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Must We Also Forget?

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Abstract

In many countries, lenders are restricted in their access to information about borrowers' past defaults. We study this provision in a model of repeated borrowing and lending with moral hazard and adverse selection. We analyze its effects on borrowers' incentives and access to credit, and identify conditions under which it is optimal. We argue that "forgetting" must be the outcome of a regulatory intervention by the government. Our model's predictions are consistent with the cross-country relationship between credit bureau regulations and provision of credit, as well as the evidence on the impact of these regulations on borrowers' and lenders' behavior.

I Introduction

In studying the "fresh start" provisions of personal bankruptcy law, economists typically focus on the *forgiveness* of debts. However, another important feature is the *forgetting* of past defaults. In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed.

In the United States, the Fair Credit Reporting Act (FCRA) prescribes that a personal bankruptcy filing may be reported by credit bureaus for up to ten years, after which it must be removed from the records made available to lenders.[1] Similar provisions exist in most other countries. In figure 1 we summarize the distribution of credit bureau regulations governing the time period of information transmission across countries.[2] Of the 113 countries with credit bureaus as of January 2007, over 90 percent of them had provisions for restricting the reporting of adverse information after a certain period of time. Also note that this fraction has remained stable over time, even as more countries have set up credit bureaus for the first time (twelve countries introduced bureaus from 2003 to 2007).[3]

Differences in information-sharing regimes across countries — whether a credit-reporting system exists, and whether there are time limits on reporting past defaults — are associated with differences in the provision of credit. In figure 2 we graph the average ratio of Private Credit to GDP according to whether the country restricts the time period of information sharing. Countries with no information sharing at all (i.e. no credit bureau), as it is now well-established, have low levels of credit. On the other hand, it is interesting to note that countries in which defaults are always reported tend to have *lower* provision of credit than those countries in which defaults are not reported ("erased") after a certain period of time.[4]

Musto (2004) studies the effect on lenders and individual borrowers of restrictions on the reporting of past defaults, using U.S. data. He shows that (i) these restrictions are binding — access to credit increases significantly when the bankruptcy "flag" is dropped from credit files;[5] and (ii) these individuals who subsequently obtain new credit are subsequently likelier

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[1]Other derogatory information can be reported for a maximum of seven years; see Hunt (2006) for a discussion of the history and regulation of consumer credit bureaus in the United States. This time period is often even shorter in other countries; Jappelli and Pagano (2004) report several specific examples.

[2]Source: Doing Business Database, World Bank, 2004 and 2008. Throughout, we use the term "credit bureau" to refer both to private credit bureaus, as well as public credit registries.

[3]See also Jappelli and Pagano (2006).

[4]Private credit/GDP is constructed from the IMF International Financial Statistics for year-end 2006. As in Djankov, McLiesh and Shleifer (2007), private credit is given by lines 22d and 42d (claims on the private sector by commercial banks and other financial institutions). The credit bureau regulations are current as of January 2007 (source: Doing Business database 2008).
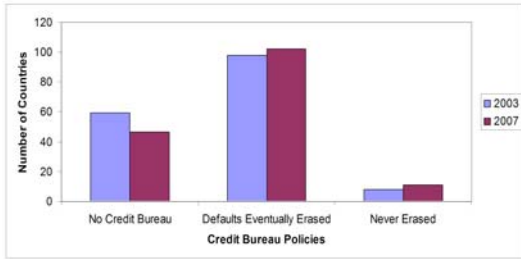
[5]That is, after 10 years.

Figure 1: Credit Bureau Policies over Time

Figure 2: Impact of Information-Sharing Regime on Provision of Credit

to default than those with similar credit scores.

In this paper we analyze these restrictions in the framework of a model of repeated borrowing and lending, and determine conditions under which they are welfare improving. In particular, we study an environment where entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. We have in mind a world of small entrepreneurs who finance their business ventures with loans for which they are personally liable.[6] In this setup, an entrepreneur's reputation, or *credit history*, as captured by the past history of successes and failures of his projects, can affect the terms at which he can get credit and hence his incentives.

In a typical equilibrium of the model, entrepreneurs whose projects fail will see a significant deterioration in their reputation, and hence in their incentives; they will thus no longer be able to obtain financing. On the other hand, success of a project improves the entrepreneur's reputation and allows him to get credit at a lower interest rate. Hence the higher an entrepreneur's reputation, the costlier a failure, and the stronger his incentives.

We then consider the impact of restricting the availability to lenders of information on entrepreneurs' past defaults. Such a restriction leads to a tradeoff in our model. On the one hand, "forgetting" a default makes incentives weaker, *ex-ante*, because it reduces the punishment from failure. On the other hand, forgetting a default improves an entrepreneur's reputation, *ex-post*. This improvement in his reputation allows him to obtain financing when he otherwise would not be able to. It also strengthens his incentives, since this improved reputation would be jeopardized by a project failure. To put it another way, those

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[6]And indeed, Avery et al (1998) use the NSSBF and SCF to show that "[l]oans with personal commitments comprise a majority of small business loans."

entrepreneurs who have their failure forgotten are pooled again with those who have not failed; as we discuss below, this plays a central role in our model.

Our key result is that, if either borrowers' incentives are sufficiently strong, or their average risk-type is not too low, welfare is higher in the presence of a limited amount of forgetting, that is, by restricting the information available to lenders on borrowers' credit history. The same result holds even if these conditions do not hold, when the output loss from poor incentives is not too large and agents are sufficiently patient. We also argue that forgetting must be the outcome of a regulatory intervention by the government — no lender would willingly agree to ignore the information available to him.

The effects of "forgetting" on lenders' and individual borrowers' behavior in our model are consistent with the empirical evidence presented by Musto (2004). However, while Musto interprets this evidence as an indication that laws imposing restrictions on memory are suboptimal we argue that these restrictions may be optimal. In addition, our results on the relation between presence of a forgetting clause and the aggregate volume of credit are consistent with the international evidence reported above.

In the Congressional debate surrounding the adoption of the FCRA (U.S. House, 1970, and U.S. Senate, 1969), the following arguments were put forward in favor of forgetting past defaults: (i) if information was not erased the stigmatized individual would not obtain a "fresh start" and so would be unable to continue as a productive member of society, (ii) old information might be less reliable or salient, and (iii) limited computer storage capacity. On the other hand, the arguments raised against forgetting this information were (i) it discourages borrowers from repaying their debts by reducing the penalty for failure, (ii) it increases the chance of costly fraud or other crimes by making it harder to identify seriously bad risks, (iii) it could lead to a tightening of credit policies (which would affect the worst risks disproportionately), and finally, (iv) it forces honest borrowers to subsidize the dishonest ones. We will show that our model, while admittedly quite stylized, allows us to capture many of these arguments and will use it to assess the tradeoffs between the positive and negative effects of forgetting.

The paper is organized as follows. In section II we present the model and the strategy sets of entrepreneurs and lenders. In the following section we show that a Markov Perfect Equilibrium of the model exists and characterize the equilibrium strategies at the most efficient MPE. In section IV we study the effects of introducing a forgetting clause on equilibrium outcomes and welfare. We derive conditions under which forgetting defaults is socially optimal, and relate them to the empirical evidence and the policy debate surrounding the adoption

of the FCRA. Section V considers an extension of the model in which the consequences of a failure are less extreme. Section VI concludes, and the proofs are in the Appendix.

Previous Literature

Our basic model is one of reputation and incentives, like those of Diamond (1989), Mailath and Samuelson (2001), and Fishman and Rob (2005). In these models, principals and agents interact repeatedly under conditions of both adverse selection and moral hazard. The equilibrium in our model shares many similarities with the ones in these papers, in that agents build reputations over time. There are nevertheless some key differences between our model and theirs — in both the setting, and in the structure of markets and information — which are discussed below (see Remarks 1 and 4).

The positive effects that a credit bureau can have through increasing the information publicly available on borrowers' histories have been widely discussed. One noteworthy paper that focuses on lenders' incentives to voluntarily share information is Pagano and Jappelli (1993). In recent empirical work, Djankov, McLiesh and Shleifer (2007) and Brown, Jappelli, and Pagano (2007) have found that credit bureaus are positively associated with increased credit.

Our main focus, however, is on the possible benefits of limiting the information available on borrowers' past histories. The paper that is closest in spirit to ours is Vercammen (1995). Like us, he studies the effect on incentives of restricting the information available on borrowers' credit histories within a model of repeated lending under moral hazard and adverse selection. In his model, however, the primary benefit of forgetting is to prevent the negative effect on incentives arising from reputation becoming too *good* (see also Mailath and Samuelson, 2001). By contrast, in our paper forgetting helps the agents who have failed — those with the worst reputations. Moreover, the characterization of forgetting seems to be closer than in Vercammen to the institutional details of credit bureau regulation in the United States (in which failures are erased, while successes may be reported forever), and allows to capture its role in giving a fresh start to those who have failed, whose importance was stressed in the Congressional debate discussed above. Finally, note that while in our paper we establish existence of an equilibrium and derive various properties of it as well as a series of conditions under which forgetting is optimal, Vercammen (1995) obtains very limited results for the model he describes and his conclusions rely on an approximated solution of a numerical example (based on a few, rather *ad-hoc* simplifications).

The benefits of limiting the availability of information on borrowers' past histories has also been explored in a few other papers. Padilla and Pagano (2000) show, in the framework of a

two-period model, that it may be optimal for the first-period lender not to share the private information he has acquired regarding the borrower with other lenders, because this allows him to sustain a long-term contractual relationship with the borrower. Also, Crémer (1995) shows that using an inefficient monitoring technology can sometimes improve incentives when the principal cannot commit not to renegotiate, because having less precise information limits the potential for renegotiation, and hence allows for stronger punishments.[7]

Finally, while our paper and the ones mentioned above consider the effect of restricting credit histories on entrepreneurs' incentives and access to credit in a production economy, Chatterjee, Corbae and Rios-Rull (2007) develop a model in which consumers borrow in order to insure themselves against income risk, and weigh the benefits of defaulting against its reputational costs. They then compute an example, and find that restricting credit histories is not beneficial.

II The Model

Consider an economy made up of a continuum (of unit mass) of risk-neutral *entrepreneurs*, who live forever and discount the future at the rate $\beta \leq 1$. In each period $t = 0, 1, \ldots$ an entrepreneur receives a new project, which requires one unit of financing in order to be undertaken. This project yields either R (success) or 0 (failure). Output is non-storable, so entrepreneurs must seek external financing in each period. In addition, there is limited liability, so if a project fails, the entrepreneur is not required to make payments on it. We assume that there are two types of entrepreneurs. There is a set of measure $p_0 \in (0, 1)$ of *safe* agents, whose projects always succeed (i.e., their return is R with probability one), and a set of *risky* agents, with measure $1 - p_0$, for whom the project may fail with some positive probability.[8] The returns on the risky agents' projects are independently and identically distributed among them. The success probability of a risky agent depends on his effort choice. He may choose to exert high effort (h), at a cost $c > 0$ (in units of the consumption good), in which case the success probability will be $\pi_h \in (0, 1)$. Alternatively, he may choose to exert low effort (l). Low effort is costless, but the success probability under low effort is only $\pi_l \in (0, \pi_h)$. We assume:

Assumption 1. $\pi_h R - 1 > c$, $\pi_l R < 1$;

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[7]By contrast, in our model forgetting facilitates financing after failures, thus making punishments *weaker*.
[8]As discussed in Remarks 1 and 4, the property that the safe types' projects never fail is not essential, while a key role is played by the fact that those of the risky types always fail with a positive probability.

i.e., the project has a positive NPV if high effort is exerted (even when the cost of exerting high effort is taken into account), while it has a negative NPV under low effort.

In addition, we require the cost of effort c to be sufficiently high, so that entrepreneurs face a nontrivial incentive problem. The following condition will imply that, when the entrepreneur is known for certain to be risky, high effort cannot be implemented in a static framework.

Assumption 2. $\frac{c}{\pi_h - \pi_l} > R - 1/\pi_h$

Finally, we introduce one further parameter restriction, requiring that π_h and π_l not be too far apart. This condition is used to ensure the existence of an equilibrium.

Assumption 3. $\pi_h^2 \leq \pi_l$

In addition to entrepreneurs, there are lenders, who provide external funding to entrepreneurs in the loan market. More specifically, we assume that in each period there are N profit-maximizing risk-neutral lenders (where N is large) who compete among themselves on the terms of the contracts offered to borrowers. Each lender lives only a single period, and is replaced by a new lender in the following period. Since lenders live only a single period, they cannot write long-term contracts. This is consistent with actual practice in U.S. unsecured credit markets, where borrowers often switch between lenders. Furthermore, as we discuss below (see Remark 2), allowing long-term contracts would result in outcomes that are both more extreme (hence less realistic), and also less efficient, than those we obtain here.

