while combating corruption reduces the shadow economy.

JEL codes: D72, H26, O17, K42.

Keywords: corruption, rent-seeking, shadow economy, law enforcement, transition.

Rules is rules. Except that in Russia rules are there to be bent - it's what makes it possible for society to function.
Robert Parsons, BBC News (2002)

Russia's businessmen are being squeezed by criminal activity on the one side... and illegal government activity on the other.
President V. Putin (2002)

After years of corruption and crony capitalism, Putin is attempting to regain control of the Russian economy by imposing the rule of law. As of mid-July, the government had launched investigations or filed charges - ranging from fraud to tax evasion - against 13 major business leaders...
The World Bank (2000)

1 Introduction

The importance of institutions and the structure of the economy for economic development is now widely recognized. The impact that governance failures, such as corruption and the shadow economy, have had upon economic growth cannot be underestimated: they affect countries all over the globe, especially in Africa and the former Soviet Union. Governance matters, and the benefits of good governance are higher per capita income, lower infant mortality rates, literacy, and less inequality; while it is the poorest people and small businesses that are most hurt by poor governance.

The shadow economy and corruption are known to always come together. However, the study of these phenomena has been generally separate form each other, with exception of empirical works (Johnson et al., 1998a, 1998b; Friedman et al., 2000). In this paper I build a theoretical model of an economy with endogenous rent-seeking and a shadow sector. It differs from the branch of the literature dealing with production/predation and allocation of talent (Acemoglu, 1995; Grossman, 1995; Murphy et al., 1993; Krueger, 1974) by the presence of a possibility for a producer to hide part of the revenues from the tax authorities and, consequently, from corrupt officials. On the other hand, it differs from the shadow economy literature (Loayza, 1996; Campos, 2000) in respect of the producer's motivation to enter the shadow economy (to "go shadow"), and by the presence of mutual interdependence of rent-seeking and shadow activities.

I will show that informal and unofficial activities matter for growth, and that their interdependence serves to alter the results of conventional economic policies aimed at decreasing the extent of the shadow economy. Most interestingly, I argue that tax law enforcement (like investing in the tax police and monitoring) has a detrimental impact on growth in a highly corrupt economy, while it is beneficial for an economy with no or little corruption. In the highly corrupt economy, an increase in law enforcement above a certain level increases the equilibrium corruption level.

The rest of the paper is organized as follows. The next section defines the general framework. Section 3 describes the assumptions made and solves the resulting model in the multiple equilibria framework. Section 4 considers policy implications. Section 5 introduces

the cost for bribe-takers, in an economy with anti-corruption campaign. The last section concludes. Appendix A is devoted to a revision of Acemoglu's (1995) model and its equilibria set. Appendix B contains proofs of the propositions. Appendix C gathers notation used in the paper.

2 Framework

This paper is an attempt to bring together two branches of literature in a theoretical study, and to show that it is the interdependence between shadow economy and corruption that matters for economic performance. In doing so, a special emphasis is made on the case of the former socialist economies, because their problems are particularly aggravated and deepened during the transition from a planned to a market economy. The specific features of transition economies are: structural disparities inherited from socialism; the tradition of rent-seeking embedded in society; lack of established property rights that are partly substituted by the Mafia; etc. The low compliance, endemic corruption and red tape create specific self-reinforcing mechanisms that lock such economies into the unproductive trajectory, where rent-seeking and corruption thrive and the state can be captured by the Mafia (North, 1990; Feige, 1997; Friedman et al., 2000; Savvateev and Polishchuk, 1998; Murphy et al., 1993; Frye and Schleifer, 1997). The following definitions are used throughout the paper:

Definition 1 *Rent-seeking is an unproductive activity, aimed at redistribution of wealth created by others (Murphy et al.,1993).*

Definition 2 *The shadow economy is a set of economic units that do not comply with government-imposed taxes and regulations (Loayza, 1996).*

The analysis that follows is inspired by Acemoglu (1995) who determines a reward structure of society as a key factor for the allocation of talent. He develops a model of the allocation of talent between productive and unproductive activities. The existence of rent-seeking creates a negative externality on productive agents and implies that relative rewards are endogenously determined. This creates multiple equilibria with different reward structures. Society may get trapped in the rent-seeking equilibrium when previous and expected allocations influence current rewards.

Another inspiration comes from the empirical findings of Johnson et al. (1998a, 1998b). Although intuition and the results of the shadow economy literature suggest exactly the opposite, these authors find that "*higher* taxation is correlated with a *lower* share of the unofficial economy". This may point out that other factors (like corruption or "social capital", see Sobel, 2002) are more important in the firm's decision to go underground. Indeed, in Johnson et al. (1998b) it is found that "countries with more corruption have higher shares of the unofficial economy", and moreover, "the extent of regulatory and bureaucratic discretion is a key determinant of underground activity". Conventionally, they also find that corruption is associated with lower growth (see also Mauro, 1995). Unfortunately, their theoretical investigation does not study the two phenomena simultaneously.

A more comprehensive theoretical model is needed in order to explore the link between the shadow economy and rent-seeking. This model will give us an understanding of another

aspect of a shadow sector, i.e. its role as a growth engine in an economy with inefficient institutions and poor provision of property rights. The model below aims to explain the following stylized facts discovered in the recent literature:

1. Positive correlation between the shadow sector size and corruption index (countries with more corruption have a larger shadow economy, Friedman et al., 2000);
2. Negative (counterintuitive, in the light of the shadow economy literature) correlation between the unofficial economy size and tax burden on the firm (Johnson et al., 1998a);
3. Negative correlation between corruption and growth (Mauro, 1995);
4. A persistent relation between the growth of the shadow sector and the output fall in the countries during transition (Johnson et al.,1997; Matveenko et al., 1998);
5. Negative relationship between firm size and the bribe paid (EBRD, 1997; Clarke and Xu, 2002).

