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Measuring Proportionality. A Systematic Approach
to World Electoral Systems in Parliamentary
Elections between 1946-2000

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Abstract

This paper proposes a new way to measure proportionality using aggregated threshold functions. Electoral systems can be summarized by a single value that shows the necessary share of the total vote to win half of the seats in parliament. This value can, then, be compared with the point of perfect proportionality. I calculate aggregate threshold values for 142 different electoral systems that were used in 525 democratic elections between 1946 and 2000. These results are also contrasted with the most commonly used indices of proportionality and turn out to be both substantively and empirically richer. Aggregated threshold functions provide both students and reformers of electoral systems with a measure based purely on institutional variables that offers an exhaustive summary of the functioning of many electoral systems.

Keywords: Aggregated threshold functions; perfect proportionality; number of seat-winning parties; comparative electoral systems

Measuring proportionality. A systematic approach to world electoral systems in parliamentary elections between 1946-2000.*

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Introduction

While the electoral system is a recurrent topic in the literature, no consensus exists on how to characterize it (G.Bingham Powell 2000; Milesi-Ferreti, Perotti, and Rostagno 2002; Persson and Tabellini 2005). In some cases, such as the literature on ethnic conflict, a relevant independent variable like the electoral system is summarized by using categorical variables (Cohen 1997; Reynal-Querol 2002; Alonso and Ruiz-Rufino 2007). Here, a distinctive feature of an electoral system, like for example the electoral formula, is the exclusive criterion used to group electoral institutions around different

*I would like to thank Adam Przeworski, Peter Mair, Mark Franklin, Umut Aydin and Jan-Hinrik Meyer-Sahling for their rich and helpful comments. I am also grateful to Rein Taagepera for his clarifications and encouragement. The responsibility of the content of this article is, however, exclusively mine.

categories of a variable. Other sets of studies that use electoral systems not only as independent variables (Cox 1997) but also as a dependent variable (Boix 1999; Blais, Dobrynska, and Agnieszka 2005) use empirical measures that produce continuous values. Here, the discussion is centered around the disproportionality index (DI) (Loosemore and Hanby 1971) and the effective threshold (Taagepera and Shugart 1989; Lijphart 1994), which possibly stand as the dominant empirical measures used in the literature (Katz 1997).

None of these approaches is completely convincing. Categorical variables are not capable of producing information about the overall functioning of electoral systems classified under the same category. If, for example, one takes the category of proportional representation (P.R.) electoral systems, it is difficult to say whether the number of districts, or assembly size, has any effect on the mechanical performance of electoral institutions. If, on the other hand, we focus on the dominant empirical measures, we may face some other problems. The effective threshold can be justified by the goodness of fit with the values empirically observed, but the method used to calculate it does not respond to a clear-cut logic; effective thresholds are also limited to district level and only refer to the electoral cost of one seat. The different indexes of disproportionality (Rae 1967; Gallagher 1991; Monroe 1994; Lijphart 1994) likewise suffer from shortcomings such as their dependency on election results; or the impossibility of measuring the mechanical effect of any electoral system using these indexes unless we have access to both the distribution of the vote among all competing parties and the corresponding seat allocation.

Aggregated threshold functions can overcome these shortcomings. These are a set of functions that show the necessary and sufficient share of votes

nationwide to win any number of seats given the institutional components of an electoral system. These functions are universal and general. That is, they can be applied to most electoral institutional designs and they can summarize in a single value the mechanical behaviour of an electoral system in the whole territory and not in a single district. They are also independent since they rely exclusively on institutional variables. Aggregated threshold functions allow us to anticipate some of the political consequences generated by electoral institutions, such as the permeability of the system to the entrance of small parties or the possibility of forming single-party cabinets. This capacity makes aggregated threshold values interesting to both students of electoral systems and electoral reformers. The former will find a straightforward measure to characterize any electoral system; the latter may find in these functions a tool for forecasting the mechanical performance they can expect from a future electoral system. Thus defined, aggregated threshold functions offer values that summarize the process of converting votes into seats in each electoral system. What they do not do is to anticipate the performance of any political party. This performance, i.e. the share of votes and the corresponding seats, will be determined by the distribution of votes that is obtained after the occurrence of each election. However, as I will show later, the trends anticipated by aggregated threshold functions match the actual electoral results that can be observed.

If in a previous article the emphasis was on defining, in a rather formal way, the aggregated threshold functions, as well as on testing their capacity of prediction (author 2007); the emphasis here, nonetheless, is placed on how these functions can serve to anticipate the functioning of most electoral

systems, and on using that information to generate a way of characterizing them. To do this, the purpose of this article is threefold. First, I explain how values from the aggregated threshold functions are obtained as well as their substantive meaning. Second, I offer the values obtained by applying aggregated threshold functions to 142 *winner-takes-all* and P.R. electoral systems used in 525 parliamentary democratic elections that were held in the world between 1946-2000. Aggregated threshold functions allow electoral systems to be characterized around one particular interesting value: the minimum share of the total vote necessary to win a majority of seats in parliament. This value will prove helpful in extracting important conclusions about the mechanical functioning of electoral systems¹. Third, I contrast aggregated threshold functions with other empirical measures such as various disproportionality indexes and the effective threshold. In this section, I show how these functions overcome some of the shortcomings linked to these existing measures and how a better understanding of the functioning of electoral institutions can be extracted if aggregated threshold functions are used to characterize electoral systems. I also show how the predicted values anticipate the major trends observed in the process of transforming votes into seats.

¹A note of caution is necessary here. As pointed out, the main purpose of this article is to measure in a single value the mechanical behaviour of any electoral system. That means that other important aspects of electoral systems like the psychological effects that they may produce in the voters are not taken into consideration (Duverger 1954). The inclusion of such effects is beyond the scope of this research. The main goal of this article is to introduce a consistent method for measuring the functioning of the different institutional variables that form an electoral system without taking into consideration any rational calculation made by the voters.

Aggregated threshold functions

Aggregated threshold functions (ATFs) calculate the range of the total vote required to win a given number of seats in any electoral system. This approach is not new in the literature. Lijphart and Gibberd (1977) calculated similar functions for electoral systems applying the most frequently used electoral formulas and Penades (2000) refined the functions so that they applied to any electoral system. These earlier approaches calculated the necessary and sufficient shares of the vote at district level to win any number of seats.

To make aggregated threshold functions understandable, a concept of electoral system is required. I define a complete electoral system as an institution consisting of the following elements: an electoral formula, the number of districts in which the country is divided, a vector that contains all district magnitudes and the number of seats in the legislative assembly. Since political parties compete under this institutional framework to win as many seats in parliament as possible, aggregated threshold functions also incorporate the number of competing parties in all districts. Once all these elements have been introduced, a general definition of aggregated threshold functions can be made. They are a set of functions that calculate the necessary and sufficient share of the total vote to win a given number of seats that are particularly distributed among all district magnitudes given any electoral formula, any number of districts, any legislative assembly size and, finally, given any number of competing parties ².

Aggregated threshold functions have two key defining features that make

²Aggregated threshold functions are mathematically defined and optimized in (author 2005)

them attractive in comparison with both Lijphart and Gibberd’s and Penadés’s approaches. First, they are universal, meaning that one can apply these functions to any conceivable electoral system regardless of the electoral formula or other institutional components used in the system. Second, aggregated threshold functions are general. Generality refers to the capacity to summarize the mechanical functioning of any electoral system taking into account all districts, and not just one, into which a country is divided. This property allow us, therefore, to observe the global mechanical functioning of particular electoral institutional designs³.