A contract is then simply described by the interest rate r at which an entrepreneur is offered one unit of financing at the beginning of a period (if the entrepreneur is not offered financing in this period then we set $r = \emptyset$). If the project succeeds, the entrepreneur makes the required interest payment r to the lender. On the other hand, if the project fails, the entrepreneur makes no payment and we assume that the debt that was incurred is forgiven, i.e., discharged. So with no loss of generality r can be taken to lie in $[0, R] \cup \emptyset$.

We assume that both an entrepreneur's type, as well as the effort he undertakes, are his private information. The loan market is thus characterized by the presence of both adverse selection and moral hazard. At the same time, in a dynamic framework such as the one we consider, the history of past financing decisions and past outcomes of the projects of an agent may convey some information regarding the agent's type, and may therefore affect the contracts he receives in the future. Hence the agent cares for his reputation as determined by his past history, and this in turn may strengthen his incentives with respect to the static contracting problem. Since lenders do not live beyond the current period, we assume that

there is a *credit bureau* that records this information in every period and makes it available to future lenders.

Let σ_t^i denote the *credit history* of agent $i \in [0, 1]$ at date t, describing for each previous period $\tau < t$ whether the agent's project was funded and if so, whether it succeeded or failed. Hence, denoting by S a success, F a failure, and $\emptyset$ the event where the project is not funded (either because the agent is not offered financing or because he does not accept any offers), σ_t^i is given by a sequence of elements out of $\{S, F, \emptyset\} : \sigma_t^i \in \Sigma_t \equiv \{S, F, \emptyset\}^t$.

We show below that only pooling equilibria can exist in this economy;[9] that is, lenders are unable to separate borrowers by offering a menu of contracts to entrepreneurs with the same credit history. Note, however, that they may optimally choose to differentiate the terms of the contracts offered on the basis of entrepreneurs' credit histories. Hence, without loss of generality we can focus our attention on the case where a lender offers a single contract $r(\sigma_t)$ to borrowers with a given credit history σ_t. We let $\mathcal{C}(\sigma_t)$ denote the set of contracts offered at date t by the N lenders to entrepreneurs with credit history σ_t, and let $\mathbb{C}_t \equiv \cup_{t,\sigma_t \in \Sigma_t} \mathcal{C}(\sigma_t)$ be the set of contracts offered by lenders for any possible history up to date t.

We assume that while lenders present at date t know $\mathbb{C}_t$, i.e., the set of contracts which were *offered* to borrowers in the past, they do not know the particular contracts which were *chosen* by an *individual* borrower. This in line with actual practice; while credit bureaus do not report the actual contracts adopted by individual borrowers, the set of contracts generally offered to borrowers is available from databases such as "Comperemedia".

As discussed earlier, the focus of our paper is the effect of restrictions on the transmission of credit bureau records. We model the *forgetting policy* in this economy as follows: when an entrepreneur's project fails, with probability q the credit bureau ignores the failure and updates the entrepreneur's record as if his project succeeded in that period.[10] That is, S now represents either a success or a failure that is forgotten, and F represents a failure that has not been forgotten. The parameter $q \in [0, 1]$ then describes the forgetting policy in the economy. Note that we take q as being fixed over time, which is in line with existing laws.

We adopt this representation of forgetting to make the analysis simpler, though it is somewhat different from existing institutions (in some inessential ways, we will argue). In practice, defaults are erased with the passage of time, rather than probabilistically. However the effects on borrowers' incentives and access to credit are similar; in particular, the consequences of higher values of q are analogous to those of allowing for a shorter period

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[9]To be precise, we focus on Markov Perfect Equilibria, and show that these must be pooling.

[10]A similar approach is also taken by Padilla and Pagano (2000).

until negative information is forgotten.[11] Also, when a default is finally erased, in practice credit bureaus report a period of no financing, rather than a success as here. As we show below, however, in our setup the informational content of no financing is the same as that of a failure; that is we only have good news and bad news. Hence the recording of a success after a failure should be viewed as the fact that the true event is replaced by one that is more favorable for the borrower, just as in practice.

One might think that forgetting is simply a way to introduce a long-term element to contracts. However, forgetting also has another effect, in that it maintains some pooling of the safe and risky entrepreneurs even after a failure. This allows access to credit at lower rates, and we show that it can therefore provide a benefit even in cases where long-term contracts could not sustain incentives (see Remarks 2 and 3).

The timeline of a single period is then as follows. Each entrepreneur must obtain a loan of 1 unit in the market in order to undertake his project. Lenders simultaneously post the rate at which they are willing to lend in this period to an agent with a given credit history, and do this for all possible credit histories at that date. If an entrepreneur is offered financing, and chooses one of the loans he is offered, he undertakes the project (funds lent cannot be diverted to consumption), and if he is risky he also chooses his effort level. The outcome of the project is then realized: if the project succeeds the entrepreneur uses the revenue R to make the required payment r to the lender, while if the project fails the entrepreneur defaults and makes no payment (since his default is forgiven). Purely for convenience, entrepreneurs are assumed to repay at the end of the same period in which they borrow.

The credit history of the entrepreneur is then updated. If the project was financed, a S is added to his history if the project succeeded in the period (or, with probability q, if it failed), and a F otherwise. If the project was not financed then a $\emptyset$ is added. Next period, the same sequence is repeated: for each updated credit history, lenders choose the contracts they will offer, each entrepreneur then freely chooses the best contract among the ones he is offered, and so on for every t.

To summarize, a lender's strategy consists in the choice of the contract to offer to entrepreneurs at any given date, for any possible credit history. The strategy of an entrepreneur specifies, in every period and for every possible credit history, the choice of the contract among the ones he is offered and, if the entrepreneur is risky, also his choice of effort.

To evaluate the expected profit of a loan offered by a lender to an entrepreneur with credit

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[11]This is exactly so for the polar cases of $q = 0$, which implies that all failures are kept in the record forever, and $q = 1$, where any failure is immediately forgotten.

history σ_t, an important role is played by the lender's belief, $p(\sigma_t)$, that the entrepreneur is a *safe* type (we will sometimes drop the reference to the borrower's credit history and refer simply to p). At the initial date such belief is given by the prior probability p_0. The belief is then updated over time on the basis of the entrepreneur's credit history σ_t, of the contracts $\mathbb{C}_t$ offered in the past, and of the entrepreneurs' borrowing and effort strategies, as we describe in detail below. We will term $p(\sigma_t)$ the *credit score* of the entrepreneur.

III   Equilibrium

A   Markov Perfect Equilibrium

In what follows we will focus on *Markov Perfect Equilibria* (MPE) in which players' strategies depend on past events only through credit scores. A key appeal of such equilibria is not only that players' strategies are simpler, but also that they resemble actual practice in consumer credit markets, where many lending decisions are conditioned on credit scores, most notably the "FICO score" developed by Fair Isaac and Company. In addition, we will discuss below the differences between MPE and other equilibria and argue that in the latter players' behavior is less plausible (see Remarks 2 and 3).

In particular, we will establish the existence and analyze the properties of *symmetric, sequential* MPE, where (i) all agents of a given type (i.e., all lenders, or all safe entrepreneurs, or all risky entrepreneurs with the same credit score) optimally choose the same strategy, (ii) beliefs are determined by Bayes' Rule whenever possible and, when this is not possible, beliefs must be *consistent*. We can now describe more formally the set of players' strategies for the Markov Perfect Equilibria that we consider.

The strategy of an entrepreneur, whatever his type, consists in the choice, for every credit score p he may have, and for any set of contracts $\mathcal{C}'$ he is offered, of accepting or not any of these contracts, and if so, which one. For the safe entrepreneurs we denote this choice by $r^s(p, \mathcal{C}') \in \mathcal{C}' \cup \emptyset$, and for the risky by $r^r(p, \mathcal{C}')$. In addition, a risky entrepreneur has to choose the effort level $e^r(p, \mathcal{C}')$ he exerts. We will allow for mixed strategies with regard to effort; hence the effort level is given by a number $e \in [0, 1]$, denoting the probability with which the entrepreneur exerts high effort.[12] Thus $e = 1$ corresponds to a pure strategy of high (h) effort, $e = 0$ to a pure strategy of low (l) effort.

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[12]This is the only form of mixed strategies that we allow; we demonstrate below that mixing only occurs for at most a single period along the equilibrium path.

Since an entrepreneur's choice depends not only on his immediate payoff (which depends on the current contract), but also on how his project outcome will affect the contracts he is offered in the future, we need to specify how lenders update their beliefs concerning the agent's type in light of the outcome of the current project. Let $p^S(p, \mathcal{C}')$ specify how lenders update their beliefs in case of success (or forgotten failure) of the project of a borrower with credit score p and facing current contracts $\mathcal{C}'$. Analogously, $p^F(p, \mathcal{C}')$ denotes the updated belief in case of a failure (which is not forgotten) and $p^\emptyset(p, \mathcal{C}')$ when the entrepreneur is not financed (we will sometimes omit the arguments and write simply $p^S, p^F, p^\emptyset$). The updated beliefs will be computed according to Bayes' rule whenever possible; when this is not possible they will be required to be *consistent* in the Sequential Perfect Equilibrium sense.

Observation 1. Since only risky agents can fail, $p^F(p, \mathcal{C}') = 0$ for any p and $\mathcal{C}' \neq \emptyset$.

We are now ready to write the formal choice problem for the entrepreneurs. Each period they have to choose which of the offered loan contracts to accept, if any, and which effort level to exert. Let $v^r(p, \mathcal{C}')$ denote the maximal discounted expected utility that a risky entrepreneur with credit score p, facing a set of contracts $\mathcal{C}'$, can obtain, given the lenders' updating rules $p^S(\cdot), p^F(\cdot), p^\emptyset(\cdot)$ and their strategies $\mathcal{C}(\cdot)$, determining future offers of contracts (to simplify the notation we do not make the dependence of v^r on these terms explicit). Observe that $v^r(\cdot)$ is recursively defined as the solution to the following problem:

$$
v^r(p, \mathcal{C}') = \max_{e \in [0,1], r \in \mathcal{C}' \cup \emptyset} \begin{cases} (e\pi_h + (1-e)\pi_l)(R - r) - ec \\ + \beta[e(\pi_h + (1-\pi_h)q) + (1-e)(\pi_l + (1-\pi_l)q)]v^r(p^S, \mathcal{C}(p^S)) \\ + \beta[(e(1-\pi_h) + (1-e)(1-\pi_l)][1-q]v^r(0, \mathcal{C}(0)), & \text{if } r \neq \emptyset; \\ \beta v^r(p^\emptyset, \mathcal{C}(p^\emptyset)), & \text{if } r = \emptyset. \end{cases}
$$
(1)

When the agent chooses to accept a loan he is offered ($r \neq \emptyset$), the first line in (1) represents the expected payoff from the current project, the second the discounted continuation utility when the project succeeds, and the third line gives the discounted continuation utility following failure. Note that in writing this expression we have used the fact that, by Observation 1, $p^F(\cdot) = 0$. When the agent is not financed ($r = \emptyset$), his utility is simply the discounted utility of being financed next period, with his credit score appropriately updated. We denote the solution of problem (1) by $e^r(p, \mathcal{C}'), r^r(p, \mathcal{C}')$, which describes the risky entrepreneur's strategy for all possible values of p and $\mathcal{C}'$.

Analogously, letting $v^s(p, \mathcal{C}')$ be the maximal discounted expected utility for a safe entrepreneur, we have:

$$v^s(p, \mathcal{C}') = \max_{r \in \mathcal{C}' \cup \emptyset} \begin{cases} R - r + \beta v^s(p^S, \mathcal{C}(p^S)) & \text{if } r \neq \emptyset; \\ \beta v^s(p^\emptyset, \mathcal{C}(p^\emptyset)), & \text{if } r = \emptyset. \end{cases} \tag{2}$$

The solution to this problem is denoted by $r^s(p, \mathcal{C}')$.

Since lenders cannot observe the specific contract chosen by an individual borrower in any given period, but only whether or not he was financed, we have:

Observation 2. Whenever an entrepreneur accepts financing, he will choose the contract with the lowest interest rate: i.e., for all $p, \mathcal{C}'$ we have $r^j(p, \mathcal{C}') \in \min(\mathcal{C}') \cup \emptyset$, for $j = s, r$.