3 The Model

Let us consider an economy with a continuum of identical firms. There are p rent-seekers in the economy, and $1-p$ producers. Entering the economy, every firm takes an irreversible decision: either to become a producer or a rent-seeker. This decision is taken on the basis of the expected net returns from each sector, V_P and V_R correspondingly. The firm observes $V = V_P - V_R$, and becomes a producer if $V > 0$ and a rent-seeker if $V < 0$. Therefore, the set of states $\{p \in [0,1] : V(p) = 0\}$ are equilibria in this economy. This particular setting is taken from the Acemoglu (1995) model, but the original form of the bribe function is changed, as to avoid the model deficiency described in Appendix A.

This model of rent-seeking is modified by assuming that each producer is able to hide a part of her output from the authorities. Following the empirical evidence, it is assumed that the primary purpose of "going shadow" is to avoid bribes and red tape and, only by extension, taxation.

Thus, a producer is coerced into bribing every rent-seeker she meets (she gives out a share z of her return as a bribe) and she also pays taxes at rate t. It is assumed here that rent-seekers take into account the amount of taxes the firm pays. This makes sense in a transition economy, where firms are often known to negotiate their taxes and bribes with the same officials (Johnson et al., 1998a). This phenomenon also takes place in developed countries (Ades and Tella, 1999). Such an assumption results in the tax rate and the bribe rate entering multiplicatively in (3). For its shadow part of the output, a firm incurs additional cost, which depends on the ability of the state to punish tax evaders.

It has been recently noticed that small firms pay a greater part of their return as a bribe to authorities (Clarke, Xu, 2002; Pradhan et al., 2000). This would imply that the bribe volume does not linearly depend on the amount of the bribe-giver's output, as it is tempting to assume. Therefore, in this model bribing is a costly process, and the bribe volume is not assumed to depend on the size of the firm. The empirical result is then endogenously obtained from the model (see stylized fact 5 and Proposition 4).

3.1 Production

A producer pays taxes and bribes on the disclosed (official) output. She does not pay taxes and bribes on the hidden (shadow) part, but does incur certain costs by "going shadow". A producer chooses the share of the declared output in her production, and the amount to invest, such that it maximizes the net return from production:

$$\max_{\gamma,x} V_P, \tag{1}$$

where V_P is a sum of official and shadow production:

$$V_P = V_O + V_S, \tag{2}$$

$$\begin{aligned} V_O &= (1-t)\left((1-pz)\,Y_O - c(Y_O)\right), &(3)\\ V_S &= Y_S - c(Y_S) - \widetilde{c}(Y_S). &(4) \end{aligned}$$

Here, p is the probability for each of the producers of meeting a rent-seeker. When this happens, the producer loses a part z of his revenues. Y_O and Y_S stand for the output of the official and shadow sectors correspondingly. It is also assumed that the tax authorities take into account the firm's production costs, while the bribe-taker does not.

We assume a quadratic cost function:

$$c(x) = \frac{cx^2}{2},$$

for production in general, and

$$\widetilde{c}(x) = \frac{kx^2}{2}, \tag{5}$$

for the additional cost the firm pays working in the shadow economy. The (exogenous) parameter k stands for the level of tax law enforcement, which can be changed by the government policies.[1] The larger k is, the larger the cost incurred by the firm that "goes shadow".

It is implicitly assumed in (3) that there are no extra public goods for the official production. This assumption is reasonable because it is *the same* firm which produces official and shadow output. Therefore, it is difficult to allow for using public goods only for its official production (see also Loayza, 1996). It has in fact been noticed that in transition countries the firms usually produce shadow output using the technology and resources from their official part (see Matveenko et al., 1998 for a review).

[1] In general, it should be that $k = k(T)$,where T is the amount of government revenues, financed by the taxes collected: $T = tY_o(1-p)$,where Y_o is a return from official production (before taxes and bribes). However, this complicates the solution without changing it much (see Johnson at al., 1998).

3.2 Rent-Seeking

The net return to rent-seeking takes the following form:

$$V_R = (1-p)R(p), \tag{6}$$

where $(1-p)$ is a probability of meeting a producer, and $R(p)$ is a monotone function of p. In Acemoglu (1995) the bribe function $R(p)$ is assumed to be decreasing in p, however its form is not uniquely described in the literature.

The decreasing function comes from the idea that the bribe-takers, if they are many, compete for bribes and this reduces the return from rent-seeking. This is not questioned here. However, when modeling this process in a search model the congestion effect is already incorporated in the search function: the more rent-seekers there are the less their chance is of meeting and bribing a producer. Therefore the net return from rent-seeking falls with p.

As the empirical evidence does not explicitly make clear whether the bribe is a decreasing or increasing function of p, it can arguably be increasing (for example, when the rent-seekers coordinate on the higher bribe volume). In this case, combined with the simplest matching function, one obtains the net return from rent-seeking of the Laffer-type curve (i.e. increasing until a certain threshold and decreasing afterwards). This would model an intuitive representation of the rent-seeking process, when the return from predation rises until the number of rent-seekers crosses a certain threshold and they start competing for bribes. The net return from rent-seeking would then start declining with p.

Clarke and Xu (2002) consider corruption in the utilities sector and argue that bribes are higher in countries where other forms of corruption are more common. They also note that, given limited enforcement resources, the possibility of being detected might be lower in more corrupt societies. To make an example, imagine an economy with a large number of rent-seekers who have a coordination device.[2] They would then form a monopolistic bribe-demanding body, and the size of each bribe would be higher than in the non-coordinated case. It is also easy to imagine that the bribe-takers create the coordination device when they are many, because of the percolation[3] effect. Moreover, it must usually be the case that the bribe for a certain service can be given to only one official due to their specialization: the bribe is attributed not to the bribe-taker personally but to his position.

When a particular form of the bribe function is sought it is usually assumed to be proportional to the firm's size, and to be equal to the amount given out by an average producer (like in Acemoglu, 1995). This paper uses a different specification for a good reason. It turns out that in case when the simple matching function is taken, this standard bribe function form yields only a single or none equilibria in the model. Appendix A shows why this is the case in the Acemoglu's model. In particular, it is shown that the high rent-seeking equilibrium always lies outside the domain $p \in [0,1]$, i.e. producers cease to exist. Because

[2] Like, for example, the centralized hierarchical structure of bribe-taking process in Waller et al. (2000), existence of which increases a volume of bribes taken.