There is, however, a third interesting property of ATFs: independence of electoral results. Independence means that one can estimate the mechanical effects of an electoral system without relying on the electoral results it generates. Independence is achieved by estimating the number of competing parties that could win seats in the legislature, as suggested by Taagepera (2007). Given an assembly size, M , and a district magnitude, M_d , Taagepera estimates that the number of parties that could win seats can be obtained using the following formula⁴

$$P = (M * M_d)^{1/4} \tag{1}$$

Since generality is a distinctive feature of aggregated threshold functions, M_d

³This new property also allows a novel approach to the study of electoral systems, since the most influential existing studies in the field (Rokkan 1968; Rae, Hanby, and Loosemore 1971; Taagepera and Shugart 1989; Lijphart 1994; Katz 1997) are focused on district-level. There are, nonetheless, a few references in the literature that attempt to calculate thresholds nationwide (Taagepera 1998; Taagepera 2002). However, these studies suffer from some of the problems mentioned above: they only focus on a limited number of electoral formulas and the calculation of the functions is done using an empirical approach.

⁴For a full development of this predictor see Taagepera (2007:133-134)

above can be substituted by the average district magnitude⁵ in order to obtain the number of parties that could win seats in the assembly nationwide. After this substitution, the number of potentially seat-winning parties is⁶

$$P = \left(\frac{M^2}{D} \right)^{1/4} \quad (2)$$

These three properties will define the essence of aggregated threshold functions: the *ex-ante* calculation of the necessary and sufficient shares of the total vote required to win a given number of seats based on purely institutional variables⁷. By necessary share of the total vote, I refer to the proportion of votes counted nationwide that a party must win to obtain a certain number of seats. In other words, let S_T be a given number of seats somehow distributed among all districts in which the country is divided; similarly, let $V_{S_T}^{nec}$ be the total share of the vote necessary to win those S_T seats. If a party fails to win a total share of the vote greater or equal to $V_{S_T}^{nec}$, then,

⁵Average district magnitude is calculated as follows

$$\widehat{M} = \frac{M}{D}$$

where D refers to the number of districts in which the territory is divided. See Lijphart (1994)

⁶In a previous article (author 2007), I tested the capacity of prediction of aggregated threshold functions using the effective number of competing parties (ENP) (Laakso and Taagepera 1979) as a proxy for the number of competing parties with a real chance of winning a seat in the legislature. The proxy worked well empirically. However a legitimate and sound criticism of using this predictor could be raised because the ENP depends, by definition, on election results produced by the electoral system in question.

By using the new indicator suggested by Rein Taagepera, this criticism is no longer valid. I have retested the capacity of predictability of aggregated threshold functions using the same data and similar satisfactory results have been obtained. Therefore, as far as aggregated threshold functions are concerned, the new indicator calculated by Taagepera does seem to be a good predictor of the number of competing parties with a real chance of winning a seat in the legislature.

⁷Conceptually, the idea is the same as the substantive meanings of threshold of inclusion (Rokkan 1968) and threshold of exclusion (Rae, Hanby, and Loosemore 1971).

it cannot get S_T seats.

By sufficient share of the total vote, I mean the proportion of votes counted nationwide that suffices to win a determined number of seats. Following a similar notation, $V_{S_T}^{suf}$ defines the total share of the votes that suffices to win a given number of seats in the legislative assembly, so, if a party wins a share of the vote greater or equal to $V_{S_T}^{suf}$, then that party will get at least S_T seats.

Lastly, to use aggregated threshold functions as a tool to characterize electoral systems, a further conceptual refinement is required. It must be decided whether to opt for the aggregated threshold function of necessary votes or that of sufficient votes. The choice between these two functions is not just a question of taste. For any measure to be parsimonious, it must combine simplicity and explanatory power. So, the simpler and the greater the substantive meaning of any measure, the more parsimonious it will be. Following this reasoning, in the rest of the article I will only use the aggregated threshold functions of necessary votes for the following reasons. First, by using this function I am adopting an exclusive criterion: any party that obtains a share of the total vote below the necessary share of the total vote will have no chance of obtaining the total number of seats for which the function was applied. Second, the aggregated threshold function of necessary votes decreases the uncertainty about the total number of seats that can be won implicit in the function of sufficient votes. Whereas the aggregated threshold function of necessary votes sets up the limit that must be reached to win a concrete number of total seats, the function of sufficient votes leaves open the possibility of winning a higher number of them.

This necessary condition allows electoral systems to be characterized by two different values. First, electoral systems can be characterized by the minimum share of the total vote that a party would require to win just one seat. This value could be used as a measure to test the flexibility of any electoral system regarding the entrance of minor parties into the legislative assembly. If the minimum value required to win one seat in the parliament is too low then one could expect a much more fragmented parliament than in the case of a higher value. Studies on the emergence of extremist parties (Veugelers and Magnan 2005; Abedi 2002) or on the capacity of success of ethnic parties in heterogenous societies (Moser 1999) could benefit from this measure.

A second, and even more appealing, criterion for characterizing electoral systems is the minimum share of the total vote needed to win half of the seats in the legislative assembly. This value would allow the characterization of complete electoral systems by locating them on a continuum and comparing them with an ideal point. If we define this ideal point as perfect proportionality⁸, we can have a continuous value that measures the distance between the minimum share of the total vote required by a party to win a majority of seats in the legislative assembly and this ideal point. This approach will then help us understand, for example, the propensity of a complete electoral system to produce coalition governments in parliamentary democracies. The higher the minimum share of the total vote needed to win half of the seats

⁸An electoral system is said to be perfectly proportional if the share of votes obtained by any given party gives it the same share of the seats in the legislative assembly. Formally, for any political party p , $V_p = S_p$ where V_p stands for the share of the total vote won by party p , and S_p stands for the share of total seats in the parliament assigned to party p .

in the legislative assembly, the less likely it will be to find single-party ruling coalitions in parliaments (King, Alt, Burns, and Laver 1990; Lupia and Strom 1995). Another case in which this value would be helpful is the analysis of the role that electoral systems have played in transforming democracies (Birch 2003).

Finally a few words about how to interpret these results. Given the institutional variables that make up a complete electoral system and the likeliest number of seat-winning parties that such an electoral system may generate, aggregated threshold values determine the minimum share of the total vote required to win a given number of seats. In the case of one seat, aggregated threshold functions must be read as determining the minimum share of the total vote that any party must win in order to get a seat⁹. Likewise, for a party to win half of the seats in parliament it must obtain at least the share of the total vote calculated. Aggregated threshold values do not establish the share of the vote which will win a given number of seats but the necessary condition for winning them.

Data

In this study, aggregated threshold functions are applied to 142 different complete electoral systems that were used in 525 parliamentary elections oc-

⁹A note of caution is necessary here. Some complete electoral systems establish a minimum share of the total vote in order to win political representation. These legal thresholds (T_L) are largely intended to prevent the entry of minority parties into parliament. This variable is, obviously, of some importance although I do not deal with it in this paper. However, the way to proceed is the following: If $T_L > V_{S_T}^{nec}$, then T_L gives us the share of the votes that are required to win the given number of votes for the which aggregated threshold function is calculated ($V_{S_T}^{nec}$)

curing between 1946 and 2000 under either a *winner-takes-all* or a P.R. electoral system. The data has been collected from Golder (2005) and has been expanded by incorporating district data information in those cases where such information was available. Given that aggregated threshold functions do not rely on electoral results, some electoral systems cannot be characterized using aggregated threshold functions¹⁰. In any case, the data presented here cover about 61% of the elections taking place in the world between 1946 and 2000 and 85% of the elections held under the electoral systems under study.

Table 1 shows the total number of elections and electoral systems used in this article. From this table we can see that *winner-takes-all* electoral systems and proportional representation electoral systems were adopted for about the same number of elections. *Winner-takes-all* electoral systems were used for 264 elections whereas 261 elections occurred under proportional representation systems.