Next, we determine the expected profits for an arbitrary lender n from a loan with interest rate r offered to the entrepreneurs with credit score p, given the entrepreneurs' strategies, $r^s(\cdot), r^r(\cdot)$, and $e^r(\cdot)$, and the contracts $\mathcal{C}^{-n}$ offered by the *other* lenders. The expression for lender n's profits will depend on which entrepreneurs accept his offer (if any):

1. No entrepreneur accepts the offer. This will be the case either if the lender offers no contract, or if his offer is higher than the lowest contract offered by other lenders (observation 2), or if both types' strategies are to reject all offers on the table. In this case his profit will be zero. More formally:

$$\Pi(r, p, \mathcal{C}^{-n}, r^s(\cdot), r^r(\cdot), e^r(\cdot)) = 0,$$
$$\text{if either } r > \min(\mathcal{C}^{-n}), \text{ or } r = \emptyset, \text{ or } r^s(p, \mathcal{C}^{-n} \cup r) = \emptyset \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) = \emptyset. \tag{3}$$

2. Only safe entrepreneurs accept. This situation arises when r is at least as low as the rates offered by all of the lenders (needed for the offer to be accepted) and the risky types reject all offers on the table. In this case the lender's profits are determined by the net payments made by the safe entrepreneurs. Since such entrepreneurs have measure p and profits have to be shared with the other lenders offering r (if any), letting $\#(r^{n'} \in \mathcal{C}^{-n} \text{ s.t. } r^{n'} = r)$ denote the number of such lenders, the lender's profits in this case are given by:

$$\Pi(r, p, \mathcal{C}^{-n}, r^s(\cdot), r^r(\cdot), e^r(\cdot)) = pr/[1 + \#(r^{n'} \in \mathcal{C}^{-n} \text{ s.t. } r^{n'} = r)],$$
$$\text{if } r \leq \min(\mathcal{C}^{-n}), r^s(p, \mathcal{C}^{-n} \cup r) \neq \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) = \emptyset \tag{4}$$

3. Only risky entrepreneurs accept. Similarly, if r is again the lowest rate offered, and the safe types reject all offers on the table, profits are given by the net payments made by risky entrepreneurs, shared among all lenders posting r. In this case, the payments made also depends on the risky entrepreneurs' effort choice $e^r(p, \mathcal{C}^{-n} \cup r))$. Recall that $e^r(\cdot) = 0$ corresponds to low effort being exerted, in which case the success probability is π_l, that $e^r(\cdot) = 1$ corresponds to high effort, with success probability π_h, and that $e^r(\cdot) \in (0, 1)$ corresponds to mixing over high and low effort with probability $e^r(\cdot)$. So we have:

$$\Pi(r, p, \mathcal{C}^{-n}, r^s(\cdot), r^r(\cdot), e^r(\cdot)) =$$
$$(1-p)\left\{e^r(p, \mathcal{C}^{-n} \cup r)\pi_h + (1 - e^r(p, \mathcal{C}^{-n} \cup r))\pi_l\right\}r/[1 + \#(r^{n'} \in \mathcal{C}^{-n} \text{ s.t. } r^{n'} = r)], \quad (5)$$
$$\text{if } r \leq \min(\mathcal{C}^{-n}) \text{ and } r^s(p, \mathcal{C}^{-n} \cup r) = \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) \neq \emptyset$$

4. All entrepreneurs accept. In this case profits will simply be the sum of (4) and (5):
$$\Pi(r, p, \mathcal{C}^{-n}, r^s(\cdot), r^r(\cdot), e^r(\cdot)) =$$
$$\left\{p + (1-p)\left[e^r(p, \mathcal{C}^{-n} \cup r)\pi_h + (1 - e^r(p, \mathcal{C}^{-n} \cup r))\pi_l\right]\right\}r/[1 + \#(r^n \in \mathcal{C}^{-n} \text{ s.t. } r^n = r)], \quad (6)$$
$$\text{if } r \leq \min(\mathcal{C}^{-n}), \text{ and } r^S(p, \mathcal{C}^{-n} \cup r) \neq \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) \neq \emptyset$$

Since a lender lives only a single period, his objective is to choose r so as to maximize his expected profits given by (3)-(6). Given our focus on symmetric MPE, we can denote the solution simply by $r(p)$; $\mathcal{C}(p)$ is then the collection of contracts r offered by lenders in equilibrium to entrepreneurs with credit score p.

We are now ready to give a formal definition of a MPE:

Definition 1. A symmetric, sequential Markov Perfect Equilibrium is a collection of lenders' and borrowers' strategies $(r(\cdot), r^s(\cdot), r^r(\cdot), e^r(\cdot))$ and beliefs $p(\cdot)$, such that:

- Lenders maximize profits, given $r^s(\cdot), r^r(\cdot), e^r(\cdot)$: for every p, $r = r(p)$ maximizes (3)-(6), when $\mathcal{C}^{-n} = r(p)$;

- Entrepreneurs' strategies are sequentially rational. That is,

    - for all $p, \mathcal{C}'$, $(r^r(p, \mathcal{C}'), e^r(p, \mathcal{C}'))$ solves (1) when $\mathcal{C}(p') = r(p')$ for all p'.
    - for all $p, \mathcal{C}'$, $r^s(p, \mathcal{C}')$ solves (2) when $\mathcal{C}(p') = r(p')$ for all p'.

- Beliefs are computed via Bayes' Rule whenever possible and are consistent otherwise.

Observe that along the equilibrium path, strategies and beliefs can be written solely as functions of the credit score p, i.e., $r(p), r^r(p), r^s(p), \mathcal{C}(p)$ and $\{p^S(p), p^F(p), p^\emptyset(p)\}$. Similarly, entrepreneurs' discounted expected utility can be written as $v^s(p), v^r(p)$.

The following notation will also be useful. Let $r_{zp}(p, e)$ denote the lowest interest rate consistent with lenders' expected profits being non-negative on a loan to entrepreneurs with credit score p, when all agents accept financing at this rate and risky entrepreneurs exert effort e. That is,

$$r_{zp}(p, e) \equiv \frac{1}{p + (1-p)(e\pi_h + (1-e)\pi_l)}. \quad (7)$$

Also let $p_{\text{NF}} \equiv \frac{1 - \pi_l R}{(1 - \pi_l)R}$ denote the lowest value of p for which this break-even rate is admissible when the risky entrepreneurs exert low effort, i.e. $r_{zp}(p_{\text{NF}}, 0) = R$.

B   Existence and Characterization of Equilibrium

The following proposition establishes that a Markov Perfect Equilibrium exists, and characterizes its properties. The proof is constructive, and we show in the subsequent proposition that this equilibrium is the MPE yielding agents the highest welfare.

Proposition 1. *Under assumptions 1-3, a (symmetric, sequential) Markov Perfect Equilibrium always exists with the following properties:*

i. *Entrepreneurs never refuse financing, and always choose the contract with the lowest interest rate offered to them: $r^s(p, \mathcal{C}') = r^r(p, \mathcal{C}') = \min(\mathcal{C}')$ whenever $\mathcal{C}' \neq \emptyset$. If a borrower does refuse financing, lenders' beliefs are that he is the risky type: $p^\emptyset(p, \mathcal{C}') = 0$ whenever $\mathcal{C}' \neq \emptyset$.*

ii. *Lenders make zero profits in equilibrium: either $r(p) = r_{zp}(p, e^r(p))$, or $r(p) = \emptyset$.*

iii. *Lenders never offer financing to entrepreneurs known to be risky with probability 1: $r(0) = \emptyset$, and so $v^r(0) = 0$.*

Furthermore, along the equilibrium path players' strategies are as follows:

a. *if $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} \leq \frac{c}{\pi_h-\pi_l}$, an entrepreneur is financed if, and only if, $p \geq p_{NF}$ and if risky exerts low effort ($e^r(p) = 0$)*

b. *if $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, there exists $0 < p_l \leq p_m \leq p_h < 1$ such that:*

   - *there is financing if and only if $p \geq p_l$*

   - *risky entrepreneurs exert high effort if $p \geq p_h$, low effort if $p \in [p_l, p_m)$, and mix between high and low effort for $p \in [p_m, p_h)$ (with $e^r(p)$ strictly increasing for $p \in [p_m, p_h)$).*

c. *if $\frac{c}{\pi_h-\pi_l} \leq \frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, there is financing for all $p > 0$, and risky entrepreneurs exert high effort ($e^r(p) = 1$).*

When c is high (region a.), incentives are weak, and risky entrepreneurs exert low effort whenever they are financed. Nevertheless, financing can still obtain as long as p is not too low ($p > p_{\text{NF}}$), since there are enough safe entrepreneurs with credit score p from which the losses made on the loans to risky agents can be recouped. By contrast, when c is low (region c.) incentives are strong enough that the risky entrepreneurs exert high effort for all

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$p > 0$. This makes financing profitable for all $p > 0$. The more interesting case occurs for intermediate values of c (region b.), where incentives depend on p. When p is sufficiently high ($p \geq p_m$), interest rates (both current and future) are low, which makes incentives strong enough that high effort can be sustained. By contrast, when $p < p_m$ interest rates are not sufficiently low to sustain high effort. Moreover, when p is particularly low ($p < p_l$) it is not feasible for lenders to break even, just as in region a.; therefore there will be no financing.

Recall that Markov Perfect Equilibrium requires that lenders use Bayes' Rule to update their beliefs whenever possible. That is,

$$p^S(p, \mathcal{C}') = \frac{p}{p + (1-p)[e^r(p, \mathcal{C}')(\pi_h + (1-\pi_h)q) + (1 - e^r(p, \mathcal{C}'))(\pi_l + (1-\pi_l)q)]},$$

for all $p, \mathcal{C}' \neq \emptyset$. From Observation 1, when agents fail they are known to be risky: $p^F(p, \mathcal{C}') = 0$, for $\mathcal{C}' \neq \emptyset$. Furthermore, when entrepreneurs are not offered any loan ($\mathcal{C}' = \emptyset$), beliefs remain unchanged: $p^\emptyset(p, \emptyset) = p$ for all p. When borrowers refuse financing, which only happens off the equilibrium path, Bayes' Rule cannot be applied. In this case, Property i. of the Proposition specifies that lenders' beliefs are that the borrower is risky, and the proof of the Proposition verifies that this is a consistent belief, and that under such beliefs refusing financing is not optimal.[13]
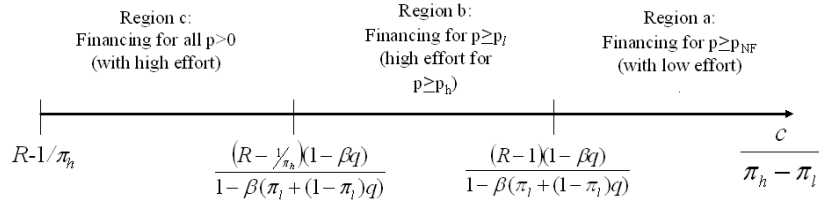
Figure 3: Equilibrium regions

In Figure 3 we plot where regions a., b., and c. lie, in the space of possible values of the effort cost c. Figure 4 then illustrates the equilibrium outcomes obtained in region b., for different values of the credit score p. Recall that $0 < p_l \leq p_m \leq p_h < 1$, so the low-effort and mixing regions may be empty, while the high-effort and no-financing regions always exist.

In proving the Proposition, we first establish property iii. — that entrepreneurs who are known to be risky are never financed — and show that this is actually a general property of

Figure 4: MPE for region b.

Markov equilibria. The basic intuition is that once an entrepreneur is known to be risky, his continuation utility in a Markov Perfect Equilibrium is not affected by the outcome of his project, which by Assumption 2 makes it impossible to provide him with incentives to exert high effort.

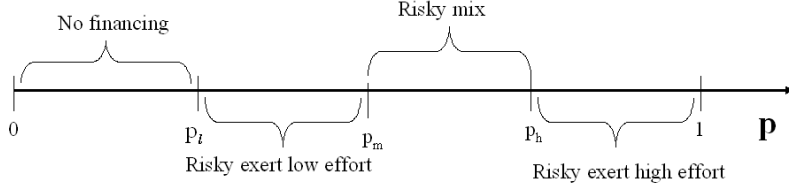**Lemma 1.** *Under Assumptions 1 and 2, any Markov Perfect Equilibrium is characterized by no financing when $p = 0$: i.e., $r(0) = \emptyset$ and hence $v^r(0) = 0$.*

This result implies that, in equilibrium, any entrepreneur who fails is excluded forever from financing (unless this failure is "forgotten").

Note that all symmetric MPE are pooling, by definition, since we have restricted lenders to offering a single contract to entrepreneurs with a given credit score p. We now show that this restriction is not binding — that separating Markov Perfect Equilibria cannot exist.

Lemma 2. *Suppose lenders may offer multiple contracts to entrepreneurs with a given credit score p. Then any (symmetric, sequential) Markov Perfect Equilibrium must be a pooling equilibrium.*

This result is a consequence of the fact that, by Lemma 1, risky entrepreneurs who are separated would not be able to obtain financing.

The rest of the proof of Proposition 1 (in the Appendix) establishes the remaining properties (i. and ii.) of the MPE, and the specific characteristics of the equilibrium we construct for the parameter regions a., b., and c.

Finally, we show that the equilibrium characterized in Proposition 1 is also the MPE that maximizes welfare. The welfare criterion we consider in this paper is the total surplus generated by entrepreneurs' projects that are financed; given agents' risk-neutrality, this is equivalent to the sum of the discounted expected utilities of all agents in the economy, including lenders.