[3] Percolation effect is found in many natural phenomena. For example it explains the spreading of contagious diseases. It is only when a significant amount of people contract the virus that it becomes an epidemic. It is said that the virus passes the percolation threshold. In the same way an endemic corruption can turn into an epidemic through the percolation effect.

the rent-seekers live at expense of producers, such economy will not be in equilibrium. Neither $p = 1$, when $V_R > V_P$, would be one, for the same reason.

Therefore, a bribing technology with cost is introduced in this model. The cost of bribing is always born: it is not only the pecuniary (or in kind) bribe that hampers the growth of the enterprise but mostly red tape, and time and effort spent to overcome it. It is, for example, reported that managers in the CIS countries spend up to almost 17% of their time on negotiations with the potential bribe-takers, and up to 60% of firms are reported to bribe frequently (Hellman and Schankerman, 2000). Therefore, apart from the bribe itself, there has to be a cost of bribing, borne either by the bribe-takers, the bribe-givers, or (most probably) by both.

The main case considered here is an increasing function $R(p)$, although in the Robustness section it is shown that the main results would not change for any continuous function, including a constant.

3.3 Solution

3.3.1 The Producer's Problem

According to the above, a producer solves the following problem.

$$\max_{\gamma,x}(1-t)((1-pz)\gamma x - \frac{c(\gamma x)^2}{2}) + (1-\gamma)x - \frac{cx^2}{2} + \frac{c(\gamma x)^2}{2} - \frac{k(1-\gamma)^2x^2}{2}. \tag{7}$$

Here $p \in [0,1]$ is the number of rent-seekers. The other variables are: investment in production, $x \geq 0$; share of the official production, $\gamma \in [0,1]$; share of return given as a bribe, $z \in [0,1]$ (practically, $z \in [0, 0.1]$); tax rate, $t \in [0,1]$; level of tax law enforcement, $k > 0$; coefficient of the production cost function, $c > 0$. The first order conditions give us the following investment and share of the declared production: either $\{x^* = 0, \gamma^* = \frac{1}{pz(1-t)+t}\}$, or

$$\begin{aligned} x^* &= x(p) = \frac{1}{c} + \frac{kpz(1-t)}{c\Theta}, & (8)\\ \gamma^* &= \gamma(p) = 1 + \frac{pzc(1-t)}{\Theta + kpz(1-t)}, & (9)\\ \text{where } \Theta &\equiv tc + tk - k < 0. & (10) \end{aligned}$$

The producer can always choose to produce nothing, but hereafter it is assumed that she does not. The following interesting properties of this solution would be useful for the analysis of policy implications.

To make sure that the definition of rent-seeking holds, (10) is obtained form (8). This implies a minimal level of law enforcement in existence: $k > c\frac{t}{1-t}$. In all that follows, it is assumed to hold.

It follows from (8) and (10) that $x'(p) < 0$, i.e. that the higher the level of rent-seeking in the economy, the lower the investments in production. If there are no rent-seekers in the

economy, it produces at its maximum, $x(0) = \frac{1}{c}$. This supports the empirical evidence of the strong negative correlation between corruption and growth/investment.

Substituting for Θ in (8), we obtain that $\frac{\partial x}{\partial k} = \frac{pzt(1-t)}{\Theta^2} > 0$, independently of the sign of Θ. Therefore, for each given number of rent-seekers, p, the more the tax law is enforced (the larger k), the higher is the investment, $x(p)$. This supports the intuitive relationship between the rent-seeking and law enforcement: $x(\underset{+}{k}, \overline{p})$, for each fixed $\overline{p}$.

It can be obtained from (9) that the optimal share of official production, $\gamma(p)$, is a decreasing function of the number of rent-seekers in the economy, p, with its maximum in the economy without rent-seekers, $\gamma^*_{\max} = \gamma^*(0) = 1$. This mirrors the stylized fact about a positive correlation between the shadow economy size and corruption: producers hide less if rent-seekers are fewer.

Thus, with an increase in the number of rent-seekers in the economy, the shadow economy size increases and output/investment falls (stylized fact 4).

Proposition 3 *The optimal net return from production, $V^*_P(p) = V_P(\gamma^*, x^*)$, is a monotonically decreasing continuous function of the number of rent-seekers in the economy, on $p \in [0, 1]$, and it is never negative.*

Proposition 4 *Everything else being equal, small firms hide more.*

Proposition 4 draws upon the empirical evidence that small firms hide a larger part of their output (stylized fact 5). This is another reason why corruption is so harmful for growth: it particularly represses small and medium-sized enterprises.

3.3.2 The Rent-Seeker

The net return from rent-seeking depends on the probability of meeting a producer (congestion effect), and on the volume of a bribe:

$$V_R = (1 - p)R(p). \tag{11}$$

Let us consider a particular case of the functional form of the bribe function, $R(p) = 1 - \alpha + \beta p$, with $\beta > 0$, and $1 - \alpha > 0$, which will assure the following properties of V_R: the net return from rent-seeking is increasing when p is small, and decreasing when p is larger than $\frac{\beta+\alpha-1}{2\beta}$. The necessary assumptions for this are that $\alpha + \beta > 1, 1 - \alpha > 0, \beta > 0$, or $\alpha < 0, 0 < \beta < 1$ (this uses the fact that V_R is a parabola in p). It also follows that the bribes are not given when everybody is rent-seeking, $V_R(1) = 0$. When everybody is producing, the net return from rent-seeking is positive: $V_R(0) = 1 - \alpha > 0$. These functional forms ensure the multiplicity of equilibria inside the domain.

This is, of course, an oversimplifying assumption. However, the model with V_R that depends on x undermines the multiplicity of equilibria (see Appendix A) and it is difficult to justify it empirically (see Section 3.2). In contradistinction, the chosen form of the bribing function has numerous advantages. It is simple but produces the necessary stylized facts.

It gives an intuitive form of the net return function (Laffer-type curve), incorporating both congestion and threshold effects. It also produces the result "small firms hide more" endogenously, without having to assume it (see Proposition 4). Moreover, this bribing function is robust to the parameter assumptions change, as long as it does not positively relate to x (see Section 4.4). In Section 5 an extension of the model is presented where the cost of bribing depends on x as, consequently, does $R(p)$.