Table 1: Number of elections and electoral systems

Type of Electoral System	Elections	Electoral Systems
<i>winner-takes-all</i>	264	59
P.R.-Divisors	170	54
P.R.-Quota	91	29
Total	525	142

Finally, given that the unit of analysis is the complete electoral system,

¹⁰Electoral systems that use single transferable vote or alternative vote, for example, cannot be characterized here given the strong dependency on a previous distribution of votes that aggregated threshold functions do not consider. The electoral systems excluded are limited vote (LV), single non-transferable vote (SNTV), Borda, alternative vote (AV) and single transferable vote (STV)

I have also omitted mixed and multi-linked electoral systems. Given the extraordinary complexity of these electoral systems, they are not the subject of this article.

two electoral systems will be considered different if any of the following criteria is observed:

1. A change of electoral formula.
2. A change in the number of districts into which the country is divided.
3. A change in the number of seats in the legislative assembly. A change is considered to have taken place when there has been a change of 20% in the size of the legislature (Lijphart 1994).
4. An electoral system established after a period of dictatorship is also considered to be a new electoral system(Golder 2005).

The characterization of electoral systems

Winner-takes-all electoral systems

Aggregated threshold functions have been applied to complete electoral systems that use First-Past-The-Post (FPTP), Block Vote (BV), Party Block (PV) and the Two Round System (TRS). As mentioned above, these electoral systems allow the application of the functions regardless of the distribution of the vote. It should be noted however that in the case of the Two Round System (TRS), aggregated threshold functions are applied to the first round only¹¹.

¹¹This restriction needs some clarification. Run-off electoral systems like TRS may invite voters to behave strategically and the overall mechanics of these systems may not be completely captured by aggregated threshold functions. While accepting this remark, I should point out that aggregated threshold functions only accounts for the mechanical effects of electoral systems and not for the psychological effect that they may produce.

Table 2 summarizes the values obtained in these *winner-takes-all* electoral systems. The first column shows descriptive data when the functions are applied to just one seat; the second column shows the same information when aggregated threshold functions are applied to half of the seats in parliament¹².

Table 2: Summary values for Winner-takes-all electoral systems

	1 seat	Half seats in parliament
Mean (%)	0.94	16.5
Std. Dev. (%)	1.1	4.9
Minimum (%)	0.04	10
Maximum (%)	3.71	27.46

The values shown in table 2 illustrate well the majoritarian nature of these electoral systems. Consider first the values obtained when the number of seats equals one. The minimum value is produced by the electoral system used between 1958 and 1981 in the French legislative elections. Here, a Two Round System electoral formula was used to distribute an average of 470 seats between approximately 6 competing parties. The minimum threshold to win a seat under this institutional setting is about 0.04% of the total vote. The maximum value was obtained in the electoral system used in the 1984 legislative election in St. Kitts and Nevis, where 11 seats were chosen using FPTP and where 1.82 parties competed in the electoral contest¹³. Under

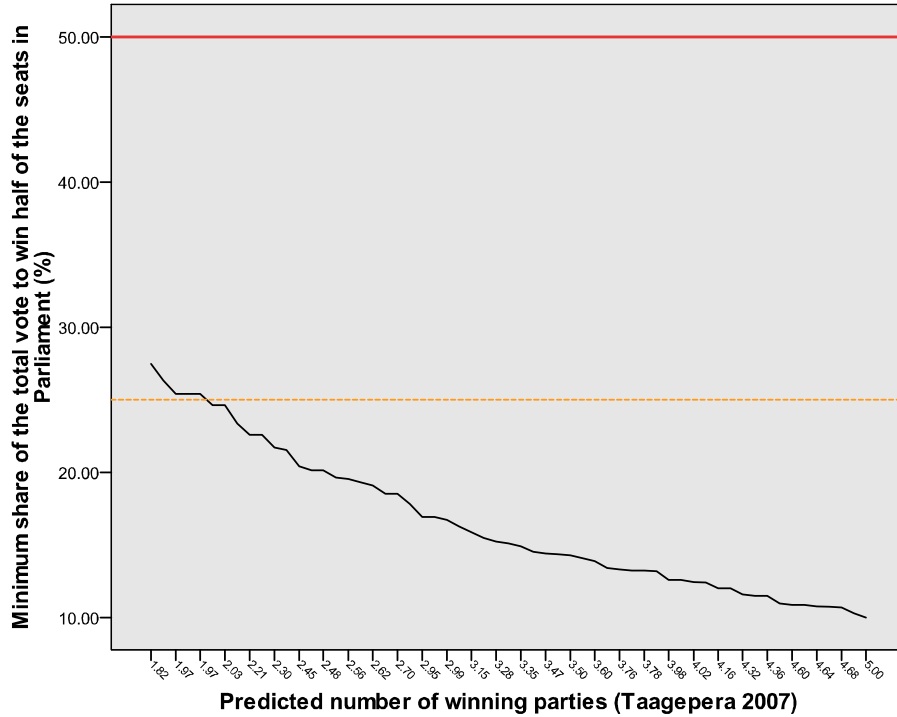
On this ground, I consider it defensible to characterize TRS by just using the first round since the second round would only apply in the case that no party obtained the majority of the votes. Aggregated threshold functions would still show, in any case, the minimum share of the total vote required to win the number of seats at stake.

¹²Appendix 1 shows the mathematical expression that has been used in this case.

¹³By definition, a *sine qua non* condition for a country to qualify as a democracy is that it holds competitive elections (Przeworski 1991). This means that at least 2 political parties must take part. Given that the number of parties is calculated here depending on the size of the assembly and average district magnitude, only those countries whose

this electoral system, the minimum share of the total vote needed to win a seat was about 3.71%.

Figure 1: Minimum share of the total vote needed to win half of the seats in parliament for winner-takes-all electoral systems



As Appendix 1 shows, when district magnitude equals 1, the minimum share necessary to win the majority of seats in the parliament depends exclusively on the seat-winning number of parties. For example, if the number of parties competing in a complete electoral system with a FPTP electoral formula equals 2, the share of votes below which a party cannot win half of the seats in parliament is 25%. This situation arises when such a number of assembly is bigger than 16 produce 2 parties with a real chance of winning seats. Figures below this may be interpreted not as meaning that the country is undemocratic, but that it produces a clear winner and a clear loser. Only 3 countries generated parties below 2: St. Kitts, St. Vincent and Grenade.

seats is won in half of the districts with the minimum number of votes, and no votes are won in the remaining districts¹⁴. This is, of course, an extreme case but it is useful because it gives us the value below which it is impossible to win half of the seats in parliament under any circumstances. The number of seat-winning parties is, then, inversely proportional to the minimum number of votes required to win the majority of the seats in the parliament. The idea is well illustrated by looking at the dotted line in figure 1.¹⁵

In terms of perfect proportionality, these values show exactly how disproportionate a *winner-takes-all* electoral system can be. If perfect proportionality is understood as implying that a party's share of the vote equals the share of seats it obtains, then the electoral systems analyzed here are far from being proportional. The thick horizontal line in figure 1 shows where the point of perfect proportionality lies. Each electoral system can, therefore, be observed in relation to this ideal point. As table 2 shows, on average the minimum value needed to win half of the seats in the legislative assembly is about 16.5% of the total vote share. That is an average distance of 33.5% away from the ideal point of perfect proportionality.