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Proposition 2. *The equilibrium constructed in Proposition 1 is the most efficient MPE.*[14]

To prove the result, we first show that the construction of the equilibrium in Proposition 1 guarantees that the equilibrium implements the highest possible effort at any p. This is clearly true for credit scores $p \geq p_h$, since high effort will be exerted in the current period, as well as in any future round of financing. The same is also true for $p < p_m$, as in the equilibrium of Proposition 1 the risky entrepreneurs exert low effort if financed, and this is the maximal effort level. The result is completed by showing this is true even when $p \in [p_m, p_h)$, i.e. in the mixing region of Proposition 1.

We conclude this section with several remarks concerning the robustness of our results to some of the assumptions and features of the model.

Remark 1. *(Only Risky Agents Fail)* In our setup, when an entrepreneur fails he is identified as risky and in that case can no longer obtain financing (since he would always exert low effort). It is a consequence of the assumption that only risky entrepreneurs can fail; this obviously simplifies the analysis. In section V, we consider an example in which the "safe" agents can also fail; in this case the posterior following a failure is above 0 and may sometimes result in continued financing (see also Mailath and Samuelson, 2001). Nevertheless, we show that the effect of forgetting is qualitatively similar to that obtained here — i.e., forgetting may still improve welfare.

Remark 2. *(Long-term Contracts)* It is also useful to compare the MPE we consider with the equilibria we would obtain if long-term contracts were feasible, that is if lenders lived forever, rather than a single period as assumed. In such case lenders only need to break-even over their life-time, and not period-by-period, and so could use the time profile of their contracts to screen for safe borrowers. This would lead to rather extreme and unrealistic contracts in equilibrium, where any net revenue to borrowers from the projects financed is postponed as far into the future as possible: that is, the interest payments would equal R in the initial periods, and subsequently zero. Since risky entrepreneurs are at risk of failing and being excluded from the pool of borrowers in the future, contracts that postpone payments to borrowers are less attractive to the risky and more attractive to the safe.

Nevertheless, in regions a. and b. a separating equilibrium could not exist, as the risky entrepreneurs' effort cost is high enough that they cannot obtain financing on their own (see Remark 3 below). Moreover, this equilibrium will be less efficient than the one with short-term contracts (and forgetting) that we study in this paper. The reason is that the postponement of payments occurring with long-term contracts decreases the cross-subsidy from

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[14]When $q \in (0, 1)$ we require an additional condition to prove this result: $\pi_l \geq \pi_h \frac{q}{1+q}$. This condition is implied by Assumption 3 when $\pi_h \geq 1/2$.

safe to risky entrepreneurs, and this will have a negative impact on the risky entrepreneurs' incentives; overall there will be fewer periods of financing where high effort is exerted. In region c., the risky entrepreneurs will be able to obtain financing on their own, and so may prefer to separate, rather than take a contract in which payments are postponed.[15] However, such a separating equilibrium will also be less efficient than the one with short-term contracts and forgetting, as the lack of any cross-subsidy means again that incentives are weaker and financing to risky entrepreneurs have to be limited.

Remark 3. *(Non-Markov Equilibria)* Observe that the Markov property of players' strategies only binds at nodes where the entrepreneur is not financed or when $p = 0$. This is because when an agent with $p > 0$ is financed, the updated belief in case of success will always be higher than the prior one, so p never hits the same value twice.

At non-Markov equilibria, by contrast, lenders' strategies may not be the same each time p equals zero. For example, the agent may continue to be financed the first time he fails, as well as at any successor node as long as his project succeeds, but permanently denied financing after a second failure. This threat of exclusion after two failures could be enough to induce high effort and hence to make financing profitable for lenders.

Since these strategies imply that the entrepreneur is not treated identically at different nodes with $p = 0$, they require some coordination among lenders. Thus such non-Markov equilibria appear rather fragile, being open to the possibility of breakdowns in coordination, or to renegotiation (which is not the case for the MPE we consider).

Moreover, while these equilibria have some similarities with the MPE with forgetting, in that a risky entrepreneur who fails may obtain additional periods of financing, they only exist when c is low and lies in region c. of Proposition 1, so that incentives are sufficiently strong. By contrast, in our MPE, efficient financing (i.e., with high effort) occurs after a failure is forgotten also for intermediate values of c (lying in region b.). This is because forgetting a failure in our setup entails pooling the risky types with the safe anew. So their reputation improves, which allows them to obtain a lower interest rate, and this further enhances incentives (see also Proposition 4 below).

IV   Optimal Forgetting

In this section we derive conditions under which forgetting entrepreneurs' failures is a socially optimal policy. That is when, in the equilibrium characterized in Proposition 1, total surplus

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[15]For instance, when β is close to 1 it is not hard to show that a separating equilibrium exists.

is higher when $q > 0$ than when $q = 0$.

What are the effects of the forgetting policy on the equilibrium properties? When we are in regions a. and c. of Proposition 1, q has no effect on the surplus generated in equilibrium by financing safe entrepreneurs. This is because, within both regions, the set of nodes for which the safe agents are financed does not depend on q: in region c. there is financing for all $p > 0$, and in region a. there is financing for $p > p_{\text{NF}}$ (recall p_{NF} does not depend on q). So in these cases the only effect of q is on the surplus generated by financing risky entrepreneurs.[16]

A first effect on this surplus due to an increase in q is that the probability that a risky entrepreneur will be excluded from financing decreases: failure of his project leads to exclusion only with probability $1 - q$. Hence forgetting gives the possibility of a fresh start after a failure. The impact of this on welfare depends on the effort choice of the risky entrepreneur *after* his failure is forgotten. If he exerts high effort (as will be the case in region c.), this extra period of financing makes a strictly positive contribution to the social surplus, given by $G \equiv \pi_h R - 1 - c > 0$. Under low effort, however (as in region a.), the contribution is strictly negative: $B \equiv \pi_l R - 1 < 0$.

But increasing q has another effect that needs to be taken into account: since exclusion after a project's failure is less likely, the incentives to exert high effort *before* a failure is forgotten will be weaker. In region a. (where low effort is always exerted when financing takes place), the weakening of incentives manifests itself in the fact that the lower bound of this region, $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, is decreasing in q, so that this region expands when q is increased. Analogously, the upper bound of parameter region c. (where high effort is always exerted), $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, also decreases in q, so that this region becomes smaller when q is higher.

Let $q(p_0)$ denote the welfare maximizing level of q (which clearly depends on the proportion p_0 of risky types in the population, as the equilibrium depends on it). From the above discussion the optimal forgetting policy when the parameters of the economy are in region a. or c. of Proposition 1 (for $q = 0$) immediately obtains:

Proposition 3. *The welfare maximizing forgetting policy respectively for high and low values of c is as follows:*

1. *If $\frac{c}{\pi_h - \pi_l} \geq \frac{R-1}{1-\beta\pi_l}$, no forgetting is optimal for all p_0: $q(p_0) = 0$.*

2. *If $\frac{c}{\pi_h - \pi_l} < \frac{R-1/\pi_h}{1-\beta\pi_l}$, for any $p_0 > 0$ some degree of forgetting is optimal: $q(p_0) > 0$.*

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[16]However, the utility the safe entrepreneurs obtain will typically be lower with forgetting, as more risky entrepreneurs remain in the pool in any given period, and hence the interest rate charged is higher.

Thus in region c., when incentives are strong and high effort is implemented everywhere, some positive level of forgetting is optimal.

We now turn our attention to region b., the intermediate values of c. Here the threshold of the value of p above which financing occurs is given by $p_l(q)$ and may thus vary with q, in which case the surplus generated by the financing of safe entrepreneurs also varies with q. However since, as shown in the proof of Proposition 1, $p_l(q) \leq p_{\text{NF}}$ for all q, as long as we restrict attention to prior probabilities $p_0 > p_{\text{NF}}$ this effect remains absent and we have financing in the initial period regardless of the level of q.

Another important feature of region b. is that the level of effort varies along the equilibrium path (switching at some point from low to high). The weakening of incentives due to forgetting now manifests itself not only in the change of the boundaries of this region, which again shift to the left as q increases, but also in the change of the points in the equilibrium paths where the switch from low to high effort takes place. Such switching points are identified by the levels of $p_h(q)$ and $p_m(q)$ introduced in Proposition 1.[17] These switching points are key to the analysis of the welfare impact of raising q, since an extra period of financing with high effort makes a positive contribution to the social surplus, while one with low effort makes a negative contribution.

Notice first that when $p_0 > p_h(0)$ high effort is always exerted by a risky entrepreneur when financed. Hence an analogous argument to that used to prove case 2. of Proposition 3 establishes that the socially optimal level of q is above 0 in this case.

On the other hand, when $p_0 \leq p_h(0)$ raising q above 0, while leading to a lower probability of exclusion, does not necessarily increase welfare. There is a tradeoff between the positive effect when high effort is exerted (this happens after a sufficiently long string of successes allows p to exceed $p_h(q)$), and the negative effect when low effort may be exerted (when $p < p_h(q)$). There are in fact two facets to the negative effect when $p < p_h(q)$. First, as discussed above, an agent whose failure is forgotten has an opportunity to exert low effort once again. In addition, however, raising q will "slow down the updating". That is, $p^S(p)$ will be closer to p, and thus a longer string of successes will be required until a risky entrepreneur switches to high effort. There is also one last effect which must be considered: the change in $p_h(q)$. In particular, we would expect that $p_h(q)$ increases with q (so that we have another negative effect on welfare), since the fact that failures are less costly can weaken incentives; this is indeed typically, though not always,[18] the case.

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[17]The dependence of these switching points on q is now highlighted.

[18]Because a higher value of q also increases the continuation utility upon success.

We will show in what follows that in region b. the positive effect of raising q prevails over the negative ones also when $p_0 \leq p_h(0)$, provided (i) p_0 is sufficiently close to $p_h(0)$, (ii) agents are sufficiently patient (β close to 1), and (iii) $|B|$ is sufficiently small relative to G. The first two conditions, in particular, are needed because the positive effect follows the negative ones along the equilibrium path. The third condition more generally limits the extent to which low effort reduces welfare.

Proposition 4. *For intermediate values of c, $\frac{R-1/\pi_h}{1-\beta\pi_l} \leq \frac{c}{\pi_h-\pi_l} < \frac{R-1}{1-\beta\pi_l}$, the optimal policy might also exhibit forgetting. More precisely, when $p_0 > p_{NF}$:*

1. *If $p_0 > p_h(0)$, welfare is always maximized at $q(p_0) > 0$.*

2. *If $p_0 \in [p_{NF}, p_h(0)]$ and $-\frac{B}{G} < \frac{p_0(1-p_h(0))(1-\pi_l)}{p_h(0)((1-\pi_h)(1+(1-p_0)\pi_h))+\pi_h^2-p_0(1-\pi_h+\pi_h^2)}$, then for β sufficiently close to 1 we also have $q(p_0) > 0$.*

While the condition in case 2. is stated in terms of $p_h(0)$, which is an endogenous variable, it is possible to show that it is not vacuous[19] (see also the example below). Figure 5 illustrates the welfare-maximizing forgetting policy, as derived in Propositions 3 and 4, as a function of the cost of effort c.
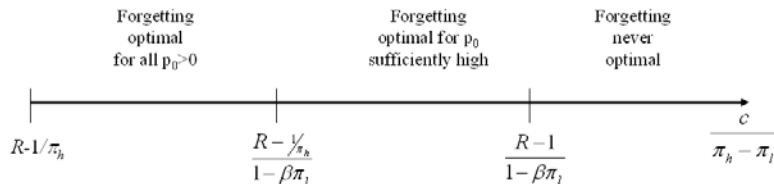
Figure 5: Welfare-maximizing forgetting policy, as a function of c

While the previous results give conditions under which some $q > 0$ maximizes total welfare, we can also determine when $q(p_0) = 1$, i.e., when it is optimal to keep no record of any failure. Since when $q = 1$ the updated belief after a success is $p^S(p) = p$ for all p, $q = 1$ is optimal if and only if $p_h(1) \leq 1$ and $p_0 \geq p_h(1)$. More precisely:

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[19]In particular, let $\pi_l \to 1/R$, so that $B \to 0$. Holding c and R fixed, it is not hard to show that $p_h(0)$ will be bounded away from 0, so that the condition will be satisfied.

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Proposition 5. $q = 1$ *maximizes total welfare if and only if* $p_0 \geq p_h(1) = \frac{1-\pi_h(R-\frac{c}{\pi_h-\pi_l})}{(1-\pi_h)(R-\frac{c}{\pi_h-\pi_l})}$ *and* $\frac{c}{\pi_h-\pi_l} \leq (R-1)$.[20]

Remark 4. (Risky Entrepreneurs Can Fail Even Under High Effort) As we discussed above, forgetting failures provides a social benefit through the additional periods of financing under high effort which it permits. In light of this, we can also understand the importance of our assumption that the risky entrepreneur can fail even when he exerts high effort, i.e., that $\pi_h < 1$. When this is not the case and we have $\pi_h = 1$ (as, for example, in Diamond, 1989) high effort ensures success, and there is no benefit from forgetting a failure, since such failures only result from low effort.