3.3.3 Existence of Equilibria

As all agents are identical by assumption, the set of equilibria in this model is described by p^*, such that $V(p^*) \equiv V_R(p^*) - V_P(p^*) = 0$. It can be seen from Fig.1 that under certain conditions multiple equilibria in this model exist.

Proposition 5 *Three equilibria exist on the domain* $[0,1]$ *if the following conditions hold:*
(1) $V_R(0) < V_P(0)$, *i.e. if* $1-\alpha < \frac{1-t}{2c}$;
(2) $V_P(\frac{\beta+\alpha-1}{2\beta}) < V_R(\frac{\beta+\alpha-1}{2\beta}) = \max V_R = \frac{(\beta+\alpha-1)^2}{4\beta}$.
Two of these equilibria are stable ($p = 0$, *and* $p = p_2$) *and one is unstable* ($p = p_1$), *where* $0 < p1 < p2$.

In Fig. 1 an upward shift of the V_P curve corresponds to an increase in the net return from production. It is shown in Section 4 that the curve moves upwards after introduction of the shadow sector, or a reduction in k. As a result, the new "bad" equilibrium has a lower level of rent-seeking than the economy with a higher tax law enforcement. It is easy to see that a decrease in law enforcement has no effect on the rent-seeking-free ("good") equilibrium. This means that producers in the economy with high rent-seeking are actually better off if the tax law enforcement is reduced. It also means that an increase in law enforcement in such economies leads to even more corruption. In the lower graph, the function $V(p) = V_P - V_R$ is drawn and the equilibria of the dynamic model are shown.

3.3.4 No Shadow Case

Let us consider the case when there is no shadow sector. It is characterized by the following conditions: $k = \infty, t = 0$. The restriction on t is needed to recreate an Acemoglu-type model as a particular case inside the more general model under consideration: $V_P^*(t = 0, k = \infty) = \lim_{t\to 0} \lim_{k\to\infty} P(p, z, t, k, c) = \lim_{t\to 0}(1-t)\frac{(1-pz)^2}{2c} = \frac{(1-pz)^2}{2c}$. Leaving t unrestricted will not change the results that follow.

Proposition 6 *The roots of* $V(p)$ *are real and belong to* $p \in [0,1]$ *if the following conditions hold:* $\frac{2(1-z)(\beta+1-\alpha)}{(1+\alpha)+2\beta+(\beta-\alpha)^2} < c < \frac{1}{2(1-\alpha)}$.

If the three equilibria exist in the no-shadow case, then in the presence of the shadow sector either three or only one equilibrium (at $p = 0$) exist. In the latter case, an introduction of the shadow sector eliminates the high-corruption equilibrium, and the economy converges to the "good" equilibrium with no corruption.

4 Policy Implications

4.1 Impact of the shadow sector

Let us compare the economy which has no possibility of shadow production to the economy where such a possibility exists. Doing a comparative statics exercise, one can trace the directions in which the equilibria move, once the possibility to hide part of the output is introduced to the purely production/rent-seeking economy.[4] The possibility of hiding part of the output has a strictly positive impact on the net return from production in the economy with high rent-seeking, and no impact in the economy with no rent-seeking. The more corrupt the economy is, the bigger an effect the shadow sector has on the aggregated net return from production.

Proposition 7 *The introduction of a possibility to "go shadow" shifts the stable high rent-seeking equilibrium ($p = p_2$) to the left. This means that the shadow economy reduces the equilibrium number of rent-seekers in the highly corrupt economies. It also extends the attractor set of the no-rent-seeking equilibrium.*

Boeri and Garibaldi (2000) point out that it is always possible to increase enforcement so as to prevent the emergence of the shadow sector; however, it is not always desirable. The propositions above give a new explanation of why it might be undesirable: the complete extermination of the shadow economy through increased law enforcement rises corruption level even more and harms all production in a rent-seeking society (more on this view in Ades and Tella, 1999, and Garoupa, 1997).

4.2 Law enforcement and output

Let us now look at how law enforcement (like investment in tax police) influences the equilibrium outcomes. It is shown on page 7 that law enforcement has a positive impact on investments, $x(p)$, for each given p. Indeed, an increase in law enforcement, k, makes the graph of $x(p)$ flatter ($x'(p) < 0$), and investment x larger for each given p. However, as will be shown below, an increase in law enforcement might have a negative impact on the equilibrium investment, by increasing an equilibrium number of rent-seekers in the economy. Even more interestingly, it is demonstrated that the impact of an increase in law enforcement on investment is *negative* in highly-corrupt economies, while it is positive in low-corrupt ones.

Proposition 8 *An increase in law enforcement shifts the high rent-seeking equilibrium to the right. It also increases the attractor set of the 'bad' equilibrium at the expense of the 'good' one. For the highly corrupt equilibrated economies, this implies that the more the law is enforced the more corruption thrives.*

[4] To evaluate the impact of the shadow sector, it is not necessary to assume that $t = 0$, as the net return from production in the no-shadow case is:

$$\widetilde{V}_P^* = \lim_{k \to \infty} V_P = \frac{(1-t)(1-pz)^2}{2c} \equiv V_P^*(k \to \infty).$$

Proposition 9 *The increase in law enforcement has a positive impact on investment, in the economy with no or little corruption, while it has a detrimental impact on the investment (and therefore output) in the highly corrupt economy. This is an equilibrium process.*

This implies that equilibrium investment increases (or does not change) if the law is *more enforced* in those economies with little or no corruption, while it *decreases* in those economies with high corruption. Note that in our model the output is a linear function of investment, and therefore results for output would mirror the results for investment.

4.3 Law enforcement and shadow economy size

The next proposition shows that, as expected, an increase in law enforcement reduces the share of the shadow economy in the economies with low corruption. Interestingly, the shadow economy grows if the law is more enforced in the highly corrupt society. Therefore, if an economy is highly corrupt, the policies of direct deterrence of the shadow sector would have an opposite effect.