P.R. electoral systems

I turn now to analyzing aggregated threshold values for those electoral systems that use a proportional representation electoral formula. These elec-

¹⁴This finding confirms mathematical and empirically a remark made by J.S.Mill (1991)

¹⁵Obviously, the complete electoral system used in St.Kitts produced the highest value needed to win half of the seats since this electoral system produced the smallest number of competing parties in the sample. The complete electoral system that produced the smallest value for winning half of the seats in the parliament was that used in the United Kingdom from 1950 onwards. According to the predictor proposed by Taagepera, about 5 competing seat-winning parties were produced by that complete electoral system.

toral systems normally use multi-member districts, and depending on the method used to distribute seats one can distinguish between divisor-based and quota-based proportional representation electoral systems. Both divisor and quota-based electoral formulas are mostly used in party-list systems. In fact, 47 countries used this type of electoral formula in 261 general elections between 1946-2000

Briefly, quota-based electoral systems distribute seats using a predetermined quota calculated using the district magnitude. Parties win as many seats as full quotas obtained and extra seats depending on whether they also have the largest remainder¹⁶. A total of 29 quota-based electoral systems were used in 91 elections between 1946-2000.

On the other hand, in divisor-based electoral systems a divisor must be found to enable the calculation of the averages needed in order to allocate the seat in each district magnitude (Balinsky and Young 1982; Lijphart 1994). Divisor-based electoral formulas were used in 170 general elections with 54 different complete electoral systems.

Aggregated threshold functions are applied considering a particular distribution of seats among all districts¹⁷. That distribution of seats minimizes the value of the function and therefore offers the value we are interested in: the minimum share of the total vote below which it is impossible to win either

¹⁶For a more detailed account of the working and formulation of quota-based electoral formulae see (Taagepera and Shugart 1989) and also (Penades 2000)

¹⁷When the distribution of district magnitudes is not known, the average district magnitude can be used instead. The correlation between the values obtained using the distribution of districts and the average district magnitude is over 0.85 which makes average district magnitude a good proxy. This is so, though, for half of the seats in Parliament. For simplicity, and in order to ease the replication of data, the values presented in this article for winning a majority in the parliament have been calculated using the average district magnitude.

one seat or half of the seats in the legislative assembly. In both quota-based and divisor-based electoral systems that distribution of seats is obtained by allocating all seats in the smallest districts. In the case of the minimum share of the total vote necessary to win just one seat, aggregated threshold functions are applied to the smallest district in the country¹⁸

Table 3: Summary values for P.R. complete electoral systems

	P.R. Quota		P.R. Divisor	
	1 seat	Half seats in parliament	1 seat	Half seats in parliament
Mean (%)	0.20	41.9	0.31	31.42
Std. Dev. (%)	0.16	6.7	0.29	6.78
Minimum (%)	0.1	19.08	0.2	20.26
Maximum (%)	0.53	49.6	0.92	46.51

Table 3 shows a summary of the values calculated for P.R. electoral systems. Looking at the values calculated for one seat, some remarks can be made. First, not surprisingly, the average is lower in quota-based than in divisor-based electoral systems. The price of winning a seat is cheaper in the former type of system. Second, despite the difference just remarked, the minimum values necessary to win a seat are close under both types of electoral

¹⁸A similar reasoning applies for half of the seats in the legislative assembly. Consider the following example. Take a complete electoral system where 100 seats are elected in the parliament using the Hare quota. These seats are distributed in 8 districts according to the following distribution $\mathbf{M}_d = [50 \ 15 \ 15 \ 4 \ 4 \ 4 \ 4 \ 4]$, where \mathbf{M}_d indicates the vector that contains all district magnitudes. Given this institutional setting, aggregated threshold functions are applied to the combination of seats that produces the minimum value needed to win a given number of seats. Since we are interested in half of the seats in parliament, the combination that produces such a value would be $\mathbf{S}_d = [1 \ 14 \ 15 \ 4 \ 4 \ 4 \ 4 \ 4]$, where \mathbf{S}_d indicates the vector that shows all seats won in each of the districts contained in \mathbf{M}_d .

For divisor-based electoral systems that use the d'Hondt electoral formula, the combination of seats that minimize the function for a total number of seats equal to half of the parliament is $\mathbf{S}_d = [0 \ 15 \ 15 \ 4 \ 4 \ 4 \ 4 \ 4]$. Note how in this case no seats are won in the biggest district. A detailed account of how aggregated threshold functions are optimized can be found in (author 2005).

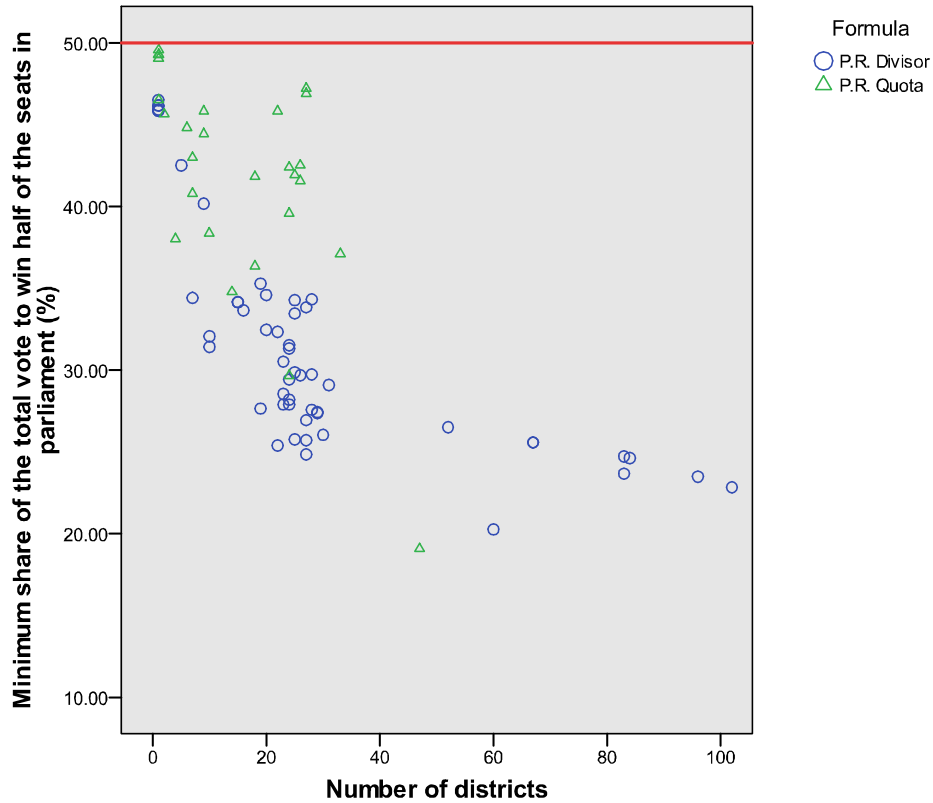
systems and they approach the values shown for winner-takes-all electoral systems. The reason for this is P.R. electoral systems distribute seats in multi-member districts according to some population criterion; this means that some electoral systems such as the one used in Brazil in 1962 or 1998 have single-member districts where the effect of the P.R. formula cannot be applied. Third, the values needed to win one seat must be interpreted with some caution because they may not show the real "price" of winning a seat. As I said before, legal thresholds should be taken into consideration at this stage¹⁹.

Similar observations about the type of electoral formula can be made if we look at the values required to win half of the seats in the parliament. On average, if a complete electoral system uses a quota-based formula, then the minimum value required to win half of the seats in parliament is about 42% whereas if a divisor-based electoral formula is used that value is about 31.5%. However, the number of districts also seems to be important: in both types of electoral systems, higher values are obtained in electoral systems with just a single district²⁰. These ideas can be better explained by looking at figure 2.

¹⁹For example, the legal threshold for winning representation under the Moldovan electoral system is 4% and the aggregated threshold value is about 0.91%. In this case, the minimum share of the total vote needed to win a seat is equivalent to the legal threshold. When legal thresholds apply not at national level but at district level instead, then aggregated threshold functions do usually show the total share of the vote necessary to win a seat. In Spain, for example, a legal threshold of 3% at district level is required to win representation; since Spain has two uninominal districts, aggregated threshold functions take the value of either of these districts as the minimum necessary to win a seat. Given the institutional components of the Spanish electoral system, the minimum value to win a seat in either of these district is 12.5% which is much higher than the legal threshold.