An Example We consider here a numerical example to illustrate the results obtained. Let $R = 3, \pi_h = 0.5$, $\pi_l = 0.32$, $c = 0.4$ and $\beta = 0.975$. For these values assumptions 1 and 2 are satisfied and we are in region b. of Proposition 1, for which high effort is implemented in equilibrium when $p \geq p_h(q)$. The threshold $p_h(0)$ above which high effort is exerted when $q = 0$ can be computed from equation (12) in the Appendix, which yields: $p_h(0) = 0.241$.

When p_0 is above this threshold (i.e., above $p_h(0) = 0.241$), from Proposition 4 we know that $q(p_0) > 0$ is optimal, because forgetting failures increases the rounds of financing to risky entrepreneurs and in these new rounds they always exert high effort. As shown in figure 7 below, the optimal forgetting policy $q(p_0)$ in this region is given by high values of q (close to 1).

When $p_0 \in [p_{\text{NF}}, p_h(0)) = [0.0196, 0.241)$ low effort is exerted.[21] For the parameters of this example the condition stated in 2. of Proposition 4 is satisfied if $p_0 > 0.205$: the increase in surplus $G = \pi_h R - 1 - c = 0.1$ from a project undertaken with high effort is high, relative to the decrease in surplus $B = -0.04$ from a project undertaken with low effort. Hence some degree of forgetting will be optimal for β sufficiently close to 1 since the additional periods of high effort provided by forgetting outweigh the cost of the extra periods of low effort at the start of the game. We will verify that this is indeed the case when $\beta = 0.975$.

Consider $p_0 = 0.206$. When $q = 0$, we have $p^S(p_0) = 0.448 > p_h(0)$, and so low effort is exerted for the first round of financing along the equilibrium path, and high effort forever after, as long as the projects succeed. On the other hand, when $q > 0$, more rounds of

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[20]The latter condition is equivalent to $p_h(1) \leq 1$ and is always satisfied for c sufficiently close to its minimal value, as defined by Assumption 2. The proof of Proposition 5 is straightforward, and can be found in Appendix B.

[21]The risky entrepreneurs never randomize in their effort choice along the equilibrium path for any of the values of p_0 and q considered in this example.
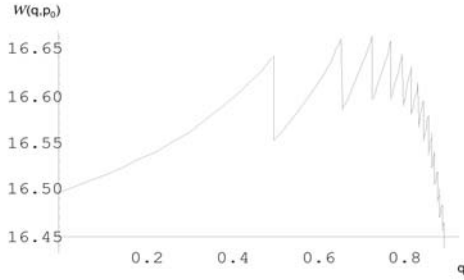
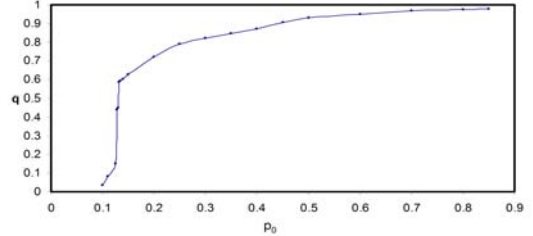Figure 6: Total welfare as a function of q (when $p_0 = 0.206$)

Figure 7: Welfare-maximizing value of q

financing with low effort may be needed before risky entrepreneurs begin to exert high effort, both because the updating is slower and because $p_h(q)$ is higher. For example, with $q = 0.735$ three periods of financing with low effort followed by success of the project are needed until the posterior exceeds $p_h(0.735) = 0.322$. By comparing welfare levels for different specification of the forgetting policy we can then find the optimal policy $q(p_0)$.[22] For instance, as we can see in figure 6, when $p_0 = 0.206$ the optimum is $q(0.206) = 0.77$. Figure 7 plots the optimal policy for all values of the prior probability $p_0 \in (0.1, 0.9)$.[23]

Discussion — Empirical Evidence and Policy Implications

Our model captures many of the key points made in the Congressional debate surrounding the adoption of the FCRA, which we summarized in the Introduction. As such, it allows us to determine conditions under which the positive arguments prevail over the negative ones.

Notice first that the main argument put forward in favor of forgetting — that it allows individuals to obtain a true fresh start and hence to continue being productive members of society — is echoed in our model, where the positive effect on welfare of forgetting is that it gives risky entrepreneurs who fail access to new financing.[24] They sometimes exert high effort, and hence this may increase aggregate surplus. Furthermore, all of the arguments

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[22]We discretize the domains of p_0 and q. For each point in the grid we compute $p_h(q)$ and then the welfare $\mathcal{W}(q, p_0)$. We assign $q(p_0)$ to be the value of q that maximizes this surplus, given p_0.

[23]Although the condition in 2. of Proposition 4 is violated for $p_0 \leq 0.205$, we can nevertheless still have $q(p_0) > 0$, since the condition is only sufficient, not necessary.

[24]Two other arguments were also made in favor of forgetting — that old information may be less relevant, and limited storage space — these do not have a role in our model. Furthermore, even if old information were less relevant (as would be the case if the type of an entrepreneur could change), lenders would take this into account and give it less weight in equilibrium.

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made against forgetting also operate in our model: (i) forgetting weakens incentives by reducing the penalty for failure — in our set-up, as we raise q, region c. shrinks, and region a. increases in size; (ii) by erasing the records of those who were bad risks in the past, there is an increased risk that frauds will be committed in the future — the analog in our model is that forgetting "slows down" the weeding out of risky entrepreneurs, hence the average quality of borrowers is lower; (iii) forgetting can lead to tighter lending standards — in our model this may be seen most sharply in the fact that raising q can shift us from region c., where there is financing for all $p > 0$, to region b. (where there is only financing for $p \geq p_l > 0$).[25] In addition, while the policy debate suggested that (iv) another negative effect of forgetting is that it forces safe agents to subsidize the risky ones, this is in fact socially optimal in our model, because it thereby improves the risky entrepreneurs' incentives.[26]

Our results are also consistent with the empirical evidence in Musto (2004). Forgetting clearly leads to higher credit scores for those who fail, and thus to more credit — in our model without forgetting they would have $p = 0$, and no credit. Moreover, Musto's second point — that those who have their failure forgotten are likelier to fail in the future than those who are observationally equivalent (i.e. with the same score) is also an implication of the model, since only the risky agents ever have their failure forgotten. However, in contrast to Musto's suggestion that these laws are inefficient, Propositions 3 and 4 show that forgetting may be optimal.

Our model can also help us understand the international evidence, and in particular the relationship between forgetting clauses and the provision of credit. An implication of our model is that, if the forgetting clause is optimally determined and economies only differ with regard to the strength of the incentive problems in them (as captured by c), there will be a positive relationship between credit volume and the degree of forgetting (as measured by q). The first reason is that forgetting is optimal when incentives are strong, i.e. for low values of c. Also, in this case, the introduction of a forgetting policy further increases the volume of credit, since it gives entrepreneurs who fail another chance at financing. This relationship is consistent with the empirical evidence reported in Figure 2 for those countries that have a credit bureau in place. Those countries in which information is only reported for a limited period of time have higher provision of credit than those which never forget defaults.

But what about those countries with no credit bureau, i.e., in which there is no information sharing? In our model this would only be optimal for very low values of c, in which

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[25]See Proposition 1. Just as suggested in the policy debate, the cohorts who are excluded from financing as a result of the introduction of such a policy are those with a low credit score p_0 — the worst risks.

[26]Since only they face a moral hazard problem.

case credit would be plentiful. However, these countries actually have the lowest provision of credit in the data. One way to understand this is that these countries typically have financial systems that are not fully developed, and that the introduction of a credit bureau would be beneficial in such a case (see, for example, Djankov, McLiesh and Shleifer 2007, and Brown, Jappelli, and Pagano 2007). And indeed, from the historical record shown in Figure 1, we can see that the fraction of countries with no credit bureau has been shrinking over time, whereas the relative shares of the other two groups have remained stable.

Finally, while we have shown that forgetting past defaults can be welfare improving, this would never arise in equilibrium as the outcome of the choice of lenders. As shown in Lemma 1, there cannot exist any Markov Perfect Equilibrium in which agents who are known to be risky (as is the case for those who failed) obtain financing. Thus forgetting can only occur through government regulation of the credit bureau's disclosure policies.

V   Extension — Both Types can Fail

The equilibria in the model considered exhibit the stark feature that, in the absence of forgetting, one single failure implies permanent exclusion from financing. This is due to the fact that the projects of safe types never fail, hence one failure entails a large drop in reputation as it perfectly identifies an entrepreneur as a risky one. In this section we extend the analysis to the case where also the projects of safe entrepreneurs can fail, though with a smaller probability (so, more precisely, they are "safer"). Hence an entrepreneur who defaults can no longer be identified for certain as risky, and so may be able to obtain additional periods of financing even without forgetting. As a result, one might think that forgetting would be superfluous.

We show here that this is not the case and that our central finding — that forgetting defaults may be welfare-enhancing — continues to hold. More precisely, we show that forgetting continues to provide a benefit even though without it agents may still obtain some financing after they fail. The reason is twofold: first, a sufficient number of failures will always result in exclusion; second, the safe types are now also at risk of exclusion, and so can also benefit from forgetting. These properties are established through the analysis of an example. We conjecture that they hold more generally, although a complete analysis of this case is beyond the scope of the current paper.

Let $\pi \in (\pi_h, 1]$ denote the probability that the project of a safe entrepreneur fails. Consider the following parameter values: $R = 3, \pi_h = 0.5, \pi_l = 0.32, \beta = 0.975, c = 0.35$, and

$p_0 = 0.1$. When $\pi = 1$ these values lie in region b. of Proposition 1, high effort is exerted for all $p \geq p_h(0) = 0.113$ and the optimal forgetting policy can be derived in the same way as in the previous example; when $p_0 = 0.1$ it is $q = 0.77$.

Next, suppose projects of safe entrepreneurs only succeed with probability $\pi = 0.99$. We find that the equilibrium strategies exhibit, in most respects, analogous properties to those found in Proposition 1.[27] In particular, when $q = 0$ there is an MPE where high effort is exerted for all $p \geq p_h(0) = 0.1065$,[28] and entrepreneurs are financed for $p \geq p_l = p_{\mathrm{NF}} = 0.0199$. We can similarly find the equilibria for positive values of q. Comparing total surplus we determine the optimal policy, which when $p_0 = 0.1$ is given by $q = 0.80$. Thus forgetting provides a benefit even when the safe entrepreneurs can fail.

There are two interesting features of this example. The first is that for $p > 0.58$ an agent will be able to obtain financing after a failure. The overall effect of this on incentives is positive, since it raises agents's continuation utility upon success. This is why $p_h(0)$ is now lower; that is, the region of p for which high effort is exerted is larger. The second feature is that the optimal value of q is higher. The reason is that forgetting now also increases the surplus generated by the projects of safe entrepreneurs who are financed, since they too are at risk of failing, and hence of being excluded (by contrast, when $\pi = 1$ this surplus was either unaffected, or decreased, by the introduction of forgetting).

VI   Conclusion

In this paper we have investigated the effects of restrictions on the information available to lenders on borrowers' past performance. These restrictions may facilitate a "fresh start" for borrowers in distress, but also clearly affect their incentives. To analyze them, we have considered an environment where borrowers need to seek funds repeatedly, and the borrower-lender relationship is characterized by the presence of both moral hazard and adverse selection. In such a framework we have determined the effects of these restrictions on borrowers' incentives as well as on lenders' behavior, and hence on access to credit and overall welfare. We found that imposing limits on the information available to lenders is desirable when either borrowers' incentives are sufficiently strong or the average quality of borrowers in the

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[27]To construct a Markov Perfect Equilibrium we must however follow a different procedure, because the continuation utility for an agent who fails will no longer be equal to zero. Hence we discretize the domain of p and, for each pair of candidates values for $p_l \geq p_{\mathrm{NF}}$ and $p_h < 1$, we compute the value function for the risky entrepreneurs, using value function iteration.

[28]For higher values of c, however, high effort might no longer be sustained for p very close to 1 — incentives deteriorate when reputation is very good. See Mailath and Samuelson (2001) for further discussion.

market is not too low. Even if neither of the conditions are satisfied, the result still holds provided the cost of bad incentives is not too high and agents are sufficiently patient. In these cases imposing such limits is welfare improving and increases credit volume, otherwise the reverse may obtain. We also show that these findings may help explain the empirical evidence.

As noted in the Introduction, there are some cross-country differences in the laws governing the memory of the credit reporting system; in general, European countries tend to allow defaults to be forgotten more quickly. In addition, bankruptcy laws, which govern the extent to which defaulting borrowers can shield assets and income, can also differ dramatically across countries. It would be interesting to study how these features of credit markets interact, and how they are related to differences in the economic environments.