Proposition 10 *The increase in law enforcement reduces the shadow sector in the economy with no or little corruption, while it has an opposite impact in the highly corrupt economy. This is an equilibrium process.*

Thus, in the highly corrupt economies, policies of direct deterrence of the shadow sector have an impact that is opposite to the desired one. This result is closely related to the literature on state capture.[5] In the light of this literature, the evident concentration of the transition governments on fighting the shadow economy, and their apparent disdain for corruption, looks almost like a deliberate policy driven by state capture. It might also be considered as a sign of the power war between the Mafia and the corrupt state, where tax law enforcement is used as an instrument.

4.4 Robustness of Results

Bribing technology was introduced at the beginning of the paper, and certain assumptions were taken about the size of the bribe requested by a corrupt agent. For obvious reasons, the bribery process from the point of view of the bribe-taker has never been studied. Because of this, there is no straightforward empirical evidence to indicate which assumption about the bribing function is correct or more realistic. It might seem crucial for the model considered here that the amount of bribe requested increases with the number of rent-seekers in the economy. However, this is not so. Almost any form of this function (including a constant, and a decreasing function, like in Acemoglu, 1995) would give the same results, as they do not depend on the derivatives of $R(p)$.[6]

[5] "In a decade of transition, fear of a leviathan state is giving way to increased focus on oligarchs who "capture the state." In the capture economy, the policy and legal environment is shaped to the captor firm's huge advantage, at the expense of the rest of the enterprise sector." (cited from Hellman et. al, 2000)

[6] Actually, only the first derivative of V_P matters, while $R(p)$, if linear in p, influences only the second derivative. An intuitive proof for this is in Fig.1. Imagine V_R being any decreasing function: a straight line or the Acemoglu's type, and move the V_P upwards. The directions where the equilibria move do not change. However, one would have to adapt the model, in order to avoid the deficiency described in Apendix A.

Moreover, if more complicated assumptions of the bribe function were made (dependence on the output of the firm, for example, as some empirical evidence suggests), it would not change the results, provided that the matching function is correctly chosen. The dependence of a bribe on the output of a firm would move the equilibria "in the right direction", and would not alter the spirit of the results of the model under consideration.

5 Costly Bribing

Let us consider briefly in this section an extension of the model above, where the government introduces an anti-corruption campaign, so that bribing becomes costly. Let us also assume that the cost of bribing is borne by the bribe-taker, independently of whether he can meet a producer. This is in line with the evidence. For example, Varese (2001) mentions an identification cost that the bribe-taker has to bear, which is that he has to signal to potential bribe-givers that he would accept the bribe, if offered. This story only makes sense if the bribe-taker can be punished, which is assumed. The eradication of corruption is assumed to be a public good, and to be paid from the taxes collected by the government:

$$C_b(p) = l(1-t)(1-pz)\gamma^*(p)x^*(p), 0 < l \leq 1.$$

Then, the net return from rent-seeking becomes:

$$V_R = (1-p)(1-\alpha+\beta p) - l(1-t)(1-pz)(\frac{1}{c} + \frac{kpz(1-t)}{c\Theta})(1 + \frac{pzc(1-t)}{\Theta + kpz(1-t)}),$$

where $\Theta = kt + ct - k$, and the value of l determines the strength of anti-corruption measures. The rest of the model remains as before.

Proposition 11 *(1) Anti-corruption measures have a strictly positive effect on the highly-corrupt economies in equilibrium.*
(2) For the economy with no corruption, anti-corruption measures have positive effect on the equilibrium $V(p)$.
(3) For the corrupt economy away from equilibrium, with $p = 1$, these measures have a positive (short-run) effect only when accompanied by tax law enforcement that fulfils the condition: $k > \frac{tc(1-z)+cz}{(1-t)(1-z)}$.

When the direct deterrence measures actually increase both shadow economy and corruption, the anti-corruption campaign reduces the equilibrium corruption level, increases the (comparative) net return from production and reduces the shadow economy share in the output all together.

6 Conclusion

We have considered a model of an economy with producers and rent-seekers, where a possibility to "go shadow" exists for producers. It was found that introducing a shadow sector into

the pure production/predation model might eliminate the "bad" equilibrium and make an economy converge to the equilibrium with low level of rent-seeking under any initial condition on p.

In line with empirical evidence, everything else being equal, small firms hide a larger part of their output.

It turns out that the policies to reduce the size of the shadow economy through better enforcement, which work in economies with low corruption, have an opposite effect in the economies locked-in to the rent-seeking equilibrium. In particular, an increase in enforcement decreases production (both total and the official share), and increases corruption level. The introduction of a possibility to "go shadow", in the economy where it was perfectly enforced before, does not have an impact on the incorrupt economy, while such a possibility is beneficial for producers the higher is corruption. These results are robust to the choice of bribing technology.

In the economy with an anti-corruption campaign, an increase in government spending on it has a strictly positive impact on comparative net return from production. An increase in the anti-corruption effort reduces the equilibrium corruption level in the rent-seeking societies. Thus, in such economies policies must be aimed at reducing corruption, and not at deterring the shadow activities directly.

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7 Appendix A. Allocation of Talent Revisited

In this appendix the Acemoglu (1995) model is briefly considered, with the forms of functions used in his example (which was to show a possible existence of a real economy with multiple equilibria). The reason for doing so is that while the rent-seeker in his model is similar to the one considered above, he obtains a result which I could not replicate. I did obtain the multiple equilibria case, however one of the equilibria would always lie outside the domain (i.e., when the number of rent-seekers is higher than the total number of agents). This outer equilibrium would also be on the increasing part of the net return functions, which does not make much economic sense.

In that case, the economy would always have one equilibrium outside the domain, and therefore the economy would seem to have a 'forced equilibrium' in the point where everyone seeks rents ($p = 1$). However, if everybody is seeking rents, the net return from rent-seeking V_R cannot be positive, because when nothing is produced nothing can be extorted by the rent-seekers. Neither it can be larger than the net return from production. Therefore, there cannot be any 'forced equilibrium' at $p = 1$, and the economy would have either one or none equilibrium.

The proof of this statement follows.