²⁰The maximum value for a quota-based system is that for the one used in Israel from 1951 to 1969. In the case of divisor-based electoral systems, the maximum is for that in use in The Netherlands from 1946 to 1998.

Figure 2: P.R. electoral systems in relation to the point of perfect proportionality



Locating all complete electoral systems along a continuum, and contrasting their minimum thresholds for winning half of the seats in parliament with the point of perfect proportionality, as I do here in figure 2, has evident advantages. First, we can draw some distinctions depending on the type of electoral formula that is being used. In this sense, P.R. complete electoral systems that use any type of quota-based formula produce results closer to the line of perfect proportionality than those electoral systems that use any of the divisor-based electoral formulas. Practically all quota-based electoral systems that have been characterized here used the Hare quota as

the electoral formula for allocating seats²¹. This electoral formula produces more proportional results than other formulas such as the d'Hondt formula, which is used in most divisor-based electoral systems (Farrell 2001).

Second, and equally interesting, is that the "proportional" label traditionally attached to these electoral systems may be challenged if one focuses on the values produced by aggregated threshold functions. As figure 2 shows, the number of districts is important when calculating the minimum number of votes required to obtain a majority of seats in the parliament. In fact, holding all other institutional variables constant, when the number of districts increases, the minimum share of the total vote for winning half of the seats in the legislature decreases²², i.e. the electoral system becomes less proportional. This is in line with earlier studies that have found that district magnitude and proportionality are negatively correlated (Taagepera and Shugart 1989; Lijphart 1994). The finding here strengthens this conclusion by testing it not at a micro-level - the district - but at a macro-level - the whole electoral territory. Therefore in considering the mechanical functioning of a complete proportional representation electoral system as a whole, the number of districts is important and that means that some so-called "proportional" electoral system may not be producing actual proportional results. The higher the number of districts the further away from the line of perfect proportionality. Furthermore, about 10% of the P.R. cases fall below the 25% line that was established for *winner-takes-all* electoral systems²³.

²¹Only Luxembourg and the Slovak Republic used a different quota, namely the Droop quota.

²²Mathematical proofs of this finding can be found in (author 2005)

²³A good example is the electoral system in used in Chile since 1993. There, 120 seats are distributed in 60 equal districts so producing two-seat districts. When district magnitude

A comparison of several indices: the supremacy of aggregated threshold values

The literature on electoral systems offers not only a variety of different indices for measuring the mechanical performance of electoral systems but also interesting discussions about them (Gallagher 1991; Monroe 1994). Why, then should aggregated threshold functions (ATFs) be used instead of the existing measures? In other words, what are the virtues of the values shown here? Some of the distinct properties of the functions have already been mentioned, however, in order to answer the previous questions, I calculate some of the major indices used in the literature and contrast them with the values produced by aggregated threshold functions that I propose here. For practical reasons, the data that I use comes from the most recent electoral systems used between 1945 and 2000 in 13 European countries plus Canada and the United States to which aggregated threshold functions are also applied. The indices calculated are Losemore and Hanby's (L-H), Rae's, Gallagher's least square and Sainte-Laguë (S-L) indices, and the effective threshold. Data is presented in table 4.

As I will show, this comparison with other electoral indices should show why aggregated threshold functions are a convincing index. Let us start with one of the most frequently used indices, the disproportionality index created by Loosemore and Hanby and its subsequent reformulations. Michael Gallagher is certainly right when he observes that indices based on largest remainders like the Loosemore and Hanby index do not survive paradoxes

 is that small, proportional representation formulae play virtually no role.

Table 4: Aggregated threshold values against other indices

Country	Year	L-H	Rae	Least square	S-L	Effec.Thre	ATF
U.K.	1997	0.212	1.274	0.168	0.223	0.375	0.100
France	1997	0.304	4.260	0.186	0.438	0.375	0.103
Canada	2000	0.171	1.025	0.137	0.151	0.375	0.124
USA	1998	0.040	0.239	0.033	0.033	0.375	0.110
France	1986	0.103	0.619	0.060	0.053	0.110	0.235
Bulgaria	1997	0.040	0.242	0.039	0.008	0.086	0.241
Spain	2000	0.076	0.455	0.061	0.045	0.097	0.265
Switzerland	1999	0.042	0.254	0.028	0.011	0.087	0.297
Portugal	1999	0.069	0.412	0.051	0.027	0.061	0.325
Finland	1999	0.059	0.351	0.031	0.020	0.052	0.342
Sweden	1968	0.037	0.223	0.029	0.014	0.081	0.343
Norway	1985	0.065	0.387	0.042	0.032	0.082	0.353
Luxembourg	1999	0.056	0.334	0.035	0.022	0.054	0.380
Latvia	1998	0.054	0.054	0.054	0.054	0.054	0.425
Moldova	1998	0.118	0.707	0.092	0.073	0.007	0.459
Slovak Rep.	1998	0.029	0.174	0.019	0.004	0.005	0.465
Netherlands	1998	0.017	0.100	0.011	0.001	0.005	0.465

Sources - Greece, (Clogg 1992)

Election Results Archive: <http://www.binghamton.edu/cdp/era/index.html>

Inter-Parliamentary Union: <http://www.ipu.org/english/home.htm>

like the so-called "new state" paradox²⁴. Gallagher's defence of the least-square method is convincing since it is a method that is not vulnerable to such paradoxes, and this index does really reflect the mechanics inside an electoral system (Gallagher and Mitchell 2005). However, Gallagher's least-square index has two shortcomings that are also found in Loosemore and Hanby's index as well as in Rae's. First, and most important, all of them rely on the occurrence of the election. This means that these indices, as well as the Sainte-Laguë index, measure the effect of the electoral system

²⁴The so-called "new state" paradox explains the variation of the Loosemore and Hanby's index when a new party enters the competition winning a seat and the existing parties maintain their vote share. Illustrations of the paradox are offered by Gallagher (1991:39) and also by Taagepera (2007:79-82)

on each individual election. They are not measuring, then, the raw effect of the institutional components of the electoral system but how electoral systems behave once a distribution of votes/seats is provided. This is a small but significant nuance since, for instance, electoral reformers could not anticipate the possible outcome of an electoral system that has not been put into practice yet. Aggregated threshold functions are not conditioned by such a problem. Instead, they anticipate the mechanical functioning of any electoral system without relying on the occurrence of an election. By calculating the necessary share of the total vote to win a given number of seats, ATFs provide electoral reformers with a tool that can be used to the forecast possible political consequences of the future electoral system.