VII   Appendix — Proofs

Lemma 1 — No financing when known to be risky

If $p = 0$, we must have $p^S(p, \mathcal{C}') = 0 = p^F(p, \mathcal{C}')$ whatever $\mathcal{C}'$, i.e., the agent will be known to be risky in the future as well. Furthermore, under assumption 1, if the agent is known to be the risky type, he can only be financed in a given period if he exerts high effort with some probability, as otherwise lenders cannot break even. But for high effort (or mixing) to be incentive compatible, the utility from high effort must be no less than that from low effort, i.e., the interest rate r offered must be such that:

$$\pi_h(R - r) - c + (\pi_h + (1 - \pi_h)q)\beta v^r(p^S(0)) + (1 - \pi_h)(1 - q)\beta v^r(p^F(0)) \geq$$

$$\pi_l(R - r) + (\pi_l + (1 - \pi_l)q)\beta v^r(p^S(0)) + (1 - \pi_l)(1 - q)\beta v^r(p^F(0)),$$

which simplifies to the static incentive compatibility condition:

$$\frac{c}{\pi_h - \pi_l} \leq R - r, \tag{8}$$

since when $p = 0$ we have $p^S = p^F = 0$.

By assumption 2, this can only be satisfied if $r < 1/\pi_h$, in which case lenders cannot break even. Thus the agent cannot be financed in equilibrium if he is known to be risky. Finally, since this agent is never financed, it is immediate that $v^r(0) = 0$.∎

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Lemma 2 — All MPE are pooling

Suppose this is not the case; consider a candidate separating equilibrium. Let r^s denote the contracts chosen by the safe types and r^r those chosen by the risky ones in such an equilibrium. From Lemma 1 we know that in a separating MPE the risky types cannot be financed, i.e. we must have $r^r = \emptyset$ for all nodes along the equilibrium path, and so their utility is $v^r = 0$. Hence for the risky entrepreneurs not to pretend to be safe, we must have either $r^s = R$ in every period, or $r^s = \emptyset$ in every period (the contract must be the same in every period by the Markov property). But if $r^s = \emptyset$ the equilibrium would in fact be pooling, contrary to the stated claim. We now argue that $r^s = R$ cannot be an equilibrium strategy for lenders, because each lender would have an incentive to undercut.

Consider, in particular, some future period $t > 0$. In such period a lender can deviate and offer $R - \epsilon$ (for ϵ small) to the safe entrepreneurs. Note that this offer can be made to the safe agents alone because the credit history of a safe agent differs from that of a risky one by virtue of the fact that only the safe are financed in the initial period in the proposed equilibrium. This deviation would clearly be profitable, thus overturning the proposed equilibrium. ∎

Proposition 1 — Characterization of the Equilibrium

To complete the proof of Proposition 1, we establish the remaining properties of the MPE, i. and ii., and the specific features of this equilibrium for parameter regions a., b., and c.

We begin by verifying property i. First note that the second part of property i. follows immediately from Observation 2. It is also easy to verify the third part of property i.: a consistent belief for lenders is that an entrepreneur is risky if he refuses financing. To see this, simply let the risky entrepreneurs refuse financing at some node with probability $\epsilon > 0$, and the safe ones with probability ϵ^2, and let $\epsilon \to 0$. Consistency of the above belief can then readily be verified using Bayes' Rule. This immediately proves the first part of property i. as well, since from Lemma 1 refusing financing would give an entrepreneur a utility of 0.

We now verify the characterization of the equilibrium strategies provided for each region, and show that there are no profitable deviations by lenders.

a. To show that the strategies specified in the Proposition constitute an MPE when $\frac{c}{\pi_h - \pi_l} \geq \frac{(R-1)(1-\beta q)}{1 - \beta(\pi_l + (1-\pi_l)q)}$, we need to demonstrate that (a-i) low effort is incentive compatible for $p \geq p_{\text{NF}}$; (a-ii) $r(p) = r_{zp}(p, 0) \leq R$ for $p \geq p_{\text{NF}}$, i.e., it is admissible; and (a-iii) there are no profitable deviations by lenders.

a-i. Given the above strategies and implied beliefs, from (1) we get:

$$v^r(p) = \pi_l(R - r_{zp}(p, 0)) + (\pi_l + (1 - \pi_l)q)\beta v^r(p^S(p)), \tag{9}$$

since from Lemma 1, $v^r(p^F(p)) = v^r(0) = 0$.

By the same argument used to derive (8) above, for low effort to be IC we need:

$$\frac{c}{\pi_h - \pi_l} \geq R - r_{zp}(p, 0) + \beta(1 - q)v^r(p^S(p)), \tag{10}$$

Since $r_{zp}(p, 0) > r_{zp}(1, 0) = 1$ for all $p < 1$,

$$v^r(p) < \frac{\pi_l(R - 1)}{1 - \beta(\pi_l + (1 - \pi_l)q)},$$

where the term on the right-hand side is the present discounted utility of a risky entrepreneur who is financed in every period (until he has a failure that is not forgotten), exerting low effort, and at the rate $r = 1$.

So for any $p \in (p_{\mathrm{NF}}, 1)$, we have

$$\begin{aligned}
R - r_{zp}(p, 0) + \beta(1 - q)v^r(p^S(p)) \quad &< R - 1 + \beta(1 - q)\frac{\pi_l(R-1)}{1 - \beta(\pi_l + (1 - \pi_l)q)} \\
= \frac{(R-1)(1 - \beta q)}{1 - \beta(\pi_l + (1 - \pi_l)q)} &< \frac{c}{\pi_h - \pi_l}
\end{aligned}$$

where the last inequality follows from the definition of region a. This verifies (10).

a-ii. $r_{zp}(p, 0) \leq R$ if and only if $\frac{1}{p + (1 - p)\pi_l} \leq R$, or equivalently $p \geq p_{\mathrm{NF}}$.

a-iii. Consider a deviation by a lender. First note that lenders make zero profits in equilibrium, so refusing to offer a contract would never be profitable. So consider a deviation consisting of the offer of a contract r' to entrepreneurs with credit score p. Without loss of generality we can restrict attention to $r' > 1$, since otherwise the deviation could never be profitable. Let the new set of contracts (which includes the deviation r') be $\mathcal{C}'$. Since $r' > 1$, by the same argument as in a-i. above we can show that the optimal response by risky entrepreneurs who accept r' is to exert low effort, i.e., $e^r(p, \mathcal{C}') = 0$. This implies that lenders cannot profit from r'. To see this, first note that if $r' \leq r_{zp}(p, 0)$, since low effort is exerted this deviation is not profitable. Alternatively, suppose that $r' > r_{zp}(p, 0)$. If $p \geq p_{\mathrm{NF}}$, this would imply $r' > r(p)$ and so no borrower would accept this contract. If $p < p_{\mathrm{NF}}$, however, by the definition of p_{NF}, we must have $r' > R$ and this deviation would not be admissible.

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b. Next, we show that for intermediate values of c, $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, an MPE exists characterized by $0 < p_l \leq p_m \leq p_h < 1$ such that: for $p \geq p_l$ entrepreneurs are always financed, $e^r(p) = 1$ for $p \geq p_h$, $e^r(p) \in (0,1)$ and is (strictly) increasing in p for $p \in [p_m, p_h)$, $e^r(p) = 0$ for $p \in [p_l, p_m)$ and $r(p) = r_{zp}(p, e^r(p))$.

We begin by characterizing the values of (b-i) p_h, (b-ii) p_m and (b-iii) p_l, showing that the effort choices specified above for the risky entrepreneurs are optimal. In (b-iv) we demonstrate that there are no profitable deviations for lenders.

b-i. Let $\tilde{p}^S(p,e) \equiv \frac{p}{p+(1-p)[e(\pi_h+(1-\pi_h)q)+(1-e)(\pi_l+(1-\pi_l)q)]}$; this is the posterior belief, following a success, that an entrepreneur is risky, when the prior belief is $p \in (0,1)$ and the effort undertaken if risky is e, calculated via Bayes' Rule. Also, let $\tilde{v}^r(p,1)$ denote the discounted expected utility for a risky entrepreneur with credit score p when he is financed in every period until experiencing a failure that is not forgotten, he exerts high effort ($e = 1$), beliefs are updated according to $\tilde{p}^S(p,1)$ and the interest rate is $r_{zp}(p',1)$ for all $p' \geq p$. Then $\tilde{v}^r(p,1)$ satisfies the following equation:[29]

$$\tilde{v}^r(p,1) = \pi_h(R - r_{zp}(p,1)) - c + \beta(\pi_h + (1-\pi_h)q)\tilde{v}^r(\tilde{p}^S(p,1),1). \qquad (11)$$

We then define p_h as the value of p that satisfies the following equality:

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h,1),1) \qquad (12)$$

Observe that, since $\tilde{p}^S(p,1)$ is strictly increasing in p, and $r_{zp}(p,1)$ is strictly decreasing, $\tilde{v}^r(p,1)$ is strictly increasing in p. Thus the term on the right-hand side of (12) is increasing in p, and so (12) has at most one solution.

By a continuity argument, it can then be verified that:

Claim 1. *A solution $p_h \in (0,1)$ to (12) always exists.*[30]

Given the monotonicity of the term on the right-hand side of (12), it is then immediate that the incentive compatibility constraint for high effort (8) is satisfied for all $p \geq p_h$.

b-ii. Next, we find p_m, the lower bound of the region where risky agents mix over effort.

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[29] Note that while $\tilde{v}^r(p,1)$ and $\tilde{p}^S(p,e)$ are well defined for all $p \in (0,1)$, they only coincide with the equilibrium values $v^r(p)$ and $p^S(p)$ when both $p \geq p_h$ and $e = e^r(p) = 1$.

[30] The proofs of claims 1-6 can be found in appendix B.

For mixing to be an equilibrium strategy at p, risky entrepreneurs must be indifferent between high and low effort, i.e.,

$$R - r_{zp}(p, e) + \beta(1-q)v^r(\tilde{p}^S(p, e)) = \frac{c}{\pi_h - \pi_l} \tag{13}$$

for some $e \in [0, 1]$. Now, let $\left(\tilde{p}^S\right)^{-1}(p_h, 1)$ denote the preimage of p_h according to the map $\tilde{p}^S(p, 1)$, i.e., $\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h, 1), 1\right) = p_h$.[31] We define p_m to be the lowest value of $p \geq \left(\tilde{p}^S\right)^{-1}(p_h, 1)$ for which a solution of (13) can be found for some e. By the construction of p_h, $e = 1$ is a solution to (13) when $p = p_h$, and so $p_m \leq p_h$. It can be shown that:

Claim 2. *A lowest value p_m always exists and, moreover, $p_m > \left(\tilde{p}^S\right)^{-1}(p_h, 1)$.*

This implies that there is at most a single period of mixing along the equilibrium path. It can also be shown that:

Claim 3. *For all $p \in [p_m, p_h]$, there exists a solution $e^r(p)$ to (13), with $e^r(p)$ strictly increasing in p.*

If there is more than one solution to (13) at p, we choose the highest.

b-iii. We determine p_l, the lower bound on the financing region, and demonstrate that low effort is incentive compatible in $[p_l, p_m)$.

♦ If $p_m \geq p_{\mathrm{NF}}$, set $p_l = p_{\mathrm{NF}}$. By construction, $r_{zp}(p, 0) \leq R$ for all $p \geq p_{\mathrm{NF}}$; hence the contract $r_{zp}(p, e^r(p))$ is admissible for all $p \geq p_{\mathrm{NF}}$.

Alternatively, if $p_m < p_{\mathrm{NF}}$ set p_l to be the lowest value of $p \geq p_m$ such that the contract $r_{zp}(p, e^r(p))$ is admissible (i.e., not greater than R). Since $r_{zp}(p, e)$ is decreasing in e, $r_{zp}(p, e^r(p)) \leq r_{zp}(p, 0)$ for all $p \in [p_m, p_{\mathrm{NF}}]$, so $p_l \leq p_{\mathrm{NF}}$. In this case we also redefine p_m, with some abuse of notation, to be equal to p_l; following this redefinition the low effort region $[p_l, p_m)$ is then empty in this case.

Observe that in either case we have $p_l > 0$. Furthermore, $p_l \leq p_{\mathrm{NF}}$, which implies that $r_{zp}(p, 0) > R$ for $p < p_l$. Finally, $p_l \leq p_m$, with p_m as defined above.

♦ To conclude, we show that low effort is incentive compatible for $p \in [p_l, p_m)$. It suffices to consider the case $p_l = p_{\mathrm{NF}}$ since when $p_l < p_{\mathrm{NF}}$, we showed immediately above that $p_l = p_m$, in which case there is no low-effort region.