An agents makes the irreversible decision of whether to become a producer or a rent-seeker on the basis of net returns from these activities:

$$V_P = (1 - pz)(\alpha + x) - c(x), \tag{12}$$

$$V_R = (1 - p)R(p) = (1 - p)j(\alpha + x). \tag{13}$$

In (12), V_P is the net return from production, where p is the number of rent-seekers in the economy, $z(\equiv 1 - q$ in the original paper) is the share or return that is taken as a bribe,

$(\alpha+x)$ is a return from production, and $c(x) = d+\frac{cx^2}{2}$ is the cost function ($d \equiv c_0$ and $c \equiv c_1$ in the original paper).

In (13), V_R is the net return from rent-seeking, where $(1-p)$ is the number of producers, and $j(\equiv \gamma$ in the original paper) is a share of output that the rent-seeker gets ($j \leq z$).

Then, after the producer maximizes his net return by choosing the amount of investment x, the equilibrium condition becomes:

$$V(p) = V_P - V_R = \frac{(1-pz)^2}{2c} + (1-pz)\alpha - d - (1-p)j(\alpha + \frac{1-pz}{c}), \tag{14}$$

where $\frac{1-pz}{c} = \arg\max_x(V_P)$.

The equation

$$V(p) = 0 \tag{15}$$

gives us the set of equilibria:

$$p_{1,2}(z,c,\alpha,d,j) = \frac{1}{z\,(z-2j)}(\alpha cz + z(1-j) - j(\alpha c + j) \pm$$

$$\pm \left(j^2(1+\alpha c)^2 - 2jz\alpha c(j+\alpha c) - 2j^2 z(1-z) + \alpha^2 c^2 z^2 + 2zdc(z-2j)\right)^{\frac{1}{2}}), \tag{16}$$

Proposition 12 *The problem (15) always has one root that is larger than* 1*, and therefore has either one or no roots on the domain interval* $p \in [0,1]$.

Proof. From (16) it follows that $p_1(0,c,\alpha,d,j) = -\frac{\alpha c+(1-j)+(j^2+\alpha^2c^2+2dc)^{\frac{1}{2}}}{2j} < 0$, $\lim_{z\to\infty} p_1(z,c,\alpha,d,j) = \lim_{z\to\infty} p_2(z,c,\alpha,d,j) = 0$, and that $\lim_{z\to 1} p_1(z,c,\alpha,d,j) = 1 + \frac{\alpha c(1-j)+(\alpha^2c^2(1-j)^2+2dc(1-2j))^{\frac{1}{2}}}{1-2j} > 1$ (if $1-2j>0$, and <1 otherwise), $\lim_{z\to 1} p_2(z,c,\alpha,d,j) = 1 + \frac{\alpha c(1-j)-(\alpha^2c^2(1-j)^2+2dc(1-2j))^{\frac{1}{2}}}{1-2j} < 1$ (if $1-2j>0$, and >1 otherwise).

Therefore, if the root is a function of z, $p_1(z)$, it turns out that there is always a root that is larger than one. If $1-2j>0$, it is p_1, and if $1-2j<0$, it is p_2.[7] ■

It is documented in various empirical studies of corruption that a firm in CIS countries, where corruption is the highest in the world, pays an amount of bribes that does not exceed 8.1% of its revenues (See Hellman and Schankerman, (2002), and also Pradhan et al. (2000), Kaufmann et al. (1999)). Consequently, z cannot be much higher than 0.08. It must be said that theoretically the size of z might be not important; however, when one starts introducing the shadow economy into the picture, it becomes such, and therefore the issue of multiple equilibria existence becomes crucial. Moreover, the next proposition shows that in the case explicitly considered by Acemoglu (1995), i.e. when $z=1$, the multiple equilibria case here never occurs, whatever the value of j.

[7] Due to the argument below, z cannot be much bigger than 0.08 in the real economies, and from the consideration that j must be smaller than or equal to z (for the balance to hold), the real economy assumption would be $1-2j>0$. Thus, it is always p_1 which is larger than one.

Proposition 13 *In the model with $z = 1$ (equivalent to Acemoglu's model with $q = 0$), the multiple equilibria case never occurs on the domain $p \in [0, 1]$.*

Proof. To the problem $V(p, z = 1, c, \alpha, d, j) = 0$, the solution is

$$p_{1,2} = 1 + \frac{1}{(1-2j)} \left(\alpha c(1-j) \mp (\alpha^2 c^2 j^2 + (1-2j)(\alpha^2 c^2 + 2cd))^{\frac{1}{2}} \right).$$

Where $p_2 > 1$ holds if $1 - 2j > 0$, i.e. if $j < 1/2$. Therefore, it would be required that $j > 1/2$ for this root to be smaller than one, i.e. $1 - 2j < 0$. For this to hold:

$$\frac{1}{(1-2j)} \left(\alpha c(1-j) - (\alpha^2 c^2 j^2 + (1-2j)(\alpha^2 c^2 + 2cd))^{\frac{1}{2}} \right) < 0.$$

In other words, it is necessary that $\alpha c(1-j) - (\alpha^2 c^2 j^2 + (1-2j)(\alpha^2 c^2 + 2cd))^{\frac{1}{2}} < 0$ as well, or that $\alpha c(1-j) < (\alpha^2 c^2 j^2 + (1-2j)(\alpha^2 c^2 + 2cd))^{\frac{1}{2}}$ and $\alpha^2 c^2 j^2 \geq (1-2j)(\alpha^2 c^2 + 2cd)$, which always holds because $1 - 2j < 0$ was assumed before and l.h.s. is always greater than zero.