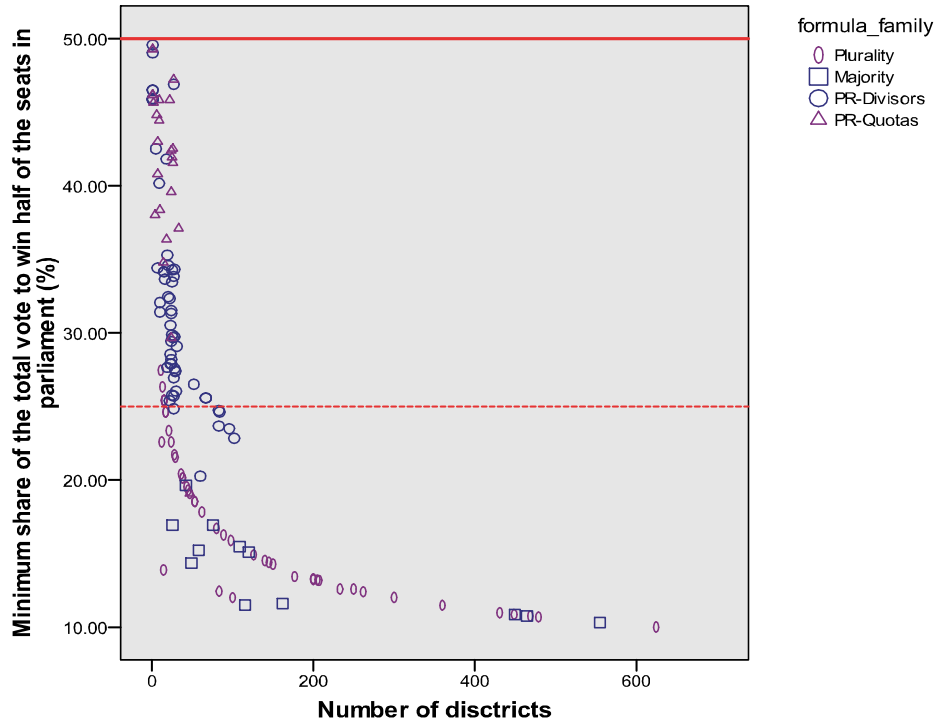
Second, these four indices, L-H, Rae's, Least-square, and S-L, fail to provide a clear interpretation of what they are measuring²⁵. How should we interpret the differences between the electoral systems used in Bulgaria in 1997 and in the Netherlands in 1998 if we look at the L-H or least square indices? Both produce a value of approximately 0.04 for the case of Bulgaria and 0.017 (L-H) or 0.011 (least square) for Netherlands in 1998. It is true that according to these indices, the election in Bulgaria in 1997 was more disproportional than the one in the Netherlands, but the question is: by how much? The same can be said of the Sainte-Laguë index; by looking at the electoral systems used in Finland and the Slovak Republic we see that the S-L index produces values of 0.02 and 0.004 respectively. Does this mean that the system used in the Slovak Republic is five times more proportional

²⁵Loosemore and Hanby's index, Rae's index and Gallagher's least square index varies from 0 (perfect proportionality) to 1 (full disproportionality). The Sainte-Laguë varies from 0 (perfect proportionality) to ∞ (full disproportionality).

than the one used in Finland? By determining the minimum share of the total vote necessary to win half of the seats in parliament, aggregated threshold functions present an intuitive and informative measure. Furthermore, these functions clearly support the argument made by Gallagher (1991) and others about, again, the negative relationship between the number of districts and the level of proportionality. Going back to the systems used in the Slovak Republic and Finland, we can say that the former is almost perfectly proportional whereas the distance of the latter from the point of perfect proportionality is much larger (approximately 0.16 points). The fact that in the Slovak Republic there is only one district and in Finland there are 15 is one key explanation of these differences. In summary, as figure 3 shows, the contrast between the values produced by aggregated threshold functions and the point of perfect proportionality provides students of electoral systems with a straightforward informative measure of the distance of each system from perfect proportionality.

Finally, the effective threshold functions calculated and developed primarily by Lijphart (1994) and Taagepera and Shugart (1989), which are shown in the seventh column of table 4, can also be criticized on several grounds. First, this effective threshold only refers to the cost of a seat in a given district magnitude, focusing then on just how electoral systems behave in a given district. In contrast, aggregated threshold functions provide information about the necessary conditions for winning any number of seats in any complete electoral system. They have, then, a much broader scope than the effective threshold. Effective thresholds, calculated as the mean of the threshold of representation and the threshold of exclusion, also fail to pro-

Figure 3: Proximity of electoral systems to the line of perfect proportionality



vide a precise meaning to what is being measured; Carles Boix refers to this value not as a concrete number but as a "range of possibilities" for obtaining representation (Boix 1999, 614). As both table 4 and figure 3 show, the data produced by aggregated threshold functions do have a clear meaning of their own. However, the most important concern about the effective threshold is the way in which it is calculated. Values are obtained using a purely empirical approach that does not correspond to a clear-cut logic. Briefly, while both Taagepera and Shugart and Lijphart calculate the exclusion threshold in a similar fashion, they disagree on how to calculate the threshold of representation: Taagepera and Shugart use the Hare quota while Lijphart uses half of this quota. Neither of them provides any logical reasoning to explain these

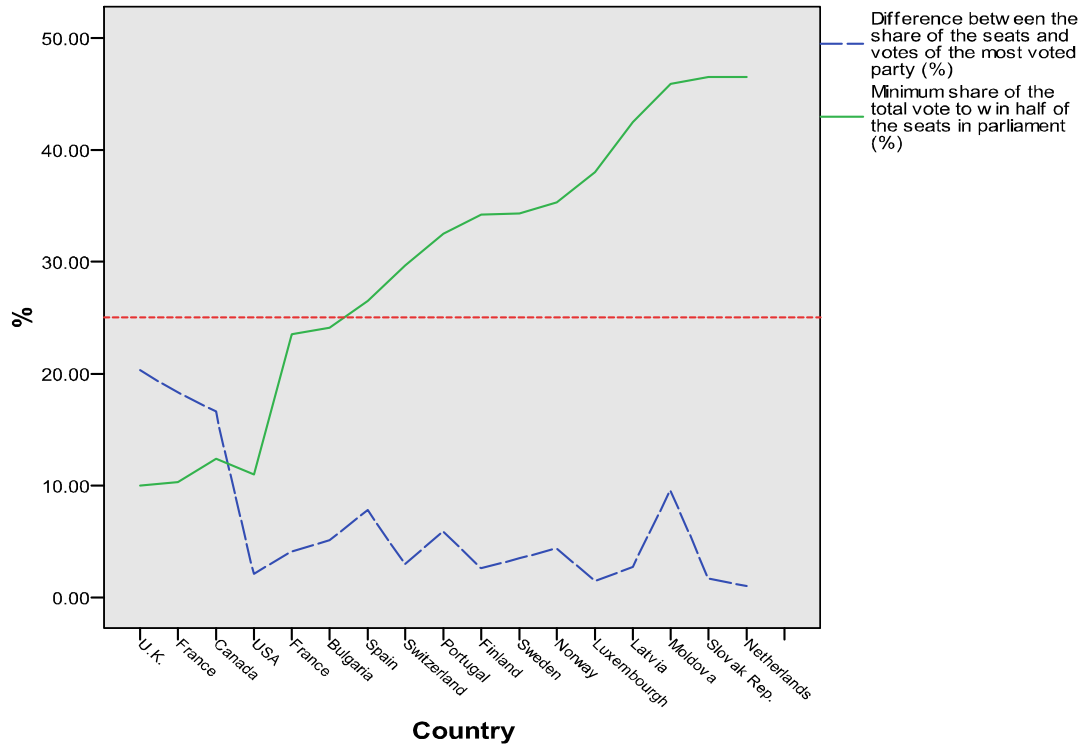
discrepancies other than the goodness of fit with their empirical results²⁶. Aggregated threshold functions in contrast provide a well-defined and logical method of calculation (Penades 2000)(author 2007) which takes into account every particular distinctive features of each complete electoral system.

A final remark must be made at this point. As I have already said, aggregated threshold functions summarize electoral systems around the minimum value needed to win half of the seats in a parliament. We know, however, that these values are rarely observed. For example, ATFs predict that no party will win a bare majority of seats in the House of Commons unless a minimum of 10% of the national vote is obtained. Yet we know that majorities of the seats in that House are obtained by a much higher percentage of votes -higher even than 25% of the vote. Likewise, ATFs predict that a minimum of 26.5% of the total vote is required to win half of the seats in the Spanish parliament and yet we know that the last party to win an overwhelming majority in that house - the conservative party (PP) in 2000 - did so after winning 44.52% of the total vote. These results show no contradictions with what has been said here and a full explanation of these disparities has been offered elsewhere (author 2007). However, one may ask why we should use the values generated by aggregated threshold functions if the empirical results may be far away from them. Figure 4 plots aggregated threshold values and the differences between the seat share and vote share for the most successful parties for all the countries in table 4.