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[31]That is, when the prior belief is $\left(\tilde{p}^S\right)^{-1}(p_h, 1)$, and the entrepreneur exerts high effort if risky, the posterior belief of lenders after observing a success is equal to p_h.

Claim 4. *The contract $r_{zp}(p, 0)$ satisfies the low-effort IC constraint for $p \in [p_{NF}, p_m)$.*

The argument is a little lengthier in this case and proceeds by induction on p, iterating the map $(\tilde{p}^S)^{-1}(p, e)$.

b-iv. As noted in a-iii. above, we can restrict attention to lenders' deviations consisting in the offer of a contract $r' > 1$ to entrepreneurs with credit score p.

Now, for r' to be accepted it must be lower than the equilibrium rate when there is financing in equilibrium. So when $p \geq p_l$, it suffices to consider $r' < r(p) \equiv r_{zp}(p, e^r(p))$. For $p < p_l$ there is no financing in equilibrium, and the deviation can be any $r' \in (1, R]$.

In the statement of the Proposition we did not describe the risky entrepreneurs' effort strategy $e^r(p, \mathcal{C}')$ off the equilibrium path. We will do so here, and show that $e^r(p, \mathcal{C}')$ renders any possible deviation r' described in the previous paragraph unprofitable.

♦ We first begin with the simplest case: $p \geq p_h$. Since high effort is implemented for these values of p, it is immediate that no deviation could be profitable, since for r' to be accepted by the entrepreneurs we would need $r' < r(p) = r_{zp}(p, 1)$.

♦ Next consider the case $p \in \left[(\tilde{p}^S)^{-1}(p_h, 1), p_h \right)$. Now if

$$R - r' + \beta(1 - q)v^r(\tilde{p}^S(p, 0)) \leq \frac{c}{\pi_h - \pi_l}, \tag{14}$$

then $e^r(p, r') = 0$ is an optimal effort choice for entrepreneurs when they are offered the rate r' and lenders' belief is that they exert low effort. If in addition $p \geq p_l$ we need $r' < r(p) \leq r_{zp}(p, 0)$ for r' to attract some entrepreneurs, and so the deviation will be unprofitable. On the other hand, if $p < p_l$, from b-iii. above we know that $r_{zp}(p, 0) > R$ (since $p_l \leq p_{NF}$), while the admissibility of the contract requires $r' \leq R$, implying $r' < r_{zp}(p, 0)$. That is, the deviation is unprofitable in this case as well.

Alternatively, suppose the reverse inequality to (14) holds. This means that low effort is not an optimal response to r'. Nevertheless, the deviation can be shown to be unprofitable. More precisely, we show in what follows that, were a profitable deviation to exist, this would contradict the construction of the equilibrium (in particular, either the definition of p_h, or of p_m, or $e^r(p)$ being maximal in the mixing region).

We begin by determining the effort level and lenders' beliefs associated with r'. First note that, for the values of p under consideration, $\tilde{p}^S(p, e) \geq p_h$ for all e. Then since $\tilde{p}^S(p, e)$ is decreasing with respect to e[32] and $v^r(p')$ is both increasing and continuous

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[32]This property can be easily verified from the expression of $\tilde{p}^S(p, e)$ and can be understood as follows: for any given p, the lower the probability e that the risky entrepreneurs exert high effort, the stronger is success a signal of a safe type.

for $p' \geq p_h$, we either have

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p,e')) \geq \frac{c}{\pi_h - \pi_l}, \text{ for } e' = 1 \qquad (15)$$

or $\qquad R - r' + \beta(1-q)v^r(\tilde{p}^S(p,e')) = \frac{c}{\pi_h - \pi_l} \text{ for some } e' \in (0,1), \qquad (16)$

so that the optimal effort choice of risky entrepreneurs when $\mathcal{C}'$ contains r' and r' is chosen, is $e^r(p, \mathcal{C}') = e'$, and lenders' beliefs $\tilde{p}^S(p,e')$ are consistent with Bayes' Rule.

We will establish that $r' \leq r_{zp}(p,e')$, implying that the deviation to r' is unprofitable. Suppose that this is not the case, i.e. that $r' > r_{zp}(p,e')$; we will prove in what follows that this implies a contradiction.

When $e' = 1$, $r' > r_{zp}(p,e') = r_{zp}(p,1)$ together with (15) imply $R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) \geq \frac{c}{\pi_h-\pi_l}$. But since, as we argued, $v^r(p')$ is increasing for $p \geq p_h$ and $r_{zp}(\cdot,1)$ strictly decreasing, this would imply that $R - r_{zp}(p_h,1) + \beta(1-q)v^r(\tilde{p}^S(p_h,1)) > \frac{c}{\pi_h-\pi_l}$, contradicting the construction of p_h in (12).

Consider next $e' < 1$. From $r' > r_{zp}(p,e')$ and equation (16) we know that $R - r_{zp}(p,e') + \beta(1-q)v^r(\tilde{p}^S(p,e')) > \frac{c}{\pi_h-\pi_l}$. On the other hand, recall that $p \in \left[(\tilde{p}^S)^{-1}(p_h,1), p_h \right)$, so that $v^r(\tilde{p}^S(p,e)) = \tilde{v}^r(\tilde{p}^S(p,e),1)$ for any e, and, from the definition of p_h, $R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) < \frac{c}{\pi_h-\pi_l}$.

So by the continuity of $\tilde{v}^r(p,1)$ it follows that there must be a solution $\tilde{e} \in (e',1)$ to (13) for the value of p under consideration. If $p < p_m$ the existence of such a solution contradicts the construction of p_m as the minimal value of p for which a solution e to (13) exists, with $r_{zp}(p,e^r(p)) \leq R$, in the region $p \in [(\tilde{p}^S)^{-1}(p_h,1), p_h]$, since $r_{zp}(p,\tilde{e}) < r_{zp}(p,e') < r' < r(p)$. Alternatively, consider $p \geq p_m$. If $\tilde{e} > e^r(p)$ this contradicts the construction of $e^r(p)$ as the highest solution of (13) at p (see the proof of Claim 3). On the other hand, if $\tilde{e} \leq e^r(p)$, this implies $e' < e^r(p)$, and thus $r' > r_{zp}(p,e') > r_{zp}(p,e^r(p)) = r(p)$, another contradiction.

♦ Now consider the remaining values: $p \in (0, (\tilde{p}^S)^{-1}(p_h,1))$. We restrict attention to deviations $r' > r_{zp}(p,1)$; this is without loss of generality, since if this were not the case the deviation could never be profitable, regardless of the risky entrepreneurs' effort choice (since no entrepreneur refuses financing). But recall that, in the proof of b-iii., we showed that for $p < (\tilde{p}^S)^{-1}(p_h,1)$, low effort will be chosen at r' whenever $r' > r(p_h) = r_{zp}(p_h,1)$. This implies then, just as in the argument immediately following (14) above, that the deviation must be unprofitable.

c. Finally, consider the low values of c: $\frac{c}{\pi_h - \pi_l} \leq \frac{(R - 1/\pi_h)(1 - \beta q)}{1 - \beta(\pi_l + (1 - \pi_l)q)}$. Note first that, by Assumption 1, $r_{zp}(p, 1) \leq R$ for all $p > 0$, so $r(p) = r_{zp}(p, 1)$ is always admissible. Also, the argument that there are no profitable deviations for lenders is the same as the one in b-iv., for the case $p \geq p_h$. So it only remains to verify that risky entrepreneurs indeed prefer to exert high rather than low effort for all $p > 0$.

For high effort to be incentive compatible for all $p > 0$, we need to show that

$$\frac{c}{\pi_h - \pi_l} \leq R - r(p) + \beta(1 - q)v^r(p^S(p)). \tag{17}$$

Notice that, for any $p > 0$, a lower bound for $v^r(p)$ is given by $\frac{\pi_h(R - 1/\pi_h) - c}{1 - \beta(\pi_h + q(1 - \pi_h))}$, which is the present discounted utility for a risky entrepreneur who is financed in every period (until a failure that is not forgotten) at $r = 1/\pi_h$ and exerts high effort.[33]

Thus since $p^S(p) > 0$ for all $p > 0$, we have

$$R - r(p) + \beta(1 - q)v^r(p^S(p)) > R - 1/\pi_h + \beta(1 - q)\frac{\pi_h(R - 1/\pi_h) - c}{1 - \beta(\pi_h + (1 - \pi_h)q)}.$$

So to verify (17) it suffices to show that

$$R - 1/\pi_h + \beta(1 - q)\frac{\pi_h(R - 1/\pi_h) - c}{1 - \beta(\pi_h + (1 - \pi_h)q)} \geq \frac{c}{\pi_h - \pi_l}.$$

But this follows immediately from the definition of region c.[34] ∎

Proposition 2 — Efficiency of Equilibrium

We begin by showing that the equilibrium constructed in Proposition 1 maximizes $e^r(p)$, the effort exerted by the risky entrepreneurs, for any p; this will play an important role in the proof of the Proposition. This result is intuitive, as the equilibrium of Proposition 1 was constructed recursively, with effort chosen to be maximal at each stage.

Claim 5. *The equilibrium constructed in Proposition 1 maximizes the risky entrepreneurs' effort $e^r(p)$, across all symmetric sequential MPE, when $q \in \{0, 1\}$. When $q \in (0, 1)$ this result holds as long as $\pi_l \geq \pi_h \frac{q}{1 + q}$.*

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[33]This follows immediately from the fact that $v^r(p)$ is the present discounted utility under the same circumstances except that the interest rate is $r(p) = r_{zp}(p, 1) < 1/\pi_h$ for all $p > 0$.

[34]Suppose this were not the case, so that $R - 1/\pi_h + \beta(1 - q)\frac{\pi_h(R - 1/\pi_h) - c}{1 - \beta(\pi_h + (1 - \pi_h)q)} < \frac{c}{\pi_h - \pi_l}$. If we multiply both sides of this inequality by $(\pi_h - \pi_l)(1 - \beta(\pi_h + (1 - \pi_h)q))$ and then simplify, this becomes $\frac{c}{\pi_h - \pi_l} > \frac{(R - 1/\pi_h)(1 - \beta q)}{1 - \beta(\pi_l + (1\pi_l)q)}$, contradicting the lower bound on c that defines region c.

The following corollary is immediate, since for lenders to break even when $p < p_l$ a higher level of effort is needed than in the equilibrium of Proposition 1, contradicting Claim 5.

Corollary 1. *No MPE can implement financing when $p < p_l$.*

From Corollary 1, we can restrict attention to $p_0 \geq p_l$, without loss of generality. Recall that welfare is given by the total surplus accruing from the agents' projects that are financed. Let $\mathcal{W}(p)$ denote the total surplus at the MPE of Proposition 1 accruing from projects of entrepreneurs with credit score p, and let $\overline{\mathcal{W}}(p)$ denote the total surplus at a different MPE. We then conclude by showing that:

Claim 6. $\mathcal{W}(p) \geq \overline{\mathcal{W}}(p)$ *for* $p \geq p_l$.

The proof of this claim is by induction on p, relying at each stage on the fact that surplus will be higher whenever effort is higher. The result then follows from Claim 5 above.∎

Proposition 3 – Optimal Forgetting (regions a. and c.)

1. When $\frac{c}{\pi_h - \pi_l} \geq \frac{R-1}{1 - \pi_l \beta}$, since $\frac{(R-1)(1-\beta q)}{1 - \beta(\pi_l + (1-\pi_l)q)}$ is decreasing in q, the condition defining region a. in Proposition 1 is satisfied for all q. At the MPE there is financing only when $p_0 \geq p_{\mathrm{NF}}$ and risky entrepreneurs never exert high effort, regardless of the value of q. Hence if $p_0 \geq p_{\mathrm{NF}}$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is $\frac{B}{1 - (\pi_l + (1-\pi_l)q)\beta}$, which is strictly decreasing in q since $B < 0$. Thus $q = 0$ is optimal. If on the other hand $p_0 < p_{\mathrm{NF}}$, such surplus is zero for all q, and so $q = 0$ is also (weakly) optimal.

2. Again notice that $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ is decreasing in q. Thus when $\frac{c}{\pi_h - \pi_l} < \frac{R-1/\pi_h}{1-\beta \pi_l}$, the condition defining region c. of Proposition 1 is satisfied for all $q \in [0, q^*]$, where $q^* = \frac{(R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\beta\pi_l)}{\beta\left((R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\pi_l)\right)} > 0$. Hence at the MPE there is always financing whatever p_0 is, and for all $q \in [0, q^*]$, and risky entrepreneurs always exert high effort. That is, for $q \in [0, q^*]$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is

$$\frac{G}{1 - (\pi_h + (1 - \pi_h)q)\beta}.$$

Now this is increasing in q since $G > 0$. Thus any $q \in (0, q^*]$ dominates $q = 0$ and the optimal value will be $q(p_0) \geq q^*$.[35]∎

Proposition 4 – Optimal Forgetting (region b.)