Then,

$\alpha^2 c^2 - 2j\alpha^2 c^2 + j^2 \alpha^2 c^2 < \alpha^2 c^2 j^2 + (1-2j)(\alpha^2 c^2 + 2cd)$,

$\alpha^2 c^2 (1-2j) < (1-2j)(\alpha^2 c^2 + 2cd)$,

$\alpha^2 c^2 > \alpha^2 c^2 + 2cd$, and

$0 > 2cd$, which never holds, as c and d are always positive. If either of them is zero, then $p_1 = p_2 = 1$. We have come to a contradiction. Thus, in this model both roots simultaneously cannot be smaller than one, and therefore there is always only one equilibrium defined as a solution to $V(p) = 0$. ■

These propositions show that the model with the bribe proportional to the return from production does not have a multiple equilibria case that would be possible to modify for a three-sector model. It is related to the form of the matching function and the net return functions, which result in the net returns being 'parallel' to each other. Therefore, I return in this paper to the more general version of the Acemoglu (1995) model, with the net return from rent-seeking being equal to $V_R = (1-p)R(p)$, where $R(p)$ is the bribe which depends on p, and not on the return of the bribe-payer. There is also anecdotal evidence from the corrupt countries that the bribe size does not change depending on the return of the payer, but the amount paid differs as a percentage of the return (therefore, becoming too big for the small firms).

8 Appendix B. Proofs of Propositions

Proof. (**Proposition 3**)
After substituting for γ and x, in (7), it is obtained: $V_P(p, z, t, k, c) = \frac{1-t}{2} \left(\frac{(1-pz)^2}{c} - \frac{(pz)^2}{tc-k+tk} \right) > 0$ always, as it is assumed that $\Theta = tc - k + tk < 0$. ■

Proof. (**Proposition 4**)
It follows from the above that $\gamma^*(x) = -\frac{pz(1-t)+t}{x(k-tc)} + \frac{k}{k-tc}$, and therefore γ is positively correlated with x, i.e. large firms have larger share of output produced officially. ■

Proof. (**Proposition 5**)
See Fig.1. It is easy to see that : (1) $V_P(0) = \frac{1-t}{2c}$ and $V_R(0) = 1-\alpha$; (2) $V_P(1) > 0$, $V_R(1) = 0$. Then, for the curves V_P and V_R to have two intersections on the domain, it is needed that $\max V_P \geq V_R(\arg\max(V_P))$, as $\arg\max(V_R) = \frac{1}{2}(1 - \frac{1-\alpha}{\beta})$ and $\max V_R = \frac{(\beta+\alpha-1)^2}{4\beta}$.

If these hold, then in the dynamic version of the model p would decrease if $\{(p < p_1) \cup (p > p_2)\}$ because then $V_P > V_R$ and it is more beneficial to become a producer than a rent-seeker. The number of rent-seekers, p, would increase if $p_1 < p < p_2$, as then $V_R > V_P$ and it is more beneficial to become a rent-seeker than a producer. Therefore, $\{p = 0, p = p_2\}$ are the stable equilibria, and $\{p = p_1\}$ is an unstable one. ■

Proof. (**Proposition 6**)
$1 - \alpha - \beta < 0$ and $1 - \alpha > 0$ hold by construction.
$\widetilde{V}_P = \lim_{k\to\infty}\lim_{t=0}(V_P - V_R) = \frac{1}{2}\frac{p^2z^2-2pz+2c\beta p^2-2cp\alpha-2c+1+2cp+2c\alpha-2c\beta p}{c} = 0$ in the equilibria.
The roots are real if the discriminant $c^2 + 2c^2\beta - 2zc - 2c^2\alpha + c^2\beta^2 + 2c\beta z - 2c^2\beta\alpha + 2zc\alpha + c^2\alpha^2 - 2c\beta + 2z^2c - 2z^2c\alpha > 0$ or $c > \frac{2(1-z)(\beta+1-\alpha)}{(1+\alpha)+2\beta+(\beta-\alpha)^2}$.
And for them to lie in the domain interval $[0, 1]$, they must both be less than one and larger than zero. For the roots to be positive, their sum and their product must be positive:
$p_1 + p_2 = -\frac{2c-2c\beta-2z-2c\alpha}{2c\beta+z^2} > 0$, holds if $1 - \alpha - \beta < 0$;
$p_1p_2 = \frac{-2c+1+2c\alpha}{2c\beta+z^2} > 0$, holds if $2c < \frac{1}{1-\alpha}$.

In general, the less strict inequality would have to hold for the roots being less than one. However, the following happens to be true under assumptions of the model:
$p_1 + p_2 < 1$. I.e. $-\frac{2c-2c\beta-2z-2c\alpha}{2c\beta+z^2} < 1 \Leftrightarrow 1 - \alpha > \frac{-z(1-z)}{2c}$ always holds, if $1 - \alpha > 0$, which holds by construction of V_R. ■

Proof. (**Proposition 7**)
Let $\Delta V = V_P^* - \widetilde{V}_P^*$, then the shadow sector would have a positive impact on the net return from production, if $\Delta V > 0$. If $k \to \infty$, then from (9) it follows that $\gamma^*(p) = 1$, i.e. all production would be declared. Then, $\Delta V = \frac{1-t}{2}\left(\frac{(1-pz)^2}{c} - \frac{(pz)^2}{tc-k+tk}\right) - \frac{1}{2}(1-t)\frac{(1-pz)^2}{c} = \frac{1-t}{2}\frac{(pz)^2}{-\Theta} > 0$ always, if $\Theta = tc - k + tk < 0$, as has been assumed throughout. In particular, $\Delta V(p = 0) = 0$, and $\Delta V(p = 1) = \frac{1}{2}(-1 + t)\frac{z^2}{tc-k+tk} > 0$.

Therefore, due to the continuity and monotonicity of V_P, the net return curve for production shifts upwards for all $p > 0$, and the shift would be larger for larger p. However, it must be mentioned that the assumption that $\Theta < 0$ implies the lower limit of law enforcement, under which this proposition holds: $k > c\frac{t}{1-t}$. This means that tax law enforcement must reach a certain minimal level for the result to hold. ■

Proof. (**Proposition 8**)
The function $V(p, z, t, k, c, \alpha, \beta) = \frac{(1-t)}{2}(\frac{(1-pz)^2}{c} - \frac{p^2z^2}{tk+tc-k}) - (1-p)(1-\alpha+\beta p)$ determines equilibrium p in this economy: $V(p, z, t, k, c, \alpha, \beta) = 0$. Let us examine how a change in k influences p. By the implicit function theorem,

$$\frac{dp}{dk} = -\frac{\frac{dV}{dk}}{\frac{dV}{dp}},$$

also $\frac{dV}{dk} = -\frac{(1-t)^2 z^2 p^2}{2(tc-k+kt)^2} < 0$, and therefore $sign(\frac{dp}{dk}) = sign(\frac{dV}{dp})$.