The difference between the seat share and the vote share of the

²⁶The different approaches used by Lijphart, and Taagepera and Shugart are nicely explained by Lijphart (1994, 25ff). See also Taagepera and Shugart (1989, 274-277).

Figure 4: Aggregated threshold values and the difference between the share of seats and votes of the most voted party.



most successful party can be used as a measure for observing to what extent electoral systems reward big parties. The higher the difference, the more disproportional the electoral system is. Figure 4 illustrates this idea. The dashed line shows the seat-vote difference and, not surprisingly, the values corresponding to the U.K, France and Canada are the highest as well as being above those obtained by aggregated threshold functions. These countries use particularly majoritarian electoral systems, which is why the aggregated threshold values represented by the solid line are far not only from the ideal 50% but also from the 25% limit (dotted line) established for winner-takes-

all electoral systems. So the empirical results produced by these electoral systems match the majoritarian nature that was anticipated by ATFs. Furthermore, figure 4 shows some other interesting things. For example, some "proportional" countries are not as "proportional" as the literature would characterize them. Spain and Portugal illustrate the idea fairly well²⁷. ATFs anticipated the majoritarian bias of the Spanish electoral system by producing a value of 26.5% as the minimum share of the total vote needed to win half of the seats in parliament, and the empirical results show that the seat-vote difference is about 8%; likewise aggregated threshold functions predicted a minimum value of 32.5% for Portugal and the seat-vote difference for the 1999 parliamentary election was about 6% for the most-voted party. Furthermore, those countries with the smallest seat-vote difference, the Slovak Republic (1.7%) and the Netherlands (1%), are also closer to perfect proportionality according to aggregated threshold functions as one can see by looking at how close the solid line is from the 50% value.

Finally, two particular cases must be explained. As figure 4 shows, the seat-vote difference for the case of the USA (1998) is rather small (2.1%) and the electoral system used to select the members of the House of Representatives is FPTP; given this institutional setting one should expect values closer to those found for the U.K rather than to the values produced by a "proportional" electoral system such as the one used in Finland in 1999 as actually happens. The explanation for this lies in the dual party system and the practically even distribution of popular support between Republicans and

²⁷In fact, as both, figures 3 and 4 show, these two countries are closer to the 25% line than to the point of perfect proportionality.

Democrats during the 1998 legislative elections. The second case relates to the parliamentary election in Moldova in 1998. Even though ATFs classify Moldova as an electoral system almost close to perfect proportionality (46%), the seat-vote difference is about 10%. The reason for this must be found in the amount of the valid vote that was used to distribute the seats. The 104 seats in parliament were distributed by approximately 76% of the valid vote. Possibly, a high legal threshold together with a high number of minor parties are the main reasons why seats were distributed only among three fourths of the total valid vote²⁸.

All these examples show, then, how the minimum share of the total vote produced by aggregated threshold functions, despite being rather unlikely in real life, does anticipate the trends that will be found when elections actually occur under the electoral systems under consideration.

Conclusion

More than three decades after the publication of Loosemore and Hanby's famous index of proportionality, the debate on how to measure electoral systems is not over (Taagepera 2007). The disproportionality index created by these scholars was the starting point for finding the best way to measure how votes are converted into seats. The present article contributes to this debate by explaining how aggregated threshold functions summarize in a unique value the mechanical functioning of any electoral system by taking

²⁸In fact, if we treat the valid vote that was used to allocate seats (76.43%) as if it were the total valid vote (100%), then the seat-vote difference would be close to 0 (0.34%) which is in close agreement with what ATFs had predicted. See also footnote 19 above where I refer to the legal threshold used in Moldova.

into account all of its institutional components. The values obtained from these functions have three properties that may attract the attention of both electoral reformers and students of electoral systems.

First, aggregated threshold functions are independent of electoral results. As opposed to most of the indices derived by Loosemore and Hanby, or Gallagher, the values obtained with these functions do not need any previous distribution of votes and seats to work. Aggregated threshold functions can calculate the necessary and sufficient conditions for winning any given number of seats just by using the institutional variables that define complete electoral systems. Electoral reformers can benefit from having such a straightforward tool to help forecast some of the political consequences of new electoral systems. Aggregated threshold functions show the effects of a variation in the number of districts, the assembly size or the electoral formula on the entry of minor parties or the probability of producing single-party cabinets. Reformers can anticipate that if a new system is far from the line of perfect proportionality, the probability of having a single-party cabinet will increase but the voice of minor parties may decrease, and that may have some implications if the number of cleavages is high. On the other hand, reformers will also be able to know in advance that systems closer to perfect proportionality will increase the probability of generating a divided government but will probably mirror society more accurately.

The second and the third properties that characterize these functions are universality and generality. Universality means that aggregated threshold functions can specifically be applied to each complete electoral system taking into account, therefore, the distinctive features that define them: elec-

toral formula, number of districts, assembly size and distribution of district magnitudes. In this article, aggregated threshold functions have been applied to 142 different complete electoral systems that have been used in 525 democratic elections between 1946-200. Generality means that the values obtained from the functions reflect the mechanical behaviour of the electoral system as a whole.

These two properties can be appealing to students of electoral systems because the information obtained thereby represents an accurate picture of how the electoral system behaves. Since the values produced by ATF's are continuous, they are richer than categorical variables that characterize electoral systems based on one distinctive feature such as the electoral formula. Scholars working on electoral systems can now benefit from a measure that explains how close or how far the complete electoral system under study is from perfect proportionality.

This way of characterizing complete electoral systems provides some initial interesting conclusions. For instance, when *winner-takes-all* electoral systems are made of single-member districts, aggregated threshold function results only depend on the number of competing parties. In this sense, the higher the number of parties, the lower the minimum share of the vote required to win half of the seats in parliament. In the case of P.R. complete electoral systems, quota-based electoral systems produce, on average, results closer to the line of perfect proportionality than divisor-based systems. The values obtained when aggregated threshold functions are applied to half of the seats in the parliament also reveal that the number of districts is an important variable. If all defining variables but the number of districts are

held constant, the minimum value necessary to win half of the seats in the legislative assembly decreases as the number of districts increases.

Aggregated threshold functions provide students of electoral systems with useful information that not only improves the information that has been produced by existing proportionality indices but also allows understanding of the functioning of electoral systems as a whole. We have now a measure that allows us to understand the relationship of electoral systems with other political phenomena. If it is argued that centre-left governments are more likely to appear in P.R electoral systems (Iversen and Soskice 2002), we can now be more precise and establish under what specific institutional designs this type of government is more common. We can also use aggregated threshold functions to understand under which particular electoral system we should expect higher levels of corruption (Chang and Golden 2007) or whether deviation from perfect proportionality explains the formation of single-party governments or coalition governments. These are the type of research questions that will be addressed in the future.

Appendix 1

Aggregated threshold functions for *winner-takes-all* electoral systems

1 Seat

The aggregated threshold function applied to those *winner-takes-all* electoral systems referred to above has the following form:

$$V_{S_T}^{nec} = \frac{M_d}{M} \left(\frac{S_d - 1 + c}{M_d - 1 + Pc} \right) * S_T, \quad (3)$$

When $M_d = S_d = 1$ and $S_T = 1$, then

$$V_{S_T=1}^{nec} = \frac{1}{MP}, \quad (4)$$

where M_d stands for district magnitude of a given district; S_d stands for the number of seats won in a given district; S_T stands for the total number of seats for which aggregated threshold functions are applied.; M stands for the number of seats elected in the legislative assembly; P stands for the number of competing parties as calculated by Taagepera Taagepera (2007) and c stands for an adjustment term as defined below.