1. When $p_0 > p_h(0)$ the proof is an immediate corollary of case 2. of Proposition 3.

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[35]The optimal value of q could be higher than q^*, which would push us out of region c., into region b.

2. Since $p_0 \geq p_{\mathrm{NF}}$, the agents will always be financed at the initial date, irrespective of q. Thus, by the argument given above, it suffices to show that we can increase the surplus generated by the risky entrepreneurs' projects. Letting $\mathcal{W}^r(q, p_0)$ denote the surplus from the risky agents' projects, when the forgetting policy is q and the prior probability of being safe is p_0, we will show that under the conditions stated in the Proposition, we can find some $\bar{q} > 0$ such that $\mathcal{W}^r(\bar{q}, p_0) > \mathcal{W}^r(0, p_0)$.

We proceed as follows. For any $q > 0$ we first find a threshold $\tilde{p}_h(q)$ for $p_h(q)$, relative to $p_h(0)$, such that *if* $p_h(q) < \tilde{p}_h(q)$ then the surplus from risky entrepreneurs' projects is higher at q than at 0. We then show that the parameter restrictions stated in the Proposition ensure the existence of $\bar{q} > 0$ such that $p_h(\bar{q}) \leq \tilde{p}_h(q)$.

Let $n(q, p_0)$ denote the number of successes (or forgotten failures), starting from the prior p_0, until the risky entrepreneurs first exert high effort, when the forgetting policy is q. Then the following upper and lower bounds for the surplus generated by lending to risky entrepreneurs can be shown to hold:[36]

$$\mathcal{W}^r(0, p_0) \leq \frac{B(1 - (\pi_l \beta)^{n(0,p_0)-1})}{1 - \pi_l \beta} + \frac{G(\pi_l \beta)^{n(0,p_0)-1}}{1 - \pi_h \beta} \tag{18}$$

and

$$\mathcal{W}^r(q, p_0) \geq \frac{B(1 - ((\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)})}{1 - (\pi_l + (1 - \pi_l)q)\beta} + \frac{G(\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)}}{1 - (\pi_h + (1 - \pi_h)q)\beta}. \tag{19}$$

So to show that $\mathcal{W}^r(q, p_0) > \mathcal{W}^r(0, p_0)$, it suffices to show that we can find $q > 0$ such that:

$$\frac{B(1 - (\pi_l \beta)^{n(0,p_0)-1})}{1 - \pi_l \beta} + \frac{G(\pi_l \beta)^{n(0,p_0)-1}}{1 - \pi_h \beta} < \frac{B(1 - ((\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)})}{1 - (\pi_l + (1 - \pi_l)q)\beta} + \frac{G(\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)}}{1 - (\pi_h + (1 - \pi_h)q)\beta}.$$

Letting $\beta \to 1$ and simplifying, the above expression reduces to:

$$\frac{\frac{B}{G}}{1 - \pi_l} + \frac{\pi_l^{n(0,p_0)-1}}{(1 - \pi_l)(1 - \pi_h)} \left[(1 - \pi_l) - \frac{B}{G}(1 - \pi_h) \right]$$

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[36]When there is no mixing in equilibrium (i.e. $p_m(q) = p_h(q)$), $\mathcal{W}^r$ is simply equal to the discounted expect surplus generated by consecutive successes of the project (the first $n(q, p_0)$ of which with low effort, the remainder with high effort):

$$\mathcal{W}^r(q, p_0) = \frac{B(1 - ((\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)})}{1 - (\pi_l + (1 - \pi_l)q)\beta} + \frac{G(\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)}}{1 - (\pi_h + (1 - \pi_h)q)\beta}.$$

With mixing in equilibrium, the exact expression of $\mathcal{W}^r$ depends on the equilibrium level of effort exerted in the mixing region. However, since there can be at most only a single period of mixing in equilibrium, an upper and lower bound for such utility is given by (18) and (19), independent of the mixing probability.

$$< \frac{\frac{B}{G}}{(1-\pi_l)(1-q)} + \frac{(\pi_l + (1-\pi_l)q)^{n(q,p_0)}}{(1-q)(1-\pi_l)(1-\pi_h)} \left[(1-\pi_l) - \frac{B}{G}(1-\pi_h) \right],$$

since $1 - (\pi_l + (1-\pi_l)q) = (1-\pi_l)(1-q)$ and $1 - (\pi_h + (1-\pi_h)q) = (1-\pi_h)(1-q)$, which is equivalent to

$$\pi_l^{n(0,p_0)-1}(1-q) - \frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l) - \frac{B}{G}(1-\pi_h)} < (\pi_l + (1-\pi_l)q)^{n(q,p_0)} \tag{20}$$

It will be useful to rewrite (20) in terms of a condition on $p_h(q)$ and $p_h(0)$. To this end, notice that $p_h(q)$ and $n(q,p_0)$ are related by the following expression — $n(q,p_0)$ is the smallest integer for which:[37]

$$\frac{p_0}{p_0 + (1-p_0)[\pi_l + (1-\pi_l)q]^{n(q,p_0)}} \geq p_h(q), \tag{21}$$

so that $\pi_l^{n(0,p_0)} \leq \frac{p_0}{1-p_0}\left(\frac{1}{p_h(0)} - 1\right)$ and $(\pi_l + (1-\pi_l)q)^{n(q,p_0)-1} \geq \frac{p_0}{1-p_0}\left(\frac{1}{p_h(q)} - 1\right)$. Thus to satisfy (20) it suffices to show that:

$$\frac{1}{\pi_l}\frac{p_0}{p_h(0)}\left(\frac{1-p_h(0)}{1-p_0}\right)(1-q) - \frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l) - \frac{B}{G}(1-\pi_h)} < (\pi_l + (1-\pi_l)q)\frac{p_0}{p_h(q)}\left(\frac{1-p_h(q)}{1-p_0}\right).$$

Simplifying, we obtain the following sufficient condition for q to implement a welfare improvement as $\beta \to 1$:

$$p_h(q) < \tilde{p}_h(q) \equiv \frac{p_0(\pi_l + (1-\pi_l)q)}{p_0(\pi_l + (1-\pi_l)q) + (1-p_0)\left[\frac{1}{\pi_l}\frac{p_0}{p_h(0)}\left(\frac{1-p_h(0)}{1-p_0}\right)(1-q) - \frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l) - \frac{B}{G}(1-\pi_h)}\right]}. \tag{22}$$

We now show that the condition on B/G stated in the Proposition ensures that we can find $\bar{q} > 0$ such that $p_h(\bar{q})$ satisfies (22) and so we can achieve a welfare improvement. We begin by providing a convenient upper bound for the level of $p_h(q)$.

For intermediate values of c, lying in the region where type b. equilibria obtain when $q = 0$, $p_h(0)$ belongs to $(0,1)$ and satisfies equation (12) above. It is then easy to see from the definition of this region in Proposition 1 that, when β is sufficiently close to 1, c will remain in the same region for any $q > 0$.[38] So for β close to 1, $p_h(q)$ also lies in $(0,1)$ and

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[37]When there is no mixing in equilibrium, i.e., $p_m(q) = p_h(q)$, the validity of (21) follows immediately from the definition of $p_h(q)$ and $n(q,p_0)$. The fact that it also holds with mixing can be seen by noticing that in such case the probability of success is greater or equal than when low effort is exerted, and so the posterior is $\tilde{p}^S(p, e^r(p)) \leq \tilde{p}^S(p, 0)$. Hence $n(q,p_0)$ will be greater or equal than the term satisfying (21). But $n(q,p_0)$ cannot be strictly greater, as this would imply that we mix for more than a single period, which we have shown (in the proof of Proposition 1) cannot happen.

[38]For β close to 1, the boundaries of the region are approximately equal to $\frac{(R-1/\pi_h)}{1-\pi_l}$ and $\frac{(R-1)}{1-\pi_l}$, both independent of q.

satisfies an expression analogous to (12):

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h(q), 1) + \beta(1 - q)\tilde{v}^r(\tilde{p}^S(p_h(q), 1), 1; q), \tag{23}$$

where, similarly to (12), $\tilde{v}^r(p, 1; q)$ denotes the discounted expected utility of a risky entrepreneur with credit score p, when he exerts high effort for all $p' > p$ and the contracts offered are $r_{zp}(p, 1)$, highlighting the dependence of the utility on the forgetting policy q. From (23) and (12) we then obtain:

$$-r_{zp}(p_h(0), 1) + \beta\tilde{v}^r(\tilde{p}^S(p_h(0), 1), 1; 0) = -r_{zp}(p_h(q), 1) + \beta(1 - q)\tilde{v}^r(\tilde{p}^S(p_h(q), 1), 1; q). \tag{24}$$

By a similar argument to that in the proof of parts a. and c. of Proposition 1, a (strict) upper bound for $\tilde{v}^r(\tilde{p}_h^S(p_h(0), 1), 1; 0)$ is given by the utility of being financed in every period at the constant rate $r = 1$ until a failure occurs, while exerting high effort, i.e., by $\frac{\pi_h(R-1)-c}{1-\beta\pi_h}$. Conversely, when the forgetting policy is q, a (strict) lower bound for $\tilde{v}^r(\tilde{p}^S(p_h(q), 1), 1; q)$ is given by $\frac{\pi_h(R-r_{zp}(p_h(q),1))-c}{1-\beta(\pi_h+(1-\pi_h)q)}$, that is, the utility of a risky agent when financed at the constant rate $r_{zp}(p_h(q), 1)$ until he experiences a failure that is not forgotten, still exerting high effort. Together with (24) this implies that:

$$-r_{zp}(p_h(0), 1) + \beta\frac{\pi_h(R - 1) - c}{1 - \beta\pi_h} > -r_{zp}(p_h(q), 1) + \beta(1 - q)\frac{\pi_h(R - r_{zp}(p_h(q), 1)) - c}{1 - \beta(\pi_h + (1 - \pi_h)q)}.$$

When $\beta \to 1$, the above inequality becomes

$$-r_{zp}(p_h(0), 1) + \frac{\pi_h(R - 1) - c}{1 - \pi_h} > -r_{zp}(p_h(q), 1) + \frac{\pi_h(R - r_{zp}(p_h(q), 1)) - c}{1 - \pi_h},$$

or, simplifying: $r_{zp}(p_h(q), 1) > (1 - \pi_h)r_{zp}(p_h(0), 1) + \pi_h$.
Using the definition of $r_{zp}(\cdot, \cdot)$ in (7), the previous expression can be rewritten as follows:

$$\frac{1}{p_h(q) + (1 - p_h(q))\pi_h} > (1 - \pi_h)\frac{1}{p_h(0) + (1 - p_h(0))\pi_h} + \pi_h,$$

or

$$p_h(0) + (1 - p_h(0))\pi_h > (1 - \pi_h)[p_h(q) + (1 - p_h(q))\pi_h] + \pi_h[p_h(q) + (1 - p_h(q))\pi_h][p_h(0) + (1 - p_h(0))\pi_h]$$
$$= [p_h(q) + (1 - p_h(q))\pi_h] [1 - \pi_h(1 - \pi_h)(1 - p_h(0))], \tag{25}$$

which is in turn equivalent to:

$$p_h(0)(1 - \pi_h) + \pi_h > [p_h(q)(1 - \pi_h) + \pi_h] [1 - \pi_h(1 - \pi_h)(1 - p_h(0))],$$

i.e.,

$$\frac{p_h(0)(1 - \pi_h) + \pi_h}{[1 - \pi_h(1 - \pi_h)(1 - p_h(0))]]} > [p_h(q)(1 - \pi_h) + \pi_h].$$

The above inequality implies that when β is close to 1 the following upper bound on the level of $p_h(q)$ must hold, for all q:

$$p_h(q) < \bar{p}_h \equiv \frac{p_h(0)(1 - \pi_h^2) + \pi_h^2}{[1 - \pi_h(1 - \pi_h)(1 - p_h(0))]]}. \tag{26}$$

Finally, note that for q close to 1, $\tilde{p}_h(q)$ is approximately equal to $p_0 \frac{(1 - \pi_l) - \frac{B}{G}(1 - \pi_h)}{p_0(1 - \pi_l) - \frac{B}{G}(1 - \pi_h)}$. Hence, under the condition on B/G stated in the Proposition we have that

$$\frac{p_h(0)(1 - \pi_h^2) + \pi_h^2}{[1 - \pi_h(1 - \pi_h)(1 - p_h(0))]]} < p_0 \frac{(1 - \pi_l) - \frac{B}{G}(1 - \pi_h)}{p_0(1 - \pi_l) - \frac{B}{G}(1 - \pi_h)},$$

or equivalently that, for q close to 1 we have $\bar{p}_h < \tilde{p}_h(q)$.

Thus on the basis of the previous discussion we can conclude that there exists $\bar{q}$ yielding a welfare improvement over $q = 0$. ∎

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