We already know that function $V(p)$ has two roots on $p \in [0,1]$, and therefore $\frac{dV}{dp} < 0$, if $p < \widehat{p}$, and $\frac{dV}{dp} > 0$, if $p > \widehat{p} : p_1 < \widehat{p} < p_2$, where p_1 and p_2 are the roots of $V(p) = 0$, i.e. the equilibria. Therefore, $\frac{dp}{dk} < 0$ for the low-corrupt economies, and $\frac{dp}{dk} > 0$ for the highly-corrupt ones. It is also obvious from the above, that increase in law enforcement, k, moves the p_2 equilibrium to the right (increasing the attractor set of the 'bad' equilibrium). ■

Proof. (**Proposition 9**)
Let us consider a formula of optimal investment (8). In the equilibrium, $p = p(k)$, and therefore $\frac{dx(p(k))}{dk} = \frac{dx}{dp}\frac{dp}{dk}$, where $\frac{dx}{dp} < 0$, is known, see pg.6. Therefore, the *equilibrium* investment, unlike the off-equilibrium one, moves in the opposite direction from equilibrium corruption, when law enforcement is increased. ■

Proof. (**Proposition 10**)
The proof is analogous to the previous proposition. Let us consider a formula of optimal investment (8). In the equilibrium, $p = p(k)$, and therefore $\frac{d\gamma(p(k))}{dk} = \frac{d\gamma}{dp}\frac{dp}{dk}$, where $\frac{d\gamma}{dp} < 0$ (see pg. 7). Therefore, $sign(\frac{d\gamma(p(k))}{dk}) = -sign(\frac{dp(k)}{dk}) = \left\{ \begin{array}{c} > 0, \text{ for low corrupt economies} \\ < 0, \text{ for highly corrupt economies} \end{array} \right\}$, by Proposition 8. Therefore, the *equilibrium* share of the official sector in the output in the highly corrupt economies decreases when tax law enforcement is increased. ■

Proof. (**Proposition 11**)
In the equilibrium with no corruption:
$\frac{dV(p=0)}{dl} = \frac{1-t}{c} > 0$.
In the equilibrium with high corruption,
$\frac{dV(p,z,t,k,c,\alpha,\beta,l)}{dl} = (1-t)(1-pz)\frac{-\Theta+pz(\Theta-c)}{-c\Theta} > 0$ if $-\Theta + pz(\Theta - c) > 0$.
It is required that $pz > \frac{\Theta}{\Theta-c}$, which always holds because r.h.s. is always less than zero, and p and z are positive. It is also known that
$\frac{dp}{dl} = -\frac{\frac{dV}{dl}}{\frac{dV}{dp}} < 0$ for $p = p_2$, as the $V(p)$ is increasing in that part of the domain interval.
Away from equilibrium, when also $p = 1$:
$\frac{dV(p=1)}{dl} = (1-t)(1-z)\frac{\Theta(1-z)+cz}{c\Theta} > 0$ if $\theta(1-z) + cz < 0$ (because $\Theta < 0$).
$(tc - k + tk)(1-z) + cz = -(1-t)(1-z)k + tc(1-z) + cz < 0$,
$k > \frac{tc(1-z)+cz}{(1-t)(1-z)}$. ■

9 Appendix C. Notation

V_O - net return from official production,

V_S - net return from shadow production,

V_R - net return from rent-seeking,

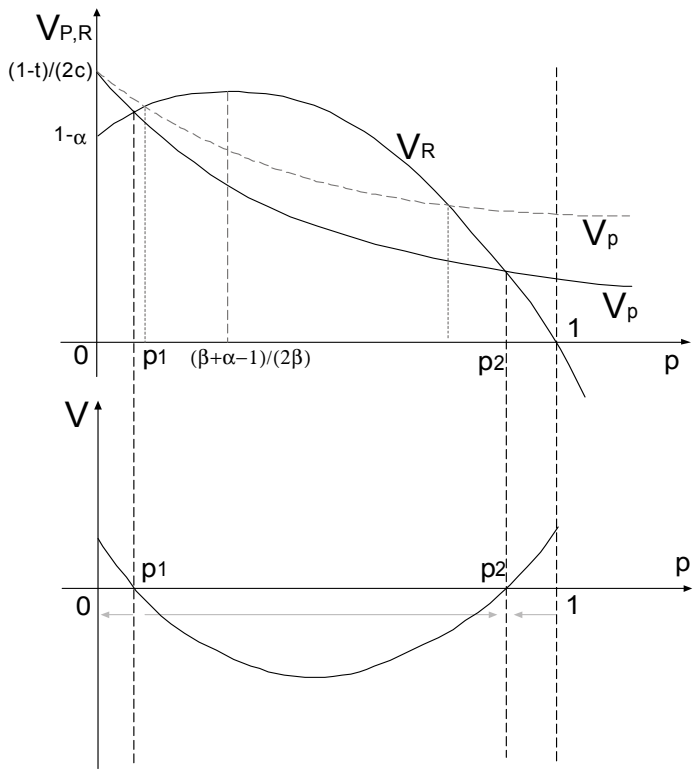

Figure 1: Equilibria of the model

V_P - net return from all production (sum of shadow and official),

$\widetilde{V}_P$ - net return from all production, when there is no possibility to "go shadow",

t - tax rate,

γ - share of the declared output in the whole production,

x - investment in a linear production function,

c - coefficient of the cost function for production,

$c(x)$ - technical cost of any production with investment x, both for shadow and declared activity,

k - ability of the state to catch tax evaders, such that $\widetilde{c}(x)$ - corresponding additional cost of going shadow is linear in k, with a positive coefficient;

p - number of rent-seekers, and $1-p$, number of producers,

$z = 1 - q$ - part of the return that a firm pays as a bribe,

$R(p)$ - bribe required by an official.

Throughout the text, a simplifying parameter is introduced:

$$\Theta \equiv tk + tc - k,$$

which stands for the relative value of the tax law enforcement and production cost of a firm.