Half of the seats in the parliament

Since $S_T = \frac{M}{2}$ and $M_d = S_d$, then according to function 3 the values shown in Table 5.3 are calculate using the function,

$$V_{S_T=\frac{M}{2}}^{nec} = \frac{M_d(M_d - 1 + c)}{2(M_d - 1 + Pc)}, \quad (5)$$

When $M_d = 1$, function 5 can be simplified as,

$$V_{S_T=\frac{M}{2}}^{nec} = \frac{1}{2P}, \quad (6)$$

Aggregated threshold functions for quota-based electoral systems

The aggregated threshold function for quota-based electoral systems has the following form:

$$V_{S_T}^{nec} = \sum_{d=1}^D \frac{M_d [P(S_d - 1) + 1 + n]}{MP(M_d + n)}, \quad (7)$$

where M_d stands for district magnitude of a given district; S_d stands for the number of seats won in a given district ; S_T stands for the total number of seats for which aggregated threshold functions are applied.; M stands for the number of seats elected in the legislative assembly; P stands for the number of competing parties; D stands for the number of districts and n stands for the modifier of the quota²⁹ (Taagepera and Shugart 1989).

²⁹When $n = 0$, then the Hare quota is defined. When $n = 1$, then the Droop quota is defined

Aggregated threshold functions for divisors-based electoral systems

The aggregated threshold function for divisor-based electoral systems has the following form:

$$V_{S_T}^{nec} = \sum_{d=1}^D \frac{M_d}{M} \left(\frac{S_d - 1 + c}{M_d - 1 + Pc} \right), \quad (8)$$

where M_d stands for district magnitude of a given district; S_d stands for the number of seats won in a given district; S_T stands for the total number of seats for which aggregated threshold functions are applied.; M stands for the number of seats elected in the legislative assembly; P stands for the number of competing parties; D stands for the number of districts and c stands for the adjustment term³⁰ (Penades 2000).

³⁰When $c = 1$, then the d'Hondt formula is defined. When $c = 0.5$, then the Sainte-Laguë is defined.

References

- Abedi, A. (2002). Challenges to established parties: The effects of party system features on the electoral fortunes of anti-political-establishment parties. *European Journal of Political Research* 41(4), 551–583.
- Alonso, S. and R. Ruiz-Rufino (2007). Political representation and ethnic conflict in new democracies. *European Journal of Political Research* 46(2), 237–267.
- Balinsky, M. L. and H. P. Young (1982). *Fair Representation: Meeting the Ideal of One Man, One Vote*. New Haven: Yale University Press.
- Birch, S. (2003). *Electoral Systems and Political Transformation in Post-Communist Europe, 'One Europe or Several?'*. Basingstoke: Palgrave-Macmillan.
- Blais, A., A. Dobrynska, and I. H. Agnieszka (2005). To adopt or not to adopt proportional representation: The politics of institutional choice. *British Journal of Political Science* 35(1), 182–190.
- Boix, C. (1999). Setting the rules of the game: The choice of electoral systems in advanced democracies. *American Political Science Review* 93(3), 609–624.
- Chang, E. C. C. and M. A. Golden (2007). Electoral systems, district magnitude and corruption. *British Journal of Political Science* 37(1), 115–137.
- Clogg, R. (1992). *A Concise History of Greece*. Cambridge: Cambridge University Press.

- Cohen, F. S. (1997). Proportional versus majoritarian ethnic conflict management in democracies. *Comparative Political Studies* 35(5), 607–30.
- Cox, G. (1997). *Making Votes Count*. New York.
- Duverger, M. (1954). *Political Parties. Their organization and activity in the modern State*. London: Methuen.
- Farrell, D. M. (2001). *Electoral Systems. A Comparative Introduction*. New York: Palgrave.
- Gallagher, M. (1991). Proportionality, disproportionality and electoral systems. *Electoral Studies* 10(1), 33–51.
- Gallagher, M. and P. Mitchell (Eds.) (2005). *The Politics of Electoral Systems*. Oxford: Oxford University Press.
- G. Binham Powell, J. (2000). *Elections as Instruments of Democracy*. New Haven: Yale University Press.
- Golder, M. (2005). Democratic electoral systems around the world 1946–2000. *Electoral Studies* 24(1), 103–121.
- Iversen, T. and D. Soskice (2002). Electoral systems and the politics of coalition: Why some democracies redistribute more than others. *American Political Science Review* 100(2), 165–181.
- Katz, R. S. (1997). *Democracy and Elections*. Oxford: Oxford University Press.
- King, G., J. Alt, N. Burns, and M. Laver (1990). A unified model of cabinet dissolution in parliamentary democracies. *American Journal of Political Science* 34(3), 846–871.

- Laakso, M. and R. Taagepera (1979). Effective number of parties: a measure with applications to western europe. *Comparative Political Studies* 12, 3–27.
- Lijphart, A. (1994). *Electoral Systems and Party Systems. A Study of Twenty-Seven Democracies 1945-1990*. Oxford: Oxford University Press.
- Lijphart, A. and R. W. Gibberd (1977). Threshold and payoffs in list systems of proportional representation. *European Journal of Political Representation* 5, 219–244.
- Loosemore, J. and V. Hanby (1971). Threshold of representation and threshold of exclusion. *Comparative Political Studies* 3, 479–488.
- Lupia, A. and K. Strom (1995). Coalition termination and the strategic timing of parliamentary elections. *The American Political Science Review* 89(3), 648–665.
- Milesi-Ferreti, G. M., R. Perotti, and M. Rostagno (2002). Electoral systems and public spending. *The Quarterly Journal of Economic* 117, 609–657.
- Mill, J. S. (1991). *Considerations on Representative Government*. New York: Prometheus Books.
- Monroe, B. L. (1994). Disproportionality and malapportionment: Measuring electoral inequity. *Electoral Studies* 13(2), 132–149.
- Moser, R. G. (1999). Electoral systems and the number of parties in post-communist states. *World Politics* 51(3), 359–384.

- Penades, A. (2000). *Los Sistemas Electorales de Representacion*. Ph.D. Dissertations. Madrid: Juan March Institute: Center for Advance Studies in Social Sciences.
- Persson, T. and G. Tabellini (2005). *The Economic Effects of Constitutions. Munich Lectures in Economics*. Cambridge, Massachusetts: The MIT Press.
- Przeworski, A. (1991). *Democracy and the Market: Political and Economic Reform in Eastern Europe and Latin America*. Cambridge: Cambridge University Press.
- Rae, D. (1967). *The Political Consequences of Electoral Law*. New Haven: Yale University Press.
- Rae, D., V. Hanby, and J. Loosemore (1971). Thresholds or representation and thresholds of exclusion. *Comparative Political Studies* 3, 479–488.
- Reynal-Querol, M. (2002). Ethnicity, political systems, and civil wars. *Journal of Conflict Resolution* 46(1), 29–54.
- Rokkan, S. (1968). Elections: Electoral systems. In D. L. Sills (Ed.), *International Encyclopedia of the Social Sciences*, Volume 5, pp. 6–21. New York: Crowell-Collier-Macmillan.
- Taagepera, R. (1998). Nationwide inclusion and exclusion thresholds of representation. *Electoral Studies* 17(4), 405–417.
- Taagepera, R. (2002). Nationwide threshold of representation. *Electoral Studies* 21, 383–401.
- Taagepera, R. (2007). *Predicting Party Sizes. The Logic of Simple Electoral*

Systems. Oxford: Oxford University Press.

Taagepera, R. and M. S. Shugart (1989). *Seats and Votes*. Yale: Yale University Press.

Veugelers, J. and A. e. Magnan (2005). Conditions of far-right strength in contemporary western europe: An application of kitschelt's theory. *European Journal of Political Research* 44(6), 837–860.